Introduction

One of the main targets of oil exploration companies nowadays are carbonate reservoirs because of the huge amounts of oil discovered in carbonates recently. Prediction of petrophysical properties of carbonates is a challenge due to the diversity of these reservoir rocks. In order to investigate main petrophysical trends such as velocity-porosity or velocity-permeability in carbonate rocks, the ultrasonic measurements are undertaken in many laboratories worldwide. However, use of Gassmann's relations to incorporate fluid saturation effects and predict velocities of saturated rock at seismic frequencies is problematic. As was shown in a number of works which reported both increase and decrease of elastic moduli with saturation, Gassmann's relations do not work well on carbonates samples. The softening of elastic moduli is assumed to be caused by chemical transformation of carbonates and/or frame dissolution with saturation. The stiffening of carbonate rock with saturation is shown to be caused by the local flow (squirt) between pores of different shapes and orientations and expected to be captured with Mavko and Jizba high frequency relations.

In this study we analyze the experimental ultrasonic measurements in dry and saturated carbonates where pore geometry is assumed to be mostly responsible for the departure from the Gassmann's relation. However, the measured moduli do not match the predictions of Mavko and Jizba (1991) relations either. This indicates that the measurement frequency is not high enough to inhibit pressure communication between compliant and stiff pores. Here we use a new squirt model recently developed by Gurevich et al. (2009b) in order to derive the moduli of saturated carbonates from "dry" moduli and to predict aspect ratios of compliant pores and attenuation of P- and S-waves.

Theoretical model

Recently, Gurevich et al. (2009a) rederived and generalized Mavko and Jizba relations for the case of arbitrary fluids (and gases) as follows:

$$\frac{1}{K_{uf}(P)} = \frac{1}{K_h} + \frac{1}{\dfrac{1}{\dfrac{1}{K_{dry}(P)} - \dfrac{1}{K_h}} + \dfrac{1}{\left(\dfrac{1}{K_f} - \dfrac{1}{K_g}\right)\phi_c(P)}} \tag{1}$$

$$\frac{1}{\mu_{dry}(P)} - \frac{1}{\mu_{uf}(P)} = \frac{4}{15}\left(\frac{1}{K_{dry}(P)} - \frac{1}{K_{uf}(P)}\right) \tag{2}$$

where $K_{dry}(P)$ and $\mu_{dry}(P)$ are the bulk and shear moduli of the dry rock at a given confining pressure P, K_f and K_g are the bulk moduli of the fluid and the material of the solid grains, $\phi_c(P)$ is the compliant porosity, and $K_h = K_{dry}(P_h)$ is the dry bulk modulus at the highest pressure. The moduli K_{uf} and μ_{uf} have been obtained assuming that the stiff pores are dry, but soft pores filled with the fluid. The saturated undrained moduli can be derived from K_{uf} and μ_{uf} by applying Gassmann's equations to stiff porosity.

Mavko and Jizba relations can be used for calculations of elastic moduli at high frequency limit. In order to obtain the elastic moduli of unrelaxed frame at intermediate frequencies Gurevich et al. (2009b) showed that the complex frequency dependent bulk modulus of fluid

$$K_f^* = \left[1 - 2J_1\left(\frac{k}{\alpha}\right)\bigg/\left(\frac{k}{\alpha}J_0\left(\frac{k}{\alpha}\right)\right)\right]K_f. \tag{3}$$

Here $k^2 = -3i\omega\eta/K_f$, ω is frequency, η is dynamic viscosity and α is pore aspect ratio.

Substitution of K_f^* for the fluid modulus K_f in equation (1) gives the final expression for the partially relaxed (frequency dependent) frame modulus K_{pf}. Then, the corresponding partially relaxed (frequency dependent) shear modulus μ_{pf} can be obtained by substituting K_{pf} for K_{uf} in equation (2). The resulting model is consistent with the Gassmann and Mavko-Jizba equations at low and high frequencies, respectively.

Note that the k parameter is complex and frequency dependent, and so are the effective fluid modulus K_f^* and partially relaxed frame moduli K_{pf} and μ_{pf}. This implies the presence of velocity dispersion and attenuation. The saturated moduli can then be computed using either Gassmann's or Biot's equations. If the frequency is low compared with Biot's characteristic frequency f_c, then the saturated bulk modulus K_{sat} can be obtained by substituting K_{pf} for the frame modulus in Gassmann's equation, while the saturated shear modulus will still be the same, $\mu_{sat} = \mu_{pf}$. If, however, the frequency is comparable with or higher than f_c, then both K_{pf} and μ_{pf} need to be substituted into Biot's dispersion equations to obtain P and S velocities.

Data and Methodology

We analyze V_P and V_S velocities in carbonates measured at pressures from 0 to 60 MPa and frequency of 750 kHz (Agersborg et al., 2008). These data are chosen because ultrasonic velocities of both dry and saturated samples are measured what allow us derivation of all the unknown parameters in equations (1) and (2).
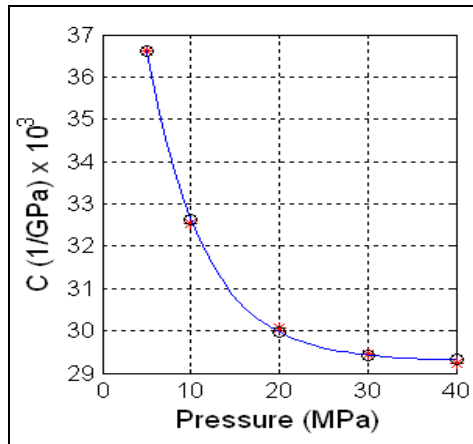

Figure 1. Exponential best fit of compressibility for sample # 5
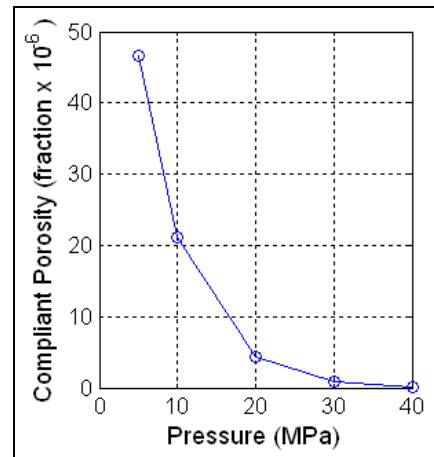

Figure 2. Estimated compliant porosity for sample #5

Our analysis consists of the next three steps: at first, we calculate compliant porosity which is a parameter in equations (1) and (2), then approximate the elastic moduli of saturated carbonates and, finally, estimate the unknown parameters of interest, for instance, aspect ratio and attenuation of P- and S-waves.

1. Compliant porosity ϕ_{c0} at the lowest confining pressure is estimated here from stress dependency of dry rock compressibility (Shapiro, 2003)

$$\Delta C_{dr}(P) = C_{drs}\theta_c \phi_{c0} \exp(-\theta_c C_{drs}P), \tag{4}$$

where $\Delta C_{dr}(P) = C_{dr}(P) - C_{drs}$ is deviation of dry compressibility $C_{dr}(P)$ from its stiff limit C_{drs} and θ_c is so-called stress sensitivity parameter introduced by Shapiro (2003). For this we use (1) calculate the compressibility at different confining pressures (Figure 1) from

the measured ultrasonic velocities of dry the samples, (2) find the exponential function coefficients of the best fits of the compressibility using non-linear Levenberg-Marquadt algorithm and (3) estimate the ϕ_{c0} as a ratio of the coefficient before the exponential to the exponent in equation (4) (for details see Pervukhina et al., 2008). Variation of compliant porosity with stress is calculated as $\phi_c = \phi_{c0} \exp\left(-\theta_c C_{drs} P\right)$ and shown in Figure 2.

2. The obtained compliant porosity as a function of pressure $\phi_c(P)$ are then used as input to the equations (1) and (2) which define generalized Mavko and Jizba model (Gurevich et al 2009a) or the new squirt model of Gurevich et al (2009b) if the equation (3) is used to obtain K_f. In order to compare the unrelaxed frame moduli with the measured moduli, the unrelaxed frame moduli are substituted into Gassmann's equations to incorporate the remaining fluid saturation effects. In the case of the squirt model, aspect ratio α is the only unknown parameter. We use α as a fitting parameter to minimize the discrepancy between the measured and approximated values.

3. Taking into account that the effective fluid modulus K_f^* is a complex module and thus the bulk and shear moduli of unrelaxed frame are complex as well. Attenuation of P- and S- wave is obtained as

$$Q_P^{-1} = -\frac{\mathrm{Im}\left(K + \frac{4}{3}\mu\right)^{1/2}}{\mathrm{Re}\left(K + \frac{4}{3}\mu\right)^{1/2}} \text{ and } Q_S^{-1} = -\frac{\mathrm{Im}(\mu)^{1/2}}{\mathrm{Re}(\mu)^{1/2}}.$$

Results

Measured bulk and shear moduli (black asterisks) in comparison with new squirt model (black dotted line), Mavko-Jizba (red line) and Gassmann (blue line) predictions for two carbonate samples are showed in Figures 3-4. For both samples, we observe a good agreement between measured elastic moduli and estimated using the new squirt model. For sample #3 (Figure 3) the measured values are between the Gassmann prediction and the Mavko-Jizba estimation. Squirt effect is obviously important for this sample and we deal with intermediate frequencies, so for neither high frequency limit (Mavko-Jizba relations) nor low frequency limit (Gassmann's relations) can explain the measurements. On the contrary for sample #4 in Figure 4, the squirt phenomenon is not important and Gassmann's relations can explain the measured data.

Estimated aspect ratios are shown in Figure 5 for all the samples. For four of five samples the aspect ratios are close to 0.01. For sample #4 the aspect ratio is much higher and estimated to be about 0.15. This result is in a good agreement with the fact that the squirt effect is not important for sample #4. Other valuable parameter estimated from the new squirt model is attenuation of P- and S-waves. Figure 6 shows the expected attenuation decay for one of the carbonate samples with pressure increase. It shows that the new squirt model is a valuable tool for the understanding of the wave propagation, including dispersion and attenuation in the carbonate rocks.

References

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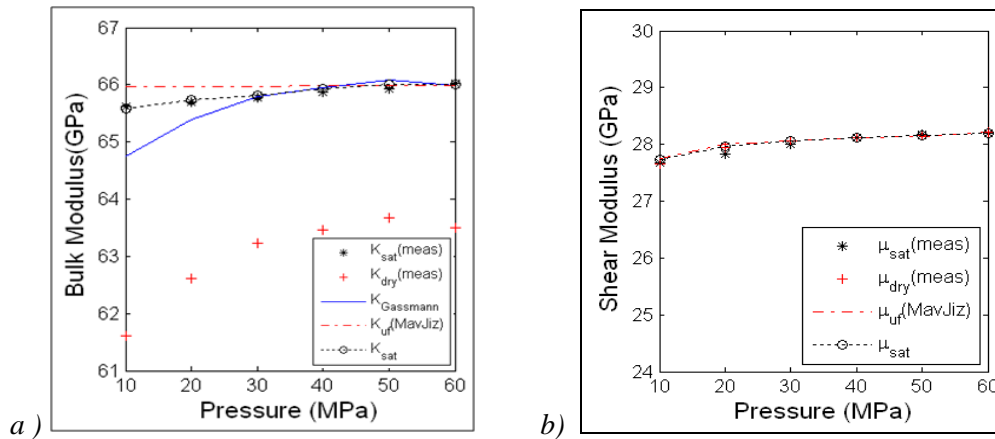Shapiro, S. A. [2003] Elastic piezosensitivity of porous and fractured rocks. Geophysics, 68:482-486.

Figure 3 . Bulk(a) and shear(b) moduli dispersion with pressure in a carbonate sample #3 where the simple estimation using Gassmann is not enough (blue line). The new squirt model can explain much better the behavior at different stress with a good fitting of the curves (dotted black line)
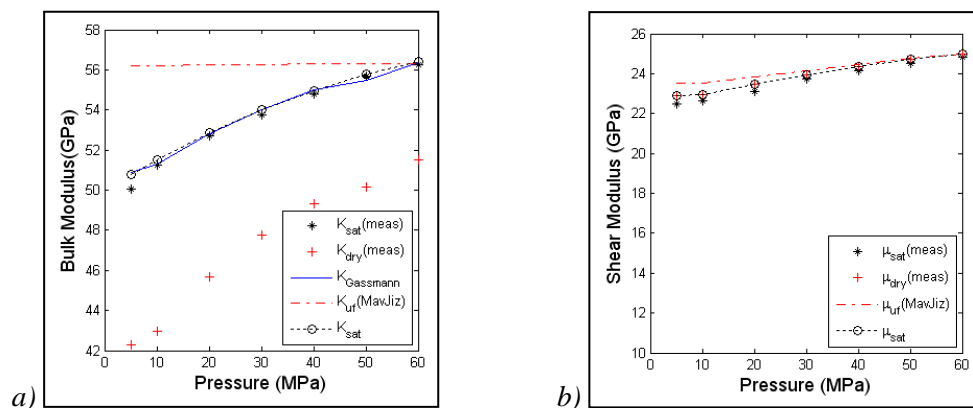

Figure 4. Bulk(a) and shear(b) moduli dispersion with pressure in a carbonate sample #4 where a simple estimation using Gassmann is enough (blue). The squirt model in this case is not needed to explain the dispersion with increasing stress, showing that the squirt fenomena is irrelevant for this sample.
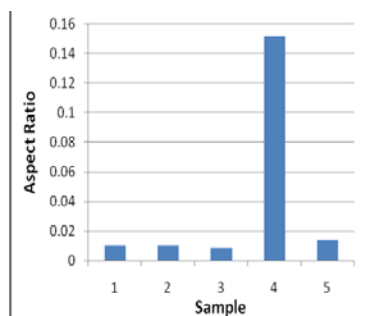

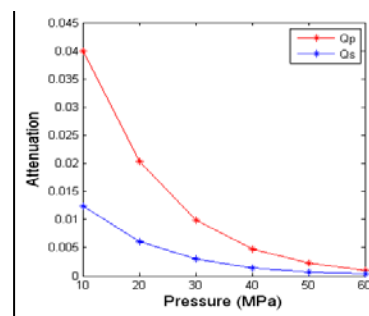*Figure 5. Estimated aspect ratios for some carbonates samples.*

Figure 6. Estimated of attenuation with increasing differential stress.