Properties of wealth distribution in network systems

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Abstract

We presents a simple model for examining the wealth distribution with agents playing evolutionary games (Prisoners' Dilemma and Snowdrift Game) on complex networks. The Pareto's power law distribution of wealth (1897) is reproduced on scale-free network, and the Gibbs or log-normal distribution for low income population is reproduced on random graph. The Pareto exponents of scale-free network are in agreement with empirical observations. The Gini coefficient of ER random graph shows a sudden increment with game parameters. We suggest that the social network of high income group is scale-free, whereas it is more like random graph for the low income group.

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1 Introduction

Complex networks can describe a wide range of systems of high importance, ranging from nature to society and biological systems. Since the discovery of small-world behavior [1] and the scale-free property [2], complex networks has attracted continuous attention [3]. By representing the agents of a given population with vertices, and the contacts between agents with edges, network theory provides a natural framework to describe the population structure [4]. For example, well-mixed populations can be represented by complete (fully-connected, regular) networks and spatially-structured populations can be associated with regular networks. Recently, many empirical evidences of real social networks have revealed that they are associated with a scale-free, power-law degree distribution, $d(k) \sim k^{-\gamma}$ with $\gamma_{actor} = 2.3 \pm 0.1$ for movie actor collaboration network [5], $\gamma_{science} = 2.1$ and 2.5 for science collaboration graph [6], $\gamma_f = 3.5 \pm 0.2$ and $\gamma_m = 3.3 \pm 0.2$ for females and males in human sexual contacts [7], etc.

It is well-known that even in developed countries, it is common that 40% of the total wealth is owned by only 10% of the population. The distribution of wealth is often described by 'Pareto'-tails (1897), which decay as a power law of large wealths [8]. The rest of low income people, follow a different distribution of either Gibbs or log-normal [10,11]:

$$P(W) \sim \begin{cases} W^{-v}, & for(W \geq W_c), \\ \exp(-\lambda W), & for(W < W_c). \end{cases} \tag{1}$$

where $P(W)$ is the probability of finding an agent with wealth greater than W. The exponent v is called Pareto exponent, and λ denotes a scaling factor. The value of Pareto exponent was found to vary between 1 and 3 for both individual wealth and company sizes [12–17]. Studies on real data show that the high-income group indeed follows the Pareto law, with $v \approx 1.6$ for USA [12], $v = 1.8 \sim 2.2$ for Japan [13,14], $v = 2.0 \sim 2.3$ for UK [11], $v \approx 1.0$ for Japanese firms [18], and $v \approx 0.9$ for India [19].

The Gini coefficient was developed by the Italian statistician Corrado Gini in 1912 to measure the inequity of income distribution. It is defined as a ratio with values between 0 and 1: the numerator is the area between the Lorenz curve of the distribution and the uniform distribution line; the denominator is the area under the uniform distribution line. Thus, a low Gini coefficient indicates more equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. Zero corresponds to perfect equality (everyone having exactly the same income) and 1 corresponds to perfect inequality (where one person has all the income, while everyone else has zero income).

In 1988, the sociologist Robert Merton used the term "Matthew Effect" to describe the phenomenon of the rich gets richer and the poor gets poorer [20]. The "Matthew Effect" are also used in some areas of life such as wealth, achievement, fame, success et al [20–23]. The Matthew Effect for Countries (MEC) was also discovered [21].

In this paper, we study the wealth distribution by using evolutionary games on different networks. The simulation results show the Pareto power-law distribution for the wealthy population and the Gibbs or log-normal distribution for the low-income group. We suggest that the the social network for high-income population and low-income group is different. The dependence of Gini coefficient on game parameters is also investigated.

2 The Model

The previous studies of wealth distribution often adopt an kinetic exchange model in which each agent is a gas molecule and each trading is a money-conserving collision [9–11,26–28]. One can refer to [10,11] for a detailed review of historical data, empirical analysis and models of wealth distribution. These models well approximate a steady economy. However, the total wealth of the system is reserved and will not vary with time [9,12].

The evolutionary games theory has been widely used to characterize some social and biological processes [29–37]. In the typical Prisoner's Dilemma (PD) and Snowdrift Game (SG), two players simultaneously decide whether to cooperate (C) or defect (D). Each player will get a payoff in each step and then the players will choose to change or keep their strategy based on some learning strategies. One can see that both games are intrinsically suitable for characterizing the economic activities such as cooperation, decision, payoff and wealth accumulation [24,25].

Our simulation starts from establishing the underlying cooperation network structure. We consider two different social networks in this paper: the Erdos-Renyi random network and the scale-free network. Starting with N disconnected nodes, the ER random graphs are generated by connecting couples of randomly selected nodes, prohibiting multiple connections, until the number of edges equals L_{max}. The scale-free social network is constructed according to the Barabási-Albert (BA) scale-free network model [2] with the "growth" and "preferential attachment" mechanisms. The BA model well reproduces power-law degree distribution which is in good agreement with the empirical evidence. In this model, starting from m_0 fully connected vertices, one vertex with $m \leq m_0$ edges is attached at each time step in such a way that the probability Π_i of being connected to the existing vertex i is proportional to the degree k_i of the vertex, i.e. $\Pi_i = \frac{k_i}{\Sigma_j k_j}$, where j runs over all existing vertices. In our simulation, we set $N = 10000$ for both kinds of graphs. And we set $m_0 = m = 5$ for BA networks, $L_{max} = 49985$ for ER random network, so that all networks have the same density of links.

In the PD or SG, each player can either 'cooperate' (invest in a common good) or 'defect' (exploit the other's investment). Initially, an equal percentage of cooperators or defectors was randomly distributed among the agents (vertices) of the population. At each time step, the agents play the game with their neighbors and get payoff according to the game rules. In the PD, a defector exploiting a cooperator gets an amount T and the exploited cooperator receives S. Two players both receive R upon mutual cooperation and P upon mutual defection, such that $T > R > P > S$. Thus in a single play of the game, each player should defect [34]. In the Snowdrift Game (SG), the order of P and S is exchanged, such that $T > R > S > P$ and thus SG is more in favor of cooperation. We rescale the games such that each depends on a single parameter [32,33]. For the PD, we choose the payoffs to have the values $T = b > 1$, $R = 1$, and $P = S = 0$, where the only parameter $1 \leq b \leq 2$ represents the advantage of defectors over cooperators. For the SG, we make $T = 1+\beta$, $R = 1$, $S = 1-\beta$, and $P = 0$ with $0 \leq \beta \leq 1$ as the only parameter.

Evolution or learning strategy is carried out by implementing the finite population analogue of replicator dynamics [29,33]. In each step, all pairs of directly connected individual x and y engage in a single round of a given game. The total payoff of agent i for the step is stored as P_i and the accumulative payoff (Wealth) of agent i since the beginning of simulation is stored as W_i. Then the strategy of each agent (Cooperate or Defect) is updated in parallel according to the Richest-Following rule: whenever a site x is updated, a neighbor y with the most payoff P_y in this time step is drawn among all its k_x neighbors (where k_x is the connectivity or degree of site x), and then the site x will copy the strategy of the chosen neighbor y. This mechanism is adopted to reflect the common practice of agents in economy that they will probably learn from richest neighbors.

3 Numerical Simulation Results

We carry out the simulation for a population of $N = 10^4$ agents occupying the vertices of a network. The distributions of wealth, the Pareto exponents and the Gini coefficients were obtained after a time period of $T = 10^5$ steps. For each value of b or β in the PD or SG, we carry out ten times of iterative simulations on ten different network realizations, and the averaged results are presented.

We first examine the wealth distribution $P(W)$ of the system. Figure 1 show the $P(W)$ for BA scale-free social networks. One can see that both PD and SG charts show power-law distribution of personal wealth which is in agreement with Pareto's law with $v = 2.23$ and $v = 2.01$ respectively. We perform different simulations by altering the values of b and β, and the results show similar wealth distributions with robust power law. For different simulations, the exponential factor v varies between 1.8 and 2.5, which is in agreement with the empirical values observed in high-income group [12–14,11]. We also note that the power law persists for both high and low cooperator's frequency

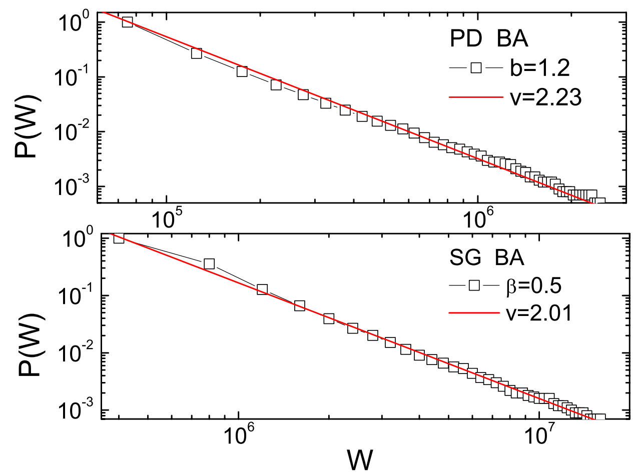

Fig. 1. (color online). Wealth distribution $P(W)$ for $N = 10^4$ agents playing PD with $b = 1.2$ (top) and SG with $\beta = 0.5$ (bottom) on BA scale-free network for 10^5 steps. The frequency of cooperators is 0.10 and 0.65 respectively. The maximum personal wealth is about 5×10^6 and 5×10^7 respectively.

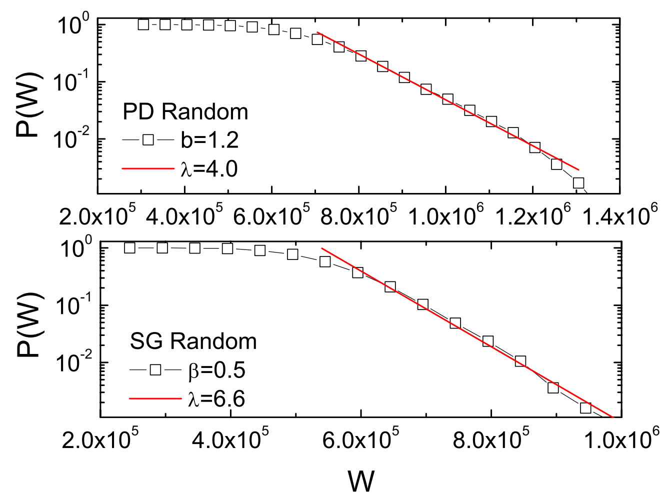

Fig. 2. (color online). Wealth distribution $P(W)$ for $N = 10^4$ agents playing PD with $b = 1.2$ and SG with $\beta = 0.5$ on ER random graph for 10^5 steps. The frequency of cooperators is 0.65 and 0.31. And the maximum personal wealth is about 1.4×10^6 and 1.0×10^6 respectively.

cases. Although the cooperation frequency changes with different parameter of b or β, the model is very robust to reproduce the Pareto Law of wealth distribution. And the wealth distribution is independent of the system size N or the simulation time T.

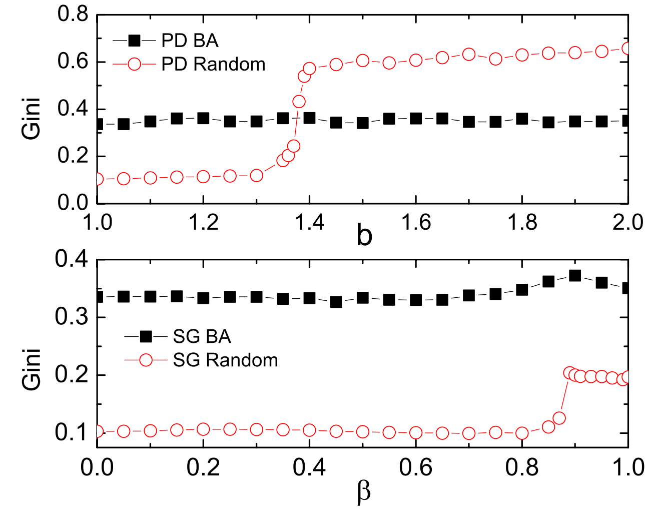

Fig. 3. (color online). Gini coefficient for PD (top) and SG (bottom).

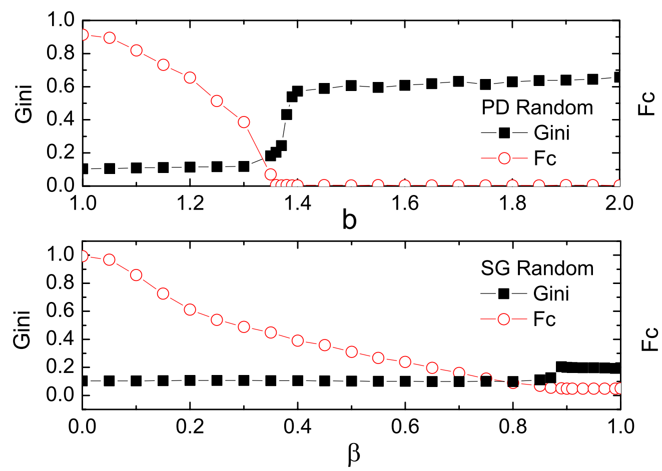

Fig. 4. (color online). The sudden increment of Gini coefficient with the decrement of cooperation frequency in ER random graphs for PD (top) and SG (bottom).

Figure 2 show the $P(W)$ of ER random social networks in Log-Normal panels. One can see that there is an exponential decay in the wealth distribution:

$$P(W) \sim \exp(-\lambda W) \tag{2}$$

with λ taking different values for different gaming parameters. This distribution is in agreement with Eq.(1) for low-income population.

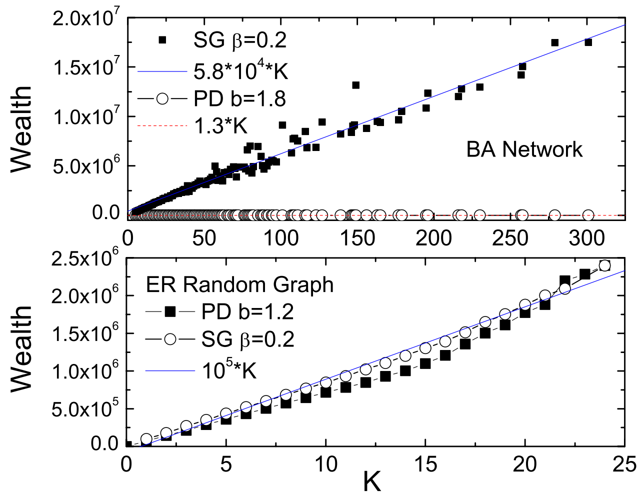

Fig. 5. (color online). K-Wealth relation. The cooperation frequency is 0.47 (SG BA), 0.01 (PD BA), 0.59 (SG Random) and 0.67 (PD random) respectively.

Next we investigate the Gini coefficient for the system. Figure 3 shows the variation of Gini coefficient with different game parameters for PD and SG. One can see that the value remains almost constant for BA network ($Gini \approx 0.35$) in both games. It is mainly because scale-free network is more heterogeneous in agents' connectivity and this will lead to the heterogeneity of wealth distribution. On the other hand, the value show a sudden increment for ER random graph in both Games. For PD, there is an sharp increment around $b = 1.35$ associated with a sudden decrement in the cooperation frequency, as shown in Fig.4. The value of Gini coefficient become even larger than in BA scale-free network when $b > 1.35$. That is, when b is large, the system become less cooperative and the social wealth will be more inequally distributed. We also note that the total social wealth for the system will decrease when $b > 1.35$. This situation is no good for the society. For SG, the increment in Gini coefficient happens at around $\beta = 0.9$. This is because the SG payoff matrix is intrinsically more in favor of cooperation. Thus the transition is more difficult to happen, and the value of Gini coefficient remains below that of BA networks.

Figure 5 shows the relation of personal wealth with its connectivity K for BA scale-free network and ER random graph in the case of PD and SG. One can see in all cases the personal wealth is roughly proportional to its connectivity. That is, the agents with more contacts will simply have more wealth. This behavior will not be affected by the learning strategy or by the strategy series that the agents take in each time step. Although everybody try their best to gain more in the game, the final score is determined by the connectivity. Since the connectivity can also represent one's information resources, this phenomenon is in accordance with the fact that in modern economy, agents with more information resources can gain more profit.

The linear relation between personal wealth and its connectivity is also a possible mechanism for the emergence of the Matthew Effect in economy. In Fig. 5, one can see that with both PD and SG, the wealth of the agent with more connectivity exceeds the agent with less connectivity. With the simulation time T, the agents with more partners will get richer and richer, while those with fewer partners will get relatively poorer. Thus the difference between the agents will be enlarged with time. Successful people (company, country etc) usually have more partners than an ordinary one, and the large relation network will make them even more successful.

4 Conclusion and Discussions

In summary, the wealth distribution in economy is modeled by evolutionary game theory on different social networks. The wealth distribution of the system is investigated together with the Pareto exponent and the Gini coefficient. The wealth of scale-free networks follows a power law with the exponent in agreement with empirical observations, while the wealth distribution of random network shows an exponential decay. One can suggest that the social network takes different structure for high and low income groups. From the presented simulation results, we suggest that the network for high income group has a heterogeneous scale-free structure, in which some people or agents have more links than others. For the low-income population, the network is more like random graph.

We also investigate the Gini coefficient for the systems. We found that the Gini coefficient of scale-free network will remain as $Gini \sim 0.35$. There is a sudden increment in the value of Gini coefficient for ER random graph.

The simulation show that the agents' personal wealth is proportional to the number of its contacts (connectivity). This mechanism can leads to the phenomenon that the rich gets richer and the poor gets relatively poorer with time (Matthew Effect). Thus, one should pay more attention on the strategy of increasing partners, because although everyone tries their best in economy, the final profit is simply determined by their connectivity of partners. This mechanism also explains why agents with more information resources can gain more profit in modern society.

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