Assessment of EGM2008 over Germany using accurate quasigeoid heights from vertical deflections, GCG05 and GPS/levelling

Summary

EGM2008 is a high-resolution global model of Earth's gravity field that allows computation of quasigeoid heights and further functionals down to a resolution of 5 arc minutes. The present paper assesses EGM2008 over Germany by means of quasigeoid heights from the German GCG05 quasigeoid model and GPS/levelling points, and quasigeoid height differences from astronomical levelling. Residual terrain model (RTM) data is used for the computation of RTM quasigeoid heights, serving to augment the resolution of EGM2008 at scales shorter than 5 arc minutes. For quasigeoid heights, the comparisons show a RMS (root mean square) agreement of ~3 cm between EGM2008 and GCG05 as well as EGM2008 and GPS/levelling. The residuals between EGM2008 (augmented with RTM) and astrogeodetic quasigeoid height differences are near or at the cm-level for two local test areas. The comparisons show the very good quality of EGM2008 over Germany, which serves as an example region where dense gravity sets were used for the model's development.

Zusammenfassung

EGM2008 ist ein hochauflösendes globales Erdschwerefeldmodell, das zur Berechnung von Quasigeoidhöhen und anderen Schwerefeldfunktionalen mit einer Auflösung von 5 Bogenminuten verwendet werden kann. Der vorliegende Beitrag bewertet EGM2008 mit Hilfe des Quasigeoidmodells GCG05, einem GPS/Nivellement Datensatz und astrogeodätisch bestimmten Differenzen von Quasigeoidhöhen. Residuale Geländedaten (RTM) werden zur Berechnung von RTM Quasigeoidhöhen genutzt, die EGM2008 auf Skalen kürzer als 5 Bogenminuten ergänzen. Die Vergleiche zeigen mittlere quadratische Abweichungen (RMS) von etwa 3 cm zwischen EGM2008 und GCG05 bzw. den GPS/Nivellementspunkten. Die Residuen zwischen EGM2008 und astrogeodätischen Differenzen von Quasigeoidhöhen in zwei lokalen Testgebieten sind auf oder nahe dem cm-Niveau. Der Beitrag zeigt die sehr gute Qualität von EGM2008 über Deutschland als Beispiel für Gebiete, in denen umfangreiche Schwerewerte für die EGM2008 Modellierung verwendet wurden.

Keywords: EGM2008, quasigeoid, GCG05, GPS/levelling, astronomical levelling, residual terrain modelling (RTM)

1. Introduction

With the computation and release of the Earth Gravitational Model EGM2008 (Pavlis et al. 2008) in April 2008, a major advancement was made in high-resolution global gravity field modelling. Developed by the U.S. National Geospatial Agency (NGA), EGM2008 is the first-ever global model that is capable of resolving the Earth's gravity field beyond spherical harmonic degree 2000. The EGM2008 set of spherical harmonic coefficients is complete to degree 2190 and order 2159. It allows computation of various gravity field functionals – such as quasigeoid heights, gravity anomalies and vertical deflections – globally with a spatial resolution of ~5 arc minutes, or ~9 km in the latitudinal direction. EGM2008 is freely available from http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html.

A number of external evaluation studies on EGM2008 have already been carried out using ‘ground-truth’ gravity field observations over several countries (Newton’s Bulletin 2009). The comparisons made in the 25 studies presented in Newton’s Bulletin (2009) provide evidence of low EGM2008 commission errors, i.e., the uncertainties of EGM2008-derived functionals, particularly over areas where EGM2008 is based on dense gravity data sets.

As a consequence of its high spatial resolution and accuracy, EGM2008 represents a large part of the gravity field spectrum. Because the residual gravity field signals are very small, EGM2008-based regional gravity field modelling encounters new challenges (e.g., Featherstone et al. 2010). Recent examples of regional gravity field modelling using EGM2008 are given by Featherstone et al. (2010) for Australia, Claessens et al. (2011) for New Zealand, Roman et al. (2010) for the United States and Denker et al. (2009) and Ihde et al. (2010) for Europe.

Beyond its resolution, that is at scales finer than ~5 arc minutes, EGM2008 is not capable of representing the high-frequency constituents of Earth’s gravity field. The neglect of high-frequency content by a harmonic model like EGM2008 is known as omission error (Torge 2001 p. 273; Gruber 2009). For quasigeoid heights derived from EGM2008, Jekeli et al. (2009) estimated the EGM2008 omission error to be ~ 4 cm. This is a global estimate which may vary for different types of terrain. Little or no attempt was made to model and account for the EGM2008-omitted high-frequency signals in the evaluation reports on EGM2008 (Newton’s Bulletin 2009).

To model and reduce the omission error, one common strategy is the remove-compute-restore (RCR) approach, where the fine-structure is sourced from residual gravity (Torge 2001, p. 286). As is known, many regional geoid or quasigeoid models are based on this method, including the above mentioned regional models based on EGM2008. Alternatively, residual terrain model (RTM) data (Forsberg 1984) can be used in elevated terrain as a source to recover parts of the omission error (Hirt 2010, Hirt et al. 2010a, 2010b). Not only is RTM-based omission error modelling advantageous for accurate prediction of functionals (e.g., Hirt 2010), but it also facilitates the assessment of EGM2008 (and other spherical harmonic models) with ground-truth observations (Hirt et al. 2010a).

The present paper assesses EGM2008 (Section 2) over Germany using the RTM augmentation technique (Section 3) and three different sources of accurate quasigeoid heights (Section 4). These are (i) the German Combined Quasigeoid GCG05 (Liebsch et al. 2006, Schirmer et al. 2006), (ii) a set of quasigeoid heights from GPS/levelling points (Ihde and Sacher 2002) and (iii) two local profiles of astrogeodetic quasigeoid differences (Hirt et al. 2008, Hirt and Flury 2008). Section 5 then presents and discusses the results of the comparisons with EGM2008. A first focus is placed on the inclusion of omission error estimates from RTM data, so as to ‘bridge’, to some extent, the spectral gap between the EGM2008 quasigeoid heights and the comparison data (Hirt et al. 2010b). A second focus is set on the role of the station heights at which EGM2008 is evaluated (Section 5). Germany was selected not only because of the sufficiently accurate comparison data sets available, but also as an example region where dense gravity data sets were available and used for the EGM2008 model construction (see Pavlis et al. 2008).

This paper is complementary to other studies comparing EGM2008 against terrestrial data sets over Germany. For example, Förste et al. (2009) and Gruber (2009) used GPS/levelling points to evaluate EGM2008 in comparison to other geopotential models with focus on the long- and medium-wavelength domain. A study by Ihde et al. (2010) used GPS/levelling points for EGM2008 evaluation while results from a comparison between astrogeodetic quasigeoid height differences and EGM2008 were reported in Berichte (2010). However, the EGM2008 omission error beyond its maximum degree of expansion was neither modelled nor reduced in these studies. For comparisons among vertical deflections and EGM2008 over Germany, see Voigt et al. (2008), Ihde et al. (2010), Hirt (2010) and Hirt et al. (2010a).

2. EGM2008

A paper outlining the details of EGM2008 has not yet become available, however a general overview of EGM2008 is given in Pavlis et al. (2008) with background information on the model’s development presented in Kenyon et al. (2007), Pavlis et al. (2007), Holmes et al. (2007), Pavlis and Saleh (2004), Pavlis et al. (2004) and Saleh and Pavlis (2003). EGM2008 consists of a total of ~4.8 million spherical harmonic coefficients complete to degree and order 2159, with additional spherical harmonic coefficients to degree 2190 and order 2159 (EGM Development Team 2008). The EGM2008 geopotential model is available free-of-charge, together with accompanying products such as a spherical harmonic model of Earth’s topography, grids of commission error estimates for different gravity field functionals and a high-degree synthesis software.

EGM2008 is based on the GRACE (Gravity Recovery and Climate Experiment)-only gravity field model ITG-GRACE03S (Mayer-Gürr 2007) which provides a highly-accurate description of the long- and medium-wavelength gravity field spectrum up to degree and order 180. The ITG-GRACE03S model incorporates almost 6 years of GRACE gravity field observations. The second input data set is a global grid of 5′×5′ area-mean gravity anomalies (band-limited to degree 2160) that was constructed from high-resolution topographic data (Pavlis et al. 2007, Pavlis and Saleh 2004), altimetry-derived gravity over the oceans (e.g., Andersen et al. 2010), and other sources of gravity data, particularly point gravity measurements (Pavlis et al. 2008).

The global grid of surface area-mean gravity anomalies was harmonically analysed to derive a set of ellipsoidal harmonic coefficients (Pavlis et al. 2004, Holmes and Pavlis 2007). The ITG-GRACE03S satellite gravity model was converted from spherical to ellipsoidal harmonics by means of Jekeli’s (1988) transformation. The ellipsoidal harmonic coefficients of both input data sets were then combined through a least-squares adjustment procedure

(Pavlis et al. 2008). The resulting ellipsoidal harmonic spectrum (complete to degree and order 2159) was finally back-converted to spherical harmonics using Jekeli's (1988) algorithm. Because this transformation preserves the maximum order, but not the maximum degree of the harmonic series expansion (see also Holmes and Pavlis 2007), some additional coefficients (to degree 2190 and order 2159) occur in spherical harmonic representation. It is recommended not to neglect these additional coefficients (cf. Holmes and Pavlis 2007). Hence, the EGM2008 spherical harmonic coefficients should be expanded to degree 2190 rather than only to 2159 or 2160 when being employed in practical applications.

The very good quality of EGM2008 in the long and medium wavelengths is mainly due to using GRACE satellite gravity field observations, supported by the spectral content implied in this band by terrestrial gravity anomalies. EGM2008's spectral band between 181 to 2159 (in terms of ellipsoidal harmonics) originates solely from the 5′×5′ area-mean gravity anomalies (see above). Because of the inhomogeneous and incomplete global coverage by surface gravity observations, the NGA 5′×5′ area-mean gravity data base is of varying quality (Pavlis et al. 2008, pp. 2-4). As a consequence, the accuracy of the EGM2008 gravity field functionals varies over different parts of Earth. EGM2008 is most accurate (i.e., lowest commission errors) in regions with high-quality terrestrial gravity data sets (i.e., dense coverage, sufficient accuracy) available for its construction.

For practical applications, EGM2008-based functionals of the gravity field are obtained through harmonic synthesis of the model coefficients. Harmonic synthesis (e.g., Torge 2001, p. 271) can be accomplished, e.g., using the publicly available high-degree harmonic synthesis software harmonic_synth (Holmes and Pavlis 2008). The software is capable of computing a variety of EGM2008 gravity field functionals (e.g., geoid and quasigeoid heights, gravity anomalies and disturbances, vertical deflections), either in terms of scattered locations or points arranged as equidistant grids.

When using the harmonic_synth software, only the scattered point option allows for (individual) ellipsoidal heights of the topography, while grid computations are carried out at some given *constant* ellipsoidal height (e.g., surface of a reference ellipsoid). The EGM2008 quasigeoid heights over Germany – computed with the harmonic_synth scattered point option at the ellipsoidal heights of the topography are shown in Fig. 1.

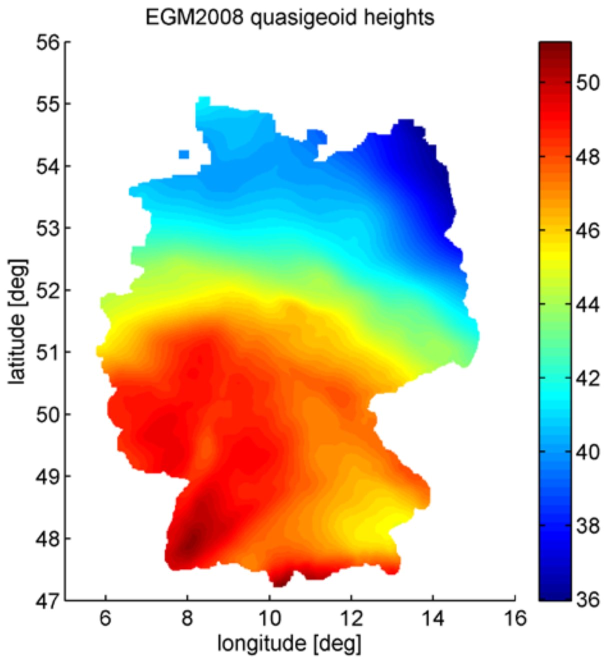

Fig. 1: EGM2008 quasigeoid heights over Germany (spectral degrees 2 to 2190, unit in metres)

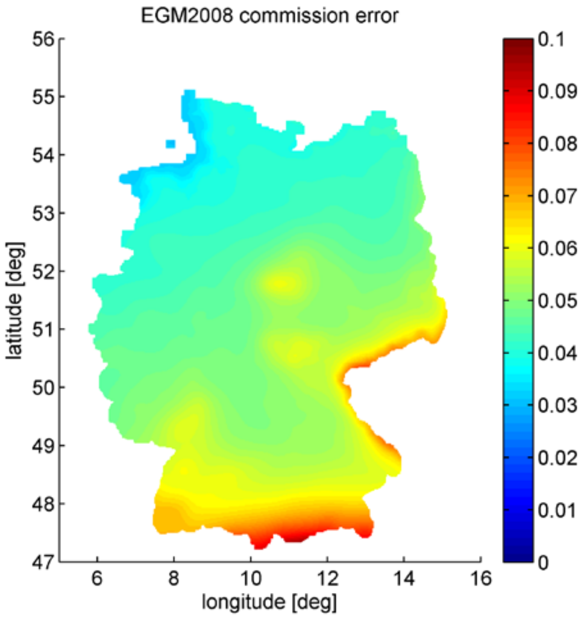

Fig. 2: EGM2008 commission errors over Germany (spectral degrees 2 to 2190, unit in metres)

Users of EGM2008 also have the option of downloading pre-computed grids of EGM2008-based functionals from the EGM2008 website (http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/TEST_RESULTS/results.html and http://earth-info.nga.mil/GandG/wgs84/ gravitymod/egm2008/index.html). These grids were computed at the surface of the reference ellipsoid using harmonic_synth's grid mode. Hence, they provide EGM2008 geoid and quasigeoid heights, gravity anomalies and vertical deflections at a constant ellipsoidal height (of 0 m) above the ellipsoid. The role of the ellipsoidal height used in the synthesis is further dealt with in Section 5.

Maps of EGM2008 commission errors were computed by the EGM2008 development team for quasi/geoid heights, gravity anomalies and vertical deflections using a dedicated error propagation technique described in Pavlis and Saleh (2004). For areas with rather scarce surface gravity coverage (for instance, parts of Africa, South America and Asia), commission errors for EGM2008 quasi/geoid undulations are estimated to be at the level of ~15cm with maximum uncertainties encountered in the mountainous parts of Asia and South America (around ~30-40 cm) and Antarctica (~100 cm). In contrast to this, the lowest commission errors are found over most parts of Europe, Oceania, North America and – because of the use of dense sets of altimetry-derived gravity – the oceans (see Pavlis et al. 2008). For those regions with high-quality surface gravity available, the EGM2008 quasi/geoid commission errors are mostly at the level of ~5 cm. A detailed map of the EGM2008 quasi/geoid commission errors over Germany is shown in Fig. 2, where the error estimates range from 3 cm to 10 cm, with an average value of 5 cm. Section 5 will demonstrate that these ‘official’ commission error estimates are rather pessimistic for Germany.

3. Residual Terrain Modelling (RTM) approach

The truncation of EGM2008 model coefficients at spherical harmonic degree 2190 produces an omission error (Torge 2001, p 273). In other words, the fine-structure of Earth’s gravity field at scales less than 5 arc minutes is not contained in the EGM2008-based gravity field functionals. As shown in Hirt (2010), residual terrain modelling (RTM) is one approach that is suited to compute and reduce this omission error. The basic idea of the RTM method (Forsberg 1984) is to construct residual elevations as the difference between a high-resolution elevation model of the topography and some long-wavelength ‘reference’ topography, which acts as a high-pass filter. The residual elevations are then used to compute RTM gravity field functionals, in order to reduce the omission error of the truncated EGM2008 model to some extent (Hirt 2010, Hirt et al. 2010a, 2010b). In the construction of EGM2008, a variant of the RTM technique was employed for the ‘prediction’ of band-limited gravity anomalies over areas with relatively poor gravity data coverage (Pavlis et al. 2007).

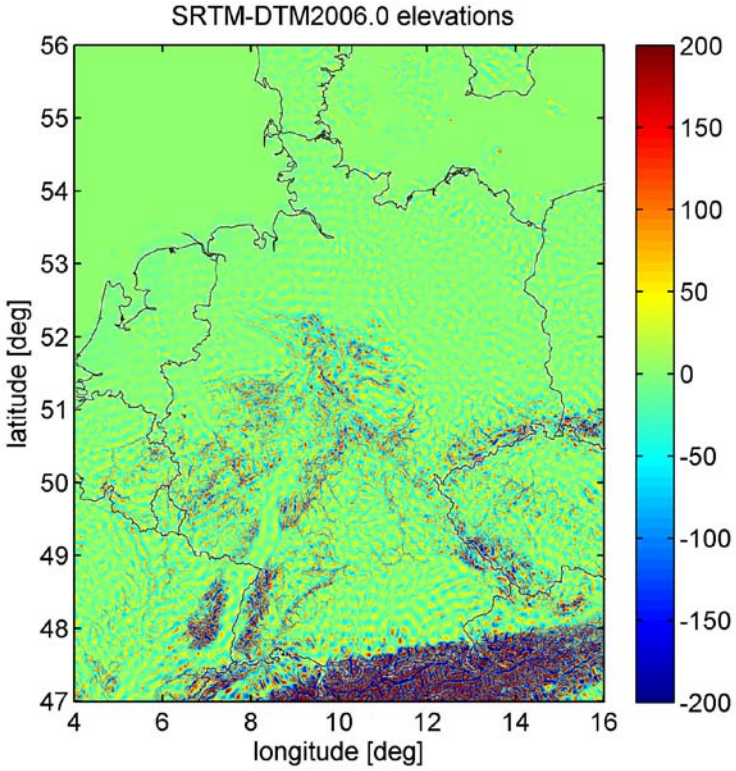

Fig. 3: RTM elevations (SRTM minus DTM2006.0) over Germany (unit in metres)

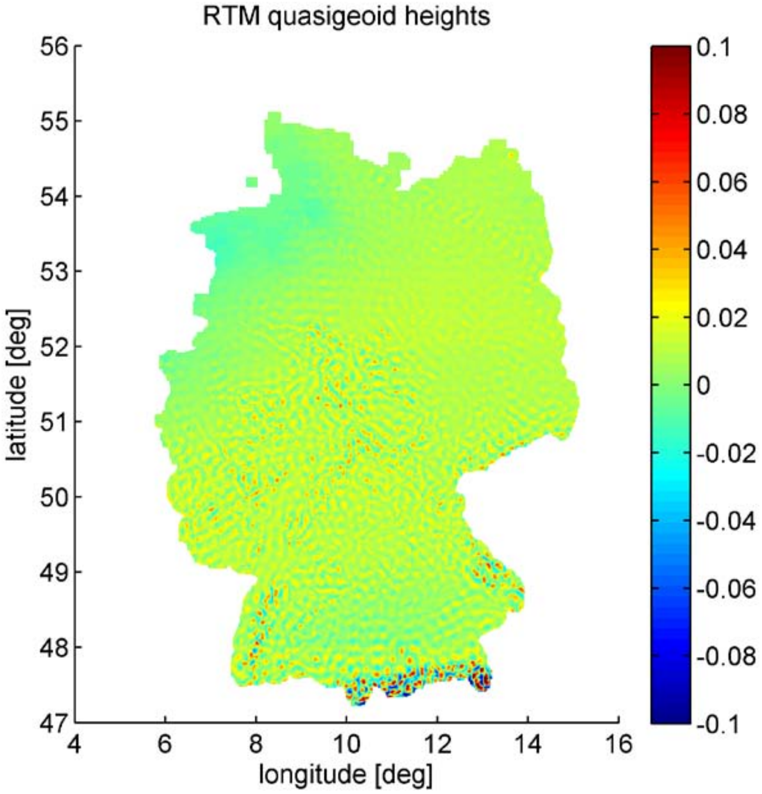

Fig. 4: RTM quasigeoid heights over Germany (unit in metres)

In this work, we use the 3 arc second SRTM (Shuttle Radar Topography Mission) data set (release 4.1 by Jarvis et al. 2008) as the high-resolution elevation model. The long-wavelength reference topography is provided by the spherical harmonic expansion of the DTM2006.0 data base (Pavlis et al. 2007, Saleh and Pavlis 2003), which is an associated EGM2008 product. Expanded to harmonic degree 2160, DTM2006.0 elevations 'remove' a large part of those gravity field signals from the SRTM topography, that are already implied by EGM2008 (Hirt 2010). The transformation of RTM elevations to RTM quasigeoid heights is accomplished using mass-density forward modelling (e.g., Torge 2001, p. 260, Forsberg 1984, Nagy et al. 2000, Hirt et al. 2010a) with software based on the TC program (Forsberg 1984) and a density assumption of constant mass-density of 2670 kg/m^3. The resulting RTM quasigeoid heights are the contribution of the RTM model topography to the EGM2008-omitted signals. Mainly because of short-scale (beyond EGM2008 resolution) mass-density anomalies in the real topography, the RTM approach only approximates the EGM2008 signal omission to some extent (Hirt 2010). To reduce the EGM2008 quasigeoid omission error, RTM quasigeoid heights are simply added to those from EGM2008 (Hirt et al. 2010b).

Fig. 3 shows the SRTM minus DTM20006.0 elevations over Germany and parts of the neighbouring countries. RTM quasigeoid heights were computed in terms of a high-resolution 0.3′×0.3′ grid (equivalent to a resolution of 550 m in latitude × 350 m in longitude) covering the whole of Germany (Fig. 4). Each RTM quasigeoid height originates from the evaluation of the SRTM-DTM2006.0 RTM data within 200 km radius around any computation point (extending the area shown in Fig. 3). Over the North Sea and Baltic Sea, DTM2006.0 and SRTM elevations were set to zero, so as to avoid artefacts coming from the bathymetry contained in DTM2006.0.

Over Germany, the RTM quasigeoid heights (Fig. 4) possess – on average – a signal strength of 1.3 cm (RMS, root mean square). In rugged terrain, such as the German Alps (South of 47.5° latitude), the amplitudes of the RTM quasigeoid are larger with maximum values of ~17 cm, while the RTM approach fails to model the omission error of EGM2008 over level terrain (Figs. 3 and 4).

4. Comparison data sets

As comparison data for an assessment of EGM2008 over Germany, this study utilizes quasigeoid heights (also denoted height anomalies) from the GCG05 quasigeoid model, from GPS/levelling and quasigeoid height differences from astronomical levelling. Because similar gravity data sets were likely used in the development of the EGM2008 and GCG05, these models are inevitably dependent to some extent. This is why GCG05 cannot be used for a truly independent assessment of EGM2008. Rather, the comparisons involving GCG05 are used to examine different EGM2008 evaluation variants, including RTM-based omission error corrections over a dense grid (Section 5). In contrast to GCG05, the GPS/levelling stations and astrogeodetic quasigeoid height differences are independent of EGM2008 and therefore a useful complement to the GCG05 comparisons. It should be noted that there exists a tight relation between the GPS/levelling set used here and GCG05 (described below).

4.1 The GCG05 quasigeoid model

The GCG05 (German Combined Quasigeoid 2005) quasigeoid model is the official height reference surface of the AdV (Arbeitsgemeinschaft der Vermessungsverwaltungen der Länder) and can be used for the conversion between ellipsoidal and physical heights over Germany (BKG 2006, Liebsch et al. 2006). GCG05 provides 120,530 quasigeoid heights on a grid of 1.0′×1.5′ (resolution of ~1.8 km in latitude × ~1.7 km in longitude). The accuracy of the GCG05 quasigeoid heights is specified to be 1-2 cm (BKG 2006). Locally, the accuracy of GCG05 quasigeoid height differences can be better than 1 cm (Hirt et al. 2007, Hirt et al. 2008), see also Sect. 5.3.

GCG05 is a gravimetric quasigeoid model that originates from two independent RCR-computations performed at Leibniz Universität Hannover (Institut für Erdmessung) and Bundesamt für Kartographie and Geodäsie (BKG). According to Liebsch et al. (2006), the model is based on ~430,000 gravity anomalies, high-resolution elevation data and ~900 GPS/levelling points. For the RCR-procedure, the EIGEN-CG01C global gravity field model (Reigber et al. 2006), expanded to degree 360, was used as reference. In addition to surface gravity data, this global model incorporates more than 2 years of CHAMP and 3.5 months of GRACE satellite gravity data (Reigber et al. 2006), conferring highly-accurate long- and medium-wavelength information to GCG05. The techniques used for the computation of the quasigeoid heights from the gravity anomalies are least-squares spectral combination (Leibniz Universität Hannover) and point mass adjustment (BKG). Both solutions were combined with GPS/levelling quasigeoid heights and arithmetically averaged to yield the GCG05 quasigeoid model (Schirmer et al. 2006, Liebsch et al. 2006).

4.2. Quasigeoid heights from GPS/levelling

A set of GPS/levelling points (Ihde and Sacher 2002) was kindly made available by BKG. This data set provides quasigeoid heights as the differences between GPS-observed ellipsoidal heights and spirit-levelled normal heights at 675 locations scattered over Germany. The GPS/levelling quasigeoid heights are independent of EGM2008 and can be assumed to be accurate to a few cm. This set was also used by Gruber (2009) for an evaluation of EGM2008 (without RTM augmentation).

The GCG05 model and the quasigeoid heights at the 675 GPS/levelling points are tightly related, but not identical, as explained next. The 675 GPS/levelling points (Ihde and Sacher 2002) form a subset of the ~900 GPS/levelling points (Liebsch et al. 2006), but were not directly used in the construction of the GCG05 model. Prior to the construction of GCG05, the ellipsoidal heights of the 675 GPS/levelling points were adapted to ETRS89 (European Terrestrial Reference System 89), as realised by the SAPOS (Satellitenpositionierungsdienst der deutschen Landesvermessungen) reference station network. The adaption of ellipsoidal heights was done in most different ways by the state survey agencies of Germany (Liebsch et al 2006, p. 135). Hence, the quasigeoid heights are different in both GPS/levelling sets (see also Liebsch et al. 2006 p. 136). As an immediate consequence, there exist small differences between the GCG05 quasigeoid heights and those of the 675 GPS/levelling points (Ihde and

Sacher 2002). Fig. 5 shows that these differences are at the level of a few cm (min/max/mean/rms: -5.3/9.1/0.5/1.5 cm), see also Liebsch et al. (2006), p. 136.

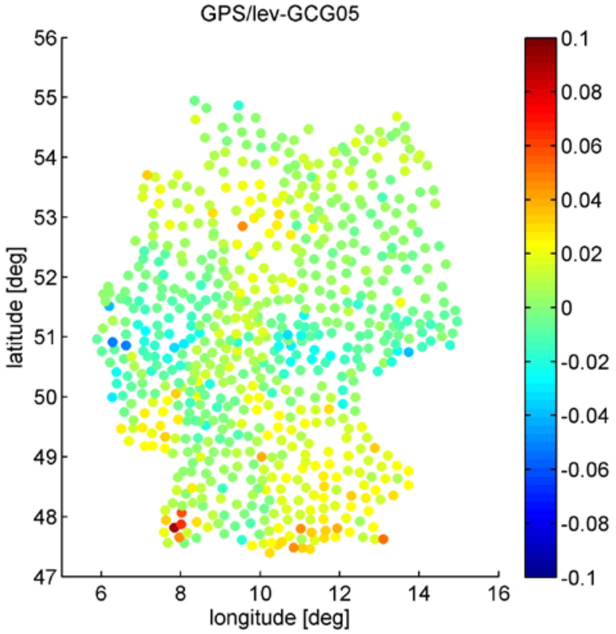

Fig. 5: Differences between quasigeoid heights of the 675 GPS/levelling points and GCG05 (unit in metres)

4.3. Astrogeodetic quasigeoid differences

Finally, this study uses two local profiles of highly-accurate quasigeoid height differences that were computed from astrogeodetic vertical deflections (Fig. 6). The vertical deflections were observed using the Hannover digital zenith camera (Hirt et al. 2010c) at densely-spaced stations. The first profile of 114 observed astrogeodetic stations over a distance of 63 km length (Fig. 6A) crosses the Harz Mountains in Northern Germany (Hirt et al. 2008). The second profile (Fig. 6B) is located in the Isar Valley, Bavaria, has a length of 23 km and consists of 103 observations (Hirt et al. 2007, Hirt and Flury 2008). In both test areas, the astrogeodetic vertical deflections were interpolated utilizing high-resolution elevation data and transformed to quasigeoid height differences by means of Helmert’s path integral (see Hirt and Flury 2008).

The accuracy of the astrogeodetic quasigeoid height differences was estimated to be 1-2 mm over the length of both profiles (Hirt et al. 2008, Hirt and Flury 2008). This makes both data sets well-suited for the local validation of EGM2008. It should be noted that both profiles were connected with additional vertical deflection observations to form a ~600 km North-South profile (Voigt et al. 2008, 2009). This data set was used for regional comparisons with EGM2008 (Berichte 2010, and Ihde et al. 2010), however without the omission error modelling as is done here.

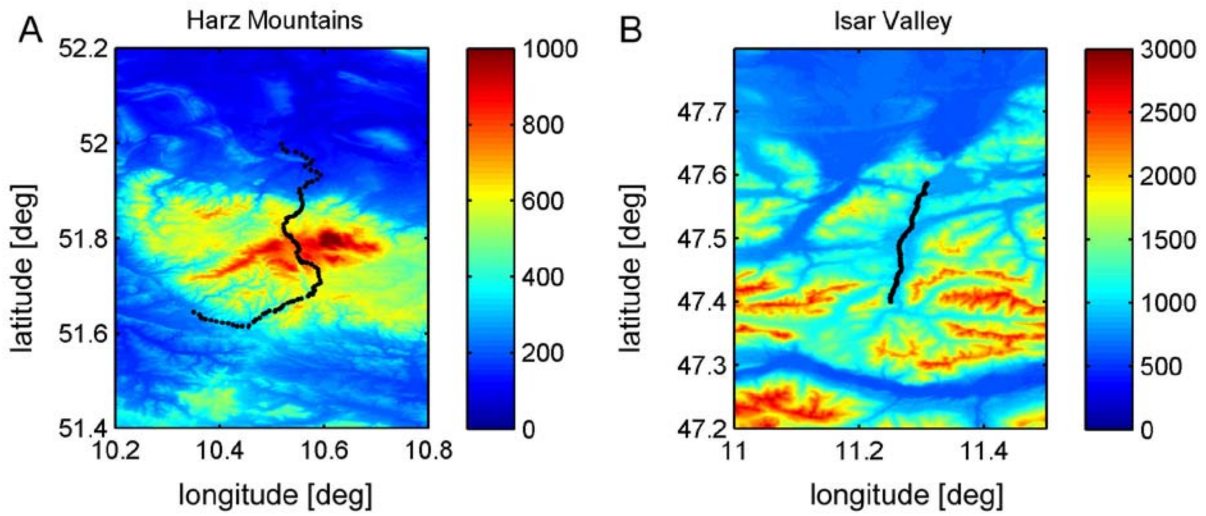

Fig. 6: Location of the astrogeodetic quasigeoid profiles. A: Harz Mountains profile, B: Isar Valley profile. The background topography are SRTM heights in metres.

5. Comparisons

5.1 EGM2008 vs. GCG05

The zero-tide[1] version of EGM2008 was evaluated with the scattered point option of the harmonic_synth software (Holmes and Pavlis 2008) over the spherical harmonic band from degree 2 to 2190 at the geodetic coordinates latitude and longitude of the 120,530 GCG05 grid points. As a first processing variant, a constant ellipsoidal height of 0 m (i.e., surface of the reference ellipsoid) was used. This replicates the case of using pre-calculated grids from the EGM2008 website. As a second processing variant, ellipsoidal heights of the topography were 'constructed' as the sum of SRTM elevations (in approximation, these are heights above mean sea level) and GCG05 quasigeoid heights and subsequently used in the synthesis procedure (cf. Claessens et al. 2009).

RTM quasigeoid heights were obtained for at the GCG05 grid points through interpolation of the 0.3′×0.3′ RTM quasigeoid grid (Fig. 3). The two synthesis variants and the optional consideration of RTM effects allow four different comparisons between GCG05 and EGM2008 (Fig. 7). The descriptive statistics of the differences are reported in Tab. 1. In each of the comparisons, the mean value of the differences was subtracted (known as 1-parameter or bias-fit) to eliminate the impact of different vertical datums (zero levels) and very long wavelength errors of the data sets (cf. Featherstone 2001 and Ihde et al. 2010).

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[1] Zero-tide means that the lunisolar permanent deformation of the Earth is included while the attraction effect is eliminated (Torge 2001, p. 77). The use of the zero-tide system follows a recommendation of the International Association of Geodesy (IAG) and is the preferred tide-system in practical quasi/geoid computations (e.g., Denker et al. 2009, Featherstone et al. 2010). Unfortunately, GCG05 cannot be considered a pure zero-tide model (Liebsch 2011, pers. comm.) which may cause small discrepancies in the comparisons. A detail discussion and analysis of the tide systems of the GCG05 input data sets is beyond the scope of the present study.

The comparison between GCG05 and EGM2008 evaluated at the ellipsoidal height = 0 m (Fig. 7A) shows RMS errors of 3.3 cm with maximum discrepancies of ~25 cm occurring in the German Alps, South of ~48°N. Evaluation of EGM2008 quasigeoid height at the ellipsoidal height of the topography (Fig. 7B) improves the agreement with GCG05 in the elevated or mountainous parts of Germany by ~5-10 cm. This is seen for the Harz Mountains (51.7°N, 10.5°E), the Black Forest (47.8°N, 8°E), and over wide areas of Bavaria. The largest improvement of up to ~20 cm is found over the German Alps.

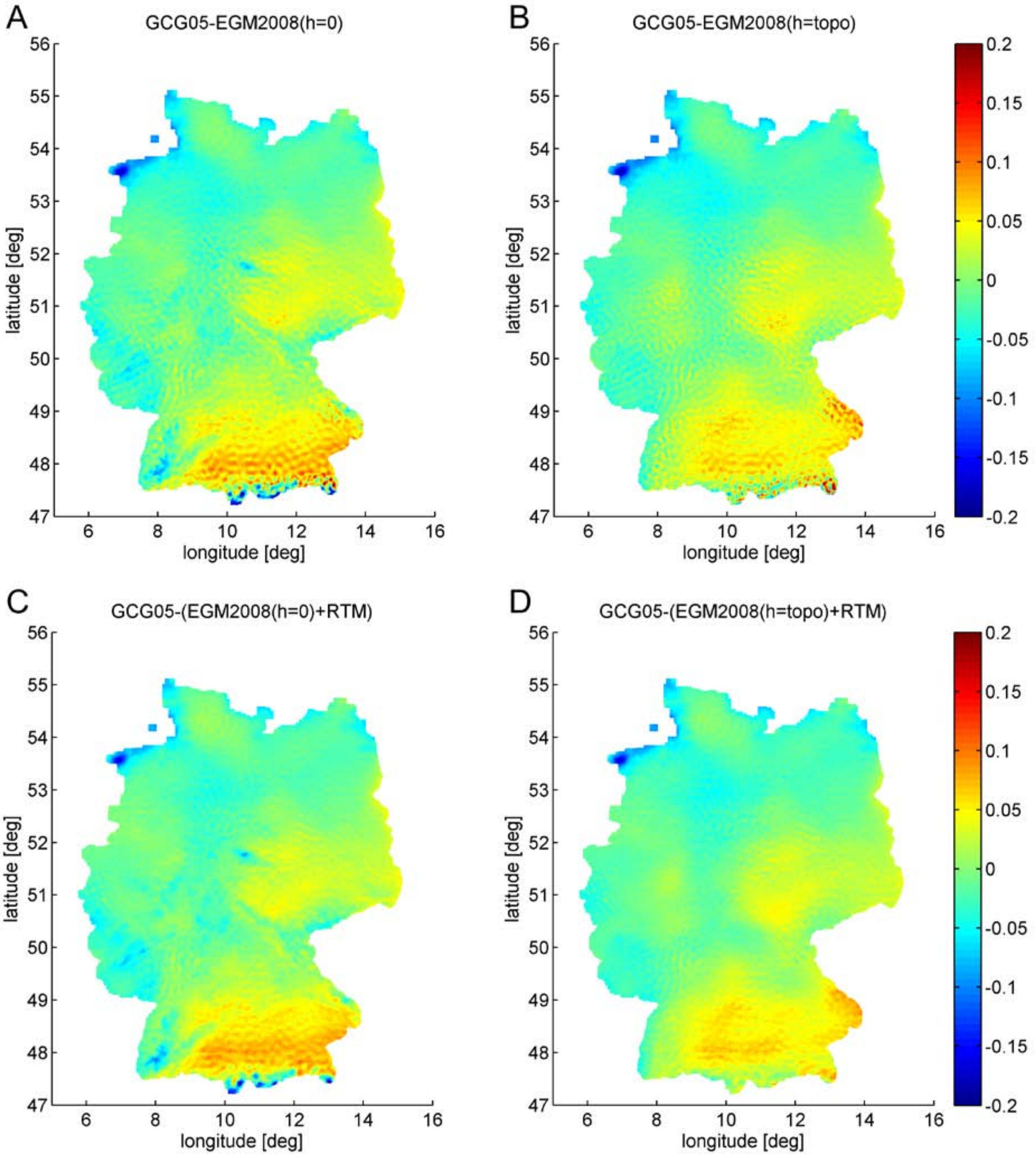

Fig. 7: Differences between the German Quasigeoid model GCG05 and variants of EGM2008. A: GCG05–EGM2008 (evaluated on the ellipsoid, h =0), B: GCG05–EGM2008

(evaluated at the ellipsoidal height of the topography), C: GCG05–[EGM2008 (evaluated on the ellipsoid, h =0) + RTM], D: GCG05–[EGM2008 (evaluated at the ellipsoidal height of the topography) + RTM]. Units in metres.

Tab. 1: Descriptive statistics of the quasigeoid differences between GCG05 and EGM2008 variants (bias-fit, 120,530 points)

Comparison	**Min [cm]**	**Max [cm]**	**RMS [cm]**
GCG05 – (EGM2008, h=0)	-24.7	24.1	3.3
GCG05 – (EGM2008, h=topo)	-19.1	23.5	3.2
GCG05 – [(EGM2008, h=0) + RTM]	-22.1	11.0	3.1
GCG05 – [(EGM2008, h= topo) + RTM]	-17.4	10.3	3.0

Additional consideration of RTM quasigeoid heights in the comparisons (Figs. 7C and 7D) reduces most of the short-wavelength (scales of ~10 km and below) error patterns seen previously. The most striking example are for the German Alps (see also Hirt et al. 2010a), but also for many other regions of Germany, except for the parts of Northern Germany with lower relief. The best model fit is observed for EGM2008 evaluated at the topography with the support of RTM quasigeoid heights beyond EGM2008's resolution (Fig. 7D). For this variant, the maximum differences are significantly reduced with respect to EGM2008-only (cf. Tab. 1), while the RMS differences only slightly improve to 3 cm. This behaviour is attributed to the medium-wavelength difference patterns (scales of ~100-200 km and larger), that are present in each of the four comparisons (Fig. 7) and are discussed in Section 5.2.

It should be noted that small-amplitude high-frequency difference patterns remain even in Fig. 7D where RTM quasigeoid heights were used to augment the EGM2008 resolution. These high-frequency effects which occur with wavelengths of ~20 km are further analysed in Section 5.3.

Given that the 3 cm RMS value reflects the commission errors of GCG05, EGM2008 and RTM quasigeoid heights, a good quality of the three data sets is indicated. However, GCG05 quasigeoid does not allow a truly independent validation of EGM2008 (see Sect. 4). Nonetheless, the comparisons provide a good feedback on the different gravimetric modelling strategies employed (harmonic analysis in case of EGM2008, and spectral combination/point mass adjustment for GCG05).

5.2 EGM2008 vs. GPS/levelling

For the comparisons with the GPS/levelling quasigeoid heights, EGM2008 and RTM quasigeoid heights were evaluated in the fashion described above. However, due to the precise ellipsoidal heights provided by the GPS component, a 'construction' of ellipsoidal heights was not necessary for the synthesis task. The descriptive statistics of the four different comparison variants among EGM2008 and the GPS/levelling set is given in Tab. 2. The difference patterns between GPS/levelling quasigeoid heights and the two selected variants 'EGM2008 (evaluated at height = 0 m)' and 'EGM2008 (evaluated at the topography) + RTM augmentation' are shown in Fig. 8. The difference patterns and behaviour of the statistics are fairly comparable to the GCG05 comparisons before (compare Tab. 1 and 2), which was expected due to the tight relation between the GPS/levelling set and the GCG05 model. Again, inclusion of RTM quasigeoid heights leads to a significant reduction in the extreme discrepancies, while there is only a small improvement in the RMS, from 3.6 cm to 3.3 cm.

Using the same GPS/levelling data set, Gruber (2010) found a similar RMS value (3.8 cm) for the EGM2008-only comparisons. For the GCG05 GPS/levelling set of ~900 points, Ihde et al. (2010) published a RMS value of 3.0 cm, reflecting the better quality of their newer GPS data.

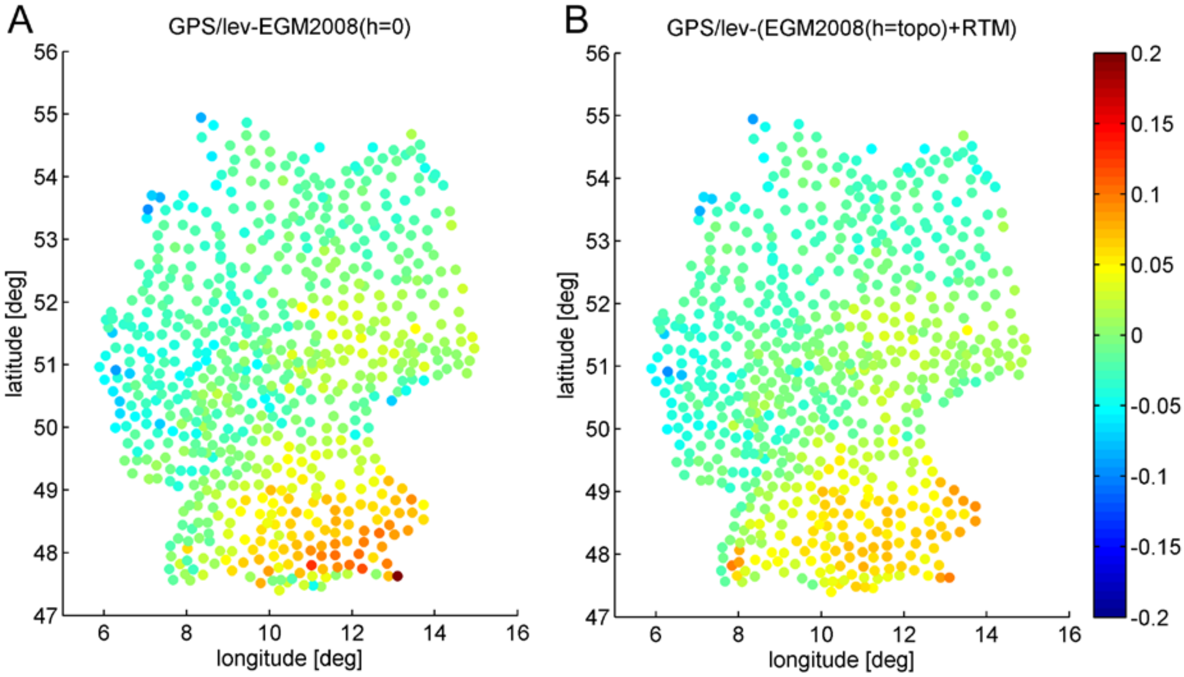

Fig. 8: Differences between GPS/levelling quasigeoid heights and variants of EGM2008. A: GPS/levelling – EGM2008 (evaluated on the ellipsoid), B: GPS/levelling – [EGM2008 (evaluated at the ellipsoidal height of the topography) + RTM]. Units in metres.

Tab. 2: Descriptive statistics of the quasigeoid differences between GPS/levelling and EGM2008 variants (bias-fit, 675 points)

Comparison	**Min [cm]**	**Max [cm]**	**RMS [cm]**
GPS/lev – (EGM2008, h=0)	-9.6	21.8	3.6
GPS/lev – (EGM2008, h= topo)	-10.5	18.0	3.6
GPS/lev – [(EGM2008, h=0) + RTM]	-8.8	13.3	3.5

GPS/lev – [(EGM2008, h= topo) + RTM]	-9.7	10.0	3.3

The comparison between EGM2008 and the truly-independent GPS/levelling heights exhibits medium-wavelength error patterns with coarsely 5 cm amplitude (e.g., yellow areas over Bavaria and Thuringia) which were similarly seen before in the GCG05 comparisons. Some correlations can be observed between the commission error map (Fig. 2) and the difference patterns in Fig. 7D. This, together with the 'official' accuracy estimates of the data sets in mind (Sect. 3 and 4), would suggest that these patterns reflect EGM2008 commission errors rather than those of the comparison data. However, there exist at least two further sources of error which may explain parts of the medium-wavelength error patterns.

First, inhomogenities in the GPS/levelling data, particularly in the ellipsoidal GPS heights, might be responsible, occurring as 'step-like effect' at some of the state boundaries, e.g. between Bavaria and Thuringia (Voigt et al. 2009; Feldmann-Westendorff 2010, pers. comm.). This effect might have propagated into the GPS/levelling set and GCG05 model, and, in turn, into the differences seen in Fig. 7 and 8. Within the framework of the current renewal of the German first-order levelling network (Deutsches Haupthöhennetz DHHN, e.g., Jahn 2010, Feldmann-Westendorff 2009, Feldmann-Westendorff and Jahn 2006), 250 high-quality GNSS/levelling stations (and 100 absolute gravity stations) will become available in the near future. The quasigeoid heights of the GNSS/levelling stations are based on state-of-the-art GNSS measurements and a re-observation of the first-order levelling lines. Owing to the expected level of accuracy (1 cm and better), this data set is likely to be suited for a future investigation of EGM2008 (and GCG05) commission errors in particular, and, of course, for evaluating future geopotential models in general.

Second, also the EIGEN-CG01C geopotential model used in the GCG05 construction might be a possible explanation for parts of the medium-wavelength differences in Fig. 7 (see also Ihde et al. 2010). However, because the difference patterns between EGM2008 and the GPS/levelling data (Fig. 8) are independent of the EIGEN-model, it can be concluded that EIGEN-CG01C is not the main contributor to the discrepancies. High-accuracy geopotential models, which are currently constructed based on the GOCE satellite gravity field mission (e.g., Rummel et al. 2009) can be expected to allow clarification of the medium-wavelength difference patterns. This is because of the expected geoid cm-accuracy over scales of ~100 km (e.g., Rummel 2005) and, importantly, because of the fact that the GOCE observations are independent of all data sets involved here (specifically EGM2008, GCG05 and the underlying geopotential models).

5.3 EGM2008 vs. astronomical levelling

Finally, the residuals between the highly-accurate astrogeodetic quasigeoid height differences and the four EGM2008 variants are shown in Fig. 9A for the Harz Mountains profile (Fig. 6A) and Fig. 9B for the Isar Valley profile (Fig. 6B). The descriptive statistics of the comparisons are given in Tab. 3 and 4. For the sake of completeness, the residuals with respect to GCG05 (cf. Hirt et al. 2007, 2008) are also displayed in Fig. 9, showing the very good agreement (better than cm-level) between the independent astrogeodetic and gravimetric quasigeoid solutions at local scales.

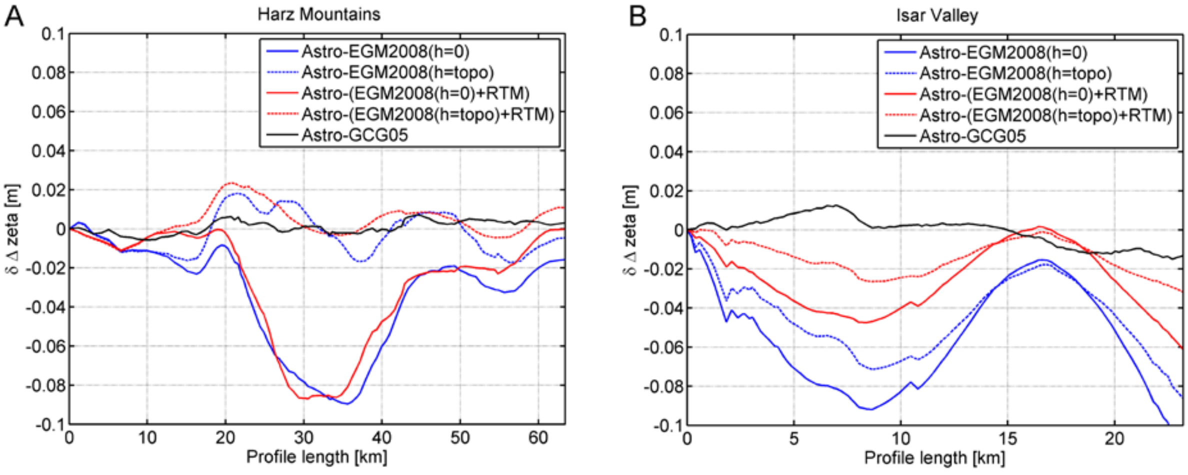

Fig. 9: Differences between the astrogeodetic quasigeoid heights and the EGM2008 variants (red, blue lines) and differences between the astrogeodetic quasigeoid heights and GCG05 (black lines). A: Harz Mountains, B: Isar Valley. The quasigeoid heights of the first profile points were set to zero for all data sets.

Tab. 3: Harz Mountains: Descriptive statistics of the quasigeoid differences between the astrogeodetic solution and EGM2008 variants (first station set to zero for any data set, 114 points)

Comparison	**Min [cm]**	**Max [cm]**	**Mean [cm]**	**RMS [cm]**
Astro – (EGM2008, h=0)	-9.0	0.3	-4.2	5.1
Astro – (EGM2008, h= topo)	-1.7	1.8	-0.0	1.1
Astro – [(EGM2008, h=0) + RTM]	-8.7	0.0	-3.7	4.8
Astro – [(EGM2008, h= topo) + RTM]	-1.2	2.4	0.5	1.0
Astro – GCG05	-0.6	0.7	0.1	0.3

Tab. 4: Isar Valley: Descriptive statistics of the quasigeoid differences between the astrogeodetic solution and EGM2008 variants (first station set to zero for any data set, 103 points)

Comparison	**Min [cm]**	**Max [cm]**	**Mean [cm]**	**RMS [cm]**
Astro – (EGM2008, h=0)	-11.5	0.0	-5.5	6.1
Astro – (EGM2008, h= topo)	-8.6	0.0	-4.4	4.8
Astro – [(EGM2008, h=0) + RTM]	-6.1	0.2	-2.5	3.0
Astro – [(EGM2008, h= topo) + RTM]	-3.2	0.0	-1.4	1.6
Astro – GCG05	-1.5	1.3	-0.1	0.8

Fig. 9 demonstrates the effect of not evaluating EGM2008 at the topography (solid versus doted lines). For both profiles, EGM2008, evaluated at the ellipsoidal height of the topography and augmented by RTM, produces the lowest RMS discrepancies of 1.0 cm (Harz) and 1.6 cm (Isar Valley). These comparisons show that EGM2008 – over well-surveyed areas and with augmentation of RTM data in mountainous terrain – is capable of delivering differences of quasigeoid heights near or at the cm-level. For other parts of Germany, this finding is corroborated by the structure of the difference patterns of the EGM2008/RTM comparisons with GCG05 and GPS/levelling (Figs. 7D and 8B). Over many regions, for instance large parts of Bavaria, the differences patterns are fairly constant, so would cancel out to some extent when quasigeoid height *differences* are computed from EGM2008. This demonstrates EGM2008 can be a source of quasigeoid height differences near the cm-level at local scales, over distances of few tens of km. However, GCG05 is an even more accurate source for quasigeoid height differences over Germany (Fig. 9).

For the comparisons involving EGM2008, oscillating differences with roughly ~20 km wavelength (i.e., the resolution of EGM2008) and amplitudes of ~2 cm are visible in Fig. 9. Similar high-frequency difference patterns occurred previously in the comparisons between GCG05 and EGM2008/RTM (Fig. 7D). This, together with the sub-cm agreement between GCG05 and the astrogeodetic solutions (Fig. 9) provides some evidence that the small high-frequency error patterns, as visible over parts of Germany (Fig. 7D) does not originate from GCG05, but from EGM2008 or from the RTM omission error corrections.

6. Conclusions

The present study evaluated the EGM2008 global geopotential model over Germany as an example region where dense gravity data sets were used for the model's development. For the EGM2008 evaluation, quasigeoid heights or quasigeoid height differences sourced from three different terrestrial data sets were used. To improve upon the short-wavelength signals, EGM2008 was augmented by quasigeoid heights from residual terrain model data. In elevated or mountainous terrain, this is efficient to reduce the omission error of the EGM2008 quasigeoid heights. The discrepancies with respect to GCG05 and GPS/levelling quasigeoid heights are at the level of 3 cm. Locally, say over distances of a few tens of km, EGM2008 (augmented by RTM) may deliver quasigeoid height differences near or at the cm-level, as was indicated by the comparisons with astrogeodetic data.

The comparisons involving the astrogeodetic data provide evidence that EGM2008, though being a good model over Germany, does not yet reach the quality of the GCG05 national quasigeoid model for quasigeoid height differences. The comparisons between EGM2008 and the three quasigeoid data sets show that the official EGM2008 commission error estimates (~5 cm for Germany) are too pessimistic. The medium-wavelength error patterns, which became visible in the comparisons between EGM2008 and GCG05 and EGM2008 and GPS/levelling could not be unambiguously attributed to one (or more) of the models used in this study. However, new data sets (quasigeoid heights from the DHHN renewal and from the GOCE mission) are expected to yield further insight into the discrepancies between EGM2008, GCG05 and GPS/levelling.

The different evaluation variants of EGM2008 used in this study have underlined the importance of using ellipsoidal heights of the topography in the synthesis of gravity field functionals. Evaluation at the ellipsoidal surface (ellipsoidal height = 0 m) may contaminate the computed quasigeoid heights by ~5-20 cm in elevated and mountainous areas of Germany. If EGM2008 is used for the prediction of quasigeoid heights or other functionals (e.g., gravity, vertical deflections) at the Earth's surface, some care should be exercised with pre-computed grids of EGM2008 functionals (and the use of the harmonic_synth software in grid mode), unless the influence of the topography is corrected otherwise.

As a general conclusion, the results of this study show the high quality of EGM2008 over densely surveyed regions and confirms the advancements made in global gravity field modelling, as demonstrated by development of EGM2008. While a similarly good quality is expected or indicated for other well-surveyed regions (see Newtons's Bulletin 2009), it should be noted that EGM2008 commission errors may be significantly higher in areas of poor gravity data coverage (cf. Pavlis et al. 2008).

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