Currency Option Pricing and Realized Volatility

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Volatility is a key parameter in currency option pricing. This paper examines alternative specifications of the volatility input to the Black-Scholes option pricing procedure. The focus is the relative performance of implied, realized, and GARCH-based models as predictors of market volatility to forecast currency options prices. Using exchange-traded, daily and intra-daily data for three major European currencies, the results indicate that the realized volatility model tends to outperform the other two specifications, both in-sample and out-of-sample. This result is intuitively appealing and expected to facilitate resolution of other problems in risk management applications.
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1. Introduction

Volatility is the key ingredient for the pricing of assets and derivative securities. It has direct implications for risk-return tradeoff driving portfolio decision-making and financial risk management (see Andersen et al., 2003). Following the seminal work of Black and Scholes (1973) and Merton (1973), volatility modeling has progressed significantly. However, its applications, particularly in currency options pricing, remain patchy. In what follows a skeletal picture of the state of play in this area is provided.

Since the volatility measure is directly unobservable in the Black-Scholes-Merton model, the spotlight was first put on the efficiency and distributional properties of implied volatility (IV) measures. Early research as reported by Latane and Rendleman (1976), Schmalensee and Trippi (1978), Chiras and Manaster (1978), Beckers (1981), among others, show that IV explains variation in future volatility better than volatility based on historical data. In subsequent research, the evidence is found to be mixed (see, for example, Day and Lewis, 1992; Canina and Figlewski, 1993; Christensen and Prabhala, 1998; Gospodinov et al., 2006; among others). Empirical research on the forecasting ability of IV for currency options pricing is limited (see Jorion, 1995).

Based on the time-varying characteristic of portfolio return distributions, Fama (1965) pioneered the construction of empirical models of exchange rate returns which allow for conditional heteroscedasticity. The work in this area is subsequently led by Engle's (1982) ARCH model, capturing the dynamics of conditional velocity, and subsequent innovations by Bollerslev (1986), Nelson (1991), and Engle and Ng (1993) (see, Bollerlev, Chou, and Kroner, 1992, for a review). ARCH-GARCH-type models are applied to options pricing by Duan (1995), Ritchken and Trevor (1999), and Duan and Wei (1999), among others. There is also considerable research on GARCH models for dealing with volatility clustering and excessive kurtosis problems (see Engle, 1982; Bollerslev, 1986; Engle and Ng, 1993; Glosten et al., 1993), but the application to currency options remains far from fully explored.

Interest in realized volatility for option pricing is relatively new. It derives from the Brownian motion, whereby asset return might have a quadratic variation process (see Baxter and Rennie, 1996). The quadratic variation process, therefore, measures the realized sample-path variation of the squared return process. The sum of intra-day squared returns under quadratic variation process is defined as realized volatility (RV). The theory of quadratic variation suggests that RV is an unbiased and highly efficient estimator of asset return volatility, as discussed in Andersen, Bollerslev, Diebold and Labys (2001, henceforth, ABDL) and Barndorff-Nielsen and Shephard (2002). ABDL (2001) derived the theoretical and empirical properties of RV for foreign exchange, based on earlier work of Taylor and Xu (1997). ABDL (2003) illustrate the idea in the context of the foreign exchange market. Further work on the econometrics of RV is found in Barndorff-Nielsen and Shephard (2002, 2004) and Maheu and McCurdy (2002). Again, there is little applied research on RV for currency options.

Set against this background, this study focuses on modeling the time varying nature of the underlying exchange rate volatility in pricing currency options. Using the data for three major European currencies, it explores the possibility of using an implied volatility model (IVM), GARCH (1,1)-based volatility model (GVM) and realized volatility model (RVM) as inputs in to the standard option pricing framework. For each model, pricing error is computed as the difference between the observed at-the-money (ATM) option price and the model-generated price. The purpose is to examine the relative performance of IVM, GVM and RVM as predictors of market volatility to forecast options prices with higher accuracy.

The paper has several attractive features. First, while IVM and GVM have been used for pricing currency options, to the best of our knowledge, the realized volatility using high-frequency (intra-day) data, has not yet been systematically explored and applied to currency options. This study will contribute to filling this void. Second, the study makes comparisons of relative performance of the competing models which enable the underlying foreign exchange return dynamics to be captured in a novel way for pricing currency options. Finally, as the currency futures markets are exchange-traded, for synchronicity, this study adopts a novel approach of using futures prices, even-handedly with spot prices, in the analysis. Note that the currency options are predominantly exchange-traded, while the spot market is an electronic, over-the-counter (OTC) market operating globally on a real time basis.

The paper is organized as follows. Section 2 specifies the models and provides the details of their empirical implementation, followed by the description of the data used in this study and the in-sample results in Section 3. The out-of-sample forecast results are presented in Sections 4, and the last section concludes the paper.

2. Methodology

The first step is to select a model for pricing currency options. Since the return to holding a foreign security is equivalent to a continuously paid dividend on a stock, the Merton (1973) version of the Black-Scholes (BS) model can be applied to foreign security. To value currency options, stock prices are substituted for exchange rates. For data synchronicity, as mentioned earlier, the BS model is used in this study in two versions: spot price version (SBS) model and futures price version (FBS) model. The spot and futures prices are used in SBS and FBS models, respectively.

In SBS model, the European type call and put options are priced as,

$$C_t^S = S_t e^{-R_t^f T} N\left(d_{1,t}\right) - X_t e^{-R_t^d T} N\left(d_{2,t}\right) \tag{1}$$

$$P_t^S = X_t e^{-R_t^d T} N\left(-d_{2,t}\right) - S_t e^{-R_t^f T} N\left(-d_{1,t}\right) \tag{2}$$

where,

$$d_{1,t} = \frac{\ln\left(S_t / X_t\right) + \left(R_t^d - R_t^f + \sigma_t^2 / 2\right) T}{\sigma_t \sqrt{T}}$$

and

$$d_{2,t} = \frac{\ln\left(S_t / X_t\right) + \left(R_t^d - R_t^f - \sigma_t^2 / 2\right) T}{\sigma_t \sqrt{T}} = d_{1,t} - \sigma_t \sqrt{T} .$$

C_t^S	=	Call option price for SBS model in domestic currency at time t.
S_t	=	Exchange rate at time t.
N	=	Cumulative normal distribution function
P_t^S	=	Put option price for SBS model in domestic currency at time t
X_t	=	Option exercise price in domestic currency at time t.
R_t^d	=	Domestic currency interest rate at time t.
R_t^f	=	Foreign currency interest rate at time t.
T	=	Option expiration time.
σ_t	=	Volatility of underlying exchange rate

For FBS versions of equations (1) and (2), we use the cost-of-carry (COC) so that futures price and spot price is

$$F_t = S_t e^{\left(R_t^d - R_t^f\right)T} \tag{3}$$

Combining equation (3) with equations (1) and (2), we have

$$C_t^F = F_t e^{-R_t^d T} N\left(d_{1,t}\right) - X_t e^{-R_t^d T} N\left(d_{2,t}\right) \tag{4}$$

$$P_t^F = X_t e^{-R_t^d T} N\left(-d_{2,t}\right) - F_t e^{-R_t^d T} N\left(-d_{1,t}\right) \tag{5}$$

where,

$$d_{1,t} = \frac{\ln\left(F_t / X_t\right) + \sigma_t^2 T / 2}{\sigma_t \sqrt{T}}$$

and

$$d_{2,t} = \frac{\ln\left(F_t / X_t\right) - \sigma_t^2 T / 2}{\sigma_t \sqrt{T}} = d_{1,t} - \sigma_t \sqrt{T}$$

C_t^F	=	Call option price for FBS model in domestic currency at time t.
P_t^F	=	Put option price for FBS model in domestic currency at time t
F_t	=	Futures price at time t

For notational convenience, let $\xi_t = e^{-R_t^f}$ and, $\eta_t = e^{-R_t^d}$ so that equations (1), (2), (4) and (5) can, respectively, be written as follows:

$$C_t^S = S_t \xi_t N\left[d_{1,t}\left(\sigma_t\right)\right] - X_t \eta_t N\left[d_{2,t}\left(\sigma_t\right)\right] \tag{6}$$

$$P_t^S = X_t \eta_t N\left[-d_{2,t}\left(\sigma_t\right)\right] - S_t \xi_t N\left[-d_{1,t}\left(\sigma_t\right)\right] \tag{7}$$

$$C_t^F = \eta_t\left[F_t N\left\{d_{1,t}\left(\sigma_t\right)\right\} - X_t N\left\{d_{2,t}\left(\sigma_t\right)\right\}\right] \tag{8}$$

$$P_t^F = \eta_t\left[X_t N\left\{-d_{2,t}\left(\sigma_t\right)\right\} - F_t N\left\{-d_{1,t}\left(\sigma_t\right)\right\}\right] \tag{9}$$

Since the volatility of underlying exchange rate (σ_t) in equations (6) to (9) is not directly observable, IVM, GVM and RVM are employed to estimate and forecast the exchange rate volatility for next trading day(σ_{t+1}).

2.1. Empirical Implementation of the Models

For simplicity, IVM is implemented using volatility measures from Datastream, given by the financial system software developed by MB Risk Management (MBRM, the developers of the widely-used UNIVERSAL Add-ins®). Following Hull's (2005) suggestion that, for a given strike price and maturity, the correct volatility to use in the BS model to price a European call should always be the same as that used to price a European put, we use a simple average of IV for call $\left(\hat{\sigma}^{IV}_{c,t}\right)$ and put $\left(\hat{\sigma}^{IV}_{p,t}\right)$ obtained from Datastream as

$$\hat{\sigma}^{IV}_{t} = \frac{\hat{\sigma}^{IV}_{c,t} + \hat{\sigma}^{IV}_{p,t}}{2} \tag{10}$$

For GVM, this study applies the GARCH (1,1) process to estimate volatility of the underlying exchange rate. If the return $\left(r_t\right)$ of the exchange rate $\left(S_t\right)$ is $\ln\left(S_t / S_{t-1}\right)$ for day t, then the autoregressive model of order 1 with normal-GARCH (1,1) error is

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t, \tag{11}$$

$$\varepsilon_t = k_t \sqrt{h_t}, \qquad k_t \approx iid\,(0,1)$$

$$h_t = \omega + \beta_1 \varepsilon^2_{t-1} + \gamma_1 h_{t-1} \tag{12}$$

Since h_t is the one-period ahead forecast variance based on the past information in equation (12), it is called conditional variance. As can be seen, the conditional variance is a function of a constant term, ω, news about volatility from the previous period, measured as the lag of the squared residual from equation (11), ε_{t-1}. (the ARCH term), and last period's conditional variance: h_{t-1}. (the GARCH term). The parameters in equations (11) and (12) are estimated by QMLE. Structural and statistical properties of the QMLE for GARCH (p,q) model can be found in Ling and McAleer (2003).

Given this specification, equation (12) is the standard deviation of h_t that estimates GARCH (1,1)-based volatility (GV) per trading day,

$$\hat{\sigma}^{GV}_{t} = \sqrt{h_t} \tag{13}$$

The days when the exchange is closed are ignored and the GV per annum is calculated as

$$\hat{\sigma}^{GV}_{t} = \sqrt{D h_t} \tag{14}$$

where, D is 252 trading days per year (as considered standard for option markets).

The RV is constructed by summing the squared intra-day returns sampled at a particular frequency. The optimal frequency for constructing RV is unknown. Following the standard practice, daily RV series is constructed using 5 minutes sampling frequency. If S_i is the exchange rate for 5 minutes sampling frequency, the underlying exchange rate return in 5 minutes interval is estimated as

$$r_{t,i} = \ln\left(\frac{S_i}{S_{i-1}}\right) \tag{15}$$

where $r_{t,i}$ is the return in interval i on day t. The realized variance of day t is computed as

$$v_t = \sum_{i=1}^{n} r^2_{t,i} \tag{16}$$

where n is the total number of intervals for an option trading day from 7:30 AM to 2:30 PM, Monday to Friday. Since RV is the standard deviation of the realized variance, the RV per trading day is

$$\hat{\sigma}^{RV}_{t} = \sqrt{v_t} \tag{17}$$

As intra-day data of trading days are used to provide RV estimate, days when the exchange is closed are ignored and the RV per annum is

$$\hat{\sigma}^{RV}_{t} = \sqrt{D v_t} \tag{18}$$

where, D is 252 trading days per year.

Since IV and RV are time-varying, the process ARMA (2,1) is introduced following Pong et al. (2004) to set up a time series model in capturing dynamics of IV or RV,

$$v_t^i = \gamma_0 + \varphi_1 v_{t-1}^i + \varphi_2 v_{t-2}^i + \theta_1 \varepsilon_{t-1} + \varepsilon_t , \quad (19)$$

for $\forall_i =$ IV and RV. The parameters in equation (19) are estimated by Quasi Maximum Likelihood Estimator (QMLE).

2.2. Generating Model Price

The procedures for generating the implied volatility model price (IVP), realized volatility model price (RVP) and GARCH (1,1)-based volatility model price (GVP) in the SBS and FBS models for in-sample and out-of-sample data are as follows. For in-sample, the whole sample (1022 observations for each currency) is used to obtain the measures of IV, GV and RV by equations (10), (14) [based on equation (12)] and (18), respectively. These volatility measures are used as inputs in SBS model (equations 6 and 7 for call and put options, respectively) and generate spot price version of implied volatility model price (SIVP), spot price version of realized volatility model price (SRVP) and spot price version of GARCH (1,1)-based volatility model price (SGVP):

$$\hat{\Pi}_{c,t+1}^m = S_{t+1}\xi_{t+1} N\left[d_{1,t}\left(\hat{\sigma}_{t+1}^v\right)\right] - X_{t+1}\eta_{t+1} N\left[d_{2,t}\left(\hat{\sigma}_{t+1}^v\right)\right] \quad (20)$$

$$\hat{\Pi}_{p,t+1}^m = X_{t+1}\eta_{t+1} N\left[-d_{2,t}\left(\hat{\sigma}_{t+1}^v\right)\right] - S_{t+1}\xi_{t+1} N\left[-d_{1,t}\left(\hat{\sigma}_{t+1}^v\right)\right] \quad (21)$$

for $\forall_v =$ IV,RV and GV; $\forall_m =$ SIVP, SRVP and SGVP.

Similarly, for FBS model (equations 8 and 9 for call and put options, respectively), we have

$$\hat{\Pi}_{c,t+1}^m = \eta_{t+1}\left[F_{t+1} N\left\{d_{1,t}\left(\hat{\sigma}_{t+1}^v\right)\right\} - X_{t+1} N\left\{d_{2,t}\left(\hat{\sigma}_{t+1}^v\right)\right\}\right] \quad (22)$$

$$\hat{\Pi}_{p,t+1}^m = \eta_{t+1}\left[X_{t+1} N\left\{-d_{2,t}\left(\hat{\sigma}_{t+1}^v\right)\right\} - F_{t+1} N\left\{-d_{1,t}\left(\hat{\sigma}_{t+1}^v\right)\right\}\right] \quad (23)$$

for $\forall_v =$ IV,RV, $\forall_m =$ FIVP,FRVP and FGVP.

For out-of-sample, the above approach is used for the first two-thirds of the whole sample (681 observations for each currency) to generate the forecast measures for the remaining one-third of the whole sample (341 observations for each currency).

2.3. Measuring the Performance of the Models

Standard statistical accuracy criteria, such as, mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) are used to evaluate the performance of the models, both in-sample and out-of-sample. Pricing errors for each model are computed as difference between the observed at-the-money (ATM) option price and the model-predicted price (estimated as described above). Thus there are six sets of errors, namely, the spot price version implied volatility model pricing error (SIVPE), spot price version realized volatility model pricing error (SRVPE), spot price version GARCH (1,1)-based volatility model pricing error (SGVPE), futures price version implied volatility model pricing error (FIVPE), futures price version realized volatility model pricing error (FRVPE) and futures price version GARCH(1,1)-based volatility model pricing error (FGVPE), for evaluation. If $\Pi_{j,t}^{ATM}$ and $\hat{\Pi}_{j,t}^m$ represent the ATM options market price and estimated options model price, respectively, the SIVPE, SGVPE, SRVPE, FIVPE, FGVPE and FRVPE are estimated for n number of observations under MSE, MAE and MAPE measures as follows, respectively,

$$MSE^k = \frac{1}{n}\sum_{t=1}^{n}\left(\Pi_{j,t}^{ATM} - \hat{\Pi}_{j,t}^m\right)^2 \quad (24)$$

$$MAE^k = \frac{1}{n}\sum_{t=1}^{n}\left[\Pi_{j,t}^{ATM} - \hat{\Pi}_{j,t}^m\right] \quad (25)$$

$$MAPE^k = \frac{1}{n}\sum_{t=1}^{n}\left[\frac{\hat{\Pi}_{j,t}^{m} - \Pi_{j,t}^{ATM}}{\Pi_{j,t}^{ATM}}\right] \quad (26)$$

for, $\forall_{\mathrm{j}} = \mathrm{C}$ and P; $\forall_{\mathrm{m}} =$ SIVP, SRVP, SGVP , FIVP, FRVP and FGVP; $\forall_{\mathrm{k}} =$ SIVPE, SRVPE, SGVPE , FIVPE, FRVPE and FGVPE,

As a further check, an adjusted version (as suggested by Harvey et al.,1997) of the test proposed by Diebold and Mariano (1995) is employed for adequacy of the results.

3. The Data and In-sample Results

3.1. The Data

The data for three major European currency options, namely, British pound, Swiss franc and Euro traded in Philadelphia Stock Exchange from 22 July 2002 to 30 June 2006 are obtained from Datastream. The options are written for 3 months and traded on Mondays through Fridays excluding public holidays from 7:30 AM to 2:30 PM. The data consist of daily (i) ATM options prices, (ii) ATM strike prices, (iii) closing spot exchange rates (all against U.S. dollar), (iv) Eurocurrency (British pound, Swiss franc, Euro) and domestic currency (U.S. dollar) interest rates, (v) ATM implied volatility of underlying exchange rates for call and put options, and (vi) futures prices on British pound, Swiss franc and Euro traded in Chicago Mercantile Exchange. These data are obtained from Datastream, and contained in a separate appendix, available on request..

In order to construct realized volatility with 5-min frequency, Reuter's intra-day exchange rate quotations for the sample currencies against the U.S. dollar are extracted from SIRCA database, using Microsoft Structured Query Language (SQL). Table 1 gives a summary of intra-daily exchange rate data.

Table 1
Intra-Daily Exchange Rate Data

Exchange rates against U.S. dollar	Sample period: 22/07/2002 - 30/06/2006; Trading hours: 0730 – 1430; Trading days: 1022				
	Total number of quotations for sample period	Average number of quotations per day	Average number of quotations per 5-min	Total number of quotations at 5-min interval for sample period	Average number of quotations at 5-min interval per day
(1)	(2)	(3)	(4)	(5)	(6)
British pound	1,723,712	1,687	20	64,700	63
Swiss franc	1,381,667	1,352	16	86,161	84
Euro	1,482,584	1,451	17	86,276	84

Notes: Average number of quotation per day (column 3) = total number of quotations for sample period (column 2) ÷ number of trading days (1022); Average number of quotations per 5-min (column 4) = Average number of quotation per day (column 3) ÷ total number of intervals (84) at 5-min frequency per trading day; Average number of quotations at 5-min interval per day (column 6) = Total number of quotations at 5-min interval for sample period (column 5)÷ number of trading days (1022).

3.2. In-sample Results

The in-sample results consist of comparison of MSE, MAE and MAPE measures for the competing models. The estimation results of GARCH (1,1) using equation (12) for whole sample of daily closing exchange rate (1022 observations for each currency) are presented in Table 2. As can be seen γ_1 is not significant for Swiss franc and Euro, but the sum of coefficients β_1 and γ_1 is less than 1 in all cases. SGVPE, SIVPE and SRVPE are calculated in the manner described in Section 2 and the results are given Table 3. For ease of comparisons, their respective differences (in percent) are calculated and presented in the last two columns of this table. Under MSE, for British pound call option, SGVPE, SIVPE, and SRVPE are 6.27E-5, 6.07E-5, and 4.81E-5, respectively. The difference between SIVPE and SGVPE (in the second last column), and SRVPE and SIVPE (in the last column) are 3.19 and 20.76 percent, respectively. The negative (positive) differences in the second-last column indicate that SIVPE is less (more) than SGVPE by the reported percentage points. Note that the differences between SIVPE and SGVPE are, on average, higher for Swiss franc than for the other two currencies in all cases. Based on the results of the second-last column, it can be seen that there is a tendency for IVM to outperform GVM, but it is not clear-cut. In the last column of the Table 3, all the numbers are negative. This indicates that RVM performs relatively better than IVM and GVM in describing underlying foreign exchange return behavior for pricing options more accurately.

Table 2
GARCH (1, 1) Estimation Results: Full Sample

Currency	Coefficients		
	ω	β_1	γ_1
British pound	7.67E-7 (1.4269)	0.0267 (2.2121)	0.9471 (36.1044)
Swiss franc	7.03E-5 (5.2063)	0.0511 (4.2929)	-0.0603 (0.2997)
Euro	2.79E-5 (1.2228)	-0.0375 (1.5948)	0.2764 (0.4475)

Notes: Equation (12) is used for whole sample of daily closing exchange rates (1022 observations), with t-ratios in the parenthesis.

Table 3
Comparison of Pricing Errors (In-sample): SBS Model

Measures	Currency	Options	Pricing errors				
			SGVPE	SIVPE	SRVPE	$\frac{\text{SIVPE}-\text{SGVPE}}{\text{SGVPE}}$ (%)	$\frac{\text{SRVPE}-\text{SIVPE}}{\text{SIVPE}}$ (%)
MSE	British pound	Call	6.27E-5	6.07E-5	4.81E-5	-3.19	-20.76
		Put	1.04E-4	1.05E-4	6.90E-5	0.96	-34.29
	Swiss franc	Call	7.00E-5	4.11E-5	2.31E-5	-41.28	-43.80
		Put	4.69E-5	2.48E-5	1.83E-5	-47.12	-26.21
	Euro	Call	5.05E-5	5.19E-5	3.36E-5	2.77	-35.26
		Put	5.65E-5	6.23E-5	4.57E-5	10.26	-26.65
MAE	British pound	Call	0.0063	0.0063	0.0056	0.00	-11.11
		Put	0.0082	0.0082	0.0065	0.00	-20.73
	Swiss franc	Call	0.0069	0.0050	0.0038	-27.53	-24.00
		Put	0.0057	0.0038	0.0033	-33.33	-13.16
	Euro	Call	0.0057	0.0057	0.0046	0.00	-19.30
		Put	0.0056	0.0057	0.0049	1.78	-14.04
MAPE	British pound	Call	0.4566	0.4540	0.3651	-0.56	-19.58
		Put	0.4912	0.4872	0.3647	-0.81	-25.14
	Swiss franc	Call	0.7802	0.5685	0.4039	-27.13	-28.95
		Put	0.9433	0.8217	0.4849	-12.89	-40.99
	Euro	Call	0.4857	0.4908	0.3609	1.05	-26.47
		Put	0.5058	0.5087	0.3960	0.57	-22.15

Notes: SGVPE, SIVPE and SRVPE represent the spot price version GARCH (1,1)-based volatility, implied volatility and realized volatility model pricing error, respectively. In the second last column, the negative (positive) differences indicate that SIVPE is less (more) than SGVPE by reported percentage. In the last column, the negative differences indicate that SRVPE is less than SIVPE by reported percentage

Table 4 is the futures price version of Table 3. The values of FGVPE, FIVPE, FRVPE, and their respective differences indicate that RV is clearly the better performer relative to IV and GV. These results are consistent with those in Table 3.

Table 4
Comparison of Pricing Errors (In-sample): FBS Model

Measures	Currency	Options	Pricing errors				
			FGVPE	FIVPE	FRVPE	(FIVPE – FGVPE) / FGVPE (%)	(FRVPE – FIVPE) / FIVPE (%)
MSE	British pound	Call	6.92E-5	6.71E-5	5.78E-5	-3.03	-13.86
		Put	9.61E-5	9.69E-5	5.27E-5	0.83	-45.61
	Swiss franc	Call	6.67E-5	3.74E-5	1.84E-5	-43.92	-50.80
		Put	5.46E-5	3.24E-5	2.59E-5	-40.70	-20.06
	Euro	Call	4.85E-5	4.93E-5	3.28E-5	1.65	-33.47
		Put	5.85E-5	6.50E-5	4.51E-5	11.11	-30.62
MAE	British pound	Call	0.0065	0.0065	0.0059	0.00	-9.23
		Put	0.0078	0.0078	0.0059	0.00	-24.36
	Swiss franc	Call	0.0066	0.0047	0.0034	-28.79	-27.66
		Put	0.0060	0.0041	0.0035	-31.67	-14.63
	Euro	Call	0.0055	0.0056	0.0046	1.82	-17.86
		Put	0.0057	0.0059	0.0048	3.51	-18.64
MAPE	British pound	Call	0.4652	0.4643	0.3834	-0.19	-17.42
		Put	0.4752	0.4706	0.3382	-0.97	-28.13
	Swiss franc	Call	0.7526	0.5392	0.3743	-28.36	-30.58
		Put	0.9738	0.8518	0.5071	-12.53	-40.47
	Euro	Call	0.4697	0.4755	0.3609	1.23	-24.10
		Put	0.5190	0.5227	0.3956	0.71	-24.32

Notes: FGVPE, FIVPE and FRVPE represent the futures price version GARCH (1,1)-based volatility, implied volatility and realized volatility model pricing error, respectively. In the second last column, the negative (positive) differences indicate that FIVPE is less (more) than FGVPE by reported percentage. In the last column, the negative differences indicate that FRVPE is less than FIVPE by percentage.

Table 5
Diebold-Mariano Equality Test (In-sample)

Models	Comparison of model pricing error	British pound		Swiss franc		Euro	
		Call	Put	Call	Put	Call	Put
IVM and GVM	SIVPE – SGVPE	-0.36	-0.19	-19.36*	-19.73*	1.49	2.30
	FIVPE – FGVPE	-0.03	-0.18	-19.87*	-19.78*	1.46	2.57
RVM and IVM	SRVPE – SIVPE	-4.24*	-7.99*	-10.39*	-4.81*	-7.22*	5.40*
	FRVPE – FIVPE	-3.82*	-9.08*	-10.53*	-5.59*	-6.37*	-6.59*
RVM and GVM	SRVPE – SGVPE	-4.38*	-8.39*	-22.85*	-16.12*	-12.29*	-4.87*
	FRVPE – FGVPE	-3.83*	-9.52*	-23.72*	-16.79*	-11.27*	-6.03*

Notes: IVM, GVM and RVM represent implied volatility model, GARCH (1,1)-based volatility model and realized volatility model, respectively. The test statistic follows a t-distribution with (n-1) degrees of freedom. * denotes 1% level of significance.

Finally, in Table 5, Diebold and Mariano adjusted test statistics indicate that SIVPE and FIVPE are statistically different from SGVPE and FGVPE at 1 percent level of significance for Swiss franc calls and puts, respectively. The negative signs indicate that SIVPE and FIVPE are less than SGVPE and FGVPE, respectively, for the Swiss franc. However, the results for British pound and Euro are not significant. SRVPE and FRVPE are statistically different and less than SIVPE and FIVPE, respectively, at 1 percent level of significance for all currencies. Finally, SRVPE and FRVPE are statistically different and less than SGVPE and FGVPE, respectively, at 1 percent level of significance for all currencies. Overall, the Diebold-Mariano test results in Table 5 are consistent with the results reported in Tables 3 and 4.

To sum up, the in-sample tests provide mixed results for comparison of IVM and GVM. Based on these results, the ability of IVM and GVM are not distinguishable to describe the underlying exchange rate return behavior that leads to pricing options more accurately. However, the tests provide a clear-cut picture for comparison of RVM and IVM, and RVM and GVM. The overall results suggest that the RVM performs relatively better than IVM and GVM in capturing the underlying exchange rate return behavior that leads to pricing options with higher accuracy.

4. Out-of-sample Results

For out-of-sample, IV and RV series are obtained using equations (10) and (18), respectively, for the first two-thirds of the whole sample (681 observations for each currency), and equation (19) to forecast for the remaining one-third of the whole sample (341 observations for each currency). Similarly, for GARCH (1,1), equation (12) is estimated using the first two-thirds of the whole sample to forecast, and then equation (14) is used to obtain GRACH variance series for the remaining one-third of the whole sample.

The estimation results of GARCH (1,1) using equation (12) for first two-third of the sample of daily closing exchange rate are presented in Table 6. As can be seen that the sum of coefficients β_1 and γ_1 is less than 1, indicating the validity of forecasting GARCH (1,1)-based volatility (FGV) for the remaining one-third of the sample.

Table 6
GARCH (1, 1) Estimation: First 681 Observations

Currency	Coefficients		
	ω	β_1	γ_1
British pound	6.95E-5 (1.4706)	0.0352 (2.3979)	0.9411 (36.8945)
Swiss franc	3.35E-5 (0.6839)	-0.0024 (0.2863)	0.9394 (10.3721)
Euro	3.82E-5 (1.4354)	-0.0511 (1.9130)	0.0842 (0.1258)

Notes: The estimations of GARCH (1,1) by equation (12) for two-third of the sample of daily closing exchange rate (681 observations for each currency) are given in Table 6. The coefficients of equation (12) with t-ratios in the parenthesis are presented.

Table 7
ARMA (2, 1) Estimation: First 681 Observations for Implied Volatility

Currency	Coefficients			
	γ_0	φ_1	φ_2	θ_1
British pound	0.0911 (49.1378)	0.8848 (6.3326)	0.0077 (0.0638)	-0.0786 (0.5446)
Swiss franc	0.1194 (24.4974)	1.5427 (13.7268)	-0.5665 (5.7560)	-0.8105 (8.4974)
Euro	0.1042 (39.8702)	1.0253 (12.1897)	-0.0707 (0.9242)	-0.4324 (5.2917)

Notes: The estimations of ARMA (2,1) by equation (19) for first two-third of the sample (681 observations for each currency) of implied volatility (IV) time series are given in Table 7. The coefficients of equation (19) with t-ratios in the parenthesis are presented.

The estimation results of ARMA(2,1) by equation (19) for first two-third of the IV time series are presented in Table 7. As can be seen, the sum of coefficients φ_1 and φ_2 is less than 1, indicating the time series is stationary. This is consistent with the results from the ADF tests so that IV is stationary and has a predicable component.

Similarly, the estimation results of ARMA(2,1) by equation (19) for first two-third of the RV time series are presented in Table 8. As can be seen, the sum of coefficients φ1 and φ2 is less than 1, indicating that this series is stationary. This is consistent with the results from the ADF tests and hence RV is stationary and has a predicable component.

Table 8
ARMA (2, 1) Estimation
First 681 Observations for Realized Volatility

Currency	Coefficients			
	γ_0	φ_1	φ_2	θ_1
British pound	0.0585 (20.1574)	1.0279 (19.8932)	-0.0548 (1.2267)	-0.8811 (26.3917)
Swiss franc	0.0711 (26.2564)	0.9493 (17.4389)	0.0102 (0.2301)	-0.8620 (22.4347)
Euro	0.0647 (31.3441)	0.9182 (14.6194)	0.0190 (0.4157)	-0.8207 (16.4365)

Notes: The estimations of ARMA (2,1) by equation (19) for first two-third of the sample (681 observations for each currency) of realized volatility (RV) are given in Table 8. The coefficients of equation (19) with t-ratios in the parenthesis are presented.

Table 9
Comparison of Pricing Errors (Out-of-sample): SBS Model

Measures	Currency	Options	Different model pricing errors (SGVPE, SIVPE, SRVPE) and their differences (in the last two columns)				
			SGVPE	SIVPE	SRVPE	(SIVPE − SGVPE) / SGVPE (%)	(SRVPE − SIVPE) / SIVPE (%)
MSE	British pound	Call	9.54E-5	7.73E-5	3.63E-5	-18.97	-53.04
		Put	9.31E-5	7.62E-5	3.90E-5	-18.15	-48.82
	Swiss franc	Call	6.30E-5	4.06E-5	2.00E-5	-35.56	-50.74
		Put	2.68E-5	1.28E-5	7.90E-6	-52.24	-38.28
	Euro	Call	8.77E-5	6.18E-5	2.89E-5	-29.53	-53.24
		Put	5.04E-5	3.15E-5	1.66E-5	-37.50	-47.30
MAE	British pound	Call	0.0081	0.0073	0.0049	-9.88	-32.88
		Put	0.0080	0.0071	0.0051	-11.25	-28.17
	Swiss franc	Call	0.0067	0.0052	0.0036	-22.39	-30.77
		Put	0.0045	0.0030	0.0022	-33.33	-26.67
	Euro	Call	0.0080	0.0065	0.0044	-18.75	-32.31
		Put	0.0061	0.0046	0.0032	-24.59	-30.43
MAPE	British pound	Call	0.6038	0.5421	0.3380	-10.22	-37.65
		Put	0.5398	0.4837	0.3274	-10.39	-32.31
	Swiss franc	Call	0.7823	0.6026	0.3967	-22.97	-34.17
		Put	0.6638	0.4324	0.2830	-34.86	-34.55
	Euro	Call	0.6824	0.5520	0.3567	-19.11	-35.38
		Put	0.6223	0.4725	0.2937	-24.07	-37.84

Notes: SGVPE, SIVPE and SRVPE represent the spot price version GARCH (1,1)-based volatility, implied volatility and realized volatility model pricing error, respectively. In the second last column, the negative differences indicate that SIVPE is less than SGVPE by reported percentage. In the last column, the negative differences indicate that SRVPE is less than SIVPE by reported percentage points.

Table 9 is the out-of-sample version of Table 4. It can be seen (from the last two columns of the table) that all the numbers are negative, implying RVM outperforms IVM as well as GVM in forecasting volatility for pricing options accurately. Similar results hold in Table 10, which is the futures price version of Table 9. As can be seen from the results in Table 11, these results pass the adjusted Diebold-Mariano tests.

Table 10
Comparison Pricing Errors (Out-of-sample): FBS Model

Measures	Currency	Options	Different model pricing errors (FGVPE, FIVPE, FRVPE) and their differences (in the last two columns)				
			FGVPE	FIVPE	FRVPE	(FIVPE – FGVPE) / FGVPE (%)	(FRVPE – FIVPE) / FIVPE (%)
MSE	British pound	Call	9.00E-5	7.27E-5	3.74E-5	-19.22	-48.56
		Put	9.97E-5	8.20E-5	3.81E-5	-17.75	-53.54
	Swiss franc	Call	5.65E-5	3.34E-5	1.07E-5	-40.88	-67.96
		Put	4.62E-5	3.25E-5	2.76E-5	-29.65	-15.08
	Euro	Call	8.07E-5	5.42E-5	1.99E-5	-32.84	-63.28
		Put	5.62E-5	3.76E-5	2.21E-5	-33.10	-41.22
MAE	British pound	Call	0.0079	0.0071	0.0049	-10.13	-30.98
		Put	0.0083	0.0075	0.0051	-9.64	-32.00
	Swiss franc	Call	0.0061	0.0045	0.0029	-26.23	-35.56
		Put	0.0051	0.0036	0.0027	-29.41	-25.00
	Euro	Call	0.0074	0.0059	0.0039	-20.27	-33.89
		Put	0.0065	0.0051	0.0035	-21.54	-31.37
MAPE	British pound	Call	0.5797	0.5194	0.3350	-10.40	-35.50
		Put	0.5592	0.5025	0.3277	-10.14	-34.79
	Swiss franc	Call	0.7251	0.5397	0.3304	-25.57	-38.78
		Put	0.7343	0.5070	0.3475	-30.95	-31.46
	Euro	Call	0.6415	0.5107	0.3188	-20.39	-37.58
		Put	0.6562	0.5082	0.3197	-22.55	-37.09

Notes: FGVPE, FIVPE and FRVPE represent the futures price version GARCH (1,1)-based volatility, implied volatility and realized volatility model pricing error, respectively. In the second last column, the negative differences indicate that FIVPE is less than FGVPE by reported percentage. In the last column, the negative differences indicate that FRVPE is less than FIVPE by reported percentage points.

Table 11
Diebold-Mariano Equality Test (Out of Sample)

Models	Comparison of model pricing error	British pound		Swiss franc		Euro	
		Call	Put	Call	Put	Call	Put
IVM and GVM	SIVPE – SGVPE	-10.14*	-10.13*	-21.52*	-23.70*	-15.86*	-15.76*
	FIVPE – FGVPE	-9.92*	-10.36*	-24.78*	-22.95*	-16.08*	-15.25*
RVM and IVM	SRVPE – SIVPE	-9.40*	-7.57*	-8.88*	-5.00*	-8.77*	-6.62*
	FRVPE – FIVPE	-8.34*	-8.86*	-9.45*	-5.70*	-8.73*	-7.26*
RVM and GVM	SRVPE – SGVPE	-10.89*	-9.23*	-13.52*	-11.50*	12.64*	-10.84*
	FRVPE – FGVPE	-9.85*	10.46*	-14.65*	-12.04	12.71*	-11.26*

Notes: IVM, GVM and RVM represent implied volatility model, GARCH (1,1)-based volatility model and realized volatility model, respectively. The test statistic follows a t-distribution with (n-1) degrees of freedom. * denotes 1% level of significance.

To sum up, the in-sample tests based on SBS and FBS models provide mixed results on the relative performance of IVM and GVM. However, out-of-sample tests for SBS and FBS models indicate that IVM performs relatively better than GVM in forecasting underlying exchange volatility for pricing options. This result is consistent with Harikumar and Boyrie (2004). Further, the overall results suggest that RVM outperforms both IVM and GVM in both in-sample and out-of-sample tests to describe the underlying exchange rate return behavior for pricing options.

5. Conclusion

This paper has provided a systematic analysis of the alternative volatility specifications in pricing foreign currency options. Using data for options on British pound, Swiss franc and Euro, the analysis pays particular attention to the highly persistent nature of the exchange rate volatility process. The results indicate that the realized volatility measures outperform the implied, and GARCH (1,1)-based measures, both in-sample and out-of-sample. This result extends ABDL (2003) to currency option pricing and is consistent with Pong et al. (2004). Since our methods are simple to implement empirically, this finding will have useful implications for business and regulators. Because the realized volatility measures used in this study do not treat the variances originating from continuous price movements as different from those originating from jumps, as discussed in ABDL (2003), the dynamic impact may differ across the two sources of variability. An interesting direction for future research is to explicitly account for non-linear features in the realized volatility and determine if this improves the volatility forecasts for currency options.

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References

Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P., 2003, Modeling and forecasting realized volatility. *Econometrica* 71, 579-625.

Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys. P., 2001, The distribution of realized exchange rate volatility. *Journal of American Statistical Association* 96,42-55.

Barndorff-Nielsen, O. E., and Shephard, N., 2004, Econometric analysis of realized covariation: high frequency based covariance, regression, and correlation in finance economics. *Econometrica* 72, 885-925.

Barndorff-Nielsen, O. E., and Shephard, N., 2002, Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of Royal Statistical Society Series B* 64, 253-280.

Baxter, M., and Rennie, A., 1996, *Financial calculus: an introduction to derivative* (Cambridge University Press).

Beckers, S., 1981, Standard deviations implied in option prices as predictors of future stock price variability. *Journal of Banking and Finance* 5, 363-381.

Black, F., and Scholes, M., 1973, The pricing of options and corporate liabilities. *Journal of Political Economy* 83, 637-705.

Bollerslev, T., 1986, Generalised Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31, 307-334.

Bollerslev, T., Chou R. Y., and Kroner, K. F., 1992, ARCH modeling in finance: a review of the theory and empirical evidence. *Journal of Econometrics* 52, 5-59.

Canina, L., and Figlewski, S., 1993, The information content of implied volatility. *Review of Financial Studies* 6, 659-681.

Chiras, D., and Manaster, S., 1978, The information content of options prices and a test of market efficiency. *Journal of Financial Economics* 213-234.

Christensen, B. J., and Prabhala, N. R., 1998, The relation between implied and realized volatility. *Journal of Financial Economics* 50, 125-150.

Day, T. E., and Lewis, C.M., 1992, Stock market volatility and the information content of stock index options. *Journal of Econometrics* 52, 267-287.

Diebold, F. X., and Mariano, R. S., 1995, Comparing predictive accuracy. *Journal of Business and Economic Statistics* 13, 253-263.

Duan, J. C., 1995, The GARCH option pricing model. *Mathematical Finance* 5, 13-32.

Duan, J. C., and Wei, J. Z., 1999, Pricing foreign currency and cross-currency options under GARCH. *Journal of Derivatives* Fall, 51-63.

Engle, R., 1982, Autoregressive conditional heteroskedasticity with Estimates of the variance of UK inflation. *Econometrica* 50, 987-1008.

Engle, R. F., and Ng, V. K 1993, Measuring and testing the impact of news on volatility. *Journal of Finance* 48, 1749-1778.

Fama, E., 1965, The behavior of stock market prices. *Journal of Business* 38, 34-105.

Glosten, L. R, Jagannathan, R., and Runkle, D. E., 1993, On the relationship between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 48, 1779-1801.

Gospodinov, N., Gavala, A., and Jiang, D., 2006, Forecasting volatility. *Journal of Forecasting* 25, 381-400.

Harikumar, T., and Boyrie, M. E. D., 2004, Evaluation of Black-Scholes and GARCH models using currency call options data. *Review of Quantitative and Accounting* 23, 299-312.

Hull, J. C., 2005, *Volatility smiles in Fundamentals of Futures and Options Markets* (Pearson Education Australia PTY, Limited).

Jorion, P., 1995, Predicting volatility in the foreign exchange market. *Journal of Finance* 50, 507-528.

Latane, H. and Rendleman, R., 1976, Standard deviation of stock prices ratios implied by option premia. *Journal of Finance* 369-382.

Ling, S. and McAleer, M., 2003, Asymptotic theory for a vector ARMA-GARCH model. *Econometric Theory* 19, 280-310.

Maheu, J. M., and McCurdy, T. H., 2002, Nonlinear features of realized FX volatility. *The Review of Economics and Statistics* 84, 668-681.

Merton, R. C. , 1973, Theory of rational option pricing. *The Bell Journal of Economics and Management Science* 4, 141-183.

Nelson, D. B., 1991, Conditional heteroskedasticity in asset pricing: a new approach. *Econometrica* 59, 347-370.

Pong, S., Shackleton, M. B., Taylor, S. J., and Xu, X., 2004, Forecasting currency volatility: a comparison of implied volatility and AR(FI)MA models. *Journal of Banking and Finance* 28, 2541-2563.

Ritchken, P., and Trevor, R, 1999, Pricing options under generalised GARCH and stochastic volatility processes. *Journal of Finance* 54, 377-402.

Schmalensee, R., and Trippi, R. R., 1978, Common stock volatility expectations implied by option premia. *Journal of Finance* 129-147.