An Investigation of Mathematics Teachers' Beliefs and Practices
Following a Professional Development Intervention Based on Constructivist Principles

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ABSTRACT

The purpose of this study was to investigate the beliefs and related classroom practices of a selected group of in-service teachers within the context of a mathematics professional development intervention for primary school teachers in the Eastern Cape of South Africa. A cohort of 34 teachers drawn from urban and rural schools in the Eastern Cape engaged in an accredited professional development intervention offered by the Rhodes University Mathematics Education Project (RUMEP). The 34 teachers were referred to as key teachers as they were expected to stimulate mathematics activities with fellow teachers in their school and in a cluster of nearby schools. The professional development intervention took place in a context of transition and transformation in education in South Africa. Curriculum transformation has been inspired by the production of a national policy document known as Curriculum 2005. This document rests on the theoretical assumptions of a learner-centred, outcomes-based approach within a constructivist framework. The professional development experiences of the RUMEP intervention were based on a strongly constructivist rationale recognising the need for key teachers to implement learner-centred, outcomes-based approaches in their classrooms.

Although the study included both qualitative and quantitative data gathering techniques the research paradigm was mainly interpretive. From the group of 34 key teachers, a purposive sample of three cases was selected for classroom observation. Two observation periods of six months each made up the First Phase and Second Phase classroom visits, interspersed with intensive professional development contact sessions. During the First Phase observations, I as the participant observer, visited the classrooms of Lulama, Makana and Ruth (pseudonyms), the three case study teachers. In the Second Phase period, a colleague and I video recorded the classroom practices of all three teachers. The videotapes were analysed by a consultant panel of observers to identify emergent themes using Yager’s (1991) Constructivist Learning Model to guide the analysis process. The panel identified a number of dominant themes and these meta-themes have possible implications for a teaching and learning approach that is based on learner-centred, constructivist strategies as advocated in the Curriculum 2005 document. The meta-themes included such challenging issues as a constructivist learning environment, learner-centredness, learner participation,
collaboration, reflection, teacher content knowledge, topic progression, and power relations. The findings of the study also suggested that the case study teachers’ beliefs did influence their classroom practices.

A significant outcome was that teachers in the field were unlikely to sustain outcomes-based, constructivist approaches without regular on-site support. Arising out of this study, I was able to isolate ten features that should usefully be incorporated into other professional development interventions in the Eastern Cape, and one of these features was the support provided to teachers in the classroom. Of further significance was the realisation that future interventions need to focus on the conceptual development of teachers’ mathematics content knowledge and the systematic planning of related activities when preparing the pace and progression of a particular mathematics topic using the National Curriculum Statement (2001) as a guide.

Quantitative data from the full cohort of 34 key teachers was collected via a mathematics Beliefs Scale, authentic assessment tests (Insight Tasks), and a School Level Environment Questionnaire (SLEQ). The results on the Belief Scale indicated significant differences in key teachers’ beliefs on two out of the three subscales. These differences were in the teaching and learning of mathematics. There was no significant difference on the sequencing of a topic subscale. The key teachers completed the Insight Tasks pre and post the intervention to measure gains in their content and pedagogic (professional) knowledge. The Insight Task results indicated that the key teachers made clear progress in their professional development.

Quantitative data was also gathered from six mathematics teachers in a selected urban school. The School Level Environment Questionnaire instrument was administered to the six teachers. The aim was to profile the teachers’ pedagogic needs within a context of curriculum transformation. The profile raised two items for discussion: Staff Freedom and Resource Adequacy. It would appear that the teachers in this particular school wanted more guidance in planning outcomes-based mathematics topics, and they highlighted the need for classroom-based resources if they were to adequately implement such a curriculum.
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CHAPTER ONE

Introduction

Crisis in mathematics education

Teacher education in South Africa faces crucial challenges to retrain and upgrade the existing corps and to instil a strong sense of professionalism... (Asmal, 2000; Minister of Education).

The Minister of Education made this statement in response to the need for improvement in the education of our teachers, particularly in the subject area of mathematics. According to a submission made to the Minister of Education by a sub-committee of The Association for Mathematics Education in South Africa (AMESA), there exists in South Africa a crisis in mathematics education (AMESA News, 2000). The AMESA submission made the point that when discussing the role of the teacher there was a tendency to blame the teacher for being inadequate in the classroom. The majority of teachers, however, were poorly equipped through no fault of their own due to the pervasive, unequal, apartheid system of the past. Apartheid education according to Crawford and Adler (1996) left in its wake “serious teacher shortages in mathematics... and for the majority of teachers a system of teaching and learning lamentably deficient” (p.1196). The AMESA submission maintained that we now have as a result, teachers who did not understand mathematics and could not make sense of the mathematics content that they had to teach. The current position was clearly presented when the submission (AMESA News, 2000) stated, “We are aware that even the majority of primary school teachers have a seriously impoverished understanding of mathematics”(p. 4).

The teacher crisis was not confined to teachers’ mathematical content knowledge alone but also to their view of mathematics. The implications of the damaging statement made by a former Prime Minister in the apartheid era, that the Bantu (black) child had no use for mathematics, influenced how most people in the country viewed mathematics. There was a general acceptance that mathematics was not for everybody. The introduction, however, of a transformed national curriculum known as Curriculum 2005 was in the process of changing this distorted perception. The AMESA sub-committee argued in their submission that the focus of any efforts in
South Africa must be on the upgrading of teachers, and that teachers required both improved mathematics content knowledge and improved pedagogical knowledge if they were to succeed in implementing Curriculum 2005.

**Key teachers**

In an attempt to help alleviate the crisis in mathematics education in the country, the Rhodes University Mathematics Education Project (RUMEP) was established as an independently funded institute of Rhodes University, and in 1993 I was appointed as its first director. The establishment of an institute geared to the upliftment of mathematics teachers in the region was a long held dream of mine and I was privileged to be given the responsibility of running the project. The main aim of the project was to develop the mathematics teaching and leaning skills of teachers in the Eastern Cape and to enhance their contribution to mathematics education in primary schools. The primary school level was considered as the level where the intervention would make the greatest impact. To achieve the declared aim of the project it was necessary to design an innovative course that would enable competent teachers to develop career paths through the University-accredited intervention, and at the same time make a positive contribution to mathematics teaching in schools where it was desperately needed. The grounded nature of the delivery of the intervention attracted attention because transforming classroom practice was recognised by educationists as being important. As an independent institute, I was able to plan the professional development activities based on the perceived needs of the teachers in the field.

The RUMEP intervention has since become a recognised and approved professional development provider for teachers in the Eastern Cape (Long, 2000). The goals and structure of the professional development intervention are described later in this chapter. The theoretical framework on which the intervention was based was firmly constructivist and fitted well with the rationale underpinning the new South African curriculum (see Curriculum 2005, following). The inception of RUMEP and the launch of Curriculum 2005 took place in a climate of transformation within the country. Before being appointed director, I was one of a team of lecturers in mathematics education in the Education Faculty of Rhodes University. It was during this time that teachers regularly approached me from township schools to deliver
mathematics workshops to upgrade their mathematics skills and concepts. Arising out of my informal activities with practicing teachers, Rhodes University appreciated that there was a need for formal mathematics upgrade courses, and as a result facilitated the establishment of RUMEP as a specialist, autonomous institute within the university.

A feature of the accredited intervention was the professional development of key teachers. The notion of a key teacher was one who was capable of implementing mathematics lessons in accordance with the mathematics learning area statements of Curriculum 2005. Although the requirements of a key teacher were to conduct school-based workshops, and facilitate seminars to discuss the new curriculum, these aspects did not form part of this investigation. I used the term ‘key teachers’ in the study because they were a carefully selected group and not a random selection of classroom teachers. They had been identified for a particular role, hence they were designated ‘key teachers’. The key teacher assumed responsibility for networking with other teachers of mathematics within the school to form a school-based support group whilst at the same time acknowledging the schools’ context and culture. Key teachers served as an immediate resource to schools by demonstrating knowledge, teaching competence and leadership in curriculum matters. A cohort of 34 key teachers enrolled in the professional development intervention and was provided opportunities to engage in mathematics teaching and learning with the aim of improving their classroom practices. The purpose of my study was to determine the key teachers’ beliefs and knowledge on the teaching and learning of mathematics and to investigate the classroom practices of three specific cases drawn from the cohort.

Context
The province of the Eastern Cape was experiencing considerable infrastructural difficulties. "The Eastern Cape is one of the poorest provinces in South Africa," assert Ota and Robinson (1999, p.1) in their survey of the province. They indicated that the former homelands of the Ciskei and the Transkei that made up a large area of the province, had a "legacy of poor infrastructure, limited employment opportunities a large youthful population of over 2.3 million in 1997, and chronic poverty." The report went on to provide startling statistics on the infrastructural shortages in the
6126 schools in the province. Ota and Robinson (1999) claimed that, "There is a need to supply running water to 34% of the schools, electricity to 77% of the schools and telecommunication services to 81% of the schools (p.2). While learner enrolment was projected to rise, the education system within the province was characterised by high dropout rates and overcrowded classrooms. Overcrowding, particularly in rural schools, was common, where primary classrooms could have as many as 100 learners. The report maintained that the poor socio-economic context contributed to the education problems in the Eastern Cape. "The low quality of education" said Ota and Robinson (1999), "is evident not only in low matriculation pass rates (42% in 2001), but also in qualitative factors such as a lack of physical resources in schools and the under-qualified and unqualified nature of a significant proportion of the teaching staff" (p.1). The official norm for a qualified teacher in 1994 was one who had a Standard 10 school certificate and a three-year professional qualification. This has now been upgraded to a four-year professional qualification. In a teacher education audit, Hofmeyser and Hall (1995) provided alarming statistics on teacher qualifications at a national level for 1994 when they found, "Most un/under-qualified teachers are African (60%) and are at primary schools" (p.31).

In the Eastern Cape, these problems were worsened according to Ota and Robinson (1999) by inadequate administrative capacity and limited teacher support mechanisms from provincial education department personnel. It was in reaction to this type of situation that Asmal (1999), the national Minister of Education, declared, "Large parts of our educational system are dysfunctional" (p.2). Despite this depressing background of infrastructural shortcomings in the Eastern Cape, there were many willing, capable, classroom practitioners eager to engage in RUMEPA's key teacher development intervention in an effort to bring about improved mathematics learning in their schools.

Curriculum 2005
The introduction in South Africa of a transformed national curriculum known as Curriculum 2005 marked a shift away from a school curriculum which could be recognised by the school subjects, each with a clearly defined syllabus. The mathematics syllabus, for instance, prescribed the content to be covered and not the learning outcomes to be achieved. The role of the teacher was one of "knowledge
transmitter” because teachers perceived their students as acquiring knowledge directly from the teacher (Fradd & Lee, 1999). The predominant role was an explicit approach where teachers had clearly defined content objectives with emphasis placed on teacher talk followed by assessment through pencil and paper tests. By contrast, Curriculum 2005 was an Outcomes Based Education (OBE) curriculum derived from national agreed-on learning area outcomes. It was based on a vision of a future South Africa and of education's vital role in transformation leading to a better life for all within a human rights framework. Asmal (1999) supported an outcomes-based approach when he stated, “Curriculum 2005 represents our best hope of transforming the retrograde inheritance of apartheid-era learning theories and obsolete teaching practices” (p.13, italics in original). The Mathematics Learning Area outcomes of the National Curriculum Statement (2001) quoted below, influenced the design and structure of RUMEP's professional development activities:

The nature of these outcomes requires an activity-based approach to learning and teaching for learners to be able to think; solve problems; collect, organise and analyse information; to work in groups, and independently; to communicate effectively and to make responsible decisions. The idea that all learners learn best when they are actively building knowledge with their classmates and their teacher underpins the critical outcomes. (p.4)

A major principle that underpinned the shift in the OBE curriculum was that of “learner-centredness.” In the revised curriculum learners were seen as agents of their own learning, capable of constructing meaning for themselves under the guidance of an ‘educator.’ The shift from teachers-as-transmitters of knowledge to teachers-as-educators was arguably the most demanding challenge facing teachers with the introduction of learner-centred education. The role of the educator, as perceived in Curriculum 2005, enabled learners to make decisions on how to work out their own solution strategies and what to accomplish in a given task. Although the educator had outcomes for the learners to achieve, the educator was less prescriptive and encouraged learner participation. The educator was one who facilitated what learners did rather than specifying and directing activities. ‘Learners’ in the new curriculum were perceived as not just students in the classroom, but included adults, out-of-school youth, and in-service teachers. The notion of ‘learner’ implied a commitment to life-long learning and the concept of life-long learning was embedded in the rationale of Curriculum 2005. In a learner-centred classroom, I assumed that the power relations between learners and the educator would be more evenly distributed
as the learners became empowered as they developed increased responsibility for building their own knowledge. My belief was that power relations within a democratic model of teaching needed to be explored. Delpit (1995) maintained that both educators and learners needed to be made aware of their respective roles and responsibilities for effective interaction and participation to take place.

A singular advantage of Curriculum 2005 was that it moved away from an apartheid curriculum that was geared to fulfil the educational aims of the apartheid system. The first President of AMESA, Mathume Bopape, (1997) asserted, “We dare not continue with the apartheid curriculum. Critically treated, the framework of the new curriculum should lead us to better outcomes” (p.4). A further advantage seen in the implementation of a transformed curriculum concerned the classroom practices of teachers and learners of mathematics. As Mngomezulu (1997) stated: “I have confidence that our teachers in all corners of South Africa are going to make Curriculum 2005 a stepping stone for bringing about a positive change in mathematics learning...” and that “learners will focus on gaining knowledge and skills as opposed to learning how to get the right answers”(p.4). The National Curriculum Statement (2001) document addressed the issues of mathematics content that acted as barriers to participation by dealing with aspects such as relevance and application. The outcomes based education approach advocated in Curriculum 2005 had gone a long way towards making mathematics accessible to many more learners than the previous traditional approach. The success of Curriculum 2005, however, depended on how well teachers implemented outcomes based principles in their classrooms. The RUMEP intervention focused on the professional development of key teachers, giving emphasis to the implementation in their classrooms of the principles and practices incorporated in Curriculum 2005.

RUMEP’s Professional Development Framework

Hewson (1999) asserted, “Professional development for teachers is essential if Curriculum 2005 is to be successfully implemented in South Africa” (p.47). The key teacher intervention supported Hewson’s belief. As the key teachers described in this study were exposed to RUMEP’s professional development activities, it was necessary to describe the goals and purposes of the intervention’s various modules. The design framework for the intervention focused on what Ball and Cohen (1999)
described as the “critical activities of the profession,” that included the “practices and teaching of learning” (p.13). The framework comprised three cycles: the first cycle was the setting of goals, which included determining learning outcomes and planning pace and progression within a given topic. The next two cycles in the framework included practice carried out in the classroom followed by reflection on the dynamic nature of the whole process. Located within the three-cycle framework, were the goals for RUMEP’s professional development intervention for key teachers. They were:

1. To promote the development of mathematical content knowledge, pedagogic knowledge, and curriculum knowledge as incorporated in Curriculum 2005.

2. To inspire a constructivist conception of teaching and learning that is learner-centred and inquiry-based.

3. To inspire ongoing critical reflection of ones’ own classroom practices, and to question one’s own assumptions as well as those of others.

4. To provide on-site support for key teachers in the field, both in the classroom and at school-based workshops, and at collegial cluster meetings.

5. To develop skills in key teachers to enable them to manage curriculum transformation without confrontation and contestation in their schools.

1 Mathematics Content Development

Arising out of the goals, I have elaborated on the RUMEP intervention structure in order to provide a perspective of how these goals were to be achieved. A priority concern expressed by many teachers in township schools was the need to develop groups of key teachers with a broader and more complex view of what it meant to think mathematically. The key teachers in the intervention were provided with opportunities to be active learners to ensure that knowledge was developed and not passively received. At every occasion, the participants discussed a problem, issue, or investigation with other members of their group, sharing and challenging each other’s ideas. The participants experienced what Hewson (1999) aptly described as “desirable teaching strategies” as they were “immersed” into intensive learning
experiences (p.51). They had opportunities to explore mathematics content and they understood how learners in the classroom learned by experiencing for themselves transformed teaching approaches. A guiding principle in the approach was that understanding developed when learners were agents of their own learning. The settings in which the learners were immersed encouraged interactive engagement and a greater repertoire of mathematical solutions, leading eventually to deeper understanding of the mathematical structure within a topic. Loucks-Horsley, Hewson, Love, and Stiles (1998) described the process of immersion in inquiry into mathematics as the "...structured opportunity to experience, firsthand, science or mathematics content and processes" (p.49). The mathematical immersion of both the specialist mathematics personnel of RUMEP with the participant key teachers became a collaborative activity where democratic power sharing became a feature of these experiences. By engaging in activities that were appropriate for adult learners, key teachers were able to explore mathematics content for their own development. This process helped key teachers to enhance their mathematical knowledge and understanding.

2 Constructivist Conceptions of Teaching and Learning
The underlying philosophy of Curriculum 2005 represented a departure for the key teachers. A purpose of the intervention was to develop an appreciation of the constructivist principles in Curriculum 2005 and its role in learner-centred education in South Africa. The broad principles of the curriculum were interpreted according to details relevant to different phase levels in schools and to different contexts. Key teachers were required to build knowledge of these details. An important principle in the curriculum was that it should be adapted to local circumstances. A further function was for the key teachers to translate the curriculum into relevant classroom practices that best suited local contexts in the Eastern Cape.

To inspire a constructivist conception of teaching and learning the key teachers were required to make sense of their mathematical and pedagogic knowledge by reviewing their existing understanding. New or changed understanding was constructed via a process of verbal interaction. The implication was that learning experiences not only presented learners (in this case key teachers) with logical arguments, but also encompassed social interaction in a process of negotiating mathematical meaning. A
focus of the professional activities was to promote informed inquiry, building on what the key teachers already knew. Inquiry took the form of posing questions, making conjectures, testing alternatives and revising in the light of new insights. Newman (1998) provided an apt description that captured the spirit of these learner-centred, professional development experiences where she believed that teachers ought to be taught in the same way as she teaches classroom learners. She described the process in these terms:

...teach a group of teachers ...in a collaborative learning classroom, where the motivation for learning comes from the learners, where students take risks and are able to build on their existing strategies, where learning is largely incidental (the result of doing something that really interests the students), in a situation where there is no fear of being graded, constrained by minimum of restrictions, where no one is coerced into complying with teacher demands but encouraged to find her own way, where everyone is a learner and everyone is a teacher. (p.40)

The emphasis at RUMEPI was on constructivist approaches of a kind described by Newman. A constructivist approach to teaching and learning was a replacement for the transmission model of learning that believed that knowledge could be transferred directly from one person to another. The theories of Vygotsky (1978) guided the intervention’s approach based on the realisation that what was taken into the mind was socially and culturally determined. While Piaget (1972) talked about the active construction of knowledge through action on the world of objects, Vygotsky believed that the construction of knowledge occurred through interaction in a social world. The intervention premised the key teacher contact sessions in a constructivist rationale drawing on the theories of Piaget, Vygotsky and the situated learning researchers (see Chapter Two).

3 Critical Reflection
A further principle that influenced the professional development experiences was that of guided reflections. Schon (1991) captured the essence of reflection by saying if practitioners were allowed to give voice to the reasons that lie behind what they do it would become what he termed a ‘reflective turn.’ What this meant in terms of the RUMEPI intervention was that instead of telling teachers what to do in their classrooms, a structure was provided for key teachers to observe and describe what it was they did themselves and with what effect. The reflective approach encouraged the key teachers to look back at their own practices and how they might use their
reflections as a basis for revised classroom action. Critical reflective practice at RUMEP could be described as making explicit key teachers’ teaching processes so that they not only asked questions routinely and deliberately, they used the answers to these questions to guide their classroom practices so that they became more aware and more effective. An important professional development function was to enable key teachers to reflect on their teaching in order to make modifications to their teaching.

4 On-site Support

In an effort to sustain key teachers in the field, a system of school-based support had to become an integral component of the intervention structure. At this time, there was limited infrastructural support for teachers provided by the Provincial Department of Education. Key teachers were encouraged to present professional development activities to teachers belonging to local collections of schools in the community, which RUMEP described as a “collegial cluster.” In a collegial cluster, a network of interested teachers came together to claim ownership of the activities planned for the benefit of fellow teachers, on the assumption that they were responsible for the mathematics professional development of their colleagues. In the clusters, key teachers were elected into leadership roles, but they also depended on RUMEP support staff in the field to stimulate sustainable transformation taking place. Although it was demanding for RUMEP in time, effort, and cost, the field-support function was perceived as an essential element that would contribute towards change taking place in previously marginalized schools.

5 Managing Transformation

For an enthusiastic key teacher willing to implement a transformed approach to mathematics teaching, particularly in a rural school in the Eastern Cape dominated by traditional teachers with traditional values, the task was awesome. To help overcome this problem the key teachers participated in a practical, change management programme. In this programme they encountered simulations based in confrontation and contestation related to transformation issues found, for example, in Curriculum 2005. Key teachers became skilled in negotiating with senior members of staff that included the principal, heads of departments, representatives of teachers’ unions and experienced teachers. For transformation within a democratic framework
to take place, an environment of cooperation at all levels was necessary if the key teachers’ activities were to have an impact in the school. In the context of a school, the school community needed to be supportive of the key teachers’ efforts. With growing numbers of principals being exposed to procedures that underpinned effective school management and governance via a government initiative, the task of the key teacher being an agent-of-change in the school became easier.

The Research Study
An issue directly related to professional development concerned the role of research and what it could do to inform future effective provision. The focus of this study was not an evaluation of the applied intervention but rather a diagnosis of factors influencing individual teacher change in classroom practice. I was concerned with the beliefs and practices of teachers following a professional development intervention as the title implies. More particularly, my study addressed the following question:

*Within the context of curriculum transformation (Curriculum 2005), how do enhanced beliefs and pedagogic knowledge of key teachers influence their classroom practices?*

The study was set in a context of transformation. Curriculum 2005 had provided a new vision for teaching and learning. The RUMEP intervention came about in response to the needs of teachers in the Eastern Cape to help transform teaching and learning. The question that arose was how had the intervention influenced the beliefs, knowledge and practices of a cohort of key teachers. The National Curriculum Statement (2001) with its focus on outcomes based processes was designed with an overall aim in mind and that was to *improve student learning*. A function of the new curriculum was for school learners to emerge with enhanced mathematical knowledge, skills and dispositions. An improvement in student learning assumed a link between learning and teaching. The professional development intervention was based on concern for student learning and in Hewson’s (1999) belief that “learning is strongly and necessarily linked to teaching” (p.48). Implicit in my research was the belief that thoughtful teaching activities would improve learning. I supported Hewson, (1999) in his belief that “good teaching activities will make it easier for the student to learn” (p.48). The structure of the intervention assumed that key teachers improved their classroom practices and the expectation was that improved
mathematics learning would follow. I required more than anecdotal evidence to support this assumption. My research focus, as a result, looked for evidence that key teachers implemented enhanced pedagogic practices in their classrooms possibly due to exposure to the interventions’ experiences.

A major methodological focus of this research was to investigate the classroom practices of three case study teachers named for the purposes of this study as Lulama, Makana and Ruth. I would gather qualitative data over a 12-month period through participant observation, interviews and video recordings. A panel of consultant observers analysed the videotapes to identify the major emerging themes (meta-themes) based on the theoretical framework of a Constructivist Learning Model, (Yager, 1991). The beliefs of a cohort of 34 key teachers were measured pre and post the intervention via a Belief Scale instrument to determine quantitative changes in their beliefs relating to the teaching and learning of mathematics, and on the sequencing of a mathematical topic. Key teachers’ pedagogic knowledge was measured using an instrument developed for the task (Insight Task). The Insight Tasks were aligned to the constructivist orientation of the professional development intervention and key teachers’ opinions of the Insight Task instrument were sampled via an open-ended questionnaire. Quantitative data on the curriculum priorities of a selected school were gathered using a structured School Level Environment Questionnaire (SLEQ).

Significance of the study

I believe this study was significant for the following important and related reasons: Firstly, there was consensus that teachers’ knowledge was major factor for improving teaching and learning mathematics (Fennema & Franke, 1992). There was little agreement however, about what knowledge would enable teachers to teach so that students learnt mathematics with understanding. Thus, a fundamental question in this study was how do teachers’ professional beliefs, mathematical content knowledge, and pedagogic knowledge relate to classroom practice. Consideration of what key teachers believed and how their beliefs related to practice provided a means of conceptualising teacher education in ways that promoted change in a rational manner.
Secondly, if the key teachers emerged with enhanced mathematics content and pedagogic skills as a result of the intervention experiences, then they would be in a better position to teach in a constructivist manner. More importantly, the research would have highlighted important meta-themes which emerged from transformed classroom practices, and these themes should be communicated and debated by other educators concerned with similar teacher upliftment interventions elsewhere. A related question concerned the learners' mathematical understanding when the educator adopted a constructivist learning approach. Moll (2002) queried how learners in South Africa learn and this study attempted to seek answers to this question.

Thirdly, using the Insight Task instrument as an integral feature of the key teachers' professional development activities, contributed towards their development of mathematical content and pedagogic knowledge. It was an authentic assessment instrument aligned to constructivist learning outcomes incorporated in the intervention as part of the key teachers' experience.

Fourthly, a significant methodological issue concerned the research technique used to analyse the videotapes in this study. This process of analysis was an approach that has possible worthwhile implications for further classroom-based research specifically within the context of South African schools.

Finally, arising out of this research, I hoped to identify strategic components of the intervention that would make it possible to develop a workable integrated framework for professional development that was sustainable and possibly transferable. Such a framework would be of value to the Provincial Department of Education as it faced the difficult task of implementing effective changes in mathematics teaching in the province in accordance with the principles advocated in Curriculum 2005.

Overview of the study
In the opening chapter, I revealed the crisis facing mathematics education in this country and I described the need for teachers to enhance their mathematical content and pedagogical knowledge in the context of the Eastern Cape. In Chapter Two, the rationale for adopting a constructivist approach to teaching is described with
attention given to the Constructivist Learning Model as a framework for identifying themes in the case study teachers’ classrooms. I showed how recent and relevant research evidence informed my approach to building teachers’ knowledge and I described the evidence that influenced my selection of activities for transforming teachers’ beliefs and practices. Principles for professional development obtained from various research studies provided me with a theoretical underpinning for the structuring of the RUMEP intervention experiences.

The research methodology is described in Chapter Three. I described my reasons for electing to use a mainly interpretive research paradigm for observing the case study teachers and I included methods for establishing quality control. I also described the instruments used for gathering quantitative data from the full cohort of key teachers. In Chapter Four, I described the classroom observations of the case study teachers. A review and analysis of these lessons is provided and the results to emerge from the quantitative details are presented. In the final chapter, I documented the dominant themes to emerge from the case study teachers’ classrooms in the form of implications for teaching and learning. I also itemised recommendations for professional development for possible inclusion in other mathematics professional development interventions. I concluded by highlighting the significant findings of this study.
CHAPTER TWO

Related Theory

Introduction
My review of theories relating to mathematics education preparatory to carrying out this study served several purposes. The overall purpose was to locate my research in the field of mathematics education and to examine the epistemological assumptions relating to constructivist teaching and learning as an area of study. Of importance was the need to set the study within the context of current curriculum reform taking place in South Africa. Another purpose was to provide a theoretical framework for the evaluation of key teachers’ knowledge and beliefs of mathematics teaching and learning from a constructivist perspective, and to evaluate the classroom practices of selected cases. A final purpose was to examine the principles that underpinned the key teacher model of professional development.

Knowledge Transmission
As a major purpose in this chapter was to locate the theoretical framework for this study, my review started by addressing the assumptions on which the theory of constructivism was based. Until recently the most widely perceived view of learning in schools in South Africa was the knowledge transmission model, implying that the mathematical knowledge acquired by past generations had to be transmitted accurately to the next generation. Another aspect of perceived practice in South Africa was that the learner was a passive receiver of mathematical knowledge. Internationally, like South Africa, data showed that learners in other countries were not being equipped with the necessary knowledge, skills and motivation to approach new problems in an efficient manner (De Corte, 1995). The situation could be summarised by Greeno (1991), as follows:

In most schools, what students mostly do is listen, watch, and mimic things that the teacher and textbook tell them and show them. If students’ epistemologies are influenced at all by the experiences they have, then most students probably learn that mathematical knowledge is a form of received knowledge, not something that is constructed either personally or socially. (p.81-82).
A significant transformation had taken place in mainstream ideas concerning the teaching and learning of mathematics around the world through the work of Piaget (1972), von Glasersfeld (1995), Ernest (1998) and others. The transformation came in the form of a shift away from the notion that knowledge is transmitted to passive learners to the notion that learners build their own knowledge. No longer were learners to be thought of as passive recipients of knowledge, but as constructors of knowledge through interaction with the environment. The theoretical shift in question came to be known as ‘constructivism.’ One of the earliest constructivists was Piaget.

A Piagetian Framework

Piaget’s (1896-1980) comprehensive theory provided a unique theoretical framework for the learning of mathematics. His research gave rise to the notion that learners acquired knowledge through inventing and reinventing their own knowledge. Especially interesting to researchers in mathematics education were the major themes that dominated Piaget’s theories. Themes to emerge were that children developed their concepts through interaction with the environment; that there was a clear relationship between thought and action, and that learner’s intellectual development progressed through clearly defined stages of cognitive development. A significant implication arising out of Piaget’s research for teaching was that knowledge could no longer be transmitted from the teacher to the learner but constructed in the mind of the learner. Piaget (1972) is acknowledged as the major inspiration for the conception of the learner as an active constructor of knowledge. The ‘learner-centred’ education movement developed on the assumption that learners had the ability to construct their own knowledge and contrasted strongly with the predominant view that knowledge needed to be ‘transmitted’ by the teacher to the learners. Learner-centeredness within a school context came to be associated with active methods as learners constructed and re-constructed knowledge in a culture of learning. The notion of ‘learner-centredness’ was a fundamental principle on which Curriculum 2005 policy was created. Although Piaget had been regarded as the pioneer of constructivist epistemology, his theories were no longer as influential as they were. A general criticism was that he underemphasized social and cultural factors. Macdonald (1990) revealed the limitations of his work when she stated, “...Piagetian theory has limits in education because he has little to say about cultural
knowledge, individual differences, the social context of education and certain modes of learning in the classroom” (p.14). Piaget’s theory included important principles on the need for learners to build up their own understanding of the mathematics that they were learning. Few would question this aspect of his theory. Criticism came from the fact that he underestimated the role of social interaction and language in the cultural context in which mathematical learning takes place.

**Vygotsky and Social Constructivism**

The term constructivism encompassed a range of theoretical positions (Good, 1993), and had been used to refer to theories concerning learning, teaching, curriculum development and the professional development of teachers (Hodson & Hodson, 1998). Many of Vygotsky’s ideas were generated in the early 1930’s, during and before Piaget’s time and only became generally known to English speaking researchers after translation much later in the 1970’s. Vygotsky’s theory, according to Wertsch (1991), was known as socio-cultural learning because development took place through the medium of culture that was founded upon special kinds of social collaboration. It assumed that the very structure of certain social exchanges functioned to promote cognitive development. Sierpinska and Lerman (1996) claimed that, “The label ‘socio-cultural’ was used to denote epistemologies which view the individual as situated within cultures and social situations” (p.846).

Research in recent years focused on the social context of mathematics classrooms (Nickson & Lerman, 1992; Lerman, 1994) and indicated a shift away from knowledge that was individually constructed to knowledge that was socially constructed and justified.

Within a social context, Vygotsky (1978) was particularly concerned with the role of language in the development of higher cognitive functions. He recognised the ways in which learners used language as a tool for problem solving as Hodson and Hodson (1998) maintained, “...the greater is the problem, Vygotsky argued, the greater is the importance of speech” (p.36). Vygotsky proposed that learning occurred under a well-known set of conditions referred to as the “zone of proximal development” or ZDP. Vygotsky (1978) defined this zone as “...the distance between the actual development level as determined by independent problem solving and the level of potential development through problem solving under adult guidance or more
capable peers" (p.76). The learning zone encompassed the gradation of what the individual could do alone to what he or she could do with the help of others. Within this zone, the learner had the opportunity to apply skills that were in the process of developing even though they were not sufficiently developed to be applied independently. Vygotsky advocated that teaching should be organised to take advantage of the dynamics of the zone of proximal development. It was in the zone of proximal development that ‘scaffolding’ (Bruner’s term) played such a crucial role. Scaffolding assisted the participation of the learners through graduated support and it was important that learners did not remain dependent on the scaffold. There had to be an effective hand over of control. Bruner (1983) summed up the gradual removal of scaffolds in saying, “Where before there was a spectator, let there now be a participant” (p.60). I as the specialist teacher in the contact sessions scaffolded my instruction, and I made these strategies explicit to the key teachers so that they could understand the reasons for implementing scaffolding strategies in their own classrooms. These procedures were not prescriptive as scaffolded instruction by its very nature was not scripted but relied to an extent on the skill of the teacher (Palincsar, 1991).

Some cultural psychologists (Tudge, 1990; Yager, 1991) for example, referred to social interaction as a commitment to collaborative learning. A ZPD perspective in my view was shallow if all it offered was that ‘two heads were better than one.’ The notion of a working pair or ‘joint activity,’ taking place in the presence of a more capable peer was referred to by Vygotsky as the ‘more capable other.’ Lerman (1998a) gave an interesting interpretation of the idea of ZPD when he referred to it as “a product of the task, the texts, the previous network of experiences of the participants, the power relationships in the classroom” (p.71). In relation to research in mathematics education, Lerman (1998a) believed that the ZPD belonged to the classroom and not to the individual learner. I agreed with Nickson (2000), who argued that Lerman’s perspective “becomes a useful tool for the researcher’s analysis of the learning interactions that take place in the classroom” (p.155). Further aspects suggested by Lerman (1998a) informed by the social nature of Vygotsky’s theory included, “actions, goals, affect, power” (p.77). In my study, I included Lerman’s four aspects into my analysis of case study teachers’ classroom observations. In
particular, I looked for evidence of shifts in power relations between the educator and the learners in the classroom.

Vygotsky described interaction with experienced members of the culture as enculturation where learners became acculturated into a community of mathematicians (Cobb, 1994). As a result, the community members developed mathematical meaning and knowledge. Within the community, the cognitive system of each participant was in a process of development, as they “appropriated” strategies from each other. In the contact sessions, I expected the key teachers to appropriate strategies in order to co-construct their mathematical knowledge through social interaction, with discussion and negotiation playing a key role. Arising out of Vygotsky’s theory, I incorporated the principles of social interaction, collaboration, and scaffolding into the structure of the key teachers’ contact sessions of the intervention.

Constructivist Teaching
Along with the term constructivist learning, the term constructivist teaching came into use. Although Vygotsky promoted a theory of learning, his constructivist views of learning had influenced procedures for teaching (Yager, 1991). Unlike Piaget’s theory, where the teacher was given a peripheral role, Vygotsky’s theory gave teachers a central role by leading learners to new levels of conceptual understanding through interaction and discussion (Hodson & Hodson, 1998). The activities that enabled the learner to participate effectively through guided and modelled participation was seen as assisted performance. Although Vygotsky’s work mainly concerned children, similar learning processes affected adults, a consideration that had important consequences for teacher education in general and my study in particular. Gallimore and Tharp (1990) made a revelatory statement concerning assisted performance and teacher education:

... identical processes of self-and other-assistance in the ZPD can be seen operating in the learning adult. Recognition of this fact allows the creation of programs for teacher training and offers guidance for organizational management of school and classroom systems of performance assistance. (p.186, italics in original).
The idea of assisted performance in a teacher intervention programme guided me in planning a learning environment for the key teachers participating in the research project. The fact that adults also required assistance within the ZPD was a strong influence on how I structured the intervention contact sessions. Gallimore and Tharp (1990) also made the point that effective assistance when it was provided did not require authority. Peers, for example, could provide useful assistance without being supervisory. Non-supervisory assistance suggested collaboration with more capable peers, which Gallimore and Tharp (1990) claimed to be “foundational to the ‘cooperative learning’ movement” (p.189). They further argued that authority should be utilised to create settings where assisted performance increased the competence of those being supervised. Where collaborative interaction and assisted performance occurred was referred to as an ‘activity setting,’ which was similar to the joint activity described above. A basic condition for workable activity settings was the notion of ‘jointness’ (Gallimore & Tharp, 1990). Jointness implied assistance from a specialist teacher. Without jointness, the specialist teacher was unable to assist performance or significantly affect the cognitive structures of the participants. The need for key teachers to help one another and the will to participate with me as the specialist teacher were necessary conditions for the positive functioning of the group.

In answer to the question of how to help key teachers become better teachers of mathematics, a rational strategy was to work together on a problem or topic that was of value both to the participating teachers and to me. By our combined efforts, I assumed a better understanding of the topic would develop. Gallimore and Tharp (1990), substantiated the point when they stated, “... the specialist will have begun to identify the teacher’s zones of proximal development (for content, pedagogy, planning, etc.) and will have some idea of what means of assistance to employ and in what order” (p.192). I believed that teachers, like all learners had zones of proximal development of professional abilities, and teachers like all learners needed to receive performance assistance to help them develop professionally. In the activity settings, this meant less presentation of ready-made solutions by me and more a matter of creating environments that supported the teachers-as-learners to build meaning for themselves. The argument so far suggested that teachers-as-learners could acquire mathematical and pedagogic knowledge through discourse both in collaborative groups and in discussion with the specialist teacher (the researcher). The notion of
scaffolded instruction in activity settings influenced my decision to incorporate these principles into the design strategy of the study. During the contact sessions, I structured my teaching via activity settings in order to enhance key teachers’ understanding of mathematics and mathematics pedagogy.

**Yager’s Constructivist Learning Model**

A purpose in this literature review was to establish a theoretical framework on which to evaluate key teachers’ classroom practices from a constructivist perspective. In selecting Yager’s (1991) Constructivist Learning Model (CLM), I was provided with a framework for connecting lines of research that related the teacher and the learner in an interactive outcome. The CLM perspective focused more on the learner than on the teacher. Yager (1991) maintained, “With the emphasis on the learner, we see that learning is an active process occurring within and influenced by the instructor and the school” (p.53). In formulating the CLM, Yager drew on the ideas of von Glasersfeld who believed that instead of assuming that knowledge had to be “...a representation of what exists”, we ought to think of knowledge as “a mapping of what turns out to be feasible, given human experience” (von Glasersfeld, 1988, cited in Yager, 1991, p.55). The consequence of this belief, von Glasersfeld suggested, would lead to profound changes in teaching. Teachers would realise that knowledge could not simply be transferred via rote learning through repeated practice, and that explaining the solution to a problem would not lead to understanding unless the learner had an internal scheme that mapped onto what was being said. The CLM assumed that knowledge was not acquired passively and that the teacher encouraged discussion of a problem in groups. Through group interaction and negotiation, an agreed solution was found and presented to the class and the whole class arrived at consensus of the various small group findings. The CLM assumed that once teachers believed in this approach they would not expect a mathematical problem to have only one solution strategy, and they would place emphasis on how the learners arrived at their solution strategy. In answer to the question of how teachers could move towards constructivist approaches, Yager (1991) offered a number of procedures, which I condensed to six for the purposes of the study. I needed a workable framework for making decisions regarding the practices of the case study teachers. Using Yager’s CLM as a guiding framework, I selected those criteria that provided a clearly understood means for analysing the cases, keeping in mind the constructivist
principles stated in the Curriculum 2005 document. As the focus of the study was to look at constructivist environments, the condensed Yager principles that follow allowed for ease of interpretation for identifying emergent meta-themes.

1 Recognising, and building on learners’ prior knowledge
This procedure assumed the learners needed knowledge to learn and it assumed that it was not possible to assimilate new knowledge (Piaget) without having some internal cognitive structure to build on. “The more we know the more we can learn” was an adage that applied here. It followed that teaching experiences must be connected to the mental structures in the learner based on the learner’s previous knowledge. It involved the teacher in seeking out and using learners’ ideas and learners’ cues to guide the progress and direction of lessons.

2 A developed, interactive learning environment
This procedure assumed that learning was a social activity. Constructivist learning environments supported the construction of knowledge through social interaction, not competition among learners for recognition. Teachers who recognised this procedure encouraged the learner’s initiation of ideas in a spirit of cooperation amongst learners, and between learners and the educator. The learning environment fostered dialogue as language was seen as playing a central role in the learning process. Language was used for making thoughts operational through observing, identifying trends in data, reasoning, making inferences, drawing conclusions, and giving reports to fellow learners in the class. Hodson and Hodson, (1998) claimed, “…the inter-mental dialogue of social interaction becomes the intra-mental dialogue of inner speech” (p.40). Language was given particular attention in the constructivist classroom, in the belief that participants appropriated the ‘cultural tools’ (Vygotsky) of mathematical thinking through social interaction.

3 Promoting learner collaboration
In this procedure, it was assumed that effective learning was collaborative in an inquiry-based context and facilitated via a skilled educator. According to Yager, (1991) this meant, “encouraging students to challenge each other’s conceptualisations and ideas” and “using cooperative learning strategies that
emphasize collaboration, respect individuality, and use division of labour tactics” (p.56). Schoenfeld (1988) argued in support of the mathematics classroom being a mathematical community and that the practice of mathematics was a human endeavour and a cultural one. He believed that it was possible to create a microcosm of mathematical culture in the classroom by providing opportunities for collaborative problem solving to take place. Vygotsky’s theoretical framework gave support to Schoenfeld’s view that mathematics was best learnt in a collaborative, interactive environment. Schoenfeld (1988) upheld the principle of small groups working together in collaborative inquiry.

4 Encouraging learners to challenge and negotiate meaning

This procedure assumed that learners actively engaged in the construction of knowledge both individually and collectively. It assumed that knowledge was not fixed but changed and developed over time. Constructivist learning environments provided opportunities for a learner’s knowledge to be challenged through negotiation with other learners and between the learner and the educator. Through the construction and reconstruction of ideas learners acquired and developed the skills of inquiry through experiencing success, making mistakes and reflecting on them, gaining feedback, advice, and encouragement from other learners and the educator, and reformulating their ideas and trying again. Yager (1991) maintained that this procedure involved the teacher in, “encouraging students to suggest causes for events and situations, and encouraging them to predict consequences,” and “encouraging students to test their own ideas, i.e., answering their questions, their guesses as to causes, and their predictions of certain consequences” (p.56).

5 Encouraging learners to reflect on and refine ideas

In this procedure, the learner engaged in a repeated cycle of planning, trying a particular solution strategy, reflecting and reviewing the strategy. This reflective cycle gave rise to self-directed and personal inquiry, sometimes with others and sometimes independently. Sierpinska and Lerman (1996) believed that reflection served two purposes: “It enables the individual to step outside the particular experience and see it as an object in its own right, and it can offer the possibility of another voice for other students, enabling them to
compare and contrast with their view” (p. 845). For significant (deep) learning to take place, learners needed to revisit ideas, ponder them, and try to apply them in a revised context. Reflecting on what was learned was the product of repeated exposure and thought. This process according to (Yager, 1991) involved the teacher in “encouraging adequate time for reflection and analysis... encouraging self-analysis, collection of real evidence to support ideas, and reformulation of ideas in light of new experiences and evidence” (p. 56). Confrey (1990) believed that reflection was the “bootstrap” for learners constructing mathematical ideas, and suggested that learners ought to solve a problem through multiple forms of representation, and then assess the success of their action in resolving the problem. In probing the learner’s actions, learner reflection was promoted.

6 Encouraging learners to create their own solution strategies

This procedure assumed that learners had the capacity to construct meaning for themselves by working out solutions to problems that made sense to them. This process indicated a shift in emphasis from the received knowledge view that mathematics comprised algorithms that had to be mastered and memorised by learners. The procedure involved the teacher in encouraging learners to build their own solution strategies to problems instead of reproducing solutions provided by the educator. The teacher emphasised realistic tasks in a meaningful setting in an effort to encourage learners to understand, and be able to communicate their various strategies for solving the problem. Confrey (1990) suggested that learners first must believe in their knowledge since “knowledge without belief is contradictory” (p. 111). The issue of learner autonomy in creating their solution strategies was crucial in a constructivist classroom asserted Confrey (1990).

The six procedures described above were selected as a guide to providing structure for the panel of observers when analysing emerging themes and they were not intended to be exclusive. Other themes to emerge would be included in the analysis. Rather than relying on grounded theory (Strauss & Corbin, 1994), I felt it better to work from a clearly stated theory of teaching and learning built on definite epistemological assumptions. There was a danger in this approach. It could be argued, for example, that the six procedures presented above could be limiting or
exaggerated, oversimplified even, and therefore misleading. I, as the researcher, needed to be aware that this might happen and be sensitive to other themes if they were raised. In analysing the emerging themes, the panel of observers had to deal with two types of knowledge: the knowledge of the learner to be transformed, and the knowledge of the teacher to be used in the transformation (Sierpinska & Lerman, 1996). The Constructivist Learning Model, based in a constructivist epistemology (Vygotsky), reduced the complexity by requiring the observers to focus less on the knowledge of the teacher and more on the knowledge of the learner. It was inevitable, however, that when reviewing the teachers’ classroom practice they would take into account both the role of the learner and the role of the teacher. Sierpinska and Lerman (1996) believed that, “A Vygotskian approach... necessarily draws teaching and learning into a unified activity” (p.867).

Constructivism and Curriculum Change

Another purpose in this chapter was to locate my study in the context of curriculum reform currently taking place in this country and to examine the theoretical links that underpinned Curriculum 2005. The national Department of Education (DoE) in South Africa looked towards constructivism to provide a framework for an Outcomes Based Education approach that had come to be known as Curriculum 2005 (Department of Education, 2000). The DoE document advocated specific features in a constructivist classroom that included a “paradigm shift” from the “old approach in the classroom” associated with apartheid education to “learner-centred education” linked to OBE. According to Moll (2002), there had been movement that placed “…constructivism at the centre of the development of teaching and learning policy for South African schools” (p.6). In an effort to clarify the differences between a traditional classroom and a constructivist classroom the Department of Education (2000, p.12) published their understanding of these differences in the form of a table:
<table>
<thead>
<tr>
<th>Traditional Classroom</th>
<th>Constructivist Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum is presented part to whole, with emphasis on basic skills.</td>
<td>Curriculum is presented whole to part with emphasis on big concepts.</td>
</tr>
<tr>
<td>Strict adherence to fixed curriculum is highly valued.</td>
<td>Pursuit of learner questions is highly valued.</td>
</tr>
<tr>
<td>Curricular activities rely heavily on textbooks.</td>
<td>Curricular activities rely heavily on primary sources of data and manipulative materials.</td>
</tr>
<tr>
<td>Students are viewed as “blank slates” onto which information is etched.</td>
<td>Learners are viewed as thinkers with emerging theories about the world.</td>
</tr>
<tr>
<td>Teachers generally behave in a didactic manner, disseminating information to students.</td>
<td>Educators generally behave in an interactive manner, mediating the environment with learners.</td>
</tr>
<tr>
<td>Teachers seek the correct answer to validate student learning.</td>
<td>Educators seek the learner’s points of view in order to understand learner’s present conceptions for use in subsequent lessons.</td>
</tr>
<tr>
<td>Assessment of student learning is viewed as separate from teaching and occurs almost entirely through testing.</td>
<td>Assessment of learner learning is interwoven with teaching and occurs through educator observations of learners, learner observation of learners at work and through learner exhibitions and portfolios.</td>
</tr>
<tr>
<td>Students primarily work alone.</td>
<td>Learners primarily work in groups.</td>
</tr>
</tbody>
</table>

Table 1: Differences between traditional and constructivist classrooms

This table illustrated the move by the Department of Education from a knowledge transmission model, to transformational practices of teaching based on a constructivist orientation. There were emphases on learners’ developing understanding, learners’ questioning and the building of cognitive structures through collaborative participation where Vygotsky’s theoretical focus on the social construction of learner knowledge had found prominence. The DoE adopted a constructivist framework asserted Moll (2002), “...since constructivist research has produced the best available, most appropriate models of how children...really learn” (p.4, italics in original). The Department of Education provided statements that described the constructivist classroom and Moll (2002) claimed, “There is a strong
case to support these initiatives on the part of the DoE, and... to develop and support constructivist approaches to teaching and learning in schools" (p.24). My interpretation of the DoE position was that they believed knowledge developed in learners as a result of social and individual activities and it was on these constructivist principles that they framed the outcomes-based curriculum. According to Moll (2002), “Constructivism is a set of psychological propositions about how children really learn, albeit one that needs more depth and breadth about how children really learn in South Africa” (p.28, italics in original). I acknowledged Moll’s concern for research into how our learners learn and the observations arising out of this study were intended to contribute to this issue.

Situated Learning

I have argued that constructivism rejected the transmission model of learning that assumed that knowledge could be transferred directly from the mind of one person to the mind of another. Within the constructivist framework there was growing interest in the ‘situated learning’ model (Brown, Collins & Duguid, 1989), where knowledge was seen as “…situated, being in part a product of the activity, the context and culture in which it is developed and used” (p.32). According to Khan and Volmink (2000), situated learning assumed that knowledge could not be transferred from one context to another and that the “…social and psychological contexts form an integral part of what is learned” (p.11). Several research studies supported a situated learning approach (Lave, 1988; Lave, & Wenger, 1991; Carraher, Carraher, & Schlieman, 1985; Saxe, 1988; Nunes, Schlieman & Carraher, 1993; Askew, 2001; Adler, 2001). These research studies revealed how context-bound mathematical competencies were acquired, although Lave (1988), and Lave and Wenger (1991) did not engage in any depth with pedagogical issues. Mathematical competencies declared De Corte, Greer, and Verschaffel (1996), “…develop through apprenticeship, an informal learning environment in which knowledge and skills are acquired in the context of authentic and situated activity through observation, scaffolding, coaching and practice” (p.513). The situated learning approach presented a challenge to the teacher in a traditional classroom because knowledge that was acquired abstractly was of little use. Rather than thinking that knowledge was abstract, the findings from research quoted above suggested that for knowledge to be useful for teaching it must be linked or situated in the contexts in which it would be used. Brown et al., (1989)
proposed cognitive apprenticeship as a promising model for teaching and learning when they said, “Cognitive apprenticeship methods try to enculturate students into authentic practices through activity and social interaction...” (p.37). Within the context of Outcomes Based Education, situated learning gave importance to making use of learners’ prior knowledge and experience and to the educator who facilitated building on that experience so that the mathematics was learned in a meaningful manner. In my study, I looked for evidence of what Brown et al., (1989) called ‘authentic’ learning taking place in case study teachers’ classrooms through such situated activities as observation, scaffolding, coaching and practice.

**Realistic Mathematics Education**

The approach known as “realistic mathematics education” (RME) was developed from Freudenthal’s view that mathematics was a process of “guided reinvention” (Freudenthal, 1983). His epistemological position involved a reaction against the traditional idea that learners should first acquire the formal skills and algorithms of mathematics with the applications to follow later. Instead, the basic idea in Freudenthal’s approach claimed (Treffers, 1991) was to put learners in touch with mathematical structure as an “...organising tool in order to let them (learners) shape these tools themselves in a process of reinvention, and learn to handle these mathematical tools in concept formation” (p.22). This meant that reality served both as the “domain of application”, but also as the source of mathematical modelling that enabled learners to implement intuitive notions that preceded conceptual understanding. A feature of this approach was that the learning environment created by the teacher had to be adaptive to the learners in order to facilitate the process of reinvention of mathematical knowledge. The fraction tasks developed by Bruce Brown at RUMEPE were based in “realistic learning environments” and were guided by specific RME principles. The RME principles are described as follows:

- The first of the RME principles came from Streefland (1991) who stated that, “…reality should serve as a source for the mathematics to be produced” (p.19, italics in original). This principle assumed that learners constructed their knowledge by investigating and modelling real situations and by using their own informal solution strategies to solve the problem. In the intervention’s fraction tasks, real-life mathematical situations were presented
to the key teachers in the form of contextual word problems, developed by Brown (2000, a & b).

- The second principle assumed that the learners would progress to higher levels of abstraction. RME made full use of mathematical tools and models to support the transition from intuitive methods to abstract symbols through the appropriate use of classroom materials. The materials provided for key teachers comprised a resource kit that included items such as prepared fraction pieces, 100-grids, magic squares and the number line.

- The third principle was derived from a constructivist view of learning that encouraged learners to formulate their own solution strategies and consequent reflection. Getting learners, whether they were key teachers or school learners, to negotiate meaning, and to reflect on the outcome was a process for attaining higher levels of abstraction. The ability of school learners to invent solution strategies to new problems, and the quality of the strategy used was an indicator of their progress in mathematical thinking. Getting school learners to create their own solution strategies followed by verbal reporting was a strategy that encouraged mental engagement as it encouraged learners to reflect on each groups’ findings.

- The fourth principle emphasised learning through social interaction and cooperation. This principle recognised discussion to be important either in small groups or as classroom discourse. De Corte et al., (1996) stated that, “Social interaction is considered essential because of the importance in learning and doing mathematics, of exchanging ideas, comparing solution strategies, and discussing arguments”(p.525). Of importance was the notion that interaction and collaboration gave rise to learner reflection.

- The fifth RME principle assumed that underlying mathematical concepts were interrelated and interconnected. A consequence of this principle had implications for the management of the classroom environment. The structure of the classroom environment needed to promote activities that allowed learners to connect mathematical ideas into a structured whole and encouraged the educator to make explicit and refine where necessary, these connections for the learners. By providing learners with opportunities to
make relevant mathematical connections, their abstract knowledge would be
deepened and strengthened.

Research from The Netherlands indicated that schools that used RME textbooks did
better than schools that did not, but the most significant outcome with respect to my
study was the impact RME had on pedagogic practice in The Netherlands through
the creation of meaningful learning environments (Treffers, 1991). The principles
underpinning RME were incorporated into the design and structure of the RUMEP
intervention and were made explicit to the key teachers attending the contact sessions
in the expectation that they in turn would create meaningful learning environments in
their schools. It was also intended that through exposure to the professional
development activities, the RME principles would become incorporated into a key
teacher’s pedagogic belief system. A RUMEP resource kit developed by Brown
(2000, a & b), with the RME principles clearly in mind, was distributed to all key
teachers for use with their learners. Also relevant from the Freudenthal School of
research were the findings of van Hiele (1999), whose research into the development
of geometric thinking strongly influenced the intervention’s approach to the teaching
of geometry. (See Ruth, Observation 3, for further discussion on van Hiele’s theory).

Teachers’ Knowledge
A further purpose of this review of the literature was to examine studies engaged in
the development and evaluation of key teachers’ pedagogic knowledge and beliefs.
In using the term knowledge, I referred to Loucks-Horsley et al., (1998) who
described knowledge as being, “… substantiated in the research, in the literature and
in the ‘wisdom of experience’” (p.27). In using the term beliefs, I defined it as what
we think we know (Ball, 1996). Loucks-Horsley et al., (1998) argued that, “Beliefs
are more than opinions: They may be less than ideal truth, but we are committed to
them” (p.27). Having made a distinction between knowledge and beliefs, I reviewed
research that indicated how to increase teachers’ knowledge and how to change
teachers’ beliefs.

The theoretical frameworks for research on teachers’ knowledge fell into the
categories of mathematical content knowledge, pedagogical knowledge, and
knowledge of learners’ understandings (Fennema & Franke, 1992; Cooney, 1994;
Schulman 1986; Fennema & Nelson, 1997). Although I discussed each type of knowledge separately, they should as Fennema and Franke (1992) suggested, be considered as part of an integrated system. It was generally accepted that the teaching of a specific mathematics topic required the educator to be conversant with the concepts and procedures pertaining to the topic. Of particular importance was the teachers' understanding of the interconnectedness of the relevant concepts and procedures asserted Fennema and Franke (1992). Lampert (1986) in the United States questioned whether this basic requirement was being fulfilled, especially amongst teachers in primary schools, and cited evidence in the domain of multiplicative structures to support this statement. Research findings in South Africa (Crawford & Adler, 1996) suggested that teachers' lack of mathematical content knowledge was a major cause for concern, due to the grossly inadequate training of teachers during the apartheid years. In an effort to address this concern, a focus of the intensive contact sessions at RUMEPE was the engagement of key teachers in mathematical exploration and problem solving activities in an attempt to raise their own conceptual understanding in a range of mathematics topics. In a study that focussed specifically on the connectedness, rather than on the correctness of teachers' knowledge of division, Simon (1993) found the teachers' concepts to be weak in respect to the connection between real-life problems and symbolic representation. Brown (2000, a & b) experienced similar difficulties when he sought solutions to real-life fraction word problems. In a case cited by Fennema and Franke (1992), descriptions were given of individual teachers' content knowledge in the topics of number and fractions. The contribution of improved content knowledge was shown in one of the teachers by the considerable differences in the richness of the type of problems she set and in the quality of her classroom discourse. Various research studies conducted by Ball (1988), Schifter, (1993), and Shulman, (1986) indicated that the content of teachers' mathematical knowledge was critical. They argued that the nature of many teachers' mathematical knowledge needed to change, with less emphasis placed on algorithmic thinking and more on conceptual thinking if teachers were to understand their learners' solution strategies and facilitated better-informed mathematical discourse in their classrooms.

I have argued that being competent in mathematical content knowledge was important for the educator, but good mathematics teaching also required developed
pedagogic knowledge. A second component of teachers’ knowledge focused on
teacher pedagogy. Of interest here were various knowledge subsystems outlined by
De Corte et al., (1996) that included the following:

- A knowledge of lesson planning, and progress within topics,
- Knowledge of the problem types and situations that were best suited to
  introducing particular lessons,
- Knowledge of resource materials available for teaching various mathematics
  topics, and knowledge about their applicability.

In my study, the inclusion of the Insight Tasks was to enhance key teachers’
pedagogic knowledge. (See Chapter 3 for further details on Insight Tasks). The
purpose of the Insight Tasks was twofold: one was to assess the key teachers’
mathematical content knowledge, and the other was to use the instrument as a
vehicle for improving the key teachers’ pedagogic knowledge. I acknowledged that
the key teachers’ mathematical and pedagogic knowledge (professional knowledge)
was a crucial area and required remediation through a structured resource in the form
of a prepared instrument. It was through the Insight Tasks that I both stimulated and
assessed the key teachers’ constructivist-aligned professional knowledge.

A third component comprised the teachers’ knowledge of the mathematical concepts
and strategies learners bring to the situation, which included their misconceptions
and any incorrect procedures they might have developed as they attempted to gain
mastery of the topic. Carpenter, Fennema, Peterson, and Carey (1988), with their
roots in cognitive science suggested that teacher change was a matter of changing the
teachers’ pedagogic content knowledge. A report by Nelson (1997) on their research
indicated, “...that teacher change occurred when teachers acquired such knowledge,
organized it into a framework that related to children’s problem-solving strategies,
and used the framework to guide their teaching” (p.5). The research showed that
teachers were good at predicting the performance of individual learners at problem
solving, but had difficulty anticipating the types of solution strategies preferred by
the learners. These findings suggested that teachers tried to plan the sequencing of a
topic based on careful assessment of learners’ understanding of the topic, although
this was not always successful. Within a South African context, I believed that
planning the sequence of a topic based on learner cognition was an important
pedagogic function required of teachers.
Teachers’ Beliefs and Practices
The relevance of this section related directly to the title of my study which stated, “An investigation into mathematics teachers’ beliefs and practices…” The evidence derived from research on teachers’ knowledge, beliefs and practices considerably influenced my selection of experiences for the key teachers. A body of research exists that suggested teachers’ beliefs about teaching and learning influenced their classroom practices (Fang, 1996; Thompson, 1992; Tracey, Perry, & Howard, 1998; Lerman, 1998b; Stipek, Givven, Salmon, & Macgyvers, 2001; van Zoest, Jones & Thornton, 1994; Howard, Perry & Keong, 2000). Franke, Fennema, and Carpenter (1997) showed that what teachers said and what they did in practice did not occur in isolation. Ernest (1989) referred to what teachers said and what they did as “espoused and enacted” theories of mathematics teaching. In reviewing the research, I found that the relationship between teachers’ beliefs and their practices was open to debate. For example, some researchers implied that change in belief preceded change in practice (Cooney, & Shealy, 1997; Shulman, 1986; Thompson, 1992), whilst some held that the reverse was true (Guskey, 1986). Others argued that change in teachers’ beliefs and practices were iterative in that with new insights teachers implemented revised teaching strategies that further challenged their beliefs (Goldsmith & Schifter, 1997).

A review of research documented by Thompson (1992) showed how teachers differed greatly in their beliefs on the nature of mathematics and the teaching and learning of mathematics. Their research revealed that there exists a relationship between teachers’ beliefs and conceptions of mathematics as well as teachers’ perceptions about mathematics teaching and learning. Thompson (1992) found teachers’ views on the nature of mathematics ranged from mathematics as absolute and a fixed body of knowledge to mathematics as fallible and an expanding human invention. These different perceptions of mathematics were related to differences in teachers’ beliefs on such mathematics education issues as teacher control and authority, evidence of mathematical understanding in learners, and the function and purpose of planning. Thompson’s research suggested that teachers’ beliefs played a significant role in shaping teachers’ normal classroom practices. One of the most striking differences Thompson observed was how teachers viewed problem solving.
One teacher neglected problem solving as an activity in the belief that her role was to transmit mathematics skills and algorithms. In her research Thompson (1992), reported discrepancies between teachers' professed beliefs and their classroom practices, suggesting that beliefs were not linked in a simple way to classroom practice. Some of the issues that complicated the relationship between beliefs and practices raised by Thompson were cited as follows:

- The social context in the form of expectations by learners, fellow teachers, parents, and education officials,
- The fact that some of the teachers' professed beliefs were more a response to the current rhetoric than of actual classroom implementation,
- The lack of teachers' strong mathematical content knowledge prevented them from maximising opportunities for applying mathematical concepts where relevant.

The issues raised by Thompson had a direct bearing on my study. Although the intervention set out to change teachers' beliefs about the nature of mathematics and how to teach it, I did not assume the teachers would implement their changed beliefs in the classroom as a matter of course. In my study I needed to know whether teachers' changed beliefs transformed into practice in the classroom, or did the beliefs remain as mere rhetoric as mentioned by Thompson? Consequently, I planned to spend time observing the classroom practices of the three case study teachers to determine whether their practices had in fact changed. I also kept in mind the social context expectations as described by Thompson, and the influence these expectations had on the practices of the individual cases. Thompson's research made me aware of the need for strong mathematical content knowledge in the key teachers, and that attention to this issue needed to be made a priority during the contact sessions of the intervention.

Carpenter and Fennema (1992) conducted research into whether it was possible to change teachers' practice in the classroom by confronting them with research-based knowledge of how learners solved word problems. The approach was known as Cognitively Guided Instruction (CGI), because teaching was interpreted as problem solving which required teachers making informed decisions in the classroom. The research produced evidence that showed significant changes in teachers' practice using CGI methods and Carpenter and Fennema, (1992), suggested the best approach
was "...by helping teachers to make informed decisions rather than by attempting to train them to perform in a specified way" (p.460). The CGI research worked with a cohort of first grade in-service teachers using various word problems for the teaching of early addition and subtraction. Of particular interest to the teachers in the CGI project, were the types of word problems given to the learners to solve and the informal solution strategies devised by the learners that arose from the word problems. From the CGI research flowed the following findings on teachers' beliefs and practices, which influenced my planning of activities for the key teachers:

- The CGI teachers believed that teachers should build on learners' existing knowledge, and that learners should be encouraged to develop their own solution strategies when solving the word problems.
- Teachers needed to observe, listen, and interpret carefully the solution strategies used by the learners when solving word problems.
- The CGI teachers placed emphasis on problem solving rather than on the teaching of number skills and despite this emphasis the CGI learners recorded better understanding of number fact knowledge, problem solving and more confidence in reporting problem solving.

Whether the positive results of the CGI research was as effective at other age levels and in other mathematical topics was not fully established although there had been further research using a CGI approach in different contexts by Cobb, Wood and Yackel (1991), and Vace and Bright (1999). The CGI findings caused me to look for specific classroom practices, which would be practical indicators of the case study teachers' beliefs carried through into practice. A belief influencing practice would become apparent if the learners were given opportunities to develop their own solution strategies to various word problems devised by the educator and the learners' themselves. Another belief influencing practice would be revealed if the key teacher placed greater emphasis on word problems and less on the acquisition of skills, and if the learners were invited to report their findings to other members in the class. A shift in teachers' pedagogic practice would be revealed if the educator carefully interpreted the learner's solution strategy and did not seek the standard algorithm as the preferred method. The CGI studies provided me with a framework that encouraged key teachers to respect their classroom-based knowledge that
emerged from what the learners were thinking and caused the educators to reflect on their own beliefs about teaching and learning. The CGI research demonstrated how teachers could change their beliefs through classroom practices that worked. Elizabeth Fennema in a personal communication (personal communication, 2000) suggested that my research should consider how the CGI research set about changing their teachers’ beliefs through practice, and consequently, I should focus my enquiry on the classroom practices of the case study teachers. Although I concurred with her suggestion, I also made use of a Belief Scale instrument to determine possible changes in the espoused beliefs of the cohort of key teachers. The Belief Scale instrument is described in Chapter 3.

In reporting the findings of a particular case study Wood, Cobb, and Yackel (1991), focused on changes in a teacher’s knowledge and beliefs about the learning and teaching of mathematics. A teacher of second grade was selected and she structured a learning environment that encouraged learner collaboration. She enabled her learners to report to the class, which allowed them to explain and justify their solution strategies. Wood et al., (1991) reported on two issues to emerge from the research. First, the creation of a constructivist classroom in which learners built mathematical meaning for themselves required fundamental changes in the teacher’s beliefs about mathematics, mathematics teaching and learning and her knowledge of how to interact with learners in such an environment. The researchers described the teacher as going through three stages whilst working in a constructivist classroom:

1. A shift in her role as the presenter of knowledge and skills to one of listening to learners’ ideas and encouraging their mathematical thinking,
2. Coming to terms with her role not to impose her own solution strategies and algorithms but to create opportunities for learners to negotiate meaning and arrive at consensus,
3. Realising that she could facilitate meaning building and play an active role in refining learners’ mathematical ideas.

(Borko & Putnam, 1996)

Second, these changes occurred when the teacher realised the difficulties her learners were experiencing and consequently changed her classroom practice towards a constructivist orientation. My own study looked for constructivist practices in the
classrooms of the three case study teachers and whether they also passed through these stages as they attempted to create opportunities for learners to negotiate meaning as they helped to refine their learners' mathematical thinking.

The research of Simon and Schifter (1991), unlike that of Wood et al., (1991) set out to change teachers' practice. They assessed changes in teachers' beliefs and classroom practices through interviews and through an instrument assessed a constructivist epistemological perspective. The study was a summer school initiative for in-service teachers with the aim of helping teachers to construct a perspective informed by constructivist principles, provided opportunities for new and deeper understandings of the mathematics they taught, and supported them as they implemented constructivist practices in the classroom. The researchers concluded that the summer school mathematics intervention had an impact on the teachers’ beliefs about mathematics teaching and learning, and that the changes in their beliefs influenced their classroom practices. The teachers expressed a greater commitment to learner understanding by focusing on learners’ ideas. They also implemented collaborative problem solving, the use of manipulative materials and the inclusion of non-routine problems in their classroom activities. The research findings of Simon and Schifter (1991) with in-service teachers suggested the following principles that I should incorporate into the key teacher intervention:

- Key teachers should develop a perspective informed by constructivist principles as a consequence of the variety of in-service activities experienced in the intervention.
- During the contact sessions, the intervention should provide opportunities for the development of key teachers' new and deeper mathematical content knowledge,
- Classroom-based support for key teachers should be included in an effort to sustain their constructivist practices in the classroom.

Summary of review of related literature

In an effort to change the beliefs and practices of the key teachers towards a constructivist perspective, the overarching principles mentioned above were drawn on to guide the design and structure of the intervention activities. In comparing the results of the CGI research with the case study of Wood et al., (1991), and the
summer school intervention of that of Simon and Schifter (1991), I saw differences in their views on how to get teachers to reconsider their beliefs on mathematics teaching and learning. The summer school, for example, concentrated on changing teachers' beliefs about mathematics learning directly, then supported teachers to adopt classroom practices consistent with their beliefs. In contrast, the solo case study teacher in the research by Wood and colleagues (1991) changed her beliefs because of her attempts to implement revised classroom strategies and through the teaching materials developed with help from the researchers. The CGI research and the summer school intervention did not present teachers with prepared materials or strategies to be used in the classroom. Instead, they supported teachers in implementing their own strategies based on their changing knowledge about mathematics, and about teaching and learning. Despite these differences in focus, the research studies revealed that teachers changed their classroom practices in accordance with their beliefs. All these studies suggested that in-service teachers can acquire richer and more appropriate pedagogic content knowledge and beliefs but they required sustained support to do so. In particular, they needed opportunities to integrate their revised pedagogic content knowledge and beliefs into their classroom practices, an idea that resonated with the importance of situated learning being applied in the contexts in which relevant knowledge was to be used.

**Professional Development of In-service Teachers**

The final purpose of this review was to establish from the research literature the principles on which the intervention was based. The aim of the professional development intervention was to facilitate ways in which the key teachers would promote learners' mathematical understanding within the framework of Outcomes Based Education. The teaching was to be based on constructivist principles and would place demands on key teachers to be knowledgeable and thoughtful in their practice. The discussion that follows provided a rationale for RUMEIP's professional development focus and expanded on the principles on which the rationale was based. The principles were derived from related research described by Borko and Putnam (1996), and Loucks-Horsley et al., (1998). They included:

1. Addressing teachers' existing knowledge and beliefs about mathematics, and about mathematics teaching and learning.
2. Providing teachers with opportunities to expand and deepen their mathematics content knowledge.

3. Working with teachers as learners in a manner that was consistent with a constructivist perspective of how teachers treated school learners as learners.


5. Offering time and support for teacher reflection, collaboration and continued learning.

Teachers in South Africa were presented with the challenge of changing the way they taught (Curriculum 2005). They had been presented with a mandate for revised curriculum statements, revised assessment techniques, and revised national standards in mathematics. All teachers were expected to implement a learner-centred outcomes-based approach to their teaching but few in-service programmes used constructivist principles to inform teachers. The Department of Education workshops on the implementation of Curriculum 2005 had been based on the assumption that teachers would understand the principles of the constructivist classroom and learner-centred teaching by instruction alone. The dominant format to date had been for an external ‘specialist’ to make a presentation, with no support or follow up. The improvement of teachers’ mathematics content knowledge and the development of revised pedagogic knowledge based on the five principles above had yet to happen at a Provincial Department of Education level. Like Borko and Putnam (1996), I firmly believed that professional development programmes should be “...designed to take advantage of what we know about the process of learning to teach” (p.702). Arising out of the five principles mentioned above, RUMEP’s intervention incorporated the following specific professional development features taking into account the history and context of the key teachers:

1. Addressing existing knowledge and beliefs

The research studies of Carpenter, Fennema, Peterson and Loef (1989), Simon and Schifter (1991), Wood et al., (1991), had shown the need to address teachers’ existing knowledge and beliefs about mathematics, mathematics teaching and learning and the critical role this knowledge played in influencing whether or not teachers implemented a revised approach to their teaching. The best method for achieving change, however, was not clear. As we had seen, Carpenter et al., (1989),
Cooney and Shealy (1997), and Shulman (1986), had all tried to promote meaningful change in classroom practices by helping teachers re-examine their beliefs. Guskey (1986) argued that changes in teachers’ beliefs occurred only when they changed their practices and found them to be successful. The RUMEP intervention addressed both the knowledge and beliefs of key teachers knowing that change in one reinforced change in the other. Clarke (1994) maintained that most teachers gauged their success in terms of student learning and not on teacher actions. He argued that when professional development activities took place away from the teacher’s classroom role, it was likely to have less impact on either the teacher’s practice or on the learners. One of Clarke’s (1994) principles for professional development was the recognition that “… changes in teachers’ beliefs about teaching and learning are derived largely from classroom practice …” (p.43). Despite Clarke’s principle, I believed the order in which beliefs and practices were addressed in a professional development intervention was not that important. Of greater concern was that classroom practices and beliefs required constant teacher reflection so that change in one would cause change in the other.

2  Mathematics content knowledge

The professional development experiences aimed to provide opportunities for key teachers to build their knowledge and skills. Clarke (1994) perceived inadequate knowledge in mathematical and pedagogical content as “impediments to teachers’ growth” (p.40). Teacher learning needed to be grounded in mathematics teaching and learning with opportunities for in-depth development of mathematical subject matter knowledge as well as pedagogical content knowledge asserted Loucks-Horsley et al., (1998) by “… listening to students’ ideas, posing questions, and recognizing common and naïve misconceptions, and … choosing and integrating curriculum and learning experiences” (p.37). Research on professional development clearly indicated the need for key teachers to engage in intensive mathematical learning experiences through participating in investigations, collecting and organizing data, making predictions and formulating solutions.

3  Teachers as learners

If teachers were to be successful in creating constructivist learning environments in which mathematics teaching and learning were treated in new ways, they needed to experience such environments themselves. Loucks-Horsley et al., (1998) asserted,
“Effective professional development experiences use or model with teachers the strategies teachers will use with their students” (p.37). If teachers were to facilitate their learners in making sense of mathematics said Borko and Putnam (1996), “...something more than a collection of predetermined computational rules... then the teachers themselves need to experience what it means to actively participate in a community of learners striving to make sense of a particular mathematics domain” (p.703). Clarke (1994) saw participation and interaction in small groups through ‘modelling’, as a key principle for professional development in which “...participants practice what they are to learn in simulated and real work settings” (p.42). It became apparent that transmission style instruction of the traditional style classroom was not likely to lead key teachers towards a constructivist perspective in knowledge and beliefs.

4 Classroom practices

Key teachers required opportunities to implement revised teaching and learning strategies situated in their classrooms practices. They needed to reconsider how to facilitate collaborative work, problem solving, and learners’ varied solution strategies. They had to reflect on a revised role for the teacher with a consequent shift in the power relations between teacher and learners. Regular feedback in the research of Carpenter et al., (1989), Simon and Schifter, (1991), Wood et al., (1991) helped the teachers in these research studies to adapt their existing teaching strategies and strengthen changes in their knowledge and beliefs. It was acknowledged that although the key teacher intensive contact sessions played an important role in teachers’ learning, without grounded classroom-based practices, key teachers were unlikely to shift towards a constructivist classroom environment.

5 Collegial clusters

Effective professional development needed to provide sustained time and support for teachers to continuously make improvements and to assess themselves to ensure positive impact and continued teacher learning. The support systems cited in the international research above were impressive. Teachers continued to grow and develop as a consequence of continuous ongoing collaboration with members of the research teams. The RUMEP intervention made a point of setting up “collegial clusters” to assist in the professional development of teachers in specific geographic areas (Mboyiya, 2001). Teachers in collegial clusters were expected to assist in their own professional development by working collaboratively on the planning and
implementation of various mathematical curriculum topics with the aim of improving the classroom practices of teachers in the clusters. Participation in the professional community of a collegial cluster would assist teachers to meet the demands of implementing Curriculum 2005.

Chapter summary
Most teachers in South Africa learned their mathematics by memorizing information through a transmission model, and this mode of teaching served as an available model for their own teaching. I believed that change occurred when beliefs were restructured through new experiences that gave rise to mathematical understandings. In my review of the literature, I provided findings from research that would help teachers to build new methods of teaching and learning that would benefit the learners in the context of a transformed curriculum. Of considerable significance was that a focus of the intervention was to base mathematical learning experiences on constructivist principles in order to get key teachers to understand why it was important for them to shift to teaching strategies that were consistent with the outcomes-based principles proposed in Curriculum 2005. I believed that the study would add considerably to the body of literature in South Africa that related to transforming classroom practice through exposure to varied, relevant and appropriate professional experiences. The review also provided a theoretical framework on which to appraise the enhanced knowledge, beliefs and practices of key teachers. In Chapter 3, I examine the quantitative and qualitative methods for collecting and analysing the data.
CHAPTER THREE

Research Methodology

Introduction
One of the issues facing me in this study was how to document a convincing methodology for determining changes in teachers’ professed beliefs, and for investigating pedagogic behaviour in the classroom. I set out to measure a cohort of key teachers’ espoused beliefs then followed a selected sample into the classroom to determine whether their beliefs gave rise to different practices. Before deciding on a relevant research paradigm, the assumptions underpinning the traditions of evaluative research methodology needed to be examined.

The content of this chapter begins with a review of the positivist and interpretive research paradigms, followed by a review of specific themes in qualitative research design based in the readings of Lincoln and Guba (1985), and Patton (1990). The research participants are described, as are the case study methods in the First and Second Phases of the qualitative data collection. A description is provided of the three structured instruments used for gathering quantitative data, and how the data was analysed. Themes for data analysis from the case study teachers, based on procedures that exemplify the Constructivist Learning Model, are described. The argument for the inclusion of ‘parallel criteria’ in the qualitative tradition as a measure of quality control is covered in this chapter. Finally, ethical aspects related to the research participants are considered in an effort to ensure that no harm befalls colleagues and their learners whilst I engaged in classroom-based research.

The positivist tradition
Two traditions dominated the field of evaluation. One of these traditions emphasized measurement and prediction. In this ‘scientific’ style of research, the emphasis was on rigorous testing of hypotheses. By changing selected features of the experimental situation whilst controlling others, it was possible to observe changes in behaviour, which were then interpreted in terms of the predetermined hypotheses. The interpretation of the results depended on the reliability and validity of the tests made
as well as the control of as many extraneous variables as possible. It had been called
the scientific or experimental research methodology and was based in the philosophy
of ‘positivism’. Von Wright (1979) argued that although there were different ways of
characterizing positivism, positivism was a philosophy of scientific method, typically
represented by such philosophers as Auguste Comte (1798-1857) and John Stuart
Mill (1806-1873). It was an intellectual tradition that extended from Comte and Mill
to the present day.

In order to understand the appeal of the scientific or experimental approach to
evaluation, the tenets of positivism needed to be reviewed. One of the tenets was the
notion that there should be one, unified, scientific method of inquiry, applicable
throughout the diversity of subjects. Another tenet was the view that the exact
sciences set a methodological ideal, which should be used as a standard for
evaluation not only in the other sciences but also in the humanities (Von Wright,
1979). Positivists held that only through experiment could conceptual truths of
genuine knowledge be obtained as that knowledge had been objectively derived and
was therefore objectively meaningful. Within these limits, the domain of knowledge
excluded any field where value judgments were made. A further tenet of positivism
was that explanations were causal under the general laws of nature as Von Wright
(1979) asserted, “explanation is, in a broad sense, ‘causal’” (p.13). Positivism saw
the task of social science research as making causal relationships and predicting
future patterns based on findings derived from this type of research. The tenets of
positivism highlighted the epistemological assumptions that underpinned the
scientific approach. A feature that, above all other, distinguished the scientific
approach was the quantification of data where precise measurement allowed for
mathematical analysis (Cohen & Manion, 1994).

An aspect of my study required me to collect data on key teachers’ espoused beliefs,
pre and post the intervention. I believed the most effective method was via a
structured Beliefs Scale instrument. An advantage in administering a Belief Scale
was in the objective collection of data, leading to statistical analysis. The logic of this
approach allowed for replication of the Beliefs Scales in various situations and with
varying target populations. It also provided me with a platform for informed
reporting, based on recognized and established statistical procedures.
Critiques of positivism

The reaction against positivism had called into question the issue that related to the study of nature and the study of man. It was the human dimension in education that was under-recognised. It was argued that the participants in a research project could not be treated as objects, similar, in principle at least, to objects studied in natural science. Opponents of positivism reacted against scientific methods in social research on the grounds that these methods were mechanistic and reductionist (Cohen & Manion, 1994). Emphasis on experimental approaches in the social sciences had brought about epistemological and ontological critiques of the positivist tradition (Constas, 1998). All research projects displayed particular features that defined the nature of the inquiry. The manner in which the research was conducted illustrated the nature of the inquiry. Constas (1998) believed that questions requiring attention concerned the nature of reality (ontology), the scope and applicability of knowledge (epistemology), and the strategies used to collect and analyse data (methodology). My study was concerned with the nature of reality found in case study teachers’ classrooms (ontology), the constructivist framework that shaped the enquiry (epistemology), and the methods for collecting and interpreting evidence (methodology).

According to Cohen & Manion (1994), the anti-positivists argued that positivism “fails to take account of our unique ability to interpret our experiences and represent them to ourselves” (p.25). A characteristic of human beings was that they constructed theories about themselves and their social environment. In not recognizing ontological reality, positivism discounted a profound difference in the study of the natural sciences with that of the social sciences. A further criticism raised by Cohen and Manion (1994), was that the very nature of experimental research in education tended to favour trivial topics being investigated, with complex human issues that warranted investigation being avoided. Experimental research was often said to be of little consequence to those whom it was intended to benefit. According to Cohen and Manion (1994), teachers and their students in the classroom gained little from scientific research that was meant to help them. If it was assumed that the grounds for criticism raised against positivism in social science research were relevant, then what were the alternatives?
The interpretive tradition

Arising out of disenchantment with the experimental tradition for the study of human behaviour arose the interpretive tradition. The implication of this alternative perspective was that people themselves needed to interpret their social situations. This approach had found favour with those interested in the sociology of knowledge. It was a style of evaluation that was best described as interpretive, because it relied on explanations and interpretations of people's actions. The interpretive approach saw educational institutions and the people in them as being 'social constructions', rather than the result of external agents developing people in ways that could be predicted, following the tenets described in the positivist research tradition. Educational issues that related to the classroom could not be adequately portrayed by statistical generalizations and explained by causal laws of the scientific tradition. Lincoln and Guba (1985), asserted that the assumptions that underpinned the positivist approach were in opposition to those which underpinned the interpretive paradigm, and that the latter approach offered greater validity when studying human behaviour. Unlike the positivistic approach that ignored the participants' perceptions of situations, interpretive researchers concentrated on ways in which participants constructed their own social reality. An interpretive approach to my research included the study of individual teachers as case studies. By questioning, observing, recording and analysing the teachers' classroom actions it was possible to build up an understanding of their perceptions and an understanding of the issues that arose out of the intervention. The individual case study teacher's classroom practice was of concern to me and I aimed throughout the study to involve them as participants in the research process. Schaller and Tobin (1998) maintained that the researcher could not be separated from the participants in the interpretive research process when they stated, "...endeavours to construct understanding that take account of the presence of the researcher and the involvement of the participants within a sociocultural setting are of critical concern" (p.42).

Quantitative and qualitative methods

Growth in the use of qualitative research had given rise to such methods as ethnology, case studies and action research, very often supplementary to and in support of the traditional quantitative techniques. The debate concerning the relative
merits of qualitative and quantitative approaches in educational research had been confused by a failure to differentiate between the considerations of epistemology and the kinds of data they generated (Wiliam, 1998). The position taken by Lincoln and Guba (2000) was that there was a clear distinction between a research paradigm based in an epistemology on the one hand, and research methods or strategies for collecting data, on the other, when they stated:

Positivist and post positivists alike argue that paradigms are, in some ways, commensurable.... We have argued that at the paradigmatic, or philosophical, level commensurability between positivists and post positivist worldviews is not possible, but that within each paradigm, mixed methodologies (strategies) may make perfectly good sense (p.169).

In my study, I used methods drawn from both traditions in an effort to gather convincing evidence. Patton (1990), considered the selection of methodologies a ‘paradigm of choices’, and argued in favour of ‘methodological appropriateness’. The notion of choices allowed for different methods according to the situation, to increase the research options and (Patton, 1990) advised, “not to replace one limited paradigm with another limited but different paradigm” (p.38). I saw my study as situated within the post positivist paradigm of interpretive research, and I included both quantitative and qualitative data gathering techniques in an attempt to strengthen the design by drawing from each tradition. In gathering data related to Belief Scales, Insight Tasks and the School Level Environment Questionnaire, quantitative methods were used, and in gathering data related to the case study teachers, qualitative methods were used.

Themes in qualitative research
Within the qualitative research tradition, there were a number of specific themes relevant to the tradition. These themes were derived from the perspectives of Lincoln and Guba (1985) and Patton (1990), and where possible were incorporated into my research design. For the classroom-based aspects of the research, I had taken the position of Patton (1990) who saw qualitative inquiry as disciplined inquiry based on the themes of naturalistic research. My research could be described as naturalistic to the extent that the participant observers (a colleague and I) did not attempt to manipulate, for purposes of the evaluation, the participating case study teachers. Instead, we saw ourselves as participant observers interested in the outcomes and processes that took place as the case study teachers engaged learners in mathematics
activities. Although the implementation of the classroom activities was in the hands of the teacher, we collaborated with the case study teachers when we made our visits. By capturing data through narrative observation and video recordings over the period of the intervention, a naturalistic inquiry was open and sensitive to changes taking place in the learners' understanding of mathematics. Qualitative research, according to Denzin and Lincoln (2000), was a "situated activity that locates the observer in the world" (p.3). In using qualitative-naturalistic inquiry it was possible to capture actions as they occurred in the classroom, because the design was not locked into looking at only predetermined variables and results.

*Inductive logic* was a theme well suited to the qualitative approach. An evaluation was inductive as far as the observers sought to make sense of the individual teacher’s practice without imposing predetermined expectations on the research outcomes. Inductive logic in this instance occurred when I made specific observations of the case study teachers that gradually developed into general themes and recognisable patterns. These patterns emerged only as the teachers themselves began to implement constructivist teaching strategies in their respective classrooms. As the focus of the investigation was on building up an overall perception of the selected case study teachers, so they needed to work in a *natural setting*. A natural setting meant that the teachers operated in schools typical of the region, and did not work under artificial conditions as was often found in experimental research. Observing the case study teachers in their own rural and urban context provided insights into the language, educational, socio-economic and cultural considerations to be found only in a natural setting.

*Fieldwork* was considered a central theme of qualitative research. By 'going into the field' meant having first-hand, personal contact with the participants in the classroom. Of particular importance was developing an awareness of the educational context in which the research was placed in order to understand the multiple realities of the situation. At one level I needed to be aware of the needs and expectations of the cohort of key teachers engaged in the intervention. At another level, I needed to get close to the case-study teachers in shared experiences, and to develop trust over the duration of the intervention. I assumed that without empathetic involvement derived from first-hand experiences I would not fully comprehend the implications of
the intervention for the key teachers in general, and of the case study teachers in particular. Patton (1990) maintained that, “understanding comes from trying to put oneself in the other person’s shoes, from trying to discern how others think, act, and feel” (p.46). An underlying assumption throughout was that I tried to discern the beliefs and actions of the participants through evidence obtained from both the cohort of 34 key teachers, and from the case study teachers.

To make a reasoned evaluation of the participants I needed to understand the intervention as a whole, which meant seeing the various components of the study as a totality. I intended looking for differences in the key teachers’ beliefs and practices against a background of educational transformation currently taking place in the country, and to foreground these changes with RUMEP’s intervention strategies firmly in mind. Patton (1990) saw advantages in understanding the local setting when he stated, “... detailed attention can be given to nuance, setting, interdependencies, complexities, idiosyncrasies and context”(p.51). Understanding changes in national educational policy with regard to a transformed curriculum in which constructivist principles were being highlighted, the reasons for policy renewal, and the intended outcomes, were important aspects to understanding the setting for the study.

Process evaluation as a theme was concerned with building up an understanding of how a project functioned taking into account all the players. According to Patton (1990), the focus in an evaluation implied an emphasis on looking at “what works and what does not”(p.53). Process evaluation required informed analysis to determine if the project had sufficient potential for possible development or expansion. Precise description of an intervention and the generation of accurate analyses of a project’s operations required the skills of sensitive and knowledgeable researchers. My task was to determine the tendencies in the intervention by searching for distinguishing characteristics as themes began to reveal themselves. It meant becoming acquainted with the details of the professional development activities at all levels of implementation. Process evaluation required awareness of the various facets of the project, and included examining the design and structure of the intervention and seeing possible implications and applications for the study. By applying the principles of process evaluation, I was able to work towards an informed position for
making recommendations concerning specific design features to be included in a framework for effective professional development.

Research Participants
A cohort of 34 key teachers of primary mathematics made up a quota sample. Quota sampling according to Cohen and Manion (1994), “…attempts to obtain representatives of the various elements of the total population in the proportions in which they occur” (p.89). In the case of this sample, the elements were all teachers selected by a democratic process of peer selection, conducted by committees in the field representing teachers of mathematics. A typical committee comprised former key teachers, members of the Association for Mathematics Education of South Africa (AMESA), and a Provincial Department of Education representative. Each committee selected representatives from the district for in-service training, as a fresh cohort of key teachers. A cohort of 34 key teachers made up a quota sample over the duration of the research. Three cases for regular observation were selected. The three cases formed a purposive sample (Cohen & Manion, 1994; Lincoln & Guba, 1985), as I selected each case with specific criteria in mind. The power of using a purposive sample in qualitative research lay in selecting ‘information-rich cases’ (Patton, 1990). Information-rich meant those cases that provided relevant information central to the issues of the research. Denzin and Lincoln (2000) affirmed that, “securing rich descriptions… of the social world are valuable…”(p.10). The three cases were selected based on gender differences, two being female and one male, and on rural/urban distribution. The Xhosa speaking male came from a rural multi-grade classroom, which included grades 5, 6, and 7 (ages 10, 11, and 12 years) in one classroom, and comprised 24 learners. One female Xhosa speaking key teacher came from an urban single-grade classroom made up of 35 grade five learners. The other female key teacher was Afrikaans mother tongue, and was drawn from an urban classroom of 36 grade two learners. For the purposes of the study, the teachers were referred to by using pseudonyms: Lulama for the Xhosa speaking female, Makana for the male, and Ruth for the Afrikaans speaking female. All were experienced teachers: Lulama with 26 years, Makana with 10 years, and Ruth with 8 years respectively. Each case represented typical classrooms found in the district.
Case Studies

A major part of the data collection was via participant observation and interviews of the three case study teachers, Lulama, Makana and Ruth. It was necessary to discuss with these teachers their classroom activities, their selection of topics, their intended outcomes, and their actual classroom practices. I elected to write individual case studies so that the uniqueness and diversity of the different contexts could be described and highlighted as each case was developed. Stake (2000) described three different types of case studies as intrinsic, where the researcher was interested in a particular case, instrumental, where a case was selected to provide a particular perception, or as a collective case study where the instrumental was extended to other cases. The cases were written in chronological sequence to show the development of the participants over time, and I selected quotations from the case study teachers and their learners to illustrate specific points. Two observation periods of 6 months each took place with the three cases. The first set of observations comprised the First Phase and was conducted via narrative inquiry. The second set of observations comprised the Second Phase and was conducted via video recordings of a sequence of lessons. The case study teachers were given the opportunity to read their ongoing narrative stories (Clandinin & Connelly, 1992). Semi-structured interviews were conducted at the completion of the videotaped observation, which were shown to the teachers for their comment. The teacher interviews focused on the clarification of teacher beliefs, and how these beliefs related to their own classroom practice as reported in chapter 4. Thandi conducted a semi-structured interview with Lulama and Makana after the initial video recorded lesson.

In each classroom, I was interested to see if there was a shift in the power relations between the educator and the learners on the one hand and between me and the case study teachers on the other hand. Constas (1998) believed that the political dimension in postmodern, qualitative research was an issue in power relations that researchers in the past either ignored or tried to control. The notion of power relations had its roots in critical theory (Habermas, 1971), and power relations in research tended to take place between the researcher and the participants. My aim was to privilege the case study teachers by giving them voice, and I treated them as
participating colleagues in the research process when Thandi and I visited their classrooms.

**Instrumentation:**

**Belief Scales**

The first of three different quantitative data collecting methods was the Belief Scale. Petersen, Fennema, Carpenter, and Loe (1989) assessed teachers' beliefs using a structured instrument designed specifically for the purpose. To assess changes in beliefs in the cohort of key teachers an adapted form of the Cognitively Guided Instruction (CGI) Belief Scales devised by Peterson et al., (1989) was administered. This Belief Scale was considered most appropriate because it focused on three constructs mentioned below, that related directly to my research. The Belief Scale questionnaires were administered pre and post the intervention at the start of the first contact period, and at the end of the second contact period with an interval of six months between the two. Although 34 key teachers answered the questionnaires, only 22 completed both pre and post-tests satisfactorily. The Belief Scale was modified to comprise 36 statements on three subscales: Role of the Learner, Role of the Teacher, and Sequencing of Topics. (See appendix A for the modified Belief Scale statements and appendix B for the score sheet). Each subscale measured interrelated but separate constructs. A high score on the Role of the Learner subscale indicated a belief that learners, instead of being receivers of knowledge, were able to construct their own knowledge. High scores on the Role of the Teacher subscale indicated a belief that mathematics teaching should facilitate learners' construction of knowledge rather than the teachers' transmission of knowledge. High scores on the Sequencing of Topics subscale indicated a belief that the progression of topics should be based on the learners' development of mathematical ideas rather than on the structure of formal mathematics, via the textbook. The key teachers rated each item on a simple agree or disagree basis. It was decided that the 5-point Likert scale of **strongly agree**, **agree**, **undecided**, **disagree**, or **strongly disagree** (Vacc & Bright, 1999) would not be used as it had proved problematic for the group of English second language speakers who were used to pilot the Belief Scale. Twelve statements distributed randomly on the questionnaire encapsulated each of the three constructs. (See Appendix C for the Belief Subscale constructs). Six of the statements in each subset of twelve were worded positively and six negatively. For example, **"Most**
children have to be shown how to do mathematical computations” (statement 11), as opposed to “Children learn mathematics best by working out their own ways of doing computations” (statement 14). The differences in responses in the sample of 22 who completed the questionnaires were calculated for each question.

Belief scale analysis

The Wilcoxon matched-pairs signed-ranks test was considered appropriate to assess the significance of difference between two samples of matched pairs of subjects. The matched pairs of subjects included two measures taken on each of the 22 satisfactorily completed questionnaires. The Wilcoxon test is a non-parametric test for correlated data (Cohen & Holliday, 1982), and was considered appropriate because of the nominal data and small sized sample (Cohen & Holliday, 1982; Downie & Heath, 1965). In the Wilcoxon signed-ranks test the difference between each pair of pre and post scores was obtained for each of the three Belief Scale constructs. In analysing the data I considered each pair of scores separately, noting whether the second score was smaller or larger than the first, and then assigning each rank a plus or minus sign of the difference it represented. When ranks were tied, I assigned the average of the tied ranks. The null hypothesis was that there was no difference in teacher beliefs pre and post the intervention. I then obtained the difference between each pair of scores, and ranked the values of these differences. The positive ranks were summed to obtain the required rank. The sample (N) was 22 key teachers. As N was less than 25, I was able to enter the table directly in Cohen and Holliday (1982, p.345) to determine the significance of T. When the observed value of T was equal or less than the critical value given in the table, the null hypothesis was rejected at that particular level of significance.

School Level Environment Questionnaire (SLEQ)

As a further aspect of the quantitative research, a sample of six mathematics teachers at a local primary school were asked to complete a set of School Level Environment Questionnaire (SLEQ) developed by Fisher and Fraser (Templeton & Jenson, 1999). The purpose of the questionnaire was to highlight what teachers’ curriculum needs were in a representative school from the area. The group of six teachers were all key teachers from current and past cohorts, and comprised a convenience sample (Cohen & Manion, 1994, p.88). All six key teachers were members of staff in the same semi-
urban school. The questionnaires came in two parts: the first examined how teachers perceived their current learning environment (Actual), and the second their ideal (Preferred) learning environment. Both versions were piloted before being used by the six teachers from the sample school. (See appendix D for the Actual and Preferred questionnaires). The SLEQ gave a picture of eight dimensions of school life: Student Support, Affiliation, Professional Interest, Staff Freedom, Participatory Decision-making, Innovation, Resource Adequacy and Work Pressure. The table in Appendix E clarified the meaning of the eight dimensions by providing a description and sample item for each. Put simply, the questionnaires were designed to help teachers assess school environments by comparing teachers’ perceptions of their actual and preferred environments. Standard deviations of the eight dimensions contained in SLEQ and the values of the mean scores were calculated based on notes of the analysis by Fisher and Fraser (Templeton & Jenson, 1999).

**Insight Tasks**

The Insight Tasks started out as a collection of examples taken from learners’ school notebooks. Many of the solution strategies provided in the examples showed errors in student working whilst others included alternative methods for finding a solution. The Insight Task required a key teacher to analyse the learner’s solution strategy, and then to make recommendations to assist the learner in refining a mathematical procedure. The Insight Task assessment set out to determine whether the key teacher was able to act as a professional, give professional advice, and interpret the writings of the learner through the eyes of a professional practitioner. The cohort of 34 key teachers that made up the sample completed Insight Tasks, pre and post the intervention. The aim of the Insight Task, as a form of assessment, was to provide evidence of gains in the key teachers’ mathematical content knowledge and pedagogical knowledge from a professional development perspective. Aligning the assessment with professional expectations was a deliberate attempt to strengthen the learning process. Webb (1997) asserted, “Through understanding the link between expectations and assessment, teachers are more likely to find ways to translate what is being advanced... into their daily work with students” (p.2). The Insight Tasks provided what Darling-Hammond and Snyder (2000) called ‘authentic’ assessment of what key teachers’ encountered in their professional capacity in their own classrooms. Darling-Hammond and Snyder (2000) believed that authentic
assessments required “...integration of different kinds of knowledge and skills as they are used in the classroom” (p.527). The Insight Tasks were designed with aligned assessment and expectations in mind. (See Appendix H for further examples of Insight Tasks). A typical Insight Task question assumed the following format:

Fikile, a grade 4 learner did her multiplication like this:

\[
46 \times 28 = □
\]

\[
46 \times 10 = 460
\]

\[
46 \times 20 = 920
\]

\[
46 \times 30 = 1380
\]

\[
46 \times 2 = 92
\]

\[
1288
\]

1. Describe what the learner has done.

2. Comment on the learner’s number sense development.

Verhage and de Lange (1997) provided a framework for the structuring of questions so that higher-order, middle-order, and lower-order thinking questions could be incorporated into the type and distribution of questions. A colleague and I devised the allocation of marks for each question on a memorandum of assessment criteria, which was arrived at through negotiation between us, and other colleagues. The assessment task format was piloted and refined before assuming its final structure. In order to establish the attitudes of key teachers towards the Insight Task as an instrument of assessment, each key teacher compiled a narrative response to a questionnaire. The written responses contributed towards assessing the merits of employing the Insight Tasks as an assessment tool.

Data Collection:

First Six-Month Phase

Wallace (1998) challenged the traditional textual form in reporting science research, and took up the challenge by using narrative story in an attempt to “deconstruct the
notion of a single classroom reality” (p.2). The use of the ‘narrative’ had now become an acceptable research focus in the analysis of teacher’s knowledge (Carter, 1993; Clandinin & Connelly, 1992; Cole & Knowles, 1992; Elbaz, 1991). Polkinghorne (1995) asserted that narrative was a particular type of discourse that had relevance in qualitative research. In narrative discourse the events taking place in the case study classrooms, for example, were described as having ‘contextual meaning’ (Polkinghorne, 1995). The advantage of using narrative description in my study as an observation technique was that the discourse was able to capture teachers’ actions in a unique manner. Bruner (1985) saw narrative as a form of reasoned knowledge, which was able to generate worthwhile, useful and valid knowledge. A narrative or story style represented a way of thinking and knowing that was particularly suited to the first phase of my research. Carter (1993) declared, “The action feature of story would seem to make it especially appropriate to the study of teaching and teacher education” (p.7). I worked closely with Lulama, Makana and Ruth during the classroom visits, which Carter (1993) believed was a necessary process for building understanding when he stated, “… through observation, conversation, and mutual construction, to help understand their practice” (p.8). Despite Polkinghorne’s (1995) concern that, “narrative cognition produces a series of anecdotal descriptions of particular incidents” (p.11), the narrative stories enabled me to describe the happenings and actions of the three cases in context, and to understand and theorise (Stake, 2000) the case study teachers’ beliefs and actions.

Second Six-Month Phase

Narrative descriptions, however, were ‘constructions’ that gave meaning to classroom action and conveyed a particular interpretation of the experience. A concern for me was the issue of subjectivity in interpreting events in a teacher’s classroom. To overcome this concern I turned to the use of videotapes as a means of capturing multiple classroom realities. The video recordings allowed for multiple viewings by a panel of ‘experienced’ observers in an effort to establish agreed meta-themes. The meta-themes were those themes identified as being most significant to emerge out of the classroom observations. Video data had been successfully used elsewhere with small, qualitative studies of mathematics teaching (Cobb, Wood, Yackel & McNeal, 1992; Schoenfeld, 1988)
Qualitative data Analysis

The video data was categorised into like themes known as meta-themes. The process included reviewing and analysing the data with the goal of transforming the video information into verifiable information. Jacobs, Kawanaka and Stigler (1999) claimed that there was an advantage to using videotaping over conventional methods when they stated, “Conventional quantitative or qualitative data must be collected and analysed linearly, but video data allow for a unique iterative process” (p.717). To validate the process of establishing meta-themes, a panel of three observers with backgrounds in mathematics education were selected to analyse the videotapes as a collaborative team. One observer was an experienced mathematics lecturer, another was experienced in qualitative research and the third was a Xhosa speaking mathematics lecturer, all currently engaged in the professional development of inservice teachers. (The observers are described in chapter 4). I made up the fourth member of the observer team. The analysis began as the observers watched and discussed the videotapes. Pirie (1996) stated, “It is at this point too that I can exploit one of the real strengths of video as data. I can have others watch episodes and suggest categories” (p.5). As a panel we kept in mind Yager’s (1991) six principles as we viewed the rich visual images. Once we were satisfied with the selected themes, we debated, commented on, and sought evidence to support our respective assertions. My goal was that, ideally, all the observers should make the same judgment about a particular theme identified on the videotape. This analytical process was repeated through viewing and reviewing until there was consensus that the final analysis was a valid interpretation of themes (LeCompte, 2000). The video data had been subjected to member checking in a process for establishing dependability (Lincoln & Guba, 1985). By employing a consultative panel of observers to layer criteria and locate footage that served to exemplify particular findings, I was making sure that the analysis procedure was open and justified. My method of analysis of the video recorded data was influenced by methods used by Pirie (1996). A flexible approach to the analysis of video data by Goldman-Segall (1995), was described by Pirie (1996) when she wrote, “… (Goldman-Segall) has created… a method of layering the interpretations of multiple viewers and enabling the search for patterns across multiple perspectives, exploiting the power of the visual to contain a variety of interpretations” (p.10). The analysis of the video data by ‘multiple viewers’ in the
form of a panel of observers in my study was free flowing, allowing for a variety of perspectives to emerge, even though the observers were guided by the central procedures pertaining to the Constructivist Learning Model (Yager, 1991).

Quality Control
An issue of concern was the trustworthiness of qualitative inquiry. Because of its apparent ‘softness’ of techniques, it had been called into question by comparison to the well-developed standards of reliability and validity and generalizing within the positivist tradition. In an effort to make qualitative research trustworthy, counterparts to the standards in the scientific tradition needed to be considered. Guba & Lincoln, (1988) proposed that certain constraints be considered, which they described as credibility, dependability, confirmability and transferability. Credibility was seen as the similarity between the observers’ perception of the situation and the participants’ perceptions of the same situation. In this study, I was seeking clarity in my interpretion of the multiple realities in the classroom and it was a question of whether I was interpreting a teacher’s beliefs, and consequent actions, in the same way as the case study teachers. Dependability was interpreted in much the same way as reliability in the positivist tradition, but allowed for developing theory to emerge. The theory to emerge in my study was the identified meta-themes that emerged from the case study teachers’ classroom practices. In an attempt to establish credibility, constraints were incorporated into the research design based on techniques advocated by Guba and Lincoln (1988). I built into the design the following techniques:

- An extended stay covering a 12 month period, on-site in the case study teachers’ classrooms to provide time to identify various themes, and to overcome possible misperceptions or misconceptions of teacher and learner behaviour,
- Regular observations via narrative and videotape as data sources to capture evidence of change towards learner-centred classrooms,
- Consultation with a Xhosa-speaking fellow observer to test shared insights, receive advice concerning different perspectives, and to give translations and interpretations of what was being said in Xhosa,
- Triangulation (see below) to analyse video data by drawing on the expertise of a team of external observers (member checks),
- Sufficient ‘slice-of-life’ data collected from the case study teachers’ classrooms for reasoned analysis and interpretation,
- The video data was viewed and discussed with the respective case study teachers themselves.

**Triangulation**

In an effort at establishing confirmability, I incorporated triangulation into the overall research design. As an alternative to using triangulation as a means of providing convincing data, Richardson (2000) and Janesick (2000), preferred the notion of ‘crystallization’ as being a better lens for viewing qualitative research. Richardson (2000) asserted, “in postmodernist mixed-genre texts, we do not triangulate; we crystallize. We recognise there are more than ‘three sides’ from which to approach the world” (p.934, italics in original). Despite Richardson’s claims for alternative techniques for validating findings in qualitative research, I felt that my research data was better suited to triangulation based on Denzin’s (1988) identities, as my research design did not focus on qualitative methods alone. Cohen & Manion (1994), supported the view that the more the methods contrasted with each other the more likely it was, through triangulation, to overcome the vulnerability of a single method, or ‘method boundness’, as it had been termed. The argument in favour of me using triangulation was that it provided a solution to the problem of relying too much on any single data source or method, and so threatening the credibility of the findings. Angrosino and Mays de Perez (2000) stated that, “the possibility of observer bias looms large” (p. 676), particularly if there was only one observer making solo classroom visits. In a deliberate attempt to limit distortion from just one observer, I called in outside observers to help identify the meta-themes to emerge from the recorded video data.

There are different types of triangulation (Denzin, 1988). The triangulation procedures that I applied in this study included *data* triangulation, *investigator* triangulation and *methodological* triangulation. Data triangulation of the 34 key teachers came in the form of Belief Scales, Insight Tasks and related open-ended questionnaires. Data triangulation of the case study teachers comprised the First Phase narrative classroom observations, and the Second Phase video recordings, with brief follow-up interviews after the initial visit. Methodological triangulation
included both quantitative and qualitative data collecting methods using structured
ingredients, and classroom based participant observation respectively. Investigator triangulation was carried out only in the Second Phase, with two different classroom observers being in the classroom whilst the video recordings were being made, one Xhosa-speaking female and one English-speaking male. As regards the interviews, Thandi conducted these in Xhosa and kept them close to the observation, making the discussion short and relevant to the classroom activities. The deeper, more far-reaching interviews that related to the case study teachers' beliefs about teaching mathematics were captured after they had reviewed a videotape of their lesson (see chapter 4). In the positivist tradition, transferability was made possible through exploiting sampling strategies to ensure that participants were representative of a wider population to which generalisations could be made. In my study, there was no attempt to generalise the results to a wider population. The results pertained only to the cohort of key teachers described in this study.

Qualitative research assumptions
Wiseman (1979) made certain assumptions about qualitative data that had been derived from her own exploratory studies in qualitative research. An assumption she made was that what people told the observer was what they believed because it made sense to them, even if others did not see it from the same perspective. She also assumed that people were rational in the decisions they made. She accepted that people did make mistakes of judgement, perception and logic, and could appear to act illogically to an outside observer. The case study teachers, for example, had difficulty explaining specific mathematics concepts to the learners, but I assumed they were acting rationally, and what was observed was a valid portrayal of teacher behaviour. Wiseman (1979) made a further assumption when she stated, "there is no such thing as absolute truth" (p.118). In making this statement, she meant that even the most objective observer could only report his or her perceptions of the participants' actions and responses seen either in the classroom or on videotape. In this study, I as the observer, tried to understand the judgements and decisions of the teachers, in the light of my own understanding of the situation. In writing-up my observations, I was aware that what I wrote was based on a subjective personal perspective. I aimed to overcome my assumptions by incorporating other perspectives through a team approach to the video analysis.
Another aspect related to validity within an interpretive framework raised by Wiseman (1979) was what she termed ‘significance’. The future of the intervention based on teacher beliefs and practices was largely dependent on how I interpreted the significance of the evidence obtained from the beliefs and actions of the key teachers. If the classroom practices were considered significant or substantive then perhaps the intervention was worthy of possible continuation or expansion. Significance was determined by posing the question “was the intervention significant in so far as it revealed something in particular, or something more general (and significant), about the professional development of mathematics teachers?” The evidence collected in this study would be significant if it shed light on the question above.

**Ethical Considerations**

In planning this research, I took into account the ethical issues of the right to privacy of the test results of all the key teachers, and the anonymity of the case study teachers. The names of the teachers and the names of their schools were replaced with pseudonyms for the protection of both from derisive comment (Fontana & Frey, 2000). Even though the three case study teachers were clearly identified in the video recordings, the tapes were never seen or discussed by anyone else other than the selected observers who upheld the confidentiality of the participants. Throughout the analysis of the video data, no stories pertaining to the teachers were revealed to others not involved in the research process.

The learners in each classroom were excited at being video taped. Initially they reacted to the presence of the camera but as the lessons proceeded, they became more focussed on the task at hand and less on the camera. They viewed themselves briefly at the end of the lesson via the video play back, and were thanked for allowing me to videotape them whilst they worked at their mathematics tasks. Having a Xhosa speaking colleague present during the recorded lessons provided opportunities to explain our interest in them as learners. At each school, principals expressed an enthusiasm for the work being done by learners in the case study classrooms, and requested us to videotape other classes in the school. We were upfront with the teachers when we said we wanted to record how they were teaching and discussed,
planned and commented as colleagues on their activities in the classroom. My Xhosa speaking colleague helped in building a positive rapport with the learners, teachers, and principals during the Second Phase classroom visits. We as the participating researchers did not pretend our research was value-free. Clearly, we were looking for evidence of effective teaching and learning, and not the objective truth. We were working regularly with the case study teachers face-to-face, and as Janesick, (2000) observed, “...qualitative researchers...are attuned to making decisions regarding ethical concerns, because this is part of life in the field” (p.385). As the researchers, we made every effort not to infringe the right to privacy, nor harm any of the teachers or learners with whom we engaged. Traditionally, researches have promised confidentiality by using pseudonyms or names that prevented participants from being identified, but the advent of video data had put these assurances in doubt (Pirie, 1996). This was particularly the case at conferences where video playback was an effective method for displaying classroom-based research. The case study teachers were assured that the video recordings would not be used for public display without their consent, and the consent of the school, and of their learners.

The quantitative data obtained from the Belief Scales and the Insight Tasks was kept confidential. The reporting of results did not refer to individuals by name but by categories or by pseudonyms. A question that I considered was how much did I disclose in writing up the report without compromising or bringing harm to participants mentioned in the study. In adopting an open approach to the research process, my intention was that the participants, whether they be key teachers, learners, or researchers, gained from a process that was transparent, fair, and of benefit to all those who were involved.

As a white male, I was aware and sensitive to the power relationship issues of gender and ethnicity raised by Ball (1990), “…that men cannot effectively research women and whites cannot effectively research blacks”(p.161). To counter this categorical viewpoint I made known to all the participants my own ideology and conceptual framework concerning the study. Janesick (2000) asserted, “The researcher owns up to his or her perspective on the study...” (p.385). In trying to give meaning to my work as a researcher, I was also articulating my own beliefs and ideology. A further step towards countering categorical agendas was to include a Xhosa speaking, female
colleague in the Second Phase video recorded classroom visits, and as a member of the videotape analysis team.

Chapter summary
In this study, I used both quantitative and qualitative methods to gather and analyse data. The qualitative methods included narrative observations and video recordings of the classroom practices of three case study teachers, Lulama, Makana and Ruth. The quantitative methods comprised the use of the Belief Scale and the Insight Task instruments to determine the espoused beliefs and pedagogic knowledge of the group of 34 key teachers. The School Level Environment Questionnaire sampled the curriculum needs of one particular school. This research was based in an interpretive paradigm with the beliefs and practices of the case study teachers being the major focus. For ethical reasons, the participants throughout the study were included in the research process and I was conscious at all times not to infringe their right to privacy.
CHAPTER FOUR

Results

Introduction

In this chapter, I present the findings from both the qualitative and quantitative data in the research study. The chapter includes the qualitative data that emerged from the observations and video recordings made in three case study teachers’ classrooms of Lulama, Makana, and Ruth. The narrative description of their observed lessons took place during the first six-month phase of the intervention. The purpose of observing the case study teachers was to determine whether they displayed constructivist practices in the classroom in accordance with their espoused beliefs, as measured on the Belief Scale. During each observation, I recorded events as they occurred and at the end of the lesson, I conducted a brief informal interview with the teacher to clarify aspects of the lesson. Thandi and I video recorded lessons in the Second Phase. I used Yager’s (1991) principles (see chapter 2) to guide me in identifying the dominant themes. Not all the themes emerged in each of the case studies, and some of the themes were included under a single heading in the study. In the final section of this chapter, I presented the quantitative data results gathered via the Belief Scale, Insight Task and SLEQ instruments.

Qualitative Data

Case study observations of Lulama, Makana, and Ruth

Case study: Lulama

Vignette:

Lulama is a female, Xhosa mother-tongue speaker of 23 years teaching experience. She taught 35 grade 5 learners, 20 girls and 15 boys, all Xhosa speaking. There were sufficient desks and chairs for all the learners with the teacher's table at the front of the room. The desks were grouped to allow six learners to work together. There were pin boards on the classroom walls but there was nothing on display at the first visit. This did change over time as both teachers' charts and learners' work were displayed. The classroom was well ventilated and there were glazed windows along the external wall. The school was one of many primary schools situated in the township, enabling the learners to walk to school with relative ease. The school drew from an
exclusively Xhosa speaking community of reasonable means. Each learner was well turned out in a school uniform and shoes. The school accommodated 700 learners from grades 5 to 7 with the principal and 24 members of staff, all Xhosa speaking. Lulama was one of the teachers responsible for teaching mathematics in the school. The school was well known to the staff of RUME, and the principal and his teachers welcomed my colleague and me on our visits. The school upheld, in the main, a traditional approach of transmission style teaching and was recognised as being committed to the well being of the learners with limited teacher and learner absenteeism.

First Phase Classroom Observations

Observation 1

The lesson started with mental mathematics. It was clear that the learners knew what was expected of them. Each learner in the group was required to ask other members of the group to do a particular calculation written on the back of the flash cards. The calculation involved addition and subtraction with single digit numbers. No multiplication or division was asked. During the mental maths activity, the learners in all but two of the groups rotated the role of the questioner quite freely, giving every individual a chance. The other two groups spent some time fighting over who would start, and then how the role of questioner would be rotated. Lulama, after ten minutes, lead a whole class activity where she asked questions from the flash cards. The language of instruction was a bilingual mixture of English and Xhosa. The learners were then given sheets of A4 (foolscap) to fold with the following instruction from the teacher, "Fold your paper into two equal parts, and then into four equal parts."

The third activity was using prepared, colourful, fraction pieces to make up wholes from halves and quarters. The learners were eager to work with the prepared fraction pieces, unlike the folded sheets of A4, and appeared frustrated when Lulama insisted on getting attention before asking: "Can you tell me what we call these parts?" The learners chorused in unison and with little enthusiasm: "a half, a quarter." They had obviously made these A4 pieces before and wanted to get busy with the next activity. Instead, Lulama asked the class how many quarters made up a whole, and repeated the question until all the learners replied in unison, "four".
The distribution of prepared fraction pieces to each group eased the tension, and brought about renewed interest. The prepared fraction pieces comprise part of a resource kit distributed to each key teacher during their contact component of the intervention. The pieces are accurately cut, brightly coloured, laminated, and form fractions of the half family (whole, halves, quarters), and the thirds family (whole, thirds, sixths), of a circle. The learners squabbled over the different coloured pieces, and were excited to be using them again. A single word problem on card was distributed to each group. The word problems were written in both Xhosa and English, and Lulama encouraged the learners to read the questions aloud in their groups, which they did. The learners worked at the following problems in groups:

1. My mother bakes 6 cakes and shares them among 4 boys. How much does each one get?
2. Mother buys a rope for Lindelwa. She shares it with her friend. How much does each girl get?
3. Use the fraction pieces to find the following:
   \[ \frac{1}{2} = \Box \]
   \[ \frac{1}{3} = \Box \]

There were six groups. Lulama explained that she intentionally produced only one card per group in an attempt to encourage the groups to work cooperatively, and not as individuals. The learners were eager and explained to each other in Xhosa. In two of the groups one or two learners tended to dominate with the rest listening. The learners clustered around the problem and did not remain in their seats. Each group had only one problem to work on and when they had finished became disruptive. Lulama spent most of her time moving from group to group asking questions, and helping the groups when they were stuck. For question three, which was more abstract, she encouraged the learners to make use of the fraction pieces, which they were eager to do. There were sufficient prepared fraction pieces for all the groups. She did not use the word 'equivalent' when discussing fraction pieces but the 'the same as'. For example, "find the fraction pieces the same as a half". I monitored the progress of each group using the fraction pieces, and found that learners were willing to explain to me in English why one third was the same as two sixths, and one half
was the same as two quarters. Using the fraction pieces, they also showed me that one whole was the same as three thirds or six sixths, and one whole was the same as two halves or four quarters. When Lulama assumed the groups were ready to report to the whole class, each group in turn came to the front of the classroom, and using the chalkboard illustrated their solution strategy to the given word problem. Although she encouraged English as the medium for reporting, few learners were able to manage on their own and turned to Lulama for assistance. Some learners found presenting to the class an ordeal, but the learners were supportive of each other, possibly because they knew their turn eventually would come.

On the chalkboard was written in both Xhosa and English a list of self-assessment questions:

- What did you learn?
- What did you enjoy about the lesson?
- What could we do better?
- What did you find difficult?

I asked Lulama how the questions were to be used in the lesson, and she explained that they would become the focus in the final lesson of the week (Friday), where time would be allocated to the learners for writing their own reflections. I probed Lulama why the learners were working with only single digit numbers, and only in addition and subtraction in the mental mathematics. She said she had not noticed this and would extend the cards for the next time. Lulama said she was pleased with the learners' overall performance, and that she had enjoyed the lesson. We spoke about the group work, and she felt that it had worked well. We discussed the need to engage all the learners in the activity, and to prevent some of the groups from being dominated by one or two individuals. We agreed that she should try using pair work to enhance interaction between learners. I spoke of the need for learners to have more problems to do. Lulama said that she thought that one problem was sufficient, and she quoted her tutor who regularly said, "a problem a day!" I smiled, as I am Lulama's tutor.
Observation 2

As a warm-up activity, Lulama asked the learners to divide themselves into groups of equal size using the numbers 2, 4 and 7. The warm-up activity was initially chaotic until one learner (a boy), took charge, and divided the class into respective groups. Once the learners were divided into groups, Lulama asked how many learners were in each group. The next activity was mental maths. Each group was given flash cards to use as in the previous observation visit. The numbers on the cards were two and three digit numbers, and the learners were required to calculate mentally using all four operations. Some division calculations produced remainders. The jump in difficulty from working with single digit numbers the previous week, to two and three digit numbers this week, was too great. A particular difficulty was had with doubling the number 586. It was done as a class activity with most of the learners working out the solution on their hands using the traditional algorithm:

\[
\begin{array}{c}
586 \\
+586 \\
\end{array}
\]

Lulama did not attempt to solicit mental methods from the learners.

In the next activity, each group was given two fraction problems from the following set:

1. *For lunch, Nosisi and her family eat six half slices of bread. How many slices of bread is this?*

2. *Zandile has a football team with 11 players. Each player gets half a cup of juice to drink. How many cups of juice does he use?*

3. *Petrus and his school class have bought 12 chocolate bars. They break each bar into quarters. How many quarters do they make?*

4. *Thabo has 36 sheep, and half of them go missing. How many have gone missing?*

The questions were written on pieces of card and were translated into Xhosa. The class was arranged in six groups, and they talked eagerly in Xhosa about the task on the card. Certain learners dominated the interactions. During the fraction activity, Lulama walked around and encouraged each group, asking questions in Xhosa. She asked the groups whether they were ready to report to the class. When they said 'no' she allowed them time to continue working, and ensured that the other groups exchanged question cards so that they kept busy. One group had a problem with the
bread question. They could not seem to understand what half slices meant. A learner from another group assisted them by drawing 12 half-slices and then matched them to 6 whole slices to illustrate the meaning.

The learners were comfortable drawing their solution strategies, and all the groups were keen to report. The first group that reported simply gave an answer without explaining how they arrived at a solution. Lulama sent them back to prepare a drawing, which they had not done. The second group reported on the sheep problem. They had drawn 36 'true-to-life' sheep. Lulama suggested that they use circles because drawing 36 sheep would take too long. The learners were comfortable with this arrangement, proceeded to complete their representation by drawing 36 circles on the board, and explained their solution strategy. Solution strategies for word problems one, two and three were all drawn. A group that attempted question four did not draw but used the following notation:

\[
36 - 18 = 18
\]

Lulama thanked the group, and this abstract representation was accepted without discussion. This surprised me, as most groups were explicit when they were reporting. For example, they counted out all 36 sheep for the class, and then showed how they marked off half of them by counting out 18. No discussion was offered for alternative solution strategies, which included:

\[
36 = 18 + 18, \text{ and } 2 \times 18 = 36
\]

I discussed with Lulama the reasons for no discussion on these alternative learner solution strategies. Her view was that the learners must take responsibility for themselves in reporting their own findings to the to the class. My interpretation of the situation was that the learners felt more comfortable explaining the realistic drawn solutions, and found it difficult to explain the abstract symbolic representation. The task was made more difficult as the explanation was required in the learners' second language. At the end of the lesson, the learners were asked to copy the following word problem into their classwork books and discuss in pairs:

Luanda is watering the mealie plants in his garden. He has 15 mealie plants growing in his garden. Each mealie plant needs a quarter of a bucket of water. How many buckets of water does he need?

The word problem questions used in the lesson were taken from the fraction booklet, compiled by the intervention's materials developer, Brown (2000a). The booklet
formed part of the resource kit distributed to all key teachers in the intervention cohort. The 'mealie plant' problem focused the attention of the learners with almost every child in the class engaged in lively discussion using mother tongue. Unfortunately, the learners were unable to present their findings due to insufficient time.

Observation 3

The lesson started with five to ten minutes of mental activities as a daily routine. The learners worked in cooperative pairs. Each pair was given a set of three cards and asked to perform the following to the stated number on the card:

+ 21
double
× 3
triple
6 less
33 more

Each card had a two-digit number, and some cards had additional three-digit numbers. The learners were to record their solutions on paper. Walking round the groups, I noticed that most learners had difficulty performing these operations. Many of them worked out the answers on their hands before the solutions were written down. An example was the sum "44 doubled." The pair wrote: 44, and underneath + 44, on their hands, and worked it out using the traditional algorithm. The recorded answers were collected with a comment from Lulama, "so that I can see your mistakes".

The next activities were fraction word problems where the learners were required to share non-unit fractions. The fraction activities were those developed by Brown (2000b). Each pair received a copy of five realistic questions and the instruction was to choose one to work on. I noticed that for the first time the questions were not written in both Xhosa and English. Lulama explained that she had asked the learners whether they needed the Xhosa translation, and they replied that they wanted to do it in English only. All the groups read their questions aloud in halting English. However, the subsequent mathematical discussions took place in Xhosa. Once the learners understood what the problem was asking them to do, they were confident in translating the words into meaningful drawings. Each pair was able to draw
something, and no numbers were used instead of drawings. Some pairs, when they had finished their chosen problem, simply sat. They needed Lulama to encourage them to try another problem, which they then did. The learners had half an hour to complete the activities, and most pairs were able to tackle at least two word problems. When the time came to report, Lulama asked each pair to join a group who had done the same problem. It was from this larger group that the reporters were chosen. There seemed to be no discussion as regards who had the "best" mathematical solution to the problems in the big groups. The learners simply selected the pair that could speak the best English.

The following are the fraction word problems and the learners' respective solution strategies in order of reporting:

1. Zandile has a football team, with 11 players.
   Each player gets half a cup of juice to drink.
   How many cups of juice does he have?

Solution:

Eleven players were drawn as stick figures, and the half-full cups were matched one-to-one with the players. The half-full cups were linked together in pairs, as follows:

P P P P P P P P P P P (11)
C+C C+C C+C C+C C+C C

Answer:

6 cups of juice

The reporter drew the 11 players and the 11 half-full cups. An explanation as to why 6 cups of juice was given as the answer, and not 5 and a half as I expected, was that in order to give out juice to each player, 6 physical cups were required. The group's interpretation of the problem was from a practical perspective. They did not interpret the problem as the volume of juice to be given out. This issue gave rise to some lively debate between learners in Xhosa, with Lulama eventually closing the discussion by saying 5 and a half was the answer, without making the reason explicit.

2. Zingiswa has a birthday party with 5 of her friends. They drink 11 cans of juice at the party. One of her friends says, "We have all had the same amount to drink." How much did each person drink?
For some reason the group did not include Zingswa as a member of the party, which meant that the party comprised five people. The reporters drew the number of stick figures. They matched two cans to one figure, and then added the symbol "1/5" to each stick figure, as follows:

Solution:
F stands for friends and C stands for cans.

\[
\begin{array}{cccccc}
F & F & F & F & F \\
C+C & C+C & C+C & C+C & C+C \\
+1/5 & +1/5 & +1/5 & +1/5 & +1/5 \\
\end{array}
\]

Answer: Each gets 2 and 1/5.
The symbol notation surprised me and no explanation was given. Lulama appeared to be satisfied with the solution and no questions were asked.

3. Lindelwa, Thobeka, Xolisa have 2 long pieces of wool. They want to play string patterns with the wool. They share the wool so they all get the same. How much wool do they get?

Solution:
The solution was drawn to illustrate a length of wool cut into thirds. Lindelwa, Thobeka, and Xolisa were drawn as stick figures. Matched to each stick figure were the fraction symbols \( \frac{1}{2} + \frac{1}{3} \). The reporters confidently explained that they gave each friend half a piece of wool, and then divided the remaining piece into thirds, using their drawing to illustrate. Looking at the remaining piece of wool, I asked whether we could not write that as taking \( \frac{1}{3} \) of \( \frac{1}{2} \). The group seemed pleased with that, but unsure.

4. Thandi buys 6 boxes of smarties for her children. She has 4 children. Her children share the smarties so they each get the same amount. How many boxes do the each get?

Solution:
The drawing comprised four stick figures. One and a half boxes (drawn as rectangles) were matched to each stick figure. The group's written explanation was presented as below:
Each one get his or her box of smarties then the two box of smarties are less and we cut it in halves so that each one can get some with all and each one get one and half box of smarties. (Group explanation)

5. A school gets 24 loaves of bread for learners to eat. They must share the bread so that each class gets the same amount of bread. There is one class for each grade from 1 to 5. How many loaves does each class get?

Solution:
Five stick figures were drawn to represent classes 1 to 5. Each class was allocated four and a half loaves. The reporters stopped here saying that there are one and a half loaves left and they did not know what to do with them. Lulama was at a loss to explain further. A learner from the class broke the remaining loaves into quarters but could make no further progress. The bell rang and Lulama abruptly ended the lesson.
In discussing this word problem with Lulama, she realised that 24 divided by 5 using fractions was too difficult a task in a realistic situation. This question was nowhere to be found in the supplied resource booklet.

Fraction understanding and realistic situations
Activities developed in the Brown (2000b) booklet had two main aims:

1. To explore different realistic contexts for which fraction understanding is useful.

2. To develop the ability to effectively apply fraction skills and concepts to each new context.

Brown (2000b) made the following observation in his fraction booklet using a realistic approach:

As they work towards an effective response to some of these situations, learners will need to work with fractions using different operations and unfamiliar ways of thinking. This will encourage them to expand and further develop their fraction understanding.

In the two fraction lessons, Lulama used word problems to develop fraction concepts informally in meaningful situations. Learners were asked to explore different realistic contexts. To work out a satisfactory solution to these realistic problems, learners needed to understand the fraction concepts within the context of the situation. As they worked out their response, they discussed informally between themselves, with
the focus being a response to the situation. Lulama encouraged learners to share their responses, but the reasons for selecting a particular solution strategy were not forthcoming, and learners were not asked how they used fractions when they worked out their responses. The learners did show evidence of developing their own more abstract understanding of fractions, but links between the realistic situation and abstract fraction concepts were not well made. In both fraction lessons, the focus was on realistic situations with learners being given the opportunity to see their fraction understanding as a way of modelling the situation. It was apparent that the learners needed further opportunities to explore different realistic situations to help build their fraction understanding.

Observation 4

The lesson started with mental mathematics. The learners compiled their own two and three-digit numbers, and asked each other the questions in pairs. They confined themselves to addition, subtraction, doubling and halving. The lesson then moved on to measurement. Two learners were called up to the front and Lulama asked, "What is the difference between them?" Learners in the class responded with: "their height is not the same; that one has big ears." Lulama requested that they look at the heights, and asked how height could be measured. The learners suggested using the RUMEPI metre rule. The metre rule forms part of the key teacher's resource kit, and is made from bright-yellow, laminated, flexible, card marked in 10-centimetre divisions on one side, and in centimetres on the other side.

The lesson moved on to a demonstration of non-standard units of measurement. Lulama asked the learners to demonstrate a hand span, a cubit and a pace. They all seemed comfortable with these units of measure. After the lesson, I spoke to a group of learners who said they had used these units of measure in their previous grade (4). The learners were then asked to measure classroom objects using non-standard units of measure. The learners were enthusiastic about what they were doing. They worked in pairs and there was much discussion on how to measure accurately, and what the appropriate unit of measure should be. For example, one couple discussed whether to use the hand span or cubit to measure the desk. They decided hand span was easier. Lulama posed questions in a mixture of Xhosa and English, and asked such questions as, "what did you measure," and "what did you use to measure it?" She followed up
with, "Show me how you did that." The learners discussed exclusively in Xhosa, but recorded their findings in English. The metre rule was much in use, either as a portable tool for measuring desktops and doors, or fixed to the wall to measure heights and book dimensions. There was conflict over the use of this resource, as the pairs using it would not allow others access to it until they were completed. There were five metre rules in the class. The learners were eager to report their findings. They reported in pairs with the one talking while the other demonstrated. The rest of the class sat quietly and listened while the pairs were reporting. The reporting was done mainly in English.

The follow-up activity was also conducted in pairs. The learners were required to use the non-standard body measurements to measure classroom objects that appeared on their worksheets. The learners were required to guess first then measure. The objects for estimation and measurement included: the desktop, the chalkboard, desk height, the door, and class book. The learners found it difficult to relate the drawings of the classroom objects on the worksheet to the actual objects in the classroom. Many learners had to ask Lulama to interpret each drawing for them before they were able to do the measuring. My interpretation of the learners' confusion was that the spatial representation of three-dimensional objects via two-dimensional line drawings was an unusual form of representation for them, and it caused difficulty. The pairs were eager to record their measurements, and they made little attempt to first estimate. When I asked selected pairs to give their estimation in hand spans, for example, they immediately began to measure. They were reluctant to make a guess possibly because they lacked the confidence to make an informed estimate. The learners indicated to Lulama that they wanted to measure the objects in centimetres using the metre rule. There was a sudden rush by the pairs to use the metre rules for measurement. When using the rulers the learners counted every marking. They did not count in tens or use any method to simplify their measuring. The learners reported their findings to the class. They only reported their actual measures and did not touch on estimation. I discussed the importance of making an informed guess or estimate with Lulama, but she said her aim was to get learners actively measuring. She was less concerned about getting learners to estimate. After the learners reported their findings, there was a reflection exercise. Lulama asked the class the following questions in both English and Xhosa:
**What did you learn?**

**What would you like to do next time?**

The learners responded to these questions verbally. They said that they enjoyed the activity and wanted to do more measuring, particularly of their own heights.

**Observation 5**

The lesson started with mental mathematics in pairs. Each pair was given a card, which they exchanged with other pairs when they were finished. They had difficulty working with three-digit numbers and several used their hands to do the working out. They made extensive use of counting on strategies when doing subtraction as a result of work done previously by Lulama, as an alternative strategy. They were not comfortable with division especially when it involved remainders. For example, they struggled with $14 \div 5$. Many pairs simply left division questions out. The multiplication was either double or triple. A learner was asked to read the following question for the whole class to calculate on paper: "185: double, triple, divide by two, add 16." Lulama provided the correct solution using traditional algorithms. There was no discussion and no questions.

The lesson moved onto measuring. Each pair was required to measure their height, hand span, pace and cubit, and then compare differences in measurement between them. Lulama had provided string, 30 cm rulers, and the metre rulers. The learners used the string successfully, measuring each other's height and using the metre rule to convert that height to numbers. There was considerable confusion whether to use centimetres or millimetres when using the 30 cm ruler, and the situation was not clarified at the time. During the report back Lulama made no comment on the differences found when using non-standard measurements. I expected her to use this as an opportunity to focus on the need for standard measurements as in millimetre, centimetre and metre. I felt that the opportunity for developing an important mathematical concept was lost.

The second part of the measuring activity was measuring accurately using 30 cm rulers and balance scales. The organisation of the groups was well managed with some groups using the rulers to measure a set of objects less than 30 centimetres. All the learners were keen to use the balance scales, which led to some pushing and
shoving. The groups using the scales had to balance stones against a standard measurement using 10gram, 20gram, and 50gram weights. It took a while for them to realise that stones had to be removed or added to a scale pan in order to get the scale to balance. Each group was eventually given an opportunity to use the balance scales, and the lesson ended with no discussion. I found this a frustrating lesson, as Lulama had not dealt adequately with the concept of standard units in measuring length. Furthermore, the scale activity appeared to be an arbitrary inclusion. Lulama did not discuss the standard units of weights with the learners and she could not tell me what she expected the outcomes of the lesson to be. She said that she just wanted the learners to weigh the stones.

Observation 6
The lesson started with mental mathematics. The learners took turns to ask the class questions, which included the following:
158 tripled; 262 times 3; 174 – 25; 428 doubled; 110 doubled; 150 – 25; 89 doubled; 130 plus 25; 281 tripled, and many more. The questions were asked in English and the learners used mathematical vocabulary such as: double, triple, less than, more than, less, more, add and subtract. One of the learners posed the question, "524 doubled." Lulama requested any learner to demonstrate a solution strategy on the chalkboard. Once the answer of 1048 was obtained, the learners were asked to triple the number. Lulama said it as "ten forty-eight". There were many gasps and intakes of breath, but they had a go at the calculation. There were many incorrect answers. A learner with a correct solution was asked to come up and show the class what she had done. Her explanation was as follows:

\[
\begin{align*}
1048 \\
+1048 \\
+1048 \\
\hline
3144
\end{align*}
\]

Another learner used an alternative algorithm:
1048 \times 3 = 3144 and explained in a mixture of English and Xhosa:
\[8 \times 3 = 24, \quad (4 \times 3) = 12 + 2 = 14, \quad (10 \times 3) = 30 + 1 = 31.\]
Lulama waited for discussion on these solution strategies, and as none was forthcoming, she moved onto measurement. She called two learners to the front of the class and asked what the difference was in centimetres between the tallest and the shortest person. The learners measured themselves using the metre rule and calculated the difference. The class was then asked to use scales to measure a bag of stones, a packet of sand and a jar of water. These were not balance scales as used previously, but kitchen spring scales. The learners became excited at the prospect of getting hands-on use of the apparatus, yet three groups had to wait while the other three groups measured stones, sand and water. As I watched the groups, it was clear they were not sure what they were trying to find out. Lulama affirmed that they were required to find the weights of the stones, sand and water, and record their findings. Some learners clustered around the group that was reporting, which made it difficult for the others to see.

Although this lesson engaged the learners in hands-on measuring, much of it appeared directionless. The jump from measuring length at the start of the lesson to measuring weights surprised me. Lulama, nonetheless, was quite clear of her outcome for the lesson. Her declared aim was for the learners to be able to measure. I discussed with Lulama the task of planning the progression of lessons within the topic of measurement. She said she needed guidance in her planning and was not sure what to do in the next lesson on measurement. We talked about possible alternatives for future lessons.

**Observation 7**

Lulama called the activity "doing mental," however it was more a practice of the four operations, with two, three and even four-digit numbers. The learners were required to provide numbers and operations for the rest of the class to do. The activity became competitive with the current learner trying to set a more challenging task than the previous learner. Some of the questions asked included: 461 tripled, 248 doubled, 1000 tripled, 294 doubled, and 5641 + 3121. The class was expected to do the calculations on paper, the main operations asked being double and triple. The 1000 tripled for example, was calculated using the standard algorithm and not done mentally. The learners worked at their tasks studiously and there were whispered shouts of "yes" when they achieved a correct answer. I noticed that learners who
gave out the number and operation did not work out the answers for themselves and relied on others in the class to do it for them. The learners seemed to enjoy having other members of the class pose the questions, and worked willingly despite the difficulty of some of the tasks. The lesson continued with Lulama setting sums on the chalkboard for the learners to find answers using their own solution strategies. Lulama asked some of the learners to show their working on the board for the following examples:

Example 1: \[ 84 - 29 = \Box \]
Example 2: \[ 370 \text{ doubled} = \Box \]

Although learners provided various solution strategies, there was no discussion on each method. Lulama's only comment was, "so many different methods." She did not draw out why the learners had chosen particular methods, nor highlight solution strategies that were more efficient. The follow-up activity was pair work using the 100-grid. The 100-grid is numbered 1 to 100 in a ten by ten square, and is mounted on bright yellow, laminated card. Using the grid learners had to make up sums for their partners in the operations of addition, subtraction, doubling and halving. The learners worked cooperatively together taking turns to pose questions. They wrote their calculations on scrap paper provided by the teacher, and counted on or back to find a solution. Once a correct solution was found, they recorded their findings on "formal" paper. I noticed that many pairs did the calculations on their hands before transferring them neatly onto the paper. This time when the pairs reported, Lulama insisted that they show the rest of the class how they arrived at a solution, using the 100-grid. For example, one pair showed 95 - 35. The pair circled 95 and counted back 35 in ones, to get to 60. Another pair had 88 + 25. They counted 25 in ones starting at 89 as follows: 89, 90, 91...113. There was no discussion of the methods used by the learners.

Lulama informed me that she was pleased with the learners' presentations, and felt encouraged because they had devised their own questions on the 100-grid, and had worked out their own solution strategies, and had presented them to the class. We discussed how willing the learners were to pose questions for themselves throughout the lesson, and she said that she wanted the learners to take responsibility for providing questions, and for finding answers. I raised the issue of refining learners' solution strategies, and cited the examples of counting on in ones on the 100-grids,
when greater multiples could have been used. Lulama indicated that she was not sure how the learners might refine their strategies.

**Review of Lulama's Classroom Observations**

On completion of the first six-month phase of observation visits, I began the review of information collected in Lulama's classroom. I kept in mind the dominant themes found in each of the three constructs that made up the Belief Scale. As the analysis took shape, I realised that the three constructs alone were too broad for meaningful interpretation, and so I turned to Yager’s (1991) principles to identify and highlight themes. I found that further analysis was possible and I was able to provide evidence of emerging themes and in so doing, I was aware of my own role as the observer in the process. I was conscious of my own interests, ideology and perceptions in writing up the review. After observing Lulama, I identified the following significant themes:

**Observer Perceptions**

Newman (1998) wrote, "...the most difficult thing about being an observer is recognising the unexpected" (p.8). As I have spent time looking over the interviews and observation reports of Lulama’s classroom, this one incident, not included in the case study narrative, caught my attention. It was an unexpected event and, I thought, more part of my own history as the researcher than the history of the observed teacher. I came to realise that perhaps it deserved further examination. The issue was based on an incident taken from the "what surprised me," section in the Observation Schedule (Appendix G):

*A grade 5 learner was given this calculation 4 - 5 = □. Her answer after a brief period of confusion was “1”. Another learner said, "there is no answer." The teacher, Lulama, pointed to the second learner and said, "you're right." At this point, I intervened and asked the first learner to explain her thinking. It took some coaxing before the learner would explain as she kept saying that she was wrong. Eventually she told me that she had taken 4 - 5 = 0 with 1 left over. I asked her what she had done with that 1. She said that she had added it but knew that it was not supposed to be like that because she was supposed to subtract. I suggested to her that she write in the subtraction sign as -1. She said that she did not know that she could do that,*
and we followed up with further examples, which she got right without
difficulty. This young learner was excited by her new discovery, and Lulama
continued the lesson without comment. No further calculations of that nature
were required.

(From Observation 2)

As I reflected on the incident, several questions came to mind. Did Lulama have the
mathematical content knowledge to be able to scaffold the learner's understanding of
negative numbers? Did she have the confidence to ask questions to which she may
not know the answer? Was Lulama listening to the learners? All the questions were
important and required investigation, but the final question troubled me most at this
stage of the research. I believed that if a teacher's teaching style did not focus on
listening to a learner's solution strategy, she would not be able to develop an
understanding of how the learner thinks, which may result in not making the learners'
understanding the primary focus in the classroom. I went back and looked at the
collected data again. I noticed that in all the observed lessons and interviews no
mention was made of listening to learner solution strategics, and trying to understand
their thinking. I looked over Lulama's Observation Schedule record to ensure that I
was explicitly observing her to see if there was evidence recorded of her learner
listening skills. The Role of the Teacher subsection on the Observation Schedule
(Appendix G) included the following three items:

How does the teacher act?

Classroom design?

Teacher's power?

It was clear from the nature of these questions on the Observation Schedule that I had
neglected to focus on the need for specific teacher listening skills. I recorded little to
show that Lulama adapted the lesson according to the learner's discourse. In all the
lessons observed, Lulama taught with her own outcomes clearly in mind. The
shortcoming of not acting on learner discourse was not Lulama's fault but mine,
because I had not stressed sufficiently the need for it in her lessons. In the Second
Phase, I made more explicit the need to understand learner's knowledge. Vace and
Bright (1999) stated, "...children's knowledge and teacher's understanding of that
knowledge are central to instructional decision making" (p.90). They argued that
teachers could not understand learners' knowledge unless they knew how to listen,
not only to the learners but to themselves as well. Furthermore, the knowledge teachers' gained from listening was not the end of the process. The data this provided needed to be reflected on to guide future lessons. I realised as the observations progressed, that Lulama needed to develop her learner listening skills, and to plan future lessons based on learner understanding. It was a significant aspect that arose out of the classroom-based observations, and I planned to make it a focus of attention with all the key teachers in the next six-month phase of the intervention.

Becoming Reflective

Carpenter, Fennema, Franke, Levi, and Empson (1999) claimed that, "developing an understanding of children's thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own students" (p.109). I saw reflection as the mechanism by which Lulama could link what she knew about her learners' content knowledge to her classroom practice. Reflection, I believed, was a way of identifying what we knew and helped us better to apply what we knew. Yager (1991) supported the notion of reflection when he stated as one of the six principles of the Constructivist Learning Model, "encouraging adequate time for reflection and analysis...and encouraging self-analysis..." (p.56). In Lulama's case, she was aware of the need for reflection and there was evidence of reflection in her own teaching file as well as from her learners. Observation lessons 1 and 4 both had written questions for the learners to reflect on. They responded in written English in Observation lesson 1, and verbally in Xhosa in Observation lesson 4. In both cases, the learners were being asked for their own comments as a form of self-assessment, which was unusual in a school where power traditionally rested solely with the teacher. The opportunity to contribute their own views was welcomed by the learners and graciously received by Lulama. I felt encouraged by the fact that Lulama was getting her learners to reflect on their own learning, and was making an effort herself at becoming a reflective teacher against a background of transmission style teaching that prevailed in the school. Lulama's own reflections were written up once a week, but were largely descriptive not reflective. I acknowledged this shortcoming as teaching was recognised as a complex activity where teachers according to Newman (1998) were, "...accustomed to many things demanding their attention at once without really thinking about them" (p.3). Understandably, the first step in reflection was simply to describe incidents that happened during the lesson. In
so doing Lulama was asked to record incidents that happened amidst the demands of the lesson. In discussing these incidents with Lulama, I suggested that they should be ones that surprised, worried, amused or even angered her as a starting point. Eisner (1991) argued that, "we learn to see, or at least we learn to see those aspects of the world that are subtle and complex" (p.17), and reflection was part of that learning to see. Eisner implied that teachers could be taught some basic skills to help them learn to see. Only isolated critical incidents appeared in Lulama's journal with most of the writing taken up with description of what she (the teacher) did in each lesson. It read like a teacher's record of work, a task she naturally found familiar.

One of the biggest hurdles, I believe, that needed to be overcome was to get the key teachers to write as Eisner (1991) stated, "to help others see and understand" (p.17), and even more importantly, help themselves see and understand so that their praxis changed. This would require key teachers, like Lulama, to write regularly, and to write with enough detail to provide a basis for reflection. Writing for almost all the key teachers was hard. Firstly, it was carried out in a second language, and secondly, writing as a process was not a regular form of communication as most of our teachers were brought up in a strong oral tradition. Many of them have had an inferior quality educational experience (Boughey, 1994), with the result that the necessary skills needed have not developed to the extent where writing was perceived as a valuable endeavour. Weekly reflective writing, as a valued but non-threatening experience was one of the aims of the professional development intervention. In Lulama, I detected the beginnings of reflective writing. Through reflection, I assumed that key teachers would discover aspects about themselves and their professional competence that might distress them (Newman, 1998). They would have to let go of assumptions and beliefs that formed much of their past teaching experience. If the reflective process was sustained, it provided a basis on which to build transformed beliefs and practices.

*Negotiating Meaning*

Another of Yager's (1991) six principles stated, "encouraging students to challenge each other's conceptualisation and ideas" (p.56). With respect to this principle, I referred specifically to the reporting of learner solution strategies to the rest of the class. My narrative account suggested that Lulama did not encourage discussion
when learners presented their own solution strategies to the class. In most of the observed lessons, learners’ varied and different solutions were not challenged either by Lulama herself, or by fellow learners. In Observation 2, for example, I discussed with Lulama the reasons for not probing the learner’s solution strategies. Her reply was that the learners must be responsible for the solutions presented, and not the teacher. I believed that Lulama acted boldly in allowing learners to take responsibility for their own learning, but she could have enhanced their learning by asking groups to explain to others how they arrived at their solutions. In Observation 4, by contrast, Lulama did seek clarification in the measuring activity by asking the groups to show her how they carried out a particular measurement. This was a practical activity and could be demonstrated, unlike the more abstract solutions presented by learners in Observation 7, for example.

During the First Phase observation period, the questioning of learner’s thinking frequently came up for discussion. Although Lulama agreed with me about the need to stimulate learner thinking through learner discourse, there was little evidence of it in each of the learner reports. The most she said was "so many methods," which was perhaps an indication of her own lack of confidence in mathematics content knowledge by not wanting to ask questions to which she did not know the answer. So much of what was observed in Lulama's class was positive, yet this remained an unobtainable step for her. It must be remembered that the narrative observations took place in the first six months of the intervention. Development in confidence in Lulama over the next six months should produce increased refinement in learners’ solution strategies when reporting to the class.

Learner Collaboration
A significant feature in Lulama’s classroom was the positive level of collaboration between learners in all the observed lessons. The learners were placed in groups of six and were required to work together in cooperative problem solving. Lulama encouraged learners to build meaning amongst themselves through discussion, by placing only one question card per group in a technique that encouraged each group to work together, in the knowledge that it was the groups’ responsibility to report their findings to the rest of the class. In all of the lessons observed, there was a high degree of learner discussion, which took place in Xhosa, and it was evident from the
learners themselves that they regularly negotiated meaning this way. Yager (1991) saw learner collaboration as an important principle for action in the Constructivist Learning Model when he stated, “Using cooperative learning strategies that emphasize collaboration, respect individuality, and use division of labour tactics” (p.56). My perception of Lulama was that she used learner discourse as a technique towards building learner understanding, in contrast to a teacher transmission style of teaching. The ease in which she facilitated learner discussion and collaboration in all the observed lessons was evidence of her strong belief in learners constructing knowledge for themselves, and not being dependent on the teacher for received information.

Sequencing of Topics

The structure and layout of the assessment standards in the National Curriculum Statement (2001) document implies a clear sequencing of mathematical topics which I refer to as “pace and progression” within a topic. There was limited evidence observed of systematic pace and progression in Lulama’s mental number work. At the beginning of the observations, learners were working with only one and two-digit numbers when doing mental work, and the operations mainly focused on the operations of addition and subtraction. There was little evidence of multiplication and division operations other than doubling and tripling activities, and no division that required remainders. As reported, I discussed these matters with Lulama who felt that her learners were not capable of operating on two and three-digit numbers. She did, however, include three and even four-digit numbers in her mental number development activities. The learners found these difficult and resorted at the outset to using standard algorithms. When learners failed to understand the application of standard algorithms, Lulama suggested using their own methods, which made sense to them. Lulama’s approach to mental was not well planned, and she jumped to extremes in respect to the difficulty of the mental activities. The lessons themselves varied in pace and progression. In the lessons observed, many of the activities she had encountered in the contact sessions of the intervention were taken without adaptation and used directly with her learners. Lulama did not see recognition of learners’ prior knowledge as a concern. She was unable to employ Yager’s (1991) principle to “use student thinking, experiences, and interest to drive lessons” (p.56). The learners were, for example, just getting into fraction material when Lulama
switched her focus to measurement. She was aware of the difficulty of planning a sequence of lessons, and expressed her concerns to me. In many ways, her concern was justified. She was trying to implement a transformed teaching style, and she did not have sufficient confidence in her own mathematical content knowledge to implement something that had not been carried out in the contact sessions. She was also concerned that by using a textbook she would be seen as a traditional teacher. Clearly, the message to emerge was that the interventions in future had to address this issue of pace and progression within a topic.

Second-Phase Classroom Observations
My colleague Thandi accompanied me on my visits to video record the case study teachers and she conducted semi-structured interviews with Lulama and Makana. Thandi was a great help in the classroom when conversing with learners in their mother tongue.

Lulama

Video Recording 1
The lesson started with Lulama writing the following numbers on the chalkboard:

\[ 325 / 785 / 437 / 924 / 639 / 536 / 794 \]

A question card was distributed to each of the six groups with the following question:

*Find the difference between the total of two big numbers and the biggest number. Use three solution strategies.*

Although the question appeared confusing, the learners were not confused and worked enthusiastically at the task in pairs, which suggested that they had done this task before. Each group received the same question. One pair of learners recorded horizontally:

\[ 924 + 794 + 785 = 2503 \]

Another pair used the place value format:

\[
\begin{array}{c}
785 \\
924 \\
794 \\
\hline
2503
\end{array}
\]
Another pair rounded up by adding 6 to one number and subtracting 6 from the other number as in:

\[
\begin{align*}
924 + 6 & = 930 + 788 \\
794 - 6 & =
\end{align*}
\]

Unfortunately, I failed to capture the completion of this mathematical process whilst video recording was taking place.

Other groups split the numbers into hundreds, tens and units as in:

\[
\begin{align*}
900 + 20 + 4 & = \\
700 + 90 + 4 & = \\
700 + 80 + 5 & =
\end{align*}
\]

In going around the class I noticed that the decomposing of numbers into HTU’s seemed to be the method preferred by most learners, which indicated that they had been taught this form of extended notation. An example of this layout, prepared by Lulama, was on display on the classroom wall, and many learners referred to it when doing their calculations.

Another pair first rounded off the numbers to the nearest ten before adding in the units:

\[
\begin{align*}
920 & \\
790 & \\
780 & \text{13 (Units)} \\
2490 & \\
2503 & \text{(The units were added separately)}
\end{align*}
\]

The next phase of the lesson was for the pairs of learners to report their preferred solution strategy to the class. I video recorded one report back only, as I still required time to interview Lulama. Two boys came to the front and recorded their strategy on the chalkboard and explained in English:

\[
924 + 794 + 785
\]

The first boy said, “round off to the nearest 10”, and then corrected himself when he said “to the nearest 100” and recorded:

\[
900 + 800 + 800
\]

The second boy clarified this statement and completed the calculation as follows:

\[
\begin{align*}
900 + 800 + 800 & = 2500 \\
4 + 4 + 5 & = \text{13} \\
2513 &
\end{align*}
\]
The boys were not asked to further refine their method once they had rounded off the hundreds and it was left to me to calculate for the class an exact answer via the differences to arrive at a correct total of 2503. The video recording demonstrated several interesting points involving learner solution strategies. Some learners added using the traditional algorithm, and another group rounded off before adding. An unexpected strategy was where the learners added 6 to 924 to make 930, an easier number, and then subtracted 6 from 794 to make 788 before continuing to subtract.

The horizontal decomposition of numbers into hundreds, tens and units was the preferred method by most learners, clear evidence that suggested that the learners had been taught this strategy at some time. The group reports provided the class with the opportunity of seeing a variety of solution strategies, with the possibility of including some of these methods into their own repertoire. The lesson ended with many pairs wanting to present their solution strategies to the class. Thandi needed time to interview Lulama in her learner-free classroom before the start of the next lesson.

**Interview with Lulama**

Thandi conducted the interview with Lulama in Xhosa. She later translated the interview into English. Thandi asked about the format of the lesson we had seen today and whether the use of work cards was normal practice for her. Lulama indicated that she regularly prepared work cards for her mathematics lessons, and that the learners received instructions from the work card. She affirmed that what we had seen today was a normal format for her lessons, and the behaviour of the learners in class confirmed that they were familiar with Lulama’s approach. (A neatly handwritten question was mounted on cardboard for each of the six groups). Thandi enquired whether the format witnessed today was her regular teaching style or had it been prepared especially for the video recording. Lulama reiterated that what we had seen today was her normal practice.

Of interest in this lesson were some of the of learner solution strategies to emerge. Thandi asked Lulama about learners using different methods. In responding Lulama was a little tense as she shifted her glasses, and said that some of the methods came from her and she referred to her charts on the wall display, and others came from the learners themselves. She said that she did not insist that the learners use any one
particular method, and this was evidenced by the alternative methods used by some learners in the lesson. Thandi said that she had seen many learners using extended notation, and said that this format could cause difficulties later on, (in division for example). Thandi asked whether there were opportunities for learners to use their own solution strategies. Lulama explained that her approach was to allow learners to use methods in which they felt comfortable. Some learners, she said, felt more secure with a method provided by the teacher, hence the chart on display to help those who were unsure of themselves. If learners wanted to use their own methods that was fine too, and she maintained that learners should be allowed to use the strategies they wanted. Thandi then referred to the wording of the question in the activity. Lulama confirmed that the learners were familiar with the question. They were able to get on easily with the task, as it was not new to them. Thandi suggested that in future she should change the wording so that the learners were always challenged. I found out later that Lulama did in fact change the activity questions. The bell for the next lesson rang and the interview ended as the learners re-entered the classroom.

**Video Recording 2**

Each group received a worksheet for two $3 \times 3$ Magic Squares to be completed, with the following instructions:

1. *Fill in the missing numbers such that the sum for the diagonal, row, and column is 246.*

   \[
   \begin{array}{ccc}
   78 & 89 & \\
   82 & & \\
   & 86 & \\
   \end{array}
   \]

2. *Find the sum for each row, column and diagonal. Fill in the missing numbers.*

   150: *(Central number in a $3 \times 3$ square, all the other squares empty).*
Learners worked on question 2 in pairs, recording their findings on laminated green cards using felt-tipped pens from the resource kit. I noticed that some learners resorted to counting on fingers whilst others used the scratch pad paper provided. There was considerable discussion in their groups as the learners worked at their calculations. Not all learners engaged in discussion. A girl, Ntash, counted on her fingers and recorded her findings as her partner, a boy, stared into space with one hand on his face showing no interest in the computations Ntash was making at the time. Later in the lesson, he was seen doing the calculations as Ntash watched and listened. The solution strategies varied with the groups. Some groups balanced the numbers on paper first before recording on the 3 x 3 square as they tackled the second task. They used the following format:

```
146  147  148  149  150  151  152  153  154
-4   -3   -2   -1   +1   +2   +3   +4
```

A group working on question 1, recorded directly onto the green card using the central cell number of 82 as the starting point, and by balancing a larger number with the respective smaller number, they were able to complete the magic square as follows:

```
82 - 4 = 78  82 + 4 = 86  +82 = 246
82 - 3 = 79  82 + 3 = 85  +82 = 246
82 - 2 = 80  82 + 2 = 84  +82 = 246
82 - 1 = 81  82 + 1 = 83  +82 = 246
```

Once the groups had worked out a solution to one or both of the questions they reported to the class. The first pair started by writing on the chalkboard the set of numbers given in the first activity on the worksheet: 78, 82, 86 as the central column, and the top right number 83. Thandi asked the group as they struggled to get started, “What should be the total?” The reporter said, “We are going to add,” and Thandi asked, “Why do they add these two numbers?” The learners replied, “We want the third number.” The reporter wrote 246 after drawing on members in the class to participate in the addition calculation. The reporter added 83 + 82 = 165, the top right number, and the central cell number given on the worksheet, and subtracted 246 - 165 = 81. The following dialogue was transcribed from the videotape:

Lulama asked, “Why do we subtract?”

The reporter replied, “To find the number.”

Thandi asked the class, “Why do they subtract?”
The reporter repeated, “To find the number.”
Thandie confirmed by saying, “We need the third number.”
The reporters proceeded to solicit the cooperation of learners in the class to calculate the addition and subtraction computations required to fill the empty cells in the first Magic Square task as follows:

<table>
<thead>
<tr>
<th>Bottom left cell: 83 + 82 = 165</th>
<th>246</th>
<th>Check: 165</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-165</td>
<td>+81</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>246</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Top left cell: 82 + 79 = 161</th>
<th>246</th>
<th>Check: 161</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-161</td>
<td>+85</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>246</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Middle left cell: 85 + 81 = 166</th>
<th>246</th>
<th>Check: 166</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-166</td>
<td>+80</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>246</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Middle right cell: 80 + 82 = 162</th>
<th>246</th>
<th>Check: 162</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-162</td>
<td>+84</td>
</tr>
<tr>
<td></td>
<td>84</td>
<td>246</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bottom right cell: 83 + 84 =167</th>
<th>246</th>
<th>Check: 167</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-167</td>
<td>+79</td>
</tr>
<tr>
<td></td>
<td>79</td>
<td>246</td>
</tr>
</tbody>
</table>

The reporters had successfully filled in all the cells of the Magic Square and they were clapped in appreciation by the rest of the class. What was noticeable was the interest shown by the other members of the class as the reporters engaged them in calculating each cell.

Another pair showed how they used the central cell of 82 to build up a balanced 3 × 3 square as follows:

\[
\begin{align*}
82 + 4 &= 86, \quad 82 - 4 = 78; \\
82 + 3 &= 85, \quad 82 - 3 = 79; \\
82 + 2 &= 84, \quad 82 - 2 = 80; \\
82 + 1 &= 83, \quad 82 - 1 = 81.
\end{align*}
\]

I collected evidence of the learners’ written recordings, and personal reflections on the weeks’ activities that included completing the Magic Square. The extract quoted below was of an un-named learner reflecting on the process of calculating the cells
for the Magic Square based on the solution strategy described above. The learner’s struggle to write correct English can be seen in the extract:

We were do magic square in that magic square we learn to add in other side and subtract in another side. We enjoy to work in group. There is nothing that make us get stuck. There’s nothing that was heavy. Our middle or centre number was 82. Our left diagonal numbers was 85 82 and 79. The something that was sure to get 85 was to add 3. Our numbers we were given first are 78, 83, and 82 and 86. Our total of the magic square is 246. Our write diagonal numbers are 83, 82 and 81.

Another learner, Ntlanjeni, recorded his Magic Square experience in these terms:

I was doing Magic Square at the first time I was then rong bur next time I then right. We where working to had Magic Sqaew where very difficult but since thandi and John cam with an grid and kip traing antill we anderstend. ... We were traing to speak English.

(Ntlanjeni).

In my opinion, Ntlanjeni, and the other learners in the class were to be admired for the effort they made in attempting to write in a second language. The quality of the written reflections were in stark contrast to their articulate verbal reports made earlier. The video recording showed learner reflections that had been placed on display for all to read. On speaking to Lulama, she said that writing reflections was an ongoing process for her and for the learners, and that all the reflections would be eventually filed for safe keeping. She found learners had difficulty expressing themselves in writing as the extracts above clearly indicate. In a sense, the activity of reporting to the class can be considered a form of learner reflection and fitted well with Lulama’s aim of establishing a reflexive climate where the teacher, the learners and parents engaged in thinking about the mathematics learning taking place.

Kholeka M. is the mother of Sindiswe, one of the learners in Lulama’s class. As a parent, she was asked by Lulama to comment on Sindiswe’s work at home.

Sindiswe’s mother wrote in English:

She is a very brilliant girl. According to her work at school she is doing well. Her maths is good. She is capable of doing her home work on her own and needs a second hand when ther’es a problem. I like the way she handles her
work.... All in all she is confident of every thing she do. That's all I can say I observe in her work.

Signed: Kholeka M. (mother).

Lulama said that the process of getting parents and learners involved in reflection was a major step forward. She believed that regular reflective writing had helped her teaching. She also believed that her learners had grown in confidence in their mathematics, and a measure of their confidence was that they were prepared to present their reflective writings for public scrutiny, despite the difficulties they found in writing up their reflections. The enthusiasm of the learners engaging in the Magic Square activity was captured on videotape.

Video Recording 3

The lesson began with Lulama posing a problem to the class:

There are five learners and five oranges. One orange is to remain in the bag. Each learner must be given an orange. How are we going to share the oranges? Anyone want to try?

A boy suggested cutting the oranges in half. Lulama’s negative response caused the boy to slump with his head down, as the rest of the class remained silent. “No one wants to try,” she said, which was not surprising as we (the learners and the observers), were a little confused by the question. After some time another boy, Temba, suggested giving 4 oranges to 4 learners, and the bag with the remaining orange to the fifth learner. Lulama confirmed the solution.

The next activity involved all the learners in their groups. Each group collected a card with one word problem prepared by Lulama. The reader in the group read aloud the problem to the group, which was written in English. Each group selected one of six cards, and all the problems involved different operations as can be seen from the question cards below:

1. Lunga is a farmer. He plants 47 trees of oranges in a row. He has 29 rows, how many trees of oranges does he have altogether?

2. Sinazo had R112 in her savings account. Her father told her that he deposited another R68 in her savings account. Sinazo calculated on a sheet of paper like this:

   R112
Is this the correct answer? If not, find each of her mistakes.

3. Chumani wants to buy a second-hand bicycle for R456. He has only saved R98. How much does he still need to buy the bicycle?

4. At your school there are 59 Grade 5 learners, 525 Grade 6 learners and 789 Grade 7 learners. How many learners does your school have altogether?

Sive solved a problem this way:

\[ 1224 \div 14 = 50 \]

His teacher marked the sum wrong. Which one of the following is the correct answer? Why do you think so?

(a) 68
(b) 151
(c) 51

5. A fruit seller sells bananas at R2, 50 per packet. One day he sold bananas for a total of R100. How many packets of bananas did he sell that day?

6. A school buys 520 exercise books with 62 pages in each book. How many pages are there altogether?

The group that worked on question 1 set out their calculation on A2 sized paper using a marker pen as follows:

\[
\begin{array}{c}
47 \times 29 = \square \\
47 \times 29 & 10 & 10 & 10 & 10 & 7 \\
10 & 290 \\
10 & 290 \\
10 & 290 & 20 \times 7 = 140 \\
10 & 290 & 9 \times 7 = 63 \\
7 & 203 & 203 \\
1363 \\
\end{array}
\]

The group that worked on question 4 provided a solution strategy for the division component based on the multiples of 18.
They wrote at the top of their recording sheet: 18, 36, 54, 72, 90, 108, and calculated $1224 \div 18$ using the following strategy:

\[
\begin{array}{c|c}
1224 & 1244 \\
-324 & \\
1224-324 = & 900 \\
-270 & \\
900-270 = & 630 \\
-216 & \\
630-216 & 414 \\
\end{array}
\]

No further recording was captured on videotape, although the group continued to be engaged in their method of calculation.

The first report was by two girls from the group on problem 1, where 47 multiplied 29 to find the number of planted orange trees. They recorded on the chalkboard the layout described above. When they came to calculate $10 \times 29$, the reporter turned to the class and asked:

“How many zero’s in this number” pointing to 290?

“One” responded the class in unison, and zero was placed in the unit’s column

“I times 29” asked the reporter?

“29” was the response, and 29 was placed in the appropriate columns. The same routine was repeated four times and Lulama did not request the reporter to refine or speed up the procedure via mental calculations. The learners appeared to want to use methods that had been drilled previously. Thandi and I were of the opinion that there was no need for the learners to repeat a mechanical procedure four times. We felt that Lulama should have intervened at this point by getting the reporter to multiply by ten mentally. When it came to multiplying $7 \times 29$, the first step was to multiply $7 \times 20$, which the class did verbally by adding the multiples of 7 in the form of 7, 14, 21, 28...to get to 140. The same process was used to work out $7 \times 9$. Both Thandi and I were impressed with the confidence of the two learners as they presented the report. Clearly, they understood their solution strategy, but we were concerned that they seemed reluctant to use quick, efficient mental methods where appropriate. We noted the learners’ approach for later discussion with Lulama.
The next report featured word problem 5. Two boys from the group presented the following three solution strategies:

4 packets = R10.00
4 packets = R10.00
4 packets = R10.00...

The pattern was repeated ten times in full to give a total of 40 packets = R100.00.

Thandi asked the presenters to provide another solution strategy. This they did by adding eight packets to give R20.00 at a time. The class participated in adding the multiples of 8 to get to $5 \times 8 = 40$, and by adding the multiples of R20 to get to $5 \times R20 = R100.00$. The reporters provided a further strategy based on 10 packets = R25.00. They explained that $4 \times R25 = R100.00$, and after probing by Thandi, that $4 \times 10$ packets gives 40 packets. This final solution strategy came about because of prompting from another teacher in the school. He asked the reporters specifically for a shorter, more efficient method, which they demonstrated they were capable of doing when challenged.

The final report to the class was question 3 where the problem was to work out the difference between R456 and R98 for the purchase of a bicycle. The group solved the problem in the following manner:

The reporter wrote on the chalkboard R456 - R98, and said we must subtract. He then told the class, “When we add on this side we ‘gonna’ add to this side,” pointing to each number in turn. He wrote:

R456 + 4 - R98 + 4
R460 - R102
R400 - R100 = R300
R60 - R2 = R58

R358

Another learner in the class asked him whether this was “a total or a difference.” The questioner had in mind that they had set out to subtract, which would give the difference. The reporter replied that it was “a total.” No probing of the calculation took place to clarify whether it was “a total” or “a difference,” or why this answer was given. I suspected that the reporter was referring to a sub-total of $R300 + R58 = R358$ when he responded.
I asked Lulama if I could quote from her reflections on the lesson once she had written them up. The extract below was taken from her portfolio:

*It was not easy for learners to see Maths in those word problems at the beginning as a result I had to give them some activities for all groups so that I could be able to work with the whole class I encourage them to: first read the problem, understand the problem, ask questions to see if they understand what is asked in the problem, work at their sums using different solution strategies in small groups. Reflections done by learners help me to understand what they could do without help and what they did not understand. As a result, we spent more time at the beginning of each lesson to discuss previous days work.* (Lulama)

Some teachers in the past believed that their learners were not capable of solving word problems. Lulama’s portfolio showed a positive rationale for getting learners to engage successfully in word problems, and the videotape clearly showed learners successfully participating in problem solving activities. The range and quality of the problems posed on the work cards was evidence that Lulama had carefully considered the task. The question of pace and progression in planning was discussed more fully with Lulama when she later reviewed this lesson on videotape. Lulama wrote in her portfolio, “*Solutions were reported and discussed with the whole class.*” The videotapes provided extensive evidence of learners reporting to the class, but discussion of a mathematical concept at a vital moment was not evident.

**Videotape Observations: Member checks**

In making the videotapes available for outside examination, I was offering an alternative theory of validity for analysing the video recordings. Support in adopting a multi-observer approach came from Goldman-Segovall (1995) when he stated, “*Building a valid multimedia research document... requires sifting through layers to find essential themes and patterns...*” (p.164). In drawing on the expertise of a consultant panel of experienced observers, my research gained strength by providing a forum for variance and critical comment.
Consultant Panel of Observers

The consultant panel of observers comprised the following members:

Thandi:
Thandi is a Xhosa speaking colleague at RUMEP with eight years experience working with in-service teachers in mathematics education. Before joining RUMEP, she was a primary school teacher in the district. She is the coordinator of the Collegial Cluster Project, which promotes a professional development programme through the activities of ten rural clusters. Thandi is currently registered for a Masters degree in mathematics education.

Sue:
Sue is a researcher in the Department of Education of Rhodes University. She has recently completed a doctorate in mathematics education for in-service teachers. Sue coordinates ongoing research in the fields of mathematics, science and technology education. Before coming to Rhodes University, Sue was a primary school teacher in the United Kingdom.

Marc:
Marc is a lecturer in the Department of Education of Rhodes University. He is a fellow student currently studying for a doctorate in mathematics education through Curtin University. His research interests lie in qualitative research. Before coming to Rhodes University, Marc was a teacher in a local private secondary school.

Panel Review Of Videotaped Lesson

Lulama

The videotape selected for review was Video Recording 3 described above. The consultation time set aside by the panel of observers allowed for the viewing of one videotape only, followed by discussion and debate, which I estimated would take 90 minutes for each of the three videotapes that were planned for review. I had the option of showing one videotape of each of the three case study teachers or three videotapes of one case study teacher. I explained to the panel of observers that I had elected the former option, as I believed that having three different cases would broaden the discussion. I chose video recording 3 for the first viewing because I believed that this videotape captured typical images of Lulama and her learners during all phases of the lesson. I made my reasons for selection clear to the panel and
said I would show any of Lulama’s other videotaped lessons if they requested them. The panel was informed that part of the research design included making a follow-up review of the same videotape. The consultant panel of observers approved the ethics of inviting the case study teachers to review their respective videotape, and in so doing providing them with the opportunity to comment on their video recorded lesson.

As I have reported earlier, I was in some dilemma whether to provide the consultant panel of observers with a theoretical framework for analysing the videotape or to allow them free range to search for ‘grounded issues’ (Strauss & Corbin, 1994). In discussion with colleagues and other researchers, I opted to provide a guiding framework based on Yager’s, (1991) Constructivist Learning Model (CLM). I also included the three constructs found in the Belief Scale as a basis for identifying emerging themes. Before viewing the videotape, the panel of observers sought clarification on the summarised procedures of the CLM that I had prepared for them.

**Constructivist Learning Model (CLM)**

We are looking for evidence of these procedures in a constructivist-learning environment:

1. Recognising, and building on learners’ prior knowledge.
2. A developed, interactive learning environment.
3. Promoting learner collaboration.
4. Encouraging learners to challenge and negotiate meaning.
5. Encouraging learners to reflect on, and reformulate ideas.

Plus other issues that might emerge in the broad categories of:

*The Role of the Teacher*

*The Role of the Learner*

*Sequencing of Topics*

I discussed the origin of Yager’s (1991) CLM procedures with the team and clarified that the purpose of their task was to look for evidence of the themes stated above, appearing in video recording 3. Having viewed the videotape, a major concern raised by Sue was whether there was evidence to show that mathematical learning had
taken place in the observed lesson. She contended that there was evidence of learners “working cooperatively” but that the learners appeared to be applying procedures that were familiar to them rather than being engaged in conceptual learning. Sue said that CLM was a learning model and there were “lovely examples” of the procedures being applied in the lesson, but she was not convinced of the learning taking place. Thandi responded by asking Sue how we could establish a convincing case that conceptual learning was happening. Sue suggested that we should see evidence of how Lulama arrived at this point, in which case we might see how the teacher developed the skills being applied in this lesson. The panel said knowing Lulama’s overall plan would help them establish what had taken place before this lesson. Thandi promised to make copies of Lulama’s weekly plans for the next panel observer session. Lulama’s preparation was not perceived as a concern as the word problems mounted on cards was evidence of her preparedness. The panel wanted to know if Lulama regularly used a prepared work card approach, and the videotaped interview of Lulama by Thandi was shown, as Thandi translated from Xhosa into English. In the interview, Lulama confirmed that her lessons were normally based on work cards.

Marc said that he had found the CLM procedures “very useful.” He said that the six principles required further clarification with respect to items 5 and 6. He debated whether item 5 would be better served by using the word “refine” in place of “reformulation” in the context of getting learners to reflect, and becoming more efficient in their solution strategies. In item 6, he was unsure of the implications of the word “consensus” as it could be applied to many different situations. I said that these were valid concerns and I would modify the wording in my summarised version of the CLM in the light of these queries before the next panel review session. The panel of observers identified seeing specific CLM procedures that included interactive learning, learner collaboration, learner reflection in the report backs, and the learners using a variety of solution strategies. Evidence of these items in Lulama’s videotaped lesson indicated that Lulama was putting into practice many of the procedures found in Yager’s (1991) Constructivist Learning Model. Sue pointed out that what was not seen was evidence of item 1, ‘building on learners’ prior knowledge.’ Marc felt he was restricted by the CLM criteria because there were no criteria that related directly to the teacher. For example, “What does the teacher do to
encourage reflection?” he asked. Other panel members agreed that the broad heading of Role of the Teacher provided limited useful guidance to the observers.

Of concern to the panel in the learner reports was the lack of probing questions by Lulama to find out how and why learners arrived at certain solutions on their way to solving a problem. It was interesting that the panel should raise this matter as I had discussed the same issue with Lulama in my earlier observational visits. At the conclusion of the 90-minute review, the panel of observers agreed that although there had been considerable debate on many issues, the procedures itemised in Yager’s Constructivist Learning Model provided a useful guiding framework for identifying themes to emerge from the videotaped lesson. Marc concluded by saying, “I like this qualitative approach. It raises so many issues.”

Case Study: Makana

Vignette

Makana taught in a farm school 30 km from Grahamstown. It was a three-teacher school and one of the teachers was the headmistress of the school. Makana taught a multi-grade class of 27 learners from grades 5, 6, and 7, and he has been teaching for 11 years. All the learners were Xhosa mother tongue speakers. The classroom was large and airy and was equipped with sufficient chairs and tables for each learner. There was provision for mounting wall displays but the room had no posters, charts or learners’ work on display in the First Phase visits. In the Second Phase, however, there was a striking display of learners’ work. The learners were drawn from farms in the area, which meant that many learners walked a distance of up to 10 kilometres daily to get to school. The school was set in arid, semi-desert terrain, hot in summer, cold in winter, and parents housed near the school lived in shacks that had neither electricity nor running water. The parents of the learners were mainly farm workers living in impoverished homes with children coming to school hungry. A daily feeding scheme helped alleviate the plight of malnourished children. The learners wore a motley collection of clothing; some with school uniforms, some with shirts donated by the local police station, and some in casual jackets to keep warm. Although the learners came from poor homes, they were polite, amenable, and friendly, and displayed a willingness to attend school. Numbers in the school were dropping as parents moved to urban areas for work.
First Phase Classroom Observations

Observation 1

I had made a prior arrangement to visit Makana, and yet he appeared surprised when I entered his classroom. The learners sat in rows and were working silently. The assignment for all the learners, irrespective of grade level, was to solve two-digit addition and subtraction calculations that he had prepared on the chalkboard. There was no discussion and no collaboration between learners as they diligently worked at their task for 35 minutes. Makana walked around speaking to individual learners in Xhosa, and assisting them where necessary. Towards the end of the lesson, Makana selected an addition and a subtraction calculation to work through aloud, using the standard algorithm in a place value format. He explained carefully the mechanics for finding the correct answer, without reference to mental methods, whilst the learners watched passively. At no time did he call on the learners to make verbal input during his bilingual Xhosa and English presentation. The 45-minute mathematics lesson ended and the learners were released for break.

I discussed with Makana whether he was comfortable with me observing his class, as he seemed distracted by my presence. He explained that the school had been one of a few schools in the district to be included in the Provincial Department of Educations’ professional development initiative called the “Imbewu Project,” and that he needed to devote his full attention to the demanding requirements of the Imbewu Project assignments. He indicated that as he was committed to attending the Imbewu workshops, he was as a consequence unable to devote time to the planning and implementation of RUMEP inspired mathematics lessons. He claimed that he did want to move away from “teacher transmission” lessons and once he had completed the Imbewu workshops, he would have more time to plan the mathematics lessons. When I enquired the nature of the Imbewu Project workshops, he replied that they were activities designed to help educators implement an “inquiry-based” approach in the classroom. We discussed his proposed plan and I informed him that RUMEP members had written much of the mathematics materials for the Imbewu Project, and these materials fitted well with the learner-centred procedures advocated by them. I intimated that it would be appropriate to integrate the Imbewu Project approach into
his mathematics lessons. Before departing, I expressed a wish that Makana contact Thandi should he not want the observational visits to continue.

**Observation 2**

I arrived at the school on the date as arranged and was met by the Headmistress of the school. She informed me that as Makana was heavily involved in the Imbewu Project he would not be available for further classroom observational visits. I told her that I realised Makana’s dilemma, and that he appeared to be under some strain. I suspected that Makana was reticent about further visits, as he was under-prepared, and was using the Imbewu Project workshops as an excuse. There had been no evidence of forward planning and there had been no portfolio available on the previous visit. Nonetheless, I agreed with the Headmistress when she said that the Provincial Department of Education’s Imbewu Project should receive Makana’s full attention and take priority. No further observational visits would be made unless I was invited by the school. Although I was disappointed at losing one of the case study teachers at this stage of my research, I was not to know that Makana would later contact Thandi to say that he was prepared to have us video record his class.

**Second Phase Classroom Observation**

**Video Recording 1**

Unlike the previous observational visit, Thandi and I found Makana to be welcoming, and willing for us to video record a series of geometry lessons. To my surprise, the class was arranged into groups with either five or six learners in each group, making up five groups. As this was a multi-grade class learners from grades 5, 6, and 7 were distributed amongst the five groups. Each group operated as a family grouping with mixed ages and mixed genders. Makana had prepared an activity card for every group, and he started the lesson by reminding the class of their group roles. Pairs of scissors and a large sheet of dotty paper were also distributed to each group. A variety of polygons had been drawn on the dotty paper in readiness for the lesson. The learner who had been allocated the role of reader read the activity task to the other members of the group. The tasks for the lesson read as follows:
You are given a collection of shapes of figures.

1. Cut out the figures and sort them into groups.
   Make up names for your groups.
2. Compare your groups to others in the class. Discuss.
3. Which two figures would you group together?
   Compare and discuss.

Below were drawn three polygons: Two were irregular quadrilaterals and the third was an irregular hexagon. The hexagon and a quadrilateral were also concave.

4. Why are the four figures below grouped together?

Below were drawn four polygons: a regular quadrilateral (square), a regular hexagon, a regular 8-pointed star, and a regular triangle (equilateral).

5. What difference is there between the figures given in question 3 and those given in this question?
   Describe each of the figures below:

Below were drawn an open quadrilateral, a circle, an irregular trapezium, and an open irregular pentagon.

6. Which two figures would you group together?
   Compare and discuss your answers with others in the class.

There was subdued discussion between members in a group before engaging in the activity. I realised that their entering behaviour would be slow as they were beginning an assignment that was new to them. All discussion in groups took place in Xhosa and when Thandi and Makana spoke to the learners, they addressed them almost exclusively in Xhosa. A learner in each group was allocated the task of cutting out the figures drawn on the dotty paper. I noticed that they took care to cut the shapes accurately. Once all the shapes were cut, the task of sorting began, which gave rise to intense discussion and negotiation. As preliminary sorting was taking place, a recorder using a marker pen recorded the findings of the group. The Tiger group, for example, had placed together a set of closed polygons and recorded the properties as follows:
It have corners
It have angles
Closed shape
Straight sides
Is a flat shape (meaning 2-dimensional)

(Tiger group)

I noticed that another group had attributed the following properties to a set of quadrilaterals:

They have 4 sides.
The sides are equal
They have 4 angles

Included in this set were a trapezium and a parallelogram that clearly did not satisfy the stated criteria that the “sides are equal.” An example of learners confirming their findings occurred when a group was intent on making sure that the regular 8-pointed star comprised 16 angles. The learners counted and recounted three times to be certain that 16 angles was the correct number of angles, which in turn gave the same number of sides and the same number of corners. Whilst the groups were involved in working through their respective tasks the headmistress visited each group and asked them to explain their findings to her. Thandi and I suspected that she was showing solidarity to Makana after the earlier disappointing observational visit, but we were pleased that she expressed such interest in what the learners were doing.

Various learner reports were made, many of them in halting English. If a reporter was not capable of giving the required explanation to the class in English, another learner came forward to complete the report. It was interesting to see that in one of the reports 14 angles was given for the 8-pointed star. The reporter then pointed to a set of cut out shapes that comprised an oval and a circle. The learner told the class, “Those shapes are round.” Underneath the two mounted shapes the group had recorded, “These are circles,” which clearly the oval was not. I expected Makana to intercede and correct these obvious errors but he remained quiet. I was left wondering why Makana was reluctant to correct the learners’ mathematical misconceptions when they arose.
Interview with Makana

Thandi started the interview by saying in Xhosa how we had seen a significant transformation in his teaching strategy with well-prepared activity cards and the learners working in cooperative groups. She enquired whether this was his normal teaching style. Makana responded by saying he now placed learners in groups. Sometimes he placed grades 5, 6, and 7 as a mixed grouping together. If on the other hand, he wanted to work with a specific grade, then he would focus on one particular grade level at a time. Thandi raised the question of Makana speaking to the class in Xhosa whilst he expected the learners to report and write in English. She asked what made the learners sufficiently confident to talk in English. Makana replied that he spent much time with the learners in the oral English lessons clarifying mathematical concepts, through the medium of English. He explained that he gave time in the oral English sessions to correcting learners’ concepts. Thandi said how we had seen learners cutting, sorting and identifying polygons, and she enquired “where next?” Makana explained how his planning included specific exploration of triangles by drawing the diagonal in a rectangle, and later to finding the midpoints of shapes, an activity conducted at the contact session of the intervention. He also intended “constructing patterns” with matchsticks. On the question of correcting learners’ mathematical misconceptions that we had seen earlier in the report back, Thandi suggested to me that a videotaped interview was an inappropriate moment for raising the matter with Makana. We agreed that we should discuss the matter informally with him when he was feeling less under threat. Both of us expressed delight in seeing Makana’s revised teaching strategy.

Video Recording 2

The Identification of Quadrilaterals was the aim of the lesson. A worksheet was distributed to each group. The instruction on the worksheet read as follows:

*What can you tell us about quadrilaterals including their midpoints?*

*Certain aspects make these shapes to be classified as quadrilaterals.*

Provided were drawings of a square, a rhombus, a rectangle, a parallelogram and a trapezium, with the instruction to “give distinct properties” of each quadrilateral. The groups concentrated on finding and recording the properties of one particular quadrilateral, although Vuyokazi, a grade 7 learner made her own individual recording of all the shapes. It was interesting to see a group that was unsure of the
parallelogram requesting the use of a class textbook as a source of reference. The dotty paper provided the learners with the opportunity of drawing their own quadrilaterals and I noticed that they concentrated on the task of accurate cutting.

The Lions group reported their properties of a set of trapeziums to the class:

- **Kinds of Trapezium**
- **Properties**
  - All have 4 sides
  - All have 4 vertices
  - They are quadrilaterals
  - Have no equal sides
  - Have 4 angles
  - No open side
  - A flat shape

(Lions)

The group did not report that a trapezium has a pair of opposite sides parallel. Makana recognised that the group had made a serious omission, and he asked them to revisit the properties of the trapezium again, using the class textbook as a reference. He also asked them if “no equal sides” was always the case. They were to report their findings to the class the next day. Thandi and I were encouraged by the fact that on this occasion Makana did not allow a mathematical misconception to go uncorrected. In our informal discussion with him, he said that previously he had been hesitant to intercede in front of visitors, but now because of our interest and support he felt more comfortable.

The next phase of the lesson required the learners to find the midpoints of the set of quadrilaterals, and later triangles. The worksheet instruction read:

- **Investigate the midpoint shapes of a trapezium and a parallelogram.**
- **Continue to find the midpoint shapes of those midpoint shapes you have just made.**
- **What do you think about the midpoint shapes of any quadrilateral?**
- **What about their areas?**
- **What about midpoint shapes of the triangles? Investigate.**
- **Reflection: What have you learnt from the activity?**
The next group to report had found the properties of the parallelogram, and the midpoints, and they recorded as below:

- *It is a parallelogram*
- *it has 4 corners*
- *4 angles*
- *2 parallel sides*
- *2 long sides are equal*
- *2 short sides are equal*
- *4 sides*
- *the sizes of the angles are not equal*
- *opposite angles are equal and opposite angles are equal (vertically opposite)*
- *in the middle point of a parallelogram you will get a parallelogram*

(Group recorder).

Underneath were drawn two parallelograms, but one figure had the midpoints joined to make another parallelogram inside the original figure. The Kudus group reported their findings on the properties of a rectangle and demonstrated an interesting sequence of quadrilaterals when the midpoints were joined:

- *Properties of rectangle*
- *4 vertices*
- *4 angles those angles are right angles*
- *4 sides*
- *2 opposite sides are equal and 2 of them are equal*
- *If you add those angles you’ve got 360° e.g. 90 x 4 = 360° those angles 90° are equal*
- *If you cut it in half it gives two triangles*
- *In the middle point of rectangle we get rombus*
- *In the midpoint of rombus we get rectangle*

(Kudus).

This list of properties was accompanied by a figure illustrating the quadrilaterals that were made when joining the midpoints of a rectangle. Also included were the measured dimensions for each figure. It was noticeable how much more Makana interacted with the learners as he made a point of going round to each group and speaking to the learners. Of significance was that he engaged with the reporters to help clarify mathematical concepts when they were presenting their findings to the
class. For example, Makana demonstrated to the group that investigated rectangles how a diagonal gave rise to two triangles, and that a rhombus was the name of the shape when the midpoints were joined. He explained to the parallelogram group that the opposite angles were equal in size and the opposite sides were equal in length. He made interesting comparisons between the midpoint results of the rectangle and the parallelogram. He pointed out that the rectangle gave a rectangle, rhombus, rectangle, rhombus, pattern whereas the parallelogram gave a continuous parallelogram pattern. Makana corrected the reporter who insisted on calling the word square, “squaar,” and the reporter who claimed that a trapezium was made up of four 90° angles. These examples were evidence that Makana was willing to intercede on behalf of the learners. I believe Makana became more confident in his teaching style as the learners gained in geometric understanding through the activities. Makana told Thandi and me that he had avoided teaching geometry in the past, as he believed that rural learners were not capable of understanding the geometric concepts involved. He was now enthusiastic about teaching geometry, because of the learners’ interest in the activities and the insights they were developing. We indicated to him that by getting learners to measure, discuss, and re-measure before cutting, was an approach that fostered learner understanding. Although the learners struggled to write correct English, they did attempt to reflect on the midpoint lesson. In answer to the question, “What have you learnt from the activity?” Mani, a learner wrote:

Reflection

I learn that if you make the midpoint first you must find the middle of the line and start to make midpoint. In this activity another one I learn is that when you make the midpoint with out the measuring you must take the opposite corners and add that corners.

(Mani).

From Mani’s last sentence, I assumed that he meant if one counted dots instead of measuring, the opposite corners of the new figure needed to be equal. Trying to express clear meaning in their reflective writing in a second language was difficult for the learners yet they continued to try. Evidence in the learner portfolios indicated that despite the difficulty, learners were prepared to write reflections on their mathematics activities, which showed a level of tenacity and commitment to the task.
"Do you now have more understanding of quadrilaterals and triangles? If no, what is your problem?" were further questions posed for learner reflection. Mani responded with the following observation:

My problem is that when we make this other learners want to see our activity. After that he goes to right your activity in his paper.

Clearly not all learners were comfortable with cooperative participation, which was perceived by Mani as an infringement of his intellectual property right. To outside observers like ourselves it appeared that the learners cooperated well, and it was only through Mani's reflective writing was it possible to detect his concern about his findings being copied by others. Thandi raised the question of individual work with Makana, and he agreed that learners in some cases should be free to work individually.

After completing the investigation on triangles a learner wrote:

I have learnt about triangles and make midpoint and patterns in the triangles. The problem is to count the perimeter of the triangles, and the shape is 2 short.

The learner had found the midpoints of an isosceles triangle by measuring the lengths of the sides. The learner had created a diminishing pattern of similar triangles, but the smallest triangle was not easy to measure accurately because of the size. The learner had also coloured the respective triangles to make a distinctive pattern.

Makana had written on the learner’s recording paper:

Nice. You have done well. Now calculate the perimeter and area. Good work.

Video Recording 3

An activity card prepared by Makana was distributed to each group. The task on the card read as follows:

Fold – cut the paper. Cut diagonal from one angle to another. What shape do you have to find. Work in pairs and discuss it. If you then with the larger group cut it again. Compare the shape you have found in your first cutting and the second one. Look for similarities and differences.

Scissors, and paper for folding and cutting, were distributed to each group. Some groups started out by converting through folding and cutting their rectangle of paper
into a square. Another group measured then cut out a parallelogram as the starting figure. The Cheetahs group cut along the diagonals of a square to produce four triangles. As they were not sure of the kind of triangles that resulted, they referred to the class textbooks for clarification. They identified the relevant figure and description for an isosceles triangle, which they incorporated into their recording of the properties of triangles. The first learner to report to the class announced "properties of isosceles" before going onto saying:

"Two sides are equal. One side is long. A triangle with two equal sides is called isosceles triangle," and Makana repeated the words "isosceles triangle."

"Why is it isosceles triangle?" asked Makana.

"Two sides are equal" responded a learner from the class.

Makana for emphasis reiterated that in an isosceles triangle, two sides are equal, and the class listened attentively.

"The sum of sides of isosceles sides is 124," said the reporter.

Makana asked, "124 what?"

The class responded, "124 millimetres," and Makana repeated "124 millimetres."

A reporter from another group made the next report. His group had studied the square. He reported as follows:

2 equal triangles, (pointing to the 2 triangles made from cutting a diagonal)
4 triangles, (pointing to the 4 triangles made from cutting 2 diagonals)
3 corners one 1 triangle. (pointing to the corners on the large triangle)
3 sides on 1 triangle, (pointing to the sides of the large triangle)
3 angles, (pointing to the angles of the large triangle)
It is isosceles triangle because 2 sides are equal, (pointing to the equal sides of the large triangle).

We found a right angle and 2 acute angles.
90°, add 45, add 45 and you get 180° total.

Makana gave a bilingual explanation using the small isosceles triangle to illustrate. He said, "Because two angles are 45°, this one is half this one", pointing to the right angle, and folding one 45° angle exactly onto the other. Makana's accurate explanation reinforced the discoveries made by the group.
A member from the group that started out with the parallelogram came to the front to present their findings. He referred to the figures clearly displayed on the recording chart as he reported to the class. He started out by saying that he was reporting on the parallelogram, which caused some surprise.

“What is a parallelogram?” asked Thandi.

The reporter replied, “A pair of opposite sides parallel, ” and then presented the groups’ findings:

- 3 sharp corners in a triangle (pointing to one of the two triangles that had been made by cutting along the diagonal)
- 2 acute angles
- 2 long sides
- 1 short side

*If you cut a parallelogram in the diagonal line you get 2 triangles.*

*The midpoint of a triangle is a triangle.* (pointing to a smaller triangle)

*The midpoint of a triangle is a small triangle.* (pointing to the smallest triangle)

*The outside triangles are equal.* (pointing to the triangles created by joining the midpoints).

*2 angles are equal, 1 is not equal.*

Thandi enquired about the two long sides of the triangle. The question was misleading, as the triangle was scalene and not isosceles, as it looked from the cut out paper. The fact that the triangle was scalene was not commented on. Had the scalene property of the triangle been pointed out perhaps the consequent misconception might not have occurred. The reporter proceeded to fold one angle on top of the other angle to show that they were equal in size. The angles were nearly equal in size, but they were not equal, as the figure was not a rhombus, and the consequent triangle was not isosceles. The reporter was incorrect in saying two angles were equal, although they looked almost the same. The previous group had started out with a square and had created isosceles triangles as a result. Perhaps their findings wrongly influenced the parallelogram group. Makana did not correct the misconception. In adopting a teaching style that encouraged learners to formulate their own solutions, Makana was placed at some risk when it came to mathematical content knowledge. To anticipate the range of mathematics concepts presented by the various groups was an expectation for which Makana was not fully prepared. If
Makana was unsure of the correctness of a finding, further checking was required. I believe mathematics understanding was a shared responsibility that included the learners and the educator, but it was up to the educator to lead the challenge of eliminating mathematical misconceptions.

**Video Recording 4**

The lesson started with a representative from each group selecting at random the name of a shape to be constructed and used in a floor-tiling pattern. The five shapes selected were isosceles triangle, rectangle, rhombus, parallelogram and square. Scissors, paper and an assignment card were distributed to each group. The assignment read as follows:

*Project: Tiling*
*The real life situation:*
*cut difference shapes*
*make tile*
*design your best floor*

The instructions were cryptic, and there was no preliminary explanation from Makana clarifying the purpose of the task for the learners. The learners waited quietly in anticipation not knowing how to start. Makana and Thandi spoke in Xhosa to each group in turn to explain the requirements of the task. They indicated that each group needed to construct a set of shapes based on the name of the shape that they had selected. The task of constructing a set of shapes involved the learners in measuring, discussing, and cutting. The group that made isosceles triangles tessellated them to make a regular hexagon. The same group went on to make a set of right-angled triangles that fitted together to make a large rectangle. The group that made parallelograms tried a variety of tiling patterns that included a chevron design, firstly with the short sides, and then the long sides making a line of symmetry. The group finally decided to display a pattern with only long sides touching to form a shape showing five clear parallel lines. The rectangle group decorated the interior of each rectangle with a symmetrically placed rectangle, and noticed that one of the rectangles was asymmetrical. They corrected the error. The squares group decorated the perimeter of each square with a design. The rhombus group joined the midpoints of their shape and drew a diminishing series of rhombuses on each of the tiles. The
learners understood that the assignment was to make a set of floor tiles, and so they proceeded to arrange their groups’ tiles on the classroom floor. The arrangements were transferred to the wall for easier comparative viewing of the different tiling patterns. The learners were pleased with the variety of tiling patterns they had constructed, as were the headmistress and the observers. The lesson had started slowly and ended with an aesthetically pleasing result. My colleague Thandi reflected on the lesson as follows:

After the design, the learners were challenged to place the tiles on the floor and see if they will join nicely. Although the learners enjoyed the activity, the teacher did not use the prevailing opportunities to introduce new concepts like tessellation and symmetry.

At the initial stage of each new activity, I noticed that learners seemed to be quiet or unsure of what is expected of them. But immediately when they understood, they worked fast towards solution.

(Thandi).

As a fellow observer, I agreed with Thandi that Makana had missed the opportunity of capitalising on the geometric concepts that emerged from these worthwhile activities. The introduction of the concepts of tessellation and symmetry would have given mathematical focus to the tiling activities. Inadequate mathematical content knowledge might have been the reason for Makana’s reticence. The second issue raised by Thandi related to the slow start made by the learners in most of the lessons we had observed together. In most authentic problem solving situations, I believe there is a period of “entering,” or “working into” the problem. Their slow start to the activities indicated to me that the task was “new,” and the learners had not been pre-prepared by their teacher for the benefit of the camera. I also believe that by speaking to the groups in their mother tongue, Makana and Thandi helped the learners realise the nature of the assignment.

The reports presented by the learners during the week were a ‘mathematical reflection’ on the tasks of the day. There were also written reflections required of the learners. The weeks’ activities provided an opportunity for learners to think back on what achievements had been accomplished in the mathematics class. The nature of the task was posed in the following manner:
Write a letter to your mom, dady, aunt or sister and tell her or him about your polygon activities, and what did you learn and how did you feel about those activities.

The letter from Vuyokazi follows:

Dear Dady
I write this letter I want tell you about those activities we do last week. On Monday we do shape and sort them in to groups and we discuss why we sort them like that. It was easy but we were afraid because we were starting. It was the first day and we answer some questions and report to other groups. On Tuesday we do activity on polygons. The topic was to say what are polygons. There were some questions we answered. It was nice. On Thursday we do activity number 3. We do different types of shape and we do midpoints and more properties. On the next week on Monday we make activity 4. We fold the paper diagonal and we get diagonal and name that triangle and make properties. Activity 5 we do matchsticks and make findings and make rule. Activity 6 we do types of shape and make as the tiles of floor... That is the end of my letter

the writer
Vuyokazi

The writer of the letter, Vuyokazi, showed a clear appreciation of the sequence of activities related to the geometric topic. The activities described in the letter followed the planning that Makana had indicated in the earlier interview with Thandi. The mathematics activities provided real life experiences on which Vuyokazi based her writing. In her letter, she conveyed some of the feelings she experienced in doing the activities even though she struggled with her English expression and spelling. By getting Vuyokazi to think back on the series of activities, and to relate how she had been developing geometric concepts, she was engaging in a reflective process. The letter indicated that Vuyokazi, despite her poor English, was becoming reflexive and by becoming reflexive, she was demonstrating an important procedure described in the Constructivist Learning Model (Yager, 1991).
Of interest in the sequence of lessons was the progress made by the learners in developing geometric concepts. I believe that learner concept development was a result of Makana’s teaching approach. Instead of a transmission style of teaching observed in the First Phase, he facilitated active participation with the learners discussing, measuring, cutting and reporting in an interactive learning environment. The video recordings did show learner mathematical misconceptions, and despite these shortcomings, the manner in which the learners actively engaged in tasks of a kind suggested by van Hiele (1999), enhanced their geometric concept development.

Panel Review of Videotaped Lesson

Makana

The lesson selected for review, in the time available, was Video Recording 4 described above. This lesson was selected for review because it was the final in the series and showed the learners taking responsibility for solving different tasks in their own way. Arising out of concerns raised at the previous panel meeting, I revised the summarised Constructivist Learning Model (CLM) statements to read as follows:

Constructivist Learning Model (CLM)

We are looking for evidence of these procedures in constructivist learning environment:

Recognising and building on learners’ prior knowledge

A developed interactive learning environment

Promoting learner collaboration

Encouraging learners to challenge and negotiate meaning

Encouraging learners to reflect on, and refine ideas

Encouraging learners to create their own solution strategies

The consultant panel of observers of Thandi, Marc, and Sue, agreed that the revised wording of the CLM principles was now clearer to them for the purpose of identifying themes emerging from the video recorded lessons. Marc commented on the similarities of teaching strategies of Makana and Lulama when he said, “I was amazed at the similarities between them, (Makana and Lulama).

“Me too,” confirmed Sue.

“The strategies he adopts are similar to hers.” Marc continued,
“You can see RUMEP all over the place, even implicitly,” and he cited the examples of learners working collaboratively in groups and coming to the front of the class to report on behalf of the group.

“But it all makes sense, which in itself is something interesting,” Marc stated in relation to the intervention.

The panel discussed the merits of immersing RUMEP’s key teachers in a constructivist learning model, which they were expected to absorb, and then implement the in their own classrooms. The panel maintained that Lulama and Makana were implementing specific aspects of a constructivist learning model, although Sue argued:

> What we are seeing is only part of the model. We are seeing pupil interaction not teacher interaction. I don’t think any of us here are advocating a radical constructivist approach where we let them get on with it and discover for themselves, and we don’t scaffold and we don’t frame the work.

The debate that followed questioned the extent to which Makana was driving the learning process in the observed lesson. Sue argued that Makana’s lesson was teacher driven only insofar, “He had chosen what he wanted the learners to achieve and how he wanted them to learn through active participation, and through an interactive process.” The consultant panel of observers concluded that they had seen evidence of some of the six principles stated in the reworded CLM, notably learner collaboration in an interactive learning environment with learners actively discussing and refining their ideas as a consequence.

**Case Study: Ruth**

**Vignette**

Ruth was a Foundation Phase teacher of 8 years experience. She taught 36 Grade 2 black and coloured learners made up of 20 girls and 16 boys whose ages ranged from 7 to 9 years. The learners’ mother tongue was either Afrikaans or Xhosa but the parents of Ruth’s learners elected to have them taught in English. In all other classes, the medium of instruction was through Afrikaans, the dominant language in the school. The school was centrally placed in an urban area and most learners walked to school or were dropped by parents on their way to work by car. The school drew learners from mixed socio-economic backgrounds, but many learners
come from extremely poor homes. All the learners wore a school uniform. The classroom was spacious with posters on the walls, which brightened the room. Ruth had access to teaching materials and made use of resources to enhance her mathematics lessons. The school had 650 learners and 23 educators from grade 1 to grade 7. The majority of the teachers were mother tongue Afrikaans speakers.

First Phase Classroom Observations

Observation 1

Ruth introduced the lesson by referring to the date saying that the class would deal with the number 4, as it was the 4th August. The learners wrote down the number as a symbol and as a word. Ruth asked the class what position the number occupied. Several hands went up and one child gave the answer “fourth.” Ruth wrote the word on the chalkboard and the learners copied it into their workbooks. The next activity required the learners to think of a sum where the answer must be 4. Many learners volunteered to give a solution. One child came up with 2 + 2 = 4, and 8 – 4 = 4. This was written on the chalkboard and the class copied the learner’s solution into their books. Ruth then asked, “what is another word for plus?” Several hands went up and the learners shouted in unison “add.” She asked for another word for minus and the learners replied “take-away.” Ruth enquired whether 4 was odd or even, and then why they thought 4 was even. A learner replied, “Because yesterday it was odd.” Another learner said it was part of an even pattern 2, 4, 6, 8. The learners wrote ‘even’ in their books and completed the activity by making a picture pattern underneath.

For the next activity, each learner was given a rectangle of plain paper made from a half-sheet of foolscap (A4). The learners were shown how to fold the rectangle to make a square. Although Ruth used the word rectangle, she did not mention the word ‘square,’ but went around to each group and asked them what shape had they made from the rectangle. The groups said that they thought they had made squares, and when asked why they replied, “they have the same lengths” and “they have the same sides.” Ruth followed up the with the question, “show me why the sides are the same?” A learner folded the two opposite sides of the square to show that they were the same in length. Another learner used his finger width to measure each side and found that each side had the same number of finger counts. In doing the paper
folding the learners were willing to help each other. For example, when folding the rectangle to make a square, one child would hold the straight sides together while the other made the crease. To close the lesson groups were called on to tell the class what they had found. Ruth accepted sensitively, her learners’ unexpected findings. Examples included one learner who said he had made a diamond, and another learner who said he had two triangles. Ruth made use of these learners by calling on them to show others in the class what they meant. During the report back many of the learners lost concentration, and Ruth quickly ended the lesson before the start of a break period.

A significant feature of the lesson was Ruth’s correct mathematical vocabulary and an emphasis placed by her on language. Ruth believed that she must be a good language role model to the learners, and she believed that she must encourage the learners to communicate their thoughts in correctly spoken English. I noticed that learner discourse in groups was either in Afrikaans, Xhosa or in bilingual English and Afrikaans, but all communication with Ruth, and all writing was carried out in English.

**Observation 2**

The learners sat as a whole class on the carpet at the rear of the classroom and Ruth sat on a small chair in front so that she was able to see each learner. In front of the learners mounted on a board was a large 100-grid from the kit. Both the large and the small 100-grids are made from bright yellow laminated card that can be written on using non-permanent felt-tipped pens. The learners were counting-on in 2’s to 20, by saying: 4 plus 2 is 6, 6 plus 2 is 8, and at the same time a learner circled the 2’s pattern on the large yellow grid for all to see. The procedure was started again with another learner standing up and counting-on from 3 to 6, and from 6 to 9 and then from 9 to 12, circling the multiples of three. The child sat down and the learners counted up to 21 without her help. Ruth asked the learners what pattern they were making and they replied “plus three”. One of the learners said there was an even-odd, even-odd pattern. The learners were asked what ‘pattern’ meant and their reply was “over and over.” The learners then moved to their desks where they were requested to use the August calendar. They were asked to find the day and the date and draw the weather for today, which was cold and windy. The learners worked with the
number 10, similar in manner to that reported in the previous observation, and two individuals were called on to say what they had worked out.

The next activity was to revise the properties of a square. Although the learners were sitting in their groups, the lesson was exclusively whole class discussion lead by Ruth, with individual learners answering questions. The first question asked by Ruth was “how do you know something is a square?” A few learners said, “equal sides.” Ruth did not let this answer pass without further probing, and it took some time to draw out that all four sides needed to be equal. She then asked the class to show how they could prove that the sides of the square were equal and did the ‘corners’ need to be square. Two learners came up to the chalkboard and measured the opposite sides of the square with their fingers to show that the sides were equal. They did not, however, measure the adjacent sides, which prompted Ruth to ask about the sides they had not measured. They measured the adjacent sides but they did not link the measurement they obtained with the previous measurement. Ruth tried to get them to show that all sides were equal, but when she asked, “show me the equal sides,” they pointed to the opposite sides. When she pointed to the adjacent sides and asked if they were equal, the learners responded hesitantly, and were not convinced that this was the case. Whilst working in groups the learners attempted to fold the adjacent sides onto each other, but needed help from Ruth and from me to accomplish the task. Discussion with two learners in particular took place in an effort to confirm that adjacent sides were equal. Many of the other learners who were not the focus of the discussion became restless and lapsed into their own private conversations. Ruth looked exhausted and ended the lesson in some frustration.

Discussion with Ruth indicated that the lesson did not go as planned. She felt that she had “lost” the majority of the learners. We discussed the possibility that she might have moved too quickly into the abstract property of shapes. We agreed that she needed to give the learners more time to explore the properties of shapes in different contexts, all the while getting learners to tell her what they had found out for themselves. We talked about the specific properties of opposite and adjacent sides, and she expressed a concern that she was not teaching “proper” geometry. I tried to reassure her that learners did not need to be pushed in to recognising the formal properties of different shapes, and it was satisfactory if they could identify a
rectangle and a square, and be able to tell her as much about them as they could. To help remedy the situation Ruth suggested that during the daily activities that involved “the number of the day”, she would get the groups to come up with their findings as a team, rather than asking one or two individuals from the whole class. She seemed enthusiastic to try this approach.

Observation 3
The learners worked in groups and their task was to make up sums with the answer 17. As I walked around, I noticed learners used different solution strategies. Two girls in one group each used fingers to count on and another group drew dots to help them count. Most learners used the 100-grid as a paper 100-grid had been pasted in each learner’s book. In another group, learners made use of the number line. There was much discussion in each group with English being the medium of conversation. In some cases learners preferred to work on their own, but in specific groups Ruth asked learners to help each other by saying, “the more you help each other, the more sums you will get and then you will get a star.” I noticed that all but one group remained involved in the activity to the very last. Ruth posed the following question to the class as a whole, “was today’s work difficult and why?” A learner responded, “it wasn’t difficult because we worked together,” and another said “it was difficult because we had to think.” A learner from the group that was less focused said, “we didn’t work together in our group.” From this discussion, Ruth stressed the importance of working together and then asked each group for one sum for the rest of the class to hear. A learner gave $5 + 12 = 17$. Ruth asked the class if they had managed to find this sum, and did it make sense to them. She called on a pair to check the sum, which they did by counting on from 5, using the hands of both children. Another group gave the sum $18 - 1 = 17$. Again, the class was asked to check to see if they had recorded the sum. They were also asked: “what else can I say instead of minus 1?” The answer came back from the class “take away”. Ruth recorded on the chalkboard several number combinations that gave the answer 17, and asked the class to check and see if they had the sum. Each sum written down was checked. $20 - 3 = 17$ was one of the sums. The group that checked it used the large 100-grid to locate 20 and then counted back 3. Ruth concluded the activity by asking whether 17 was odd or even, and asked a learner to circle on the 100 grid the pattern
of odd numbers that included 17. The learner made the pattern: 11, 13, 15, 17, and 19.

The next part of the lesson required learners to take on particular roles within the groups. Each group had to choose a leader, a recorder, a collector and two reporters. Before selecting, the class was asked to reiterate the duties of each role so that everyone was aware of the respective roles. For example, Ruth asked what must the leader do. The class responded, “The leader must make everybody work together.” Conflict erupted in all the groups over the selection of roles. Ruth did not intervene in disputes but always referred them back to the group. Individuals in a group would rush to her and ask her to make them leader, or not make them a reporter. It seemed that learners wanted to be elected the leader of the group, and the least preferred role was that of reporter. The lesson then moved onto making rectangles using boxes. The collectors in each group had to get scissors and boxes from the teacher’s desk. Ruth used empty cornflakes boxes for the activity. Each group was required to cut out the rectangles that made up the faces of the box. The rectangles had to be checked to make sure they were rectangles by using matchsticks or their fingers to measure the sides. The learners within in their groups sorted different sized rectangles and called them large, medium and small. Ruth then called for all the large rectangles to be brought to the front, followed by the medium and the small rectangles. Once the class was settled she asked a learner to tell the class what she had found out about her box. The girl said it was hard to cut, and she needed her friend to help her, and “a box has lots of rectangles.”

Ruth’s comments after the lesson were that she felt that she had negotiated a successful lesson in that the learners were actively involved at each stage of the lesson. She was encouraged with the manner in which learners negotiated meaning with other members of the group, and the fact that they were challenged by the number activity itself. The level of sustained participation even by learners who were normally easily distracted pleased her. Ruth expressed particular pleasure over the rectangles activity.
Observation 4
The calendar was used to find the day and there was a discussion on the weather. A weather chart was filled in. The number of the day was 7. A learner was asked to show the rest of the class who was sitting in 7th position. She did this by counting aloud and other learners joined in. The task of the day was to write out sums where the answer would be 7. Ruth observed each group in turn to determine the level of cooperative learning taking place. She used a checklist with the categories “good, satisfactory, and needs help.” The learners worked mainly in pairs, even though they were arranged in groups of between four and six. Most pairs used “counting on” strategies with fingers being used more frequently than the 100-grid and the number line. Each pair produced no less than 16 different number combinations. One pair had 24, all of which were correct. The groups were called on to report their findings, and learners were eager to show the others what they had found. The activity ended with Ruth asking whether 7 were odd or even. “Odd” came the reply, “because you can’t share 7 equally, one group would get 3, the other 4.” Ruth asked that the learner to share this information with the whole class.

The next activity was a geometric one. Matchsticks were distributed to all the groups with the instruction to work together to make squares. In visiting each group Ruth asked how they would know if they had made a square. The learners said all the sides would be the same. The learners were involved in the activity with every one sticking matches. There was considerable discussion on how long the sides should be. One group tried to make a square as long as a double A4 page. They discovered that they could not because they could only make a rectangle. Ruth provided guidance when she asked if what they had made was a square. The learners knew that it was a rectangle and not a square, but did not know how to “fix” what they had done. Ruth helped them make one square using four matchsticks, and said that the square did not have to take up the whole piece of paper. They understood Ruth’s demonstration, and set about making a square. A group with only 3 learners was most cooperative and produced a successful result. One group with 6 learners only allowed 4 learners to actively participate. The other two played by themselves with their own matches, not interested in the activities or discussion of the others in the group. Some groups finished before others and became disruptive. The reporting was done in pairs. The learners found listening to the reporters difficult and were generally restless. Some of
the reporters only spoke to the teacher, and Ruth had to ask them to speak to the class, which they were reluctant to do. One particular group indicated that they made a group of 4 small equal-sized squares inside a big square.

After the lesson, we discussed features that emerged from the activities. She said that in all her mathematics lessons she insisted that learners build their own meaning for themselves, and not wait to be told a solution by the teacher. She said she found this difficult because learners expected her to always provide a correct answer. Of great pleasure to her was when a group surprised her, as in the last group’s square innovation. We also discussed why she kept to the routine of always starting the lesson by analysing the date, for example. Ruth said that she was conscious of the fact that many of the learners came from extremely poor economic homes where there was little stimulation and limited stability in the lives of these children. She believed that by starting the day with a regular activity to which the learners could relate and not feel threatened, they would settle better into the activities of the day.

Observation 5

The learners worked in pairs to find as many sums as they could with an answer of 14. Once again, a variety of ‘counting on’ strategies were used. All the learners were involved in the activity, bustling off to use the number line or large yellow 100-grid, drawing dots or using their fingers. It appeared that more learners were using the number grid than previously. Ruth went to each group asking questions and giving encouragement. One pair had written $16 - 2 = 14$ and $16 - 1 = 14$. She asked them to check which one was correct. The learners discussed the question using the number line, counting back from 16. They “discovered” that $16 - 1$ was 15 and crossed out the incorrect sum.

One group had written the following:

\[
\begin{align*}
15 - 1 & = 14 \\
16 - 2 & = 14 \\
17 - 3 & = 14 \\
18 - 4 & = 14 \\
19 - 5 & = 14 \\
20 - 6 & = 14 \ldots \\
25 - 11 & = 14
\end{align*}
\]
The group said they had $15 - 1 = 14$, then the next step had to be $16 - 2 = 14$. They were able to see a pattern but could not explain the pattern to others. After the lesson, I discussed the importance of pattern building with Ruth. She said that she did some sort of pattern building everyday in the form of clapping patterns, drawing patterns and number patterns. I noticed that during a lesson, she would clap a pattern and the learners would join in until she had the attention of the entire class before continuing. The clapping pattern served as an effective non-verbal management strategy. We discussed the question of pattern recognition and how difficult it was for some learners to “see” number patterns. We agreed that the only way was for learners to build a pattern on a daily basis, and for them to describe the pattern they had made in their own words.

The next stage in the lesson was “using only 2’s to make 14.” After considerable negotiation, the groups established that using many twos was an option, and they busied themselves by adding 2’s. A girl was called on to report on the work achieved by her group. She wrote on the chalkboard:

\[ 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14. \]

Ruth asked her “how many 2’s did you use?”

She replied, “7.” Ruth asked, “What do you see?”

“A pattern,” she said.

“What is the pattern?”

“Plus 2” she said, and the class recited in unison “plus 2”.

Ruth expanded the concept of adding 2’s by writing on the chalkboard and saying:

“2 plus 2 plus 2 ... equals 14.

We can say 7 2’s are 14. Say after me 7 2’s are 14.”

The learners repeated this after her. When Ruth asked whether 14 were odd or even, a learner responded, “even, because we can share 14 between two groups the same.”

The learners copied the chalkboard summary of learner contributions into their workbooks.

Ruth and I discussed the introduction of pattern recognition into the Foundation Phase Curriculum. She said the revised curriculum required learners to copy and expand patterns and to describe such patterns informally. According to the National Curriculum Statement (2001), learners at this level were also expected to create their
own number sequence at least up to 200. Ruth believed pattern building created
lively opportunities for teachers and learners, but she said other Foundation Phase
teachers in the school tended to avoid pattern building in their classroom activities,
and they expressed a lack of confidence in their learners’ ability to cope with the
such activities. Ruth and I discussed a quote from Kerslake (1994) when she claimed,
“... looking for or finding pattern is not mathematics but explaining pattern is”
(p.24). We agreed that at this level looking for pattern was fundamental because it
was not possible to explain a pattern before it was found.

Review of Ruth’s Classroom Observations

Becoming reflective

Ruth expressed an interest in our discussion on developing geometric concepts. She
said that she was concerned that she was not getting her learners to the ‘right’ level
in geometry. She felt she needed to ‘push’ them into identifying properties of shapes
once they had had experience with drawing, cutting, and exploring everyday shapes.
She wanted to move from playing with shapes to defining them. She expressed
concern that she was not being effective if the learners could not identify different
shapes without visual help. In discussing the van Hiele (1999) article on levels of
geometry, Ruth said that it helped her understand why such exposure was important
to learners for the further development of mathematical thinking. She quoted van
Hiele (1999) when he said, “We must provide teaching that is appropriate to the level
of children’s thinking” (p.311). We discussed the type of activities most suited to van
Hiele’s levels of geometric thinking at the visual, descriptive, and informal deductive
levels. At the visual level, shapes were judged by their appearance. For example, a
rectangle looked like a box. At the descriptive level, shapes had properties. An
equilateral triangle, for example, had three equal sides and three equal angles, and
both line and rotational symmetry. At the highest level, the informal deductive level,
properties were deduced from one another; one property preceded or followed
another property. Ruth said that she now recognised the importance of reflecting on
learner activities, and to use reflection as a vehicle for planning activities that
developed a learner’s level of thinking aimed particularly at the visual and
descriptive levels, for her age group of learners. She said that she believed that many
of the learners were just getting to the descriptive level, and she was aware that she
was trying to push them into the informal deductive level when they were not yet ready for it. Her reading of the van Hiele (1999) article followed by our discussion had caused her to reflect on her approach to developing geometric thinking. She said she now had a clearer idea for planning pace and progression at a level that was more appropriate to the learners’ experience and development.

Concerning reflection, Ruth maintained that ‘reflection’ was the single most significant aspect to influence her classroom practice and the process had become a daily routine, and an ongoing story in her portfolio. In response to the question, “how do you think reflection has helped you as a teacher and as a person?” Ruth wrote:

As a teacher, I am more aware of what I do and how I teach. I look at and think critically about my teaching and the way my learners find meaning and understanding in what I do. I realise that my way of teaching has an impact on the way they learn and because their learning is important to me, I try to give of my best in my teaching so that they ultimately become high achievers in their learning. Reflection has helped me to always be aware of how I can improve. I constantly think of ideas or methods of getting the necessary knowledge, skills and attitudes across to the learners in ways that are meaningful to them. I have also become aware of my different learners abilities and try to plan and prepare my lessons with this in mind. As a person I have learnt to think about what I do; I have become more aware of myself and my abilities; I have learnt to be more tolerant of others and what they have to offer; to listen to other people’s ideas; I think I have gained a bit more self-confidence. I am more open to try out new ideas and more willing to share ideas with others.

(Ruth)

The essence of reflective practice could be described as making explicit teaching activities so that they became the object of critical scrutiny. It has been argued that through such critical enquiry by educators, teaching developed (Schon, 1983). Ruth mentioned her commitment to thinking critically about her teaching and reflecting on her teaching was a way of making her aware of how she taught. She believed it was a method of self-assessment, and without reflection she would be “teaching in the dark” (interview), not knowing if she was effective and how she should plan her
teaching. As a reflective teacher, Ruth not only asked questions routinely and deliberately, she used the answers to these questions to guide her classroom practices so that she became more aware and more effective.

When questioned about the marked improvement in the selection and pacing in her lessons, Ruth said that she was developing a growing belief in her own ability, and a strong desire to do the best she could for her learners. She said that ongoing reflection had caused her to give greater thought to her lesson planning. She wrote, “I have become more aware of my teaching, and am able to do thorough preparation and planning of lessons. I have become a reflective educator and my teaching has improved because of this.” (Questionnaire). Ruth’s practice gave rise to characteristics that Mulholland and Wallace (2001) described as belief in ‘self-efficacy’, when they stated, “...people's beliefs in their own ability to produce desired effects by their actions” (p.245). If, as Mulholland and Wallace (2001) asserted, that a strong self-efficacy belief was linked to increased learner achievement and desirable teacher behaviour, then professional activities should aim to enhance a belief in teacher self-efficacy. Mathematics had been identified as a subject in which a low perceived teacher efficacy impacted negatively on student results. Despite efforts made by the National Department of Education to raise the profile of mathematics teaching, results had not improved in respect to the number of black matriculating students passing at the higher grade (Business Day, 2002). Ruth’s commitment to mathematics teaching was seen in contrast to the general position, and I needed to analyse her lived experiences to determine her sources of self-efficacy, in an effort to highlight ways in which the key teacher intervention might assist other teachers develop these beliefs.

*Negotiating meaning*

Sparks-Langer & Colton (1991) described the differences between novice and expert teachers’ interpretations of classroom events. They maintained that experts had rich, deep schemata (Piaget) to draw upon when making a decision. They argued that schemata did not automatically appear in the teacher’s mind but they were built up through experience. I have argued earlier that constructivist theory supported the notion that individuals were constantly building meaning for themselves in the contexts in which they found themselves and that ‘context’ influenced learner
development through "situated learning" (Brown, Collins & Duguid, 1989; Lave, 1988; Lave & Wenger, 1991). As noted in chapter two, the situated learning researchers suggested that knowledge was constructed through interaction between the individual and the context surrounding the problem. Instead of memorised rules or algorithms, Ruth anchored learners’ knowledge and experience in rich educational contexts. She was aware of the disadvantaged background from which her learners were drawn, and she was conscious of the need to provide rich educational contexts in which they could construct meaning for themselves. I noticed that in all the observed lessons, learners were provided with opportunities to engage with appropriate resources to aid their meaning building. Added to this, each lesson was structured around extensive discourse. Learners were regularly asked to clarify or expand on a statement in order to build understanding, and Ruth herself consistently facilitated language as a tool for building meaning.

Language and learner collaboration
In many ways, the demographic makeup of the class was atypical in that the learners comprised Afrikaans and Xhosa mother tongue speakers, and yet all communication was carried out through the medium of English. National policy before democracy was for all instruction to take place in mother tongue in the first three years of schooling, with a gradual transfer to a bilingual approach in the then official languages of English and Afrikaans. There are now eleven official languages, which allow instruction to be carried out in any of these languages from day one of a child attending school. The School Governing Body in Ruth’s school opted to have one class of learners entering school taught solely in English. The other entry class was taught in Afrikaans. The learners in Ruth’s class were only in their second year of English instruction, which explained why they felt more comfortable speaking Xhosa or Afrikaans when conversing with each other in groups and were reluctant giving verbal reports in English. All discourse between Ruth and her learners took place in English, as well as all written recording. Ruth had made a firm decision not to use Afrikaans to clarify concepts because she believed that within the context of the classroom learners must build understanding for themselves through English. At home, most of the learners would speak very little English despite the fact that their parents had elected to have them taught in English in a mainly Afrikaans speaking school.
Ruth encouraged cooperative learning in groups whenever learners were given tasks to complete. The learners worked more efficiently in pairs and tended to lose concentration when not directly engaged in an activity. Classroom activities were structured around cooperative group or pair work. Not every member of a group collaborated well as some learners told other learners in the group “not to copy.” Ruth was relaxed and encouraging as she attempted to ‘scaffold’ learner’s understanding through probing questions and by constantly getting the learners to explain their own solution strategies. As I observed Ruth interacting with her learners, I was aware that scaffolding served to support the learners in new and more challenging tasks that could not have been achieved without her sensitive assistance. Scaffolding of the kind observed in Ruth’s classroom was ‘mediated’ through language interactions (Vygotsky, cited in Wertsch, 1991). The claim here was that language was a tool of immense influence in the development of cognitive functioning. In observing Ruth, I was aware of the role discourse played in her classroom, both in peer group discussion and in dialogue between herself and her learners. Scaffolding took place in the context of social interaction either with a ‘sensitive expert’ (Ruth) or in a peer group with individual learners’ influencing the pace and direction of the interaction as well as the way the mathematics task was carried out. I believe Ruth encouraged collaborative learning as a means of stimulating language interaction in an effort to develop in her learners new varieties of memory and to create new processes of thought.

Second Phase Classroom Observation

Video Recording 1

The video recordings of Ruth’s class did not take place as planned. I had to delay the Second Phase visits because Ruth was absent from school due to ill health. The class that I observed in the First Phase moved up to the next grade on completion of Grade 2, which meant that I had to videotape a fresh intake of Grade 2 learners who were beginning a new school year. Having to observe Ruth with a different group of learners was unexpected but was not detrimental to the overall research design. The sequence of lessons captured on videotape was carried out shortly after the start of a new school year.
The lesson started outside with learners on the school field getting into groups of two on instruction from Ruth. A learner counted aloud the number of groups, which amounted to 17. The learners returned to the classroom for the continuation of the lesson that focused on the building up of equal sized groups. Before starting the activity, Ruth clarified with the learners the respective roles of the leader, the collector, and the reporter. The learners were required to make their own decisions in respect to their role within a group, and when an individual approached Ruth to make a decision on the group’s behalf, she declined. The collectors then gathered a set of strings for each group, and counters (bottle tops) were distributed. The first task was to make 4 groups using the pieces of string. The next task was to make 4 groups of 2 counters each. Some learners cooperated and worked together and others worked in pairs or individually. Ruth visited each group in turn and encouraged them to collaborate. She told a group:

“You are supposed to work together, not alone. If you work alone it won’t work.”

Learners at one table made 2 groups of 4, and not 4 groups of 2 as requested. In most cases, the learners made circles of string into which they placed the counters, whilst some learners threaded the counters onto open lengths of string. The next step was to make 5 groups of 2 using the counters.

As they proceeded, the learners carefully recorded their findings in their books as follows:

\[ 2 + 2 + 2 + 2 = 8 \]
\[ 4 \text{ groups of } 2 = 8 \]
\[ 4 \times 2 = 8 \]

and,

\[ 2 + 2 + 2 + 2 + 2 = 10 \]
\[ 5 \text{ groups of } 2 = 10 \]
\[ 5 \times 2 = 10 \]

As a summary, Ruth asked a series of questions related to the day’s tasks:

“What number were we working with today?”

“2” came the response.

“What were we doing?”

“We were counting 2, 2, 2...”

“If we were doing that every time, what do we call that?”

“We were doing a pattern.”
“What pattern were we doing?”
“Plus 2, plus 2, plus 2.”
Ruth concluded the lesson by saying,
“Learn your plus 2 pattern at home.”

After seeing this lesson, I was struck by Ruth’s consistent teaching style. Ruth had sustained a similar teaching style in all the lessons I observed. Of particular interest was the emphasis she placed on verbal interaction with learners that she believed was beneficial to her bilingual learners. The video recording showed Ruth participated by speaking to the learners throughout the lesson, and by drawing on the learners’ contributions, and this was particularly apparent in the end-of-lesson summary. Ruth believed that she must scaffold the learners thinking by constantly interacting with them and she worked hard to engage the learners and keep their attention without pushing them too fast. In discussing these issues with Ruth, she said that she strived to pace the lesson according to the needs of the learners. She perceived that she must constantly interact with the learners by speaking only in English. Ruth believed that learner conversations conducted in mother tongue, (related to the tasks in hand), assisted concept development and she was tolerant of these discussions taking place during the lesson. Another feature was her pacing through each phase of the lesson, driven mostly by her, but influenced significantly by the learners’ contributions.

Ruth’s consistent standard of teaching was based in a belief that she must work from the needs of the learners and not be driven by external factors. Ruth had maintained a richly documented portfolio that indicated thoughtful planning and insightful reflective comments on past lessons.

**Video Recording 2**
I was delayed in getting to the start of Ruth’s lesson, which resulted in the video recording starting midway through the lesson with Ruth building the “Plus 3 pattern” with the class. Referring to the large number grid on which the multiples of 3 had been coloured in, she posed the question,
“Does anyone see a pattern in the numbers that were coloured in and the numbers that were not coloured in?”
Ruth further clarified by asking,
“What can you tell me about the colours?”
"We are jumping 3 every time," responded a girl, and Ruth added the learner’s response to the chalkboard summary. Ruth followed up by asking for a description of the pattern and a learner responded saying,

"Colour, blank, blank," and the class repeated the pattern aloud saying, "colour, blank, blank; colour, blank, blank," and so on, as they referred directly to the completed multiples of 3 pattern on their individual counting grids in their books. Ruth added the "colour, blank, blank" response to the chalkboard summary.

Ruth directed the learners to look specifically at the coloured in numbers on the large counting card, and asked the question,

“What do you notice how the numbers go?”

Using a ruler a boy came to the front and pointed out to the class the diagonal pattern of coloured in numbers that made up the multiples of 3 saying,

“They are going across like this.”

Another learner added, “Making a pattern every time.”

“What kind of pattern,” asked Ruth?

Other learners responded by saying,

“A number pattern.”

“Across pattern.”

“Plus 3 pattern”

The boy again came to the front again and pointed out the diagonal pattern of non-coloured numbers to which Ruth responded in genuine surprise exclaiming, “Oh wow! Oh wow!” Clearly, she had not expected a learner to identify this particular pattern, hence her surprise. Ruth introduced the term “diagonal” into their vocabulary and documented the chalkboard summary as follows:

\[
\begin{align*}
1 & + 3 + 3 + 3 \\
2 & \text{jump 3 every time} \\
3 & \text{colour, blank, blank} \\
3 & \text{colour, blank, blank} \\
4 & \text{The numbers are coloured in in a diagonal pattern.}
\end{align*}
\]

Ruth required the learners to write in their books “what we noticed about the plus 3 pattern,” under the heading of “Plus 3 pattern.” Before this point in the lesson, the learners had become restless through the extended pattern searching session, and now they settled quietly to writing up their findings in their books. Ruth gained the attention of the class via a clapping pattern, and the lesson ended with a prayer.
In discussion with Ruth, I drew attention to the manner in which she developed the lesson based on the contributions learners made, and how she built on learner knowledge via a systematic chalkboard summary made up of learner findings. Making use of student prior knowledge was one of the important procedures in the Constructivist Learning Model in which Yager (1991) stated, “Using student thinking, experiences and interests to drive lessons (this means frequently altering teachers’ plans)” (p.56). Ruth provided ample evidence of this particular principle that we had not seen in the observed lessons of the other case study teachers. The videotape showed the learners engaged in a meaningful, pattern building approach to understanding the 3 times table, very different in teaching style to a rote learned approach.

An issue for Ruth was the level of restlessness in the class whilst she or other individual learners were speaking. She believed that the role of discourse was fundamental to her teaching style. Ball (1991) asserted that the NCTM (1991) document gave, “unprecedented attention to discourse in the mathematics classroom where discourse is used to highlight the ways in which knowledge is constructed and exchanged in classrooms” (p.44). Ruth maintained that the discourse that focused on the number pattern was necessary as pattern finding was critical for the learners’ concept development, and that was the reason she probed for relevant learner contributions. In doing so, many learners lost attention and became fidgety, and Ruth had to strain to overcome an increase in the noise level. Ruth said that whilst she was teaching she was not distracted by the restlessness of the learners and her tone and manner captured on the videotape confirmed this to be the case. Ruth said she was aware of the role she played in shaping the discourse through the signals she sent about the learners’ knowledge and the ways of thinking and knowing that were to be valued. Ruth said she would discuss the noise level issue with her learners in an effort to promote a positive discourse that benefited all the learners. Ruth recognised that she must continue to work at the pace of her learners when she wrote in her portfolio:

*I felt that the learners that did make an effort really worked well and showed steady progression. At the moment I feel progress is slow, but I think I need to give them the time and allow them the opportunity to work at this pace so that*
they can find real meaning with grouping, as they will need the basic understanding to move on to more complex multiples and multiplication.

(Ruth)

**Video Recording 3**

The lesson started with all the learners sitting on the mat. On a chair in front Ruth held up flash cards for learners to see and the task was to “jump 1 back” or “count 2 more.” A large counting grid was on display behind Ruth, and learners were able to refer to it when necessary. During this activity, individual learners were called on by name to answer questions whilst the others observed. Although the learners were quiet, there was constant movement as if they were incapable of sitting still. I noted the learners’ restlessness in the previous videotaped lesson, and it would appear that the class has a limited oral attention span. In this lesson as in the previous one, they became distracted when they were required to engage in making individual verbal responses to Ruth’s questions. The learners became more attentive when they collectively built up the “plus 2 pattern” to 30, followed by the “plus 3 pattern” to 30 with Ruth recording the patterns on a chalkboard in front of the learners. Before returning to their desks, the class was instructed to sit in groups of 4.

The next phase of the lesson involved Ruth working with each group of 4 in turn. She asked the questions,

“How many groups?” and,

“How many people do we have in the groups?”

To verify the number of people in the groups of 4 Ruth called on an individual learner to count aloud and point to each learner in turn. At the same time, the learners coloured in the multiples of 4 on their personal 100-grids, once the numbers in the groups of 4 had been systematically confirmed. Ruth orchestrated the whole procedure as she interacted with the learners as they participated in building understanding of the “plus 4 pattern.” She smoothly managed the “counting of heads” in groups in a real-life situation, and linked the results to the large 100-grid, followed by getting the learners to record the pattern on their individual 100-grids. The process was connected and gave the learners the opportunity to make sense of the constant function of adding 4. Once all the groups of learners had been counted, Ruth posed the question:

“We have counted all the groups, how are we going to count?”
Various methods were volunteered that included starting counting the groups a second time, using counters or fingers, or continue colouring in the individual 100-grid. Ruth gave the learners the option of using any of these methods to complete the pattern of 4 on the 100-grid. The learners tackled the task with a will. No learners opted to use the counters. Instead, they elected to colour in the grid by counting on in 4's. I noticed that learners in two groups went astray in their pattern colouring. One of these groups refused to cooperate with each other and had placed barriers around their books so that the pattern could not be seen. Perhaps if they had collaborated more they would not have made errors in their pattern building. Ruth visited all the groups and was able to assist those learners who had lost the pattern. Meanwhile, other groups forged ahead and successfully filled in their plus 4 pattern. Ruth gained the attention of the class via a clapping pattern. By way of a summary, she wrote on the chalkboard the first steps of the plus 4 pattern:

1 group of 4
1 x 4 = 4

2 groups of 4
2 x 4 = 8

Ruth ended the lesson by saying they would continue the recording the next day.

The aspect that I found particularly interesting in this lesson was the manner in which Ruth linked the 'realistic' situation to the number pattern. In this instance, the realistic situation was the counting of learners sitting in groups of 4, and relating the situation to the constant pattern of adding multiples of 4. A test of their understanding came when they were challenged to complete the 100-grid on their own. Most of the class were able to continue unaided by counting on 4 each time, but some learners were unable to see the pattern, and possibly not able to make links between the realistic process of counting groups of 4 people at a time, and the pattern they were trying to build. Ruth reflected:

...I think they are much more confident in working with groups, whether it be on the number card, using counters or using themselves. The way in which they work shows that they have developed a good understanding of 'groups of sums.' There are still learners who need assistance in adding groups of numbers together, but they have also showed progress and this is important to them. I have to show that I am just as proud of their progress as they are,
and encourage them to improve more. I think the learners did very well today.

(Ruth).

It was also interesting to see that given the option of using concrete materials to assist their understanding, almost all the learners elected to work directly from the number card, and not make use of the counters. I perceived that the learners were able to relate more easily to the 100-grid as the card provided direct number solutions. My belief was that learners developed number sense through working regularly with number, and being able to refer quickly and easily to a structured number system such as that found on the 100-grid.

Analysis Of Emerging Themes

This was the third and final panel review session and with the research question firmly in mind, I was anxious for the panel of observers to focus specifically on the major themes to emerge from all three case study teachers’ observed video recordings, which included comments on Ruth’s videotaped lesson, which they had just seen. They had seen Video Recording 2, in which the learners looked for patterns whilst building the multiples of 3. I was disappointed that Thandi was unable to attend this concluding session as she was called away on an urgent family matter. I was not able to arrange an alternative date, as the other panel members were committed elsewhere.

To give focus and direction to the final panel discussion, I reflected on what had taken place at the previous panel review sessions. At the first review session, discussion had been wide-ranging, and tended to centre not only on the wording of the summarised Constructivist Learning Model (CLM) principles but also on other issues of interest or concern to the panel of observers. At the second review session, I provided a refined set of principles to guide the discussion. Although the discourse focused on the principles located in the CLM, other issues did arise for debate. As the leader of the panel, I used the CLM as a focus for the discussion. I did not adhere to Yagers’ six points rigidly as I wanted the members to freely discuss emerging issues and not be constrained by the specific six points. For the final session, I made a list of the dominant themes that emerged from the observer discussions in the previous review sessions. I also reviewed the observations that I had made after
describing all the video recordings, noted the themes to emerge from these observations, and added these to the list. I wanted the panel of observers in this final session to provide supporting argument for what they perceived to be significant themes they had identified in the videotapes they had observed. The following themes emerged through a discursive process of analysis, informed by the CLM, refined by me and finally revisited again by the panel of observers. This process could be perceived as overlapping layers of analyses. The term ‘overlapping layer’ implied more than a linear process but one of sequence and depth. The layers comprised:

Layer 1 the initial analysis framed by the CLM procedures,
Layer 2 discursive observational analyses,
Layer 3 the identification of major themes,
Layer 4 the observers revisit the major themes.

The following were the major themes to emerge:

*Collaboration and Cooperation*

Marc and Sue reported seeing learner collaboration in all three videotaped lessons but more particularly in the cases of Lulama and Makana. “Teacher–learner collaboration” was seen to be much stronger in Ruth’s lesson as she interacted with the learners whilst they were working. Although she appeared more physically distant, Ruth’s collaboration was seen as a “prototype” of her teacher-learner interaction. Marc observed that in reference to collaboration there was “a definite RUMEP theme that comes through in all three cases,” and, “collaboration is seen as part and parcel of their belief structure.” He concluded that the theme of collaboration “from an outsider’s perspective comes through incredibly strongly.”

The issue of whether we were seeing evidence of collaboration or cooperation was debated. The panel agreed that collaboration was “a mutual coming together with a shared vision” whereas it was possible to cooperate reluctantly, without collaboration. The videotapes had shown collaborative groups in action, but in the case of Ruth, there was a difference. Sue explained it well when she said that in the cases of Lulama and Makana there was “collaboration amongst groups to cooperate with the teacher.” In Ruth’s case, it was not merely cooperating with the teacher. In Sue’s words, “They were involved, enjoying what they were doing, although the little ones were not focused all the time, there was no apathy or reluctance.” The
panel approved the preference of the term collaboration over that of cooperation as a major theme noting that the concepts were linked. They also recognised that the nature of the activity determined the degree of collaboration. Ruth's activity, for example, was seen as being "less complex," and required learner cooperation more than collaboration or negotiation.

Negotiation and Participation
I raised the question of whether there was negotiation coming out of the collaboration that we had seen and I questioned Marc whether learners of this age had the mental skills to be able to negotiate meaning. Marc asked, "What does negotiation mean, even at a more advanced level? To what extent do we negotiate meaning?" He perceived that negotiation implied some sort of contestation, followed by a conceding and accepting of ideas in a spirit of "give and take." The panel agreed that they had seen collaboration in groups in all three cases, but they were not convinced that they had seen evidence of negotiation of meaning taking place particularly in the classes where learners discussed in Xhosa. Marc reconsidered the view that there was a lack of convincing evidence by citing the case of Makana, and the lesson in which the learners generated their own tiling patterns. He referred to the group that made various arrangements using parallelograms, and arising out of perceived "insecurity", they changed the tile pattern possibly through negotiation by saying, "let's try this." He said the learners in this case might well have negotiated. Sue questioned whether this was "negotiation of meaning or negotiation of procedure."

The panel was of the opinion that in Ruth's lesson, for example, instead of negotiation there was evidence of much "learner participation," and although they had noticed that the learners were participating the question remained, "to what extent were they participating in the actual focus of the lesson or not?" The observers were emphatic that they had seen participation in all three cases but they raised a concern: "participation in what?" In response to the question of whether learners of this age group were able to negotiate meaning, Sue argued that it was possible as long as the teacher scaffolded the process for the learner.
Learning and Progression

I raised the notion of situated learning with the panel and received little response. Instead, the panel preferred to focus on whether they had seen evidence of learning taking place. Earlier, Sue had expressed concern on the same issue when reviewing Lulama’s videotape. In answer to the question was there learning taking place, Marc made the following cogent observation:

_to me there needs to be a sense of coherence, starting somewhere and ending somewhere, and the process in-between is where learning is supposed to take place. In the last video, there was a definite sense of coherence; the way she documented on the board, a kind of progression, one thing leads to another. That to me is evidence of learning. In the others, it could be argued they were not sure where they were heading and where they were heading for._ (Marc).

Sue agreed with Marc that there was definite evidence of progression, but asked whether that implied learning. Marc believed that this was the case, “because the progression was determined by the learners. She took the cues from the learners.” The panel agreed that learning had taken place in Ruth’s classroom, and remained less convinced about the other two cases. In selecting the themes for the panel discussion, I had placed “progression” as a theme for discussion on its own as I had not anticipated it becoming linked to learning. The emergence of “learning and progression” as a connected theme was interesting as it was unexpected.

Learners’ Prior Knowledge

The panel confirmed that there was evidence of prior knowledge in Ruth’s lesson but they had seen little evidence of it in the other lessons. Sue had previously raised doubts over how well the learners progressed in Makana’s tiling lesson without seeing Makana first establish direction and purpose for the tasks. The panel assumed that the learners in his class had prior knowledge or were familiar with “something in the process,” and knew what to do. The observers noted that the learners had prior content knowledge because they understood concepts related to triangles and quadrilaterals. Ruth was commended for building the lesson on what the learners contributed, and not working from a prescribed text. The question was raised
whether Ruth would have achieved the same outcome if she had given this lesson to another class. The panel members believed that the term “diagonal” would not have appeared on the board summary if the boy had not suggested it, and, “for her it was revelation, and got so excited” at seeing the diagonal pattern of numbers that were not coloured in. Sue likened the event to looking at a picture where one sees certain images and not others until they are pointed out. Marc enquired how the learners’ "colour, blank, blank," technique had emerged. Sue believed it was the repetition of a technique applied differently because of the context. The technique was based on clapping rhythms, and on a game of 1, 2, stand up 3. The class had made the colour, blank, blank, rhythm uniquely their own, and it was evidence of appropriate application that was important for Sue, not just repeating a technique suggested at RUMEP.

Reflection
The panel had seen learner reflection in all three cases. In the cases of Lulama and Makana, the learners reflected through “summing up” in the report backs. In the case of Ruth, teacher reflection took place through her daily writings and she based her preparation on her reflections. The observers noted Ruth’s personal comment that constant reflection had positively influenced her teaching.

Sustainability and Momentum
Having reviewed Ruth’s lesson Marc believed that she could have continued in the same vain for another 15 minutes, whereas with the other two cases “one gets a sense that it’s getting laborious.” With hindsight, he believed that Sue’s concerns in connection with the amount of learning taking place in Lulama’s lesson now placed Sue’s misgivings into perspective. Sue said the notion of sustainability viewed from a teaching perspective was interesting, as she had not thought about it in that way before, and asked, “What is it that you are trying to sustain?” Sue responded to her own query with the following insightful observation:

In my mind, this approach is more sustainable because it involves a more holistic approach. In one extract, (Ruth’s lesson) we have seen teacher-learner interaction, learner-focused work, reflection, and feedback. There were more elements that were sustainable. (Sue).
Marc maintained that another criteria for sustainability were if there was “momentum” in a lesson. The panel approved the notion of momentum, particularly if applied to the learners in a class. They also recognised in Ruth’s lesson a daily “routine”, and how routine helped sustain momentum, particularly in the lives of children where there was a need for comfort and stability. Marc concluded, “Routine in a lesson is crucial, routine which sustains momentum.” The observers affirmed evidence of routine and momentum in Ruth’s lesson.

Professional Content Knowledge
The observers rightfully explained that they had not seen enough of the case study teachers to comment fairly. In Ruth’s Grade 2 class there was a low level of content knowledge, but the observers believed that Ruth was “comfortable and flexible” not just having the knowledge, but approaching the knowledge in different ways with the learners. In the other two cases, the observers would like to have seen greater clarification of mathematical concepts by the teachers. Sue made a valid point when saying that they had not seen the other teachers in a more teacher-learner interactive role because she was sure they did interact with the learners to a degree. Evidence from Makana’s videotapes 2 and 3 showed him clarifying content knowledge with the learners but these videotapes were not seen by the panel because of time constraints. Another issue that concerned the panel in relation to content knowledge was the use of a second language in which both educators and learners had to express themselves. Second language instruction gave rise to incorrectly worded assignment cards and misspellings as seen in Makana’s final video recording.

Power Relations
Sue believed there was evidence of “power differentiation” in Ruth’s lesson. Ruth was observed as being “the authority.” This showed in the manner in which “she stands, walks, and speaks”, and then “she handed over” power to the learners. In her classroom management, she encouraged “negotiation of power” by facilitating learner decision-making in respect to the learners’ group roles. In the other two cases, power was “devolved more” with the learners taking responsibility for their own learning progress. Makana, for example, had “set up” the activity initially, and then shifted the power to the learners. Lulama shifted the power to the learners in her
class when they became responsible for reporting their group findings to others in the class.

Marc looked at the issue of power from a research perspective. He said that in all three cases the learners were aware of my presence and the camera in the classroom, but he was not sure to what extent that had an influence on “the discourse, collaboration, and participation” of the learners. From the videotapes he felt the teachers looked relaxed and “at ease” and were not unduly affected by the camera. Of greater concern to the panel were how I intended to use the videotapes and to whom they were being shown. Sue enquired whether the case study teachers were aware that a panel of observers were shown them. I assured the panel that the video data was confidential, and that the case study teachers had been informed that anonymous observers would discuss their video recorded lessons. The panel approved the ethics in the next phase of the research where the case study teachers reviewed their own lessons, and analysed the lesson from their perspective.

Methodology
An unintended outcome was the panels’ comments on the use of capturing data via the video camera in the classroom. Marc believed that unlike classroom observation “you only capture what you film,” whereas using the technique of observation it was possible to capture more. Sue countered this position by saying that the same argument applied to observation techniques because the observer tended to focus on aspects of interest or of concern to the observer. The best option they suggested would entail the use of two cameras, one focused on the teacher and the other on the learners. The panel concurred that an advantage of a video recording technique was that external observers like themselves had “opened up the process” through member checking in analysing the video data. The panel of observers appreciated the additional strategy of being able to revisit my analysis of their observations. They approved the option of being able to review my interpretation of their comments and they welcomed being given the opportunity to comment on my analysis of the meta-themes. I incorporated their comments into the review of emerging themes, the final layer of the multi-layered process of video data analyses.
Analysis of Video Recordings by Case Study Teachers

In the First Phase, I reviewed all the lessons I had observed from my perspective alone. In the Second Phase, I again documented reviews on all the lessons Thandi and I observed. In addition, each lesson in the Second Phase was video recorded for review and analysis in an effort to broaden the perspective. A further dimension in the analysis cycle was to include the perspective of the case study teachers themselves. Each case study teacher reviewed his or her own videotaped lesson and responded to my questions relating to issues that emerged from the video recording. The videotapes selected for review were the same as those seen by the panel of observers. My first thought was to structure the interviews on Yager’s six points as I had done with the panel of consultant observers. After consultation with Thandi, I decided on a more grounded and less theoretical approach by getting the case study teachers to comment directly on the videotape in question. The interview questions can be found in Appendix F.

The purpose for including a case study teacher’s perspective was to determine their beliefs as they related to their practices as viewed on the videotape. Although the panel of observers had identified specific themes to emerge from the video recorded lessons, I needed to know from the case study teachers their beliefs and values as observed in their teaching strategies. Thandi indicated that a one-on-one interview by me with a case study teacher alone might be threatening and suggested inviting fellow teachers of mathematics from the same school to participate in discussion with the case study teacher following the viewing of the lesson. In each case, the viewing of the videotape and the related discussion lasted approximately 75 minutes.

_Lulama_

To give opinion and provide support, other colleagues attended Lulama’s review session. The support team for Lulama comprised two mathematics teachers from her school, Kholoela and Mthuthuzeli (both male), and included Thandi, RUMEPS’s field support person. As I was seeking confirmation to belief-related questions, it fell to me to pose the questions but these I distributed amongst the team so as not to place too much pressure on Lulama. The format was informal and relaxed, which enabled all the participants to contribute freely on issues where appropriate.
Having viewed the videotape, I asked Lulama why she had placed the learners in groups. Lulama responded by saying at first the learners were not confident in their groups, and so they worked in pairs. As pairs, they would “discuss and participate.” Now she said “they discuss all the time” and were working in collaborative groups “where everybody is expected to say something.” The videotapes confirmed learner participation in groups. Lulama was asked why she prepared only one work card for a group of four or five learners. Lulama explained the logic of this strategy. Every learner was assigned a group role and one of the roles was “reader” whose responsibility was to read aloud the task to the group. Lulama approved of this strategy because she said it promoted group collaboration. She explained that although there was only one card per group “all the cards are different,” in that each group had a different problem to solve. At first, the cards were written in Xhosa and English, and now they were written only in English arising out of a joint decision with her learners to improve English communication in the class. The other members of the support team commented on the confidence of the learners speaking English in the verbal reports and said that as a policy they were encouraged to see how well it was working. They noted that the learners understood their mathematics and they were “not afraid” to report their findings in English.

The next question referred to Lulama’s encouragement of varied learner solution strategies. She said that she “wanted to move away from the way I was taught. I was not given the chance.” Her belief was that her learners should be given the chance that she never had. She noted that the process started slowly with some learners “reserved and not active” at first. Thandi observed that Lulama’s approach was “a way of challenging creative thinking” as she saw learners “reading themselves, take control, and think for themselves.” Kholela said there was evidence of this in the subtraction solution strategy where he was impressed to see the reporter round up by adding 4 to each number before subtracting. Mthuthuzeli commented on the fact that the learner and not the teacher posed the question, “is it a total or a difference?” which implied that the learner was thinking about the reporter’s solution strategy. Mthuthuzeli asked Lulama whether the strategies used by the “packets” group were their own. Lulama confirmed that they were and that she had had to seek assistance from her colleagues because she was confused by the solution strategies used. Thandi enquired the origin of the multiplication strategy where the learners set out 47 as 10
+ 10 + 10 + 10 + 7, and proceeded to repeat 29 x 10 four times in a laboured, repetitive manner. The team members believed that the learners had been "drilled" at some stage and agreed there was "no reason for repetition" as the procedure should have been "refined." It was noted that the group that had engaged in the division solution strategy offset this mechanical strategy by counting in multiples of 18.

Lulama was asked why she requested learners to present reports to the class. Her response was that it gave opportunity "to share his or her solution strategy" with members of the class. Lulama maintained that in a "normal" lesson she also would "correct vocabulary and clarify." The team expressed concern at not seeing Lulama correcting and clarifying mathematical concepts in the videotape. Lulama was questioned whether she was threatened by the variety of solution strategies presented by her learners. Lulama said that she would turn to her colleagues in the school when she was unsure, and admitted that using a learner-centred approach could be a threat to the teacher. Thandi observed that in many classes that she had visited as the RUMEPI field support person she had seen "learners more advanced than their own teachers." Thandi added that teachers were often "shocked" by their own mathematical misconceptions but that "if we reflect it will clarify our misconceptions." The team members added that "to revisit is powerful" if it causes the teacher to think about the mathematics in the solution strategy and how to advise the learner accordingly. Lulama was asked why she allowed learners to take their own actions in problem solving situations. She concluded by saying she believed it was important "to give opportunities for her Grade 5 learners to be challenged so that the learners are producing."

Concerning her planning of pace and progression Lulama stated that her first step was to ask, "What do I want the learners to know?" From there, she selected the topics and the activities within the topic. To know how to plan the term's topics she consulted with her school's Learning Area Committee comprising Mthuthuzeli, Kholela and herself. To prepare the carefully compiled set of work cards Lulama said that she consulted textbooks, and references she had collected from RUMEPI and AMESA. Mthuthuzeli and Kholela both agreed that the work cards were appropriately selected for the particular class. They remarked that within the school there was now a "spirit of consultation" which encouraged teachers to share ideas that had been gathered from workshops, and from "training days" organised by the
Provincial Department of Education. On the question of evaluation, Lulama reported that she used a variety of assessment techniques that included peer assessment, group assessment and “times when I observe.” She said that she also gave them “tests related to the topic” as one of the assessment procedures. According to Lulama, she included learner reflection as a part of the evaluation process. Reflections provided insight into “where they are” and provided opportunities for learners to “express their views.” There was evidence that they now “like to reflect.”

When asked why her learners were so good at problem solving, Lulama replied, “I think we were wrong to think learners can’t do anything, now we are giving them a chance.” Lulama explained that to start with (in the First Phase) “I was not sure what I was doing,” and now that she had implemented a learner-centred approach, “I tried it and it works.” Lulama had continued with a learner-centred teaching style because as she said, “I know it works.” Thandi pointed out that the observed lesson was clearly not a “once off experience” for the learners, and she argued that change was an “ongoing process.” I posed the question, “What made teachers want to change?”

Mthuthuzeli made the following pertinent observation:

_I think that one changes because one sees that now my way of teaching is outdated. In fact, it means that you accept that what you have been doing is outdated. Once you don’t accept what I’m doing is right you will never, ever change... One has to see the need to change within yourself, the teacher inside you... You need to be exposed (in-service courses). If a teacher doesn’t want to change you will never see that teacher here._

(Mthuthuzeli)

Thandi spoke to the issue of power relations by explaining her manner of operating with teachers in the field. Her approach to professional development was to get teachers to plan their own programme through the democratic functioning of the school clusters. Team members identified power sharing in Lulama’s class when the learners participated in collaborative groups, and when the reporters presented the groups’ findings. A cause for confusion was the start of the lesson where Lulama posed a verbal problem to the class that gave rise to her comment “nobody wants to try.” The team interpreted the opening of the lesson in traditional terms with power firmly in the hands of the teacher. The power relationship later shifted to the learners
as they became responsible for finding their own solution strategies. Kholela
believed that the learners came with the attitude that they were part of the learning
process and did not expect Lulama to provide answers to the problems. He observed
that Lulama’s learners were interested in learning mathematics and contributed to a
“culture of mathematics in the school.” Lulama closed by saying she believed in the
power of her learners to solve problems and she would “continue to teach this way.”

Makana
Makana’s support team comprised Lulama, Mthuthuzeli and Thandi, whilst I posed
most of the questions. The video recordings reviewed by the team included the report
back component of Videotape 3 and the complete Videotape 4. The first question put
to Makana after viewing the videotape was why he placed learners into groups. He
responded by saying that “learners assist each other” and that it fostered
“communication” between learners. He referred to policy advocated in Outcomes
Based Education (OBE) where “we talk about cooperation.” Being a multigrade
class, he placed learners in “family groupings” of learners of mixed ages and mixed
abilities. He believed that the more able learners should help the less able learners
within a cooperative framework through scaffolding, yet he had to be aware of the
person who “tends to dominate” a group. In Thandi’s opinion, the application of an
“integrated curriculum” across Grades 5, 6, and 7 worked well and it would have
been less beneficial if they had been taught “according to their grade and not in
family groups.” She argued that the learners “grew stronger and became more
advanced” not being taught topics in “isolation.” Despite the case made for
integrated family groupings, Makana made the point that on some occasions he had
to provide learners with more challenging grade level activities, or challenging
individual activities. On being questioned, why he distributed only one work card per
group his immediate response was “it works for me.” After consideration, he
explained that through the allocation of group roles, each learner had the opportunity
to become a scribe, reporter, reader, or leader. Team members noted that instead of a
quiet class with only the teacher talking, the learners “helped each other” and
Makana “helped learners take responsibility for themselves” in a setting of social
cooperation.
Makana was asked why he wanted learners to reflect and he responded, “to check strengths and weaknesses.” He believed by getting learners to regularly reflect the teacher built a better understanding of the learner’s thinking. As an example, the case was cited of the learner who recorded in her portfolio that she enjoyed participating in a group, but was resentful of her work “being copied.” Mthuthuzeli reacted by saying:

*Having heard the learners comment, when they work as a group they help each other but individually, she is saying, I don’t want someone to take it, particularly not knowing what is taking place, instead of copying. Group work is right but they need the chance to work individually. They can participate in a group and get it correct, not even understanding the work.*

(Mthuthuzeli)

The team acknowledged that a learner’s verbal report was a form of mathematical reflection where learners “share findings” and learners gain from hearing alternative solution strategies from other group reporters. Team members commented on the interest shown by the class as reporters made their presentations and how well Makana clarified mathematical concepts, misconceptions notwithstanding. Thandi enquired whether Makana felt “unsettled” when using a learner-centred approach where the learners were encouraged to make “self-discoveries” and they came up with “fresh,” unexpected solution strategies. Thandi referred specifically to the incident where the reporter found two angles of a scalene triangle equal. Makana admitted that it was threatening, as there was no colleague able to assist him in his three-teacher school. Mthuthuzeli reminded the team that a “hands-on investigation” style of teaching was more demanding for the teacher than using a textbook for the “transmission of knowledge.” Makana believed in giving his learners a chance that he never had “to explore” even if they presented findings he had “not seen before.” Makana was asked how he planned the pace and progression in teaching this series of geometry lessons. He explained that in his rural school, there were no resources for teaching art or technology and he used these times for consolidating concepts in mathematics through the medium of English as a means of assisting the preparation of learners. He said that he developed the geometry topic by referring to school textbooks for ideas and gathered relevant activities at RUMEPI, but the rural school cluster provided the guidance and support that gave him the confidence to teach
geometry for the first time. He admitted that he was surprised how easy it had been for the learners and he believed that by “getting learners cutting, discussing, measuring” had enabled them to build understanding. Mthuthuzeli commented on the lesson by saying, “The learners enjoyed a lot” and Lulama commented, “You can see by the way they were doing.” Thandi noticed that when learners required clarification on specific mathematical vocabulary such as ‘isosceles and trapezium’, they referred to textbooks in the class.

Arising out of remarks made previously by the consultant panel of observers, I asked whether Makana had pre-prepared the learners for the tiling lesson just seen. He was adamant that no discussion had taken place before the lesson and the learners had worked directly from the prepared assignment card and from verbal encouragement from Thandi and himself. The support team acknowledged that the learners achieved satisfactory tiling results by their own actions. Thandi commented that Makana could have derived further clarification of mathematical concepts such as “symmetry and tessellation.” Makana acknowledged that he had missed an opportunity for consolidating mathematics concepts derived from the activities. In response to my question whether his mathematical content knowledge had improved, he believed that he had developed particularly in geometry because previously, “I didn’t care much about that.” Team members noted the “geometry and pattern work” displayed on the walls of the classroom, which indicated the range of interesting mathematics activities that Makana was now investigating with his learners. In response to the question of how his teaching had changed Makana said that the support provided by the school cluster had “helped me a lot” and that they now “planned together.” He claimed that without the support of the cluster members he would not have used an investigative approach. He also mentioned how he had benefited from inter-school visits to see how other teachers organised and managed mathematics teaching in a multigrade class.

I raised the question of power relations in the class. Thandi asserted that Makana had moved from where the teacher had the power and “the learners remained passive. Instead of him dominating the lesson the learners made decisions about their own work.” The allocation of group roles provided learners with a “sense of responsibility.” Lulama noted that the learners were “allowed to express themselves”
and that “he just made activities and gave them to the groups.” Although Thandi accepted that power was not “one-sided,” she believed an important role of the teacher was to “eliminate maths misconceptions” through ongoing reflection. Makana confirmed the need for constant reflection from the learners and from himself. His final comment was that the videotape had enabled him to “see our mistakes.” At the same time, he said that he felt encouraged by the comments from the support team who had appreciated what his learners were able to achieve without being “drilled.”

Thandi rounded off the discussion by praising farm school teachers like Makana, and what they had achieved despite the difficulties of teaching in a rural, isolated environment. She believed that they had achieved “a balance” because they placed emphasis on the curriculum and the learners and had not used a “lack of facilities” as an excuse for not implementing learner-centred teaching. She informed the team that the Minister for Education (MEC) in the Provincial Department of Education and his advisors had recently visited a farm school in a neighbouring cluster to witness mathematics teaching in a multigrade class. The MEC and his entourage came away impressed by the quality of learning taking place with rural learners, and the influence the support clusters were having on mathematics teaching and learning. She observed that the videotape had showed that farm school learners “are not afraid of geometry.” Thandi was pleased to report that Makana had presented an “inquiry-based” approach to teaching geometry at a workshop for other members of his cluster and had been well received by the teachers.

*Ruth*

There was no support team available at the time Ruth reviewed her video recording, as she was only available at a time when her colleagues could not attend. Videotape 2 was selected for discussion and analysis. My first question to Ruth concerned the language of communication and why she chose not to use a bilingual approach. Her reason for speaking only English to the learners was that “it helps them to hear English. I can see they are comfortable speaking English to me which means there must be progress.” Ruth pointed out that she allowed the learners to use their mother tongue language when conversing in their groups. She said, “I don’t really mind that.” Ruth appreciated that the only English likely to be heard in a day by her
learners would be in her classroom. She believed they had come “to expect” to speak English and they were “confident enough” to try. I asked Ruth why she encouraged group collaboration and she responded, “There’s no doubt about it, they learn better,” but that it did depend on the type of activity. She said that her learners did “a lot of group work” and cited as an example an incident when they were building the “add 3 pattern” in groups. A boy wanted her to develop the pattern for him and when she declined she was “amazed to see” another member in the group scaffold the boy’s learning. She noted that learners negotiated in their groups and there were “sometimes arguments when they are trying to make their point” and “others will say if something is wrong.”

Ruth believed a strength in her teaching style was that she accepted learner contributions, and developed the lesson using their prior knowledge to dictate the direction of the lesson. The documented summary based on learners’ thinking was quoted as an example. “Colour, blank, blank” to describe the multiples of 3 number pattern was a typical inclusion of original learner thinking. I asked Ruth if the verbal teacher-learner interaction routine as depicted in the videotape was her normal teaching style. She replied,

> I actually feel comfortable doing it this way. I get more out of them. It’s not me telling them what to do. It’s a two-way thing. And the fact they are so keen to give their ideas. They feel comfortable with the set-up as it is. I don’t really know. I haven’t thought about it consciously.

(Ruth)

As an aspect of her teaching style, I asked Ruth if she encouraged learners to develop their own solution strategies. She indicated that although some learners were doing well and learning from “each other,” others felt insecure unless a method came from the teacher. In reference to building group solution strategies, Ruth believed it to be a “good thing” that not all strategies emanated from her. Once learners had developed a solution strategy of their own, they were “eager” to share their findings with the class.

With regard to power sharing, I asked Ruth how she disseminated power in her class. Ruth believed that her first priority was “to make sure that everybody is happy
including me.” As an example, Ruth cited the formulation of class rules where the learners were part of the decision making process and they “decided together.” She indicated that when it came to discipline she discussed with the learners an appropriate action. Not doing homework was quoted as a case for debate. Ruth discouraged punishments, as she believed that “school must not be a gaol” for her learners. She allowed them “to have a say at what is being done.” Ruth voiced her belief in power sharing with the following statement:

*I don’t stand there and say I’m the one who knows everything, tells everything. I try to let things go the way they are at the moment. If I planned a lesson and it doesn’t go accordingly, it’s best to go that way. If they lead somewhere else, I go along with it. They learn from that as well.*

(Ruth)

I asked Ruth about her own mathematics development. She believed that she had “definitely grown” and was more confident as a result. She referred particularly to her teaching of geometry, which she now felt “so good about it.” Ruth believed that as her confidence had grown so her attitude changed. She felt that previously the younger learners “did not enjoy school, and now I can see the learners liking coming to school.” Ruth believed that she provided a secure learning environment to learners from “poverty-stricken” homes and how aware she had become of the learners’ socio-economic “context.” Having seen the videotape Ruth said she was now more aware of the noise level in the class and that she would discuss “ground rules” relating to the matter with her learners. In closing, Ruth stated that she was “feeling confident” and was committed to “do a lot “ with this age group of learners. She believed analysis of the videotape had been useful to her and she was pleased to be included in the review process.

**Quantitative Data**

*Belief Scales*

To assess the changes in the beliefs in the full cohort of key teachers an adapted Beliefs Scale (Carpenter, et al., 1989) was administered at the beginning and end of the First Phase of the research. Out of the cohort of 34 key teachers, only 22 completed the questionnaires. The three case study teachers completed the questionnaires satisfactorily. The Belief Scale consisted of 36 questions designed to
assess key teachers’ beliefs. Twelve statements per construct, centred on each of the three constructs, six of them positive statements and six of them negative statements. I set out to establish a baseline to determine whether there was a significant difference in the beliefs of key teachers on each of the constructs pre and post the intervention. The null hypothesis was that there was no change in the espoused beliefs of the key teachers between the pre and post-test results. Because the sample was small, I used the Wilcoxon matched pairs signed ranks, non-parametric test to find a significant difference (T) in the pre and post test results (Cohen & Holiday, 1982). T was calculated for two correlated samples in a one tailed test. A one tailed test was chosen as I expected directionality resulting from the influence of the intervention. The significant differences for each of the three constructs, and the combined constructs, were computed and the results tabled as shown:

### Results

Critical value on a one-tailed test for sample size 22: T ≤ 75

<table>
<thead>
<tr>
<th>Subscales</th>
<th>N</th>
<th>T</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Role of the Learner</td>
<td>N = 22</td>
<td>T = 42.0</td>
<td>p &lt; 0.05*</td>
</tr>
<tr>
<td>Role of the Teacher</td>
<td>N = 22</td>
<td>T = 41.0</td>
<td>p &lt; 0.05*</td>
</tr>
<tr>
<td>Sequencing of Topics</td>
<td>N = 22</td>
<td>T = 83.0</td>
<td>p &gt; 0.05</td>
</tr>
<tr>
<td>All Constructs</td>
<td>N = 22</td>
<td>T = 62.5</td>
<td>p &lt; 0.05*</td>
</tr>
</tbody>
</table>

*significant at the 95% confidence level

Table 2: Significant differences on the Belief Scale subscales.

The null hypothesis was rejected for Constructs 1 and 2, and for All Constructs. The Belief Scale results indicated that in the Role of the Learner, the Role of the Teacher, and in All Constructs, there was statistically significant change in the key teachers’ espoused beliefs over the first six months phase of the study. In the Sequencing of Topics, there was no significant difference. The task of how to plan a sequence of lessons within a topic remained difficult. The results indicated that greater attention needed to have been placed on this important curriculum aspect. During the intensive contact sessions of the course, the key teachers had the opportunity as in the study reported by researchers Vacc and Bright (1999), to “change their beliefs to a more constructivist orientation about the teaching and learning of mathematics” (p.103).
Although the key teachers’ professed beliefs had changed significantly on two out of the three constructs, there was no evidence to support that the change was due to the RUMEP intervention alone. The results of the case study teachers showed mainly positive changes in their professed beliefs. On the Role of the Learner, the three cases Lulama, Makana and Ruth, changed their beliefs positively. On the Role of the Teacher, Lulama and Makana changed positively. There was no change in Ruth’s beliefs on this construct. Lulama was the only case study teacher to record a change on the Sequencing of Topics construct. Overall, Lulama, Makana and Ruth positively changed their professed beliefs.

**School Level Environment Questionnaire (SLEQ)**

The group of teachers that participated in the questionnaire came from the same school and many of them had been involved in RUMEP programmes. Of the school mathematics staff of six, two were female. The table following provided the means and standard deviations of each of the eight dimensions contained within the SLEQ. (See Appendix D for the Actual and Preferred questions). The table following also provided the statistics for the Actual and Preferred versions of SLEQ for the group of six mathematics teachers. The values of the mean scores of all eight scales ranged from 13.0 to 24.0 for the Actual sample and 12.9 to 26.1 for the Preferred sample.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Preferred</td>
</tr>
<tr>
<td>Student Support</td>
<td>23.3</td>
<td>24.3</td>
</tr>
<tr>
<td>Affiliation</td>
<td>22.6</td>
<td>22.4</td>
</tr>
<tr>
<td>Professional Interest</td>
<td>24.0</td>
<td>25.4</td>
</tr>
<tr>
<td>Staff Freedom</td>
<td>17.0</td>
<td>12.9</td>
</tr>
<tr>
<td>Participatory Decisions</td>
<td>21.3</td>
<td>23.3</td>
</tr>
<tr>
<td>Innovation</td>
<td>23.4</td>
<td>25.6</td>
</tr>
<tr>
<td>Resource Adequacy</td>
<td>13.0</td>
<td>26.1</td>
</tr>
<tr>
<td>Work Pressure</td>
<td>18.3</td>
<td>17.9</td>
</tr>
</tbody>
</table>

N=6

Table 3: Scores of Actual and Preferred values on all eight scales.

The standard deviations from the questionnaire regarding how teachers perceived their actual environment appeared to be spread rather wide, indicating that the individual participants perceived their school environments very differently to their
colleagues; especially when it came to Student Support and Affiliation. This might have indicated the different personalities amongst members of the mathematics staff as some individuals may have found developing personal relationships between staff and learners easier than others. It could also have been an indication that certain individuals did not value personal relationships as highly as others. Another aspect that came to mind was whether the dimensions of Student Support and Affiliation were more important to female staff than to male. I would have required a larger sample of teachers for this assumption to be verified. The teachers in the group seemed to agree that their professional interests were currently being met (std = 1.8) and that they were generally open to trying new teaching strategies (std = 2.4).

The profiles featured below showed interesting results. Given the history of neglect in the previously disadvantaged schools, I was not surprised to see sizable score differences between actual and preferred forms in the area of Resource Adequacy. There was also a substantial difference when it came to Staff Freedom. The following figure provided mean profiles of all eight dimensions.

![Profiles of mean school environment scores on the SLEQ](image)

Figure 1: Profiles of mean school environment scores on the SLEQ.

Arising out of the SLEQ Profile I selected for comment the two dimensions of Staff Freedom and Resource Adequacy. The profile indicated that these were the two extreme dimensions. The teachers in the sample appeared to want more structure and control in their classrooms particularly in the areas of the mathematics syllabus and
lesson plans. This was despite the fact that the policy in South Africa was to provide teachers with mathematical guidelines rather than prescriptions (Department of Education, 1997). Preliminary discussions with the six teachers revealed that there was uncertainty around implementing the new curriculum. They were unsure of how to plan units of work, assess their learners and use the curriculum guidelines as set down in the policy documents. An issue related to this research concerned the extent to which key teachers supported themselves through contact with other teachers in implementing the revised curriculum. Newman (1998) asserted that there could be a “culture of isolationism” that exists in school practice (117). There could be, for example, competition between members of a school staff that lead to non-collaboration. A school environment could foster isolationism by promoting excessive competition between it and other schools through cultural, sporting and academic endeavours. Through the case study teachers, I was able to get some measure of how teachers in a cluster of schools assisted the planning and practice of colleagues in the cluster.

Concerning the dimension of Resource Adequacy, Adler (1998) asserted that a resourced teacher was one that used the resources that were available “…and not simply a teacher surrounded (or not) by resources” (p.1). Adler, (1998) argued that the emphasis must shift from focusing on “more” resources to how “…we use the resources we have, how this changes over time, how we integrate new resources into our practices, and with what consequences”(p.2). I believe the need was to develop reflective practice in all the key teachers so that generating meaningful classroom based practice became the primary focus for the teachers rather than the lack of resources.

Insight Tasks
The Insight Tasks set out to be a close approximation of what the key teachers would see in their professional capacity in their classrooms. Darling-Hammond and Snyder (2000) supported the incorporation of assessment into the curriculum of teacher preparation on the grounds that it may well increase the chance of knowledge and skills being better integrated and applied in the classroom. The Insight Tasks were incorporated into the research design in an effort to build an image of the professional knowledge of the case study teachers. The cohort of key teachers that
included the case study teachers completed two Insight Tasks, the pre-test at the start of the First Phase, and the post-test at the end of the Second Phase of the research. The Insight Task results for the three case study teachers were as follows:

<table>
<thead>
<tr>
<th>Insight Task Results (in percentages).</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lulama</td>
<td>32</td>
<td>59</td>
</tr>
<tr>
<td>Makana</td>
<td>45</td>
<td>68</td>
</tr>
<tr>
<td>Ruth</td>
<td>83</td>
<td>91</td>
</tr>
<tr>
<td>Cohort average</td>
<td>46</td>
<td>64</td>
</tr>
</tbody>
</table>

I was not surprised to see the substantial gains for the cohort as a whole, and in Lulama and Makana in particular. The poor marks in the pre-test were expected as the concept of an assessment test that was aligned to a constructivist orientation in the form of an Insight Task was a new and challenging experience for the cohort as a whole. Ruth was the exception. Her results showed that she had a clear professional understanding when she entered the course and her competence had grown at the end. Over the duration of the research, Lulama and Makana made gains of 27 and 23 percentage points respectively. Although Lulama’s mathematical content knowledge developed over the period of the research, her classroom practice suggested shortcomings when exposed to learners’ different solution strategies. I detected from the Insight Tasks that most members of the cohort experienced similar difficulties in providing precise professional advice. Having discussed these issues with colleagues, I perceived that the ability to provide relevant professional guidance was dependent on the mathematical “insights” that a teacher was capable of bringing to the question. I further considered whether teachers from other contexts, or other countries even, might be equally challenged to provide appropriate professional guidance. Ruth, on the other hand, had shown in her response to the Insight Task that she had the professional knowledge to be able to give perceptive guidance.

I incorporated the Insight Tasks into the research design as a means of using the assessment to broaden and deepen the professional knowledge of the key teachers. The Insight Tasks were not only a means of assessment but created opportunities for learning. My concern was to devise a form of assessment more compatible with an
enhanced vision of mathematics learning. In drawing on learner solution strategies as a framework for creating the Insight Tasks, I aimed to align mathematics teacher assessment to principles found in the Constructivist Learning Model (CLM). A shortcoming I perceived in the Insight Tasks was that the questions tended to focus on the content area of number and number operations. To become a valid assessment instrument, I needed to broaden the range of questions to include the content areas of geometry, data handling, patterns, and algebra.

The results on the final Insight Task showed Lulama was below average and Makana was slightly above the cohort’s average level of professional competence in answering the questions posed in the test. Ruth demonstrated the insights of a competent professional in that she had understood the learners' thinking and met the demands of the mathematical subject matter. The post-test results revealed that the majority of the key teachers had demonstrated acceptable levels of mathematical pedagogical competence. To gain the views of the key teachers towards the Insight Tasks as a means of assessment, they all completed a questionnaire. The responses of Lulama, Makana and Ruth quoted below were in answer to the question: “You were given Insight Tasks to do instead of a traditional mathematics test. Give your comments on the Insight Tasks as you see them.”

Makana wrote:

_The Insight Tasks were of help and broaden my mind to know what mistakes learners do when solving problems. I also learn the way to remedy the situation. These insight tasks helped me to deal immediately with the situations not to mark the learner right or wrong answers. These tasks assisted me to know different strategies used by the learners._

Lulama wrote:

_They are more challenging compared to traditional approaches. A learner is not compelled to know one method to solve a problem. He is free to use whatever strategy he is comfortable with as long as he is able to understand his solution. Learners are given a chance to construct what they think in a meaningful way. The way to solve a problem is important and not the answer that we should always look at. Different strategies are welcomed._
The final comment was by Ruth:

The tasks give me insight into a variety of strategies that can be used to solve problems, thus emphasising there is never only one way of finding a solution. The tasks have made me move away from the idea that only one method (and that is usually the educator’s method) can be used and therefore any method I should come across would be wrong. The tasks have also made me aware of taking more of what learners do, why, and how they do something - to actually look into it and ask them questions to understand their thinking, their understanding and what meaning they have developed through what they have done.

The insight tasks have made me improve and develop my thinking skills and refine my problem-solving strategies. What I have experienced with the insight tasks I have also experienced with my learners in that learners come up with different ideas and strategies and I have had to work with these as the learners had done them or guide learners to help them improve their own strategies. Fortunately, the insight tasks have prepared me well for this and I am able to guide and encourage my learners confidently. My learners also feel pleased that their ideas and methods are listened to and 'accepted' and therefore are prepared to try out other ideas and suggestions.

An analysis of the comments above indicated that the Insight Tasks had caused the case study teachers to look closely at the learners’ solution strategies and had required them to consider carefully their advice to the learners. Their comments suggested that they were sensitised to the thinking and reasoning of the learners and that the learners were striving to make sense of their mathematics and not simply following a prescribed procedure. The questionnaire revealed that the case study teachers believed that the Insight Tasks helped them in the classroom.

Chapter summary:

Qualitative results

Arising out of Lulama’s responses and from supporting comments by the team, specific beliefs emerged relating to Lulama’s practice seen in the video recording. Lulama’s strategy of placing learners in groups and assigning group roles revealed a
belief in active participation, collaboration, and learner discourse in an interactive social setting. The videotape produced clear evidence of her learners using a variety of solution strategies, some of which the learners themselves devised, and others that had been derived. Although Lulama encouraged creative thinking in the learners, many of their solution strategies required refining by her to make learners’ mathematical thinking more efficient. Well prepared, and well-chosen work cards was evidence of careful planning. As an aspect of her preparation, Lulama had considered the need for a repertoire of assessment techniques. Included in the repertoire were regular learner reflections, some as group reports and some as written reflections. Lulama believed that power was shared in her classroom and that learners should take responsibility for their own learning and not depend entirely on the teacher. She firmly believed that she had changed as a teacher because she had found that a learner-centred approach had worked well for her.

The setting for Makana was different in that he taught in a rural multigrade class in a farm school. His comments indicated that he believed in an interactive, participative approach to the teaching of geometry. Learner collaboration, and group discussion was a feature of his lessons. Apart from the verbal reports, the learners conversed amongst themselves in their home language, Xhosa. Makana used a bilingual Xhosa/English strategy when wanting to consolidate mathematical concepts. Makana believed in his learners investigating and taking responsibility for their own learning. He preferred a “hands-on” approach where the groups explored and then presented their findings to the class, even if he was sometimes unsettled by a learner’s unexpected solution strategy. He was aware that mathematical misconceptions in learners needed to be rectified and that he was at fault in not recognising the learner’s error. He recognised that teacher reflection was a means to remedy such errors and he believed that his own mathematical content knowledge had improved, particularly in geometry. He promoted learner reflection as a means of building learner understanding. He believed that other members of the cluster had been a motivating influence on him in terms of planning, management, content knowledge and teaching style, and it was largely due to their willingness to support him that he had become a learner-centred teacher.
Ruth adopted a teacher-learner participation teaching style that she believed worked well for her learners, and that a regular teaching routine was a comfort to them. She believed that her verbal teaching style was not one-way transmission but two-way interaction that benefited the learners. As a teaching strategy, she provided space for learners to develop their own solution strategies but some were reluctant to try, as they required the security of methods advocated by the teacher. Power was distributed in her class through a democratic process of making decisions that included the participation of all learners and the teacher. A feature of Ruth’s teaching was the manner in which she structured the lesson around the learners’ prior knowledge, which she believed was her teaching strength. She remained aware of the needs of her learners and sensitive to their socio-economic environment.

*Quantitative results*

The Belief Scales set out to determine changes in the beliefs of the cohort of key teachers. The results indicated significant changes in the espoused beliefs on the Role of the Learner and the Role of the Teacher subscales for 22 key teachers. It was possible that the shift in their beliefs was due to their exposure to RUMEPI’s mathematics education experiences, but I was not able to substantiate the claim without further evidence. On the evidence available, I assumed that 22 key teachers shifted their professed beliefs on two subscales of the Belief Scale. To determine whether the professed beliefs of Lulama, Makana and Ruth were translated into practice, a more fine-grained investigation of classroom practices was required via observation and video recording.

The SLEQ instrument provided me as the researcher with a structured tool for looking at a school environment to help determine the needs of the school. Importantly, the teachers were given a voice so that their own concerns could be addressed, rather than the concerns of the researcher. The SLEQ was easy to administer and could be used by teachers themselves in small groups, such as the one sampled, or in larger groups. In the context of a school, it provided a starting point for discussions on staff professional development. The questionnaire provided useful supplementary data that informed the research. It was necessary, however, to follow up with classroom observations and discussions with the case study teachers, to build up an image of how mathematics teaching and learning as practice was perceived.
Within the context of a university environment, it was considered a bold step to include the Insight Tasks as an assessment procedure instead of a more traditional mathematics examination. The written responses made by the case study teachers suggested evidence of integration of mathematical knowledge and skills through analysis and synthesis of learners' varied solution strategies. The information that emerged provided me with additional data relating to the professional knowledge, beliefs, and values of the case study teachers.
CHAPTER FIVE

Conclusions

Introduction
In this chapter, I draw together the results of the study. The reader will recall that the aim of this study based on my research question was within the context of curriculum change how do enhanced beliefs and pedagogic knowledge of key teachers influence their classroom practices. In attempting to answer the research question I focused on the professional development of the teacher in a South African context, because I believed it was through empowered teachers that transformed mathematics teaching and learning could take place in our schools. Despite the limitations, I identified themes that emerged from this study that have relevance for teaching and learning in the context of curriculum change in this country, and I recommended ten aspects for possible inclusion in effective professional development interventions. Of particular significance were the findings that either added to published research or findings that were new to the South African research context.

The teacher as agent of change
If I were asked to describe what was typical in most primary mathematics classes in South Africa I would say that I had seen learners spend their time practising procedures presented by teachers, performing calculations, carrying out corrections or going over completed work. The mathematics in which the learners individually engaged consisted of standard algorithms, mechanical procedures and terminology. Most teachers would be using a transmission of knowledge teaching approach, but learners would only be expected to copy and practice through “drill and practice” methods (Stoker, 1991). In most South African schools, learning mathematics had not been exciting, challenging and relevant. Mathematics classes had not typically abounded with dialogue, participation and mental stimulation. Additionally, mathematics had acted as a barrier for most learners, disconnected from everyday worlds and realities of students (Adler, 1991).

South Africa was in a state of transition characterised by transformation in many spheres including education. Nakabugo and Sieborger (2001) confirmed the position
when they stated, "Embraced in this transformation is the significant 'paradigm shift' in the way people think about learning..." (p.54). Differences between the old and new curriculum were expressed in the Curriculum 2005 documents. A major difference stressed in this document concerned the shift from transmission style teaching to a learner-centred outcomes based approach. According to the Department of Education (1997) policy document the old approach was characterised by passive learners, rote-learning, textbook/worksheet bound, and teachers being responsible for learning. Curriculum 2005, on the other hand, was characterised by active learners, learner-centredness, reasoning, reflection, and interaction. The National Curriculum Statement (2001) clearly stated where the emphasis should be placed in the transformed curriculum:

The teaching and learning of mathematics should be interactive and provide opportunities for learners to engage in mathematical discussions and to develop communication reasoning and problem solving skills. It should not be limited only to the transmission and acquisition of mathematical knowledge. (p.12)

The results of this study suggested that the case study teachers did not suddenly abandon one approach for another. There was progress towards an increased use of strategies that were regarded as desirable. Strategies were not adopted unless the educator was confident of its success. The notion of a paradigm shift, or as the Department of Education (1997) described, "from one way of looking to another" (p.6) was questionable if it implied an easy, rapid change in teaching approach. The change that was required to conceptualise teaching as an interactive, learner-centred, teacher facilitated activity required time and support to establish. Setting old and new practices in opposition to each other obscured the reality that there was a gradual movement from one towards the other. If Curriculum 2005 was to make the desired impact in classrooms in the Eastern Cape, much might be gained from paying attention to the issues that emerged from my research concerning the beliefs and classroom practices of key teachers.

Motala, (2001), posed two important questions concerning education in South Africa: "What is quality education and how do we achieve it?" (p.62). She believed that there were strategic areas for enhancing quality within the school setting and one of these was by improving teacher quality. As over 92% of the budget for the Eastern Cape’s Department of Education was allocated to salaries of education
personnel alone, I like Motala, believed that time and energy needed to be concentrated on the professional development of educators. Shulman (1997) maintained that being professional embodied specific principles. He argued that to call something professional was a claim that assumed “grounded knowledge” based in research and was not professional unless the new understandings had been tried and tested in the classroom. It was in schools where professionals applied their practice and it was in the “crucible” of the classroom that professionals demonstrated their understanding of theory and practice.

The Minister of Education (Asmal, 1999), also recognised that teachers were a strong influence for bringing about change in schools when he stated: “It is time to reassert the dignity of the teaching profession, because teachers at their best are vital agents of change” (p.13, my italics). This research study did more than theorise about Curriculum 2005 policy. It was firmly grounded in investigating key teachers’ beliefs and practices in the classroom. I was seeking to make explicit the themes that were most evident in their classrooms. The nature of the professional development intervention was an ongoing series of mathematical and pedagogic experiences designed to bring about change from within. Hargreaves (1994), in similar vein, made a distinction between ‘root’ and ‘branch’ changes. Branch changes represented significant changes in classroom practice and root changes happened when teachers’ beliefs were affected and their understandings underwent redefinition thus resulting in change at a deeper level that was sustainable. It was my contention that root changes occurred only when the teachers were able to articulate reasons for their classroom practices. In this study, the three case study teachers were provided opportunities to voice their beliefs relating to their practices.

**Case study teachers**

The three case study teachers Lulama, Makana and Ruth had similar opportunities to enhance their mathematics and pedagogic beliefs and to implement their beliefs in the classroom. They participated in the same contact sessions, they cooperated with Thandi and me, and they interacted with their own learners over the period of the study. They engaged, however, in varying levels of constructivist practice and consequently different themes emerged from the practice. What distinguished Ruth from the other two cases was how well she conceptualised her knowledge. Lulama
and Makana, in the main, applied the knowledge that they had acquired in the professional development intervention in a relatively static way. Lulama appreciated that her learners were able to construct solution strategies to solve a variety of word problems. She used carefully prepared work cards for her groups and found that her learners solved them using their own methods and were impressive in reporting their findings to the rest of the class. She also used a variety of activities that involved the learners inventing their own solutions, but she was reluctant to help her learners refine their strategies despite the coaching she received from Thandi and me. Lulama admitted that using a learner-centred approach was threatening when learners presented strategies that she herself did not understand.

Makana did respond to coaching and made an effort to refine the learners’ thinking. Neither Lulama nor Makana were always confident of their own mathematics content ability, despite priority given to this aspect in the professional development activities. Makana was also reluctant to implement transformed pedagogic practices during the first six months of the research study and refused me opportunities to visit his classroom, a situation that suggested change in some teachers takes more time than others. It was only in the second six months that he was sufficiently confident for Thandi and me to video record his lessons. All his videotaped lessons were on geometry and included those topics encountered in the contact sessions of the intervention. He now had the confidence to implement a transformed teaching strategy in a topic he had not previously taught. This I considered an example of progress.

A major focus of the intervention was helping key teachers to understand learners’ mathematical thinking based on the conviction that this understanding provided a basis for fundamental shifts in teacher’ beliefs and practices. Similarly, I believed that understanding how teachers changed their beliefs and practices assisted in the planning of professional development activities for the key teachers. The key teachers did not simply assimilate research-based knowledge about teaching and learning mathematics. As with the findings of Cognitively Guided Instruction research (Franke, Fennema, Ansell, & Behrend, 1998), the case study teachers who were most perceptive in determining the learners’ mathematical thinking processes were best able to reposition their own beliefs and knowledge. When I asked what
caused Lulama to change, she said that she tried out a particular practice and found that “it worked.” The learners responded to the work card activity and she decided the practice was worthwhile and should be continued. Makana tried practical geometry activities with learners who had no experience of measuring, cutting and sorting and because they engaged in the activity so well, he continued with the approach. Ruth, unlike Lulama and Makana was able to explain why her practices worked for her and under what conditions and thus provided a basis for her to sustain her development that enabled her to grow. Ruth understood why her practice worked, whereas Lulama and Makana only knew their practices worked and were able to continue the practice. My research supported the assertion by Franke et al., (1998) when they said:

> Whether change for a particular teacher becomes self-sustaining and generative depends not just on the principles inherent in the professional development program but on the understanding of the principles constructed by the teachers. Thus, teacher change may not be captured in the experiences the teachers have engaged in but in the meanings they have constructed. (p.68)

Ruth’s classroom practices provided rich evidence of how she built on learners’ mathematical knowledge and in so doing, brought content knowledge and pedagogic practice closer together (Ball, 2000). Ruth had built for herself a clear understanding of the principles of reformed practice.

**Limitations of the research**

A shortcoming of this research was that I observed the classroom practices of only three key teachers. Ideally, I would like to have made in-depth observations in the classrooms of all 34 key teachers in the cohort. As the key teachers were drawn from a variety of contexts, I expect the results would have been equally varied. During the First Phase of the research, I as an English speaker alone carried out observations in the case study teachers’ classrooms. As I later realised it was more efficient to have Thandi, a Xhosa speaking colleague, on each visit to interact with the learners and their teacher. In formulating the implications for teaching and learning, I was conscious that the meta-themes needed to be treated as tentative as they were derived from my interpretation of the evidence in the First Phase. A further methodological consideration might have been to include the learners’ perspective on constructivist-style, outcomes-based teaching, which would have added another dimension to the
data. Capturing such data, however, from primary school learners would have been a challenge.

Having had the benefit of external observers to review the videotapes in the Second Phase, I believe in retrospect, it might have been a better research design if I had used video recordings in all the classroom observations. I also believe that the meta-themes to emerge from the panel observations of the videotapes would have been strengthened if the panel had been able to allocate more of their time to reviewing all the videotapes and not just one from each of the cases. A procedural difficulty arose with the panel of observers in interpreting the Constructivist Learning Model. It involved lengthy discussion before the panel was satisfied with the wording of the final format. A further shortcoming was the inability of Thandi to attend the final video review session. From a technical perspective, if I had had the technological facilities to transfer the video data to C-D Rom, the reader would have benefited from being able to see the case study teachers and their learners in their specific classrooms contexts. This facility would have added to my written descriptions and would have provided a visual image, which would have been an added resource for the reader. The reader will find the original observation reports and videotapes used to capture data in the case study teachers’ classrooms in the Rhodes University Mathematics Education Project library (see final entry under References).

With hindsight, I realised that the choice of the Belief Scale was problematic. The CGI subscale Sequence of Topic did not fulfill the requirement I was hoping for. It did not adequately measure the notion of ‘pace and progression,’ which had become so important with the introduction of an outcomes based curriculum. I believe that it would have been more appropriate to use a scale for primary teachers of the type developed by Howard, Perry and Keong (2000). Perhaps the Belief Scale and the SLEQ instruments might have been more easily interpreted if they had been translated into Xhosa.

Finally, it would be of interest to extend the observations of the key teachers to establish how well they had sustained a constructivist teaching approach in their classrooms. My research study took place over a 12-month period. Given the fact that support structures via the clusters were still in place, it would be informative to
see how committed teachers were to a transformed approach to teaching when the stimulation of the video camera was no longer present.

**Implications for teaching and learning:**

As a research methodology, the videotaping of case study teachers’ lessons followed by interpretation by a panel of consultant observers (member checks) was a strength in the research design, as it provided a different perspective to the emerging themes other than my own. The follow-up review by the case study teachers of their videotapes was also an added strength to the design as it provided a further perspective on the emerging themes. The different case study teachers whose work I had reviewed spent time sharing understanding of their classroom experiences, which involved consideration of the detailed practices of teaching. Although the major research methodology was the participant observation in the case study teachers’ classrooms, useful data was gathered on the full cohort of key teachers using quantitative methods. The Belief Scale indicated changes in key teachers’ beliefs over the course of the intervention on the subscales of The Role of the Learner and the Role of the Teacher. The SLEQ instrument highlighted a selected school’s curriculum need for improved resources and the teachers’ uncertainties about implementing the new curriculum.

The Insight Tasks showed improvement in the key teachers’ content and pedagogic knowledge and revealed areas of weakness in certain individuals. The key teachers responded positively to the authentic tests based on learner solution strategies. Analysis of all the key teachers’ comments indicated that the Insight Tasks caused them to look closely at learners’ solution strategies and required the teachers to consider the advice given to the learner. It made the key teachers aware that learners were trying to make sense of their mathematics and not simply follow a procedure laid down by the teacher. Alde, one of the most competent key teachers, summed up the Insight Task experience in these terms:
It stimulated my interest because it forced me to look at how knowledge was constructed within the learners and how I can augment that process. I really had to dig deep in my own resources and understanding to provide the correct guidance after the analysis of the strategies. The refinement of strategies had a big influence on me because one needs to put it into proper perspective i.e., whether it is too elementary or too intricate. One other aspect of the insight task was the fact that one really gained insight into the way learners made sense of knowledge!

(Alde: a key teacher. Underlining in original.)

Darling-Hammond and Snyder (2000) made the point when they said, “If teacher education is professional education, it should prepare candidates to take into account the different needs of students...when making decisions,” (p.528). Through the Insight Tasks, I integrated learning and assessment and stimulated a revised expectation of assessment. It was a strategy designed to develop professional competence in key teachers and not just measure results. I believe that by incorporating Insight Tasks as a component of the curriculum, the professional knowledge of key teachers was broadened and deepened. The Insight Tasks were not seen as ‘add-ons’ at the end of a course but as an integral part of the intervention.

Qualitative Themes

The following meta-themes emerged from the qualitative data:

- **Concerning the constructivist learning environment.** Crawford and Deer (1993) asserted that most teachers continue to teach as they were taught and that educational theory had little influence on teachers’ beliefs and limited impact on their professional practice. A first requirement for changing professional practice argued Ball (1988) was to ‘unlearn to learn,’ which meant unlearning discredited theories of teaching and learning mathematics. Provision was made for key teachers to ‘unlearn old theories’ and experience learning about teaching through experience. Through the influence of Piaget, Vygotsky and the situated learning researchers, the key teachers were exposed to experiences that included working in cooperative groups, reflection, self-directed enquiry, engaging in mathematical activities, negotiating meaning, reporting back, and they gained experience of
educational decision-making. I believe that for key teachers to translate theory into effective practice required the RUMEP personnel like Crawford and Deer, (1993), to “practise what we preach” by implementing constructivist approaches in our preparation of the key teachers. According to Richardson (1999):

Constructivism in teacher education had only recently entered the writing and thinking of practitioners and scholars in the field. This has come about in large part because reformers interested in changing the nature of pedagogy in the schools from a transfer-of-knowledge model to constructivism began to realise the necessity of preparing teachers in the constructivist forms. (p.149)

The evidence from the three case study teachers was that they were implementing constructivist approaches to teaching to varying degrees in their classrooms and the videotape analysis confirmed this finding. I considered this a measure of their belief in constructivist learning theory.

- Concerning learner-centredness. As has been noted, a significant feature of Curriculum 2005 was the transformation from teacher-centred to learner-centred instruction. Learner-centred practice could be interpreted as a principle of constructivism, the epistemology that underpinned the RUMEP intervention. In the three case study teachers’ classrooms, there was a belief in learner-centred teaching and not as Jessop and Penny (1998) reported, “the rhetoric of child-centred learning,” as is frequently found in a “rules without reasons, instrumental frame” (p.397). Evidence from this study revealed that learners actively participated, discussed, made cooperative decisions and took responsibility for their own understanding. Vavrus (2001) quoted Darling Hammond who was adamant about the need for professional development programmes to support “practices that are learner-centred and knowledge-based rather than procedure-oriented and rule-based” (p.646, italics in original). The key teacher intervention used the Curriculum 2005 document to drive the policy of learner-centredness as a means of transforming classroom practice.

- Concerning participation. A constructivist framework for learning suggested that students actively participate in building their own understandings and that the educator was required to carefully select appropriate activities. In the classrooms of Lulama and Makana, there was evidence of well-chosen
activities in which the learners willingly and actively participated. The learners themselves influenced the activities in Ruth’s class as she steadily built on their own understandings. The learners of all three teachers were challenged to engage their higher mental capacities as they worked through the prepared activities and sought to find solutions for themselves. Activity by itself was not considered enough, as the activities demanded a degree of mental involvement. Active learner participation was a firmly held belief in the three cases.

- **Concerning collaboration.** Learner collaboration was not seen as an end in itself but as a context within which learners were encouraged to question, rethink, and revise their mathematical understandings. In the three cases, opportunities were provided to learners for group planning, related discussion and reflection, and making joint decisions with partners in the group. From a constructivist point of view, knowledge implied understanding, and that knowledge could be shared with others in a community of learners. Although I hoped to see negotiation between learners, there was more evidence of discussion taking place in their groups. Discussion helped learners to escape from their own individual perceptions of the world. It added to the richness of their understanding and enabled members in a group to make contact with the minds of others in a direct way. Learners in the three classes were encouraged to communicate their thoughts on a mathematical activity resulting in a richer environment for each individual.

- **Concerning reflection.** On completion of an activity, a review of the learning achieved was perceived as being useful to the learners. When learners presented their reports in Lulama and Makana’s class, a mechanism was in place for making sense of the activity and locating it in a wider framework of meaning and purpose. For the reporting learner it was an opportunity to clarify personal theories and for the rest of the class it was an opportunity to create new meanings. In Ruth’s class, there was the added incentive that learner presentations would be incorporated into the fabric of the lesson often in the form of a documented chalkboard summary. The written reflections from the learners in Lulama and Makana’s classes were touching in their attempts to describe in disjointed English their mathematical findings.
Concerning teacher content knowledge. It was noted earlier that the Association for Mathematics Education of South Africa (AMESA, 2000) submission to the Minister of Education stated there was a crisis in mathematics education in South Africa, and that teachers’ content and pedagogic knowledge required upgrading in an effort to alleviate the situation. The RUMEPI intervention identified with the AMESA submission and made the development of teachers’ content knowledge a priority. Key teachers were immersed in mathematical experiences that gave them opportunities to see how ideas in mathematics were connected. The key teachers came with widely varying knowledge of mathematics and the link between concepts embedded in an investigation was not obvious to many teachers in the group. The “why” questions to those teachers with limited mathematics knowledge made them feel insecure. A lack of adequate content knowledge revealed itself when learners presented their various findings. Although Lulama and Makana listened to learner explanations, they did not see how to build on these explanations or to help learners refine procedures used in the inquiry. Lulama, particularly, was uncertain of multiple alternatives for solving problems. I recognised that an open-ended, learner-centred style of teaching was disconcerting to the mathematically less confident teacher. Lulama and Makana felt insecure but they believed that a learner-centred approach was better in the end for their learners. The reality in South Africa was that teachers’ limited content knowledge contributed to the crisis in mathematics education. Taylor and Vinjevold (1999) highlighted the issue when they stated, “Teachers’ poor conceptual knowledge of the subjects they are teaching is a fundamental constraint on the quality of teaching and learning activities…” (p.230). Techniques employed by key teachers to assist them develop content knowledge whilst in the field were through constant written reflection of the mathematics activities in a personal portfolio, and through consultative discussions with fellow teachers in collegial clusters. The key teachers were aware of the need to constantly improve their mathematical content knowledge.

Concerning pace and progression. The evidence from Ruth’s classroom was that there was momentum and direction in her teaching. I believed that she would sustain this momentum with her current class or with any other class
that she taught as her teaching reflected clear decision-making and progression. Within a particular topic, she demonstrated vision and coherence as she steadily built on learner’s prior knowledge and facilitated their making of mathematical connections. Lulama and Makana struggled with pace and progression and were haphazard in their sequencing of topics. They tended to skip from one activity to another without due regard for the needs of the learners. The shift from content knowledge to be covered in a prescribed syllabus to the achievement of learning outcomes in an outcomes-based curriculum was too big a step. Added to this, the insufficient emphasis given to this issue in the contact sessions of the intervention contributed to their apparent lack of direction. The need to provide opportunities for planning systematic pace and progression guided by the assessment standards of the National Curriculum Statement emerged as an important requirement for future interventions.

- Concerning power relations. The image that most key teachers held before engaging in the intervention was that of a classroom with a teacher in charge of students who conformed to the teacher’s rules, regulations and standards. In a classroom such as this, the teacher had the power and the learners were compliant and powerless. A power hierarchy exemplified by the authoritative role of the teacher and the passive role of the learner characterised a traditional classroom. According to Crawford and Deer (1993), most people did not think of the classroom “...as a kind of political organisation” and “...the power relationships within them influence the kinds of learning that occur” (p.112). Before the advent of Curriculum 2005, the authoritative role of the teacher and the passive role of the learner were unquestioned in South African schools. Because of the influence of constructivism, knowledge was now recognised as a socially constructed outcome. Like Boaler (2002), I believed in a pedagogy that distributed power and I agreed with her when she said we needed to help “...mathematics teachers replace ‘pedagogies of poverty’...with pedagogies of power” (p.256). The strategies implemented by the three case study teachers clearly indicated that power was shared between the educator and their learners. Power sharing encouraged learners to take ownership of their own solution strategies, present verbal reports, write personal reflections and scaffold the learning of fellow learners.
**Recommendations for Professional Development**

In this study, I investigated the classroom practices of key teachers who had participated in the RUMEP intervention. I assumed that the intervention’s professional development activities influenced the case study teachers’ beliefs and practices. I was made aware that if transformation was to take place in a classroom, teachers needed to construct their own professional knowledge and not simply rely on knowledge received from others. Although this study focused on the classroom practices of three selected teachers, specific aspects concerning the intervention were significant and I recommend the following features be incorporated into effective future professional development interventions:

1. Allow teachers to construct their own knowledge through immersion in mathematical processes. Teachers, like classroom learners, learn mathematics better by investigating for themselves and building their own understanding as opposed to being required to memorize known knowledge (Hewson, 1999). Teachers require an in-depth knowledge of mathematics concepts, not just coverage of the national curriculum statements.

2. Strengthen teachers’ knowledge of how learners best learn mathematics. While deep thorough knowledge of mathematics content is important, teachers must also know about constructivist learning. They must know how to listen to learners’ ideas, pose questions that move them further, and help them develop stronger concepts. Teachers require relevant up-to-date pedagogic knowledge (Curriculum 2005).

3. Assist teachers to understand how they might make informed decisions on curriculum content, planning, pace and progression and implementation within the context of a transformed education system (National Curriculum Statement, 2001).

4. Provide teachers with opportunities to work collaboratively in groups, to engage in discourse about mathematics teaching and learning and to observe specialist teachers modelling constructivist strategies. Teachers need situated experiences with such strategies as group roles, inquiry-based learning, negotiating, reflecting and reporting (Loucks-Horsley et al., 1998).
5. Provide mathematics teachers with opportunities to develop, reflect and
practise new knowledge in their own context. Be aware that deep learning
requires sustained effort and takes time.

6. Into the design of the professional development interventions, build in
opportunities for regular follow-up activities. Professional development
should provide teachers with a structure of ongoing support for reflecting
on their learning, and gaining feedback on the changes they make.

7. Professional development in South Africa is often via accredited courses.
Many of these courses are distance-based with limited potential to impact
on teaching and learning. Effective courses require a design structure that
includes face-to-face contact as well as regular on-site follow up
arrangements.

8. Explore opportunities to learn mathematical and pedagogic content that
are situated in the context in which the subject matter is used. Use
learners’ authentic recorded solution strategies as a site for teachers to
analyse, interpret, and give professional advice in order to refine and
improve the learners’ solution strategies (Insight Tasks).

Points 9 and 10 were not an outcome of the study. I included them as
suggestions derived from the review of the literature.

9. Promote collaborative professional exchange through collegiality
(collegial school clusters). With limited external support being available
from official sources for individual teachers, teachers need to support
each other and enrich each other’s work. Teachers in collaboration with
Department of Education personnel should work together, to inquire into
questions of common interest and to share what has been gained from
workshops and conferences.

10. Develop an ‘agents of change’ approach. The school’s culture should
reflect an expectation that teachers will strive for transformation and
improvement. Teachers should have opportunities to develop the
knowledge and skills they require as change agents, so that they can work
competently, and effectively take on a broader mission with others in the
school and in a cluster of schools (Key teacher development).
Significance of the findings
The results of my study have either added to the literature or have contributed new information on the professional development of mathematics educators. A significant outcome of the study was the ten itemised components that might be incorporated, either in part or as a whole, into professional development interventions within the Eastern Cape, or further afield. These aspects were significant because they were not theoretical. They arose out of situated practice, which included the link between the social context and the acquisition of knowledge (Brown, Collins, & Duguid, 1989). The focus was on key teachers developing pedagogic and mathematical content understanding through the authentic activities of seeing how others tackle problems, scaffolding, coaching and ‘participation within a community of practice’ (Lave & Wenger, 1991). I agreed with Hargreaves (1994), when he argued that collaboration required “teacher development beyond personal, idiosyncratic reflection, or dependence on outside experts, to a point where teachers can learn from each other, sharing and developing their expertise together” (p.183).

The Insight Tasks emerged from this study as a unique assessment activity related to professional development. They have the potential to act as a tool to stimulate professional development of teachers in the South African context. The idea of using Insight Tasks was stimulated by Darling-Hammond and Snyder’s (2000) notion of authentic assessment. Carefully aligned assessments (Webb, 1997) with learner-centred outcomes, based in a constructivist approach added value to the key teachers’ professional development over the period of the intervention. The key teachers were more willing to attend to a learner’s mathematical solution strategy when they knew they would be using an instrument that gave them feedback on their own understanding of mathematics and pedagogic content knowledge. As Ruth commented, “What I have experienced with the Insight Tasks I have also experienced with my learners in that learners come up with different ideas and strategies…” I believe the Insight Tasks developed in this study, contributed to the research literature on authentic assessment.

A fundamental finding arising out of the intervention was to acknowledge that as with the findings of (Franke, Fennema, & Carpenter, 1997; Thompson, 1992; Wood,
Cobb, & Yackel, 1991), key teachers’ beliefs influenced their practices as they shifted to a learner-centred view of learning. It was clear that the task of transforming key teachers’ classroom practices was not a matter of adding to existing techniques but providing teachers with the opportunity to construct ways of thinking and seeing and having a set of beliefs pertaining to a revised image of the nature of teaching and learning itself. The notion of ‘enculturation through appropriation’ (Vygotsky, 1978), in the professional development activities was an element in the process of building key teachers’ content and pedagogic knowledge. The qualitative meta-themes that emerged from the teacher observations added to the debate on how to change teachers’ beliefs and transform their classroom practices.

Earlier, Moll (2002) posed a significant question when he asked how children learn in South Africa. Observation in three classrooms showed that South African learners displayed the same characteristics as those described in international research (Wood et al., 1991; Simon & Schifter, 1991; Carpenter & Fennema, 1992), depending on the beliefs and practices of the teacher. The nature and content of classroom activities was determined by ‘provisions made by the teacher’ (Askew, 2001). If the teacher believed in a constructivist learning approach of the kind described in Yager’s Constructivist Learning Model (1991), and was able to implement constructivist practices in the classroom, then the learners responded accordingly.

A related question was what did learners in school learn if a teacher adopted a constructivist teaching style. The evidence from this study suggested that if learners were presented problems within a ‘realistic mathematics’ framework (Streefland, 1991), they were capable of creating their own solution strategies that made sense to them, and collaborated in ‘joint activities’ (Vygotsky, 1978) in trying to find an answer. The learners took responsibility for their own understanding through exchanging ideas, discussion and reflection when participating with others in a group (De Corte, et al., 1996). Participation metaphors, however, could be misleading when they equated learning with doing. Holt-Reynolds (2000), maintained that there was a perception amongst teachers that learners who were “actively doing things in classrooms are learning while learners who are passive in classrooms are not learning”(p.22). The activity-based tasks in Lulama and Makana’s classroom
suggested that they interpreted good teaching as engaging learners actively and thus overlooked some of the wider implications of constructivist pedagogy.

Of particular significance was the question of how teachers' mathematical content knowledge in South Africa related to their classroom practice (Crawford & Adler, 1996; AMESA News, 2000). The study showed that when mathematics knowledge was not sufficiently deep or flexible the key teacher was unable to help learners construct a deeper understanding of their own knowledge. An important outcome of the study was the need to provide teachers in any in-service intervention, with opportunities to reconstruct for themselves a conceptual understanding of the mathematics found in the National Curriculum Statement (2001). The need for understanding systematic pace and progression within a mathematics topic had relevance for further professional development interventions.

A further significant outcome concerned my research methodology. Although the use of classroom video recordings had been used with small studies of mathematics teaching (Cobb et al., 1992; Schoenfeld, 1988), my treatment of the data was innovative within a context of rural and semi-urban schools in South Africa. Based on the notion of multiple viewers (Pirie, 1996; Goldman-Segall, 1995) I employed a layering technique in analysing the video data. The reviewers revisited the data for further reflection and deeper analysis. I maintained a firmly qualitative approach throughout by allowing themes to emerge based on the perceptions of the various reviewers, without trying to quantify the data. The reviewers in the analysis cycle included a panel of consultant observers, the case study teachers and myself. In using a layered technique, the panel of observers had the opportunity of reviewing my report of their observations, and the case study teachers were given the opportunity to respond to my questions based on a joint viewing of their lessons on videotape. In drawing together these various perceptions of the teachers' classroom practices, I was in a strong position to build a composite picture of the dominant qualitative themes. I believe that my methodology for analysing the video data contributed to a better understanding of the data and is a technique well suited to classroom-based research in this country.
Finally, at the outset, I had preconceived ideas of what I expected from the key teachers and their learners when I first visited their classrooms. These ideas changed as I built up a rapport with the participants. I became more aware of the difficulties learners faced in their various contexts and how the teachers were trying to cope with these difficulties. I came to realise how demanding it was for teachers to implement curriculum reform in a context where learners lived in impoverished home environments and came hungry to school. I also had growing admiration for the teachers as they worked hard at facilitating learner-centred methods with limited resources. I acknowledged that it was easy to be critical of learners’ limited standards of achievement and I had to remind myself of the very low levels of achievement the learners had come from. At the same time, I became increasingly aware of the importance of situating teacher knowledge in classroom practice. Like Lampert and Ball (1998), I was not merely an observer in the teachers’ classrooms but I helped the teachers to plan, teach and reflect on their lessons. By working closely with the teachers, I linked my research to their classroom practices. I agreed with Malone (2000) when he asked for more research on how teachers themselves developed “mathematics content, students’ learning styles, lesson planning, instruction, assessment, and reflection” (p.33). The shared goal between the case study teachers and me was to provide opportunities for them to become conscious of practices that would help them to create theory to sustain their future classroom practices.

Ultimately, the goal was for the teachers to apply these generalisations to uphold a transformed teaching approach. My initial research focus was more on teachers’ beliefs and less on teachers’ classroom practices. As the study progressed, I was drawn more and more towards understanding how the teachers used their professional knowledge to transform their classroom practices.

**Conclusion**

Promoting transformation in schools required an understanding of how teachers’ beliefs influenced how they taught. This study highlighted that transformation of teaching to learner-centred, constructivist practices was neither simple nor straightforward. My findings indicated that the integration of transformed teaching and learning imposed different demands on the teacher, and that carefully considered professional development experiences were required to meet that challenge. Throughout the study, I strived for a disciplined approach with the overall aim of
raising levels of classroom practice. Like Hatch and Shiu (1998), quoted in a review of research in mathematics education by Sierpinska and Kilpatrick, I believed in the statement “research in mathematics education is of limited value unless it affects classroom practice and experience” (p. 297).
REFERENCES


Note:
Observation reports and videotapes will be found in the Rhodes University Mathematics Project library via (m.j.japp@ru.ac.za).
APPENDIX A

Mathematics Belief Scale

Note: For copyright reasons Appendix A has not been reproduced.

(Co-ordinator, ADT Program (Bibliographic Services), Curtin University of Technology, 21/11/03)
APPENDIX B

Belief Scale: Score Sheet

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APPENDIX C

Belief Subscales

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APPENDIX D

School-Level Environment Questionnaire: Actual

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APPENDIX E

Description of eight different dimensions in the SLEQ

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APPENDIX F

Interviews II

We see much group work. Why?
How does working collaboratively help the learners?
Do you give new work cards every day? Why?
Do you believe learners must challenge and negotiate meaning?
How has reflective writing helped you? The learners?
How did discussion and interaction help the learners?
Don’t you prefer a silent classroom?
Why did you get them to record their findings on paper?
Why did you get them to present report backs?
How do you encourage mental methods?
How did you clarify maths concepts after the report back?
Why did you display learners’ work on the walls?
What is your normal teaching style?
Why did you let the learners take their own actions?
Why did you encourage different learner solution strategies?
Did you find learner-centred teaching threatening?
How did you do your planning?
Why are the learner report backs so good in English?
What did you do if you were unsure of the maths content?
How did you encourage scaffolding?
Do you believe in power sharing in your class?
How do you evaluate your learners?
You are a different teacher now. Why?
What has helped you most?
APPENDIX G

Observation Sheet

Classroom focus: How teachers are changing their practice

Name: __________________ Date: __________________

1 Nature of the task:

Mathematical content:
Pace
Progression
Pedagogy:
Situatedness of the task
Cooperative learning
Interaction

2 Role of the learners:
Solution strategies employed
Learner discussion
Learner reflection

3 Role of the teacher:
How does the teacher act?
Classroom design
Teacher’s power

4 Other Issues:
Evidence of collegiality
Classroom atmosphere
What surprised me?
APPENDIX H

Examples of Insight Task Questions

Question 1
Look at the following strategy used by a grade 2 learner:

\[ 46 - 28 = \square \]

0000000000
0000000000
0000000000
0000000000
000000
46 - 28 = 18

How would you help the learner to refine this strategy?

Question 2
Look at the following pattern:

\[ \begin{align*}
2 \times 2 &= 4 \\
2 \times 2 \times 2 &= 8 \\
2 \times 2 \times 2 \times 2 &= 16 \\
2 \times 2 \times 2 \times 2 \times 2 &= 32
\end{align*} \]

If the pattern continues, could 375 be one of the products in the pattern? The following are the learners’ explanations:

Learner One says: ‘yes, because enough 2’s make 375’
Learner Two says: ‘no, 2 is even and even times even equals even’
Learner Three says: ‘no, because 375 is not divisible by 2’

What would you say to each learner?

Question 3

\[
\begin{array}{c}
1 \\
\frac{1}{2} + \frac{1}{2} = 1 \\
6 & \div \frac{5}{5} \\
2 & \div 1
\end{array}
\]

What has the learner done?

What steps would you take to help this learner?

Question 4
Ayanda, a grade 5 learner was given the following word problem to solve:
There are 147 seats in a row at the local soccer stadium and there are 26 rows. How many seats are there?

Ayanda solved the problem like this:

\[
\begin{align*}
147 \times 26 &= □ \\
140 \times 20 &= 2800 \\
140 \times 6 &= 840 \\
2800 + 840 &= 3640 \text{ seats}
\end{align*}
\]

Comment on Ayanda’s strategy.

What advice would you give to Ayanda?

Show two other strategies for solving this problem.

Use the same numbers (147 and 26) to write a suitable, realistic problem for Ayanda to solve.

**Question 5**

Andile solved for \( x \) like this:

\[
\begin{align*}
x - 4 &= 6 \\
4 \\
\frac{x - 2}{2} &= \frac{6}{2} \\
x - 2 &= 6 \\
x &= 6 + 2 \\
x &= 8
\end{align*}
\]

What has Andile done?

Solve it correctly.

**Question 6**

An open-ended problem should allow for more than one solution (or path leading to the solution) without being vague.

Write an open-ended, realistic word problem for the grade that you teach. Give this problem to your learners and photocopy two different learner solution strategies. Comment on the different strategies used by your learners.

**Question 7**

A grade 4 learner subtracted like this:

\[
\begin{align*}
768 - 89 &= □ \\
89 + (1 + 10 + 600 + 60 + 8) \\
89 + (9 + 670) &= 679
\end{align*}
\]

Comment on this strategy.

Record 3 other strategies learners might have used to solve this subtraction situation.
Question 8

8027 ÷ 7 = □

Comment on the two strategies below used by different learners. Which strategy do you prefer and why? Devise your own division strategy for this task.

\[
\begin{align*}
7000 ÷ 7 &= 1000 \\
700 ÷ 7 &= 100 \\
280 ÷ 7 &= 40 \\
42 ÷ 7 &= 6 \\
5 ÷ 7 &= 0 r 5 \\
8027 ÷ 7 &= 1146 r 5
\end{align*}
\]

Question 9

\[
\begin{array}{c}
\begin{array}{cccc}
\array{11} & \array{11} & \array{11} & \array{11} \\
\array{11} & \array{11} & \array{11} & \array{11} \\
\array{11} & \array{11} & \array{11} & \array{11} \\
\array{11} & \array{11} & \array{11} & \array{11} \\
\end{array}
\end{array}
\]

96 ÷ 6 = □

A grade 3 learner divided using a drawing. (See above.)

What has the learner done?

What suggestions would you give the child to refine her strategy?