# Trends in microcrack properties and reconstruction of TI elasticity tensors of shales

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#### **Summary**

Stress dependency of the TI elastic tensor of shales is important for seismic interpretation, overpressure prediction, 4D monitoring, etc. Using Sayers-Kachanov formalism, we develop a new model for transversely isotropic (TI) media which predicts stress sensitivity behaviour of all five elastic coefficients. The model is used to parameterize elastic properties of about 20 shales obtained from our laboratory measurements and also from a literature survey. The four fitting parameters (namely, tangential and normal compliances of a single crack, characteristic pressure, and crack orientation anisotropy parameter) show correlations with the depth from which the shale was extracted. With increasing depth, the tangential compliance broadly decreases exponentially and the ratio of normal to tangential compliance generally increases linearly. The crack orientation anisotropy parameter exponentially increases with the depth for most of the shales indicating that cracks may be more aligned in the bedding plane. The characteristic pressure shows no simple correlation with depth. The suggested model allows prediction of stress dependency of all five elastic coefficients if only three of them are known. This can be useful, for instance, for the reconstruction of all five elastic coefficients of shale from log data.

## Introduction

Modeling stress dependency of the full elastic tensor of shales is important for seismic interpretation, overpressure prediction, 4D monitoring, etc. In spite of the importance of the problem, there is no widely accepted theory for stress dependency of elastic properties of shales. Shapiro and Kaselow (2005) suggested a stress dependency model for orthorhombic media based on dual porosity approach. Their model is based on the bimodal distribution of pore compliances and superposition of deformation fields caused by closure/shape change of these two group of pores under applied stress. Ciz and Shapiro (2009) applied this model to shales. However, their approach implies that normal and tangential compliances of each grain contact are equal. We believe that this assumption may not be accurate for shales, where normal compliance may be much smaller than the tangential compliance. This assumption may be the reason for the fact that the model of Ciz and Shapiro (2009) predicts the elastic coefficient c<sub>13</sub> to be independent of confining stress.

The aim of this paper is to develop a model of stress dependency of all five elastic coefficients of TI shales without predefining the compliance ratio of grain contacts (but still with the minimum number of parameters). To this end, we combine the dual porosity approach of Shapiro and Kaselow (2005) with the non-interaction approximation of Sayers and Kachanov (1995).

## Theoretical model

We assume that variation of elastic properties of a shale with pressure up to 60 MPa can be explained by closure of soft (compliant) porosity. Soft porosity comprises a small part of total porosity and consists of pores with high compliances, such as fractures, cracks and grain contacts. Figure 1 shows an SEM image which illustrates existence of such type of pores in shales.

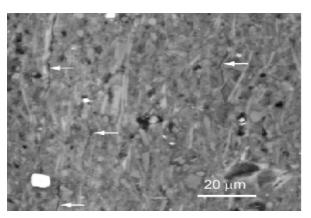


Figure 1: An Officer Basin shale showing particle alignment and the presence of microfractures (white arrows).

Sayers and Kachanov (1995) provided a formalism that allows calculation of an excess of compliances  $\Delta S_{iikl}$  due to the introduction of the compliant porosity to a rock matrix with compliance of  $S_{iikl}^0$  as follows,

$$\Delta S_{ijkl} \equiv \frac{1}{4} \left( \delta_{ik} \alpha_{jl} + \delta_{il} \alpha_{jk} + \delta_{jk} \alpha_{il} + \delta_{jl} \alpha_{ik} \right) + \beta_{ijkl}$$
 (1)

$$\Delta S_{ijkl} = \frac{1}{4} \left( \delta_{ik} \alpha_{jl} + \delta_{il} \alpha_{jk} + \delta_{jk} \alpha_{il} + \delta_{jl} \alpha_{ik} \right) + \beta_{ijkl}$$

$$\alpha_{ij} = \frac{1}{V} \sum_{r} B_{T}^{(r)} n_{i}^{(r)} n_{j}^{(r)} A^{(r)}$$
(2)

$$\beta_{ijkl} = \frac{1}{V} \sum_{r} \left( B_N^{(r)} - B_T^{(r)} \right) n_i^{(r)} n_j^{(r)} n_k^{(r)} n_l^{(r)} A^{(r)}. \tag{3}$$

# Trends in microcrack properties of shales

Here,  $\delta_{ii}$  is the Kronecker delta, r is the number of planar discontinuities with surface area  $A^{(r)}$  and surface-normal vectors  $n_i^{(r)}$  in volume V and  $B_N$  and  $B_T$  are the normal and tangential compliances of an individual crack.

To parameterize the stress dependency of shales, we use four parameters, namely  $B_T$ , tangential compliance; B, ratio of normal to tangential compliance;  $\eta$ , crack orientation anisotropy parameter that characterizes angular crack distribution; and  $P_0$ , the characteristic pressure at which compliant pores close. At zero pressure, the number of cracks with normals oriented at angle  $\theta$  is determined as  $N_0(\theta) = 1 + \eta \cos^2 \theta$ . Following Shapiro and Kaselow (2005) and Vlastos et al. (2006), we assume that the number of cracks decreases exponentially with pressure as follows  $N(P, \theta) = N_0(\theta) \exp(-P/P_0)$ . Substituting these formulas into equations (1-3), we obtain variation in compliances introduced by the compliant porosity.

$$\Delta S_{11} = \frac{B_{70} \exp(-P/P_0)}{105} (14 + 4\eta + 21B + 3B\eta)$$
 (4)

$$\Delta S_{33} = \frac{B_{T0} \exp(-P/P_0)}{105} (14 + 6\eta + 21B + 15B\eta)$$
 (5)

$$\Delta S_{11} = \frac{I_{10} + I_{10} + I_{10}}{105} (14 + 4\eta + 21B + 3B\eta)$$

$$\Delta S_{33} = \frac{B_{T0} \exp(-P/P_0)}{105} (14 + 6\eta + 21B + 15B\eta)$$

$$\Delta S_{44} = \frac{B_{T0} \exp(-P/P_0)}{105} (42 + 16\eta + 28B + 12B\eta)$$

$$\Delta S_{66} = \frac{B_{T0} \exp(-P/P_0)}{105} (42 + 10\eta + 28B + 4B\eta)$$

$$\Delta S_{13} = \frac{B_{T0} \exp(-P/P_0)}{105} (7B + 3B\eta - 7 - 3\eta)$$
(8)

$$\Delta S_{66} = \frac{B_{70} \exp(-P/P_0)}{105} (42 + 10\eta + 28B + 4B\eta)$$
 (7)

$$\Delta S_{13} = \frac{B_{70} \exp(-P/P_0)}{105} (7B + 3B\eta - 7 - 3\eta). \tag{8}$$

If we assume  $B_N = B_T$ , then equations (4-8) can be simplified as follows

$$\Delta S_{11} = \frac{B_{T0} \exp(-P/P_0)}{35} (5+\eta) \tag{9}$$

$$\Delta S_{11} = \frac{B_{T0} \exp(-P/P_0)}{35} (5+\eta)$$

$$\Delta S_{33} = \frac{B_{T0} \exp(-P/P_0)}{35} (5+3\eta)$$

$$\Delta S_{44} = \frac{B_{T0} \exp(-P/P_0)}{35} (10+4\eta)$$

$$\Delta S_{66} = \frac{2B_{T0} \exp(-P/P_0)}{35} (5+\eta)$$
(12)

$$\Delta S_{44} = \frac{B_{70} \exp(-P/P_0)}{35} (10 + 4\eta) \tag{11}$$

$$\Delta S_{66} = \frac{2B_{T0} \exp(-P/P_0)}{35} (5+\eta)$$
 (12)

$$\Delta S_{12} = 0. \tag{13}$$

# Data

We tested our model using a number of shales from the Officer Basin, Bass Basin, Carnarvon Basin (offshore Australia), Africa, and the North Sea (Pervukhina et al, 2008), as well Gulf Coast shale, hard shales and siliceous shale from Wang (2002). The shales were recovered from depths between 200 and 3604 m and vary in their physical properties and, most probably, in their mineralogical

content. Unfortunately, only few publications report information about overburden pressure, diagenesis, geological history, clay content and mineralogy of the investigated shales. The depth of origin of the shale samples is the only environmental parameter known for all the shales.

# Results

We use equations (4-8) to fit experimentally obtained stress dependencies of elastic coefficients of shales considering  $B_{T0}$ , B,  $P_0$  and  $\eta$  as fitting parameters. A histogram of the ratio of tangential over normal compliances, B, obtained by the fitting is shown in Figure 2. The values are distributed in three distinct groups, namely, small values less than 0.2, normal values from 0.7-1.1 and large values of about 2. Departure from unity is observed even in the group with the values form 0.7 to 1.1. This suggests that the ratio B=1implied by Ciz and Shapiro (2009), is not applicable for modeling of shales.

To understand the excess in misfit of the experimental stress dependencies caused by assumption that  $B_N = B_T$ , we fit the experimental data using equations (9-13). The misfits obtained by the use of the equation sets (4-8) and (9-13) are shown in Figure 3a by dark blue dots and pink circles, respectively. Figure 3b shows relative excess of misfit caused by assumption that  $B_N = B_T$ . The misfits are almost the same when B is close to unity. However in other cases, misfit by equations (9-13) noticeably exceeds the one obtained by use of equations (4-8) (note that the plot in Figure 3a is in logarithmic scale). Below we use equations (4-8) to fit the experimental stress dependencies of elastic properties of shales to avoid additional errors caused by assumption that  $B_N = B_T$ .

All four fitting parameters are plotted versus depth of shale extraction in Figure 4. Crack orientation anisotropy parameter,  $\eta$ , reveals a very general exponential growth with the depth of origin (Figure 4a) although there is significant scattering of the fitting values. Higher values of  $\eta$  mean better alignment of the cracks in the bedding plane; consequently, an increase of  $\eta$  with the depth indicates fast growth of anisotropy of shale with increase of overburden pressure.

Tangential compliance of a single crack normalized on the area of the crack  $B_T$  exponentially decreases with the depth (Figure 4b). This implies that, as expected, cracks tend to be sufficiently stiffer in the shales that are recovered from larger depths that in the shales extracted from the shallower

# Trends in microcrack properties of shales

The characteristic pressure,  $P_0$ , shows no obvious trend with the depth of origin (Figure 4c).  $P_0$  is equal to 20 MPa for the shallow and deep depth of less than 1500 m and more than 2500 m. For the intermediate depth of 1500-2500 m,  $P_0$  drops to 10 MPa.

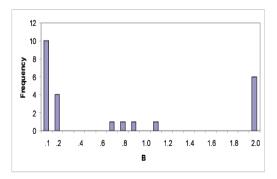
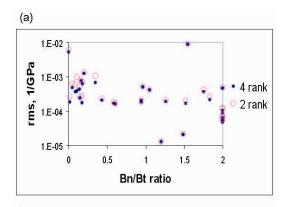


Figure 2: Histogram of the ratio of normal to tangential compliance for all the shale samples. Most of the values are far from unity.



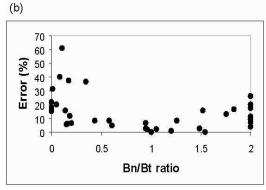


Figure 3: Quality of fitting of the experimental stress dependencies of elastic coefficients. (a) Misfits from equations (4-8) and equations (9-13) are shown be dark blue dots and pink circles, respectively. (b) Relative excess in misfit caused by use of equations (9-13).

The ratio of normal to tangential compliance, *B*, shows positive trend with the depth (Figure 4d), implying that normal stiffness of shales grows slower with the overburden pressure than the tangential one. In other words with increase of the depth, cracks become relatively stiffer in the plane of the crack than in the normal direction. While it is generally accepted that individual shales become stiffer and more anisotropic with depth, the general increase of the ratio of normal to tangential compliance and the decrease of the characteristic pressure in the depth range of 1500-2500 m are new findings, which have to be explained.

The proposed model can be used for prediction of the unknown elastic parameters from the known ones. This problem is practically important either for laboratory measurements in shales, where the  $c_{13}$  coefficient is often unreliable, or for field data analysis where log data give only two of five elastic coefficients of TI media. Note that the fitting problem described by equations (4-8) is over determined. Thus, if the experimental data are obtained for several effective stresses, the four fitting parameters might be determined from experimental stress dependencies of an incomplete set of elastic coefficients. Then the fitting parameters can be used to predict stress dependencies of all five elastic coefficients of the TI medium. For one of the shales, Figure 5 shows the prediction of stress dependencies of the elastic coefficients using as an input stress dependencies of five, four and three elastic coefficients. The predictions obtained from input of four elastic coefficients (c11, c33, c44, c66) are almost indistinguishable from those obtained for the input of five; the differences for the case of input of three elastic coefficients (c11, c33, c66) are very small as well.

## Conclusions

A new stress dependency model for TI media was developed and used to parameterize stress dependencies of the elastic properties of about 20 shales. The four fitting parameters (namely, tangential and normal compliances of a single discontinuity, characteristic pressure, and crack orientation anisotropy parameter) show moderate correlations with the depth from which the shale was extracted. With increasing depth, the tangential compliance broadly decreases exponentially and the ratio of normal to tangential compliance increases linearly. The crack orientation anisotropy parameter exponentially increases with the depth for most of the shales, indicating that cracks are getting more aligned in the bedding plane. The characteristic pressure shows no correlation with the extraction depth. The suggested model allows predicting of stress dependency of all five elastic coefficients if only two of them are known what can be used, for instance, for the reconstruction of all five elastic coefficients of shale from log data.

# Trends in microcrack properties of shales

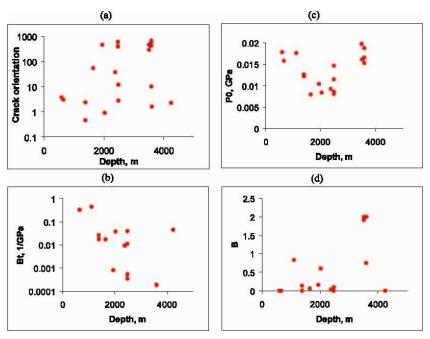


Figure 4: Variations with depth of (a) crack orientation distribution  $\theta$ , (b) tangential compliance  $B_T$ , (c) characteristic pressure  $P_0$ , (d) ratio of normal to tangential compliance B

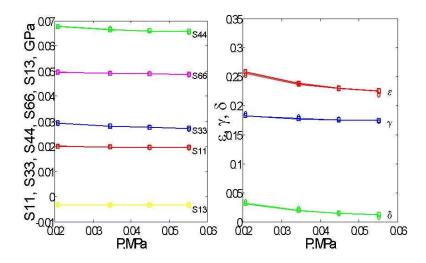


Figure 5: Compliances (left) and anisotropy parameters (right). Values calculated from ultrasonic measurements are shown by circles. Fits using full set of five compliances is shown by thick lines. Thin lines show fits using incomplete sets of compliances. In most of the cases the thin and thick lines coincide.

## **EDITED REFERENCES**

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

## REFERENCES

- Ciz, R., and S. A. Shapiro, 2009, Stress-dependent anisotropy in transversely isotropic rocks: Comparison between theory and laboratory experiment on shale: Geophysics, **74**, no. 1, D7–D12, doi:10.1190/1.3008546.
- Sayers, C., and M. Kachanov, 1995, Microcrack-indused elastic wave anisotropy of brittle rock: Journal of Geophysical Research, **100**, B3, 4149–4156, doi:10.1029/94JB03134.
- Shapiro, C. A., and A. Kaselow, 2005, Porosity and elastic anisotropy of rocks under tectonic-stress and pore-pressure changes: Geophysics, **70**, no. 5, N27–N38, doi:10.1190/1.2073884.
- Wang, Z., 2002, Seismic anisotropy in sedimentary rocks, part 2: Laboratory data: Geophysics, **67**, 1423–1440, doi:10.1190/1.1512743.