SHEAR WAVE DISPERSION AND ATTENUATION IN PERIODIC SYSTEMS OF ALTERNATING SOLID AND VISCOUS FLUID LAYERS

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Abstract

The attenuation and dispersion of elastic waves in fluid-saturated rocks due to the viscosity of the pore fluid is investigated using an idealized exactly solvable example of a system of alternating solid and viscous fluid layers. Waves in periodic layered systems at low frequencies are studied using an asymptotic analysis of Rytov's exact dispersion equations. Since the wavelength of shear waves in fluids (viscous skin depth) is much smaller than the wavelength of shear or compressional waves in solids, the presence of viscous fluid layers necessitates the inclusion of higher terms in the long-wavelength asymptotic expansion. This expansion allows for the derivation of explicit analytical expressions for the attenuation and dispersion of shear waves, with the directions of propagation and of particle motion being in the bedding plane. The attenuation (dispersion) is controlled by the parameter which represents the ratio of Biot's characteristic frequency to the viscoelastic characteristic frequency. If Biot's characteristic frequency is small compared with the viscoelastic characteristic frequency, the solution is identical to that derived from an anisotropic version of the Frenkel-Biot theory of poroelasticity. In the opposite case when Biot's characteristic frequency is greater than the viscoelastic characteristic frequency, the attenuation/dispersion is dominated by the classical viscoelastic absorption due to the shear stiffening effect of the viscous fluid layers. The product of these two characteristic frequencies is equal to the squared resonant frequency of the layered system, times a dimensionless proportionality constant of the order 1. This explains why the visco-elastic and poroelastic mechanisms are usually treated separately in the context of macroscopic (effective medium) theories, as these theories imply that frequency is small compared to the resonant (scattering) frequency of individual pores.

Keywords: Attenuation; Dispersion; Viscous fluid; Shear wave; Poroelasticity

1. Introduction

Phenomena associated with the viscosity of the pore fluid represent one of the main causes of the attenuation and dispersion of elastic waves in reservoir rocks and other fluid-saturated porous materials. However, there is some uncertainty as to the relationship between the two viscosity-related mechanisms: viscoelastic mechanism (viscous shear relaxation) and Frenkel-Biot poroelastic mechanism (Frenkel, 1944, Biot, 1956, 1962).

In this paper we investigate the effect of these two mechanisms using an idealized exactly solvable example of a system of periodically alternating fluid and solid layers. Although such a configuration is obviously unrealistic, it possesses a number of the key features of real porous materials. So far, most of the research on such a periodic layered system has been focused either on ideal and low viscosity fluids and relatively high frequencies (Schoenberg, 1984, Schoenberg and Sen, 1986, Molotkov and Bakulin, 1996), or on the low-frequency asymptotes (Gurevich, 1999, 2002). In this paper we present a new approach that enables an explicit analysis of solid/viscous fluid layers in a broad range of frequencies and of fluid viscosities.

The properties of waves in periodic layered systems at low frequencies can be studied using a long-wave asymptotic analysis of the known exact dispersion equations (Rytov, 1956, Brekhovskikh, 1981, Christensen, 1979). For the asymptotic analysis to be valid, the wavelengths of all waves must be greater than the spatial period of the periodic system. Note that the wavelength of shear waves in the fluid (viscous skin depth) is much smaller than the wavelength of shear waves in the solid, or of acoustic waves in the fluid. This means that the presence of viscous fluid layers requires a careful evaluation of all terms in asymptotic expansions as functions of frequency, layer thickness and fluid viscosity.

2. Long-wave dispersion equation

Figure 1 shows a system of periodically alternating solid and fluid layers of period d. The elastic solid has density ρ_s, bulk modulus K_s and shear modulus μ_s. The fluid is assumed Newtonian with density ρ_f, bulk modulus (inverse compressibility) K_f, and dynamic viscosity η. The solid and fluid layer thicknesses are h_s and h_f, respectively, so that $h_s + h_f = d$.

We analyze the propagation of a shear wave in the x direction parallel to the layering, with the displacement in the direction y normal to x but also parallel to the bedding (SH wave). For a given frequency ω the solution can be sought in the form of a plane wave $u_y = u_{y0} \exp i(ax - \omega t)$. We seek to obtain the phase velocity $c = \omega / a$ as a function of ω for long waves, i.e., for frequencies where $|a(\omega)d| \ll 1$. To do this, one can utilize known results for solid layered systems, regarding the viscous fluid as another solid with a complex shear modulus $\mu_f = -i\omega\eta$. Propagation of an SH wave in a periodic system of solid layers denoted by s and f is governed by Rytov's exact dispersion equation (Rytov, 1956, Brekhovskikh, 1981):

$$p\left[\tan^2\frac{\beta_s h_s}{2} + \tan^2\frac{\beta_f h_f}{2}\right] + (1+p^2)\tan\frac{\beta_s h_s}{2}\tan\frac{\beta_f h_f}{2} = 0., \qquad (1\ )$$

Here $\beta_s^2 = \omega^2\left(1/c_s^2 - 1/c^2\right)$, $\beta_s^2 = \omega^2\left(1/c_s^2 - 1/c^2\right)$, $\beta_f^2 = \omega^2\left(1/c_f^2 - 1/c^2\right)$, where $c_s = \left(\mu_s / \rho_s\right)^{1/2}$, and $c_f = \left(\mu_f / \rho_f\right)^{1/2}$ are shear velocities in the materials s and f, respectively, and $p = \mu_f \beta_f / \mu_s \beta_s$.

Our aim is to solve the dispersion equation (1) on a macroscale, that is for long waves. In this case the arguments of the tangents in equation (1) are small, and the tangents can be replaced by their respective arguments. The resulting equation

$$p\left[\left(\frac{\beta_s h_s}{2}\right)^2 + \left(\frac{\beta_f h_f}{2}\right)^2\right] + (1+p^2)\frac{\beta_s h_s}{2}\frac{\beta_f h_f}{2} = 0 \qquad (2\ )$$

can be solved analytically to give a simple averaging formula

$$c^2 = \frac{h_s\mu_s + h_f\mu_f}{h_s\rho_s + h_f\rho_f}. \qquad (3\ )$$

Assuming that layers of type f are composed of Newtonian fluid, we can write its shear modulus as $\mu_f = -i\omega\eta$ this yields

$$c^2 = \frac{(1-\phi)\mu_s - i\omega\eta\phi}{\rho} = c_0^2\left(1 - \frac{\phi}{1-\phi}\frac{i\omega\eta}{\mu_s}\right), \tag{4 }$$

where $\phi = h_f / d$ is the volume fraction of the fluid layers (porosity), $\rho = (1-\phi)\rho_s + \phi\rho_f$ is the average density, and

$$c_0 = \lim_{\omega\to 0} c = \sqrt{\frac{(1-\phi)\mu_s}{\rho}} \tag{5 }$$

is the static shear velocity in the system. Due to the effect of viscosity, the velocity is now complex, implying that attenuation will occur.

Equation (4) is the result given in textbooks and is termed as the low-frequency, or long-wavelength approximation (see e.g., Brekhovskikh, 1981) with the obvious requirement that $\beta_s h_s$ and $\beta_f h_f$ must be small. However, the wavelength of the viscous wave in the fluid is much shorter than that of the shear wave in the solid. Thus, the decrease of frequency ω also increases the relative magnitude of the terms containing β_f, and thus simple replacement of tangents by their arguments is no longer possible. Furthermore, the replacement of $\tan\left(\beta_f h_f / 2\right)$ by $\beta_f h_f / 2$ implies that thickness of the fluid layers is small compared with the viscous skin depth in the fluid (wavelength of the viscous wave). This unnecessarily restricts the range of frequencies or fluid viscosities. To avoid these restrictions, it is necessary to carefully evaluate all the terms in asymptotic expansions as functions of frequency, layer thickness, and fluid viscosity. Therefore, we define two characteristic frequencies: the viscoelastic characteristic frequency

$$\omega_V = \frac{\mu_s}{\eta}, \tag{6 }$$

which is the frequency at which the absolute value of the complex shear modulus of the viscous fluid equals the solid shear modulus. In turn,

$$\omega_B = \frac{\eta}{\rho_f h_f^2}, \tag{7 }$$

defines a so-called Biot's characteristic frequency, at which the wavelength of the viscous wave (viscous skin depth in the fluid) equals the thickness of the fluid layers h_f. By introducing the permeability of the system of parallel slits (Biot, 1956, Bedford, 1986):

$$\kappa = \frac{\phi h_f^2}{12}, \tag{8 }$$

ω_B can be also written as

$$\omega_B = \frac{\eta\phi}{12\rho_f \kappa}. \tag{9 }$$

The expressions for the two characteristic frequencies may be multiplied to give

$$\omega_V \omega_B = \frac{\mu_s}{h_f^2 \rho_f} = A\omega_r^2, \tag{10 }$$

where $A = \rho / 4\pi^2\phi^2(1-\phi)\rho_f$ is a dimensionless parameter of order 1 depending only on the porosity and the ratio of solid-to-fluid densities, while ω_r is the fundamental frequency of the layered periodic system,

$$\omega_r = \frac{2\pi c_0}{d}, \tag{11 }$$

at which the wavelength of the shear wave equals the period d of the system.

The three characteristic frequencies ω_V, ω_r and ω_B lead to the introduction of three dimensionless frequencies:

$$\Omega_V = \omega / \omega_V = \omega\eta / \mu_s, \tag{12 }$$

$$\Omega_r = \omega / \omega_r = \omega d / 2\pi c_0, \tag{13 }$$

and

$$\Omega_B = \omega / \omega_B = \omega h_f^2 \rho_f / \eta. \tag{14 }$$

Note that for our long-wave analysis, Ω_r is small since the period d is small compared with the wavelength $\lambda = 2\pi c_0 / \omega$. Furthermore, Newtonian fluid model can only be valid if $i\omega\eta \ll K_f$ (Landau and Lifshitz, 1987). Assuming that K_f is of the same order as the shear modulus μ_s of the solid layers, we can conclude that $\Omega_V = \omega\eta / \mu_s$ must also be small. Seeking a solution to the dispersion equation (1) as a Taylor series in Ω_V and Ω_r, we obtain the following expression for the complex velocity of the shear waves propagating parallel to the layering (Appendix A)

$$\frac{1}{c^2} = \frac{\rho_s(1-\phi) + \rho_f \phi L^{-1} \tan L}{\mu_s(1-\phi)} \left[1 + i\Lambda\Omega_V \frac{\phi}{1-\phi}\right], \tag{15 }$$

or

$$\frac{1}{c^2} = \frac{1}{c_0^2}\left[1 + \frac{\phi\rho_f}{\rho}\left(\frac{\tan L}{L} - 1\right)\right]\left(1 + i\frac{\phi}{1-\phi}\Lambda\Omega_V\right). \tag{16 }$$

where

$$L = \frac{1}{2}\left(i\Omega_B\right)^{1/2} = \frac{h_f}{2}\left(\frac{i\omega\rho_f}{\eta}\right)^{1/2} \tag{17 }$$

and

$$\Lambda = \frac{1 + L^{-1}\tan L + \tan^2 L}{2}. \tag{18 }$$

Equation (15) is the central result of this paper. It expresses the long-wave velocity

$$v = (\mathrm{Re}\, c^{-1})^{-1} \tag{19 }$$

and attenuation

$$Q^{-1} = \frac{\mathrm{Im}\, c^{-2}}{\mathrm{Re}\, c^{-2}} \tag{20 }$$

of the shear wave propagating parallel to the layering, as a function of the frequency ω. This frequency dependence is controlled by two physical mechanisms associated with two characteristic frequencies ω_V and ω_B. The long-wave approximation means equation (15) is valid for frequencies both below the fundamental frequency, $\omega \ll \omega_r$, and the characteristic viscoelastic frequency, $\omega \ll \omega_V$. As mentioned earlier, this last condition must be satisfied for the fluid to be considered Newtonian.

3. Asymptotic analysis

To analyze the dispersion equation (15), we define a fundamental parameter of the layered solid/fluid system

$$B = \frac{\omega_B}{\omega_V} = \frac{\eta^2 \phi}{12 \mu_s \kappa \rho_f}. \tag{21 }$$

This parameter shows which of the two viscosity-related dissipation mechanisms dominates at frequencies $\omega \ll \omega_r$, when the macroscopic description is realistic. We emphasize that the parameter B does not depend on the frequency, but only on the physical and geometrical properties of the layered system (or a porous rock). For a permeability of 1 darcy and a viscosity of that of water, the parameter B is about 10^{-8}. However, this may be much larger for more viscous fluids (heavy oil, bitumen) and/or lower permeabilities.

Note that $L \to 0$, $\tan L \to 0$, $L^{-1} \tan L \sim 1 + L^2/3$ for low frequencies, $\omega \ll \omega_B$, and $L^{-1} \to 0$, $\tan L \to i$, $L^{-1} \tan L \to 0$ at higher frequencies $\omega \gg \omega_B$.

3.1. Poroelastic case

For high-permeability reservoir rocks and soils, $B \ll 1$,

$$\omega_B < \sqrt{A}\omega_r < \omega_V \tag{22 }$$

so that Ω_V is negligible compared to Ω_B. Thus equation (15) reduces to

$$\frac{1}{c^2} = \frac{\rho_s (1-\phi) + \rho_f \phi L^{-1} \tan L}{\mu_s (1-\phi)}. \tag{23 }$$

Equation (23) can be rewritten in the form

$$\frac{1}{c^2} = \frac{1}{\mu}\left[\rho - \frac{\rho_f^2\phi^2}{\tilde{\rho}_{12}(\omega)}\right]. \tag{24 }$$

In equation (24)

$$\mu = \mu_s\left(1-\phi\right) = \rho c_0^2 \tag{25 }$$

is the static shear modulus of the system, and $\tilde{\rho}_{12}$ is the generalized virtual mass coefficient of the porous medium, which, for a system of plane slits, is given by Bedford (1986). In our notation $\tilde{\rho}_{12}$ can be written in the form

$$\tilde{\rho}_{12}(\omega) = \phi\rho_f\left(1 - L^{-1}\tan L\right)^{-1}. \tag{26 }$$

Equation (24) is identical to the dispersion equation for shear waves in a saturated porous medium with the virtual mass (26) and frame shear modulus (25), as obtained from the theory of poroelasticity (see e.g. Berryman, 1985). Thus, when viscoelastic effects are negligible, $\Omega_V \to 0$, the shear wave attenuation and dispersion as described by equation (15) are identical to the attenuation and dispersion predicted by the theory of poroelasticity.

In particular, the attenuation $1/Q$ corresponding to the dispersion equation (23) is

$$Q^{-1} = \frac{\rho_f\phi}{\mu_s\left(1-\phi\right)}\operatorname{Im}\left(L^{-1}\tan L\right) \tag{27 }$$

Note that the characteristic frequency of this attenuation mechanism is controlled by the frequency dependence of $\tilde{\rho}_{12}$, and equals ω_B.

In the theory of poroelasticity the viscoelastic phenomenon is ignored as the fluid shear stress is neglected in the microscopic (pore-scale) constitutive equations. Pride et al. (1992) analyzed the effect of this approximation and showed that it requires the parameter $\Omega_V = \omega\eta / \mu_s$ to be small. Indeed, if Ω_V is very small, the viscoelastic attenuation is also very small, (equation (16)). However, if at the same time the parameter Ω_B is even smaller than Ω_V, i.e., $\Omega_B < \Omega_V \ll 1$, the poroelastic effects on attenuation would be even less pronounced than the viscoelastic ones. The condition on which the viscoelastic attenuation can be neglected *relative* to the poroelastic attenuation is $B = \Omega_V / \Omega_B \ll 1$. And, most importantly, this condition involves parameters pertaining to the medium only, and is independent of the frequency. If this condition holds for a particular medium, poroelasticity theory would apply for all frequencies below the resonant frequency of the individual pores. This is consistent with the observations of Bedford (1986), who compared numerically the solutions of the exact dispersion equation for a layered solid/fluid system (with very small parameter B) with the prediction of Frenkel-Biot theory of poroelasticity, and found an excellent agreement in a wide frequency range. This is not surprising. Schoenberg and Sen (1986) and Molotkov and Bakulin (1996) showed analytically that in the case of low viscosity $\Omega_V \ll \Omega_B$ and small viscous skin depth $\Omega_B \ll 1$, the exact constitutive equations for a solid/fluid layered medium represent a partial case of anisotropic equations of poroelasticity.

3.2. Viscoelastic case

For low-permeability reservoir rocks and soils, such as clays, and for porous rocks saturated with very viscous fluids, such as bitumen, $B \gg 1$, and

$$\omega_V < \sqrt{A}\omega_r < \omega_B \qquad (28\ )$$

so that at frequencies below ω_r the ratio Ω_B is negligible compared to Ω_V. Then equation (15) reduces to

$$\frac{1}{c^2} = \frac{\rho}{\mu_s\left(1-\phi\right)}\left[1 + i\Omega_V \frac{\phi}{\left(1-\phi\right)}\right]. \qquad (29\ )$$

Equation (29) is equivalent to the classical viscoelastic solution (4) with attenuation given by

$$Q^{-1} = \Omega_V \frac{\phi}{1-\phi} = \frac{\omega\eta\phi}{\mu_s(1-\phi)}. \qquad (30\ )$$

3.3. Low frequencies

Suppose that both characteristic frequencies ω_B and ω_V are large and are comparable to each other. Then, taking in equation (15) the limit of low frequencies, we obtain

$$\frac{1}{c^2} = \frac{\rho}{\mu_s(1-\phi)}\left(1 + i\Omega_V \frac{\phi}{1-\phi} + i\Omega_B \frac{\phi\rho_f}{12\rho}\right). \qquad (31\ )$$

This equation extends the viscoelastic result (29) to low but non-zero Ω_B, and is equivalent to the low-frequency asymptotic analysis of Gurevich (1999). In particular, it shows that, at low frequencies, the viscoelastic and poroelastic attenuation effects are additive,

$$Q^{-1} = \Omega_V \frac{\phi}{1-\phi} + \Omega_B \frac{\phi\rho_f}{12\rho} = \frac{\omega\eta\phi}{\mu_s(1-\phi)} + \frac{\omega\rho_f^2\kappa}{\rho\eta}. \qquad (32\ )$$

3.4. Static limit

In the limit of zero frequency ($\omega = \Omega_V = \Omega_B = 0$), equation (15) gives

$$c_0^2 = \frac{(1-\phi)\mu_s}{\rho}, \qquad (33\ )$$

which is the same as the static result (4).

3.5. General case

Finally, suppose that the parameter B is of order 1. This is an intermediate situation, when all three frequencies ω_B, ω_r and ω_V are of the same order of magnitude. The parameter ω_r is primarily controlled by the dominant grain size of the rock, and thus is very high (1 MHz for grains smaller than 1mm size). As the poroelastic and viscoelastic effects are controlled by the ratios ω / ω_B, ω / ω_V, it is clear that at typical seismic exploration frequencies both effects are negligible. These effects may become important, however, at the ultrasonic frequencies used in sonic logs and laboratory experiments. In these cases the contributions of the viscoelastic and poroelastic effects are comparable, and the general relation (15) which accommodates both, should be used. However, as noted before, due to the limitations of the Newtonian fluid model, $\omega_V \ll 1$, and thus $\omega_B \ll 1$ as well. Therefore, Λ as given by equation (17) can be replaced by its low-frequency value,

$$\Lambda = 1, \tag{34 }$$

and equation (15) reduces to

$$\frac{1}{c^2} = \frac{\rho_s (1-\phi) + \rho_f \phi L^{-1} \tan L}{\mu_s (1-\phi)} \left[1 + i\Omega_V \frac{\phi}{1-\phi} \right]. \tag{35 }$$

Moreover, the simplified equation (35) is valid for any relation between the characteristic frequencies (that is, for any value of B). Indeed, it has just been shown to be valid for $B \sim 1$. For $B \gg 1$, $\Lambda = 1$ and equations (15) and (35) both reduce to the same asymptotic result (29). For $B \ll 1$ the dimensionless viscoelastic frequency Ω_V is negligible, and thus the term containing Λ has no effect on the velocity or attenuation, which are given by equation (23). Therefore, equation (35) can be considered as a simplified version of equation (15) for any B and for any frequencies for which the model considered in this paper is valid.

The interplay between poroelastic and viscoelastic attenuation for different values of parameter B is shown in Figures 2-4, where inverse quality factor is a function of normalized frequency $\Omega_r = \omega / \omega_r$. Because of this normalization, zero on the horizontal axis corresponds to $\Omega_r = 1$, or $\omega = \omega_r$. The solid line correspond to the full solution, as derived from equation (15) or (35), the dashed line to the prediction of Frenkel-Biot theory, equation (24), and the dash-dotted line to the pure viscoelastic solution, equation (29). For comparison, the circles in these plots show the direct numerical solution of the exact Rytov's dispersion equation (1). In Figure 2 we show the deviation of the derived approximation from the exact numerical solution of Rytov's equation. This appears at frequencies higher then ω_r, which validates the assumption of derived equation (15). Thus we conclude that our approximate solution is valid at all frequencies much smaller than ω_r and ω_V. At low values of B attenuation is dominated by the poroelastic mechanism (Figures 2-3), with viscoelastic attenuation beginning to build up at frequencies close to ω_V. However, at larger values of B (Figure 4) the viscoelastic attenuation dominates.

4. Discussion and conclusions

The shear wave attenuation and dispersion studied in this paper are controlled by three dimensionless frequencies Ω_V, Ω_r and Ω_B. Asymptotic dispersion equations for both compressional and shear wave at low frequencies $\Omega_V \ll 1$, and $\Omega_B \ll 1$ are given in (Gurevich, 2002). The present paper extends these results for the shear waves to arbitrary values of Ω_B. Apart from the macroscopic assumption $\omega \ll \omega_r$, the only limitation is that the viscoelastic attenuation is small, $\Omega_V = \omega\eta / \mu_0 \ll 1$, but this is not restrictive since this condition must be satisfied for the fluid to be considered Newtonian (Landau and Lifshitz, 1987). Although an assumption of a Newtonian pore fluid is quite common for porous media, it does impose a limitation on the type of fluid, especially if a large frequency range is considered (Bird et al. 1987). Non-Newtonian fluid effects can be incorporated using the approach of Tsiklauri and Beresnev (2003). Note also that the original dispersion equation (1) was derived for a stack of solid layers with no-slip condition on interfaces between the layers. This condition is thus also implied when one of the solids is replaced by a Newtonian fluid. Although recent experiments show that slip may occur on a boundary between a real fluid and solid, indications are that this is a nonlinear effect which may only become significant at finite displacement amplitudes (Craig et al., 2001).

The highly idealized model of a periodic system of flat parallel layers considered in this paper embeds two mechanisms of attenuation. Which of the two mechanisms, viscoelastic or poroelastic, is dominant for a given material depends on the parameter B, which is independent of frequency. In other words, if for a certain material at a given frequency poroelastic or viscoelastic effects are dominant, the same effects would be dominant at all frequencies below the resonant frequency of the individual pores. This is in contrast with a common perception that, for given material, poroelastic and viscoelastic effects may dominate at different frequencies.

The fact that the dominant mechanism of attenuation is controlled by the frequency independent parameter B has been demonstrated in the present paper for an idealized porous medium consisting of solid and fluid layers. For a general three-dimensional periodic porous medium with a single characteristic pore size, this fact was proved mathematically by Boutin and Auriault (1990). Their approach is based on the theory of asymptotic homogenization of periodic structures, the theory which explicitly uses the ratio ω / ω_r as a small parameter (Levy, 1979, Auriault, 1980, Burridge and Keller, 1981). It should be emphasised however that real porous materials usually have a wide range of pore sizes, and therefore for real media the scaling behaviour of the type derived here may at best serve as zero-order approximation.

Explicit expressions for the frequency dependence of the velocity and attenuation of shear waves in a periodic system of flat solid and viscous fluid layers have been derived by solving the exact Rytov's dispersion equation in the long-wavelength approximation. Dispersion and attenuation are related to the well known mechanisms of wave attenuation in porous media: viscoelastic mechanism (viscous shear relaxation) and poroelastic visco-inertial mechanism. This is the first time the expressions describing the effects of these two mechanisms in a broad frequency range have been derived from the same standpoint. The asymptotic expressions for various limiting cases coincide well with the results of previous studies. The validity of the derived approximate dispersion equation has been demonstrated by comparison with the numerical solution of the exact Rytov's dispersion equation.

The procedure described above can also be applied to longitudinal waves propagating in the direction of layering. This will be the subject of a future paper.

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Appendix A: Derivation of the long-wave dispersion equation

For long waves $\beta_s h_s$ is always small, and equation (1) reduces to

$$p\left[\left(\frac{\beta_s h_s}{2}\right)^2 + \tan^2\frac{\beta_f h_f}{2}\right] + (1+p^2)\frac{\beta_s h_s}{2}\tan\frac{\beta_f h_f}{2} = 0, \tag{36 }$$

where

$$\beta_s h_s = \omega h_s\left(\frac{1}{c_s^2} - \frac{1}{c^2}\right)^{1/2}, \tag{37 }$$

$$\beta_f h_f = \omega h_f\left(\frac{1}{c_f^2} - \frac{1}{c^2}\right)^{1/2}, \tag{38 }$$

and

$$p = \frac{\mu_f \beta_f}{\mu_s \beta_s}. \tag{39 }$$

Equation (36) must be solved for the phase velocity c, which must be close to its static value c_0 (small attenuation and dispersion), and always enters equation (36) in the form $1/c^2$. Therefore, it is convenient to make a substitution

$$\frac{1}{c^2} = \alpha\frac{1}{c_0^2}, \tag{40 }$$

where $\alpha = O(1)$ is dimensionless quadratic slowness to be determined.

To find the asymptotic solution of equation (36), it is necessary to represent each of its terms in the non-dimensional form. Using definitions (12) and (13), equations (37) and (38) may be written in the form

$$\beta_s h_s = 2\pi\Omega_r (1-\phi)\left[\frac{(1-\phi)\rho_s}{\rho} - \alpha\right]^{1/2}, \tag{41 }$$

$$\beta_f h_f = 2\pi\Omega_r \phi\left[i\frac{(1-\phi)\rho_f}{\rho}\Omega_V^{-1} - \alpha\right]^{1/2}, \tag{42 }$$

and

$$p = -i\Omega_V \frac{\beta_f}{\beta_s}. \tag{43 }$$

With the definitions (40)-(43) equation (36) becomes non-dimensional. This equation has to be solved for $\Omega_V \ll 1$ and $\Omega_r \ll 1$ while allowing for arbitrary value of Ω_B, so that

$$\Omega_B = O(1). \tag{44 }$$

Taking into account equation (10), the condition (44) can be written in the form

$$\Omega_V = O(\Omega_r^2), \tag{45 }$$

or

$$\Omega_V = \gamma\Omega_r^2, \tag{46 }$$

where γ is a proportionality constant. Substitution of equation (46) into equations (42) and (43) yields

$$\beta_f h_f = 2\pi\phi\left[i\frac{(1-\phi)\rho_f}{\gamma\rho} - \Omega_r^2\alpha\right]^{1/2}, \tag{47 }$$

and

$$p = -i\gamma\Omega_r^2\frac{\beta_f}{\beta_s}. \tag{48 }$$

Substitution of equations (41), (47), and (48) into equation (36) results in an equation for unknown α with a single small parameter Ω_r. Seeking its solution in the form of a power series in Ω_r,

$$\alpha = \alpha_0 + \alpha_1\Omega_r^2 + ..., \tag{49 }$$

yields the following result

$$\alpha = \frac{\rho_s\left(1-\phi\right)+\rho_f\phi L^{-1}\tan L}{\rho}\left[1+i\gamma\Omega_r^2\frac{\phi}{2\left(1-\phi\right)}\left(1+L^{-1}\tan L+\tan^2 L\right)\right]. \tag{50 }$$

Substituting this result back into (40) and taking into account equation (46), one obtains equation (15).

Figure Captions

Fig. 1. Medium of alternating solid and viscous fluid layers.

Fig. 2. Attenuation (Q^{-1}) versus normalized frequency $\Omega_r = \omega/\omega_r$ for $B = 10^{-6}$. The solid line correspond to the full solution (derived equation 15), the dashed line to the prediction of Biot's theory (equation 23), the dash-dotted line to the pure viscoelastic solution (equation 29) and the circles to the numerical solution of exact Rytov's dispersion equation 1.

Fig. 3. The same as Figure 2 for $B = 10^{-3}$. The difference between poroelastic and viscoelastic solutions decreases.

Fig. 4. The same as Figure 2 for $B = 10^{2}$. The viscoelastic solution coincides with the full solution and dominates over the poroelastic one.

FIGURE 1

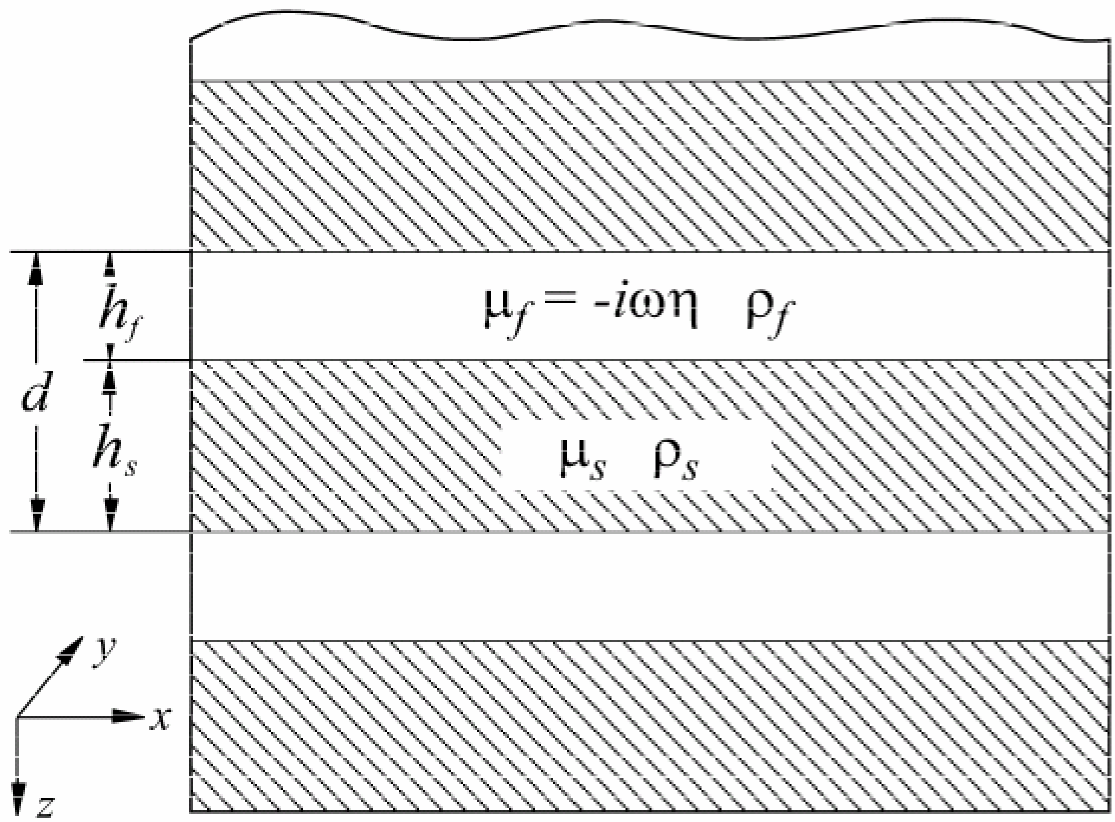


FIGURE 2

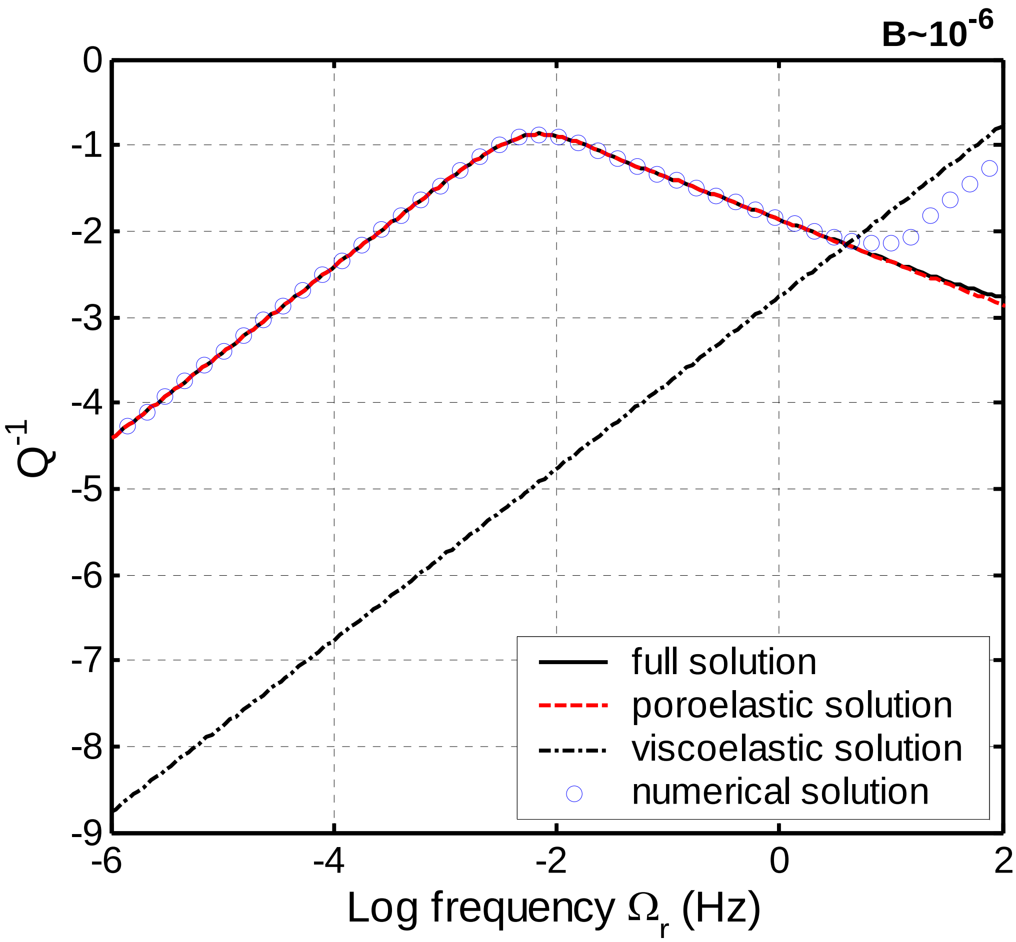

B~10-6
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
Q-1
-6
-4
-2
0
2
Log frequency Ωr (Hz)
full solution
poroelastic solution
viscoelastic solution
numerical solution

FIGURE 3

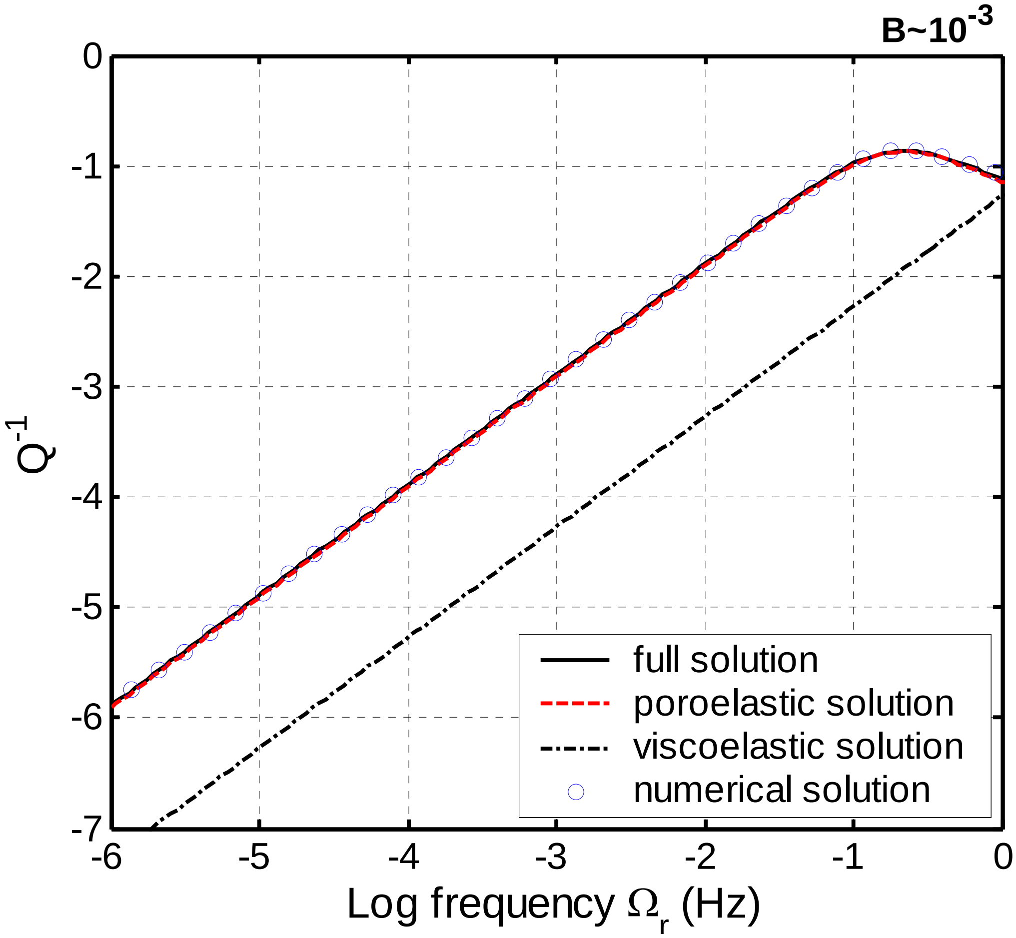
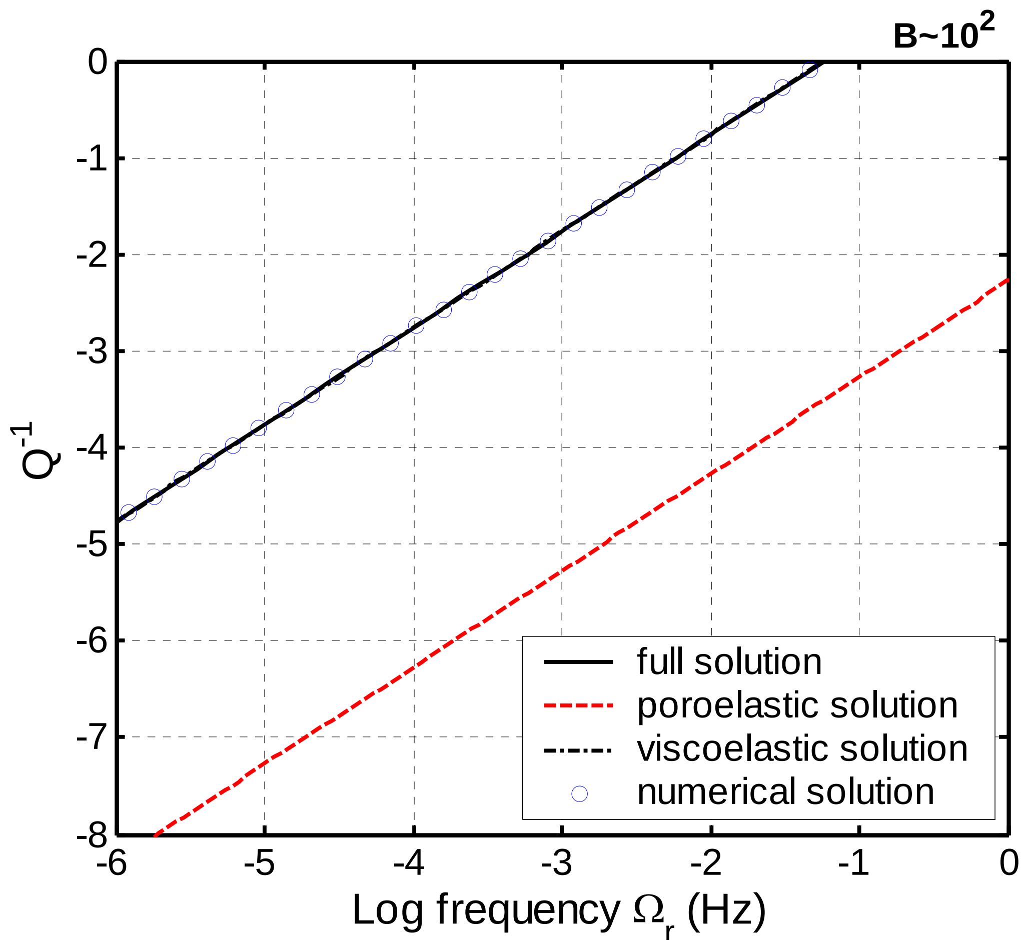

B~10^-3
0
-1
-2
-3
-4
-5
-6
-7
Q^-1
-6
-5
-4
-3
-2
-1
0
Log frequency Ω_r (Hz)
full solution
poroelastic solution
viscoelastic solution
numerical solution

FIGURE 4


B~10^2
0
-1
-2
-3
-4
-5
-6
-7
-8
Q^-1
-6
-5
-4
-3
-2
-1
0
Log frequency Ωr (Hz)
full solution
poroelastic solution
viscoelastic solution
numerical solution