Survey Review, **34**, 267 (January 1998)

STRATEGIES FOR THE ACCURATE DETERMINATION OF ORTHOMETRIC HEIGHTS FROM GPS

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ABSTRACT

In order for Global Positioning System (GPS) derived ellipsoidal heights to have any physical meaning in a surveying or engineering application, they must be transformed to orthometric heights. A practical approach is presented where gravimetrically and geometrically derived geoid heights are combined to accurately recover orthometric heights from GPS. This approach is supported by results from three GPS controlled gravity surveys conducted in Western Australia.

INTRODUCTION

The Global Positioning System (GPS) provides the surveyor with three-dimensional co-ordinates with respect to the geocentric World Geodetic System 1984 (WGS84). Before these GPS-derived positions can be used in the local reference frame however, two distinctly different co-ordinate transformations must be applied. The horizontal transformation from WGS84 latitude and longitude to the local horizontal datum is relatively straight-forward when using conformal or projective transformation models. In the future however, these horizontal co-ordinate transformations may become unnecessary as different countries move to the use of a geocentric datum for surveying and mapping, as is the case in Australia for example **[10]**.

However, it is unlikely that ellipsoidal heights will ever be used for practical surveying, engineering or geophysical applications as they have no physical meaning. For instance, when using ellipsoidal heights there is the possibility that water will appear to flow up-hill because the physical force of gravity is not considered. Therefore, it will always be necessary to transform GPS-derived ellipsoidal heights to orthometric heights, using a knowledge of the position of the geoid with respect to the WGS84 ellipsoid.

The geoid is used as the reference surface to which orthometric heights are referred, and the vertical datum is usually established using tide gauge measurements of mean sea-level in conjunction with geodetic levelling. In Australia, which is used to illustrate the methods presented here, elevations are generally referred to the Australian Height Datum, which will be assumed to coincide with the geoid and represent an orthometric height system, although not strictly true [8, 17, 22] but adequate for the purpose of this discussion. Most surveying measurements are made in relation to the geoid because the equipment is aligned with the local gravity vector, usually through the use of a spirit bubble. As such, geodesists have chosen an oblate ellipsoid of revolution, flattened at the poles, to approximate the geoid in order to simplify survey data reduction and mapping. On a global scale, the geoid departs from the WGS84 ellipsoid by approximately ± 100 metres. This geoid-ellipsoid separation varies from approximately -30 metres in south-western Australia to approximately $+70$ metres in north-eastern Australia.

A GPS-derived ellipsoidal height is converted to an orthometric height using a knowledge of the undulations in the geoid-WGS84-ellipsoid separation. This vertical co-ordinate transformation has already been discussed by many authors. To summarise, a WGS84 ellipsoidal height (h) is transformed to an orthometric height (H) by *algebraically* subtracting the geoid-WGS84-ellipsoid separation, viz

$$H = h - N \tag{1}$$

However, equation (1) is not directly relevant in surveying and geodesy, which routinely uses relative or differential GPS that provides ellipsoidal height differences with respect to a fixed base-station. Instead, the change in orthometric height over a GPS baseline (A to B) is determined by using the corresponding change in geoid-ellipsoid separation, viz

$$H_A - H_B = h_A - h_B - (N_A - N_B) \tag{2}$$

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \tag{3}$$

For example, an ellipsoidal height difference of $+1.78$ metres over a baseline where the geoid-ellipsoid separation *increases* by 2.34 metres, yields an orthometric height difference of -0.56 metres. Therefore, if the geoid is neglected, there is the possibility that water would appear to flow up-hill, and a knowledge of the undulations in the geoid is essential to transform the geometrical GPS ellipsoidal heights to the physically meaningful orthometric heights.

The conversion of GPS ellipsoidal heights to orthometric heights for surveying and engineering purposes still poses many practical problems however. Firstly, the ellipsoidal height is the least accurately determined GPS co-ordinate, predominantly due to inherent geometric weakness and atmospheric errors. Secondly, the position of the geoid with respect to WGS84 is often known less accurately than the GPS height. Moreover, the errors in the geoid are difficult to predict and can vary considerably according to location. Simply because a geoid model behaves well in one region does not necessarily mean that it will in another. This is probably the most significant problem facing the determination of orthometric heights from GPS using the geoid.

There are two principal approaches with which to transform GPS ellipsoidal heights to orthometric heights. These comprise a gravimetrically determined geoid model, and interpolation between geometrically derived geoid heights where GPS measurements have been co-located with benchmarks. To overcome the limitation mentioned in the previous paragraph, a combined approach is proposed in this paper where the gravimetric geoid model is constrained by using geometrically derived geoid heights that surround the survey area. This procedure can be used to transform GPS heights more accurately than either method alone, and this is supported by results from three GPS campaigns in Western Australia.

Obtaining the Geoid Height

The gravimetric method

The geoid-WGS84-ellipsoid separation can be computed from terrestrial gravity measurements in conjunction with a global geopotential model and digital terrain model; see, for example **[7]**. The gravimetric method offers the benefit of a regional or nation-wide geoid model, provided that there exists a homogeneous coverage of terrestrial data. However, gravimetric geoid determination is an involved mathematical and computational procedure, which is still under refinement. As such, gravimetric geoid models are usually only computed through research projects at universities or government surveying and mapping authorities. Gravimetric determinations of the geoid of Australia have been of interest to many geodesists over the years; see **[13]** for a review. The current gravimetric geoid currently in use is called AUSGEOID93 **[24]**. It was computed by the Australian Surveying and Land Information Group (AUSLIG) using the techniques and software of Kearsley (1988). AUSGEOID93 is readily available to all Australian GPS surveyors through the following:

1) *Winter* is a Microsoft Windows-based package, which can be used to interpolate AUSGEOID93 geoid heights from a pre-computed grid. The resolution of this grid is 10′ by 10′ on WGS84 and digital geoid files are available for 1:25,000 Australian Map Grid (AMG) sheets.

2) AUSLIG provides interpolation of geoid heights from the AUSGEOID93 grid via the World Wide Web (Internet). Simply use a search tool to locate AUSLIG and follow the paths to the section on computing a geoid height.

3) Rigorously computed geoid heights will be computed by AUSLIG at discrete

points specified by the user, if required. This avoids interpolation from the published grid and forms the best available AUSGEOID93 heights. However, the resolution of AUSGEOID93 is inevitably limited by the spacing of the Australian gravity data, which is approximately 11 kilometres over most of the continent.

If a gravimetric geoid model is unavailable in the region to be surveyed, or if the GPS height requirements are of a lower accuracy, the EGM96 global geopotential model can be used instead **[16]**. A computer file of the 130,676 geopotential coefficients is freely available, and geoid-WGS84-separations can be computed on a desktop computer using the routines of Rapp **[20]**.

The geometric method

Just as GPS can be used with the geoid to provide orthometric heights, GPS and orthometric heights can be used to provide an estimate of the position of the geoid at discrete points through a simple rearrangement of equation (1).

If the geoid is assumed to be approximated by a flat surface, which is usually sufficient over quite small areas (typically a few kilometres), linear interpolation can be used to estimate the geoid-WGS84-ellipsoid separation. Using two benchmarks which have both been occupied with GPS, the ellipsoidal height at an intermediate station X *between* these can be transformed to an orthometric height using

$$H_X = H_A + \Delta h_{AX} - \frac{l_{AX}}{s_{AB}} \Delta N_{AB} \tag{4}$$

where s_{AB} is the separation of the benchmarks A and B and l_{AB} is the distance of the unknown point X from benchmark A. These two distances are given automatically by most GPS manufacturer's processing software.

This geometric determination of the orthometric height is trivial for a short profile. However, GPS surveys are rarely conducted along a profile. Therefore, equation (4) is re-written for a plane

$$h - H = N = N_0 + N_1 e + N_2 n \tag{5}$$

where the coefficient N_0 represents a bias, N_1 and N_2 represent tilts of the geoid plane with respect to WGS84, and e is the easting and n is the northing in some plane coordinate system. In order to determine these three coefficients, requires that GPS observations are made at a minimum of three benchmarks that *surround* the survey area. Equation (5) is then formed at each of these benchmarks and solved using the following simple matrix inversion:

$$\begin{pmatrix} N_0 \\ N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} 1 & e_A & n_A \\ 1 & e_B & n_B \\ 1 & e_C & n_C \end{pmatrix}^{-1} \begin{pmatrix} (h-H)_A \\ (h-H)_B \\ (h-H)_C \end{pmatrix} \tag{6}$$

where the subscripts A, B and C refer to the three benchmarks. Once the coefficients N_0, N_1 and N_2 are determined, they are used in equation (7), which is a combination of equations (1) and (5), given the easting (e_X) and northing (n_X) of the intermediate points at which the orthometric heights are desired.

$$H_X = H_A + \Delta h_{AB} - N_A + N_0 + N_1 e_X + N_2 n_X \tag{7}$$

If station A is selected such that $N_A = N_0$, equation (7) simplifies to

$$H_X = H_A + \Delta h_{AB} + N_1 e_X + N_2 n_X \tag{8}$$

If more than three benchmarks are used, equation (6) becomes over-determined and a least squares solution should be used in preference. Alternatively, however, if more GPS surveyed benchmarks are available, the geoid can be modelled using a low-order polynomial surface **[4]**. However, the latter approach should be treated with some caution because spurious features can result as an artefact of using higher order polynomial surfaces, which do not accurately model the geoid. Another option is to use an interpolation based upon multiple regression equations **[1]**.

There are some important precautions that must be taken when using geometrical interpolation to determine orthometric heights from GPS. These comprise:

1. The geometrically derived geoid-WGS84-ellipsoid separation is limited to the combined accuracy of the GPS and orthometric heights. If errors exist in one or both of these measurements, the geometrical geoid model will be distorted, which will degrade any subsequently interpolated GPS-derived orthometric heights.
2. Interpolation of the geometrically derived geoid can prove superior to a gravimetric geoid for some survey areas that are smaller than the resolution of the gravimetric geoid. Examples of the use of the geometric geoid modelling method in Australia are given by Collier and Croft **[4]** and Gilliland **[11]**.
3. Interpolation, by definition, can only be applied in the area bound by the benchmarks used to geometrically define the transformation surface, otherwise less accurate and unreliable extrapolation, again by definition, is used for those areas outside the control points.

4. Independent checks should always be used to ensure that the geoid is accurately modelled by the type of surface chosen in the area of interest. In the case of a plane, four or more benchmarks should be used; three to define the plane and the remainder to provide an independent check. This will give the most realistic estimate of how well orthometric heights can be recovered from GPS for each particular survey area.

ERROR SOURCES AFFECTING GPS-DETERMINED HEIGHTS

As the objective is to determine accurate orthometric heights from GPS, the error sources that affect each of the GPS ellipsoidal height, geoid height, orthometric height, and their combination must all be considered.

Errors in GPS ellipsoidal heights

The most important consideration is the mode in which the GPS survey is conducted, due to the resolution of the GPS observables used.

Table 1. *Summary of approximate accuracy of GPS positioning (in metres), where C/A denotes the C/A code only, L1 denotes the L1 carrier-phase only, L1 & L2 denotes dual-frequency carrier phase and Rt is real time. These values are estimates only and must not be relied upon.*

GPS survey method	observables	horizontal (m)	vertical (m)
single point	C/A	100	140
static (differential)	C/A	0.5-2	1-3
static (relative)	L1	0.02	0.03
static (relative)	L1 & L2	0.005	0.02
rapid-static	L1 & L2	0.02	0.03
pure kinematic	C/A	2-5	3-8
pure kinematic	L1	0.03	0.05
pure kinematic	L1 & L2	0.01	0.02
semi-kinematic	L1 & L2	0.01	0.02
RT differential	C/A	3-5	4-8
RT pure kinematic	C/A	2-5	3-8
RT pure kinematic	L1	0.1	0.2
RT pure kinematic	L1 & L2	0.05	0.1
RT semi-kinematic	L1	0.03	0.05
RT semi-kinematic	L1 & L2	0.02	0.03

Single-point, code-based GPS positioning is currently restricted by the effects of selective availability and thus provides absolute ellipsoidal heights with respect to WGS84 accurate to only ±140 metres. Differential GPS, which uses the code observables only, improves vertical positioning and can provide ellipsoidal height differences typically to between ±0.5 and ±8 metres. Relative GPS, which uses a combination of the code and carrier-phase observables, can provide ellipsoidal height differences at the centimetre level over baselines up to 20-30 kilometres in length. The *approximate* accuracy levels of positioning for current GPS surveying techniques are summarised in Table 1.

In addition to the mode of GPS survey and observables used, other errors inherent to GPS surveying affect the GPS heights. When combined, these dictate that GPS ellipsoidal heights are typically 1½ to 2 times less accurate than GPS horizontal positions. As GPS is principally a navigation system, the designers of GPS did not intend, or expect, for it to be used to determinate heights in a surveying environment. Nevertheless, advances in GPS technology, most notably the use of the carrier-phase observables, have meant that it is indeed a practical tool for the determination of height (e.g.**[2]**,**[5]**,**[11]**).

The major error sources that affect GPS ellipsoidal heights are as follows:

Vertical dilution of precision

The geometry of GPS positioning is intrinsically weak in elevation because the satellites are usually situated above the antenna. This geometrical weakness is described by the vertical dilution of precision (VDOP) such that the error in ellipsoidal height has been magnified by the VDOP value. Therefore, it is important to schedule and conduct surveys at times and in areas where the VDOP is as low as possible.

Satellite ephemeris and GPS baseline length

Another geometrical error that affects GPS ellipsoidal heights is the approximate correlation between ephemeris errors as a function of the baseline length. This "rule-of-thumb" relation is given by Wells *et al* **[25]** as

$$\frac{\sigma_b}{b} \approx \frac{\sigma_\rho}{\rho} \tag{9}$$

where b is the GPS baseline length, σ_b its associated error, and ρ is the satellite altitude with an error of σ_r. Assuming $\rho \approx$ 20200 kilometres and $\sigma_\rho \approx$ ±20-40 metres for the broadcast ephemeris, the approximate height error is ±1-1.5 parts per million (mm/km) of the baseline length. Therefore, it is important to observe as short as possible GPS baselines when using the broadcast ephemeris.

For longer GPS baselines, it is preferable to use the precise ephemeris produced by the International GPS Service for Geodynamics (IGS). These data are available over the Internet usually a few days after the survey, and can be used with most GPS manufacturers' post-processing software. Simply use a search tool to locate the IGS Central Bureau and follow the paths to the section on precise orbits. These ephemerides are usually one order of magnitude more precise than the broadcast ephemeris, thus giving an order of magnitude improvement in the relative GPS baseline.

The atmosphere

The ionosphere and troposphere are the layers within the Earth's atmosphere that adversely effect GPS measurements. This has the greatest impact upon GPS height determination because the signals pass down through the atmosphere.

Dual-frequency GPS observations can remove almost all of the ionospheric refraction effect, which is dispersive, using the ionosphere-free observable. However, single-frequency GPS observations must rely upon atmospheric model parameters, which are broadcast as part of the navigation message, and only reduce approximately 50-60% of the ionospheric effect **[15]**.

The refractive effect of the troposphere is non-dispersive and thus can not be eliminated with the use of dual-frequency data. Instead, atmospheric models must be used for all GPS measurements **[3,5]**.

Fortunately for the GPS surveyor, however, the majority of the atmospheric effects is reduced further when using relative or differential GPS observations, especially over shorter baselines. Nevertheless, the treatment of tropospheric refraction is one of the few remaining obstacles to very precise GPS height determination.

Multipath

Multipath occurs when the GPS signals are reflected from a nearby object or the ground before reaching the antenna. Multipath can cause errors in ellipsoidal heights of a few metres for code observations and a few centimetres for carrier phase observations. However, the exact size of multipath errors is almost impossible to quantify because they are site-specific. Therefore, the only solution to reduce multipath at present is to select sites well away from potentially reflective areas and to use ground planes or choke rings attached to the antennae. Multipath effects are of most concern for short station occupations, such as in real-time kinematic GPS surveys, as the multipath effect manifests as a bias in the GPS position.

Antenna orientation and phase centre

When surveying with GPS, the position is actually computed at the phase centre of the antenna, then the software makes corrections to the survey mark. This phase centre does not necessarily coincide with the geometrical centre of the antenna, nor is it fixed. It can vary according to each satellite's elevation and azimuth. Most commercial software is pre-programmed to automatically correct for this using factory-based or other independent calibrations. However, careful survey procedures should always be employed in order to reduce the phase centre error as much as possible. It is always preferable to use the same make and model of antennae from the same manufacturer. Also, when collecting GPS data, all antennae should be oriented in the same direction (for example, due North) in order for any systematic phase centre offsets to cancel to a large extent on differencing.

Measurement of the antenna height

Errors in, or the complete omission of, the measured antenna height above the survey mark is probably the most common human error committed during GPS surveying. Obviously, this is critical for GPS height surveys. If there is an error in the measurement of the antenna height, the station should be re-observed, which can be especially frustrating if this error was committed at the GPS base station. It is, therefore, prudent to take care when measuring and recording the GPS antenna height. Effective strategies include:

The use of multiple measurements in both imperial and metric systems;
Measurements taken to different parts of the antenna (eg. both base and top);
Calculate the true vertical height whilst in the field;
Optical levelling; or even,
Photogrammetric methods

Fixing the integer ambiguities

For relative carrier-phase GPS surveys, it is essential to fix the integer ambiguities to their correct values in order to achieve accurate vertical positioning (e.g. **[9]**). Integer float solutions are usually only accurate to $\pm$1-2 metres, or worse, in the vertical. Of more concern is that the statistical tests utilised by the GPS processing software can sometimes be satisfied for the incorrect set of integer ambiguities. Therefore, it is essential to observe as much GPS data as possible, and to use closed networks to provide redundancy.

Errors in the existing orthometric heights

When using the geometrical method to model the geoid or using benchmarks to assess the accuracy of the geoid in a survey area, the highest order benchmarks should be used. Systematic and random errors are known to exist in local and regional levelling networks by virtue of the methods used in geodetic levelling. These errors can be due to instrumental, atmospheric and data reduction errors.

The Australian Height Datum (AHD) has recently been evaluated by Morgan **[19]** who concludes that it serves as a reasonably homogeneous vertical datum to third-order specifications. In Australia, these specifications allow a misclose of $12\sqrt{k}$ millimetres, where the k is the direct distance traversed in kilometres. This loosely implies an accuracy of a few centimetres in most areas. However, gross errors and regional distortions may degrade this estimate of accuracy. Also, not all benchmarks have been spirit levelled and trigonometric height determinations are inherently less accurate.

Therefore, the highest order benchmarks in the survey area, that have been spirit levelled, should only be used for geometrical geoid modelling or quality assurance of the GPS derived orthometric heights. If only trigonometric heights are available, these should not be used for geoid modelling nor quality assurance. In many instances, GPS in conjunction with a global geopotential model can provide a more accurate approach than the trigonometric heights.

Errors in gravimetric geoid heights

As was stated in the introduction, the major disadvantage with a gravimetrically determined geoid is that its accuracy is not well understood, and is known to vary depending upon location. Moreover, this accuracy is difficult to quantify and can not be assumed to be homogeneous. The geoid may be accurately modelled in one region and thus provide accurate orthometric heights, but this may not be the case in another. Unfortunately, the principal method used to verify a gravimetric geoid at present is to compare it with GPS and levelled heights, which are themselves in error. Moreover, some authors tend to apply biases and tilts to gravimetric solutions such that they fit the GPS and levelling data, thus producing somewhat misleading accuracy estimates of the gravimetric geoid solution when used in other areas.

The primary errors in a gravimetric geoid are due to: 1) the quality and spatial resolution of the gravity data, geopotential coefficients and digital terrain data used; 2) the formulation of the boundary value problem of physical geodesy and the approximations made during its solution; and, 3) the computational procedures and numerical methods used. Ongoing research at many institutions around the world aims to reduce the errors in the gravimetric geoid to a similar level to those in GPS baselines.

The accuracy of the Australian gravimetric geoid has been studied by many geodesists using GPS and AHD data to verify its accuracy in specific test areas; see, for example, **[12]**, **[18]**. The accuracy of the most recent national gravimetric geoid, AUSGEOID93, has been studied by its authors, Steed and Holtznagel **[24]**, and independently by Featherstone and Alexander **[9]**. Its absolute accuracy is estimated to be ± 0.3-0.5 metres and its relative accuracy is typically ± 2-3mm/km, which reduces to ± 0.1-1mm/km in areas where the geoid is smooth or well known, but increases to ± 4-5mm/km in rugged geoid areas (notably the mountains). However, it is essential to point out that these analyses can be misleading as they are subject to possibly erroneous AHD and GPS data.

The Advantages and Disadvantages of the Gravimetric and Geometric Methods of Geoid Modelling

For convenience of both presentation and interpretation, the advantages and disadvantages of the gravimetric and geometric approaches to recover orthometric heights from GPS are summarised in Tables 2 and 3, respectively.

It can be seen that each approach has its own benefits and limitations. Therefore, it will be proposed later in this paper that a combination of the gravimetric and geometric methods be used. This makes use of the advantages of each method to a large extent, in order to accurately determine orthometric heights from GPS. The combined approach can reduce the errors inherent to each method, but is inevitably restricted by the quality of the orthometric, geoid and GPS heights.

Table 2. *Summary of advantages and disadvantages of using only a gravimetric geoid in conjunction with GPS to determine heights.*

Advantages	Disadvantages
1. Gravimetry provides a geoid model over a wide area, given a complete and homogeneous coverage of terrestrial gravity and terrain data;	1. The accuracy of the gravimetric geoid is not well understood and varies from place to place;
2. Geoid heights are usually readily available from national surveying and mapping authorities or universities; the former is the case in Australia;	2. Errors in GPS heights combine additively with errors in gravimetric geoid heights;
3. There is no prerequisite for existing vertical geodetic control in the GPS survey area.	3. Biases may exist between the local orthometric heights and GPS-gravimetric-geoid derived orthometric heights;
	4. The spatial resolution of a gravimetric geoid is limited by that of the input gravity and terrain data.

Table 3. *Summary of advantages and disadvantages of using only the geometrical method todetermine orthometric heights.*

Advantages	Disadvantages
1. The geometric method ensures that the GPS-derived heights are compatible with local orthometric heights and thus absorbs any biases in the survey area;	1. The geometric method is subject to errors in orthometric and GPS heights, which combine additively and propogate through the interpolation process;
2. It can model the geoid accurately through interpolation over small areas by using simple polynomial surfaces.	2. It assumes that the geoid is accurately modelled by a plane or low-order polynomial surface.

THE COMBINED GRAVIMETRIC-GEOMETRIC METHOD

Assuming that the objective is to use GPS to provide orthometric heights that are compatible with benchmarks that define the local vertical datum, a combination of the gravimetric and geometric interpolation approaches can prove superior to either method alone. This is because the geometrically derived geoid heights can account for any local biases in the gravimetric geoid solution with respect to the local benchmarks for each particular survey area. Moreover, it ensures that the GPS-derived orthometric heights are compatible with the local control, which ensures easier integration of GPS heights with the existing orthometric heights. The strategy is outlined in the following scheme:

1) Observe GPS ellipsoidal heights at a minimum of three benchmarks that *surround* the survey area. This provides sufficient control with which to geometrically model the transformation using a plane. However, more than three benchmarks should be observed with GPS in order to provide a check on this method, or to model the transformation using a low-order polynomial surface.

2) Obtain the geoid height from a gravimetric geoid model at all GPS surveyed points. In Australia, this can be achieved using *Winter*, the Internet, or by requesting rigorously computed values from AUSLIG.

3) Algebraically subtract these gravimetric geoid heights from all GPS ellipsoidal heights via equations (1) or (3).

4) Calculate the differences (residuals) between the gravimetric geoid heights and the geometrically derived geoid heights at the benchmarks.

5) Use the geometrical method to determine a surface that models the residuals using the control stations that surround the survey area (ie. use the residuals in equations 4 through 6).

6) Apply the interpolated residuals to all the heights that have been derived from GPS in conjunction with the gravimetric geoid alone, as described in stage 3 (ie. using equations 7 and 8).

7) Use the additional control points for quality assurance and to determine whether this combined method provides improved geoid heights (over the gravimetric geoid alone or geometrical interpolation alone) in the survey area.

The major advantage of this combined method is that any localised biases between the gravimetrically and geometrically derived geoid models are removed such that the GPS-derived orthometric heights are fully compatible with the local vertical control. This essentially combines the advantages of the two methods.

The gravimetric geoid is assumed to provide geoid information of wavelength greater than the terrain and gravity data used in its computation. The geometrical method controls the errors in these wavelengths and can add geoid information that corresponds to the spacing of the control stations, together with a bias, if the GPS station spacing is less than the resolution of the gravimetric geoid.

However, as for the geometrical method alone, this combined approach is always susceptible to errors in the GPS and orthometric heights, which if in error can act to distort the gravimetric geoid heights. This is why it is essential to observe GPS at as many vertical control points as possible to attempt to identify any such errors before they propagate into the combined geoid solution and thus the orthometric heights.

A General Strategy for GPS Height Surveys

Firstly, the accuracy specifications of the GPS height survey must be identified. These dictate, in part, the GPS survey technique and observables that must be used. Also, it is important to bear in mind that the accuracy of the GPS technique is near-proportional to the cost of the GPS equipment and survey logistics involved. Table 4 lists the three main classes of GPS survey technique and the approaches that should be used to model the geoid. This table should be used together with Table 1 which lists the *approximate* accuracy of each GPS survey mode.

Table 4. *A list of the main GPS surveying modes and appropriate methods to determine the geoid with which to recover orthometric heights.*

GPS survey method	Geoid determination method
Single-point code	There is no real need to use geoid heights because the maximum geoid-WGS84-ellipsoid separation is ~ 100 metres, which is less than the error introduced by selective availability (±140m)
Code differential	It is sufficient to use a global geopotential model such as EGM96 **[16]**. In Australia EGM96 alone has been estimated to provide AHD heights to less than 0.5 metre in many cases **[14]**.
Carrier-phase relative (integer fixed solution)	As this is the most accurate GPS survey method, the most accurate geoid modelling method should be used. The options include a gravimetric geoid alone, geometrical interpolation alone, or the combined method. It is strongly recommended that each is tested to determine the optimal approach in each particular survey area.

A regional gravimetric geoid alone, geometrical interpolation alone or the combined method can be used for all modes of GPS survey. There is one additional consideration concerning the horizontal extent of the GPS survey, which implies that one geoid modelling approach may prove superior to another. For example, geometrical interpolation is usually less accurate than the gravimetric method over areas much larger than the resolution of the gravimetric geoid model. However, this assertion can only be ascertained by using additional benchmarks to provide independent checks on the recovery of orthometric heights from GPS by each approach in each survey area.

In Australia, AUSGEOID93 can prove superior to geometrical interpolation over distances greater than approximately 20 kilometres, which is the minimum resolution of the pre-computed AUSGEOID93 grid. Alternatively, the rigorously computed AUSGEOID93 geoid heights can give improved results for GPS baselines greater than ~ 11 kilometres, but the exact resolution is strongly dependent on the resolution of gravity observations used to compute this geoid solution. This is approximately 11 kilometres in most regions, but can decrease to a couple of kilometres in some areas where geophysical exploration has increased the gravity coverage.

As the resolution of AUSGEOID93 is limited by the resolution of the gravity data, geometrical interpolation has the potential to give improved results over GPS baselines less than ~ 11 kilometres in length over many areas. However, this depends on whether the geoid can be accurately modelled by a simple plane or polynomial surface over these distances. Gilliland [11] and Collier and Croft [4] have demonstrated that this is not always the case.

However, the combination approach is acceptable for all scales of survey. The advantage is that the combination method can effectively account for any biases in the gravimetric geoid solution with respect to the vertical control in the survey area and refine any short wavelength geoid undulations that have not been provided due to the lack of gravity data. Therefore, the GPS surveyor can gain the benefits of both the gravimetric geoid and geometrical interpolation when using this combined method. The general strategy for GPS orthometric height determination is as follows:

1) Choose the appropriate GPS survey mode and observables to be used according to the survey tolerance. (Table 1 can *assist* in making this decision.)

2) When using the precise carrier-phase relative GPS method, observe a closed network of independent GPS baselines and ensure that sufficient GPS data are collected to allow a reliable determination of the integer ambiguities, thus providing integer-fixed GPS baseline solutions.

3) Observe and compute GPS ellipsoidal heights at a minimum of three benchmarks that surround the survey area. Also, observe as many additional benchmarks as possible within the survey area so as to allow quality assurance of the geoid modelling approach chosen.

4) Take care with the GPS survey procedures. This includes choosing an observation session with a low VDOP; carefully measuring the GPS antenna

height; orienting all antennae in the same direction to reduce phase centre offsets; choosing baselines as short as possible to reduce atmospheric and satellite orbital effects; observing data for as long as possible to ensure integer fixed solutions; and, avoiding other sources of GPS error such as multipath and electrical interference.

5) Process the GPS data, ensuring that fixed integer ambiguity solutions are achieved. This is important for real-time kinematic GPS methods. Beware, however, because the GPS software can occasionally fix the incorrect integer ambiguities, especially for short station occupations **[26]**. Identify and correct any outliers and re-observe these baselines if necessary, then network adjust the GPS baselines.

6) Apply the co-ordinate transformation to the GPS ellipsoidal heights via any one of:

a) a gravimetric geoid model alone;
b) geometrical interpolation alone;
c) the combination approach proposed herein.

7) Assess the GPS-derived orthometric heights by comparing them to the additional GPS surveyed benchmarks inside the survey area. This is the best form of quality assurance as it gives a near-independent estimate of the accuracy with which orthometric heights are recovered from GPS and the geoid. However, recall that the orthometric and GPS heights used may themselves be in error.

EXAMPLES OF GPS HEIGHT SURVEYS IN WESTERN AUSTRALIA

The GPS surveys described were conducted to provide three-dimensional geodetic control for gravity surveys to be used in geophysical studies **[6]**, **[23]**. The Australian Height Datum (AHD) elevation tolerance is dictated by the vertical gravity gradient and gravimeter sensitivity. The *Sodin* gravimeter used has a reading sensitivity of approximately 0.01mgal, which equates to approximately ± 3 centimetres in orthometric height, and required that relative carrier-phase GPS was used. Two dual-frequency *Trimble 4000SSE Geodetic System Surveyor* GPS receivers were used in all the surveys described.

The GPS baselines were observed in a closed network and code and carrier-phase data were collected at a 15-second epoch for between 10 and 25 minutes depending upon the geometry and number of satellites available and the baseline length. This configuration was expected to be sufficient in order to allow for the detection of outliers and to fix the integer ambiguities to their correct values. If integer float solutions were encountered, these baselines were simply re-observed before the end of the campaign.

All existing benchmarks and standard survey marks of the Western Australian geodetic network that surround and lie within the survey areas were occupied with GPS in order to provide the best possible quality control. (The foresight of the Western Australian surveying authorities has been of great benefit to the GPS surveyor in this part of the world. By providing three-dimensional control at the majority of standard survey marks through spirit levelling to most horizontal geodetic control stations in the State has certainly simplified GPS survey procedures). Minimally, partially and fully constrained network adjustments were used to identify any outliers and the most appropriate control, bearing in mind that the precision of GPS is usually greater than the surveys used to establish the geodetic control in the first place. Those baselines that were considered poor were also re-observed before the end of the campaign.

The GPS ellipsoidal heights were initially transformed to AHD heights using AUSGEOID93 via *Winter*, and the residuals between these and the existing AHD heights computed for all benchmarks in the survey area. Only a small number of benchmarks are available in these remote parts of Western Australia. Also, many of these are situated in locations that are unsuitable for occupation by GPS (eg. under trees, which were used by the original surveyors to provide shade from the sun). This reduces the number of control data that can be used in these analyses. Nevertheless, the total of 24 control points in different locations across the State of Western Australia are sufficient to give a fair indication of the relative merits of each transformation approach.

In all three survey areas, the combined gravimetric-geometric approach produced results that met the survey tolerance. Moreover, the differences between the GPS-derived orthometric heights and spirit-levelled AHD heights were superior to either of the AUSGEOID93 or geometrical approaches when used alone. In the smaller survey area (Tallering), the geometrical-only method was marginally better than the gravimetric-only method for determining the orthometric heights. This is because the 10′ AUSGEOID93 grid is of insufficient resolution to model the short wavelength geoid undulations when compared to the geometrical method. In the larger survey areas (Yallalie and Southern Cross) the opposite was found, which is because the simple plane surface is unable to model the geoid undulations over such an area, whereas they are modelled to a greater extent by AUSGEOID93.

In the Southern Cross survey area, the GPS-derived heights are between 19.7 and 23.0 centimetres below the AHD benchmark heights, which is well outside the 3 centimetre tolerance. This bias caused the Bouguer gravity anomalies to be over-estimated and are thus incompatible with previous gravity surveys in the area [23]. Therefore, the combination approach was essential so that the biased GPS-derived orthometric heights are compatible with the existing AHD control and gravity anomalies in the survey area.

Table 5. *Differences between GPS-dervived and spirit-levelled AHD at existing benchmarks using different geoid modelling procedures for three survey areas in Western Australia.*

Survey area	Dimension (km)	No.	Geoid modelling method	Mean residual(m)	σ residual(m)
Tallering	25 by 40	7	AUSGEOID93 only	0.053	0.059
"	"	"	linear interpolation only	0.002	0.052
"	"	"	combination	0.001	0.047
Yallalie	50 by 50	9	AUSGEOID93 only	0.007	0.096
"	"	"	linear interpolation only	0.002	0.209
"	"	"	combination	0.000	0.011
Southern Cross	50 by 60	8	AUSGEOID93 only	0.213	0.011
"	"	"	linear interpolation only	0.013	0.089
"	"	"	combination	0.000	0.010

Concluding Remarks

There are three approaches to determine the geoid-WGS84-ellipsoid separation that enables the transformation of GPS-derived ellipsoidal heights to orthometric heights. These comprise:

1) A gravimetric geoid, which is usually supplied by government surveying and mapping authorities or universities.

2) Geometrical interpolation, where GPS ellipsoidal heights are observed at benchmarks to determine the geoid at discrete points which are then interpolated using a surface to provide orthometric heights at the intermediate GPS points.

3) A combination of a gravimetric geoid and geometrical interpolation.

Of these three methods, the combination approach is the most robust and allows any localised distortions, systematic errors or datum differences between the gravimetric geoid and the local vertical datum to be modelled in the survey area. However, some caution must be exercised in order to ensure that any errors in the GPS and orthometric heights do not contaminate the transformation model, and thus decrease the accuracy with which subsequent GPS-derived orthometric heights are determined.

This can only be achieved by applying a quality assurance procedure to the GPS-derived orthometric heights by observing GPS at as many benchmarks as possible in and around the survey area. These data allow subsequent checks to be made on the typical accuracy of GPS-derived orthometric heights for each particular survey in each particular area, thus accounting for errors in the gravimetric geoid which are known to exist, but can not be accurately quantified spatially.

Other considerations for the accurate determination of orthometric heights from GPS include the mode of GPS survey and observables used, careful data collection and processing procedures, and network analysis. If all the simple procedures outlined in this paper are followed, then the GPS surveyor should be able to assure that every precaution was taken to determine the best possible orthometric heights during a GPS survey.

ACKNOWLEDGEMENTS

This paper was completed whilst the first author was a Visiting Professor at the Department of Geodesy and Geomatics Engineering, University of New Brunswick, Canada. The hospitality of Professor Petr Vanicek and colleagues is therefore gratefully acknowledged, as is financial assistance from the first author's employer, Curtin University of Technology. The geometrical geoid interpolation used software was kindly supplied by Dr Phil Collier of The University of Melbourne; the gravimetric geoid heights were determined from AUSGEOID93 and the Winter software kindly supplied by Mr Jim Steed of the Australian Surveying and Land Information Group; the *Sodin* gravimeter was provided by the Department of Geology and Geophysics at the University of Western Australia; and, the GPS receivers and processing software were provided by the School of Surveying and Land Information at Curtin University of Technology. We would also like to thank David Rout, Kate Crossing ad Tom Ridsdill-Smith for their assistance with the data acquisition in Southern Cross and Tallering. This paper has resulted from the research projects funded by Australian Research Council grants A49331318, A44391011 and A7348564 to Featherstone and Dentith.

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