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Study of the Earth's short-scale gravity field using the ERTM2160 gravity model

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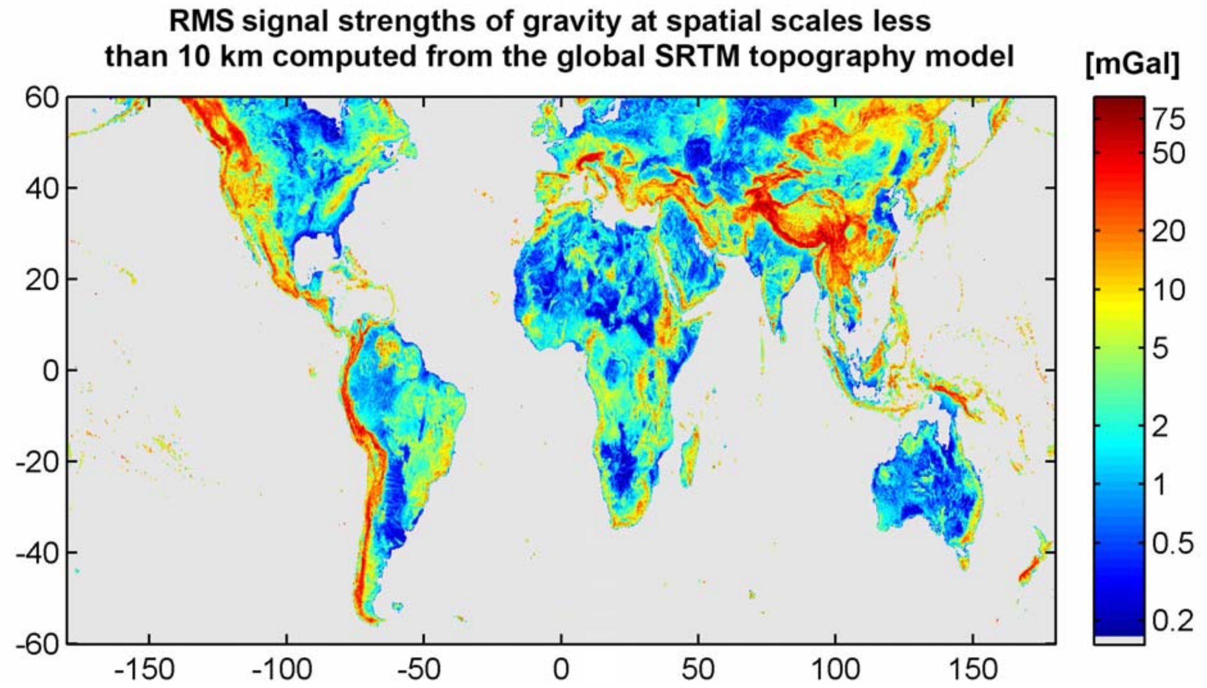
Abstract

This paper describes the computation and analysis of the Earth's short-scale gravity field through high-resolution gravity forward modelling using the Shuttle Radar Topography Mission (SRTM) global topography model. We use the established residual terrain modelling technique along with advanced computational resources and massive parallelisation to convert the high-pass filtered SRTM topography – complemented with bathymetric information in coastal zones – to implied short-scale gravity effects. The result is the ERTM2160 model (Earth Residual Terrain modelled-gravity field with the spatial scales equivalent to spherical-harmonic coefficients up to degree 2160 removed). ERTM2160, used successfully for the construction of the GGMplus gravity maps, approximates the short-scale (i.e., ~10 km down to ~250 m) gravity field in terms of gravity disturbances, quasi/geoid heights and vertical deflections at ~3 billion gridded points within ±60 latitude. ERTM2160 reaches maximum values for the quasi/geoid height of ~30 cm, gravity disturbance in excess of 100 mGal, and vertical deflections of ~30 arc-seconds over the Himalaya mountains. Analysis of the ERTM2160 field as a function of terrain roughness shows in good approximation a linear relationship between terrain roughness and gravity effects, with values of ~1.7 cm (quasi/geoid heights), ~11 mGal (gravity disturbances) and 1.5 arc-seconds (vertical deflections) signal strength per 100 m standard deviation of the terrain. These statistics can be used to assess the magnitude of omitted gravity signals over various types of terrain when using degree-2160 gravity models such as EGM2008. Applications for ERTM2160 are outlined including its use in gravity smoothing procedures, augmentation of EGM2008, fill-in for future ultra-high resolution gravity models in spherical harmonics, or calculation of localized or global power spectra of Earth's short-scale gravity field. ERTM2160 is freely available via http://ddfe.curtin.edu.au/gravitymodels/ERTM2160.

Graphical Abstract



Highlights

- Residual gravity model ERTM2160 computed from the SRTM topography at 250 m resolution
- Supercomputing resources used for forward gravity modelling at ~3 billion points
- Global short-scale RMS signal magnitudes are 1.6 cm for geoid, 11 mGal for gravity
- Linear relation between terrain roughness and RMS gravity signal magnitudes found

Key words

Gravity field, forward-modelling, gravity, quasi/geoid, vertical deflection, supercomputing

1 Introduction

Forward-modelling of the gravity field from topographic mass models is central to physical geodesy and potential field geophysics (e.g., Forsberg, 1984; Jacoby and Smilde, 2009). All gravity forward modelling techniques are based on the evaluation of Newton's integral (Kuhn and Seitz, 2005) which can be done either in the spectral domain (Rummel et al., 1988; Balmino et al., 2012), or in the space domain (Forsberg, 1984; Nagy et al., 2000). For gravity forward modelling in the space domain, the topographic masses are usually represented through gridded digital elevation models decomposing the terrain into discrete geometrical mass-bodies (i.e., point masses, prisms or tesseroids), cf. Heck and Seitz (2007). The practical evaluation of Newton's integral at a single computation point *P* involves numerical integration (summation) of gravity effects generated by each geometrical mass-body to some distance around *P* (Tziavos and Sideris, 2013) when evaluating short-scale gravity effects and global numerical integration when evaluating full-scale gravity effects (e.g. Kuhn et al., 2009).

Until recently, one of the limiting factors for the application of space domain techniques in ultra-high resolution forward modelling on regional to global scales was their enormous computational demand. This is due to the fact that Newton's integral has to be evaluated separately for each computation point without drawing information from other already evaluated gravity effects. Therefore, the required number of operations increases linearly with the number of computation points, which is why ultra-high resolution (i.e., spatial density of *P* commensurate to the elevation data resolution, say ~100-200 m) gravity forward modelling on a global scale is a computationally demanding task. However, this drawback can also be used as advantage when employing parallel computation techniques as the gravitational effect at each computation point can be obtained independently of all other points. This advantage has been exploited in this study through the use of advanced computational resources along with parallelization of the computations.

This study focuses on (i) gravity forward-modelling of the Earth's short-scale gravity field from the high-resolution SRTM topography (augmented with bathymetry in coastal zones) in the space domain, and (ii) analysis of gravity signal magnitudes with spatially varying statistics. The term "short-scale" is defined here as spatial scales of ~10 km (or beyond spherical harmonic degree 2160) down to ~250 m. The target area for our ultra-high resolution gravity forward-modelling are all continents between ±60° geodetic latitude as represented through the Shuttle Radar Topography Mission (SRTM) global elevation model, including adjoining coastal zones, Earth's major lakes and numerous islands. Using a dense grid spacing of 7.2 arc-sec, there are more than 3 billion computation points in our near-global target area, which necessitates the use of advanced computational resources and parallelization of the forward-modelling task. The main result of the gravity forward modelling is a model that describes Earth's short-scale gravity field (over our target area) in terms of quasi/geoid heights, gravity disturbances and vertical deflections: ERTM2160 (Earth Residual Terrain Modelled - gravity field with the 2160 indicating that spatial scales up to spherical-harmonic degree and order 2160 were removed).

ERTM2160 was created in the context of the GGMplus (Global Gravity Maps plus) initiative (Hirt et al., 2013) to deliver the short-scale constituents for the GGMplus gravity maps (http://geodesy.curtin.edu.au/research/GGMplus). While Hirt et al. (2013) give a general description of the gravity forward modelling and the combination of forward-modelled gravity with observed gravity data used to construct GGMplus, we here provide a full account of the conversion of the global SRTM topography to short-scale ERTM2160 gravity effects (Sect. 2), and present an entirely new analysis of their statistical characteristics (Sect 3). In order to provide a complete description of the methods deployed, the methods and data summary (Sect. 2) has deliberately some overlap with previously reported research (Hirt et al., 2013; Hirt, 2013).

Regarding the gravity forward-modelling applied with ultra-high resolution on a near-global scale, new research presented in this study includes (i) the role of accurate high-pass filtering for short-scale gravity forward modelling, (ii) the treatment of major lakes in the forward modelling and (iii) identification and removal of low-quality and bad-data areas in the topography models (data cleaning) cf. Sect 2. The main focus is placed in this paper on studying the characteristics of the ERTM2160 short-scale gravity field. New results presented include (i) magnitude statistics of gravity anomalies, geoid heights and vertical deflections, (ii) a first comparison with estimates from degree-variances models, and (iii) the investigation of the functional relationship among gravity signal strengths and terrain roughness (Sect. 3). We further summarize application examples (Sect. 4) and outline limitations for ERTM2160 (Sect. 5), before making some concluding remarks (Sect. 6).

Apart from Hirt et al. (2013), results from ultra-high resolution (say few 100 m) gravity forward-modelling on a near-global scale were not yet reported in the literature. Thus far, gravity forward-modelling is either limited in spatial resolution (say 1-2 arc-min, or ~2-4 km) when done globally, e.g., Gruber et al. (2013); Balmino et al. (2012); Bonvalot et al. (2012), or limited to regional areas when done with ultra-high resolution (say around 250 m), e.g., Kuhn et al. (2009). It is only through the computation of ERTM2160 that the study of the short scale gravity field characteristics has become possible at a near-global scale and with ultra-high resolution.

2. Data and methods

2.1 Data sets and combination

As high-resolution representations of the topographic masses over land, we selected the ~250 m (7.5 arc-sec) resolution SRTM V4.1 topography model provided by Jarvis et al. (2008). This data set is based on the second (research-grade) release of the SRTM mapping mission (Farr et al. 2007), with improved interpolation methods often based on auxiliary data sets used for filling of no-data areas (‘holes’), as described by Reuter et al. (2007). The resolution of the V4.1 250 m version, derived by Jarvis et al. (2008) from the 90 m SRTM basis resolution, is commensurate with the ERTM2160 target resolution of 7.2 arc-sec. The SRTM V4.1 topography model is available within the ± 60° latitude SRTM coverage, and incorporates coastline information through the SRTM water body data set. The SRTM elevation model is referred to the EGM96 geoid model (resolution of degree and order 360).

In order to avoid ‘edge effects’ of the SRTM-based forward-modelling along coast lines and at ± 60° latitude, we included – outside the V4.1 coverage – bathymetric depth information as available through the 30 arc-sec resolution V7 SRTM30_PLUS topography/bathymetry model (Becker et al. 2009). The bathymetric component of the SRTM30_PLUS data set is based on altimetry and – where available – depth soundings (Becker et al., 2009). SRTM30_PLUS also contains bathymetric information for Earth’s major lakes (Great Lakes, Caspian Sea, Baikal) which is taken into account in ERTM2160 (Sect. 2.2). According to Becker et al. (2009) SRTM30_PLUS provides GTOPO30 data (USGS 1996) in high northern latitudes, which is a relevant data source for forward-modelling at ERTM2160 computation points near or at 60° latitude.

Following a case study by Hirt (2013) both data sets are combined at 7.5 arc-sec resolution whereby SRTM30_PLUS data is used everywhere outside the V4.1 data coverage. This ensures a mostly smooth transition from land to oceans and land to interior lakes, as well as at the northern and southern extent of the SRTM coverage. SRTM V4.1 and its combination with SRTM30_PLUS have proven suitable for short-scale gravity forward-modelling over local and regionally limited land areas (e.g., Hirt, 2012) as well as along some coastal zones (Hirt, 2013). Notwithstanding it is important to note that at a global scale both data sets are not free of errors and artefacts, necessitating some data cleaning as described in Sect 2.5.

2.2 Treatment of water bodies

We make use of the concept of rock-equivalent topography (RET; Rummel et al. 1988), allowing convenient treatment of topographic and water masses in forward-modelling with a single constant mass-density. In the RET concept, the lake and ocean water masses are condensed ('compressed') into layers of rock. With the standard rock mass-density ρ = 2670 kg m^{-3}, and ocean water mass-density ρ_O = 1030 kg m^{-3}, RET-heights $H_{RET}^{(sea)}$ are obtained over the oceans

$$H_{RET}^{(sea)} = H_{BED}\left(1-\frac{\rho_O}{\rho}\right), \tag{1}$$

whereby H_{BED} (<0) is the bathymetric depth with respect to mean sea level (MSL) from SRTM30_PLUS. For inland water bodies, RET-heights $H_{RET}^{(lakes)}$ are calculated from

$$H_{RET}^{(lakes)} = H_{BED} + \frac{\rho_L}{\rho}\left(H_{SUR} - H_{BED}\right), \tag{2}$$

where ρ_L = 1000 kg m^{-3} is the lake water mass-density, H_{SUR} is the height of the water body above MSL (as implied by the SRTM V4.1 model), and H_{BED} is the height of the lake bottom, taken from SRTM30_PLUS (H_{SUR} - H_{BED} is the water column height). Table 1 lists the water bodies considered in the present work at 30 arc-sec resolution. We acknowledge recent work by Balmino et al. (2012) who have forward-modelled gravity effects implied by the water-masses of several great lakes at 1 arc-min resolution, and Grombein et al. (2014) at 5 arc-min resolution.

Table 1 Water bodies modelled in ERTM2160, and surface heights (extracted from SRTM V4.1)

Water body	**Surface height H_{SUR} [m]**
Oceans	0
Caspian Sea	–29
Lake Baikal	+449
Lake Superior	+179
Lake Michigan and Huron	+175
Lake Erie	+172
Lake Ontario	+73

2.3 High-pass filtering

In short-scale gravity forward-modelling based on the well-established residual terrain modelling (RTM) technique (Forsberg and Tscherning, 1981; Forsberg, 1984), accurate high-pass filtering of the elevation data is crucial. Subtraction of a long-wavelength spherical harmonic reference surface is suitable to extract the short-scale information from elevation models, particularly when gravity forward-modelling is used for augmentation of GGMs beyond their nominal resolution (e.g., Forsberg, 1984; Hirt, 2010). For the generation of the ERTM2160 short-scale gravity model, a spherical harmonic reference surface (denoted with RET2012 in the sequel) has been developed that is rigorously consistent to the input topography (SRTM V4.1 over (dry) land and rock-equivalent heights from SRTM30_PLUS over the oceans and lakes) everywhere over our target area with the procedure described in Hirt (2013). The fully-normalized RET2012 spherical harmonic coefficients (SHCs) $\overline{HC}_{nm}$, $\overline{HS}_{nm}$ are evaluated here to degree and order 2160

$$H = \sum_{n=0}^{2160}\sum_{m=0}^{n}(\overline{HC}_{nm}\cos m\lambda + \overline{HS}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi) \qquad (3)$$

with φ and λ are the geocentric latitude and longitude, and $\bar{P}_{nm}(\sin\varphi)$ are the fully-normalized associated Legendre functions. Subtracting the synthesized heights H from the high-resolution RET-topography provides the high-pass filtered SRTM data for the ERTM2160 forward-modelling.

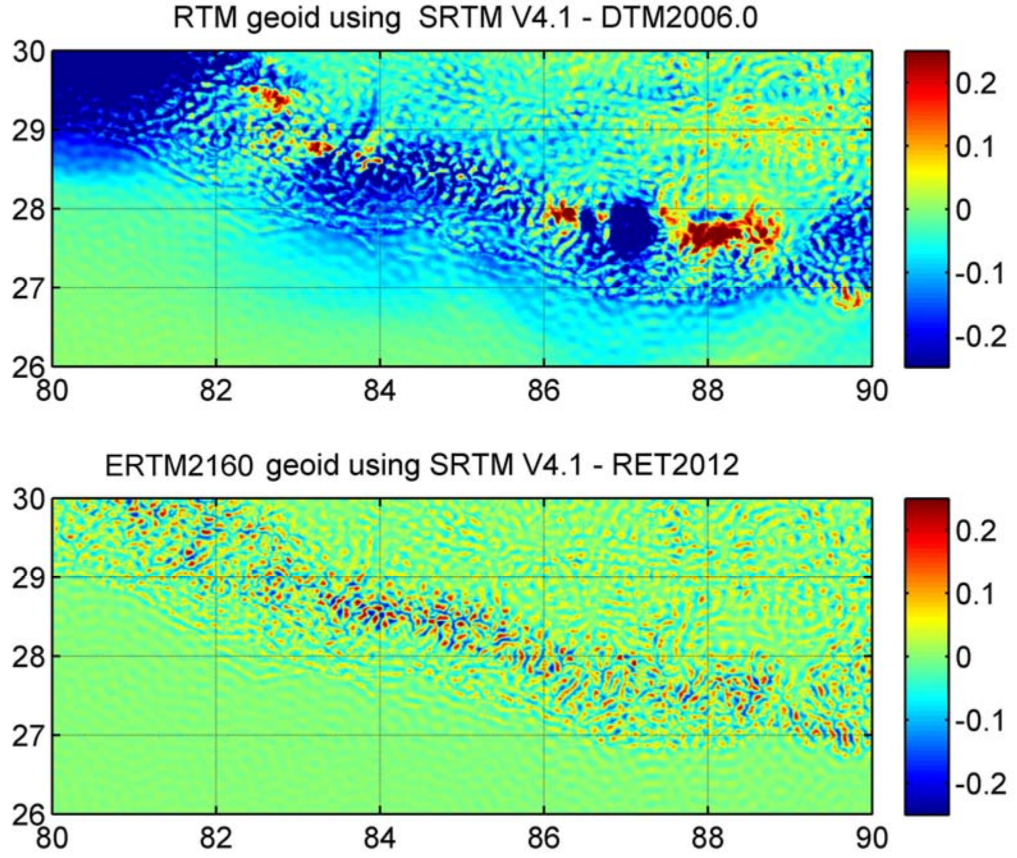


Fig. 1 Top: RTM geoid effects based on SRTMV4.1 and DTM2006.0 as long-wavelength reference surface over a 4°x10° test area in the Himalayas, Bottom: ERTM2160 quasi/geoid calculated based on SRTM V4.1 and RET2012 as rigorously consistent long-wavelength reference surface, units in m. All maps are coordinated in terms of geodetic latitudes and longitudes, unit degree.

We have tested the spherical harmonic expansion of the DTM2006.0 data created by the EGM2008 development team (Pavlis et al., 2007; 2012) as alternative reference surface for high-pass filtering of the SRTM V4.1 topography and SRTM30_PLUS V7 rock-equivalent bathymetry. Fig. 1 compares forward-modelled quasi/geoid effects over the Himalaya using synthesized heights from DTM2006 (top) and RET2012 (bottom) for high-pass filtering of the SRTM V4.1 topography. From Fig. 1, the combination SRTM V4.1 minus DTM2006.0 produces regional-scale offsets with amplitudes at the dm-level over the Himalayas, while SRTMV4.1 minus RET2012 does not show such effects.

Fig. 1 shows indirectly that SRTM V4.1 and the SRTM release used for the DTM2006.0 data base (Pavlis et al., 2007) are not compatible, with likely differences in the hole-filling procedures used. A similar behaviour as displayed in Fig. 1 is visible over other parts of the Himalayas and parts of the Andes, suggesting inconsistencies between the elevation data bases. While DTM2006.0 was used successfully in earlier studies on forward-modelling over European test areas (e.g., Hirt et al., 2010; Hirt, 2012), DTM2006.0 cannot be used along with SRTM V4.1 over some rugged land areas for accurate high-pass filtering and short-scale forward-modelling. Further inconsistencies would occur over marine areas, even if DTM2006.0 depths were made rock-equivalent. This is because the bathymetry grids used for creating DTM2006.0 and RET2012 are different as well. In the remainder of this paper we therefore only use RET2012 as rigorously consistent long-wavelength reference for our topography/bathymetry-combined RET input grid.

2.4 Forward-modelling and use of supercomputing facilities

The short-scale gravity forward-modelling, i.e., the conversion of the high-pass filtered and rock-equivalent SRTM topography to gravitational effects, relies on the RTM technique. The gravity field functionals computed are (i) quasi/geoid heights, (ii) gravity disturbances, (iii) North-South vertical deflections, and (iv) East-West vertical deflections. Using regularly-spaced 7.2 arc-sec grids of computation points *P* over all continents, and adjoining marine areas within ±60° latitude, the numerical integration needs to be carried out at more than 3 billion locations.

We used software based on Forsberg's TC-program that deploys mass-prisms (e.g. Nagy et al., 2000) in the ~5 km near-zone, point-masses and McMillan expansions in the far-zone (Forsberg, 1984). Different to the original TC-approach, we do not distinguish between different mass-densities over land and oceans in the forward-modelling. Instead we use the high-pass filtered SRTM land topography (over dry land) and SRTM30_PLUS rock-equivalent topography (over water bodies) as input data (Sect. 2.3), along with a single uniform mass-density of 2670 kg m^{-3} (Hirt, 2013). In the RTM technique, the forward-modelling needs to be carried out only to some distance around *P* (Forsberg, 1984). When high-pass filtering the topography with a degree-2160 spherical harmonic reference surface (equivalent to ~10 km), it is sufficient for all gravity functionals computed to take into account mass-effects only within ~200 km radius (Hirt et al., 2010). Beyond this radius, mass-prism effects largely cancel out because of the oscillating nature of RTM elevations (see also Forsberg and Tscherning, 1981).

We divided the gravity forward-modelling task in 1° × 1° regions over land and sea, which can be processed in parallel, i.e., independent of each other. This straightforward and efficient approach of parallelization is taken here because computation points *P* can be computed without dependencies from each other. The resolution of the input topography is down-sampled from 7.5 arc-sec to 30 arc-sec outside ~100 km radius around *P* (using a 4 x 4 box means), reducing the number of mass-elements and thus the required computation time (two-grid approach, cf. Forsberg, 1984). Using a standard desktop PC (e.g. Intel Q9400 central processing unit CPU @ 2.66 GHz) and a single CPU we observed a forward-modelling speed of about 5-6 points per second. For a total of ~18300 1° × 1° tiles within the SRTM coverage and adjoining marine zones, this translates into a total computation time of about 20 years, underlining the demanding nature of near-global ultra-high resolution forward-modelling and necessitating the use of advanced computational resources and massive parallelization.

We acknowledge some technique optimizations are possible, e.g., based on efficient tesseroid formulae in place of prims (Grombein et al., 2013), which however, will not circumvent the need for supercomputing. Alternatively, Fast Forier Transform (FFT) methods (e.g., Forsberg 1985) could be used for a more efficient calculation of gravity effects from RTM data, while the application of FFT for the accurate calculation of RTM vertical deflections is "rather complicated" (Forsberg, 1985, p359). FFT techniques were not deployed in this study.

To accomplish the forward-modelling we used the Epic supercomputer that is part of Western Australia's iVEC supercomputing initiative (www.ivec.org) and Pawsey centre, providing advanced resources to Western Australian researchers, particularly in Earth Sciences. Epic is a Linux cluster system that operates a total of 9600 Intel Xeon X5660 CPUs along with 18 TB of RAM. With up to 1153 CPUs (or a ~12 % share) simultaneously available to us, we completed the gravity forward-modelling task as described before within a period of less than three weeks time, or ~30,000 CPU-hours. This demonstrates the pivotal role of parallelization and supercomputer deployment for ultra-high resolution forward-modelling at a global scale.

2.5 Detection and removal of artefacts

Global inspection of the forward-modelling results over our target area showed a number of locations with unrealistically large negative gravity disturbances as small as -1040 mGal. At the locations of these suspicious gravity minima, we identified spike-like depressions in the input topography, both over land areas (SRTM V4.1) and over coastal zones (SRTM30_PLUS V7). We analysed all locations with forward-modelled gravity disturbances smaller than an arbitrary threshold of –400 mGal, and found by visual inspection of the forward-modelled gravity further artefacts present in both elevation data sets. These artefacts are cautiously attributed to

- Unfilled holes in the SRTM V4.1 data and interpolation errors along the seams of 1-degree tiles over parts of Asia.
- Discrepancies between ship depth-soundings and altimetric depths (SRTM30_PLUS V7) resulting in 'sea-floor holes' of up to 5 km.

From inspection of all forward-modelled functionals, particularly local minima of gravity disturbances turned out to be very sensitive for unnaturally steep gradients in topography models (which occur at spike-like depressions). In a similar context, this sensitivity was indicated by Kirby and Featherstone (2001, 2002) who detected bad data areas in a national elevation model via gravimetric terrain corrections. We decided to clean the input topography by masking out the affected locations, before filling them with bicubic interpolation. Though this procedure does not recover any information of the terrain shape, it satisfactorily removes the identified artefacts from the input topography. We iteratively repeated all steps of the forward modelling (Sects. 2.1 to 2.4) for computation points within a

~200 km radius around affected areas. From Table 2, the fraction of points with removed artefacts is 0.001 % for land points and larger for ocean points (0.03 %), suggesting the overall impact to be comparatively small.

We communicated the ‘bad-data’ locations to the producers of SRTM V4.1 and SRTM30_PLUS V7, confirming the presence of artefacts in their data set. Importantly, these problems had not necessarily become evident had we restricted the modelling to a regionally limited area, e.g., European Alps, as done in past research (e.g., Hirt, 2012). Fig. 2 illustrates the effect of unfilled holes in the SRTM input topography on forward-modelled gravity disturbances over a moderately affected region, and shows both data sets after hole-filling of the SRTM data. While we made an attempt to remove notable or striking artefacts from the input topography through testing against thresholds and visual inspection, less spurious effects are likely to be present in the forward-modelled gravity (cf. Sect. 5). Complete cleaning of the input elevation data at 3 billion points remains a challenge, seemingly also for providers of elevation data sets.

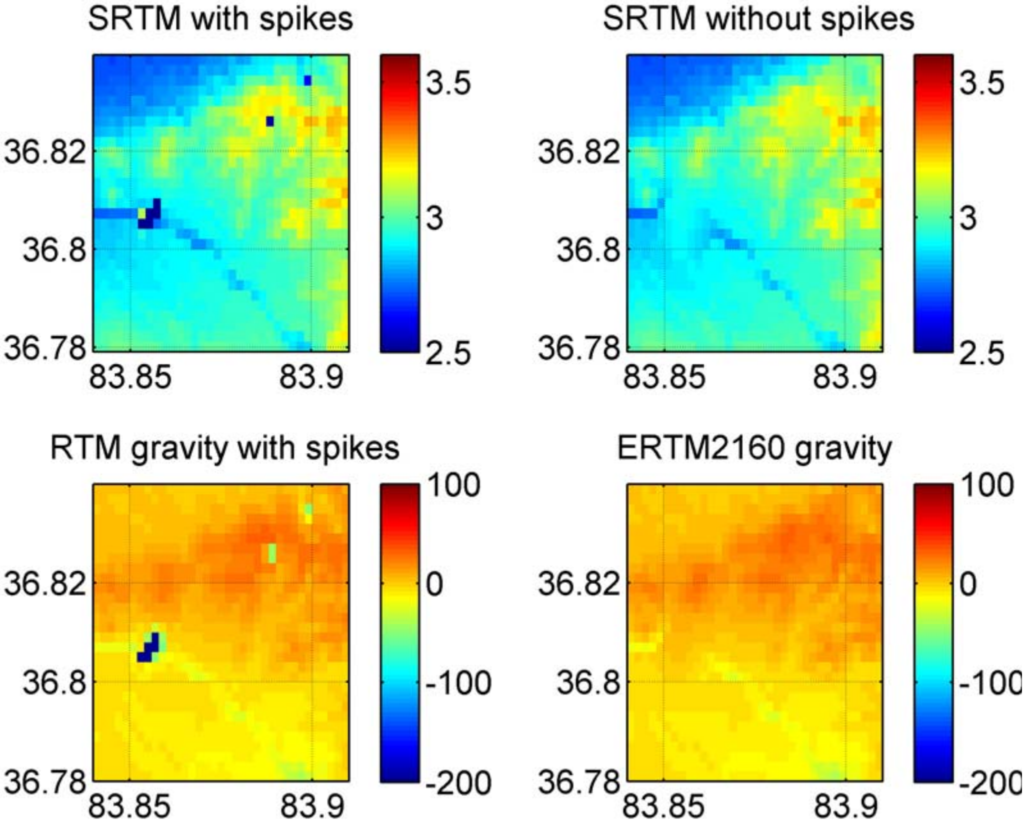


Fig. 2 RTM gravity disturbances before (left) and after spike removal (right). Top: SRTM V4.1 elevations in km, Bottom: short-scale RTM gravity disturbances in mGal.

Table 2 Summary of elevation data sets used, and artefacts replaced

Elevation data set	Model resolution (arc-sec)	#ERTM2160 computation points (billion)	#Elevations replaced	Fraction of directly affected ERTM2160 points
SRTM250m V4.1	7.5	~2.9	2913	~0.001%
SRTM30_PLUS	30	~1.7	2977	~0.03 %

3 Results, comparisons and analyses

3.1 ERTM2160 characteristics

The main outcome of the gravity forward-modelling procedures described in Sect. 2 is the ERTM2160 short-scale gravity field model. It provides numerical values for the four functionals quasi/geoid height, gravity disturbances, North-South and East-West vertical deflections at 3,062,677,383 locations over the SRTM data area (extended with a ~10 km buffer over sea) at a spatial resolution of 7.2 arc-sec. The descriptive statistics of ERTM2160 (Table 3) provide for the first time near-global topography-based estimates of Earth's short-scale gravity field signal strength (half wavelength of ~10 km down to ~250 m), which are omitted by degree-2160 spherical harmonic potential models. ERTM2160 can be used to augment –in approximation– any degree-2160 geopotential model (e.g., EGM2008; Pavlis et al., 2012) or topographic potential model (e.g., dV_ELL_RET2012, Claessens and Hirt, 2013) beyond harmonic degree 2160, thus reducing the signal omission error (e.g., Gruber, 2009) to some extent. Note that the RTM-technique does not augment the spherical harmonic model rigorously because the underlying filtering in the topography domain does not exactly correspond to the filtering in the gravity domain (cf. Section 5).

Table 3 Descriptive statistics of the ERTM2160 gravity field functionals at 3,062,677,383 land and near-coastal points between ± 60° latitude

Functional	Unit	Min	Max	Mean	RMS
Quasi/geoid	m	- 0.280	0.304	0.000	0.016
Gravity disturbance	mGal	-362.4	139.9	-1.050	10.59
North-South vertical deflection	arc-sec	-29.1	31.3	0.000	1.43
East-West vertical deflection	arc-sec	-32.3	29.1	0.000	1.46

From Table 3, the ERTM2160 quasi/geoid has a RMS (root-mean-square) signal strength of 1.6 cm (maximum of ~30 cm over the Himalayas), the RMS of gravity disturbances is 10.6 mGal (variation between –360 to +140 mGal), and the RMS signal strength of vertical deflections is 1.4 arc-sec (maximum ~30 arc-sec). Because of the coverage and point density reached, we consider these estimates to be robust and globally representative 'average values' over land areas of all topography types (flat to high mountains) and adjoining coastal zones.

Fig. 3 shows the local variability of the RMS signal strengths as computed over 0.1° × 0.1° regions (each covering 10,000 ERTM2160 data points) for geoid effects and gravity disturbances. The RMS quasi/geoid signals (Fig. 3 top) are mostly below or at the cm-level over regions with flat topography (i.e., most of Australia), reach 3-4 cm over mountain ranges such as parts of the Rocky Mountains and Andes, and a maximum RMS strength of ~10 cm over the Himalaya region. The signal strengths of gravity disturbances (Fig. 3 bottom) and vertical deflections (shown in Fig. 4) vary qualitatively in a similar way, with maximum RMS signal strengths of ~70 mGal and ~10 arc-sec present over the Himalayas (Fig. 3 bottom and Fig. 4).

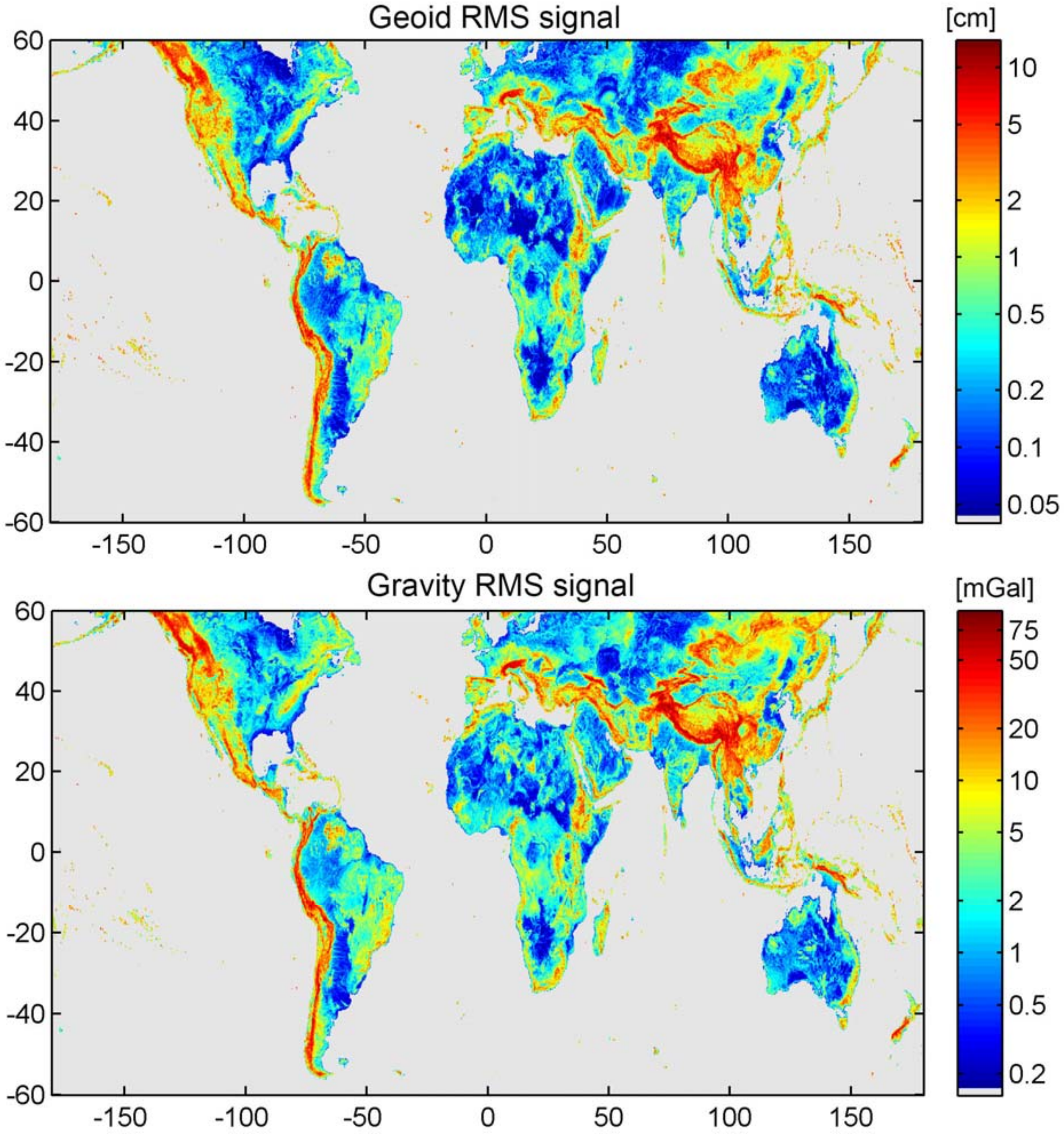


Fig. 3 Spatially varying ERTM2160 RMS signal strengths. Top: RMS of ERTM2160 quasi/geoid effects in centimeters, Bottom: RMS of ERTM2160 gravity disturbances in mGal

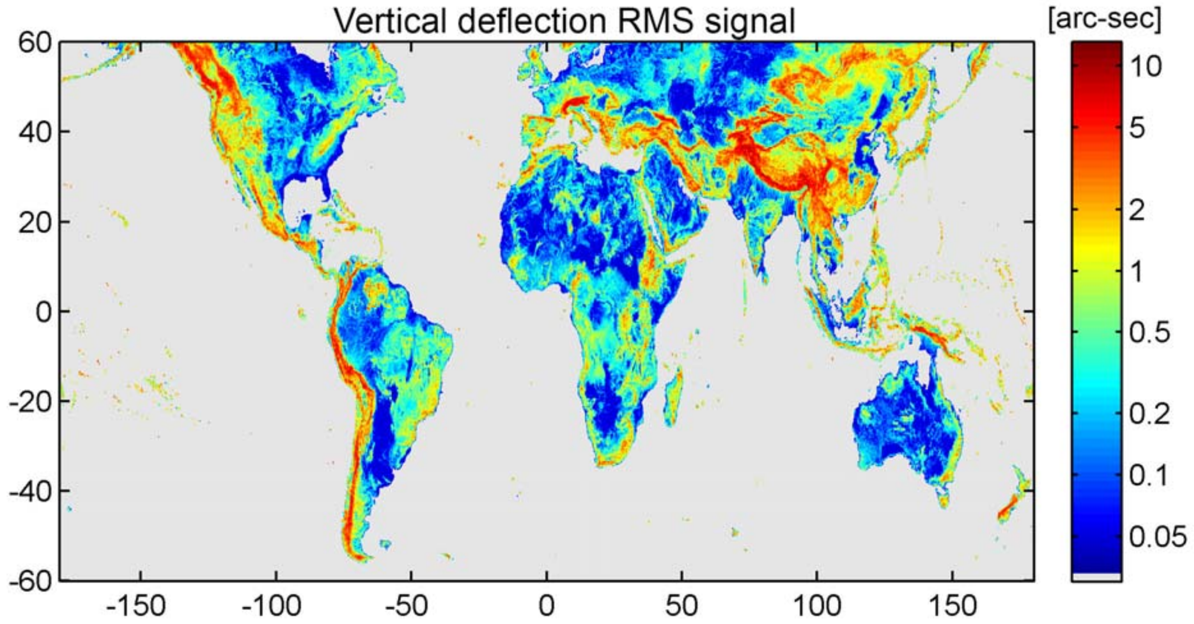


Fig. 4 Spatially varying ERTM2160 RMS signal strengths. RMS of ERTM2160 North-South vertical deflection in arc-sec

3.2 Signal strength as a function of terrain roughness

As a refinement of the global ERTM2160 statistics, signal strengths for the four gravity functionals were computed as a function of the terrain roughness. A reasonable measure for the local terrain roughness (variability of heights) is the standard deviation (STD) of the RTM elevations calculated within sufficiently small regions. The entire ERTM2160 data area was subdivided into 0.1° × 0.1° tiles, and terrain roughness values were assigned to each tile. Fig. 5 shows the spatially varying RMS signal strengths for geoid, gravity and the two vertical deflection components as a function of the terrain roughness (blue dots). There is marked correlation between terrain roughness and gravity signal strengths which varies between 0.976 and 0.995 depending on the functional (Table 4).

It is useful to form classes of different terrain roughness, e.g., variability of heights < 100 m, 100 m to 200 m, and so on, and to calculate the gravity statistics within these classes. A generalisation of this idea leads to classes of small class widths (e.g., 20 m) and a subsequent least squares fit of the gravity signal strengths in order to establish the relationship between terrain roughness and gravity signal strengths. The RMS gravity signal strengths were thus calculated over all areas with the same terrain roughness in classes of 20 m width (from 0 to 500 m terrain roughness, and larger class width of 100 m from 500 m to 800 m roughness because of the reduced number of data points). The classified RMS signal strengths (red curves in Fig. 5) reveal in fairly good approximation a linear relationship between RMS gravity signal strengths and terrain roughness values. We then fitted the RMS signal strengths (blue points) through least-squares regression lines (without intercept terms/bias fit).

From a linear regression (green straight lines in Fig. 5), the RMS signal strength per 100 m terrain roughness are ~1.7 cm (geoid heights), ~11 mGal (gravity disturbances) and ~1.5 arc-sec (vertical deflections in North-South and East-West direction), cf. Table 4. These numbers can be used as a "rule of thumb" to easily estimate the magnitude of signals omitted by degree-2160 (or 10 km resolution) potential models for various types of hilly or mountainous terrain anywhere on Earth. For instance, over a rugged terrain with ± 200 m STD, an omission error of ~3.4 cm (geoid height), ~22 mGal (gravity disturbances) and ~ 3 arc-sec (vertical deflections) is to be expected when using the full expansion of the EGM2008 geopotential model. As an aside, the RMS signal strengths per 100 m terrain roughness (Table 4) are very similar to the (global) RMS signal strengths (Table 3). This is because globally the mean terrain roughness is 92.6 m which is close to 100 m too.

Table 4 Correlation coefficients (between terrain roughness and gravity signal strengths) and gravity signal strengths per 100 m terrain roughness) for the ERTM2160 gravity field functionals

Functional	Correlation coefficient	RMS signal strength per 100 m terrain roughness
Quasi/geoid	0.976	1.74 cm
Gravity disturbance	0.995	11.5 mGal
North-South vertical deflection	0.981	1.53 arc-sec
East-West vertical deflection	0.982	1.57 arc-sec

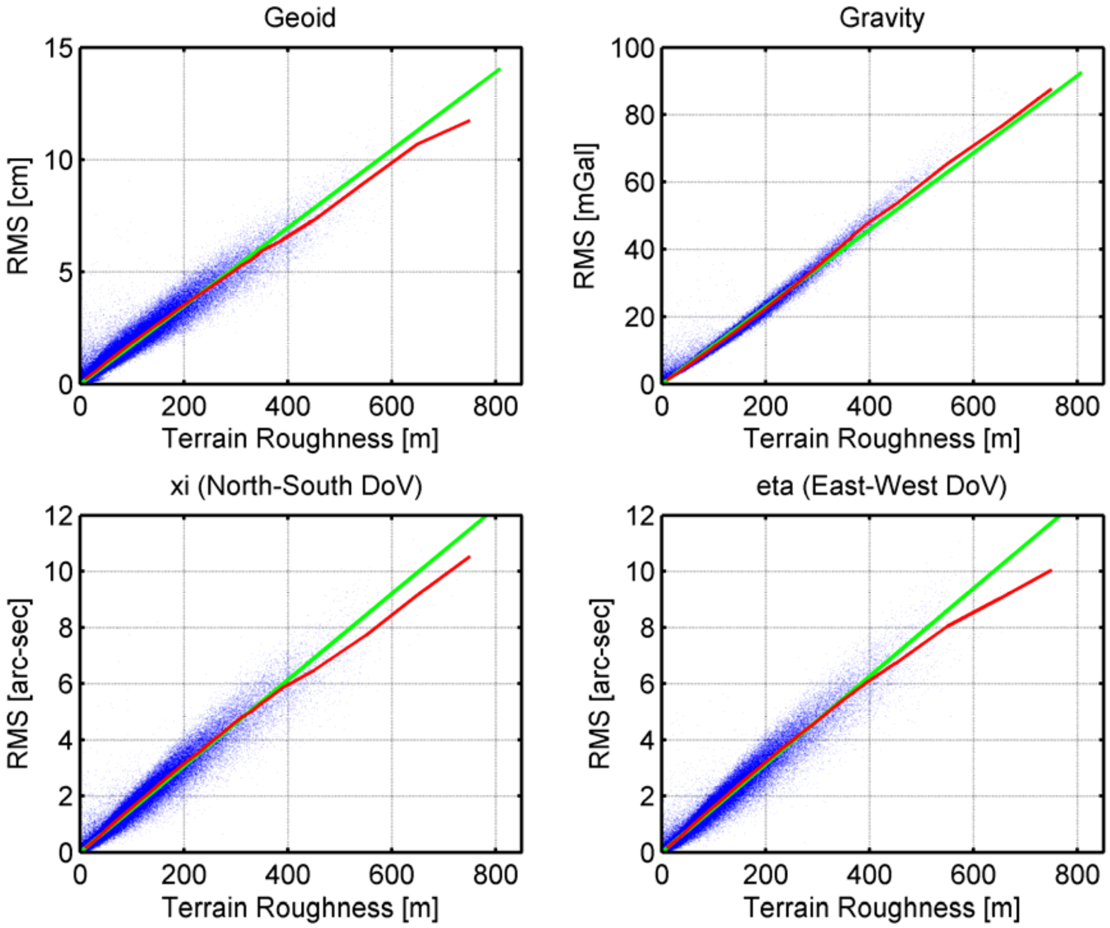


Fig. 5 Spatially varying ERTM2160 signal strengths (RMS) as a function of the terrain roughness (standard deviation of heights) computed over 0.1 degree cells (blue). RMS signal strengths for terrain roughness classes (20 m class width from 0 to 500 m, 100 m class width beyond) shown in red, and regression curve (linear model) shown in green. Signal strengths shown for geoid height and gravity disturbances (top row), and North-South and East-West deflection of the vertical (bottom row).

3.3 Comparisons with degree-variance models

For comparison purposes, we have compiled estimates for short-scale signal strength from the literature, which are based on frequently used degree-variance models or modifications thereof (Table 5). We include estimated RMS signal components from

(i) the Tscherning-Rapp (1974) model at spatial scales of ~10 to ~1 km, or harmonic degrees of ~2,000 to 20,000 (numerical values from Torge, 1981; Roland, 2005),
(ii) the rule of thumb by Kaula (1966), with numerical values from equations provided by Jekeli et al. (2009), Sanso and Sideris (2013)
(iii) Jekeli et al. (2009) who fitted a power law model through the EGM2008 power spectral density between degrees 120 and 1,200, and
(iv) Sanso and Sideris (2013) who fitted a modified version of the Tscherning-Rapp model through the EGM2008 signal between degrees 180 and 1,800,

which we compare against those from ERTM2160 (Table 3).

RMS signal strengths estimated from the classical Tscherning-Rapp model (that relies on free-air gravity anomalies) are in good agreement with ERTM2160. For all functionals, the ERTM2160 signal strengths are somewhat smaller than implied by Tscherning-Rapp (1.6 vs. 2.6 cm, 10.6 vs. 11.8 mGal, and 1.45 vs. 1.75 arc-sec). This could potentially be attributed to the influence of un-modelled mass-density variations in ERTM2160, but it may also suggest that the Tscherning-Rapp model slightly overestimates the short-scale signal strength. The quasi/geoid signal strength estimate from ERTM2160 (1.6 cm) is in between recent estimates by Jekeli et al. (2009) (4.1 cm) and Sanso and Sideris (2013) (0.5 cm), cautiously suggesting that the former overestimates and the latter underestimates the quasi/geoid omission error of degree-2160 geopotential models. Note that ERTM2160 essentially reflects the topography-implied gravity field characteristics over land, while the power law models are (partially) based on gravity data over the oceans too, where the gravity field is often smoother. ERTM2160 signal strengths are found to be mostly smaller than those implied by the power laws (Table 5). This either indicates underestimation of signal strengths by ERTM2160, or overestimation through the power laws. Ongoing research attempts to clarify this observation.

Table 5 Estimates for short-scale gravity field signals from a cursory literature survey

Model	**Functional**	**Spatial scales**	**Signal strength**	**Reference**
Tscherning-Rapp	quasi/geoid	~10 to 1 km	2.6 cm	Roland (2005, p7) based on Torge (1981)
	gravity		11.8 mGal	
	vertical deflection		1.75 arc-sec	
Kaula	geoid	< ~10 km	2.9 cm	Jekeli et al. (2009, Eq. 19), Sanso and Sideris (2013, Eq. 3.179)
Power law model based on EGM2008	geoid	<~10 km	4.1 cm	Jekeli et al. (2009, Eq. 23)
Modified Tscherning-Rapp based on EGM2008	geoid	<~10 km	~0.5 cm	Sanso and Sideris (2013, Fig 3.9)

4 Application examples

The following geodetic applications could benefit from the availability of ERTM2160.

- As a central application, ERTM2160 spectrally enhances degree-2160 geopotential models (e.g., EGM2008) by simple addition of synthesized gravity effects and ERTM2160 gravity. This combination provides spectrally more complete gravity knowledge than provided by degree-2160 models alone, as could be demonstrated by comparisons against ground-truth gravity field observations in several studies (e.g., Hirt, 2010; Hirt et al., 2010; Guimarães et al., 2012; Šprlák et al., 2012; Zhang and Xuebao, 2012; Filmer et al., 2013). This technique can serve a number of applications such as improved GNSS height transfer, in-situ-computation of height system corrections and screening of gravity data bases (Hirt, 2012). ERTM2160 could also be of some utility in the construction of future geopotential models of degree higher than 2160, in analogy to Pavlis et al. (2007; 2012) who used topography to forward model gravity effects at spatial scales of ~10 to ~24 km, and utilized these “fill-in gravity” as additional input for EGM2008.
- For Remove-Compute-Restore (RCR)-based regional gravimetric quasi/geoid computations (e.g., Forsberg and Tscherning, 1981; Tscherning, 2013; Denker, 2013), ERTM2160 gravity disturbances could prove useful as in-situ data source to smooth observed gravity anomalies. Analogously, in astrogeodetic geoid determination based on astronomical-topographic levelling (Hirt and Flury, 2008), ERTM2160 vertical deflections could be used to smooth observed vertical deflection before interpolation.
- Flury (2006) described and applied a range of methods for transforming regional gravity data sets from the space into the frequency domain, and studied the resulting power spectra. While Flury (2006) worked with topographically-reduced gravity anomalies, he pointed out the need to analyse the spectral constituents of topographic gravity signals as well. By applying Flury’s methods on ERTM2160 gravity effects, ‘localised’ or – through averaging – global short-scale power spectra could be obtained, which are useful for further verification or refinement of existing degree-variance models at very short spatial scales.

5 Limitations

For the application of the ERTM2160 topography-implied gravity field model, e.g., as a proxy over regions with scarce gravity data coverage, or as an aid to smooth gravity field observations before interpolation, it is important to be aware of limitations originating from the modelling techniques and topography/bathymetry data used.

First and foremost, the ERTM2160 gravity forward-modelling is based on the assumption of constant mass-density for the residual topography. While the mass-density of major water-bodies (Table 1) has been taken into account as rock-equivalent topography (Sect 2.2), no attempt was made to model local mass-density anomalies as associated with, e.g., salt-domes, valley fillings in the mountains, oceanic sediments. This is mainly because a global digital density data base that would provide 3D information on local mass-density anomalies with sufficient spatial resolution (e.g. Tsoulis, 2013) was not available for this work. ERTM2160 implicitly relies on the assumption of isostatically uncompensated residual topography. Given Earth’s lithosphere thickness often reaches several tens of km (e.g., Watts, 2011), it is reasonable to assume the topographic masses supported at spatial scales less than 10 km.

A weakness of the RTM gravity forward-modelling technique, though widely used in practice, is the fact that the spectral characteristics of high-pass filtered elevation data and implied RTM gravity effects are different. In other words, the residual gravity field is not consistent with the residual topography because the relationship between gravity and topography is non-linear (e.g., Rummel et al., 1988). The spectral inconsistency caused by the non-linear relationship can produce additional errors as large as ~6% of the RTM gravity signal (cf. Hirt and Kuhn, 2014, Sect. 4 ibid) in case a degree-2160 spherical harmonic topography is used as filter. The investigation of pathways for a correction or reduction (e.g., filtering in the gravity domain instead of the topography domain, e.g. Baran et al. (2006); Pavlis et al. (2007)) of this issue is a future task.

While the 7.2 arc-sec spatial resolution of the ERTM2160 short-scale gravity field investigated in our study is much higher than that of any previous global forward-modelling efforts (mostly 1 arc-min in the past), there is still a representation error involved. This is because the very fine structure of the terrain at spatial scales of few metres to ~220 m is not represented by the 7.2 arc-sec topography data used. In rapidly undulating and steep mountainous terrain (e.g., 45° inclination) as an extreme case, the topography representation error associated with 7.2 arc-sec resolution is estimated to reach values as large as ~100 m, which translates into a gravity representation error of ~10 mGal. Use of higher-resolution topography data in future forward modelling efforts will reduce this effect.

Finally, it is important to note that topography and bathymetry models only ever approximate the geometry of the actual terrain and sea bed only to some extent. While any large-scale (i.e, half-wavelengths of 10 km or more) errors in the elevation data are filtered out in the RTM-approach, short-scale errors will have entered unfiltered in the ERTM2160 gravity field. Although an attempt was made to remove obvious small-scale bad-data areas from the input topography and bathymetry (Sect 2.5), there may be smaller artefacts present in ERTM2160. Particularly along the coastlines of the several hundreds of Pacific islands, the high-pass filtered bathymetry often exhibits peak-like or circular depressions, with an associated ~10-20 mGal gravity effect, in some cases possibly exceeding ~100 mGal. In the absence of independent control (reliable bathymetry or gravimetric observations) over these regions, it is difficult to decide whether these depressions are real or artificial. ERTM2160 may therefore have limitations in coastal zones surrounding islands.

6 Concluding remarks

The successful development of the ERTM2160 short-scale gravity model demonstrates that ultra-high resolution gravity forward-modelling has become possible at a global scale based on massive parallel computation. As such, ERTM2160 is the first of a new kind of topography-based gravity field representations, which combine localized ultra-high resolution information and near-global coverage. ERTM2160 gravity functionals can be used to augment any degree-2160 harmonic model at spatial scales of ~10 km to ~250 m. This enhances the spatial resolution of EGM2008 or other degree-2160 models by a factor of 40.

The ERTM2160 model was used to study the characteristics of Earth's short-scale gravity field based on near-global coverage over land areas and ultra-high resolution. Spatially varying statistics were applied to calculate global maps of RTM gravity signal strengths and their dependency on the terrain roughness. The relationship between the RTM gravity signal strengths and terrain roughness values was found to be linear with a correlation of 0.995 for gravity, and slightly lesser correlation for geoid heights and vertical deflections. This was used to establish a new rule of thumb that per 100 m variation in terrain height (standard deviation) gravity field signals of 1.7 cm (geoid), 11 mGal (gravity) and 1.5 arc-sec may be expected at spatial scales of ~10 km to ~250 m. This new rule of thumb may be of value to easily estimate the magnitude of the omission error in gravity signals by degree-2160 geopotential models, notably EGM2008 over various types of terrain.

While a forward-modelling grid-resolution of 7.2 arc-secs – commensurate with the 250 m elevation data – was chosen for this work, a further increase in forward-modelling resolution is likely based on the ever-increasing performance of supercomputing resources. The global calculation of gravity effects at the ~3 arc-sec SRTM basis resolution is foreseeable, as is a further increase to 1 arc-sec (ASTER basis resolution, Tachikawa et al., 2011). The availability of largely clean elevation data – free of artefacts – is crucial in this context.

Postscript

Bad data areas, which were detected in the SRTM30_PLUS bathymetry via analysis of ERTM2160 gravity effects and reported to the data producers (Scripps Institution of Oceanography, Prof. Sandwell), have now been rectified in the latest SRTM30_PLUS releases (v9 and v10).

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