TECHNICAL NOTE

A Note on Void Ratio of Fibre-Reinforced Soils

Sanjay Kumar Shukla[1] • Mohamed A. Shahin[*2] • Hazim Abu-Taleb[3]

Abstract This technical note extends the concept of void ratio, presented traditionally in soil mechanics, for fibre-reinforced soils. Phase relationships related to the void ratio of fibre-reinforced soils are presented along with their definitions. A simple analytical model verified with experimental data for estimating the void ratio of fibre-reinforced soils is developed which can be used to express the compressibility of fibre-reinforced soils in geotechnical engineering applications. The results indicate that the void ratio of fibre-reinforced soils is dependent on the volume ratio of fibre-soil solid.

Keywords Fibre • Soil • Void ratio • Reinforcement

[*]Corresponding author, Mohamed A. Shahin
m.shahin@curtin.edu.au

[1]Discipline of Civil and Environmental Engineering, School of Engineering, Edith Cowan University, 270 Joondalup Drive, Joondalup, Perth, WA 6027, Australia, E-mail: s.shukla@ecu.edu.au, sanjaykshukla1@gmail.com

[2]Department of Civil Engineering, Curtin University, Perth, WA 6845, Australia, E-mail: m.shahin@curtin.edu.au

[3]Graduate Engineer, Perth, Australia

Introduction

The concept of phase relationships of soil mechanics is extensively used in geotechnical engineering to quantify the engineering behavior of soils due to the fact that the relative proportions of each phase in an element of soil have a significant impact on its engineering behavior. An element of soil mass can be represented as shown in Fig. 1 by a three-phase system containing soil solid (mineral particles), liquid (usually water), and gas (usually air). The space occupied by the air and/or water in a soil mass represents the volume of soil void. The ratio of the volume of soil void to the volume of soil solid in a given soil mass is called the void ratio of soil. The importance of the void ratio lies in the fact that it significantly affects the soil behavior, such as compressibility, which is generally studied in terms of the change in the soil void ratio while ignoring the change in the volume of the individual solid particles due to the fact that crushing and fracturing of the solid particles under applied stresses are insignificant [1-3].

In recent years, synthetic fibres have been developed from several wastes such as plastic and old tires, and have been used for improving the engineering behavior of soils by the addition of random distribution of fibres within the soil mass or by placing one or more layers of fibres in the form of geosynthetic products within the soil mass [4]. An element of fibre mass can be represented as shown in Fig. 2 by a three-phase system containing fibre solid, liquid and gas. When fibre is added to the soil matrix, a similar phase system to that used for soils can be proposed for fibre-reinforced soils. However, it is well expected that the addition of fibre to unreinforced soils, forming the fibre-reinforced soils, affects the phase relationships of fibre-reinforced soils. The properties of added fibre under applied stresses are expected to behave differently from those of the soil matrix alone; thus, the soil solid and fibre solid should be dealt with as two different components of the phase system of fibre-reinforced soils. Consequently, an additional phase can be embedded into the phase system of fibre-reinforced soils to represent the solid volume of fibre, leading to the four-phase system shown in Fig. 3. Diambra et al. [5] and Ibrahim et al. [6] briefly discussed the fibre content in terms of the total dry unit weight and total volume of fibre-reinforced soil to study the behavior of fibre-reinforced sands, without focusing on the void ratio. When dealing with the behavior of fibre-reinforced soils in most studies found in the literature on the fibre-reinforced soils, both the fibre solid and soil solid are combined together as one component and no special attention has been paid to the volume of fibre (i.e., fibre void volume + fibre solid volume) to develop the phase relationships for fibre-reinforced soils.

Keeping the need for developing phase relationships in mind, the concept of void ratio needs to be extended in order to make it possible to explain the behavior of fibre-reinforced soils, following the approach used widely in soil mechanics for analyzing the unreinforced soils. In the present work, an attempt is made to extend the traditional concept of the phase relationships for the void ratio of fibre-reinforced soils, which has not received due attention in the past but is considered to be an important factor in geotechnical engineering applications of fibre-reinforced soils.

Void Ratio of Fibre-Reinforced Soils

In the context of the phase diagram of an element of fibre-reinforced soil, as shown in Fig. 3, the following phase relationships are proposed along with their definitions:

Void ratio of reinforced soil mass,

$$e_r = \frac{V_v}{V_s} \tag{1}$$

Void ratio of soil mass,

$$e_s = \frac{V_{vs}}{V_{ss}} \tag{2}$$

Void ratio of fibre mass,

$$e_f = \frac{V_{vf}}{V_{sf}} \tag{3}$$

Volume ratio of fibre-soil solid,

$$m = \frac{V_{sf}}{V_{ss}} \tag{4}$$

where V_v is the volume of void of fibre-reinforced soil, V_s is the volume of solid of fibre-reinforced soil, V_{vs} is the volume of soil void, V_{ss} is the volume of soil solid, V_{vf} is the volume of fiber void, and V_{sf} is the volume of fibre solid.

A derivation of an expression for the void ratio of fiber-reinforced soil, e_r, can be obtained in terms of the void ratio of soil, e_s, void ratio of fibre, e_f, and volume ratio of fiber-soil solid, m, as follows:

$$e_r = \frac{V_v}{V_s} = \frac{V_{vs} + V_{vf}}{V_{ss} + V_{sf}} = \frac{\frac{V_{vs}}{V_{ss}} + \frac{V_{vf}}{V_{sf}} \times \frac{V_{sf}}{V_{ss}}}{1 + \frac{V_{sf}}{V_{ss}}} = \frac{e_s + me_f}{1 + m} \qquad (5)$$

From Eq. (5), the following two extreme special cases may be observed:

(a) As the volume ratio of fibre-soil solid, m, tends to 0, the void ratio of fibre-reinforced soil, e_r, becomes equal to the void ratio of soil, e_s; and

(b) As the volume ratio of fibre-soil solid, m, tends to ∞, the void ratio of fibre-reinforced soil, e_r, becomes equal to the void ratio of fibre, e_f.

Variation of the void ratio of fibre-reinforced soil, e_r, with the volume ratio of fibre-soil solid, m, can be observed graphically, as shown in Fig. 4, for typical randomly selected values of void ratio of soil, e_s, and void ratio of fibre, e_f, equal to 0.8 and 1.5, respectively. It can be noticed that the void ratio of fibre-reinforced soil, e_r, is greatly dependent on the volume ratio of fibre-soil solid, m. In the following section, a variation of e_r with m will also be observed experimentally so that an expression that correlates e_r with m can be established, which can be used for prediction of e_r.

Experimental Programme

Testing materials

The soil used in this study is Brickies sand, classified in accordance with the Unified Soil Classification System (USCS) as poorly graded sand (SP). Fig. 5 shows the particle-size distribution curve of the sand used and Table 1 summarizes its basic properties. The fibre used as reinforcement is virgin homo-polymer polypropylene, with the properties given in Table 2 and structure shown in Fig. 6.

Preparation of specimens

In this section, a series of laboratory tests were conducted on fibre-reinforced soils in an attempt to establish an empirical expression to predict e_r from m. Before conducting the tests, it was important to relate both e_r and m to some other properties that can be measured in the laboratory. To do so, m was expressed in terms of the weights of fibre solid and soil solid as well as the specific gravity of fibre and soil instead of the volumes of fibre solid and soil solid, as follows:

$$m = \frac{V_{sf}}{V_{ss}} = \frac{(W_{sf} / \gamma_w G_{sf})}{(W_{ss} / \gamma_w G_{ss})} = \frac{W_{sf} G_{ss}}{W_{ss} G_{sf}} = p \frac{G_{ss}}{G_{sf}} \tag{6}$$

where W_{sf} is the weight of fibre solid; W_{ss} is the weight of soil solid; p is the ratio of W_{sf} to W_{ss}, that is, the fibre content by dry weight of soil; γ_w is the unit weight of water; G_{ss} is the specific gravity of soil; and G_{sf} is the specific gravity of fibre.

On the other hand, using the soil mechanics phase relationships, e_r can be calculated as follows:

$$e_r = \frac{G_{sr} \gamma_w}{\gamma_{dr}} - 1 \tag{7}$$

where: G_{sr} is the specific gravity of fibre-reinforced soil mixture; γ_w is the unit weight of water; and γ_{dr} is the dry unit weight of fibre-reinforced soil mixture. In geotechnical engineering applications of fibre-reinforced soils, the mixture of soil and fibre should be compacted to reach its optimum compaction, hence, γ_{dr} in Eq. (7) is that corresponds to the maximum dry unit weight, $\gamma_{dr(\max)}$.

In order to develop the relationship between the void ratio of fibre-reinforced soil, e_r, and volume ratio of fibre-soil solid, m, a number of specimens of sand-fibre mixtures were prepared at seven different values of p = 0.25%, 0.5%, 1.0%, 2.0%, 3.0%, 4.0%, and 5.0%. For each sand-fibre mixture, a specific gravity test was performed to obtain G_{sr} and the standard Proctor compaction test was also carried out to obtain $\gamma_{dr(\max)}$. The fibre-reinforced soil samples were prepared by mixing dry sand with fibre using Hobart mixer, and water was added to the mixture to achieve required moisture content. The prepared samples were mixed for 90 seconds to allow homogeneity and were then placed in polyethylene bags, tied up and left overnight for water to be uniformly distributed prior to testing. Using the experimental data obtained together with Eqs. (6) and (7), seven different values of m and corresponding e_r were calculated and used to develop the relationship between m and e_r, as explained below.

Test Results ad Discussion

The results of specific gravity tests carried out on the sand-fibre mixtures for different values of fiber content p are presented in Fig. 7. It can be seen that the specific gravity of mixtures decreases with an increase of p, as expected. In the current work, it was also possible to develop an analytical derivation for determination of the specific gravity of the sand-fibre mixture based on the specific gravity of both the soil and fibre, as follows:

$$
\begin{aligned}
G_{sr} &= \frac{W_s}{V_s \times \gamma_w} = \frac{W_{ss} + W_{sf}}{(V_{ss} + V_{sf}) \times \gamma_w} = \frac{W_{ss} + p \times W_{ss}}{\left[\left(\dfrac{W_{ss}}{G_{ss} \times \gamma_w}\right) + \left(\dfrac{W_{sf}}{G_{sf} \times \gamma_w}\right)\right] \times \gamma_w} \\
&= \frac{W_{ss} + p \times W_{ss}}{\left(\dfrac{W_{ss}}{G_{ss}}\right) + \left(\dfrac{p \times W_{ss}}{G_{sf}}\right)} = \frac{1 + p}{\left(\dfrac{1}{G_{ss}}\right) + \left(\dfrac{p}{G_{sf}}\right)} = (1 + p)\left(\frac{1}{G_{ss}} + \frac{p}{G_{sf}}\right)^{-1}
\end{aligned}
\tag{8}
$$

The validity of Eq. (8) was verified by the experimental data in Fig. 7 and the results show an excellent agreement. The results of the compaction tests are presented in Fig. 8, which indicates that the maximum dry density of the sand-fibre mixtures decreases with an increase of fibre content, as expected.

Using the results of the above tests, the values of m and corresponding e_r were calculated and plotted in Fig. 9, which shows that the void ratio of the sand-fibre mixture increases with an increase of the volume ratio m of the mixture. The trend in Fig. 9 obtained from the experimental results is in a good agreement with that observed in Fig. 4 obtained analytically using Eq. (5). The data shown in Fig. 9 can be re-plotted on log scale in the horizontal axis, as shown in Fig. 10. It can be seen from Fig. 10 that m was adjusted to be $(100m + 1)$ to make the value of m fits more adequately on the log scale and to avoid the problem of having log of zero. Based on the data shown in Fig. 10, a linear model is proposed for the relationship between m and e_r, as follows:

$$e_r = a\ln(100m+1) + b \tag{9}$$

where: $a = 0.0333$ is the slope of the linear relationship and represent the type of fibre used (i.e., virgin homo-polymer polypropylene) and $b = 0.4913$ is the value of e_r at $m = 0$ and represents the type of sand used (i.e., poorly graded silica sand). For other types of fibre and/or sand, a and b can be obtained from the data obtained from a minimum of four compaction tests on the fibre-reinforced soil at hand from which the measured values of m and the corresponding e_r can be calculated and used to determine a and b, then Eq. (9) can be used for future prediction the void ratio of that fibre-reinforced soil.

Concluding Remarks

The concept of phase relationships, being adopted widely in soil mechanics for unreinforced soils, was utilized for developing the phase relationships related to the void ratio of fibre-reinforced soils. The phase relationships are fairly of general nature and can be utilized in several applications of fibre-reinforced soils especially in design of foundations. An analytical linear regression model for the relationship between the void ratio of fibre-reinforced soil and logarithmic of volume ratio of fibre-soil solid was derived, which can be useful for expressing the fibre-reinforced soil compressibility, relative density and permeability in a way similar to that traditionally used in soil mechanics for analysis of unreinforced soils. The developed relationship includes two dependent empirical parameters, one for the soil type and the other for the fibre type. These two parameters can be determined for any soil-fiber mixture from the data obtained from a minimum of four compaction tests on the fibre-reinforced soil of interest. The developed void ratio of sand-fibre mixture reduces the gap knowledge in soil mechanics of fibre-reinforced soils. It should be noted that other researchers may carry out experimental studies with several fibre types and also for cemented fibre-reinforced soils to compare their results with the developed phase relationships.

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Table 1 Properties of the sand used

Property	Value
Specific gravity	2.65
Maximum dry unit weight* (kN/m^3)	17.4
Particle size	
D_{10} (mm)	0.18
D_{30} (mm)	0.32
D_{60} (mm)	0.50
Coefficient of uniformity, C_u	2.78
Coefficient of curvature, C_c	1.14

***** Standard Proctor.

Table 2 Properties of the fiber used

Property	Value
Specific gravity	0.91
Length (mm)	19
Tensile strength (MPa)	620–758

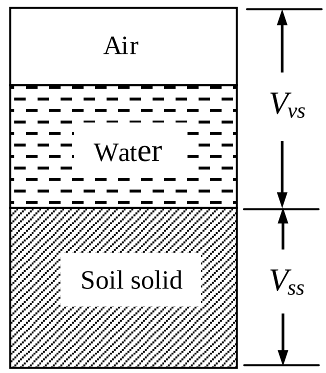

Fig. 1 An element of soil mass represented by a three-phase system

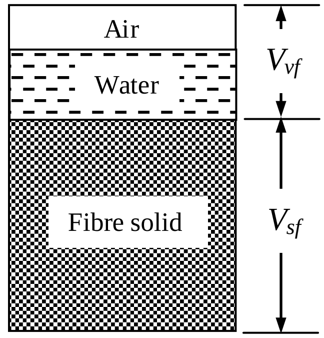

Fig. 2 An element of fiber mass represented by a three-phase system

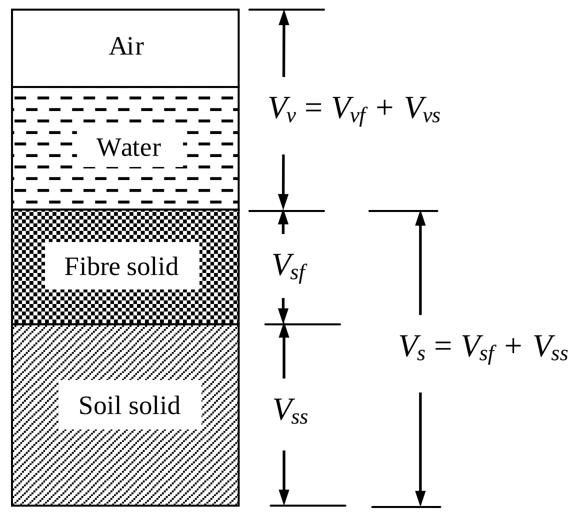

Fig. 3 An element of fiber-reinforced soil mass represented by a three-phase system

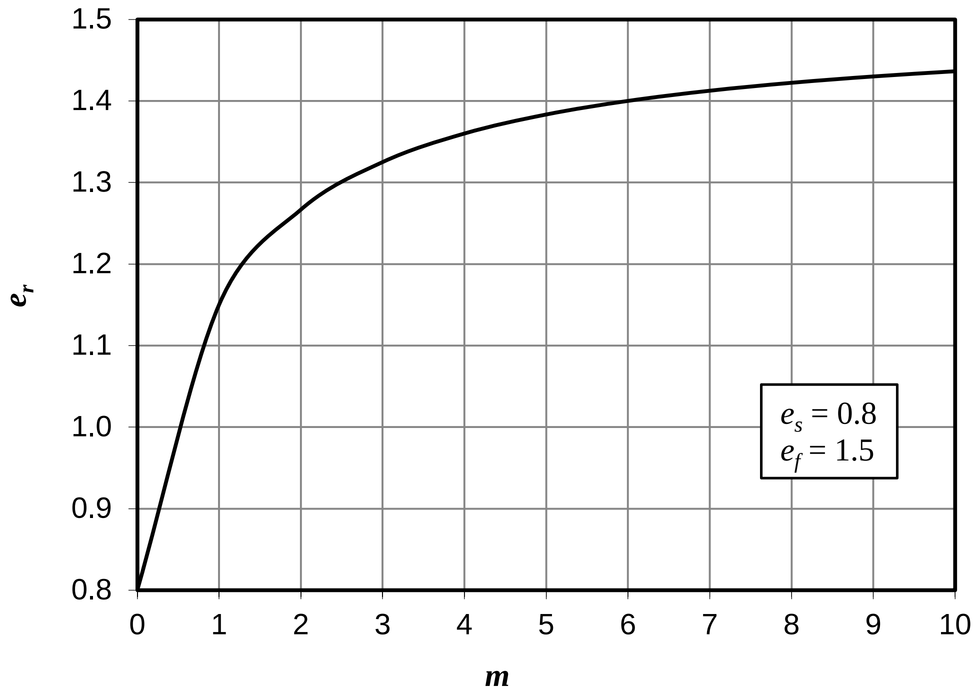

Fig. 4 Effect of volume ratio of fibre-soil solid, m, on void ratio of fibre-reinforced soil, e_r based on Eq. (5)

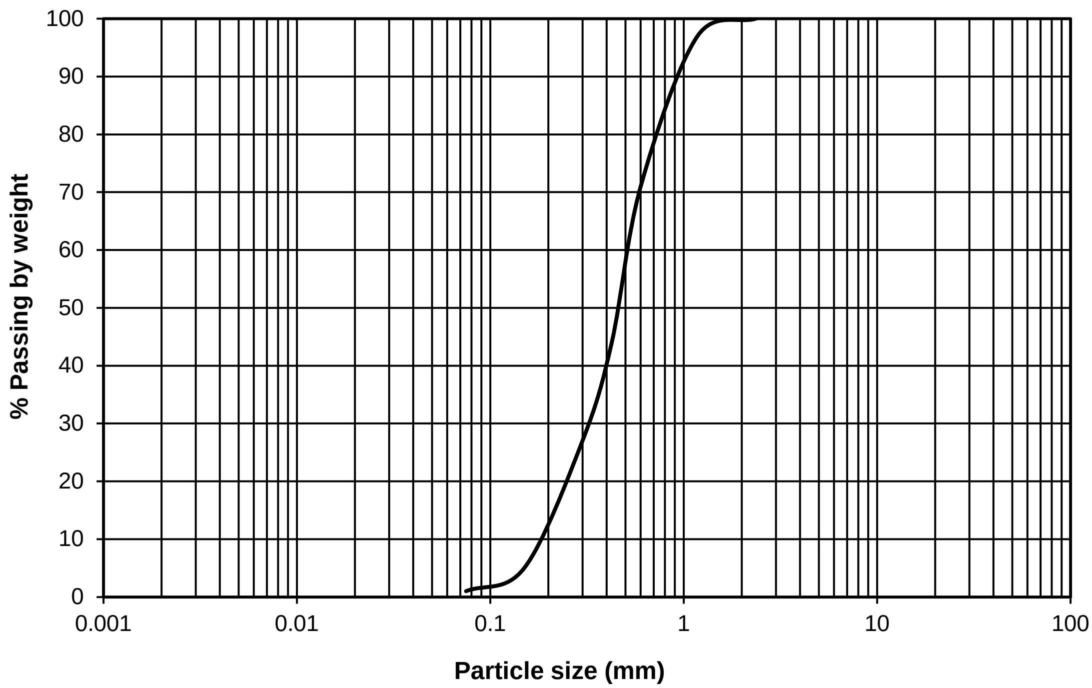

Fig. 5 Particle-size distribution of the sand used

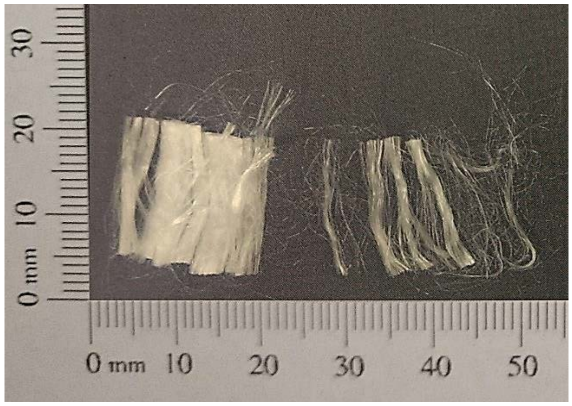

Fig. 6 Virgin homo-polymer polypropylene fiber

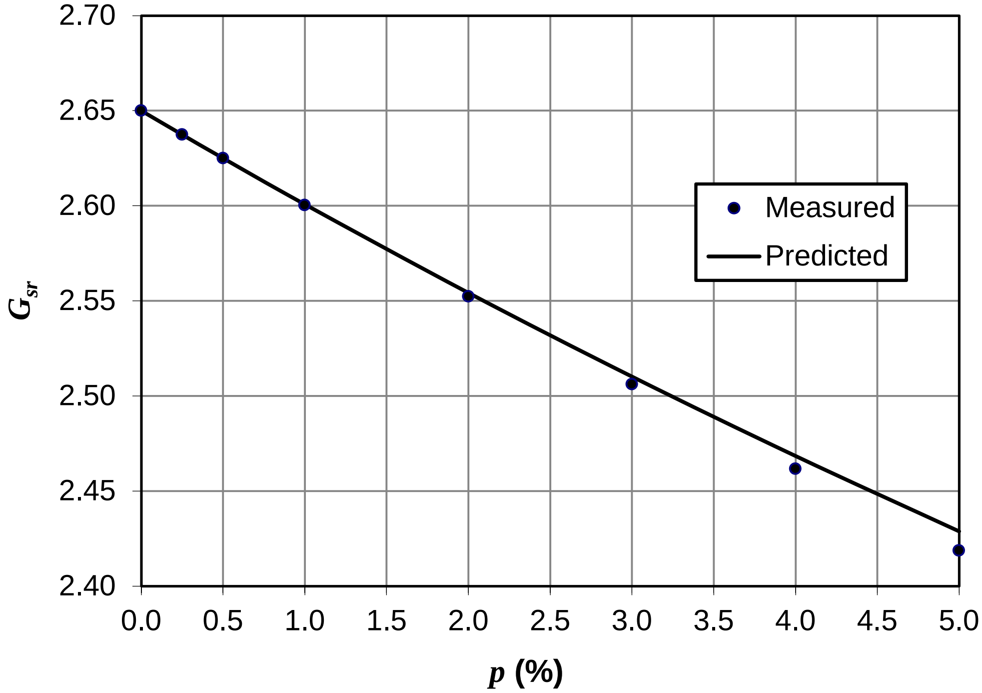

Fig. 7 Variation of the specific gravity G_{sr} of sand-fibre mixtures with the fibre content p

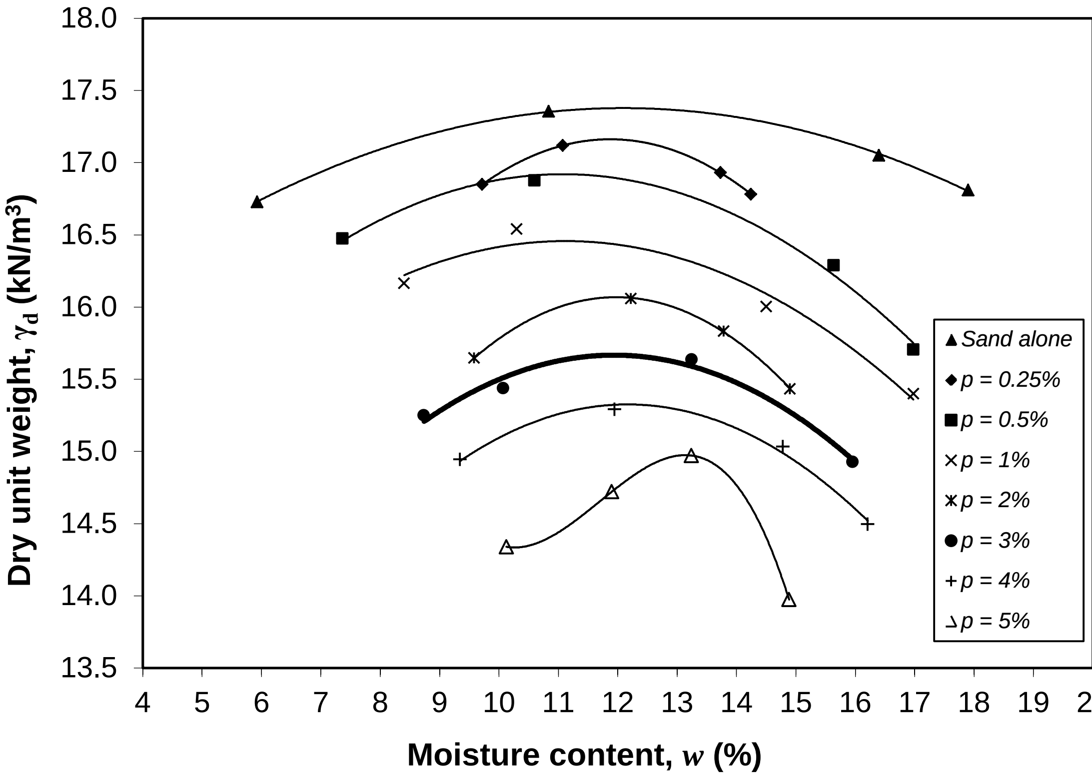

Fig. 8 Compaction curves for various sand-fibre mixtures

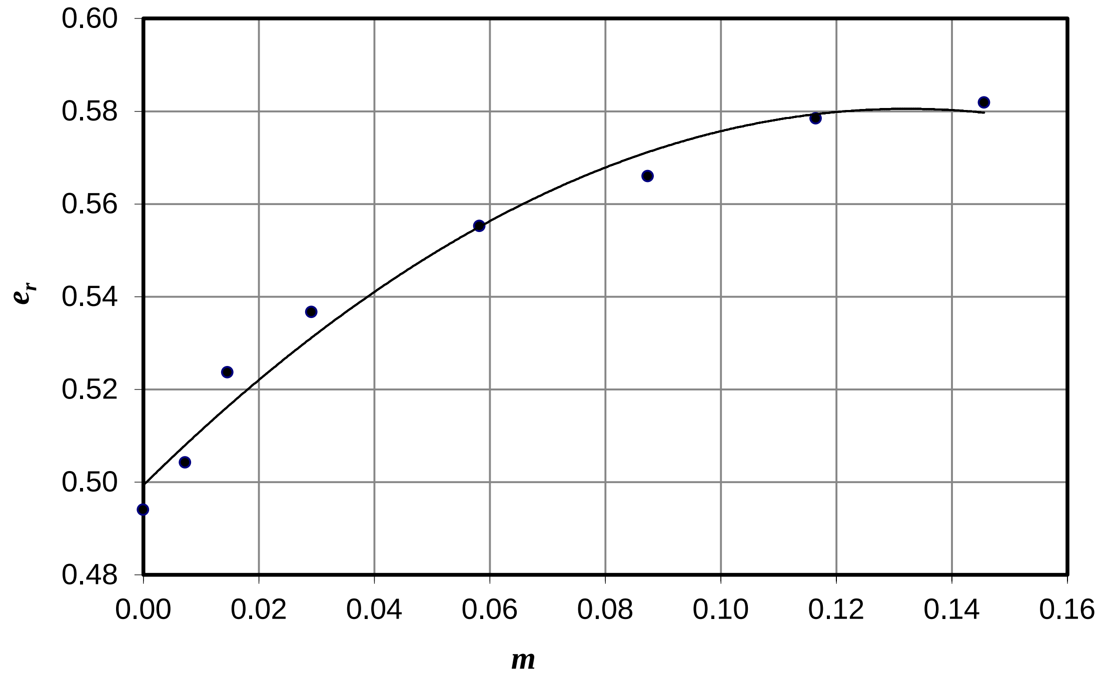

Fig. 9 Effect of volume ratio of fibre-soil solid, m, on void ratio of fibre-reinforced soil, e_r, based on the experimental data

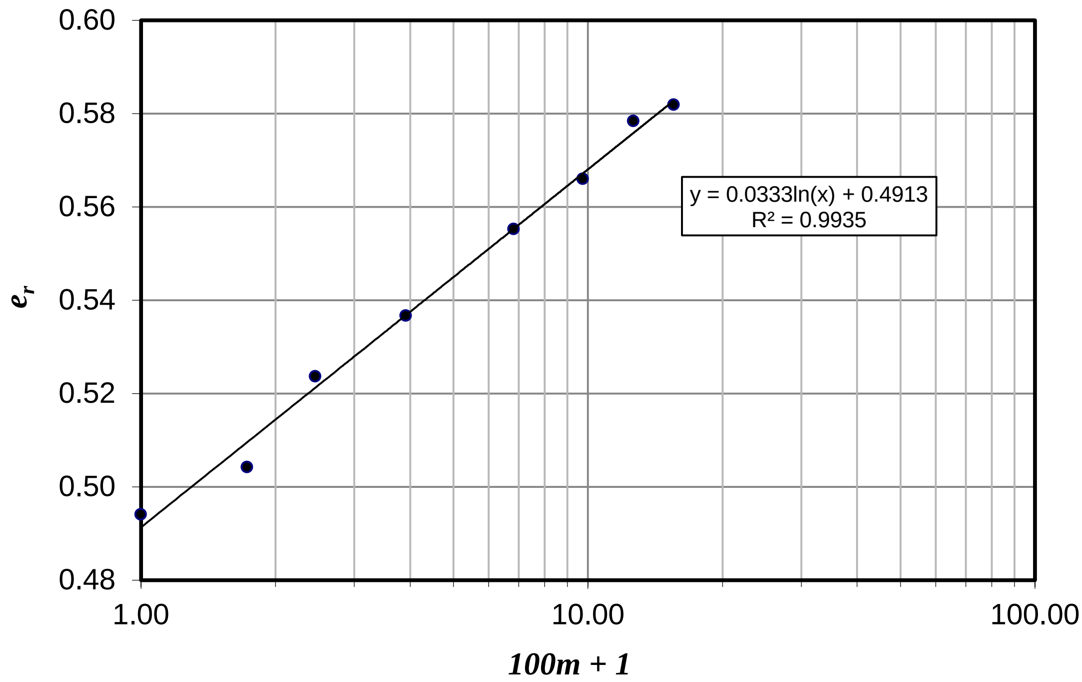

Fig. 10 Variation of void ratio, e_r, of the sand-fibre mixture used with logarithm of $(100m+1)$