Performance Measure of Multi-User Detection Algorithms for MIMO Relay Network

M. R. A. Khandaker [1], Yue Rong [2]

[1,2] *Department of Electrical and Computer Engineering, Curtin University of Technology*
Bentley, WA, Australia
[1] m.khandaker@postgrad.curtin.edu.au
[2] y.rong@curtin.edu.au

***Abstract*—Due to the emerging demand on new multimedia applications, next generation wireless communication systems will need to support data rates much greater than 3G systems. This will require more efficient utilization of the radio resources. Relay and multiple-input multiple-output (MIMO) techniques can significantly enhance system power efficiency, extend system coverage and are considered promising candidates for next generation wireless communication systems. In this paper, we study the zero-forcing (ZF) and minimum mean-squared error (MMSE) algorithms for a MIMO relay network and compare their performance in terms of bit-error-rate (BER). In particular, we show their performance in a single-user as well as in a multi-user scenario. Multi-user MIMO relay algorithms have significant performance improvement over single-user algorithms.**

I. Introduction

Next-generation wireless systems are expected to support higher data rates. The bandwidth limitation of wireless communication prompts the need for spectrally efficient methods of communication. Multiple-input multiple-output (MIMO) system which uses multiple antennas at transmitting and receiving ends can be an effective means to increase spectral efficiency [1], [2]. MIMO systems can be used to transmit multiple data streams that can be separated using receiver signal processing, as in [3] and [4]. Thus a multiple-antenna system can provide higher data transmission rate than a single-antenna system in a scattered environment.

In the previous research works with MIMO, emphasis was given on the point-to-point (P-P) single-user (SU) case. The capacity of single-user point-to-point MIMO channel was well studied in [5] and [6]. The extensive capacity offered by the single-user MIMO systems encourages the use of multiple antennas in shared multi-access channels, which we call multi-user (MU) MIMO systems. In next generation wireless systems, multiple users equipped with multiple antennas will transmit simultaneously to the base station with multiple receive antennas and vice versa. Transceiver design for such systems has been studied and performance enhancing transmission schemes are proposed in [7], [8]. The capacity of multiuser MIMO systems is investigated for flat-fading channels in [9].

Relays are low cost and low transmit power elements that receive and forward data from source to destination. Since they implement a subset of base station functions, relays are a low cost and low complexity solution to meet the requirement of higher capacity, extended coverage and improved link reliability [10]-[12]. In a cellular environment, a relay can be deployed in areas with significant shadowing effects [13], such as inside buildings, mines and tunnels as well as far from the base station at the cell edge. Wang, et. al. considered a regenerative MIMO relay in [14]. Multi-user MIMO relay systems have been studied in [15] in which the relay does not decode the received signals from the source but simply processes them with matrix multiplications (amplify and forward). In [16], a fair scheduling scheme has been proposed, called the strongest-weakest-normalized-subchannel-first (SWNSF) scheduling, which can significantly increase the coverage of the multiuser MIMO system while further improving the system capacity. In some recent research works, such as [17] and [18], optimization of MIMO relays have been studied which has much better performance in terms of bit-error-rate (BER).

In this paper, we compare the performance of zero-forcing (ZF) and minimum mean-squared error (MMSE) algorithms in a MIMO relay network in terms of BER. Although these two algorithms have been studied for single-user MIMO and single-user MIMO relay channels, they have not yet been considered for multi-user MIMO relay systems. In this paper, we study the algorithms in a single-user as well as in a multi-user scenario.

Our results show that multi-user MIMO relay algorithms have significant performance improvement over single-user algorithms.

The rest of the paper is organized as follows: the system model is described in Section II; Section III introduces single-user and multi-user detection algorithms for MIMO relay systems; Section IV shows the simulation results which justify the significant performance gain of multi-user detection algorithms under various system scenarios and the conclusion is given in Section V.

II. System Model

Our proposed multi-user MIMO relay system is illustrated in Fig. 1. For simplicity, we only plot a system with two users. In a multi-user MIMO relay system, users are transmitting signals to the same destination through relays where a relay only communicates with a single user. The users are far apart to interfere each other. The users, the relays and the destination are equipped with $N_{s,i}$, $N_{r,i}$, $i = 1, \cdots, N_u$, and N_d antennas respectively. Here N_u is the number of users. All data streams are transmitted through the relays and there is no direct link between the source and the destination. The MIMO channel between the ith mobile user and the ith relay is $\mathbf{H}_i$ and that between the ith relay and the destination is denoted by $\mathbf{G}_i$. The noise added to the received signal at the ith relay is $\mathbf{n}_i$ and the noise at the destination is $\mathbf{n}_d$. We use a linear signal processing unit denoted by relaying matrix $\mathbf{F}_i$ at the ith relay. Hence, the signal received at the ith relay is $\mathbf{r}_i = \mathbf{H}_i\mathbf{s}_i + \mathbf{n}_i$ where $\mathbf{s}_i$ is the transmit signal vector of the ith user. The received signal vector at the destination is

$$\mathbf{y} = \sum_{i=1}^{N_u} \mathbf{G}_i\mathbf{F}_i\mathbf{r}_i + \mathbf{n}_d = \sum_{i=1}^{N_u} \mathbf{G}_i\mathbf{F}_i(\mathbf{H}_i\mathbf{s}_i + \mathbf{n}_i) + \mathbf{n}_d. \quad (1)$$

The system is considered to operate in a flat-fading Rayleigh environment.

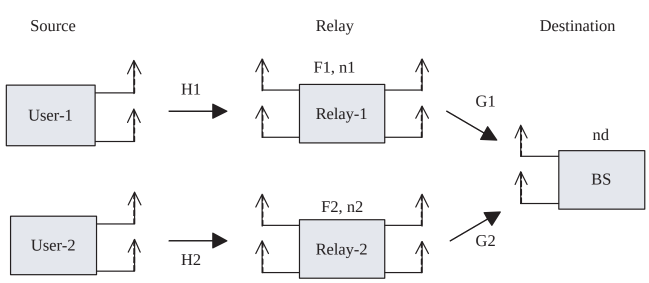

Fig. 1. Two-user MIMO relay system model

A simple approach to design the relay is to treat it as an all-pass amplify-and-forward (AF) unit, which we construct as $\mathbf{F}_i = \alpha_i\mathbf{I}_{N_{r,i}}$, where α_i is the amplifying factor of relay i and $\mathbf{I}_{N_{r,i}}$ is an identity matrix of dimension $N_{r,i}$. We can find α_i from $p_i = \alpha_i^2 tr\{\mathbf{F}_i(\mathbf{H}_i\mathbf{H}_i^H + \mathbf{I}_{N_{r,i}})\mathbf{F}_i^H\}$. Here $p_i > 0$ is the transmit power available at relay i, $(\cdot)^H$ denotes matrix Hermitian and $tr\{\cdot\}$ indicates trace of a matrix.

III. Signal Detection Algorithms for MIMO Relay Systems

In this section we develop two detection algorithms for MIMO relay systems namely the zero-forcing (ZF) and the minimum mean-squared error (MMSE) algorithms for single-user as well as multi-user detection.

The single-user detection algorithms treat the signals from other users as noise. Thus the total noise incorporated to the received signal becomes higher which results in higher BER. For the ith user, the equivalent channel matrix is $\mathbf{H} = \mathbf{G}_i\mathbf{F}_i\mathbf{H}_i$, the signal vector is $\mathbf{s} = \mathbf{s}_i$, and the equivalent noise vector is $\mathbf{n} = \sum_{j=1, j\neq i}^{N_u} \mathbf{G}_j\mathbf{F}_j\mathbf{H}_j\mathbf{s}_j + \sum_{i=1}^{N_u} \mathbf{G}_i\mathbf{F}_i\mathbf{n}_i + \mathbf{n}_d$. Thus we see from (1) that the received signal vector at the destination can be equivalently written as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}. \quad (2)$$

For multi-user detection algorithms the equivalent channel matrix becomes

$$\mathbf{H} = [\mathbf{G}_1\mathbf{F}_1\mathbf{H}_1, \mathbf{G}_2\mathbf{F}_2\mathbf{H}_2, \cdots, \mathbf{G}_{N_u}\mathbf{F}_{N_u}\mathbf{H}_{N_u}].$$

The equivalent signal vector $\mathbf{s} = \left[\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_{N_u}^T\right]^T$ is assumed to have zero-mean, uncorrelated random entries with unit variance, and the equivalent noise vector is $\mathbf{n} = \sum_{i=1}^{N_u} \mathbf{G}_i\mathbf{F}_i\mathbf{n}_i + \mathbf{n}_d$. Here $(\cdot)^T$ indicates transpose of a matrix.

A. The ZF Algorithm

The first decoding technique to be described in this paper is ZF. The ZF linear detector meeting the constraint $\mathbf{W}^H\mathbf{H} = \mathbf{I}_{N_s}$ is given by,

$$\mathbf{W} = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}$$

$\mathbf{W}$ is also known as the pseudo-inverse for a general $m \times n$ matrix and $(\cdot)^{-1}$ indicates simple matrix inversion. In order for a pseudo-inverse to exist, N_d must be greater than or equal to $\sum_{i=1}^{N_u} N_{s,i}$. Then the estimate for the transmit signal vector will be $\hat{\mathbf{s}} = \mathbf{W}^H\mathbf{y}$.

B. MMSE Algorithm

To solve $\mathbf{s}$ in (2) the MMSE approach tries to find a coefficient matrix $\mathbf{W}$ that minimizes the statistical expectation of the signal estimation error given by

$$E\{[\mathbf{W}^H\mathbf{y}-\mathbf{s}][\mathbf{W}^H\mathbf{y}-\mathbf{s}]^H\} \tag{3}$$

where $E\{\cdot\}$ denotes statistical expectation. Substituting (2) into (3), we find that the $\mathbf{W}$ which minimizes (3) can be written as

$$\mathbf{W}=(\mathbf{H}\mathbf{H}^H+\mathbf{C})^{-1}\mathbf{H} \tag{4}$$

where $\mathbf{C}$ is the noise covariance matrix given by $\mathbf{C}=E\left[\mathbf{n}\mathbf{n}^H\right]$.

For detecting ith user (SU),

$$\begin{aligned}\mathbf{C}&=E\left[\mathbf{n}\mathbf{n}^H\right]\\ &=E\Bigg[\left(\sum_{j=1,j\neq i}^{N_u}\mathbf{G}_j\mathbf{F}_j\mathbf{H}_j\mathbf{s}_j+\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{n}_i+\mathbf{n}_d\right)\\ &\quad\times\left(\sum_{j=1,j\neq i}^{N_u}\mathbf{G}_j\mathbf{F}_j\mathbf{H}_j\mathbf{s}_j+\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{n}_i+\mathbf{n}_d\right)^H\Bigg]\\ &=\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{F}_i^H\mathbf{G}_i^H\\ &\quad+\sum_{j=1,j\neq i}^{N_u}\mathbf{G}_j\mathbf{F}_j\mathbf{H}_j\mathbf{H}_j^H\mathbf{F}_j^H\mathbf{G}_j^H+\mathbf{I}_{N_d}\end{aligned}$$

and for multi-user (MU) detection,

$$\begin{aligned}\mathbf{C}&=E\left[\mathbf{n}\mathbf{n}^H\right]\\ &=E\left[\left(\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{n}_i+\mathbf{n}_d\right)\left(\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{n}_i+\mathbf{n}_d\right)^H\right]\\ &=\sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{F}_i\mathbf{F}_i^H\mathbf{G}_i^H+\mathbf{I}_{N_d}.\end{aligned}$$

When comparing to the ZF equalizer, apart from the noise covariance matrix term both the equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to the ZF equalizer.

IV. Simulation Results and Discussions

For simplicity, we considered a two-user MIMO relay system with $N_{s,1}=N_{s,2}=2$, $N_{r,1}=N_{r,2}=2$ and $N_d=4$ where a relay decodes the signals only from the user transmitting data through it. BPSK signal constellation is used to modulate the transmitting signals. We transmitted 10^3 randomly generated bits and the BER results are averaged through 200 channel realizations. We used two schemes of signal-to-noise ratio (SNR). Firstly, we varied the SNR in the source-to-relay link fixing the SNR in the relay-to-destination link (see Fig.2).

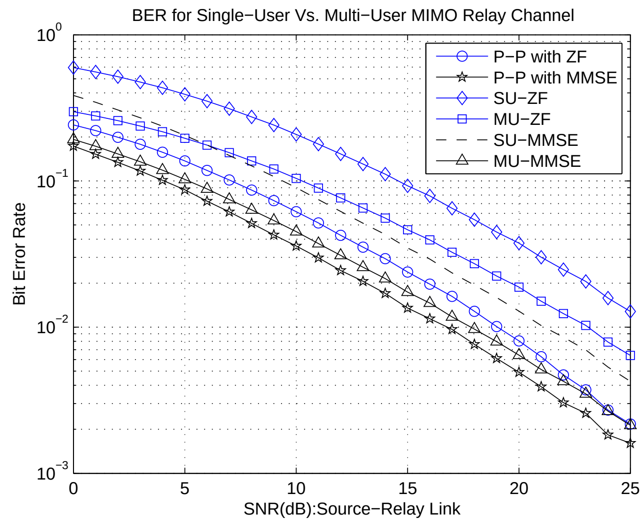

Fig. 2. BER for single-user vs. multi-user MIMO relay channel with varying SNR in the source-to-relay link and fixed SNR in the relay-to-destination link

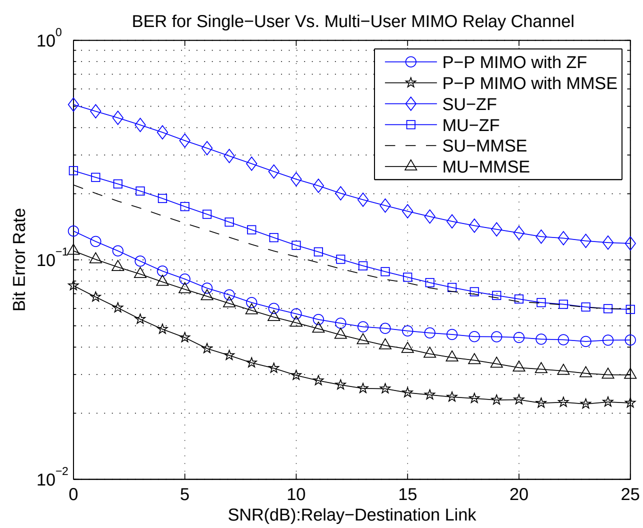

Fig. 3. BER for single-user vs. multi-user MIMO relay channel with varying SNR in the relay-to-destination link and fixed SNR in the source-to-relay link

Then we varied the SNR in the relay-to-destination link keeping the SNR in the source-to-relay link fixed (see Fig.3). We also considered a single-user case as well as a point-to-point MIMO communication system with relays to demonstrate the diversity gain obtained in multi-user MIMO relay system. In case of a single-user detection, the receiver treats the signals from the other users as noise whereas, in a point-to-point link, only one user communicates with the receiver. Our results show that multi-user detection techniques perform better than the single-user ones in both schemes of SNR. The point-to-point communication system has the lowest BER with both algorithms as it encounters less noise. It is also demonstrated that MMSE algorithm has lower BER compared to the zero-forcing technique in all the cases, but zero-forcing detector is comparatively easier to implement and efficient to reduce inter-symbol-interference (ISI).

V. Conclusions

In conclusion, we have demonstrated the advantage of using multi-user diversity in MIMO relay network. We designed relays as all-pass amplify-and-forward (AF) units which are simpler to implement. In the future, it is worth further investigating MIMO relay networks with multiple users interfering each other, optimizing the relay matrix to allocate transmit power efficiently, users sharing the relays in a cooperative MIMO relay network and designing the relays as decode-and-forward ones.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *in AT&T Bell Labs Memo.*, Murray Hill, NJ, 1995.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers.Commun.*, vol. 6, no. 3, pp. 311-335, Mar. 1998.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41-59, Autumn 1996.

[4] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, pp. 2198-2206, Dec. 2001.

[5] E. Telatar, "Capacity of multi-antenna gaussian channels," *Technical Report*, Bell Labs, 1995.

[6] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, March 1998.

[7] S. Serbetli and A. Yener, "Iterative transceiver optimization for multiuser MIMO systems," *Proc. 40th Annual Allerton Conference on Communication, Control and Computing*, Monticello, Ill, USA, October 2002.

[8] S. Serbetli and A. Yener, "Transceiver optimization for multiuser MIMO systems," *IEEE Trans. Signal Processing*, vol. 52, no. 1, pp. 214-226, 2004.

[9] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative waterfilling for Gaussian vector multiple-access channels," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 145-152, 2004.

[10] R. Pabst, B. H. Walke, D. C. Schultz, D. C. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80-89, Sept. 2004.

[11] H. Hu, H. Yanikomeroglu, D. D. Falconer, and S. Periyalwar, "Range extension without capacity penalty in cellular networks with digital fixed relays," *Proc. Glob. Telecom. Conf.*, Nov. 29 - Dec. 3 2004, vol. 5, pp. 3053-3057

[12] H. Bolukbasi, H. Yanikomeroglu, H. Falconer, and S. Periyalwar, "On the capacity of cellular fixed relay networks," *Electrical and Computer Engr., 2004. Canadian Conf.*, May 2004, vol. 4, pp. 2217-2220.

[13] I. Kang, W. Sheen, R. Chen, and S. Lin C Hsiao, "Throughput improvement with relay-augmented cellular architecture," *IEEE 802.16mmr-05 008*, http://www.wirelessman.org, Sep. 2005.

[14] B.Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29-43, Jan. 2005.

[15] T. Tang, C. B. Chae, R. W. Heath. Jr., "On achievable sum rates of a multiuser MIMO relay channel," *Proceedings of IEEE ISIT'06*, Seattle, WA, Jul. 2006.

[16] C.-J. Chen and L.-C. Wang, "Enhancing coverage and capacity for multiuser MIMO systems by utilizing scheduling," *IEEE Trans. On Wireless Commun*, VOL. 5, NO. 5, pp. 1148 -1157, MAY 2006.

[17] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Processing*, to appear, 2009.

[18] Y. Rong and Y. Hua, "Optimality of diagonalization of multi-hop MIMO relays," *IEEE Trans. Wireless Commun.*, to appear.