School of Electrical and Computer Engineering

Control of Feedback for Assistive Listening Devices

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This thesis is presented for the Degree of
Doctor of Philosophy
of
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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Carlos Renato Calcada Nakagawa
June 2014
To my loving wife.
Abstract

Acoustic feedback is used to refer to the undesired acoustic coupling between the loudspeaker and microphone in audio reinforcement systems. Systems susceptible to feedback problems include public address systems and hearing aids. It is common practice to use adaptive filters as acoustic feedback cancellers to compensate for the feedback signal. The main challenge of using adaptive filters for feedback cancellation is that the filter estimates may become biased whenever the correlation between the incoming and loudspeaker signals is non-zero.

There are two main contributions in this thesis which deals with the biased estimation problem. First, we propose a two microphone method for feedback cancellation where an additional microphone is employed to enhance the canceller’s performance. The second microphone is spatially located further away from the loudspeaker compared to the main microphone so that the feedback signal received is more attenuated. The additional microphone is used to obtain an incoming signal estimate which is then subtracted from the primary microphone signal to create the error signal prior to adapting the canceller’s coefficients. With this method, the biased solution is no longer dependent on the correlation between the incoming and loudspeaker signals, but on the second feedback path. Accordingly, by doing a proper acoustic design based on near field properties of the feedback path and far field properties of the impinging signals significant system benefits are obtained.

This thesis also makes a contribution to probe signal injection methods. It is accepted that an unbiased solution is obtained if the canceller bases its estimation solely on the probe signal. However, we show that the solution is biased even if the probe signal is white noise. From this insight, we then derive conditions for obtaining an unbiased estimation. To reduce signal quality degradation probe signals are usually shaped to provide some level of perceptual masking. Thus, it is important to know the impact the shaping filter has on system performance. We present analytically that the shaping filter has a detrimental effect on system performance in terms of convergence rate. Accordingly, we propose a new method which restores convergence rate of the canceller while maintaining the benefits of spectrally shaping the noise.
Completing my PhD degree is probably the most challenging activity so far in my life, full of ups and downs. It was truly a roller-coaster ride. The best and worst moments of my doctoral journey have been shared with many people. It has been a great privilege to spend several years in the Department of Electrical and Computer Engineering at Curtin University, and its members will always remain dear to me.

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Renato Nakagawa
Perth, Australia - 2014
Publications


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Statement of Contribution

The candidate was responsible for the original idea for each manuscript, the software implementation, and the conduction of reported experiments. The candidate also wrote the manuscripts, which was commented on by the co-authors.
Contents

I General Introduction 1

1 Introduction 3
    1.1 Motivation ................................................. 3
    1.2 Scope of Research ........................................... 4
    1.3 Thesis Overview ............................................. 4
    References ...................................................... 6

2 Acoustic Feedback Problem and Control 9
    2.1 The Acoustic Feedback Problem ............................. 9
        2.1.1 Acoustic Coupling ...................................... 10
        2.1.2 Stability Problem ....................................... 12
        2.1.3 Amplification Limitation ................................. 12
        2.1.4 Sound Quality Degradation ............................... 13
        2.1.5 Feedback Path Characteristics .......................... 13
    2.2 Acoustic Feedback Control Techniques ....................... 16
        2.2.1 Feedforward Suppression ................................. 17
        2.2.2 Feedback Cancellation ................................. 19
        2.2.3 Evaluation of Feedback Cancellation Systems .......... 23
    2.3 Summary ...................................................... 25
    References ...................................................... 25

3 Biased Estimation Problem and Proposed Solutions 35
    3.1 The Biased Estimation Problem .............................. 35
    3.2 Towards Unbiased Estimation ............................... 37
## 3.2.1 Prior-knowledge of feedback path

37

## 3.2.2 Decorrelation Methods

37

## 3.3 Summary

45

References

46

### 4 Thesis Contributions and Conclusion

51

#### 4.1 Summary of Contributions

51

##### 4.1.1 Two Microphone Approach

51

##### 4.1.2 Probe Signal Investigations and Improvements

53

#### 4.2 Future Direction

54

#### 4.3 Conclusion

55

References

56

## II Publications

57

### A Dual microphone solution for acoustic feedback cancellation for assistive listening

A.1

I Introduction

A.3

II Background

A.5

III Proposed dual microphone method

A.6

IV Experiment Results

A.8

V Evaluation based on experimental data

A.9

VI Conclusion

A.11

References

A.11

### B Closed-loop feedback cancellation utilizing two microphones and transform domain processing

B.1

I Introduction and Contribution

B.3

II Review TM-AFC

B.5

III Transform Domain Filtered Error TM-AFC

B.7

IV Simulation results

B.10

V Conclusion

B.12

References

B.13

### C Analysis of Two Microphone Method for Feedback Cancellation

C.1

I Introduction

C.3

II System description

C.5

III TM-AFC Optimal Solution

C.7

IV Simulation Results

C.9

V Conclusion

C.13
References .......................................................... C.13

D  New Insights Into Optimal Acoustic Feedback Cancellation  D.1
   I  Introduction .................................................. D.3
   II  System description ........................................ D.4
   III New insights into the bias problem using probe signal injection . D.6
       A  Conditions for identifiability ......................... D.8
   IV  Simulation Verification ................................. D.9
   V  Conclusion ................................................ D.10
   References ............................................... D.11

E  Feedback Cancellation with Probe Shaping Compensation  E.1
   I  Introduction ................................................ E.3
   II  System Description ....................................... E.4
   III Delay condition for unbiased solution ............... E.6
   IV  Probe shaping impact on system behavior .......... E.6
   V  Probe shaping compensation .......................... E.8
       A  Delay condition for proposed approach ........... E.9
       B  Comment on more general $M(q)$ ................. E.9
   VI  Simulation results ..................................... E.9
   VII Conclusion ............................................. E.12
   References ............................................... E.12

III  Appendices  ............................................. I

Appendix A  Papers contribution statements ............ III

Appendix B  Copyright permission statements .......... V
List of Figures

1.1 Scope of research. .......................................... 5
1.2 Thesis Overview. ........................................ 7

2.1 Audio system with acoustic feedback. ......................... 11
2.2 Microphone location for different hearing aid microphone arrangement. .................................. 14
2.3 Feedback path’s characteristics. ............................ 15
2.4 Feedback control: feedforward suppression techniques. .... 16
2.5 Feedback control: feedback cancellation techniques. .... 16
2.6 Traditional feedback canceller. ............................. 19
2.7 Traditional echo canceller. ................................. 22

3.1 Delay in the forward path and/or in the adaptive filter estimation path. ...................... 38
3.2 Phase modification methods. ................................ 39
3.3 Pre-whitening method. ...................................... 39
3.4 Probe signal injection with non-continuous adaptation. .......... 40
3.5 Feedback canceller with probe signal injection and continuous adaptation. ............... 41
3.6 Feedback canceller’s estimation based on probe signal. .. 42
3.7 Redraw of feedback canceller’s estimation based on probe signal. 
   Incoming signal can be treated as uncorrelated noise. .......... 42
3.8 Probe signal enhancement. .................................. 44
3.9 Two microphone approach in dealing with biased feedback canceller. ..................... 44

4.1 Overview of relations between papers. ....................... 52

A.1 Adaptive Canceller ........................................ A.4
A.2 Microphone Placement ..................................... A.7
A.3 Proposed Block Diagram .................................. A.8
A.4 Misalignment proposed method vs classic adaptation .......... A.10
A.5 Error signals - proposed vs classic

B.1 General AFC set-up.
B.2 TM-AFC set-up.
B.3 Proposed microphone location.
B.5 Feedback paths’ characteristics.
B.6 Instantaneous misalignment and ASG plots for varying \( M \).

C.1 TM-AFC feedback canceler.
C.2 Proposed microphone location.
C.3 Feedback path characteristics for proposed microphone placement.
C.4 Feedback paths are varied at time 40 seconds.
C.5 Source location is varied at time 40 seconds.
C.6 PEM comparison: source location is varied at time 40 seconds.

D.1 Injected probe signal approach in acoustic feedback cancellation.
D.2 Feedback path characteristics.
D.3 Misalignment between \( G(q) \) and \( A(q) \) with varying gain \( K \) with \( d_k = 1 \).
D.4 Misalignment between \( G(q) \) and \( A(q) \) with varying delay \( d_k \) with \( K = 30 \) dB.

E.1 Traditional feedback canceler with probe signal injection.
E.2 Proposed approach.
E.3 Frequency and impulse responses for \( M_1(q) \) and \( M_2(q) \).
E.4 Estimated and true PTF curves.
List of Tables

A.1 PESQ measure - proposed vs classic . . . . . . . . . . . . . . . . . . . . A.11
E.1 PTF estimate, convergence rate (dB/iteration). . . . . . . . . . . E.11
E.2 PTF estimate, steady state error (dB). . . . . . . . . . . . . . . . E.11
Part I

General Introduction
Chapter 1

Introduction

1.1 Motivation

Hearing loss is one of the most prevalent chronic health conditions. Some 16% of adults suffer from hearing loss great enough to adversely affect their daily life [1]. According to many surveys, one out of six people suffers from hearing loss and would benefit from using hearing aids [1]. There are two main reasons that causes hearing loss; one is the increased exposure to noise in daily life and the other is ageing. The world’s population is ageing rapidly at an unprecedented rate. The proportion of people aged over 60 years will double from about 11% to 22% between 2000 and 2050 [2]. It is expected that the number of people suffering from hearing loss will continue to grow over time.

There exist different ways for helping people with hearing impairments. One well known and possibly the most commonly used method is by means of a hearing aid. A hearing aid is an electronic device that makes listening easier for people with a hearing loss. A hearing aid consists of a microphone, an amplifier and a loudspeaker (receiver). The microphone picks up sounds in an acoustic environment and turns them into electric signals. The amplifier selectively amplifies the signals. The loudspeaker then changes the electric signals back to sounds and delivers the sound to the ear. A modern hearing aid is small in size, and it typically fits behind the ear or even in the ear canal of its user.

As hearing aids become smaller and smaller, acoustic feedback, i.e., the acoustic coupling between the loudspeaker and the microphone(s) of the hearing aid, poses a major problem to hearing aid users. Acoustic feedback results in severe distortion of the desired signal and howling if the hearing aid gain is increased [3–5]. As a result, the maximum amplification that can be used in a commercial hearing aid is often too small to compensate for the hearing loss. Therefore, an urgent need exists for efficient and well working signal processing algorithms for
1.2 Scope of Research

This thesis studies the acoustic feedback problem and state-of-the-art feedback control techniques. Acoustic feedback control techniques aim to cancel the effect of the feedback on the performance of audio reinforcement systems. Acoustic feedback control is defined in [5] as the process of attempting to solve the problem either completely, i.e., to remove the acoustic coupling, or partially, e.g., to remove the howling artefacts from the loudspeaker signal. Many feedback control methods have been proposed in the literature, however, there is still a lack of reliability in the available automatic acoustic feedback control solutions [5].

Proposed techniques in the literature can be generally classified into feedforward suppression and feedback cancellation techniques [4]. This thesis focuses on the study of automatic continuous feedback cancellation techniques for hearing aids. The use of feedback cancellation techniques is currently a preferred option to tackling the feedback problem as it is able to provide the system with higher added stable gains [4, 6].

The main challenge with traditional feedback cancellers is the bias estimation problem. The biased solution in a traditional canceller’s estimate is caused by the correlation between the loudspeaker and incoming signal [4, 7–9]. It generally leads to poor system performance, results in signal distortion (canceller cancels portion of the desired signal), and, in worst case, causes the cancellation system to fail.

Different techniques have been proposed in the literature to deal with the biased estimation problem [4, 5]. However, none of these methods is a straightforward solution to the given problem, since many problems occur while implementing the proposals. Here, future hearing aids still offer room for improvements.

The scope of research is illustrated in Fig. 1.1. At a high level we present the acoustic feedback problem and the state-of-the-art feedback control techniques currently presented in the literature. We then focus on adaptive feedback cancellation techniques and, more specifically, we study the biased estimation problem. We then make contributions by proposing new methods and algorithms to reduce and even remove the bias term in the optimal solution.

1.3 Thesis Overview

This thesis is composed of three parts. This is illustrated in Fig. 1.2.
Part I serves as a general introduction, in which we state and define the research problem. We present a literature review and the main contributions of this thesis. Chapter 2 defines the acoustic feedback problem in more detail and presents state-of-the-art acoustic feedback control techniques. Chapter 3 then focuses on the biased estimation problem encountered with adaptive feedback cancellation techniques. Methods which aim at reducing, and even removing, the bias term is also presented in Chapter 3. Chapter 4 then summarises the main contribution and concludes.

Part II is the main part of this thesis. It consists of a collection of papers, contributing to the development of acoustic feedback cancellation systems. This thesis makes two main contributions: the development and analysis of a two microphone approach in dealing with the biased problem; and investigations and improvements to the probe signal approach to remove the biased estimation altogether.

The two microphone approach is presented and developed in Papers A, B and C [10–12]. This feedback cancellation technique employs an additional microphone to obtain an incoming signal estimate. This estimate is then subtracted from the error signal prior to adapting the canceller’s coefficients. Thus, with this method, the biased solution is no longer dependent on the correlation between
the incoming and loudspeaker signals.

Papers D and E [13, 14] makes a contribution to the probe signal injection method. In Paper D we show that the solution is biased even if white noise is used to drive the canceller’s adaptation. From this insight, we then derive conditions for obtaining an unbiased estimation. To reduce signal quality degradation probe signals are usually shaped to provide some level of perceptual masking. Thus, it is important to know the impact the shaping filter has on system performance. In Paper E we study the impact shaping the probe signal has on system performance and propose a method to restore it while still maintaining benefits that come with spectrally shaping the probe noise.

Part III consists of appendices which provide necessary information to fulfil required formalities in the compilation of this thesis.

Parts I and III are written in Australian English, whereas Part II is written in American English to comply with IEEE requirements.

References

Fig. 1.2 Thesis Overview.


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Chapter 2

Acoustic Feedback Problem and Control

Acoustic feedback poses a problem in the normal operation of audio reinforcement systems, such as public address (PA) systems and hearing aids. The main aim for these types of systems is to amplify the desired incoming sound. However, the acoustic feedback problem limits the maximum achievable amplification possible by the audio system. Also, it deteriorates the sound quality by producing a distortion of the desired signal, and is a cause of instability in such systems [1, 2].

Acoustic feedback control techniques tries to minimise the effect of the feedback on the performance of hearing aids. This is a challenging problem and many years of research have been spent on the topic. From the research, many feedback control methods have been proposed in the literature, however, there is still a lack of reliability in the available automatic acoustic feedback control solutions [2]. Thus, there is still a need and demand for effective feedback control techniques [3].

Historically, studies have focused on controlling the effects of acoustic feedback in public address systems [2, 4–6] and more attention has specifically been paid to the hearing aid application over the last couple of decades, see example studies in [7–14].

2.1 The Acoustic Feedback Problem

Acoustic feedback occurs in audio reinforcement systems where the microphone picks up part of the loudspeaker output signal and re-amplifies it creating a feedback loop of amplification. The amplified sound is continuously re-amplified to the point at which a tonal squeal, or “howling” occurs. The term acoustic
feedback has been used to refer to the undesired acoustic coupling between a loudspeaker and a microphone as well as to the howling effect that results from the coupling [2]. In precise terms the objectionable audible sound produced by a sound reinforcement system due to acoustic feedback should be called audible oscillation due to acoustic feedback [1, 2]. In this thesis, the term acoustic feedback will be used consistently throughout the text to refer to the undesired acoustic coupling between a loudspeaker and a microphone.

Systems susceptible to acoustic feedback includes PA systems and hearing aids. With PA systems, a loud and obnoxious squeal may occur if a speaker in a conference room or an auditorium stands too close to the loudspeaker. A portion of the sound from the loudspeaker has been picked up by the microphone, has been amplified, and then broadcast into the room. This repeating cycle of sound amplification, broadcasting and pick up continues until the system is no longer stable and oscillation occurs. An audible manifestation of this instability is a loud and overwhelming squeal. This sound is annoying to both the speaker and the audience.

The same principle applies on a smaller scale to a hearing aid. Amplified sound transmitted to the ear canal from the loudspeaker is radiated out through the vent, or via various other pathways (such as acoustic leakage between the hearing aid shell and the wall of the ear canal via a pathway), back to the microphone [1]. Then it is amplified and re-radiated out of the ear canal, where it is picked up again by the microphone, re/amplified and so forth. With a hearing aid, it is generally not possible to move the microphone further away from the receiver to prevent feedback, as may be done with a PA system.

It is noted that the two applications mentioned here are quite different in nature [15]. For instance, in hearing aid applications usually one loudspeaker and one, or two, microphones are used, whereas in PA systems multichannel configurations are common use. The acoustic scenario inherent to these applications also defer somewhat and this determines the preferred acoustic feedback control method. For instance, in hearing aid applications, the feedback path impulse response is much shorter than in PA systems while, on the other hand, the computational power is much smaller than in PA systems. Therefore, it seems natural that different acoustic feedback control methods have been developed for these different applications.

### 2.1.1 Acoustic Coupling

The acoustic coupling between the loudspeaker and microphone is illustrated in Fig. 2.1 for a single microphone single loudspeaker audio system. From Fig. 2.1,
y(n) is the loudspeaker signal and m(n) is the received microphone signal. The forward path \( K(q) \) represents the regular signal processing path of the device (i.e., a frequency-specific gain, compression and/or noise reduction) [3] and the feedback path is represented as \( G(q) \). Normally, the feedback path \( G(q) \) between the loudspeaker and the microphone is assumed to be a discrete-time finite impulse response (FIR) filter [2] with coefficient vector

\[
g = [g_0, g_1, \ldots, g_{L_g-1}]^T \tag{2.1}
\]

and filter length \( L_g \). This can be represented as a polynomial transfer function \( G(q) \) in \( q \) as \( G(q) = g^T q \) with

\[
q = [1, q^{-1}, \ldots, q^{-L_g+1}]^T. \tag{2.2}
\]

This representation allows the following notation, for the filtering of \( y(n) \) by \( G(q) \) [16],

\[
G(q) y(n) = g^T y(n). \tag{2.3}
\]

Column vectors are emphasised using lower letters in bold, the superscript \( T \) denote vector transpose, the discrete-time index is denoted by \( n \), and the symbol \( q^{-1} \) denotes the discrete-time delay operator \( q^{-1} u(n) = u(n - 1) \). All signals are real-valued, and we denote all signals as discrete-time signals with time index \( n \) for convenience, although in practice the feedback \( f(n) \) and incoming signals \( u(n) \) are continuous-time signals.

From Fig. 2.1, it can be seen that

\[
m(n) = u(n) + G(q) y(n) \tag{2.4}
\]
and

\[ y(n) = C(q)u(n) \]  \hspace{1cm} (2.5)

where \( C(q) \) is the closed-loop system

\[ C(q) = \frac{K(q)}{1 - K(q)G(q)}. \]  \hspace{1cm} (2.6)

### 2.1.2 Stability Problem

A measure to determine stability in a linear and time-invariant closed-loop system is the open-loop transfer function

\[ \Theta(\Omega) = K(\Omega)G(\Omega), \]  \hspace{1cm} (2.7)

where the short-term frequency spectra of \( K(q) \) and \( G(q) \) is denoted by \( K(\Omega) \) and \( G(\Omega) \), respectively, and \( \Omega \) is the discrete time frequency. The open-loop function in (2.7) plays a crucial role in acoustic feedback control.

The Nyquist criterion [17, 18] states that the system is unstable if the open-loop gain is

\[ |\Theta(\Omega)| \geq 1, \]  \hspace{1cm} (2.8)

and open-loop phase is

\[ \angle \Theta(\Omega) = \ell 2\pi, \ \ell = \mathbb{Z}. \]  \hspace{1cm} (2.9)

In cases where the amplification is larger than the attenuation of the feedback path, and the feedback signal is in phase, instabilities occur and howling is provoked [19]. That is, if the magnitude response of the loop-gain is greater than unity and the loop-phase is a multiple of \( 2\pi \). The criterion in (2.8) and (2.9) is essential in acoustic feedback control, since any acoustic feedback control method effectively attempts at preventing either one or both of these conditions from being met [2].

### 2.1.3 Amplification Limitation

The main functionality of \( K(\Omega) \) in an audio reinforcement system is to amplify sound signals. Thus, \( K(\Omega) \) typically has a value larger than one for a wide range of \( \Omega \). Hence, depending on \( G(\Omega) \), there is a potential risk to violate the condition stated in (2.8), and system instability would then occur at the frequencies \( \Omega \) for
which the condition stated in (2.9) is fulfilled. To avoid this, the amount of gain \( K(\Omega) \) has to be limited at critical frequencies \( \Omega \) where the signals are in phase. Thus, the maximum amplification achievable is limited.

### 2.1.4 Sound Quality Degradation

Furthermore, the acoustic feedback can significantly degrade the sound quality of the loudspeaker signal \( y(n) \) [20]. From (2.5) and (2.6), the magnitude of the input-output transfer function from microphone to loudspeaker of the audio system shown in Fig. 2.1 is determined by

\[
|C(\Omega)| = \frac{|Y(\Omega)|}{|U(\Omega)|} = \frac{|K(\Omega)|}{|1 - \Theta(\Omega)|} \tag{2.10}
\]

where \( Y(\Omega) \) and \( U(\Omega) \) are the spectra of the loudspeaker and incoming signal, respectively. In an ideal situation, for a system without feedback, the magnitude of the loudspeaker signal spectrum \( |Y(\Omega)| \) is the incoming signal spectrum \( |U(\Omega)| \) shaped by the forward path magnitude function \( |K(\Omega)| \), i.e., it is desired that

\[
|Y(\Omega)| = |K(\Omega)| \cdot |U(\Omega)|. \tag{2.11}
\]

Otherwise, even for a stable system with feedback, i.e. \( 0 < \Theta(\Omega) < 1 \), undesired modifications of the loudspeaker signal may be introduced [20]. For instance, in the limit as \( |\Theta(\Omega)| \to 1 \), we get

\[
|C(\Omega)| \to \begin{cases} 
\infty & \text{for } |\Theta(\Omega)| \to 1, \text{ and } \angle \Theta(\Omega) = l2\pi, \ l = \mathbb{Z}, \\
\frac{|K(\Omega)|}{2} & \text{for } |\Theta(\Omega)| \to 1, \text{ and } \angle \Theta(\Omega) = \pi + l2\pi, \ l = \mathbb{Z}.
\end{cases} \tag{2.12}
\]

This corresponds to an undesired shaping of the loudspeaker signal depending on the values of \( \Theta(\Omega) \) across frequencies \( \Omega \). This undesired signal shaping may lead to a significant sound distortion. To avoid significant audible distortion, a gain margin (the difference between the maximum stable gain and the actual system gain of the system) of at least 6 dB is advisable for hearing aids, i.e. the gain is set so that \( |\Theta(\Omega)| \leq 0.5 \) [21]. For PA systems, the gain margin recommended is 2 dB [6].

### 2.1.5 Feedback Path Characteristics

Now we will present some of feedback path’s general characteristics for hearing aids.
The coupling between the loudspeaker and microphone consists of several pathways. In most cases, the main contributor to the acoustic feedback stems from the vent, i.e., the hole in the earmold of the hearing aid [21, 22]. Venting is essential to reduce the occlusion effect [23, 24]. This effect refers to the increase in loudness of one’s own voice and the low frequency boost that hearing aid users experience when the ear canal is completely blocked [25]. The unnatural perception of their own voice is disturbing to hearing aid users [25]. Thus, removing the vent is not an option to help solve the feedback problem.

The geometric configuration of the hearing aid, the ear canal and the acoustics outside the ear also help determine the feedback path characteristics [26]. As a result of the shorter distances between loudspeaker and microphone, the attenuation of the feedback path is smaller for in-the-ear (ITE) and in-the-canal (ITC) hearing aids than for behind-the-ear (BTE) hearing aids. This is illustrated in Figs. 2.2 and 2.3, where $G_1(q)$ is the feedback path to an ITE microphone and $G_2(q)$ is the feedback path to a BTE microphone. Also, since the ear canal shape differs among hearing aid users, the feedback path is user-dependent [20, 25].

Another characteristic of feedback paths in hearing aids is that oscillations is more probable to occur at higher frequencies [26], typically above 3 kHz, due to the hearing aid styles and the surrounding geometry of hearing aids, see Fig. 2.3. Unfortunately, the desired amplification in the hearing aid forward path is often higher at high frequencies, as hearing loss is common at these frequencies, making feedback problems even more probable to occur [1].

In addition, the acoustic feedback path can vary significantly under different conditions and acoustic environments. There have been studies in the literature
2.1 The Acoustic Feedback Problem

on the variability of the feedback path [26–28] and its causes are mentioned in [1, 19, 21, 26, 27, 29]. All these factors influence the occurrence of acoustic feedback and the frequency at which it occurs. Coughing, chewing, sneezing, yawning, talking, tilting the head, bringing a hand up to the face, use of the telephone, the proximity of reflective surfaces and placing a hat on the head can also initiate oscillations in a hearing aid. As a result of these variables, acoustic feedback can be a very elusive phenomenon. It can occur at different frequencies with the same hearing aid at different times and under different acoustic conditions [1].

Studies on modelling the impulse response for the feedback path can be found here [21, 30–37]. In general, the impulse response of a feedback path is short in duration, in the order of a few milliseconds, especially when compared to the feedback paths of PA systems, in which the length of the impulse response could easily be hundreds and even thousands of milliseconds depending on the room acoustics [20]. Also, generally included in the feedback path is the characteristic of the loudspeaker, the microphone, the analogue-to-digital converter (ADC), the digital-to-analogue converter (DAC), and low-pass filters [3, 38]. Moreover, loudspeakers and microphones are essentially non-linear devices, which become part of the acoustic feedback path. This makes the feedback control even more challenging. However, the non-linearity can often be modelled and compensated as discussed in [39]. In this thesis, we do not investigate the non-linearity in acoustic feedback path.

Fig. 2.3 Feedback path’s characteristics.
2.2 Acoustic Feedback Control Techniques

Acoustic feedback control techniques aim to cancel the effect of the feedback on the performance of audio reinforcement systems. Acoustic feedback control is defined in [2] as the process of attempting to solve the problem either completely, i.e., to remove the acoustic coupling, or partially, e.g., to remove the howling artefacts from the loudspeaker signal. Many feedback control methods have been proposed in the literature, however, there is still a lack of reliability in the available automatic acoustic feedback control solutions [2]. Thus, there is still a need and demand for improved feedback control techniques [3]. An overview of signal processing techniques used to deal with the feedback problem is provided in this section.

Proposed techniques in the literature can be generally classified into feedforward suppression and feedback cancellation techniques [3]. With feedforward suppression techniques, illustrated in Fig. 2.4, the aim is to manipulate the term $K(\Omega)$, in the open-loop function (2.7), to avoid the Nyquist criterion in (2.8) and (2.9) being met. One example of this is to reduce the gain at critical frequencies.

Fig. 2.4 Feedback control: feedforward suppression techniques.

Fig. 2.5 Feedback control: feedback cancellation techniques.
For feedback cancellation techniques, illustrated in Fig. 2.5, the feedback path $G(q)$ is modelled with an internal filter $\hat{G}(q)$ placed in parallel to the feedback path. An estimate $\hat{f}(n)$ of the feedback signal $f(n)$ is produced and subtracted from the microphone signal $m(n)$. Thus, the hearing aid gain is not affected by this method. Additionally, it even allows hearing aid gain settings with closed-loop gains larger than one [19]. With feedback cancellation techniques, the open-loop equation presented in (2.7) is now defined as

$$\Theta(\Omega) = K(\Omega) \left( G(\Omega) - \hat{G}(\Omega) \right), \quad (2.13)$$

where $\hat{G}(\Omega)$ is the frequency response for $\hat{G}(q)$. As can be seen from (2.13), if $\hat{G}(\Omega)$ can perfectly model $G(\Omega)$ then the feedback signal is completely cancelled and the feedback problem is eliminated.

Feedback cancellation techniques is a preferred method over feedforward suppression as it has the potential to remove the feedback contribution and provide the system with the desired system response $K(\Omega)$. This method is currently the state-of-the-art for hearing aids.

Hybrid approaches which combine feedback cancellation and feedforward suppression techniques have also been proposed in the literature, see for instance [27, 40–46].

Next we present some more details on the different techniques.

### 2.2.1 Feedforward Suppression

In feedforward suppression techniques, the signal processing forward path $K(\Omega)$ is modified in such a way that it is stable in conjunction with the feedback path [3]. Based on the Nyquist stability criterion, these techniques can be divided into two categories: gain reduction and phase modification methods. Thus, with feedforward suppression techniques, the aim is to limit $|K(\Omega)|$ so that $|\Theta(\Omega)| \ll 1$ for all critical frequencies and/or have $\angle \Theta(\Omega) \neq l2\pi \forall \Omega, l \in \mathbb{Z}$.

**Gain Reduction**

Since acoustic feedback is caused by a combination of phase angle and excessive gain at a critical frequency, one solution is to reduce the overall gain until the howling ceases. This can be as simple as the user reducing the volume control when oscillations become noticeable. However, more sophisticated automatic gain reduction methods exist [47, 48]. Unfortunately, while this may eliminate the effects of acoustic feedback, the overall gain may be reduced to the point that the gain provided to the wearer is inadequate to allow speech to be audible.
and intelligible. Furthermore, fullband gain reduction is often not necessary for stabilising the system, but instead, gain reduction can be applied to the critical subbands where the open-loop gain is close to unity [49].

A more common gain reduction technique is notch-filter-based howling suppression methods. Notch-filter-based howling suppression methods generally perform some howling detection to first find the instability frequency and then suppress it by notch filtering [2, 15]. Thus, the gain is reduced in narrow frequency bands around the critical frequencies [11, 13, 15, 50–55]. These gain reduction methods tend to be reactive, in the sense that howling needs to occur before it is can be detected and suppressed [2].

An attempt to reduce undesired gain reductions is carried out in [56] using spatial filters by assuming that the feedback and desired signals are coming from different spatial directions. Spatial filtering methods for acoustic feedback control aim at altering the open-loop response (2.7) by using microphone/loudspeaker arrays of which the received/transmitted signals are processed by beamforming filters. The general objective is then to design a beamformer that has its main lobe in the direction of the desired source signal while having a zero in the direction of the loudspeaker, the source of feedback signal. Hence, the gain provided to the desired signals is ideally unchanged, whereas the feedback signal is attenuated.

**Phase Modification**

The second type of feedforward suppression is phase modification techniques, which includes frequency shifting [6, 57], delay and phase modulation [58]. The goal of phase modulation feedback control is to control the phase of the microphone signal in such a way that every frequency component in the feedback signal has a different phase each time it arrives at the microphone after each cycle around the closed-loop [58]. In this way, the phase condition in the Nyquist criterion (2.9) can be guaranteed not to hold for the same frequency at two successive instants, hence the closed-loop system stability can be improved, regardless of the magnitude condition in (2.8) [2]. Strictly speaking, phase modification techniques causes the system to become time-varying, thus the Nyquist stability criterion does not apply any more. Phase modification can be performed by modulating $K (\Omega)$ with an exponential function $e^{j\phi(\Omega)}$, as

$$K_m (\Omega) = K (\Omega) e^{j\phi(\Omega)}$$

to form the modified forward path frequency response $K_m (\Omega)$ [20]. The downside of this type of approach is that it compromises the basic frequency response and limited added gain is achievable [58].
Therefore, the increase in maximum stable gain with feedforward suppression techniques has generally been found to be limited. In addition, feedforward suppression techniques compromise the basic frequency response of the hearing aid and may seriously affect the sound quality [3]. Nevertheless, feedforward suppression techniques are an effective way to keep the system from going unstable.

### 2.2.2 Feedback Cancellation

With acoustic feedback cancellation (AFC) methods for feedback control, a model of the acoustic feedback path is identified either offline (during initialisation or when instabilities are detected) or online (during operation of the device). In AFC, the acoustic feedback path model is used to predict the feedback signal component \( f(n) \) in the microphone signal \( m(n) \), refer to Fig. 2.5. The predicted feedback signal \( \hat{f}(n) \) is then subtracted from the microphone signal, hence resulting in a feedback-compensated signal, which is in fact an estimate of the source signal component \( \hat{u}(n) \) in the microphone signal. If an accurate model of the acoustic feedback path can be identified, then the AFC method achieves a nearly complete elimination of the acoustic coupling, and consequently very large gains may be obtained. Thus, no modifications are required to the forward path \( K(\Omega) \).

As seen in Sec. 2.1.5, the acoustic path \( G(\Omega) \) between the loudspeaker and the microphone can vary significantly depending on the acoustical environment. Hence, adaptive feedback cancellers \( \hat{G}(\Omega) \) are called for.

### AFC Using Adaptive Filters

Fig. 2.6 illustrates a traditional feedback canceller using an adaptive filter. The goal of the adaptive filter \( \hat{G}(\Omega) \) is to estimate and track variations to the feedback
path $G(\Omega)$. There are different ways to estimate the coefficient vector

$$\hat{g} = [\hat{g}_0, \hat{g}_1, \ldots, \hat{g}_{L_g-1}]^T \quad (2.14)$$

of the adaptive filter $\hat{G}(\Omega)$ with length $L_g$. Generally, the length $L_g$ of the feedback path $G(\Omega)$ is assumed to be fixed and known a priori. There has been some research, however, which proposed to recursively find the FIR filter of proper order, see [10].

A general class of adaptive filters, known as Wiener filters, minimises the cost function $J_{\text{MSE}}(n)$ in terms of the mean square error of $e(n)$,

$$J_{\text{MSE}}(n) = E\{e^2(n)\} \quad (2.15)$$

where

$$e(n) = m(n) - \hat{g}^T y(n), \quad (2.16)$$

$y(n) = [y(n), y(n-1), \ldots, y(n-L_g+1)]^T$ is the loudspeaker signal vector, $E\{\cdot\}$ is the expectation operator, and the signals $u(n)$ and $y(n)$ are considered realisations of the underlying stochastic processes. The Wiener filter is derived based on ensemble averages, so that the filter is statistically optimal on average across all realisations of the underlying stochastic processes [59]. Minimising (2.15) with respect to $\hat{g}$, we obtain

$$\hat{g}_o = R_{yy}^{-1}r_{ym} \quad (2.17)$$

where $\hat{g}_o$ is the Wiener-Hopf solution, $R_{yy} = E\{y(n)y^T(n)\}$ represents a correlation matrix, and $r_{ym} = E\{y(n)m(n)\}$ represents a correlation vector.

A deterministic gradient approach, such as the steepest decent algorithm, can be used to recursively compute the Wiener-Hopf solution so that inversion of the correlation matrix $R_{yy}$ is not required. The gradient with respect to $\hat{g}$ can be shown to be

$$\delta J_{\text{MSE}}(n) = -2E\{y(n)e(n)\} \quad (2.18)$$

and the update of $\hat{g}$ is given as

$$\hat{g}_{n+1} = \hat{g}_n + \mu(n) E\{y(n)e(n)\} \quad (2.19)$$

where $\mu(n)$ is the step size parameter.

A widely used stochastic gradient approach is the least mean square (LMS)
algorithm [60]. It is popular due to its simplicity and it does not require the knowledge of $R_{yy}$ and $r_{ym}$. The LMS adaptive filter estimation of $\hat{g}$ is carried out using the stochastic gradient vector $y(n)e(n)$ and the step size parameter $\mu(n)$, as

$$\hat{g}_{n+1} = \hat{g}_n + \mu(n)y(n)e(n).$$  \hspace{1cm} (2.20)

Other stochastic gradient algorithms include the normalised least mean square (NLMS) and the affine projection (AP) algorithms [59]. The NLMS differs from the LMS algorithm by utilising a step size parameter normalised by the signal power estimate of $y(n)$. The AP algorithm can be considered as a generalisation of the NLMS algorithm, which involves the loudspeaker signal matrix

$$A(n) = [y(n), y(n-1), \ldots, y(n-N+1)]^T$$  \hspace{1cm} (2.21)

of order $N - 1$, instead of the loudspeaker signal vector. In this way, the NLMS algorithm is a specific case of an AP algorithm when $N = 1$. Both algorithms improve the convergence rate of the original LMS algorithm at the cost of increased computational complexity.

A deterministic approach referred to as the method of least squares (LS) can be considered to be another class of adaptive filters. The LS approach is based on averages of deterministic data samples over time. More specifically, it minimises the cost function in terms of the sum of squares of the error signal as $J_{LS} = \sum_{i=0}^{n} e^2(i)$. The basic LS approach requires a potentially computationally complex matrix inversion. Accordingly, the recursive least squares (RLS) algorithm was developed based on the matrix inversion lemma to bypass the matrix inversion [59]. The RLS algorithm typically achieves improved convergence rate compared to the AP and NLMS algorithms, depending on the signal properties of the input sequence. Furthermore, the RLS can be considered to be a special case of the Kalman filter framework, which has a recursive solution based on the latest data samples and its state estimate [59].

There has been a lot of work presented in the literature which improves on the performance of adaptive filters for different applications. Such works include choosing optimal step size and regularisation parameters as presented in [61–66], the filtered-X algorithms with a fixed filter to model a known part of the unknown impulse response in series with the adaptive filter [67–70], the proportionate algorithms for long and sparse impulse response estimations [71–75], the robust algorithms with slow divergence properties as discussed in [76, 77], and other computationally efficient algorithms such as [78, 79]. Another technique to improve the performance of adaptive filters is to estimate its coefficients in
subbands [80, 81], in transform domain [82–84], such as the frequency domain [85–87], or in a band-limited frequency region [88]. The main advantages are typically increased convergence rate, more control flexibilities, and computational complexity reductions. Delayless subband structure can be found here [89, 90].

Feedback cancellation using adaptive filters is generally more effective to control feedback than feedforward suppression methods, and it provides better sound quality [2, 3]. Nevertheless, one of the biggest challenges for AFC is the biased estimation problem. We study this problem in more detail in Chapter 3.

**Contrast with Acoustic Echo Canceller**

Quite often in the literature the AFC problem is treated to be fairly similar to the acoustic echo cancellation (AEC) problem, see for instance some recent work [83, 91, 92]. The acoustic echo problem generally occurs during hands-free telephony, e.g. in car communication systems. The structure of the AFC and AEC problems are very similar. Fig. 2.7 illustrates a typical single microphone single loudspeaker echo environment where an adaptive filter \( \hat{G}(q) \) is used to estimate the echo signal \( f(n) \) and remove it from the microphone signal \( m(n) \). The far-end and near-end denote the transmitting and receiving ends over a communication channel. Just as with the feedback path, the echo path is time varying (e.g. people moving around the room, doors opening/closing) and adaptive filters are used to track its variations [93]. Ideally, only the near-end signal \( u(n) \) should be transmitted to the far-end. Practically, however, the microphone picks up part of the loudspeaker signal which is then transmitted and perceived at the far-end as an echo.

One main difference in the structure between AFC and AEC problems is that AFC is a closed-loop system (with amplification), whereas AEC is generally considered to be an open-loop problem (the unc cancelled far-end echo is assumed to be very attenuated). Note from Fig. 2.7 the absence of the forward path \( K(q) \).
Echo cancellation using adaptive filters faces two main problems, the double-talk problem and the non-uniqueness problem (in stereo or multichannel audio systems). Double-talk occurs when the speech of the two talkers arrives simultaneously at the canceller, i.e. \( u(n) \neq 0 \) and \( y(n) \neq 0 \). In the double-talk situation, the near-end speech acts as a strong uncorrelated noise to the adaptive algorithm. The disturbing near-end speech may cause the adaptive filter to diverge, allowing annoying audible echo to pass through to the far-end [94]. The usual way to alleviate this problem is to slow down or completely halt the filter adaptation when near-end speech is detected. To accomplish this a double-talk detector (DTD) is employed, see [76, 95, 96] and references within for more information about and challenges faced by DTDs.

Although the double-talk situation is difficult to handle, it is typically not always present in an echo cancellation situation, and it is possible to carry out a normal adaptation of adaptive filters during those times. In contrast, in an audio system with feedback, the loudspeaker signal \( y(n) \) is a processed version of the incoming signal \( u(n) \), i.e. \( y(n) = C(q)u(n) \), thus double-talk is always present. Hence, in AFC, the canceller needs to adapt during a continuous double-talk situation, thus freezing the filter’s coefficients when double-talk is detected is generally not an option. From the double-talk problem’s point of view, the AFC problem is more difficult to solve than the AEC problem [20].

Regarding the non-uniqueness problem, the loudspeaker signals are highly correlated to each other in stereo or multichannel audio systems [97]. This results in an infinite number of solutions for the echo cancellers. One effective solution is to attempt to decorrelate the loudspeaker signals by using non-linear methods [97], such as half-wave rectification. Although the underlying reasons are different, decorrelation techniques are useful in AEC as well as in AFC (to deal with the biased estimation problem). However, methods which are effective for decorrelation for one system might not be effective for the other [98]. A practical implementation of multichannel AEC can be found here [99].

Therefore, the two problems have their similarities, however, the solutions to one problem may not always be effective to the other. As such, each solution should be designed and verified specifically for each system.

### 2.2.3 Evaluation of Feedback Cancellation Systems

As presented earlier, the acoustic feedback poses problems to the normal operation of audio reinforcement systems as it may cause system instability, it limits the maximum achievable amplification, and it deteriorates the sound quality by producing a distortion of the desired incoming signal. As presented so far, a mul-
titude of different approaches have been proposed in the literature to deal with the acoustic feedback problem. Different measures have also been used to quantify the performance of each method differently. However, the ultimate objectives of all these methods are the same: to improve sound quality, to increase the amount of achievable amplification, and to operate in a reliable way [2]. Hence, evaluations and comparisons between different methods should be carried out with these three objectives in mind.

An objective evaluation can be carried out either in computer simulations or by physical measurements. Generally, it is straightforward to evaluate different algorithms on computer simulations rather than on real-time physical measurements. In computer simulations all the required information is available, such as, the knowledge of the true feedback path. Also, it is easily reproducible for the different algorithms and complicated test scenarios, with different test situations/parameters, can be quickly compared. Estimates of the true feedback path and some of its variations are generally obtained through measurements performed on a mannequin which is then used in computer simulations.

In this thesis we use three main measures: misalignment, maximum stable gain (MSG), and the Perceptual Evaluation of Speech Quality (PESQ) measure.

The normalized misalignment is the distance measure between the true and estimated feedback path, such as the mean square deviation $E\{\|g - \hat{g}\|^2\}$ [59]. This measure can be computed in the frequency domain as

$$\xi = 20 \log_{10} \frac{\|G(\Omega) - \hat{G}(\Omega)\|}{\|G(\Omega)\|}.$$  

The distance between the true and estimated feedback path is closely related to the open-loop equation $\Theta(\Omega) = K(\Omega)(G(\Omega) - \hat{G}(\Omega))$ presented in (2.13) and thus, closely related to the system stability. In [92] a power transfer function (PTF) was introduced, defined as $E\{\|G(\Omega) - \hat{G}(\Omega)\|^2\}$ and an approximation was proposed. The work in [92, 100, 101] shows how the approximate PTF can be used to predict the convergence rate, system stability bound and the steady-state behaviour of the entire cancellation system across frequency and time.

Another distance measure is the maximum stable gain (MSG) [102]. There are different ways to define the MSG, for instance see [2, 3, 14, 29]. One way of defining it is

$$\text{MSG} = 20 \log_{10} \left( \frac{1}{\Omega} \left\| \frac{1}{G(\Omega) - \hat{G}(\Omega)} \right\| \right),$$  

i.e., the MSG is determined by the frequency where the mismatch between the
actual and estimated path is greatest [14]. However, the system will only be unstable when the phase at that frequency equals a multiple of $2\pi$. Closely related to the MSG is the added stable gain (ASG), which is defined as the additional gain that is possible by using the feedback canceller, i.e.

$$\text{ASG} = \text{MSG} - 20 \log_{10} \left( \min_{\Omega} \frac{1}{\|G(\Omega)\|} \right).$$  \hspace{1cm} (2.24)

To measure speech quality we use the PESQ measure [103]. The PESQ is a model that delivers an estimated mean opinion score (MOS) [104] that is highly correlated with the MOS obtained from subjective listening experiments. Its algorithm requires two input signals for the computation of speech quality, the original speech signal and its degraded version. The output value of the voice quality returned by the PESQ is in the range $[-0.5 \ldots 4.5]$, where the values around $-0.5$ indicate a very poor quality signal and the values around 4.5 indicate a high quality signal.

There have been many works in the literature which evaluate and compare different acoustic feedback control methods and measures. For more information, we refer to [2, 3, 87, 105–107].

### 2.3 Summary

This chapter presented that the acoustic feedback poses problems in the normal operation of audio reinforcement systems as it may cause system instability, it limits the maximum achievable amplification, and it deteriorates the sound quality by producing a distortion of the desired incoming signal. Furthermore, the feedback path’s characteristics and variations make the feedback control in hearing aids unique and difficult to solve.

State-of-the-art acoustic feedback control techniques were then presented and categorised into two categories: feedforward suppression techniques and feedback cancellation methods.

Feedback cancellation using adaptive filters is generally more effective to control feedback than feedforward suppression methods. However, one of the biggest challenges for AFC is the biased estimation problem which is presented in the next chapter in detail.

### References


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Chapter 3

Biased Estimation Problem and Proposed Solutions

3.1 The Biased Estimation Problem

The use of feedback cancellation techniques is currently a preferred option to tackling the feedback problem [1, 2]. However, a straightforward use of cancellation techniques results in a bias term in the coefficients of the canceller. The biased solution, in a traditional canceller’s estimate, is caused by the correlation between the loudspeaker and incoming signal [1, 3–5]. It generally leads to poor system performance, results in signal distortion (canceller cancels portion of the desired signal), and, in worst case, causes the cancellation system to fail.

Minimising the MSE function in (2.15) results in the Wiener filter presented in (2.17). Substituting \( m(n) = u(n) + y^T(n)g \) in (2.17) we obtain the following biased solution

\[
\hat{g}_o = g + R_{yy}^{-1}r_{yu} \quad \text{(bias term)}
\]  \hfill (3.1)

The bias term is related to the correlation between \( y(n) \) and \( u(n) \). If \( r_{yu} = 0 \), then the feedback path estimate is unbiased. However, as a result of the closed-loop system, \( y(n) = C(q)u(n) \) and \( y(n) \) and \( u(n) \) are thus correlated, i.e., \( r_{yu} \neq 0 \).

If we assume that the forward path is just a delay (system processing delay) and amplification, i.e. \( K(q) = q^{-d_k} \tilde{K}(q) \) and \( \tilde{K}(q) = K \), then we can write \( y(n) \) as

\[
y(n) = K \cdot u_s(n - d_k)
\]  \hfill (3.2)
where \( u_s(n) = S(q)u(n) \) and
\[
S(q) = \frac{1}{1 - K(q)G(q)} \tag{3.3}
\]
is the sensitivity function. Using (3.2) we can re-write the term \( r_{yu} \) from (3.1) as
\[
r_{yu} = K \cdot E \{ u_s(n - d_k)u(n) \}
= K \cdot r_{u su}(d_k)
= K \cdot \begin{bmatrix}
    r_{u u}(d_k) \\
    r_{u u}(d_k + 1) \\
    \vdots \\
    r_{u u}(d_k + L - 1)
\end{bmatrix} \tag{3.4}
\]
where \( r_{u u}(d_k) = E \{ u_s(n)u(n - d_k) \} \) and
\[
u_s(n) = [u_s(n), u_s(n - 1), \ldots, u_s(n - L + 1)]^T. \tag{3.5}
\]

Expanding the correlation matrix \( R_{yy} \) using (3.2) we obtain
\[
R_{yy} = K^2 \cdot E \{ u_s(n - d_k)u_s^T(n - d_k) \}
= K^2 \cdot R_{u su} \tag{3.6}
\]
thus, the bias term in (3.1) can be written as
\[
R_{yy}^{-1}r_{yu} = \frac{1}{K} \cdot R_{u su}^{-1}r_{u u}(d_k). \tag{3.7}
\]

It can be seen from (3.7) that the correlation vector \( r_{u u}(d_k) \) plays a key role in obtaining an unbiased estimation of the feedback path. If the incoming signal has short correlation time compared to the audio processing delay \( d_k \), i.e. \( r_{u u}(k) = 0 \forall k \geq d_k \), then the correlation vector \( r_{u u}(d_k) \) will be zero and an unbiased estimation is obtained. Otherwise, the adaptive filter coefficients converge towards a biased coefficient vector. Thus, the system delay \( d_k \) helps to decorrelate \( y(n) \) and \( u(n) \). Also, (3.7) reveals that higher gain levels \( K \) reduces the impact of the bias term in the optimal solution [6]. Unfortunately, for many common sound signals like speech and tonal (music) signals, the signal correlation time is longer than the audio system latency, especially in a hearing aid application, where the system latency is typically between 4–8 ms [7]. Consequently, the estimate becomes biased, and the cancellation performance is hindered.

Therefore, as a result of the presence of a closed signal loop, standard adaptive filtering techniques for open-loop systems (as used in echo cancellation) fail to
provide a reliable feedback path estimate if the desired signal is spectrally coloured [1]. The challenge of this approach is to properly estimate the external feedback path with an adaptive filter. This is challenging to realise due to the correlation of the input signal and the signal which is acoustically fed back to the microphone. For reliable estimates of the feedback path, the adaptation has to be controlled by sophisticated methods.

3.2 Towards Unbiased Estimation

Different techniques have been proposed in the literature to deal with the biased estimation problem [1, 8]. However, none of these methods is a straightforward solution to the given problem, since many problems occur while implementing the proposed methods. Accordingly, there is still room for improvements of methods and designs to be integrated in future hearing aids. In this section we present some of the traditional methods for obtaining an unbiased estimation.

3.2.1 Prior-knowledge of feedback path

One solution to deal with the bias problem is to use prior knowledge of the feedback path to constrain the canceller’s adaptation [9, 10]. The canceller’s coefficients are not allowed to deviate too much from a reference filter. The reference filter coefficients are measured during start-up or fitting. This method, however, limits the tracking capability of the canceller during feedback path variations.

In [11] the feedback cancellation is restricted to the frequency band that encompasses the unstable frequencies by limiting the adaptation to critical frequencies known to cause instability. Typically, the acoustic feedback path of a hearing aid provides less attenuation at high frequencies, as a result, the risk for instability is often highest in the high frequency range. Whereas, most of the incoming signal correlation is concentrated at lower frequencies, for instance, speech signals. Thus, by limiting the adaptation of the canceller on higher frequencies, the bias problem may be reduced.

3.2.2 Decorrelation Methods

Unbiased estimation of the feedback path using a feedback canceller can be obtained through methods which decorrelate the loudspeaker from the incoming signal. The decorrelation can be performed either in the forward path of the hearing aid or in the adaptive filter estimation path in the cancellation system. Such techniques include introducing delays, phase modifications and other non-linearities, use of probe signal injection, use of synthetic signals [12], and use
Fig. 3.1 Delay in the forward path and/or in the adaptive filter estimation path.

Delays

In [4] it was suggested to insert a delay in the forward path and/or in the adaptive filter estimation path to decorrelate the loudspeaker from the incoming signal. Fig. 3.1 illustrates the use of these delays where the forward path delay is represented by \( d_k \), i.e. \( K(q) = q^{-d_k} \hat{K}(q) \), and the feedback path delay is represented by \( d_g \), where \( d_g \) is used to model the acoustic feedback path impulse response due to propagation time of the sound from the loudspeaker to the microphone and the processing delay by the ADC and DAC, i.e. \( G(q) = q^{-d_g} \hat{G}(q) \) and \( \hat{G}(q) \) models \( \bar{G}(q) \). As presented in Sec. 3.1, the system delays aid in bypassing the strong signal correlation at lower time lags. Also, the delay in the adaptive filter estimation path can better model the initial delay in the acoustic feedback path impulse response. However, the delay in the forward path \( d_k \) of the hearing aid should be kept small in order not to degrade intelligibility and sound quality [5, 14]. Also, relatively short delays can be used to correctly model the initial delay in feedback paths.

Therefore, although delay is effective to decorrelate many signals with relatively short correlation times, its use may be limited in practice.

Phase Modification and Use of Non-Linearities

Phase modification and introduction of non-linearities are decorrelation methods applied to the forward signal processing path of the hearing aid. This is similar to the feedforward suppression techniques mentioned in Sec. 2.2.1, but with the purpose of decorrelating \( y(n) \) from \( u(n) \) instead. This can be seen in Fig. 3.2 where \( K(q) \) is varied to reduce the bias term in the canceller’s \( \hat{G}(q) \) optimal solution.
3.2 Towards Unbiased Estimation

Frequency compression and shifting [15, 16] as well as phase modification [17, 18] have been studied in the literature with the aim of reducing the undesired signal correlation. The downside for phase modification methods is that modifications to the forward path results in undesirable audible artifacts.

Pre-Whitening

In the pre-whitening approach, the decorrelation is carried out on the signals used for the adaptive filter estimation, refer to Fig. 3.3. In this way, the forward path is unmodified, and no artefacts are introduced to the loudspeaker signal as a result of the decorrelation process.

One important method is the prediction error method (PEM), which is based on closed-loop identification theory [3, 19]. The PEM was analysed and proposed
to deal with the feedback problem in hearing aids in [5, 20–22] and in PA systems in [23, 24].

With this method, the bias term is reduced by incorporating a stationary or time-varying model of the desired incoming signal in the identification process. The incoming signal $u(n)$ is assumed to be a white noise sequence $\epsilon(n)$ filtered by an all-pole model $A(q)$, i.e., $u(n) = A(q)\epsilon(n)$ where $A(q) = \frac{1}{1 + q^{-1}P(q)}$ is a monic and inversely stable. The system then utilises pre-filters (inverse signal model) that are used to approximately whiten the incoming signal components and thereby compensate for the biased estimation.

Some recent work involving voiced-unvoiced detection to improve the PEM can be found here [25]. Several other modifications to the PEM are presented in [26–28].

In practice the pre-whitening approach works well for unvoiced parts of speech signals which can be modelled adequately as a white noise sequence filtered through the all-pole model. However, these methods do not perform as well for music signals or some voiced speech segments where the inverse signal model cannot be adequately estimated.

**Probe Signal Injection**

Training signals, such as probe noise signals can also be used to reduce the bias problem. These signals, which are designed to be uncorrelated to the incoming signal, are injected into the loudspeaker signal [20, 29, 30].

In the literature there are many different configurations for the use of the probe signal injection. Fig. 3.4 presents a non-continuous adaptation set up, see [29, 31, 32], where the system updates the estimated feedback path whenever changes in the feedback path, instabilities, or quiet intervals are detected. When a positive detection occurs, the normal hearing aid processing is interrupted and
Fig. 3.5 Feedback canceller with probe signal injection and continuous adaptation.

A probe signal is injected into the system to adjust the filter coefficients to give an estimate of the feedback path. The hearing aid is then returned to normal operation with the feedback cancellation filter as part of the system. As the estimation happens in open-loop, an unbiased estimate of the filter's coefficients is obtained. However, due to the reactive nature of this approach and issues with the reliability of the detectors, such systems may be objectionable.

Another approach is to use a mixture of the loudspeaker signal and probe signal for the continuous adaptation of the canceller [8]. This is illustrated in Fig. 3.5 where the loudspeaker signal is now defined as

\[ y(n) = K \cdot u_s(n - d_k) + w_s(n) \]

with \( w_s(n) = S(q)w(n) \). From expanding the correlation matrix \( R_{yy} \) we now obtain

\[
R_{yy} = K^2 \cdot E \left\{ u_s(n - d_k) u_s^T(n - d_k) \right\} + E \left\{ w_s(n) w_s^T(n) \right\}
\]

\[
= K^2 \cdot R_{u_s u_s} + R_{w_s w_s}\tag{3.8}
\]

where \( u_s(n) \) and \( w_s(n) \) are uncorrelated.

Therefore, the traditional bias term presented in (3.1), using this particular probe signal injection, can be written as

\[
R_{yy}^{-1} r_{yu} = \frac{1}{K} \left( R_{u_s u_s} + \frac{1}{K^2} R_{w_s w_s} \right)^{-1} r_{u_s u}(d_k).\tag{3.9}
\]

It can be seen from (3.9) that the probe signal \( w(n) \) will aid in reducing the bias term. The stronger the probe signal, the more the bias term is reduced. Another consideration from (3.8) and (3.9) is that the probe signal can guarantee a persistently exiting signal so that the inverse correlation matrix \( R_{yy}^{-1} \) exists for the identification [19]. However, there will still be some bias in the solution with this approach, especially when the probe signal is weak relative to the incoming signal.
Biased Estimation Problem and Proposed Solutions

Fig. 3.6 Feedback canceller’s estimation based on probe signal.

Fig. 3.7 Redraw of feedback canceller’s estimation based on probe signal. Incoming signal can be treated as uncorrelated noise.
Yet another approach is to base the canceller’s adaptation solely on the probe signal as seen in Fig. 3.6. There seems to be some misunderstanding in the literature with this method. It is accepted that this approach produces an unbiased solution, see for e.g. [2, 8]. However, as we presented in [33], the canceller’s optimal solution will still be biased, even if the probe signal is considered to be a white noise sequence. We then showed that an adequate forward path delay is sufficient to obtain an unbiased solution, see Paper D [33] for more details.

This approach, as presented in Fig. 3.6, has more similarities to the acoustic echo cancellation problem than other feedback control methods. As with echo cancellation, the incoming signal (near-end) is considered to be a strong uncorrelated noise, which disturbs the adaptive filter’s adaptation. This can be seen in Fig. 3.7 where the block diagram from Fig. 3.6 is rearranged to highlight this. In the presence of a disturbing incoming signal, and if a proper delay is used as per [33], the canceller’s optimal solution is an unbiased estimate of the feedback path.

In the absence of the incoming signal, and if a proper delay is used as per [33], the canceller’s optimal solution is an unbiased estimate of the feedback path.

In the presence of a disturbing incoming signal, the probe noise signal must often be powerful compared to the loudspeaker signal to achieve a noticeable improvement in performance. Unfortunately, powerful probe noise signals are clearly audible and undesirable to the user [20, 30]. In [34], it was shown theoretically that when the probe noise level is adjusted to be inaudible, the probe noise to disturbing signal ratio is generally low and the convergence rate of the adaptive system is decreased. Another drawback is that system performance is hindered, as we show in Paper E [35], when the probe signal is spectrally shaped to make it perceptually inaudible [8, 36, 37].

There have been studies to improve the system performance when the canceller’s adaptation is based solely on the probe signal. In [35] we showed theoretically that masking the probe signal compromises system behaviour. We then proposed a method to restore system behaviour by making use of the inverse of the shaping filter to pre-filter the microphone signal. In [34] a novel approach was proposed which employs probe enhancement filters $A(q)$, see Fig. 3.8. These enhancement filters reduce the influences of the disturbing incoming signal on the estimation of the canceller’s coefficients. The filters $A(q)$ increases the probe noise to disturbing signal ratio, and it leads to an increased convergence rate compared to the traditional probe signal approach.

Therefore, the use of probe signals is a promising method that effectively deals with the biased estimation problem in adaptive feedback cancellation.

**Use of Multiple Microphones**

With the advancement of digital signal processing and miniaturisation it has become more common to use multiple microphones techniques in hearing aids. Some
Fig. 3.8 Probe signal enhancement.

Fig. 3.9 Two microphone approach in dealing with biased feedback canceller.
early ideas on using a second microphone in feedback cancellation techniques can be found here [38, 39]. In these inventions, the feedback signal is detected by a second microphone which is placed inside the hearing aid and close to the loudspeaker. The resulting signal from the second microphone is added out-of-phase to the main signal to attempt to remove the feedback signal.

In Papers A, B, and C [40–42], we proposed an additional microphone to be employed to deal with bias estimation problem, refer to Fig. 3.9. The second microphone is spatially located at a location further away from the loudspeaker (feedback source) compared to the main microphone so that the feedback signal received is more attenuated. The additional microphone is used to obtain an incoming signal estimate which is then subtracted from \( \tilde{u}_1(n) \) to create the error signal prior to adapting the canceller’s coefficients, thus removing the undesired signal correlation.

The challenge with the proposed two microphone approach is the presence of a second feedback path. In [42] analytic expressions was presented which showed the second feedback path’s impact on the canceller’s optimal solution. From the obtained solution, it can be seen that the second feedback path is the main limitation to the system’s performance improvement. Thus, the biased solution is no longer dependent on the correlation between the incoming and loudspeaker signals, but on the second feedback path. Accordingly, by doing a proper acoustic design based on near field properties of the feedback path and far field properties of the impinging signals significant system benefits is obtained.

In [43] an external wireless microphone is employed as the additional microphone to obtain an incoming signal estimate. With this approach, the second feedback path can be ignored. However, the challenge now is to obtain an adequate estimate of the incoming signal as acoustic channels lose coherence over larger distances [44].

In [45] a binaural approach, where the microphones from one ear is used to obtain the incoming signal arriving at the other ear and remove it from the error signal prior to adapting the canceller. However, this approach will not be very robust and will probably only work when the two microphones pick up the same incoming signal (for instance, when the speaker is right in front of the hearing aid user in an open environment).

### 3.3 Summary

The main challenge with traditional feedback cancellers is the bias estimation problem. The biased solution in a traditional canceller’s estimate is caused by the correlation between the loudspeaker and incoming signal. It can result in
poor system performance, signal distortion, and, may even causes the cancellation system to fail.

This chapter reviewed state-of-the-art techniques proposed in the literature that deals with the biased estimation problem.

References


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Chapter 4

Thesis Contributions and Conclusion

This chapter provides a summary of contributions achieved from the conducted research, followed by a discussion on future research direction for this work, and ends with a conclusion.

4.1 Summary of Contributions

There are two main topics in this work which is illustrated in Fig. 4.1: the development and analysis of a two microphone approach in dealing with the biased problem; and investigations and improvements to the probe signal approach.

4.1.1 Two Microphone Approach

Firstly, we proposed and analysed a two microphone method with the aim of dealing with the bias estimation problem (Papers A, B and C). An additional microphone is used to obtain an incoming signal estimate which is then subtracted from the error signal prior to adapting the canceller’s coefficients. With this method, the biased solution is no longer dependent on the correlation between the incoming and loudspeaker signals.

Paper A - Dual microphone solution for acoustic feedback cancellation for assistive listening

In this work [1] we originally presented the two microphone approach for acoustic feedback cancellation (TM-AFC). An additional microphone is employed to enhance the canceller’s performance for hearing aids. The second microphone is spatially located at a location further away from the loudspeaker compared to the main microphone so that the feedback signal received is more attenuated.
The additional microphone is used to obtain an incoming signal estimate which is then subtracted from the primary microphone signal to create the error signal prior to adapting the canceler’s coefficients. The work in [1] suggested the use of two microphones and two adaptive filters arranged in such a way that allows the speech signal to be identified and removed from the adaptation process. This results in a more stable solution when compared with the traditional canceler which was verified by experiments and evaluations. The PESQ measure was also used to show that the proposed method results in better signal quality compared with a traditional canceler.

**Paper B - Closed-loop feedback cancellation utilising two microphones and transform domain processing**

In Paper B [2], we proposed to use orthogonal transforms with the TM-AFC method. The discrete Fourier transform (DFT) and the discrete cosine transform (DCT) are implemented to transform the adaptive filter signals. The intention is to further enhance the overall TM-AFC performance. A bank of adaptive filters is employed, each adapting to different portions of the spectrum. This enables for a finer control of the adaptation process. The full band filter’s coefficients are synthesised and used to provide the necessary signal estimates. Furthermore, this work does not make use of probe signal injection as in [1] which benefits signal quality.

Also, Paper B proposed to have one microphone in the ear canal and the additional microphone behind the ear. The microphone in the ear canal is the main microphone, which signal is amplified and broadcast through the loudspeaker. By
having such an arrangement, the natural position for signal pick-up is maintained with the aim of providing the user with a more natural hearing. In the single microphone scenario, such placement may limit the amount of gain achievable due to a stronger coupling between the loudspeaker and microphone signals. For higher gains, the main microphone may be placed behind the ear, however, this may affect the auditory cues and natural hearing. Thus, by using the TM-AFC approach, natural hearing and higher gains can be obtained.

From the simulation results presented, we see improvements in convergence rates and stable solutions using real speech signals when transform domain techniques are employed.

**Paper C - Analysis of Two Microphones Method for Feedback Cancellation**

Paper C [3] is a continuation of the work from Papers A and B. In this work we expanded on the theoretical analysis for the TM-AFC method by making use of the near-field and far-field models. We then presented analytic expressions showing the second feedback path’s impact on the canceller’s optimal solution. From the obtained solution, it can be seen that the second feedback path is the main limitation to the system’s performance improvement. It can also be seen that the biased solution is no longer dependent on the correlation between the incoming and loudspeaker signals.

Accordingly, by doing a proper acoustic design based on near field properties of the feedback path and far field properties of the impinging signals significant system benefits is obtained. We demonstrate this with simulation results and compare the TM-AFC method with the PEM in terms of misalignment and MSG. The results show that a more stable solution is obtained with the TM-AFC compared to the PEM.

**4.1.2 Probe Signal Investigations and Improvements**

Secondly, we studied the probe signal injection method and showed that the solution is biased even if white noise is used to drive the canceller’s adaptation. From this insight, we then derived conditions for obtaining an unbiased estimation (Paper D). To reduce signal quality degradation probe signals are usually shaped to provide some level of perceptual masking. Thus, it is important to know the impact the shaping filter has on system performance. In Paper E we studied the impact shaping the probe signal has on system performance and proposed a method to restore it while still maintaining benefits that come with spectrally shaping the probe noise.
Paper D - New Insights Into Optimal Acoustic Feedback Cancellation

In this paper [4] we presented new insights into the bias problem for acoustic feedback cancellation when a probe signal approach is used. We showed that the optimum solution of the feedback canceller is not the feedback path but the product of the feedback path and the sensitivity function and hence, the solution is biased.

The novelty of this paper also consists of the derivation of the conditions for unbiased feedback cancellation when a probe signal is used as input to the canceller. An adequate delay in the forward path is necessary to reduce, or remove the bias term. The theoretical analysis is verified with simulation results.

Paper E - Feedback Cancellation With Probe Shaping Compensation

Paper E is a continuation of the work in Paper D. In Paper E [5] we continued to analyse the particular method of using an injected probe signal as the input to the canceller, where the canceller bases the estimation of the feedback path on the probe signal.

Paper D suggested that if the probe signal is spectrally shaped, then the solution may still be biased. Paper E thus extends the delay condition from Paper D to obtain an unbiased solution when spectrally shaped probe signals are used. We then showed the detrimental impact a shaping filter has on system behaviour, specifically on convergence rate.

Finally, we presented a new approach which employs a filter that compensates for the use of a shaping filter. This improves system performance while maintaining the benefits which arise from perceptually shaping the probe signal.

4.2 Future Direction

For future work, we propose further investigations into quantifying the perceptibility and signal quality improvements obtainable for the TM-AFC method. With the TM-AFC approach, we suggested to have the main microphone inside the ear canal, to provide a more natural hearing experience to the user, and a second microphone behind the ear, to enable higher gains to be achieved. It would be interesting to quantify the improvement of the user’s natural pick-up of the incoming signal to further motivate the benefits of the proposed TM-AFC. Further work can also be conducted to characterise the sensitivity of the placement of the microphones. One approach would be to develop a model which allows the simulation of the different microphone locations, cf. [6]. However, some other aspects need to be taken into account such as the ear shape and size.
4.3 Conclusion

Some further work could also be conducted with the probe signal investigations. There has been much advancement and interest in the use of perceptually shaped probe signals that drive the cancellers estimation. This method effectively solves the biased estimation problem, however, at the cost of compromising signal quality. Thus, the design of shaping filters and its compensator, as presented in Paper E, has much potential for providing new designs of hearing aids where the probe signal is shaped according to perceptual masking principles.

4.3 Conclusion

This research investigated the biased estimation problem encountered when adaptive filters are used for acoustic feedback cancellation techniques.

The first main contribution of this thesis is the development of a two microphone approach which is presented in Papers A, B and C. The proposed approach removes the undesired biased term from the canceller’s solution. We showed that by doing a proper acoustic design based on near field properties of the feedback path and far field properties of the impinging signals system benefits, such as, increased achievable gain levels, stable solutions, and natural hearing can be obtained.

The second main contribution is the investigation on the use of a probe signal which drives the adaptation of the canceller. We showed in Paper D that the canceller’s converges to a biased solution even when the probe signal is white noise. This is important as there has been some misunderstanding in the literature which accepts that the solution is unbiased when the probe signal is designed to be uncorrelated with the incoming signal. Furthermore, in Paper D, we derived conditions for unbiased feedback cancellation. This effectively deals with the biased estimation problem.

We continued our probe signal investigations with Paper E. Generally, a shaping filter is used to mask the probe signal with the aim of improving sound quality. In Paper E we investigate the impact this shaping/masking filter has on system performance. We showed that system performance may be hindered when the probe signal is spectrally shaped. Based on this, we then presented a new approach which employs a filter that compensates for the use of a shaping filter. This improves system performance in terms of convergence rate while maintaining the benefits which arise from perceptually shaping the probe signal.
References


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Part II

Publications
Paper A

Dual microphone solution for acoustic feedback cancellation for assistive listening

@IEEE

The layout has been revised.

Includes corrections to some typographical error.
Dual microphone solution for acoustic feedback cancellation for assistive listening

C. R. C. Nakagawa, S. Nordholm, and W.-Y. Yan

Abstract

The method proposed in this paper improves the identification and cancellation of the feedback path by making the adaptive canceler robust against the impact of the desired speech signal. The proposed method allows for the canceler’s coefficients to be continuously adapted allowing it to track variations in the feedback path even in the presence of the desired signal. It suggests the use of dual microphones and dual adaptive filters arranged in such a way that allows the speech signal to be identified and removed from the adaptation process. This results in a more robust solution which was verified by our experiments and evaluations. The perceptual evaluation of speech quality (PESQ) measure was also used to show that the proposed method results in better signal quality.

I Introduction

With the advance of technology, such as, advances in digital signal processing, hearing aids are becoming smaller and smaller in size. Many hearing devices today can be fitted completely inside the ear canal of the user [1]. This reduction in size leads to a decreasing distance between the loudspeaker and the microphone. As a result, acoustic feedback occurs due to the acoustic coupling between the loudspeaker and the microphone.

Acoustic feedback poses a problem in the normal operation of hearing aids. The feedback limits the maximum achievable amplification possible by the hearing device, deteriorates the sound quality by producing a distortion of the desired signal, and is a cause of instability in hearing aids [2, 3]. The feedback path possesses some general characteristics. One characteristic is that the feedback varies under different conditions and environments. There has been some study in the literature of the variability of the feedback path [4, 5]. Causes of the feedback path and its variations are mentioned in [1, 2, 4–6]. The general observation is that the feedback path tend to show less attenuation at high frequencies than at low. Thus, oscillations due to feedback often occurs at higher frequencies [2].
Acoustic feedback control techniques try to minimize the effect of the feedback on the performance of hearing aids. [7] defines acoustic feedback control as the process of attempting to solve the acoustic feedback problem either completely (i.e., to remove the acoustic coupling) or partially (e.g., to remove the howling artifacts from the loudspeaker signal). Many feedback control methods have been proposed in the literature, however, there is still a lack of reliability in the available automatic acoustic feedback control solutions [7]. Thus, there is still a need and demand for improved feedforward suppression and/or cancellation techniques [3]. Proposed techniques in the literature can be generally classified into feedforward suppression and feedback cancellation techniques [3].

The use of feedback cancellation techniques in the acoustic feedback control is a preferred option as it is able to be made adaptive to track the variations in the feedback path [3, 8]. Fig. A.1 illustrates a classic feedback canceler. One main challenge with adaptive feedback cancelers is that the unobservable desired input signal $u_1(n)$ acts as a disturbance to the adaptation to the canceler. If the feedback estimate $\hat{f}_1(n) = f_1(n)$ then the error signal $e_c(n) = u_1(n)$. Therefore, if this error signal is used to adapt the filter’s coefficients it will result in the cancellation of the desired signal leading to degraded signal quality.

This paper proposes a method to identify the desired input signal $u_1(n)$ and remove it from the error signal prior to adapting the feedback canceler’s coefficients. Thus, making the adaptation more robust against the disturbance. The proposed idea also allows for the filter to be continuously adapted even in the presence of the desired input signal. This method results in better signal quality than the classic approach based on the PESQ method.
II Background

One main challenge with adaptive feedback cancelers is that the desired input signal \( u_1(n) \) acts as a disturbance to the canceler’s adaptation. The presence of the closed-signal loop gain \( K(\omega) \) introduces signal correlation when the desired signal is spectrally colored (e.g. speech or music signal) [3]. If the loudspeaker signal \( y(n) \) and the speech signal \( u_1(n) \) are not correlated, then the feedback path estimate is said to be unbiased. As a result of the bias term, the adaptive feedback canceler fails to provide a reliable feedback estimate and even cancels the desired signal instead.

The adaptive filter continuously adapts the coefficients \( \hat{g}_1 = [\hat{g}_0, \hat{g}_1, \ldots, \hat{g}_{L-1}]^T \) of the feedback canceler based on standard adaptive filtering procedures (Wiener filtering) where \( L \) is the length of the impulse response of the feedback path.

The adaptive filter tries to minimize the error signal \( e(n) \) using the cost function

\[
J(\hat{g}_1) = E\{|m_1(n) - \hat{g}_1^T y(n)|^2\}. \tag{A.1}
\]

With \( y(n) = [y(n) \ y(n-1) \ \ldots \ y(n-L+1)]^T \) then (A.1) results in the Wiener filter

\[
\hat{g}_1 = R_{yy}^{-1}(n) r_{ym_1}(n). \tag{A.2}
\]

where \( R_{\alpha\beta}(n) \) is the cross-correlation (autocorrelation when \( \alpha = \beta \)).

Assuming a sufficient-order \( L \), and using \( m_1(n) = g^T y(n) + u_1(n) \) then, (A.2) can be written as

\[
\hat{g}_1 = g_1 + R_{yy}^{-1}(n) r_{yu_1}(n)_{\text{bias}}. \tag{A.3}
\]

Ideally, \( \hat{g}_1 = g_1 \), however, from (A.3) it can be seen that the desired signal \( u_1(n) \) acts as a disturbance to the adaptation of the feedback canceler.

In the literature, several solutions have been proposed to reduce the bias problem. One solution is to incorporate signal decorrelating operations in the signal processing path of the hearing aid, such as introducing delays, probe signals, and non-linearities [3, 9, 10]. However, decorrelation tends to degrade the sound quality, making full decorrelation impossible [3]. Another attempt to minimize the bias is to reduce the adaptation speed of the adaptive feedback canceler [11] or constrain its adaptation based on prior knowledge of the feedback path [3]. Yet another approach, is to do a closed-loop system identification [8, 11, 12]. A more recent approach is to use dual microphones for feedback cancellation where the
coefficients of feedback canceler are updated after subtracting the speech signal from the input signal by dual microphones [13].

The proposed method here differentiates itself by using dual microphones and dual adaptive filters on each ear plug, it also allows for continuous adaptation of the feedback canceler, and more flexibility with the desired input source location.

III Proposed dual microphone method

This paper proposes an alternative way to improve the identification and cancellation of the feedback path $G_1(\omega)$ in the presence of the desired speech signal $u_1(n)$ by reducing the impact of the desired signal on the adaptation of the feedback canceler $\hat{G}_1(\omega)$. This method also allows for the canceler’s coefficients to continuously adapt allowing it to track variations in the feedback path.

The proposed idea is to use a second microphone in the assistive listening device. The location of such microphone is important. On one hand, the two microphones should be located as close as possible to each other so that the desired signal picked up by the microphones be as similar as possible. On the other hand, the two microphones should be as far as possible from each other, so that the feedback picked up by the second microphone be more attenuated than the first microphone. By this, the second microphone is able to capture the desired speech signal, with minimum presence of the feedback signal. This new signal is then removed from the error signal prior to adapting the feedback canceler, thus, removing the bias term from the adaptation.

Fig. A.2 illustrates an assistive listening device with two microphones. The device shown is plugged into the user’s right ear. Microphone 1 faces forward from the head and microphone 2 faces outward. The desired input wave $u(t)$ travels through two separate channels, $H_1(\omega)$ and $H_2(\omega)$ to reach each of the microphones. The signal picked up from microphone 1 is amplified and played out through the device’s loudspeaker. The amplified signal is fed back into the microphones through two separate channels $G_1(\omega)$ and $G_2(\omega)$.

It is desired that channel $H_1(\omega)$ be as similar as possible to $H_2(\omega)$ and that the feedback channel $G_2(\omega)$ have high attenuation. To achieve this, the placement of the microphones is crucial. The distance between the microphones compared to the distance from the microphones to the desired signal source should be relatively small. Also, the distance between microphone 2 and the feedback source should be relatively large.

Fig. A.3 illustrates the block diagram of the proposed dual microphones method with two adaptive filters, $\hat{G}_1(\omega)$ and $\hat{H}(\omega)$. The first filter, $\hat{G}_1(\omega)$ is adapted to match the feedback channel $G_1(\omega)$. The second filter $\hat{H}(\omega)$ is adapted
to match the channel $H(\omega)$ which is the transfer function from $H_2(\omega)$ to $H_1(\omega)$ in Fig. A.2. $K(\omega)$ is the signal processing path of the assistive listening device, which is generally some selective frequency gain.

The error equation $e_p(n)$ is given as

$$e_p(n) = g_1^T y(n) - \hat{g}_1^T y(n) + h^T u_2(n) - \hat{h}^T u_2(n) - \hat{h}^T f_2(n) \quad (A.4)$$

where $u_1(n) = h^T u_2(n)$, $g_1 = [g_0 \ g_1 \ ... \ g_{L-1}]^T$ is the coefficients of the feedback channel $G_1(\omega)$, $\hat{h} = [\hat{h}_0 \ \hat{h}_1 \ ... \ \hat{h}_{L-1}]^T$ is the coefficients of the estimate $\hat{H}(\omega)$, $y(n) = [y(n) \ y(n-1) \ ... \ y(n-L+1)]^T$ is the loudspeaker signal, and $f_2(n) = [f_2(n) \ f_2(n-1) \ ... \ f_2(n-L+1)]^T$ is the feedback signal picked up by the second microphone.

Let $a(n) = [g_1^T \ h^T]^T$, $z(n) = [y^T(n) \ u_2^T(n)]^T$, and $\xi(n) = [0^T \ f_2^T(n)]^T$, then (A.4) becomes

$$e_p(n) = [a - \hat{a}]^T z(n) - \hat{a}^T \xi(n). \quad (A.5)$$

We wish to minimise the cost function

$$J \left\{ |e_p(n)|^2 \right\} = E \left\{ |[a - \hat{a}]^T z(n) - \hat{a}^T \xi(n)|^2 \right\}. \quad (A.6)$$
By differentiating (A.6) with respect to $\hat{a}^T$ leads to

$$\hat{a} = \left[ R_{zz}(n) + R_{z\xi}(n) + R_{\xi z}(n) + R_{\xi\xi}(n) \right]^{-1} \cdot \left[ R_{zz}(n) + R_{\xi z}(n) \right] a. \quad (A.7)$$

If $\|R_{zx}(n)\| \gg \|R_{\xi\xi}(n)\|$ and the correlation between $u_2(n)$ and $f_2(n)$ is small, then the terms $R_{\xi\xi}(n)$, $R_{zx}(n)$, and $R_{\xi z}(n)$ in (A.7) can be ignored resulting in

$$\hat{a} = a. \quad (A.8)$$

Such assumptions can be made due to the microphone arrangement proposed. The greater the attenuation in the feedback channel $G_2(\omega)$, the weaker the signal $f_2(n)$ becomes and the less impact it will have in the system.

**IV Experiment Results**

Experiments were conducted in order to verify the performance of the proposed method and to validate our assumptions. The assistive listening device used was Sensear’s ear plug SP1x with 16 kHz sampling rate and with modified firmware to suit our real time experiment requirements. The layout of the microphone placement is illustrated in Fig. A.2 where one microphone faces forward from the head and the second microphone faces outward and is further away from the feedback source.

To measure the feedback path, a Gaussian white noise signal $w(n)$ was injected
into the loudspeaker and both microphones were set to record. When no speech
signal is present, the feedback path signals $f_1(n)$ and $f_2(n)$ can be measured
and used to estimate the channels $G_1(\omega)$ and $G_2(\omega)$. When the ear plug is
properly fitted into the user’s ears, we found that the $\|G_1(\omega)\| \gg \|G_2(\omega)\|$ for
most frequencies. At some frequency locations $\|G_1(\omega)\|$ is over 32 dB higher than
$\|G_2(\omega)\|$. The further away the second microphone is from the feedback source
the more attenuation there will be in $G_2(\omega)$.

Included in the feedback paths $G_1(\omega)$ and $G_2(\omega)$ is the characteristic of
the loudspeaker, the microphone, the analogue-to-digital converter (ADC), the
digital-to-analogue converter (DAC), and low-pass filters [3, 14].

Speech signals were also recorded during the experiments using dual micro-
phones. Three locations for the speech signal source was used: speaker placed in
front, side, and back of the head. These signals were recorded when no feedback
was present and were used in our evaluations.

V Evaluation based on experimental data

This section presents some of our evaluations based on experimental data. Fig.
A.4 compares the misalignment, defined as $\Delta = \frac{|g_1 - \hat{g}_1|^2}{|g_1|^2}$, of the proposed method
verses the classic adaptive filter illustrated in Fig. A.1. These plots were obtained
using 256 tap filters, with a modified LMS (MLMS) algorithm - where the step
size is normalized with respect to both the filter’s input and error signal. The
value of the step size was set to 0.1. The speaker was located facing the side of the
head and the injected noise variance was set to a value of 0.1. The forward path
gain $K(\omega)$ was set to 0 dB. All adaptive filters were set to the same parameter
values and algorithm.

Fig. A.4 and Fig. A.5 shows that the proposed method is more robust in
the presence of disturbance caused by the desired input signal. Fig. A.4 shows
that the misalignment does not diverge as wildly as the classic approach does and
Fig. A.5 presents the errors signals for the classic and proposed approach. With
the classic approach, $e_c(n)$ is very similar to $u_1(n)$ whereas in $e_p(n)$ there is less
impact from $u_1(n)$.

Another objective measure used is the PESQ measure. The classic adaptive
canceler tends to degrade the desired signal quality as it cancels the speech due
to the bias term. Therefore, the use of the PESQ measure quantifies the speech
quality and is an appropriate measure to compare the proposed method against
the classic approach. PESQ provides a score in the range of 1 to 5 where 1 is
unacceptable and 5 is excellent. Table A.1 presents the results. The reference
signal used is $u_1(n)$ and the degraded signal used is the loudspeaker signal $y(n)$.
Fig. A.4 Misalignment proposed method vs classic adaptation

Fig. A.5 Error signals - proposed vs classic
VI Conclusion

This paper proposed an approach that improves the identification and cancellation of the feedback path by reducing the impact of the desired signal on the adaptation of the feedback canceler. This method allows for the canceler’s coefficients to continuously adapt allowing it to track variations in the feedback path. The suggested microphone layout assumes that the speech signal received by both microphones are similar, but the feedback received by the second microphone has greater attenuation than the first. Two adaptive filters were used, the first was used as the feedback canceler and the second was used to match the desired speech signal recorded by the dual microphones. With such arrangement, the speech signal from the second microphone is subtracted from the error signal before adapting the canceler. This results in a more robust solution which was verified by our experiments and evaluations. The perceptual evaluation of speech quality (PESQ) measure was also used to show that the proposed method results in better signal quality.

References


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Paper B

Closed-loop feedback cancellation utilizing two microphones and transform domain processing

The layout has been revised.
Closed-loop feedback cancellation utilizing two microphones and transform domain processing

C. R. C. Nakagawa, S. Nordholm, F. Albu, and W.-Y. Yan

Abstract

In this paper we are studying the use of two microphones for acoustic feedback cancellation in hearing aids. With the two microphones approach, an additional microphone is employed to provide added information about the signals which is then utilized to obtain an incoming signal estimate. This estimate is removed from the error signal prior to adapting the canceller, thus removing the undesired signal correlation. In this paper, we propose to use orthogonal transforms with the two microphones approach. The discrete Fourier transform and the discrete cosine transform are implemented to transform the adaptive filter signals. Also, a bank of adaptive filters is employed, each adapting to different portions of the spectrum for a finer control of the adaptation process. Simulation results based on real measured feedback paths and speech signals show improved convergence rates and stable solutions.

I Introduction and Contribution

Sound reinforcement systems such as public address systems and hearing aids suffer from acoustic feedback problems. Acoustic feedback results from the acoustic coupling between the loudspeaker and microphone. The microphone(s) picks up the loudspeaker’s signal and re-amplifies it creating an acoustic loop. For each round trip the signal traveling around this loop gets re-amplified potentially causing system instability. The feedback problem limits the maximum stable gain (MSG) achievable, it deteriorates the sound quality by producing a distortion of the incoming signal, and it is a cause of instability in acoustic systems working in closed-loop [1].

The use of acoustic feedback cancelers (AFC) is currently a preferred option in feedback control techniques [2]. The purpose of AFC is essentially to identify a model of the feedback path and to estimate the feedback signal. The feedback estimate is then subtracted from the microphone signal. However one of the challenges with feedback cancelers, as a result of the closed-loop signals, is
the bias problem where the canceler’s coefficients become biased when the correlation between the loudspeaker and incoming signal is non-zero [1, 3]. This correlation generally leads to a poor system performance and in the worst-case scenario, it may cause the cancellation system to fail. Different techniques have been proposed to reduce this correlation including phase modification, frequency shifting, decorrelating pre-filters, adaptive filters in tandem, use of synthesized signals, and probe noise injection [2–10]. The use of orthogonal transforms to transform the adaptive filter signals can also be used to reduce signal correlation. Originally, the use of orthogonal transform was proposed to increase convergence rates in stochastic gradient algorithms such as the least mean squares (LMS) algorithm [11–13]. In [14] the discrete cosine transform (DCT) is applied to the prediction error method (PEM) to boost the PEM performance for acoustic echo cancellation (AEC) and AFC. In [15], an additional microphone was employed to provide added information which was utilized to obtain an incoming signal estimate. This estimate is removed from the error signal prior to adapting the canceler, thus removing the undesired signal correlation. We refer to this method as the two microphone acoustic feedback canceler (TM-AFC).

In this paper, we propose to use orthogonal transforms with the TM-AFC method. The discrete Fourier transform (DFT) and the DCT are implemented to transform the adaptive filter signals. The DCT is used because it is the closest transform to the optimal Karhunen-Loeve Transform (KLT) for low-pass signals, like speech signals [14]. We use the DFT as a reference. The motivation is to further enhance the overall TM-AFC performance. In this work, the transform is not only applied to the input signal of the canceler as in [14], but also to the error signal. Another differentiator is that a bank of adaptive filters is employed, each adapting to different portions of the spectrum. This enables for a finer control of the adaptation process. The full band filter’s coefficients are synthesized and used to provide the necessary signal estimates. The proposed structure is similar to delayless subband filtering but without decimation [16, 17]. Furthermore, this work does not make use of probe signal injection as in [15] which benefits signal quality [3]. From the simulation results, we see improvements in convergence rates and stable solutions using real speech signals.

This paper is structured as follows. First, we review the TM-AFC approach. Then, the proposed transform domain with filtered error version of the TM-AFC method is presented followed by simulation results.
Fig. B.1 illustrates a feedback canceler for an hearing aid with a single microphone. The feedback path between the loudspeaker and the microphone is assumed to be a discrete-time finite impulse response (FIR) filter with coefficient vector \( g_1 = [g_{10} \ldots g_{1,L_\phi - 1}]^T \) with filter length \( L_\phi \) which is represented as a polynomial transfer function \( G_1(q) \) in \( q \) as \( G_1(q) = g_1^T q \) with \( q = [1 \ q^{-1} \ldots q^{-L_\phi + 1}]^T \). This representation allows the following notation, for the filtering of \( y(n) \) by \( G(q) \), \( G_1(q)y(n) = g_1^T(n)y(n) \) [18]. Column vectors are emphasized using lower letters in bold, the superscript \( T \) denote vector transpose, the discrete-time index is denoted by \( n \), and the symbol \( q^{-1} \) denotes the discrete-time delay operator \( q^{-1}u(n) = u(n - 1) \). All signals are real-valued, and we denote all signals as discrete-time signals with time index \( n \) for convenience.

The forward path \( K(q) \) represents the regular signal processing path of the device. In this paper, \( K(q) \) has a delay \( d_k \geq 1 \) and provides the system with a constant gain i.e., \( K(q) = q^{-d_k}K \). The adaptive filter \( \hat{G}_1(q) \), with coefficient vector \( \hat{g}_1 = [\hat{g}_{10} \ldots \hat{g}_{1,L_\phi - 1}]^T \) and filter length \( L_\hat{\phi} = L_\phi \), identifies and tracks changes to the feedback path by producing an estimate \( \hat{f}_1(n) \) of the feedback signal \( f(n) \). The loudspeaker and microphone signals are \( y(n) \) and \( m_1(n) \), respectively. The incoming signal is denoted by \( u_1(n) \) and the feedback signal is denoted by \( f_1(n) = G_1(q)y(n) \). The estimate \( \hat{f}_1(n) \) is subtracted from the microphone signal \( m_1(n) \). The error signal \( e_1(n) \) is used to update the canceler’s coefficients and is also amplified by the forward path and played out through the loudspeaker. As a result of the non-zero correlation between the incoming and loudspeaker signal, the canceler’s optimal solution is biased [3].

To remove the undesired signal correlation in the canceler’s optimal solution an additional microphone was used in [15] to obtain an incoming signal estimate,
which is subtracted from the error signal prior to adapting the feedback canceler. The two microphones are placed rather close but not in the same position which means that the received signals have high correlation. The TM-AFC configuration is presented in Fig. B.2. We write the relationship between the incoming signals $u_1(n)$ and $u_2(n)$ as

$$u_1(n) = H(q)u_2(n) + \zeta(n) \tag{B.1}$$

where $\zeta(n)$ is the part of $u_1(n)$ that is not predictable from $u_2(n)$ and $H(q)$ is a FIR filter with length $L_h$. The delay $d_m$ in the first microphone signal path is to avoid having a non-causal system. $\hat{H}(q)$ is an adaptive FIR filter of length $L_h \geq L_h + d_m$ which filters the second microphone signal $m_2(n)$ producing the incoming signal estimate $\hat{u}_1(n)$ which is subtracted from the error signal $e_1(n)$.

It is required that $|G_1(q)| > |G_2(q)|$. A possible location for the microphones would be to have one microphone in the ear canal and an additional microphone behind the ear, for instance, refer to Fig. C.2. The microphone in the ear canal is the main microphone, which signal is amplified and played out through the loudspeaker. By having such an arrangement, the natural position for signal pick-up is maintained providing the user with a more natural hearing [19]. Thus, having the main microphone placed in the ear canal is desirable. In the single microphone scenario, such placement may limit the amount of gain possible due
to a stronger coupling between the loudspeaker and microphone signals. For higher gains, the microphone may be placed behind the ear. This may affect the auditory cues and natural hearing. Thus, by using the TM-AFC approach, natural hearing and higher gains can be obtained.

A challenge with the TM-AFC approach is the presence of a second feedback channel $G_2(q)$. In [15] it was shown that $G_2(q)$ introduces a bias to the solution. However, with the proposed microphone arrangement, it can be assumed that $|G_2(q)|$ is weak.

### III Transform Domain Filtered Error TM-AFC

In this section we present an extended version of the TM-AFC. The intention in using orthonormal transformation is to further improve on the performance of the TM-AFC approach.

The orthonormal transformations, $\mathbf{T}$, used in this paper are the DFT and DCT. The $M \times M$ DCT and DFT matrix coefficients $\mathbf{T}_{DCT}[k,l]$ and $\mathbf{T}_{DFT}[k,l]$ are given as in (B.2)-(B.3), respectively. Note that there may be several other orthogonal transforms suitable for adaptive filtering algorithms, please refer to [12].
Closed-loop feedback cancellation utilizing two microphones

\[
T_{\text{DCT}}[k,l] = \begin{cases} 
\frac{1}{\sqrt{M}}, & k = 0 \& l = 0 \ldots M - 1 \\
\left(\frac{2}{M}\right)^{\frac{1}{2}} \cos \frac{\pi (2l+1)k}{2M}, & k = 1 \ldots M - 1 \& l = 0 \ldots M - 1.
\end{cases}
\]  

(B.2)

\[
T_{\text{DFT}}[k,l] = \frac{1}{\sqrt{M}} e^{-2\pi kl/M}, k, l = 0 \ldots M - 1.
\]  

(B.3)

We refer to this proposed approach as the transform domain (TD) with filtered error (Fe) TM-AFC and is presented in Fig. (B.4). The inputs \(y(n)\) and \(m_2(n)\), of \(\hat{G}_1(q)\) and \(\hat{H}(q)\), respectively, are transformed by \(T\), which can be any suitable orthogonal transform. The transform matrix \(T\) can be thought of as a bank of \(M\) parallel filters tuned to different portions of the spectrum of the input sequence [12]. The components of the transformed input vectors appear to be approximately decorrelated with one another. Moreover, an appropriate power normalization can convert the input autocorrelation matrix to a normalized matrix whose eigenvalue spread will be smaller than that of the original input signal, thereby improving the convergence behavior of the system in the transform domain [12, 14]. A difference between the proposed approach and the one used in [14], to improve the PEM, is that the error signal is also filtered by \(T\) and a bank of adaptive filters is used.

\(M\) adaptive filters (AF) are used to adapt the different portions of the spectrum. Then, the full band filters, \(\hat{G}_1(q)\) and \(\hat{H}(q)\), are synthesized by adding the estimated coefficients of the \(M\) filters together. The feedback estimate \(\hat{f}_1(n)\) is produced by filtering the loudspeaker signal \(y(n)\) by this full band feedback canceler \(\hat{G}_1(q)\). The same procedure is applied to \(\hat{H}(q)\). This structure is similar to delayless subband filtering but without decimation [16, 17].

The improvement in performance comes at the cost of an increase in computational complexity. Three transform domain operations are required as well as an additional \(M - 1\) adaptive filters for each identification. Nevertheless, when the DFT is used, we can make use of the complex conjugate symmetry to reduce complexity, thus reducing the number of filters used (only \(M/2 + 1\) filters are required). Also, fast versions of the algorithm for the DCT and DFT are available, which reduces the complexity from \(O(M^2)\) to \(O(M \log M)\) operations [20, 21].
Fig. B.4 Proposed TD-Fe-TM-AFC method.
IV Simulation results

In order to perform simulations, experiments were first conducted to obtain the feedback path’s characteristics and variations. Measurements were conducted in a recording studio using a Brüel & Kjær (B&K) head and torso simulator type 4128C. Fig. B.5 presents the feedback path’s characteristics with a normal fit in the ear with and without obstruction. Obstruction refers to a flat surfaced object placed very close to the ear to simulate the use of a mobile phone. In our simulations this will be used simulate a path change to analyze the tracking performance of the algorithm. Note that the second feedback path’s magnitude response is much weaker than the first feedback path. Speech signals were also recorded using the two microphones. The input sequence used for the speech signals was real speech segments from NOIZEUS database which contains 30 IEEE sentences spoken by 3 male and 3 female speakers [22]. The speech signals were concatenated together and played out back to back.

To assess the performance of the algorithm, the misalignment between the true and estimated feedback path and the added stable gain (ASG) measures are used. The misalignment is used to represent the accuracy of the feedback path estimation and is defined as

$$\text{Misalignment} = 20 \log_{10} \frac{\int_0^\pi \| G(\omega) - \hat{G}(\omega) \|_2 \, d\omega}{\int_0^\pi \| G(\omega) \|_2 \, d\omega}. \quad (B.4)$$

To quantify the added achievable amplification the ASG is defined as

$$\text{ASG} = \text{MSG} - 20 \log_{10} \left[ \min_\omega \frac{1}{|G(\omega)|} \right] \quad (B.5)$$
Fig. B.6 Instantaneous misalignment and ASG plots for varying $M$. 

(a) Misalignment when $M = 2$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 2\mu$. 

(b) ASG when $M = 2$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 2\mu$. 

(c) Misalignment when $M = 4$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 5\mu$. 

(d) ASG when $M = 4$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 5\mu$. 

(e) Misalignment when $M = 8$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 10\mu$. 

(f) ASG when $M = 8$, $\mu = 0.0001$, and $\mu_{TM-NLMS} = 10\mu$. 

IV Simulation results | B.11
where MSG is defined as

$$\text{MSG} = 20 \log_{10} \left[ \min_{\omega} \frac{1}{|G(\omega) - \hat{G}(\omega)|} \right].$$ \hspace{1cm} (B.6)

The MSG and ASG is determined by the frequency where the mismatch between the actual and estimated path is greatest. However, the system will only be unstable when the phase at that frequency equals a multiple of $2\pi$.

In the simulations the following parameters were used. The length of the actual feedback path is $L_g = 32$ samples. The simulation run lasts for 80 seconds with a instantaneous change of feedback path occurring at time 40 seconds. Speech is used as the incoming signal. The complex normalized least mean squares (NLMS) algorithm is used for adapting the $M$ filters for $\hat{G}_1(q)$ and $\hat{H}(q)$ with step size $\mu = 0.0001$. The filter length for $\hat{H}(q)$ is $L_h = 8$ with $d_m = 3$. The sampling frequency is 16 kHz, and the forward path gain $K = 30$ dB with a forward path delay of $d_k = 32$ samples.

Fig. B.6 presents the misalignment and ASG curves for varying values of $M$. We compare the transform domain version of the algorithms with the two microphone NLMS (TM-NLMS) with a step-size which gives similar initial convergence. The TD-Fe method is also applied to the traditional NLMS filter and is labeled TD-Fe-NLMS-DFT. Figs. B.6a-B.6b presents the case where $M = 2$. With $M = 2$, both the DCT and DFT transform result in similar performance in terms of misalignment and ASG. The step size $\mu_{\text{TM-NLMS}} = 2\mu$ is used to give similar initial convergence. In Figs. B.6c-B.6d $M = 4$, and $\mu_{\text{TM-NLMS}} = 5\mu$. Finally, in Figs. B.6e-B.6f $M = 8$, and $\mu_{\text{TM-NLMS}} = 10\mu$.

As the value of $M$ is increased, the convergence rate is improved at the cost of higher complexity. It is interesting to note that for higher values of $M$, the DFT transform starts to give greater improvements in performance than the DCT. Also note that TD-Fe-NLMS is very sensitive to the incoming signal, whereas, the TM-NLMS and TD-Fe-TM-NLMS methods are more robust to the incoming signal variations.

V Conclusion

In this paper we extended the TM-AFC method. We proposed to improve on the TM-AFC performance by utilizing orthogonal transforms. Both the adaptive filter’s input and error signals are transformed and a bank of adaptive filters used. The full band filter’s coefficients are then synthesized and used to provide the necessary signal estimates. Simulation results based on real measured feedback
paths and speech signals showed improved convergence rates and stable solutions.

References


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Paper C

Analysis of Two Microphone Method for Feedback Cancellation

The layout has been revised.
Analysis of Two Microphone Method for Feedback Cancellation

C. R. C. Nakagawa, S. Nordholm, and W.-Y. Yan

Abstract

Acoustic feedback cancellation in hearing aids makes use of adaptive filters to continuously identify and track variations to the feedback path. One of the biggest problems remaining in using adaptive filters for feedback cancellation is the biased estimation of the filter’s coefficients. In order to remove the undesired correlation between the loudspeaker and incoming signal, a recent alternative scheme proposed to employ an additional microphone. This microphone can provide added information to obtain an incoming signal estimate. This estimate is used to create the error signal which adapts the canceler’s coefficients. This letter provides the theoretical analysis for the two microphone method. It presents analytic expressions showing that the optimal solution is no longer dependent on the signal correlation aforementioned but is now mainly determined by the additional feedback path. Finally, it demonstrates simulation results with the prediction error method in terms of misalignment and maximum gain for a proposed microphone placement. The results show that a more stable solution is obtained with the proposed two microphone approach.

I Introduction

Acoustic feedback is used to refer to the undesired acoustic coupling between the loudspeaker and microphone. Systems susceptible to problems resulting from the acoustic coupling include public address systems and assistive listening devices, such as hearing aids. It is common practice to use adaptive filters as acoustic feedback cancelers (AFC) to compensate for the feedback signal. The main challenge of using adaptive filters for AFC is that the filter estimates become biased whenever there is correlation between the incoming and loudspeaker signals [1]. This results in the well known bias problem [2], which generally leads to poor cancellation performance. Different techniques have been proposed to reduce this correlation including phase modification, frequency shifting, decorrelating pre-filters, adaptive filters in tandem, use of synthesized signals, and probe noise injection [3–11].
In [12, 13], an additional microphone is employed to enhance the canceler’s performance for assistive listening devices. The second microphone is spatially located at a location further away from the loudspeaker compared to the main microphone so that the feedback signal received is more attenuated. The purpose of using the additional microphone is used to obtain an incoming signal estimate. If the incoming signal is known, the bias term can be removed. This estimate is used to create the error signal prior to adapting the canceler’s coefficients. In [14] it was proposed to have one microphone in the ear canal and the additional microphone behind the ear, see Fig. C.2. The microphone in the ear canal is the main microphone. This microphone’s signal is amplified and played out through the loudspeaker. By having such an arrangement, the natural position for signal pick-up is maintained with the aim of providing the user with a more natural hearing. In the single microphone scenario, such placement may limit the amount of gain achievable due to a stronger coupling between the loudspeaker and microphone signals. For higher gains, the main microphone may be placed behind the ear, however, this may affect the auditory cues and natural hearing. Thus, by using the two microphone AFC (TM-AFC) approach, both natural hearing and higher gains may be obtained. In [13] it was proposed to use of an external wireless microphone as the additional microphone. That will resolve the problem of feedback but will provide a challenge to predicting the incoming signal over larger distances since acoustic channels usually lose coherence over distance [15].

The challenge with the TM-AFC approach is the presence of a second feedback path. In this letter we provide a theoretical analysis for the TM-AFC method [12]. We present analytic expressions showing the impact of the second feedback path’s on the canceler’s optimal solution. We show that the biased solution is no longer dependent on the correlation between the incoming and loudspeaker signals, but on the second feedback path. Accordingly, by doing a proper acoustic design based on near field properties of the feedback path and far field properties of the impinging signals significant system benefits have been obtained. We demonstrate this with simulation results and compare the TM-AFC method with the prediction error method (PEM) [1] in terms of misalignment and maximum stable gain (MSG). The results show that a more stable solution is obtained with the TM-AFC.

Section II of this work expands on theoretical analysis for the TM-AFC by presenting the TM signal model. Section III then presents the optimal solution for the TM-AFC method. Finally, Section IV presents simulation results to validate the theoretical solution.
II System description

Fig. C.1 illustrates the TM-AFC method. The feedback paths between the loudspeaker and the microphones are assumed to be a discrete-time finite impulse response (FIR) filter with coefficient vectors $\mathbf{g}_i = [g_{i0}, \ldots, g_{iL_q-1}]^T$, where $i = 1, 2$, with filter length $L_q$ which is represented as a polynomial transfer function $G_i(q)$ in $q$ as $G_i(q) = \mathbf{g}_i^T \mathbf{q}$ with $\mathbf{q} = [1, \ldots, q^{-L_q+1}]^T$ [16]. The adaptive filter $\hat{G}_i(q)$ with coefficient vector $\hat{\mathbf{g}}_i = [\hat{g}_{i0}, \ldots, \hat{g}_{iL_q-1}]^T$ identifies and tracks changes to the feedback path producing an estimate $\hat{f}_i(n)$ of the feedback signal $f_i(n)$. The incoming signals are denoted by $u_i(n)$ and the feedback signals are denoted by $f_i(n) = G_i(q)y_i(n)$. The estimate $\hat{f}_i(n)$ is subtracted from the microphone signal $m_i(n)$. A probe noise signal $w(n)$, that is designed to be uncorrelated to $u_i(n)$, may be injected into the loudspeaker signal $y_1(n)$. The forward path $K(q)$ is assumed to have a delay $d_k \geq 1$ and constant gain $K$.

From Fig. C.1, it can be seen that $y_1(n) = S(q)(q^{-d_m}K(q)u_1(n) + w(n))$ where $S(q)$ is the sensitivity function $S(q) = (1 - q^{-d_m}K(q)(G_1(q) - \hat{G}_1(q))))^{-1}$. The open-loop frequency function $e^{-j\omega d_m}K(\omega)(G_1(\omega) - \hat{G}_1(\omega))$ plays a central part in acoustic feedback control, where the spectrum of $K(q)$ and $G_1(q)$ is denoted by $K(\omega)$ and $G_1(\omega)$, respectively, and $\omega = [0, 2\pi)$. The Nyquist criterion states that oscillations may occur if the magnitude response of the open-loop function is greater than unity and the phase response is a multiple of $2\pi$ [17].

To obtain desired system benefits, the two microphones are arranged in such a way to make use of the far-field properties of the incoming signals and near-field properties of the feedback signals. Accordingly, the incoming source signal received by the first microphone $u_1(n)$, in the discrete time domain, can be obtained from $u_2(n)$ as

$$u_1(n - d_m) = H(q)u_2(n) + \zeta(n) \quad (C.1)$$

where $H(q)$ is assumed to be a FIR filter with length $L_h$, $\zeta(n)$ is the component of $u_1(n)$ that is not predictable from $u_2(n)$, and to avoid having a non-causal system a delay $d_m$ is added to the first microphone path. From (C.1) it is assumed that $u_1(n)$ and $u_2(n)$ are coherent signals which is the case with our proposed microphone location. Practically, the proposed microphone arrangement illustrated in Fig. C.2 provides the greatest distance between the microphones for the one device. If an external microphone is used instead as the secondary microphone, the model in (C.1) may not be valid. For instance, if the microphone of the other ear is used. In this case, employing a head related transfer function (HRTF) might be a more adequate approach in estimating the incoming signal. Furthermore, strong wind noise may pose a challenge to the model in (C.1). Thus, a wind
Fig. C.1 TM-AFC feedback canceler.

detection algorithm may be used to control the filter’s adaptation [18].

With the proposed microphone arrangement, based on the near field model, the feedback signal received by the second microphone \( f_2(n) = G_2(q)y_1(n) \) is more attenuated than the feedback received by the first microphone \( f_1(n) = G_1(q)y_1(n) \), as the main microphone is placed closer to the feedback source, i.e., \( |G_2(\omega)| < |G_1(\omega)| \).

An adaptive FIR filter \( \hat{H}(q) \) with coefficient vector \( \hat{h} = [\hat{h}_0, \ldots, \hat{h}_{L_{\hat{h}}-1}]^T \) of length \( L_{\hat{h}} \) is employed to identify and track changes to \( H(q) \). If far-field model is assumed, we expect that \( u_1(n) \) is a delayed version of \( u_2(n) \). However, in practice this might not be the case, especially at higher frequencies as a result of reflections from the head. Therefore, an adaptive filter \( \hat{H}(q) \) is used instead of merely a delay. Thus, \( \hat{H}(q) \) filters the second microphone signal to produce an incoming signal estimate \( \hat{u}_1(n) \) which is then removed from the signal \( \tilde{u}_1(n) \) to create the error signal \( e_1(n) \). Note, from Fig. C.1, that the error signal is not amplified and played back through the loudspeaker as with the traditional method.

A challenge with the TM-AFC approach is the presence of a second feedback path \( G_2(q) \). This is presented in detail next.
III TM-AFC Optimal Solution

In this section we derive the optimal solution for the TM-AFC. The error for the TM-AFC is defined as

\[
e_1(n) = \tilde{a}_1(n) - \hat{a}_1(n)
\]

\[
e_1(n) = u_1(n - d_m) + \tilde{g}_1^T y_1(n - d_m) - \hat{h}^T (u_2(n) + f_2(n))
\]

\[
= \hat{h}^T u_2(n) + \zeta(n) + \tilde{g}_1^T y_1(n - d_m) - \tilde{g}_2^T y_1(n)
\]  \hspace{1cm} (C.2)

where \( \tilde{g}_1 = g_1 - \tilde{g}_1, \tilde{h} = h - \hat{h}, y_1(n) = [y_1(n), \ldots, y_1(n - L_g + 1)]^T, u_2(n) = [u_2(n), \ldots, u_2(n - L_h + 1)]^T, f_2(n) = [f_2(n), \ldots, f_2(n - L_h + 1)]^T, \) and \( g_2 = [g_{2_{n0}}, \ldots, g_{2_{nk_L-1}}] \) is the coefficient vector for \( \hat{H}(q)G_2(q) \).

Typically, an acoustic feedback path \( G_i(q) \) contains a delay \( d_g \) that arises from the processing delay of the analog-to-digital converter (ADC), digital-to-analog converter (DAC) and the distance between the microphone and loudspeaker, i.e., \( G_i(q) = q^{-d_g} \tilde{G}_i(q) \) with \( L_g = d_g + L_g [1] \). Now, we assume that \( G_2(q) \) contains an initial delay \( d_g \) that is greater than \( d_m \). We can then represent \( G_2(q) \) as \( G_2(q) = q^{-d_m} \tilde{G}_2(q) \) resulting in

\[
e_1(n) = \tilde{h}^T u_2(n) + \zeta(n)
\]

\[
+ \tilde{g}_1^T y_1(n - d_m) - \tilde{g}_2^T y_1(n - d_m)
\]  \hspace{1cm} (C.3)

where \( \tilde{g}_2 = [\tilde{g}_{n0}, \ldots, \tilde{g}_{nL_g-1}]^T \) is the coefficient vector for \( \hat{H}(q)\tilde{G}_2(q) \).

Let \( a = \left[ \tilde{g}_1^T \tilde{h}^T \right]^T, b = \left[ \tilde{g}_2^T \tilde{y}^T \right]^T, x(n) = \left[ y_1^T(n - d_m) \ u_2^T(n) \right]^T, \) and \( z(n) = \left[ y_2^T(n - d_m) \ 0^T \right]^T \) where the dimension of the null vectors are \( L_h \). Then \( e_1(n) \) can be written as

\[
e_1(n) = a^T x(n) - b^T z(n) + \zeta(n).
\]  \hspace{1cm} (C.4)

Minimizing the mean-squared error (MSE) results in \( a_o \), the optimal solution,

\[
a_o = R_{xx}^{-1} R_{xz} b - R_{xx}^{-1} r_x \zeta
\]  \hspace{1cm} (C.5)

which can be expressed as

\[
\begin{bmatrix}
\tilde{g}_{1_o} \\
\tilde{h}_o
\end{bmatrix} = R_{xx}^{-1} \begin{bmatrix}
R_{y_1 y_1} \tilde{g}_{2_o} \\
R_{u_2 y_1} \tilde{g}_{2_o}
\end{bmatrix} - R_{xx}^{-1} \begin{bmatrix}
r_{y_1} \zeta \\
r_{u_2} \zeta
\end{bmatrix}
\]  \hspace{1cm} (C.6)

where \( R_{\alpha\beta} \) and \( r_{\alpha\beta} \) represent a correlation matrix and vector, respectively, \( \tilde{g}_{1_o} = \).
\( g - \hat{g}_1, \tilde{h}_o = h - \hat{h}_o, \) and

\[
R_{xx} = \begin{bmatrix}
R_{y_1y_1} & R_{y_1u_2} \\
R_{u_2y_1} & R_{u_2u_2}
\end{bmatrix}.
\] (C.7)

The inverse for \( R_{xx} \) can be obtained by using block-wise inversion

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{-1} = \begin{bmatrix}
A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} \\
(D - CA^{-1}B)^{-1}
\end{bmatrix}.
\] (C.8)

This results in

\[
R_{xx}^{-1} = \begin{bmatrix}
E & F \\
G & H
\end{bmatrix}
\]

where

\[
E = R_{y_1y_1}^{-1} + R_{y_1u_1}^{-1}R_{y_1u_2}
\]

\[ \cdot \left( R_{u_2u_2} - R_{u_2y_1}R_{y_1y_1}^{-1}R_{y_1u_2} \right)^{-1} R_{u_2y_1}R_{y_1y_1}^{-1} \];

(C.9)

\[
F = -R_{y_1y_1}^{-1}R_{y_1u_2} \left( R_{u_2u_2} - R_{u_2y_1}R_{y_1y_1}^{-1}R_{y_1u_2} \right)^{-1} R_{u_2y_1}R_{y_1y_1}^{-1} \];

(C.10)

\[
G = - \left( R_{u_2u_2} - R_{u_2y_1}R_{y_1y_1}^{-1}R_{y_1u_2} \right)^{-1} R_{u_2y_1}R_{y_1y_1}^{-1} \];

(C.11)

\[
H = \left( R_{u_2u_2} - R_{u_2y_1}R_{y_1y_1}^{-1}R_{y_1u_2} \right)^{-1} .
\]

Therefore, the solution in (C.6) can be expanded as

\[
\begin{bmatrix}
\tilde{g}_1 \\
\tilde{h}_o
\end{bmatrix}
= \begin{bmatrix}
E \cdot R_{y_1y_1} \tilde{g}_2 + F \cdot R_{y_1u_1} \tilde{g}_2 \\
G \cdot R_{y_1y_1} \tilde{g}_2 + H \cdot R_{y_1u_1} \tilde{g}_2
\end{bmatrix}
- \begin{bmatrix}
E \cdot r_{y_1\zeta} + F \cdot r_{u_2\zeta} \\
G \cdot r_{y_1\zeta} + H \cdot r_{u_2\zeta}
\end{bmatrix}.
\] (C.12)

If \( \zeta(n) \) is assumed to be uncorrelated with \( y_1(n) \) and \( u_2(n) \), and expanding \( E \),
\[ \hat{g}_{1o} = R_{y_1y_1}^{-1} R_{y_1y_1} \hat{g}_2 \]

\[ + R_{y_1y_1}^{-1} R_{y_1u_2} (R_{u_2y_1} - R_{u_2y_1} R_{y_1y_1}^{-1} R_{y_1u_2})^{-1} 
\cdot R_{u_2y_1} R_{y_1y_1} R_{y_1y_1} \hat{g}_2 \]

\[ - R_{y_1y_1}^{-1} R_{y_1u_2} (R_{u_2y_1} - R_{u_2y_1} R_{y_1y_1}^{-1} R_{y_1u_2})^{-1} 
\cdot R_{u_2y_1} \hat{g}_2 \] (C.13)

and

\[ \tilde{h}_o = - (R_{u_2u_2} - R_{u_2y_1} R_{y_1y_1}^{-1} R_{y_1u_2})^{-1} R_{u_2y_1} R_{y_1y_1} \hat{g}_2 \]

\[ + (R_{u_2u_2} - R_{u_2y_1} R_{y_1y_1}^{-1} R_{y_1u_2})^{-1} R_{u_2y_1} \hat{g}_2 \] (C.14)

which reduces to \( \hat{g}_{1o} = \hat{g}_2 \) and \( \tilde{h}_o = 0 \), i.e.,

\[ \hat{g}_{1o} = g_1 - \hat{g}_2 \] and \( \tilde{h}_o = h \). (C.15)

Therefore, it can also be seen that the optimal solution is no longer dependent on the undesired signal correlation. The bias term is now dependent on \( \hat{H}(q) \hat{G}_2(q) \).

Simulation results showing the limitation imposed by the second feedback path in (C.15) in terms of misalignment and MSG are presented in the next section.

**IV Simulation Results**

In order to perform simulations, experiments were first conducted to obtain the feedback path’s characteristics. As in [14] it is proposed that the main microphone be placed in the ear and the second microphone placed behind the ear as illustrated in Fig. C.2. By having such an arrangement, the natural position for signal pick-up is maintained providing the user with a more natural hearing. Fig. C.3 presents the feedback path’s characteristics with a normal fit in the ear and when a flat object is placed near the ear. Note that the second feedback path’s magnitude response is much weaker than the first feedback path.

Speech signals were also recorded using the two microphones. The input sequence used for the speech signals was real speech segments from NOIZEUS database which contains 30 IEEE sentences spoken by 3 male and 3 female speakers [19]. The speech signals were concatenated together and played out back to
Performance measure used are the misalignment and MSG. The misalignment and MSG are defined as \( \Delta \left( G_1(\omega), \hat{G}_1(\omega) \right) = 20 \log_{10} \int_0^\pi \frac{|G_1(\omega) - \hat{G}_1(\omega)| d\omega}{\int_0^\pi |G_1(\omega)| d\omega} \) and \( \text{MSG} \left( G_1(\omega), \hat{G}_1(\omega) \right) = 20 \log_{10} \left( \max_\omega \left| G_1(\omega) - \hat{G}_1(\omega) \right| \right) \), respectively. The MSG is determined by the frequency where the mismatch between the actual and estimated path is greatest. However, the system will only be unstable when the phase at that frequency equals a multiple of \( 2\pi \).

In the simulations the delay \( d_g = 16 \) samples and \( L_g = 38 \) samples. The simulation run lasts for 80 seconds with concatenated speech signals as incoming signal. The normalized least mean squares (NLMS) algorithm is used for all adaptive filters. The feedback canceler \( \hat{G}_1(q) \) step size \( \mu_g = 0.001 \) and \( L_\hat{g} = 22 \) where \( G_1(q) = q^{-d_g} \hat{G}_1(q) \) and \( \hat{G}_1(q) \) models \( G_1(q) \). The filter \( \hat{H}(q) \) step size
Fig. C.4 Feedback paths are varied at time 40 seconds.

\[ \mu_h = 0.001 \] and the filter length \( L_h = 10 \) and \( d_m = 1 \). The injected probe signal \( w(n) \) used to excite the channel is a white Gaussian noise sequence with a level of \(-20 \) dB with respect to \( K(q)u_1(n) \). The sampling frequency is 16 kHz, and the forward path gain \( K = 35 \) dB with a forward path delay of \( d_k = 64 \) samples.

Fig. C.4 presents the misalignment and MSG curves for the TM-AFC where at time 40 seconds the feedback paths are varied. The theoretical solution, \( G_1(q) - \tilde{G}_{1w}(q) = \tilde{G}_{2h}(q) \), presented in (C.15) is also plotted as means of validating the theoretical analysis.

In Fig. C.5 the incoming signal source location is varied. In this scenario, the source is initially facing the right side of the head (device is placed on the right ear). Then, at time 40 seconds, the source is located to face the left side of the head. This simulates a meeting scenario where different speakers may speak at different times from different locations. As can be seen from the plots that the adaptation of \( \tilde{H}(q) \) is fast enough to track the changes without compromising system performance.

Finally, Fig. C.6 presents the misalignment and MSG curves for the PEM for both microphones \( m_1 \) and \( m_2 \). For the PEM setup, a 21 order AR model is used to estimate the incoming signal using a frame size of 160 samples through the Levison-Durbin algorithm. The NLMS algorithm with same parameters values as presented previously is used to adapt the canceler. It can be seen that the performance curves for the PEM fluctuate over time depending on the incoming signal. The reason for this fluctuation might be from using different speech signals concatenated together. Therefore, some speech sounds are better mod-
Fig. C.5 Source location is varied at time 40 seconds.

Fig. C.6 PEM comparison: source location is varied at time 40 seconds.
eled than others by the PEM, hence the variations. With the TM-AFC approach, knowledge of the incoming signal model is not required and a more stable solution is obtained. The reason the PEM performs worst with the second microphone $m_2(n)$ is that the feedback to incoming signal ratio is weak. Higher gain levels can improve the performance, but would not allow the comparison with $m_1(n)$ as the system would become unstable.

Note that the steady-state values for the TM-AFC revolves around $|\hat{H}(q)\hat{G}_2(q)|$. If $|G_2(q)|$ is stronger, then the achievable steady-state values for the misalignment and MSG will be worst. Else, if $|G_2(q)|$ is weaker, then better performance is achievable. Thus, the TM-AFC’s performance is dependent on $G_2(q)$ which in turn is dependent on microphone placement.

V Conclusion

With the TM-AFC approach, an incoming signal estimate is obtained to produce a more adequate error signal. This letter showed that the TM-AFC method removes the bias term caused by the undesired signal correlation from the filter’s optimal solution. Nevertheless, the new optimal solution is now dependent on the additional feedback path. Accordingly, by doing a proper acoustic design based on near field properties of the feedback channel and far field properties of the impinging signals significant system benefits can be achieved. Simulations results showed that a more stable steady-state solution is obtained when compared with the PEM in terms of misalignment and MSG.

References


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Paper D

New Insights Into Optimal Acoustic Feedback Cancellation

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The layout has been revised.

Includes corrections to some typographical errors.
New Insights Into Optimal Acoustic Feedback Cancellation

C. R. C. Nakagawa, S. Nordholm, and W.-Y. Yan

Abstract

In this paper we present new insights into the bias problem for acoustic feedback cancellation when a probe signal approach is used. The optimum solution of the feedback canceler is not the feedback path but the product of the feedback path and the sensitivity function and hence, the solution is biased. The novelty of this paper also consists of the derivation of the conditions for unbiased feedback cancellation when a probe signal is used as input to the canceler. An adequate delay in the forward path is necessary to reduce, or remove the bias term. The theoretical analysis is verified with simulation results.

I Introduction

Acoustic feedback poses a problem in the normal operation of assistive listening devices due to the acoustic coupling between the loudspeaker and microphone. The microphone picks up the loudspeaker signal and re-amplifies it creating an acoustic loop, thus the signal traveling around this loop gets stronger for each round trip potentially causing stability problems. The feedback limits the maximum stable gain (MSG) achievable, it deteriorates the sound quality by producing a distortion of the incoming signal, and it is a cause of instability in acoustic systems working in closed-loop [1].

The use of feedback cancellation techniques is currently a preferred option to tackling the feedback problem [1]. The main challenge with traditional feedback cancelers is the well known bias problem. The biased solution in the canceler’s estimate is caused by the correlation between the loudspeaker and incoming signal [1, 2]. It generally leads to a poor system performance and in the worst-case scenario, it causes the cancellation system to fail. Different techniques have been proposed to reduce this correlation including phase modification, frequency shifting, non-linear processing, decorrelating pre-filters, probe noise injection, and the use of multiple microphones to estimate the incoming signal and remove it prior to adapting the canceler [1, 3–6].

In this paper we study the adaptive feedback canceler’s optimal solution when a probe signal is injected into the system. With a probe signal injection approach,
either the probe or the loudspeaker signal can be used as an input to the adaptive canceler [3]. If the loudspeaker signal is used as an input to the canceler then the optimal solution results in the well known bias problem, where the correlation between the loudspeaker signal and incoming signal is of interest. However, if the probe signal, designed to be uncorrelated with the incoming signal, is used to drive the adaptive canceler, then it is accepted that the solution is an unbiased estimate of the feedback path see [3, 7] and references therein. However, we show theoretically that the injection of a probe signal, with the probe signal used as an input to the adaptive canceler, does not guarantee an unbiased solution. Thus, new insights into the bias problem is presented.

Section II presents the system description for acoustic feedback cancellation using a probe signal approach. Then, Section III establishes theoretical expressions for the optimum solution and presents new insights into the bias problem. Once the biased solution is recognized, conditions under which it can be reduced, or removed, is proposed. Finally, Section IV produces simulations results that verify the derived theoretical expressions.

In this paper, column vectors are emphasized using lower letters in bold. The superscript $T$ denote vector transpose, the expectation operator is denoted by $E \{ \cdot \}$, the discrete-time index is denoted by $n$, and the symbol $q^{-1}$ denotes the discrete-time delay operator $q^{-1}u(n) = u(n - 1)$. All signals are real-valued, and we denote all signals as discrete-time signals with time index $n$ for convenience.

II System description

Fig. D.1 illustrates a feedback canceler for an assistive listening device with a single microphone.

The feedback path between the loudspeaker and the microphone is assumed to be a discrete-time finite impulse response (FIR) filter with coefficient vector $g = [ g_0 \ g_1 \ \ldots \ g_{L_g-1} ]^T$ with filter length $L_g$ which is represented as a polynomial transfer function $G(q)$ in $q$ as $G(q) = g^Tq$ with $q = [ 1 \ q^{-1} \ \ldots \ q^{-L_g+1} ]^T$. This representation allows the following notation, for the filtering of $y(n)$ by $G(q)$ [8],

$$G(q)y(n) = g^T y(n). \quad (D.1)$$

Typically, the acoustic feedback path $G(q)$ contains a delay $d_q$ that arises from the processing delay of the analogue-to-digital converter (ADC) and digital-to-analogue converter (DAC), i.e., $G(q) = q^{-d_q} \hat{G}(q)$ with $L_g = d_q + L_{\hat{g}}$ [9]. The feedback path $G(q)$ is therefore modeled as a cascade of a delay $d_q$ and a feedback canceler $\hat{G}(q)$. The adaptive filter $\hat{G}(q)$ identifies and tracks changes to the
feedback path producing an estimate $\hat{f}(n)$ of the feedback signal $f(n)$. The loudspeaker and microphone signals are $y(n)$ and $m(n)$, respectively. The incoming signal is denoted by $u(n)$ and the feedback signal is denoted by $f(n) = G(q)y(n)$. The estimate $\hat{f}(n)$ is subtracted from the microphone signal $m(n)$.

A probe noise signal $w(n)$, that is designed to be uncorrelated to $u(n)$, is injected into the loudspeaker signal $y(n)$ and used as the input to the feedback canceler $\hat{G}(q)$.

The forward path $K(q)$ represents the regular signal processing path of the device (i.e., a frequency-specific gain, compression and/or noise reduction). In this paper, $K(q)$ has a delay $d_k$ of at least one sample and provides the system with a constant gain $\bar{K}(q) = \bar{K}$, i.e., $K(q) = q^{-d_k}\bar{K}(q)$.

A standard criterion to find an optimal set of coefficients is to minimize the mean square error (MSE) cost function

$$J(\hat{g}) = E\{|e(n)|^2\}. \quad (D.2)$$

From Fig. D.1, it can be seen that

$$m(n) = u(n) + G(q)y(n), \quad (D.3)$$

and

$$y(n) = K(q) \left( m(n) - \hat{G}(q)y(n) \right) + w(n). \quad (D.4)$$
where we set $d_g = 0$ to simplify equations but it does not impact the results. Nevertheless, the delay $d_g$ will be used in our simulations presented in Section IV.

Substituting (D.3) into (D.4)

$$y(n) = S(q)K(q)u(n) + S(q)w(n)$$

$$= S(q)\bar{y}(n)$$

where $S(q)$ is the sensitivity function

$$S(q) = \frac{1}{1 - K(q)\left(G(q) - \hat{G}(q)\right)}$$

$$\bar{y}(n) = K(q)u(n) + w(n),$$

and

$$e(n) = m(n) - \hat{G}(q)w(n).$$

The frequency function

$$K(\omega)\left(G(\omega) - \hat{G}(\omega)\right)$$

in (D.6) is often referred to as the “loop-response”, where the spectrum of $K(q)$ and $G(q)$ is denoted by $K(\omega)$ and $G(\omega)$, respectively, with $\omega = [0, 2\pi]$. It plays a central part in acoustic feedback control [1, 3]. The Nyquist criterion states that oscillations may occur if the magnitude response of the loop-gain is greater than unity and the loop-phase is a multiple of $2\pi$ [10]. It can be seen in (D.6) that the channel $G(q)$ may lead to system instability. To avoid this, the amount of gain $K(q)$ has to be limited. However, if the feedback canceler $\hat{G}(q)$ can resemble $G(q)$, then the system is brought closer to its desired response $S(q) = 1$. Ideally, $\hat{G}(q) = G(q)$ which results in $S(q) = 1$.

III New insights into the bias problem using probe signal injection

This section highlights the fact that there is some bias in the optimal solution when a probe signal is used to drive the feedback canceler. Furthermore, it presents conditions in which the solution’s bias can be reduced or even removed.
III New insights into the bias problem using probe signal injection | D.7

completely.

Fig. D.1 illustrates the case where $w(n)$ is the input to the feedback canceler. Minimizing the mean square error (MSE) cost function in (D.2), i.e.,

$$\frac{\delta E \{|e(n)|^2\}}{\delta \hat{g}^T} = 0$$

(D.10)

results in the Wiener filter [11]

$$\hat{g}_o = E \left\{ w(n)w^T(n) \right\}^{-1} E \{ w(n)m(n) \}$$

(D.11)

where

$$w(n) = \left[ w(n) \ w(n-1) \ \ldots \ w(n-L_\hat{g}+1) \right]^T$$

(D.12)

and $\hat{g}_o$ is the set of optimal coefficients.

Using (D.5)-(D.7) we expand (D.3) as

$$m(n) = u(n) + A_{FIR}(q)\bar{y}(n) + (A(q) - A_{FIR}(q)) \bar{y}(n)$$

$$= u(n) + \bar{K} \cdot A_{FIR}(q)u(n-d_k) + A_{FIR}(q)w(n) + \xi(n)$$

(D.13)

where

$$A(q) = G(q)S(q)$$

(D.14)

is a causal IIR filter, which may be specified as $A(q) = a_0 + q^{-1}a_1 + \ldots$. The filter $A_{FIR}(q)$ corresponds to the first $L_\hat{g}$ coefficients of $A(q)$ and

$$\xi(n) = (A(q) - A_{FIR}(q)) \bar{y}(n)$$

$$= q^{-L_\hat{g}}A_r(q)\bar{y}(n)$$

(D.15)

represents the residual impulse response $q^{-L_\hat{g}}A_r(q)$.

Assuming a sufficient-order filter and using (D.13) in (D.11) results in

$$\hat{g}_o = a_{FIR} + E \left\{ w(n)w^T(n) \right\}^{-1} E \{ w(n)\xi(n) \}$$

$$= a_{FIR} + E \left\{ w(n)w^T(n) \right\}^{-1} E \{ w(n)w_{a_r}^T(n-L_\hat{g}) \}$$

(D.16)

as $w(n)$ and $u(n)$ are uncorrelated by construction of $w(n)$. In this paper we assume that $w(n)$ is a white Gaussian noise sequence and, as a result, $E\{ w(n)w_{a_r}^T(n-L_\hat{g}) \} = 0$ and (D.16) becomes

...
\[ \hat{g}_0 = a_{\text{FIR}}. \] (D.17)

However, if the probe noise signal \( w(n) \) is masked/shaped to reduce its influence on sound quality, then \( E \{ w(n)w^T(n - L) \} \) may not be zero and, as a result, will contribute to the bias term in (D.16).

Therefore, assuming that \( w(n) \) is white Gaussian noise, it is shown that the optimal solution of the feedback canceler \( \hat{G}(q) \) is not the feedback path \( G(q) \) but the product of the feedback path \( G(q) \) and the sensitivity function \( S(q) \) and hence, the solution is biased.

### A Conditions for identifiability

Now, we present conditions for identifiability where the desired solution \( \hat{G}(q) = G(q) \) can be obtained from (D.17).

If we write (D.14) as

\[ A(q) = G(q) + q^{-d_k}A(q)K(q)E(q) \] (D.18)

where

\[ E(q) = (G(q) - \hat{G}(q)) \] (D.19)

then, it can be seen from (D.18) that as a delay, \( d_k \), is contained in \( K(q)E(q) \) the first \( d_k \) coefficients of \( A(q) \) coincide with the impulse response of the feedback path \( [g_0 \ldots g_{d_k-1}] \), i.e., the first \( d_k \) coefficients will not be biased. If \( d_k \geq L_g \) then \( G(q) \) can be completely obtained from the first \( L_g \) coefficients of \( A(q) \), such as,

\[
A(q) = g_0 + q^{-1}g_1 + \ldots + q^{-L_g+1}g_{L_g-1} + \ldots \\
+ q^{-d_k+1}a_{d_k-1} + \ldots
\] (D.20)

As we have influence over the design of the forward path, we can vary \( d_k \) to reduce, or even remove, the bias term. Thus, by using an adequate delay, \( d_k \geq L_g \), the solution is decoupled and an unbiased optimal solution for \( G(q) \) can be obtained from using the first \( L_g \) coefficients of \( A(q) \), assuming sufficient filter order.

Lower gain values for \( K(q) \) could also be used to reduce the bias term, however, this goes against assistive listening devices’ main objective which is to provide its users with an amplified signal to compensate for their hearing impairment.
If we define
\[
\hat{A}(q) = \hat{G}(q)S(q) \tag{D.21}
\]
as the canceler, then \(\hat{A}(q)\) is an unbiased estimate of \(A(q)\). It is interesting to consider what happens to the biased solution in (D.18) as \(\hat{A}(q) \to A(q)\). From multiplying both sides of (D.19) by \(S(q)\) we can write \(E(q)\) as
\[
E(q) = \frac{A(q) - \hat{A}(q)}{1 + K(q) \left( A(q) - \hat{A}(q) \right)} \tag{D.22}
\]
then as \(\hat{A}(q) \to A(q)\) it can be seen from (D.22) that \(E(q) \to 0\) and, as such, the bias term is reduced over time if the system converges, i.e. \(A(q) \to G(q)\).

### IV Simulation Verification

The goal of the simulations is to verify the derived theoretical expression in (D.18). To assess the performance of the algorithm, the misalignment between the true feedback path \(G(q)\) and \(A(q)\) is used. The normalized misalignment curve is defined, in the frequency domain, as
\[
\Delta(G(q), A(q)) = 10 \log_{10} \frac{\int_0^{\pi} |G(\omega) - A(\omega)|^2 d\omega}{\int_0^{\pi} |G(\omega)|^2 d\omega}. \tag{D.23}
\]

In order to perform simulations, experiments were first conducted to obtain the feedback path’s characteristics and variations. The assistive listening device used in our experiments was a Sensear ear plug SP1x with 16 kHz sampling rate with a modified firmware to suit our real time experiment requirements. Measurements were conducted in an anechoic chamber on a Brüel & Kjær head and torso simulator type 4128C. The device’s microphone was set to record while a Gaussian white noise probe signal \(w(n)\) was being injected into the loudspeaker to excite the feedback path. With such recordings we were able to identify the path \(G(q)\). Included in the feedback path are the characteristics of the loudspeaker, the microphone, the ADC, the DAC.

To reduce complexity, the feedback path is therefore modeled as a cascade of a delay \(d_g\) and a shorter feedback canceler. The delay \(d_g\) was set to 32 samples, \(L_g = 96\), and \(L_{\hat{g}} = 48\). The last 16 samples of \(G(q)\) is not modeled as the main impulse is contained within the first 80 samples, see Fig. D.2 for the feedback path characteristics. The incoming signal \(u(n) = 0\).

The update of the feedback canceler’s coefficients, \(\hat{g}\), with step size \(\mu = 0.01\)
is performed using the normalized least-mean-square (NLMS) algorithm

\[ \hat{g}(n) = \hat{g}(n - 1) + \frac{\mu}{w(n)^T w(n)} w(n)e(n). \]  

(D.24)

It can be seen from (D.18) that \( K(q) = q^{-d_k}\bar{K}(q) \) has an influence on the amount of bias in the solution. It is expected that the higher the gain \( \bar{K}(q) \) the more the solution will be biased. Also, the longer the delay \( d_k \) in \( K(q) \), the less the solution will be biased. With this in mind, we present two plots. In the first plot, the delay is fixed to its lowest value \( d_k = 1 \) and the gain is varied. In the second plot, the gain \( \bar{K}(q) \) is fixed and the delay \( d_k \) varied. Each misalignment curve presented is an average of 50 simulations run where in each run a new realization of white Gaussian noise sequence is drawn for \( w(n) \).

Fig. D.3 presents the first plot where the gain is varied from 0 dB to 30 dB in 10 dB increments. The delay \( d_k \) is kept constant \( d_k = 1 \). As the gain is increased, the misalignment between \( G(q) \) and \( A(q) \) increases. If \( K(q) \) were to be an open circuit, \( A(q) = G(q) \) as per (D.18).

Fig. D.4 presents the second plot where the delay \( d_k \) is varied. Here the gain is kept constant \( \bar{K}(q) = 30 \) dB. As the delay is increased, the misalignment curves shifts downwards as \( K(q)E(q) \) and \( G(q) \) are decoupled. If \( d_k \geq L_g \) the misalignment value is \( -\infty \), which is not shown in the plot.

With both plots, it can be seen that the misalignment between \( G(q) \) and \( A(q) \) is reduced over time as \( E(q) \to 0 \) resulting in \( A(q) \to G(q) \).

V Conclusion

This paper presented new insights into the bias problem for acoustic feedback cancellation when a probe signal is used. It was presented, using theoretical results, that the feedback canceler’s optimum solution is not the feedback path \( G(q) \) but the product of the feedback path \( G(q) \) and the sensitivity function \( S(q) \) and hence, the solution is biased.

The novelty of this paper also consists of the derivation of the conditions for unbiased feedback cancellation when a probe signal is used as input to the canceler. It was demonstrated that by manipulating the forward path \( K(q) \) the bias term resulting from the sensitivity function \( S(q) \) can be reduced, and even removed. Lower gains and/or higher delays in the forward path results in a reduction of the solution’s bias. However, assistive listening devices normally require higher gains, so it is recommended to add an adequate delay, ideally \( d_k \geq L_g \), to deal with the biased solution. Thus, by adding an adequate delay in the forward path an unbiased solution can be obtained.
Fig. D.2 Feedback path characteristics.

The theoretical analysis was verified with simulation results.

References


Fig. D.3 Misalignment between $G(q)$ and $A(q)$ with varying gain $\bar{K}$ with $d_k = 1$. As the gain is increased, the misalignment between $G(q)$ and $A(q)$ increases. If $K(q)$ were to be an open circuit, $A(q) = G(q)$.


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Fig. D.4 Misalignment between $G(q)$ and $A(q)$ with varying delay $d_k$ with $\bar{K} = 30$ dB. As the delay is increased, the misalignment curves shifts downwards as $K(q)E(q)$ and $G(q)$ are decoupled. If $d_k \geq L_g$ the misalignment value is $-\infty$ (not shown in the plot). Also note that $d_g = 32$. Plot with $d_k = 1$ is presented in Fig. D.3.
Paper E

Feedback Cancellation with Probe Shaping Compensation

The layout has been revised.
Feedback Cancellation With Probe Shaping Compensation

C. R. C. Nakagawa, S. Nordholm, and W.-Y. Yan

Abstract

Adaptive feedback cancellation methods may integrate the use of probe signals to assist with the biased optimal solution in acoustic systems working in closed-loop. However, injecting a probe noise in the loudspeaker decreases the signal quality perceived by users of assistive listening devices. To counter this, probe signals are usually shaped to provide some level of perceptual masking. In this letter we show the impact of using a shaping filter on the system behavior in terms of convergence rate and steady state error. From this study, it can be concluded that shaping the probe signal may have detrimental influence in terms of system performance. Accordingly, we propose to use the unshaped probe signal combined with an inverse filter of the shaping filter to identify the feedback channel. This restructure of the problem restores convergence rate of LMS type algorithms. Furthermore, we also show that an adequate forward path delay is required to obtain an unbiased solution and that the suggested scheme reduces this delay.

I Introduction

Acoustic feedback occurs when part of the loudspeaker signal from an audio system is picked up by its microphone creating an acoustic loop. The signal traveling around this loop gets re-amplified for each round trip potentially causing audible artifacts. The feedback limits the maximum stable gain achievable, it deteriorates the sound quality by producing a distortion of the incoming signal, and it is a cause of instability in acoustic systems working in closed-loop [1].

The use of adaptive filters as feedback cancelers is a common method to compensate for the feedback signal. However, one of the main challenges with these cancelers is the well known bias problem [1–3]. The correlation between the loudspeaker and incoming signal generally leads to a poor system performance and it may cause the cancellation system to fail. Different techniques have been proposed to reduce this correlation including phase modification, frequency shifting, non-linear processing, decorrelating pre-filters, probe noise injection, and the use of multiple microphones [1, 2, 4–6].
In this letter we analyze a particular method of using an injected probe signal as the input to the canceler, where the canceler bases the estimation of the feedback path on the probe signal. The feedback canceler’s optimum solution is biased [3], even if the probe signal is white noise [7]. To obtain an unbiased solution, [7] suggested the use of an adequate delay in the forward path. However, if the probe signal is spectrally shaped, then the solution may still be biased. The work in [2] notes that “shaping the probe signal decreases the decorrelation effect, making the noise injection less effective in removing the bias”, nevertheless, [2] does not present any further details. Thus, there is a need to gain more insights on system performance as a result of shaping the probe signal.

To study the impact on the canceler’s performance we use the methodology presented in [8] and show the influence a shaping filter has on system behavior. The work in [8, 9] presents the notion of a frequency domain measure, called the power transfer function (PTF), which is used to predict system behavior such as convergence rate and steady-state error. Then, we extend the delay condition from [7] to obtain an unbiased solution. Finally, we present a new approach which employs a filter that compensates for the use of a shaping filter. This improves system performance while maintaining the benefits which arise from perceptually shaping the probe signal.

The proposed approach has a similar structure to that of the prediction error method in [5] and the work in [10] where pre-filters are used as enhancement filters to increase the probe to disturbing signal ratio. However, the aim of the proposed approach is to remove the negative impact of the shaping filter on system performance. Furthermore, an adequate forward path delay is sufficient to decorrelated the signals.

This paper is organized as follows. Sec. II gives an overall system description. Sec. III extends the delay condition from [7] by taking into account the probe signal correlation. Sec. IV shows the impact the shaping filter has on system behavior. Sec. V presents the proposed method where it is validated in Sec. VI with simulation results. Sec. VII concludes the paper.

II System Description

Fig. E.1 illustrates a feedback canceler for a device with a single microphone. The feedback path between the loudspeaker and the microphone is assumed to be a discrete-time finite impulse response (FIR) filter with coefficient vector $g = [g_0 \ g_1 \ \ldots \ g_{L_g - 1}]^T$ with filter length $L_g$ which is represented as a polynomial transfer function $G(q)$ in $q$ as $G(q) = g^T q$ with $q = [1 \ q^{-1} \ \ldots \ q^{-L_g + 1}]^T$. This representation allows the following notation, for the filtering of $y(n)$ by $G(q)$,
$G(q)y(n) = g^T y(n)$ [11]. Column vectors are emphasized using lower letters in bold, the superscript T denotes vector transpose, the discrete-time index is denoted by $n$, and the symbol $q^{-1}$ denotes the discrete-time delay operator. All signals are real-valued, and we denote all signals as discrete-time signals with time index $n$ for convenience.

The forward path $K(q)$ represents the regular signal processing path of the device. In this work, $K(q)$ has a delay $d_k \geq 1$ and provides the system with a constant gain i.e., $K(q) = q^{-d_k} K$. The adaptive filter $\hat{G}(q)$, with coefficient vector $\hat{g} = [\hat{g}_0 \hat{g}_1 \ldots \hat{g}_{L_g-1}]^T$, identifies and tracks changes to the feedback path, where we assume sufficient order with $L_{\hat{g}} = L_g$. If $L_{\hat{g}} < L_g$ then the system is undermodelled and the canceler’s solution will be biased.

A shaped probe signal $w_m(n)$ is used as the input to the feedback canceler $\hat{G}(q)$ and injected into the loudspeaker signal $y(n)$. The probe signal $w_m(n)$ is generated as $w_m(n) = M(q)w(n)$, where $w(n)$ is a white noise sequence, and $M(q)$ is a known spectral shaping filter which is designed to provide some kind of perceptual masking of the noise signal.

The loudspeaker signal is defined as $y(n) = K \cdot S(q)u(n-d_k) + w_m(n) + K \cdot \left( G(q) - \hat{G}(q) \right) S(q)w_m(n-d_k)$ where $u(n)$ is the incoming signal which, we assume in our analysis, is a zero-mean stationary stochastic signal with correlation function $r_u(k) = E\{u(n)u(n-k)\}$, $E\{\cdot\}$ denotes the expectation operator, and the sensitivity function $S(q) = \frac{1}{1 - K(q)(\hat{G}(q) - G(q))}$. The Nyquist criterion states that oscillations may occur if the magnitude response of the loop-gain is greater
than unity and the loop-phase is a multiple of $2\pi$ [12]. The error signal is defined as

$$e(n) = u(n) + K \cdot G(q)S(q)u(n - d_k) + (G(q) - \hat{G}(q))w_m(n) + K \cdot G(q)(G(q) - \hat{G}(q))S(q)w_m(n - d_k). \quad (E.1)$$

As presented in [7], the optimal solution in the mean square sense is biased even if $w_m(n)$ is white noise as a result of the last term in (E.1), where it is assumed that $w_m(n)$ and $u(n)$ are uncorrelated. However, [7] showed that if an adequate forward path delay is in place $d_k \geq L_g$ then an unbiased solution is obtained when $w_m(n)$ is white noise. Next we study the impact of shaping the probe signal on the cancellation system.

### III Delay condition for unbiased solution

Section II presented that if the probe signal is shaped, then a bias term may arise. In such a case, the forward path delay has to be sufficiently long so that the correlation introduced by the shaping filter does not contribute to a bias term. We assume that the shaped probe signal $w_m(n)$ will have a finite correlation function, i.e. $r_{w_m}(k) = 0 \forall |k| > k_{w_m}$, where $k_{w_m}$ is a finite integer number. If we also take into account the delay condition presented in [7], then the delay condition for an unbiased solution is given by $d_k \geq L_g + k_{w_m}$.

Note that if we assume that $K(q) = q^{-d_k}\hat{K}(q)$, a more general forward path with $L_{\hat{k}}$ the length of $\hat{K}(q)$, then $\hat{K}(q)$ will add correlation to the solution. Thus, the delay condition needs to include $L_{\hat{k}}$, i.e., in this more general case the delay condition is $d_k \geq L_g + k_{w_m} + L_{\hat{k}}$.

### IV Probe shaping impact on system behavior

We are interested in studying the impact that shaping the probe signal has on the system’s convergence rate and steady-state behavior. To accomplish this, we follow the methodology presented in [9] where the notion of a PTF measure is presented and used to give insights into the system’s performance. The work in [9] uses open-loop signals and the closed-loop effects are ignored, nevertheless, it provides a reasonable estimate for system behavior in closed-loop without requiring knowledge of the feedback path.

In [9] the estimate of the PTF is defined as $\hat{\xi}(\Omega, n) \approx E\{\hat{G}(\Omega, n)\hat{G}^*(\Omega, n)\}$ where the feedback path is assumed to be time-varying, $\hat{G}(n) = E\{\hat{g}(n)\hat{g}^T(n)\}$.
\[ \tilde{g}(n) = g(n) - g(n-1), \text{ the path variation vector is } \tilde{g}(n) = g(n) - g(n-1), \text{ and } \Omega \text{ is the discrete frequency bin. The estimate } \hat{\xi}(\Omega, n), \text{ which is the diagonal elements of the DFT of } \tilde{G}(n) \text{ (assumed to be a Toeplitz matrix), for a single microphone setup can be written as } \hat{\xi}(\Omega, n) = (1 - 2\mu(n)S_{wn}(\Omega))\hat{\xi}(\Omega, n-1) + Lg\mu^2(n) \cdot (S_{wn}(\Omega)S_u(\Omega)) + S_\tilde{g}(\Omega) \text{ where } S_{wn}(\Omega) \text{ denotes the power spectrum density (PSD) of the shaped probe noise signal } w_m(n), S_u(\Omega) \text{ denotes the auto PSD of the incoming signal } u(n), \text{ and } S_\tilde{g}(\Omega) \text{ is the covariance of the feedback path changes. The PTF equation was derived for the least-mean square (LMS) algorithm under the assumptions of sufficiently small step size } \mu(n) \text{ and large model. The assumption of } \tilde{G}(n) \text{ being a Toeplitz matrix is valid if the feedback path is assumed to be a stationary stochastic variable.}

The PTF approximate expression can be viewed as a first-order difference equation in } \hat{\xi}(\Omega, n) \text{ described by the transfer function } Z(q) = \frac{\beta}{1 - \alpha q^{-1}}. \text{ The coefficient } \alpha \text{ determines the pole location in } Z(q) \text{ and thus the decay rate of } \hat{\xi}(\Omega, n) \text{ [8]}

\[ \alpha = 1 - 2\mu(n)|M(\Omega)|^2S_w(\Omega) \quad (E.2) \]

and convergence rate (CR) in dB/iteration is given by \( \text{CR} = 10\log_{10}(|\alpha|) \), where \( S_w(\Omega) \) is the PSD for \( w(n) \), and \( M(\Omega) \) is the frequency response for \( M(q) \). The steady-state (SS) behavior, \( \hat{\xi}(\Omega, n) = \lim_{n \to \infty} \hat{\xi}(\Omega, n) \), is presented as

\[ \text{SS} = \lim_{n \to \infty} Lg\mu(n)\frac{2}{2\mu(n)|M(\Omega)|^2S_w(\Omega)} + \lim_{n \to \infty} S_\tilde{g}(\Omega) \cdot \frac{S_u(\Omega)}{2\mu(n)|M(\Omega)|^2S_w(\Omega)}. \quad (E.3) \]

Using (E.2) and (E.3) it can be seen that the shaping filter \( M(q) \) impacts the system behavior, more specifically, the convergence rate and tracking error. For instance, a probe signal may be shaped to have a long-term average speech spectrum as seen in simulations in [5]. Considering this scenario we may intuitively interpret (E.2) and (E.3). From (E.2) it can be observed that at higher frequencies, the frequencies of interest, the convergence rate will be slower and tracking error higher as a result of a small \( |M(\Omega)|^2 \). At the same time, lower frequencies will carry a higher weight which may lead to an unstable system. Thus, a very small step size may be required to achieve convergence depending on the level of the incoming signal. Therefore, it can be seen that introducing a shaping filter may negatively impact system behavior.
We now propose a method to improve system performance when using a shaped probe signal. This method, presented in Fig. E.2, can be viewed as an extension of the traditional probe driven system according to Fig. E.1. The main observations to be made from Fig. E.2 is that $w(n)$ is used as the input into the adaptive algorithm instead of $w_m(n)$. Also, the definition of the error signal is modified, where the microphone signal is filtered by $M^{-1}(q)$ prior to calculating $e_p(n)$. Thus, $M(q)$ is designed so that its inverse $M^{-1}(q)$ exists. Note that the shaped probe signal $w_m(n)$ is still injected to the loudspeaker signal with the aim to render the injected noise less perceptual. The new error signal $e_p(n)$ is defined as

$$e_p(n) = M^{-1}(q) (u(n) + K \cdot G(q) S(q) u(n - d_k)) + (G(q) - \hat{G}(q)) w(n) + K \cdot G(q) (G(q) - \hat{G}(q)) S(q) w(n - d_k). \quad (E.4)$$

The convergence rate and the tracking error for the proposed approach will now be both independent of $|M(\Omega)|^2$. However, $|M^{-1}(\Omega)|^2$ will now influence the steady-state error. The convergence rate for the proposed approach can be approximated by $CR_p = 10 \log_{10}(|\alpha_p|)$ in dB/iteration where

$$\alpha_p = 1 - 2\mu(n)S_w(\Omega) \quad (E.5)$$
and the steady-state behavior is presented as

$$SS_p = \lim_{n \to \infty} L_g \frac{\mu(n)}{2} \frac{S_u(\Omega)}{|M(\Omega)|^2} + \lim_{n \to \infty} \frac{S_g(\Omega)}{2\mu(n)S_w(\Omega)}.$$  \hspace{1cm} (E.6)

By comparing (E.2) and (E.3) with (E.5) and (E.6) a trade-off between convergence rate and steady-state error can be seen. That is, the cost for higher convergence rate is a higher value for the steady-state error. In the particular case where the incoming signal is considered to be speech signals, we can expect that at lower frequencies, where the incoming signal is most dominant, the convergence rate will be slower with a much lower steady-state value. At higher frequencies, frequencies of interest, the convergence rate will be higher with a small degradation in steady-state performance. This is presented and verified in more details in Section VI.

A Delay condition for proposed approach

Another benefit with the proposed approach is that it reduces the forward path delay $d_k$ required to decorrelate the closed-loop signals to produce an unbiased solution, especially if $k_{wm}$ is large. This is most beneficial with open fitted assistive listening devices where the forward path delay must be sufficiently small. Thus, the condition on the forward path delay with the proposed approach is $d_k \geq L_g$ and is no longer dependent on $k_{wm}$. It can be shown that with an adequate forward path delay, and assuming that $w(n)$ is uncorrelated with $M^{-1}(q)u(n)$, an unbiased solution is still obtainable.

B Comment on more general $M(q)$

It must be pointed out that $M(q)$ may not always have an inverse, especially if $M(q)$ is designed based on statistical information of $u(n)$, $w(n)$, and some masking threshold to perceptually mask the noise. The design of $M(q)$ is beyond the scope of this work. Nevertheless, in the case where $M(q)$ is not invertible, we wish to design a filter which estimates an equalizer for $M(q)$. One potential solution is to carry out a least-squares fit between the known signals $w_m(n)$ and $w(n)$ to obtain the coefficients for a compensation filter. Then, both the microphone signal and the input signal into the canceler are filtered prior to adaptation.

VI Simulation results

In this section we verify some of the theoretical analysis presented. Let $M(q)$ shape the probe signal with a long-term average speech spectrum which can
be modeled with a low-order, autoregressive (AR) random process. Two fixed, invertible models, were used to shape the probe signal. A first order model $M_1(q)$, as used in [13] to generate a sequence with a long term speech-like spectrum, is defined as $M_1(q) = (1 - 0.9q^{-1})^{-1}$ and a second order model $M_2(q)$, based on [4], is presented as $M_2(q) = \left(1 - 2 \times 0.92\cos\left(\frac{200 \times 2 \times \pi}{15750}\right)q^{-1} + 0.92^2q^{-2}\right)^{-1}$.

Fig. E.3 presents the frequency and impulse response for $M_1(q)$ and $M_2(q)$. From observing the plots, some of the system behavior may be deduced. We obtain $k_{wm}$ for the delay condition from the impulse response, where a forward path delay of around 20 or 60 samples, in addition to $L_g$, for $M_1(q)$ or $M_2(q)$ respectively may be required to obtain an unbiased solution.

From the frequency response in Fig. E.3 it can be inferred, using (E.2), that for frequencies over 3 kHz the convergence rate will be relatively slower when the shaping filters $M_1(q)$ or $M_2(q)$ are used. And at lower frequencies, where an incoming speech would be most dominant, the convergence rate will be higher (especially with $M_2(q)$), potentially resulting in system instability. To quantify this, we present the PTF estimate for convergence rate and steady-state error in Tables E.1 and E.2 respectively. The LMS algorithm with the following parameters were used, step size $\mu = 0.00002$, $L_g = 32$, $h_u = [1 \ 0.3]^T$ shapes the incoming signal which is a white Gaussian noise (WGN) sequence with unit variance, and $M(q)$ shapes a WGN unit variance probe signal.

The convergence rate for the odd numbered frequency bins for the traditional case with $M_1(q)$ and $M_2(q)$, and the proposed case (same values for both shaping filters) are presented in Table E.1. The convergence rate is higher at lower frequencies and slower at higher ones as a result of shaping the probe signal, see
Table E.1 PTF estimate, convergence rate (dB/iteration).

<table>
<thead>
<tr>
<th>Ω</th>
<th>I (500Hz)</th>
<th>3 (1.5kHz)</th>
<th>5 (2.5kHz)</th>
<th>7 (3.5kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1(q)$</td>
<td>-1.74e-02</td>
<td>-3.90e-03</td>
<td>-1.18e-03</td>
<td>-5.54e-04</td>
</tr>
<tr>
<td>$M_2(q)$</td>
<td>-1.35e+00</td>
<td>-1.23e-01</td>
<td>-8.73e-03</td>
<td>-1.80e-03</td>
</tr>
<tr>
<td>Prop.</td>
<td>-1.74e-04</td>
<td>-1.74e-04</td>
<td>-1.74e-04</td>
<td>-1.74e-04</td>
</tr>
</tbody>
</table>

Table E.2 PTF estimate, steady state error (dB).

<table>
<thead>
<tr>
<th>Ω</th>
<th>I (500Hz)</th>
<th>3 (1.5kHz)</th>
<th>5 (2.5kHz)</th>
<th>7 (3.5kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>-32.67</td>
<td>-32.70</td>
<td>-32.79</td>
<td>-32.94</td>
</tr>
<tr>
<td>Prop. $M_1^{-1}(q)$</td>
<td>-52.67</td>
<td>-46.21</td>
<td>-41.11</td>
<td>-37.98</td>
</tr>
<tr>
<td>Prop. $M_2^{-1}(q)$</td>
<td>-70.90</td>
<td>-61.12</td>
<td>-49.80</td>
<td>-43.09</td>
</tr>
</tbody>
</table>

(E.2). It can also be seen that the proposed approach restores the convergence rate achieved as if WGN is used instead of the shaped noise, refer to (E.5). The estimated steady-state values achievable using (E.3) and (E.6) are presented in Table E.2 where the probe shaping filter $M(q)$ does not affect the steady-state values for the traditional approach, however, it comes into effect with the proposed approach based on (E.6).

Next, to validate the estimates from Tables E.1 and E.2, we present in Fig. E.4 three sub-figures (low, mid, and high frequencies) comparing the traditional probe shaped approach with the proposed showing the estimated and true PTF curves. To obtain the true PTF curves, the LMS algorithm is used in closed-loop, with step size $\mu = 0.00002$, and a forward path gain of $K = 0$ dB. The incoming signal is a WGN sequence with unit variance and shaped by $h_u = [1 \ 0.3]^T$, and the probe signal is also WGN with unit variance but filtered by $M_1(q)$. A forward path delay of $d_k = 64$ samples (4 ms) was used and $L_g = L_g = 32$ samples. The feedback is considered to be a random Gaussian channel with variance $\sigma_g^2 = 0.001$. In each simulation run, new realizations of Gaussian stochastic sequences are drawn. Fig. E.4 presents the true PTF averaged values for 100 simulation runs.

From the plots in Fig. E.4 the influence of the probe shaping filter can be seen. It is observed that the convergence rate for the traditional approach is faster at lower frequencies than those at higher ones, and the steady-state values are not
affected by $M_1(q)$. With the proposed approach, the convergence rate is recovered to that as if in the absence of $M_1(q)$, and is constant at all frequency bins, which agrees with (E.5). The trade-off between convergence rate and steady-state error can also be seen. For instance, from observing the plot for bin 3, it can be seen that with the proposed approach, slower convergence rates is achieved while obtaining a lower steady-state error when compared to the traditional approach, whereas faster convergence rates is obtainable at higher frequencies (bin 15) at the cost of slightly higher steady-state error.

**VII Conclusion**

Feedback cancellation systems which employ the use of probe signal injection may introduce a shaping filter to perceptually mask the probe signal. This letter studied the impact on system behavior as a result of using a shaping filter. It was found that the shaping filter changes the adaptation speed and may also introduce bias in the solution. To combat those limitations, we have proposed a scheme which restores convergence speed. The suggested method uses the unshaped probe signal combined with a filtered version of the microphone signal to identify the feedback channel. By employing these signals in the identification, the adaptive canceler has a restored convergence. Furthermore, we have showed that an adequate forward path delay is sufficient to obtain an unbiased solution and also that the proposed scheme reduces this delay.

**References**


Fig. E.4 Estimated and true PTF curves for: (a) bin 3 (1.5 kHz), (b) bin 7 (3.5 kHz), and (c) bin 15 (7.5 kHz). Where $M_1(q)$ was used as the shaping filter.


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Part III

Appendices
Appendix A

Papers contribution statements

**Paper A**  I, Carlos Renato Calcada Nakagawa contributed the original idea, the software implementation, conducted reported experiments, wrote the manuscript, which was commented on by the co-authors, to the paper entitled Dual microphone solution for acoustic feedback cancellation for assistive listening.

I, Prof. Sven Nordholm as a co-author and main supervisor, endorse that this level of contribution by the candidate indicated above is appropriate.

**Paper B**  I, Carlos Renato Calcada Nakagawa contributed the original idea, the software implementation, conducted reported experiments, wrote the manuscript, which was commented on by the co-authors, to the paper entitled Closed-loop feedback cancellation utilizing two microphones and transform domain processing.

I, Prof. Sven Nordholm as a co-author and main supervisor, endorse that this level of contribution by the candidate indicated above is appropriate.

**Paper C**  I, Carlos Renato Calcada Nakagawa contributed the original idea, the software implementation, conducted reported experiments, wrote the manuscript, which was commented on by the co-authors, to the paper entitled Analysis of Two Microphones Method for Feedback Cancellation.

I, Prof. Sven Nordholm as a co-author and main supervisor, endorse that this level of contribution by the candidate indicated above is appropriate.
**Paper D**  I, Carlos Renato Calcada Nakagawa contributed the original idea, the software implementation, conducted reported experiments, wrote the manuscript, which was commented on by the co-authors, to the paper entitled New Insights Into Optimal Acoustic Feedback Cancellation.

I, Prof. Sven Nordholm as a co-author and main supervisor, endorse that this level of contribution by the candidate indicated above is appropriate.

**Paper E**  I, Carlos Renato Calcada Nakagawa contributed the original idea, the software implementation, conducted reported experiments, wrote the manuscript, which was commented on by the co-authors, to the paper entitled Feedback Cancellation With Probe Shaping Compensation.

I, Prof. Sven Nordholm as a co-author and main supervisor, endorse that this level of contribution by the candidate indicated above is appropriate.
Appendix B

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