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The impact of fiscal transfers induced by emissions taxes in stock-flow pollution problems with regions and population mobility

Felix Chan and Jeffrey D Petchey[1]

Abstract

There is a long-standing literature on the potential for the fiscal transfers induced by taxes on emissions to produce a double dividend. This has taken place within the context of static models without regions or population mobility. In this paper, we examine the potential effects of fiscal transfers induced by emissions taxes in a stock-flow pollutant problem over multiple regions in the presence of population mobility. We show that the induced fiscal transfers influence the time chosen by a social welfare maximising decision maker to converge to sustainable pollutant stock targets. The precise impact of the induced fiscal transfers on the convergence time is shown to depend upon the nature of the stock-flow problem and in particular whether the current pollutant stock exceeds or is less than the sustainable level.

JEL Codes: Q53, R11, H23, H41, H73.

Keywords: air pollution, water pollution, regional economic activity, environmental issues, externalities, environmental taxes and subsidies, public goods, inter-jurisdictional differentials and their effects.

1. Introduction

A number of large developed economies, including Germany, the United Kingdom, the Netherlands, Sweden and Denmark have introduced national taxes on carbon emissions. The aim is to provide a shadow price for the negative externalities created by polluting inputs that contribute to the stock of carbon concentration in the atmosphere. The United States and other countries have by contrast tended to focus more on regulation and direct action in attempting to mitigate various forms of atmospheric emissions.

The economic costs and benefits of the different policy instruments available for mitigation have been exhaustively debated in the economics literature over a long period of time. From this research, there is a tendency to favour market based approaches such as Pigouvian type environmental taxes or permit markets to provide shadow prices for pollutant emissions. It is well-known that by providing a single price for emissions and equalizing marginal abatement costs across firms, taxes and market solutions can provide the required mitigation at least cost [Baumol (1972), Baumol and Oates (1988)].

Apart from their role in establishing efficient prices for negative externalities, environmental taxes may also create a "fiscal dividend" if the revenue raised from taxes is recycled through government budgets and used, for example, to reduce

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distortionary taxes elsewhere, counteract adverse distributional consequences or create other benefits such as the provision of more public services. There is a long standing and extensive literature on the fiscal compensation effects of environmental taxes. It is beyond the scope of this paper to provide an exhaustive critique of this literature though the interested reader can consult Goulder (1995), Bovenburg (1999), Bovenberg and Goulder (2002) and Callan et al (2009) for surveys.

Fiscal recycling and compensation are notable features of emissions tax schemes adopted in the countries noted above. In Germany, for example, revenue from the eco-tax directly offsets pension contributions [see Habler and Roeder (2012)] while the Australian emissions tax included a compensation package of revenue recycling to compensate low income households for the direct effect of the tax on energy prices [Meng et al (2013].

The question of whether there is a fiscal dividend induced by environmental taxes has been considered within the context of static models which focus on pollutant flows. In practice, many environmental problems have a stock *and* a flow aspect to them. For example, the flow of pollutants into the atmosphere from economic activities in individual countries adds to the common stock of existing pollutants (e.g. carbon). For river and lake systems, such as the Murray-Darling basin in Australia or the Great Lakes of North America, the flow of pollutants from agricultural and industrial activities adds to the existing stock of pollutants in natural systems. Balanced against this, pollutants are naturally sequestered and this tends to reduce the pollutant stock. When the flow of new pollutants exceeds the rate of absorption, then the stock, which matters for welfare, increases over time.

Stock-flow problems have a well-known theoretical structure [see Perman and McGilvray (2003)]. The pollutant stock is treated as a pure public bad generating negative externalities that reduce social welfare. The current quantity of the public bad is assumed to be given. There is also some desired future quantity which is estimated from scientific rather than economic decision models. Important questions of economic interest are then as follows: *(i)* which mitigation instruments should governments use; *(ii)* what is the nature of the optimal transition path for the pollutant stock; and *(iii)* how is the optimal convergence time determined and what should it depend upon?

Given the discussion above, the purpose of our paper is to consider the social welfare effects of fiscal recycling in stock flow pollution problems when emissions taxes are used for mitigation. It is also supposed that the emissions flow is constant and the transition path can be determined at the beginning of the transition phase and does not require any adjustment during the transition period. We then focus on question *(iii)* above; namely, determining the optimal convergence time in stock-flow pollutant problems when there is fiscal recycling. We also consider this problem in a regional context with population mobility across regions in response to differences in per capita social welfare arising from underlying production technologies. This means any solution to the optimal convergence time problem cannot make people in one region better or worse off than their neighbours in other regions. As far as we know, this is the first time the fiscal transfers induced by emissions taxes have been examined in a regional context with population mobility and a stock-flow pollutant.

Our model is of a particular stock-flow pollutant; the concentration of carbon in the atmosphere, though the results generalize to any stock-flow problem. It assumes that the global economy consists of two distinct regions which differ only in terms of their production technology for generating emissions from an energy input. These region-specific emissions contribute to the global stock of carbon in the atmosphere which is modelled as a pure public bad. The world population has homogenous preferences over a local public good provided by regions and a private consumption good. Since regional production technologies differ per capita incomes can vary across regions. People are mobile and make their location choices to equate per capita inter-temporal welfare. There is a known current quantity for the concentration of carbon in the atmosphere and a desirable (sustainable) quantity, both exogenously given. Given the application to atmospheric carbon pollution, the current stock quantity is less than the target (sustainable) quantity.

With this basic set up, we suppose there is a mythical social planner who chooses the time period, *T*, in which the actual global pollutant stock should converge to its desired level. The planner makes this choice to maximize inter-temporal global social welfare which is increasing in the social welfare of the two regions. This maximization is undertaken subject to the migration constraint and on the assumption that the time path for the global pollutant stock in the transition phase has constant emissions. Once convergence to the sustainable stock is achieved, regional economies enter a steady state in which sequestration is equal to global emissions and the pollutant stock is stable at its target from then on. The mythical planner also levies an emissions tax in each region. These taxes play an important role in our model: they are the tax rates needed to ensure that mitigation in each region is consistent with achieving the global pollutant goal. Tax rates are set once the convergence time is chosen by the planner - hence they are not separate decision variables. Importantly for our purposes, revenue from the emissions taxes is recycled to residents within each region on an equal per capita lump sum basis.

From the solution to the social welfare maximisation problem we obtain the first order necessary condition for what we call a constrained optimal convergence time. We are able to present this condition in terms of the inter-temporal marginal benefits and costs of an incremental change in the convergence time. From this, we show it is optimal for the fiscal transfers induced by emissions taxes to affect the convergence time choice. For the atmospheric problem considered, the induced fiscal transfers are shown to delay convergence, while for stock flow problems where the current stock exceeds the target, such as pollution of rivers or lakes, the induced fiscal transfers from emissions taxes lead to faster convergence and a shorter transition phase.

The paper is organized as follows. Section 2 sets out the basic structure of the model, including specification of the stock-flow pollution dynamics, regional production technologies and emissions taxes, preferences/feasibility, regional social welfare and population mobility. Section 3 finds the first order condition for the convergence time and presents the main result in relation to the role of induced fiscal transfers in a stock-flow problem with regions and population mobility. Conclusions are presented in Section 4 and mathematical details are in the Appendices.

2. Regional economy with population mobility

Let us assume the global economy is made up of two regions, $i = 1,2$, differentiated by the intensity of emissions generated by their production process. In other respects, regions have the same production technology. The regions can be thought of as two blocs, for example, one high income and the other low income, or literally as two countries. Let H be the given world population and H_i be the population of region *i*, for $i = 1,2$, where $H = H_1 + H_2$. The total world population is perfectly mobile across regions and makes its location choice to satisfy a migration constraint which equates per capita inter-temporal indirect utility. Regions also provide a local public good. An emissions tax is also levied on emissions in each region to provide a shadow price for pollutant emissions. The world population has homogeneous preferences though per capita incomes differ across regions as a result of different production technologies. The analysis is, therefore, conducted in terms of a representative citizen from each region. We now describe the dynamics associated with global pollutant emissions.

2.1 Global pollutant dynamics

Define $G(t)$ as the global stock of CO_2eq, a pure public bad, which adversely affects the welfare of people in both regions equally. The value of the pollutant stock is a function of time, t, where $t = 0,...,T,...,\infty$ is divided into two periods. In the first, time runs from some base year, $t = 0$, to a convergence year, $t = T$, where the stock becomes equal to $\overline{G}$, which we think of the sustainable quantity of the pollutant stock assumed to be exogenously given from the perspective of our economic model. We think of $\overline{G}$ as determined by scientific analysis of the sustainable quantity of CO_2eq in the atmosphere to which the global economy must adjust over time. For purposes of discussion here, we assume this to be 450 ppm of CO_2eq consistent with one of the main scenarios in IPCC (2013), but in general it can be any quantity. Also, denote G_0 as the current stock of CO_2eq at time $t = 0$. We assume this to be 400 ppm CO_2eq based on Mauna Loa Observatory data for 2015 though it can be any given quantity less than the target. The second period is a steady-state where the CO_2eq pollutant stock stabilizes at the sustainable quantity and remains there from the convergence year to infinity.

Let $G_{,t}(t) = g - \delta G(t)$ be the differential equation for the global CO_2eq stock in the transition period where g is the total flow of carbon emissions and δ is the rate of sequestration. We assume the emissions flow to be constant in the transition period while the rate of sequestration, δ, is a parameter in the transition phase and the steady-state. An alternative to assuming a constant emissions flow is to explore the optimal path for emissions using optimal control. This would allow us to examine the dynamic optimality of the trade-off between accumulation of the pollutant stock and emissions in the transition phase. This is an extremely important and interesting normative economic problem: to find the optimal time path for emissions consistent with maximising inter-temporal welfare. We do not tackle this in our paper for the following reasons. In practice policy-makers tend not to follow optimal paths. This would require frequent adjustments in emissions, both up and down, throughout the transition phase as optimality necessitates that emissions adjust inter-temporally. The reality of policy-making is that the political process has difficulty agreeing to any transition path, let alone one that is optimal. In this environment, a transition path with constant emissions is just as likely to be chosen by policy-makers as any other. In this sense, our analysis can be thought of as positive rather than normative.

Our model also incorporates the complexity of local public goods, taxes and population mobility and the assumption of constant emissions greatly assists tractability. More specifically, a constant emissions flow in the transition phase allows us to collapse the dynamics into a quasi-static problem. This can be seen by noting that the solution to the differential equation for the pollutant stock is:

$$G(t) = \left(G_0 - \frac{g}{\delta} \right) e^{-\delta t} + \frac{g}{\delta}. \qquad (2.1)$$

A given sustainable CO_2eq stock, combined with the constancy of emissions during the transition phase, creates a unique relation between emissions and the time when the actual stock of CO_2eq converges to its exogenously given environmentally sustainable quantity. This relation is a result of the model set-up and is not due to the form of the first order differential equation. The relationship between g and T is:[2]

$$g(T) = \delta\left(\overline{G} - G_0 e^{-\delta T}\right)\frac{1}{1 - e^{-\delta T}} . \tag{2.2}$$

Hence, during the transition period from G_0 to $\overline{G}$ the total emissions flow $g(T)$ is a function of T, conditional on the rate of sequestration. Given this, the solution to the differential equation for the global stock of CO_2eq becomes:

$$G\left(g(T),t\right) = \left(G_0 - \frac{g(T)}{\delta}\right)e^{-\delta t} + \frac{g(T)}{\delta} . \tag{2.3}$$

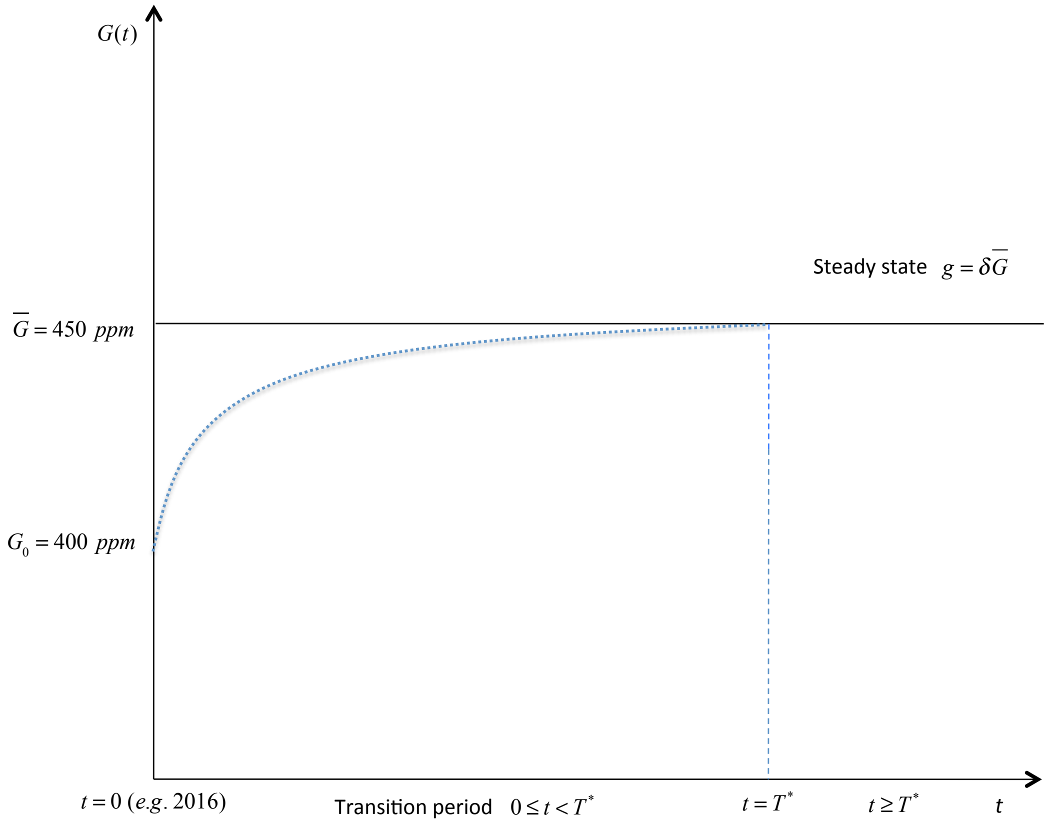
From equation (2.3), once T is chosen so too is $G(g(T),t)$ in the transition period. As will be discussed in Section 3, we assume that T is chosen by a social planner to maximize global social welfare. In selecting T, we are choosing the inter-temporal quantity of the global public bad. With constant transition period emissions, the pollutant stock is an exponential function of time, as shown in Figure 1 below, where $G_0 = 400$ is the current quantity of the CO_2eq stock and $G_0 = 450$ is the given sustainable quantity to be reached by some convergence time T^* [quantities drawn from United Nations (2013)].

Hence, the current quantity of the pollutant stock is lower than the target quantity, that is, $G_0 < \overline{G}$. This makes it clear that the aim of policy is to increase the stock of CO_2eq from its current level to the target level. The target level is of course lower than it would otherwise be absent policy action over emissions. When we think of achieving a sustainable CO_2eq stock it is not immediately apparent that our goal is to increase the current stock to the sustainable quantity, but this is the aim of policy as made clear by Figure 1.

On the other hand, if $G_0 > \overline{G}$, as would be the case with river basin or lake pollution, then the transition path path depicted in Figure 1 is inverted with the actual pollutant stock decreasing in t during the transition phase. In that type of stock-flow problem, the policy problem is to decrease the current pollutant stock to its sustainable quantity over some period of time. As will be noted, our results apply to both types of problem even though our focus is on the case depicted in Figure 1.

[2] It is assumed that $\overline{G} > G_0 e^{-\delta T}$, otherwise emissions are zero or negative.

Figure 1: Pollutant Stock Dynamics



We allocate the total emissions flow required during the transition period to meet the sustainable global CO_2eq stock target between the two regions on an equal per capita basis. Implicit here is the assumption of an exogenously given policy rule which allocates the burden of achieving the sustainable global CO_2eq stock equally across the two regions on a per capita basis. Other sharing rules are possible and our results do not depend on what is adopted. However, it seems equitable to assume such an allocation rule. This means region-specific emissions, $g_i(T)$, consistent with the global sustainable CO_2eq stock target, are also a function of *T* during the transition period and we can define:

$$g_i(T) = \xi_i g(T), \qquad i = 1,2 \qquad (2.4)$$

where $\xi_i = H_i / H$ for $i = 1,2$. This implies that the per capita emissions flow in each region is equal to the global per capita emissions flow; hence $g_i(T)/H_i = \xi_i g(T)/H$ for $i = 1,2$. For the CO_2eq stock to be constant in the steady-state global emissions must equal sequestration, that is, $\overline{g} = \delta\overline{G}$. Steady state emissions from region *i* are:

$$\overline{g}_i = \xi_i \delta\overline{G}. \qquad i = 1,2 \qquad (2.5)$$

In the steady state, per capita emissions in each region are equal to the global per capita emission flow, namely, $\overline{g}_i / H_i = \overline{g} / H$ for $i = 1,2$.

Having described the pollutant dynamics, we now develop the structure of production technologies in each region. This is the key source of asymmetry in our model. In addition to this, we develop expressions for the emissions taxes and explain their role.

2.2 Production technologies and emissions taxes

Output per capita in region i is defined by the continuous, differentiable and quasi-concave production function,

$$y_i = A_i(t)k_i^{\alpha}m_i^{\beta} \qquad i = 1,2 \qquad (2.6)$$

where $A_i(t)$ is a technical change variable, a function of time. We define k_i as a non-polluting input and m_i as an energy input generating emissions. The creation of emissions is proportional to the usage of the polluting enery input as captured by the relationship $m_i = \mu_i g_i$ for $i = 1,2$ where $0 \leq \mu_i$ is the given region-specific rate at which emissions are converted into units of the polluting input. With this set-up, the higher is μ_i the lower the emissions created by a given usage of the input. Thus, production in region i is cleaner with higher μ_i, and dirtier with a lower μ_i. Regional variation in μ_i is a source of difference in production technologies of the two regions. For example, a region with a service based economy is more likely to have a larger μ_i than a region with a manufacturing economy.

Denote ω and r as given world prices for the polluting and non-polluting inputs respectively. There is also a region-specific per unit emission abatement tax, s_i, where $i = 1,2$, levied on the polluting input. The constraint $rk_i + (\omega + s_i)m_i = E_i$, for $i = 1,2$, defines per capita spending on inputs by firms in region i where E_i is total spending. In Appendix A, we derive the demand function for the polluting and non-polluting inputs from a problem in which firms choose the two inputs to maximize output, subject to the spending constraint. From Appendix A, per capita (least cost) demand for the polluting input in region i is defined by the expression,

$$m_i = \varepsilon\left(\frac{1}{\omega + s_i}\right) \qquad i = 1,2, \qquad (2.7)$$

where $\varepsilon = \beta/(\alpha+\beta)$. Equation (2.7) implies that per capita demand for the input is continuous and monotonically decreasing in its own-price *and* the emission tax.

To derive the emission taxes that need to apply in each region, recall, first, that equation (2.7) expresses least cost per capita demand for the polluting input as a function of the tax for given values of the input price and production parameters. Notice, next, that from the discussion of equation (2.4) the allowable per capita usage of the polluting input in each region consistent with the chosen compliance time is defined by $\mu_i g(T)/H$. Now set the least cost demand for the polluting equal to the allowable per capita usage consistent with the chosen compliance time as follows:

$$m_i = \mu_i \frac{g(T)}{H}. \qquad i = 1,2 \qquad (2.8)$$

This leads us to:

Definition 1: The transition period emissions tax is the value of s_i which satisfies equation (2.8). That is:

$$s_i = \varepsilon\left(\frac{H}{\mu_i g(T)}\right) - \omega. \qquad i = 1,2 \qquad (2.9)$$

The tax defined at equation (2.9) plays a special role in our model. Specifically, once T is chosen by the social planner, it ensures that per capita usage of the polluting input in the region, and hence the region's emissions, are consistent with convergence of the global CO_2eq stock to its sustainable quantity, $\overline{G}$, by period *T*. In principle, s_i can be positive (a tax) or negative (a subsidy) depending on the value of ω relative to the first term on the right side of equation (2.9). However, for reasonable choices of the parameters in equation (2.9) s_i is positive and this is what we suppose henceforth.

The emissions tax differs across regions based on differences in μ_i where $i = 1,2$. If $\mu_2 < \mu_1$, implying region 2 produces more emissions for given energy input usage, then $s_2 > s_1$ and region 2 requires a relatively higher emissions tax to ensure its input usage and emissions flow are consistent with achieving the global sustainable stock target by the chosen compliance time. The reverse logic holds if $\mu_2 > \mu_1$; region 2 will require a relatively lower emissions tax to ensure its energy input usage and emissions flow are consistent with achieving the sustainable stock.

The emissions taxes act like Pigouvian taxes [see Pigou (1936)]. The tax inclusive price for the polluting input is defined as $\omega + s_i$ where i=1,2. Notice that while the underlying price for the input is the same across regions the tax inclusive prices differ because the emission tax is different. The tax inclusive price is higher in the region with the larger per unit emissions. Also, once T is chosen so too are the emissions taxes and the tax inclusive energy prices consistent with ensuring regional emissions are consistent with achieving the global pollutant stock target.

Similarly, the following condition sets the least cost demand for the polluting energy input equal to the allowable per capita usage consistent with the steady-state per capita emissions flow:

$$m_i = \mu_i \frac{\delta \overline{G}}{H}. \qquad i = 1,2 \qquad (2.10)$$

Definition 2: The steady-state emissions tax is the value of s_i which satisfies equation (2.10). That is:

$$\overline{s}_i = \varepsilon \left(\frac{H}{\mu_i \delta \overline{G}} \right) - \omega . \qquad i = 1,2 \qquad (2.11)$$

The steady-state emissions tax ensures per capita demand for the energy input in a region is consistent with the per capita emissions flow required in the steady-state to keep the global CO_2eq stock constant (e.g., at $\overline{G} = 450$ ppm from Figure 1).

Using equations (2.8) and (2.9) in the firm's optimization problem from Appendix A, total output from region *i* in the transition period is:[3]

$$y_i\big(g(T)\big) = A_i(t) \left\{ \frac{1}{r}(1-\varepsilon) \right\}^{\alpha} \left\{ \mu_i \frac{g(T)}{H} \right\}^{\beta}. \qquad \text{i=1,2.} \qquad (2.12)$$

If $\mu_1 > \mu_2$, then $y_1 > y_2$ and vice-versa during the transition period. The region with relatively lower emissions can have higher per capita output in the transition period and still have the same per capita emissions as the relatively higher emission region.

Utilizing equations (2.10) and (2.11) in the firms' optimization problem in Appendix A, steady-state per capita output in region i is:

$$\overline{y}_i = A_i(t) \left\{ \frac{1}{r}(1-\varepsilon) \right\}^{\alpha} \left\{ \mu_i \frac{\delta \overline{G}}{H} \right\}^{\beta} \qquad i = 1,2 . \qquad (2.13)$$

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[3] See (A.3) in Appendix A.

As with the transition period, the region with the cleaner production technology has higher per capita output in the steady state while still complying with the sustainable stock.

2.3 Preferences and regional feasibility

As noted earlier, we assume the world population has homogeneous preferences though as we shall see per capita incomes can differ across regions. However, since per capita incomes are the same within a region, the analysis can proceed in terms of a representative person from each region. This person is assumed to have a continuous, differentiable and concave utility function with the following form;

$$u_i = \log(x_i q_i) - G(g(T), t) \qquad i = 1,2\,. \qquad (2.14)$$

Here we define x_i as per capita consumption of a private good, q_i as consumption of a pure local public good and $G(g(T), t)$ as the global CO_2eq stock from equation (2.3) - a pure public bad. The pure local public good provides a benefit only to the residents of the region in which it is provided and hence has no inter-regional spillovers. It can be thought of as any one of the services provided by governments such as education, health or public transport which benefits only the residents of the region. The pure public bad, the global CO_2eq stock, affects everyone in the world negatively and equally. Thus, it can also be interpreted as a local public bad with perfect inter-regional spillovers. With this functional form, utility is separable in the consumption goods and the global CO_2eq stock. It is assumed that the price of the private good is fixed at one while the price of the local public good is a parameter denoted as c_i. Hence, apart from the tax inclusive price of the polluting input, $\omega + s_i$, which varies with the emissions tax, all prices in our model are fixed. Similarly, steady-state per person utility in region i is defined by:

$$\bar{u}_i = \log(x_i q_i) - \bar{G}\,. \qquad i = 1,2 \qquad (2.15)$$

Per capita revenue from the emissions tax, R_i, in the transition period is

$$R_i(g(T)) = s_i \mu_i \frac{g(T)}{H}, \qquad i = 1,2 \qquad (2.16)$$

where s_i is the emissions tax defined by equation (2.9) and $\mu_i g(T)/H$ is the per capita polluting input usage consistent with the choice of convergence time. Similarly, steady-state per capita tax revenue in region *i* is:

$$\bar{R}_i = \bar{s}_i \mu_i \frac{\delta \bar{G}}{H} \qquad i = 1,2\,. \qquad (2.17)$$

In the transition period and the steady-state, we assume that the emissions tax revenue in each region is recycled to the population of the region on an equal per capita lump sum basis by the government of the region. Per capita income in region i can then be defined as,

$$\pi_i(g(T)) = y_i(g(T)) + R_i(g(T)), \qquad i = 1,2 \qquad (2.18)$$

while for the steady state per capita income becomes:

$$\bar{\pi}_i = \bar{y}_i + \bar{R}_i\,. \qquad i = 1,2\,. \qquad (2.19)$$

With the revenue from the emissions tax distributed in this way, the feasible constraint in region i during the transition period is

$$x_i H_i + c_i q_i = \pi_i(g(T)) H_i \qquad i = 1,2 \qquad (2.20)$$

where $x_i H_i$ is total expenditure on private consumption, $c_i q_i$ is total spending on the local public good and $\pi_i(g(T))H_i$ is total income which, from equation (2.18), is equal to produced income plus revenue from the emissions tax. With this set up, revenue from the emissions tax is added to produced output to form an intermediate (numeraire) good which is then converted into private consumption and the local public good through equation (2.20). This assumption is a simple device to proxy all the different ways that revenue from emissions taxes is recycled to fund the provision of government services and private good consumption. Since it is provided to citizens lump sum this recycling has no efficiency costs in our model.

Similarly, in the steady-state the feasible constraint is

$$x_i H_i + c_i q_i = \bar{\pi}_i H_i . \qquad i = 1,2 \qquad (2.21)$$

Since $\bar{\pi}_i$ is fixed and H_i is independent of *T* in the steady-state, citizen income $\bar{\pi}_i H_i$ and hence public and private consumption, are given in the steady-state.

We assume for the transition period and the steady state that provision of the local public good in both regions is consistent with an efficiency rule. Expressions for local public good provision consistent with this rule are derived in Appendix B. Substituting them into the direct utility function allows us to derive indirect utility functions for a representative person in each region. For the transition period, indirect utility for a representative person in region i is given by

$$V_i = \log H_i \frac{\pi_i^2}{4c_i} - G(g(T),t) \qquad i = 1,2 , \qquad (2.22)$$

while in the steady state it is

$$V_i = \log H_i \frac{\bar{\pi}_i^2}{4c_i} - \bar{G} . \qquad i = 1,2 \qquad (2.23)$$

2.4 Regional social welfare and population mobility

Suppose the welfare of a representative person in region i is the sum of their indirect utility in the transition period time and the steady-state, as defined at equations (2.22) and (2.23) above. We think of this sum as defining a social welfare function, W_i, for region i, where $i = 1,2$. A convenient presentation of the social welfare function for a region is as follows,

$$\begin{aligned} W_i(g(T)) = \int_0^\infty e^{-\rho t} \log H_i dt + \int_0^T e^{-\rho t} \log\left[\pi_i(g(T))\right]^2 dt + \\ \int_T^\infty e^{-\rho t} \log \bar{\pi}_i^2 \, dt - \int_0^\infty e^{-\rho t} G(g(T),t)\, dt - \int_T^\infty e^{-\rho t} \bar{G}\, dt. \end{aligned} \qquad i = 1,2 . \quad (2.24)$$

Recall that we also allow the global population to be freely mobile between the two regions. With free inter-regional population mobility per capita inter-temporal social welfare must be the same across the two regions in any equilibrium. This implies that the convergence choice, *T*, must be consistent with the following migration constraint:

$$W_1(g(T)) - W_2(g(T)) = 0 . \qquad (2.25)$$

From the free mobility condition, during the transition period the supply of mobile citizens in each region is, implicitly, a function of the choice of convergence time. For the transition period we can, therefore, define $H_i(T)$, for $i = 1,2$, so the spatial allocation of mobile citizens in the global economy is dependent on the transition time. However, in the steady-state the supply of mobile citizens to each region is given since there is never any reason for a person to move in the steady state.

3. Optimal convergence time and induced fiscal transfers

We now turn to the main result of the paper; that on social welfare grounds the fiscal transfers induced by the use of emissions taxes to achieve a sustainable pollutant stock target affects the time taken to converge to the target. This holds for stock flow problems whether the current stock is lower or higher than the target. We begin by assuming the world social welfare function, $W(T)$, is the sum of the two regional per capita social welfare functions as follows,

$$W(T) = \Delta W_1\big(g(T)\big) + (1-\Delta)W_2\big(g(T)\big), \tag{3.1}$$

where $0 \le \Delta \le 1$ is the weight placed on region 1 in the global social welfare function, $0 \le \Delta \le 1$ is the weight on region 2 while $W_1\big(g(T)\big)$ and $W_2\big(g(T)\big)$ are the regional welfare functions, defined at equation (2.23). To find the value of *T* that maximizes global social welfare, given that we have an exponential CO_2eq stock transition path (see Figure 1), we solve the following maximization problem:[4]

$$\begin{aligned} &\underset{T}{Max}\ W = \Delta W_1\big(g(T)\big) + (1-\Delta)W_2\big(g(T)\big) \\ &Sto: W_1\big(g(T)\big) - W_2(g(T)) = 0. \end{aligned} \tag{3.2}$$

The solution proceeds by differentiating the objective function in this maximization problem with respect to *T*. For the case where $\Delta = 0.5$ this yields

$$\begin{aligned} &\int_0^\infty e^{-\rho t}\frac{H_{1,T}}{H_1(T)}dt + \int_0^T e^{-\rho t}\frac{2\pi_{1,T}}{\pi_1(g(T))}dt + \int_0^\infty e^{-\rho t}\frac{H_{2,T}}{H_2(T)}dt \\ &\qquad + \int_0^T e^{-\rho t}\frac{2\pi_{2,T}}{\pi_2(g(T))}dt - 2\int_0^T e^{-\rho t}G_{,g}g_{,T}dt = 0 \end{aligned} \tag{3.3}$$

where

$$\frac{dG}{dg} = G_{,g} > 0 \text{; and} \qquad \frac{dg}{dT} = g_{,T} < 0$$

capture how the global CO_2eq stock responds to changes in emissions and emissions respond to changes in the transition time. An expression for the emissions response to a change in the convergence time is derived in Appendix D.

The terms $H_{1,T}$ and $H_{2,T}$ in equation (3.3) capture the population supply response in each region to a change in the transition time. Expressions for these population responses can be found by totally differentiating the equal utility constraint with respect to H_1, H_2 and *T* to obtain:

$$H_{1,T} = 2\rho H_1 \int_0^T e^{-\rho t}\left(\frac{\pi_{2,T}}{\pi_2} - \frac{\pi_{1,T}}{\pi_1}\right)dt \tag{3.4}$$

$$H_{2,T} = -\frac{H_2}{H_1}H_{1,T}. \tag{3.5}$$

Combining equations (3.4) and (3.5) with (3.3) yields the first order necessary condition for the convergence time which takes account of the migration constraint, feasibility and fiscal transfers from the emissions taxes as follows:

$$\int_0^T e^{-\rho t}\frac{2\pi_{1,T}}{\pi_1(g(T))}dt + \int_0^T e^{-\rho t}\frac{2\pi_{2,T}}{\pi_2(g(T))}dt - 2\int_0^T e^{-\rho t}G_{,g}g_{,T}dt = 0. \tag{3.6}$$

[4] Feasibility is incorporated into the objective function in the welfare maximization problem.

A solution to the global social welfare maximization problem is a value, T^*, that solves equation (3.6). Notice that because we have allowed the world population to be mobile across regions, T^* must also satisfy the equal utility constraint. It must be a convergence time in which a person in region 1 has the same indirect utility as her counterpart in region 2. The outcome is equitable if we place the same welfare weight on people who live in different regions in the social welfare function.[5] However, a T that solves equation (3.6) can only be thought of as 'constrained optimal' since we have assumed an exponential transition path with a constant global emissions flow.

Once the convergence time is chosen by the social planner so too is the constant global emissions flow consistent with achieving the target pollutant stock. This also means regional emissions taxes that ensure regional emissions are consistent with achieving the global target by T^* are also determined. Though the social planner has one instrument, in choosing this she determines the dynamics of the model as set out in Section 2.1 and illustrated in Figure 1. Given that regions provide local public goods efficiently she also determines local public good provision, income and private good consumption in each region. Therefore, once T^* is chosen, all the endogenous variables of the model are determined.

The first two terms in equation (3.6) capture the inter-temporal effect of an incremental change in T on numeraire income in regions 1 and 2 respectively. The last term captures the impact of an incremental change in T on the CO_2eq pollutant stock. It is helpful for our results below to break up the impact of an incremental change in T on numeraire income into its impact on emissions tax revenue and produced output (the two components that make up numeraire income). This means we can express the first order necessary condition in terms of marginal costs and benefits of a change in T as follows:

$$\begin{aligned}&\underbrace{\int_0^T e^{-\rho t}\frac{2R_{1,T}}{\pi_1(g(T))}dt}_{FE_1=\textit{fiscal}\text{ effect in state 1}}+\underbrace{\int_0^T e^{-\rho t}\frac{2y_{1,T}}{\pi_1(g(T))}dt}_{OE_1=\text{output effect in state 1}}\\ &+\underbrace{\int_0^T e^{-\rho t}\frac{2R_{2,T}}{\pi_2(g(T))}dt}_{FE_2=\textit{fiscal}\text{ effect in state 2}}+\underbrace{\int_0^T e^{-\rho t}\frac{2y_{2,T}}{\pi_2(g(T))}dt}_{OE_2=\text{output effect in state 2}}-\underbrace{2\int_0^T e^{-\rho t}G_{,g}g_{,T}dt}_{SE=\text{stock effect}}=0\end{aligned} \tag{3.7}$$

The first term captures the impact of an incremental change in T on the emissions tax revenue raised in region 1 during the transition phase. We call this the fiscal effect of an incremental change in T experienced by region 1. It captures the change in fiscal transfers induced in region 1 by an incremental change in T in the transition period. The second term captures the effect of an incremental change in T on produced output in region 1. We think of this as an output effect. The next two terms are analogous fiscal and output effects for region 2. The last term captures the impact of incremental increase in T on the CO_2eq pollutant stock. The first order necessary condition for T sums these terms across regions and ensures they are equal to zero in net terms.

We can sign the fiscal, output and CO_2eq stock effects of an incremental change in T appearing in equation (3.7) for the case where the current stock is less

[5] A set of second order sufficient conditions for existence of a solution is provided in Appendix C. We assume these conditions hold, assuring us of the existence of at least one solution. If the conditions are not satisfied equilibria may still exist as our conditions are sufficient and not necessary. The question of what are the least restrictive sufficient conditions is beyond the scope of the paper.

than the sustainable quantity. We can then say which terms are considered by the planner to be marginal benefits or costs of incremental increases in *T*. Start by noticing from Appendix D that for this case the emissions flow in the transition period is decreasing in *T*; that is, $g_{,T} < 0$. Note also that $R_{i,T} = R_{i,g} g_{,T}$ where,

$$R_{i,g} = -\frac{\omega}{H}\mu_i < 0 \qquad\qquad i = 1,2. \qquad (3.8)$$

This means that $R_{i,T} > 0$, and the induced fiscal effects of an incremental increase in *T* are positive in each region. That is, $FE_1 > 0$ and $FE_2 > 0$ in equation (3.7). The social planner considers the fiscal effects of an incremental change in *T* to be marginal benefits. It is also the case that $y_{i,T} = y_{i,g} g_{,T}$. From this, we can deduce that:

$$y_{i,g} = \beta\left\{\frac{1}{r}(1-\varepsilon)\right\}^{\alpha}\left(\frac{\mu_i}{H}\right)^{\beta}\left(g(T)\right)^{\beta-1} > 0. \qquad i = 1,2 \qquad (3.9)$$

Since $g_{,T} < 0$, then $y_{i,T} = y_{i,g} g_{,T} < 0$ and the output effects of an incremental increase in *T* are negative. This means that $OE_1 < 0$ and $OE_2 < 0$ in equation (3.7). The output effects can be interpreted as marginal costs of an increment in *T*. Lastly, we know that $G_{,g} > 0$, so if $g_{,T} < 0$ we have $G_{,g} g_{,T} < 0$. This implies that the stock effect is a marginal benefit of an incremental increase in *T* since *G* is a pure public bad; that is, $SE > 0$.

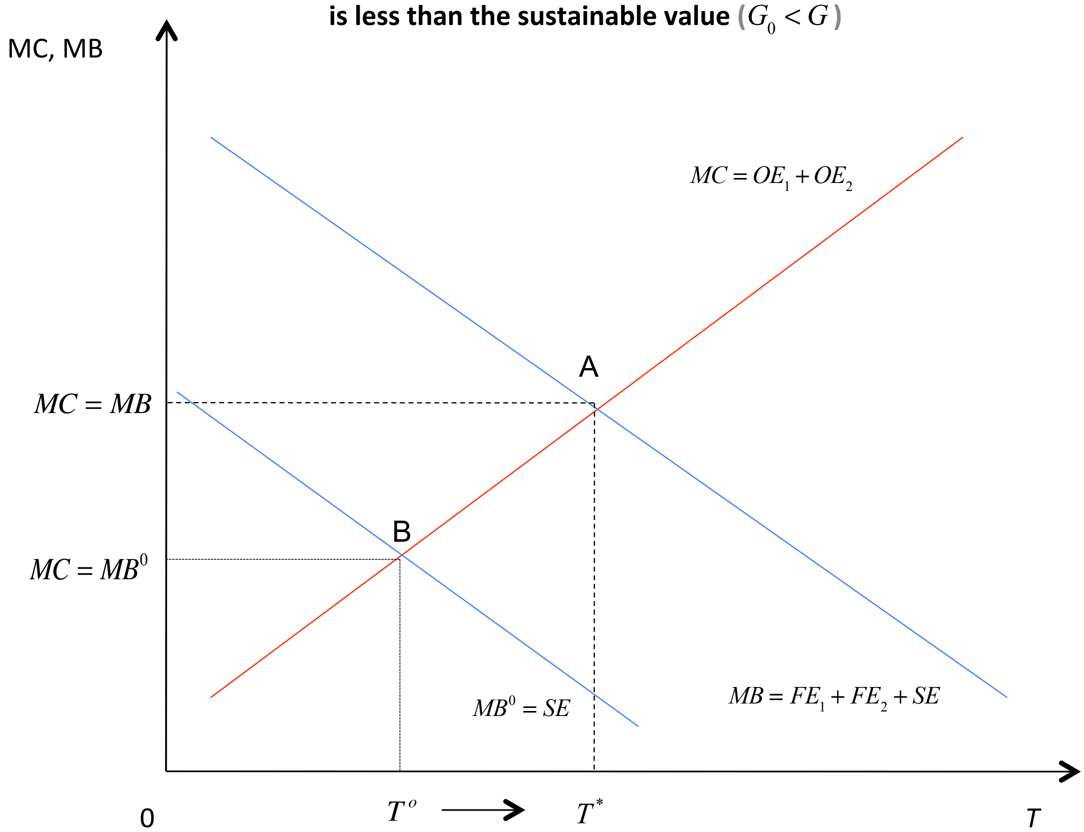
Given the above signs, the first order necessary condition, equation (3.7), can be expressed as an equality between marginal benefit and marginal cost when the current pollutant stock is less than the sustainable quantity as follows:

$$\underbrace{FE_1 + FE_2 + SE}_{marginal\,benefit} = \underbrace{OE_1 + OE_2}_{marginal\,cost}. \qquad (3.10)$$

By increasing the marginal benefit associated with any given *T*, without affecting the marginal cost, the fiscal effects induced by recycling the revenue from emissions taxes result in a larger *T* in any solution to the global welfare maximization problem. Convergence to the target sustainable global CO_2eq stock is delayed by the fiscal consequences of using emissions taxes when the current pollutant stock is less than the sustainable quantity. This is illustrated in Figure 2 below.

In the Figure, there is a single marginal cost curve which is the sum of the two output effects; that is, $MC = OE_1 + OE_2$. If there is no revenue recycling from the emissions tax, the marginal benefit curve captures only the stock effect of increased *T*; that is, $MB^0 = SE$. An equilibrium convergence time, T^0, occurs at point B where $MB^0 = MC$. However, if there is revenue recycling as we suppose the marginal benefit curve is $MB = FE_1 + FE_2 + SE$ and the equilibrium convergence time, T^*, is at point A where $T^* > T^0$. As shown, the impact of revenue recycling when the current pollutant stock is less than the sustainable target, is to delay convergence.

Figure 2: Transition choice when the current pollutant stock is less than the sustainable value $(G_0 < \overline{G})$



For the case where the current pollutant stock is greater than the sustainable stock, as with river or lake pollution, the fiscal transfers induced by emissions taxes reduce the convergence time. From Appendix D we can see the emissions flow in the transition period is increasing in *T*, that is, $g_{,T} > 0$. Note also that $R_{i,T} = R_{i,g} g_{,T}$ where,

$$R_{i,g} = -\frac{\omega}{H}\mu_i < 0 \qquad i = 1,2. \qquad (3.11)$$

This means that $R_{i,T} < 0$ when the current pollutant stock is more than the target and the induced fiscal effects in region *i* of an incremental increase in *T* are negative. That is, $FE_1 < 0$ and $FE_2 < 0$. The social planner now considers the fiscal effects of an incremental increase in *T* to be marginal costs. As before, we know that $y_{i,T} = y_{i,g} g_{,T}$ where $y_{i,g} > 0$. Since $g_{,T}$ is now positive, we know that $y_{i,T} = y_{i,g} g_{,T} > 0$ and the output effects of an incremental increase in *T* are now positive. This means that $OE_1 > 0$ and $OE_2 > 0$. The output effects can be interpreted as marginal benefits of an increment in *T*. Lastly, we know that $G_{,g} > 0$, so if $g_{,T} > 0$ we have $G_{,g} g_{,T} > 0$. The social planner now perceives the stock effect to be a marginal cost of an incremental increase in *T*; that is, $SE < 0$ in equation (3.7).

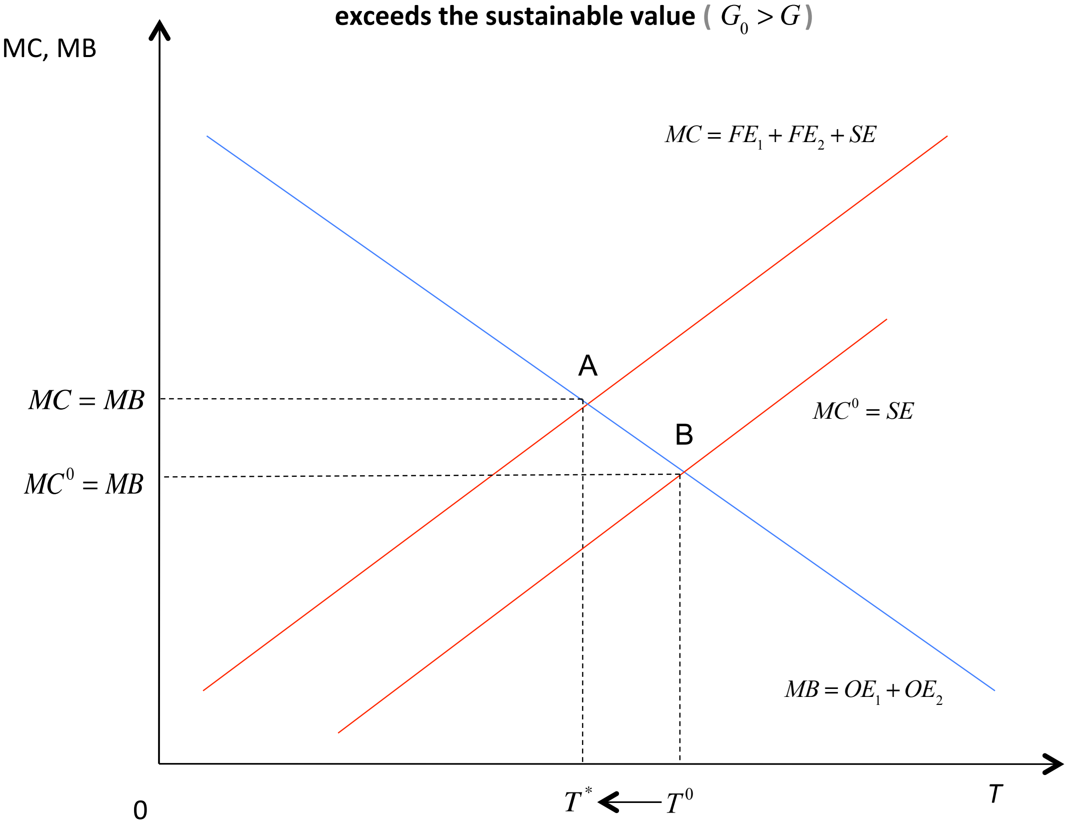
Therefore, the first order necessary condition for *T* when the current pollutant stock exceeds the target can be expressed as

$$\underbrace{OE_1 + OE_2}_{marginal\ benefit} = \underbrace{FE_1 + FE_2 + SE}_{marginal\ cost}. \tag{3.12}$$

By increasing the marginal cost for a given *T*, without affecting the marginal benefit, the fiscal effects induced by recycling the revenue from emissions taxes result in a smaller *T* in any solution to the global welfare maximization problem.[6] Therefore, convergence to the target sustainable global CO_2eq stock is quickened by the fiscal consequences of using emissions taxes when the current pollutant stock is greater than the sustainable quantity. This is illustrated in Figure 3.

In the Figure, there is one marginal benefit curve which is the sum of the two output effects; that is, $MB = OE_1 + OE_2$. If there is no revenue recycling from the emissions tax, the marginal cost curve captures only the stock effect of increased *T*; that is, $MC^0 = SE$. An equilibrium convergence time, T^0, occurs at point B where $MB = MC^0$. However, if there is revenue recycling as we suppose the marginal cost curve is $MC = FE_1 + FE_2 + SE$ and the equilibrium convergence time, T^*, is at point A where $T^* < T^0$. The impact of revenue recycling when the current pollutant stock exceeds the sustainable target, is to hasten convergence.

Figure 3: Transition choice when the current pollutant stock exceeds the sustainable value ($G_0 > \overline{G}$)



[6] Comparing equation (3.12) with equation (3.11), a symmetry between the choice of *T* for each pollutant problem is apparent. When we switch between the case where the current stock is less than the sustainable value (Figure 2) and the case where the current stock exceeds the sustainable value (Figure 3) the marginal benefits and costs of a change in *T* interchange in the first order necessary condition.

This completes our presentation of the results. The key point we wish to make is that when emissions taxes are used for mitigation and we consider an economy made up of regions where population mobility imposes an equal utility constraint on any solution, the time it takes to converge to a sustainable target in stock-flow pollutant problems should take account of the induced fiscal effects of revenue recycling. What is more, when the current stock is less than the sustainable target, the fiscal effects are marginal benefits of an incremental increase in T and result in delayed convergence relative to a case of no revenue recycling. However, if the current stock exceeds the sustainable target, the fiscal effects are marginal costs and result in faster convergence to the sustainable target relative to a case with no recycling. Hence, the precise impact of recycling on the choice of convergence time depends on the nature of the stock flow problem being considered.

4. Conclusion

In this paper, we have noted that the effects of fiscal transfers induced by emissions taxes have been extensively examined within the context of static models with no regions or population mobility. Therefore, we have set out to examine the impact of these transfers in a stock flow pollutant problem with regions and population mobility. To do this, we have constructed a model of a stock flow pollutant where the current stock is less than the future sustainable stock. We have thought of this as capturing the particular case of atmospheric carbon pollution. Though we have focused on this case, we have also presented our results for a case in which the current pollutant stock exceeds the sustainable quantity.

We consider an economy consisting of two regions which generate emissions contributing to a common stock of pollutant (carbon in the atmosphere). The total population has homogenous preferences over a local public good provided by regions and a private consumption good. This population is also mobile across regions in response to the choice of convergence time. We suppose a social planner chooses the time it takes for the actual global pollutant stock to converge to its desired level using emissions taxes in each region as mitigation instruments. The planner makes this choice to maximize inter-temporal global social welfare with equal weights on the two regions. The maximization is undertaken subject to the migration constraint and on the assumption that the time path for the global pollutant stock in the transition phase has constant emissions. Revenue from the emissions taxes is recycled to residents within each region on an equal per capita lump sum basis.

The main finding from this analysis is that the optimal convergence time in a world with a stock flow pollutant, emissions taxes, regions and free mobility is in part determined by the fiscal transfers induced by the emissions taxes. On social welfare grounds the time taken to converge to sustainable stock targets should take account of the fiscal effects of transfers induced by emissions taxes. This is so whether we are dealing with a stock flow pollutant problem where the current stock exceeds the sustainable quantity (river or lake pollution) or one in which the current stock is less than the sustainable quantity (atmospheric carbon pollution). We also demonstrate a symmetry between the two types of pollutant stock problems. When the current stock exceeds the sustainable level, the fiscal effects are a cost of delayed convergence and tend to shorten the convergence time. If the current stock is less than the sustainable quantity the fiscal effects induced by emissions taxes are a marginal benefit from delayed convergence, tending to lengthen the convergence time.

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Appendix A
Polluting input demand

Per capita demands for k_i and m_i are solved from

$$\begin{aligned} &\underset{k_i, m_i}{Max} \;\; y_i = A_i k_i^{\alpha} m_i^{\beta} \\ &s.t. \quad rk_i + (\omega + s_i) m_i = E_i \end{aligned} \qquad i = 1,2 \qquad \text{(A.1)}$$

For $E_i = 1$, the expenditure constraint implies

$$k_i = (1/r) \cdot \left[1 - \left(\omega + s_i\right) m_i\right], \qquad \text{(A.2)}$$

so per capita demand for the polluting input in a state is found by solving

$$\underset{m_i}{Max} \;\; y_i = \frac{A_i}{r^{\alpha}} \left[1 - \left(\omega + s_i\right) m_i\right]^{\alpha} m_i^{\beta} \qquad i = 1,2 \qquad \text{(A.3)}$$

This yields least cost, per capita, demand for the polluting input as

$$m_i = \varepsilon \frac{1}{\omega + s_i}, \qquad i = 1,2 \qquad \text{(A.4)}$$

where

$$\varepsilon = \frac{\beta}{\alpha + \beta}.$$

Appendix B

Provision of local public goods and regional welfare

We suppose that the government of region i is benevolent and makes its choice of local public good provision in the region by solving the following maximization problem:

$$\underset{q_i}{Max}\ u_i = \log\left\{\left(\pi_i - \frac{c_i q_i}{H_i}\right)q_i\right\} - G(g(T),t) \qquad i = 1,2 \qquad \text{(B.1)}$$

From this, public and private good provision in the transition period are

$$q_i = H_i \frac{\pi_i}{2c_i}, \qquad x_i = \frac{\pi_i}{2} \qquad i = 1,2 \qquad \text{(B.2)}$$

Using (B.2) in the objective of (B.1) yields the indirect (maximum value) function for region i in the transition period as:

$$V_i = \log H_i \frac{\pi_i^2}{4c_i} - G(g(T),t)\,. \qquad i = 1,2 \qquad \text{(B.3)}$$

This can be expressed more conveniently as

$$V_i = \log H_i + \log \pi_i^2 - G(g(T),t) - \overline{C}_i\,, \qquad i = 1,2 \qquad \text{(B.4)}$$

where $\overline{C}_i = \log 4c_i$ is a constant, independent of the convergence time.

From the main text, $H_i(T)$ and π_i are functions of the convergence time chosen by the social planner. Hence, during the transition period we have:

$$q_i(T) = H_i(T)\frac{\pi_i(T)}{2c_i}, \qquad x_i(T) = \frac{\pi_i(T)}{2}\,. \qquad \text{(B.5)}$$

Given efficient regional government behaviour, local public good provision and private consumption in each region are chosen by the social planner once she chooses the convergence time *T*.

In the *steady-state*, the pollutant stock is $\overline{G}$ and household utility becomes

$$\overline{u}_i = \log(x_i q_i) - \overline{G}\,. \qquad i = 1,2 \qquad \text{(B.6)}$$

The steady state household budget constraint is $H_i x_i + c_i q_i = H_i \overline{\pi}_i$ where $\overline{\pi}_i$ is steady-state household income, as defined in the main text. Solving an analogous problem to (B.1) above, indirect utility in the steady-state is

$$V_i = \log H_i \frac{\overline{\pi}_i^2}{4c_i} - \overline{G}\ . \qquad i = 1,2 \qquad \text{(B.7)}$$

This can be expressed as

$$V_i = \log H_i + \log \overline{\pi}_i^2 - \overline{G} - \overline{C}_i\,. \qquad \text{(B.8)}$$

The *welfare* of a representative household in state *i* (the sum of indirect utility in the transition period and steady state), which we also think of as the regional social welfare function, is

$$\begin{aligned} W_i(T) = &\int_0^T e^{-\rho t}\left[\log H_i + \log \pi_i^2 - G(g(T),t)\right]dt \\ &+ \int_T^\infty e^{-\rho t}\left[\log H_i + \log \overline{\pi}_i^2 - \overline{G}\right]dt \end{aligned} \qquad \text{(B.9)}$$

This has a more convenient representation, namely,

$$
\begin{aligned}
W_i(T) = \int_0^{\infty} e^{-\rho t} \log H_i dt + \int_0^{T} e^{-\rho t} \log \pi_i^2\, dt + \\
\int_T^{\infty} e^{-\rho t} \log \overline{\pi}_i^2\, dt - \int_0^{\infty} e^{-\rho t} G(g(T),t)\, dt - \int_T^{\infty} e^{-\rho t}\, \overline{G}\, dt.
\end{aligned}
\tag{B.10}
$$

Appendix C
Second order sufficient conditions

If $W_i(g(T))$, the social welfare function in region i, for $i=1,2$, is concave in T then so too is the global social welfare function. To check this, differentiate the first order necessary condition, (3.5), with respect to T. This yields:

$$2\int_0^T e^{-\rho t}\left[\frac{\pi_{-i,T}^2}{\pi_{-i}^2}-\frac{\pi_{-i,TT}}{\pi_{-i}}\right]dt+2e^{-\rho t}\frac{\pi_{-i,T}(T)}{\pi_{-i}(T)}$$

$$-\int_0^T e^{-\rho t}\left(G_{,gg}g_{,T}^2+G_{,g}g_{,TT}\right)dt-e^{-\rho t}G_{,g}g_{,T}$$

where $-i$ denotes the neighbor of state i. Noting that $\pi_{i,T}=\pi_{i,g}g_{,T}$ this becomes:

$$W_{i,TT}(T)=-2\int_0^T e^{-\rho t}\left[\frac{(\pi_{-i,g}g_T)^2}{\pi_{-i}^2}-\frac{\left(\pi_{-i,gg}g_T^2+\pi_{-i,g}g_{,TT}\right)}{\pi_{-i}}\right]dt$$

$$+2e^{-\rho t}\frac{\pi_{-i,g}g_T}{\pi_{-i}(T)}-\int_0^T e^{-\rho t}\left(G_{,gg}g_{,T}^2+G_{,g}g_{,TT}\right)dt-e^{-\rho t}G_{,g}(T)g_{,T}(T)$$

One set of sufficient conditions which ensure that $W_{i,TT}<0$ is as follows:

B1. $\pi_{i,g}^2g_{,T}^2-\pi_i\pi_{i,gg}g_{,T}^2\ge 0$

B2. $\pi_{i,g}\left[g_{,TT}\left(1-e^{-\rho t}\right)+g_{,T}e^{-\rho t}\right]\le 0$

B3. $\dfrac{g_{,TT}}{\delta}\left[\dfrac{1-e^{-\rho t}}{\rho}-\dfrac{1-e^{-(\rho+\delta)}}{\rho+\delta}\right]+e^{-\rho t}G_{,g}(T)g_{,T}(T)\ge 0$

Note that B.3 can be further simplified to:

$$\frac{g_{,TT}}{\delta}A+\frac{g_{,T}}{\delta}B\ge 0,$$

where:

$$A=\left[\frac{1-e^{-\rho t}}{\rho}-\frac{1-e^{-(\rho+\delta)}}{\rho+\delta}\right]$$

$$B=\left[e^{-\rho t}-e^{-(\rho+\delta)}\right].$$

Remark 1: B1-B3 is one of many possible sets of sufficient conditions that ensure concavity of the objective function of state i in the choice variable T. Any conditions that ensure $W_{i,TT}<0$ can be used. Though the issue of the least restrictive set of conditions is interesting, it is beyond the scope of the paper.

Remark 2: Condition B1 represents the derivative of income effect whereas B2-B3 jointly represent the derivative of the pollutant stock effect.

It is useful to see if the sufficient condition holds when the current stock is less than the sustainable stock. Let $\rho=\delta=0.1$ and $t=20$. With these parameter values

$$\frac{1-e^{-\rho t}}{\rho}=8.647, \qquad \frac{1-e^{-(\rho+\delta)}}{\rho+\delta}=0.9065.$$

And hence

$$A=\left[\frac{1-e^{-\rho t}}{\rho}-\frac{1-e^{-(\rho+\delta)}}{\rho+\delta}\right]=7.739>0$$

With the same parameter values

$$B = \left[e^{-\rho t} - e^{-(\rho+\delta)} \right] = -0.6834 < 0 .$$

From *Annex E*, $g_{,T} < 0, g_{,TT} > 0$ and hence B3 holds, except for small values of *t*. For B2 to hold requires

$$\left[g_{,TT} \left(1 - e^{-\rho t}\right) + g_{,T} e^{-\rho t} \right] \leq 0$$

since $\pi_{2,g} > 0$. Noting that $(1 - e^{-\rho t}) > 0$ and $e^{-\rho t} > 0$, given that $g_{,T} < 0, g_{,TT} > 0$ there is no reason why this might not be satisfied. For the parameter values it will hold if

$$\frac{g_{,T}}{g_{,TT}} \geq \frac{1 - e^{-\rho t}}{e^{-\rho t}} = 6.3909$$

which is plausible. Now consider condition B1. For this to hold requires

$$\pi_{2,g}^2 \geq \pi_2 \pi_{2,g,g} .$$

Once again, it is plausible that this is satisfied. Thus, there are no severe restrictions required for the sufficient condition to be satisfied.

Appendix D
Emission flow responses

From (2.2) in the main text

$$\frac{dg}{dT} = g_{,T} = \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}}\left[\frac{1}{1-e^{-\delta T}}\right]\left[G_0 - \overline{G}\right].$$

$$\frac{d^2 g}{dT^2} = g_{,TT} = -\frac{\delta^3 e^{-\delta T}}{1-e^{-\delta T}}\left[\frac{1}{1-e^{-\delta T}}\right]^2\left[\frac{1}{1-e^{-\delta T}} + \frac{e^{-\delta T}}{1-e^{-\delta T}}\right]\left[G_0 - \overline{G}\right].$$

From this

$$g_{,T} < 0,\ g_{,TT} > 0 \ \textit{if}\ G_0 < \overline{G} \ \text{(atmospheric pollution)}$$

and

$$g_{,T} > 0,\ g_{,TT} < 0 \ \textit{if}\ G_0 > \overline{G} \ \text{(river/lake pollution).}$$

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