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Australian Coal Mining: Estimating Technical Change and Resource Rents in a Translog Cost Function*

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Abstract

This paper estimates a translog cost function for the Australian coal industry from 1968/69 to 2004/05. We use a variable measuring the shift to open-pit mining to capture the impact of embodied technical change, while using a time trend to capture the impact of other technical change and changing resource rents. The cost function is estimated with Zellner's SUR procedure. The shift to open-cut mining is shown to be important in lowering cost during the 1970s and 1980s, but more recently cost reduction is captured by the time trend.

Keywords: Coal mining, Translog, Cost functions, Technical change, Resource rents

JEL Classifications: D24, L71

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I. Introduction

Australia is currently the largest global exporter of black (hard) coal and has been since 1986 when its exports surpassed those of the United States. Black coal is also Australia's largest export industry and accounted for around 10% in value of exports in 2005 (ABS Cat 5368.0). The prominence of the industry is due to Australia's many natural advantages, including its abundant supplies of easily accessible good quality coal located relatively close to established rail and port facilities.

Australia's natural advantages mean that at least some producers can earn substantial resource rents. Hotelling's (1931) seminal contribution to the analysis of non-renewable resource production demonstrates that such resource rents can be expected to increase over time for a homogenous resource. If there is heterogeneity of deposits an optimal movement from the exploitation of superior to inferior reserves over time. This implies that unit production costs (including resource rent) can be expected to rise over time unless there are new discoveries or improvements in the technology of coal mining. In this paper we use estimation of a translog cost function for the Australian coal mining industry to examine the effects of resource rents and technical progress on the cost of production in Australian coal mining over the period 1968/69 to 2004/05.

There have been a variety of methods used to model coal supply. Being a resource industry predominantly managed by engineers, attempts to understand cost relationships have typically been couched in terms of physical variables such as coal seam conditions and thickness (Zimmerman, 1977; Lev and Murphy, 1983; Gordon, 1983; Steenblik, 1992). Although they are often used to assess the impact of a variety of government policies, these engineering-based models do not provide an understanding of the fundamental economic relationships. Similarly, logistic functions and surveys have been used to forecast capacity and future production based on current technology and practices,

again without recognising the underlying production processes (Hotard, Liu and Ristroph, 1983; ABARE, 1997).

Production and cost functions provide greater clarity in understanding the relationships between inputs and their impact on output as well as providing a basis for assessing technical change. In seminal studies, Rhodes (1945) and Lomax (1950) model coal mining in Great Britain using a Cobb-Douglas production function. Chakravarty and Hojman (1982) use a non-homogenous production function, allowing variable elasticities and variable returns to scale, to assess productivity improvements. They find significant returns to scale and variable elasticities of substitution over the 16-year period examined. More recently, Ellerman, et. al. (2001) find evidence of long-term increasing labor productivity in US coal mines, except during a period of rapidly rising output in the mid 1970s when high prices encouraged exploitation of more marginal coal reserves.

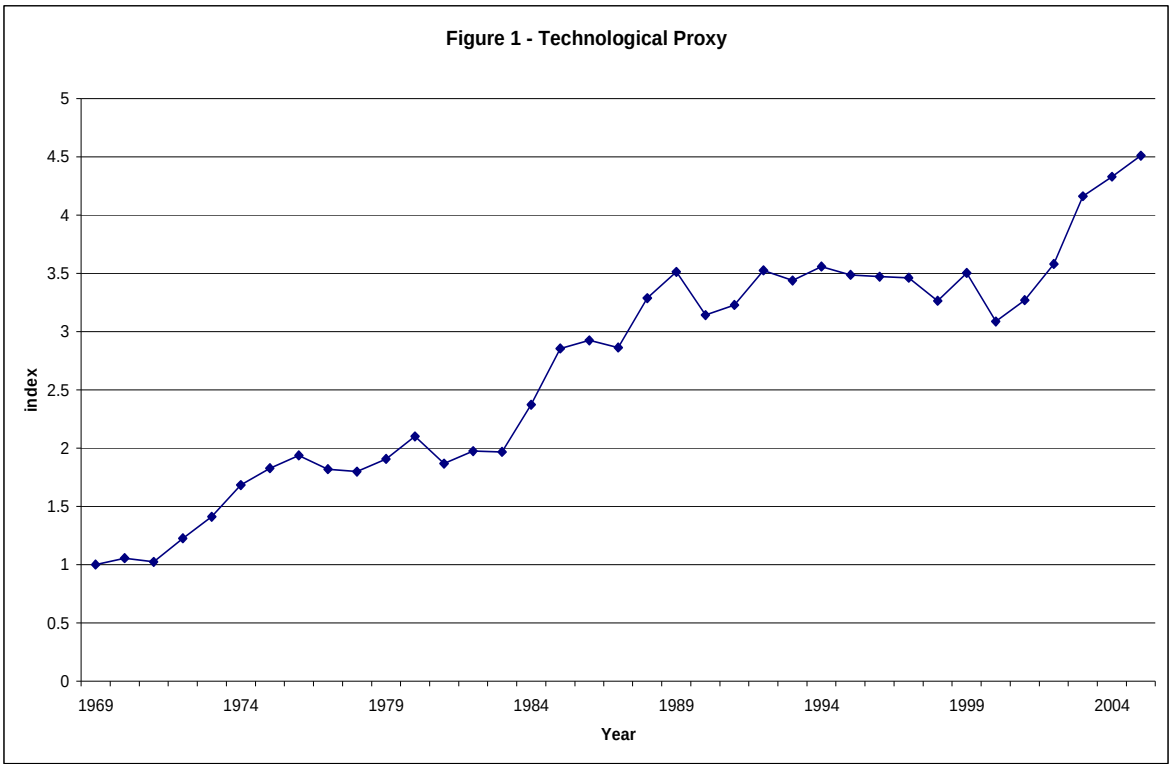
Donnelly and Dragun (1984) model the Australian coal industry with a homogenous, constant returns to scale, translog cost function. Their interest is in assessing differences in elasticities of substitution between production processes. However, their findings are limited by a small data set and the failure of the estimates to satisfy the required regularity conditions of a proper cost function. We follow Donnelly and Dragun in using a translog cost function to model the Australian coal industry over the 37 years, 1968/69 through 2004/05. Unfortunately, changes in data collection make it impossible to separate coal-mining processes, so our estimates are for the aggregate coal industry.

Standard practice in production or cost function estimation is to use a time trend as a proxy for technical change. While this may provide a reasonable approximation in the case of manufacturing or renewable resource industries, there is a substantial difficulty in the case of a non-renewable resource, such as coal, due to the cost-increasing effects of rising scarcity rents or resource exhaustion. A further complication is that new discoveries may offset the impact of resource exhaustion by providing new low-cost mining

opportunities. As a result, the coefficient of a time trend conflates the influence of changing resource rents or resource exhaustion with the opposing effects of new discoveries and technical change.

We separate the effect of at least that portion of technical progress embodied in the switch to open-cut mining by including a variable measuring the relative importance of open-pit mines as a proxy for embodied technical progress. This allows the time trend to capture the residual influence of rising resource rents (or resource exhaustion) and disembodied technical change. The shift in Australian coal mining from a primarily underground mining industry to a substantially open-cut mining industry over the sample period reflects technical advance, in that input requirements for open-pit mining have been lower for the most accessible deposits. The transition has occurred gradually due to the fixed capital committed to underground mining prior to the development of modern open-pit mining methods. The ratio of open-cut production to underground production is illustrated in Figure 1, where the production ratio is converted to an index equal to 1.0 in 1968.

We use the rise in the ratio of open-pit mine output to underground mine output as a measure of embodied technical progress. We allow for the possibility that this technical progress is biased, particularly toward saving labor, by including in the cost function variables that multiply each input price by the technology proxy. The same treatment is extended to the time trend to allow for the possibility that the impact of either disembodied technical change or resource exhaustion is biased.



The translog cost function model used for our estimates is described in Section 2. The variables and data sources used in estimation are described in Section 3. Empirical results are reviewed in Section 4, which also includes tests of various restrictions on the translog form to determine whether coal mining can be better described with a less flexible functional form. We conclude with our observations on the interplay of technical change and resource rents in influencing production costs in Australian coal mining.

II. The Model

Australian coal mining is modeled using a translog cost function.[1] In comparison to other specifications, such as the Cobb-Douglas and CES functional forms, a feature of the translog functional form is that input substitution elasticities can change over the range of values of the independent variables. We also allow technical change and resource

[1] We utilise a cost function rather than a production function, as it is more likely that the input prices faced by the industry, rather than input quantities, are exogenous under the assumption of perfectly competitive markets.

exhaustion to alter input elasticity for each input in that the cost function is not restricted to be homothetic in either the technology proxy or the time trend. Finally we impose constant returns to scale and estimate a unit cost function to remove output quantity from the right-hand-side of the function. [2]

A generalised translog unit cost function with the technology proxy, a time trend and prices for three inputs, the rental price of capital, (R) labor, (L) and materials (M), is:

$$\ln UC = \alpha_0 + \sum_i \alpha_i \ln P_i + \alpha_t t + \alpha_{tp} \ln tp + \frac{1}{2}\sum_i \sum_j \alpha_{ij} \ln P_i \ln P_j + \sum_i \alpha_{ti} t \bullet \ln P_i + \sum_i \alpha_{tpi} \ln tp \bullet \ln P_i + \alpha_{tpt} \ln(tp) \bullet t + \frac{1}{2}\alpha_{tt} t^2 + \frac{1}{2}\alpha_{tptp} (\ln tp)^2 , \quad \forall \text{i} \tag{1}$$

where i, j = R, L and M; t = time; tp = technical proxy; q = output and $\alpha_{ij} = \alpha_{ji}$, $\alpha_{ti} = \alpha_{it}$, $\alpha_{tpi} = \alpha_{itp}$, $\alpha_{qi} = \alpha_{iq}$, $\alpha_{qt} = \alpha_{tq}$, $\alpha_{qtp} = \alpha_{tpq}$ and $\alpha_{tpt} = \alpha_{ttp}$. The technology proxy is defined as the ratio of open-cut mines to underground mines. As is typical in cost function analyses, the function is restricted to be homogeneous of degree one in prices, so that: [3]

$$\sum_i \alpha_i = 1 \ , \ \alpha_{\text{ii}} = -\sum_j \alpha_{ij} \ , \ \forall\, i, \quad j \neq i \ , \ \sum_i \alpha_{ti} = 0 \text{ and } \sum_i \alpha_{tpi} = 0$$

Using Shephard's Lemma (Shephard, 1953), we derive share equations for each input by taking the first derivative of equation (1) with respect to each of the input prices as follows:

[2] In a competitive industry individual firms maximise profits in long-run equilibrium by operating at a point of local constant returns to scale. However, there may be external effects of firm expansion on other firms, leading to non-constant returns to scale at the industry level. Also, the cost function is estimated with a disturbance term, implying disequilibrium at some level. Experimentation with cost functions that allow for non-constant returns by including output variables does show some evidence of non-constant returns, but multicolinearity with time and the technical progress variables makes interpretation of the coefficients problematic. Results including quantity variables are available from the authors.

[3] This implies that if all prices double, total cost will also double.

$$S_i = \frac{P_i x_i}{C} = \frac{\partial C}{\partial P_i} \bullet \frac{P_i}{C} = \frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial \ln UC}{\partial \ln P_i} = \alpha_i + \alpha_{ii} \ln p_i + \sum_j \alpha_{ij} \ln P_j + \alpha_{ti} t + \alpha_{tpi} \ln tp, \quad \forall i \tag{2}$$

These input share equations, when estimated simultaneously with the cost function, increase the degrees of freedom and the efficiency of the estimates. By construction, the share equations sum up to one. Therefore, to prevent singularity in the covariance matrix when estimating the system, one of the share equations must be dropped. As discussed in Berndt (1991) the parameter estimates, log-likelihood values and estimated standard errors are invariant to which equation is dropped when the system is estimated by maximum likelihood methods.

Investigating productivity and technological progress within a cost function framework is based on duality theory.[4] Duality theory (Shephard, 1953 and then Uzawa, 1962) says that a well behaved cost function satisfying the following conditions: non-negative; linearly homogeneous in input prices; non-decreasing in output and input prices; concave in input prices and; continuous in output and input prices can be used to derive the technology exhibited by a well behaved production function. In other words, it is possible to use a cost function to identify the economically meaningful features of the underlying production technology. A further condition, that the cost function is differentiable, allows the application of Shephard's Lemma and the derivation of share equations.

Compliance with the regularity condition that the cost function be linearly homogenous in input prices is imposed prior to estimation. The regularity condition that the cost function is concave in input prices requires that the n x n matrix of second-order

[4] A comprehensive description of the theory can be found in Chambers (1988).

derivatives from the unit cost function $\left(\partial^2 \ln UC / \partial \ln p_i \, \partial \ln p_j\right)$ is negative semi-definite at each observation. The flexibility of the translog form means that concavity is not arbitrarily imposed but is explicitly tested.

Substitution elasticities describe the shape of the production isoquants, so that shallow curve isoquants have large substitution effects and sharply curved isoquants have small substitution effects. In the two-variable case, Hicks (1963) defines the direct elasticity of substitution between two inputs i and j, σ_{ij}^{D}, as the percentage change in the input ratio following a percentage change in the marginal rate of technical substitution, where two inputs are substitutes in production if $\sigma_{ij}^{D} > 0$ and complements in production if $\sigma_{ij}^{D} < 0$. Allen (1938) uses a measure for the n-input case by allowing for cross affects between inputs, σ_{ij}^{A}, which is interpreted in the same way as the Hicks elasticity and is equal to the Hicks elasticity when there are only two inputs. The Allen partial elasticity of substitution shows how input demands change in response to a change in input prices. It can also be shown that the Allen partial elasticity of substitution can be rewritten as $\sigma_{ij}^{D} = \varepsilon_{ij}/S_j$ where ε_{ij} is the elasticity of derived demand and S_j is the cost share of input j.

Uzawa (1962) shows that the Allen partial elasticities of substitution for a general dual cost function can be calculated as:

$$\sigma_{ij} = \frac{C \bullet C_{ij}}{C_i \bullet C_j} \qquad (3)$$

where $C_{i,,}$ C_j and C_{ij} are the first and second partial derivatives respectively of the cost function or unit cost function with respect to input prices P_i and P_j. For the translog cost function, the Allen partial elasticities of substitution are (see Berndt 1991):

$$\sigma_{ij} = \frac{\alpha_{ij} + S_i S_j}{S_i S_j} \quad \text{for } i \neq j \qquad \text{and} \qquad \sigma_{ii} = \frac{\alpha_{ii} + S_i^{\,2} - S_i}{S_i^{\,2}} \quad \text{for } i = j \qquad (4)$$

where S_i and S_j are the cost shares of inputs i and j, respectively. Berndt (1991) suggests that in deriving the elasticities fitted shares should be used and that these should be evaluated at the midpoint of the dataset.

Within a cost framework, technical progress is described by a decrease in costs holding all input prices and output constant (also known as cost diminution). This concept of technical change includes the effect of improvements embodied in new mining techniques. The shift from underground to open-cut operations in Australian coal mining reflects a perception that saving labor is cost effective in an environment of steadily rising wage rates. Open-cut mining has had higher labor productivity than underground mining, although the capital embodied in site development is often huge. In this study we use the ratio of output from open-cut coal mines to that from underground mines as a proxy for technical change embodied in the move from underground to open-cut mining.

We use estimates of the unit cost function in (1) to calculate the rate of cost change associated with the shift from underground to open-cut operations by taking the first derivative with respect to the natural log of technical proxy (tp):

$$\theta_{tp} = \frac{\partial \ln UC}{\partial \ln tp} = \alpha_{tp} + \alpha_{tptp} \ln tp + \alpha_{tpt} t + \sum_i \alpha_{itp} \ln P_i \qquad (5)$$

A cost function characterised by $\theta_{tp} < 0$ indicates cost savings from the shift to open-cut operations, $\theta_{tp} = 0$ indicates no cost savings and cost increases occur when $\theta_{tp} > 0$. The impact of the technological proxy is Hicks-neutral when $\alpha_{itp} = 0$ for all i, reflecting a parallel shift of the isoquants.

We also calculate a residual rate of cost change, which we attribute to changing resource rents or resource exhaustion combined with residual technical change not tied to

the shift of mining from underground to open cut, by taking the first derivative of the unit cost function with respect to time:

$$\theta_t = \frac{\partial \ln UC}{\partial t} = \alpha_t + \alpha_{tt} t + \alpha_{tpt} tp + \sum_i \alpha_{it} \ln P_i \qquad (6)$$

A value of $\theta_t < 0$ indicates cost reduction over time, implying that the impact of residual technical progress or discovery exceeds the impact of rising resource rents or resource exhaustion. A value of $\theta_t > 0$ indicates cost increase over time, implying that the impact of rising rents or exhaustion exceeds that of residual technical progress or discovery.

Suitable cost data covering the total cost of all inputs, especially capital input, used in the coal industry are not available. However, under long-run competitive equilibrium price is equal to cost. To this end, we specify that the long-run equilibrium price of coal, $\ln P^*_{coal,t}$, is equal to the natural log of unit cost, which implies,

$$\ln P^*_{coal,t} = \ln UC_t \qquad (7)$$

and that the logarithm of the actual price of coal, $\ln P_{coal,t}$, partially adjusts to differences between the actual price and long-run equilibrium price. Thus,

$$\ln P_{coal,t} - \ln P_{coal,t-1} = \lambda(\ln P^*_{coal,t} - \ln P_{coal,t-1}) + \varepsilon_t \qquad (8)$$

Coal and oil are substitute energy fuels, so that shocks to the price of oil will impact on the disturbance term in (8) such that:

$$\varepsilon_t = \beta(\ln P_{oil,t} - \ln P_{oil}^{*}) + \upsilon_t \quad (9)$$

where υ_t is a well-behaved error term. Combining equations (7), (8) and (9) defines the estimating equation as:

$$\ln P_{coal,t} = \lambda(\ln UC_t) + (1-\lambda)\ln P_{coal,t-1} + \beta(\ln P_{oil,t} - \ln P_{oil}^{*}) + \varepsilon_t \quad (10)$$

In equation (10) the rate of adjustment of coal prices to unit cost is measured by λ, whilst the sensitivity of coal prices to oil prices is captured by β. This equation is estimated along with the system of equations defined at (2). The parameter estimates are then used to calculate the rate of cost change with respect to the ratio of open-cut to underground mining and with respect to time, given by equations (5) and (6), respectively.

III. Data

This study uses Australian annual coal data on all coal types (black and brown) for the 37 year period, 1968-69 to 2004-05. Price indices for labor and materials (P_L, P_M) are taken from various issues of ABS Catalogue 8415.0, *Mining Operations*. The rental price of capital, P_R, is defined as, $P_R = (1/m + i)P_k$. Where m is the average age of the gross capital stock, i is the 10-year bond rate (opportunity cost) and P_k is the price of new capital. These data are taken from ABS Catalogue 5204, *Capital Stock by Industry* and various *RBA Bulletins*. Coal prices are derived from revenue data, which is found in ABS Catalogue 8415.0, *Mining Operations*. Labor and materials shares are calculated from data given in various issues of ABS Catalogue 8415.0, *Mining Operations*, and 8221.0, *Manufacturing Industry, Australia* and tables from the Electricity Suppliers Association (www.esaa.com.au).

The technology index is defined as the ratio of output from open-cut mines to output from underground mines. Data to construct the technology index are taken from Coal Services Pty Ltd, *Australian Black Coal Statistics*, 2002. Finally, the index for crude oil prices in Australian dollars is derived from crude oil prices reported by the Department of Energy, http://www.eia.doe.gov and the OECD, *Main Economic Indicators*.

IV. Results

We estimate 3 different models to calculate rates of cost diminution:

A. The simplest model assumes that prices are in perfectly competitive long-run equilibrium aside from a random disturbance term, so that there is no distinction between unit cost and prices. We estimate equation (11) along with the set of equations specified at (2).

$$\ln P_{coal,t} = \ln UC_t + \varepsilon_t \qquad (11)$$

B. The second model has the divergence between the unit cost of coal and the price of coal related to oil price shocks, modifying (11) with $\varepsilon_t = \beta(\ln P_{oil,t} - \ln P^*_{oil}) + \upsilon_t$, where υ_t is a well-behaved error term and $\ln P^*_{oil}$ is the long-run average oil price over the sample period. This yields

$$\ln P_{coal,t} = \ln UC_t + \beta(\ln P_{oil,t} - \ln P^*_{oil}) + \upsilon \qquad (12)$$

We then estimate equation (12) along with the set of equations specified at (2), assuming that the share equations are not altered by oil price shocks.

C. The final model we estimate incorporates oil price shocks as in (12), while allowing coal prices to only partially adjust to deviations from the long-run coal price. We then estimate the equation given by (10) along with the set of share equations described in (2).

In each model we impose the restrictions implied by assuming the cost function is homogenous of degree one in input prices and exhibiting constant returns to scale. A stochastic framework is specified where additive error terms are appended to each of the factor share equations to reflect unexplained factors that impact on cost shares (such as measurement error by the data collectors and/or the possibility that firms make random errors in choosing their cost-minimising input bundles). Ordinary Least Squares (OLS) could be used to estimate the coal price equation and each factor share equation separately. However, this ignores the additional information available from imposing cross-equation restrictions. Equation by equation estimation by OLS also ignores the additional information available when error terms are correlated across observations. This is likely to be the case when share equations are dependent on the same industry conditions. To take into account these factors, Seemingly Unrelated Regression (SUR) estimation is used to jointly estimate the systems of equations described by Models (A), (B) and (C). (Zellner, 1962).[5]

By construction, the share equations add up to one and therefore one of the share equations is a linear function of the others. To prevent singularity in the residuals, we drop the capital share equation. Joint estimation is then based on the factor share equations for labor and materials along with the coal price equation. We assume that the error terms are

[5] As discussed in Barten (1969) the parameter estimates are invariant to which share equation is dropped, as long as the estimates are indeed maximum likelihood estimates (or, equivalently, iterative generalised feasible least squares estimation is used).

normally distributed, that they have a constant variance over time, and that there is contemporaneous correlation between equations (but impose zero covariance over time).

When the system of equations is estimated, we are unable to reject the hypothesis of autocorrelation for the coal price equation or for the share equations in any of the models. The system is therefore re-estimated, taking into account of various levels of auto-correlation. This leads to the results for the three models given in Table 1. The results for Models A and B are based on a second-order auto-correlation adjustment, while the results for Model C are based on a first-order adjustment.[6]

Models A, B and C differ in terms of assumptions regarding the relationship between the coal price and the unit cost of producing coal. In Model A, any difference between price and unit cost is assumed to be randomly distributed, while in Model B the difference is related to the deviation of the price of crude oil from its long-run trend. In Model C the assumption from Model B regarding oil price is retained and it is further assumed that the price of coal only partially adjusts to disturbances in each year. The estimates in Table 1 show that the coefficient of the oil price deviation is not statistically significant in Model B. However in Model C, this coefficient is significant and of the expected positive sign, suggesting that each one percent deviation of the oil price from trend leads to about an eighth of a percent change in the coal price in the same direction. Further in Model C, the coefficient of the lagged coal price is statistically significant and between zero and one as expected, suggesting that approximately one half of the any difference between the price and unit cost of coal is made up in the following year.

[6] Choice of the order of auto-correlation is based on testing for a significant improvement in the log-likelihood ratio. Full results are available from the authors.

Table 1 - Results for Models A, B and C

Variable	Estimated coefficient		
	Model A	**Model B**	**Model C**
Constant	0.1026 [0.88]	0.2308 [1.25]	0.9699 [4.99]
Capital Price	0.5141 [13.91]	0.5111 [14.09]	0.5029 [14.92]
Labor Price	0.2804 [16.87]	0.2818 [16.43]	0.2861 [17.72]
Tech Proxy	0.0148 [0.04]	-0.0565 [0.13]	0.5331 [0.85]
Time	0.0386 [2.01]	0.0290 [1.37]	-0.0634 [1.71]
Labor*Capital	-0.0284 [1.02]	-0.0331 [1.22]	-0.0229 [0.97]
Labor*Materials	-0.0275 [1.19]	-0.0295 [1.31]	-0.0434 [2.12]
Capital*Materials	-0.0373 [1.02]	-0.0379 [1.07]	-0.0241 [0.71]
(Tech Proxy)^2	-3.7584 [3.09]	-3.9619 [3.57]	-7.9365 [3.86]
(Time)^2	-0.0090 [4.33]	-0.0094 [4.66]	-0.0147 [4.50]
Tech Proxy*Time	0.1657 [3.40]	0.1797 [3.82]	0.3600 [4.37]
Labor*Tech Proxy	0.0013 [0.04]	0.0029 [0.09]	-0.0315 [1.14]
Materials*Tech Proxy	-0.0331 [0.97]	-0.0358 [1.10]	-0.0635 [2.10]
Labor*Time	-0.0058 [3.05]	-0.0061 [3.32]	-0.0050 [2.90]
Materials*Time	-0.0002 [0.09]	-0.0000 [0.02]	0.0015 [0.92]
Oil Price		0.0515 [0.91]	0.1258 [3.74]
Lambda			0.4705 [4.99]
Auto-correlation order	AR2	AR2	AR1
Log-likelihood ratio	258.75	259.15	256.53

Notes: Figures in brackets are t-ratios

Models A, B and C yield broadly similar results in terms of the impact of input prices on the coal price. The coefficients of the capital and labor prices are positive and statistically significant at slightly more than .5 for capital and slightly less than .3 for labor. The sum of the coefficients for all inputs is restricted to equal one to guarantee that the conditions for linear homogeneity with respect to input prices are satisfied, so the implied coefficient for the omitted materials input is about .2. At the start of the sample period each coefficient is approximately equal to the elasticity of cost with respect to the

respective input price.[7] These elasticities change over the sample period, especially the impact of labor, due to the negative and statistically significant coefficient on the interaction between the labor price and the time trend. Materials price has no clear interaction with time, but the cost elasticity with respect to materials price falls in each model with the shift to open-cut mining. The sum of coefficients of the interaction of time with each of the inputs prices is restricted to equal zero to satisfy the requirements of linear homogeneity in input prices. Likewise, the interaction of input prices with the technology proxy sums to zero. Thus, by implication, the elasticity of cost with respect to capital price is rising both directly with time and with the general shift over time to open-cut mining.[8]

Fundamental requirements of a well-behaved cost function include monotonicity (so that an increase in input prices does not decrease cost) and concavity in input prices. For a cost function to be monotonic, the first derivative of cost with respect to input prices must be positive. Even with the changes in the implied values of these derivatives over the sample period, each derivative is positive in every year of the sample for each of the models with results reported in Table 1. As discussed in Berndt (1991), concavity requires that the matrix of substitution elasticities be negative semi-definite. The coefficients on the interaction of input prices in Table 1 are all negative (although not generally by statistically significant amounts). Together with the positive first derivatives, this leads to a matrix of substitution elasticites for each model with results reported in Table 1 that is negative semi-definite.[9]

[7] Each of the input prices and the technology proxy are measured by index values that are set equal to 1.0 in the first period and the logarithm of 1 equals zero.
[8] The implied elasticity of cost with respect to capital at the end of the sample period is slightly less than three quarters using coefficients for any of Models A, B and C, while the elasticities for labor and materials are slightly more and slightly less than one eighth, respectively.
[9] We evaluate the determinants of the matrices at the mid-point of the sample but the values of the regression coefficients are such that the same result would obtain at all sample values. Details of the tests are available from the authors.

The influences of time and the technological proxy (the ratio of output from open-cut mining to the output of underground mining) on costs are similar in Models A, B and C, but in each case there is a complicated pattern for the estimated impact. The coefficients of the first-order terms for both time and the technological proxy are generally not statistically significant and vary in sign across the three models. However, the coefficients of the square of both time and the technological proxy are negative and statistically significant in each model, while the coefficients of the interaction between time and the technological proxy are positive and statistically significant in each model. The opposing signs for the coefficients of the second-order terms in the regression lead to varying rates of cost change associated with time and the technological proxy over the sample period.

Figure 2 shows the values of the first derivative of the logarithm of unit cost with respect to time, labeled CDT, for each year in the sample period as calculated from the coefficients for Model A and for Model C in Table 1 using equation (6)[10]. These first derivatives give the rate of cost change with respect to time measured as a proportion of the unit cost of production. The variation over the sample period is striking and similar for both models, with rates of cost change ranging from close to plus 10% to greater than minus 10%. The cost changes are generally positive from the mid-70s through the mid-90s and then increasingly negative until about 2000 before leveling off at minus 5% to minus 8% per year. As discussed above, production costs tend to increase over time for a non-renewable resource due to either rising resource rents for a homogenous non-renewable resource or exhaustion of superior deposits when the resource is of heterogeneous quality. However, new discoveries and technical progress (excluding the separately measured technical change associated with the shift to open-cut mining) reduce costs. The pattern in Figure 2 suggests resource exhaustion was the dominant influence for at least two

decades, while the combination of new discoveries and technical progress has dominated for the last decade.

Figure 2 - Cost changes with time

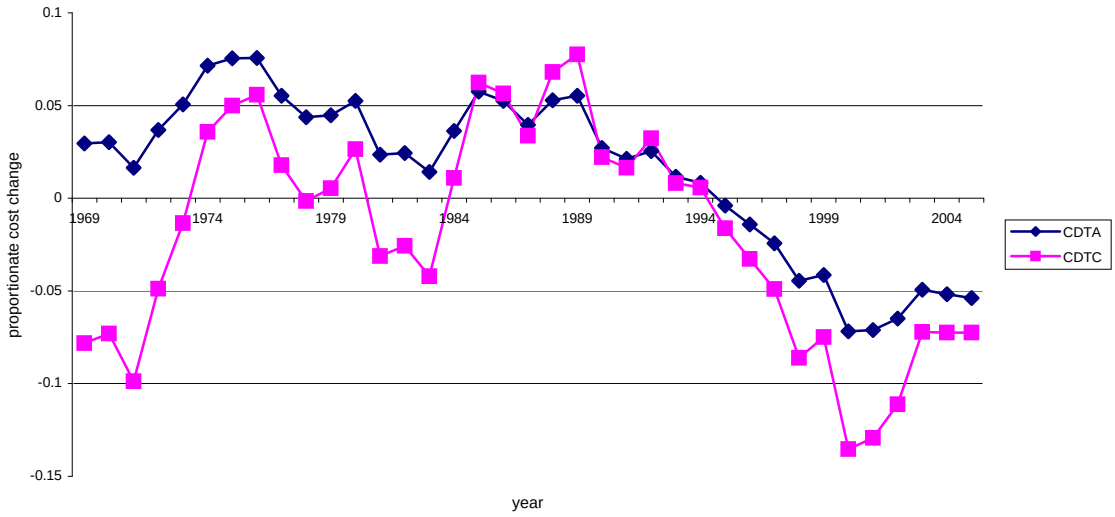
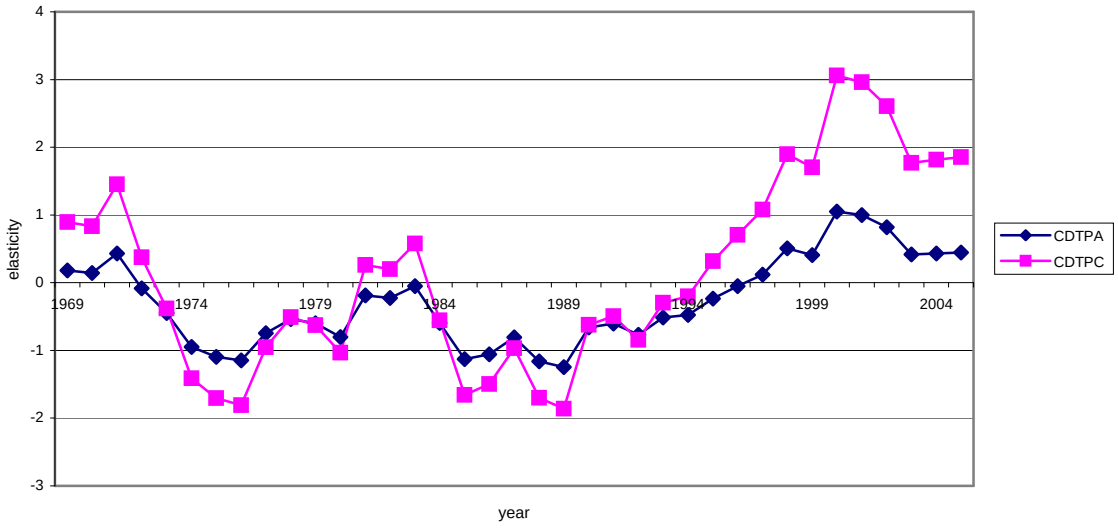


Figure 3 shows the values of the first derivative of the logarithm of unit cost with respect to the logarithm of the technological proxy, labeled CDTP, for each year in the sample period as calculated from the coefficients for Model A and for Model C in Table 1 using equation (5). These values are estimates of the elasticity of cost with respect to the technological proxy (measured by the ratio of output from open-cut mining to the output from underground mining). The shift from underground to open-cut mining was an outstanding feature of the Australian coal industry in the 1970s and 1980s as shown previously in Figure 1. Figure 3 shows that for two decades from the early 1970s, aside from three years in the early 80s, the effect of the shift was substantially cost reducing, with each ten percent increase in the ratio of open-cut to underground coal leading to a decrease in average unit production cost of between ten and twenty percent. However, after the early 1990s the cost advantage of open-cut mining apparently disappeared and was replaced with a cost disadvantage. This corresponds to a period in which the ratio of

[10] Cost changes for Model B are not shown, as they are virtually identical to those for Model A.

output from open-cut mines to underground mines fluctuated in a narrow band, at least until the last few years when open-cut output has risen relative to underground output.

Figure 3 - Cost elasticity with technological proxy



Combining the pattern in Figure 2 with that in Figure 3 suggests the nature of technical change in Australian coal mining has altered over the sample period. In the 1970s and 1980s, productivity improvements and cost reductions were achieved through shifting production methods from underground mining to open-cut mining. Rising nominal and real prices for Australian coal during this period also encouraged the continued exploitation of mines with reserves of dwindling quality so production costs in these mines increased while resource rents in mines of superior quality rose. From the late 1980s until recently, coal prices fluctuated in a narrow band against rising cost pressures, particularly higher wage rates, encouraging the abandonment of inferior deposits.[11] Figure 2 suggests this has generated cost savings of five percent or more a year in aggregate production costs.

[11] Ellerman et al (2001) use data on individual US coal mines and find that relatively less productive mines operated throughout a period of high prices in the 1980s, but tended to shut down once prices declined in the 1990s.

Production methods in Australian coal mining have also adapted to changing relative input prices. Figure 4 shows that wage rates have risen substantially relative to the prices of capital and materials. The Allen partial own-elasticities of substitution (σ_{ij}) calculated from the results in Table 1 using the formula in (5) are each negative, while the cross-price elasticities are almost all positive at all sample values.[12] The relative price changes have thus encouraged substitution away from labor. Furthermore, isoquant shifts associated with technical change through the technological proxy and the time trend have been biased against labor and, to a lesser extent against materials.

Figure 4 - Log coal price and input prices

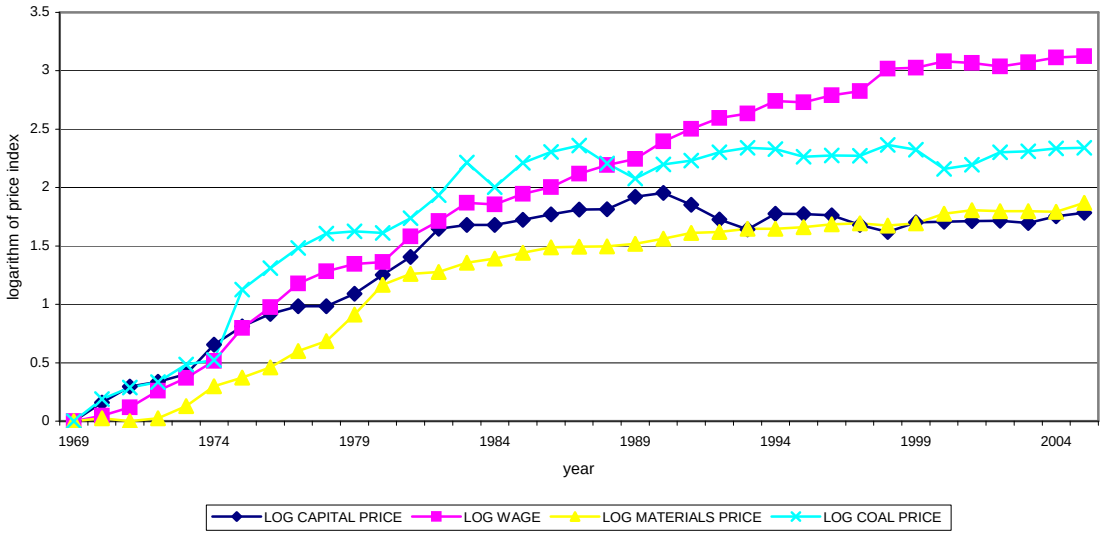
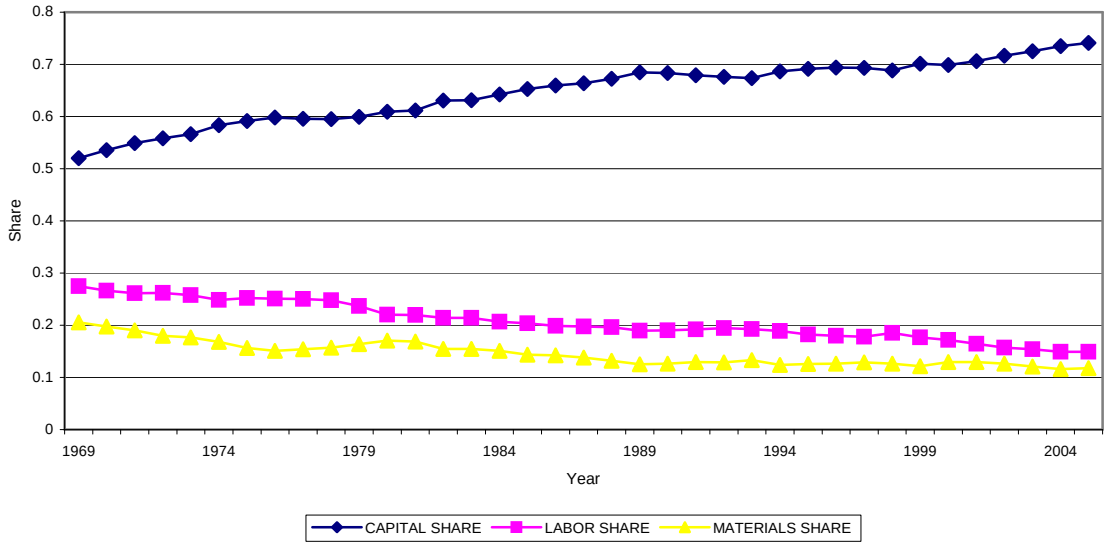


Figure 5 shows the pattern of fitted cost shares calculated by applying the formula in (3) to the regression coefficients in Table 1 for Model A.[13] Capital's share is generally increasing throughout the period, while the shares of labor and, to a lesser extent, materials are generally falling. The rising share of capital relative to labor demonstrates the degree to which coal miners are economising on the use of labor that has become relatively more expensive over the sample period.

[12] An exception is that the cross-price elasticities between labor and materials are generally negative for later years in the sample period. Full details of the Allen partial elasticities are available from the authors.
[13] The corresponding patterns for the other models are very similar and are omitted to provide a clearer diagram.

Figure 5 - Fitted Cost Shares from Model C



V. Conclusions

The Australian coal mining industry has seen major developments over the last four decades years, particularly with growth in large-scale, open-cut mines and improved technology that has been generally labor-saving. In this study, a flexible, non-homothetic translog cost function is shown to provide a satisfactory econometric model of the changing conditions in the industry. In particular, we separate cost changes associated with the shift from underground to open-cut mining from cost changes due to changing input prices and residual changes occurring with the passage of time, including changes due to rising resource rents or higher production costs with resource exhaustion.

We find that the shift from underground to open-cut mining led to substantial cost reductions during the 1970s and 80s, but that since then a cost disadvantage has developed for open-cut relative to underground. In the 1970s and 1980s rising coal prices increased resource rents and encouraged production from relatively inefficient mines with high unit costs. Since then technical progress across underground and open-cut mining reduced

production costs by at least five percent a year as prices fluctuated without trend up to the end of the sample period. We also find that there has been substantial substitution of capital for labor in Australian coal mining, driven by both a rising relative price of labor and by a labor-saving bias in technical change. As a result, the optimal cost share of capital has increased from about one half to about three quarters, while the share of labor has dropped from about three tenths to about one eighth and the share of materials from about two tenths to about one eighth.

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