Fluid substitution, dispersion, and attenuation in fractured and porous reservoirs—insights from new rock physics models

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The importance of natural fractures for development and production of hydrocarbon reservoirs requires little justification. While in clastic reservoirs fractures can cause permeability anisotropy and thus affect field development, in carbonates and tight sands they are often critical for reservoir production. If open fractures have a preferential direction (which is almost always the case), they cause azimuthal seismic anisotropy, making seismic a powerful tool for the characterization of fractured reservoirs.

For the purposes of seismic characterization, fractures are distinguished from other void space (such as pores, caverns, vugs) by the fact that their effect on the elastic properties is disproportional to their relatively small total volume, or fracture porosity ($\phi_f \ll 1$). From this definition it is clear that reservoirs whose entire void space consists solely (or primarily) of fractures are likely to be of little interest, since they would not have enough volume of voids to hold significant quantities of hydrocarbons. Reservoirs of greatest interest contain voids of two types: stiff voids such as pores or vugs (so called equant porosity) which contain most of the fluids, and fractures whose overall volume (fracture porosity) is small (often less than 0.1%).

In reservoirs with equant porosity, the effect of fractures depends on the hydraulic connectivity between fractures and pores. Furthermore, the same connectivity also affects the seismic response of such reservoirs. Indeed, when a liquid-filled fracture is hydraulically isolated, its effect on the axial compliance of the rock in the direction normal to the fracture plane is minimal, as the liquid effectively seals the fracture. But when the fracture is connected to a network of stiff pores having much larger volume, the liquid can escape into these pores making the fracture much more compliant. Thus wave-induced fluid flow between pores and fractures has a first-order effect on the seismic response of fractured reservoirs and opens hitherto largely unexplored potential for characterization not only of fractures as such but their connectivity with the rest of the void space (pores, caverns, vugs).

In this paper, we show how elastic properties of a fractured rock with equant porosity can be modeled on the consistent basis of the Biot-Gassmann theory of poroelasticity. For simplicity and clarity, we limit the analysis to a single system of aligned and rotationally symmetric fractures in an otherwise isotropic porous rock. We first discuss the low-frequency limit where the analysis is quite general and independent of both size and shape of fractures and pores. We then proceed to the more challenging problem of modeling

Figure 1. Fractured and porous reservoir rock: (a) cartoon representation, (b) model of planar fractures, (c) model of finite penny-shaped cracks (pattern courtesy of M. Schoenberg).

frequency-dependent effects, notably attenuation, velocity dispersion and frequency-dependent anisotropy. In contrast to the quasi-static case, these effects depend critically on the geometry of the fracture system. Having in mind a general picture of a fractured reservoir with equant porosity as shown in Figure 1a, we present the results for two simplified geometrical configurations, which may be regarded as its representative end members. In the first configuration (shown schematically in Figure 1b), fractures are modeled as thin highly porous layers of infinite extent in a low- or medium-porosity background. In the second configuration, the fractures are modeled as sparsely distributed thin penny-shaped cracks of a given (finite) diameter which is much larger than the pore size (Figure 1c). The latter assumption of fractures larger than pores is consistent with the geologic terminology: All reservoirs contain microcracks comparable to, or smaller than, the pores, but these reservoirs are not considered as fractured reservoirs by geologists or engineers.

Background. Studies of the effect of natural fractures on elastic properties of the reservoir have a long history because of their importance for optimal reservoir development. Most studies performed in the 1980s and 1990s centered on seismic anisotropy caused by the presence of fully or partially aligned fractures and the resultant effects on the properties of an otherwise isotropic elastic solid material. In the simplest case of fully aligned, hydraulically isolated and rotationally symmetric fractures, their effect can be quantified using two parameters, normal and tangential excess fracture compliances Z_N and Z_T . When fractures are dry or filled with gas, Z_N and Z_T are of the same order of magnitude. When they are filled with a liquid (say, brine or oil), $Z_N \ll Z_T$, which means that in this case P-wave velocities parallel and perpendicular to fractures are almost equal. Such behavior is caused by the fact that the liquid effectively seals otherwise very compliant fractures: as they are hydraulically isolated, the liquid simply has nowhere to go.

It was recognized early on that fully liquid-saturated fractured rocks with equant porosity represent an intermediate case between dry and liquid-filled rocks with isolated

fractures: when such a rock is compressed perpendicular to the fracture plane, the fluid can escape into the equant pores, thus increasing the normal fracture compliance. This is a first-order effect and hence any model of the fractured reservoir with equant porosity must take this fluid flow into account. The effect was first quantified by Thomsen (1995) for a specific case of sparse penny-shaped cracks and spherical equant pores. More recently, Cardona (2002) and Gurevich (2003) showed that exact static elastic moduli of a fluid-saturated fractured rock with equant porosity can be obtained directly from the properties of unfractured porous rock and fracture compliances of the dry fractured rock using anisotropic Gassmann (1951) or Brown-Korrington (1975) equations. This approach is completely general and does not require one to specify the shape of pores or fractures as long as their cumulative effect on the properties of the dry rock is known.

Thomsen's equant porosity theory and the anisotropic Gassmann approach assume that the frequency of the elastic wave is sufficiently low to attain so-called relaxed conditions where fluid pressure is equal in pores and fractures. This occurs when the fluid diffusion length is large compared to the characteristic dimensions of the fracture system. Conversely, at the other end of the frequency spectrum, the fluid diffusion length is small compared to characteristic fracture size, and the fluid has no time to move between pores and fractures during a wave cycle. Thus the fractures behave as if they were isolated (unrelaxed regime). At intermediate frequencies the wave-induced fluid flow causes attenuation, dispersion and frequency-dependent anisotropy. To estimate the significance of these effects, let's assume that in the relaxed regime our fluid-saturated fractured reservoir has P-wave anisotropy of 10% (P-wave propagating perpendicular to the fractures is 10% slower than parallel to fractures). In the unrelaxed regime where fractures behave as if they were isolated, these two velocities become equal. Assuming that dispersion of P-waves parallel to fractures is small, the dispersion of waves perpendicular to the fractures can be estimated to be about 0.1. A similar value can be obtained for the peak value of the dimensionless attenuation (reciprocal quality factor) Q^{-1} . This shows that seismic dispersion and attenuation caused by wave-induced fluid flow between pores and fractures can be significant and deserves careful analysis.

In recent years, a number of models of attenuation and dispersion due to flow between pores and fractures have been proposed, all limited to penny-shaped fractures of finite size. Hudson et al. (1996) model fractures as thin penny-shaped voids, and account for fluid-flow effects by applying the diffusion equation to a single crack and ignoring interaction between cracks. This approximation, however, leads to some unphysical effects, such as the result that the anisotropy of the fluid-saturated fractured and porous rock in the low-frequency limit is the same as for the dry rock. Chapman (2003) and Maultzsch et al. (2003) analyze frequency-dependent anisotropy caused by the presence of penny-shaped fractures in a porous rock, by considering the connectivity of individual fractures, pores and microcracks. A more general computational model which can take account of pores and fractures of any size and shape was proposed by Jakobsen et al. (2003) using the T-matrix approximation, commonly used to study effective properties of heterogeneous media. In the T-matrix approximation the effect of voids (pores, fractures) is introduced as a perturbation of the solution for the elastic background medium.

An alternative approach is to model the effect of fractures as a perturbation with respect to an isotropic porous

background medium. This approach seems attractive because it allows us to use all the machinery of the theory of wave propagation in fluid-saturated porous media, known as the theory of poroelasticity (Biot, 1962), without specifying individual shapes of grains or pores. It also seems logical to assume that the perturbation of the porous medium caused by the introduction of fractures will be much smaller than the perturbation caused by putting all the pores and fractures into an elastic solid. In the following section, we show how this approach can be applied to the study of both static (or low-frequency) properties of the fractured reservoir (which in particular gives a simple recipe for fluid substitution) and dynamic properties, which allows one to model attenuation, dispersion and frequency-dependent anisotropy.

Fluid substitution at low frequencies. In his seminal 1951 paper, Gassmann derived his famous equations which form the backbone of modeling fluid effects on seismic data. Gassmann showed that the bulk modulus of an isotropic fluid-saturated porous rock K^{sat} can be expressed as a sum of the bulk modulus of the dry matrix K plus an additional term responsible for the stiffening effect of the fluid:

$$K^{sat} = K + \alpha^2 M, \quad (1)$$

where $\alpha = 1 - K/K_g$ is the Biot-Willis coefficient,

$$M = \frac{K_g}{\left(1 - \frac{K}{K_g}\right) - \phi \left(1 - \frac{K_g}{K_f}\right)} \quad (2)$$

is the so-called pore-space modulus, ϕ is porosity, K_g and K_f are the bulk moduli of the solid grain material and fluid respectively. In contrast to the bulk modulus, the shear modulus is not affected by the presence of fluid, $\mu^{sat} = \mu_0$.

The Gassmann equation is routinely used in all quantitative analysis of seismic and sonic log data. However, little known is the fact that in the same 1951 paper Gassmann presented a similar result for an anisotropic porous matrix, which looks remarkably similar to the isotropic equations; but now instead of bulk modulus, they express the stiffness matrix of the saturated rock c_{ij}^{sat} as a sum of the dry stiffness matrix c_{ij} plus the stiffening term caused by the action of fluid:

$$c_{ij}^{sat} = c_{ij} + \alpha_i \alpha_j M, \quad i, j = 1, \dots, 6 \quad (3)$$

where $\alpha_m = 1 - K_m/K_g$, $K_m = (c_{m1} + c_{m2} + c_{m3})/3$ for $m=1, 2$, and 3 , $\alpha_4 = \alpha_5 = \alpha_6 = 0$, and the scalar M is the direct analog of Gassmann's pore space modulus and is given by Equation 2 with K replaced by $K^* = (K_1 + K_2 + K_3)/3$. Note that while the rock is anisotropic, the solid grain material is still assumed isotropic (and homogeneous). This means that anisotropy is caused by the geometrical configuration of the grains and pores. Extensions to microheterogeneous and anisotropic grains have been derived by Brown and Korrington (1975); however, they found limited use in practice because of a large number of parameters required to characterize the rock.

Equations 3 apply to any anisotropic rock regardless of a particular geometry of the matrix. To use them for a fractured medium all we need to do is specify the effect of fractures on the dry stiffness matrix. Since dry rock is an elastic material, any standard model of fractures in an elastic solid can be used here. This effect can be best quantified by two

excess fracture compliance parameters Z_N and Z_T . Details of this procedure as well as explicit expressions for saturated stiffnesses as functions of the properties of dry unfractured rock, fractures and fluid are given in Gurevich (2002). Anisotropic Gassmann Equations 3 are not limited to a single fracture set; they can also be used to study the effect of fluid properties in porous rock permeated by a number of fracture sets. This was done, for example, by Galvin et al. (2007) who studied the fluid effect on shear-wave splitting in porous reservoirs permeated by two "conjugate" fracture sets.

The crucial assumption in the Gassmann equations is that fluid pressure is spatially constant in a representative volume. This requires that all pore space be interconnected (if there are isolated pores they can be treated as part of the solid). But even if the pore space is all interconnected, it takes time for the fluid pressure to equilibrate. Thus the Gassmann equations are said to represent the low-frequency limit of the elastic moduli. But how low should the frequency be? Obviously it should be smaller than any of the characteristic frequencies of the system.

An idealized rock containing only equant (stiff) porosity filled with a viscous fluid and no fractures or microcracks possesses three characteristic frequencies corresponding to three potential mechanisms of attenuation/dispersion: scattering frequency ω_{sc} where the wavelength is comparable to the grain size, viscoelastic frequency ω_{VE} where complex fluid shear modulus becomes comparable (in absolute value) to its bulk modulus, and Biot's characteristic frequency ω_B , where the viscous skin depth equals the pore channel size. For the Gassmann equations to be valid, the wave frequency must be small compared to the smallest of these three characteristic frequencies, which is usually Biot's frequency and is between 10^1 and 10^6 Hz. This means that the Gassmann equations can be safely applied at seismic and sonic frequencies.

Condition $\omega \ll \omega_B$ ensures that fluid pressure has enough time to equilibrate between the peaks and troughs of the wave. However, if the rock also contains fractures (oriented or not), it will take time for the fluid pressure to equilibrate between these fractures and adjacent pores. Depending on the fracture size this is known as either local flow or squirt (for grain-size cracks) or mesoscopic flow (for fractures or other heterogeneities much larger than the pores size but smaller than the wavelength). Thus the presence of fractures introduces an additional characteristic frequency ω_M associated with this flow. For mesoscopic fractures which are the subject of this paper, this characteristic frequency is such that the fluid diffusion length equals the characteristic length associated with the fractures (depending on the particular geometry, this can be fracture spacing, fracture diameter or some other characteristic). This frequency depends on the matrix permeability, the fluid viscosity and the characteristic fracture dimension, and, as shown both theoretically (e.g., by Pride et al., 2004) and experimentally (Batzle et al., 2006), the condition $\omega \ll \omega_M$ may not be satisfied even at seismic frequencies, making an analysis of the frequency dependency of elastic properties of such rocks even more important. The above analysis suggests that it is reasonable to assume that the frequency is low compared to Biot's characteristic frequency (ensuring the validity of the Gassmann equations for the host (unfractured) rock), but not with respect to the mesoscopic flow frequency ω_M . This is the assumption we make in all subsequent analysis. To avoid any confusion, here and below low frequency or relaxed regime refers to the situations where $\omega \ll \omega_M$, and high frequency or unrelaxed regime means $\omega \gg \omega_M$.

Fractures as planes of weakness. If the fracture size is large compared to the fracture spacing, then fractures can be modeled

as planes of weakness or linear-slip interfaces. For fractures in an isotropic elastic solid, this was done by Schoenberg and Douma (1988) who showed that such fractures can be modeled as thin and highly compliant layers with bulk and shear moduli proportional to their thickness. For a porous material it is logical to model such planes of weakness as thin and soft layers with very high porosity $\phi \rightarrow 1$ (Figure 1b). When both pores and fractures are dry, this material is elastic and equivalent to an elastic material with linear-slip interfaces. When saturated with a fluid, this can be studied using Biot's theory of poroelasticity (Biot, 1962). Consider a set of periodically alternating layers, with spatial period H and the volume fractions h_b and $h_c \ll h_b$ (where subscript b refers to the host rock, and c to fractures), so that $h_b + h_c = 1$. The host rock and fractures are assumed to be made of the same isotropic grain material with bulk modulus K_g , shear modulus μ_g and density ρ_g , and saturated with the same fluid of bulk modulus K_f , density ρ_f and dynamic viscosity η . This layered system represents a particular case of a periodically layered porous medium governed by Biot's equations of poroelasticity with periodically varying coefficients. Such equations were studied by White et al. (1975) and Norris (1993) who derived a dispersion equation for arbitrary relative thicknesses h_b and h_c . The result for fractures can be obtained by taking the limits $h_c \rightarrow 0$, $K_c \rightarrow 0$ and $\mu_c \rightarrow 0$ (where K_c and μ_c are the dry bulk and shear moduli of the fracture material). This was done by Brajanovski et al. (2005) who showed that the dynamic-equivalent saturated P-wave modulus c_{11}^{sat} of such a system is given by:

$$\frac{1}{c_{11}^{sat}} = \frac{1}{C} \left[1 + \frac{(C - \alpha M)^2}{CL} \frac{\Delta_N}{1 - \Delta_N + \Delta_N \sqrt{i\Omega} \cot\left(\frac{C}{M} \sqrt{i\Omega}\right)} \right], \quad (4)$$

where $L = K + 4\mu/3$ and $C = K^{sat} + 4\mu/3 = L + \alpha^2 M$ are the dry and saturated P-wave moduli of the host rock, K and μ are its dry bulk and shear moduli, α and M are the effective stress coefficient and the pore space modulus defined earlier, $\Omega = \omega H^2 M / 4\kappa CL$ is normalized frequency, and $\Delta_N = LZ_N / (1 + LZ_N)$ is the dry normal fracture weakness (Schoenberg and Douma, 1988; Bakulin et al., 2000). Equation 4 can be used to evaluate the frequency dependence of the P-wave attenuation (inverse quality factor) $Q^{-1} = \text{Im}c_{11}^{sat} / \text{Re}c_{11}^{sat}$ and phase velocity $V = (\text{Re}c_{11}^{sat} / \rho)^{1/2}$ where $\rho_b = \rho_g(1 - \phi) + \rho_f \phi$ is the density of the fluid-saturated rock. Figure 2 illustrates the behavior of attenuation and dispersion at different background porosity for constant fracture weakness. We see that the level of both the attenuation and dispersion caused by wave-induced flow between pores and fractures increases with increasing background porosity as expected.

Despite the idealized fracture geometry of this 1D model, we can infer some important physical aspects of wave attenuation in the presence of (quasi-)periodic and random fracture sets. The fully periodic 1D model contains two very distinct length scales; one that is associated with the fracture thickness and another one that represents the distance between two fracture planes. This high contrast of length scales is paired with high contrasts of material properties as fractures are modeled as highly compliant layers. Figure 3 shows variation of $1/Q$ with frequency in bilogarithmic scale, and reveals three different frequency regions where $1/Q$ is proportional to ω to the power of 1, $1/2$, and $-1/2$, respectively. These regimes are separated by two crossover frequencies ω_P and ω_M corresponding to points P and M given by Brajanovski et al. (2006):

Figure 2. P-wave velocity and attenuation as a function of frequency in a water-saturated sandstone with quartz as the grain material ($K_g=37$ GPa, $\mu_g=44$ GPa, $\rho_g=2.65$ g/cm³), fracture weakness $\Delta_N=0.2$ and background porosity varying from 0.001% to 30%.

Figure 3. Log-log plot of attenuation versus circular frequency for water-saturated sandstone ($K_g=37$ GPa, $\mu_g=44$ GPa, $\rho_g=2.65$ g/cm³) with porosity $\phi=0.2$ and fracture weakness Δ_N ranging from 0.05 to 0.2.

$$\omega_P = 9 \frac{D}{H^2}, \quad \omega_M = 4\sqrt{2} \left(\frac{C}{M} \right)^2 \Delta_N^{-2} \frac{D}{H^2}, \quad (5)$$

where $D=\kappa N/\eta$ is the hydraulic diffusivity of the background rock and $N=ML/C$.

The attenuation behavior described above can be interpreted as a superposition of two coupled diffusion processes. Although each layer alone does not produce any attenuation (the layer itself is homogeneous), when they are connected together, attenuation takes place because a pore pressure gradient across the interface is induced. In order to equilibrate pressure, fluid diffuses between layers (background and fracture). Mirror symmetry of the system results in a no-flow condition in the middle of each layer. One can

Figure 4. Numerical simulations (lines) and theoretical predictions (circles) of phase velocity dispersion and attenuation for periodically spaced (blue) and randomly spaced (red) planar fractures with an average spacing $H=0.4$ m and fracture weakness $\Delta_N=0.25$.

say that maximum attenuation occurs when fluid penetrates a layer to the maximum possible depth. Therefore, the condition of *diffusion resonance* for background and fractures is given by the equality of the diffusion length λ_d and half-thicknesses of these respective layers, and by the hydraulic coupling as a consequence of fluid-mass conservation. The crossover frequency ω_P is independent of fracture weakness and depends solely on the ratio between the diffusivity and thickness of the background layer. This is understandable since the diffusion length in a fracture at low frequencies is several orders of magnitude larger than the fracture thickness; such that it has a negligible impact on the frequency dependency of the diffusion process in the background. If it were possible for a diffusion process in the background to exist alone, then its resonant frequency would be defined by the equality of the diffusion length and the half-thickness so that $\omega_P=8D_b/H^2$. This estimate is very similar to the expression for the crossover frequency ω_P as given by Equation 5.

On the other hand, maximum frequency ω_M is independent of the fracture diffusivity (and hence permeability). This is a consequence of the fact that in Equation 4 all fracture properties are contained in a single parameter: fracture weakness. For a weakly consolidated fracture matrix, the maximum frequency ω_M depends primarily on fracture thickness (weakness). Using the definition of fracture weakness $\Delta_N=LZ_N/(1+LZ_N)$, we can rewrite ω_M directly in the properties of the host and fracture materials so that the ratio of the two crossover frequencies is

$$\frac{\omega_P}{\omega_M} \propto \left(\frac{N_b}{N_c} \right)^2 \left(\frac{h}{H} \right)^2, \quad (6)$$

where h is the fracture thickness. We see that the separation of these characteristic frequencies depends on the ratios of the spatial scales and of the poroelastic moduli.

The theoretical results presented above have all been obtained from Equation 4 derived for periodically spaced fractures. In real reservoirs fractures are more likely to be distributed in a random fashion. In order to examine the significance of the periodicity assumption, Lambert et al. (2006) performed 1D numerical simulations for a porous medium with fractures modeled as layers of small but finite thickness and small but finite elastic moduli. The numerical results for a periodic system of fractures (blue lines in Figure 4) show very good agreement with the theoretical predictions (blue circles). However, when the same fractures are placed at random intervals (red lines), the attenuation behavior is different as it shows only two of the three attenuation regimes, with the asymptote $Q^{-1} \propto \omega$ absent, and the intermediate scaling law $Q^{-1} \propto \omega^{1/2}$ becoming the low-frequency asymptote. This numerical result for a random distribution of fractures is consistent with both theoretical and numerical results for randomly layered porous media with small contrast between layers (Gurevich and Lopatnikov 1995, Gelinsky et al. 1998).

We explain this observation as follows (a detailed analysis can be found in Müller and Rothert, 2006). Consider a randomly layered poroelastic medium with hydraulic diffusivity as a fluctuating quantity. During the course of pore pressure diffusion neighboring layers with relatively small diffusivity contrasts melt together and behave effectively like a single but thicker layer whereas pressure gradients still persist in the vicinity of layers with relatively high diffusivity contrast. As the diffusion process evolves with time the thicknesses of these effective layers increases without limit. If we identify the thickness of the effective layer with the distance between two fracture planes, then according to Equation 5 for ω_p the crossover frequency shifts toward zero frequency. We conclude that the low frequency asymptotic behavior $Q^{-1} \propto \sqrt{\omega}$ is a feature of an infinite 1D random media irrespective of the material contrasts. We approximate this behavior by the equation

$$\frac{1}{c_{11}^{sat}} = \frac{1}{C^*} + \frac{1}{C_0} - \frac{1}{C} \left(1 + \sqrt{-i\omega / \omega_M} \right). \quad (7)$$

This approximation (plotted as red circles in Figure 4) shows excellent agreement with numerical simulations.

Fractures as penny-shaped inclusions. Fractures whose size is much smaller than their spacing can be modeled as penny-shaped cracks. This leads to the problem of the interaction of a plane longitudinal elastic wave with an open oblate spheroidal crack of radius a and thickness $2b \ll a$ placed perpendicular to the direction of wave propagation. This problem was considered by Galvin and Gurevich (2006) who considered the case of so-called mesoscopic cracks whose radius is small compared to the wavelength $2\pi/k_1$ of the normal compressional wave, but large compared to the individual pore size. Furthermore, crack thickness $2b$ (but not crack radius!) was assumed smaller than the fluid diffusion length. As shown by Gurevich and Lopatnikov (1995), the interaction of propagating waves in heterogeneous poroelastic media and the resulting attenuation can be treated as a scattering problem from fast into slow P-waves, which can be mathematically posed as a mixed boundary value problem for Biot's equations of poroelasticity with boundary conditions. It was also assumed that the crack is in hydraulic communication with the host rock. Together with the small thickness assumption this allows one to neglect the volume change of the crack-fill-

ing fluid.

As a consequence, the sum of total and relative displacement normal to the crack surface equals zero, in addition to standard conditions of the continuity of total stress and pore pressure, while outside of the crack bulk and relative fluid displacements are assumed continuous. Using these boundary conditions and making use of the cylindrical crack symmetry the scattering problem can be transformed into a single integral equation (Fredholm equation of the second kind) in an unknown wave-amplitude function (Galvin and Gurevich, 2006). This theory can be used to estimate attenuation and dispersion of an elastic wave propagating in a medium with a random distribution of aligned cracks by using a Foldy-type approximation of multiple scattering which expresses the effective wavenumber k^* for the medium with cracks in terms of the number of scatterers per unit volume n_0 and the far-field forward scattering amplitude for a single scatterer. For a small concentration of aligned cracks one can compute effective wave velocity $V(\omega) = \omega / \text{Re}k^*$ and attenuation $Q^{-1} = 2\text{Im}k^* / \text{Re}k^*$.

This theory presented by Galvin and Gurevich (2006) does not provide explicit analytical expressions for the complete frequency range (in general, the integral equation must be solved numerically); however, simple expressions for the low- and high-frequency asymptotic behavior can be found. Figure 5 shows the results based on numerical solution of the integral equation along with low and high-frequency asymptotes. The solution for attenuation and dispersion of elastic waves in a porous fluid-saturated medium with aligned fractures of finite size exhibits a relaxation peak at a frequency where the fluid diffusion length is of the order of the crack radius a .

In the low-frequency limit, the effective saturated P-wave modulus agrees exactly with the anisotropic Gassmann result. The high-frequency asymptote of the attenuation is exactly the same as the one for planar fractures with the same specific surface per unit volume which can be expressed through crack density for finite cracks as $S = \pi a / n$, or through the distance between infinite planar fractures as $S = 1/H$. This agreement reflects the fact that at high frequencies the fluid diffusion length is small compared with both the fracture size and spacing, and therefore the diffusion is taking place in the immediate vicinity of the fracture surface. At low frequencies however the attenuation and dispersion in the two models are different. This is the result of the fact that for infinite fractures (both with periodic and random spacing) the pattern of conversion scattering and interference is one-dimensional, whereas finite fractures at low frequencies act essentially as 3D point scatterers (from normal P-wave into the diffusion-type slow wave).

Conclusions. The wave-induced fluid flow between pores and fractures considered in this paper has a similar physical nature to the so-called squirt flow (Mavko and Nur 1975), and hence, the present models could be viewed as new models of squirt-flow attenuation, consistent with Biot's theory of poroelasticity. These models can also be viewed as a special case of double porosity models of so-called mesoscopic flow attenuation (Pride et al., 2004), a variant designed specifically for open fractures in a poroelastic background. The concept of mesoscopic flow refers to wave-induced flow caused by the presence of mesoscopic heterogeneities, that is, heterogeneities small compared to the wavelength but much larger than the size of individual pores or grains.

Fractures in a porous rock can be modeled in two ways:

As very thin and highly porous layers in a porous background. When such a medium is saturated with the liquid, mode con-

Figure 5. P-wave velocity and attenuation versus frequency due to wave-induced flow between pores and penny-shaped fractures: numerical solution of the integral equation (asterisks), low- and high-frequency asymptotes (dashed lines).

version and interference between normal P-wave and Biot's slow waves results in frequency-dependent velocity dispersion and attenuation. This frequency-dependency is controlled by the ratio of fluid diffusion length to fracture spacing. At high frequencies, when fluid diffusion length is small compared to fracture spacing, the fractures behave as if they were isolated and the modulus approaches that of the unfractured fluid-saturated rock, while attenuation/dispersion is controlled by the fluid diffusion in the vicinity of fracture surfaces. Conversely, at low frequencies, when fluid diffusion length is larger than fracture spacing, attenuation and dispersion is controlled by the interference pattern of wave-induced fluid diffusion (or Biot's slow waves), and thus depends on the pattern of fracture spacing (periodic or random).

As a random system of aligned penny-shaped cracks. In this case, P-wave attenuation and dispersion is controlled by the ratio of the fluid diffusion length to the crack radius. At high frequencies, that is, when fracture radius is larger than the fluid diffusion length, the attenuation and dispersion is very similar to that caused by planar fractures. However, at low frequencies when the fluid diffusion length is large compared with fracture size, the pattern produced is very different, with fractures acting essentially as independent point scatterers.

Note that the fluid diffusion length (wavelength of Biot's slow wave) is usually much smaller than the wavelength of the normal compressional or shear wave. Thus the presence of fractures in a fluid-saturated porous medium can cause significant attenuation and dispersion at low frequencies, well before the onset of elastic scattering. In the low-frequency limit both models are consistent with anisotropic Gassmann (or Brown-Korrington) theory which provides a convenient

recipe for fluid substitution in fractured reservoirs. The main conclusion of this work is that Biot's theory of poroelasticity provides the most versatile and convenient basis for modeling elastic properties, attenuation and dispersion not only in isotropic porous rocks but also in highly heterogeneous and anisotropic materials such as fractured reservoirs.

Suggested reading. "Elastic anisotropy due to aligned cracks in porous rock" by Thomsen (*Geophysical Prospecting*, 1995). "Two theories for fluid substitution in porous rocks with aligned cracks" by Cardona (*Journal of Applied Geophysics*, 2002). "Elastic properties of saturated porous rocks with aligned fractures" by Gurevich (*Journal of Applied Geophysics*, 2003). "Über die elastizität poröser medien" by Gassmann (*Viertel. Naturforsch. Ges. Zurich*, 1951). "On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid" by Brown and Korrington (*Geophysics*, 1975). "The mechanical properties of materials with interconnected cracks and pores" by Hudson et al. (*Geophysical Journal International*, 1996). "Frequency dependent anisotropy due to meso-scale fractures in the presence of equant porosity" by Chapman (*Geophysical Prospecting*, 2003). "Modeling frequency-dependent seismic anisotropy in fluid-saturated rock with aligned fractures: Implication of fracture size estimation from anisotropic measurements" by Maultzsch et al. (*Geophysical Prospecting*, 2003). "The acoustic signature of fluid flow in complex porous media" by Jakobsen et al. (*Journal of Applied Geophysics*, 2003). "Mechanics of deformation and acoustic propagation in porous media" by Biot (*Journal of Applied Physics*, 1962). "Fluid-dependent shear-wave splitting in a poroelastic medium with conjugate fracture sets" by Galvin et al. (*Geophysical Prospecting*, 2007). "Seismic attenuation due to wave-induced flow" by Pride et al. (*Journal of Geophysical Research*, 2004). "Fluid mobility and frequency-dependent seismic velocity—Direct measurements" by Baztle et al. (*Geophysics*, 2006). "Elastic-wave propagation in media with parallel fractures and aligned cracks" by Schoenberg and Douma (*Geophysical Prospecting*, 1988). "Low-frequency dispersion and attenuation in partially saturated rocks" by Norris (*Journal of the Acoustical Society of America*, 1993). "Low-frequency seismic waves in fluid saturated layered rocks" by White et al. (*Izvestiya Academy of Sciences USSR, Physics of Solid Earth*, 1975). "A model for P-wave attenuation and dispersion in a porous medium permeated by aligned fractures" by Brajanovski et al. (*Geophysical Journal International*, 2005). "Estimation of fracture parameters from reflection seismic data-part I: HTI model due to a single fracture set" by Bakulin et al. (*Geophysics*, 2000). "Characteristic frequencies of seismic attenuation due to wave-induced fluid flow in fractured porous media" by Brajanovski et al. (*Geophysical Journal International*, 2006). "Velocity and attenuation of elastic waves in finely layered porous rocks" by Gurevich and Lopatnikov (*Geophysical Journal International*, 1995). "Dynamic poroelasticity of thinly layered structures" by Gelinsky et al. (*International Journal of Solids Structures*, 1998). "Seismic attenuation due to wave-induced flow: Why Q in random structures scales differently" by Müller and Rotherth (*Geophysical Research Letters*, 2006). "Interaction of an elastic wave with a circular crack in a fluid-saturated porous medium" by Galvin and Gurevich (*Applied Physics Letters*, 2006). "Melt squirt in the asthenosphere" by Mavko and Nur (*Journal of Geophysical Research*, 1975). TJE

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