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A New Model for Squirt-flow Attenuation and Dispersion in Fluid-saturated Rocks

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SUMMARY

We develop a new simple model of squirt-flow model attenuation and dispersion, in which most parameters can be independently measured or estimated from measurements. The pore space of the rock is assumed to consist of stiff porosity and compliant (or soft) pores present at grain contacts. The effect of isotropically distributed soft pores is modeled by considering pressure relaxation in a disk-shaped gap between adjacent grains. This derivation gives the complex and frequency-dependent effective bulk and shear moduli of a rock, in which the soft pores are liquid-saturated and stiff pores are dry. The resulting squirt model is consistent with Gassmann s and Mavko-Jizba equations at low and high frequencies, respectively.

Introduction

Elastic wave attenuation and dispersion are observed in fluid saturated rocks in all ranges of frequencies, from seismic to sonic and ultrasonic measurements. The main cause of these phenomena is attributed to the flow of the pore fluid induced by the passing wave. Wave-induced fluid flow occurs as a passing wave creates local pressure gradients within the fluid phase and the resulting fluid flow is accompanied with internal friction until the pore pressure is equilibrated. At sonic and ultrasonic frequencies attenuation is believed to be dominated by the local flow (squirt) between pores of different shapes and orientations.

First theoretical models of squirt-flow attenuation were mostly based on the analysis of aspect ratio distributions (Mavko and Nur, 1975, Jones, 1986). More recently, an alternative approach has emerged based on the recognition of the fact that the pore space of many rocks has a binary structure (Shapiro, 2003): relatively stiff pores, which form the majority of the pore space, and relatively compliant (or soft) pores, which are responsible for the pressure dependency of the elastic moduli.

In this paper, we propose a new model of squirt-flow dispersion and attenuation, which uses a pressure relaxation approach of Murphy et al. (1986) in conjunction with the extension of Mavko and Jizba (1991) equations to arbitrary fluids. The resulting model is consistent with the Gassmann and Mavko-Jizba equations at low and high frequencies, respectively, and with the piezosensitivity model of Shapiro (2003). It can also naturally be incorporated into Biot's poroelasticity theory to obtain velocity and attenuation prediction in a broad frequency range.

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Low and high-frequency moduli

At low frequency, the fluid pressure has sufficient time to equilibrate between stiff and compliant pores during half wave cycle. Thus the bulk and shear moduli of the saturated rock (called relaxed moduli) are given by Gassmann’s equations.

At high frequency, we assume that the fluid pressure has no time to equilibrated between stiff and soft pores. Thus the soft pores can be considered hydraulically isolated from stiff pores and from each other. Another important assumption is that the dry rock at pressure $P > P_h$ contains stiff pores only, and is an isotropic linearly elastic solid with a bulk modulus K_h . At a pressure $P < P_h$ the rock is weakened by the presence of compliant porosity. For the case where the pore fluid is liquid, expressions for the high frequency bulk and shear moduli (unrelaxed) were derived by Mavko and Jizba (1991). A more general version of these equations, which allows the fluid modulus to be small or even zero is (Gurevich et al., 2009)

$$\frac{1}{K_{uf}(P)} = \frac{1}{K_h} + \frac{1}{\dfrac{1}{\dfrac{1}{K_{dry}(P)} - \dfrac{1}{K_h}} + \dfrac{1}{\left(\dfrac{1}{K_f} - \dfrac{1}{K_g}\right)\phi_c(P)}} \tag{1}$$

$$\frac{1}{\mu_{dry}(P)} - \frac{1}{\mu_{uf}(P)} = \frac{4}{15}\left(\frac{1}{K_{dry}(P)} - \frac{1}{K_{uf}(P)}\right) \tag{2}$$

where $K_{dry}(P)$ and $\mu_{dry}(P)$ are the bulk and shear moduli of the dry rock at a given confining pressure P, K_f and K_g are the bulk moduli of the fluid and the material of the solid grains, $\phi_c(P)$ is the compliant porosity, and $K_h = K_{dry}(P_h)$ is the dry bulk modulus at the highest pressure. The moduli K_{uf} and μ_{uf} have been obtained assuming that the stiff pores are dry, but soft pores filled with the fluid. The saturated undrained moduli can be derived from K_{uf} and μ_{uf} by applying Gassmann's equations to stiff porosity.

Frequency-dependent moduli

In order to model the frequency dependency of the moduli, we need to assume a particular geometrical configuration. Here we assume a particular geometry proposed by Murphy et al. (1986): a compliant pore forms a disk-shaped gap between two grains, and its edge opens into a toroidal stiff pore. It is assumed that the gap also has asperities, and thus its stiffness is finite even when the gap is empty. However these asperities are assumed (somewhat arbitrarily) not to affect the geometry of the gap, as far as fluid movement is concerned.

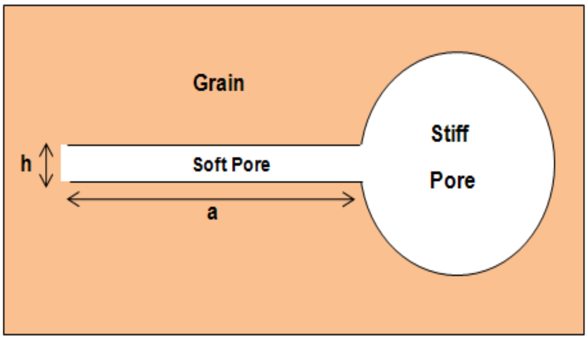

Figure 1 *Sketch of the model configuration: Axial symmetric section through the model.*

The gap has radius a and thickness h. The additional effective stiffness K^* of the gap due to the presence of fluid can be defined as a ratio of the force ΔF (acoustic force) exerted by the fluid onto the gap wall, to the uniaxial dynamic loading (displacement) Δh: $K^* = -\Delta F/\Delta h$. The force ΔF is the integral of pressure over the surface S_g of the gap, $\Delta F = \int_{S_g} p(r)dS$. For sinusoidal loading $\Delta h \exp(i\omega t)$ with frequency ω, pressure p can be obtained as a solution of the ordinary differential equation

$$\frac{d^2p}{dr^2} + \frac{1}{r}\frac{dp}{dr} + k^2 p = C\,, \qquad (3)$$

where r is radial coordinate, k is the wavenumber of the pressure diffusion wave in the gap defined by $k^2 = -i\omega h_0 D/K_f$, $D = 12\eta/h_0^3$ is the viscous resistance, η is dynamic viscosity of the fluid and $C = i\omega D \Delta h$. The boundary condition at the edge of the gap ($r = a$) is $p|_{r=a} = 0$.

At sufficiently low frequencies, the pressure in the gap will be equilibrated, and thus will be zero throughout the gap. Equation (3) is an inhomogeneous Bessel equation with a constant right-hand side. The solution of equation (3) with boundary condition $p|_{r=a} = 0$ is $p = (C/k^2)\left[1 - J_0(kr)/J_0(ka)\right]$. This gives the following expression for the fluid-related gap stiffness

$$K^* = \left(\pi a^2 K_f/h_0\right)\left[1 - 2J_1(ka)/kaJ_0(ka)\right].. \qquad (4)$$

In the low frequency limit $k \to 0$ and thus K^* vanishes. This corresponds to the fact that at low frequencies the fluid poses no resistance to gap deformation. Conversely, in the limit of high frequency, equation (4) gives

$$K^* = \pi a^2 K_f / h_0 \tag{5}$$

This is the gap resistance in the unrelaxed state, when fluid has no time to escape from the gap within the half-period of the wave. Comparison of equations (4) and (5) shows that at any frequency the gap stiffness is the same as unrelaxed stiffness computed for a modified fluid with an effective frequency dependent bulk modulus

$$K_f^* = \left[1 - 2J_1(ka)/kaJ_0(ka)\right] K_f . \tag{6}$$

Substitution of K_f^* for the fluid modulus K_f in equation (1) gives the final expression for the partially relaxed (frequency dependent) frame modulus K_{pf}. Then, the corresponding partially relaxed (frequency dependent) shear modulus μ_{pf} can be obtained by substituting K_{pf} for K_{uf} in equation (2)

Note that the wavenumber k of the pressure wave is complex and frequency dependent, and so are the effective fluid modulus K_f^* and partially relaxed frame moduli K_{pf} and μ_{pf}. This implies the presence of velocity dispersion and attenuation. The saturated moduli can then be computed using either Gassmann's or Biot's equations. If the frequency is low compared with Biot's characteristic frequency f_c, then the saturated bulk modulus K_{sat} can be obtained by substituting K_{pf} for the frame modulus in Gassmann's equation, while the saturated shear modulus will still be the same, $\mu_{sat} = \mu_{pf}$. If, however, the frequency is comparable with or higher than f_c, then both K_{pf} and μ_{pf} need to be substituted into Biot's dispersion equations to obtain P and S velocities.

Example

Figure 2a,b shows a comparison between measured (Agersborg et al., 2008) and predicted bulk and shear moduli for a water-saturated carbonate sample as a function of pressure. Compliant porosity was obtained from velocity in dry sample using the theory of Shapiro (2003). The aspect ratio of the grain contact h_0 / a was obtained by the best fit of the prediction to the data, and was estimated to be 0.01. The partially relaxed moduli corresponding to this aspect ratio are shown in Figure 2a,b as dashed lines. We see that the model describes the observed shape of the pressure dependency reasonably well. Figures 3 and 4 show the predicted bulk modulus and P-wave attenuation as functions of pressure and frequency. We see that both dispersion and attenuation decrease with increasing pressure, as they should.

Conclusions

We have developed a new simple model of squirt-flow dispersion and attenuation in granular fluid-saturated media. The model has only one freely adjustable parameter (effective aspect ratio of the compliant grain contacts). The results are exactly consistent with Gassmann theory in the low frequency limit, and with Mavko-Jizba unrelaxed moduli in the high-frequency limit.

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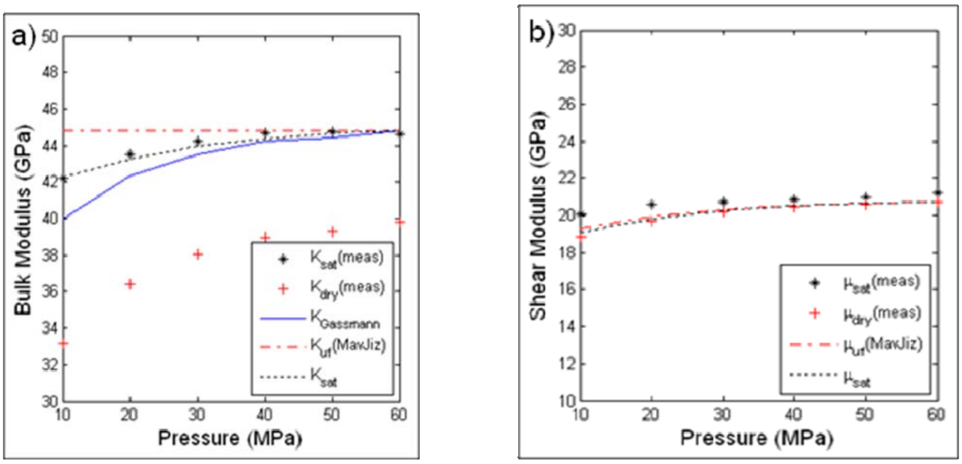

Figure 2 *Comparison of model prediction and ultrasonic measurements for bulk (a) and shear (b) moduli of fluid saturated rock. K_{pf} and μ_{pf} are the obtained partially relaxed frame moduli.*

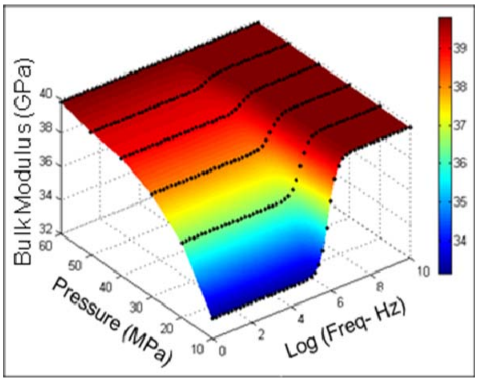

Figure 3 *Model prediction of bulk modulus as a function of frequency and pressure.*

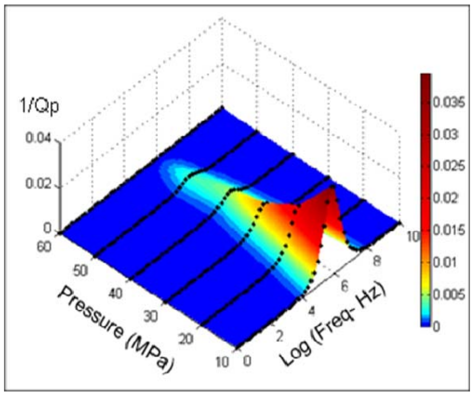

Figure 4 *Model prediction of P wave attenuation as a function of frequency and pressure.*

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