

**A reassessment of spectral T_e estimation in continental interiors: the case of
North America**

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Abstract

Conventional spectral T_e studies use the real part of the admittance between gravity anomalies and topography or alternatively the square of the magnitude of the coherency (i.e., coherence). Here we show the utility of treating both the admittance and coherency as complex quantities. Inverting the real parts to estimate T_e , we use the imaginary parts to tell if the inversion is biased by noise. One method inverts the square of the real coherency, with the internal-to-total load ratio F derived (as a function of wave number) directly from the gravity and topography. The other method inverts the real part of the admittance assuming that F is wave number-independent. We test the methods using synthetic elastic plate models loaded at the surface and Moho in such a way that the final relief is the actual North American topography. In some of the models, we add gravity noise generated by a model having both surface and internal loads such that the final topography is zero, and find that both methods are susceptible to noise. Application of the two methods to North America gives T_e maps showing substantial agreement except in regions affected by noise, but these are not a dominant part of the total area. Given the suggested mechanisms by which noise might arise, it is not surprising that it is not a more widespread feature of the North American craton. Importantly, both methods show that large parts of the Canadian Shield are characterised by $T_e > 100$ km.

1. Introduction

The effective elastic thickness (T_e) of the lithosphere is a convenient measure of the flexural rigidity (D), which is the resistance to bending under applied loads. The two are related by the equation:

$$T_e = \sqrt[3]{\frac{12(1-\sigma^2)D}{E}} \quad (1)$$

where the elastic constants E and σ are Young's modulus and Poisson's ratio, respectively (see Table 1). In general, T_e does not correspond to a physical depth, but rather represents the integrated brittle, elastic and ductile strength of the lithosphere (Watts and Burov, 2003). That is, while the real lithosphere comprises materials of varying rheologies, Burov and Diament (1995) demonstrated that T_e is a valid measure of the flexural rigidity for any rheology. Since T_e governs many dynamic properties of the Earth, for instance subduction and orogenesis, glacial isostatic adjustment, and stratigraphy (e.g., Watts, 2001), it is an important parameter; however its magnitude over the continents is currently the subject of much controversy.

T_e is commonly estimated through spectral analysis of gravity and topography data. In one approach, the admittance (Q) is used, while another employs the coherence (γ^2). The admittance is the wave number (k) domain transfer function between gravity (G) and topography (H). Here capitals indicate either Fourier or wavelet transforms. Thus:

$$Q(k) = \frac{\langle GH^* \rangle}{\langle HH^* \rangle} \quad (2)$$

where the * indicates complex conjugation, and the angular brackets indicate an averaging process described in Section 4.1. The coherence between gravity and

topography gives an indication of their statistical relationship, and is computed through the formula:

$$\gamma^2(k) = \frac{\langle GH^* \rangle \langle GH^* \rangle^*}{\langle GG^* \rangle \langle HH^* \rangle} \quad (3)$$

The gravity anomaly in Eqs (2) and (3) can either be the free-air or Bouguer anomaly. In order to determine T_e from these quantities, they are computed from observed data, and then compared against the predictions from a thin elastic plate model. The predicted coherence/admittance will depend upon (amongst other parameters) T_e , so the best-fitting predicted model gives the T_e for the region under consideration.

Historically, while Lewis and Dorman (1970) were the first to employ the admittance, they computed an Airy isostatic anomaly, and did not consider the flexural rigidity. The first spectral T_e estimates were obtained from the admittance by McKenzie and Bowin (1976) in the oceans, and Banks et al. (1977) over the continents, by comparison of the observed admittances with those predicted by a thin plate model with topographic (surface) loading only. However, subsequent studies (e.g., Cochran, 1980; McNutt, 1980), found that the admittance tends to be biased towards regions of high (continental) topography, where the plate is likely to be weaker.

This problem was addressed by Forsyth (1985), who developed the Bouguer coherence method. Although the coherence had been employed in earlier studies (e.g., McKenzie and Bowin, 1976; Watts, 1978), it was used merely in terms of quality control of the data. Forsyth's advance was to estimate T_e by fitting the observed coherence with a predicted coherence based on a model involving initial subsurface (internal) loading at some depth, in addition to the initial surface load. Importantly, by

solving a pair of linear wave number domain equations, both initial loads (and their wave number-dependent amplitude ratio, f) can be determined directly from the observed gravity and topography. This process has been termed “load deconvolution” (Lowry and Smith, 1994) and because the solution fits the observed gravity and topography it automatically fits the admittance. Thereafter, most continental T_e estimates were obtained with the coherence method (e.g., Watts, 2001), until papers by McKenzie and Fairhead (1997) and McKenzie (2003) raised doubts regarding its efficacy. These papers generated some controversy, which we describe in detail in Section 2, but give a brief overview of below.

McKenzie and Fairhead (1997) and McKenzie (2003) wrote that the problem with Forsyth’s deconvolution method lay in the way it models the initial internal load, particularly in continental shields with heavily eroded topography. Load deconvolution, they said, only correctly models those initial internal loads that produce some degree of measurable signal in the final topography after flexure (“expressed loads”). It cannot model, they said, a separate type of initial internal load that, together with an initial surface load, produces flat topography (“unexpressed loads”); or more to the point, load deconvolution will always treat large unexpressed loads as showing evidence of a strong plate. Unfortunately McKenzie and Fairhead (1997) used the term ‘noise’, without being explicit, to describe the gravity anomalies caused by unexpressed initial internal loads, which is probably why the controversy arose. This particular wording caused numerous authors (e.g., Simons et al., 2000; Banks et al., 2001; Armstrong and Watts, 2001; Swain and Kirby, 2003a) to miss the real issue, and the matter was thought by some people to have been put to rest. However, in McKenzie (2003) the arguments against load deconvolution methods,

and particularly the coherence method, were made clearer. Although Pérez-Gussinyé et al. (2004) and Pérez-Gussinyé and Watts (2005) responded to many of McKenzie's claims, we feel that more work is still required to address the most important of his issues, namely that in regions of subdued, eroded topography, T_e estimates obtained from the coherence method are upper bounds which may be many times larger than the true values.

We tackle the problem by following McKenzie (2003)'s definition of 'noise' as those gravity anomalies caused by unexpressed initial internal loads. Therefore, 'noise' in this paper constitutes that part of the observed gravity field not predicted by the load deconvolution method. Although we simulate noise (using the formulations in Appendix A2) and add it to synthetic data, we do not explicitly model it during inversion of observed admittance/coherence data. We do, however, present a method for its detection, via the imaginary components of the coherency (from which the coherence is derived) and admittance. As shown in Appendix B1, finite values of the imaginary component of the cross-spectrum, GH^* in Eqs (2) and (3), indicate parts of the spectrum where two signals are out of phase (or have random phase). When the wavelet transform is used, the spatial locations of such regions are revealed as well. Hence, if features in one signal are not expressed to any degree in a second signal, then $\text{Im}(GH^*)$ is non-zero. In terms of flexure, then, those internal loads that produce no topographic expression, and which cannot be accounted for by the deconvolution method, can be detected by analysis of $\text{Im}(GH^*)$, via the complex admittance or coherency.

A related issue is that the load deconvolution method assumes that the two initial loads must have a random phase difference (i.e., be “statistically uncorrelated”). McKenzie (2003)’s model of unexpressed loading, however, requires that the initial loads must have a phase difference of zero (i.e., be correlated or coherent) in order to produce a flat final surface topography after flexure. The reverse, that 100%-correlated initial loads will always produce flat topography after flexure, is not necessarily true though, and in Appendix A1 we present a theoretical formulation of loading on an elastic plate by including varying degrees of correlation between initial loads. While this model does not describe ‘noise’ under our definition (and hence is not used for that purpose), it is useful for: 1) explaining how initial load correlations can bias T_e estimates upwards or downwards, as was found in practice by Macario et al. (1995) and Kirby and Swain (2008); and 2) demonstrating that even 100%-correlated initial loads do not always result in an unexpressed loading regime. McKenzie’s “zero-final-topography” model, therefore, is a special case of the more general model presented in Appendix A1.

To investigate the efficacy of T_e -recovery methods in regions where ‘noise’ is likely to bias results, we perform synthetic modelling of a typical cratonic region, that of North America (Section 5): we add noise to some of the synthetic models, but, as mentioned, do not invert for it and merely note its effect on recovered T_e estimates. We then present results of the actual T_e distribution over this continent (Section 6). The method we use is an improved version of the ‘fan’ wavelet coherence and admittance method (Kirby and Swain, 2004; Swain and Kirby, 2006; Kirby and Swain, 2008). While an intensive ‘calibration’ of the method has already been performed (Kirby and Swain, 2008), this was undertaken using an earlier version of

our fan wavelet method. The new method presented here uses the square of the real part of the coherency rather than the conventional coherence, which turns out to be (a) consistent with most Fourier methods and (b) less sensitive to correlations between initial loads (Section 4 and Appendix B1).

2. The Controversy

According to McKenzie and Fairhead (1997) and McKenzie (2003), continental elastic thickness should rarely exceed 25 km, even in Precambrian cratons. This value was based on: 1) estimates they derived from the free-air admittance without using load deconvolution; and 2) an investigation of Forsyth's coherence method which, they said, would often yield upper bounds on T_e , especially in eroded continental interiors.

The ability of Forsyth's coherence method to accurately estimate T_e rests on two assumptions. First, that the loads that initially deform the plate are statistically uncorrelated or incoherent. This assumption was acknowledged by Forsyth (1985) and all other studies that used the method. The second assumption is that the initial internal load should produce a measurable response in the final surface topography.

The arguments of McKenzie and Fairhead (1997) and McKenzie (2003) mainly focussed on the second assumption, but also involved the first. Ancient shield regions, they said, are predominantly characterised by eroded, and hence subdued, topography: the coherence between any two signals will necessarily be zero if one of these signals is uniformly zero. Of course, one interpretation, mentioned by Swain and Kirby (2003a), is that if topography is almost flat and there is a large Bouguer anomaly

(indicating substantial internal loads), then the plate must be strong in order to resist the buoyancy force of the internal loads.

However, McKenzie (2003) advanced a different interpretation, which allows for low T_e values in such cases. Where Forsyth (1985) described a plate model with two initial loads (surface and internal), McKenzie (2003) split the initial internal load into two components: initial internal loads that have a surface topographic expression (“expressed loads”), and initial internal loads that don’t (“unexpressed loads”). It is the latter type of load, he said, that is not only unaccounted for in Forsyth’s model, but is also the most prevalent in shields. [We should point out here that in McKenzie and Fairhead (1997), this distinction was not made obvious, and the unexpressed loads were referred to as just “noise”. It was probably this shortcoming that caused so much confusion. Nevertheless, the matter was addressed properly in McKenzie (2003). So as mentioned in the Introduction, the term ‘noise’ in this article refers to the gravity effect of unexpressed internal loads.] Under McKenzie’s model, a flat topography can result if: the initial surface and internal loads are completely correlated (i.e., with a 0° phase difference); surface topography is then produced by flexure; erosion or sedimentation then takes place which reduces the topography; further rebound may take place, but further erosion removes this; eventually the sheer weight of the present-day internal loads resists rebound and a flat peneplain results (McKenzie, 2003). Under this scenario, T_e can be low, but the reasoning does not discount a high T_e : as mentioned, the strength of the plate would resist rebound, letting erosion do its work.

An argument against this proposal was made by Pérez-Gussinyé et al. (2004). They suggested that as erosion removes the surface topography, isostatic adjustment would reduce the amplitude of the internal load, if it were manifested as relief on some internal interface. That is, both surface topography and the gravity anomalies due to the internal load would decrease. However, they said, this is not observed in shields, where the gravity anomalies are still large, indicating a substantial internal load that must be mechanically maintained by a strong plate.

Aside from the mechanical considerations, McKenzie and Fairhead (1997) and McKenzie (2003) also stated that Forsyth's method only gives accurate T_e estimates when the power in the free-air anomaly is of the same order of magnitude as the power in the "uncompensated topography" (which is $2\pi\Gamma\rho h$, the simple planar Bouguer correction). McKenzie and Fairhead (1997) reasoned that the dominant signal in free-air anomalies is the gravity effect of the topography, so if the topography is almost flat yet there are still substantial free-air anomalies (due to unexpressed internal loads), then Forsyth's second assumption (above) is not satisfied. In addition to the comparison of power spectra, McKenzie and Fairhead (1997) and McKenzie (2003) proposed that a measure of such violation would be that the free-air coherence (i.e., the coherence between free-air anomalies and the topography) is close to zero at short wavelengths. If the free-air coherence is low, they said, then the Bouguer coherence must also be low, but not because of mechanical support of the shorter-wavelength topography: it would be low because of "noise" (i.e., unexpressed internal loads) in the gravity field. In such cases, the T_e calculated from the Bouguer coherence method must therefore be an upper limit, and not a true estimate, according to them.

Showing the results from some analyses over shields, using the multitaper method of spectral estimation, McKenzie and Fairhead (1997) and McKenzie (2003) suggested that most of the Earth's shield regions fall into the low free-air coherence category, indicating that unexpressed loads were widespread there, and meaning that the high T_e estimates computed from load deconvolution methods were upper bounds. Instead, they proposed that T_e is more reliably recovered by inversion of the observed free-air admittance using an analytic formula where both T_e and the loading ratio, f , are adjustable, independent variables. Importantly, f in this method is not a function of wave number, because it is not estimated from the data. For this reason, we call this approach the "uniform- f " method, to distinguish it from the load deconvolution method in which f is estimated from the data and is generally wave number-dependent.

In summary, the reasons behind McKenzie and Fairhead (1997) and McKenzie (2003)'s suggestion of using the uniform- f free-air admittance instead of the load-deconvolution Bouguer coherence method were: 1) Forsyth's equations do not take account of unexpressed internal loads; 2) admittance rather than coherence should be used because the latter is biased by gravitational noise, whereas the former is not (although the error estimates increase); and, 3) that the free-air rather than Bouguer anomaly should be used because low free-air coherence at short wavelengths indicates the presence of unexpressed internal loads, which cannot be identified if the Bouguer anomaly is used.

As a final note, we choose to display the loading ratio results in terms of the F parameter introduced by McKenzie (2003). The relationship between the f introduced by Forsyth (1985) and F is:

$$F = \frac{f}{1+f} \quad (4)$$

Thus, whereas f is the ratio of the initial internal load amplitude to the initial surface load amplitude, F is the ratio of the initial internal load amplitude to the total amplitude of both initial loads. Purely surface loading gives $f = F = 0$, purely internal loading gives $f = \infty$ and $F = 1$, while equal loading gives $f = 1$ and $F = 0.5$. Remember that f and F are, in general, wave number-dependent parameters. However, while the load-deconvolution methods estimate a wave number-dependent f from the data and model, the uniform- f methods assume that f has the same value at all wave numbers.

3. The Wavelet Transform

The method we use for effective elastic thickness estimation using ‘fan’ wavelets is explained in detail in Kirby and Swain (2008) and references therein. The reader should consult these references for detailed information about the method. Briefly, the fan wavelet method computes 2D Morlet wavelet coefficients of gravity and topography at a number of azimuths spanning 180° , which ensures isotropy but does not average the imaginary components of the coefficients to zero (see discussion in Section 4.1). The wavelet admittance and coherence between gravity and topography are then formed via Eqs (2) and (3), and provide estimates of these quantities at each grid node of the study area. The scale of the wavelet is adjusted so that small-scale wavelets resolve the shorter wavelengths, while large-scale wavelets resolve the

longer wavelengths. Scale may then be directly mapped to an equivalent Fourier wave number using a simple algebraic expression (Kirby, 2005).

The advantage over methods that use the windowed Fourier transform (e.g., multitapers or the Gabor transform) is that the data need not be windowed. Since the wavelet, which spans the whole study area, is convolved in the space domain with the whole data area, the full spectrum is recovered. In contrast, the choice of window size in multitaper methods limits the bandwidth of the spectrum, meaning that, sometimes, large flexural wavelengths cannot be recovered.

4. T_e Estimation by Spectral Methods

In this section we review the coherence method and show that, as conventionally implemented, it actually uses the square of the real part of the coherency rather than the square of its modulus. We therefore reformulate our wavelet version of the method in terms of this quantity and point out the utility of its counterpart, the squared imaginary coherency.

4.1 Averaging

In the Bouguer coherence method of Forsyth (1985), and its wavelet adaptation by Swain and Kirby (2006), the observed coherence between Bouguer anomaly and topography data is calculated in the wavenumber domain through Eq. (3). If the spectra are computed using the classical Fourier periodogram technique, the angular brackets in Eq. (3) indicate an averaging process that is performed around concentric, isotropic annuli spanning 360° in the wavenumber domain (e.g., Banks et al., 1977).

In this approach, the coherence becomes a function of 1-D radial wave number ($k =$

$|\mathbf{k}|$): $\gamma^2(k)$. With the multitaper method, the averaging is performed over a number of tapers (e.g., McKenzie and Fairhead, 1997; Simons et al., 2000), and the coherence is a function of 2-D wave number, $\gamma^2(\mathbf{k})$, thus revealing its anisotropy. However, most multitaper implementations that use isotropic coherence (e.g., Pérez-Gussinyé et al., 2004) also use annular averaging. In the fan wavelet method, the averaging is performed over a series of azimuths, and the coherence is a function of (equivalent Fourier) wave number and spatial location (\mathbf{x}) , $\gamma^2(k, \mathbf{x})$. In contrast to annular averaging, the rotation of the wavelets spans only the upper two quadrants of the wavenumber domain (i.e., over 180°) in order to make the wavelets complex as well as isotropic. As Kirby (2005) showed, there is no loss of information incurred when restricting the angular range, because the lower two quadrants of a signal's Fourier transform contain duplicated, and hence redundant, information.

4.2 Coherency and Coherence

The coherency is defined in the wavelet or Fourier domains by:

$$\Gamma = \frac{\langle GH^* \rangle}{\langle GG^* \rangle^{\frac{1}{2}} \langle HH^* \rangle^{\frac{1}{2}}} \quad (5)$$

Note that the terms “coherency” and “coherence” for this function and its modulus-squared appear to have been introduced by Weiner (1930), but were also used by Tukey (1984) and this usage is followed widely in geophysics (e.g., Claerbout, 1976; Kanasewich, 1981). In the wavelet case, the term “coherency” is also used by Liu (1994). On the other hand, Wieczorek (2007) uses the term “degree-correlation”, though for a spherical harmonic analysis. [Our reasons for not using the symbol γ for the coherency will shortly become apparent.]

Because the product GH^* is complex, the coherency is a complex variable, $\Gamma = \Gamma_R + i\Gamma_I$, and therefore contains important information that the coherence does not. [Throughout this article, subscripts R and I indicate real and imaginary parts, respectively.] Now, comparison of Eqs (3) and (5) shows that $\gamma^2 = \Gamma\Gamma^* = |\Gamma|^2 = \Gamma_R^2 + \Gamma_I^2$. That is, the classical coherence should actually be the sum of two components, which we call the ‘squared real coherency’ (SRC, Γ_R^2) and ‘squared imaginary coherency’ (SIC, Γ_I^2).

Several authors (e.g., Forsyth, 1985) have noted that observed admittances are real, though since the admittance and coherency have the same numerator, this would not be expected always to be the case. The explanation for it lies in a consideration of how the averaging of the numerator of Eq. (5) is performed:

$\langle GH^* \rangle = \langle (GH^*)_R \rangle + i \langle (GH^*)_I \rangle$, and similarly for $\langle (GH^*)^* \rangle$. Since both G and H are Hermitian, GH^* is also Hermitian, and its lower two quadrants contain the same information as its upper two quadrants: both the real and imaginary parts are 180°-rotated versions of the upper two quadrants, but the imaginary part is further multiplied by -1 . Hence, if a 360° averaging is performed over concentric annuli in the wavenumber domain, as in the periodogram and isotropic multitaper methods, then $\langle (GH^*)_I \rangle_{360} = 0$ and the imaginary information vanishes. Thus the “coherence” in this conventional implementation, although real, is in practice the square of the real coherency and not its modulus squared, i.e. $\gamma^2 \square \langle (GH^*)_R \rangle_{360}^2$.

In the fan wavelet method, the averaging is performed over the two upper quadrants only (i.e., over 180°). Hence $\langle (GH^*)_I \rangle_{180} \neq 0$, and $\gamma^2 \square \langle (GH^*)_R \rangle_{180}^2 + \langle (GH^*)_I \rangle_{180}^2$.

Therefore, to make the fan wavelet method consistent with 360° annular-averaging methods, we therefore use the square of the real part of the coherency:

$$\Gamma_R^2 = \frac{(\text{Re} \langle GH^* \rangle)^2}{\langle GG^* \rangle \langle HH^* \rangle} \quad (6)$$

from Eq. (5), instead of the coherence.

4.3 The Bouguer Squared Real Coherency Deconvolution (BCD) Method

In the Bouguer coherence method of Forsyth (1985), referred to from now on as the “BCD method”, T_e is estimated by comparing the coherence between observed Bouguer anomalies and topography (the “observed coherence”) with that predicted from the loading and flexure of a thin elastic plate (the “predicted coherence”). Load-deconvolution allows for the estimation of the two initial loads from the observed gravity and topography data, assuming an initial value for T_e , from which the (wave number-dependent) initial internal-to-surface loading ratio, f , is determined and used in the computation of the predicted coherence (see Appendix A1.1). The value of T_e that provides the best fit between observed and predicted coherence is then chosen.

In our modification of Forsyth’s method we use the wavelet, instead of Fourier, transform, and the SRC instead of the coherence. The χ^2 misfit between the observed and predicted Bouguer SRC is minimised using Brent’s method of 1-D minimisation (Press et al., 1992), at each grid node of the study area, and is calculated through:

$$\chi^2 = \sum \left(\frac{\Gamma_{B,o,R}^2 - \Gamma_{B,p,R}^2}{\varepsilon_{\Gamma_{B,o,R}^2}} \right)^2 \quad (7)$$

(e.g., Press et al., 1992), where $\Gamma_{B,o,R}^2$ is the observed Bouguer SRC, $\Gamma_{B,p,R}^2$ is the predicted Bouguer SRC, and $\varepsilon_{\Gamma_{B,o,R}^2}$ are the errors on the observed Bouguer SRC.

While we have previously weighted χ^2 by the inverse of wave number when using the coherence (Kirby and Swain, 2006; Swain and Kirby, 2006; Kirby and Swain, 2008), we now use jackknifed error estimates with the SRC (Thomson and Chave, 1991; Swain and Kirby, 2006). Inverse wave number weighting has the effect of damping the spurious high, short-wavelength Bouguer coherence due to random correlations between initial synthetic loads (Kirby and Swain, 2008). However, since many of these correlations propagate through to both the observed Bouguer SRC and SIC, they have a greater effect upon the coherence than upon the SRC. With the multitaper method, however, the coherence should not be sensitive to initial load correlation as long as the auto- and cross-spectra are annular-averaged over 360° to remove the imaginary part, as discussed above. This has been found to be the case with synthetic data by Pérez-Gussinyé et al. (2009).

We note here that Forsyth (1985) assumed that the surface and subsurface loading processes are independent, or statistically uncorrelated, which allows for solution of the flexure equation for predicted coherence (and admittance). However, in Appendix A1 we derive an expression for the predicted Bouguer SRC without forcing the assumption of independent loads. While we do not invert the correlated-load SRC, the model provides useful insights into correlated-load flexure. Most notably, this

modelling has shown that cases of positive observed real Bouguer coherency, $\Gamma_{B,o,R}$, should be set to zero before inversion, because such values are not admitted by the uncorrelated-loads plate model (Figure A2a). This step further helps to reduce the effects of load correlation.

We also compute confidence limits on T_e , defined by $\chi_{\min}^2 + \Delta\chi^2$; for one degree of freedom, 95% limits correspond to $\Delta\chi^2 = 4$, while 99.99% limits correspond to $\Delta\chi^2 = 15.1$ (Press et al., 1992). Since the loading ratio is not an independent variable in this method, but is estimated from deconvolution of the observed gravity and topography data, we can not estimate its confidence interval.

4.4 The Free-air Admittance Uniform- f (FQU) Method

An alternative spectral method used in T_e -estimation, and the one recommended by McKenzie and Fairhead (1997) and McKenzie (2003), is to fit analytic admittance curves to the observed admittances. In contrast to load-deconvolution methods, this “uniform- f ” method assumes that the loading ratio is independent of wave number. From now on we refer to this method as the “FQU method”. Here, the observed (complex) wavelet admittance between the free-air anomaly and topography is computed, but only the real part is inverted. We use an iterative least squares method (Tarantola, 1987; Swain and Kirby, 2003b) to find those values of T_e and F that give an analytic admittance curve [the real part of Eq. (A21) with $\delta = 90^\circ$] that provides the best χ^2 fit to the observed admittance, as in Eq. (7). This process is repeated for all grid nodes in the study area. Again, confidence limits are computed (see Section 4.3), but the method now allows for the estimation of limits on F as well as T_e .

5. Synthetic Modelling

5.1 Synthetic Data Generation

As discussed in Section 2, a main criticism of Forsyth's Bouguer coherence method is that it does not perform well when: 1) the power in the uncompensated topography is much less than the power in the free-air anomaly; and 2) the short-wavelength free-air coherence is close to zero (McKenzie and Fairhead, 1997; McKenzie, 2003). The synthetic modelling performed in Kirby and Swain (2008), and in many other papers (e.g., Macario et al., 1995; Stark et al, 2003; Pérez-Gussinyé et al., 2004; Audet and Mareschal, 2007), uses fractal topography whose power spectrum is of the same order of magnitude as the free-air anomaly at all wavelengths. Therefore, we develop synthetic models with subdued topography in the following manner. First, the space-domain flexure equation describing the flexure (v) of a thin elastic plate subject to an initial surface load h_i , and an initial internal load at the Moho w_i , is (where all vertical displacements are positive upwards):

$$\square^4(D, v) + (\rho_m - \rho_f)gv = -(\rho_c - \rho_f)gh_i - (\rho_m - \rho_c)gw_i \quad (8)$$

(e.g., Kirby and Swain, 2008) where:

$$\square^4(D, v) \equiv \nabla^2(D\nabla^2v) - (1-\sigma)\left(\frac{\partial^2 D}{\partial x^2} \frac{\partial^2 v}{\partial y^2} - 2\frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 v}{\partial x^2}\right) \quad (9)$$

ρ_f is the density of the overlying fluid (water or air), and other constants are given in Table 1. Instead of using two initial loads to generate final topography and Bouguer anomaly as is commonly done, we now rearrange the flexural equation so that the inputs are final topography (h) and a random fractal initial internal load, and the outputs are initial topography (which is not used further) and final Moho topography. This involves substitution of:

$$h_i = h - v \quad (10)$$

into Eq. (8), giving:

$$\square^4(D, v) + (\rho_m - \rho_c)gv = -(\rho_c - \rho_f)gh - (\rho_m - \rho_c)gw_i \quad (11)$$

We use finite difference equations in the space domain to determine the flexure (v) (Appendix A of Kirby and Swain, 2008) because one of the model plates we use has spatially-variable T_e (see below). Then, the final Moho topography (w) is obtained from:

$$w = w_i + v \quad (12)$$

which is then upward continued via the formula of Parker (1972) to obtain a final Bouguer anomaly.

The input final topography we used was the actual topography over North America (Figure 1a), which contains extensive regions of both subdued and mountainous topography (clearly visible in a plot of its variance – Figure 1b). The data were taken from the recently-released EGM2008 harmonic coefficients model (Pavlis et al., 2008), expanded to degree and order 1000, and represented on a 20 km grid of the Lambert conic conformal projection. See Section 6.1 for further information. We then generated an initial random fractal internal load of fractal dimension 2.5 at depth 35 km, and used Eqs (11) and (12) (with a certain T_e distribution, described below) to determine the corresponding final Bouguer anomaly. We did this 100 times using 100 different internal loads, then averaged the 100 final Bouguer anomalies to give a single final Bouguer anomaly that would be used, together with the North American topography, in the wavelet T_e -recovery algorithm. Our motivation for averaging the 100 final Bouguer anomalies was to avoid biasing the result toward one particular (random fractal) internal load geometry. Finally, we computed a synthetic free-air anomaly by adding the simple Bouguer correction to the averaged synthetic Bouguer

anomaly. An approximate initial loading ratio (calculated as an average in the space domain between the 100 pairs of h_i and w_i) for the model is $f \approx 0.16$, or $F \approx 0.14$.

We chose two T_e distributions by which to generate synthetic gravity anomalies. The first was uniformly 10 km, while the second was a superposition of 2-D Gaussian functions, rising from 10 km to 140 km over the subducted topography regions of North America (Figure 1c). The dimensions of these grids are 6380×6380 km, on a 20 km grid spacing. The free-air anomalies from the two plates are shown in Figure 2.

5.2 Modelling Unexpressed Internal Loads

As discussed in Section 2, McKenzie's chief criticism of the coherence method was that the predicted coherence is determined using a plate model where all internal loads are expressed, to some degree, in the topography. Therefore, in his Appendix A, McKenzie (2003) presented a method by which to determine the gravity anomaly due to surface and internal loads that, after flexure, produce zero topography. As discussed in Section 1, we refer to this gravity anomaly as 'noise', as did McKenzie and Fairhead (1997). The technique was subsequently applied to synthetic data by Crosby (2007), who added this 'noise' to the final, flexed, gravity anomaly from surface and expressed internal loading without altering the topography. Hence, this noise gives the effect of unexpressed internal loading.

Unfortunately, the "zero-topography transfer functions" (between the unexpressed internal load and its gravity field) derived by McKenzie (2003) and Crosby (2007) are both incorrect (for different reasons). In our Appendix A2 we derive a correct equation and explain their mistakes. [We also note that our models are two-layer with

loading at the surface and Moho, while McKenzie and Crosby use a three-layer model with loading at the surface and mid-crust. Nevertheless, we have computed the correct transfer function for the three-layer case and find it has the same spectral characteristics as the two-layer model.] As discussed in Appendix A2.1, and shown in Figure A3, the zero-topography transfer function has the properties of a bandpass filter, suppressing both long and short wavelengths, but preferentially passing wavelengths close to the Bouguer coherence transition wavelength (which we now refer to as the “Bouguer rollover”). Thus, we would expect that the position of this rollover is shifted by the noise, resulting in biased T_e values. However, the magnitude of the bias will depend upon the shape of the noise spectrum, and as we show in Appendix A2.2, can actually result in very little change to T_e .

Since our loading model (Section 5.1) does not include unexpressed internal loads, we simulated these in the following manner. We first generated a random fractal surface with fractal dimension 2.5 to use as the initial internal load (called a “type-I” load, and shown in Figure 3a), then solved Eq. (11) with the constraint that $h = 0$, for both of our T_e distributions. The final gravity anomalies after application of Eq. (12) and upward continuation (called type-I noise), are shown in Figure 3b for the uniform- T_e plate, and 3c for the Gaussian- T_e plate. These were then added to the gravity anomalies due to expressed loads (Section 5.1), with the topography left unaltered.

Crosby (2007) used high pass-filtered noise in his study (cutoff wavelength of 750 km), though this high pass filter may have been needed because his zero-topography transfer function is wrong and remains high at long wavelengths. To maintain

consistency, we also high pass filtered our type-I load (but using a cutoff wavelength of 250 km) to produce a “type-II” load (Figure 3d), and then followed the same procedure as above to generate type-II noise (Figures 3e and 3f). Our choice of lower cutoff wavelength will be made clear in the remainder of this section.

Note that we had to use space-domain modelling due to the variable T_e distribution shown in Figure 1c, but we have verified that this result is equivalent to the result obtained from a uniform- T_e plate and the Fourier solution to Eq. (11). We also checked that the initial loads give a final topography of zero when used in Eq. (8).

5.3 T_e Results

The fan wavelet method was only applied over a subset of the total region, shown in Figure 1 and measuring 5100×5100 km, because the synthetic gravity anomaly was generated with periodic boundary conditions in the finite difference algorithm, and real data are not periodic. Furthermore, since a primary issue of the controversy outlined in Section 2 concerns the merits of the Bouguer coherence/SRC load-deconvolution (BCD) method versus the free-air admittance uniform- f (FQU) method, we restrict our figures to those two sets of results.

Our first significant finding was that, even in the absence of added noise, subdued topography causes moderate T_e overestimates. Moreover, this overestimation affects both the BCD and FQU methods, with the maximum difference with respect to the uniform $T_e = 10$ km plate being ~ 14 km (BCD), and ~ 18 km (FQU). Comparison of Figures 4a and 4b with the topographic variance map (Figure 1b) shows that the regions of T_e overestimation approximately correspond with regions of low

topographic variance ($<10^{3.5}-10^4 \text{ m}^2$), whereas when this variance is large T_e is faithfully recovered or even slightly underestimated (by up to 4 km for both methods). When T_e is high however (Figures 4c and 4d), the amount of overestimation is difficult to ascertain. This is due to the relatively large width of the transition wavelet (Figure 1c) compared to the width of the Gaussian bump, causing smoothing of the wavelet coefficients between high- and low- T_e regions, as described in Kirby and Swain (2008). Hence from now on, the no-noise T_e values will be used as benchmarks for comparison with results from noisy data.

When type-I (i.e., full-spectrum) noise is added to the gravity, the overestimation increases with respect to the no-noise results, with considerable differences mainly over the subdued topography regions (Figures 4e–h). While the bias is most noticeable in the BCD results (Figures 4e and 4g), the FQU results are also affected by the noise, occasionally to a greater extent than the BCD results.

McKenzie and Fairhead (1997), McKenzie (2003) and Crosby (2007) state that noise will increase the variance of the free-air admittance, but will not bias it. Indeed, theory would suggest that they are correct. If the gravity Fourier transform (G) in Eqs (2) and (3) is replaced by $G + N$, where N is the Fourier transform of some noise which is statistically uncorrelated with the topography, then the admittance should be unchanged because $\langle NH^* \rangle = 0$, while the coherence becomes biased (see Appendix A2.2 for a full discussion). However, our results show that this is not true in practice, and that the FQU T_e can occasionally be just as biased as the BCD T_e . It is important to note that McKenzie (2003) also implies that if the errors in the admittance are large enough then it may not be possible to distinguish between high and low T_e models, if

their confidence intervals overlap – so the fact that the best-fit T_e is biased might not be significant. However, as shown in Section 5.7, we have carefully estimated the confidence intervals to show that for both methods the bias with type-I noise and subdued topography can be significant.

When we add the type-II (high-pass filtered) noise to the gravity, however, the results change considerably (Figures 4i–l), with much less additional T_e bias caused by this type of noise. Compared to the FQU results, the BCD results tend to have a more systematic bias over subdued topography, though this bias is small (<10 km on average). The FQU bias, however, while lower on average, has larger extremes and does not appear to correlate that well with subdued topography. These results imply that the presence of band-limited noise does not greatly affect the BCD method. This is especially true when T_e is large, considering that estimation errors increase with increasing T_e (Kirby and Swain, 2008).

5.4 Amplitude vs Phase

We next turn to the claim by McKenzie and Fairhead (1997) that the Bouguer coherence method can only give reliable T_e estimates when the power spectrum of the free-air anomaly is of the same order of magnitude as that of the Bouguer correction (Section 2). This is incorrect. Forsyth’s method is mainly concerned with the phase relationships between loads, so one should not compare amplitude or power spectra, which are largely phase-independent. Consider two arbitrary signals with Fourier/wavelet transforms G and H . If the amplitude and phase of each of these signals are independent, then the coherency can be written as the product of two terms, the “amplitude-coherency” and the “phase-coherency”:

$$\Gamma = \frac{\langle |G||H| \rangle}{\langle |G|^2 \rangle^{\frac{1}{2}} \langle |H|^2 \rangle^{\frac{1}{2}}} \langle e^{i(\varphi_G - \varphi_H)} \rangle \quad (12)$$

(e.g., Nolte et al., 2004), where φ is the phase of the signal. Figure 5 shows the relationship between these three for one of the synthetic models (which model is immaterial: the relationship holds for all data, real and synthetic). It can be seen that the coherency variation is dominated by the phase-coherency for both Bouguer and free-air anomalies, and not by the amplitude-coherency which is mostly in the range 0.5–1.

The flaw in the argument can be demonstrated using a third type of noise, which is simply a filtered fractal surface added directly to the final gravity anomalies, i.e., without using the zero final topography procedure outlined in Section 5.2. For this surface we used the field shown in Figure 3d, scaled to have a range of ± 20 mGal, called type-IIa noise. This was then added to the Gaussian- T_e Bouguer anomaly, and T_e recovered using the BCD method.

Figure 6 shows plots of the wavelet power spectra of the “uncompensated topography” (i.e., $2\pi\Gamma\rho h$), and three free-air anomalies (with no noise, type-II noise, and type-IIa noise), from the Gaussian- T_e plate. In the synthetic Cordillera where the topography is highly variable, the four signals have very similar power at wavelengths less than ~ 220 km, indicating that most of the free-air power comes from the attraction of the topography in this wavelength band. T_e is recovered by the BCD method very well in all cases.

However, in the synthetic shield where the topography is subdued, while the power in the noiseless free-air anomaly is very similar to that in the uncompensated topography, the noisy free-air anomalies have much greater power at wavelengths less than ~ 400 km. While the type-II noise T_e is slightly overestimated (relative to the no-noise result), the type-IIa noise T_e is no different from the no-noise value, even though the power in its free-air anomaly is much greater than the power in the uncompensated topography at short wavelengths. If McKenzie's argument were correct, then T_e from the type-IIa noisy data should be overestimated relative to the no-noise value, which it isn't.

We have used a different noise type here to illustrate a point: that having low relative topographic power does not necessarily imply that T_e will always be overestimated by the BCD method in the presence of noise. Whether type-II or type-IIa noise is an accurate model of actual unexpressed loading we return to in Section 7, but here we note that the sharp decrease of the type-II free-air anomaly spectrum at the shortest wavelengths (due to the shape of the zero-topography transfer function in Figure A3) is not normally observed in nature.

In summary, we conclude that comparing the power spectra of free-air anomaly and uncompensated topography tells us very little about whether or not a method based on the coherence can work. [Swain and Kirby (2003a) reached a similar conclusion, though using synthetic topography that was not 'subdued'.]

5.5 SRC Results

Figure 7 shows the observed Bouguer and free-air SRCs and the T_e results of the BCD method across the green profiles in Figure 4, for the uniform- T_e plate. Remember that incidences of positive Bouguer real coherency are excluded from the inversion (Section 4.3). Looking at the Bouguer SRC (third row), we see first that when there is no added noise, the Bouguer SRC rollover is pushed to longer wavelengths than predicted by theory over the subdued topography, resulting in T_e overestimates (as discussed in Section 5.3). This contour will be used as a reference in the following discussion.

When type-I noise is added, the rollover is pushed to even longer wavelengths, mimicking the effect of a high T_e . This is what Crosby (2007) found in his experiment 4, and could support McKenzie's proposal that the Bouguer coherence method can not give reliable T_e estimates when topography is subdued and there exist unexpressed internal loads. However, this phenomenon occurs predominantly over the subdued topography, and is less marked for type-II noise, as discussed later.

While the identification of subdued topography is easy, identification of noise (unexpressed internal loading) is not. McKenzie and Fairhead (1997) suggested that analysis of the free-air coherence reveals zones where the free-air anomaly is incoherent with the topography, which he said shows where the gravity field is dominated by unexpressed internal loads. As expected, the free-air SRC ($\Gamma_{F,R}^2$, fourth row in Figure 7) over subdued topography has very low values for the noisy models, though not for the no-noise model. However, a confounding factor is that when $F > 0$, theory predicts a dip in $\Gamma_{F,R}^2$ at wavelengths slightly longer than the Bouguer SRC rollover wavelength, with the dip deepening with increasing F , increasing T_e , or

decreasing depth to internal loading. Thus the dip is not noticeable in the low- T_e regions of the two models (Figures 7 and 8).

The question then arises as to how to separate the noise-induced low free-air SRC from the flexure-induced signal. We address this problem by using the complex number nature of the coherency, and compute the normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$), shown in the fifth row of panels in Figures 7 and 8. As shown in Appendix B1, this quantity can provide a better measure of the signal-to-noise ratio than can the free-air SRC because 1) the imaginary part of the coherency holds information about the uncorrelated (or unexpressed) harmonics of two signals, and 2) the normalisation amplifies zones with a small imaginary part when the total coherence (i.e., $|\Gamma|^2$) is low. As noted above, $\Gamma_{F,R}^2$ for the Gaussian- T_e plate (no-noise panels in Figure 8) shows lower values around the flexural wavelength as predicted by theory, which do not show up to such an extent in $\bar{\Gamma}_{F,I}^2$. So, in Appendix B2 we show results of additional synthetic modelling that demonstrate that when the free-air SRC dips in response to flexure, $\bar{\Gamma}_{F,I}^2$ remains zero, implying that it is a measure of noise only.

Incidentally, McKenzie (2003) postulated that, when expressed loads are absent, a measure of the fraction of the total load due to unexpressed loads is $1 - \Gamma_{F,R}^2$ (although he used $1 - \gamma_F^2$), where the free-air SRC is averaged over a large wave number range (and his use of multitapers has averaged out the imaginary part to zero). This is not strictly true, because it assumes that the total free-air coherency-squared, $|\Gamma_F|^2 = 1$, $\forall k$, when in fact we have shown that $|\Gamma_F|^2 \leq 1$ (Appendices A1 and B1). His

postulated measure will therefore overestimate the proportion of unexpressed internal loads. Furthermore, this approach requires use of analytic expressions for the free-air coherence, which makes them model-dependent. In contrast, the normalised free-air SIC is determined from observed data only and also accounts for low total coherence.

Finally, it is important to note that the wavelength of the type-I noise rollover in the Gaussian plate (Figure 8) correlates with the long wavelength extent of high $\bar{\Gamma}_{F,I}^2$, and not with the long wavelength extent of low $\Gamma_{F,R}^2$, supporting use of $\bar{\Gamma}_{F,I}^2$ over $\Gamma_{F,R}^2$ in noise identification. This is not as apparent in the uniform plate plots because the noise is of lower amplitude and the theoretical free-air SRC dip is small, as discussed above.

While type-I noise has been shown to bias recovered T_e above its no-noise values in subdued topography, the type-II noise models show that the addition of this kind of noise reduces the Bouguer SRC only within the wave number band where the noise is considerable. If the rollover does not fall within this band, then T_e can be reliably estimated. This is particularly apparent in the Gaussian- T_e plate, where the no-noise Bouguer rollover does not fall within the noise band, so the type-II noise rollover is not biased, and there is very little difference between the no-noise and type-II noise T_e .

In summary, interpretation of $\Gamma_{F,R}^2$ (or γ_F^2) alone can lead to the false conclusion that noise is present. So McKenzie's statement that the presence of low short-wavelength free-air coherence indicates failure of the Bouguer coherence method is wrong. His proposal assumes that low free-air coherence at short wavelengths implies low

coherence at all wavelengths. Rather, Forsyth’s method fails when $\bar{\Gamma}_{F,I}^2$ is high around the true Bouguer rollover. Inspection of $\bar{\Gamma}_{F,I}^2$ with real data should allow us to determine when gravity noise is likely to be a problem.

5.6 Admittance Results

The problem of reliably estimating T_e in the presence of subdued topography and unexpressed loading still stands, though. McKenzie and Fairhead (1997), McKenzie (2003) and Crosby (2007) proposed that the free-air admittance can resolve the issue, because it is less sensitive to noise than the Bouguer SRC (coherence). However, for the uniform- T_e plate (Figure 9), it can be seen that the FQU T_e is still overestimated in subdued topography even when no noise is present.

Since we know the noise field, we can compute a “noise-admittance” via

$\langle NH^* \rangle / \langle HH^* \rangle$, shown in the 4th row of panels in Figures 9 and 10. Immediately apparent is the high degree of correlation between the observed/theoretical admittance difference (3rd row of panels) and the real part of the noise admittance (4th row of panels). Hence, noise does bias admittance, which in turn biases T_e . Note that it is possible for T_e to be underestimated (from its no-noise value) in the presence of noise (Figures 4 and 10), whereas with the BCD method noise would always cause either T_e overestimates or no change at all.

However, as for the BCD method, type-II noise affects the FQU method very little. Again, this is due to the relative positions of the peak of the noise spectrum and the admittance transition wavelength. [As a proxy for this latter quantity, we use the longest wavelength at which the admittance is 60 mGal/km, because this represents

the midpoint between the theoretical longest and shortest wavelength admittance values, for the densities we use (Table 1), and as long as f is not much greater than 1.] Looking at the Gaussian plate (Figure 10), the type-II noise admittance has high values at wavelengths below the no-noise admittance transition, and T_e is not much different to the no-noise results. The type-I noise admittance, however, has high values around the no-noise transition, and T_e is biased (downwards in this case).

Whereas identification of SRC-biasing noise can be achieved with $\bar{\Gamma}_{F,I}^2$, identification of admittance-biasing noise is more difficult. Nevertheless, some insight can be gained by analysis of the imaginary component of the observed admittance ($Q_{F,I}$). First, note the resemblance between the imaginary components of the observed and noise admittances ($Q_{F,I}$ and $Q_{N,I}$) in Figures 9 and 10. While there appears to be very little similarity in the details between the real and imaginary components of the noise admittance, there is a gross correlation. That is, when the real noise admittance ($Q_{N,R}$) has high variance, so does $Q_{F,I}$, and when $Q_{N,R}$ has low variance, so too does $Q_{F,I}$. Hence, the imaginary component of the observed admittance can provide an indication of where the real observed admittance is likely to be altered and T_e biased.

In summary, these results demonstrate that the FQU method can be biased by noise, despite McKenzie (2003) and Crosby (2007) stating that noise will only increase the variance of the free-air admittance without biasing it. The variance of the noisy model admittances in Figures 9 and 10 is indeed larger than that of the no-noise admittances, but this increase in variance tricks the FQU method into producing incorrect T_e estimates upon inversion.

5.7 Misfit Results

Another good indicator of where noise has biased the FQU best-fitting T_e and F is the χ^2 misfit surface (Figure 11). This becomes broader when the noise has a significant impact upon the admittance, which is most noticeable in the subdued topography regions for type-I noise. Note though, that when T_e is high, all spectral methods will give larger errors due to the larger multitaper-window or wavelet sizes needed to resolve the flexural wavelength. Nevertheless, the 99% FQU T_e confidence intervals for type-II noise almost always contain the true value, whereas the same cannot be said for the type-I noise results. Hence, type-I noise, if it is present, will bias FQU results.

The BCD χ^2 misfit curves, however, do not provide useful information about the presence of noise, because when noise reduces the Bouguer SRC in subdued topography, the inversion fits the resulting SRC curves well, but with an overestimated T_e value, just as McKenzie and Crosby have argued.

6. North America

6.1 Data

Figure 12 shows the gravity and topography grids used in this study, both being computed from the EGM2008 harmonic coefficients model (Pavlis et al., 2008). Although the model provides coefficients up to degree and order 2160 (10 arc-minute minimum wavelength, or ~20 km at the equator), our analysis is undertaken on a 20 km grid, so we avoid spatial aliasing and data redundancy by only evaluating up to degree and order 1000 (~40 km minimum wavelength at the equator). In any case, we detected the presence of the Gibbs phenomenon in the gravity coefficients at high

latitudes using the full expansion, which were greatly reduced with the lower-degree expansion. In order to reduce the effects of distortion due to Earth curvature, the data were computed on a Lambert conic conformal projection grid (origin at 100°W, 0°N; standard parallels at 35°N and 65°N). The study area has dimensions of 5980 km (easting) × 8620 km (northing).

The gravity disturbance, rather than the gravity anomaly, was computed from the harmonic coefficients, for the following reason. The BCD method uses Eq. (4) of Parker (1972) in order to derive Moho topography from the observed Bouguer anomaly. However, in deriving his formula, Parker assumes that the gravity anomaly is the vertical derivative of the gravitational potential (U), when in fact it is the gravity disturbance that is given by this vertical derivative: $\delta g_B = \partial U / \partial z$ (Heiskanen and Moritz, 1967). The gravity disturbance is thus more compatible with Parker's method than is the anomaly. The free-air disturbances from EGM2008 were then converted to Bouguer disturbances using the Bouguer plate correction from the EGM2008 topography and a reduction density of 2670 kg.m^{-3} . Terrain corrections on land were computed from the EGM2008 topography (with the same reduction density) using the planar FFT method of Li and Sideris (1994), as described in Kirby and Featherstone (1999). Marine terrain corrections were computed using the formula of Parker (1972) from the EGM2008 topography.

Finally, the bathymetry and ice sheet surfaces were converted to an equivalent topography before wavelet transformation (Kirby and Swain, 2008), using the ice cap thickness over Greenland supplied by the Arctic Gravity Project (see Figure 12 caption).

The inversion of the observed wavelet SRC and admittance includes a term for the depth to the Moho, z_m (assuming that subsurface loading occurs here). Rather than assuming a constant depth we used the Moho depths given by the CRUST2.0 model (Bassin et al., 2000), interpolated to the 20 km Lambert grid. While other Moho depth models exist within the study area (e.g., van der Lee and Frederiksen, 2005), they do not cover the whole area, and in any case, we found that the recovered T_e was rather insensitive to small changes in z_m .

6.2 T_e and F Results

The effective elastic thickness results from the BCD and FQU methods are shown in Figure 13, while the corresponding best-fitting F values appear in Figure 14. These results were obtained by inversion of the observed SRC/admittance using a two-layer loading model with initial loading at the surface and Moho, and the densities and elastic constants given in Table 1. Both methods give very large T_e values (>100 km) over the Precambrian Shield regions of North America, with the FQU T_e consistently higher and covering a greater area. As Kirby and Swain (2008) found, T_e from the inversion of admittance generally has a better spatial resolution than that from the coherence, which is smoother. This is because the admittance transition wavelength is smaller than that of the SRC, requiring smaller-scale wavelets for its detection and affording better spatial resolution. Both methods give low values (<20 km) over the Cordillera, with a steep T_e gradient separating this province from the Shield. The southerly regions of the Interior Platform are characterised by intermediate values (~ 50 – 70 km), except over the Mid-continental Rift (MCR) region, where the BCD method gives much higher T_e values (~ 90 – 100 km) than the FQU method (~ 30 – 50

km) – this region is discussed in more detail in Section 6.6. The southern Superior province also shows large differences between the methods, with the FQU method returning relatively low values (~20–40 km) while the BCD method results are much higher (~80–100 km). Another region of high T_e occupies the northern half of the Greenland Shield, and in both results we note a corridor of high T_e over the Innuitian orogen, connecting the Greenland Shield with the Arctic platform, and low T_e over the failed Atlantic spreading centre between Greenland and north-east Canada, in Baffin Bay and the Labrador Sea. Finally, both methods recover T_e of ~50 km over the Appalachian Mountains. The 95% confidence limits do not suggest excessive errors, though we recall (Section 5.7) that the BCD intervals are not a fair indicator of the true T_e over subdued topography when noise is present. The FQU errors (which are better at indicating bias) in the Interior Platform are high, but are low over the Shield, suggesting that T_e here appears to be very high.

McKenzie and Fairhead (1997) and McKenzie (2003) contend that surface, rather than expressed internal, loads should dominate continental interiors. They postulate, therefore, that the loading ratio should have values very close to zero in the eroded topography regions, especially when determined by the FQU method. Figure 14 suggests otherwise, and shows that expressed subsurface loading has dominated continental tectonics in North America, or at least been equal in magnitude to surface loading. Again, the 95% FQU confidence limits do not suggest excessive errors on F , except over parts of the Interior Platform. This area will be discussed further in Sections 6.6 and 7.

However, McKenzie (2003) used a three-layer loading model for his inversions of the free-air admittance, with loading occurring within the crust at 15 km depth. Hence, we also inverted our observed Bouguer SRC and free-air admittance using such a model, with loading occurring at the upper-crust/middle-crust interface (as given by the CRUST2.0 model, which has a mean depth of ~ 15 km over the continent), and the densities shown in Table 1. The T_e and F results are shown in Figures 15 and 16, respectively. It is well known that decreasing the depth of internal loading reduces the recovered T_e upon inversion, and this is shown in a comparison of Figures 13 and 15, but it appears that the change in model affects the FQU T_e more than that from the BCD. Very noticeable in the three-layer FQU result is the absence of the sharp T_e discontinuity at the western edge of the Shield that exists in the two-layer FQU result. While the characteristics of the observed admittance do change markedly over many of the boundaries of the Shield (Section 6.4), we believe that the aforementioned two-layer T_e discontinuity is an artefact of the inversion, since the wavelets we use cannot resolve such a sharp spatial gradient in either the SRC or admittance (Kirby and Swain, 2008).

Hence, many of McKenzie's low T_e values can be explained by his choice of inversion model, but importantly, the Shield shows high (>100 km) values from both two- and three-layer models. Use of this model has also decreased the recovered F values from the two-layer FQU results, but not greatly altered those from the BCD method, at least in the Precambrian provinces. This sensitivity of admittance methods to loading ratio is well-known (e.g., Forsyth, 1985; Kirby and Swain, 2008), so the change is not surprising. Nevertheless, the F values still suggest equal loading over much of the Shield. The overall misfit is similar for the two models but we note that

the upper 95% confidence limits in T_e are somewhat smaller for the two-layer model over the Shield, though larger further south (Figures 13e, 13f, 15e, 15f).

6.3 SRC Results

Figure 17 shows coherency slices across the three transects indicated in Figure 13. Discussing the transect that crosses the Mid-continental Rift (MCR) first, the Bouguer SRC rollover passes through a zone of very low free-air SRC over the subdued topography region. The recovery of the BCD method here is thrown into question though by the occurrence of high noise levels (high $\bar{\Gamma}_{F,I}^2$) at just slightly shorter wavelengths than the rollover, which suggest that the noise could be pushing it to longer wavelengths. Slightly to the west in the Interior Platform, however, the Bouguer rollover is still within the high free-air coherence and low $\bar{\Gamma}_{F,I}^2$ zone, suggesting a reliable (and fairly large, 50 km) T_e estimate in that province.

The BCD method gives even larger T_e values over the Shield. Even though the transect crossing the Shield exhibits more noise over subdued topography than does the MCR transect, the noise lies at longer wavelengths than the Bouguer rollover, so it cannot be noise that is pushing the rollover to longer wavelengths. Recall from Figures 7, 8 and A3 that it was noise lying at just shorter wavelengths than the rollover that caused decoherence and pushed it to longer wavelengths. Over the Canadian Shield this is not the case, and the high BCD-recovered T_e is most likely correct. Further evidence can be seen in the south-north transect panels in Figure 17. In the Interior Platform and Superior province the noise lies at shorter wavelengths than the Bouguer rollover (suggesting T_e overestimation), while over Shield the noise lies at longer wavelengths. We believe that this longer-wavelength noise is not of

sufficient amplitude to degrade the Bouguer SRC, and in model C of Figure A4 we use theoretical modelling to show that such noise does not move the Bouguer rollover.

Remember, though, that the synthetic results suggested that subdued topography alone can cause T_e overestimates, for both BCD and FQU methods. Even though most of the Shield is characterised by low topographic variance (Figure 1b), synthetic modelling of the region showed that if the true T_e were 10 km, the maximum increase would be ~14 km from the BCD method, and ~18 km from the FQU method, in the absence of noise (Section 5.3). These overestimates are not sufficient to cause great concern. Moreover, if the true North American T_e were given by the synthetic Gaussian model, then T_e would actually be underestimated by both methods due to the smoothing effect of the wavelet.

These observations point to the preliminary conclusion that the BCD method might be unreliable over the Superior province and the Mid-continental Rift region, but it is most likely reliable in the Canadian Shield to the north. This conclusion is supported by an analysis of the FQU method results, following.

6.4 Admittance Results

Figure 18 shows cross-sections through the admittances over the three transects.

Using the imaginary observed admittance as an indicator of noise (Section 5.6), it is seen that in the Shield most of the admittance-biasing noise (i.e., a high variance in $Q_{F,I}$) occurs either at wavelengths much shorter than the admittance transition, or not at all, suggesting accurate T_e recovery in these regions. This is not the case over the Mid-continental Rift and southern Interior Platform, though, where the transition

coincides with high $Q_{F,I}$ variance. Since our synthetic modelling shows that such noise can cause both T_e over- and underestimates, it is difficult to place a true value on T_e here. We return to this region in Section 6.6.

6.5 Misfit Results

The FQU χ^2 misfit surface for the location in the actual Shield (Figure 19) shows a similar pattern to those in the synthetic Shield with both no added noise and type-II noise (Figure 11), suggesting again that noise is not biasing the admittance here.

While the T_e misfits are broad, 99.99% confidence limits indicate values of at least 100 km. The Cordilleran location shows low, but relatively well-constrained, T_e values.

6.6 Mid-continental Rift

The large difference in T_e and F values from the BCD and FQU methods over the Mid-continental Rift region warrants further analysis (Figure 20). Since McKenzie (2003) investigated this area using an FQU multitaper method with a window size of ~ 1000 km, we spatially-averaged the auto- and cross-spectra over cells of dimension 500×500 km, after azimuthally-averaging them. This procedure is described in more detail in Kirby and Swain (2006), and has the advantage that the 1-D profiles are representative of the mean admittance/coherency over a region and are more directly comparable with multitaper estimates which average the spectra over a finite window size.

The two-layer BCD result gives a T_e of ~ 98 km, with F around 0.5 ($f = 1$), while the FQU result gives a T_e of ~ 37 km with $F \sim 0.7$ ($f = 2.33$). However, Figure 20 shows

that $\bar{\Gamma}_{F,I}^2$ is high at wavelengths just shorter than the Bouguer SRC rollover, which appears to be destroying the coherence and causing a T_e overestimate. We also note that there is an oscillation in the observed free-air admittance between $10^{2.9}$ km and $10^{2.6}$ km wavelength, which correlates with the imaginary admittance oscillation and indicates a degree of noise-admittance that might also bias the recovered FQU T_e and F values. It is interesting that if the actual T_e is 37 km as predicted by the FQU method, then a remnant of the true roll-over (coinciding with the green curve) is visible in the observed Bouguer SRC at a wavelength of $10^{2.4}$ km. These observations lead us to favour our FQU results in the MCR region over the BCD results.

While our BCD and FQU T_e values are very different, they are also very different to the values of $T_e = 11.8$ km and $F = 0$ obtained by McKenzie (2003) in the region. He used the multitaper method with a single window of size approximately 1000 km square, and inverted the free-air admittance over a bandwidth of 120–1000 km only (we invert the complete spectrum). McKenzie ascribes the positive admittance at his longest wavelength (only 1000 km, even though he plots up to 2000 km) to dynamic support of the topography by mantle convection. The complete spectrum (Figure 20), however, shows that the admittance falls to zero at long wavelengths, as predicted by flexural theory. Finally, McKenzie’s use of a shallower loading depth in his inversion inevitably produces lower T_e and F estimates – when we invert the observed admittance using the same three-layer model as McKenzie (2003), we retrieve similar values to him: $T_e = 9.6$ km and $F = 0.03$.

However, the MCR region has a very broad FQU misfit surface (Figure 19), similar to the type-I noise misfit surfaces for the synthetic plates (Figure 11), suggesting again

that noise is causing admittance bias here. We therefore conclude that neither of the methods gives reliable T_e estimates in this region.

7. Discussion

Our studies of real North American data have shown that some kind of noise indeed affects the gravity field in this continent, and it is an undoubted fact that noise causes decoherence between any two signals. It is called ‘noise’ because it is not predicted by any general model of loading of an elastic plate, which implies that the loads causing it have no topographic expression. The noise is not simply that part of the gravity field that produces low free-air coherence, however, because our theoretical and synthetic modelling has shown that there are other causes of this. To model the noise one could simply add a fractal surface directly to the gravity (as in the type-IIa noise in Section 5.4), but this would imply zero flexure which is physically unrealistic unless T_e is uniformly very large and the ‘noise’ is supported by the plate. Therefore we have chosen to model it by the method proposed by McKenzie (2003) and add it to some of our synthetic data sets. This model can be questioned (e.g., Pérez-Gussinyé et al., 2004), but it gives results that mimic in some ways the results from real data.

We have found two model-independent ways to detect noise and determine whether or not it is likely to bias T_e estimates. Figure 21a shows a map of the value of $\bar{\Gamma}_{F,I}^2$ at wavelengths just below the observed Bouguer SRC rollover, while Figure 21b shows the value of $|Q_{F,I}|$ at wavelengths just around the real admittance transition. The agreement between these maps is very good, pointing to a consistent identification of T_e -biasing noise. So if unexpressed internal loads are indeed the cause of this noise, then Figures 21a and 21b show locations where such loading might have occurred,

and where load-deconvolution methods fail. If we mask out regions in Figures 13a and 13d where the corresponding noise levels are high and are likely to grossly bias T_e , then in Figures 21c and 21d we see that high (>100 km) T_e values still remain in the northern Shield, with approximately equal (expressed) loading here (compare Figures 14b, 16b and 21a,b).

It is interesting that these regions occur mainly in the Superior Province, the southern Interior Platform and the southern Appalachians. They include a number of intracratonic basins (e.g., the Illinois and Michigan Basins) as well as the Mid-continental Rift. McKenzie (2003) gives an account of the evolution of the Mid-continental Rift, stating that the loads associated with the large gravity anomalies there have no topographic expression, which implies that the initial loads correlated perfectly. In the case of intracratonic basins, a common model (e.g., Allen and Allen, 1990) is that of a sag basin caused by down-flexure of the crust during a thermal contraction phase following the emplacement of dense rocks in the lower crust. This intrusion constitutes an initial internal load. After flexural adjustment the sag will fill with sediments, which constitute a surface load and cause further down flexure, but ultimately stabilising to produce a flat topography. The loads representing the intrusion and the sedimentary pile must be in phase, or fully coherent, for the latter to be true.

Other mechanisms could perhaps be invoked to explain the occurrence of noise in the southern Superior Province and southern Appalachians, but we see no reason to expect such mechanisms to be ubiquitous in cratons.

8. Conclusions

Through theoretical and synthetic modelling we have shown that important information about the admittance and coherency between gravity anomalies and topography can be extracted when these quantities are treated as complex variables. This has not been noticed before (except perhaps in one or two studies) because the averaging of the auto- and cross-spectra has effectively removed their imaginary parts. While we use the real parts to estimate T_e through inversion, analysis of the imaginary terms tells us whether or not the inversion is being biased by noise in the data. Such noise has been the subject of much controversy in the past decade, and has been attributed, by McKenzie (2003), to a particular loading regime that produces no final topography after flexure. The nature of the noise, though, is not the primary subject of this paper: we do not dispute that noise exists and can bias T_e estimates, but merely provide methods for its detection.

These new methods improve upon those suggested by McKenzie and Fairhead (1997) and McKenzie (2003), who advocated use of the free-air coherence and free-air anomaly power spectrum, though importantly, just focussing on their short-wavelength characteristics (McKenzie and Fairhead, 1997), or their average over a large bandwidth (McKenzie, 2003). We have found, though, that these two parameters give misleading information about the presence of noise that can result in bona fide T_e estimates being rejected. Instead, our theoretical and synthetic modelling has shown that it is high values of noise only at immediately shorter wavelengths than the observed Bouguer SRC transition wavelength, or immediately surrounding the observed real free-air admittance transition wavelength, that are important.

Despite the claims of McKenzie and Fairhead (1997), McKenzie (2003) and Crosby (2007), our results show that noise can affect the admittance just as much as the coherence, but both quantities are usually unbiased when the topographic variance is high. Although noise will result in systematic overestimates of T_e from the BCD method, it can lead to both under- and overestimates of T_e from the FQU method. Nonetheless, if noise is not present but the topographic variance is low, the BCD method will give T_e overestimates and the FQU method both under- and overestimates. While the bias is not extremely large, spectral methods for T_e -estimation should be used with care in regions where the topography is subdued.

We have found that noise is widespread in the southern Superior province of North America, the southern Interior Platform, and the southern Appalachians, and go so far as to say that T_e and loading ratio cannot be reliably estimated by any method in these regions, whether from spectral analysis or forward modelling, without additional information. Elsewhere, though, the noise levels are not large enough to bias either the BCD or FQU results, and suggest that expressed internal loads have played a significant role in shaping at least the Precambrian regions of the continent.

Most importantly, though, both the BCD and FQU methods imply that the elastic thickness of the Shield is very high, well over 100 km, and that its boundaries are characterised by a rapid and large T_e decrease. The results also suggest that the Canadian and Greenland Shields are fused. Even when a three-layer inversion is performed, and the lower 95% (and even 99.99%) confidence interval is chosen, the T_e values are much higher than the 25 km maximum postulated by McKenzie and

Fairhead (1997) and McKenzie (2003). One can only conclude, then, that most of the strength of the Shield is borne by the upper mantle.

Appendix A: The Complex Coherency and Admittance

A1. Expressed Internal Loading Regime

A1.1 Predicted Quantities

Consider the Fourier/wavelet transforms of two initial loads H_i and W_i applied at the surface and Moho (at depth z_m from sea level), respectively, of a thin elastic plate of known T_e . Here we assume that there are no unexpressed loads present (though see Appendix A2). These initial loads are related to the final, observed gravity anomaly and topography after flexure (G and H) by:

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \mu_W & \mu_H \\ \kappa_W & \kappa_H \end{pmatrix} \begin{pmatrix} W_i \\ H_i \end{pmatrix} \quad (\text{A1})$$

[c.f. Eq. (18) of Forsyth (1985)], where the wave number-dependent deconvolution coefficients are:

$$\mu_W = \frac{2\pi G \Delta\rho_2}{\Phi} \left[-A\Delta\rho_1 e^{-kd} + (\Phi - \Delta\rho_2) e^{-kz_m} \right] \quad (\text{A2})$$

$$\mu_H = \frac{2\pi G \Delta\rho_1}{\Phi} \left[A(\Phi - \Delta\rho_1) e^{-kd} - \Delta\rho_2 e^{-kz_m} \right] \quad (\text{A3})$$

$$\kappa_W = \frac{-\Delta\rho_2}{\Phi} \quad (\text{A4})$$

$$\kappa_H = 1 - \frac{\Delta\rho_1}{\Phi} \quad (\text{A5})$$

where k is wave number, A is a parameter that has the value 1 if the gravity anomaly (G) is free-air or 0 if it is Bouguer, Γ is the Newtonian gravitational constant, d is ocean depth (set to zero if on land), and where:

$$\Phi = \frac{Dk^4}{g} + \rho_m - \rho_f \quad (\text{A6})$$

where D is flexural rigidity, g is the gravitational acceleration, ρ_c and ρ_m are crust and mantle densities, respectively, ρ_f is the density of the overlying fluid (air or water), $\Delta\rho_1 = \rho_c - \rho_f$, and $\Delta\rho_2 = \rho_m - \rho_c$ (see Table 1).

A predicted coherency (Γ_p) formula for the plate is found by substituting G and H from Eq. (A1) into Eq. (5). Alternatively, a predicted admittance is found by substituting Eq. (A1) into the admittance formula, Eq. (2). The auto- and cross-spectra are then:

$$\langle GH^* \rangle = \langle \mu_w \kappa_w W_i W_i^* + \mu_H \kappa_H H_i H_i^* + \mu_w \kappa_H W_i H_i^* + \mu_H \kappa_w H_i W_i^* \rangle \quad (\text{A7})$$

$$\langle GG^* \rangle = \langle \mu_w^2 W_i W_i^* + \mu_H^2 H_i H_i^* + \mu_w \mu_H (W_i H_i^* + H_i W_i^*) \rangle \quad (\text{A8})$$

$$\langle HH^* \rangle = \langle \kappa_w^2 W_i W_i^* + \kappa_H^2 H_i H_i^* + \kappa_w \kappa_H (W_i H_i^* + H_i W_i^*) \rangle \quad (\text{A9})$$

In the classical method of Forsyth (1985), the restriction of having statistically uncorrelated initial loads is expressed by setting the average of terms containing the product of surface and subsurface loads to zero, i.e., $\langle W_i H_i^* \rangle = \langle H_i W_i^* \rangle = 0$. Here, we choose not to make this restriction, and instead let $H_i = |H_i| e^{i\alpha_H}$ and $W_i = |W_i| e^{i\alpha_W}$, where α is the phase of the load, and $\delta = \alpha_W - \alpha_H$ is the phase angle or lag between the two initial loads and is a wave number-dependent parameter. Here, we limit δ to values in a range 0° to 90° , and as will be shown in Appendix A1.2, $\delta = 90^\circ$ corresponds to randomly correlated initial loads, while $\delta = 0^\circ$ corresponds to

correlated initial loads. Thus, the auto- and cross-spectra become, after expansion of the complex exponentials, and separation of real and imaginary parts:

$$\text{Re}\langle GH^* \rangle = \mu_w \kappa_w \langle |W_i|^2 \rangle + \mu_H \kappa_H \langle |H_i|^2 \rangle + (\mu_w \kappa_H + \mu_H \kappa_w) \langle |W_i| |H_i| \cos \delta \rangle \quad (\text{A10})$$

$$\text{Im}\langle GH^* \rangle = (\mu_w \kappa_H - \mu_H \kappa_w) \langle |W_i| |H_i| \sin \delta \rangle \quad (\text{A11})$$

$$\langle GG^* \rangle = \mu_w^2 \langle |W_i|^2 \rangle + \mu_H^2 \langle |H_i|^2 \rangle + 2\mu_w \mu_H \langle |W_i| |H_i| \cos \delta \rangle \quad (\text{A12})$$

$$\langle HH^* \rangle = \kappa_w^2 \langle |W_i|^2 \rangle + \kappa_H^2 \langle |H_i|^2 \rangle + 2\kappa_w \kappa_H \langle |W_i| |H_i| \cos \delta \rangle \quad (\text{A13})$$

where the deconvolution coefficients can be taken out of the averaging because they are constants at a given wave number.

In Forsyth's method, the ratio between the initial subsurface and surface load amplitudes, f , is wave number-dependent, and is computed through:

$$f^2 = \frac{1}{r^2} \frac{\langle |W_i|^2 \rangle}{\langle |H_i|^2 \rangle} \quad (\text{A14})$$

where we define $r = \Delta\rho_1/\Delta\rho_2$.

When used in the admittance or coherency, the auto- and cross-spectra appear as ratios, which means that when we substitute Eqs (A10)–(A13) into Eqs (2) and (5) and divide numerator and denominator by $\langle |H_i|^2 \rangle$, we get, using Eq. (A14):

$$Q_p = \frac{\mu_w \kappa_w f^2 r^2 + \mu_H \kappa_H + (\mu_w \kappa_H + \mu_H \kappa_w) X + i(\mu_w \kappa_H - \mu_H \kappa_w) Y}{\kappa_w^2 f^2 r^2 + \kappa_H^2 + 2\kappa_w \kappa_H X} \quad (\text{A15})$$

$$\Gamma_p = \frac{\mu_w \kappa_w f^2 r^2 + \mu_H \kappa_H + (\mu_w \kappa_H + \mu_H \kappa_w) X + i(\mu_w \kappa_H - \mu_H \kappa_w) Y}{\left[\mu_w^2 f^2 r^2 + \mu_H^2 + 2\mu_w \mu_H X \right]^{\frac{1}{2}} \left[\kappa_w^2 f^2 r^2 + \kappa_H^2 + 2\kappa_w \kappa_H X \right]^{\frac{1}{2}}} \quad (\text{A16})$$

where:

$$X = \frac{\langle |W_i| |H_i| \cos \delta \rangle}{\langle |H_i|^2 \rangle} \quad (\text{A17})$$

$$Y = \frac{\langle |W_i| |H_i| \sin \delta \rangle}{\langle |H_i|^2 \rangle} \quad (\text{A18})$$

In practice, we find that the Y term (i.e., the imaginary component of the predicted admittance/coherency) is vanishingly small for all degrees of load correlation, because the surface/internal load plate model does not generate out-of-phase final loads. Hence $\Gamma_{p,I}^2 \rightarrow 0$ and $\gamma_p^2 \rightarrow \Gamma_{p,R}^2$, and similarly for the predicted admittance, which is why we only invert the real part of the observed SRC/admittance.

In the classical load-deconvolution method of Forsyth (1985) $X = 0$, which, as we shall show, corresponds to $\delta = 90^\circ$, $\forall k$. Incidentally, the method we present (when $X = Y = 0$) gives identical results to the method that Forsyth (1985) originally proposed, whereby G , H , W_i and H_i are used to separate the surface and internal components of the loads, which are then used in his Eq. (25).

A1.2 Analytic Quantities

We cannot yet determine a reasonable method by which to compute a predicted Bouguer SRC/admittance when taking into account correlated initial loads, due to the difficulty in evaluating the X and Y terms in Eqs (A17) and (A18). Indeed, this may be impossible without some form of additional independent constraint. We can, however, achieve a theoretical understanding of their significance by deriving an analytic (rather than predicted) formula when making certain assumptions in Eqs (A17) and (A18). Note that these assumptions are made only to derive a useful analytic equation, and since we do not invert for load correlation in this study, they do not impact upon

our T_e results. First, if we assume that the amplitudes and phases of a given load are independent, then $\langle |W_i||H_i| \cos \delta \rangle = \langle |W_i||H_i| \rangle \langle \cos \delta \rangle$ in the X term in Eq. (A17), and similarly for the Y expression. Second, if we then assume that the amplitudes of the two initial loads are independent, then we may approximate $\langle |W_i||H_i| \rangle / \langle |H_i|^2 \rangle$ by fr , using Eq. (A14). And, third, if we further assume that δ is independent of azimuth (an assumption addressed below), then $\langle \cos \delta \rangle = \cos \delta$ and $\langle \sin \delta \rangle = \sin \delta$. Hence we find:

$$X \approx fr \cos \delta \quad (\text{A19})$$

$$Y \approx fr \sin \delta \quad (\text{A20})$$

which may be substituted in Eqs (A15) and (A16), to give the analytic, rather than predicted, admittance and coherency, Q_a and Γ_a :

$$Q_a = \frac{\mu_w \kappa_w f^2 r^2 + \mu_H \kappa_H + (\mu_w \kappa_H + \mu_H \kappa_w) fr \cos \delta + i(\mu_w \kappa_H - \mu_H \kappa_w) fr \sin \delta}{\kappa_w^2 f^2 r^2 + \kappa_H^2 + 2\kappa_w \kappa_H fr \cos \delta} \quad (\text{A21})$$

$$\Gamma_a = \frac{\mu_w \kappa_w f^2 r^2 + \mu_H \kappa_H + (\mu_w \kappa_H + \mu_H \kappa_w) fr \cos \delta + i(\mu_w \kappa_H - \mu_H \kappa_w) fr \sin \delta}{\left[\mu_w^2 f^2 r^2 + \mu_H^2 + 2\mu_w \mu_H fr \cos \delta \right]^{\frac{1}{2}} \left[\kappa_w^2 f^2 r^2 + \kappa_H^2 + 2\kappa_w \kappa_H fr \cos \delta \right]^{\frac{1}{2}}} \quad (\text{A22})$$

Recall that the free-air and Bouguer versions are determined by the value of the parameter A in the deconvolution coefficients, Eqs (A2) and (A3).

What is the relationship, then, between coherency and coherence? From Eq. (A22) it can be shown through algebra that $|\Gamma_a|^2 = \Gamma_{a,R}^2 + \Gamma_{a,I}^2 \equiv 1$ at all wave numbers for both Bouguer and free-air coherencies. Hence, let us consider the square of just the real part of the Bouguer coherency at $\delta = 90^\circ$. Using Forsyth's expressions

$$\xi = 1 + Dk^4/g\Delta\rho_2 \quad \text{and} \quad \phi = 1 + Dk^4/g\Delta\rho_1 \quad \text{we can derive from Eq. (A22):}$$

$$\Gamma_{B,a,R}^2(\delta = 90^\circ) = \frac{(-\phi f^2 r^2 - \xi)^2}{(\phi^2 f^2 r^2 + 1)(f^2 r^2 + \xi^2)} \quad (\text{A23})$$

This expression is identical to the analytic Bouguer coherence formula from the classical Forsyth method (e.g., Kirby and Swain, 2006), implying that $\delta = 90^\circ$ corresponds to the case of randomly correlated loads. Of course, in reality δ here will not be uniformly 90° at all wave numbers and all azimuths, but from tests on data sets with varying degrees of correlation, we have found that randomly correlated loads exhibit the property that $\langle \cos \delta \rangle \approx 0$, while loads with a correlation coefficient of 0.999 exhibit the property that $\langle \cos \delta \rangle \approx 1$. This, then, explains why we assign $\delta = 90^\circ$ to randomly correlated loads, and $\delta = 0^\circ$ to correlated loads, in the analytic equations.

Furthermore, Eq. (A23) also implies that the analytic Bouguer coherence is actually the square of the real part of the analytic Bouguer coherency, rather than its modulus squared, i.e., $\gamma_{B,a}^2 = \Gamma_{B,a,R}^2$. Eq. (A22) also reveals that when $\delta = 0^\circ$, we have

$\Gamma_{B,a,R} = \pm 1$ and $\Gamma_{B,a,I} = 0$ at all wave numbers, in keeping with the outcomes of having perfectly correlated loads.

Finally, the significance of the imaginary component of the coherency and admittance is illustrated by consideration of their phases. If this phase is computed via:

$$\varphi = \tan^{-1} \left(\frac{\Gamma_{a,I}}{\Gamma_{a,R}} \right) \quad (\text{A24})$$

then it is evident that the phase of the admittance and coherency are identical, from Eqs (A21) and (A22). Figure A1 shows plots of the phase for the Bouguer

admittance/coherency, for various values of wavelength-independent initial load phase. At the longest wavelengths the phase is always 180° no matter what the phase relationship of the initial loads. That is, very wide initial loads of any phase difference will always flex the plate so as to produce anti-phase final loads, regardless of the plate strength; such anti-phase final loads are akin to the compensation mechanism of Airy isostasy. At the shortest wavelengths, conversely, φ asymptotically approaches δ . That is, very narrow initial loads will not flex the plate at all, hence preserving their existing phase relationship. These observations suggest therefore, that while δ is the phase difference of the initial loads, φ measures the load phase difference after flexure. Note that if the imaginary component were to be zero at all wavelengths, then the final loads would always be perfectly in phase, which evidently is not true.

A1.3 Wavelength-dependent Load Correlation

In the real world, the phase relationship between the initial loads would most likely be wave number-dependent, $\delta(k)$. In one scenario, qualitative reasoning suggests that long-wavelength initial loads, generated by large-scale tectonic events, could be correlated, while short-wavelength loads could be randomly correlated or uncorrelated. In Figure A2 we show analytic curves when δ follows this reasoning, and linearly increases from 0° (correlated) at long wavelengths to 90° (randomly correlated) at short wavelengths.

Figure A2a shows that the correlated-load Bouguer coherency departs from the classical (i.e., $\delta = 90^\circ$) curve and becomes positive, but as wavelength decreases and the initial loads become less correlated, it falls back to the classical curve. Hence,

when squared to give the Bouguer SRC, the solid red curve in Figure A2b emerges. This second rollover appears in many observed coherence profiles and corresponds to positive values of the real coherency. While Simons et al. (2003) proposed that such occurrences could arise from a “short-wavelength anisotropic coherence”, we see here that such a signature can also arise from some degree of initial load correlation.

It is also noticeable in Figure A2b that the half-SRC transition wavelength of the red curve is larger than that predicted from the classical coherence for the same T_e , leading to a T_e overestimation in this case. However, one can construct scenarios where the best-fitting SRC follows the second (shorter wavelength) rollover of the red curve, leading to T_e underestimation. Such scenarios can arise when the initial loads are only correlated in a narrow wave number band, as often occurs with synthetic data (Kirby and Swain, 2008). If high initial load correlation occurs at wave numbers close to the Bouguer rollover, the possibility arises that a classical coherence inversion will lead to either T_e overestimates or underestimates.

With the free-air SRC (Figure A2c), the characteristic dip observed in the classical curve is amplified under initial-load correlation. Not only does this mimic the effect of a shallower subsurface load, it could also give the misleading impression that noise is present in the gravity data if a classical, uncorrelated initial-loads model is used to interpret the free-air coherence.

With regard to the admittance (Figure A2d), the initial-load correlation gives rise to a “dipolar” admittance profile (red curve). The inversion fits this as a curve that gives

T_e and F overestimations, but it is also possible that underestimates result if the model F is different (see caption).

A2. Unexpressed Internal Loading Regime

A2.1 Gravity

According to McKenzie (2003), unexpressed initial internal loads are those that produce no surface topography after flexure. If there are no expressed internal loads present we may still use Eq. (A1) to derive equations for gravity and topography. In this case, the equation for H in Eq. (A1) yields:

$$H = \kappa_W W_i' + \kappa_H H_i' = 0 \quad (\text{A25})$$

or:

$$H_i' = -\frac{\kappa_W}{\kappa_H} W_i' \quad (\text{A26})$$

where W_i' is the initial internal load that produces no final topography, and H_i' is the corresponding initial surface load. Since the ratio of the coefficients is purely a wave number-dependent parameter, the two loads are in phase. Then, substitution of Eq. (A26) into Eq. (A1) for G gives the final gravity anomaly due to unexpressed internal loading, G' , which, again, is in phase with both initial loads:

$$G' = \eta_0 W_i' \quad (\text{A27})$$

where the “zero-topography transfer function” is:

$$\eta_0(k) = \mu_W - \frac{\mu_H \kappa_W}{\kappa_H} \quad (\text{A28})$$

Since there is no final topography, the Bouguer and free-air anomalies are equal. So, from Eqs (A2)–(A5), Eq. (A28) becomes, for the two-layer Moho-loading model:

$$\eta_0(k) = 2\pi G \Delta\rho_2 e^{-kz_m} \left(1 + \frac{g\Delta\rho_2}{Dk^4}\right)^{-1} \quad (\text{A29})$$

This transfer function, shown in Figures A2a and A2b, exhibits the characteristics of a band-pass filter, its peak and bandwidth dependent upon T_e . Interestingly, both very long and very short wavelengths in W_i' are suppressed in the final gravity field, but disturbingly, the filter maximises those wavelengths close to the coherence transition wavelength (“rollover”). Indeed, Figure A3c shows that the coherence rollover will always lie within the half-width of the transfer function no matter what the T_e value. This implies that, if W_i' contains significant power at wavelengths close to the rollover, then the coherence may be biased, and T_e may be incorrectly estimated. This need not be the case though, as discussed in Appendix A2.2.

In McKenzie (2003)’s implementation of this “zero-topography” model, he correctly derives the three-layer equivalent of our Eq. (A26) [his Eqs (A26) and (A27)], but then computes the gravity effect of this flexed internal load using only the internal load equations [his Eq. (A31), which is merely a rearrangement of his Eqs (A7), (A9) and (A10)]. That is, he does not add the gravity effect of the flexed initial surface load that must be present in order to balance the initial internal load. Crosby (2007), using a slightly different notation, gives a correct version of this equation [his Eq. (14)], but unfortunately his Eq. (12) contains an error (the second ρ_w should be ρ_u) which also occurs in his Eq. (13).

A2.2 Coherency

We may now combine the expressed and unexpressed loading regimes. Eq. (A1) now becomes:

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \mu_W & \mu_H \\ \kappa_W & \kappa_H \end{pmatrix} \begin{pmatrix} W_i \\ H_i \end{pmatrix} + \begin{pmatrix} \eta_0 W_i' \\ 0 \end{pmatrix} \quad (\text{A30})$$

When computing the auto- and cross-spectra for use in the admittance and coherency, we can make certain assumptions. The unexpressed internal load must be, by definition, statistically uncorrelated with both expressed internal and expressed surface loads; if it were not then it would generate a measurable final surface topography because it would share characteristics with the expressed load. That is, the expressed and unexpressed loading regimes must be independent. Therefore, terms

such as $\langle W_i' H_i^* \rangle$ and $\langle W_i W_i'^* \rangle$ vanish, leaving only the auto-spectrum $\langle |W_i'|^2 \rangle$. Thus

Eqs (A10), (A11) and (A13) remain unchanged, but Eq. (A12) becomes:

$$\langle GG^* \rangle = \mu_W^2 \langle |W_i|^2 \rangle + \mu_H^2 \langle |H_i|^2 \rangle + 2\mu_W \mu_H \langle |W_i| |H_i| \cos \delta \rangle + \eta_0^2 \langle |W_i'|^2 \rangle \quad (\text{A31})$$

Since the admittance does not contain a $\langle GG^* \rangle$ term, it should, in theory, be unaffected by unexpressed loading. The coherency, however, becomes:

$$\Gamma_a = \frac{\mu_W \kappa_W f^2 r^2 + \mu_H \kappa_H + (\mu_W \kappa_H + \mu_H \kappa_W) f r \cos \delta + i(\mu_W \kappa_H - \mu_H \kappa_W) f r \sin \delta}{\left[\mu_W^2 f^2 r^2 + \mu_H^2 + 2\mu_W \mu_H f r \cos \delta + \eta_0^2 f_u^2 r^2 \right]^{\frac{1}{2}} \left[\kappa_W^2 f^2 r^2 + \kappa_H^2 + 2\kappa_W \kappa_H f r \cos \delta \right]^{\frac{1}{2}}} \quad (\text{A32})$$

from Eq. (A22), where we have introduced an “unexpressed loading ratio”:

$$f_u^2 = \frac{1}{r^2} \frac{\langle |W_i'|^2 \rangle}{\langle |H_i|^2 \rangle} \quad (\text{A33})$$

and the case $f_u = 0$ corresponds to no unexpressed loading.

Since W_i' cannot be known if both expressed and unexpressed loading are present, there is no possibility of estimating f_u , and thence the coherency. We can, however,

guess at realistic values of f_u , but would not expect it to reach very large values. Synthetic modelling shows that h_i , the initial (expressed) surface load in the space domain, can have very high relief, of the order of tens of kilometres. Using Eq. (A33) as a guide, with crust and mantle densities of 2800 and 3300 kg.m⁻³, even $f_u = 1$ suggests that the unexpressed initial internal load would have a relief of over five times the initial expressed surface load. Therefore f_u is probably much less than 1 in nature, and if so, the noise it represents would probably not bias T_e very much.

Supposing that the noise is band-limited, we can construct theoretical SRC curves, shown in Figure A4. In model A (which corresponds to the type-II noise synthetic simulations in Section 5), the presence of short-wavelength noise does not disturb the position of the Bouguer SRC rollover, and T_e will not be biased. In model B (similar to type-I noise), the noise has a bandwidth that includes the rollover, which is pushed to longer wavelengths, and T_e will be overestimated. Model C is an attempt to represent the noise regime in the Canadian Shield of the actual North American study (Section 6.3). The noise is long-wavelength but fades at the Bouguer rollover, which is not shifted from its no-noise position: T_e will not be biased, even though the free-air SRC is low here. Finally, model D shows that very long-wavelength noise will not affect the position of the rollover, nor reduce the long-wavelength Bouguer SRC from 1.

Appendix B: Significance of the Imaginary Coherency

B1. Signal Correlation

Although theory shows that $\Gamma_{a,R}^2 + \Gamma_{a,I}^2 = 1$ (see Appendix A1.2), we have shown that, in practice for observed data $\Gamma_{o,R}^2 + \Gamma_{o,I}^2 = \gamma_o^2 \leq 1$ due to the averaging process. So

when the total coherence is low at a given wavelength, it becomes difficult to interpret $\Gamma_{o,l}^2$. Here we define the normalised coherency-squared, which effectively represents the relative power in the real and imaginary parts of the coherency:

$$\bar{\Gamma}_R^2 = \frac{(\text{Re}\Gamma)^2}{|\Gamma|^2} \quad \bar{\Gamma}_I^2 = \frac{(\text{Im}\Gamma)^2}{|\Gamma|^2} \quad (\text{B1})$$

This normalisation now means that $\bar{\Gamma}_R^2 + \bar{\Gamma}_I^2 = 1$, and these quantities make it easier to interpret the relative contribution of the real and imaginary parts to the total coherence.

The importance of the imaginary coherency may be demonstrated by considering two random, fractal surfaces in the space (\mathbf{x}) domain, $u(\mathbf{x})$ and $v(\mathbf{x})$, of fractal dimension 2.5, generated on a grid of dimensions 5100×5100 km with a 20 km grid spacing. A further surface, v' , was then generated from u and v by varying the correlation coefficient (R) between them, as did Macario et al. (1995), via:

$$v' = Ru + v\sqrt{1-R^2} \quad (\text{B2})$$

In our simulations, we used values of $R = 0, -0.33$ and -0.67 , these values representing an increase of correlation.

Figure B1 shows slices through the 3-D arrays of observed SRC/SIC and normalised-SRC/SIC between the two surfaces. In Figure B1, the SRC and SIC are both very low when $R = 0$, though isolated spikes of high coherency can be seen. As $|R|$ increases, the SRC increases as one would expect, given that the surfaces are now partially-correlated, and the isolated high-SRC values for $R = 0$ are amplified. Also, the SIC decreases, though it is not easy to identify variations in this parameter. The

normalised-SRC and -SIC plots reinforce this trend however, and show the relative distribution of real and imaginary components of the coherency. Thus, from this analysis, we can conclude that the imaginary component of the coherency reveals the locations, in both space and wave number domains, where part of one signal is not expressed in the other signal, manifested as a high value of $\bar{\Gamma}_7^2$, the normalised-SIC. Conversely, a low value of $\bar{\Gamma}_7^2$ shows where the signals are correlated to some degree.

This property of the imaginary component of the cross-spectrum (and hence coherency and admittance for the flexural case), that it is zero for correlated signals, may be shown mathematically as follows. Taking the Fourier (or Morlet-wavelet) transform of Eq. (B2) (where U and V are the transforms of u and v), the cross-spectrum between the two surfaces is:

$$VU^* = RUU^* + VU^*\sqrt{1-R^2} \quad (\text{B3})$$

When $R = 1$ (i.e., correlated surfaces), the imaginary component of the cross-spectrum is:

$$\text{Im}(VU^*) = \text{Im}(UU^*) = 0 \quad (\text{B4})$$

because UU^* is real-valued. Conversely, when $R = 0$ (i.e., randomly correlated surfaces), the imaginary component of the cross-spectrum is:

$$\text{Im}(VU^*) = \text{Im}(VU^*) = V_I U_R - V_R U_I \quad (\text{B5})$$

The only way in which this imaginary component can be zero, in general, is if $V_I U_R = V_R U_I$, or:

$$\frac{V_I}{V_R} = \frac{U_I}{U_R} \quad (\text{B5})$$

Taking the inverse tangent of both sides gives:

$$\varphi_V = \varphi_U \tag{B6}$$

where the φ 's are the phases of the signals. That is, the imaginary component is only zero when the signals are in phase at all wave numbers, i.e., correlated. If the phases differ, then the imaginary component will be non-zero.

B2. Free-air Coherence

Theory predicts a dip in the free-air coherence (actually its SRC) between post-loading gravity and topography when $F > 0$. This dip deepens for larger T_e and larger F . In this section we determine the corresponding effect upon the observed normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$).

To do this we chose four values of T_e and three values of F (i.e., 12 plates, Figure B2), and computed 100 free-air anomaly/topography pairs after loading (at the surface and at a Moho of 35 km depth) by random, fractal surfaces on each of the 12 plates (dimensions 5100×5100 km with a 20 km grid spacing). Note the topography in this simulation is “regular”, and not subdued. We then used the wavelet method to compute the observed free-air SRC and normalised-SIC in the following manner. For each plate and for each of its 100 simulations, we computed the gravity and topography auto-spectra and the gravity/topography cross-spectra at every grid node (i.e., local wavelet spectra), and stored these separately. After computing the remaining 99 simulations, we averaged the 100 auto- and cross-spectra separately, and then averaged the local spectra over all space-domain grid nodes at each wave number to give global spectra (e.g., Kirby and Swain, 2004). We then used Eq. (5) for the coherency to give global (i.e., 1-D) mean profiles of the free-air coherency for that plate. We can perform the spatial averaging because the plates are of uniform T_e .

The results in Figure B2 show that, even when the free-air SRC falls to low values, the normalised-SIC remains zero and does not increase. That is, any high values of $\bar{\Gamma}_{F,I}^2$ seen in the data will not be due to flexure.

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Figure Captions

Figure 1. a) EGM2008 topography over North America, used as the final topography in the synthetic modelling. b) The logarithm of the variance of the topography, calculated in a moving square window of side 100 km. c) The Gaussian T_e distribution used to generate synthetic gravity anomalies from the topography in (a). In the other synthetic tests, a uniform T_e of 10 km was used. The green circles show the half-width of the space-domain Morlet ‘transition wavelet’ needed to resolve the coherence transition wavelength for $T_e = 140$ km (larger circle, diameter 2160 km), and $T_e = 10$ km (smaller circle, diameter 300 km). The black-outlined box shows the area over which the wavelet analysis was performed.

Figure 2. Synthetic (noise-free) free-air anomalies from (a) the uniform $T_e = 10$ km plate, and (b) the plate with the Gaussian T_e distribution shown in Figure 1c. The black-outlined box shows the area over which the wavelet analysis was performed. Map axes are grid eastings and northings in km.

Figure 3. The top two panels show the initial unexpressed internal loads (w_i') used to give zero final topography after flexure: a) type-I (range ± 3100 m), d) type-II (range ± 1040 m). The bottom four panels show the final gravity anomalies after flexure of: (b) type-I load on the uniform $T_e = 10$ km plate (range ± 20 mGal); (c) type-I load on the Gaussian T_e plate (range ± 90 mGal); (e) type-II load on the uniform $T_e = 10$ km plate (range ± 20 mGal); (f) type-II load on the Gaussian T_e plate (range ± 30 mGal). Axes are as for Figure 1.

Figure 4. Recovered T_e differences with respect to: (a)–(d) the synthetic models, i.e., recovered T_e minus model T_e , where the models are a uniform T_e (10 km) plate and the Gaussian T_e distribution shown in Figure 1c; and (e)–(l) the no-noise results, e.g., Figure 4k is the type-II noise Gaussian BCD T_e minus the no-noise Gaussian BCD T_e . The green lines show the locations of the cross-sections in Figures 7–10. Axes are as for Figure 1, though over the smaller area shown in the black box in Figure 1.

Figure 5. Relationship between the observed SRC, amplitude-SRC and phase-SRC, for the uniform- T_e synthetic plate with type-I added noise (Bouguer and free-air). Cross-sections through the 3-D SRC arrays are from west to east along the green line in Figure 4e. If the amplitude- and phase-coherency are truly independent, then their product should equal the total coherency. This condition is mostly satisfied, but there are some zones where it is not.

Figure 6. Isotropic fan wavelet power spectra of the Bouguer plate correction (i.e., $2\pi\Gamma\rho h$, green), and the free-air anomalies with no added noise (red), type-II (blue), and type-IIa noise (pink, see Section 5.4) at two locations in the Gaussian- T_e plate: (a) in the synthetic Cordillera, coordinates E–1000 km, N5000 km; (b) at the peak of the Gaussian, coordinates E500 km, N6460 km). In each panel the true (i.e., model) T_e at the location is indicated, together with the estimates from the BCD method (in km).

Figure 7. Cross-sections through various 3-D coherency/squared-coherency arrays, from west to east along the green line in Figure 4, for the uniform- T_e plate (BCD method). The first row of panels show the T_e recovered by the BCD method (red) and its 95% confidence limits (grey shading – not visible in this particular figure),

together with the model T_e (thick black line), and the topography across the transect (thin black line, scaled by a factor of 0.3). The second row shows the real part of the observed Bouguer coherency. The third, fourth and fifth rows show the Bouguer SRC, the free-air SRC, and the normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$), respectively; the black line in these plots is the SRC rollover ($\Gamma_{B,R}^2 = 0.5$ contour) from the no-noise result; the thinner grey line is the SRC rollover from the given noise type. The thick grey line in the no-noise Bouguer SRC panel is the analytic Bouguer SRC transition wavelength for the plate (Kirby and Swain, 2008).

Figure 8. As Figure 7, except for the Gaussian- T_e plate.

Figure 9. Cross-sections through various 3-D admittance arrays, from west to east along the green line in Figure 4, for the uniform- T_e plate (FQU method). The first row of panels show the T_e recovered by the FQU method (red) and its 95% confidence limits (grey shading), together with the model T_e (thick black line), and the topography across the transect (thin black line, scaled by a factor of 0.3). The second row shows the real part of the observed free-air admittance. The third row shows the difference between $Q_{F,R}$ and the analytic admittance for the plate. The fourth row shows the real part of the noise admittance. The fifth and sixth rows show the imaginary part of the observed free-air admittance, and noise admittance, respectively. The black line in these plots is the 60 mGal/km contour from the no-noise result, used as a proxy for the free-air admittance transition wavelength (valid for $f < 2$, approximately); the thinner grey line is the 60 mGal/km contour from the given noise type. The thick grey line in the no-noise $Q_{F,R}$ panel is the 60 mGal/km contour from the analytic admittance for the plate.

Figure 10. As Figure 9, except for the Gaussian- T_e plate.

Figure 11. FQU and BCD χ^2 misfits at two locations in the uniform- T_e and Gaussian- T_e plates. The Cordilleran point is at coordinates E-1000 km, N5000 km; the Shield point is at coordinates E260 km, N7000 km. The true T_e for the uniform- T_e plate is 10 km at both locations; that for the Gaussian- T_e plate in the Shield is 116 km; for both plates, the true $F \approx 0.14$. The top three rows of panels show the χ^2 misfit surfaces from the FQU method as a function of T_e and F ; the red crosses mark the location of the best-fitting T_e and F values, while the thick black and thin black contours show the 95% and 99.99% confidence limits, respectively, on these values. The bottom three rows of panels show the χ^2 misfit curves from the BCD method (recall F is not an independent variable here but is estimated from the data).

Figure 12. EGM2008 data used for the computation of the North American elastic thickness. a) Complete Bouguer gravity disturbance. b) Topography/bathymetry, with equivalent topography shown over Greenland computed using $h' = h - t_i(\rho_c - \rho_i)/\rho_c$, where t_i is the ice-sheet thickness, ρ_c is mean crustal density (2800 kg.m⁻³), and ρ_i is the density of ice (900 kg.m⁻³). The major provinces are bounded with the grey lines in (b): Su, Superior; Ch, Churchill; Sl, Slave; Be, Bear; Gr, Grenville; In, Innuitian orogen; Ar, Arctic platform; Hu, Hudson platform; Ap, Appalachian orogen; At, Atlantic plain; Ou, Ouachita orogen.

Figure 13. The effective elastic thickness of North America and Greenland from: (a)–(c) the BCD method; and (d)–(f) the FQU method. The two-layer model of

Forsyth (1985) was used to invert the observed SRC and admittance. (b) and (e) show the lower 95% confidence limits, while (c) and (f) show the upper limits. In both images topography shaded relief is superimposed (illumination from north-west). The major provinces are bounded with grey lines; see Figure 12 for province names. The green lines in show the location of the cross-sections in Figures 17 and 18: the northern west-east line is the “Shield” transect; the southern west-east line is the “MCR” (Mid-continental rift) transect.

Figure 14. Loading ratio, F , corresponding to the T_e results in Figure 13. (a) shows F recovered from the BCD method: this value is the transition- F (i.e., the value of F around the Bouguer SRC rollover), and no confidence limits are computed. (b)–(d) show F recovered from the FQU method: this value is the value at all wavelengths; (c) and (d) show the lower and upper 95% confidence limits, respectively. The green lines are explained in Figure 13.

Figure 15. T_e of North America, except using the three-layer model of McKenzie (2003) to invert the SRC and admittance. See Figure 13.

Figure 16. Loading ratio, F , corresponding to the T_e results in Figure 15 (loading model of McKenzie, 2003).

Figure 17. Cross-sections through various 3-D coherency/squared-coherency arrays, along the three green lines in Figure 13. The first row of panels show the T_e recovered by the BCD method (red) and its 95% confidence limits (grey shading), together with the topography across the transect (thin black line, scaled by a factor of 0.3). The

second row shows the real part of the observed Bouguer coherency. The third, fourth and fifth rows show the Bouguer SRC, the free-air SRC, and the normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$), respectively; the black line in these plots is the $\Gamma_{B,R}^2 = 0.5$ contour (Bouguer rollover). The blue arrows mark the approximate locations of the boundaries between the following provinces: Cord, Cordillera; Int, Interior Platform; App, Appalachians; Sup, Superior. The arrow labelled MCR shows the location of the Mid-continental Rift.

Figure 18. Cross-sections through various 3-D admittance arrays, from west to east along the three green lines in Figure 13. The first row of panels show the T_e recovered by the FQU method (red) and its 95% confidence limits (grey shading), together with the topography across the transect (thin black line, scaled by a factor of 0.3). The second row shows the real part of the observed free-air admittance. The third row shows the predicted admittance from the FQU inversion. The fourth row shows the imaginary part of the observed free-air admittance. The black line in the spectral plots is the $Q_{F,R} = 60$ mGal/km contour, used as a proxy for the free-air admittance transition wavelength (valid for $f < 2$, approximately). The blue arrows are described in the Figure 17 caption.

Figure 19. FQU and BCD χ^2 misfits at three locations in the North American continent. The Cordilleran point is at coordinates W112° 05', N40° 31'; the Shield point is at coordinates W90° 56', N59° 32'; and the Mid-continental Rift point is at coordinates W93° 22', N45° 41'. The top row of panels show the χ^2 misfit surfaces from the FQU method as a function of T_e and F ; the red crosses mark the location of the best-fitting T_e and F values, while the thick black and thin black contours show the

95% and 99.99% confidence limits, respectively, on these values. The bottom row of panels show the χ^2 misfit curves from the BCD method (recall F is not an independent variable here but is estimated from the data).

Figure 20. 1-D profiles of the squared-coherency and admittance over the Mid-continental Rift (coordinates W93° 22', N45° 41'). The auto- and cross-spectra were spatially averaged into 500×500 km bins before coherency/admittance computation. The first row shows the observed Bouguer SRC (black line and black/blue circles with error bars; the blue circles/error bars show those values that have a positive real Bouguer coherency, and were thus excluded from the inversion); the best-fitting predicted SRC (BCD method, red line); the values of the best-fitting T_e and transition- F from the BCD method (red); and the analytic Bouguer SRC for $T_e = 37$ km and $F = 0.7$ ($f = 2.33$) (green) which are the best-fitting values from the FQU method in the MCR region (below). The second row shows the observed free-air SRC (black line and circles with error bars); and the observed normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$, pink). The third row shows the observed real free-air admittance in mGal/km (black line and circles with error bars); the best-fitting analytic admittance (FQU method, red line); the values of the best-fitting T_e and F from the FQU method (red); the analytic free-air admittance for $T_e = 11.8$ km and $F = 0$ ($f = 0$) (green), which were the values obtained by McKenzie (2003) for the MCR region; and the observed imaginary free-air admittance (dotted blue line). Note that we used a two-layer Moho-loading model with depth to Moho of $z_m = 35$ km in the inversion, while McKenzie used a three-layer model, with mid-crustal loading at 15 km; densities for both models are given in Table 1.

Figure 21. (a) The maximum value of the normalised free-air SIC ($\bar{\Gamma}_{F,I}^2$) at the three wavelet scales just shorter than the observed Bouguer SRC rollover wavelength. (b) The maximum absolute value of the imaginary free-air admittance ($|Q_{F,I}|$) at the five wavelet scales around the observed real free-air admittance transition wavelength. (c) The BCD T_e with regions of $\bar{\Gamma}_{F,I}^2 > 0.5$ masked in grey. (d) The FQU T_e with regions of $|Q_{F,I}| > 50$ mGal/km masked in grey. While the threshold values in (c) and (d) might seem arbitrary, we note the very sharp changes from low values of $\bar{\Gamma}_{F,I}^2$ and $|Q_{F,I}|$ to high values.

Figure A1. The phase of the Bouguer admittance/coherency from Eq. (A24), for a range of values of the wavelength-independent initial load phase (δ). The $\delta = 90^\circ$ curve is shown in red, and corresponds to initial loads with a random correlation. In all cases $T_e = 30$ km, $F = 0.33$ ($f = 0.5$), and $z_m = 35$ km (two-layer model).

Figure A2. (a) Analytic Bouguer real coherency; (b) Bouguer squared real coherency (SRC); (c) free-air SRC; and (d) free-air real admittance – from Eqs (A21) and (A22). The black curves show the classical (i.e., $\delta = 90^\circ$, $\forall k$) values expected for this model, while the red curves show the functions dependent on an initial load phase (δ) linearly increasing from 0° at long wavelengths to 90° at short wavelengths. In all cases $T_e = 30$ km, $F = 0.33$ ($f = 0.5$), and $z_m = 35$ km (two-layer model). The dashed red curves show the best-fitting classical SRC/admittance if the red curves are treated as observed data: in (b), the best-fitting $T_e = 45.2$ km and $F = 0.50$; in (d) the best-fitting $T_e = 61.6$ km and $F = 0.61$. If, however, we change the model F to 0.5, then the best-

fitting Bouguer SRC $T_e = 34.5$ km with $F = 0.52$; while the best-fitting free-air admittance $T_e < 0.01$ km with $F = 0$.

Figure A3. (a) and (b) The wave number domain “zero-topography transfer function”, η_0 [red, from Eq. (A29)] for two different T_e values, that relates the spectra of unexpressed initial internal loads, W'_i , to their resulting gravity field, G' , i.e., when it is assumed that the final topography is zero. The figures also show the analytic Bouguer SRC (black) for the corresponding T_e ($F = 0.5$), scaled to match the peak amplitude of η_0 . (c) The locus of the peak wavelength of η_0 (red) as a function of T_e , with its half-width shaded in grey. The black line is the locus of the coherence transition wavelength ($F = 0.5$).

Figure A4. Four models showing the effect of unexpressed internal loads (“noise”) upon the Bouguer and free-air squared real coherencies. The dotted curves show the Bouguer SRC (red) and free-air SRC (blue) with no noise (where they are not visible they coincide with the solid curves). The green curves show the values of f_u (the “unexpressed loading ratio”) that were used to generate the noise-influenced Bouguer (solid red) and free-air (solid blue) SRCs; the green curves are hence an indicator of $\bar{\Gamma}_{F,I}^2$, and $f_u = 0$ corresponds to no added noise; these curves have also been scaled – the actual maximum $f_u = 1$. Parameters for the four models are: A and B, $T_e = 90$ km, $F = 0.14$; C and D, $T_e = 150$ km, $F = 0.6$. In all cases the initial loads were randomly correlated ($\delta = 90^\circ$), and a two-layer loading model was used, with loading at a Moho of 35 km depth and the densities given in Table 1. The red and blue curves were generated using Eq. (A32).

Figure B1. Cross-sections (at 2500 km northing) through the coherency-squared (rows 2 and 3) and normalised coherency-squared (rows 4 and 5) between two random, fractal surfaces (u and v') of varying correlation (R). The top row shows cross-sections through u (blue) and v' (red). Note how $\bar{\Gamma}_R^2$ increases and $\bar{\Gamma}_I^2$ decreases, as $|R|$ increases, but that the corresponding decrease in Γ_I^2 is not so obvious.

Figure B2. Observed free-air SRC (black circles) and its normalised-SIC (pink circles) from synthetic modelling using regular topography, averaged over 100 models, then averaged over the space domain at each wave number, which yields global (1-D) profiles. The green curves show the analytic free-air SRC for each model. $F = 0.09$ corresponds to $f = 0.1$; $F = 0.5$ to $f = 1$, and $F = 0.8$ to $f = 4$.

Constant	Symbol	Value	Units
Young's modulus	E	100	GPa
Newtonian gravitational constant	Γ	6.67259×10^{-11}	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Poisson's ratio	σ	0.25	
Gravity acceleration	g	9.79	ms^{-2}
Mantle density ^c	ρ_m	3300	kg m^{-3}
Crust density ^a	ρ_c	2800	kg m^{-3}
Upper crust density ^b	ρ_u	2600	kg m^{-3}
Lower crust density ^b	ρ_l	2900	kg m^{-3}
Sea water density ^c	ρ_w	1030	kg m^{-3}

Table 1. Symbols and values of constants. Densities used in inversion are: ^a two-layer loading model; ^b three-layer loading model; ^c both two- and three-layer models. Note that McKenzie and Fairhead (1997) use $E = 100$ GPa and $\sigma = 0.5$ (which will yield 7% underestimated T_e compared to our values), while McKenzie (2003) uses $E = 95$ GPa and $\sigma = 0.295$ (which yields 1% overestimates).





















































