Drinking-Straw Microbalance and Seesaw: Stability and Instability

Peter Chapman and Leslie Glasser

Citation: The Physics Teacher **53**, 165 (2015); doi: 10.1119/1.4908087
View online: http://dx.doi.org/10.1119/1.4908087
View Table of Contents: http://scitation.aip.org/content/aapt/journal/tpt/53/3?ver=pdfcov
Published by the American Association of Physics Teachers

Articles you may be interested in

Stabilization of sample temperature in a surface-science vacuum chamber to 0.03 K and quartz-crystal microbalance frequency to 0.06 Hz over 0.5 h
Rev. Sci. Instrum. **81**, 103901 (2010); 10.1063/1.3488368

Development of a magnetic quartz crystal microbalance
Rev. Sci. Instrum. **78**, 065111 (2007); 10.1063/1.2749448

Convert Your Common Physical Balance into a Microbalance
Phys. Teach. **44**, 118 (2006); 10.1119/1.2165445

High-stability quartz-crystal microbalance for investigations in surface science
Rev. Sci. Instrum. **74**, 3039 (2003); 10.1063/1.1574395

Novel needle microbalance for adsorption studies
Rev. Sci. Instrum. **68**, 4536 (1997); 10.1063/1.1148427

This article is copyrighted as indicated in the article. Reuse of AAPT content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 134.7.93.129 On: Thu, 09 Jun 2016 01:42:22

Drinking-Straw Microbalance and Seesaw: Stability and Instability

Peter Chapman and Leslie Glasser, Nanochemistry Research Institute, Curtin University, Perth, Australia

The mechanics of a beam balance are little appreciated and seldom understood. We here consider the conditions that result in a stable balance, with center of gravity below the fulcrum (pivot point), while an unstable balance results when the center of gravity is above the fulcrum. The highly sensitive drinking-straw microbalance, which uses a plastic drinking straw as a rigid beam, is briefly described with some slight convenient modifications. Different placements of the center of gravity are considered analytically to explain the equilibrium neutrality, stability, and instability of such beam balances as the microbalance, the playground "seesaw" or "teeter-totter," the "dipping bird," and other toys and magic tricks.

The analytical two-pan beam balance[1] has long been basic equipment in a laboratory (although often now superseded by electronic devices) and appears elsewhere in many different disguises, such as in playground seesaws or toys. In essence, it consists of a rigid beam supported on a fulcrum (or pivot point); weights can be placed along opposing lengths of the beam and adjusted (in position or in weight) to balance the beam in a more or less horizontal position. On the other hand, the now-ubiquitous single pan substitution balance maintains a constant load on the internal beam and fulcrum by *subtracting* weight from the same side of the beam to compensate for the added weight of the sample, thus avoiding errors from any flexing of the beam and fulcrum.

Balances are devices that compare the masses of samples while under the influence of a uniform gravitational field; hence, the measure used is weight, not mass—a beam balance will not operate in the absence of gravitational or centripetal fields. In the diagrams that follow, we thus use w to represent the weights, although much of the more recent literature is couched in terms of the less directly relevant mass, m, and we mention the center of gravity (CofG) rather than center of mass.

While analysis of the mechanical equilibrium of the beam seems only an undergraduate exercise, the issue of the stability or instability of the balance is little appreciated and even less understood. An examination of a well-stocked academic library finds few modern discussions of the mechanics of the balance and almost no consideration of the stability requirements;[2] one must examine older texts[3] or scour the Internet[4] for detailed analysis.

Making and using a practical drinking-straw microbalance is a simple yet valuable experience. The microbalance provides an inexpensive tool whereby very small weights (< 1 mg) and their changes can be measured as an aid in laboratory investigations, while an analysis of the statics of this and like devices provides valuable insights into the behavior

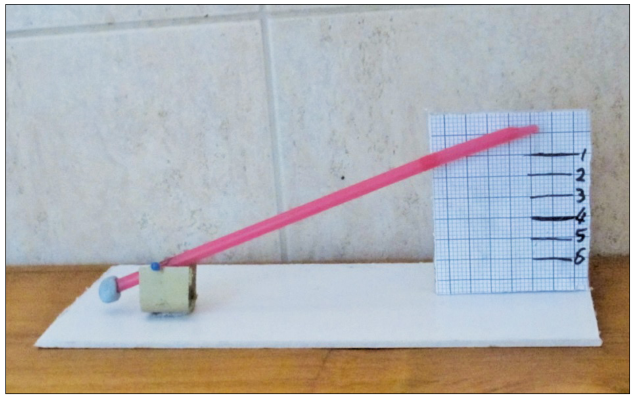


Fig. 1. A drinking-straw microbalance. The U-shaped support is a cutoff from a plastic curtain rod but could be a bent aluminium sheet, for example. The gradations (1-6) 1 cm apart on the scale in the background have been drawn to ease measurement of the deflection of the straw. Calibration (see final sections of the text) permits these gradations to be converted to weights.

of various mechanical systems. The drinking-straw microbalance[5,6] consists of a plastic drinking straw acting as a nearly weightless rigid beam. One end of the straw is appropriately weighted, and a fine pin or needle (which pierces the straw perpendicular to the long axis) is placed at the center of gravity of the beam to act as pivot point (the fulcrum). The fulcrum needle rolls freely on a horizontal surface. The opposite end of the straw is adapted to accept objects to be weighed. If desired, the whole can be placed in a sealed transparent container so that the effects of an applied atmosphere on a sample may be studied.

We have constructed such a microbalance (Fig. 1) by combining published instructions,[5,6] using a plastic channel for the fulcrum support and an easily adjustable ball of malleable material (Blu-Tack, Plasticine, putty, or similar) as the fixed weight. This malleability means that the position of the fulcrum needle along the straw is not critical since the position of the center of gravity is easily manipulated by adjusting the fixed weight. An important issue to note is that the published instructions require that the needle be "put through just above the long axis of the straw."[6] This requirement is not explained in the sources but is crucial to the operation of the microbalance and to other balanced objects, as is analyzed below.

Microbalance statics

A weightless beam balance that is supported by a fulcrum at its center of gravity (CofG) is in *neutral equilibrium* when the Archimedian lever rule equality applies:

DOI: 10.1119/1.4908087

This article is copyrighted as indicated in the article. Reuse of AAPT content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 134.7.93.129 On: Thu, 09 Jun 2016 01:42:22

$$w_a \times l_a = w_b \times l_b\,, \tag{1}$$

where the weights w_i are distant l_i from the fulcrum (Fig. 2). The angle of the beam to the horizontal can take on any value and the system remains in balance. If, however, the center of gravity is placed above the fulcrum (Fig. 3), the beam will be in unstable equilibrium and will tend to rotate toward the vertical, from either side, following any slight imbalance. This follows since gravity acts vertically and so the position of the center of gravity in the diagram is displaced to the right side of the fulcrum when the right-hand side is depressed.

It is interesting to note that this instability is the situation of a playground seesaw (UK) or teeter-totter (US), which bounces from end to end. This instability has, perhaps, led to this balance form being replaced by a safer spring-based version. The dipping bird,[7,8] which has two extreme positions, must also have its pivot above or close to its center of weight, but this seems generally to be ignored in analysis, with focus on its properties as a heat engine.

If the center of gravity is placed below the fulcrum, as depicted in Fig. 4, we have a stable situation, where an increase in the depression of the right-hand side caused by an increase in weight w_b results in the position of the center of gravity moving up the left-hand arm, compensating the weight change by an increase in effective length l_a of the opposite end. In this way, the rotation of the beam is limited. A system with an extended right-hand arm can be utilized as a microbalance with the rotation corresponding to the weight change, as discussed below. A number of balancing toys and magic tricks also rely on the center of gravity lying below the fulcrum.[7,9]

Quantitative beam balance analysis

An equivalent quantitative explanation of stable equilibrium for a beam where the center of gravity (CofG) lies below the pivot point, and a weight W has been added to the beam (as occurs in commercial balances to increase stability—see Fig. 5) may be undertaken in terms of torques. Addition of weight to one end provides torque, which depresses that end but also raises the CofG toward the opposite end, to provide an opposing and balancing torque.

Taking moments about the fulcrum with the weight W of the beam system centered at a perpendicular distance h below the fulcrum yields

$$w_a l_a \cos\theta + Wh\sin\theta = w_b l_b \cos\theta, \tag{2}$$

so that

$$\frac{w_b l_b - w_a l_a}{Wh} = \tan\theta. \tag{3}$$

If the beam is symmetrical, with $l_a = l_b = l$, then the balance sensitivity S is defined by[3]

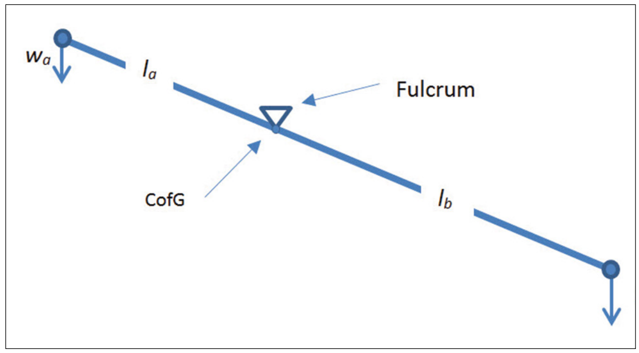


Fig. 2. Balance in neutral equilibrium when $w_a \times l_a = w_b \times l_b$, with the fulcrum placed at the center of gravity (CofG) of the system.

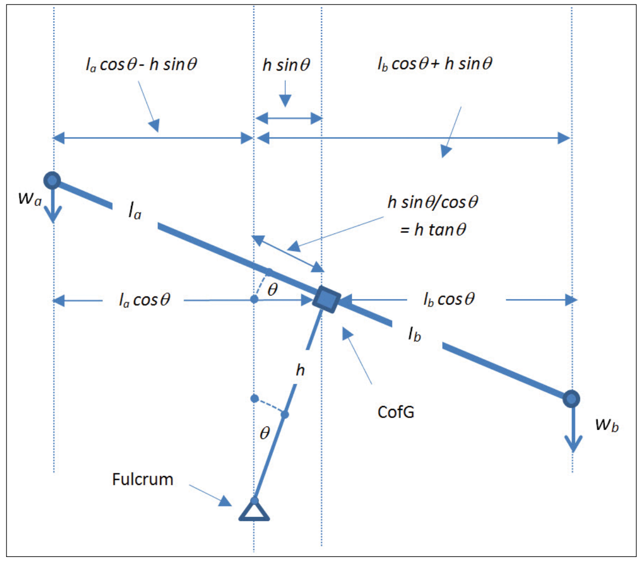


Fig. 3. With the the center of gravity displaced by *h above* the fulcrum, the beam is in unstable equilibrium since the effective length of the lower arm increases by *h* tan θ as the beam rotates by an angle θ about the fixed fulcrum, thus exaggerating the effect of the rotation.

$$S = \tan\theta/(w_b - w_a) = l/Wh \approx \theta/(w_b - w_a) \tag{4}$$

since, for angles less than about 25°, $\tan\theta \approx \theta$ so that S represents the rotation angle per unit weight difference. Thus, the rotation of a pointer lying along the axis of the CofG is proportional to a small weight difference ($w_b - w_a$). A sensitive balance has long arms l and a small overall weight W located close to the axis of the beam (i. e., h is small).

For an asymmetrical balance, using the same symbols,

$$\frac{w_b l_b - w_a l_a}{Wh} = \tan\theta = \frac{w_b l_b}{Wh} - \frac{w_a l_a}{Wh} = c\frac{l_a}{Wh}w_a\,, \tag{5}$$

where c is a constant and l_a/Wh represents the sensitivity S.

Calibrating the microbalance

The microbalance is readily calibrated by finding the deflection caused by a small piece of copy paper, as the following example shows. A metric size A4 sheet of standard copy

This article is copyrighted as indicated in the article. Reuse of AAPT content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 134.7.93.129 On: Thu, 09 Jun 2016 01:42:22

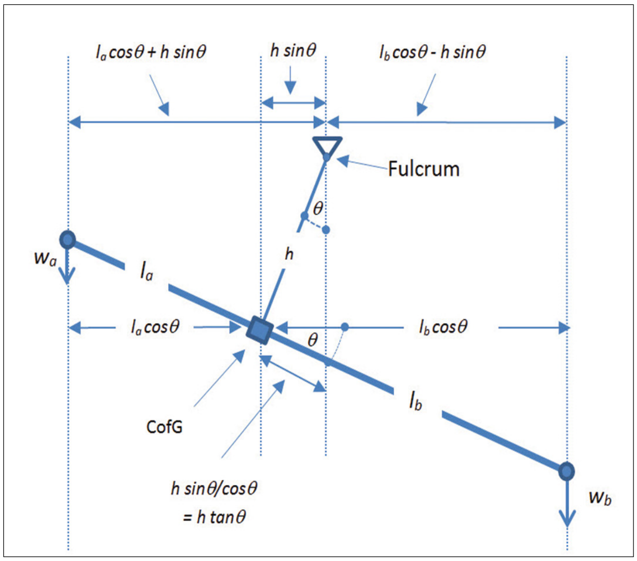


Fig. 4. With the center of gravity displaced by *h below* the fulcrum, the beam is in stable equilibrium since the effective length of the upper arm increases by *h* tan θ as the beam rotates by an angle θ about the fixed fulcrum resulting from the increase in weight w_b.

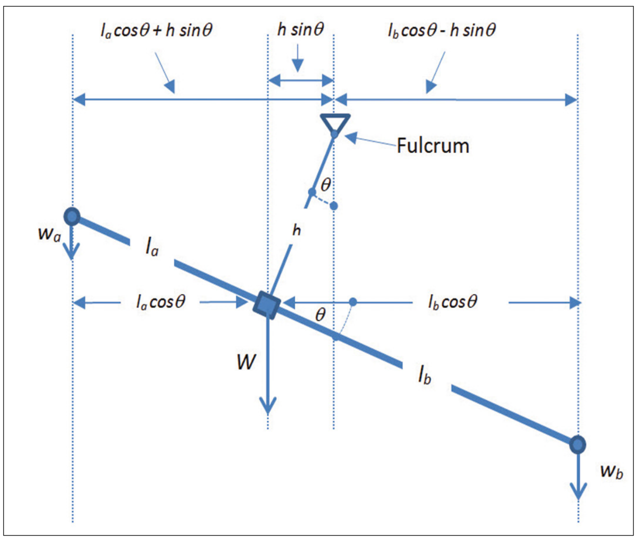


Fig. 5. Quantitative torque analysis of a stable balance beam with stabilizing weight *W*. The beam system (independent of the added weights) has a total weight *W* centered at a perpendicular distance *h* below the fulcrum when the beam is horizontal.

paper has a nominal weight of 80 g $\cdot$ m^{-2} and a nominal area 210 $\times$297 mm^2, that is 1/16th of a square meter. A grid of 10 mm square is easily printed[10] on the paper. An area of 10$\times$ 4 mm^2 carefully cut out of a sheet had a measured weight of 2.85 mg (nominal weight 3.2 mg, which would be the value used in the absence of an electronic microbalance) and deflected the beam end by 4.5 cm. Thus, a deflection of 1 cm corresponds to an added weight of 0.63 mg. It is thus possible to estimate a weight to about 0.5 mg. Students may easily construct the microbalance and use it to determine weights of small objects such as grains of sand, insects and their parts, seeds, etc., as an aid in a laboratory investigation.

Table I. Weights of 10 white sesame seeds, as measured on a microbalance, the deflection of the straw microbalance, and the deflection converted to a weight, using the calibration factor of 1.58 cm/mg, which assumes that deflection is strictly proportional to the added weight.

Sample #	Microbalance (mg)	Straw balance (cm)	Straw balance (mg)
1	3.72	5.7	3.6
2	3.66	6.4	4.1
3	3.32	5.3	3.4
4	5.13	6.7	4.2
5	3.95	5.8	3.7
6	4.25	5.5	3.5
7	4.35	6.1	3.9
8	4.57	5.7	3.6
9	3.39	4.0	2.5
10	4.42	5.9	3.7
Mean	**4.08**		**3.62**
Std. Dev.	**0.57**		**0.46**
Std. Dev./Mean	**0.14**		**0.13**

Using the microbalance

Seeds have historically been used as measures of weight since they are readily available, convenient in size, rather uniform, and constant in weight once dried. Indeed, the carat unit of weight is believed[11] to be named after the carob bean, with a similar weight of 200 mg. A standard measure of seeds is the weight of one thousand.[12] It is thus a useful and appropriate exercise for students to check the variability of a sample of seeds. In the present case, we select the very small sesame seeds.

Table I lists the weights of 10 white sesame seeds weighed on an electronic microbalance and the same seeds weighed on the straw microbalance, with the latter weights calculated with

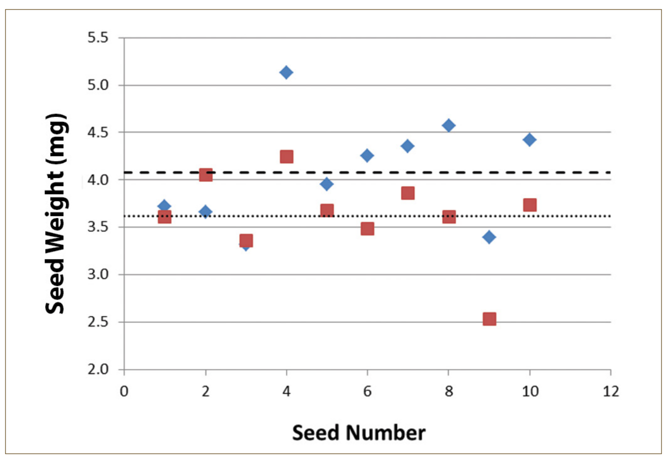


Fig. 6. Plot of sesame seed weights. The diamonds represent the electronic microbalance values with the mean as a broken horizontal line, while the squares represent the straw microbalance values with the mean as a dotted horizontal line.

This article is copyrighted as indicated in the article. Reuse of AAPT content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 134.7.93.129 On: Thu, 09 Jun 2016 01:42:22

Look What's in The Physics Store!

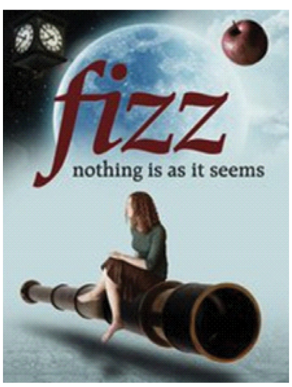
Fizz: Nothing is as it seems

by Zvi Schreiber

A YOUNG WOMAN'S QUEST TO UNRAVEL THE UNIVERSE

The future. In response to environmental degradation, the Eco-community sect eschews science and technology, returning to an austere agricultural life of nature-worship. But one young member, Fizz, struggles to reconcile these doctrines with her own burning curiosity. Risking life and social standing, Fizz embarks on a quest that brings her face-to-face with the often-eccentric giants of physics, from Aristotle and Galileo to Einstein and Hawking. One encounter at a time, Fizz pieces together the intricate workings of our universe, while struggling with the resulting intellectual, moral, and personal challenges.

All proceeds will be used to support AAPT's Student Fund, which primarily goes to the Outstanding Student program!



Members: $7.50
Non-Members: $9.50

AAPT
PHYSICS EDUCATION

Order yours now at
www.aapt.org/store

the single-point calibration established above. The ratio of the standard deviation to the mean of the weights is nearly identical for the two data sets, demonstrating the usability of the straw microbalance. The data are plotted as Fig. 6.

Conclusion

The conditions for equilibrium, instability, and stability of beam balances are here considered both qualitatively and quantitatively. The application of a straw microbalance in the study of the weights of the very lightweight sesame seeds has been demonstrated, yielding a mean weight and standard deviation for a set of 10 seeds. The results from a commercial electronic microbalance are also provided, for reference purposes, but access to such an instrument is not required for use of the straw microbalance.

References

1. "Analytical Balance," Wikipedia, http://en.wikipedia.org/wiki/Analytical_balance.
2. Louis A. Bloomfield, *How Things Work: The Physics of Everyday Life*, 2nd ed. (Wiley, Inc., New York, 2001).
3. Henry Margenau, William W. Watson, and Carol G. Montgomery, *Physics: Principles and Applications* (McGraw-Hill Book Co., Inc., New York, 1949).
4. John Rennie, "Physics Stack Exchange," http://physics.stackexchange.com/questions/34138.
5. Don Rathjen, "Drinking-Straw Microbalance," http://www.exo.net/~donr/activities/Microbalance.pdf.
6. Nuffield Foundation, "Making a Microbalance," http://www.nuffieldfoundation.org/practical-physics/making-microbalance.
7. Arlyne M. Sarquis and Jerry L. Sarquis, "Toys in the classroom," *J. Chem. Educ.* **82** (10), 1450–1453 (2005); Catharine H. Colwell, "Resource Lesson: Center of Mass," http://dev.physicslab.org/Document.aspx?doctype=3&filename=RotaryMotion_CenterMass.xml.
8. Erick Castellón, "Application of the second law of thermodynamics to explain the working of toys," *J. Chem. Educ.* **91** (5), 687–691 (2014).
9. Larry Kirkpatrick and Gregory Francis, *Physics: A Conceptual World View*, 6th ed. (Thomson Brooks/Cole, Pacific Grove, CA, 2007).
10. Incompetech, "Plain Graph Paper PDF Generator," http://incompetech.com/graphpaper/custom/a4-2-10mm.pdf.
11. Lindsay A. Turnbull, Luis Santamaria, Toni Martorell, Joan Rallo, and Andy Hector, "Seed size variability: From carob to carats," *Biol. Lett.* **2** (3), 397–400 (2006).
12. Kew Royal Botanic Gardens, "Seed Information Database," http://data.kew.org/sid/weight.html.

Peter Chapman *is a senior technician in the Department of Chemistry at Curtin University, Western Australia.*

Leslie Glasser *is adjunct research professor at Curtin University and emeritus professor of physical chemistry from the University of the Witwatersrand, Johannesburg, South Africa.*
Nanochemistry Research Institute, Department of Chemistry, Curtin University, GPO Box U1987, Perth, WA 6845, Australia; l.glasser@curtin.edu.au

This article is copyrighted as indicated in the article. Reuse of AAPT content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 134.7.93.129 On: Thu, 09 Jun 2016 01:42:22