GNSS-based heighting in Australia: current, emerging and future issues

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ABSTRACT

Ellipsoidal heights, i.e., w.r.t. a geometrical Earth figure, determined from Global Navigation Satellite Systems (GNSS) are inherently their least accurate coordinate, due mainly to satellite geometry and atmospheric refraction. For most practical purposes, however, these GNSS-derived ellipsoidal heights have to be transformed to heights that relate to the Earth's gravity field, which generally adds further uncertainty. The reduction in accuracy of the transformed height is due to errors in gravimetric quasi/geoid models, but this is compounded yet further in Australia – and elsewhere – because of the imperfect realisation of local vertical datums. This paper comments upon current, emerging and future issues with height determination on the Australian Height Datum (AHD) using GNSS. This comprises the reference frame used for GNSS ellipsoidal heights, theory- and data-driven inaccuracies in modelling the quasi/geoid, and deficiencies in the realisation of the AHD. While some of these issues will be redressed, in part, by the production of AUSGeoid2008 that is fitted to the AHD, there will always be the need to routinely apply checks on GNSS-derived heights in Australia, and elsewhere.

INTRODUCTION AND BACKGROUND

Global Navigation Satellite Systems (GNSS), notably the Global Positioning System (GPS), yield ellipsoidal (geodetic) heights relative to the surface of a geodetic reference ellipsoid, which are transformed from the Cartesian coordinates used in the GNSS data processing. Typically, WGS84 is the geodetic reference ellipsoid used since this is embedded in GPS and is generally the default in GPS processing software, but which is geometrically practically identical to the GRS80 geodetic reference ellipsoid (to less than 0.1mm!). The ellipsoidal height (h) is measured positively above the ellipsoid (away from the Earth) and negatively below, and along the ellipsoidal surface normal.

In south western Australia, for example, GPS-derived ellipsoidal heights on low-lying coastal land can be around 30 m *below* WGS84 or GRS80 (i.e., h = –30m). Similarly, in northern Queensland, GPS-only heights on low-lying land can be around 70 m *above* WGS84 or GRS80 (i.e., h = +70m). To most lay users, these GPS-only heights will be counterintuitive. Therefore, ellipsoidal heights have to be transformed to physically and intuitively meaningful heights related to gravity, especially when concerning fluid flows. For consistency with existing heights, they should be transformed to heights that are compatible with the local vertical datum (LVD) in use and thus also relate to local mean sea-level (MSL).

The use of GPS for such height determination was first discussed by Engelis et al. (1984; 1985). Several reviews and descriptions have already been published on GPS-based heighting in Australia (e.g., Gilliland, 1986; Kearsley, 1988b; Mitchell, 1988, 1990; Jaksa et al., 1991; Kearsley et al., 1993; Featherstone and Alexander, 1996; Steed and Hotznagel, 1994; Featherstone, 1998; Featherstone and Kuhn, 2006), so these will not be duplicated here. Instead, the focus of this paper is to examine the issues surrounding the problems that GNSS users now encounter when determining AHD heights.

If the LVD is defined in terms of [approximated; see later] orthometric heights, then a geoid model is needed for the transformation (cf. Meyer *et al.*, 2006); if the LVD is defined in terms of normal or normal-orthometric heights, then a quasigeoid model is needed (Featherstone and Kuhn, 2006). This is an algebraic transform, where the geoid-ellipsoid separation (N) is subtracted from the ellipsoidal height (h) to give the orthometric height (H), or the quasigeoid-ellipsoid separation or height anomaly (ζ) is subtracted from the ellipsoidal height to give the normal or normal-orthometric height (H_N). Since geoid and quasigeoid models contain errors, as do LVDs, then additional transformations are needed to account for these inconsistencies, both of which will be discussed later.

Another limitation to GNSS-based height determination is that the ellipsoidal height is inherently less accurate than horizontal position. This is caused by a combination of error sources, but the major contributors are inaccurately modelled atmospheric refraction and the geometry of the resection where satellites are always situated above the GPS receiver (for ground-based applications). As such, GNSS-based height determination will always be poorer than horizontal positioning because of 1) errors inherent in the GPS-derived ellipsoidal heights and 2) the subsequent coordinate transformation(s) to get heights that are compatible with the LVD.

Despite these inadequacies, GPS has established itself as a competitor to low-order spirit-levelling over long distances, and is generally superior to long-range trigonometric heighting, provided that a sufficiently accurate coordinate transformation can be achieved. For instance, to spirit-level 50 km takes around one working week on reasonably flat ground with good visibility, whereas it takes only a few hours with dual-frequency carrier-phase GPS. Therefore, GPS offers an attractive alternative height determination tool, but only provided that it is sufficiently accurate for the application at hand.

Since November 1998, GPS users in Australia have had access to the AUSGeoid98 model (Featherstone *et al*., 2001). The term quasigeoid is more appropriate than geoid because the Australian Height Datum (AHD) is an approximation of a normal-orthometric height system (Roelse *et al*., 1971; 1975; Holloway, 1988; Featherstone and Kuhn, 2006), so is more compatible with the quasigeoid than the geoid. The quasigeoid makes no assumptions about the Earth's internal mass-density distribution (Molodensky *et al*., 1962; Heiskanen and Moritz, 1967). AUSGeoid98 is not strictly a quasigeoid model, however, as the full Molodensky theory was not used. However, the difference between the geoid and quasigeoid over Australia is probably less than 15 cm (Featherstone and Kirby, 1998).

Despite its proven utility in many cases, AUSGeoid98 still does not meet expected accuracy requirements in all areas of Australia as a complete replacement for class LC (Intergovernmental Committee on Surveying and Mapping, 2007) spirit-levelling on the AHD (e.g., Featherstone and Guo, 2001). This has become exacerbated in an absolute sense (cf. Featherstone, 2001a) when using single-point GPS techniques, such as precise point positioning (PPP) (Zumberge et al., 1997; Kouba and Héroux, 2001; Castleden *et al.*, 2004), or relative carrier-phase GPS over very long baselines, such as from the AUSPOS (http://www.ga.gov.au/geodesy/sgc/wwwgps/; Dawson *et al.*, 2001) service (Featherstone and Dent, 2002). As such, it is often necessary to apply post-survey adjustments to the heights (cf. Collier and Croft, 1997a,b; Featherstone *et al.*, 1998; Iliffe *et al.*, 2000), which is particularly inconvenient for real-time kinematic (RTK) GNSS surveying (Featherstone and Stewart, 2001).

While it is difficult to isolate the source of the error between AUSGeoid98 and the AHD – the so-called inseparability problem (Featherstone, 2004) – there is now a body of rather compelling evidence of fundamental problems with the practical realisation of the AHD (e.g., Roelse *et al.*, 1971; 1975; Featherstone, 1998, 2002a, 2004; Featherstone and Stewart, 1998; Featherstone and Kuhn, 2006; Filmer and Featherstone, 2008; Featherstone and Filmer, 2008). These problems were confirmed independently from a comparison with astrogeodetic vertical deflections across Australia (Featherstone, 2006) and from a simulated error-free gravity field model (Baran *et al.*, 2006). In short, there is a north-south slope of about 1.5 m and higher order distortions of around 50 cm.

The Intergovernmental Committee on Surveying and Mapping has chosen to retain the AHD for the "foreseeable future". Therefore, it is necessary to address the practical problems now caused by absolute, long-baseline and real-time AHD height determination from GNSS. The new quasigeoid model of Australia (being computed at present) will therefore comprise two solutions: a scientific gravimetric-only quasigeoid model from improved data, theories and computational techniques; and a practical 'geoid-type' product for the more direct transformation of GNSS heights to the AHD and *vice versa* (cf. Featherstone, 1998). This approach has been used in the USA for many years (e.g., Milbert, 1995; Smith and Milbert, 1999; Smith and Roman, 2001), in the UK (Iliffe *et al*., 2003), and in many other counties (too many to cite here).

The 'geoid-type' product, probably to be called AUSGeoid2008 to avoid confusion, will result from fitting the new gravimetric quasigeoid model to the pointwise-defined reference surface of the AHD at GPS-levelling stations (cf. Featherstone, 1998, 2000a; Fotopoulos *et al*., 2003; Featherstone and Sproule, 2006; Soltanpour et al., 2006). Both models will refer to the GRS80 ellipsoid (Moritz, 1980a), so will be compatible with the Geocentric Datum of Australia 1994 (GDA94). A new grid of Pizzetti vertical deflections (cf. Featherstone and Rüeger, 2000; Featherstone, 2006) will also be computed from the gravimetric quasigeoid gradients and released with the geoid-type model, as was the case for AUSGeoid98.

As for AUSGeoid98, the Western Australian Centre for Geodesy and our collaborators will supply software and techniques for AUSGeoid2008 to Geoscience Australia (GA). In order to avoid user confusion and data management problems, only the fitted 'geoid-type' product (cf. Vermeer, 1998) and Pizzetti vertical deflections will be released over the web free-of-charge. The scientific gravimetric-only quasigeoid will only be released on a user-requested basis, but with clear *caveat emptors* so that the two models do not become mixed. Unrestricted release of the two solutions would cause confusion for GNSS users and serious problems for managers of geodetic databases.

This paper examines the current, emerging and future issues surrounding height determination on the AHD using GNSS, comprising the reference frame used for GNSS-derived ellipsoidal heights, theory- and data-driven inaccuracies in modelling the quasi/geoid, and deficiencies in the realisation of the AHD. Some consideration will be given to the methods currently being considered to compute the new Australian gravimetric quasigeoid model and the 'geoid-type' product for more direct GNSS heighting on the AHD. It is essential to point out that these are entirely different surfaces: one is theoretically exact regarding the Earth's gravity field, and the other is a pragmatic product to ease the activities of GNSS users in Australia.

GNSS ELLIPSOIDAL HEIGHTS

As stated, the ellipsoidal height is the least accurately GNSS-determined coordinate, mainly because of atmospheric refraction coupled with the geometry of the resection (e.g., Dodson, 1995; Rothacher, 2002), and thus will probably never reach the same accuracy as GNSS-determined horizontal positions. Unmodelled atmospheric refraction affects the pseudoranges to the satellites, which is then exacerbated by geometry where all satellites are above the receiver for Earth-bound applications. However, GNSS-height determination can be made more reliable by considering the following practical issues.

In the sequel, we will only assume dual-frequency carrier-phase observations, as the "Rolls Royce" of GPS positioning methods. First, the data span should be as long as feasibly possible. This allows for more redundancy in the least-squares position solution. To partly counter the geometry problem, the cut-off elevation can be reduced from the usual 15 degrees to 10 or even 5 degrees. However, this is at the risk of increased multipath. If multipath affects the solution (usually found from a cyclical pattern in the carrier-phase residuals, especially for low-elevation satellites), then the cut-off angle can be increased in the software or the offending satellite removed from the solution. Good quality commercial GNSS data processing packages normally offer these options.

Though seemingly simple, the measurement and specification of the antenna height is probably the 'weakest link' in the determination of GNSS ellipsoidal heights. The position solution is actually computed at the electrical phase centre of the antenna, which is often slightly different for the L1 and L2 GPS frequencies (e.g., Rothacher, 2001). It also varies as a function of the elevation of the satellite. Specifying the wrong antenna type will mean that the software-based phase-centre variation correction and phase centre offset to the antenna reference point (ARP) will be wrong, up to 15cm in some cases (e.g., Ebner and Featherstone, 2008). Given that a long-as-possible occupation should be used, there is sufficient time for the field operator to carefully calculate the true vertical height of the ARP as a check. The antenna height should also be measured at the start and end of the occupation.

Also, several national geodetic agencies (e.g., Zilkoski *et al.*, 1997; Land Information New Zealand, 2003; Intergovernmental Committee on Surveying and Mapping, 2007) provide standards and recommended practices (SARPs) for the determination of GNSS heights. These are normally based on collective experience of practicing geodesists in these agencies. However, SARPs offer no guarantee that the position will be accurate; instead, they are only probabilistic. That is, even if the SARPs are followed they will not necessarily guarantee correct results, but it is more likely than if they are not followed.

It is also important to consider the reference frame / datum used for the GNSS-derived ellipsoidal heights (cf. Kotsakis, 2008), as different reference frame realisations can cause discrepancies of several centimetres (e.g., Smith and Roman, 2001). Johnston (GA, 2008, pers. comm.) advises that the International Terrestrial Reference Frame 2005 (ITRF2005; Altamimi *et al.*, 2007) will be used for ellipsoidal heights in Australia. Therefore, the AUSGeoid2008 model will be fitted to GPS-AHD heights on this datum. However, great care will be needed to ensure that GDA94 ellipsoidal heights are not used with the new model. A comparison of around 200 ITRF2005 and GDA94 ellipsoidal heights across Western Australia shows a mean difference of ~3 cm, but it reached ~18cm in one case.

There is the related consideration of the 'purity' of the ellipsoidal heights used at the GPS base station for a relative GPS survey. At present, the GDA94 coordinates are entered for the GPS base station, and then the baseline vector used to calculate the ellipsoidal height at the remote station. Of course, if there is an error at the base station (e.g., from a different realisation of ITRF or the GDA94), this will contaminate ellipsoidal heights at all remote stations. If the base station has previously been occupied with GPS and tied geodetically to the ITRF, then the ellipsoidal height will be 'pure'.

However, pure ellipsoidal heights might not be available at the base station, making it necessary to derive an ellipsoidal height from an AHD height and a quasi/geoid model. The problem is that this generates an 'impure' ellipsoidal height (it is derived and not observed). The amount of error in this impure ellipsoidal height is difficult to quantify because errors in the quasi/geoid model and AHD height vary spatially (discussed later) and combine. This problem will be alleviated slightly when AUSGeoid2008 is released because it will be aligned more with the AHD (cf. Featherstone, 1998, and see later).

NEW AUSTRALIAN GRAVIMETRIC QUASIGEOID

Computation of Australian quasi/geoid models has occupied geodesists for over four decades, which has been reviewed by Kearsley and Govind (1991) and extended by Featherstone et al. (2001). As well as these national geoid models, regional geoid models have been computed for experimental purposes (Featherstone *et al*., 1996, 1997; Freund *et al*., 1997; Kirby *et al*., 1997; Featherstone and Sideris, 1998; Forsberg and Featherstone, 1998; Higgins *et al*., 1998; Vella and Featherstone, 1999; Claessens *et al*., 2001; Featherstone et al., 2004; Kirby, 2003; Featherstone, 2007). However, only three national standards of model have been released by GA (and its predecessor agencies): AUSGeoid91 (Kearsley and Govind, 1991), AUSGeoid93 (Steed and Hotznagel, 1994) and AUSGeoid98 (Featherstone *et al*., 2001).

We are now in the process of computing a new Australian gravimetric quasigeoid model, which will then be fitted to the AHD via GPS at benchmarks. We deliberately awaited the April 2008 release of the EGM2008 global geopotential model (Pavlis *et al*., 2008) and satellite-altimeter-derived gravity anomalies from re-tracked waveform data

(Andersen *et al.*, 2008). Given the timeframe for this article in a special issue, we can therefore only speculate on the methods that will ultimately be used for AUSGeoid2008. While we could have released a new Australian quasigeoid model several years ago, we felt that it was preferable to wait until the latest datasets became available. Hopefully, this will lead to a 'product' that may match the longevity of AUSGeoid98.

Regional gravimetric quasigeoid models are generally based on some adaptation of Stokes's integral, which can be altered to compute the quasigeoid via the Molodensky *et al.* (1962) theory (e.g., Heiskanen and Moritz, 1967). Essentially, there are two main schools of thought (cf. Sjöberg, 2005): some choose the remove-compute-restore (RCR) technique, and others choose the modified kernel approach. Neither has been proven unequivocally superior, and results vary from region to region. This is why it is important to continue to test both approaches in the Australian context (Featherstone, 2002c; Featherstone *et al.*, 2004). Given that we know that the AHD is based on an [approximated] normal-orthometric height system (Featherstone and Kuhn, 2006), a quasigeoid computation appears the more appropriate.

The approach that was found to be the most effective for AUSGeoid98 was a hybrid combination of the RCR technique with a low-degree deterministically modified kernel (Featherstone *et al.*, 1998b, 2001) and a limited spherical cap about the computation points (Forsberg and Featherstone, 1998). These have been implemented in the one-dimensional FFT (Haagmans *et al.*, 1993) so that the computations are numerically very efficient (Featherstone and Sideris, 1998). For instance, an Australia-wide gravimetric quasigeoid model at a one-arc-minute grid-spacing can be computed in a few days on a medium-performance workstation.

We have previously verified that our computer software and mathematical models are working correctly (Featherstone and Olliver, 1997; Novàk *et al.*, 2001; Featherstone, 2002c). We plan to use our realistic synthetic gravity field model of Australia (Baran *et al.*, 2006) as yet another validation, which we will try to run simultaneously with the computation of AUSGeoid2008. This will give a better indication of the errors in

AUSGeoid2008 that come from observational data. It could be feasible to provide an error map to accompany AUSGeoid2008, but this is a considerable task so may only be released at a later date.

Since AUSGeoid98 was computed, physical geodesists have provided seemingly improved mathematical models for the computation of the quasigeoid (too many to cite here). While these new approaches appear theoretically sound, it is essential to continue to test them in the Australian context. The new theoretical developments that we have implemented so far include downward-continuation corrections to the satellite-derived gravity data (cf. Nsombo and Sjöberg, 1996; Sjöberg, 1999), ellipsoidal corrections to the spherical boundary-value problem (Claessens, 2006; Hipkin, 2004), and implementation of filters by way of modified Stokes's integration kernels (Vaníček and Featherstone, 1998; Featherstone, 2003a).

The long- and medium-wavelength components of AUSGeoid2008 will most probably come from EGM2008. From our initial analysis as part of an International Association of Geodesy (IAG) study group (Claessens and Featherstone, 2008) to evaluate preliminary solutions of this new model, EGM2008 shows a significant improvement on its predecessor, EGM96 (Lemoine *et al*., 1998), as well as upon AUSGeoid98 in several regions. EGM2008 uses data from the Gravity Recovery and Climate Experiment (GRACE) satellite gravimetry mission (Tapley *et al*., 2004), terrestrial gravity data, a digital elevation model (DEM) derived from the Shuttle Radar Topography Mission (SRTM). It extends to spherical harmonic degree and order 2160, which corresponds to a grid spacing of 5 arc-minutes on the Earth's surface (~8 km at Australian latitudes).

To compute a global geopotential model to spherical harmonic degree and order 2160 is a massive computational undertaking, but this only really became possible because of high-degree spherical harmonic analysis and synthesis routines (Holmes and Featherstone, 2002a,b). As with our previous studies (e.g., Amos and Featherstone, 2003), EGM2008 is currently being compared with Australian gravity anomalies, GPS-levelling and vertical deflections (cf. Featherstone, 2006). This will supersede the study in Claessens and Featherstone (2008).

We also ensured that recent Australian datasets were supplied to the EGM2008 development team. EGM2008 will also include new gravity and terrain data from previously unsurveyed parts of the world. For instance, the Arctic, Mongolia, Ethiopia and Malaysia have been covered with airborne gravity measurements. The SRTM DEM has also provided terrain data in previously unsurveyed areas. The marine gravity data comes from re-tracked satellite altimetry (cf. Deng and Featherstone, 2006; Sandwell and Smith, 2005), which makes some improvements in the notoriously problematic coastal zone (Deng et al., 2002, 2005; Hipkin, 2000; Andersen and Knudsen, 2000; Hwang et al., 2006).

Although EGM2008 is a large improvement on EGM96, the Australian gravity and terrain data will be used twice: once to compute EGM2008, then again to compute AUSGeoid2008. This introduces unwanted correlations of the data errors, which are not yet well understood. Using a satellite-only global geopotential model avoids such correlations (Vaníček and Sjöberg, 1991). The truncation bias can be computed explicitly for a modified Stokes kernel and the EGM2008 model (e.g., Featherstone et al., 2004). In this scenario, the satellite-only solution is used to avoid correlations, but EGM2008 is used to add medium-frequency information. These alternative approaches are currently being tested numerically in Australia.

AUSGeoid2008 will use a more accurate treatment of the degree-zero term in EGM2008, where the difference in potential is now taken into account to better define the scale of the quasigeoid model (cf. Kirby and Featherstone, 1997). The degree-one term remains inadmissible assuming that both EGM2008 and GRS80 are co-located at the geocentre. An ellipsoidal correction will be applied to the gravity anomalies computed from EGM2008 (cf. Hipkin, 2004). As these ellipsoidal corrections only apply to the global geopotential model, additional corrections may be needed to the quasigeoid contribution from the terrestrial gravity data (cf. Claessens, 2006).

Since AUSGeoid98 was computed, approximately a quarter of a million land gravity observations have been added to GA's land gravity database (Murray, 1997). These are mainly in the form of spatially dense regional surveys for resource exploration. Most of these new gravity surveys have been positioned with GPS and an unspecified geoid or quasigeoid model, which gives rise to a 'circular argument' in that the same data will be used to compute a quasigeoid model. However, the GPS-derived heights are probably more accurate than the barometric heighting used for most of the national gravity database (Murray, 1997), and most of the benefit will come from more data being used to compute mean gravity anomalies for the Stokes integration.

The land gravity data will be processed in largely the same way as for AUSGeoid98 (cf. Featherstone et al., 2001), but the terrain corrections (described later) will be of much higher spatial resolution from an improved DEM. We anticipate a version 3 DEM soon. We will also apply more advanced data cleaning procedures. This has been fruitful, because Sproule et al. (2006) show that only a couple of hundred land gravity measurements are probably in gross error (0.018% of the whole database), which bodes well for previous Australian quasigeoid models in that errors have not contaminated them too much. Naturally, these newly found erroneous data will be removed.

We will use independent GRACE data to detect the more serious long-wavelength systematic errors in the land gravity anomalies. Long-wavelength terrestrial gravity anomaly errors can degrade the gravimetric quasigeoid model, because quasigeoid computation from gravity data in Stokes's integral is a shift-filter process (Vaníček and Featherstone, 1998). Any long-wavelength errors will be accounted for through the use of modified integration kernels as high-pass digital filters (Featherstone et al., 1998b; Featherstone, 2003a), or other filters could be used in a pre-processing stage. Again, this will be tested in the Australian context.

Featherstone (2003b, 2008) showed, *post facto*, that the marine gravity data used in AUSGeoid98 had not all been crossover adjusted, even though we applied some coarse data screening (Featherstone *et al*., 2001). A crossover adjustment is needed to account for temporal drift in the marine gravimeters (e.g., Wessel and Watts, 1988). We attempted a crossover adjustment in 2004, but it was not successful because of the relatively low number of crossovers versus the length of the ship-tracks. This caused the adjustment to become ill-conditioned. As such, it will be necessary to ignore the ship-track data totally. In fact, GA has now removed the ship-track gravity records from the national gravity database (cf. Featherstone, 2008).

Instead, marine gravity anomalies will be derived from satellite radar altimetry after coastal re-tracking (described next). However, there will always be the problem of a lack of gravity data in the coastal zone until (expensive) airborne gravity surveys are flown around the whole continent. Such a programme is currently underway in the USA, and is showing promising results. Meanwhile, there will be the problem of how best to merge the satellite altimeter data and land gravity data at the coastal zone. It is likely that least-squares collocation (LSC; Moritz, 1980b) will be used to ‘drape’ the altimeter data onto the land data (cf. Kirby and Forsberg, 1998).

Marine gravity anomalies can be deduced from sea-surface heights measured by echoed radar signals transmitted from a variety of satellite radar altimetry missions. A variety of techniques exist (e.g., Featherstone, 2003b), each of which – disturbingly – yield slightly different results from largely the same data sources, especially near the coast. The new grid from the Danish National Space Research Centre (DNSC), which uses waveform re-tracking, was released commensurately with EGM2008 in April 2008. We expect some significant improvements over AUSGeoid98 in marine areas (shown later), extending onshore in the populated coastal areas. However, the lack of coastal data will remain.

AUSGeoid98 used topographic corrections computed from the version 1 DEM of Australia. This DEM had to be generalised from a 9"×9" grid to a 27"×27" grid to avoid some spuriously large terrain correction values (Kirby and Featherstone, 1999). Kirby and Featherstone (2001) later showed that this was due to incorrect stream-flow data in the version 1 DEM. The version 1 Australian DEM has since been corrected and revised to give the version 2 DEM-9S model This has permitted the computation of a new grid of gravimetric terrain corrections at the full 9"×9" spatial resolution (Kirby and Featherstone, 2002; Featherstone and Kirby, 2002). We anticipate a version 3 DEM sometime soon, which will be used to recompute terrain corrections, and to reconstruct mean gravity anomalies to reduce aliasing (cf. Featherstone and Kirby, 2000).

These new terrain corrections will use Moritz's (1968) algorithm as an approximation of the Molodensky G1 an G2 terms, since this was used in AUSGeoid98 and the software is readily available. Computing these terms from a 9"×9" DEM and the Australian gravity anomalies, as demanded by the full Molodensky theory, and then evaluating them will probably needlessly delay the release of AUSGeoid2008. Given the ~15 cm maximum difference between geoid and quasigeoid over Australia (Featherstone and Kirby, 1997) in comparison to the errors in the AHD (discussed next), the fitting to GPS-AHD data (described later) will [partially] account or this theoretical deficiency. Of course, it should be dealt with in the future.

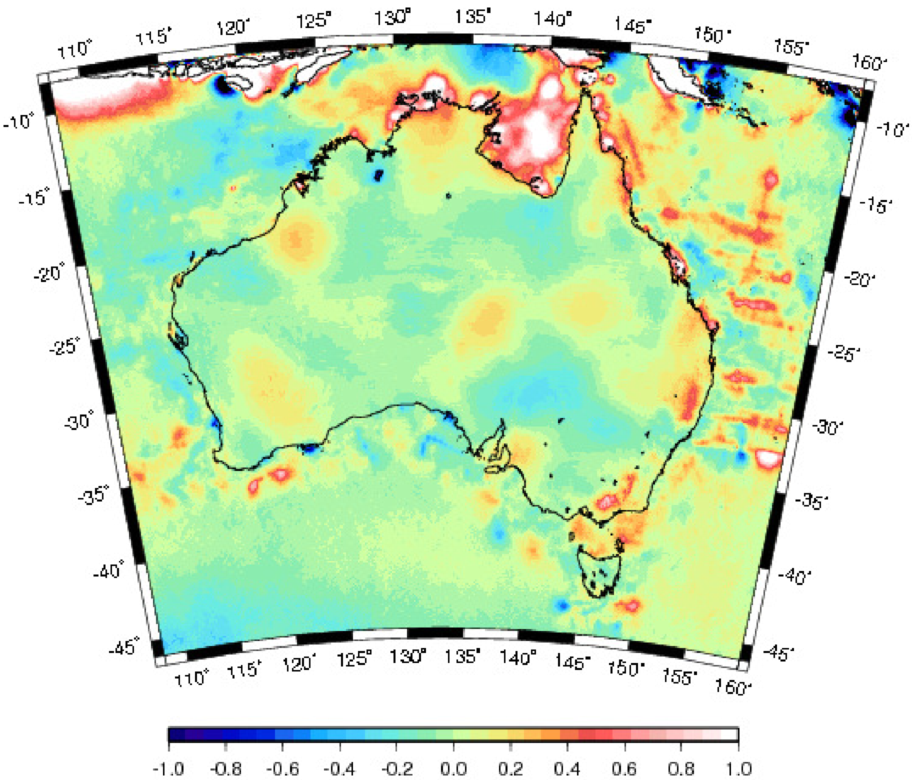

Figure 1: Differences (in metres) between EGM2008 and AUSGeoid98 (Lambert projection)

Figure 1 shows the differences between EGM2008 and AUSGeoid98, highlighting the known, and some unknown, problems in AUSGeoid98. The long-wavelength differences of around 20 cm in magnitude between the models on the mainland are due to improved data from the GRACE mission (cf. Featherstone, 2007). The striped differences offshore, particularly north east of Queensland, are due to the use of unadjusted ship-track gravity data in AUSGeoid98. There are also differences very close to the coasts that are due to a combination of the ship-track data and altimeter gravity anomalies that did not use re-tracked data in AUSGeoid98, so are less accurate in the coastal zone (cf. Andersen and Knudsen, 2000).

The large difference of up to a metre over most of the Gulf of Carpentaria (centred at ~15°S, 140°E) is more enigmatic. Initially, it was thought that the altimeter-derived gravity anomalies were in error in this shallow sea. However, Tregoning *et al*. (2008) show that a weather-driven annual sea surface height variation of ~40 cm amplitude affects the GRACE solutions. Therefore, the differences in this region are more likely due to aliasing in the global geopotential models, but errors in the altimeter data cannot be ruled out. Clearly, this needs further attention.

THE AUSTRALIAN HEIGHT DATUM

Given the recent review in this journal by Featherstone and Kuhn (2006), this section is relatively brief, assuming that the reader has read it as a primer. However, we will try to emphasise the issues relevant to GNSS-based height determination.

Since 1971, the Australian Height Datum (AHD) (Roelse *et al*., 1971; 1975; Granger, 1972; Inter-governmental Committee on Surveying and Mapping, 2004) has formed the framework for precise heights as the gazetted [legal] vertical geodetic datum. It was established in 1983 for Tasmania. AHD heights were realised in staged least-squares adjustments of spirit levelling observations from the Australian National Levelling

Network (ANLN) because of limited computer power at that time. A sparser subset of this network called the 'basic' levelling was adjusted to define the AHD, and then the supplementary levelling was tied to this to propagate AHD heights further to users.

In the 1971 adjustment of the basic levelling, the AHD height was held fixed to zero for mean sea level (MSL) at 30 tide gauges on the mainland, likewise for two tide gauges on Tasmania in 1983. There are several objections to this approach: vertical datums in many overseas countries are established from only one tide gauge; most of the MSL observations used in the AHD were observed over roughly a three-year period that does not properly sample the longest 18.6-year luni-solar tide; and the extra constraints due to unmodelled sea surface topography applied 'strain' to the network adjustment, but this was countered by the [then] desire to have zero height at MSL. This fixing has caused the AHD to become distorted by a metre or so, mainly in a north-south direction, but other distortions exist (Featherstone and Filmer, 2008).

The height system chosen for the AHD was a normal-orthometric height system (Roelse *et al*., 1971; 1975; Holloway, 1988; Featherstone and Kuhn, 2006), but this was because gravity was not observed along the levelling lines. It was also based on a truncated form of Rapp's (1961) formula, which does not give a true normal-orthometric height system. This is not an ideal situation because geopotential numbers should be converted to a height system that better describes fluid flows. However, without gravity along the levelling lines, this is difficult to achieve. Though not yet quantified all over Australia, studies in Western Australia (Allister and Featherstone, 2001) and overseas (Tenzer *et al*., 2005; Santos *et al*., 2006) and simulations (Dennis and Featherstone, 2003) indicate that this could be 10-20 cm, or more.

However, subtleties of height systems and tide-gauge fixing strategies cannot outweigh the quality of levelling data. They were observed over a reasonably short timeframe so as to provide control for national mapping (e.g., Lines, 1992), and typically used third-order techniques. While some traverses are claimed as first-order, many of these do not meet the current class-LC closure tolerance (Intergovernmental Committee on

Surveying and Mapping, 2007). Morgan (1992) estimates that, overall, the AHD is a third-order datum. Filmer and Featherstone (2008) use GPS-AUSGeoid98 height differences to isolate sections in loops that have misclosures of ~40 cm, but there is one loop that miscloses by over a metre, well outside the class-LC closure tolerance.

Mainly from the above considerations, the integrity of the AHD has continually attracted the interest of scientists before and after its realisation (e.g., Leppert, 1967; Leppert *et al*., 1975; Angus-Leppan, 1975; Hamon and Greig, 1972; Mitchell, 1973a,b,c, 1988, 1990; Coleman *et al*., 1979; Bretreger, 1986; Gilliland, 1986; Holloway, 1988; Kearsley *et al*., 1988; Macleod *et al*., 1988; Morgan, 1992; Featherstone and Stewart, 1998; Featherstone, 1998, 2001b, 2004, 2006; Johnston and Luton, 2001; Featherstone and Kuhn, 2006; Featherstone and Sproule, 2006; Soltanpour *et al*., 2006; Featherstone and Filmer, 2008; Filmer and Featherstone, 2008). There is also a small (~10-20 cm) offset between the mainland and Tasmania (Rizos *et al*., 1991, Featherstone, 2000b), though this value is still open to debate. Fundamentally, they are separate vertical datums, though both called AHD in most of the literature.

The above causes for the deficiencies in the AHD now show rather convincingly that there is a north-south slope of ~1.5 m due to the MSL constraints applied (e.g., Featherstone, 2004, 2006), but the omission of rigorous normal/orthometric corrections and the limited quality of the spirit-levelling observations remain key contributing factors (Kearsley *et al*., 1988; Morgan, 1992; Featherstone and Filmer, 2008; Filmer and Featherstone, 2008). While this north-south slope and distortions are seemingly small, they cause problems for GNSS heighting if class-LC standards are to be reached from GNSS with respect to the AHD.

Another issue is the time variation of heights (Biró, 1983; Ekman, 1989), which can be caused by vertical tectonic motion (Wellman and Tracey, 1987), extraction of groundwater or hydrocarbons, soil compaction or expansion, and disturbance of the benchmarks. As such, the AHD height expressed on a coordinate summary sheet may have changed from when the observations were made over three decades ago. Actually, the AHD is not strictly a static datum because State/Territory geodetic agencies have re-levelled and re-adjusted sections of the AHD, yet designated the re-adjusted heights AHD. An example is in Western Australia, where a benchmark changed in AHD height by about 3 cm from a re-levelling and re-adjustment (Featherstone and Galvin, 2008).

THE AUSGEOID2008 GEOID-TYPE SURFACE

It is conceivable that the gravimetric AUSGeoid2008 will a better reflection of the gravity field than the AHD. However, the ultimate desire is to recover AHD heights more directly from GNSS (Featherstone, 1998). As the Intergovernmental Committee on Surveying and Mapping has decided to retain the AHD for the "foreseeable future", we need to seek an interim solution, where the gravimetric quasigeoid model is warped and distorted to fit the AHD using GPS-levelling data (cf. Featherstone, 2000a; Fotopoulos *et al*., 2003; Featherstone and Sproule, 2006; Soltanpour *et al*., 2006).

This approach has been used in several other countries, such as the USA (Milbert, 1995; Smith and Milbert, 1999; Smith and Roman, 2001) and the UK (Iliffe *et al*., 2003). However, it acts to hide the issue of distortions in the AHD, which will ultimately have to be addressed, especially when the GRACE and GOCE (Gravity field and steady-state Ocean Circulation Explorer) satellite gravity missions start to deliver 1 cm quasigeoid models at distances of ~100 km (e.g., Rummel *et al*., 2002; Arabelos and Tscherning, 2001). It may come about that GNSS users will ultimately demand a new vertical datum in Australia because of the deficiencies in the AHD when used with future gravity field models.

Another issue that has arisen over the last few years is that absolute GNSS positioning techniques have become popular, notably because of the availability of precise point positioning (PPP) (Zumberge *et al*., 1997; Kouba and Héroux, 2001; Castleden *et al*., 2004), or relative carrier-phase GPS over very long baselines, such as from the AUSPOS service (http://www.ga.gov.au/geodesy/sgc/wwwgps/; Dawson *et al*., 2001).

The previous use of relative GNSS over short baselines meant that the geoid model was applied differentially and common errors cancelled (Kearsley, 1988a,b). However, the absolute-type GNSS positioning, when used with AUSGeoid98 can show 1-2 m discrepancies at AHD benchmarks (cf. Featherstone and Dent, 2002). Therefore, there is now a more pressing need to produce a surface for the more *direct* transformation of GNSS ellipsoidal heights to the AHD (Featherstone, 1998).

The Intergovernmental Committee on Surveying and Mapping undertook a nation-wide programme to AUSPOS GPS-survey the junction points and 32 tide gauges of the AHD, dubbed height modernisation (Johnston and Luton, 2001; cf. National Geodetic Survey, 2003). More localised surveys are also being conducted by State and Territory geodetic agencies. At present, GA is compiling all geodetic-quality GNSS data in a SINEX file, which will be reprocessed in ITRF2005 (Johnston, 2008, pers. comm.). For example, Western Australia has provided a SINEX file, at which 254 are at AHD benchmarks, and this number is expected to increase.

These nation-wide co-located GPS and AHD data will be used in two stages: first to test the gravimetric-only quasigeoid model on land, which will also involve a minimally constrained readjustment of the AHD to avoid distortions introduced by fixing all tide gauges to zero height; and second to produce the 'geoid-type' surface designed specifically for the direct transformation of GPS ellipsoidal heights to the AHD and *vice versa*. For the fitted AUSGeoid2008 (Featherstone and Sproule, 2006), we adapted existing software for fitting the gravimetric quasigeoid model to the AHD via GNSS using LSC interpolation.

We used LSC in a cross-validation mode to empirically determine the correlation length (2,500 km) and data noise (14 mm) to optimally interpolate the residuals between AUSGeoid98 and 254 new GPS-AHD data to generate a 'geoid-type' model. Table 1 gives the descriptive statistics showing that the fitted quasigeoid gives better height transformation accuracy, though some large differences remain where the GPS-AHD data are sparse. This will be improved further by the use of the new gravimetric quasigeoid model and the addition of more GPS-AHD data that have a better/denser spatial distribution.

	Mean	Max	Min	STD
AUSGeoid98 quasigeoid only	7.6	86.5	-72.1	28.6
LSC-fitted ‘geoid-type’ model	0.0	52.5	-60.3	15.6

Table 1. Descriptive statistics (in cm) of the fit of AUSGeoid98 and the fitted models to 254 GPS-AHD data (from Featherstone and Sproule, 2006)

Importantly, the fitted AUSGeoid2008 model will not be as good for the direct transformation of GNSS ellipsoidal heights to the AHD in areas of sparse GNSS observations at benchmarks. Therefore, it is in the interest of all State/Territory geodetic agencies to ensure that all their geodetic-quality GNSS data are forwarded to GA, ideally for reprocessing on ITRF2005. Dense GNSS networks at AHD benchmarks (preferably from the basic, not supplementary, ANLN) in populated and coastal areas will be particularly advantageous. The concentration on the coastal land will also help alleviate the problems of a lack of good quality gravity data in the coastal offshore.

At this time, it is difficult to ascertain whether the new AUSGeoid2008 (fitted) model will deliver AHD heights that match class-LC spirit levelling closure tolerances (Inter-governmental Committee on Surveying and Mapping, 2007). However, given that it will be based on newer data and methods and fitted to the AHD, it is very likely that it will outperform AUSGeoid98, especially over long distances.

CONCLUDING REMARKS

This paper has discussed the current, emerging and future issues with GNSS-based height determination on the AHD, comprising the reference frames chosen for GNSS-derived ellipsoidal heights, theory- and data-driven inaccuracies in modelling the quasi/geoid, and deficiencies in the realisation of the AHD. Since the AHD will not be revised in the foreseeable future, it will be necessary to warp the new gravimetric quasigeoid model (currently being computed) to fit the AHD. This will produce a 'geoid-type' model that allows for the direct transformation of GNSS heights to the AHD, provided that good-quality GNSS-AHD data have been used in its construction. The term 'geoid-type' model reflects the fact that this is neither a geoid nor a quasigeoid, but a surface designed to model the base of the distorted AHD (cf. Featherstone, 1998). The issue of the future of the AHD is left for debate.

Postscript: This invited paper was written for this special issue during the time that AUSGeoid2008 was being computed (submitted in April 2008, revised after review in July 2008). As such, there are potentially speculative comments on the production of AUSGeoid2008 that may not be incorporated in the published model.

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