Optimal Operation of Multiple Unbalanced Distributed Generation Sources in Three-Phase Four-Wire LV Distribution Networks

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Abstract—**Distributed Generation (DG) in the form of residential roof top photovoltaic installations is driven by consumer action. The placement of DG in the distribution network is not controlled by the network operator. Power quality issues, especially voltage rise and unbalance, is restricting ability of networks to accommodate further connections. There is a growing interest in utilizing the latent capacity of DG inverters to provide reactive power, or to integrate storage into DG systems, to increase the renewable power fraction. This paper presents an optimization method that is able to simultaneously manage the operation of many arbitrary located residential DG sources to reduce system losses and improve power quality. The optimization model is solved by a Sequential Quadratic Programming (SQP) based approach and the validity is tested on an accurate three-phase four-wire unbalanced distribution network model developed during the Perth Solar City trial.**

Key words--Sequential Quadratic Programming, distribution network, three-phase four-wire, balance

I. INTRODUCTION

Distribution networks service large numbers of customers through complex substation and feeder networks. An Australian zone substation can typically supply up to a dozen medium voltage feeders. A typical metropolitan 22kV feeder could supply as many as 100 connected 400/230Vac low voltage feeders and 5,000 homes. Voltage drops and network losses have always been a challenging problem. Unbalance is emerging as a growing power quality concern at the low voltage level. Large numbers of single phase roof top photovoltaic residential systems are adding to this issue [1-3]. Unbalance is expressed in terms of negative- and zero-sequence components and restricted by power quality standards IEC61000, IEEE, [4-6] to minimize adverse effects for consumer equipment and the distribution networks, including supplementary network losses [7].

Increasing numbers of distributed generators (DGs), such as wind power plants and photo-voltaic systems are being connected to the distribution networks [8-11]. The proper management and operation of DGs can bring technical, economic and environmental benefits to the distribution network, which also provides us an effective alternative for network performance improvement.

Many studies have been devoted to optimizing DG control to get a better network performance [12-22]. Among them, a significant number are limited to the planning stage [12-16], that is, benefits are attained by optimal sitting and sizing of DGs. Although DG allocation has an important impact, in many instances, such as the installation of roof top PV systems, the location will be driven by consumer actions. Network performance then mainly depends on operating conditions of DGs. The optimization effect of optimal DGs placement may deteriorate or disappear when the network loading conditions suffer significant changes, which is very normal in practice.

A few studies have focused on optimal operation of the existing DG units by optimizing their reactive power generation. In [17], a linearization based constrained optimization method is applied to find the operating points for all DGs that optimize the voltage profile. Test results show that voltage can be improved by reactive power control of DGs. Reference [18] aims to minimize grid losses by optimally controlling reactive power of DGs at lower voltage levels. To ensure the global optima is found, a commercial global optimization solver is used. However, both of them have some drawbacks. As the distribution network has a higher R/X ratio, the optimization effects by only controlling reactive power are limited. Moreover, DG control only impacts one aspect of the network performance, which may lead to more serious problems in other aspects of the performance.

For example, residential DGs are normally single phase and the network balance may deteriorate when improving the voltage profile by DGs control. In addition, to achieve economical operation of distribution network in a market-oriented environment, modern heuristic methods, such as Particle Swarm Optimization (PSO) and Genetic Algorithm (GA), are applied to find the optimal operational strategies of DGs in [19-22]. As optimization objective is cost minimization or profit maximization, it is not easy to ensure operational improvements of network after optimization.

In this paper a novel and comprehensive operational strategy is proposed which finds the optimal operating points of residential DGs in the low voltage (LV) distribution network. The search for optimal DGs generation is determined by the network losses, balance and voltage profile.

Specifically, a three-step procedure based on the application of sequential quadratic programming (SQP) method is proposed to study the viability of DG control in a residential system.

Initially a residential system is studied where the DGs are initially sized for the PV array real power requirement. At any time the PV generation is less than the array rating, the DG inverters will have a capability to supply reactive power, at a cost of incremental inverter loss. The reactive power of DGs is controlled to optimize the objective function. A second system is then proposed where DGs are installed with an additional capacity factor to allow for the active control of the distribution network performance. In the example presented the apparent power rating of the DGs is twice the array rating.

As optimization by only reactive power management is limited because of high ratio of R/X of distribution network, a final system is presented where distributed storage is added to each inverter to allow both active and reactive power control. They are both optimized to get better performance.

This paper is organized as follows: Section II presents motivation, related work and some background about this study. In Section III network optimization modeling and the solution algorithm are described. Some experimental results obtained with the application of the proposed procedure to an actual LV distribution network are reported and analyzed in Section IV. Finally, the conclusions are provided in Section V.

II. PERTH SOLAR CITY PROJECT

The network under study is contained within Perth Solar City which is a research program funded by the Australian Government Department of Climate Change and Energy Efficiency. As part of the Perth Solar City program, Western Power, the regional transmission and distribution network services provider, is undertaking a technical trial to determine the power quality impacts of a large number of PV systems on the distribution network.

A typical LV network, “Pavetta 1”, in the suburb of Forrestfield was selected by Western Power after desktop audits and site visits for the high penetration trial. The 400/230V network, shown in Figure 1, is supplied from a 200kVA 22kV/400V distribution transformer and includes 101 buses and 77 consumers. Of these 31 consumers have roof top PV systems which have typical ratings of 1.88kW. Total rated PV installation capacity is 58kW representing a branch penetration of 29%.

Figure 1. Pavetta 1 LV Distribution Network

The network under study is an aerial, three-phase four-wire, construction (Figure 2). As the consumers have a mixture of single- and three-phase house connections the loading is inherently unbalanced. Load data, voltage and current are recorded by smart meters on the Western Power network at the point of connection to each consumer switchboard.

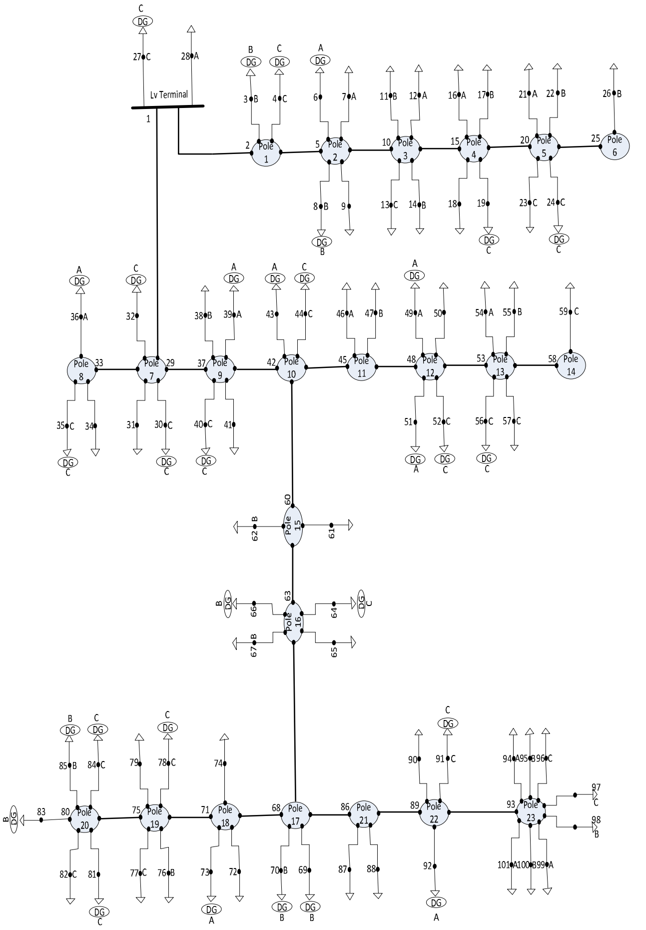

Figure 2. Pavetta 1 LV Network Diagram

III. OPTIMIZATION MODEL AND ALGORITHM

A. Optimization Model

From the mathematical point of view, the optimal operation of distribution network with regard to DGs is a nonlinear optimization problem with equality and inequality constraints. In this paper, to reduce the losses and improve the voltage magnitude/balance profile in the LV network, the optimization problem is formulated as:

$$\min\sum_{p=1}^{4}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} I_{ij}^{p2}R_{ij}^{p}+\sum_{p=1}^{3}\sum_{i\in\delta}\left(k_{i1}^{p}S_{DGi}^{p}{}^{2}+k_{i2}^{p}S_{DGi}^{p}+k_{i3}^{p}\right)+$$

$$\sum_{i\in\omega}\left[k_{i+}\left(V_{iopt+}-V_{i+}\right)^{2}+k_{i-}V_{i-}^{2}+k_{i0}V_{i0}^{2}\right]+\sum_{p=1}^{3}\sum_{i\in\gamma}\left(V_{iN}^{p}-V_{i}^{p}\right)^{2} \qquad (1)$$

Subject to:

$$\begin{aligned}P_{DGi}^{p}-P_{Li}^{p}-P_{i}^{p}&=0\\ Q_{DGi}^{p}-Q_{Li}^{p}-Q_{i}^{p}&=0\end{aligned} \qquad (2)$$

$$P_{DGi}^{p}{}^{2}+Q_{DGi}^{p}{}^{2}\le S_{i}^{p2} \qquad (3)$$

$$\begin{gathered} V^{p}_{ilower} \le V^{p}_{i} \le V^{p}_{iupper} \\ \theta^{p}_{ilower} \le \theta^{p}_{i} \le \theta^{p}_{iupper} \end{gathered} \tag{4}$$

Where *i,j=1,2,...n*, is the bus number and *p=1,2,3,4* represents the phase designations a, b, c and n respectively. In this study, as the network is unbalanced, both the bus number i and the phase designation p are needed to identify a node i^p. Accordingly, I^p_{ij} and R^p_{ij} are the current through and resistance of the branch between nodes $i^p$and j^p while V^p_i, $S^p_{DGi} = \sqrt{{P^p_{DGi}}^2 + {Q^p_{DGi}}^2}$ and S^p_i are the voltage magnitude, DG apparent power and DG capacity at node i^p respectively.

ω and γ are defined as the sets of three phase buses and single phase buses separately while δ is a set of buses where DGs are connected. In this study, the DG inverter loss is modeled as the quadratic polynomial of the DG apparent power S^p_{DGi} with coefficients k^p_{i1}, k^p_{i2} and k^p_{i3}. To optimize the voltage magnitude and balance profile of the three phase nodes simultaneously, voltage symmetrical components are used, which consist of positive-, negative- and zero-sequence voltages respectively. Sequence components can be obtained from the following transformation:

$$\begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{5}$$

Where $\alpha = \exp\,(j\frac{2\pi}{3})$.

On this basis, voltage unbalance factors have been defined separately by the IEC [7, 23-24], IEEE [23-24] and National Equipment Manufacturer's Association (NEMA) [23-25]. In this study, the negative sequence based percentage Voltage Unbalance Factor, developed by the IEC, is chosen as a measure to evaluate the voltage unbalance. To mitigate the zero-sequence components in the three-phase four-wire network, a similar, zero sequence based voltage unbalance factor is also defined here.

$$\%\mathrm{VUF}_- = \frac{V_-}{V_+} \times 100\text{；}\quad \%\mathrm{VUF}_0 = \frac{V_0}{V_+} \times 100 \tag{6}$$

In line with the IEC standard, in medium or low voltage network voltage unbalance factors should not exceed 2% [4]. To coordinate the voltage magnitude and balance improvements, weighting coefficients k^p_{i+}, k^p_{i-} and k^p_{i0} are used for the different sequence voltage deviations from their optimal values in the balanced case separately. In addition, to simultaneously improve the voltage level at the single phase nodes, nodal voltage deviation from the rated value V^p_{iN} is applied.

Thus, the objective function in (1) is defined as the sum of losses caused by both the distribution lines and DG inverters and voltage deviations at both three-phase nodes and single-phase nodes. Equality constraints in (2) are the power balance equations. Of these P^p_{DGi} (Q^p_{DGi}), P^p_{Li} (Q^p_{Li}) and P^p_i (Q^p_i) are the DG, load and network active (reactive) power respectively. P^p_i and Q^p_i can be derived as follows:

$$\begin{aligned} P^p_i &= V^p_i \sum_{j=1}^{n} \left[V^p_j \left(G^p_{ij} \cos\theta^p_{ij} + B^p_{ij} \sin\theta^p_{ij} \right) \right] \\ Q^p_i &= V^p_i \sum_{j=1}^{n} \left[V^p_j \left(G^p_{ij} \sin\theta^p_{ij} - B^p_{ij} \cos\theta^p_{ij} \right) \right] \end{aligned} \tag{7}$$

Inequality constraints described by (3) demonstrate the limits on DG generation. In line with IEC 60038 for nominal voltages and associated permissible tolerances for voltage derivation within lower voltage networks, the boundary constraints on voltage magnitude and phase angle are also given in (4).

B. Sequential Quadratic Programming

The optimization problem in last section is generally referred as a constrained nonlinear programming (CNLP) problem. This paper proposes to use the Sequential Quadratic Programming (SQP) to solve it. The earliest reference to SQP-type methods seems to have been in the Ph.D. thesis of Wilson [26] in 1963. CNLP is widely used in varying power system optimization applications [27-29].

IV. RESULTS AND DISCUSSION

A. Test Network

In this section the proposed three-step DG control strategy is applied to optimize the operation of Pavetta 1 LV network (Figure 2), which is three-phase four-wire and unbalanced. The lines and loads specification are provided by Western Power. In this case we consider the peak demand load case, 172kW, which occurred in the early evening, 6.45pm, on 25th January, 2012. Perth has a summer peaking load dominated by air-conditioning. The aerial network has four equally sized conductors on a mixture of 0.9m and 1.2m cross arms. The consumer mains are 6mm^2 copper with R=3.7Ω/km and X=0.369Ω/km. The aerial mains are constructed with two seven strand, all aluminum conductor types:

- 7/4.50 AAC – R=0.316Ω/km; X=0.292Ω/km;
- 7/3.75 AAC – R=0.542Ω/km; X=0.304Ω/km.

To better show the optimization effects of the three-step method, two DG control groups are chosen. One is referred as 'DG control based on single capacity' and the other is named as 'DG control based on double capacity'. The former control group is based on single DG capacity and includes three cases which are 'NOQ1S' with no DG reactive power outputs, 'OPTQ1S' with optimal DG reactive power control and 'MAXQ1S' with maximum DG reactive power generation. To meet the reactive power demand of consumers, active power outputs of DGs are given and kept constant for the above three cases. To further improve the network performance, more DG reactive power is optimally controlled by doubling the size of DG, which is referred as 'OPTQ2S'. As active power control is more effective than reactive power control in the Pavetta 1 LV network with high R/X ratio, finally, both DG reactive power and active power are controlled to get better performance of network, which is referred as ‘OPTPQ2S’. The last two control cases are based on double DG capacity, which, thus, form the second control group. For the different control cases, the objective function, system losses, voltage magnitudes and unbalance factors are calculated and compared respectively in Figure 3 to Figure 13.

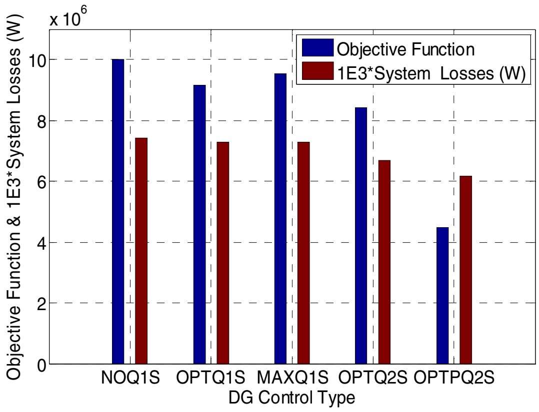

Figure 3. Objective Function and System Losses

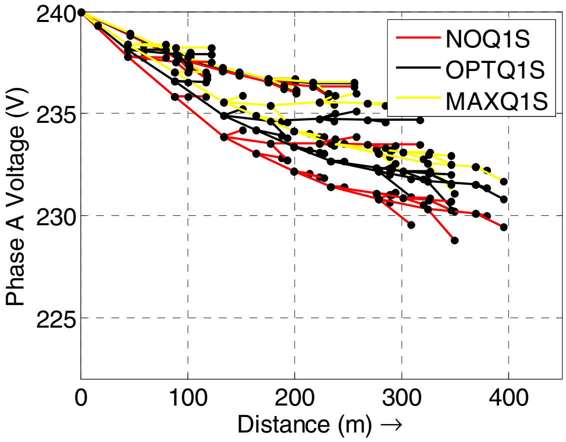

Figure 4. Phase A Voltage by Node

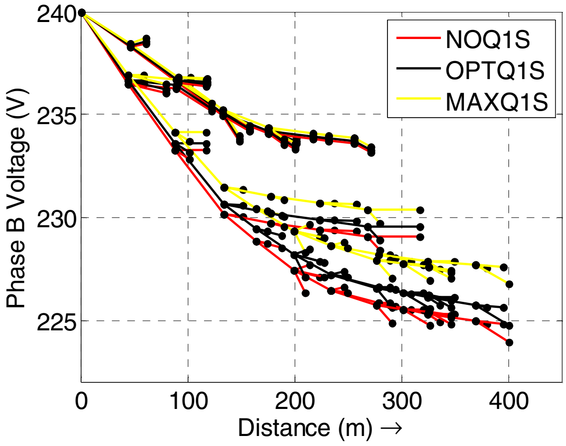

Figure 5. Phase B Voltage by Node

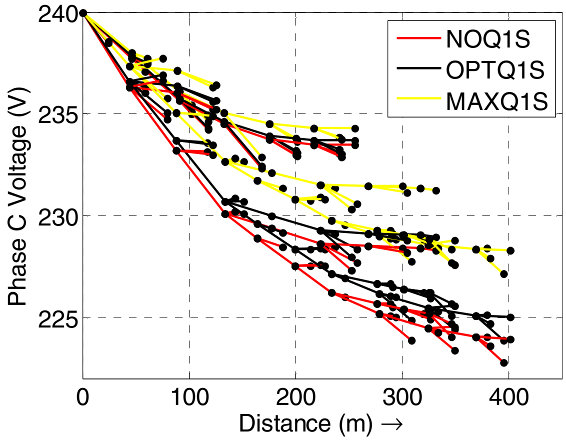

Figure 6. Phase C Voltage by Node

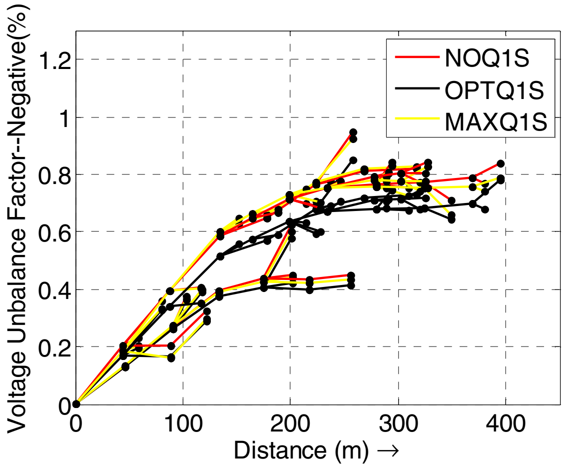

Figure 7. Negative Sequence Voltage Unbalance Factor by Node

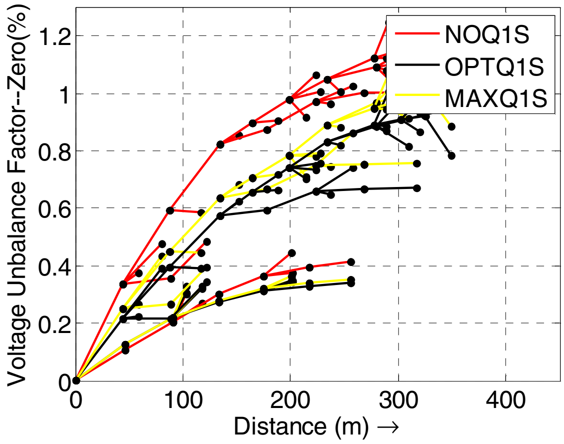

Figure 8. Zero Sequence Voltage Unbalance Factor by Node

B. DG Control Based on Single Capacity

As indicated in the objective function, one of the optimization goals is to reduce the system losses caused by conductor lines and inverters. Figure 3 demonstrates that both the objective function and system losses show a downward trend from $10.01e^6$ and 7.41kW for ‘NOQ1S’ and $9.54e^6$ and 7.29kW for ‘MAXQ1S’ to $9.17e^6$ and 7.28kW for ‘OPTQ1S’ separately. However, the loss improvement of 0.13kW is not so significant because of the limited impacts of DG reactive power control on the LV network with a high R/X ratio.

Improvement of voltage magnitude/unbalance profile is shown in Figures 4 to 8. We find that a more balanced system with a higher voltage level is achieved after optimization of the DG reactive power outputs. Specially, Figures 4 to 6 demonstrate that the phase voltages for 'OPTQ1S' are higher than for 'NOQ1S'. Here 'MAXQ1S' has the highest voltage level because of maximum reactive power generation. Furthermore, we can see from Figures 7 to 8 that both the negative sequence and zero sequence voltage unbalance factor for 'OPTQ1S' are the smallest for majority of the nodes, that is, the system is more balanced after DG reactive optimization.

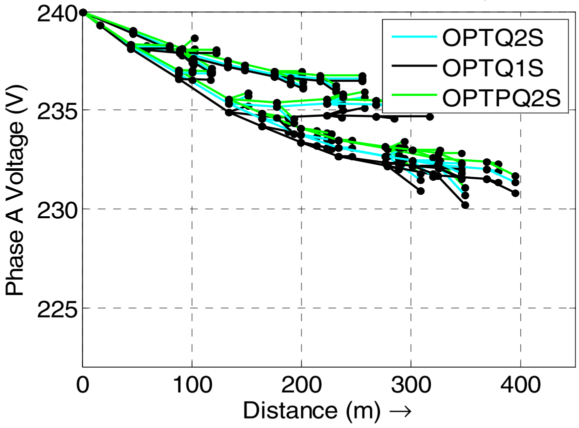

Figure 9. Phase A Voltage by Node

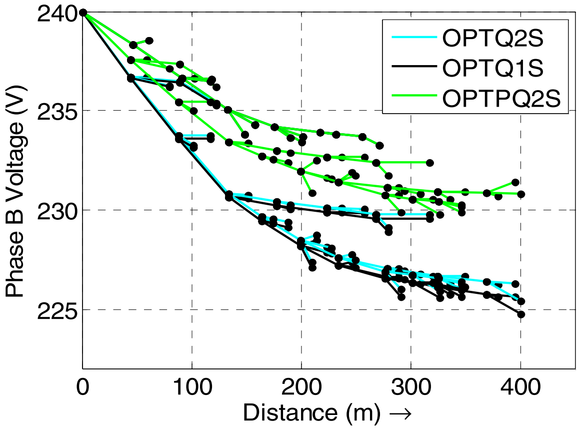

Figure 10. Phase B Voltage by Node

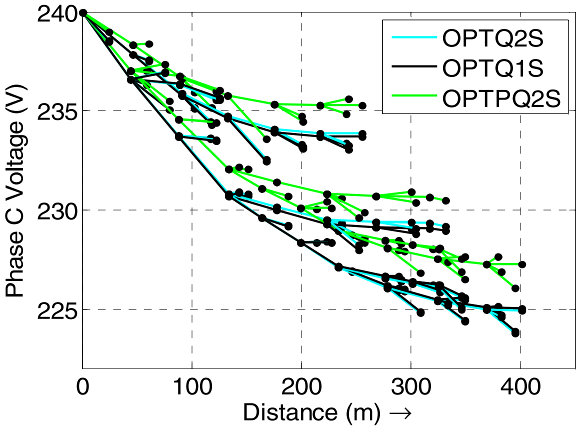

Figure 11. Phase C Voltage by Node

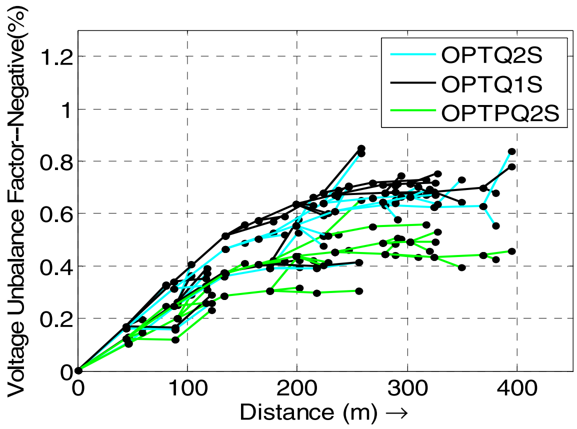

Figure 12. Negative Sequence Voltage Unbalance Factor by Node

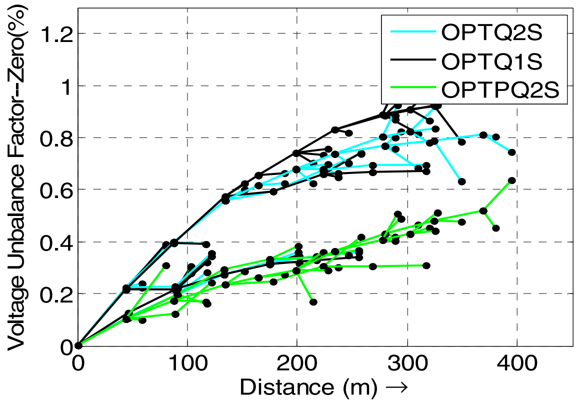

Figure 13. Zero Sequence Voltage Unbalance Factor by Node

C. DG Control Based on Double Capacity

In Figures 9 to 13, we can clearly see that network performance with regard to system losses and voltage magnitude/balance profile are further improved by both DG reactive power and active power control which is based on double DG inverter size. Specifically, by doubling the reactive power limits 'OPTQ2S' improves relative to 'OPTQ1S'. The objective function and system losses decline from $9.17e^6$ and 7.28kW for 'OPTQ1S' to $8.44e^6$ and 6.68kW for'OPTQ2S'. The voltage improvement by only DG reactive power control is rather slight because of high R/X ratio. To break through this bottleneck, DG active power control is applied under the premise of active power supply. Figures above indicate that power quality can be significantly improved with DG active power control in the case of 'OPTPQ2S' with objective function and system losses of $4.48e^6$ and 6.16kW respectively.

Table 1 summarizes the improvement in losses and the improvement in voltage quality, across each network node. In this case the values are averaged across each network node.

TABLE I. LOSS AND VOLTAGE QUALITY FACTORS

Scheme	**NOQ 1S**	**MAXQ 1S**	**OPTQ 1S**	**OPTQ 2S**	**OPTPQ 2S**
Losses	7.41kW	7.29kW	7.28kW	6.68kW	6.16kW
$V_{average}$	231.04	232.82	231.69	231.87	233.27
VUF_{-}	0.61%	0.60%	0.54%	0.51%	0.38%
VUF_{0}	0.79%	0.67%	0.62%	0.57%	0.33%

V. CONCLUSION

This paper has made a comprehensive study on the performance improvement possible in a practical four wire three phase distribution network with X/R ratios that are close to one. Three case studies where considered. The first case allowed DG inverters with a normal rating determined by the connected PV panel load to be reactively controlled when spare capacity was available. The second case proposed a doubling of the nominal inverter capacity to achieve a network control benefit. The final case allowed the inverters to provide real power from embedded storage.

An optimal DG control procedure, which weights network loss, positive, negative and zero sequence voltages was proposed and implemented using sequential quadratic programming (SQP). The viability of this approach has been verified by the optimization of an actual LV distribution network. It was shown that the optimal dispatch of the inverter capacity did not operate all inverters at their maximum output. As the inverters are single phase, and not uniformly distributed across the phases, the inverter outputs needed to be managed to limit the impact on the system voltage balance.

The results demonstrate that controlling both DG active power and reactive power is a feasible and effective method to improve the operational voltage quality of distribution network, which in turn increases the network load capacity and the capacity to increase higher fractions of renewable energy.

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