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Quantification of Crack Formation Using Image Analysis and its Relationship with Permeability

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Abstract

In this study a relationship between permeability of concrete and fractal dimension of crack is established. For this purpose four series of specimens of fiber reinforced cementitious composites are prepared. Specimens are subjected to uniaxial tension in order to create targeted damage (cracking) prior to permeability test. Image analysis is done on the cracked surface and fractal dimension of cracks are calculated using box counting method. Maximum crack width is found to have correlation with the coefficient of permeability. However, such correlation was observed neither between coefficient of permeability and crack area nor between coefficient of permeability and crack density. Relationships of fractal dimension of cracks is established with the maximum crack width, crack area and crack density. Trilateral relationship among coefficient of permeability, the maximum crack width and fractal dimension are established.

Keywords: Permeability, fractal dimension, box counting method, crack area, crack distribution.

Quantifizieren der Rissbildung mit Hilfe der Bildanalyse und deren Einfluss auf die Permeabilität

Zusammenfassung

In diesem Beitrag wird eine Verbindung zwischen der Permeabilität und der fraktalen Dimension von Rissen hergestellt. Zu diesem Zweck wurden vier Serien von Faser bewehrten Zement gebundenen zusammen gesetzten Werkstoffen hergestellt. Die Proben wurden einer einachsigen Zugspannung ausgesetzt, um sie gezielt durch Rissbildung zu schädigen, bevor die Permeabilität bestimmt wurde. Die Bruchflächen wurden mit Hilfe der Bildanalyse untersucht und die fraktale Dimension der Risse wurde durch Zählen von Kästchen in einem vorgegebenen Raster bestimmt. Es konnte festgestellt werden, dass die maximale Rissweite mit dem Koeffizienten der Permeabilität korreliert. Keine Korrelation wurde dagegen gefunden zwischen dem Koeffizienten der Permeabilität einerseits und der Rissfläche oder der Rissdichte andererseits. Beziehungen zwischen der fraktalen Dimension der Risse und der maximalen Rissweite, der Rissfläche und der Rissdichte konnten nachgewiesen werden. Eine dreifache Beziehung zwischen dem Koeffizienten der Permeabilität, der maximalen Rissweite und der fraktalen Dimension konnte hergestellt werden.

Stichwörter: *Permeabilität, Fraktale Dimension, Methode des Zählens der Kästchen, Rissfläche, Rissverteilung.*

1 Introduction

Transport properties of concrete, especially permeability, affect the durability and integrity of a structure. High permeability, due to porosity or cracking, provides ingress for water, chlorides, and other corrosive agents. If such agents reach reinforcing bars within the structure, the bars corrode, thus compromising the ability of the structure to withstand loads, which eventually leads to structural failure. Generally, maximum crack width is used to evaluate the resistance against permeability of aggressive substances into concrete. Because of the surface roughness and tortuosity of the cracks in concrete, in addition to local crack information such as the maximum crack width, the global crack information such as crack area, crack density and fractal dimension of cracks can be used to evaluate the permeability. Image analysis of crack maps can be used to obtain the global crack information such as crack area, crack density and fractal dimension of cracks. In this study, an attempt is made to establish relationship between global crack information and permeability of concrete. Image analysis suggested by Nishiwaki and Mihashi [1] is performed on observed crack maps of tested specimens. Fractal dimension of cracks is found to have correlation with the permeability.

2 Concept of Fractal Dimension

Fractal is derived from the Latin word ***frangere***, which means `to break` [2]. Fractals are of fragmented geometric shape that can be subdivided in parts. The fractal geometry is a mathematical concept that describes objects of irregular shape. Some natural geometrical shapes, that can be irregular, tortuous, rough or fragmented, can be described using concepts of fractal geometry as long as the requirement of self-similarity is satisfied. The latter term implies that the geometrical features of an object are independent of the magnification or observation scale. “Fractal dimension” is a non-integer number and it measures the degree of fractal boundary fragmentation or irregularity over multiple scales. Fractal dimension of cracks contains the whole crack information where the tortuosity of crack as well as the crack area and crack width are accounted for in the calculation using the box counting method. Fractal dimension is also influenced by the crack density and crack distribution. A wider crack or in another word larger crack area will have higher fractal dimension than that of thinner crack and smaller crack area. Fractal dimensions of cracks with different crack distributions will have different values even if their crack areas are same.

3 Fractal Dimension Using Box Counting Method

In this study the fractal dimension of cracks is calculated using the Box Counting method [2] from the image analysis of crack map. First, an image of a crack map whose fractal dimension is to be calculated is considered (Fig. 1(a)). Then the image is broken up into boxes of a given size (r) and counted number of those boxes containing the cracks (N(r)) (Fig. 1(b)). Then the same procedure is repeated with several different box sizes (Figs. 1(c) - 1(d)). A plot of logarithm of the number of boxes through which the crack passed (log(N(r))) and the logarithm of the size of the boxes (log(r)) is constructed (Fig. 1(e)). The fractal dimension is related to the slope of the relationship between the logarithm of the number of boxes through which the cracks passes and the logarithm of the size of the boxes. The fractal dimension D is given by the slope of the linear portion of a log(N(r)) versus log(r). This is from,

$$D = \frac{Log(N(r))}{Log(1/r)} \tag{1}$$

The crack area and the crack density are calculated from the image analysis of crack map. Image analysis combines techniques that compute statistics and measurements based on the gray-level intensities of the image pixels. In this technique the image of specimen’s crack surface is recorded by high resolution digital camera. The digitized image of cracked specimen is used to calculate the crack area and crack distribution using a computer program. Crack density is defined as the ratio of total number of pixels of crack to total number of pixels in crack band. Fig. 2 shows the distribution of crack area of a typical cracked specimen obtained from the image analysis.

4 Experiments

High performance fiber reinforced cementitious composites (HPFRCC) containing different volume fractions of Polyvinyl Alcohol (PVA) fiber is considered in this study. Properties of cement, sand and PVA fibers are shown in Table 1. Mix proportions are shown in Table 2. Four series of speci-

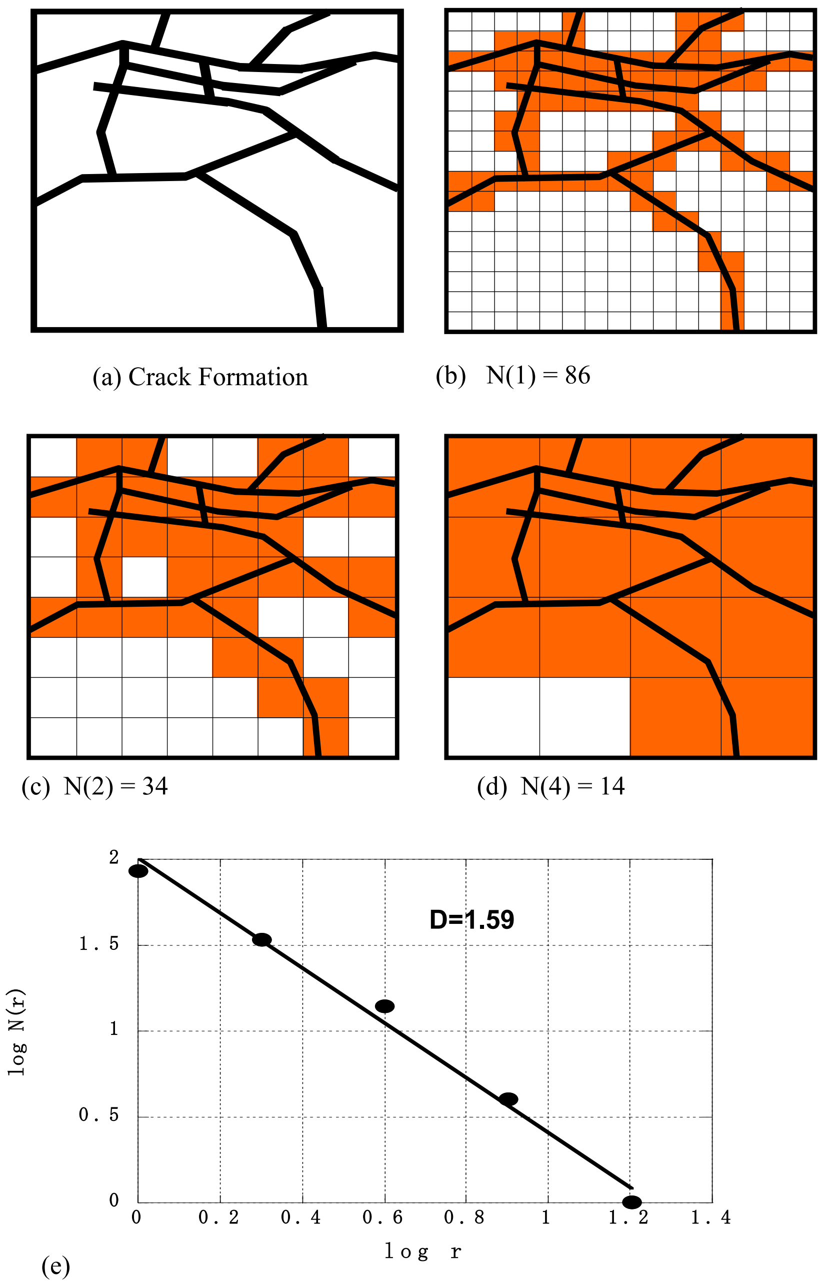

Figure 1: Fractal Dimension by Box Counting Method

Table 1: Properties of Materials

Name	Description	Properties
Cement	High-early strength Portland cement	Density= 3.14 gm/cc
Fine Aggregate	Silica sand No. 8	Density= 2.61 gm/cc, Size= 105-120µm
Fiber	Polyvinyl Alcohol (PVA)	Density= 1.3 gm/cc, Diameter= 37 µm, Length= 6 mm, Tensile Strength= 1620 MPa
Loading device	Screw bars	Diameter= 6 mm (M6)

Table 2: Mix Proportions

Series	Water/Cement (wt.%)	Sand/Cement (wt.%)	Viscuss agent/Cement (wt.%)	PVA Fiber (Vol.%)
1	62	40	1.4	1.0
2				1.5
3				2.0
4				3.0

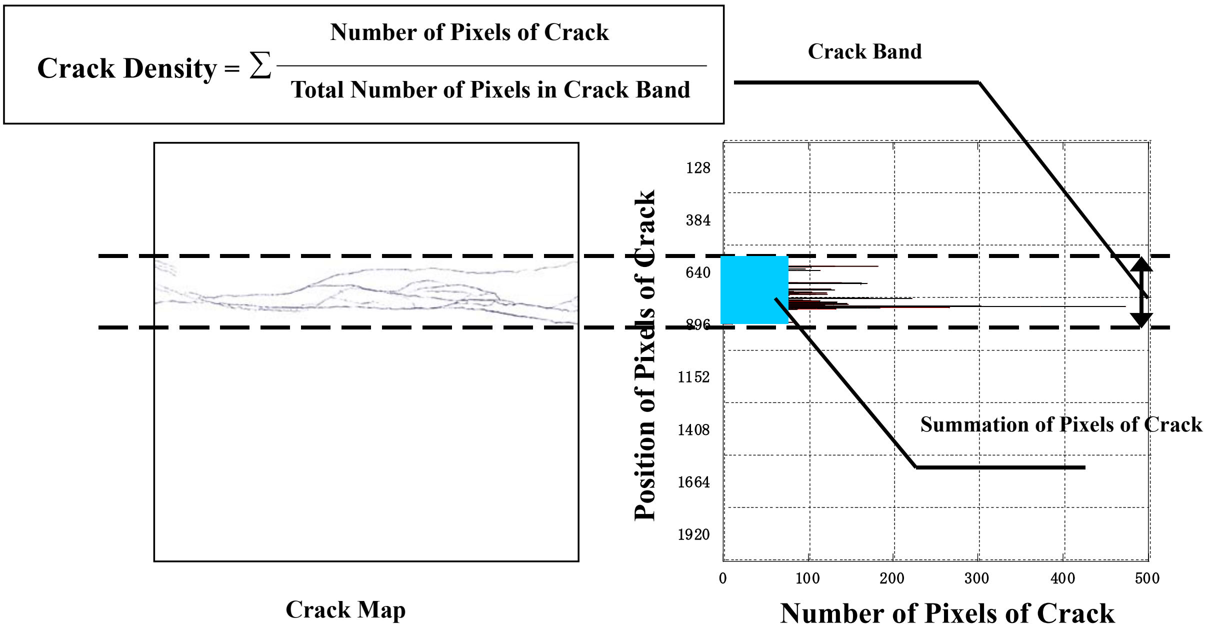

Figure 2: Distribution of Crack Area of a Typical Cracked Specimen.

mens were cast. Volume fractions of PVA fibers used were 1 %, 1.5 %, 2 % and 3 % in the first, second, third and forth series, respectively. Fiber reinforced cementitious composite containing 2 % PVA exhibited excellent mechanical properties as reported by Mihashi et al. [3]. In each series, square plate specimens of 75 x 75 x 25 mm were cast and tested.

The testing procedure was carried out in two steps. In the first step, the specimens were subjected to uniaxial tension in order to damage them at different levels. Tension tests were carried out on the specimens at an age of 7 days. Details of uniaxial tension test setup are shown in Fig. 3. After the specimens were damaged, the cracks were observed and image of crack surface was recorded by digital camera. The crack widths were measured using a microscope.

In the second step, water permeability test was performed on the cracked specimens. The permeability test setup is outlined in Fig. 4. Four Plexiglas plates were used to prepare square boxes of dimension 75 x 75 mm. Two Plexiglas boxes, were mounted on the top and bottom surface of the specimen with silicon rubber. A Plexiglas plate with a pipe was mounted on the top Plexiglas box and another was placed on the bottom Plexiglas box. Four long threaded bars were used to clamp the cell together. The outer surface of the specimen and all connections were sealed with silicon rubber to prevent leaking of water. After the silicon rubber set, the cells were filled with distilled water both above and below the specimen. The water permeability test started with the filling of the upper pipe with water. The water permeated from the top to the bottom due to the pressure head. The water drop was measured at regular time intervals, depending on the water flow rate of the specimen.

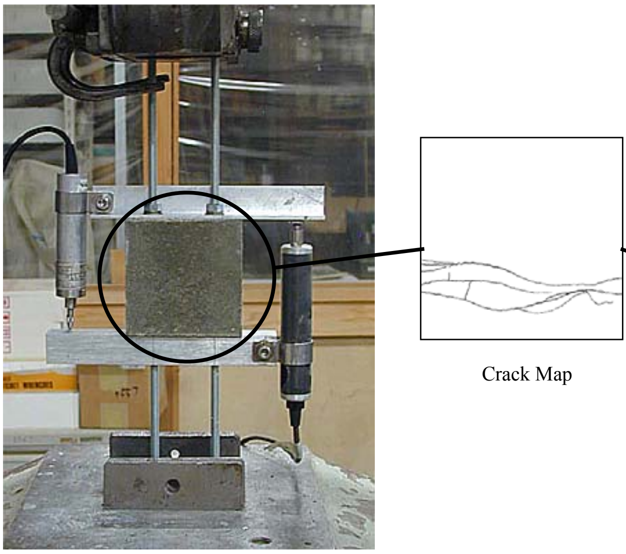

Figure 3: Tension Test

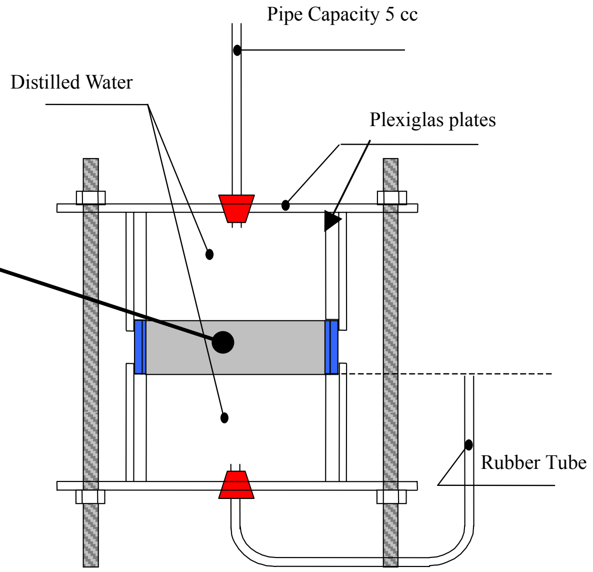

Figure 4: Permeability Test

The calculations to determine the coefficient of permeability are described as follows.

The water flow through the system is assumed to be continuous and laminar; therefore, Darcy's law can be applied. Because the flow is continuous, the amount of water flowing out of the pipe is shown to be:

$$dV = -A'\left(\frac{dh}{dt}\right) \tag{2}$$

where V is the total volume of water that travels through the sample, dh is the differential of pressure between the initial pressure head h_0 and the remaining pressure head h_1 at the measured time t; A' is the cross-sectional area of the pipe.

Darcy's law states:

$$Q = kA\frac{h}{l} \tag{3}$$

where Q is the flow rate through the specimen (dV/dt), k is the permeability coefficient and the parameter under study, l is the thickness of the specimen, and A is the cross-sectional area of the concrete.

By combining equations (2) and (3),

$$k\frac{h}{l}A = -\frac{A'dh}{dt} \tag{4}$$

By integrating equation (4)

$$k\int_0^t dt = -l\frac{A'}{A}\int_{h_0}^{h_1}\frac{dh}{h} \tag{5}$$

The coefficient of permeability is finally given by

$$k = l\frac{A'}{At}\ln\frac{h_0}{h_1} \tag{6}$$

5 Results and Discussion

Typical cracking behaviour of HPFRCC specimens observed in all four series is shown in Fig. 5. This figure also shows that number of cracks increase with increasing fiber content. The relationships of coefficient of permeability with local crack information such as maximum crack width, as well as global crack information such as fractal dimension of cracks, crack area and crack density are shown in Figs. 6 to 9. Fig. 6 shows the relationship between coefficient of permeability and the maximum crack width. Generally, in all series except the series that contains 1.5 % PVA fiber, the coefficient of permeability increases with increase in the maximum crack width. A correlation between coefficient of permeability and the maximum crack width is established. This also coincides with the results reported by other researchers [5, 6]. The relationship between coefficient of permeability and fractal dimension of cracks is shown in Fig. 7. However, no correlation between coefficient of permeability and fractal dimension can be

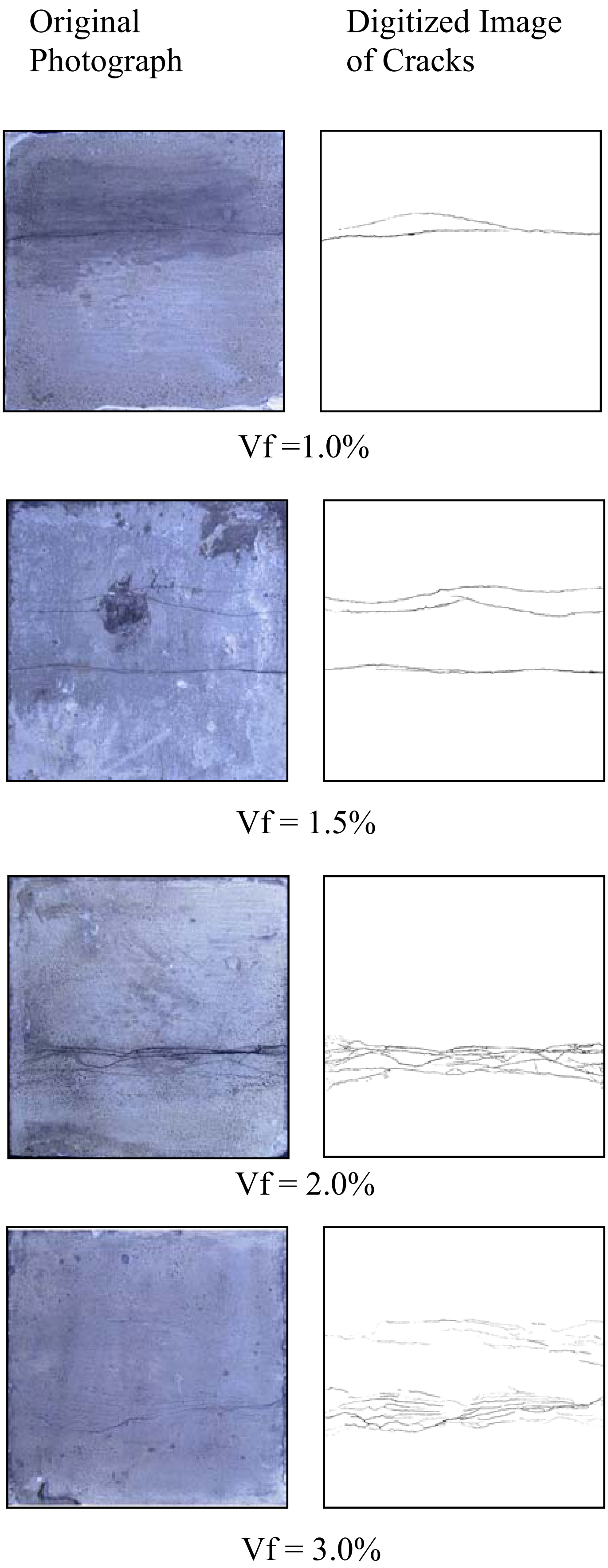

Figure 5: Typical Multiple Cracking Behaviour of HPFRCC Specimens

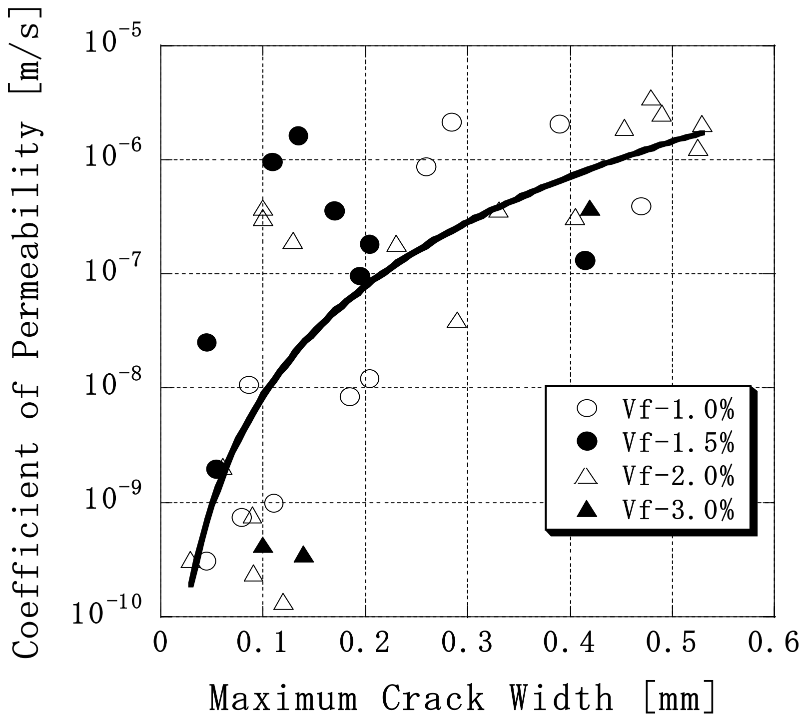

Figure 6: Relationship between Coefficient of Permeability and Maximum Crack Width

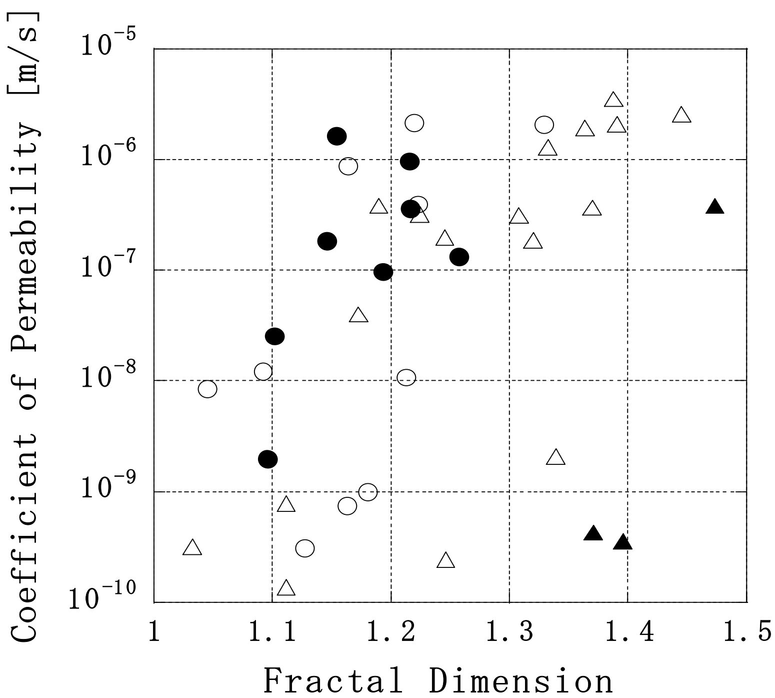

Figure 7: Relationship between Coefficient of Permeability and Fractal Dimension

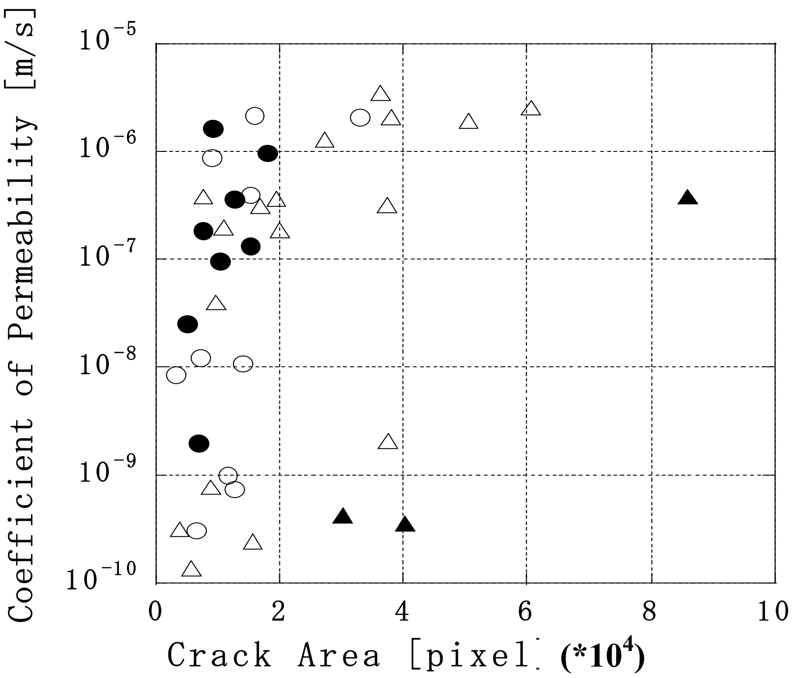

Figure 8: Relationship between Coefficient of Permeability and Crack Area

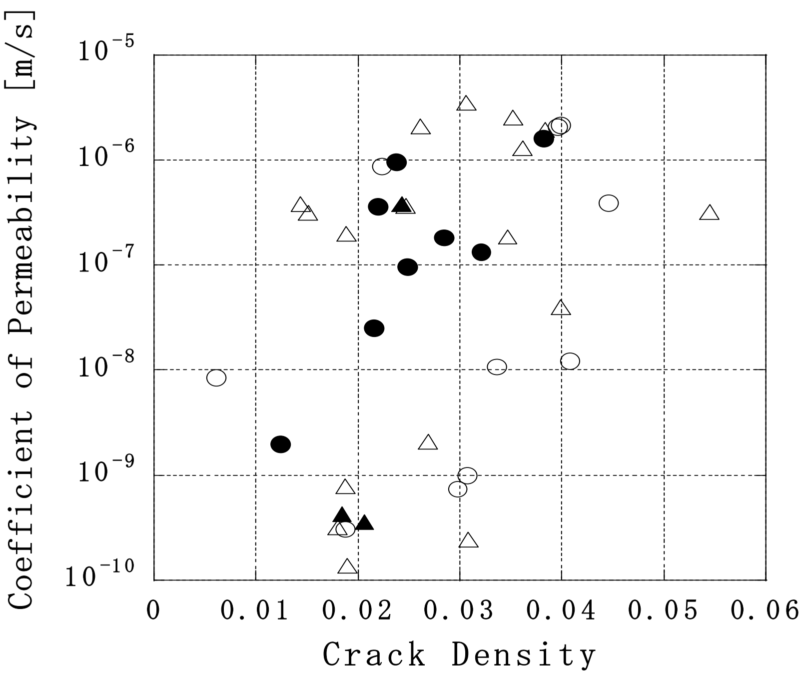

Figure 9: Relationship between Coefficient of Permeability and Crack Density

established. Furthermore, neither correlations between coefficient of permeability and crack area nor between coefficient of permeability and crack density are observed as shown in Figs. 8 and 9, respectively.

In this section the relationships of fractal dimension of cracks with the maximum crack width, the crack area, and the crack density are established. Fractal dimension of cracks and crack area are plotted in Fig. 10 where a good correlation is observed. The

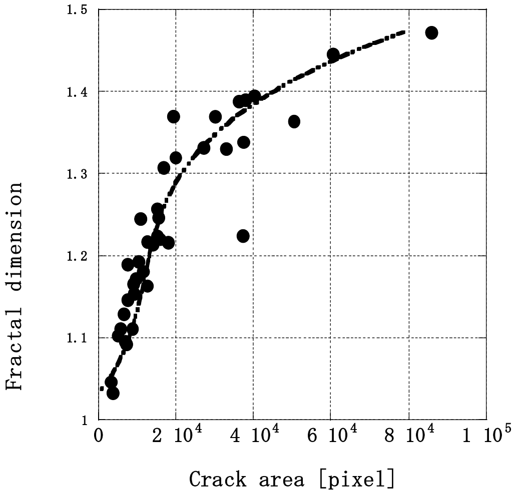

Figure 10: Relationship between fractal dimension and crack area.

figure shows that the fractal dimension of cracks increases with increase in crack area. Relationships between fractal dimension and maximum crack width and between fractal dimension and crack density are also observed as shown in Figs. 11 and 12, respectively despite large scattering of results.

In the previous section a bilateral relationship between coefficient of permeability and local crack information such as the maximum crack width is observed. However, no relationship between coefficient of permeability and global crack information such as crack area, fractal dimension and crack density is observed. In this case if an additional parameter is considered where a trilateral relationship of studied parameters can be observed. Fig. 13 shows the relationship of coefficient of permeability with the maximum crack width and crack density for all series. No clear relationship among coefficient of permeability, the maximum crack width and the crack density can be established. This is because no clear relationship between coefficient of permeability and crack density was observed as discussed earlier. However, if coefficient of permeability, the maximum crack width and crack area are related as shown in Fig. 14, a trend of increase in coefficient of permeability with increase in the maximum crack width and crack area can be observed except the crack area between 15,000 and 40,000 pixels where the coefficient of permeability is extremely large between crack width of 0.1 and

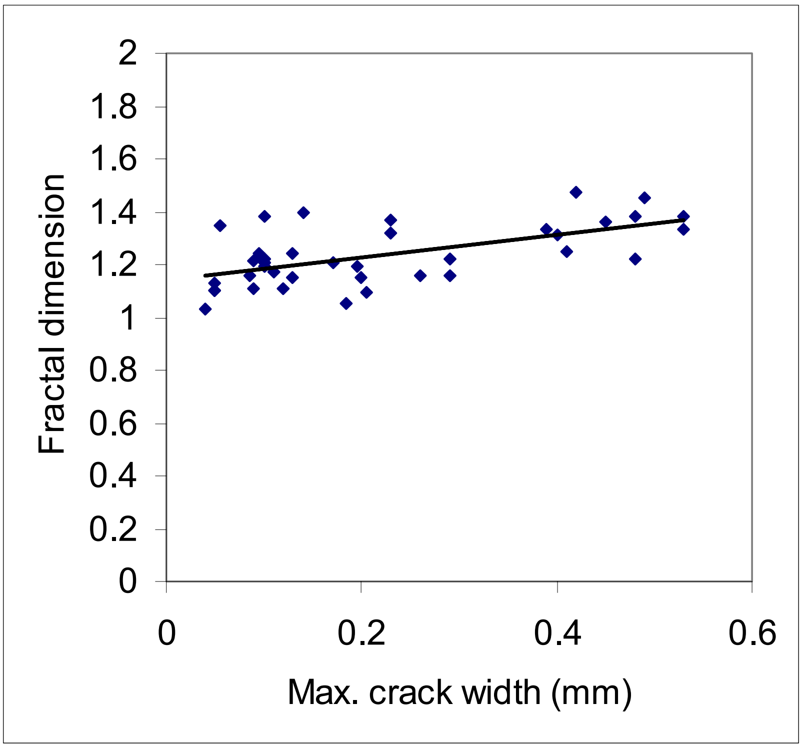

Figure 11: Relationship between fractal dimension and maximum crack width.

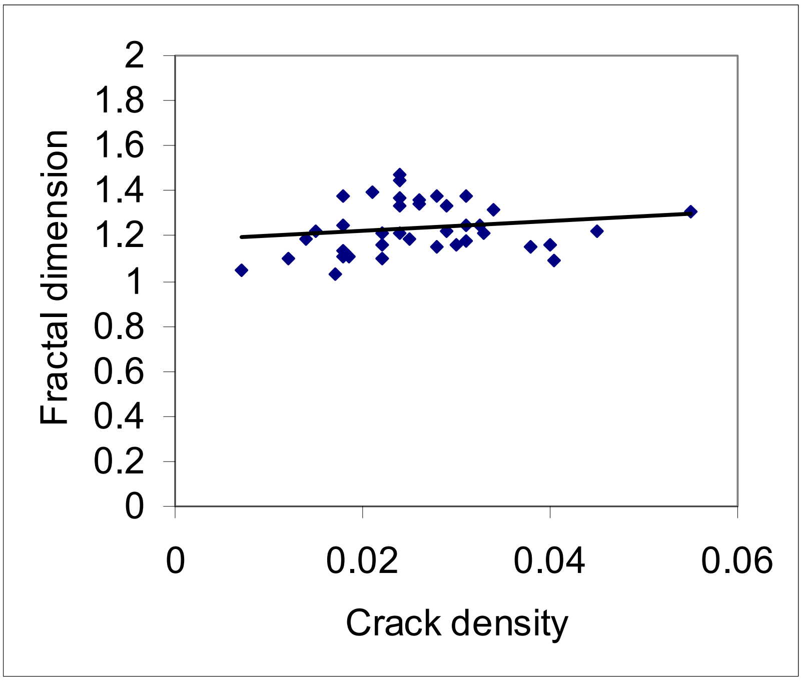

Figure 12: Relationship between fractal dimension and crack density.

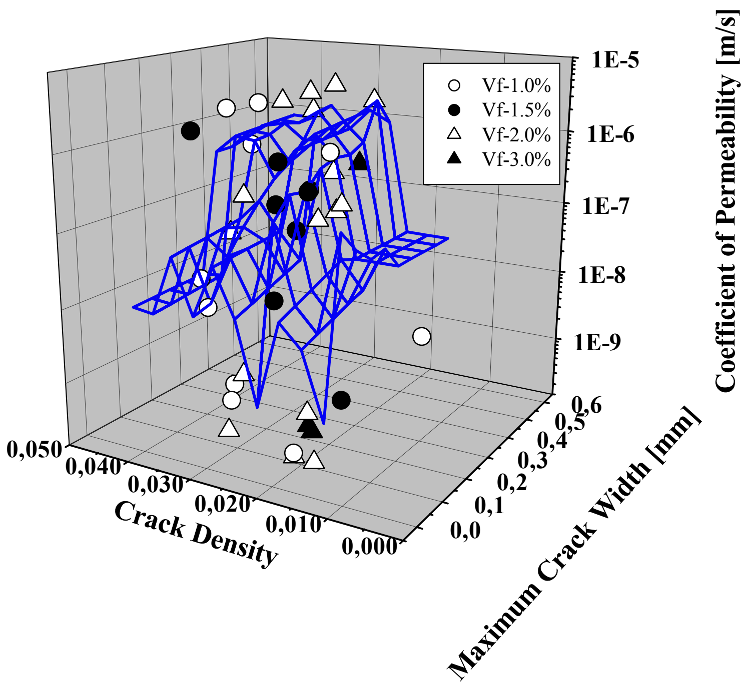

Figure 13: Relationship between coefficient of permeability, crack density and maximum crack width.

0.2 mm. A similar type of behaviour can also be observed among coefficient of permeability, the maximum crack width and fractal dimension of crack as shown in Fig. 15. Fig. 15 clearly shows that the coefficient of permeability increases with increase in fractal dimension of cracks.

The quantitative evaluation of crack formation obtained from image analysis was carried out in this study. The relationship between the crack information and coefficient of permeability was presented. The results show that the maximum crack width alone to evaluate the resistance against permeabili-

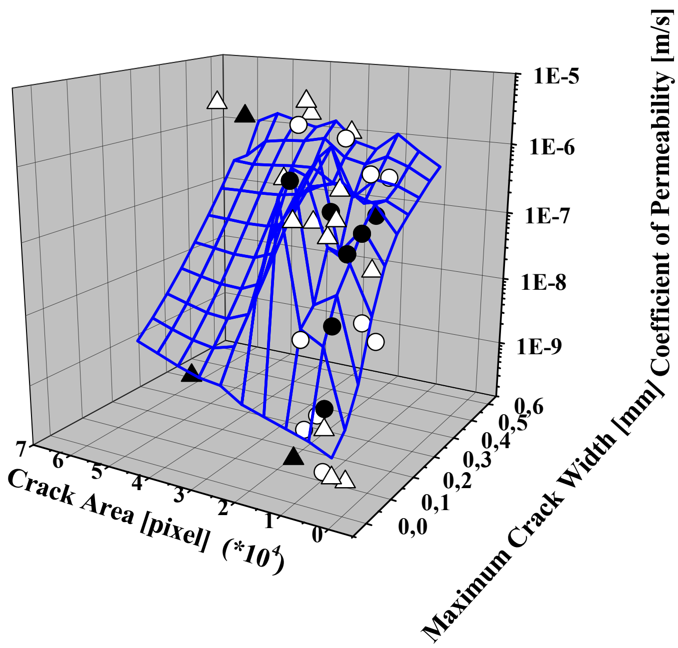

Figure 14: Relationship between coefficient of permeability, crack area and maximum crack width.

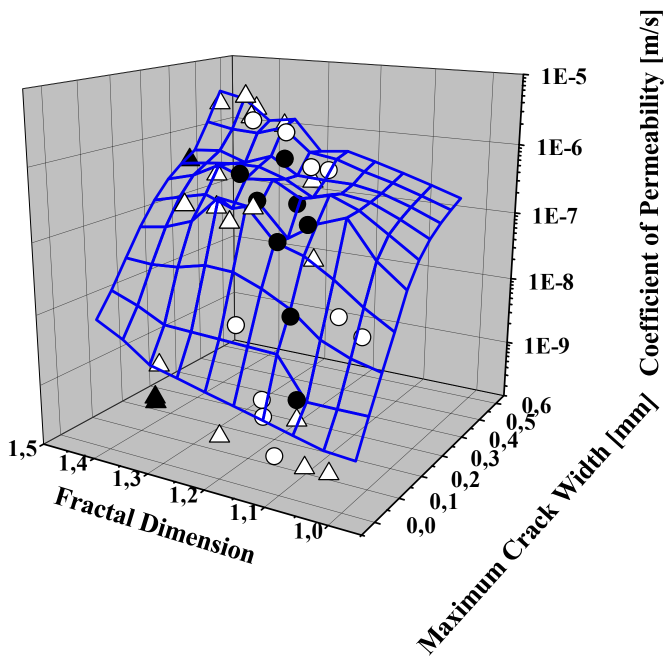

Figure 15: Relationship between coefficient of permeability, fractal dimension and maximum crack width.

ty of concrete is not enough as can be seen in Fig. 6 where the scattering is very large. Therefore, the evaluation needs to be done not only for the local crack information such as the maximum crack width but also for the global crack information. In case of global crack information, fractal dimension of cracks, crack area and crack density are obtained from image analysis of crack maps and are presented earlier. However, the coefficient of permeability could not be related to crack density. On the other hand, if the fractal dimension is related to the maximum crack width and coefficient of permeability a good trilateral relationship is established where the coefficient of permeability increases with increase in fractal dimension and maximum crack width.

6 Conclusions

In this study an attempt was made to establish a relationship between coefficient of permeability and local crack information as well as global crack information. A relationship between coefficient of permeability and the maximum crack width was established though a large scatter was observed. Fractal dimension of cracks is found to have relationship with crack area, crack density and maximum crack width. A very good trilateral relationship among fractal dimension of cracks, maximum crack width and coefficient of permeability was established.

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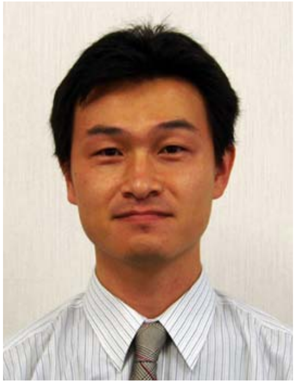

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