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Uncertainties in Local Anisotropy Estimation from Multi-offset VSP Data

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SUMMARY

We have quantified the errors associated with VTI parameter estimation using multi-offset VSP data. Two common methods, P-wave slownesses only and slowness-polarization are investigated. Estimation errors are expressed in terms of the magnitude of the earth anisotropy, uncertainties related to first break pickings and maximum available source offset. For homogeneous overburden, P-wave slownesses technique can be used to estimate VTI parameters. We demonstrate that estimation errors of using only P-wave slownesses are significantly decreased as longer source offsets are included in the inversion algorithm. Larger offsets involve P-waves which propagate near horizontal at the receiver level and enhance the method's efficiency. An example synthetic VSP is presented next where P-wave slownesses technique successfully recovers VTI model parameters. In case of heterogeneous overburden, P-wave slowness-polarization technique seems to be a solution as there is no need to compute P-wave horizontal slownesses. However, we demonstrate that the errors of VTI parameter estimation using this technique are small only where the anisotropy is very weak (below 5%) and they are not improved by increasing the offset. Furthermore, wave interference effect on polarizations makes the method impractical even on noise free synthetic data.

Introduction

We estimate VTI parameters from multi-offset VSP data. We use two methods: P-wave slowness technique, and P-wave slowness - polarization technique. The accuracy of the methods is dependent on availability of data from large offsets (this translates to higher angles of wave propagation at the receiver location), errors in estimating slownesses and polarizations (P-wave's first arrival picking and wave interferences effect on polarizations), and magnitude of the anisotropy itself. The aim of this paper is to clarify the influence of each of these factors on the accuracy of the anisotropy estimation.

VTI parameter estimation - P-wave slownesses technique

Miller and Spencer (1994) derived a phase dispersion relation for P or SV waves from Kelvin-Christoffel equation in terms of the horizontal slowness ($p = p_X$), the vertical slowness ($q = p_Z$) and four of the five density normalized elastic coefficients of the VTI medium, a_{11}, a_{13}, a_{33} and a_{55} :

$$
\begin{aligned}
&a_{11}a_{55}p^4 + (-a_{11} - a_{55} + Aq^2)p^2 + a_{33}(a_{55}q^4 - q^2) + 1 - a_{55}q^2 = 0 \\
&A = a_{11}a_{33} + a_{55}^2 - (a_{13} + a_{55})^2
\end{aligned}
\tag{1}
$$

Where an independent measurement of density exists, dispersion relation can be inverted for a given set of slownesses to estimate the elastic stiffnesses of the VTI medium and hence VTI anisotropy parameters δ and ε as defined by Thomsen (1986).

To quantify the errors of P-wave slownesses technique, we generated synthetic slownesses of P-wave using Kelvin-Christoffel equation over a range of anisotropy values common in sedimentary basins (Tsvankin, 2001). Since measurements are always erroneous, we add 1% error to the slowness vector. Then, we use the horizontal and vertical components of this vector in the inversion algorithm. To quantify the effect of offset range, we inspect various ranges for maximum P-wave propagation angle with vertical axis. The inversion algorithm uses equation 1 as forward model and minimizes a least square objective function to estimate the Thomsen (1986) δ and ε anisotropy parameters. Figure 1 shows errors in estimating the Thomsen anisotropy parameters where P-wave phase angle ranges from -30^o to 30^o in subfigures A, B, and C; -45^o to 45^o in subfigures D, E, and F; -80^o to 80^o in subfigures G, H, and I. A total number of 100 realizations is generated and averaged to produce each grid point on the error maps. Figures 1C, 1F and 1I show three example-realizations for the model parameters $\delta = -0.14$ and $\varepsilon = 0.08$ at various P-wave maximum propagation angles. In all the cases, P-wave vertical velocity, S-wave vertical velocity, medium density and anisotropy parameter γ are 3000 m/s, 1500 m/s, 2000 kg/m^3 and 0.07, respectively.

VTI parameter estimation - P-wave slownesses technique, Synthetic Walkaway VSP example

Figure 2 displays the geometry of the earth model where a walkaway VSP survey was acquired using a finite difference wave propagation algorithm. The geometry and layer properties (table 1) are taken from the Naylor field, Otway Basin in Victoria, Australia. Based on the other studies in the Naylor field, a constant VTI anisotropy $\delta = -0.14$ and $\varepsilon = 0.08$ is defined for all the intervals below layer number 3. The VSP survey comprises of 301 source positions at every 20 meters interval on the surface and distributed symmetrically on both sides of the well. 201 three-component receivers are positioned from the surface down to 2 km depth at every 10 meters. Both vertical and horizontal components of the wave-field are recorded at each receiver location. P-wave vertical slowness, *q*, is calculated in shot domain as the gradient of the recorded P-wave first arrivals. As there is not any array of horizontally positioned receivers in the well, lateral homogeneity assumption and Snell's law (horizontal slowness, *p* is conserved with depth along each ray) allow us to estimate *p* at the surface and transfer it to the receiver level. Therefore, the estimates of the horizontal slowness, *p*, are made in the receiver domain as the gradient of the recorded P-wave first arrivals. Figure 3 demonstrates an example of plotting P-wave q versus p for the receiver located at depth 1250 m (circles). Equation 1 is then fitted to these data points (red curve) and δ and ε are estimated. Figure 4 shows the results of the inversion over a large depth interval of the model.

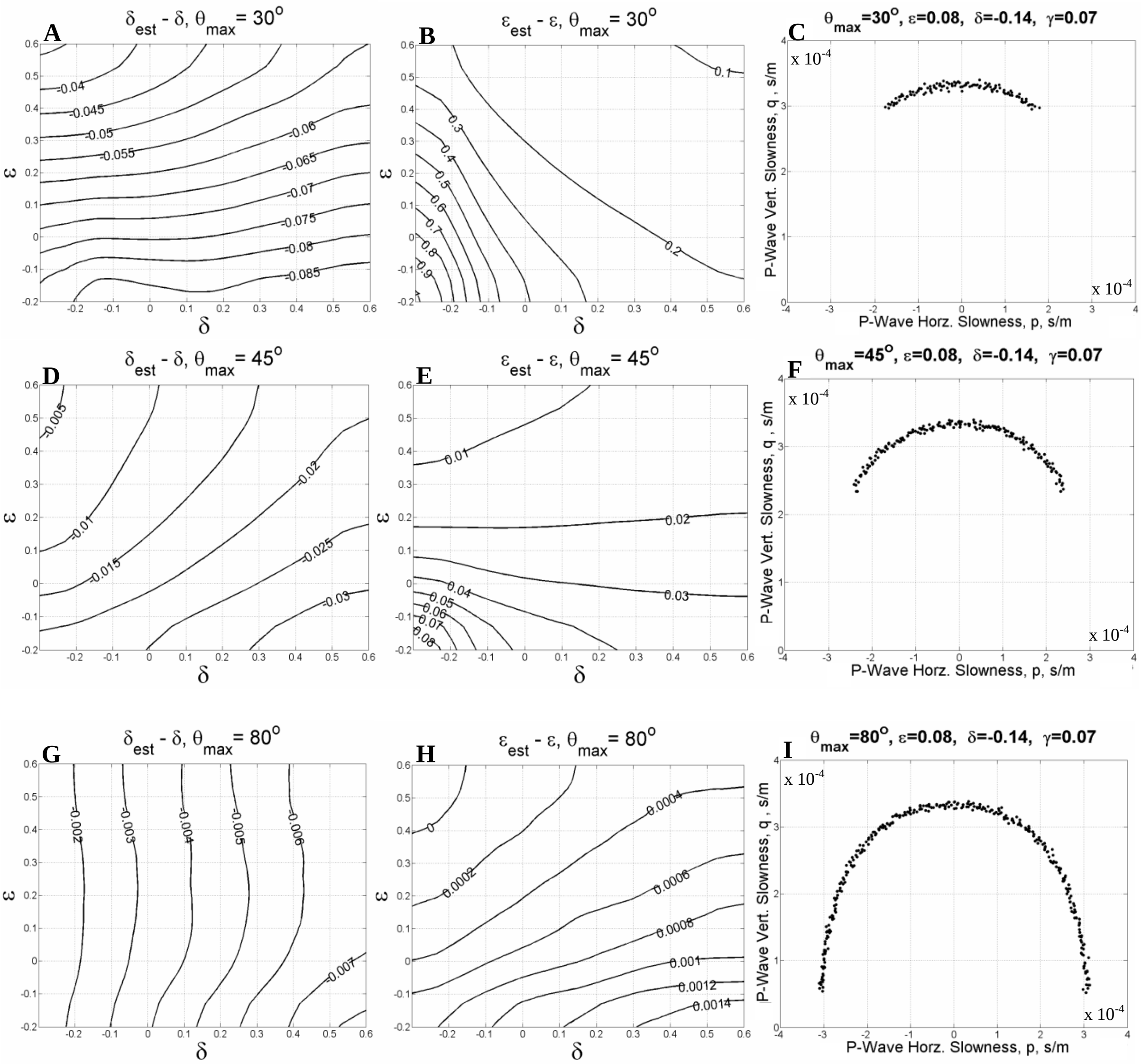

Figure 1 *Left and middle columns are error maps associated with the inversion of synthetic P-wave slownesses. Right column, is an example of P-wave synthetic slownesses generated at point with* δ *and* ε *equal to -0.14 and 0.08, respectively.*

VTI parameter estimation - P-wave slowness - polarization technique

Where there is lateral heterogeneity in the overburden, horizontal slowness measured on the surface cannot be transferred to the receiver level. Grechka and Mateeva (2007) propose an equation for VTI anisotropy parameter estimation which relates P-wave vertical slowness, q to the angle between the P-wave's polarization vector, ψ and the vertical axis:

$$q(\psi) \approx \cos\psi\,(1+\delta_{vsp}\sin^2\psi+\eta_{VSP}\sin^4\psi)\big/V_{P0} \quad \textbf{(2)}$$

Where $\delta_{VSP} = (f_0 - 1)\delta$, $\eta_{VSP} = (2f_0 - 1)\eta$, are newly defined anisotropy parameters for VSP applications, $f_0 = \left(1 - V_{S0}^2 / V_{P0}^2\right)^{-1}$ and $\eta = \dfrac{\varepsilon - \delta}{1 + 2\delta}$ is Alkhalifah-Tsvankin (1995) anellipticity coefficient.

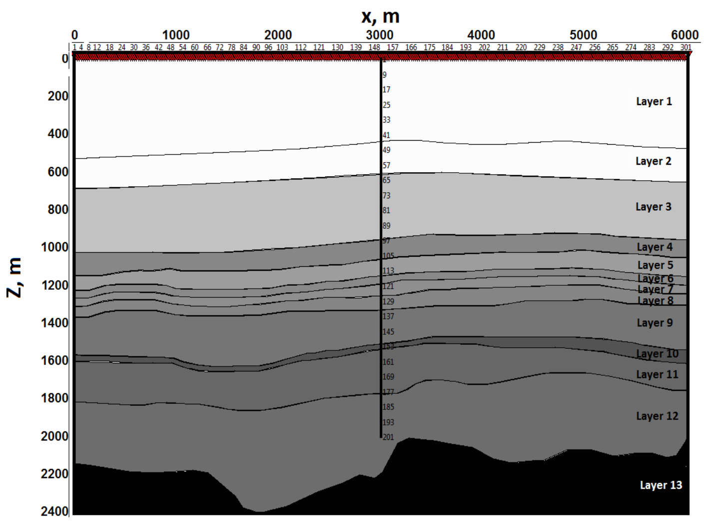

Figure 2 *Geometry of the earth model used to generate synthetic walkaway VSP. Anisotropy starts from layer number four and remains constant with depth.*

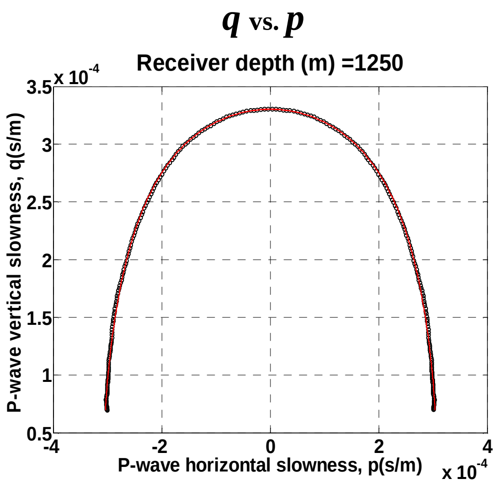

Figure 3 *Vertical component of the slowness vector plotted versus the horizontal component for the receiver located at the depth 1250 m.*

	Layer 1	*Layer 2*	*Layer 3*	*Layer 4*	*Layer 5*	*Layer 6*	*Layer 7*	*Layer 8*	*Layer 9*	*Layer 10*	*Layer 11*	*Layer 12*	*Layer 13*
Vp_0 (m/s)	2329	2329	2689	3051	2915	3040	3007	3063	3164	3375	3251	3214	3878
Vs_0 (m/s)	1089	1089	1279	1596	1514	1595	1618	1657	1735	1801	1779	1635	2287
ρ (kg/m^3)	2141	2141	2212	2287	2215	2260	2258	2282	2309	2322	2420	2358	2491

Table 1 *Layer properties of the geological model displayed on figure 2. Anisotropy starts from layer number 4 and remains constant with depth ($\delta = -0.14$ and $\varepsilon = 0.08$).*

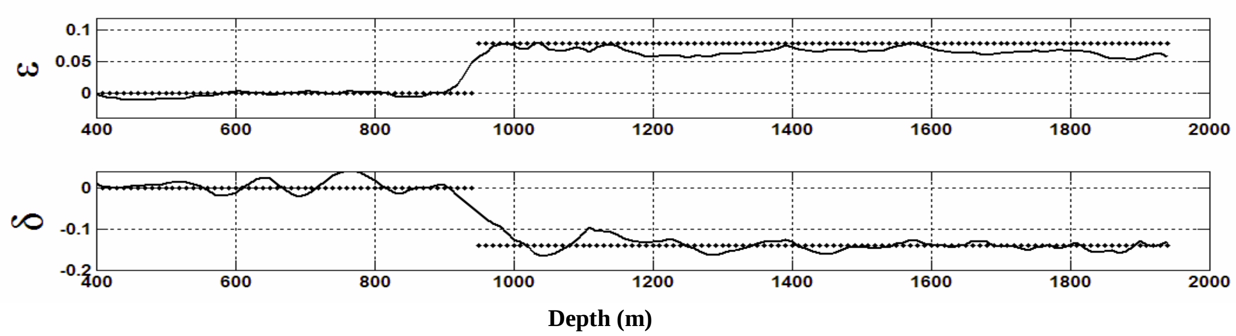

Figure 4 *Estimated Thomsen anisotropy parameters δ and ε (solid curves) versus the values used to build the model $\delta = -0.14$ and $\varepsilon = 0.08$ (dotted curve).*

Similar to the previous method, we produced error maps of VTI anisotropy parameter estimation using equation 2. In order to study the accuracy of the equation, we did not add any errors to the data generated by Kelvin-Christoffel equation. Figure 5 shows the absolute values of the errors in estimating the Thomsen anisotropy parameters δ and ε using Grechka and Mateeva (2007) approximation given by equation 2. P and S wave vertical velocities, density and anisotropy parameter γ are the same as before.

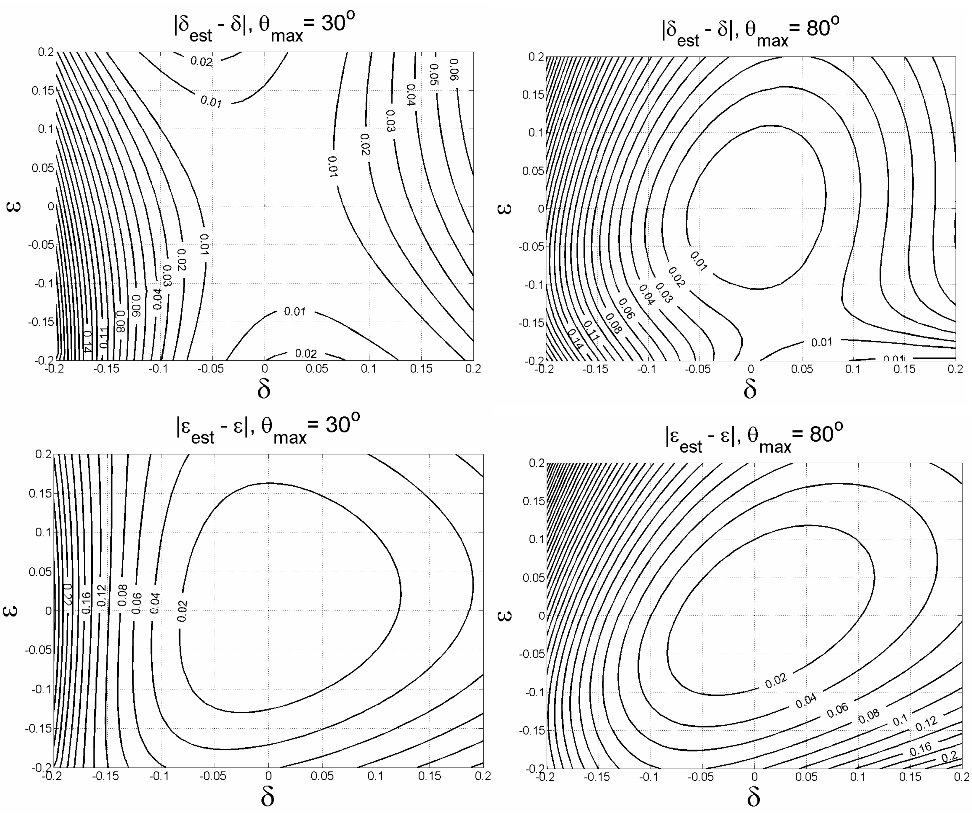

Figure 5 *Absolute errors associated with the inversion of synthetic P-wave's slowness and polarization using equation 2 as forward model.*

Results and conclusions

We studied the accuracy of the two most common VTI parameter estimation techniques based on P-wave measurements. For moderate offset ranges (or $\theta_{MAX} = \pm 45^{o}$), estimation errors based on only P-wave's slownesses are satisfactory (0.5-3% for δ in figure 1D and 1-8% for ε in figure 1E) and for longer offsets (or $\theta_{MAX} \geq 45^{0}$) are very good (almost zero in figure 1G and 1H). Therefore, if the assumption of lateral homogeneity is valid, P-wave slownesses technique is a robust method for VTI parameter estimation. On the other hand, VTI anisotropy estimation using P-wave's slowness-polarization technique as introduced by Grechka and Mateeva's (2007) fails beyond weak anisotropies as small as 5%, and does not improve by involving longer offsets (figure 5). Furthermore, the effect of wave interference on P-wave polarization makes the method impractical even on noise free synthetic data. Taking into account this interference effect for anisotropy estimation techniques that rely on polarization measurements is a part of our future research.

References

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