

# Multi-Bernoulli Filter Based Sensor Selection with Limited Sensing Range for Multi-Target Tracking

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**Abstract**—In this paper, we consider a sensor network with limited sensing range and present a sensor selection algorithm for multi-target tracking problem. The proposed algorithm is based on the multi-Bernoulli filtering and a collection of sub-selection problems for individual target. A sub-selection problem for each target is investigated under the framework of partially observed Markov decision process. Each sub-selection problem is solved using a combination of information theoretic method and limited sensing range. Numerical studies validate the effectiveness of our method for multi-target tracking scenario in a sensor network.

**Keywords**—multi-target tracking, sensor network, hierarchical sensor selection, multi-Bernoulli filter

## I. INTRODUCTION

Sensor management is one of important tasks for efficient and accurate data processing in sensor networks [1]. Particularly, for multi-target tracking in sensor networks, intelligent sensor selection algorithm is necessary because the original problem is naturally combinatorial with respect to the number of targets and number of sensors. This paper aims at developing a sensor selection algorithm for multi-target tracking in order to achieve computational efficiency and desirable performance [4], [6]. To this end, a proper formulation of optimisation problem is required and it is closely related to underlying multi-object tracking algorithm and how to define the cost to optimise with respect to the tracking algorithm and sensor network topology.

Multi-target filtering has been independently investigated in literature and involves the joint estimation of the number of targets and their individual states from a sequence of observations in the presence of association uncertainty, detection uncertainty and clutter [2], [3], [13]. Conventional multi-target tracking

approach used in literature is a combination of single-target trackers, such as JPDA and MHT. However, these methods are not suitable for principled sensor selection because there is no way of formulating the selection cost that accommodates multi-target in a mathematical description.

Due to Mahler's Finite Set Statistic (FISST) [13], it is able to treat multi-sensor multi-target system in a single framework. FISST treats the collection of target states, referred to as the *multi-target state*, as a finite set. The probability hypothesis density (PHD) and cardinalized PHD (CPHD) filters have been proposed as approximate solutions to optimal Bayes multi-target filtering [10], [12]. Sequential Monte Carlo (SMC) and Gaussian mixture (GM) implementations of these filters [14], [15], [16] have opened the doors to many areas of applications and further research [17], [18], and [19]. Another type of approximation on FISST filter is the multi-Bernoulli filter, which propagates the parameters of a multi-Bernoulli RFS that approximates the posterior multi-target density [20]. A generalization of the filter known as the generalized labeled multi-Bernoulli filter, or simply the Vo-Vo filter, is a closed form solution to the Bayes multi-target filter that produces target tracks [21], [22].

Sensor selection for multi-target tracking in sensor networks has been recently investigated in a more principled way due to the advanced multi-target tracking methodology using random finite set that accommodates the multi-object filtering density. In recent literature the sensor management problem for multi-target tracking is posed as a Partially Observed Markov Decision Process (POMDP) using FISST [9]. There were several attempts to solve this problem in recent studies as follows. In [9], [13] the Kullback-Leibler discrimination and posterior expected number

of targets were proposed as objective functions for POMDP. Rényi divergence was utilized as the cost function in [23] the particle multi-object Bayes filter is employed to propagate the multi-object posterior, while in [24] the particle probability hypothesis density (PHD) filter is used for multi-target state estimation. In [25], [26] an efficient sensor management solution was proposed using the Rényi divergence and posterior cardinality as cost functions, while the multi-Bernoulli filter [20] was used for multi-target state estimation. For the closed form of implementation, Cauchy-Swarz divergence for poisson point process [27] was used to design the reward function for multi-target tracking [28]. In [29], the multi-Bernoulli filter also used with a different objective function. More recently in [30], sensor control using RFS has been investigated for more difficult scenarios where filter parameters are not known.

The problem addressed in this paper is a sensor management problem for multi-target systems with limited sensing range (LRS). When a sensor network monitors multiple objects with LRS, not all sensors but active sensors should be involved in the selection process in order to achieve computational efficiency and guarantee desirable performance. We adopt the multi-Bernoulli filter [20] as a multi-target filter because of its parametric structure that is suitable for proposed sensor selection algorithm in this paper. The objective function for the optimisation is parameterised using the expected probability of detection and the decomposition of the original problem using individual Bernoulli components obtained from multi-Bernoulli filter.

## II. MULTI-BERNOULLI FILTER WITH SEQUENTIAL MULTI-SENSOR UPDATE

In a physical sensor network, each sensor has a finite field of view (FoV) which represents its sensing range. Typically, the sensing range for distance range and bearing sensor is described as a region  $[-\phi_0, +\phi_0] \times [0, d_0]$  in polar coordinates. Each target moves according to the nearly constant velocity model:

$$\mathbf{x}_k = I_2 \otimes \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix} \mathbf{x}_{k-1} + I_2 \otimes \begin{bmatrix} \Delta^2/2 \\ \Delta \end{bmatrix} \mathbf{v}_k \quad (1)$$

where  $\mathbf{x}_k = [p_{x,k}, v_{x,k}, p_{y,k}, v_{y,k}]^T$ ,  $I_2$  is the  $2 \times 2$  identity matrix,  $\otimes$  denotes the Kronecker product;  $\Delta$  is the sampling period, and  $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$  is i.i.d. Gaussian noise. Without loss of generality, we assume that process noise is time-invariant zero-mean Gaussian, and identically distributed with  $\mathbf{Q}_k = \sigma_v^2 I_2$  where  $\sigma_v$  is the given standard deviation. Target generated observations from sensor  $l$  are noisy vectors of range and bearing  $\mathbf{z}_k^l$  given by

$$\mathbf{z}_k^l = h_k^l(\mathbf{x}) + w^l \quad (2)$$

where  $h_k^l(\mathbf{x}) = \left[ \|\mathbf{p}_k - \mathbf{s}_l\|, \arctan \frac{y_l - p_{y,k}}{x_l - p_{x,k}} \right]^T$ ,  $w^l$  is zero mean Gaussian noise, i.e.  $\mathcal{N}(w^l; 0, R^l)$ , and  $\mathbf{s}_l = [x_l$

$y_l]^T$  is the position of the sensor. The measurement noise  $w^l$  is composed of range noise  $w_r^l$  and bearing noise  $w_\phi^l$ . The standard deviation of  $w_r^l$  and  $w_\phi^l$  are respectively given by  $\sigma_r^l$  and  $\sigma_\phi^l$ , and  $R^l = \text{diag}([\sigma_r^l]^2, [\sigma_\phi^l]^2)$ .

Contrast to a single-object state system where objects are represented by random vectors, a multi-object system describes multiple objects by using a random finite set (RFS) of vectors. It is a natural representation of multi-object system because RFS constrains not only random vectors as elements of the set but also the number of objects as cardinality information of the set.

Suppose the multi-target state set and measurement set of all sensors at time  $k$ , are given by

$$\begin{aligned} X_k &= \{\mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,N(k)}\} \\ Z_k &= \{Z_k^1, \dots, Z_k^l, \dots, Z_k^m\} \end{aligned}$$

where  $N(k)$  is the cardinality of  $X_k$ ,  $m$  is the number of sensors,  $Z_k^l = \{\mathbf{z}_{k,1}^l, \dots, \mathbf{z}_{k,M^l(k)}^l\}$  is the measurement set of the  $l$ th sensor, with  $M^l(k)$  denoting the cardinality of  $Z_k^l$ .

The multi-target state  $X_k$  is composed of persisting targets  $S_k$  that survived from the previous time step and new births  $\Gamma_k$ , hence  $X_k = S_k \cup \Gamma_k$ . For each  $l = 1, \dots, m$ , each measurement set is represented by the union of two sets  $Z_k^l = \Theta_k^l \cup K_k^l$ , where  $\Theta_k^l$  is a set of observations generated by real targets and  $K_k^l$  is a set of false measurements, i.e., clutter.

We follow the notation from [20] for the multi-Bernoulli RFS as  $\pi = \{(r^{(j)}, p^{(j)})\}_{j=1}^M$ , where multi-target density  $\pi$  is represented by a set of pair of parameters: existence probability  $r$  and spatial density  $p$ . Thus, at time  $k$  the posterior multi-target density is a multi-Bernoulli RFS given by  $\pi_k = \{(r_k^{(j)}, p_k^{(j)})\}_{j=1}^{M_k}$ , and the density of new births is also multi-Bernoulli RFS given by  $\pi_{\Gamma,k+1} = \{(r_{\Gamma,k+1}^{(j)}, p_{\Gamma,k+1}^{(j)})\}_{j=1}^{M_{\Gamma,k+1}}$ , then the predicted density  $\pi_{k+1|k}$  is a union of two multi-Bernoulli RFSs as

$$\{(r_{S,k+1|k}^{(j)}, p_{S,k+1|k}^{(j)})\}_{j=1}^{M_k} \cup \{(r_{\Gamma,k+1}^{(j)}, p_{\Gamma,k+1}^{(j)})\}_{j=1}^{M_{\Gamma,k+1}} \quad (3)$$

At time  $k+1$ , if the predicted multi-target density is a multi-Bernoulli RFS  $\pi_{k+1|k} = \{(r_{k+1|k}^{(j)}, p_{k+1|k}^{(j)})\}_{j=1}^{M_{k+1|k}}$ , then the updated multi-Bernoulli density  $\pi_{k+1}$  is composed of legacy tracks  $\{(r_{L,k+1}^{(j)}, p_{L,k+1}^{(j)})\}_{j=1}^{M_{k+1|k}}$  and measurement-updated tracks  $\{(r_{U,k+1}(\mathbf{z}), p_{U,k+1}(\cdot|\mathbf{z}))\}_{\mathbf{z} \in Z_k}$ ,

$$\{(r_{L,k+1}^{(j)}, p_{L,k+1}^{(j)})\}_{j=1}^{M_{k+1|k}} \cup \{(r_{U,k+1}(\mathbf{z}), p_{U,k+1}(\cdot|\mathbf{z}))\}_{\mathbf{z} \in Z_k} \quad (4)$$

The multi-Bernoulli prediction (3) and update (4) can be implemented using Gaussian mixture or particle approximations as analogous to Gaussian mixture PHD [15] and particle filtering (i.e., SMC) [14] for PHD filters. In this paper we consider SMC approximation due to the nonlinearity of measurement model appeared in (2). For the details of the SMC

implementation, we refer the readers to subsection IV-A of [20]. In SMC implementations, the predicted multi-target density  $\{r_{k+1|k}^{(j)}, p_{k+1|k}^{(j)}\}_{j=1}^{M_{k+1|k}}$  is given as a weighted sum of dirac delta mass at particle locations

$$p_{k+1|k}^{(j)}(\mathbf{x}) = \sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} \delta_{\mathbf{x}_{i,k+1|k}^{(j)}}(\mathbf{x}). \quad (5)$$

where  $L_{k+1|k}^{(j)}$  is the number of particles for  $j$ th Bernoulli component;  $\omega_{i,k+1|k}^{(j)}$  represents the associate weight of  $i$ th particle from  $j$ th Bernoulli component. This equation will be used in next section for the expectation of detection probability.

The multi-Bernoulli density update (4) for sensor network applications can be implemented with different schemes considering given network topology and communication constraints. For example, measurements from all sensors in the network can be augmented as one single large vector measurement and processed at once. It is called a centralized scheme because all the measurements are transferred to the processing center and execute one multi-Bernoulli update. However, as a size of network grows and due to the non-fully connected network topology, it is necessary to develop efficient implementation for the multi-Bernoulli update.

Multiple multi-Bernoulli measurement update can be implemented using various schemes considering geographical sensor locations or communication bandwidth related to LSR. Several measurement update schemes such as parallel, sequential, and random update have been introduced and compared in [7]. Among them, we adopt the sequential update scheme due to the simple implementation. It is summarized in pseudo code in Algorithm 1.

### III. SENSOR SELECTION WITH DECOMPOSED POMDP

In this section, we briefly explain the POMDP framework and propose a decomposed approximation using multi-Bernoulli tracks and LSR network topology. Then, objective function based on the Rény divergence is subsequently explained.

#### A. Decomposed POMDP

The POMDP problem can be defined as the optimisation problem with expectation cost as follows.

$$\mathcal{S}_k = \arg \max_{U \in \mathbb{S}} \mathbf{E}[\mathcal{D}(U, f(X_{k-1}|Z_{0:k-1}, \mathcal{S}_{0:k-1}), Z_k)]. \quad (6)$$

where  $\mathcal{D}(U, f, Z)$  is the individual cost given  $U$ ,  $f$  and  $Z$ , where  $U$ ,  $f$ , and  $Z$  denote a possible set of sensors, the multi-target posterior density, and the associated measurement set to the set of sensors  $U$ , respectively. Notice that the expectation is with respect to the multi-object density and the general formulation of POMDP is a  $p$ -step future decision process, whereas, in this paper we only consider one-step future decision.

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#### Algorithm 1 Multi-sensor multi-Bernoulli filter with sequential update

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**Require:** multi-target density  $\pi_k = \{r_k^{(j)}, p_k^{(j)}\}_{j=1}^{M_k}$ , selected sensor set  $\mathbf{s}'_l (l = 1, \dots, m')$

- 1: **for**  $l = 1, \dots, m'$  **do**
- 2:     **if**  $l = 1$  **then**
- 3:         compute  $r_{S,k+1|k}^{(j,l)}, p_{S,k+1|k}^{(j,l)}(\mathbf{x}), r_{\Gamma,k+1}^{(j,l)}$   
 $p_{\Gamma,k+1}^{(j,l)}(\mathbf{x})$
- 4:         to obtain  $\pi_{k+1|k}^l$
- 5:         **else**
- 6:             pseudo-predict  $\pi_{k+1|k}^l = \pi_{k+1}^{l-1}$
- 7:         **end if**
- 8:         compute  $r_{L,k+1}^{(j,l)}, p_{L,k+1}^{(j,l)}(\mathbf{x})$  to obtain  $\pi_{L,k+1}^l$
- 9:         **for each**  $\mathbf{z}_{k+1}^l \in Z_k^l$  **do**
- 10:             compute  $r_{U,k+1}^{(j)}(\mathbf{z}_{k+1}^l), p_{U,k+1}^{(j)}(\mathbf{x}; \mathbf{z}_{k+1}^l)$
- 11:             to obtain  $\pi_{U,k+1}^l(\mathbf{z}_{k+1}^l)$
- 12:             **end for**
- 13:          $\pi_{k+1}^l = \pi_{L,k+1}^l \cup \pi_{U,k+1}^l(\mathbf{z}_{k+1}^l)$
- 14:     **end for**

**Ensure:**  $\pi_{k+1} = \pi_{k+1}^{m'}$

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In general solving (6) is intractable because all possible sensor combinations need to be explored, which is an NP-hard problem [31]. As a tractable suboptimal solution, we propose an approximated sensor selection scheme by decomposing the global POMDP with multi-target density into multiple POMDPs with independent track densities. A similar idea has been shown to be effective in robot navigation [32], [33].

Given the multi-target density  $\pi = \{(r^{(j)}, p^{(j)})\}_{j=1}^M$ , the objective function  $\mathbf{E}[\mathcal{D}(U, \pi, Z)]$  can be approximated by

$$\mathbf{E}[\mathcal{D}(U, \pi, Z)] \simeq \prod_{j=1}^M \mathbf{E}[\mathcal{D}(U, \{(r^{(j)}, p^{(j)})\}, Z)] \quad (7)$$

where  $M$  is a distinct number of Bernoulli components that represents independent target tracks. Thus, note that individual Bernoulli component  $\{(r^{(j)}, p^{(j)})\}$  is updated using a set of sensors with respect to the target  $j$ . Hence,  $\mathcal{S}_k \simeq \cup_{j=1}^M \mathcal{S}_k^{(j)}$ , where  $\mathcal{S}_k^{(j)}$  is the selected sensor set for the single target density of target  $j$ . The underlying rationale behind this decomposition is the assumption that the expectation with respect to the multi-Bernoulli density can be approximated by the product of expectations with respect to independent Bernoulli components if target states are well separated in the LSR sensor network. In order to obtain distinct Bernoulli components for independent target tracks, pruning and merging of multi-Bernoulli components are used and k-mean clustering with estimated cardinality that represents  $M$  target tracks.

#### B. Information Theoretic Method

The objective function plays a crucial role in POMDP based sensor selection problem. Information

theoretic method is a typical objective function for sensor management. Here, we propose to maximize the information gain of single object density as the subgoal of sensor selection, which is the product of existence probability and state distribution given in [13] as following

$$v_k^{(j)} = r_k^{(j)} \cdot p_k^{(j)} \quad (8)$$

for Bernoulli set  $\{r_k^{(j)}, p_k^{(j)}\}$ . Notice that the density is the product of a scalar and a probability distribution.

In this paper, Rényi divergence is used to design an objective function by measuring an information theoretic distance between the future updated posterior density and the predicted density. The Rényi divergence, also known as alpha divergence, measures the information gain between any two probability densities. Specifically, the objective function for target  $j$  with sensor  $l$  is defined as the Rényi divergence between the target information gain and the predicted information gain given as the follows.

$$\mathcal{D}_{k+1}^{(j),l} \triangleq \mathcal{R}^l(v_{k+1}^{(j),l} || v_{k+1|k}^{(j)}) = \frac{r_{k+1}^{(j),l}}{r_{k+1|k}^{(j)}} \mathcal{R}^l(p_{k+1}^{(j),l} || p_{k+1|k}^{(j)}) \quad (9)$$

where  $\mathcal{R}^l(p_{k+1}^{(j),l} || p_{k+1|k}^{(j)})$  is the Rényi divergence between the future measurement updated posterior and predicted distribution of target  $j$  denoted by  $\mathcal{R}^{(j),l}$  for short. Given the Bayesian recursion,  $\mathcal{R}^{(j),l}$  is given as following

$$\mathcal{R}^{(j),l} = \frac{1}{\alpha - 1} \log \frac{\int [g_{k+1}^l(\mathbf{z} | \mathbf{x}_{k+1}^{(j)})]^\alpha p_{k+1|k}^{(j)}(\mathbf{x}) d\mathbf{x}_{k+1}^{(j)}}{[p(\mathbf{z} | Z_k)]^\alpha} \quad (10)$$

where  $g_{k+1}^l(z|x)$  denotes the measurement likelihood function of the sensor  $l$  where the specific form is given in (2),  $p(\mathbf{z} | Z_k) = \int g_{k+1}^l(\mathbf{z} | \mathbf{x}_{k+1}^{(j)}) p_{k+1|k}^{(j)}(\mathbf{x}) d\mathbf{x}_{k+1}^{(j)}$  and  $\alpha$  is a parameter that determines how much we emphasize the tails of two density in the metric. The Rényi divergence becomes the Kullback-Leibler discrimination and Hellinger affinity respectively when  $\alpha \rightarrow 1$  and  $\alpha = 0.5$  [35].

In order to compute the expectation of (9), we adopt a predicted ideal measurement set proposed by Mahler [11] generate one future measurement  $\mathbf{z}$  for sensor  $l$  based on the predicted state assuming no clutter and unity detection rate as illustrated in [26], and  $r_{k+1}^{(j),l} = r_{U,k+1}^{(j),l}(\mathbf{z})$ , i.e., unity detection rate.

In the SMC implementation, the objective function is represented using the predicted multi-Bernoulli density as given in (5). Substitute (5) into (10), then, we obtain

$$\mathbf{E}[\mathcal{D}_{k+1}^{(j),l}] = \frac{r_{U,k+1}^{(j),l}(\mathbf{z})}{(\alpha-1)r_{k+1|k}^{(j)}} \log \frac{\sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} [g_{k+1}^l(\mathbf{z} | \mathbf{x}_{k+1}^{(j)})]^\alpha}{[\sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} g_{k+1}^l(\mathbf{z} | \mathbf{x}_{k+1}^{(j)})]^\alpha} \quad (11)$$

The benefits of maximising the information gain as objective function are two-fold: first, maximising the measurement-updated existence probability tends

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### Algorithm 2 Maximise Rényi divergence

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**Require:**  $\pi_{k+1|k}, \mathbf{s}_l (l = 1, \dots, m)$   
1: **for**  $j = 1, \dots, M_{k+1|k}$  **do**  
2:     **for**  $l = 1, \dots, m$  **do**  
3:         predict  $\mathbf{z} = \sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} h^l(\mathbf{x}_{i,k+1|k}^{(j)})$   
4:         compute  $\mathbf{E}[\mathcal{D}_{k+1}^{(j),l}]$  given by (11)  
5:     **end for**  
6:      $\mathcal{S}_{k+1}^{(j)} = \{\arg \max_{l \in \{1, \dots, m\}} \mathbf{E}[\mathcal{D}_{k+1}^{(j),l}]\}$   
7: **end for**  
**Ensure:**  $\mathcal{S}_{k+1} = \text{unique}(\cup_{j=1}^M \mathcal{S}_{k+1}^{(j)})$

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### Algorithm 3 Sensor selection by maximisation of detection probability

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**Require:**  $\pi_{k+1|k}, \mathbf{s}_l (l = 1, \dots, m)$   
1: **for**  $j = 1, \dots, M_{k+1|k}$  **do**  
2:     **for**  $l = 1, \dots, m$  **do**  
3:          $\mathbf{E}[\mathcal{P}_D^{l,(j)}] = \sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} p_{D,k}^l(\mathbf{x}_{i,k+1|k}^{(j)})$   
4:     **end for**  
5:      $\mathcal{S}_{k+1}^{(j)} = \{\arg \max_{l \in \{1, \dots, m\}} \mathbf{E}[\mathcal{P}_D^{l,(j)}]\}$   
6: **end for**  
**Ensure:**  $\mathcal{S}_{k+1} = \cup_{j=1}^M \mathcal{S}_{k+1}^{(j)}$

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to avoid losing targets; second, Rényi divergence between the predicted and updated distribution obtains more information from future measurements and makes target state estimation more accurate. Assume the sensor network contains  $m$  candidate sensors with fixed and known position  $\mathbf{s}_l (l = 1, \dots, m)$ , then Algorithm 2 provides the SMC implementation of sensor selection by maximising the proposed objective function. The *unique()* function eliminates repeatedly chosen sensor to ensure each sensor will be used in the update only once during information fusion process.

### C. Limited Sensing Range-based Method

As we consider the sensor network with LSR, the individual cost is defined using the probability of detection that is dependent on the pair of each track and each sensor. Thus, we optimise the expected cost of the probability of detection  $p_{D,k+1|k}$  for each pair of target track and sensor. Typically, the probability of detection for a sensor with bearing and range measurements decreases proportional to the target-sensor distance and reduces to zero if the target-

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### Algorithm 4 Sensor selection by information theoretic method and LSR

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**Require:**  $\pi_{k+1|k}, \mathbf{s}_l (l = 1, \dots, m)$   
1: Algorithm 3  
**Ensure:**  $\tilde{\mathcal{S}}_{k+1}$   
**Require:**  $\tilde{\mathcal{S}}_{k+1}$   
2: Algorithm 2  
**Ensure:**  $\mathcal{S}_{k+1}$

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sensor distance is larger than a threshold. Thus, we use a simple state dependent probability of detection model with minimum  $R_1$  and maximum  $R_2$  distance thresholds as

$$p_D^l(\mathbf{x}_k) = \begin{cases} q_{\max}, & \|\mathbf{p}_k - \mathbf{s}_l\| \leq R_1 \\ q_{\max} - \lambda \|\mathbf{p}_k - \mathbf{s}_l\|, & R_1 < \|\mathbf{p}_k - \mathbf{s}_l\| \leq R_2 \\ 0, & \|\mathbf{p}_k - \mathbf{s}_l\| > R_2 \end{cases} \quad (12)$$

where  $\lambda$  controls the shape of the profile. Then, the pseudo-code for the proposed sensor selection is given in the following. Here, the expectation of the probability of detection is approximated by SMC method as follows which is a convolution between the predicted spatial density (5) and the state dependent detection profile given in (12).

$$\mathbf{E}[P_D^{l,(j)}] = \sum_{i=1}^{L_{k+1|k}^{(j)}} \omega_{i,k+1|k}^{(j)} p_{D,k}^l(\mathbf{x}_{i,k+1|k}^{(j)}) \quad (13)$$

where  $L_{k+1|k}^{(j)}$  is the number of particles for  $j$ th target in the prediction step;  $(\omega_{i,k+1|k}^{(j)}, \mathbf{x}_{i,k+1|k}^{(j)})$  is the  $i$ th associate weight and particle for  $j$ th target.

#### D. Sensor Selection with Two Schemes

Aforementioned two selection schemes can be simply combined to enhance the performance. As the sensor selection scheme based on the probability of detection gives rough choices, it can be used as an initial solution for the information theoretic method. This combination of two schemes is described in Algorithm 4.

## IV. NUMERICAL STUDIES

In this section we present numerical results for a multi-target tracking scenario in an LSR sensor network, with  $9 \times 9$  sensors laid out uniformly over a square of size  $[-1000m, 1000m]^2$ .

Unknown time-varying number of targets are observed in clutter. Each target has a survival probability  $p_S = 0.95$ . New born targets appear spontaneously according to a Poisson RFS with intensity  $\gamma_k = \sum_{i=1}^3 0.2\mathcal{N}(\cdot; \bar{\mathbf{x}}_i, Q)$ , where  $Q = \text{diag}([(50m, 2m/s, 50m, 2m/s]^2)$  and

$$\begin{aligned} \bar{\mathbf{x}}_1 &= [200m; -10m/s; 600m; -20m/s]^T, \\ \bar{\mathbf{x}}_2 &= [-300m; 15m/s; 200m; -10m/s]^T, \\ \bar{\mathbf{x}}_3 &= [-850m; 25m/s; -450m; 5m/s]^T. \end{aligned}$$

Each target moves according to the constant velocity model given by (1) with standard derivation for the process noise  $\sigma_v = 1m/s$  for both  $v_{x,k}$  and  $v_{y,k}$ .

The sensing range of each node is  $[-\pi/2, +\pi/2] \times [0, 300m]$ , with clutter uniformly distributed over this area with rate  $\lambda_c = 5$  per scan. The detection profile (12) for each sensor is specified by  $q_{\max} = 0.99$ ,  $R_1 = 100m$ ,  $R_2 = 300m$ , and  $\lambda = 0.002m^{-1}$ . The measurement noise covariance for sensor  $l$  at time  $k$  is  $R_k^l = \text{diag}([\sigma_{r,k}^l]^2, [\sigma_{\phi,k}^l]^2)$ , where  $(\sigma_{r,k}^l)^2 = \sigma_0 +$

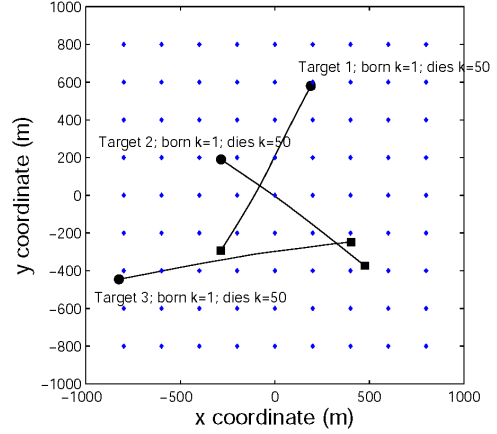


Fig.1. Target tracks. Start/stop positions for each track are shown with  $\bullet/\blacksquare$

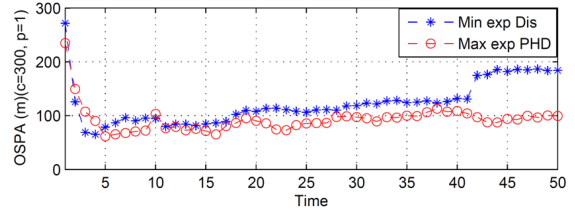


Fig.2. Comparison of OSPA distance

$\beta_r \|\mathbf{p}_k - \mathbf{s}_l\|^2$ , and  $(\sigma_{\phi,k}^l)^2 = \sigma_1 + \beta_\phi \|\mathbf{p}_k - \mathbf{s}_l\|$ , with  $\sigma_0 = 1m$ ,  $\beta_r = 5 \times 10^{-5}m^{-1}$ ,  $\sigma_1 = \pi/180rad$ ,  $\beta_\phi = 10^{-5}rad \cdot m^{-1}$ . Three targets appear in the scene as illustrated in Fig. 1. Since nearest-neighbouring strategy has been widely used in sensor selection, we implement minimum expected distance as another objective function for comparison. It is intuitive for this scenario that to select the proper sensor with respect to a target should choose the nearest sensor that target falls in its FoV. The Optimal Subpattern Assignment (OSPA) metric composed of location error and cardinality error, is adopted for tracking performance evaluation [34]. Fig. 2 shows the OSPA distance ( $c = 300m$ ,  $p = 1$ ) comparison from 500 Monte Carlo runs. It is obvious that the proposed tracker with maximizing the expected information gain of PHD is better than nearest-neighbouring strategy. Our hierarchical selection with maximizing the PHD can choose proper sensors to perform effective multi-target tracking. However, the nearest-neighbouring strategy ignores sensor's FoV and may choose sensors cannot observe targets instead.

Both algorithms were implemented in MATLAB R2012a on a computer with an IntelCore E5500 CPU and 2GB of RAM. A comparison of the two objective functions with regard to the total sample size and computation time shows that the probability of detection based selection is about 20% cheaper.

## V. CONCLUSION

This paper represents an efficient sensor selection approach using decomposed global objective function. The proposed decomposed sensor selection approach considers suboptimization problems for individual targets to avoid combinatorial search over all sensor combinations in the original global problem. The multi-sensor multi-target tracking is performed using the sequential multi-Bernoulli filter. A combination of information theoretic method and LSR based method is proposed. Simulations shows encouraging results.

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