Nonovershooting and nonundershooting exact output regulation*

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Abstract

We consider the classic problem of exact output regulation for a linear time invariant plant. Under the assumption that either a state feedback or measurement feedback output regulator exists, we give design methods to obtain a regulator that avoids overshoot and undershoot in the transient response.

1 Introduction

The problem of output regulation is central to modern control theory. The basic problem considers a multivariable linear time invariant (LTI) plant that is subject to known external disturbances, and which is desired to track a known reference signal. The reference signals and external disturbances are modelled by two independent exosystems. The aim of the problem is to design a feedback controller which internally stabilises the plant while rejecting the disturbances and ensuring the output converges asymptotically to the desired reference signal. The problem has a long history, and extensive compilations of results are given in [2] and [3].

A special case of the output regulation problem is that of designing a control law to ensure the plant output takes a known constant reference value, and also exhibits a desirable transient response, in particular the absence of overshoot or undershoot. Much of the literature for this problem has concerned single-input single-output systems (SISO). Dharba and Bhattacharyya [4] showed how to design a two parameter feedback controller for an LTI continuous-time plant that renders the step response nonovershooting. Bement and Jayasuriya [5] gave an eigenvalue assignment method to obtain a nonovershooting LTI state feedback controller for continuous-time plants with one nonminimum phase zero. In [6] conditions are given for the existence of a controller to achieve a sign invariant

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impulse response, and hence also a nonovershooting step response. However such an approach is inherently conservative, because a sign invariant impulse response (and hence also a monotonic step response) is not necessary to avoid undershoot or overshoot.

To date there have been few papers offering analysis or design methods for undershoot or overshoot in the step response of multi-input multi-output (MIMO) systems. A recent contribution in this area is [7], which gave conditions under which a state feedback controller could be obtained to yield a nonovershooting step response for LTI MIMO systems; this design method is applicable to nonminimum phase systems, and could be applied to both continuous-time and discrete-time systems. In [8] it was shown that the state feedback law can be implemented in conjunction with a dynamic observer; the nonovershooting property was seen to be preserved if the initial observer error is sufficiently small. In [9] the design method of [7] was modified to achieve a step response for MIMO systems that is both nonovershooting and nonundershooting.

For the general problem of exact output regulation, there have been only a few papers offering design methods to deliver a desirable transient response. Saberi *et al* [10] gave a general framework for optimising transient performance in regulation problems. By defining a performance index involving the energy of the error signal, they introduced several optimal and suboptimal control problems to find control laws that achieve output regulation and also obtain the infimum of this performance index. The authors noted that in some problems it was necessary to employ high-gain feedback controllers. More recently, Zhang and Lan [11] considered output regulation for SISO systems and employed the composite nonlinear feedback (CNF) technique of [12]-[13] to obtain a nonlinear state feedback control law that offered improved transient response, relative to that achievable with a linear control law. We note however that neither the methods of [10] nor [11] were able to avoid overshoot or undershoot in the transient response of the tracking signal.

In this paper we seek to adapt the multivariable design methods of [7] and [9] to the general problem of exact output regulation. We assume the problem of output regulation by state feedback is solvable, i.e. there exists a linear state feedback controller that internally stabilizes the plant and achieves output regulation. In this case we show that if there exists a state feedback controller that yields a nonovershooting response for a step reference, then a state feedback output regulator can be obtained to deliver a nonovershooting output regulation. Secondly, for the problem of output regulation by measurement feedback, we show that if the problem is solvable and if there exists a dynamic observer that yields a nonovershooting response for a constant step reference, then a measurement feedback output regulator can be found to deliver nonovershooting output regulation. To the best of the authors' knowledge, this is the first design method that achieves multivariable exact output regulation with a nonovershooting (or nonundershooting) transient response.

Notation. Throughout this paper, the symbol 0_n represents the zero vector of length n, and I_n is the n-dimensional identity matrix. For a square matrix A, we use $\sigma(A)$ to denote its spectrum. We say that a square matrix A is *Hurwitz-stable* if $\sigma(A)$ lies within the open left-hand complex plane, and it is *anti-Hurwitz-stable* if $\sigma(A)$ lies within the open right-hand complex plane. For any real or complex scalar λ and vector v, we say that (λ, v) form an *eigenpair* of a square matrix A if $Av = \lambda v$. For any matrix A with $2n$ rows, we define $\overline{\pi}\{A\}$ and $\underline{\pi}\{A\}$ by taking the upper n and lower n rows of A, respectively. If α is a vector of length n, we use $\mathrm{diag}(\alpha)$ to denote the $n \times n$ diagonal matrix whose leading diagonal contains the entries of α.

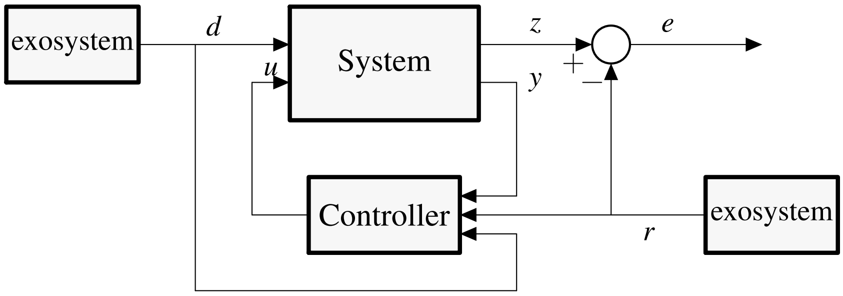


Figure 1: Output feedback control architecture

2 Problem formulation

We consider a linear multivariable plant ruled by the equation

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Hd(t) \\ y(t) = C_y x(t) + D_y u(t) + G_y d(t) \\ z(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

where, for all $t \geq 0$, the signal $x(t) \in \mathbb{R}^n$ represents the state, $u(t) \in \mathbb{R}^m$ represents the control input, $y(t) \in \mathbb{R}^p$ represents the measured output, $z(t) \in \mathbb{R}^q$ represents the controlled output, $r(t) \in \mathbb{R}^\rho$ represents a reference signal and $d(t) \in \mathbb{R}^\delta$ represents a disturbance signal, as shown in Figure 1. All the matrices appearing in (1) are appropriate dimensional constant matrices.

The disturbance input d and the reference input r are generated by two autonomous exosystems ruled respectively by

$$\Sigma_{exo,1} : \begin{cases} \dot{\eta}(t) = S_1 \eta(t), \quad \eta(0) = \eta_0 \\ d(t) = L_1 \eta(t) \end{cases} \quad \text{and} \quad \Sigma_{exo,2} : \begin{cases} \dot{\zeta}(t) = S_2 \zeta(t), \quad \zeta(0) = \zeta_0 \\ r(t) = L_2 \zeta(t) \end{cases}$$

where, for all $t \geq 0$, $\eta(t) \in \mathbb{R}^{n_1}$ and $\zeta(t) \in \mathbb{R}^{n_2}$, and S_1, S_2, L_1, L_2 are also appropriate dimensional constant matrices. We assume that all the eigenvalues of S_1 and S_2 are anti-Hurwitz-stable, i.e., they all have non-negative real part. This assumption does not cause any loss of generality, see [3, p. 18]; indeed, if the closed-loop system (excluding the exosystems) is internally stable, the vanishing modes of the exosystem do not affect the regulation of the output. We also assume that the states of the exosystems η and ζ are measurable, i.e., they are available to be used to generate a feedforward action in the control law.

We design a controller with measurement signal y which generates the control input signal u. Our design objective is for the reference signal r to be asymptotically tracked by the output z of the system, while minimising or eliminating the effect of the disturbance. As such, by defining the error signal

$$e(t) \stackrel{\text{def}}{=} z(t) - r(t),$$

our objective is to achieve $\lim_{t\to\infty} e(t) = 0$. We then consider a new system Σ_e obtained from Σ by considering the new output e instead of z:

$$\Sigma_e : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Hd(t), \quad x(0) = x_0 \\ y(t) = C_y x(t) + D_y u(t) + G_y d(t) \\ e(t) = Cx(t) + Du(t) - r(t) \end{cases}$$

It is convenient to incorporate the two exosystems into a single exosystem whose state w is defined as

$$w(t) \stackrel{\text{def}}{=} \begin{bmatrix} \eta(t) \\ \zeta(t) \end{bmatrix}$$

so that

$$\Sigma_{exo}: \begin{cases} \dot{w}(t) &= Sw(t), \quad w(0) = w_0 \\ \begin{bmatrix} d(t) \\ r(t) \end{bmatrix} &= \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} w(t) \end{cases}$$

where $S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$. By defining

$$\begin{aligned} E_w &\stackrel{\text{def}}{=} \begin{bmatrix} HL_1 & 0 \end{bmatrix} \\ D_{yw} &\stackrel{\text{def}}{=} \begin{bmatrix} G_y L_1 & 0 \end{bmatrix} \\ D_{ew} &\stackrel{\text{def}}{=} \begin{bmatrix} 0 & -L_2 \end{bmatrix} \end{aligned}$$

we can re-write Σ_e as

$$\Sigma_e: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E_w w(t), & x(0) = x_0 \\ \dot{w}(t) = Sw(t), & w(0) = w_0 \\ y(t) = C_y x(t) + D_y u(t) + D_{yw} w(t) \\ e(t) = C_e x(t) + D_{eu} u(t) + D_{ew} w(t) \end{cases} \tag{2}$$

In order to avoid issues of well-posedness of output dynamic architectures, and in order to simplify the derivations of the tracking control law, we assume $D_y = 0$. This assumption does not lead to a significant loss of generality, as shown in [3, p. 16]. For design purposes we will also consider the *nominal plant* Σ_{nom} which arises when both exosystems are excluded from consideration. In this case Σ_e simplifies to the homogenous system

$$\Sigma_{nom}: \begin{cases} \dot{\tilde{x}}(t) = A\tilde{x}(t) + B\tilde{u}(t), \quad \tilde{x}(0) = \tilde{x}_0 \\ \tilde{e}(t) = C_e\tilde{x}(t) + D_{eu}\tilde{u}(t) \end{cases} \tag{3}$$

For this system, the problem of exact output regulation consists of driving the system state to the origin from some arbitrary non-zero initial condition. In later sections we will consider control methodologies that regulate the nominal plant with desirable transient performance, and then consider the conditions under which these control methods can also be used to achieve the same desirable transient performance when applied to Σ_e. Next we briefly revisit some classic results on output regulation by state feedback and measurement feedback. The discussion that follows is based on [3, Chapter 2]. We will only consider the continuous-time case. However, all the results presented here can be adapted to the discrete-time case with only minor modifications.

2.1 Exact output regulation with state feedback

In the case where the controller has access to the state of the system, as well as to the reference and the disturbance, we have $p = n$,

$$C_y = I, \qquad D_y = 0, \qquad D_{yw} = 0,$$

and the control input has the form

$$u(t) = F\,x(t) + G\,w(t), \tag{4}$$

which is given by a static state-feedback component $F\,x(t)$ and a static feedforward component $G\,w(t)$ that uses the states of the exosystems. The closed-loop system is

$$\Sigma_{CL}: \begin{cases} \dot{x}(t) &= (A+BF)\,x(t) + (BG+E_w)\,w(t), & x(0) = x_0 \\ \dot{w}(t) &= S\,w(t), & w(0) = w_0 \\ e(t) &= (C_e + D_{eu}F)\,x(t) + (D_{ew}G + D_{yw})\,w(t) \end{cases} \tag{5}$$

Definition 2.1 *(a) A state feedback controller u of the form (4) is said to achieve* exact output feedback regulation *if both the following conditions hold*

(I) Internal Stability: The system $\dot{x}(t) = (A+BF)\,x(t)$ is asymptotically stable, and

(II) Output Regulation: For all $x_0 \in \mathbb{R}^n$, and $w_0 \in \mathbb{R}^{n_1+n_2}$, the system Σ_{CL} satisfies $\lim_{t\to\infty} e(t) = 0$.

(b) For a given initial condition (x_0, w_0) of Σ_e, the control law u in (4) is said to achieve nonovershooting exact output feedback regulation *for Σ_e from (x_0, w_0) if the output $e \to 0$ without changing sign in any component, i.e., for each $i \in \{1, \ldots, q\}$, $sgn(e_i(t))$ is constant for all $t \geq 0$. Finally, we say the controller achieves* globally nonovershooting exact output feedback regulation *if e is nonovershooting for all initial conditions (x_0, w_0).*

The following theorem gives conditions under which exact output feedback regulation can be achieved by a state feedback control law of the form (4).

Theorem 2.1 *([3], Theorem 2.3.1) Assume system Σ_e in (1) satisfies the following assumptions*

(A.1) The pair (A, B) is stabilizable.

(A.2) The matrix S is anti-Hurwitz-stable.

(A.3) There exists matrices Γ and Π satisfying

$$\Pi S = A\Pi + B\Gamma + E_w \tag{6}$$

$$0 = C\Pi + D\Gamma + D_{ew} \tag{7}$$

Let F be any matrix such that $A+BF$ is Hurwitz-stable, and let $G = \Gamma - F\,\Pi$. Then u as in (4) achieves exact output feedback regulation for Σ_e.

These last two equations are known as the *regulator equation*. Solvability conditions for these equations are given in [3, Chapter 2].

2.2 Exact output regulation problem with measurement feedback

In the general case of any measurement y being available for feedback, we consider measurement feedback controllers of the form

$$\Sigma_c : \begin{cases} \dot{v}(t) &= A_c v(t) + B_c y(t) \\ u(t) &= C_c v(t) + D_c y(t) \end{cases} \tag{8}$$

Definition 2.2 *A measurement feedback control law* Σ_c *of the form (8) is said to achieve* exact output feedback regulation *if both the following conditions hold*

(I) Internal Stability: The system

$$\dot{x}(t) = (A + BD_cC_y)\,x(t) + BC_cv(t) \tag{9}$$

$$\dot{v}(t) = B_cC_yx(t) + A_cv(t) \tag{10}$$

is Hurwitz-stable

(II) Output Regulation: For all $x(0) \in \mathbb{R}^n$, $v(0) \in \mathbb{R}^\delta$ *and* $w(0) \in \mathbb{R}^\rho$, *the closed-loop system* Σ_{CL} *satisfies* $\lim_{t\to\infty} e(t) = 0$.

The following theorem gives conditions under which exact output feedback regulation can be achieved by a measurement feedback control law of the form (8).

Theorem 2.2 *([3], Theorem 2.4.1) Assume system* Σ_e *in (2) satisfies the following assumptions (A.1), (A.2) as well as the following assumption*

(A.3) the pair $\left(\begin{bmatrix} C_y & D_{yw} \end{bmatrix}, \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \right)$ *is detectable.*

Then, the exact output regulation problem is solvable by measurement feedback if and only if there exist matrices Γ *and* Π *satisfying (6)-(7). A suitable measurement feedback controller is then given by*

$$\Sigma_c \;:\; \left\{ \begin{array}{rcl} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{w}}(t) \end{bmatrix} & = & \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ & & + \begin{bmatrix} K_A \\ K_S \end{bmatrix} \left(\begin{bmatrix} C_y & D_{yw} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} - y(t) \right) \\ u(t) & = & F\,\hat{x}(t) + (\Gamma - F\Pi)\,\hat{w}(t) \end{array} \right. \tag{11}$$

where F, K_A *and* K_S *are such that*

$$A + BF \quad \text{and} \quad \begin{bmatrix} A + K_AC_y & E_w + K_AD_{yw} \\ K_SC_y & S + K_SD_{yw} \end{bmatrix}$$

are both Hurwitz stable.

2.3 Nonovershooting and nonundershooting tracking controller design methods

The paper [7] gave several methods for the design of a linear state feedback control law to deliver a nonovershooting step response for systems in the form Σ_{nom}, and [9] extended the design methods to deliver a nonundershooting step response system, and also a monotonic step response. Our aim in this paper is to consider how these methods may be employed to achieve exact output regulation with a nonovershooting (or nonundershooting) transient response. We now briefly review these methods.

The design methods assume the system Σ_{nom} is at a known initial equilibrium (u_o, x_0, y_0), and that the closed loop poles are to be selected from within a user-specified interval $[a, b]$ of the negative real line. The algorithm selects candidate sets $\mathscr{L}$ of distinct closed-loop eigenvalues within the specified interval and then associates them with candidate sets of eigenvectors $\mathscr{V}$ and eigendirections $\mathscr{W}$. These are obtained in terms of the system matrix pencil

$$P_\Sigma(s) \stackrel{\text{def}}{=} \begin{bmatrix} A - sI_n & B \\ C & D \end{bmatrix} \tag{12}$$

in such a way that only a small number (generally one or two, or at most three) of the closed-loop modes contribute to each output component. The error function $e(t)$ is then formulated in terms of the candidate set of eigenvectors and a test is used to determine if the system response is nonovershooting (or nonundershooting) in all components. If the test is not successful, then a new candidate set $\mathscr{L}$ is chosen, and the process is repeated. If it succeeds, then the desired matrix F can be obtained by applying Moore's algorithm [14] to the sets $\mathscr{V}$ and $\mathscr{W}$. The tests are analytic in nature, and do not require simulating the system response to test for overshoot or undershoot.

The method exploits any minimum phase zeros that may exist; these modes are then chosen among the closed-loop poles and rendered invisible at the outputs. Recently the design method was incorporated into a public domain MATLAB® toolbox, known as **NOUS** [15]. The toolbox asks the user to specify their LTI system in state space form, together with a specified initial condition and desired step reference. The user is also asked to nominate a subinterval $[a, b]$ of the negative real line within which the poles of the closed loop system are to be located. The User also specifies whether a nonovershooting, or nonundershooting, or monotonic, response, is desired. The **NOUS** toolbox then seeks to obtain a gain matrix that will deliver these closed loop poles, and also the desired transient response.

Testing by the present authors using the **NOUS** toolbox has provided enough evidence that shows that the search method is likely to be successful if the number of system states, less the number of minimum phase zeros, is not more than three times the number of control inputs, i.e. the inequality

$$n - z \leq lp \tag{13}$$

holds true for some $l \leq 3$, where z is the number of minimum phase zeros. Interestingly, for MIMO systems the presence of non-minimum phase zeros does not negatively impact upon the success of the search, and [16] gave several examples of systems for which nonovershooting and nonundershooting responses could be obtained, notwithstanding the presence of several real nonminimum phase zeros. Moreover, the search algorithm can some times be successful even where (13) requires $l = 4$ or more.

3 Nonovershooting and nonundershooting output regulation

In this section we present the main results of our paper. We extend the classic problem of output regulation to also consider the design of linear control laws of the form (4) and (11) to deliver a desirable transient response. Specifically, we consider the problem of choosing the control laws for Σ_e such that e is nonovershooting, for any given (x_0, w_0). Our first result indicates that if we can obtain a state feedback control law $\tilde{u}(t) = F\tilde{x}(t)$ that achieves nonovershooting exact output feedback regulation for Σ_{nom}, then the state feedback law u in (4) with this F will achieve nonovershooting exact output feedback regulation for Σ_e.

Theorem 3.1 *Let (x_0, w_0) be any initial condition for Σ_e in (2). Assume that (A.1)-(A.2) hold and that Π and Γ satisfy (6)-(7). Assume there exists F such that $\tilde{u}(t) = F\,\tilde{x}(t)$ yields $\tilde{e}(t) \to 0$ without overshoot from initial condition $\tilde{x}_0 = x_0 - \Pi w_0$, and let $G \stackrel{def}{=} \Gamma - F\,\Pi$. Then u in (4), with this F and G, yields nonovershooting exact output feedback regulation for system Σ_e from the initial condition (x_0, w_0).*

Proof: The closed-loop system arising from applying $\tilde{u} = F\,\tilde{x}$ to Σ_{nom} is

$$\left\{\begin{array}{lcl} \dot{\tilde{x}}(t) &=& (A+B\,F)\,\tilde{x}(t), \quad \tilde{x}(0) = \tilde{x}_0 \\ \tilde{e}(t) &=& (C_e + D_{eu}\,F)\,\tilde{x}(t) \end{array}\right. \tag{14}$$

and by assumption we have $\tilde{e} \to 0$ without overshoot. Next we consider Σ_e and introduce the change of coordinates $\xi(t) = x(t) - \Pi\,w(t)$. Then $\xi(0) = x(0) - \Pi\,w(0) = \tilde{x}_0$, and since $\dot{w} = Sw$, we obtain

$$\begin{aligned} \dot{\xi}(t) &= \dot{x}(t) - \Pi S w(t) \\ &= (A+BF)x(t) + (E_w + B(\Gamma - F\Pi) - \Pi S)w(t) \\ &= (A+BF)x(t) - (A+BF)\Pi w(t), \quad \text{using (6)} \\ &= (A+BF)\xi(t). \end{aligned} \tag{15}$$

$$\begin{aligned} (C_e + D_{eu}F)x(t) + (D_{eu}(\Gamma - F\Pi) + D_{ew})w(t) &= (C_e + D_{eu}F)x(t) + (D_{eu}\Gamma - D_{eu}F\Pi + D_{ew})w(t) \\ &= C_e x + D_{eu}F(x(t) - \Pi w(t)) + (D_{eu}\Gamma + D_{ew})w(t) \\ &= C_e\xi(t) + D_{eu}F\xi(t) + (C_e\Pi + D_{eu}\Gamma + D_{ew})w(t) \\ &= (C_e + D_{eu}F)\xi(t) \end{aligned} \tag{16}$$

by (7). Hence the closed loop system arising from Σ_e under u in (4) with initial condition (x_0, w_0) is

$$\left\{\begin{array}{lcl} \dot{\xi}(t) &=& (A+B\,F)\,\xi(t), \quad \xi(0) = \xi_0, \\ e(t) &=& (C_e + D_{eu}\,F)\,\xi(t) \end{array}\right. \tag{17}$$

which is identical to (14), and so $e \to 0$ without overshoot. Hence u achieves nonovershooting exact output regulation for Σ_e from (x_0, w_0). ■

The significance of Theorem 3.1 is that if the exact output regulation of Σ_e can be achieved by state feedback, then the design methods of [7] can be utilised to obtain nonovershooting exact output regulation. It is an immediate corollary to Theorem 3.1 that if $\tilde{u} = F\tilde{x}$ delivers a nonundershooting, or monotonic, step response for Σ_{nom}, then u with this same F will deliver nonundershooting, or monotonic, output regulation for Σ_e.

Our second result says that if we can obtain a state feedback control law $\tilde{u}(t) = F\,\tilde{x}(t)$ that achieves nonovershooting exact output feedback regulation for Σ_{nom}, then the measurement feedback control law u in (11) with this F will achieve nonovershooting exact output feedback regulation for Σ_e, provided the initial estimator error is sufficiently small. First we require a technical lemma.

Lemma 3.1 *Let $\{\mu_1, \ldots, \mu_n\}$ and $\{\lambda_1, \ldots, \lambda_n\}$ be sets of distinct negative real numbers such that for all $i, j \in \{1, \ldots, n\}$, we have $\mu_i < \lambda_j$. Let $\{\alpha_1, \ldots, \alpha_n\}$ and $\{\beta_1, \ldots, \beta_n\}$ be arbitrary sets of real numbers, with $\alpha_n \neq 0$ and $\beta_n \neq 0$. Define*

$$f(t) = \sum_{i=1}^{n} \alpha_i e^{\mu_i t}, \quad g(t) = \sum_{i=1}^{n} \beta_i e^{\lambda_i t} \tag{18}$$

and assume $g(t) \neq 0$ for all $t \geq 0$. Then there exists a positive real number δ such that $g(t) + \delta f(t) \neq 0$ for all $t \geq 0$.

Proof: Let the μ_i and λ_i are ordered so that $\mu_1 < \cdots < \mu_n$ and $\lambda_1 < \cdots < \lambda_n$, and assume that $g(t) > 0$ for all $t \geq 0$. As f and g are the sums of finitely many negative real exponential functions, they have finitely many local extrema. Hence there exists a $\bar{t} > 0$ such that all the extrema of f and g lie to the left of $\bar{t}$, both f and g are monotonic on the interval $t \geq \bar{t}$, and $f(t) \to 0$ and $g(t) \to 0$ as $t \to \infty$. Hence we can find a $\delta_1 > 0$ such that

$$\frac{1}{\delta_1} > \sup\left\{\frac{|f(t)|}{g(t)} : 0 \leq t \leq \bar{t}\right\} \tag{19}$$

Then $\delta_1|f(t)| < g(t)$ for all $0 \leq t \leq \bar{t}$, and we have $0 < g(t) + \delta_1 f(t)$. Next we seek $\delta_2 > 0$ such that $0 < g(t) + \delta_2 f(t)$ for all $t > \bar{t}$. If $f(\bar{t}) > 0$ then we may simply choose $\delta_2 = 1$. Next assume $f(\bar{t}) < 0$; noting that $\alpha_n e^{\mu_n t}$ is the dominant term of f for large t, we define sets $S_1 \stackrel{\text{def}}{=} \{i \in \{1, \ldots, n\} : \alpha_i \alpha_n > 0\}$ and $S_2 \stackrel{\text{def}}{=} \{i \in \{1, \ldots, n\} : \alpha_i \alpha_n < 0\}$. We introduce functions f_1 and f_2 comprising those exponential components of f whose coefficients α_i are of the same sign as α_n, and opposite sign to α_n, respectively:

$$f_1(t) = \sum_{i \in S_1} \alpha_i e^{\mu_i t}, \quad f_2(t) = \sum_{i \in S_2} \alpha_i e^{\mu_i t}, \tag{20}$$

Then $f(t) = f_1(t) + f_2(t)$, and, since $f(t) \to \alpha_n e^{\mu_n t}$ as $t \to 0$, we conclude that $\alpha_n < 0$ because $f(t) < 0$ for $t \geq \bar{t}$. Hence $f_1(t) < 0$, and $f_2(t) > 0$, yielding $|f_1(t)| > |f_2(t)|$. We have, for all $t > \bar{t}$,

$$|f(t)| \leq |f_1(t)| \leq |f_1(\bar{t})| e^{\mu_n(t-\bar{t})} \tag{21}$$

Noting that $\beta_n e^{\lambda_n t}$ is the dominant term of g for large t, we introduce sets T_1 and T_2 with $T_1 \stackrel{\text{def}}{=} \{i \in \{1, \ldots, n\} : \beta_i \beta_n > 0\}$ and $T_2 \stackrel{\text{def}}{=} \{i \in \{1, \ldots, n\} : \beta_i \beta_n < 0\}$, and introduce functions g_1 and g_2 comprising those exponential terms of g whose coefficients β_i are of the same sign as β_n, and opposite sign to β_n, respectively:

$$g_1(t) = \sum_{i \in T_1} \beta_i e^{\lambda_i t}, \quad g_2(t) = \sum_{i \in T_2} \beta_i e^{\lambda_i t}, \tag{22}$$

Then $g(t) = g_1(t) + g_2(t)$, and since $g(t) \to \beta_n e^{\lambda_n t}$ as $t \to 0$, then $\beta_n > 0$ because $g(t) > 0$. Hence $g_1(t) > 0$, and $g_2(t) < 0$, yielding $g_1(t) > |g_2(t)|$, and $\frac{|g_2(t)|}{g_1(t)} \to 0$ as $t \to 0$. Hence if we introduce the function $\gamma(t)$ such that $g(t) = \gamma(t) g_1(t)$, we have that $0 < \gamma(t) < 1$ for all $t \geq \bar{t}$, and $\gamma(t) \to 1$ as $t \to \infty$. So let $\gamma = \inf\{\gamma(t) : t \geq \bar{t}\}$; we then have for all $t \geq \bar{t}$

$$g(t) \geq \gamma g_1(t) \geq \gamma g_1(\bar{t}) e^{\lambda_1(t-\bar{t})} \tag{23}$$

Hence from (21) and (23), we obtain

$$\frac{|f(t)|}{g(t)} \leq \frac{|f_1(\bar{t})|}{\gamma g_1(\bar{t})} \tag{24}$$

as $\mu_n < \lambda_1 < 0$. Defining $\delta_2 \stackrel{\text{def}}{=} \frac{\gamma g_1(\bar{t})}{|f_1(\bar{t})|}$ and $\delta = \min\{\delta_1, \delta_2\}$, we have $g(t) + \delta f(t) > 0$ for all $t \geq 0$. A similar argument can be used if $g(t) < 0$ for all $t \geq 0$. ■

Theorem 3.2 *Let (x_0, w_0) be any initial condition for Σ_e in (2). Assume that (A.1)-(A.3) hold and that Π and Γ satisfy (6)-(7). Assume there exists F such that*

(i) The eigenvalues of $A + BF$ are real and asymptotically stable, and

(ii) $\tilde{u}(t) = F\,\tilde{x}(t)$ *yields nonovershooting exact output feedback regulation for* Σ_{hom} *from initial condition* $\tilde{x}_0 = x_0 - \Pi\, w_0$.

Let K_A *and* K_S *be chosen such that the matrix*

$$A_{cc} \stackrel{def}{=} \begin{bmatrix} A + K_A C_y & E_w + K_A D_{yw} \\ K_S C_y & S + K_S D_{yw} \end{bmatrix} \tag{25}$$

has real stable eigenvalues all lying to the left of all the eigenvalues of $A + BF$, *i.e., for any* $\lambda \in \sigma(A+BF)$ *and* $\mu \in \sigma(A_{cc})$, *we have* $\mu < \lambda$. *Then (11), with this same* F, K_A *and* K_S, *yields nonovershooting exact output feedback regulation for system* Σ_e *from the initial condition* (x_0, w_0), *provided the initial estimation error* $\left[\begin{smallmatrix} \hat{x}(0)-x_0 \\ \hat{w}(0)-w_0 \end{smallmatrix}\right]$ *is sufficiently small.*

Proof: Our first task is to find the form of the closed loop system when controller Σ_c in (11) is applied to Σ_e. Let us denote the estimation error as

$$\varepsilon(t) \stackrel{\text{def}}{=} \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} \hat{x}(t) - x(t) \\ \hat{w}(t) - w(t) \end{bmatrix}$$

so that

$$\begin{aligned}
\dot{\varepsilon}(t) &= \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} - \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \\
&\quad - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \left(\begin{bmatrix} C_y & D_{yw} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} - \begin{bmatrix} C_y & D_{yw} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} \right) \\
&= \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \varepsilon(t) + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} C_y & D_{yw} \end{bmatrix} \varepsilon(t) \\
&= \begin{bmatrix} A + K_1 C_y & E_w + K_1 D_{yw} \\ K_2 C_y & S + K_2 D_{yw} \end{bmatrix} \varepsilon(t)
\end{aligned}$$

Now,

$$\begin{aligned}
\begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} &= \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\
&= \begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} (F\,\hat{x}(t) + B\,(\Gamma - F\Pi)\,\hat{w}(t)) \\
&= \begin{bmatrix} A + BF & E_w + B\,(\Gamma - F\Pi) \\ 0 & S \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} BF & B\,(\Gamma - F\Pi) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix}
\end{aligned}$$

and the closed-loop system is

$$\left\{ \begin{aligned}
\left[\begin{array}{c} \dot{x}(t) \\ \dot{w}(t) \\ \hline \dot{\varepsilon}_1(t) \\ \dot{\varepsilon}_2(t) \end{array}\right] &= \left[\begin{array}{cc|cc} A + BF & E_w + B\,(\Gamma - F\Pi) & BF & B\,(\Gamma - F\Pi) \\ 0 & S & 0 & 0 \\ \hline 0 & 0 & A + K_A\,C_y & E_w + K_A\,D_{yw} \\ 0 & 0 & K_S\,C_y & S + K_S\,D_{yw} \end{array}\right] \left[\begin{array}{c} x(t) \\ w(t) \\ \hline \varepsilon_1(t) \\ \varepsilon_2(t) \end{array}\right] \\
e(t) &= \left[\begin{array}{cc|cc} C_e + D_{eu}F & D_{eu}[\Gamma - F\Pi] + D_{ew} & D_{eu}F & D_e(\Gamma - F\Pi) \end{array}\right] \left[\begin{array}{c} x(t) \\ w(t) \\ \hline \varepsilon_1(t) \\ \varepsilon_2(t) \end{array}\right]
\end{aligned} \right.$$

Introducing the change of coordinates $\xi(t) = x(t) - \Pi w(t)$, and recalling that $\dot{w} = Sw$, we obtain

$$\begin{aligned}
\dot{\xi}(t) &= \dot{x}(t) - \Pi S w(t) \\
&= (A+BF)x(t) + (E_w + B(\Gamma - F\Pi) - \Pi S)w(t) + BF\varepsilon_1(t) + B(\Gamma - F\Pi)\varepsilon_2(t) \\
&= (A+BF)x(t) - (A+BF)w(t) + BF\varepsilon_1(t) + B(\Gamma - F\Pi)\varepsilon_2(t), \quad \text{using (6)} \\
&= (A+BF)\xi(t) + BF\varepsilon_1(t) + B(\Gamma - F\Pi)\varepsilon_2(t). \qquad (26)
\end{aligned}$$

Also

$$\begin{aligned}
(C_e + D_{eu}F)x(t) + (D_{eu}(\Gamma - F\Pi) + D_{ew})w(t) &= (C_e + D_{eu}F)x(t) + (D_{eu}\Gamma - D_{eu}F\Pi + D_{ew})w(t) \\
&= C_e x(t) + D_{eu}F(x(t) - \Pi w(t)) + D_{eu}\Gamma + D_{ew})w(t) \\
&= C_e\xi(t) + D_{eu}F\xi(t) + (C_e\Pi + D_{eu}\Gamma + D_{ew})w(t) \\
&= (C_e + D_{eu}F)\xi(t) \qquad (27)
\end{aligned}$$

by (7). Hence we may write the closed loop system as

$$\left\{ \begin{array}{lcll}
\dot{\xi}(t) &=& (A+BF)\,\xi(t) + [BF\ B(\Gamma - F\Pi)]\varepsilon(t), & \xi(0) = \xi_0, \\
\dot{\varepsilon}(t) &=& A_{cc}\varepsilon(t), & \varepsilon(0) = \varepsilon_0 \\
e(t) &=& (C_e + D_{eu}F)\,\xi(t) + [D_{eu}F\ D_e(\Gamma - F\Pi)]\varepsilon(t) &
\end{array} \right. \qquad (28)$$

Let

$$\bar{A} \stackrel{\text{def}}{=} \begin{bmatrix} A+BF & [BF\ B(\Gamma - F\Pi)] \\ 0 & A_{cc} \end{bmatrix} \qquad (29)$$

and $\sigma(A+BF) \stackrel{\text{def}}{=} \{\lambda_i : 1 \le i \le n\}$ and $\sigma(A_{cc}) \stackrel{\text{def}}{=} \{\mu_i : 1 \le i \le n\}$. Then $\sigma(\bar{A}) = \sigma(A+BF) \cup \sigma(A_{cc})$, due to the diagonal structure of $\bar{A}$. Let $\{(\lambda_i, v_i) : 1 \le i \le n\}$ be the eigenpairs of $A+BF$, let $V = [v_1|\ldots|v_n]$ and introduce $\alpha(\tilde{x}_0) \stackrel{\text{def}}{=} V^{-1}\tilde{x}_0$. Then the output $\tilde{e}$ of the nominal plant arising from $\tilde{u} = F\tilde{x}$ is

$$\tilde{e}(t) = (C+DF)[v_1 e^{\lambda_1 t}|\ldots|v_n e^{\lambda_n t}]\,\mathrm{diag}(\alpha(\tilde{x}_0)). \qquad (30)$$

In each component we have $\bar{e}_i(t) \ne 0$ for all $t \ge 0$, because by assumption Σ_{nom} is nonovershooting from $\tilde{x}_0$. If we denote $\{w_1, \ldots, w_{2n}\}$ as the eigenvectors of $\bar{A}$, then the eigenpairs of $\bar{A}$ are (λ_i, w_i) for $1 \le i \le n$, and (μ_{i-n}, w_n) for $n+1 \le i \le 2n$. Recalling that (λ_i, v_i) are eigenpairs of $A+BF$ for $1 \le i \le n$, we note that $w_i = \begin{bmatrix} v_i \\ 0_n \end{bmatrix}$, because

$$\begin{aligned}
(\bar{A} - \lambda_i I_{2n}) \begin{bmatrix} v_i \\ 0_n \end{bmatrix} &= \begin{bmatrix} (A+BF-\lambda_i I_n)v_i \\ 0_n \end{bmatrix} \\
&= 0_{2n}. \qquad (31)
\end{aligned}$$

Let $W \stackrel{\text{def}}{=} [w_1|\ldots|w_{2n}]$, and introduce $W_{11} \stackrel{\text{def}}{=} \overline{\pi}\{[w_1|\ldots|w_n]\} = V$, $W_{12} \stackrel{\text{def}}{=} \overline{\pi}\{[w_{n+1}|\ldots|w_{2n}]\}$, $W_{21} \stackrel{\text{def}}{=} \underline{\pi}\{[w_1|\ldots|w_n]\} = 0$, $W_{22} \stackrel{\text{def}}{=} \underline{\pi}\{[w_{n+1}|\ldots|w_{2n}]\}$. We may then decompose W as

$$W = \begin{bmatrix} W_{11} & W_{12} \\ 0 & W_{22} \end{bmatrix}. \qquad (32)$$

Then

$$W^{-1} \begin{bmatrix} \xi_0 \\ \varepsilon_0 \end{bmatrix} = \begin{bmatrix} W_{11}^{-1}\xi_0 - W_{11}^{-1}W_{12}W_{22}^{-1}\varepsilon_0 \\ W_{22}^{-1}\varepsilon_0 \end{bmatrix} \qquad (33)$$

and after introducing $\beta(\varepsilon_0) \stackrel{\text{def}}{=} -W_{11}^{-1}W_{12}W_{22}^{-1}\varepsilon_0$, $\gamma(\varepsilon_0) \stackrel{\text{def}}{=} W_{22}^{-1}\varepsilon_0$, the state and estimator trajectories of the closed loop system (28) are given by

$$\begin{bmatrix} \xi(t) \\ \varepsilon(t) \end{bmatrix} = \begin{bmatrix} [v_1 e^{\lambda_1 t}|\dots|v_n e^{\lambda_n t}]\,\mathrm{diag}(\alpha(\xi_0)+\beta(\varepsilon_0)) \\ \underline{\pi}\{[w_{n+1}e^{\mu_1 t}|\dots|w_{2n}e^{\mu_n t}]\}\,\mathrm{diag}(\gamma(\varepsilon_0)) \end{bmatrix}, \tag{34}$$

Hence the output of (28) is

$$\begin{aligned} e(t) &= (C_e + D_{eu}F)[v_1 e^{\lambda_1 t}|\dots|v_n e^{\lambda_n t}]\,\mathrm{diag}(\alpha(\xi_0)+\beta(\varepsilon_0)) \\ &\quad +[D_{eu}F\ D_e(\Gamma - F\Pi)]\underline{\pi}\{[w_{n+1}e^{\mu_1 t}|\dots|w_{2n}e^{\mu_n t}]\}\,\mathrm{diag}(\gamma(\varepsilon_0)) \\ &\stackrel{\text{def}}{=} (1+\delta(\varepsilon_0))\tilde{e}(t) + N(t,\varepsilon_0), \end{aligned} \tag{35}$$

where $\tilde{e}(t)$ is defined in (30), and $\delta(\varepsilon_0)\tilde{e}(t)$ and $N(t,\varepsilon_0)$ are the terms depending upon $\beta(\varepsilon_0)$ and $\gamma(\varepsilon_0)$ respectively. From (30) we know that $\tilde{e}_i(t) \neq 0$ for all $t \geq 0$ in all output components, and hence for sufficiently small $\|\varepsilon_0\|$, we can ensure that $(1+\delta(\varepsilon_0))\tilde{e}(t) \neq 0$ in all components. Since $\|\gamma(\varepsilon_0)\| \leq \|W_{22}^{-1}\|\|\varepsilon_0\|$, we see that $\|N(t,\varepsilon_0)\| \to 0$ for all $t \geq 0$ as $\|\varepsilon_0\| \to 0$. We may then apply Lemma 3.1 with $f_i(t) = N_i(t,\varepsilon_0)$ and $g_i(t) = (1+\delta(\varepsilon_0))\tilde{e}_i(t)$, noting that the eigenvalues in $\sigma(A_{cc})$ and $\sigma(A+BF)$ satisfy the assumptions of Lemma 3.1 for f and g, respectively. Hence there exists a sufficiently small $\|\varepsilon_0\|$ that ensures $(1+\delta(\varepsilon_0))\tilde{e}_i(t) + N_i(t,\varepsilon_0) \neq 0$ for all $t \geq 0$. We conclude that for sufficiently small $\|\varepsilon_0\|$, $e(t)$ will not change sign for all $t \geq 0$ in any component and hence Σ_e subject to the control Σ_c in (11) has a nonovershooting response from the initial condition $(x_0, w_0, \varepsilon_0)$. ■

4 Example

Example 4.1 Consider the system Σ_e in (2) with $n = 5$ and $m = p = 3$ whose matrices are

$$A = \begin{bmatrix} -3 & -8 & 2 & 6 & 0 \\ 2 & 0 & 0 & 0 & 9 \\ -3 & -7 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 3 & -8 & 0 \\ 0 & 0 & 3 \\ 1 & -1 & 0 \\ 2 & 0 & 0 \end{bmatrix} \quad E_w = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 1 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_e = \begin{bmatrix} -9 & -10 & 3 & 0 & 0 \\ 0 & -3 & -1 & -8 & -4 \\ -4 & 2 & 0 & 0 & -7 \end{bmatrix} \quad D_{eu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -7 & 0 \\ -9 & 0 & 0 \end{bmatrix}.$$

The disturbance $d(t) \in \mathbb{R}^2$ and the reference signal $r(t) \in \mathbb{R}^3$ are generated by two exosystems whose matrices are

$$S_1 = \begin{bmatrix} 0 & -14 \\ 14 & 0 \end{bmatrix} \quad L_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \quad L_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

The pair (A,B) is reachable, and therefore also stabilisable. Our first task is to find a non-overshooting stabilising static state feedback matrix for the nominal system Σ_{nom} in (3) given by the quadruple (A,B,C_e,D_{eu}). Firstly we note that invariant zeros of Σ_{nom} are all real, and equal to $z_1 = -5.8946$, $z_2 = -0.2271$, $z_3 = 5.1790$ and $z_4 = 14.4348$. Thus, the system is nonminimum phase, invertible and has $n-p = 2$ minimum-phase zeros. Thus (13) holds with $l = 1$ and we may use the design method of Theorem 3.1 of [7]; we will obtain a globally nonovershooting control law of the form $\tilde{u} = F\tilde{x}$ for Σ_{nom}. We choose a set of desired closed-loop poles $\mathscr{L} = \{-5.8946,\ -0.2271, -1,\ -2,\ -3\}$, to include the two minimum phase zeros; the remaining three poles may be chosen to be any three real and distinct stable poles. Following (9) of [7], we choose target vector $s_1 = 0$, $s_2 = 0$, $s_3 = e_1$, $s_4 = e_2$ and $s_5 = e_3$, where 0, e_1, e_2 and e_3 are respectively the zero vector and the canonical basis vectors of the output space $\mathbb{R}^3$. For each $i \in \{1,\ldots,5\}$, we then solve the matrix equation [7, eq. (6)]:

$$\begin{bmatrix} A-\lambda_i I & B \\ C & D \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ s_i \end{bmatrix} \tag{36}$$

and obtain solutions sets $\mathscr{V} = \{v_1,\ldots,v_5\} \subset \mathbb{R}^5$ and $\mathscr{W} = \{w_1,\ldots,w_5\} \subset \mathbb{R}^3$. We construct matrices $V = [v_1\ \ v_2\ \ v_3\ \ v_4\ \ v_5]$ and $W = [w_1\ \ w_2\ \ w_3\ \ w_4\ \ w_5]$. Since V is non-singular, we can compute the non-overshooting stabilising static state feedback matrix F as

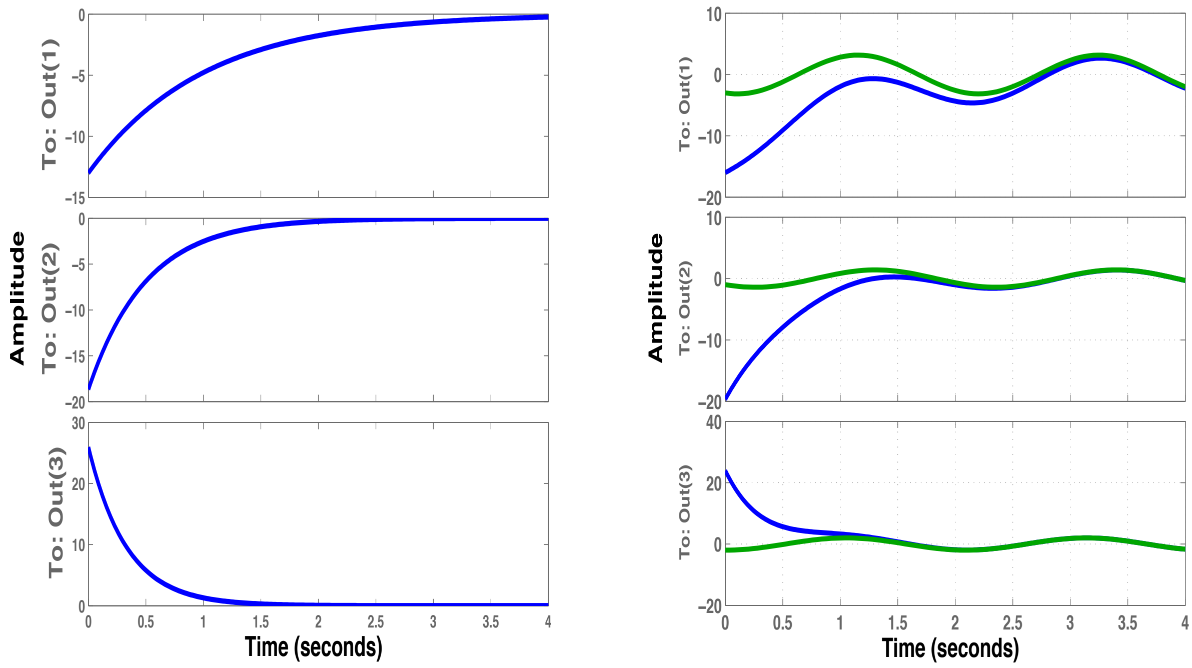
$$\begin{aligned} F &= WV^{-1} \\ &= \begin{bmatrix} -2.2051 & -10.0062 & 3.6242 & 7.1567 & -4.6382 \\ -1.3220 & -6.7630 & 2.6339 & 5.0909 & -1.3492 \\ 5.6227 & 22.2055 & -9.6653 & -15.3973 & 6.5325 \end{bmatrix}. \end{aligned} \tag{37}$$

By Theorem 3.1 of [7], the control law $\tilde{u} = F\tilde{x}$ yields a globally nonovershooting response for Σ_{nom}. The closed-loop eigenstructure is such that the closed-loop poles λ_1, λ_2 associated with the minimum phase zeros are invisible at the outputs. The poles λ_3, λ_4 and λ_5 contribute only to the first, second and third outputs, respectively. Thus each error component is governed by a single negative exponential, and hence vanishes without overshoot.

We now turn our attention to the feedforward part of the control law. Solution of the regulator equations (6)-(7) is given by the matrices

$$\Pi = \begin{bmatrix} -0.2060 & 0.0496 & -0.1979 & -0.1042 \\ -0.0757 & -0.1318 & 0.0560 & -0.0847 \\ -0.8704 & -0.2906 & -0.0738 & 0.0719 \\ 0.0352 & 0.0933 & -0.0518 & -0.0900 \\ -0.0645 & 0.0040 & 0.0481 & 0.0160 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.1249 & -0.0545 & -0.0481 & -0.0961 \\ 0.1534 & -0.0110 & 0.0183 & -0.0231 \\ -1.7387 & 3.4705 & 0.0045 & -0.2280 \end{bmatrix}.$$

Then $G = \Gamma - F\Pi$, and we may apply the state feedback law u in (4) to Σ_e. In Figure 2 we show the system response in the case where the initial condition for Σ is $x_0 = [1\ \ 1\ \ 1\ \ 1\ \ 1]^\top$, and the initial states of the exosystems are $\eta_0 = [1\ \ 1]^\top$ and $\zeta_0 = [1\ \ 1]^\top$. Figure 2(a) shows the tracking error term in each of the three output components. We see that each of these vanish without changing sign in all components, at exponential rates $e^{\lambda t}$ for $\lambda = -1$, -2 and -3, respectively. In Figure 2(b) the reference signal $r(t)$ and the output $z(t)$ are shown, and it is clear that the reference is tracked asymptotically without overshoot in all components.



(a) tracking error $\varepsilon(t)$ (b) reference $r(t)$ and regulated output $z(t)$

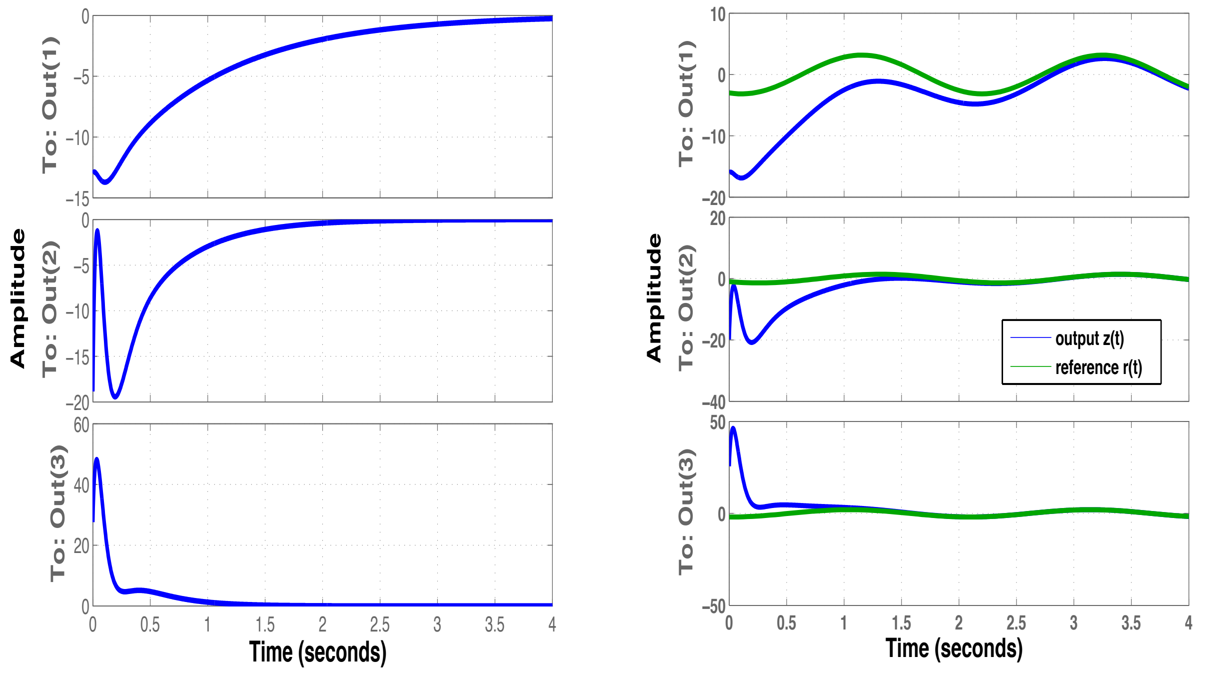
Figure 2: System responses using state feedback.

Next we consider the problem of output regulation via measurement feedback. We will implement the controller (11) on Σ_e; this requires us to obtain gain matrices K_A and K_S such that A_{cc} in (25) has poles to the left of all the poles of $A+BF$. A_{cc} has dimension $n+n_1+n_2=9$, so we choose poles at $\{-11,\ -12,\ -13,\ -14,\ -15, 16,\ -17,\ -18,\ -19\}$. Using MATLAB®'s `place` command on the pair $\left(\begin{bmatrix} A & E_w \\ 0 & S \end{bmatrix}^\top, \begin{bmatrix} C_y & D_{yw} \end{bmatrix}^\top\right)$, we obtain the matrices

$$K_A = \begin{bmatrix} 48.0035 & 64.8235 & 521.3166 \\ -23.8073 & -39.2259 & -221.5731 \\ -25.6893 & -46.7218 & -322.1780 \\ -34.4213 & -43.3481 & -137.2139 \\ -10.3497 & -21.9117 & -110.2701 \end{bmatrix},\ K_s = \begin{bmatrix} -5.2539 & 1.6886 & 82.2003 \\ 29.4410 & 15.7723 & -26.4926 \\ 0.0046 & -0.0264 & -73.9012 \\ 0.0012 & -0.0050 & -30.5597 \end{bmatrix}. \tag{38}$$

Figure 3 shows the system response from Σ_e under the measurement feedback law (8) with initial conditions of $x_0 = [1\ \ 1\ \ 1\ \ 1\ \ 1]^\top$, $\eta_0 = [1\ \ 1]^\top$ and $\zeta_0 = [1\ \ 1]^\top$ and initial estimator error $\varepsilon(0) = \delta[x_0^\top\ \eta_0^\top\ \zeta_0^\top]^\top$, with $\delta = 0.05$.

We observe from Figure 3(a) that the tracking errors converge to zero without changing sign in all components, although the convergence is no longer monotonic, as was the case with state feedback. Similarly in Figure 3(b) we have $z(t)$ converging to $r(t)$ in all three outputs, without overshoot. We note that if we apply the measurement feedback control with initial estimator error given by $\delta = 0.06$, the error signal changes sign in the second output component, confirming our expectation that



(a) tracking error $\varepsilon(t)$ (b) reference $r(t)$ and regulated output $z(t)$

Figure 3: System responses using measurement feedback.

the nonovershooting response of the state feedback controller is only preserved under measurement feedback if the initial estimator error is sufficiently small.

5 Conclusion

We have revisited the design method for a linear state feedback tracking controllers given in [7]-[9] and used the Internal Model Principle to extend it to accommodate the nonovershooting and nonundershooting tracking of time-varying signals in the presence of known time varying disturbances. To the best of the authors' knowledge, this paper presents the first multivariable linear control scheme for the tracking of time-varying signals without overshoot or undershoot.

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