Exploratory Bi-factor Analysis in Sport, Exercise, and Performance Psychology: A Substantive-Methodological Synergy

Abstract

The first purpose of this study was to provide a brief and general case for the possible utility of the bi-factor model in sport, exercise, and performance (SEP) psychology. The second purpose of this study was to demonstrate how exploratory and confirmatory forms of the bi-factor model may be compared to each other and to more commonly used factor models in SEP psychology within a substantive-methodological synergy format. The substantive focus was the consideration of the bi-factor model for the Psychological Need Thwarting Scale (PNTS). The methodological focus was the bi-factor model which has a general factor, group factors, and a pattern matrix with a bi-factor structure. The synergy was a demonstration of how exploratory (EBFA) and confirmatory bi-factor (CBFA) analysis may be compared to each other and to more commonly used and more restrictive analyses (e.g., correlated first-order factor analysis, a second-order factor analysis), by reanalyzing existing data. A four-factor EBFA on PNTS data produced an approximate bi-factor structure that may offer a viable conceptualization for future research in this area. More broadly, when the underlying measurement theory is consistent with (a) a conceptually broad general factor, as well as (b) conceptually narrower group factors, there may be utility in comparing bi-factor analysis to more commonly used and more restrictive factor analyses in SEP psychology.

Keywords: psychological need thwarting, bi-factor model, target rotation, psychometric testing

Exploratory Bi-factor Analysis in Sport, Exercise, and Performance Psychology: A Substantive-Methodological Synergy

The bi-factor model (Holzinger & Swineford, 1937) has a general factor, group factors, and a pattern (or “loading”) matrix with a bi-factor structure. Theory-based multidimensional scales in sport, exercise, and performance (SEP) psychology often are developed in a way that is consistent with bi-factor structure: a general continuous latent construct along with several more narrowly defined continuous latent sub-domains often are hypothesized (Tenenbaum, Eklund, & Kamata, 2012). Theory-based multidimensional scales in SEP psychology often are fitted to parameterizations of the factor model (e.g., correlated first-order factors model, second-order factor model) that do not specify a bi-factor structure. In such instances there may be a type of misfit between the theory behind the instrument development and the model imposed on the data (from responses to the instrument) that (a) may be alleviated through bi-factor analysis and (b) assessed relative to competing parameterizations of the factor model. Prior to providing a substantive example (i.e., Psychological Needs Thwarting), more general reviews of (a) the role of competing theories/models and (b) the possible utility of the bi-factor model are provided.

The importance of theory in sport psychology research and practice has been advocated since the infancy of the modern era of sport psychology (e.g., Dishman, 1983; Landers, 1983). Today, few scholars in SEP psychology would dispute the value of theory, and especially the testing of competing theories, in their work. Structural equation modeling (SEM) makes explicit the presence of a priori theory through the requirement of model specification (Kline, 2010). As detailed in Bollen (1989), in SEM the latent variable equation describes relationships between latent variables (i.e., a “conceptual” theory) and the measurement equation describes relationships between latent variables and the indicators of latent variables (i.e., a "measurement" theory). Competing theories (e.g., different measurement models) often can be formally tested on a given dataset in SEM (Jöreskog, 1993).

The utility of the bi-factor model, which can be imposed in SEM, has been recently rediscovered in psychology to good effect – including for the purpose of comparing the bi-factor model (and the theory implied by it) to more commonly used models (and the theory implied by those models) by re-analyzing existing datasets (Reise, 2012). We, however, are unaware of a single application of bi-factor analysis in SEP psychology despite the possible fit between the bi-factor model, and, how theory driven multidimensional scales often are developed in SEP psychology. Therefore, the first purpose of this study was to provide a brief and general case for the possible utility of the bi-factor model in SEP psychology. The second purpose of this study was to explore the possible utility of the bi-factor model in SEP psychology by demonstrating how the bi-factor model may be compared to more commonly used models within a substantive-methodological synergy format (Marsh & Hau, 2007). The second purpose of this study can be viewed as a general way to formally conceptualize and test competing measurement theories on a given dataset. Competing theories often differ on the strength of a priori theory which may be conceptualized (though imperfectly) as comparing a weaker theory (e.g., a less restrictive model) based on a model generation (e.g., exploratory) approach to a stronger theory (e.g., a more restrictive model) based on a model dis-confirmatory approach (Jöreskog, 1993).

According to Reise, Moore, and Haviland (2010) there may not be a single reason for why the bi-factor model has been used infrequently in psychology until recently. One practical reason may have been the lack of a "bi-factor" command in some statistical software packages. More substantive explanations may include the conceptualization of a general factor and group factors that are orthogonal (Reise, 2012). Whatever the reason(s) for the infrequent use of the bi-factor model in psychology over the last few decades, it seems clear that the potential utility of both (a) the model itself has been (re-)discovered by an increasing number of scholars in psychology (e.g., Chen, Hayes, Carver, Laurenceau, & Zhang, 2012; Gibbons, Rush, & Immekus, 2009) and (b) comparing the model to other parameterizations of the factor model (e.g., Chen, West, & Sousa, 2006; Reise, 2012).

The Possible Utility of the Bi-factor Model in SEP Psychology

Theory-based scales in SEP psychology often are developed to measure a general continuous latent construct along with several more narrowly defined continuous latent sub-domains (Tenenbaum, Eklund, & Kamata, 2012).[1] The definition of the general construct typically serves to demarcate a sufficiently broad conceptual space. The definition of each sub-domain normally is intended to more precisely define a particular area within the broader conceptual space of the more general construct. Item development characteristically flows from the definition of the general construct and/or the definition of a sub-domain. In either case, given that the sub-domain resides within the broader conceptual space of the more general construct, responses to items may be directly influenced by a general latent construct as well as the relevant latent sub-domain. Stated differently, in the bi-factor model items load on two latent variables (i.e., general construct and the relevant sub-domain).

The common factor model has been closely linked with investigations of construct validity in SEP psychology for several decades (Zhu, 2012).[2] Two commonly observed parameterizations of the confirmatory factor model in SEP psychology are the correlated first-order factors model and/or a higher-order factor (e.g., second-order) model (Myers, Ahn, & Jin, 2011). A second-order factor model, where a higher-order factor exerts direct effects on the first-order factors (but not on the items) is a common parameterization of a higher-order factor model (see panel a of Figure 1 for an example).[3] The correlated first-order factors model typically does not specify a general factor (see panel b of Figure 1 for an example); though the possible presence of a general factor often is provided as a post-hoc conceptual explanation for why the first-order factors co-vary. Thus, neither of the two commonly observed parameterizations of the factor model in SEP psychology specify that at least some items may be directly influenced by a general factor as well as a more narrowly defined factor based on a particular sub-domain. The bi-factor model may rectify this short-coming by requiring researchers to more carefully consider if theory stipulates that both general and sub-domain factors influence responses to items in a confirmatory sense. Alternatively, if theory fails to inform such decisions then bi-factor models can prompt researchers to refine theory be requiring researchers to test potential paths linking general and sub-domain factors to items in an exploratory fashion.

The bi-factor model (Holzinger & Swineford, 1937) has a general factor, group factors, and a pattern (or “loading”) matrix with a bi-factor structure (see panel c in Figure 1 for an example). The general factor is analogous to the “general” factor, and the group factors are analogous to “sub-domains” noted in earlier paragraphs. A pattern matrix with bi-factor structure was originally described as having freely estimated parameters in the first column (i.e., general factor) and, a freely estimated parameter in an additional column (i.e., group factors) for each row (e.g., item).[4] The general factor and the group factors were all orthogonal. Fitting the bi-factor model to data was an early form of a confirmatory factor analysis (CFA), and for this reason, the modern approach to fitting this model will be referred to as confirmatory bi-factor analysis (CBFA) from this point forward. In CBFA researchers are required to specify an explicit bi-factor structure a priori based on substantive measurement theory.

Exploratory bi-factor analysis (EBFA; Jennrich & Bentler, 2011) is exploratory factor analysis (EFA) with a bi-factor rotation criterion (e.g., bi-quartimin). The requirement for a priori specification of an explicit bi-factor structure in CBFA is relaxed in EBFA (see panel d in Figure 1 for an example). Key components of the bi-factor model (a general factor, group factors, and a pattern matrix with a bi-factor structure) are largely unchanged in EBFA. The definition of a pattern matrix with bi-factor structure was, however, expanded slightly by Jennrich and Bentler. The expanded definition was adopted in this study and is as follows: A pattern matrix with bi-factor structure has freely estimated parameters in the first column (i.e., general factor) and, *at most*, a freely estimated parameter in an additional column (i.e., group factors) for each row. Thus, an item can meaningfully load on only the general factor under this expanded definition. The general factor and the group factors were all orthogonal in Jennrich and Bentler but the group factors were free to correlate with each other but not with the general factor in Jennrich and Bentler (2012).[5]

The bi-factor model may make an important contribution to SEP psychology because many theory-driven multidimensional scales developed in SEP are often more theoretically consistent with a bi-factor structure than with structures imposed by more commonly used factor models. An example of possible utility of the bi-factor model may exist with multidimensional self-efficacy instruments where both a general self-efficacy and more domain-specific self-efficacies often are of simultaneous interest (e.g., Humphries, Hebert, Daigle, & Martin, 2012). Further, as noted earlier, recent methodological developments that allow EBFA in addition to CBFA provide flexibility to accommodate the incomplete (e.g., unsure if an item truly cross-loads on an unintended factor) substantive measurement theory that is often observed in SEP psychology (e.g., Myers, Chase, Pierce, & Martin, 2011). The usefulness of the bi-factor model in SEP psychology may be more persuasive to many researchers in SEP psychology if relevant empirically-based examples illustrate the potential value of CBFA and/or EBFA.

The second purpose of this study was to explore the possible utility of the bi-factor model in SEP psychology by demonstrating how the bi-factor model may be compared to more commonly used models within a substantive-methodological synergy format (Marsh & Hau, 2007). The substantive focus was considering the possible utility of the bi-factor model for responses to the Psychological Need Thwarting Scale (Bartholomew, Ntoumanis, Ryan, & Thøgersen-Ntoumani, 2011). The methodological focus was to outline how exploratory and confirmatory forms of the bi-factor model may be compared to each other and to more commonly used factor models in SEP psychology. The synergy was a demonstration of how EBFA and CBFA may be compared to each other and to more commonly used and more restrictive analyses by reanalyzing existing data.

The Substance

Psychological Need Thwarting

A sub-theory within self-determination theory (SDT; Deci & Ryan, 1985) is basic psychological needs theory. Basic psychological needs theory (BPNT; Deci & Ryan, 2000) posits that individuals function and develop most effectively as a consequence of social environmental supports for the satisfaction of their basic psychological needs (i.e., autonomy, competence, and relatedness). Need satisfaction represents a bright side of human existence within BPNT (Ryan & Deci, 2000). Within the sport context, satisfaction of basic needs has been linked to theoretically-relevant variables such as well-being and motivation (e.g., Adie, Duda, & Ntoumanis, 2008; Reinboth, Duda, & Ntoumanis, 2004).[6]

Psychological need thwarting represents a dark side of human existence in BPNT (Ryan & Deci, 2000). This dark side of human existence can be characterized by an individual adopting sub-optimal patterns of behavior, goals, and affect (e.g., Ryan, Deci, Grolnick, & La Guardia, 2006). While the potential negative outcomes of need thwarting are intuitive (e.g., ill-being; Adie et al., 2008), systematic investigation of this phenomenon has, until recently, been hampered by measurement limitations (Vallerand, Pelletier, & Koestner, 2008). Simply, low need satisfaction may not be (but at times has been treated as) a sufficiently valid indicator of high need thwarting (Bartholomew et al., 2011).

The Psychological Need Thwarting Scale (PNTS; Bartholomew et al., 2011) is "…a SDT-based multidimensional questionnaire designed to tap the negative experiential state that occurs when athletes' perceive their psychological needs to be actively undermined in the sport environment" (p. 79). The multi-study report within which the PNTS was developed provides an example of how theory driven multidimensional scales in SEP psychology often are developed. Study 1 "generated a pool of items designed to tap the negative experiential state that occurs when athletes perceive their needs for autonomy, competence, and relatedness to be actively undermined" (p. 75). Notice that in the previous two quotations there seems to be reference to both a general construct (i.e., psychological need thwarting) and to more specific sub-domains (i.e., autonomy, competence, and relatedness).

The multi-study report by Bartholomew et al. (2011) within which the PNTS was developed also provides an example of two commonly observed parameterizations of the factor model in SEP psychology. Study 2 imposed a correlated first-order confirmatory factors model on the 12 accepted items. The factors were labeled autonomy, competence, and relatedness and the bivariate correlations between these factors ranged from .52 (autonomy and relatedness) to

.85 (competence and relatedness). Study 3 initially imposed the same correlated first-order confirmatory factors model as in Study 2 but subsequently imposed a higher-order factor model where a second-order factor, psychological need thwarting, exerted direct effects on the first-order factors only. A general psychological needs thwarting factor was present in the second-order model but it was restricted to have only indirect effects on the items through autonomy, competence, and relatedness (see panel a in Figure 1). A general psychological needs thwarting factor was absent (as is common under an independent cluster model, ICM, CFA approach) in the correlated first-order factors model (see panel b in Figure 1).

In Bartholomew et al. (2011) the transition from the correlated first-order factors models to the second-order factors model occurred when switching from a focus on factorial validity to a focus on predictive validity with regard to the PNTS. A rationale provided for the second-order model was "such a model would be particularly useful for researchers who are interested in obtaining an overall measure of need thwarting (e.g., when such a measure is used in complex SEM) and is justifiable from a theoretical perspective…" (Bartholomew et al., 2011, p. 96). A rationale for when the correlated first-order factor model may be preferred followed the full sentence from which the previous quote was extracted: "On the other hand, if researchers are interested in examining whether the thwarting of specific needs predict specific outcomes, we would recommend the use of the three-factor model…" (p. 97). There may be instances when a researcher is interested in the predictive validity of both a general psychological needs thwarting factor as well as the group factors proposed by the PNTS (autonomy, competence, and relatedness) and the bi-factor model provides a direct parameterization of the measurement part of this fuller model within either a confirmatory (see panel c in Figure 1) or an exploratory (see panel d in Figure 1) framework. Common factor models currently employed in SEP psychology do not support such a conceptualization and can be viewed as competing theories and compared to the bi-factor model as summarized in the subsequent Methodology section.

In the Bartholomew et al. (2011) study the transition from the correlated first-order factors models to the second-order factor model demonstrates a very common approach in SEP psychology that may be improved upon in future research under a bi-factor approach. For instance, had an exploratory bi-factor model been imposed in Study 2, an initial factorial investigation of autonomy, competence, and relatedness as well as general psychological need thwarting would have been possible instead of omitting general psychological need thwarting from the factor model. The results from the exploratory bi-factor model in Study 2 could have then informed a confirmatory bi-factor model in Study 3 instead of switching from the correlated first-order factors models to the second-order factor model. Had a bi-factor model been imposed in Study 3 of Bartholomew et al. the relation of autonomy, competence, and relatedness, with theoretically relevant variables (i.e., need satisfaction, subjective vitality, and exhaustion) could have also been explored (instead of only relations between theoretically relevant variables and psychological need thwarting). Beyond the approach taken in the Bartholomew et al. (2011) study, it is important to note that it is generally desirable that the accepted measurement model from the most relevant validity study is held constant in subsequent predictive validity studies (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999) and the utilization of the bi-factor model may allow such consistency to appear more frequently.

The Methodology

A brief and general case for the possible utility of the bi-factor model in SEP psychology was provided in a previous section. The narrow focus of the current section was to outline how, in general and under a competing theories (i.e., manifest as statistical models) approach (Jöreskog, 1993), exploratory and confirmatory forms of the bi-factor model may be compared to each other and to more commonly used factor models in SEP psychology with examples from the PNTS (see Figure 1).[7] Clearly there are instances when a substantive theory cannot be fully represented in a statistical model, but in this manuscript, competing theories are manifest as competing statistical models. Theoretical and empirical information should be considered when comparing models (e.g., Morin & Maïano, 2011). The current section, however, focused more on structural relationships between models and less on the veracity of conceptual arguments.

The exploratory form of the bi-factor model was put forth because the complete a priori knowledge that is required under a confirmatory bi-factor model (for separating items into groups) often is incomplete in practice (Jennrich & Bentler, 2011). The difference between how the group factors were specified in panel d as compared to panel c illustrated this point (see Figure 1). The exploratory bi-factor model allowed all three group factors - autonomy, competence, and relatedness - to directly influence each of the 12 items. The confirmatory bi-factor model allowed each group factor to directly influence only those items that were intended to indicate a particular group factor (e.g., aut_1 loads on autonomy only). Thus, the confirmatory model imposed a pattern matrix with bi-factor structure whereas the exploratory model allowed a pattern matrix with bi-factor structure to emerge but did not force it to emerge. The model depicted in panel c, as compared to the model depicted in panel d, better illustrated the a priori theory because it included only those pattern coefficients that were intended. Stated differently and from a bi-factor perspective, the theory depicted in panel c is more consistent with how PNTS items were developed (e.g., aut_1 loads on autonomy and psychological needs thwarting only) but also is more restrictive than the theory depicted in panel d (e.g., does not force aut_1 to have zero loadings on competence and relatedness).

The model depicted in panel c was nested within the model depicted in panel d (i.e., it estimated 18 more unrotated pattern coefficients), and therefore cannot empirically fit data better. Stated differently, the measurement theory depicted in panel c is a more parsimonious version (where the additional restrictions can be viewed as imposing a stronger a priori theory and expending fewer degrees of freedom) of the measurement theory depicted in panel d (where the fewer restrictions can be viewed as imposing a weaker a priori theory and expending more degrees of freedom). An advantage of the theory depicted in panel c is greater fidelity to the a priori measurement theory, while a potential weakness of the theory depicted in panel c is placing too many restrictions on the data. An advantage of the theory depicted in panel d is the flexibility to accommodate unintended cross-loadings, while a potential weakness of the theory depicted in panel d is less fidelity to the a priori measurement theory.

The nesting of the model depicted in panel c within the model depicted in panel d relied upon a general relationship between EFA and CFA. Recall that exploratory bi-factor analysis is EFA with a bi-factor rotation criterion (Jennrich & Bentler, 2011). Because rotation occurs after estimation of the unrotated model, the model depicted in panel d was mathematically equivalent to an EFA where the number of factors, m, = 4. The confirmatory form of the bi-factor model depicted in panel c was equivalent to a particular CFA with $m = 4$. Exploratory structural equation modeling (ESEM; Asparouhov & Muthén, 2009) has recently been put forth as a way to place EFA within the broader structural equation modeling (SEM) framework in SEP psychology (e.g., Gucciardi, Hanton, & Mallett, 2012). A convenience of imposing EFA within the ESEM framework is the ability to easily compare a particular EFA to a more constrained version of that model imposed within the CFA framework. This convenience was adopted in the current study to allow for comparison of an exploratory bi-factor model (e.g., panel d) to a more constrained confirmatory version of this model (e.g., see panel c).

The correlated first-order factors model depicted in panel b was nested within the confirmatory form of the bi-factor model depicted in panel c (i.e., it estimated 12 more pattern coefficients and 3 fewer correlations) and therefore cannot empirically fit data better (e.g., Rindskopf & Rose, 1988). To assist in conceptualizing the nesting of these models, we let the correlated first-order factors model be renamed the correlated group factors - autonomy, competence, and relatedness - model. Notice the absence of a general psychological need thwarting factor, and the presence of three correlations between the group factors, in the correlated group factors model. Notice the presence of a general psychological need thwarting factor and its many pattern coefficients, and the absence of correlations between the group factors, in the confirmatory bi-factor model. The confirmatory bi-factor model, then, essentially replaced the three correlations between the group factors with a general factor and its 12 pattern coefficients. From a conceptual perspective, in the confirmatory bi-factor model the group factors - autonomy, competence, and relatedness - were hypothesized as unrelated to each other after removing the effect of a general psychological need thwarting from each PNTS item.

The second-order factor model depicted in panel a was mathematically equivalent to the correlated first-order factors model depicted in panel b. In general, however, when the number of first-order factors exceeds three the second-order factor model is nested within the correlated first-order factors model (e.g., Rindskopf & Rose, 1988). To assist in conceptualizing the relationship between these two models, let the correlated first-order factors model once again be renamed the correlated group factors model. Notice again the absence of a general psychological need thwarting factor, and the presence of three correlations between autonomy, competence, and relatedness, in the correlated group factors model. Notice the presence of a general psychological need thwarting factor and its pattern coefficients on the first-order factors only, and the absence of residual correlations between the group factors, in the second-order factor model. The second-order factor model, then, essentially replaced the three correlations between the group factors (i.e., the model imposed in Study 2 of Bartholomew et al., 2011) with a second-order general factor and its three pattern coefficients (i.e., the model imposed in Study 3 of Bartholomew et al., 2011). From a conceptual perspective, in the second-order factor model the first-order factors -autonomy, competence, and relatedness - were hypothesized as unrelated to each other after removing a second-order psychological need thwarting factor from each of the first-order factors. Both of these models can be viewed as competing theories in relation to the theory implied by CBFA (i.e., panel c) and/or the theory implied by EBFA (i.e., panel d).

The Synergy

A conceptual case for the possible utility of the bi-factor model for the PNTS was provided in a previous section. The purpose of this section was to demonstrate how EBFA and CBFA may be compared to each other and to more commonly used and more restrictive analyses, by reanalyzing existing data from the PNTS. Data were fit to increasingly restrictive models as presented in the methodology section. Consistent with the publication manual of the American Psychological Association (2010), the results can be viewed as original research derived from “secondary analyses that test hypotheses by presenting novel analyses of data not considered or addressed in previous reports” (p.10). An advantage of secondary analyses is that the idiosyncrasies within the dataset are constant across the novel and former analyses.

Consistent with a competing theories approach on a given dataset, however, replication of the subsequent findings with new datasets would also be of value (Jöreskog, 1993).

The least restrictive analysis was EBFA. Given that EBFA is EFA with a bi-factor rotation criterion a few issues related to EFA arose. The first EFA-related issue was: how many factors were warranted to explain responses to the PNTS? The approach taken in this study has been referred to as "comparing_*m*s": "a series of nested model comparisons manipulating the number of factors, *m*, while considering the interpretability of the solution with regard to a priori theory, and, possible empirical limitations of the nested model comparisons…" (Myers, 2013, p. 716-717). The approach taken was considered reasonable given that the PNTS is a relatively new instrument and that a strictly confirmatory approach (e.g., specifying a fixed number of factors based on theory alone) may be too restrictive. Further, EBFA was not readily available during initial data analysis for the parent study (Bartholomew et al., 2011) on which the secondary analyses focused on.

The second EFA-related issue was selection of a rotation criterion. The authors of this manuscript are aware of three rotation criterion that have been used in EBFA: bi-quartimin (Jennrich & Bentler, 2011), bi-geomin (Jennrich & Bentler, 2012), and target (Reise, Moore, & Maydeu-Olivares, 2011). In each case the rotation criterion can be orthogonal or oblique (but in the oblique case the general factor remains orthogonal to the group factors). For a given model, model-data fit was equivalent under each rotation criterion because rotation occurs after the model is estimated. Thus, results from various rotation criteria were considered with regard to a priori measurement theory. Target rotation fully specified the target matrix (available by request) for the group factors in accord with a priori theory (e.g., aut_1 was targeted to have a pattern coefficient = 0 on competence and relatedness and = .50 on autonomy) because simulation research suggests that EFA factors may be defined more consistent with a well-developed a priori theory with an increasing number of targets (Myers, Ahn, & Jin, 2013).

Models were fit in M*plus* (Muthén & Muthén, 1998-2012) under maximum-likelihood estimation with a correction for non-normality so that standard errors for parameter estimates and the test statistic for exact fit could be appropriately adjusted.[8] Nested models were compared with the change in the likelihood ratio χ^2 (robust) test, $\Delta\chi^2_R$. The approach taken to determine *m* was susceptible to over-factoring and inflated Type I error (Hayashi, Bentler, & Yuan, 2007); therefore α was set to .01 for these comparisons (α = .05 otherwise). Consistent with weaknesses of the $\Delta\chi^2_R$ with real data and imperfect theories (Yuan & Bentler, 2004), and the utility of strict adherence to the null hypothesis testing framework with regard to the assessment of model-data fit in general (Marsh, Hau, & Wen, 2004), a set of guidelines were also used to judge the magnitude of change in model-data fit for nested models (e.g., $CFI_{simple} - CFI_{complex} = \Delta CFI$). Consistent with Marsh et al. (2010), ΔCFI ≤ -.01, ΔTLI ≤ .00, and ΔRMSEA ≥ .015, was interpreted as evidence in favor of the more complex model. Heuristic classifications model-data fit (e.g., likelihood-based “exact” fit test, RMSEA-based “close” fit test, etc.) were relatively consistent both with Hu and Bentler (1999) and with cautions against overreliance on model-data fit indices at the expense of substantive considerations (Kline, 2010; Marsh, Hau, & Wen, 2004).

Effect size was considered in two ways after a model was accepted. First, an approach from CBFA for determining the percentage of common variance in the items accounted for by the group factor and the set of general factors (e.g., Reise, 2012) was applied to EBFA. Second, a pattern coefficient had to meet two criteria to be determined as meaningfully large: (a) statistically significant and (b) the absolute value of the standardized coefficient ≥ .20. The latter criterion is, of course, somewhat arbitrary but was consistent with Jennrich and Bentler (2012).

PNTS

Data from Study 2 (N = 354) and Study 3 (N = 289) in Bartholomew et al. (2011) were merged (N = 643) and re-analyzed. The merging of these two datasets was done to maximize sample size and was consistent with an assumption in the parent study: that these two samples were drawn from the same population. Participant age ranged from 12-17 years. Data were collected from females (n = 276) and males (n = 354). Athletes with a wide range of competitive experience were sampled from a variety of sports. Fuller demographic information was provided in Bartholomew et al.

EBFA. The null hypothesis for exact fit was rejected until four factors were extracted (i.e., see Model 4 in Table 1).[9] Extracting four factors was consistent with the exploratory form of the bi-factor model depicted in panel d of Figure 1. The EBFA that extracted three factors (i.e., Model 3) fit the data significantly worse than the EBFA that extracted four factors, $\Delta\chi^2_R(9) = 35$, $p < .001$. Extracting three exploratory factors was consistent with an exploratory form of the correlated first-order factors model depicted in panel b of Figure 1. The EBFA that specified four factors was accepted. Total variance explained in each item by the accepted EBFA ranged from 26% to 66% with a mean of 50% (see Table 2).

Orthogonal target rotation produced a pattern matrix that generally was consistent with a priori expectations (compare Table 2 to panel c in Figure 1).[10] Standardized pattern coefficients from the general factor, labeled psychological need thwarting, ranged from .31 to .69 and this factor accounted for 56% of the common variance in the items. Each of the items also had a meaningful pattern coefficient from the most relevant group factor. Standardized pattern coefficients from the first group factor, labeled autonomy, on the items intended to measure this factor, labeled aut_#, ranged from .37 to .52. Standardized pattern coefficients from the second group factor, labeled competence, on the items intended to measure this factor, labeled com_#, ranged from .27 to .47. Standardized pattern coefficients from the third group factor, labeled relatedness, on the items intended to measure this factor, labeled rel_#, ranged from .20 to .54. The group factors combined to account for 44% of the common variance in the items. One-third (4 of 12) of the items also had a meaningful pattern coefficient on a non-intended group factor and these cases are reviewed in the Discussion section.

CBFA versus EBFA. The accepted, but not very parsimonious, EBFA was then compared to a more parsimonious confirmatory form of the bi-factor model based on theory alone (see panel c in Figure 1). This CBFA (i.e., Model 5 in Table 1) exhibited significantly worse fit than the accepted EBFA (i.e., Model 4), $\Delta\chi^2_R(18) = 61$, $p < .001$. There was evidence, however, for close fit of the CBFA: $\chi^2_R(42) = 103$, $p < .001$, RMSEA = .048 ($CI_{90\%}$ =.036-.059), SRMR = .036, CFI = .966, and TLI = .946. Thus, when the PNTS was represented in a confirmatory form of the bi-factor model the a priori measurement theory appeared to be close, but incomplete, and handled more effectively in an exploratory form of the bi-factor model. Thus, the key difference appears to be that there may be some non-zero loadings on non-intended group factors (see Table 2 for results and Figure 1, panel c and panel d, for a visual) that the CBFA (but not the EBFA) forced to equal zero.

Simpler CFA versus CBFA. The CBFA based on theory alone was then compared to more commonly used and more restrictive analyses. In this case the correlated first-order factors model depicted in panel b, and the second-order factor model depicted in panel a, of Figure 1 were mathematically equivalent and are referred to collectively as the simpler CFA. The simpler CFA (i.e., Model 6 and Model 7 in Table 1) exhibited significantly worse fit than the CBFA (i.e., Model 5), $\Delta\chi^2_R(9) = 20$, $p = .016$. There was evidence, however, for close fit of the simpler CFA:

$\chi^2_R(51) = 123$, $p < .001$, RMSEA = .047 ($CI_{90\%}$ =.037-.058), SRMR = .042, CFI = .960, and TLI = .948. Thus, while the factor models used in Bartholomew et al. (2011) exhibited close fit, fit was at least as good, but generally similar, under a CBFA based on theory alone. Thus, the key difference appears to be that the simpler CFA may be too restrictive (e.g., the absence of a general factor exerting direct effects on the PNTS items) as compared to the CBFA. While the magnitude of the difference between the simpler CFA and the CBFA may be considered modest, recall that the difference between the CBFA and the EBFA appeared to be more substantial, and thus, support for EBFA was provided.

Discussion

We believe there may be a strong conceptual fit between the bi-factor model and how theory-driven multidimensional scales often are developed in SEP psychology. In many cases the a priori measurement theory that guides instrument and item development may support, or at least not preclude the possibility that, responses to items may be directly influenced by both (a) a general latent construct as well as (b) the relevant latent group factors. Using bi-factor analysis to determine if items are influenced by both general and group factors has the additional benefit of refining common theories (e.g., SDT) where such questions may have previously gone unarticulated. Neither of the more commonly used parameterizations of the factor model, the correlated first-order factors model and the second-order factor model, typically specify that at least some items may be directly influenced by a general factor as well as a more narrowly defined group factor. This misfit between instrument development and parameterization of the factor model has likely contributed to instances where the accepted measurement model from the most relevant validity study (e.g., correlated first-order factors model) fails to be implemented in a subsequent study that is focused on the predictive validity of a general latent construct (e.g., second-order factor model).

The possibility for a conceptual fit between the bi-factor model and how a particular multidimensional scale is developed may initially be unclear in many cases because researchers have not addressed such issues. Such uncertainty was present in the example detailed in this manuscript. In these instances, considering (and providing summaries of the results from) multiple plausible parameterizations of the factor model may be useful for both the authors, and the readers, of subsequent research reports. For example, as illustrated in Figure 1, the more commonly used parameterizations of the factor model may frequently be viewed as more restricted models as compared to confirmatory and exploratory forms of the bi-factor model. The confirmatory and exploratory forms of the bi-factor model can be considered increasingly flexible forms of the factor model and a potentially interesting question then becomes: is the extra complexity worthwhile? Theoretical and empirical information should be considered and provided when deciding which model to accept. Predictive validity is a facet of construct validity that may be considered when deciding which model to eventually accept for a given purpose. Predictive validity was not considered in the examples in the current study and future research in this area is warranted prior to more strongly advocating a particular factor model.

A four-factor exploratory bi-factor analysis on the psychological needs thwarting scale data produced a well-fitting model (in general and as compared to a confirmatory bi-factor analysis) with an approximate bi-factor structure that was relatively consistent with the a priori measurement theory as manifest within a bi-factor perspective. It is interesting to note that the general psychological need thwarting factor - which is not modeled in the correlated first-order factors model and is specified in a fairly restricted way in second-order factor model - accounted for more (56% versus 44%) of the common variance than the set of group factors (autonomy, competence, and relatedness). Competence was implicated in some way for each item that did not follow a bi-factor structure (e.g., had a meaningful pattern coefficient on a non-intended group factor in addition to the intended group factor and the general group factor). Three items not intended to measure competence had a meaningful pattern coefficient on the competence group factor: aut_1, *I feel prevented from making choices with regard to the way I train*; aut_2, *I feel pushed to behave in certain ways*; rel_2, *I feel others can be dismissive of me*. One of the competence items, com_1, *I feel inadequate because I am not given opportunities to fulfill my potential*, had a meaningful pattern coefficient on the autonomy group factor.

Future researchers should consider whether each of the items identified in the previous paragraph should be revised in a way to omit the observed complexity, or, whether the initially unexpected complexity is theoretically meaningful (and the item should not be revised). For example, there may well be various situations where the thwarting of one need (e.g., competence) may have direct implications for the thwarting of one or more indicators designed to measure a different need (e.g., autonomy). As Asparouhov and Muthén (2009) stated more generally, having "pure" items that load on a single factor often is not a requirement of a well-defined factor structure and such a hypothesis can be too restrictive. Whatever the ultimate outcome with regard to the revising (or not) of these particular items, Table 1 and Table 2 combine to provide evidence for the consideration of the bi-factor model for future research in an emerging area of research in SEP psychology: psychological need thwarting (e.g., Gunnell, Crocker, Wilson, Mack & Zumbo, 2013).

Theory-based measurement in SEP psychology recently has returned to more frequent consideration of EFA as advances in software have situated the exploratory factor model within a broader latent variable model. An analyst who imposes an exploratory factor model often confronts the issue of how to select a rotation criterion. For the examples in the current study we selected orthogonal target rotation after considering output from other rotation criteria in relation to our a priori expectations. Given that there is no right or wrong rotation criterion from a mathematical perspective with regard to model-data fit, defense of our selection relies, in part, upon the interpretability of rotated pattern matrix in each case (see Table 2). We do not know if our selection was optimal but we think that it was reasonable. The fact that we chose an orthogonal over an oblique solution, while consistent with original tenets of the bi-factor model, can be debated. Much like the second-order factor model which often specifies orthogonal first-order factors after removing the second-order factor from each first-order factor, our accepted exploratory bi-factor model specified orthogonal group factors (e.g., autonomy, competence, and relatedness) after removing the general factor (e.g., psychological need thwarting) from each item. Selection of an oblique solution among the group factors would have been reasonable too – though interpretation became slightly more complicated (see Jennrich & Bentler, 2012). Should exploratory bi-factor analysis begin to appear in SEP psychology it may be useful for the justification of the selected rotation criterion to focus on the degree to which the selected rotation criterion appears to produce a reasonable solution with regard to a priori measurement theory and less on the degree to which the solution is correct. We cannot know if a selected rotation criterion produces a correct solution unless the population values are known, and if these values are known, then a confirmatory bi-factor analysis (requiring no rotation criterion) may be more appropriate than an exploratory bi-factor analysis. Should bi-factor analysis become more common in SEP psychology, however, the results from this study suggest that exploratory (as opposed to confirmatory) bi-factor analysis frequently may offer a viable alternative, so choice of rotation criterion may be an issue that requires further research.

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Footnotes

[1] Not all measurement problems in SEP psychology take, or need to take, a latent variable approach. A latent variable approach, however, was the emphasis of this manuscript and frequently is taken in SEP psychology (Myers, Ahn, & Jin, 2011) Reviews of other approaches (e.g., Classical Test Theory and derivation of composite scores) in SEP psychology are available elsewhere (e.g., Petscher & Schatschneider, 2012).

[2] Item response theory has also been linked with investigations of construct validity in SEP psychology (e.g., Kang, Zhu, Ragan, & Frogley, 2007; Zhu, Timm, & Ainsworth, 2001). A full information bi-factor analysis framework has been put forth (Cai, Yang, & Hansen, 2011).

[3] This parameterization is of interest in the current study but it is not the only possible parameterization of a higher-order factor model. Other parameterizations, including a model with direct effects from a general second-order factor, have been proposed (Yung, Thissen, & McLeod, 1999). Yung et al. formally demonstrate the general relationship between the higher-order factor model and the hierarchical factor model (which the bi-factor is a special case of).

[4] For simplicity, when referring to a pattern matrix let row = item, from this point forward.

[5] Oblique rotation to exact bi-factor structure failed but this is unlikely to be a problem in practice because exact bi-factor structure is unlikely to be present (Jennrich & Bentler, 2012).

[6] It should be noted that while the BPNT acronym used in this manuscript was consistent with work in this general area (http://www.selfdeterminationtheory.org/theory), the BNT acronym has been used in some work in SEP psychology (e.g., Adie et al., 2008) for the same purpose.

[7] Identification constraints generally were not discussed in this section to aid in accessibility of presentation. For the same reason not all nested model relationships were discussed.

[8] This approach was paralleled the approach taken in the parent study (Bartholomew et al., 2011) where a modest degree of non-normality was observed with regard to respondents' usage of the seven ordered response options (see Bartholomew et al., for item-level skew and kurtosis values). The data could have been modeled as ordered and under a different estimation method.

[9] A model with five exploratory factors encountered problems that resulted in the inability to compute relevant information (e.g., standard errors; chi-square value).

[10] Specific inter-factor correlations under oblique target rotation generally were small to moderate (M =.36) and ranged from .13 (relatedness with autonomy) to .62 (relatedness with competence). Had the oblique solution been the accepted solution in the current manuscript then consideration of the structure coefficients in addition to the pattern coefficients would have been important (Graham, Guthrie, & Thompson, 2003).

Table 1

Factor Models and the Psychological Needs Thwarting Scale: Some Key Results from Question 1 through Question 4

	Question 1: number of factors (*m*)						Nested model comparison	
Model	$\chi^2(df)$, *p*	*Par*	*RMSEA*	*SRMR*	*CFI*	*TLI*	Complex	$\Delta\chi^2(\Delta df)$, *p*
Model 1: *m*=1	328(54),<.001	36	.089(.080,.098)	.069	.847	.813	Model 2	188(11), <.001
Model 2: *m*=2, EBFA	120(43),<.001	47	.053(.042,.064)	.031	.957	.934	Model 3	40(10), <.001
Model 3: *m*=3, EBFA	74(33),<.001	57	.044(.030,.057)	.023	.977	.955	Model 4	35(9), <.001
Model 4: *m*=4, EBFA	35(24),.062	66	.027(.000,.045)	.014	.994	.982	-----	-----
	Question 2: confirmatory bi-factor analysis (CBFA)						CBFA versus EBFA	
Model	$\chi^2(df)$, *p*	*Par*	*RMSEA*	*SRMR*	*CFI*	*TLI*	Complex	$\Delta\chi^2(\Delta df)$, *p*
Model 5: *m*=4, CBFA	103(42),<.001	48	.048(.036,.059)	.036	.966	.946	Model 4	61(18), <.001
	Question 3: first-order correlated factors (1st-order)						1st-order versus CBFA	
Model	$\chi^2(df)$, *p*	*Par*	*RMSEA*	*SRMR*	*CFI*	*TLI*	Complex	$\Delta\chi^2(\Delta df)$, *p*

Model 6: *m*=4, 1st-order	123(51),<.001	39	.047(.037,.058)	.042	.960	.948	Model 5	20(9), .016

	Question 4: second-order (2nd-order)						2nd-order versus 1st-order	
Model	χ^2(*df*), *p*	*Par*	*RMSEA*	*SRMR*	*CFI*	*TLI*	Complex	$\Delta\chi^2$(Δ*df*), *p*
Model 7: *m*=4, 2nd-order	123(51),<.001	39	.047(.037,.058)	.042	.960	.948	Model 6	-----

Note. m = number of factors, EBFA = exploratory bi-factor analysis; PAR = number of parameters estimated; Complex = more complex model that a nested simpler model was compared to; Model 4 corresponds to panel d in Figure 1; Model 5 corresponds to panel c in Figure 1; Model 6 corresponds to panel b in Figure 1; Model 7 corresponds to panel a in Figure 1.

Table 2

Psychological Needs Thwarting Scale under Exploratory Bi-Factor Analysis with Target Rotation

	General Factor			Group Factors									
	Psychological Need Thwarting			Autonomy (aut)			Competence (com)			Relatedness (rel)			
Item	λ	*SE*	λ^0	λ	*SE*	λ^0	λ	*SE*	λ^0	λ	*SE*	λ^0	R^2
aut_1	0.52	.08	.31	0.80	.10	.48	0.38	.10	.23	----	----	----	39%
aut_2	0.64	.13	.35	0.96	.17	.52	0.43	.14	.23	----	----	----	46%
aut_3	0.95	.12	.53	0.92	.11	.51	----	----	----	----	----	----	56%
aut_4	0.96	.14	.61	0.58	.08	.37	----	----	----	----	----	----	52%
com_1	0.63	.12	.40	0.50	.12	.32	0.74	.22	.47	----	----	----	49%
com_2	0.88	.07	.62	----	----	----	0.38	.09	.27	----	----	----	47%
com_3	0.92	.08	.66	----	----	----	0.57	.09	.40	----	----	----	60%
com_4	0.99	.06	.69	----	----	----	0.58	.07	.40	----	----	----	66%
rel_1	0.60	.10	.48	----	----	----	----	----	----	0.58	.12	.46	45%
rel_2	0.85	.07	.60	----	----	----	0.31	.07	.22	0.62	.10	.44	61%
rel_3	0.70	.08	.48	----	----	----	----	----	----	0.79	.15	.54	53%
rel_4	0.84	.13	.46	----	----	----	----	----	----	0.37	.14	.20	26%
PCVE		56%						44%					

Note. λ = pattern coefficient; λ^0 = standardized pattern coefficient; PCVE = percentage of common variance explained; Estimated coefficients that were not statistically significant ($\alpha = .05$) and/or $|\lambda^0| < .20$ were omitted from the table.

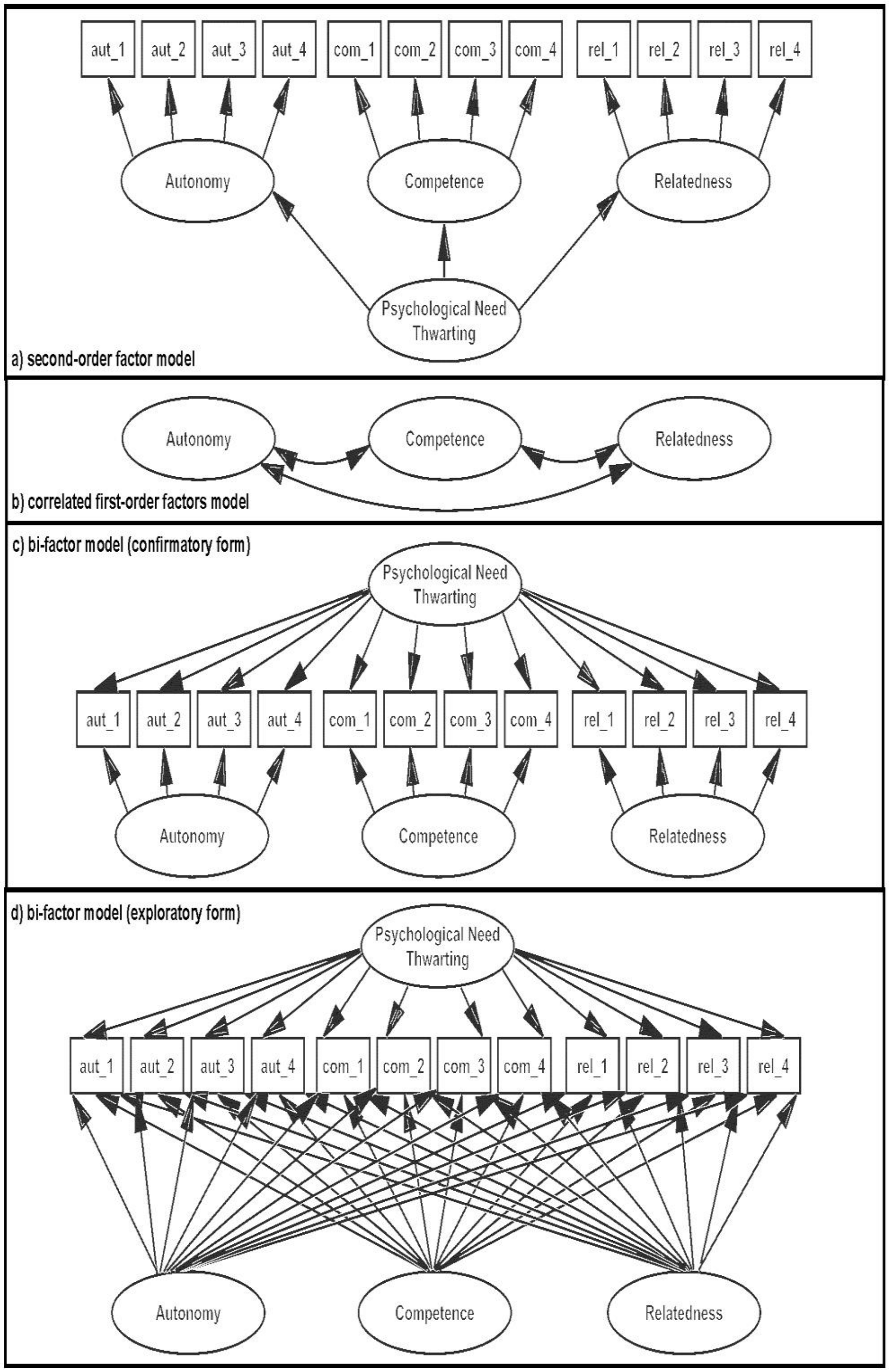

Figure 1. Factor models and the psychological needs thwarting scale. Model parameters (e.g., variances), identification constraints, and items sometimes were omitted to reduce clutter. The sequence of the panels follows, in general, a nested order: from a) simplest to d) most complex.