

1 **Low-Cost, High-Precision, Single-Frequency GPS-BDS RTK Positioning**

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3

4 **Abstract** The integration of the Chinese BDS with other systems, such as the American GPS,
5 makes precise RTK positioning possible with low-cost receivers. We investigate the performance
6 of low-cost ublox receivers, which cost a few hundred USDs, while making use of L1 GPS + B1
7 BDS data in Dunedin, New Zealand. Comparisons will be made to L1 + L2 GPS and survey-grade
8 receivers which cost several thousand USDs. The least-squares variance component estimation
9 (LS-VCE) procedure is used to determine the code and phase variances and covariances of the
10 receivers and thus formulate a realistic stochastic model. Otherwise the ambiguity resolution and
11 hence positioning performance would deteriorate. For the same reasons, the existence of receiver-
12 induced time-correlation is also investigated. The low-cost RTK performance is then evaluated by
13 formal and empirical ambiguity success-rates and positioning precisions. It will be shown that the
14 code and phase precision of the low-cost receivers can be significantly improved by using survey-
15 grade antennas, since they have better signal reception and multipath suppression abilities in
16 comparison to low-cost patch antennas. It will also be demonstrated that the low-cost receivers can
17 achieve competitive ambiguity resolution and positioning performance to survey-grade dual-
18 frequency GPS receivers.

19 **Keywords** Low-cost Receiver, Multi-GNSS, Real Time Kinematic (RTK) positioning, Least-
20 squares variance component estimation (LS-VCE), Time-correlation

21

22 **Introduction**

23 In the past few decades, the American Global Positioning System (GPS) has been the primary
24 positioning tool for many applications. The integration with the emerging Global Navigation
25 Satellite Systems (GNSSs), such as the Chinese BeiDou Navigation Satellite System (BDS), can

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26 give improved precise real-time kinematic (RTK) positioning. Some first results using BDS
27 outside of China are reported in Montenbruck et al. (2013) and Nadarajah et al. (2013). When BDS
28 is combined with GPS about double the number of satellites will be visible in the Asia-pacific
29 region, which can make single-frequency RTK (Verhagen et al. 2012; He et al. 2014; Teunissen
30 et al. 2014; Odolinski et al. 2015) and low-cost receiver RTK positioning possible (Odolinski and
31 Teunissen 2016; Mongredien et al. 2016). Other studies on GPS RTK positioning and ambiguity
32 resolution using low-cost receivers can be found in Takasu and Yasuda (2008;2009), Wisniewski
33 et al. (2013) and Pesyna et al. (2014). We will analyze the performance of L1 GPS + B1 BDS in
34 Dunedin, New Zealand using low-cost ublox receivers. We will compare the performance to that
35 of dual-frequency GPS survey-grade receivers.

36 We describe first the GPS+BDS functional/stochastic model, and then the real data used
37 for our evaluations. Least-squares variance component estimation (LS-VCE) results are then
38 presented as a way to determine the code and phase (co)variances and formulate a realistic
39 stochastic model. An incorrect stochastic model deteriorates the ambiguity resolution performance
40 and consequently impacts the achievable positioning precisions. For that same reasons, receiver-
41 induced time-correlation will also be investigated. Once the stochastic model has been correctly
42 defined, the ambiguity resolution and positioning performance is investigated. The performance is
43 evaluated formally and empirically, for customary and high elevation cut-off angles. The high cut-
44 off angles are used to mimic situations when low-elevation multipath is to be avoided. All results
45 will be compared between using low-cost and survey-grade antennas. We then further extend our
46 short-baseline analysis and investigate the performance for a baseline where small residual
47 ionospheric delays are present. A summary with conclusions is finally given.

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49 **GPS+BDS model**

50 The model that will be used for the LS-VCE, as introduced in Teunissen (1988), is given as
51 follows. Assume that $s_G + 1$ GPS satellites are tracked on f_G frequencies and $s_B + 1$ BDS
52 satellites on f_B frequencies. As we apply system-specific double-differencing (DD) one pivot
53 satellite is used per system. The total number of DD phase and code observations per epoch equals
54 then $2f_G s_G + 2f_B s_B$. We assume for now that cross-correlation between frequencies, and code
55 and phase is absent. Following Teunissen et al. (2014), the combined multi-frequency short-

56 baseline GPS+BDS model is then defined as follows.

57 The system-specific DD phase and code observation vectors are denoted as ϕ_* and p_* ,
 58 respectively, with $* = \{G, B\}$ where $G = \text{GPS}$ and $B = \text{BDS}$. The single-epoch GNSS model of
 59 the combined system is given as

$$60 \begin{bmatrix} \phi \\ p \end{bmatrix} - \begin{bmatrix} \Lambda & I \\ 0 & I \end{bmatrix} \begin{bmatrix} a \\ \rho \end{bmatrix} = \begin{bmatrix} \varepsilon \\ e \end{bmatrix} \quad (1)$$

61 and

$$62 D \begin{bmatrix} \phi \\ p \end{bmatrix} = \begin{bmatrix} Q_{\phi\phi} & 0 \\ 0 & Q_{pp} \end{bmatrix} \quad (2)$$

63 in which $\phi = [\phi_G^T, \phi_B^T]^T \in \mathbb{R}^{f_{GS} + f_{BS}}$ is the combined phase vector, $p = [p_G^T, p_B^T]^T \in$
 64 $\mathbb{R}^{f_{GS} + f_{BS}}$ the combined code vector, $a = [a_G^T, a_B^T]^T \in \mathbb{Z}^{f_{GS} + f_{BS}}$ the combined integer
 65 ambiguity vector, $\rho = [\rho_G^T, \rho_B^T]^T \in \mathbb{R}^{s_G + s_B}$ the receiver-satellite range, $\varepsilon = [\varepsilon_G^T, \varepsilon_B^T]^T \in$
 66 $\mathbb{R}^{f_{GS} + f_{BS}}$ the combined phase random observation noise vector, $e = [e_G^T, e_B^T]^T \in \mathbb{R}^{f_{GS} + f_{BS}}$
 67 the combined code random observation noise vector, $D[\cdot]$ denotes the dispersion operator, and
 68 with the entries of the wavelength matrix given as

$$69 \Lambda = \text{blkdiag}[\Lambda_G, \Lambda_B], \quad \Lambda_* = \text{diag}[\lambda_{1_*}, \dots, \lambda_{f_*}] \otimes I_{s_*}$$

70 where λ_{j_*} is the wavelength of frequency j_* , \otimes denotes the Kronecker product, I_{s_*} is the
 71 $s_* \times s_*$ unit matrix, ‘diag’ is a diagonal and ‘blkdiag’ a blockdiagonal matrix, respectively. The
 72 entries of the positive definite variance matrices are given as

$$73 \begin{aligned} Q_{\phi\phi} &= \text{blkdiag}[Q_{\phi_G\phi_G}, Q_{\phi_B\phi_B}], Q_{\phi_*\phi_*} = C_{\phi_*\phi_*} \otimes 2Q_* \\ Q_{pp} &= \text{blkdiag}[Q_{p_Gp_G}, Q_{p_Bp_B}], Q_{p_*p_*} = C_{p_*p_*} \otimes 2Q_* \\ C_{\phi_*\phi_*} &= \text{diag}[\sigma_{\phi_{1_*}}^2, \dots, \sigma_{\phi_{f_*}}^2], C_{p_*p_*} = \text{diag}[\sigma_{p_{1_*}}^2, \dots, \sigma_{p_{f_*}}^2] \\ Q_* &= D_{s_*}^T W_*^{-1} D_{s_*}, W_* = \text{diag}[w_{1_*}, \dots, w_{s_*+1}] \end{aligned} \quad (3)$$

74 where $D_{s_*}^T = [-e_{s_*}, I_{s_*}]$ is the $s_* \times (s_* + 1)$ differencing matrix, e_{s_*} is the $s_* \times 1$ vector of
 75 values of 1, $\sigma_{\phi_{j_*}}$, $\sigma_{p_{j_*}}$ the phase and code standard deviation, respectively, and w_{i_*} denotes the
 76 satellite elevation dependent weight.

77 The model (1) applies to short baselines, and thus the ionospheric and tropospheric delays
 78 are assumed absent. The broadcast ephemerides are used to obtain the satellite coordinates and the
 79 baseline is precisely known. Hence the receiver-satellite range ρ is known, which is referred to as

80 the "geometry-fixed" model. The Least-squares AMBIGUITY Decorrelation Adjustment
81 (LAMBDA) (Teunissen 1995) is furthermore used to estimate the integer ambiguities a and treat
82 them as time-constant parameters over a large observation time span, so that their uncertainty can
83 be assumed to be negligible, which is referred to as the "ambiguity-fixed" model. The observation
84 noise vectors ε and e , respectively, are zero-mean vectors provided that no multipath is present
85 in (1).

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87 **Experiment setup**

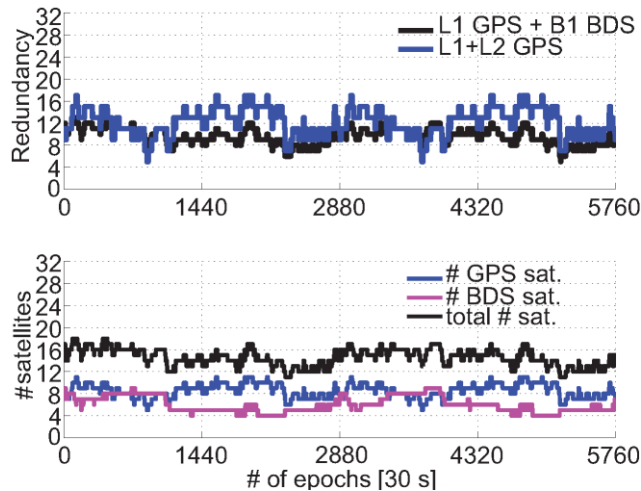
88 The GNSS receivers used to collect GNSS data are depicted in Figure 1. Two ublox EVK-M8T
89 receivers were setup to collect L1+B1 GPS+BDS data for two days with a measurement interval
90 of 1 s. These low-cost receivers cost a few hundred USDs. Since patch antennas have been shown
91 to have less effective signal reception and multipath suppression in comparison to survey-grade
92 antennas (Pesyna et al. 2014), the receivers that collected data for two days were also connected
93 to Trimble Zephyr 2 antennas. These antennas have a cost of slightly more than one thousand
94 USDs per antenna. To compare the low-cost solution to a survey-grade solution, two Trimble
95 NetRS receivers, which cost several thousand USDs, were connected to the same Zephyr antennas
96 through a splitter and collected L1+L2 GPS data. Another receiver setup, with Zephyr antennas,
97 was also analyzed with a baseline length of 7 km, so as to evaluate the low-cost receiver
98 performance when small residual ionospheric delays are present. The detection, identification, and
99 adaption (DIA) procedure (Teunissen 1990) was furthermore used to eliminate any outliers.

100 The model we will use for positioning is obtained by linearizing the observation equations
101 in (1) with respect to the unknown receiver coordinates. Figure 2 depicts the corresponding
102 redundancy of the two models when also including ambiguities as unknown parameters, together
103 with the number of satellites over 48 h with a 30 s epoch interval.

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 106 **Fig. 1** GNSS ublox EVK-M8T receivers collecting data for GPS + BDS single-baseline RTK, with
 107 patch antennas (top, left) and Trimble Zephyr 2 antennas (top, right) in Jan 4-6 and Jan 6-8, 2016,
 108 respectively. Survey-grade Trimble NetRS dual-frequency GPS receivers have been connected to
 109 the same Zephyr 2 antennas to truly track the same GPS constellation. A 7 km baseline setup (Jan
 110 18-19, 2016) also used to assess the positioning performance is shown at bottom
 111



112
 113 **Fig. 2** Number (#) of satellites (bottom) and redundancy (top) of L1+B1 GPS+BDS and L1+L2
 114 GPS in Jan 6-8, 2016 (48 h) for an elevation cut-off angle of 10°
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116 While the number of BDS satellites (magenta lines) is overall smaller than when compared

117 to GPS (blue lines) in Dunedin, Figure 2 shows that the redundancy of L1+B1 GPS+BDS is almost
118 similar to that of L1+L2 GPS except for some hours at the middle of the two days. This implies,
119 for instance, that the two models can potentially give competitive RTK ambiguity resolution and
120 positioning performance, see the extensive overviews in Teunissen et al. (2014) and Odolinski and
121 Teunissen (2016). This is however only true if the receiver-satellite geometry strength and receiver
122 code and phase observation noise would be of similar magnitude between the receivers used, hence
123 the need for an analysis of the observation precision.

124 In the following receiver evaluations, a set of reference ambiguities were determined using
125 a known baseline and treated as time-constant parameters over the two days in a dynamic model.
126 Note also that the Geostationary Orbit (GEO) C03 BDS satellite had to be excluded from all the
127 results as it yielded many incorrectly fixed instances caused by low-elevation multipath (Wang et
128 al. 2015a; 2015b), due to it being almost stationary and having a low elevation angle of around
129 12° with respect to the receivers (Odolinski and Teunissen 2016).

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131 **Least-squares Variance Component Estimation for low-cost receivers**

132 We make use of the LS-VCE procedure (Teunissen 1988; Teunissen and Amiri-Simkooei 2008)
133 to determine the code and phase precision of the low-cost receivers. The estimated (co)variances
134 are needed to formulate a realistic stochastic model for precise RTK positioning. The ambiguity
135 resolution performance would otherwise deteriorate and hence the achievable positioning
136 precisions as well.

137 The noise vectors for phase ε and code e in (1) serve as an input into the LS-VCE. To
138 capture the elevation dependency, an exponential weighting function (Euler and Goad 1991) is
139 used in (3) as follows

$$140 \quad w_{i_*} = \left[1 + 10 \exp \left(-\frac{\theta^{i_*}}{10} \right) \right]^{-2} \quad (4)$$

141 where θ^{i_*} is the elevation of the satellite i_* in degrees. This formulation allows us to estimate
142 zenith-referenced standard deviations (STDs) of the undifferenced phase and code observations,
143 $\sigma_{\phi_{j_*}}$ and $\sigma_{p_{j_*}}$, respectively. Table 1 depicts the corresponding code and phase STDs for the short-
144 baseline setups at top of Figure 1 for instantaneous, i.e. epoch-by-epoch, RTK. The presented
145 STDs are the mean of the single-epoch STDs over the entire observation time span. The STDs

146 within parentheses are based on single-epoch multipath (MP) corrected DD residuals. These
147 residuals were obtained by subtracting the DD residuals (1) from the previous day of data (Jan 6
148 minus Jan 5 for patch and Jan 9 minus Jan 8 for Zephyr antennas), while taking into account the
149 approximate satellite constellation repeatability of 23 h and 56 min and using the elevation
150 weighting (4) similar to Zaminpardaz et al. (2016). This procedure was applied since the code e
151 and phase ε vectors in (1) are believed to be contaminated by multipath effects. The MP correction
152 was only applied to the GPS Medium Earth Orbit (MEO) and BDS GEO/Inclined Geosynchronous
153 Satellite Orbit (IGSO), since these satellites have a constellation repeatability of about one sidereal
154 day (Axelrad et al. 2005; Jiang et al. 2011), and the GEO satellites are almost stationary
155 (Montenbruck et al. 2013). One should, however, also be aware of that different satellites can have
156 different repeatability periods that differ several seconds from each other, as was shown for GPS
157 (Axelrad et al. 2005). We use the same time-separation between days for all satellites when
158 applying the MP corrections, which consequently can result in some residual multipath effects to
159 remain in the presented results. The doubling in noise that enters through the epoch-by-epoch MP
160 correction has been accounted for in all the results presented by dividing the day-differenced
161 residuals by square root of two.

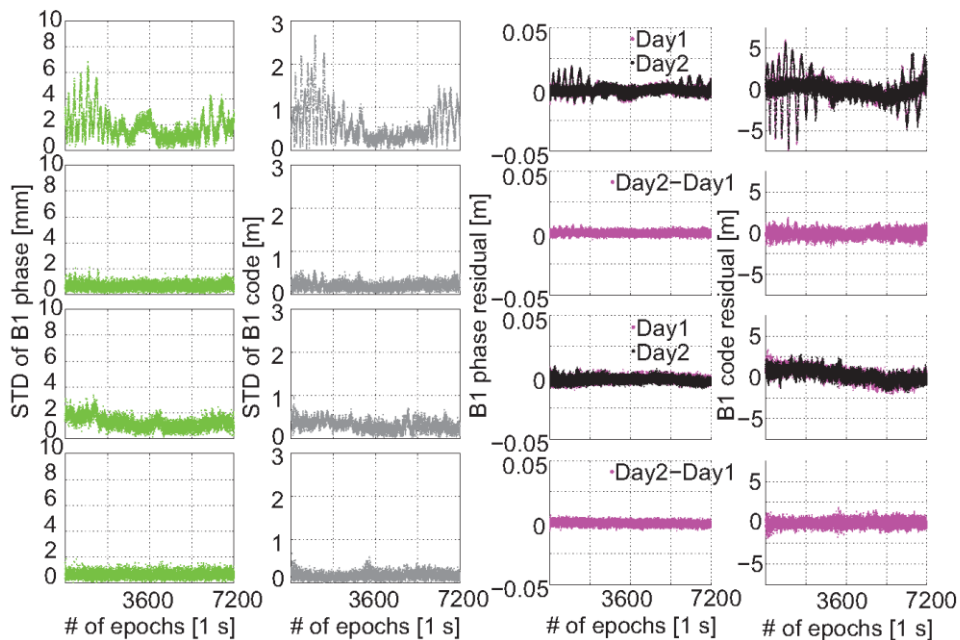
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163 **Table 1** LS-VCE of zenith-referenced and undifferenced code $\hat{\sigma}_{p_{j^*}}$ and phase $\hat{\sigma}_{\phi_{j^*}}$ STDs for
164 Dunedin. The STDs based on MP-corrected DD residuals are shown within parentheses. The STDs
165 have been estimated based on models including all satellites unless otherwise stated, and always
166 including GPS. The 24 h (30 s) period is 22:00-22:00 (hh:mm) UTC Jan 4-5 for ublox+patch,
167 21:48-21:48 UTC Jan 7-8 for ublox/Trimble NetRS+Zephyr. The 2 h (1 s) period for GPS is 04:00-
168 06:00 UTC Jan 5 for patch and 03:48-05:48 UTC Jan 8 for Zephyr. The corresponding BDS STDs
169 are based on 18:00-20:00 UTC Jan 5 for patch and 17:48-19:48 UTC Jan 8, 2016 for Zephyr

Receiver/antenna	System	Frequency	2 h [1 s interval]		24 h [30 s interval]	
			$\hat{\sigma}_{p_{j^*}}$ [cm]	$\hat{\sigma}_{\phi_{j^*}}$ [mm]	$\hat{\sigma}_{p_{j^*}}$ [cm]	$\hat{\sigma}_{\phi_{j^*}}$ [mm]
Trimble NetRS +Zephyr	GPS	L1	18(15)	2(1)	18	2
		L2	20(15)	2(1)	20	2
ublox EVK-M8T +Zephyr	GPS	L1	31(20)	2(1)	34	2
	BDS	B1	30(-)	2(-)	29	2
		GEO + IGSO		33(20)	2(1)	29
ublox EVK-M8T +patch	GPS	L1	53(24)	2(1)	49	2
	BDS	B1	73(-)	2(-)	49	2
		GEO + IGSO		74(22)	2(1)	47

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In support of understanding Table 1 better, Figure 3 shows the single-epoch B1 BDS code and phase STDs (left two columns) corresponding to a period of two hours. The code and phase DD residuals are also shown for two consecutive days (right two columns), where elevation-weighting (4) is applied since the observations are highly dependent on the elevation of the satellites. The patch-antenna residuals and corresponding single-epoch estimated STDs show very large fluctuations at times likely due to multipath or poor signal reception since it repeats over both days. The code and phase STDs based on elevation-weighted day-differenced DD residuals are thus depicted as well at the second and fourth row respectively.



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Fig. 3 Single-epoch LS-VCE of zenith-referenced and undifferenced B1 BDS code and phase STDs (left two columns) for Day1 (2h [1 s]) using ublox EVK-M8T receivers and L1+B1 GPS+BDS. The elevation-weighted code/phase DD residuals are also shown (right two columns), all in the first and third row. The corresponding STDs and code and phase DD residuals when applying MP corrections are shown in the second and fourth row, using a repeatability period of approximately one sidereal day for the GPS MEO and BDS GEO+IGSO satellites. In the first two rows results are given for patch on Jan 5 at 18:00 and Jan 6 at 17:56 (hh:mm) UTC, and in the last two rows when Zephyr antennas are used on Jan 8 at 17:48 and Jan 9, 2016 at 17:44 (hh:mm) UTC

190 Figure 3 shows the significant improved patch antenna STDs when the MP corrections
 191 have been applied (second row left), which is also reflected by the corresponding much smaller
 192 residuals (second row right). Table 1 shows consequently the more similar magnitude of STDs
 193 between using Zephyr and patch antennas for the u-blox receivers when MP corrections have been
 194 applied. For example, the B1 code STD decreases from 74 cm for the patch-antenna model to 22
 195 cm when day-differencing has been applied. Corresponding values for the Zephyr-antenna model
 196 are 33 cm, which decreases to 20 cm. Since the non-differenced STDs for the patch antenna
 197 model are much larger than the corresponding Zephyr ones, this indicates the potentially better
 198 multipath suppression by the Zephyr antennas which is also reflected by the much smaller code
 199 and phase residuals in the figure. We also note in Table 1 that the Trimble NetRS non-differenced
 200 L1 and L2 code STDs of 18 cm and 20 cm, respectively, decrease to 15 cm for both frequencies
 201 when day-differencing has been applied.

202 The covariances between code and phase, and frequencies, were also estimated. Table 2
 203 shows the corresponding cross-correlations together with the STDs based on the 2 h (1 s) data in
 204 Table 1, whereas the patch-antenna model is based on 24 h (30 s) due to the earlier-referenced
 205 possibility of multipath so that these effects are more likely to average out over time.

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 207 **Table 2** LS-VCE of zenith-referenced cross-correlations, which are the non-diagonal values, and
 208 undifferenced code $\hat{\sigma}_{p_{j_*}}$ and phase $\hat{\sigma}_{\phi_{j_*}}$ STDs, which are the diagonal values. This is all given
 209 for Dunedin on $j_* = 1_*, \dots, f_*$. The estimation is based on 2 h of data (1 s) whereas the patch
 210 antenna model is based on 24 h (30 s), while making use of the same periods as specified in
 211 Table 1. The BDS values have been estimated based on using all GPS+BDS satellites

Receiver/antenna	System		C1C	L1C	C2W	L2W
Trimble NetRS +Zephyr 2 h [1 s]	GPS	C1C	18 cm	0.1	0.1	
		L1C	0.1	2 mm		0.35
		C2W	0.1		20 cm	-0.1
		L2W		0.35	-0.1	2 mm
			C1C	L1C	C2I	L2I
ublox EVK-M8T +Zephyr 2 h [1 s]	GPS	C1C	31 cm	-0.1		
		L1C	-0.1	2 mm		
	BDS	C2I			30 cm	0.0
		L2I			0.0	2 mm
			C1C	L1C	C2I	L2I
ublox EVK-M8T +patch 24 h [30 s]	GPS	C1C	49 cm	-0.1		
		L1C	-0.1	2 mm		
	BDS	C2I			49 cm	0.0
		L2I			0.0	2 mm

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Table 2 shows that the Trimble NetRS receivers have an estimated correlation coefficient of 0.35 between L1C and L2W phase, which is significant since the corresponding STD is 0.007 that is not explicitly shown herein. That the GPS L1 and L2 phase observables can be highly correlated for some receivers has also been found in Teunissen et al. (1998), Bona and Tiberius (2000), Amiri-Simkooei and Tiberius (2007), and Amiri-Simkooei et al. (2009), and specifically for a Trimble NetRS receiver in Amiri-Simkooei et al. (2016). More importantly the ublox receivers show close to zero cross-correlation between GPS C1C code and L1C phase, as well as BDS C2I code and L2I phase.

222 **Time-correlation of low-cost RTK positioning**

223 In this section we examine the time-correlation of the low-cost receivers, which, if neglected in a
224 multi-epoch model, could negatively influence the ambiguity resolution and hence positioning
225 performance. Time-correlation can be caused by internal effects in the receivers, for instance
226 filtering to reduce the observation noise level (Bona 2000). External effects can also cause time-
227 correlation, which include atmospheric delays and multipath. We use a short-baseline (SB) setup
228 (Figure 1, top), and thus multipath is the main external source herein that can cause time-
229 correlation in addition to the internal effects within the receivers. We will also compare the results
230 to a zero-baseline (ZB) setup, where two receivers are connected to the same antenna at the same
231 location as the setups shown at the top of Figure 1. As a result of the zero-baseline set-up, the
232 contributions of multipath are largely eliminated. The remaining small effects would mainly be
233 due to the non-simultaneity of sampling between the receivers. One should finally be aware that
234 internal errors due to the noise from the common Low Noise Amplifier (LNA) between the
235 receivers largely extent cancel when a zero-baseline is used (Amiri-Simkooei and Tiberius 2007).
236 This is the reason that the receiver noise thus far has only been assessed based on the short baseline
237 setups in Figure 1, where the receivers are connected to their own antenna and LNA, and thus give
238 more realistic noise estimates.

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240 **Time-correlation without reducing multipath**

241 The model we will now use is obtained by linearizing the observation equations in (1) with respect

242 to the unknown receiver coordinates. In this step, the code and phase variances that were estimated
 243 by LS-VCE in Table 2 are fixed. We consider estimating a functionally known quantity, like local
 244 North/East/Up ambiguity-float or ambiguity-fixed components respectively, and m is the number
 245 of times. These positioning errors were obtained by comparing the estimated positions to precise
 246 benchmark coordinates. The autocovariance function can then be determined by (Teunissen and
 247 Amiri-Simkooei 2008)

$$248 \hat{\sigma}_\tau = \frac{\sum_{i=1}^{m-\tau} \hat{e}_i \hat{e}_{i+\tau}}{m-\tau}, \tau = 1, 2, \dots, m-1 \quad (5)$$

249 where τ is the time-lag and \hat{e}_i the least-squares positioning errors. The variance of the noise
 250 process follows as,

$$251 \hat{\sigma}^2 = \frac{\sum_{i=1}^m \hat{e}_i \hat{e}_i}{m} \quad (6)$$

252 The autocorrelation function can then be defined by,

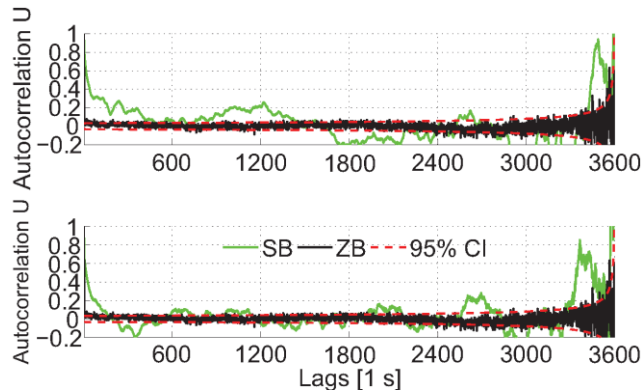
$$253 \hat{\rho}_\tau = \frac{\hat{\sigma}_\tau}{\hat{\sigma}^2}, \tau = 1, 2, \dots, m-1 \quad (7)$$

254 where $\hat{\rho}_{\tau=0} = 1$ by definition. An approximation of the STD of the autocorrelation function is
 255 given through linearization and the error propagation law of (7) as follows under the assumption
 256 of white noise, i.e. that consecutive observations are uncorrelated (Amiri-Simkooei and Tiberius
 257 2007),

$$258 \sigma_{\hat{\rho}_\tau} \approx \sqrt{\frac{1}{m-\tau} + \frac{2\hat{\rho}_\tau^2}{m}}, \tau = 1, 2, \dots, m-1 \quad (8)$$

259 Equation (8) shows that with increasing time-lags τ the precision of the estimated time-correlation
 260 (7) becomes poorer, which makes sense since the number of positions used to estimate the
 261 autocovariance $\hat{\sigma}_\tau$ (5) is $m - \tau$. To see this at work, Figure 4 depicts the autocorrelation function
 262 (7) in green for instantaneous ambiguity-fixed L1+B1 GPS+BDS RTK positioning for the local
 263 Up-component. This is based on 1 h (1 s) of data for ublox+patch at top and ublox+Zephyr at
 264 bottom for the short-baselines in Figure 1. In order to separate the contribution of multipath on the
 265 time-correlation estimates we depict in black color the corresponding autocorrelation for a zero-
 266 baseline ublox+patch antenna setup, which collects independent data with approximately the same
 267 satellite configuration to the short-baselines but about 100 days later. This is given together with
 268 the 95% confidence interval (CI) in red as computed from the STD of the autocorrelation in (8).

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 271 **Fig. 4** Autocorrelation coefficients (green) for L1+B1 GPS+BDS instantaneous short-baseline
 272 ambiguity-fixed positioning errors during 1 h (1 s). The results are based on data on Jan 5 at
 273 02:50 for ublox+patch at top and Jan 8, 2016 at 02:38 (hh:mm) UTC for ublox+Zephyr at bottom.
 274 Autocorrelation coefficients are also depicted for a ublox+patch zero-baseline setup in black color,
 275 on Apr 15, 2016 at 19:39 (hh:mm) UTC. Their corresponding 95% confidence intervals are
 276 depicted as dashed red lines

277
 278 Figure 4 shows that the precision gets poorer with increasing time-lags since the
 279 autocorrelation function is then based on less data. We can also see time-correlation estimates with
 280 a periodic behavior that differ significantly from zero for the short baseline setups, which indicates
 281 that multipath might be present. The short-baseline patch-antenna setup (top) experiences time-
 282 correlation estimates that take several hundreds of seconds to reach about zero, with a reduction
 283 in time to reach zero when the Zephyr-antenna is used (bottom). These decorrelation times are
 284 similar to the phase-multipath time-correlation results found in Ray and Cannon (1999), Miller et
 285 al. (2012), and Pesyna et al. (2014). However the zero-baseline time-correlation estimates in black
 286 color, without multipath, fall reasonably well within the 95% CI and quickly drop to about zero
 287 after a time-lag of one second. This can also be seen in the zoom-in depicted in Figure 5.

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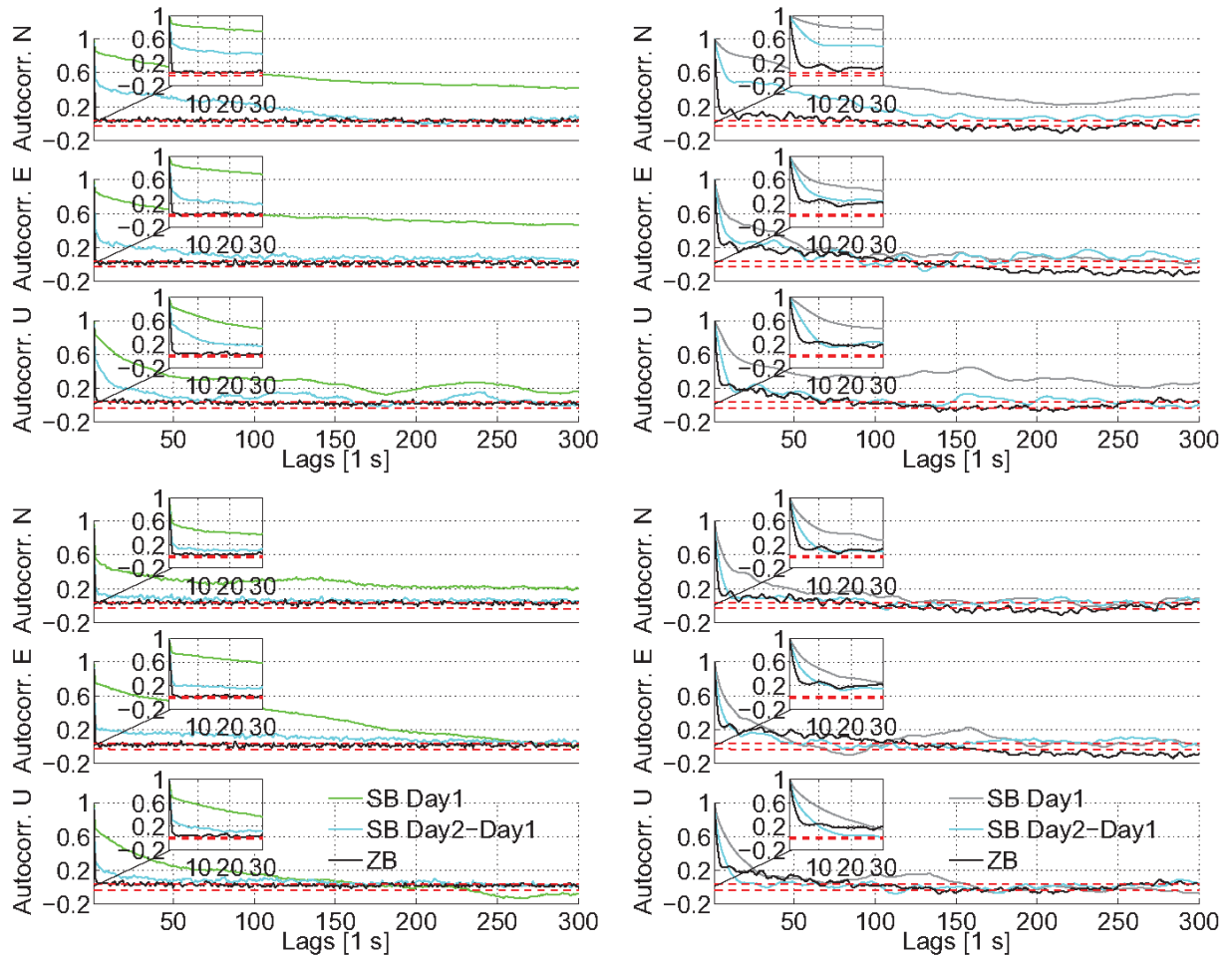
289 Time-correlation when reducing multipath

290 We will now further compare the zero-baseline results, based on independent data, to the short
 291 baselines that were likely influenced by multipath effects (Figure 4). We will attempt to reduce
 292 multipath for the short baselines by performing day-differences on the estimated positions while
 293 taking into account the satellite repeatability period of approximately 23 h and 56 min, similar
 294 to the DD residuals used to compute the code and phase STDs in Table 1.

295 Figure 5 depicts the autocorrelation functions for instantaneous short-baseline ambiguity-
296 fixed (left) and ambiguity-float (right) North/East/Up positioning, for the ublox+patch (top three
297 rows) and ublox+Zephyr (bottom three rows) models based on 1 h (1 s) of data. The
298 corresponding estimates based on ublox+patch zero-baseline data is depicted in black color. This
299 is given together with the 95% confidence interval (CI) for the autocorrelation function. A zoom-
300 in is used to depict the first five minutes (300 s) and 30 s only.

301 Figure 5 shows the overall larger time-correlation estimates for the short-baseline patch
302 antenna model over the 300 s in comparison to the Zephyr antenna. There is a relatively similar
303 magnitude of time-correlation estimates for the zero-baseline (black color) and the day-differenced
304 short-baseline (cyan) positioning time-series between the Zephyr and patch antenna (shown at
305 bottom three rows), particularly after a time-lag of about 10 s. The differences to the zero-baseline
306 are however larger when the patch antenna model is used for the short-baseline (top), which is
307 likely due to larger magnitudes of residual multipath that remain in the day-differenced positioning
308 solutions. Most importantly the zero-baseline L1+B1 phase-driven instantaneous ambiguity-fixed
309 positioning shows time-correlation estimates that drop down close to zero after a time-lag of one
310 second. The code-driven ambiguity-float positioning, however, shows time-correlation
311 coefficients for time-lags up to a few seconds that exceed values of about 0.2 (Table 3). This can,
312 for instance, be caused by receiver-related filtering to bring down the code observation noise level,
313 see also Bona (2000) and Li et al. (2008; 2016).

314 Table 3 summarizes the zero-baseline time-correlation estimates for different time lags
315 between $\tau = 0, \dots, 4$ s, where the Trimble NetRS L1+L2 GPS time-correlation estimates are based
316 on day-differenced short-baseline data. In support of Table 3, Figure 6 shows the corresponding
317 survey-grade Trimble NetRS receiver L1+L2 GPS results. For both L1 and L2 code and phase one
318 can see that the time-correlation drops down to close to zero after one second for the day-
319 differenced positions, where multipath has been largely reduced.



320

321 **Fig. 5** Autocorrelation coefficients for L1+B1 GPS+BDS instantaneous short-baseline ambiguity-
 322 fixed (left) and ambiguity-float (right) positioning errors based on 1 h (1 s). Day1 is Jan 5, at
 323 02:50 (hh:mm) UTC for patch in the top three rows and Jan 8, 2016 at 02:38 (hh:mm) UTC for
 324 Zephyr in bottom three rows. The corresponding autocorrelation coefficients for day-differenced,
 325 Jan 6 - Jan 5 for patch and Jan 9 - Jan 8 for Zephyr, and zero-baseline, ublox+patch at Apr 15 of
 326 2016 at 19:39 (hh:mm) UTC, positioning are also shown. Their corresponding 95% confidence
 327 intervals are depicted as dashed red lines. A zoom-in is given for the first 300 s and 30 s,
 328 respectively

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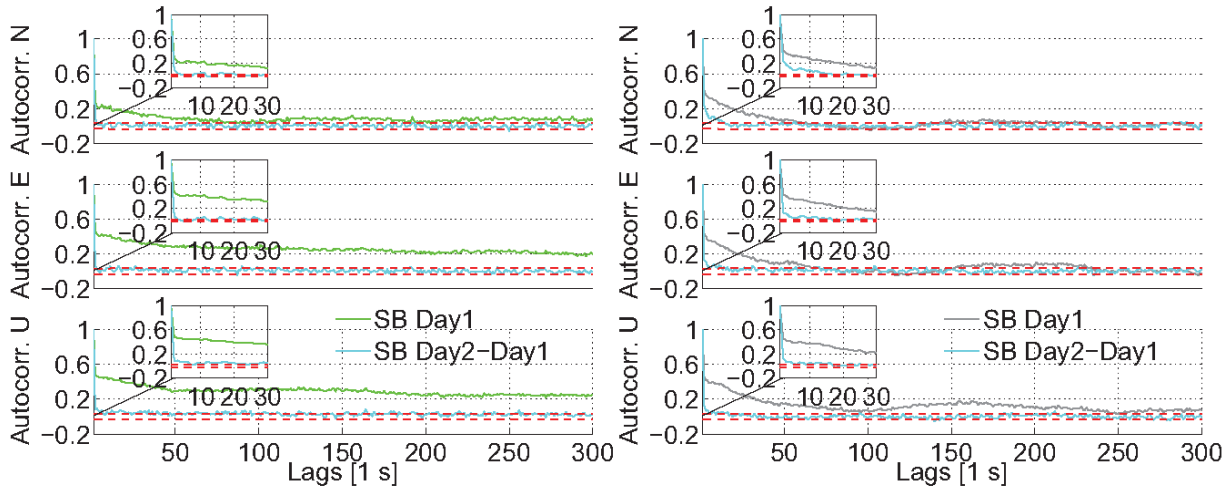
333

334

335 **Table 3** Time-correlation estimates ($\hat{\rho}_\tau$) for different time lags $\tau = 0, \dots, 4$ s of the day-
 336 differenced N/E/U ambiguity-float short-baseline (day-differenced) positioning for Trimble
 337 NetRS L1+L2 GPS, and zero-baseline for the L1+B1 GPS+BDS ublox receivers with patch
 338 antennas. Ambiguity-fixed solutions are shown within parentheses. All depicted results are from
 339 Figures 5 and 6

Receiver/antenna and model	Positioning	Time lag	Autocorrelation coefficient $\hat{\rho}_\tau$		
			N	E	U
Ublox+patch	Float (fixed)	$\tau = 0$ (s)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
L1+B1 GPS+BDS (zero-baseline)	Float (fixed)	$\tau = 1$ (s)	0.634 (0.055)	0.592 (0.055)	0.646 (0.107)
		$\tau = 2$ (s)	0.409 (0.022)	0.322 (0.043)	0.446 (0.056)
		$\tau = 3$ (s)	0.278 (0.019)	0.176 (0.039)	0.311 (0.035)
		$\tau = 4$ (s)	0.245 (-0.017)	0.137 (0.025)	0.254 (0.047)
Trimble	Float (fixed)	$\tau = 0$ (s)	1.000 (1.000)	1.000 (1.000)	1.000 (1.000)
NetRS+Zephyr L1+L2 GPS (day-differenced short-baseline)	Float (fixed)	$\tau = 1$ (s)	0.131 (0.073)	0.236 (0.089)	0.125 (0.081)
		$\tau = 2$ (s)	0.114 (0.015)	0.173 (0.037)	0.065 (0.093)
		$\tau = 3$ (s)	0.063 (0.008)	0.129 (0.014)	0.075 (0.066)
		$\tau = 4$ (s)	0.047 (-0.011)	0.089 (0.018)	0.035 (0.066)

340
341



342
 343 **Fig. 6** Autocorrelation coefficients for Trimble NetRS+Zephyr L1+L2 GPS instantaneous short-
 344 baseline ambiguity-fixed (left) and ambiguity-float (right) positioning errors, based 1 h (1 s).
 345 Day1 is Jan 8, 2016 at 02:38 (hh:mm) UTC. The corresponding autocorrelation coefficients for
 346 day-differenced (Jan 9 - Jan 8, 2016) positioning is also shown. Their corresponding 95%
 347 confidence intervals are depicted as dashed red lines. A zoom-in window is given for the first 300
 348 s and 30 s, respectively

349
350

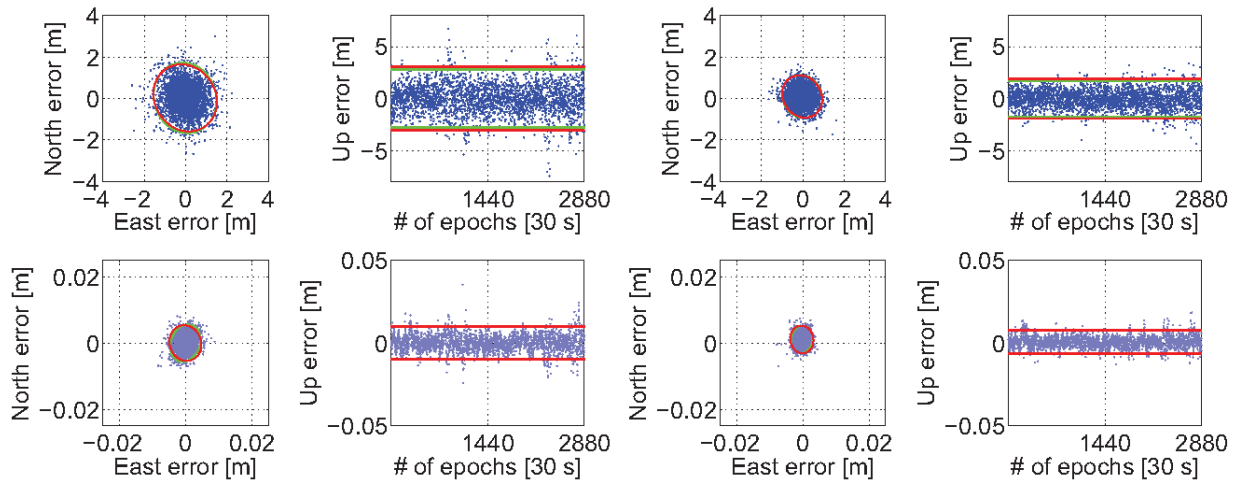
351 **Low-cost RTK positioning**

352 We are now in the position to test our (co)variances that were estimated by LS-VCE. For this
353 purpose, we will use data that is independent of data used to estimate the STDs in Tables 1-2. The
354 positioning stochastic models will be based on STDs estimated from 2 h (1 s) data for the Zephyr
355 antenna models, whereas for the patch antenna model a 24 h period is used so that any multipath
356 effects are more likely to average out over time (Table 1). In the evaluations we neglect the cross-
357 correlation coefficients. To avoid the earlier-referenced receiver-induced time-correlation a 30 s
358 measurement interval will also be used.

360 **Positioning performance**

361 The instantaneous RTK positioning results for 24 h data are shown in Figure 7. The 95%
362 empirical and formal confidence ellipse and interval are shown in green and red, respectively.
363 They were computed from the empirical and formal position variance matrices. The empirical
364 variance matrix was estimated from the positioning errors as obtained from comparing the
365 estimated positions to precise benchmark coordinates. The formal variance matrix used is
366 determined from the mean of all single-epoch formal variance-matrices.

367



368 **Fig. 7** Few-meter baseline horizontal (N, E) position scatter and corresponding vertical (U) time
369 series of the float (top) and correctly fixed (bottom) L1+B1 GPS+BDS single-epoch RTK
370 solutions for an elevation cut-off angle of 10°. The 95% empirical and formal confidence ellipse
371 and interval is shown in green and red, respectively. The 24 h (30 s) period is 22:00-22:00 UTC
372

373 Jan 5-6 for patch in the left two columns and 21:48-21:48 (hh:mm) UTC Jan 8-9, 2016 for Zephyr
374 in the right two columns, which are periods independent of the periods used to determine the
375 stochastic model through the code/phase STDs in Table 2

376

377 Figure 7 shows a good fit between the formal and empirical confidence ellipses and
378 intervals, which thus illustrates realistic LS-VCE STDs in Table 2 that were used in the stochastic
379 model. Note, the improvement by two orders of magnitude when going from float to fixed
380 solutions, and that the ublox+Zephyr antenna model has significantly more precise ambiguity-float
381 positioning solutions in comparison to when patch antennas are used.

382

383 Ambiguity resolution and positioning performance for higher cut-off angles

384 We will investigate the low-cost L1+B1 GPS+BDS positioning performance for high elevation
385 cut-off angles to mimic situations in urban canyon environments or when low-elevation multipath
386 is to be avoided. Comparisons will be made to the survey-grade Trimble NetRS L1+L2 GPS
387 receivers. In Teunissen et al. (2014) and Odolinski and Teunissen (2016) it was shown that a good
388 ambiguity resolution performance does not necessarily imply a good positioning performance,
389 hence we will also investigate what effect this will have on our models at hand.

390 To investigate the ambiguity resolution performance, we consider the ambiguity success-
391 rates (SRs). We can make use of the SR formula of Teunissen (1998),

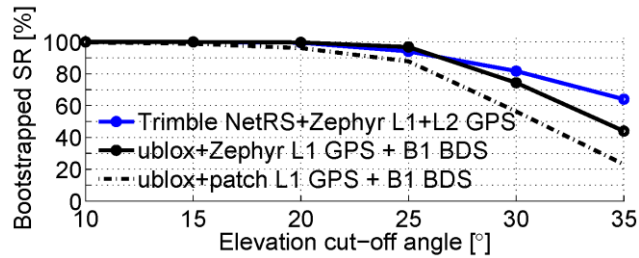
$$392 \quad P[\check{z}_{IB} = z] = \prod_{i=1}^n \left[2\Phi\left(\frac{1}{2\sigma_{\check{z}_{iI}}}\right) - 1 \right] \quad (9)$$

393 where $P[\check{z}_{IB} = z]$ denotes the probability of correct integer estimation of the integer bootstrapped
394 (IB) estimator \check{z}_{IB} and $\sigma_{\check{z}_{iI}}$, $i = 1, \dots, n$, $I = \{1, \dots, (i-1)\}$, denote the conditional STDs of the
395 LAMBDA decorrelated ambiguities. The bootstrapped SR (9) is easy to compute and is also a
396 sharp lower bound of the integer least-squares (ILS) SR (Teunissen 1999).

397 Figure 8 depicts the single-epoch SRs of the two RTK models over two days of data. The
398 SRs are computed based on epochs with the condition of Positional Dilution of Precision (PDOP)
399 ≤ 10 and averaged over all epochs. By including and excluding epochs with large PDOPs, we
400 will show how the positioning performance of the different models are affected by poor receiver-

401 satellite geometries. The results reveal that the L1+B1 ublox+Zephyr (full black lines) has similar
 402 PDOP-conditioned formal ambiguity SRs for cut-off angles up to 25° when compared to the
 403 Trimble NetRS L1+L2 GPS model (blue lines). The L1+B1 ublox+patch model (dotted black
 404 lines) has also a similar performance to the two other models for a cut-off angle of 10°, whereas
 405 the performance gets worse with increasing cut-off angles.

406
 407



408 **Fig. 8** Single-epoch bootstrapped (BS) success-rate (SR) for L1 + L2 GPS and L1 + B1 GPS+BDS,
 409 as function of the elevation cut-off angle. The results are based on data in Jan 4-6 and Jan 6-8,
 410 2016 for the patch and Zephyr antenna models, respectively. The BS SRs are taken as a mean of
 411 all single-epoch SRs over two days, and conditioned on $PDOP \leq 10$
 412

413
 414 The formal results in Figure 8 are very promising. Hence in the next step, we will
 415 investigate the corresponding empirical performance of the different models. To better understand
 416 how this exclusion of epochs with large PDOPs influence the empirical ambiguity-fixed and
 417 ambiguity-float positioning performance, Table 4 shows the corresponding positioning STDs for
 418 two days of data. These STDs were computed by comparing the estimated positions to very precise
 419 benchmarks. In addition to the positioning performance we depict in Table 4 the corresponding
 420 empirical ILS SRs for full ambiguity-resolution, which is given by,

$$421 \quad P_{SE} = \frac{\text{\# of correctly fixed epochs}}{\text{total \# of epochs}} \quad (10)$$

422 In addition and within parentheses the corresponding bootstrapped SRs are given.

423 Table 4 shows that all PDOP-conditioned ILS SRs are consistent with the BS SRs, which
 424 again shows that the STDs in Table 2 used for the stochastic models are realistic. It also shows
 425 that the L1+B1 ublox+patch model has, as expected, smaller SRs in comparison to when the
 426 survey-grade Zephyr antenna is used. Moreover, this latter ublox+Zephyr model has comparable
 427 SRs to the PDOP-conditioned SRs of the Trimble NetRS L1+L2 GPS model for cut-off angles up

428 to 25°, similar to what we showed in the previous formal analysis (Figure 8).

429

430

431

432

433

434 **Table 4** Single-epoch empirical STDs (N, E, U) of correctly ambiguity-fixed positions for
 435 Trimble NetRS L1+L2 GPS, L1+B1 ublox+Zephyr and ublox+patch, respectively, together with
 436 their ILS SR, for 20° – 35° elevation cut-off and 48 hours of data in Jan 4-6 and Jan 6-8 2016,
 437 respectively. The empirical STDs and ILS SRs are also shown when conditioned on PDOP≤10,
 438 and bootstrapped SRs are shown within parentheses

Model	Empirical STDs [mm], ILS SR (%)																
	Cut-off (°):		20				25				30				35		
	N	E	U	SR	N	E	U	SR	N	E	U	SR	N	E	U	SR	
L1+L2 Trimble NetRS	3	2	7	99.9 (99.9)	90	23	114	99.4 (99.3)	73	27	195	93.0 (92.9)	75	37	229	80.2 (80.2)	
PDOP≤ 10	3	2	6	99.5 (99.5)	3	3	8	94.1 (94.0)	4	3	9	81.8 (81.7)	5	3	11	64.1 (64.0)	
L1+B1 ublox+Zephyr	2	1	4	99.8 (99.7)	2	2	5	97.8 (96.8)	2	2	6	77.3 (74.7)	2	2	7	50.3 (45.0)	
PDOP≤ 10									2	2	6	76.7 (74.4)	2	2	6	48.8 (44.0)	
L1+B1 ublox+patch	2	2	6	96.9 (96.1)	3	2	7	89.8 (87.8)	3	2	8	57.3 (56.4)	3	4	12	25.0 (23.2)	
PDOP≤ 10									3	2	8	57.0 (56.3)	3	3	10	24.9 (23.1)	

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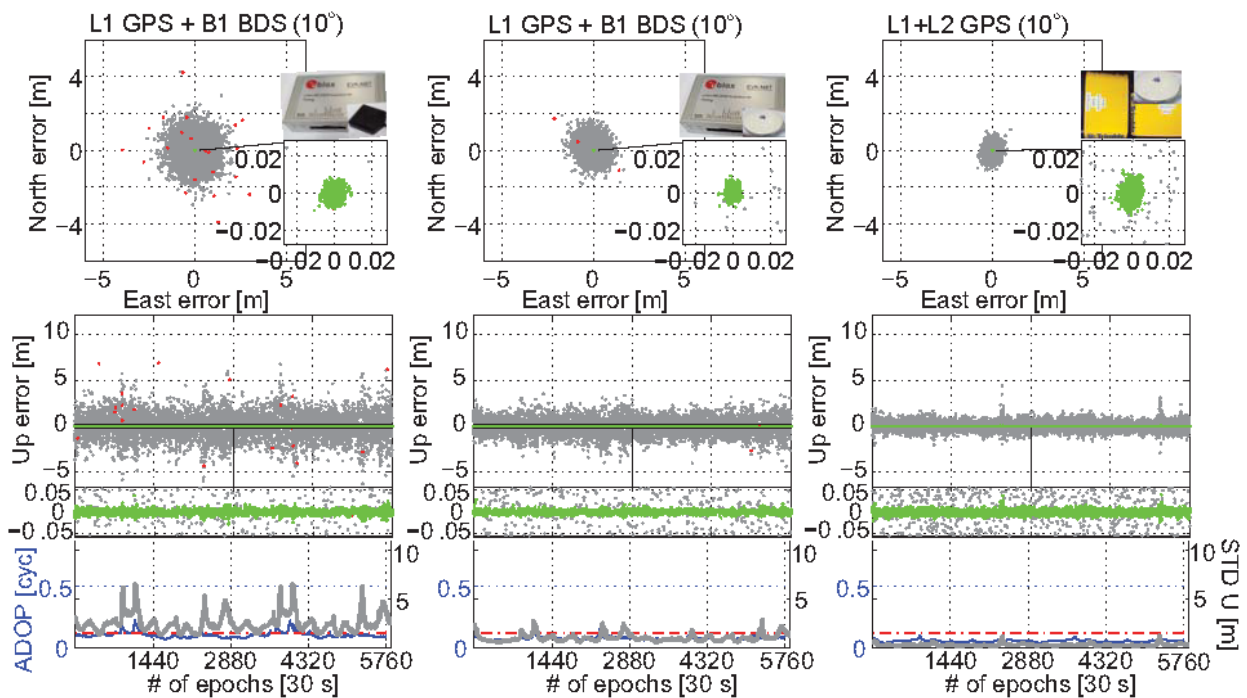
441 In support of a better understanding Table 4, Figures 9-10 shows typical positioning
 442 examples of the different models for an elevation cut-off angle of 10° and 25°, respectively. The
 443 first row shows the local horizontal (N, E) positioning scatterplots and the second row the vertical
 444 (U) time series over two days of data. The float solutions are depicted in gray, incorrectly and
 445 correctly fixed solutions in red and green, respectively. The zoomed-in is given to better show the
 446 spread of the correctly fixed solutions with millimeter-centimeter level precisions. The formal
 447 ambiguity-float STDs are also shown under the Up time series as to reflect consistency between
 448 the empirical and formal positioning results. In addition, we depict in Figure 9 the Ambiguity
 449 Dilution of Precision (ADOP) as introduced by Teunissen (1997), as an easy-to-compute scalar
 450 diagnostic to measure the intrinsic model strength for successful ambiguity resolution. The ADOP

451 is defined as

452
$$ADOP = \sqrt{|Q_{\hat{a}\hat{a}}|}^{\frac{1}{n}} \quad (\text{cycle}) \quad (11)$$

453 with n the dimension of the ambiguity vector, $Q_{\hat{a}\hat{a}}$ is the ambiguity variance matrix, and $|\cdot|$
454 denotes the determinant. ADOP gives a good approximation to the average precision of the
455 ambiguities, and it also provides for a good approximation to the ILS SR (Verhagen 2005). The
456 rule-of-thumb is that an ADOP smaller than about 0.12 cycles corresponds to an ambiguity SR
457 larger than 99.9% (Odijk and Teunissen 2008).

458

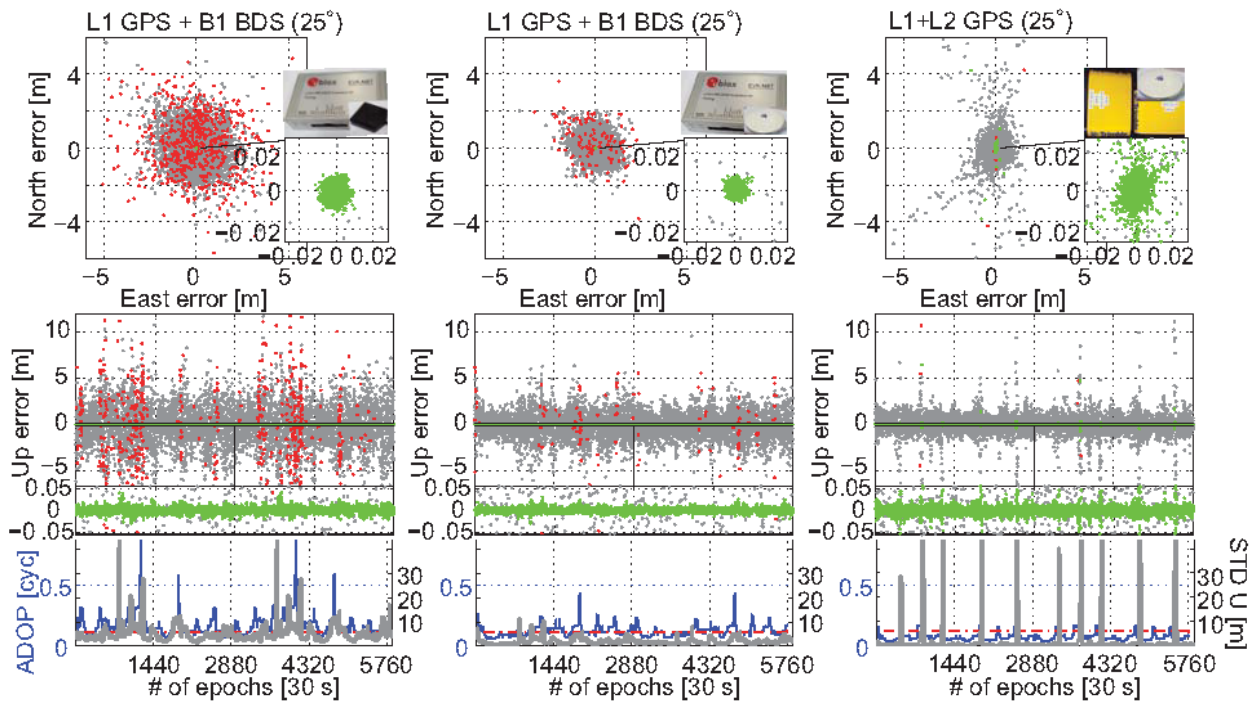


459

460 **Fig. 9** Few-meter baseline horizontal (N, E) scatterplots and vertical (U) time series for L1 + B1
461 ublox+patch (first column) with 99.5% ILS SR, L1 + B1 ublox+Zephyr (second column) with
462 100% ILS SR, and Trimble L1+L2 GPS (third column) with 100% ILS SR, using 10° cut-off.
463 The results are based on data in Jan 4-6 for ublox+patch and Jan 7-8, 2016 for
464 ublox/Trimble+Zephyr. The SRs are conditioned on $PDOP \leq 10$ and computed based on all
465 epochs. The correctly fixed solutions are depicted in green, incorrectly fixed in red, and ambiguity-
466 float in gray. Below the vertical time series the ADOP is depicted in blue color, the 0.12 cycles
467 level as red, and ambiguity-float Up formal STDs are shown in gray

468

469 Figures 9-10 reveal that more solutions are likely to be incorrectly fixed (red dots) when
 470 the ADOPs (blue lines) are larger than the 0.12 cycle level (red dashed lines). Figure 9 also shows
 471 that the L1+B1 ublox+patch model achieves an ILS SR similar to that of the survey-grade L1+L2
 472 GPS model for the cut-off angle of 10°. The SR is namely 99.5% for the L1+B1 ublox+patch
 473 model and 100% for the L1+L2 GPS model. This ILS SR corresponds to the availability of
 474 correctly fixed solutions (green dots) with millimeter to centimeter level positioning precision over
 475 the two days.
 476



477
 478 **Fig. 10** Few-meter baseline horizontal (N, E) scatterplots and vertical (U) time series for L1 + B1
 479 ublox+patch (first column) with 89.8% ILS SR, L1 + B1 ublox+Zephyr (second column) with
 480 97.8% ILS SR, and Trimble L1+L2 GPS (third column) with 94.1% ILS SR, using 25° cut-off.
 481 The results are based on data in Jan 4-6 for ublox+patch and Jan 7-8, 2016 for
 482 ublox/Trimble+Zephyr. The SRs are conditioned on PDOP ≤ 10 and computed based on all
 483 epochs. The correctly fixed solutions are depicted in green, incorrectly fixed in red, and ambiguity-
 484 float in gray. Below the vertical time-series the ADOP is depicted in blue color, the 0.12 cycles
 485 level as red, and ambiguity-float Up formal STDs are shown in gray
 486

487 Figure 10 shows that the L1+B1 ublox patch antenna model has some instances with

488 significantly different ADOPs in comparison to the corresponding Zephyr antenna model for the
489 cut-off angle of 25° , for example just before epochs 1440 and 4320, respectively. This is mainly
490 because the two models are based on different days of data where the BDS MEO satellite
491 constellation does not repeat each day and thus the patch antenna model has a smaller number of
492 satellites for some epochs that result in larger ADOPs. The L1+L2 GPS model has moreover large
493 ambiguity-fixed positioning excursions at the same time as the formal STDs are large for the cut-
494 off angle of 25° , due the poor GPS-only receiver-satellite geometry for this high cut-off angle.
495 This is also reflected by the corresponding relatively large ambiguity-fixed STDs depicted in Table
496 4 that are improved from decimeter- to millimeter-level when the $\text{PDOP} \leq 10$ condition is applied.
497 Finally, Figure 10 shows that L1+B1 ublox+Zephyr has a larger SR of 97.8% when compared to
498 the PDOP-conditioned SR for L1+L2 GPS of 94.1% for the cut-off angle of 25° (Table 4),
499 owing to the use of BDS that improves the receiver-satellite geometry.
500

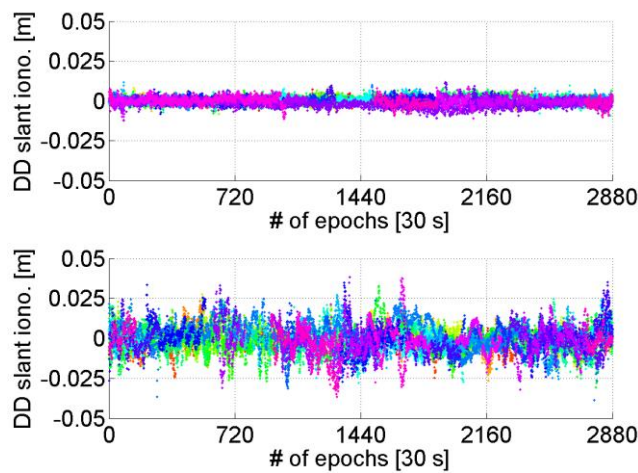
501 **Low-cost RTK positioning for a 7 km baseline**

502 So far we have considered baseline lengths of a few meters only. One might wonder whether the
503 low-cost L1+B1 GPS+BDS ublox RTK solution would perform similarly well for baselines where
504 small residual ionospheric delays are present. Hence in the following we analyze the RTK
505 positioning performance for a 7 km baseline in Dunedin for 24 h (30 s) during Jan 18-19, 2016
506 (Figure 1). The Australian Space Forecast centre (<http://www.sws.bom.gov.au/Geophysical/2/1>)
507 warned for an increased geomagnetic disturbance during this period, which implies that residual
508 ionospheric delays can indeed be present. We make use of the same stochastic model settings as
509 in the previous Section (Table 2), which is based on an independent baseline.

510 To deal with any residual ionospheric delays we also add to our model (1) a vector with
511 slant ionosphere pseudo-observations $\iota = [\iota_G^T, \iota_B^T]^T \in \mathbb{R}^{s_G + s_B}$ and parameterize the ionospheric
512 delays. The inclusions of these pseudo-observables can provide us with stochastic information of
513 the ionospheric delays between stations, which is also referred to as the "ionosphere-weighted"
514 model. The stochastic model settings of these observables are set by following the rule of thumb
515 by Schaffrin and Bock (1988), where the double-differenced (DD) slant ionospheric delay zenith-
516 referenced STD can be modeled as a function of the baseline length as 1.4 mm/km. For a baseline
517 length of 7 km this corresponds to a DD STD of almost 10 mm.

518 As to verify whether slant ionospheric delays are indeed present, we depict in Figure 11
519 the DD slant ionospheric delays with the ionosphere-weighted stochastic model settings mentioned
520 above, while making use of a L1+B1 ublox+Zephyr geometry-fixed and ambiguity-fixed model.
521 The DD slant ionospheric delays are shown for all satellites (GPS+BDS) with an elevation cut-off
522 angle of 20°, as to avoid any low-elevation multipath on our estimates. At top of Figure 11 the
523 few-meter baseline setup is shown to illustrate the situation when ionospheric delays are not
524 present, whereas at bottom the corresponding 7 km baseline results are depicted.

525
526



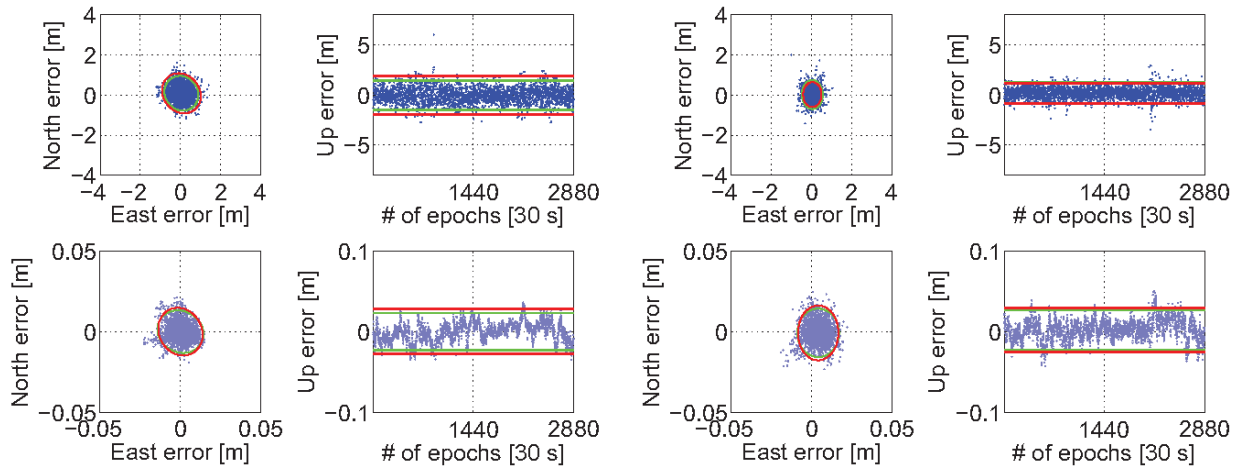
527
528 **Fig. 11** DD ambiguity-fixed slant ionospheric delays using L1+B1 GPS+BDS ublox+Zephyr for
529 a few meter baseline (top) on Jan 7 and seven kilometer baseline (bottom) on Jan 18, 2016 for an
530 elevation cut-off angle of 20°. The ionosphere-weighted model has been used with the DD slant
531 ionospheric delay zenith-referenced STD set to approximately 10 mm for both models

532
533 Figure 11 shows, as expected, that the magnitudes of the slant ionospheric delays for the
534 few-meter baseline resemble that of the phase noise precision (Figure 3). The 7 km baseline,
535 however, show significantly larger magnitudes of the slant ionospheric delays that exceeds the
536 phase noise precision, which indicates that the slant ionospheric delays should indeed be modeled.

537 To investigate whether the stochastic model settings as determined from a few meter
538 baseline is also applicable to the 7 km baseline, we show in Figure 12 the corresponding
539 instantaneous RTK positioning results for ublox+Zephyr L1+B1 GPS+BDS and Trimble
540 NetRS+Zephyr L1+L2 GPS, while making use of an elevation cut-off angle of 10°. There is a
541 good fit between the formal and empirical positioning confidence ellipses and intervals, which

542 thus again illustrates realistic LS-VCE STDs in Table 2 that were used in the stochastic model.
543 This in addition to the STD used for the DD slant ionosphere pseudo-observations.

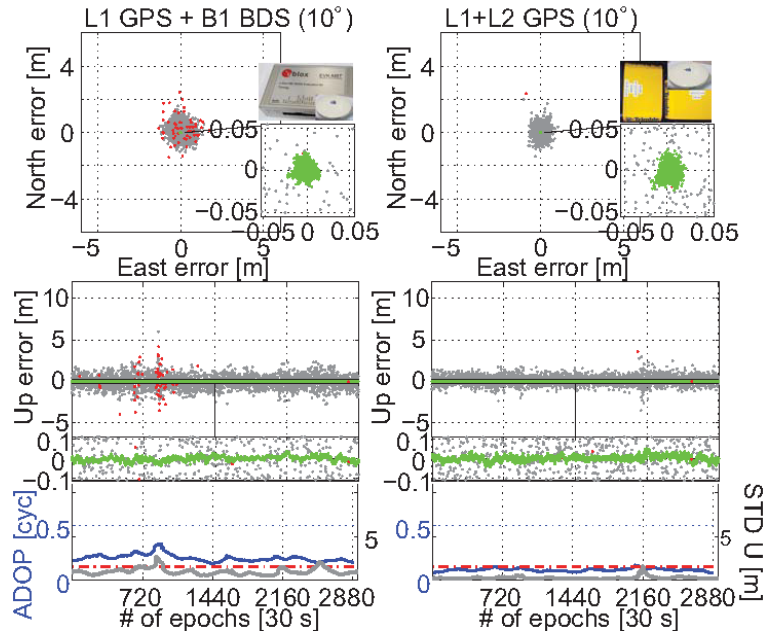
544



545 **Fig. 12** Seven kilometer baseline horizontal (N, E) position scatter and corresponding vertical (U)
546 time-series of the float (top) and correctly fixed (bottom) L1+B1 GPS+BDS (left two columns)
547 and L1+L2 GPS (right two columns) single-epoch RTK solutions for an elevation cut-off angle of
548 10° . The 95% empirical and formal confidence ellipse and interval is shown in green and red,
549 respectively. The 24 h (30 s) period is 23:0-23:00 UTC Jan 18-19, 2016, where the baseline is
550 independent of the few meter baseline used to determine the stochastic model through the
551 code/phase STDs in Table 2

553
554 Figure 13 depicts the corresponding float, incorrectly and correctly fixed positioning
555 solutions at top, respectively, together with the ADOPs and ambiguity-float Up formal STDs at
556 the bottom. The figure reveals that the L1+B1 GPS+BDS model has some instances around and
557 after epoch 720 when the ADOP (blue color) is relatively large which consequently yields
558 incorrectly fixed solutions (red). The instantaneous ILS SR for the L1+B1 ublox+Zephyr model is
559 97%, whereas the corresponding ILS SR for the Trimble NetRS L1+L2 model is 99.8%. We have
560 thus illustrated that the low-cost receiver-solution still has the potential to perform very well even
561 for a baseline length of 7 km, where small residual ionospheric delays are present.

562



563

564 **Fig. 13** Seven kilometer baseline horizontal (N, E) scatterplots and vertical (U) time series for L1
 565 + B1 ublox+Zephyr (first column) with 97.0% ILS SR, and Trimble L1+L2 GPS (second
 566 column) with 99.8% ILS SR, using 10° cut-off during Jan 18-19, 2016. The correctly fixed
 567 solutions are depicted in green, incorrectly fixed in red, and ambiguity-float in gray. Below the
 568 vertical time series the ADOP is depicted in blue color, the 0.12 cycles level as red, and ambiguity-
 569 float Up formal STDs are shown in gray

570

571 Conclusions

572 We evaluated a low-cost ublox L1+B1 GPS+BDS RTK model and compared its ambiguity
 573 resolution and positioning performance to a survey-grade receiver L1+L2 GPS solution, in
 574 Dunedin, New Zealand. The least-squares variance component estimation (LS-VCE) procedure
 575 was initially used to determine the (co)variances of the low-cost receivers. The estimated
 576 (co)variances are needed so as to formulate a realistic stochastic model. Otherwise the ambiguity
 577 resolution performance and hence the achievable positioning precisions would deteriorate. For the
 578 same reasons, we also investigated the presence of receiver-induced time-correlation. Since we
 579 analyzed a short baseline, the LS-VCE and time-correlation estimates were shown to likely be
 580 affected by multipath. To mitigate multipath we connected the low-cost receivers to survey-grade
 581 antennas and compared the performance to a zero-baseline setup.

582 It was shown that the survey-grade antennas can significantly improve the performance for

583 the low-cost receivers so that the code and phase noise estimates more resemble that of survey-
584 grade receivers. The cross-correlations between GPS L1 code and phase and BDS B1 code and
585 phase were all shown to be close to zero for the low-cost receivers at hand, whereas significant
586 cross-correlation was present for the L1 and L2 phase observables of the survey-grade receivers.
587 The receiver-induced time correlation was also shown to be close to zero for phase in both receiver
588 types, whereas some time-correlation existed for the code observables in the low-cost receivers.
589 The LS-VCE STDs were shown to be realistically estimated for both an independent period and
590 baseline. We also demonstrated that the low-cost receivers, which cost a few hundred USDs, can
591 give competitive instantaneous ambiguity resolution and positioning performance to the survey-
592 grade receivers, which cost several thousand USDs. This was shown both formally and
593 empirically, and is particularly true when the low-cost receivers are connected to survey-grade
594 antennas. It was finally shown that the low-cost receiver-solution with survey-grade antennas still
595 has the potential to achieve competitive ambiguity resolution and positioning performance to the
596 survey-grade receiver solution for a baseline length of 7 km, where small residual slant
597 ionospheric delays are present.

598

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603

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719 **Authors Biographies**



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Robert Odolinski conducted his Ph.D. studies at Curtin University, Australia in 2011 to 2014. His research focus is next generation multi-GNSS integer ambiguity resolution enabled precise positioning. In 2015 Robert started his position as a Lecturer/Research Fellow in Geodesy/GNSS at the School of Surveying, University of Otago, New Zealand. He is a member of the Joint Study Group in Multi-GNSS theory and algorithms of the International Association of Geodesy (IAG).

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