Analysis of the RLMS Adaptive Beamforming Algorithm Implemented with Finite Precision

Jalal Abdulsayed SRAR[1,2], Kah-Seng CHUNG[2] and Ali MANSOUR[2]
[1]Dept. of Electrical and Electronic Engineering, The 7th October University, Misurata, Libya
[2]Dept. of Electrical and Computer Engineering, Curtin University of Technology, Perth, Australia
jalal.srar@gmail.com, k.chung@curtin.edu.au, and a.mansour@curtin.edu.au

Abstract — **This paper studies the influence of the use of finite wordlength on the operation of the RLMS adaptive beamforming algorithm. The convergence behavior of RLMS, based on the minimum mean square error (MSE), is analyzed for operation with finite precision. Computer simulation results verify that a wordlength of nine bits is sufficient for the RLMS algorithm to achieve performance close to that provided by full precision. The performance measures used include residual MSE, rate of convergence, error vector magnitude (EVM), and beam pattern. Based on all these measures, it is shown that the RLMS algorithm outperforms other earlier algorithms, such as least mean square (LMS), recursive least square (RLS), modified robust variable step size (MRVSS) and constrained stability LMS (CSLMS).**

Keywords-component; RLMS algorithm; array beamforming; fixed-point arithmetic.

I. INTRODUCTION

Spatial division multiple access is gaining popularity as a mean for enhancing greater capacity in wireless communications to meet the ever growing demand for higher data rates and wider coverage without a corresponding increase in spectrum allocation. Adaptive or smart antennas are used to exploit the spatial domain for minimizing interferences. These antennas are required to have fast convergence with low mean square error (MSE), good channel tracking and low complexity.

Various adaptive algorithms, including least mean square (LMS) and recursive least square (RLS), have been introduced for use in adaptive beamforming [1]. The former has good tracking performance with low computational complexity, and is robust against numerical errors. On the other hand, the RLS algorithm can achieve a faster convergence which is independent of the eigen-value spread variations of the input signal correlation matrix [1]. However, LMS algorithm is slow to converge while RLS algorithm suffers from numerical stability problem when implemented with finite precision arithmetic [2, 3] and tracking problem [4].

Lately, several variants of LMS and RLS algorithms have been investigated to enhance the convergence and tracking ability in time varying environments. For example, Affine LMS [5], variable step size LMS (VSSLMS) [6], constrained stability LMS [7] and modified robust variable step size LMS (MRVSS) [8] are based on the use of adaptive step size to improve the convergence speed of LMS algorithm. On the hand, the adaptive forgetting factor RLS algorithm (AFF-RLS) has been proposed for enhancing the tracking ability of RLS algorithm [9].

Recently, a different approach has been adopted to overcome the drawbacks of both the LMS and RLS algorithms. The new RLMS algorithm converges rapidly and is able to track properly in time varying environments [10]. As shown in Fig. 1, the RLMS algorithm consists of an RLS stage followed by an LMS stage via an array image factor ($\mathcal{F}$). It is shown in [4, 10] that the RLMS algorithm converges faster than either the RLS or LMS algorithm operating on its own, whereas this convergence is less sensitive to variations in the input signal to noise ratio (SNR). Furthermore, RLMS can operate with a noisy reference signal. As described in, [11, 12] RLMS can be used to achieve an accurate fixed beam by prior setting the elements of $\mathcal{F}$ with prescribed values for the required direction. On the other hand, the beam direction can also be made adaptive to automatically track the target signal. The algorithm operating in the former mode will from hereon be referred to as RLMS_1, while that in the adaptive mode is called RLMS.

In this paper, we extend our study on the RLMS algorithm [4, 10-12] by analyzing the effect on the convergence behavior due to the use of finite precision arithmetic for its implementation. Some approximations are introduced in order to simplify the analysis. In Section II, expressions are derived for the estimation of the quantization error associated with the overall error signal, e_{RLMS}. A description on the basis for computer simulations is given in Section III, while the simulation results are presented in Section IV. Finally, in Section V, we conclude the paper.

II. CONVERGENCE WITH FINITE PRECISION ARITHMETIC

The RLMS algorithm is depicted in Fig. 1, which shows the overall error signal, e_{RLMS} at the j^{th} iteration is given by

$$e_{\text{RLMS}}(j) = e_{\text{RLS}}(j) - e_{\text{LMS}}(j-1) \tag{1}$$

where $e_{\text{RLS}}(j) = d(j) - \boldsymbol{W}_{\text{RLS}}^{H}(j)\boldsymbol{X}(j)$ is the error signal of the RLS stage, while that of the LMS stage is $e_{\text{LMS}}(j) = d(j) - \boldsymbol{W}_{\text{LMS}}^{H}(j)\boldsymbol{X}_{\text{LMS}}(j)$. $\boldsymbol{X}$ and $\boldsymbol{X}_{\text{LMS}}$ are the input signal vectors of the RLS and LMS stage, respectively. $d(j)$ is the external reference signal.

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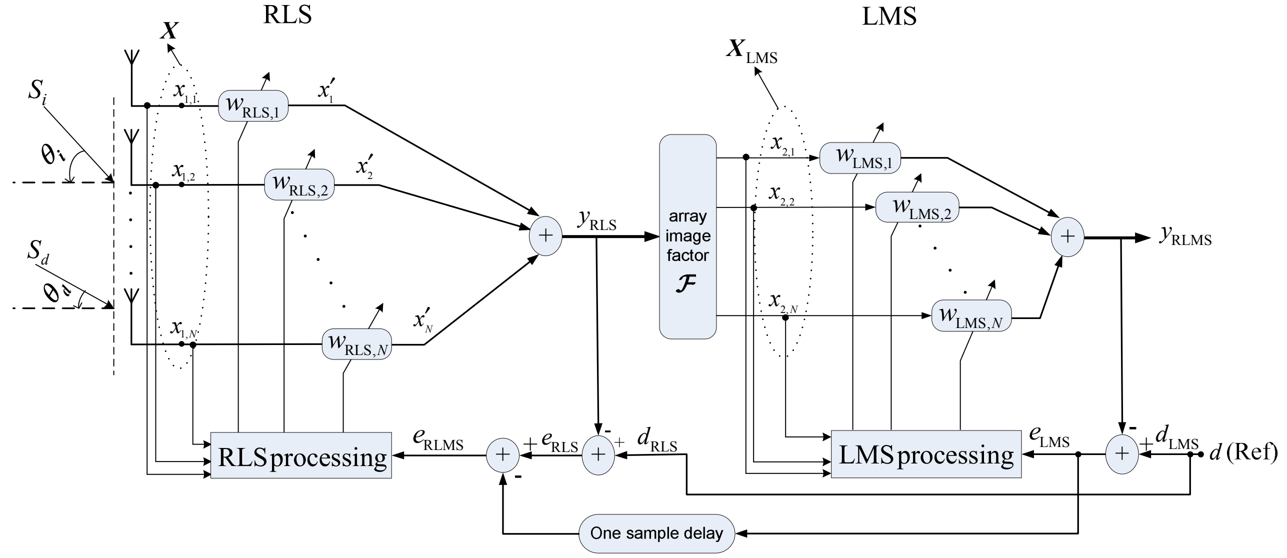


Figure 1. The RLMS algorithm with an external reference signal.

It is shown in [4] that for the case with external referencing, the mean square error $\xi(j)$ of the overall error signal, e_{RLMS}, at the j^{th} iteration is given by

$$\begin{aligned}\xi(j) &= \sum_{i=1}^{j} \lambda^{j-i} E\left[\left|d(i)\right|^2 + \left|d(i-1)\right|^2\right] \\ &+ \boldsymbol{W}_{\text{RLMS}}^{H}(j-1)\boldsymbol{Q}(j-1)\boldsymbol{W}_{\text{RLMS}}(j-1) \\ &+ \boldsymbol{W}_{\text{RLS}}^{H}(j)\boldsymbol{Q}(j)\boldsymbol{W}_{\text{RLS}}(j) - \boldsymbol{W}_{\text{RLS}}^{H}(j)\boldsymbol{Z}(j) \\ &- \boldsymbol{W}_{\text{RLMS}}^{H}(j-1)\boldsymbol{Z}(j-1) - \boldsymbol{Z}^{H}(j)\boldsymbol{W}_{\text{RLS}}(j) \\ &- \boldsymbol{Z}^{H}(j-1)\boldsymbol{W}_{\text{RLMS}}(j-1)\end{aligned} \tag{2}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator; $\boldsymbol{W}_{\text{RLS}}$ and $\boldsymbol{W}_{\text{LMS}}$ are the weight vectors of the RLS and LMS stage, respectively; λ is the forgetting factor of the RLS stage; $(\bullet)^H$ denotes the Hermitian matrix of (•), and

$$\boldsymbol{W}_{\text{RLMS}}^{H} = \boldsymbol{W}_{\text{LMS}}^{H} \mathcal{F} \boldsymbol{W}_{\text{RLS}}^{H}.$$

The correlation matrix of the input signals, $\boldsymbol{Q}$, is defined as

$$\boldsymbol{Q}(j) = \sum_{i=1}^{j} \lambda^{j-i} \mathrm{E}\left[\boldsymbol{X}(j)\boldsymbol{X}^{H}(j)\right],$$

and $\boldsymbol{Z}(j)$ corresponds to the input signal cross-correlation vector given by

$$\boldsymbol{Z}(j) = \sum_{i=1}^{j} \lambda^{j-i} \mathrm{E}\left[\boldsymbol{X}(j)d^{*}(j)\right].$$

As shown in [4], the mean square error $\xi(j)$ converges to a minimum error given by

$$\begin{aligned}\xi_{\min} &= \sum_{i=1}^{j} \lambda^{j-i} E\left[\left|d(i)\right|^2 + \left|d(i-1)\right|^2\right] - \boldsymbol{Z}^{H}(j-1)\boldsymbol{W}_{\text{RLMS}}(j-1) \\ &- \boldsymbol{Z}^{H}(j)\boldsymbol{W}_{\text{opt}_{\text{RLS}}}(j) + \boldsymbol{W}_{\text{RLMS}}^{H}(j-1)\boldsymbol{Z}(j-1)\left\{\mathcal{F}^{H}\boldsymbol{W}_{\text{LMS}}(j-1) - 1\right\}\end{aligned} \tag{3}$$

where the optimum weight vector of the RLS stage is

$$\boldsymbol{W}_{opt_{RLS}}(j) = \boldsymbol{Q}^{-1}(j)\boldsymbol{Z}(j),$$

It is to be noted that the above analysis assumes the followings:

- The propagation environment is time invariant.
- The components of the signal vector $\boldsymbol{X}(j)$ are independent identically distributed (iid).
- All signals are zero mean and statistically stationary at least to the second order.

Now, with the RLMS algorithm being implemented using finite precision arithmetic, the results of the various mathematical calculations will be affected by round-off and truncation errors. The influence of these errors on the operation of the RLMS algorithm is analyzed as follows.

First, the signal terms expressed in finite precision are represented by primed symbols to differentiate them from their corresponding counterparts in full precision. For example, the input signal and weight vectors in finite precision can be expressed as

$$\boldsymbol{X}'(j) = \boldsymbol{X}(j) + \boldsymbol{\alpha}(j) \tag{4}$$

$$\boldsymbol{W}'(j) = \boldsymbol{W}(j) + \boldsymbol{\rho}(j) \tag{5}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\rho}$ are the corresponding quantization error vectors. The elements of $\boldsymbol{\alpha}$ and $\boldsymbol{\rho}$ are assumed to be independent of $\boldsymbol{X}$ and $\boldsymbol{W}$ respectively, and both are white sequences with zero mean and variance of σ_q^2. For a signal of ±1 V amplitude range represented by an N_b–bit wordlength, the resulting variance of the quantization error is given by [13]

$$\sigma_q^2 = 2^{2(1-N_b)}/12 \tag{6}$$

Substitute (4) and (5) in (1), we obtain the overall error signal in finite precision e'_{RLMS} as

$$\begin{aligned} e'_{\mathrm{RLMS}}(j) = d(j) - d(j-1) \\ -\boldsymbol{W}_{\mathrm{RLS}}^H(j)\boldsymbol{X}(j) + \boldsymbol{W}_{\mathrm{LMS}}^H(j-1)\boldsymbol{X}_{\mathrm{LMS}}(j-1) \\ -\boldsymbol{\rho}_{\mathrm{RLS}}^H(j)\boldsymbol{X}(j) + \boldsymbol{\rho}_{\mathrm{LMS}}^H(j-1)\boldsymbol{X}_{\mathrm{LMS}}(j-1) \\ -\boldsymbol{W}_{\mathrm{RLS}}^H(j)\boldsymbol{\alpha}_{\mathrm{RLS}}(j) + \boldsymbol{W}_{\mathrm{LMS}}^H(j-1)\boldsymbol{\alpha}_{\mathrm{LMS}}(j-1) \\ -\eta_{\mathrm{RLS}}(j) + \eta_{\mathrm{LMS}}(j-1) \end{aligned} \tag{7}$$

where η_{RLS} and η_{LMS} are truncation and round-off errors associated with the RLS and LMS stages respectively. Both η_{RLS} and η_{LMS} have approximately the same variance, i.e., $\sigma_\eta^2 = c\sigma_q^2$, where the constant c depends on how the inner product of a vector manipulation is implemented. In our case, the inner product is performed with both the signal and weights vectors quantized. In this case, $c = N$, where N is the number of array elements [13].

Since the first four terms on the right hand side of (7) are the same as those for e_{RLMS}, we can rewrite (7) as

$$\begin{aligned} e'_{\mathrm{RLMS}}(j) = e_{\mathrm{RLMS}}(j) - \boldsymbol{\rho}_{\mathrm{RLS}}^H(j)\boldsymbol{X}(j) + \boldsymbol{\rho}_{\mathrm{LMS}}^H(j-1)\boldsymbol{X}_{\mathrm{LMS}}(j-1) \\ -\boldsymbol{W}_{\mathrm{RLS}}^H(j)\boldsymbol{\alpha}_{\mathrm{RLS}}(j) + \boldsymbol{W}_{\mathrm{LMS}}^H(j-1)\boldsymbol{\alpha}_{\mathrm{LMS}}(j-1) \\ -\eta_{\mathrm{RLS}}(j) + \eta_{\mathrm{LMS}}(j-1) \end{aligned} \tag{8}$$

Rearrange (8), and perform the expectation for both sides yields

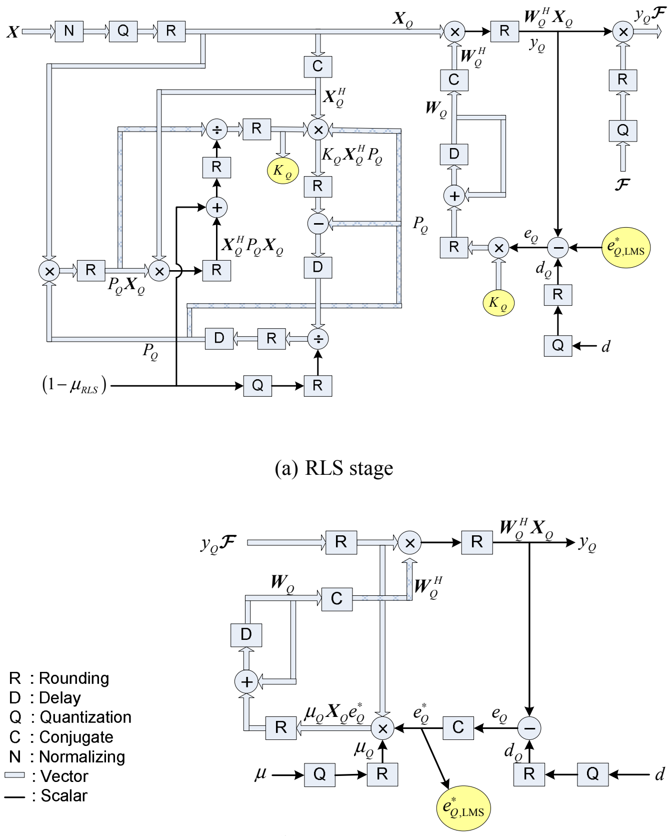
$$\begin{aligned} \mathrm{E}\left[e'_{\mathrm{RLMS}}(j) - e_{\mathrm{RLMS}}(j)\right] = \\ \mathrm{E}\Big[\boldsymbol{\rho}_{\mathrm{LMS}}^H(j-1)\boldsymbol{X}_{\mathrm{LMS}}(j-1) - \boldsymbol{\rho}_{\mathrm{RLS}}^H(j)\boldsymbol{X}(j) \\ -\boldsymbol{W}_{\mathrm{RLS}}^H(j)\boldsymbol{\alpha}_{\mathrm{RLS}}(j) + \boldsymbol{W}_{\mathrm{LMS}}^H(j-1)\boldsymbol{\alpha}_{\mathrm{LMS}}(j-1) \\ -\eta_{\mathrm{RLS}}(j) + \eta_{\mathrm{LMS}}(j-1)\Big] \end{aligned} \tag{9}$$

From (9), it can be observed that those terms related to the LMS algorithm stage will contribute in such a way to decrease the overall error. For an asymptotic behavior of the RLMS algorithm after reaching convergence, we can approximate, that $\boldsymbol{X}_{\mathrm{LMS}} \to \boldsymbol{X}$ and $\boldsymbol{W}_{\mathrm{LMS}} \to \boldsymbol{W}_{\mathrm{RLS}}$. In this case, $\boldsymbol{\rho}$ and $\boldsymbol{\alpha}$ should also reach their asymptotic limits, so that $\boldsymbol{\rho}_{\mathrm{LMS}} \to \boldsymbol{\rho}_{\mathrm{RLS}}$ and $\boldsymbol{\alpha}_{\mathrm{LMS}} \to \boldsymbol{\alpha}_{\mathrm{RLS}}$. If we consider a convergence in probability [14] for η_{RLS} and η_{LMS}, then (9) will ultimately takes on a value equivalent to one quantizing step size.

III. COMPUTER SIMULATION

A. Quantization Model

To simulate the RLMS adaptation process in finite precision arithmetic, the quantization model of Fig. 2 has been adopted. The input signal vector is first normalized to an amplitude range of ±1 so as to make full use of a given word length. Furthermore, the inputs to every arithmetic function, such as multiplication and addition, are expressed with the same specified numerical precision. The result of each arithmetical operation is then rounded to the specified word length.



(a) RLS stage

(b) LMS stage

Figure 2. Quantization model used for evaluating the RLMS algorithm with fixed point arithmetic.

B. Simulated Algorithms

The performance of the RLMS beamforming algorithm has previously been presented in [4, 10-12]. These earlier results are based on the RLMS algorithm being implemented with full precision for operation under various channel conditions, including additive white Gaussian noise, and cochannel interference. In this paper, we examine the influence on the performance of the RLMS algorithm when it is implemented using finite precision arithmetic. Also, we consider the two modes of operation of the RLMS algorithm, namely operating with adaptive array factor $\mathcal{F}$ (refer to as RLMS), and with fixed prescribed $\mathcal{F}$ (refer to as RLMS$_1$).

For comparison purposes, the conventional LMS, RLS, CSLMS and MRVSS algorithms have also been simulated using the same finite numerical precision. Table I tabulates the steps and parameters used in the simulations of the RLMS$_1$, RLMS CSLMS, and MRVSS algorithms. The parameter ε is a small constant which has been adjusted to yield the best possible performance for CSLMS. For the MRVSS algorithm, its step size μ is updated with upper and lower bounds of $\mu_{\max}$ and $\mu_{\min}$, respectively. Also, $\alpha > 0$, $\eta > 1$, $(\gamma, \upsilon) > 0$, and $P(j)$ is the time averaged error correlation over two consecutive values. The time averaged error square signal β has its upper and lower bounds given by $\beta_{\max}$ and $\beta_{\min}$, respectively.

TABLE I. CSLMS, MRVSS AND RLMS ALGORITHMS AS PRESENTED IN [7], [8] AND [11], RESPECTIVELY

RLMS$_1$ Algorithm	$\boldsymbol{K}(j)=\dfrac{\boldsymbol{P}(j-1)\boldsymbol{X}(j)}{1-\mu_{\mathrm{RLS}}+\boldsymbol{X}^H(j)\boldsymbol{P}(j-1)\boldsymbol{X}(j)}$ $\boldsymbol{P}(j)=\dfrac{1}{1-\mu_{\mathrm{RLS}}}\left[\boldsymbol{P}(j-1)-\boldsymbol{K}(j)\boldsymbol{X}^H(j)\boldsymbol{P}(j-1)\right]$ $e_{\mathrm{RLMS}}(j)=d(j)-\boldsymbol{W}^H_{\mathrm{RLS}}(j)\boldsymbol{X}(j)-e_{\mathrm{LMS}}(j-1)$ $\boldsymbol{W}_{\mathrm{RLS}}(j)=\boldsymbol{W}_{\mathrm{RLS}}(j-1)+\boldsymbol{K}(j)e_{\mathrm{RLMS}}(j)$
	$\boldsymbol{X}_{\mathrm{LMS}}(j)=\boldsymbol{\mathcal{F}}\boldsymbol{W}^H_{\mathrm{RLS}}(j)\boldsymbol{X}(j)$
	$e_{\mathrm{LMS}}(j)=d(j)-\boldsymbol{W}^H_{\mathrm{LMS}}(j)\boldsymbol{X}_{\mathrm{LMS}}(j)$ $\boldsymbol{W}_{\mathrm{LMS}}(j+1)=\boldsymbol{W}_{\mathrm{LMS}}(j)+\mu_{\mathrm{LMS}}\boldsymbol{X}_{\mathrm{LMS}}(j)e^*_{\mathrm{LMS}}(j)$
RLMS Algorithm	Same as RLMS$_1$ except $\boldsymbol{\mathcal{F}}$ is updated as $\mathcal{F}_k(j)\simeq\dfrac{x_{\mathrm{LMS}}(j)}{w_{\mathrm{RLS}}(j)y_{\mathrm{RLS}}(j)}$, k is the element of $\boldsymbol{\mathcal{F}}$ index
CSLMS Algorithm	$\boldsymbol{W}(j+1)=\boldsymbol{W}(j)+\dfrac{\mu}{\lVert\delta\boldsymbol{W}(j)\rVert^2+\varepsilon}\delta\boldsymbol{X}(j)\left(\delta e^{[j]}(j)\right)^*$
	$\delta\boldsymbol{W}(j)=\boldsymbol{W}(j)-\boldsymbol{W}(j-1)$, $\delta\boldsymbol{X}(j)=\boldsymbol{X}(j)-\boldsymbol{X}(j-1)$, $\delta e^{[j]}(j)=e^{[j]}(j)-e^{[j]}(j-1)$, and $e^{[k]}(j)=d(j)-\boldsymbol{W}^H(k)\boldsymbol{X}(j)$.
MRVSS Algorithm	$\boldsymbol{W}(j+1)=\boldsymbol{W}(j)+\mu(j)\boldsymbol{X}(j)e(j)$
	$\mu(j+1)=\begin{cases}\mu_{\max}\ ;\ \text{if}\ \mu(j+1)>\mu_{\max}\\ \mu_{\min}\ ;\ \text{if}\ \mu(j+1)<\mu_{\min}\\ \alpha\mu(j)+\gamma P^2(j)\end{cases}$,
	with $P(j+1)=(1-\beta(j))P(j)+\beta(j)e(j)e(j-1)$,
	and $\beta(j+1)=\begin{cases}\beta_{\max}\ ;\ \text{if}\ \beta(j+1)>\beta_{\max}\\ \beta_{\min}\ ;\ \text{if}\ \beta(j+1)<\beta_{\min}\\ \eta\beta(j)+\upsilon P^2(j)\end{cases}$,

C. *Performance Measures*

The effect of finite wordlength on the overall error signal, e'_{RLMS}, of the RLMS algorithm in terms of the mean square error (MSE) is first examined in a noise free condition. This is followed by a comparison on the rate of convergence with other adaptive beamforming schemes, namely LMS, RLS, CSLMS and MRVSS algorithms. Furthermore, the influence of finite wordlength on the fidelity of the received signal is investigated based on the error vector magnitude (EVM), as an accurate measure of any distortion introduced by the adaptive scheme on the received signal at a given signal-to-noise ratio (SNR). It is shown in [15] that EVM is more sensitive to variations in SNR than bit error rate (BER).EVM is defined in [16] as

$$EVM_{\mathrm{RMS}}=\sqrt{\frac{1}{KP_{\mathrm{o}}}\sum_{j=1}^{K}\left|S_r(j)-S_t(j)\right|^2}, \qquad (10)$$

where K is the number of symbols used, $S_r(j)$ is the j^{th} output of the beamformer, and $S_t(j)$ is the j^{th} transmit symbol. P_{o} is the average symbol power for the given modulation.

IV. SIMULATION RESULTS

An adaptive uniform linear array consisting of eight isotropic antenna elements spaced half a wavelength apart is simulated. A desired binary phase shift keyed (BPSK) signal arrives at an angle θ_d of 0°. All weight vectors of a given algorithm are initially set to zero.

Table II shows the values of the various constants adopted for the simulations of the six different adaptive algorithms. The values adopted here for the MRVSS and CSLMS algorithms are chosen to yield good performance under the given conditions.

TABLE II. VALUES OF THE CONSTANTS USED IN SIMULATION

Algorithm	**Noise Free Channel**
RLS	$\mu_{\mathrm{RLS}}=0.05$
LMS	$\mu=0.05$
RLMS$_1$	$\mu_{\mathrm{RLS}}=0.05$, $\mu_{\mathrm{LMS}}=0.2$
RLMS	$\mu_{\mathrm{RLS}}=0.025$, $\mu_{\mathrm{LMS}}=0.3$
CSLMS	$\varepsilon=0.05$, $\mu=0.05$
MRVSS	$\beta_{\max}=1$, $\beta_{\min}=0$, $\upsilon=5\times10^{-4}$, $\mu_{\max}=0.2$, $\mu_{\min}=10^{-4}$ Initial $\mu=\mu_{\max}$, $\alpha=0.97$, $\gamma=4.8\times10^{-4}$, $\eta=0.97$

A. *MSE Performance*

In an attempt to determine the numerical precision required for the implementation of the RLMS algorithm, we consider how its convergence is affected through the use of a different wordlength in a noise free condition. First, the values of MSE of the overall error signal, e'_{RLMS}, obtained with the RLMS and RLMS$_1$ algorithms for a given wordlength have been measured after 300 iterations to ensure complete convergence. These MSE values have been obtained for wordlengths N_b from 6 to 12 bits and plotted in Fig. 3. From the results, it is observed that for N_b equal to or greater than 9 bits, the resultant MSE becomes sufficiently small to be acceptable for both the RLMS and RLMS$_1$ algorithms.

Based on a wordlength of 9 bits, the theoretical overall error signal e'_{RLMS} for RLMS$_1$ has been computed from (7) and plotted in Fig. 4. For comparison, the overall error signal e_{RLMS} computed from (2) with full numerical precision is also plotted in Fig. 4. It shows that the convergence speed of the RLMS$_1$ algorithm achieved with a 9-bit precision is only marginally slower than the version implemented with full precision.

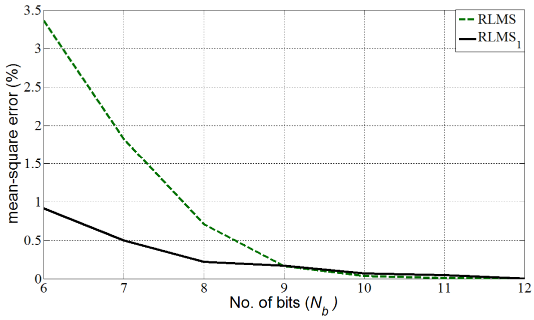


Figure 3. The variations of residual MSE as a function of the wordlength used to implement the RLMS and $RLMS_1$ algorithms in a noise free channel.

Next, the rates of convergence of the RLMS and $RLMS_1$ algorithms have been simulated using a 9-bit wordlength. The resulting curves are plotted as shown in Fig. 5. The simulated results compared well with the theoretical curve presented in Fig. 4 for the $RLMS_1$ algorithm. It can be observed from Fig. 5 that both the RLMS and $RLMS_1$ algorithms converge much quicker than the other four algorithms, which have also been implemented with the same numerical precision.

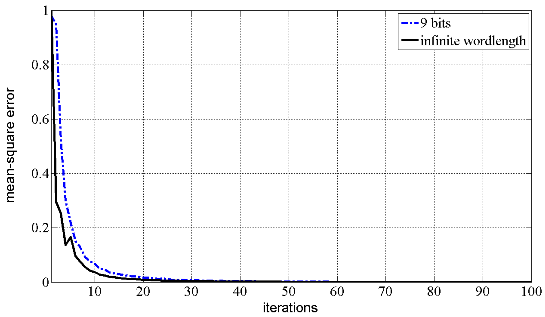


Figure 4. The theoretical values of MSE of the $RLMS_1$ algorithm obtained with full precision and 9-bit precision.

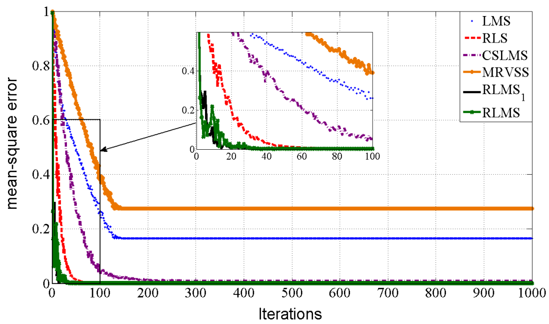


Figure 5. The rates of convergence of the RLMS, $RLMS_1$, CSLMS, MRVSS, RLS and LMS algorithms based on 9-bit precision.

B. EVM and Scatter Plots

The influence of finite precision on the fidelity of the received signal is investigated based on the EVM as expressed in (10). The EVM values are calculated after 512 iterations to make sure that final convergence is achieved for a given algorithm. This has been carried out for $RLMS_1$, RLMS, LMS, RLS, MRVSS and CSLMS algorithms with different precision ranging from 6 to 12 bits. The results are plotted in Fig. 6, which clearly shows that both the proposed RLMS and $RLMS_1$ algorithms are more tolerant to finite precision among the six schemes considered.

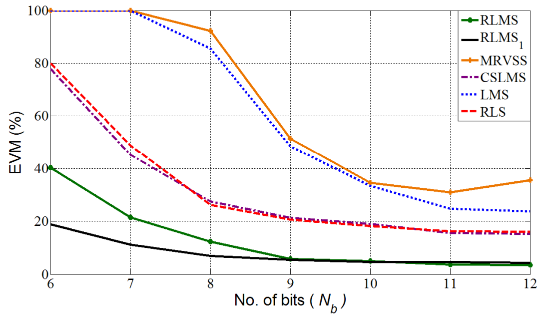
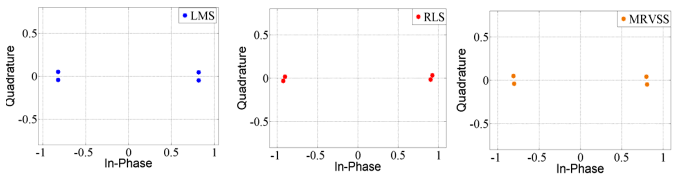
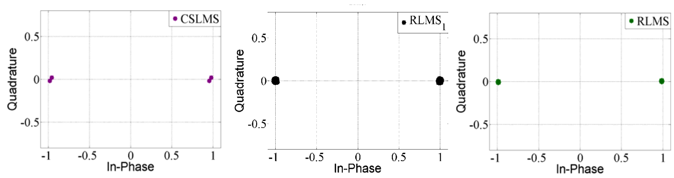


Figure 6. The EVM values of the $RLMS_1$, RLMS, LMS, CSLMS and MRVSS algorithms implemented with different wordlengths.

To demonstrate how well the signal fidelity is retained, the scattered plots of the recovered BPSK signal obtained for the six algorithms, based on a 9-bit implementation, are shown in Fig. 7. Each of these scatter plots is obtained from 2048 signal samples after the convergence of a given algorithm. Ideally, a BPSK signal has only two states, namely, -1 and +1. It is observed that the use of finite precision is causing spreading of these two states. Among the six algorithms considered, the scattered plots of the RLMS and $RLMS_1$ algorithms show the least spreading. This observation is verified by the low values of EVM achieved with these two algorithms.



(a) LMS algorithm (b) RLS algorithm (c) MRVSS algorithm



(d) CSLMS algorithm (e) $RLMS_1$ algorithm (f) RLMS algorithm

Figure 7. The scatter plots of the recovered BPSK signal obtained with all the six algorithms being implemented in 9-bit precision.

C. Beam Pattern Performance

Fig. 6 shows the beam patterns obtained through the use of $RLMS_1$, RLMS, LMS, RLS, CSLMS and MRVSS algorithms implemented with 9-bit accuracy. The desired signal arrives at $\theta_d = 10^\circ$, while the two cochannel interfering signals of equal amplitude as the desired signal are coming from $\theta_i = -30^\circ$ and $\theta_i = 45^\circ$. It is observed that RLMS$_1$, RLMS, RLS, CSLMS algorithms achieve similar gain in the direction of the desired signal. Moreover, both the RLMS and RLMS$_1$ algorithms provide greater rejection to the interfering signals at $\theta_i = -30^\circ$ and $\theta_i = 45^\circ$. This suggests that the use of 9-bit precision is sufficient to maintain the effectiveness of the RLMS and RLMS$_1$ algorithms in rejecting interfering signals emanating outside their mainlobes.

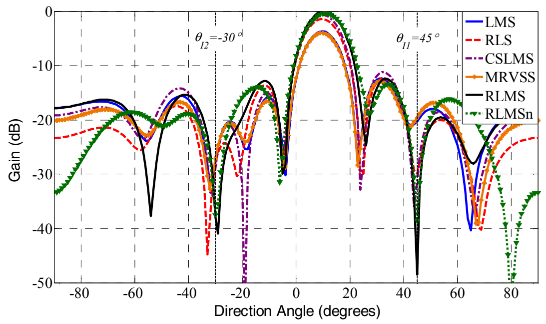


Figure 7. The beam patterns obtained with the LMS, RLS, CSLMS, MRVSS, RLMS$_1$ and RLMS algorithms using a 9-bit wordlength.

V. Conclusion

In this paper, the convergence behavior of the RLMS algorithm, based on the minimum mean square error, is analyzed for operation with finite numerical precision. It is shown that the implementation of an eight element uniform linear array using the RLMS algorithm with a wordlength of nine bits is sufficient to achieve performance close to that provided by full precision. Comparisons based on various performance measures, such as residual MSE, rate of convergence, error vector magnitude, and beam pattern, show that the RLMS and RLMS$_1$ algorithms outperform four other previously published algorithms, namely, least mean square (LMS), recursive least square (RLS), modified robust variable step size (MRVSS) and constrained stability LMS (CSLMS).

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