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A new GPS-based evaluation of distortions in the Australian Height Datum in Western Australia

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Abstract

Previous work on assessing the errors in the Australian Height Datum (AHD) across Western Australia used fewer and older global positioning system (GPS) data and a global quasigeoid model. A larger and improved State-wide set of 243 GPS-derived ellipsoidal heights and a regional gravimetric quasigeoid model are now available. Therefore, it is possible to re-evaluate the north-south tilt in the AHD and look for regional systematic distortions with some more confidence in Western Australia. This new analysis shows an apparent north-south tilt of ~0.27 mm/km in the existing AHD over the whole of the State, but which increases to ~0.6 mm/km over smaller regions, showing regional systematic distortions. When mean sea-level constraints are removed from the AHD by a minimally constrained least-squares adjustment of the spirit-levelling observations that is less prone to the effect of sea-surface topography, the north-south tilt reduces to ~0.18 mm/km, but the regional distortions remain, showing that errors are present in the spirit-levelling observations.

Keywords: Heights, geodesy, GPS, quasigeoid, AHD, levelling, sea surface topography

Introduction and Background

Featherstone and Stewart (1998) first suggested the presence of distortions in the Australian Height Datum (AHD) across Western Australia (WA) based on Global Positioning System (GPS) and quasigeoid data from a global model. However, their analysis used a reasonably old (1996) GPS dataset of 63 points and the EGM96 global quasigeoid model (Lemoine *et al.*, 1998). The spatial resolution of EGM96 is only ~55 km. Later, Featherstone (2004) used a 2002-observed GPS dataset of 48 points (Featherstone *et al.*, 2004) and the AUSGeoid98 regional gravimetric quasigeoid model (Featherstone *et al.*, 2001) to indicate a north-south tilt of ~0.81 mm/km in the AHD, but only over part of south-western WA. The spatial resolution of AUSGeoid98 is ~4 km through the addition of regional gravity and terrain data in a modified Stokes integral (cf. Featherstone, 1999).

The study reported here uses a larger and improved (through more sophisticated processing techniques and models) WA-wide GPS dataset of 243 points and a modified version of AUSGeoid98 to look for a north-south-tilt in the AHD (cf. Featherstone, 2004; 2006), as well as for higher-order distortions (cf. Featherstone and Stewart, 1998). The modified version of AUSGeoid98 used was 'augmented' (Featherstone, 2007) using gravity field data from the Gravity Recovery And Climate Experiment (GRACE) satellite mission (*e.g.*, Tapley *et al.*, 2004). As such, this quasigeoid model is expected to be less prone to long- and medium-wavelength errors, thus strengthening its power to detect distortions in the AHD.

Methods, Data and Results

Errors in the AHD (ε) can be assessed using GPS and a quasigeoid model via:

$$\varepsilon = h - \zeta - H_N \qquad (1)$$

where h is the GPS-derived ellipsoidal height, ζ is the gravimetric quasigeoid height (cf. Featherstone *et al.*, 2001; Featherstone, 1999, 2007), and H_N is the spirit-levelled AHD height (cf. Roelse *et al.*, 1975). All quantities must refer to the same ground point.

The AHD uses the normal-orthometric height system (*e.g.*, Roelse *et al.*, 1975; Heck, 2005; Featherstone and Kuhn, 2006), which is the distance measured along the normal gravity plumbline from the quasigeoid to the point of interest (Figure 1). The advantage over other types of heights (*e.g.*, orthometric or normal) is that it does not require gravity observations along the levelling traverses, which are typically not available in Australia. However, AHD heights are not fully normal-orthometric because they were derived from a cumulative correction to levelled height differences for the GRS67 ellipsoid (IAG 1967) using a truncated form of Rapp's (1961) formulas (Roelse *et al.*, 1975).

The 243 GPS-derived ellipsoidal heights used in this study came from a GPS SINEX (system independent exchange format) file created by the WA geodetic agency, *Landgate*, comprising most of its geodetic-quality GPS data archives (L. Morgan, 2007, pers. comm.). The term 'geodetic-quality' means dual-frequency carrier-phase GPS data collected continuously for at least six hours at each point. These data were processed by Hu (2007, unpublished) using the Bernese v5 scientific GPS analysis software (Hugentobler *et al.*, 2006) and products from the International GNSS Service (Moore and Neilan, 2005).

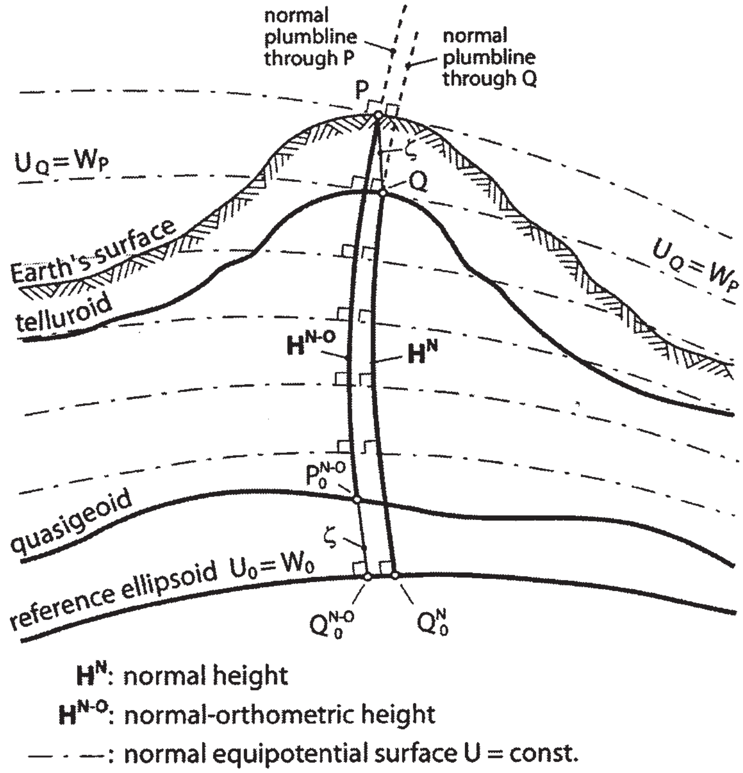

Figure 1. The normal-orthometric height $H^{N\text{-}O}$ is reckoned along the normal gravity plumbline from the point on the quasigeoid to the point $P_0^{N\text{-}O}$ on the Earth's surface. The quasigeoid height ζ is reckoned along the ellipsoidal surface normal from point $Q_0^{N\text{-}O}$ on the ellipsoid to point on the quasigeoid. The ellipsoidal height h is reckoned along the ellipsoidal surface normal from the surface of the ellipsoid to the point $P_0^{N\text{-}O}$ on the Earth's surface (from Featherstone and Kuhn, 2006).

All GPS-derived ellipsoidal heights are in terms of the latest ITRF2005 reference frame (Altamimi *et al.*, 2007) and expressed relative to the surface of the GRS80 reference ellipsoid (Moritz, 1980).

The officially published AHD heights, derived from mainly class-C (ICSM, 2002) sprit-levelling observations, were also provided by *Landgate* (L. Morgan, 2007, pers. comm.). The GRACE-augmented version of AUSGeoid98 (Featherstone, 2007) was bi-cubically interpolated to the GPS positions using the software in Featherstone (2001).

The advancement offered by this study over previous GPS-quasigeoid-based studies on the AHD (Featherstone and Stewart, 1998; Featherstone et al., 2001; Featherstone and Guo, 2001; Featherstone, 2004, 2006; Baran *et al.*, 2006) is the use of more, better spaced (cf. Figures 3 and 7) and reprocessed GPS data (Hu, 2007, unpublished),

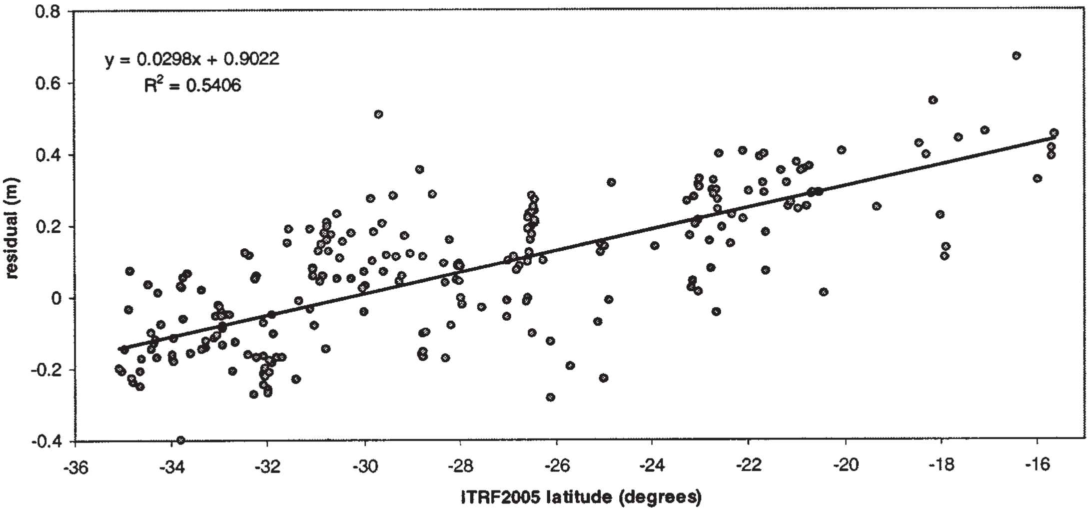

Figure 2. Linear regression of the GPS-quasigeoid-AHD residuals (ε) in metres versus latitude in degrees. From the gradient in degrees, this gives an apparent tilt of ~0.27 mm/km.

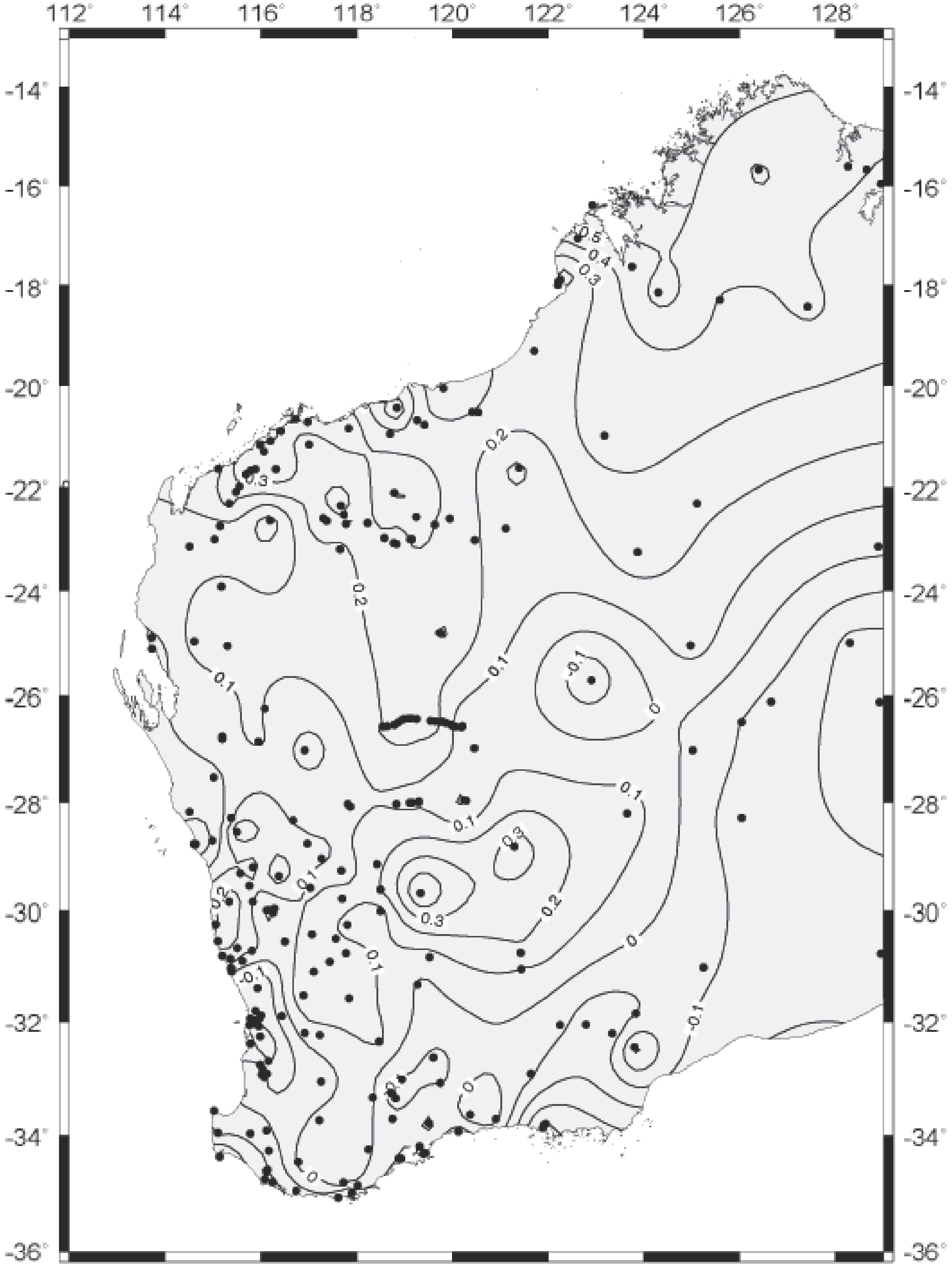

Figure 3. Contour plot (in metres) of the GPS-quasigeoid-AHD residuals (ε) and the 234 GPS-AHD points (Mercator projection)

and the GRACE-augmented AUSGeoid98 (Featherstone, 2007).

The average of the one-sigma precision (*i.e.*, standard deviation) of the 243 GPS-derived ellipsoidal heights is ~2 mm, which comes from internal error propagation in the Bernese software (Hu, 2007, unpublished). However, internally propagated GPS errors are typically overoptimistic by an order of magnitude, so ~20 mm is a more reasonable error estimate for these data. The precision of the GRACE-augmented AUSGeoid98 is much harder to estimate. Indeed, no-one has yet come up with the full error-propagation formulas for regional gravimetric quasigeoid models. Therefore, it is cautiously estimated to be ~100–200 mm, but this value remains open to debate.

Some other *caveat emptors* are necessary before presentation and discussion of the results. Distortions in GPS-derived ellipsoidal heights have never been reported, and the precision estimates for the data used here are all consistently around the same level and not spatially correlated. As such, this is the most reliable data source used here. On the other hand, it is impossible to reliably isolate what proportion of the residuals ε comes from the quasigeoid model or from the AHD; the so-called inseparability problem (Featherstone, 2004). This creates a problem when trying to correctly attribute the residuals ε solely to tilts and distortions in the AHD, but this will be circumvented in part by also using readjusted spirit-levelling heights (described later).

Results for official AHD heights in WA

To look for latitudinal (north-south) and longitudinal (east-west) trends in the residuals ε (Eq. 1), and particularly for a north-south tilt in the AHD across WA, we performed linear regressions in latitude (Figure 2) and longitude (Figure 4).

A north-south apparent tilt is shown in Figure 2, which is reasonably significant with an R^2 value of ~0.54. This tilt is equivalent to ~0.27 mm/km when converting degrees to kilometres (one degree is ~111 km at the equator), which is less than the value of ~0.81 mm/km determined in southwest WA (Featherstone, 2004). This indicates further higher-order distortions in the AHD, where apparent tilts will be different in different regions. Indeed, this is seen from Figure 2, where a linear regression of the residuals ε south of ~29°S latitude, which includes the area studied by Featherstone (2004), would give a larger tilt of ~0.60 mm/km.

These non-linear distortions are mapped in Figure 3 using tensioned spline interpolation from the Generic Mapping Tools (Wessel and Smith, 1998; http://gmt.soest.hawaii.edu/), together with the locations of the 243 GPS-AHD points. Most of the high and low contours of the residuals ε correlate spatially with those in Featherstone and Stewart (1998). The largest residuals are persistent in the western Goldfields region (centred at ~30°S, ~120°E) and in the northern Kimberley region (centred at ~16°S, ~126°E). As will be shown later, these are most probably due to spirit-levelling errors. The north-south trend (cf. Figure 2) is due to the constants applied to the AHD, which are described later.

Contrary to expectation based on previous studies (Featherstone and Stewart, 1998; Featherstone *et al.*, 2001; Featherstone and Guo, 2001; Featherstone, 2004; 2006; Baran *et al.*, 2006), there is a small east-west tilt of ~0.07 mm/km in Figure 4, but it is not statistically significant with an R^2 value of ~0.02; there is a large longitudinal scatter in the residuals. Again, this indicates distortions in the AHD, but is less reliably determined than the north-south trend in Figure 2.

Results for unofficial heights in WA

Next, we sought the source of the above distortions in the AHD in WA using another set of normal-orthometric heights. A large proportion of the north-south tilt in the AHD has been previously attributed to sea-surface topography (SST) effects (*e.g.*, Featherstone, 2001; Featherstone and Kuhn, 2006). This is because mean sea-level (MSL) was fixed to zero height at 30 tide-gauges to realise the AHD on the mainland, and the distorting effect of this approach was recognised at the time (Roelse *et al.*, 1975). SST is the difference between the geoid and MSL, caused by ocean currents and other oceanographic phenomena (*e.g.*, Pugh, 1987).

Around Australia, SST is dominantly north-south-trending (*e.g.*, Tapley *et al.*, 2003), so gives the most plausible explanation for the north-south tilt in the AHD observed in Figures 2 and 3. However, the east-west tilt in the AHD observed in Figure 4, albeit less significant, is enigmatic. One factor could be the presence of the Leeuwin Current, which is a narrow (~50 km), long (~5500 km) and meandering coastal eastern boundary current along the Australian continental shelf slope. It moves southward west of WA's coast, then heads eastwards into the Great Australian Bight south of WA, and finally to the southern tip of Tasmania (Ridgway and Condie, 2004). Such boundary currents will cause near-coastal SST effects, so could account for some of the east-west apparent tilt.

In order to determine the SST contribution to the apparent tilts and distortions in the AHD, we least-squares adjusted the normal-orthometric spirit-levelling observations used to establish the AHD without the effects of SST. These spirit-levelling data were supplied by the national geodetic agency, *Geoscience Australia* (G. Johnston, 2006, pers. comm.). We performed a minimally constrained adjustment (*i.e.*, by fixing only one tide-gauge at Albany) of these spirit-levelling data to determine new normal-orthometric heights for the 243 GPS stations, and then repeated the earlier experiments using these spirit-levelled heights that are uncontaminated by SST. SST at Albany will still affect these readjusted heights, but this is a constant value that will not affect the identification of tilts and higher order distortions, so is unimportant.

This second analysis will indicate how much the MSL constraints cause the apparent tilts and distortions in the AHD. Ten tide-gauges were held fixed to zero height at MSL in the original AHD around WA (Roelse *et al.*, 1975); Eucla, Esperance, Albany, Bunbury, Fremantle, Geraldton, Carnarvon, Port Hedland, Broome and Wyndham).

Figure 5 is equivalent to Figure 2 and Figure 6 is equivalent to Figure 4, but now use heights from the minimally constrained least-squares adjusted spirit-levelling observations without SST contamination. These

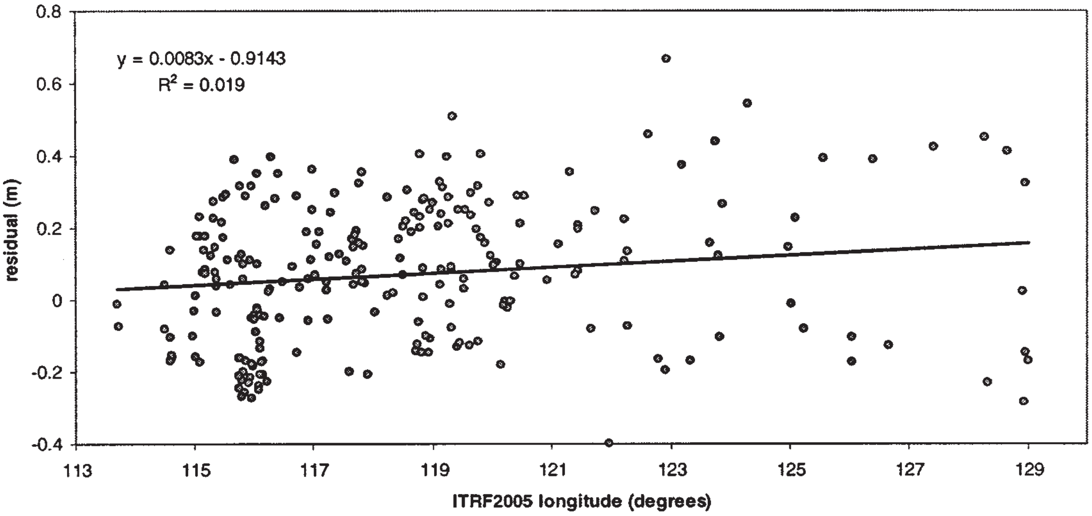

Figure 4. Linear regression of the GPS-quasigeoid-AHD residuals (ε) in metres versus longitude in degrees. From the gradient in degrees, this gives an apparent tilt of ~0.07 mm/km.

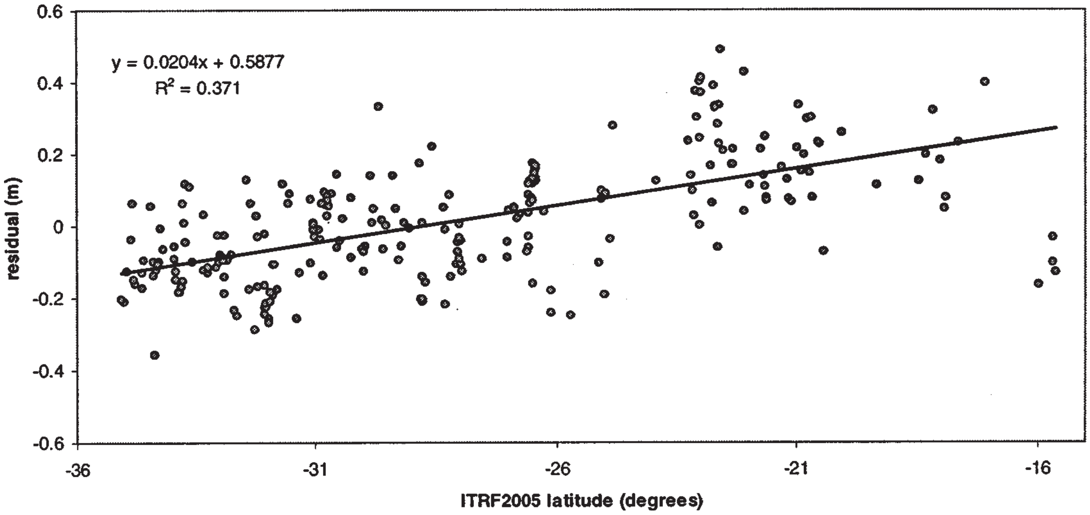

Figure 5. Linear regression of the GPS-quasigeoid-AHDnoSST residuals (ε) in metres versus latitude in degrees. From the gradient in degrees, this gives a tilt of ~0.18 mm/km.

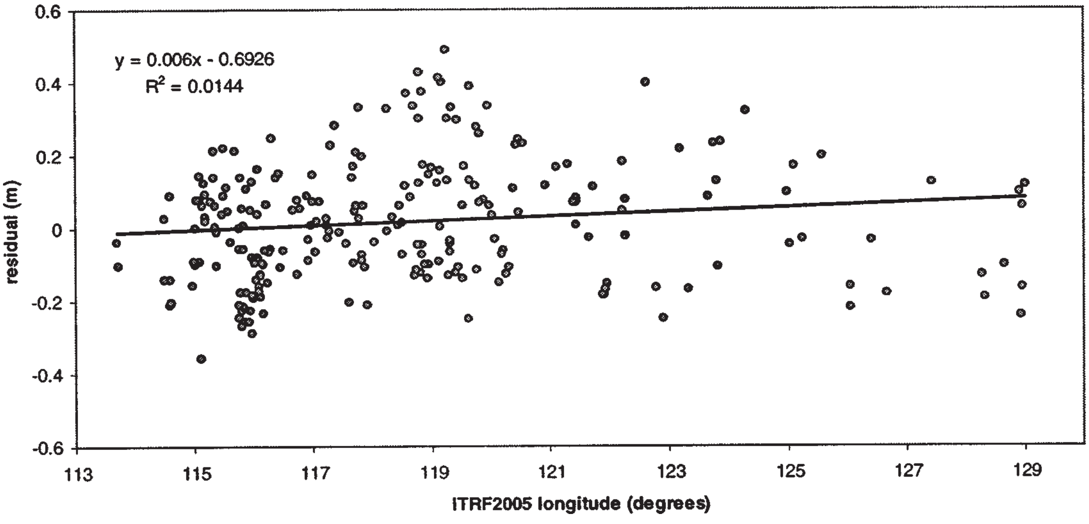

Figure 6. Linear regression of the GPS-quasigeoid-AHDnoSST residuals (ε) in metres versus longitude in degrees. From the gradient in degrees, this gives a tilt of ~0.01 mm/km.

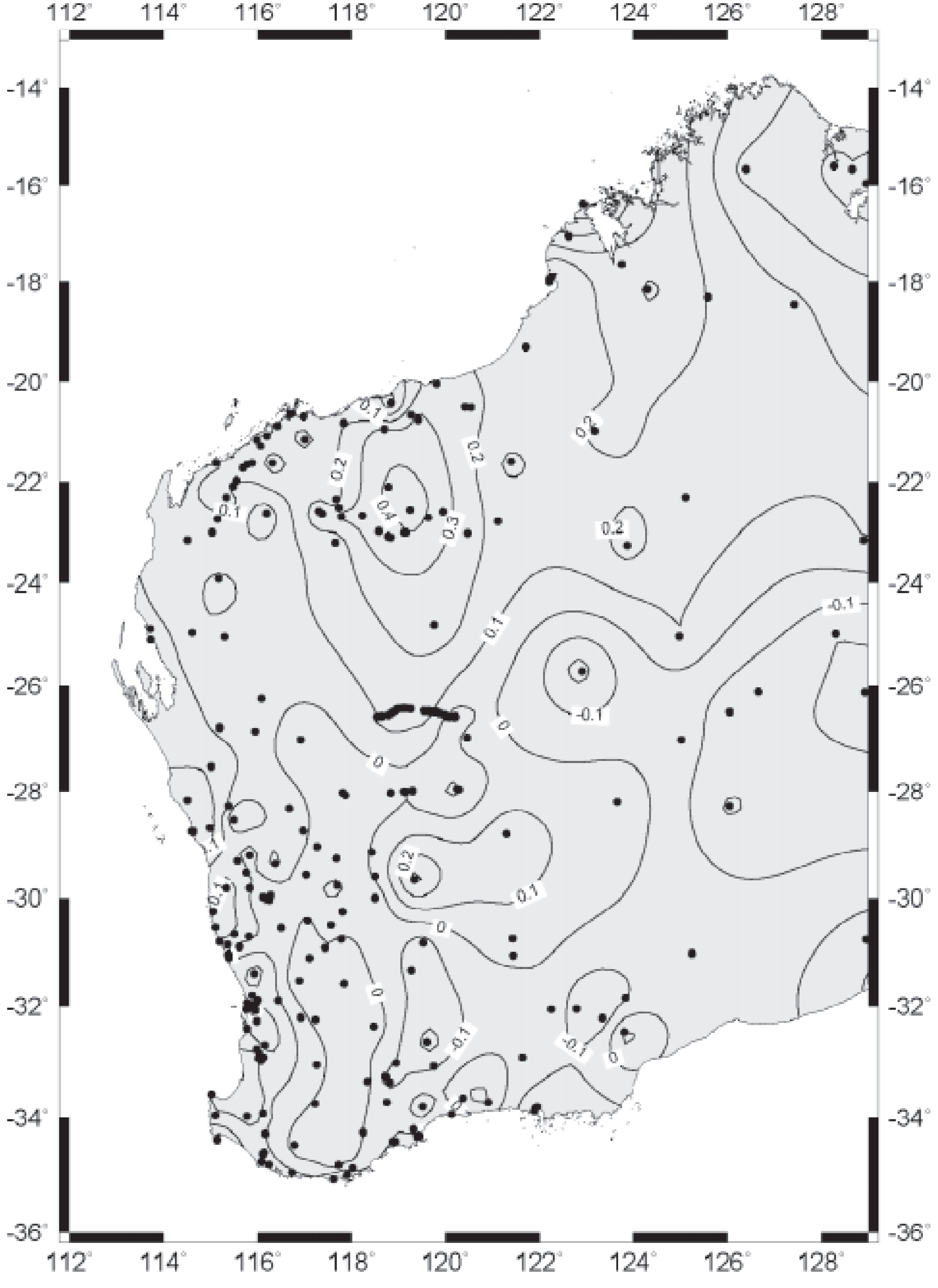

Figure 7. Contour plot (in metres) of the GPS-quasigeoid-AHDnoSST residuals (ε) and the 234 GPS-AHD points (Mercator projection)

Table 1

Descriptive statistics of the residuals (ε) in metres

Residual	Maximum	Minimum	Mean	STD
GPS-quasigeoid-AHD	0.666	-0.437	0.071	±0.203
GPS-quasigeoid-AHDnoSST	0.606	-0.359	0.018	±0.168

readjusted and SST-free height data will be termed AHDnoSST to distinguish them from the officially published AHD heights used earlier.

The contour plot in Figure 7, compared to Figure 3, confirms that the residuals ε have been reduced in magnitude after the minimally constrained readjustment of the spirit-levelling observations without the SST-affected constraints used to realise the official AHD. However, parts of the general pattern in Figure 3 remain (notably in the western Goldfields and northern Kimberley), though the magnitudes are reduced. This strongly suggests that the spirit-levelling observations are the cause of the regional distortions, but the SST effects on the constrained AHD adjustment also act to exacerbate them.

Table 1 summarises the descriptive statistics of the residuals ε for the 'official' AHD heights and 'unofficial' readjusted (AHDnoSST) heights that are not affected by SST constraints. The summary by descriptive statistics is permitted because the residuals are reasonably normally distributed. First, the mean values should be neglected because of the unimportant 53 mm constant bias caused by the SST at Albany. The range (maximum minus minimum) is reduced by 138 mm when the SST constraints are removed, and the standard deviation decreases by 35 mm. Therefore, SST accounts for a reasonable proportion of the larger residuals in Figures 2 to 4 versus Figures 5 to 7. However, the quality of the spirit-levelling data remains a contributor to the distortions in the AHD.

The summary of the tilts and their significance (via the R^2 value) in Table 2 are more telling. Both the north-south and east-west tilts decrease when the SST constraints are removed. The east-west tilts are both insignificant. More importantly, however, the north-south tilt and its significance are reduced substantially. This shows that SST is indeed a major contributor to the north-south tilt in the AHD over WA, as well as exaggerating the higher-order distortions. However, there are other distorting effects on the AHD that are not caused by the SST constraints. The quality, processing and geometry of the spirit-levelling observations is thus a remaining cause, and work is currently underway to try to isolate these.

Summary and Conclusion

We have used a newer and reprocessed set of 243 GPS-derived ellipsoidal heights in a consistent reference frame and a GRACE-augmented version of AUSGeoid98 to assess the north-south tilt and higher-order distortions in the AHD over WA. Using published (official) AHD heights, the north-south apparent tilt is ~0.27 mm/km (R^2=0.54), but which reduces to ~0.018 mm/km (R^2=0.37) when the SST constraints are removed from a readjustment of the same spirit-levelling data. The east-west apparent tilts are insignificant (R^2=~0.02).

This shows that the MSL constraints applied to the AHD (1971) allow SST, which was not modelled at the time, to cause a north-south tilt in the AHD, as well as exacerbating the effect of spirit-levelling errors. While the tilt is seemingly small, it is now a known error that can be corrected in any future redefinition of the AHD. However, higher-order (regional) distortions remain, which must be due to other errors in the spirit-levelling data, which we are currently investigating.

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Table 2

Summary of latitudinal and longitudinal tilts of the residuals (ε) and their significance by way of the R-squared value

Tilt	North-south	R-squared	East-west	R-squared
GPS-quasigeoid-AHD	~0.27 mm/km	0.54	~0.07 mm/km	0.02
GPS-quasigeoid-AHDnoSST	~0.18 mm/km	0.37	~0.01 mm/km	0.01

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