

**Science and Mathematics Education Centre**

**Enhancement of Learning Capabilities of Students of All Ability  
Levels Through a Constructivist Approach**

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**This thesis is presented for the Degree of  
Doctor of Mathematics Education  
of  
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## DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

A handwritten signature in black ink, appearing to read 'Jawad', written on a light blue rectangular background.

Signature:

Date: 25/06/2013

## DEDICATION

**This analytical story of my journey of education is dedicated to**

*Each and every stranger*  
That I learn from

*Each and every friend in my past and present*  
For teaching me by learning from me  
And learning from me by teaching me

*Each and every teacher who inspired me*  
To think critically by being critical about me

*Mr John Cousins*, Former Principal of Robinvale Secondary College  
*Mr Jim Long*, Former Vice Principal of Yarrawonga Secondary College  
For their contributions to my professional life

*My dear wife, Warna*  
My darling Daughter, Ranshikha and  
*My beloved Son, Subharthe*  
For teaching and offering me family life

*Minaret College and*  
Its Director Mr Mohamed Hassan OAM  
For teaching me Islam

*Former Principal of Dharmasoka College*  
Late Mr Wimalasuriya  
For not throwing me out from Year 9C Class

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For his patience and guidance in this work

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For being the greatest teachers in my life  
By persuading me about the value of education

*And in particular*

My Year 11 and 12 Physics Teacher Mr Mahindarathne and  
*My Year 9 Physics Teacher Mrs Kiribandhane*  
For teaching me what learning and teaching really is

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Even a minute project cannot be completed without the assistance of many other people. This current work cannot stand above this maxim. To think otherwise is delusional and deceitful. Firstly, I pay attention to the foremost backbone of this work.

Ever since I enrolled in this doctoral program, I have been strongly resolved to carry out my enrolment to its logical end successfully. There has never been even an iota of vacillation of this resolve.

However, regardless of my steadfast determination and dedication, I was thrice close to abandoning the project because I needed to be fair to my thesis advisor, Associate Professor Dr Peter Taylor. When I simply did not have time to engage in the research I felt guilty and remorseful. On these occasions, I sought permission from Peter to withdraw from the course if he wished me to do so. Each time he made it absolutely clear that if my goal was to continue in the course he would support it. The ultimate outcome of this assurance of support and understanding of my predicament is this thesis. There were plenty of encouragements from him when I needed them.

Also, early in my writing I displayed a writing style of a confrontational nature. This confrontational aspect could have been a remnant of my Marxist past. Peter first thought this would be suitable for my thesis writing. To an extent, he encouraged this confrontational slant. At some point, he questioned my style. This objection clearly matched with my strengthening sense of epistemological plurality. As a result, my writing style became more inclusive and less confrontational. At that point I was convinced that whatever writing style I choose to use my aim is to include and engage my audience; not to exclude or chase them off. I hope that my thesis contains aspects of both inclusiveness and ‘confrontationism’ in a coherent synthesis.

My advisor’s critical comments also contributed to the worldview I hold, plurality of epistemology. For instance, his criticisms ‘confirmed’ for me that there is ‘objectivity’ in the way we perceive reality. That is why his critical comments

resonated with me. The comments poked at my underlying ‘uncertainties’ which were somewhat obvious to me and also which I deliberately ignored looking at. These issues were mostly centred on my understanding of research methodology. That is, I was suffering from an almost chosen lack of awareness of the research methods that I was employing on my own volition. This aspect of awareness is a central theme of my current work. How ironical! I felt.

Apart from these issues, Peter painlessly, tirelessly and patiently repeatedly edited my work. That helped me to express my ideas in a much clearer way. Therefore, this experience has been a workshop in writing too.

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One of the most beautiful moments of my life eventuated due to this study. This is how it happened. My choice of autobiography as a research method was only possible because Peter sanctioned and encouraged it. To help me feel comfortable with my own choice of research methods, he provided me many research articles on autobiography and other related research methods. During the days of the final curtain fall of my mother's life, I told her about the many stories I have been writing about my parents' contribution to my life. Responding to this, she told me that "Now I can close my eyes in peace and happiness knowing that I have fulfilled my life mission." Memory of this moment will always be a moment of profound endearment and fulfilment that I will cherish forever.

## ABSTRACT

This is an inquiry into academic constructivism which is considered to be appealing to students of all ability levels, especially students of low ability. For this reason, many regard academic constructivism as an ocean of aspiration and a flame of inspiration for learners and educators. For instance, Tobin and Tippins (1993) proposed constructivism as a major referent for learning, teaching and curriculum design.

However, there are many others who critically contest the practices and strategies of constructivism. For example, in a friendly critique, Airasian and Walsh (1997) argued that there are discrepancies between the theory of constructivism and its applications in the classroom. Further, they opined that implementing constructivism in the classroom in an effective manner may be considerably more challenging than the simple slogans suggest. Similar criticisms were made in Elkind (2004).

Moreover, there are others who disdain constructivism as an educational fad festered with many deficiencies. They condemn constructivism as a disaster (See Ziffer (2006) and Hirsch (2001)). In addition to the dynamics of these three camps, there are my own life experiences of epistemological and academic constructivism, both as a student and a teacher.

Even though the noun ‘constructivism’ was added to my vocabulary just a decade and a half ago, constructivist ideas have been playing in my mind, in one form or another, since I was a Year 9 student. My association with constructivism was initiated by the practice of one particular teacher, Mrs Kiribandhane, during the short three months that she was my physics teacher. This teacher might not have been aware of the ideas of constructivism. Even after I moved into a class that offered a more advanced mathematics subject, the influence of her teaching seemed to be growing much stronger. Later on, in Years 11 and 12, my attachment to constructivist ideas was nurtured by another brilliant physics teacher, Mr Mahindaratne, who had a BSc degree in physics and a Graduate Diploma in

Education. This teacher also might not have been aware of the formal movement of constructivism.

In this study, I examine ideas of constructivism in relation to current teaching practices and inquire critically into the development of my own practice as a learner-educator, in the hope that this might help me to improve my own professional practice. Since this study involves academic constructivism, invariably it involves the epistemological foundations of constructivism. Therefore, this inquiry involves the evolution of my epistemological belief system and its effect on my practice as a learner-educator. I also hope that insights developed in relation to the epistemological and academic aspects of my ‘constructivist’ evolution will make a contribution to other learners and educators. One significant quality judgement for this inquiry needs to be the extent to which it raises pedagogical thoughtfulness and mindfulness in both me and my audience.

# TABLE OF CONTENTS

<b>ACKNOWLEDGEMENTS.....</b>	<b>iv</b>
<b>ABSTRACT .....</b>	<b>vii</b>
<b>TABLE OF CONTENTS .....</b>	<b>ix</b>
<b>ARC ONE-TWO: ORIENTATION OF MY INQUIRY .....</b>	<b>1</b>
FOREWORD: A PROFESSION OF MANY CONTRADICTIONS .....	2
CHAPTER 1: INTRODUCTION .....	6
<i>Background .....</i>	<i>6</i>
<i>Objectives .....</i>	<i>8</i>
<i>Significance.....</i>	<i>13</i>
<i>Research Methods .....</i>	<i>14</i>
<i>Quality Standards .....</i>	<i>24</i>
<i>Thesis Structure .....</i>	<i>30</i>
CHAPTER 2: JOURNEY OF MY EDUCATION, JOURNEYING BY WANDERING OR WANDERING BY JOURNEYING .....	35
<i>Beginning.....</i>	<i>35</i>
<i>My First Bold Step.....</i>	<i>38</i>
<i>My First Two Teachers .....</i>	<i>40</i>
<i>Wandering.....</i>	<i>45</i>
<i>My Journey with Academic Constructivism .....</i>	<i>49</i>
<i>Effect of the New Teaching Approach .....</i>	<i>51</i>
<i>My First Success as a Teacher .....</i>	<i>55</i>
<i>Debunking Myths.....</i>	<i>56</i>
<i>Harvest of Intellectual Arrogance .....</i>	<i>58</i>
<i>My Fate .....</i>	<i>60</i>
<i>The Dream Which Came True and Then Vanished.....</i>	<i>64</i>
<i>Learning to Heal From Ill-Health .....</i>	<i>66</i>
<i>Aspects of My Adolescent Constructivism .....</i>	<i>72</i>
<i>Summing Up .....</i>	<i>74</i>

**ARC TWO-THREE: THE REVOLVING AXIS OF PARADIGM ROTATION; THE  
JOURNEY THROUGH MANY ISMS .....75**

CHAPTER 3: FROM LOGICAL POSITIVISM TO CONSTRUCTIVIST EPISTEMOLOGY, THE JOURNEY I NEVER  
EXPECTED TO BE IN ..... 77

<i>A Logical Positivist in Metamorphism</i> .....	77
<i>My Path to Constructivist Epistemology</i> .....	78
<i>To Epistemological Constructivism in the Vehicle of Academic Constructivism</i> .....	81
<i>Aristotle and Newton on Motion</i> .....	85
<i>Is Knowledge Discovery or Creation?</i> .....	89
<i>Nature of Scientific Knowledge and the Dark Story of Light</i> .....	91
<i>A Rainbow of Epistemologies</i> .....	110
<i>Colours of the Rainbow</i> .....	113

CHAPTER 4: CONSTRUCTIVIST VIEWS, MANY FACETS OF A SINGLE FACE OR A SINGLE FACE MADE OF  
MANY FACETS ..... 115

<i>Is Knowledge Non-Transferable?</i> .....	117
<i>Is Knowledge Transferrable?</i> .....	119
<i>Transfer of Knowledge</i> .....	120
<i>Cognising as Activity</i> .....	123
<i>Constructivism as Post-Epistemological Theory</i> .....	124
<i>Radical Constructivism</i> .....	126
<i>Social Constructivism</i> .....	129
<i>Implications of Social Constructivism for Education</i> .....	132
<i>Society and Education</i> .....	136
<i>Independence of Mathematics Traditions of Teachers and Students</i> .....	141
<i>Parochialism and Internationalism</i> .....	144
<i>Commonality between Various Forms of Constructivism</i> .....	144
<i>Two Major Forms of Constructivism: Radical and Trivial</i> .....	146
<i>Correspondence between Different Versions</i> .....	149
<i>Constructivism as a Method</i> .....	152
<i>Systems of Beliefs</i> .....	154
<i>Why Constructivism?</i> .....	157
<i>Mirrors of Isms (Ism in Isms)</i> .....	158
SUMMING UP.....	161

**ARC THREE-FOUR: REFLECTING THROUGH PARADIGMS AND  
CONSTRUCTING PARADIGMS THROUGH REFLECTIONS .....163**

CHAPTER 5: THE FUTILITY OF SINGLE FLAVOURED SLOGANS, THE ACTUAL MESSAGES SENT OUT AND  
SUNK IN ..... 165

*Purpose*..... 165  
*Inadequacy of Simple Slogans*..... 166  
*Doing Personal Best or Being Perseverant and Resilient* ..... 169  
*Irony of the Irony* ..... 170  
*Food for Thought* ..... 173  
*Missing Equivalent Fractions* ..... 188

CHAPTER 6: TEACHERS AND HOW THEY TEACH ..... 193

*Reductionism* ..... 193  
*Wave-Particle Duality* ..... 194  
*Paradoxes of Duality*..... 197  
*Teaching: Instructivism or Constructivism?* ..... 199  
*Buddha as a Teacher*..... 200  
*Jesus as a Teacher: Stoning the Sinner* ..... 207  
*Socrates: The Celebrated Teacher*..... 208  
*My Teachers and the Teacher Me*..... 211  
*Storytelling* ..... 212  
*Storytelling of My Teacher*..... 214

**ARC FOUR-ONE: CULMINATION; A BRIEF MOMENT ON A DWARF SUMMIT  
.....217**

CHAPTER 7: INSTRUCTIVIST CONSTRUCTIVISM OR CONSTRUCTIVIST INSTRUCTIVISM? COINING MANY  
FACETS INTO ONE OR SPLITTING ONE INTO MANY ..... 218

*First Example: An Autistic Child*..... 221  
*Second Example: Vanishing Fractions* ..... 222  
*Third Example: Lost Giftedness* ..... 223  
*Fourth Example: A Driving Lesson of a Strange Kind* ..... 224  
*Teaching as a Coin with Many Sides* ..... 225  
*My Pedagogical Commandments* ..... 228  
*Foundational Aspirations of My Teaching*..... 231  
*Do's and Do not's*..... 233  
*A way of Thinking Versus a Way of Doing*..... 234  
*Pedagogical Awareness* ..... 238  
*An Example of Dream Walking* ..... 240

<i>Failures of Assimilation and Osmosis</i> .....	243
<i>Answers, Answers, Nothing but the Answers</i> .....	244
CHAPTER 8: ENHANCEMENT OF LEARNING CAPABILITIES OF STUDENTS OF ALL ABILITY LEVELS, THROUGH A CONSTRUCTIVIST APPROACH.....	246
BACKGROUND .....	246
<i>The Thesis Title</i> .....	246
<i>My Passion</i> .....	246
<i>An Example of Choice</i> .....	247
MY MATHEMATICS COINS .....	250
<i>Algorithm of Long Division</i> .....	250
<i>Division</i> .....	252
<i>Long Division</i> .....	253
<i>Division of 1 by 7</i> .....	258
<i>The Number Half</i> .....	259
<i>Directed Numbers</i> .....	264
<i>A Special Line</i> .....	269
<i>Why Are Straight Lines Straight? I</i> .....	270
<i>Relations and Functions: For Year Ten or Eleven</i> .....	275
<i>Division by Zero</i> .....	278
<i>The focus of our next lesson idea is how to recognise the 'shape' of a function from its rule</i> .280	
<i>Pictures of Mathematical Functions through the Conceptual Camera of the Mind</i> .....	280
MY SCIENCE COINS .....	285
<i>How Do We See</i> .....	285
<i>Atoms</i> .....	287
<i>Magnets</i> .....	294
SUMMING UP.....	295
CHAPTER 9: CONCLUSION; STEPPING-STONES TO STEPPING-STONES, LOOKING BACK OVER MY SHOULDER .....	296
<i>Themes of the Inquiry</i> .....	297
<i>Educators as Change Agents</i> .....	301
<i>What is Next?</i> .....	302
<b>References</b> .....	<b>306</b>
<b>APPENDIX</b> .....	<b>311</b>
<i>Fractions I</i> .....	311
<i>Fractions II</i> .....	314

<i>Equivalent Fractions</i> .....	315
<i>Addition and Subtraction of Fractions</i> .....	319
<i>Multiplication of Fractions</i> .....	321
<i>Division of Fractions</i> .....	324
<i>A Little More Formal Method of Looking At the Operation of Division</i> .....	327
<i>The Number Zero and Negative Numbers</i> .....	329
<i>Why Straight Lines Are Straight? II</i> .....	337
<i>Functions</i> .....	340
<i>Evaluating a Function at a Given x Value</i> .....	343
SOME OTHER MISCELLANEOUS EXAMPLES .....	345
<i>A Perimeter Problem</i> .....	345
<i>Some Area Problems</i> .....	346
CALCULUS .....	347
<i>The Chain Rule</i> .....	347
<i>Integrals Which Are a Prelude to Change of Variable Transformation</i> .....	349
TRIANGLE PROPERTIES: .....	349
<b>GLOSSARY OF TERMS</b> .....	<b>351</b>

## **ARC ONE-TWO: ORIENTATION OF MY INQUIRY**

This section consists of three elements, Foreword, Chapter 1: Introduction, and Chapter 2: Journey of My Education. To signify the underlying cyclic nature of this inquiry, I have chosen to name the sections as 'arcs'. The reader may think of the chapters as sub-arcs that together make a section arc. This metaphor of arc links to the metaphor of rainbow of epistemology which I have presented in the inquiry.

Being a wanderer, I wonder whether it is appropriate to name this section as 'orientation'. In a sense, all of us are wanderers and pilgrims, simultaneously. This is not different even for me. Thus I am also a wanderer and a pilgrim. When we are wanderers we do not have an orientation. When we are pilgrims we cannot do without an orientation. I hope that I have managed this duality in a coherent manner throughout this inquiry.

The way we view reality affects the way we see our profession. The way we see our profession also contributes to the ways we view reality. The inner conflicts between the opposing views, and the underlying dialectic nature of the current inquiry is epitomised in the Foreword. Therefore, the Foreword signals one of the axes of orientation of my inquiry.

In Chapter 1: Introduction, I explain my motives and the background of this inquiry. It is quite appropriate to think that I was situating myself for this inquiry since I was a Year 9 student even though I did not have any idea what was in offing.

## **Foreword: A Profession of Many Contradictions**

Years of my learning and teaching have convinced me that teaching is a profession of many contradictions. The task of delicately balancing and synthesising these contradictory aspects is a significant challenge for any teacher. This is an analytical story of my perceptions of how some teachers of mine overcame this challenge thereby helping me to become an excellent teacher to myself. I hope that my experiences with those teachers have helped me to become an excellent teacher for others too.

Here are some examples of contradictions that a teacher has to deal with on a daily basis, in order to perform with competence.

Sometimes I wonder, can I teach someone else other than myself? I can talk to others, I can instruct or demonstrate to others, even if they do not pay attention or just pretend to pay attention. Nonetheless, can I really teach others, even if they pay complete attention? By teaching, if I just mean talking, demonstrating or instructing others, then, yes I can teach. On the other hand, by teaching, if I mean not what I am doing, but how it is affecting the student, then certainly I cannot teach anybody other than myself, since I do not have direct access to the mental processes of the student even when they give me undivided attention.

Then why are there teachers and how do we reconcile the fact that most of us are influenced by great teachers of ours, such as Buddha, Jesus and the Prophet Mohammed?

Consider the claim that learning cannot occur without understanding. If this statement is true I wonder how I can walk, since I have only a minimal understanding of the physiological process of walking. Similarly, even though I understand what I talk, I do not understand the complex act of talking. Should I wait to walk or talk until I understand the processes of walking and talking?

Then consider the proposition that as long as one keeps learning, understanding is not necessary. If this is the case then the question arises, are we mindless machines, who perform according to someone's program or are we donkeys or monkeys that dance to someone else's tune?

Do the above two paragraphs indicate that we should learn only reflexive tasks (such as walking) without requiring to understand them because reflexive processes are wired by evolutionary process, and all other reflective tasks (reading, counting, addition etc.) should be only learnt with understanding? An instance against this is that I understood what counting really is when I was taking an Abstract Algebra Unit at Graduate School. Does this mean that to learn counting I should have waited until I got to Graduate School? Also, I understood the place value system, multiplication and division algorithms only when I began to teach my daughter. That was a decade later than when I obtained a Ph.D. in Mathematics. Does this mean that I should have waited to learn the place value system, multiplication and division until I got to teach my daughter?

If I have understood then does it mean that I have learned? There are many things that I have understood but not learned. For instance, I have understood the principles of piano playing, but I cannot play the piano. Next consider the statement, if I have learned then I have understood. I know a teacher who had trained a parrot to recite the two times table.

Should we use visual aids in teaching? If yes, then how do the students learn to make mental images of concepts without having to associate with a concrete object? If we do not use visual aids at all then how will they come across the effectiveness of visual images?

Consider the notion that not having to remember formulas such as Newton's Second Law,  $F=ma$ , enhances students' learning. If we subscribe to this view we would also advocate that students do not need to remember Newton's Second Law. They just need to understand the

concept. They can refer to the formula in a book if they want to. This raises the following question. If students understand Newton's Second Law, is not that understanding itself the formula?

On the other hand if a student remembers the formula then has the student learned or understood Newton's Second Law in any meaningful way? An image of a parrot comes to my mind repeatedly saying,  $F$  equals  $m$  times  $a$ ,  $F$  equals  $m$  times  $a$ .

Should teachers help students? Again whichever way I answer, a contradiction arises. A person can help another person in so many ways, but how can a teacher help a student to learn or understand, if the purpose of Education is producing independent learners? If teachers cannot help students then why do some students claim that their teacher helped them to become independent learners?

Teachers should promote students' self-esteem. I wonder if I promote the self-esteem of a student how it can be self-esteem. Does this mean teachers cannot ever influence one's self-esteem?

This list can go on and on. If one looks carefully then it can be seen that each of these ideas has clear validity. At the same time, if one bases his or her practice just on one side of these contradictions then the practice is bound to be dull and counter-educative.

This current work is about synthesis of opposite philosophies, ideas, strategies and practices. These contradictions spring up due to the very nature of teaching and learning. Teaching is not a one-way street; rather it is the epitome of the ultimate interaction between two independent minds, the one of the teacher and the other of the student. Anything related to mind interactions cannot be explained or realised by basing oneself on a single paradigm or a simple strategy. Then again this last statement brings up another contradiction. Whatever our teaching is, isn't

it based on the single aspiration that we all want our future generation to be much better than us?

# Chapter 1

## Introduction

### Background

Since I was a Year 9 student in Sri Lanka, I have been enthralled by ideas similar to those of [constructivism](#)<sup>1</sup>. This profound effect on my learning and teaching (precisely life itself) was initiated by the inspiring teaching practices of my Year 9 physics teacher. It was only decades later that I became aware that this teacher's approach was compatible with the ideals of constructivism. The teacher herself may never have been aware of the movement of constructivism in her whole life. Nevertheless, I felt the stark difference in her approach in comparison with the practices of many other teachers I knew at that time.

She might or might not have continued with her education beyond Year 10. If she did, of course, she had been unable to gain University admission at the end of Year 12. Possibly after several years of service as a teacher she had taken a two-year teacher training course. Regardless of her academic qualifications the contribution she made to my life has been invaluable and long lasting. Since then I have been endeavouring to emulate my learning and teaching approaches in her mould. This teacher has shown me the power of mind, teaching and learning. If I can do the same to just a single student then I have fulfilled my life mission, I feel.

Later in Australia, through my Graduate Diploma in Education, I was formally introduced to constructivism. To learn more about constructivism, several years later I enrolled in this graduate course. Through wide reading I discovered that the opinions about constructivism are not as simply one sided as I had once thought. I also came to know that current education practices are considered to be motivated and moulded by the movement of constructivism.

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<sup>1</sup> Note that the blue underlined terms are e-linked to the glossary.

As Kruckberg (2006) noted, a major challenge of educators is to bridge the two worlds of the educator and the learner by generating an interest in the learner in the subject taught. As I distinctively remember, the two teachers I mentioned above did not have any difficulty in achieving this goal. The teachers and I were in one world, travelling on the same journey. Granted, it was a journey they had already made several times. Nevertheless, the journey was always fresh; for each time they made the journey they did so with a different batch of students. Although they were in charge of the journey I never had to relinquish control of my mind; in fact, their approach inspired me to be in **right mindfulness**. And it was my own journey. When I review their practice with today's eyes I am certain that they believed that their students were very much capable of **owning** their knowledge. Their approach could have been motivated by Socrates' method which has been widely reviewed in literature. For instance, see (Seeskin, 1987).

I vividly remember the intellectual entertainment that the teachers and I experienced. At that time I termed their teaching approach as "teaching with a philosophy ('meaning'). I used this term to signify that the learning process made sense to me. Consequently, I felt ownership of my knowledge. Even though my teachers never relinquished control of the class, I was in the driving seat of the vehicle of my own learning. Basically, this is the core of **academic constructivism** (Taylor, 1998; Confrey, 1990; Ernst, 2001; Driver, Leach, Mortimer and Scott (1994). Thereafter, whenever I met a new teacher I measured him or her against these two unique teachers, and I asked myself "Is the new teacher teaching with a philosophy?"

Education has grown into an industry. For this reason and because of the nature of cognition and teaching-learning education involves social issues. Powerful economic, social and political forces exert strong influence on education systems worldwide. An inquiry into the issues involving teaching and learning naturally encompasses social critique. This sentiment is compatible with what has been noted in the literature; constructivism has many faces. See Ernest (1995), Geelan (1997) and von Glasersfeld (1990). Consequently, I feel that it is essential to involve these different and diverse aspects in my inquiry.

## **Objectives**

My purpose in this research has been to

- (i) understand opposing views on constructivism,
- (ii) critically assess and analyse my epistemological and pedagogical journey into the domain of academic constructivism,
- (iii) develop a framework of academic approaches based on various epistemological and academic paradigms that can enhance my professional practice as a learner-educator, and
- (iv) reach students of different ability levels more effectively.

Some argue that, as stated in Richardson (2003), academic constructivism is a theory of learning, not a theory of teaching. But, for me, whether it is self-learning (teacher is the student herself) or teacher guided or directed learning, the learning side is always coined with the teaching side and vice versa. Therefore it can be argued that even if we concede that it is merely a theory of learning, academic constructivism always radiates implications for effective teaching. In the same paper, it is stated that even though a teacher may identify himself as an academic constructivist his teaching may not be constructivist. Some may explain this discrepancy by hypothesising that the teacher still carries the baggage of non-constructivist beliefs. Then it can be argued that there may be teachers who disdain constructivism or who are not aware of constructivism (as were my two teachers) and yet practise as exemplary constructivists. As my thesis advisor, P.C. Taylor, communicated to me, discrepancy between a teacher's constructivist convictions and his or her teaching practice can be due to system constraints. This contradiction could also occur due to naive interpretation of constructivism.

The incompatibility between the practices and the belief systems of some teachers discussed above led me to ask: what could have been my two teachers' epistemological belief system? Could it be that their unexpressed epistemology was constructivism, or something else? What can I surmise about this aspect from my perceptions of their practice?

One of my key objectives in this study has been to critically inquire into what effective teaching means to me as a student and an educator. Since we share a

common humanity, if my efforts are sincere and intensive enough, other learners and educators may be able to identify with the understanding that I generate in my work. von Glasersfeld (1995) recognised that there had been a decline in education in the past two to three decades as manifested by falling literacy and numeracy skills of the general population. This decline is attributed to the influence of behaviourist theories. Then many others contend that the dominant academic theory of the day is constructivism, and therefore this decline is the failure of constructivism. Can this deterioration be the result of both constructivism and [behaviourism](#)? Do not many of us try to exclude one approach at the expense of the other? This interesting view, in some form, is discussed in Taylor and Willison (2006) who argue that both views should be held in a dialectical relationship serving as a key referent for teaching whose viability depends on pedagogical purpose.

Even though when I was young my learning approaches were quite in the mould of constructivism, my epistemological worldview had been one of a quite [logical positivist](#) nature. By the time I was introduced to academic constructivism I had developed into an epistemological constructivist. Nevertheless, during this introductory period I sometimes felt grave objections to the terminologies used in constructivism. This tension and cohesion fuelled my curiosity and inspired my inquisitiveness. This resulted in this thesis.

### *Stage One*

On the one hand, due to my own experience with and notions of academic constructivism I was deeply attracted towards it. On the other hand, by reading about my own experience with and notions of the education crisis in developed countries, I held strong objections to it. This diametrically opposite tension motivated me to examine my own practice as a learner-educator and the practices of other past and present teachers known to me. In doing so, I paid attention to the following aspects, which became a basic research model for my work and the first stage of my inquiry:

- How do the approaches of those teachers differ from or agree with constructivism?
- What do these differences or similarities mean to the teachers' practices, including my own?

- How do these constructivist approaches and framework affect the quality of the learning and teaching process?

Although I did not believe that there could be definite answers to the questions above my inquiry was guided by and based on these attributes and I hope that I have generated insightful opinions. To form these insights I have focussed on the following questions:

- How do I perceive the development of my own academic constructivism as a learner and an educator?
- How did my epistemological belief system evolve into epistemological constructivism and how did this evolution influence further development of my academic constructivism?
- How did my exposure to epistemological constructivism and academic constructivism, as documented in the literature through the Graduate Diploma in Education and recent graduate studies, impact on my views and practices as a learner-educator?

### *Stage Two*

In the second stage of my inquiry, I appraised the similarities and incompatibilities between my own epistemological and academic views and practices with generally prevailing constructivist views and practices. In this deeper level of my inquiry I reflected on the following issues:

- What kind of epistemological views were indicated in my past teachers' practices?
- How might these epistemological views have contributed to their academic approaches and successes or failures as teachers? Then the natural extension of this analysis is to reflect on my own practice as a learner-educator.
- How did becoming a teacher in Australia impact on my professional practice, both as a student and a teacher?

In this analysis, I cogitated on the following aspects:

- Differences and similarities between my practices and views and those of constructivism as documented in the literature and evident from the prevailing practices.

- Identification of myself with the camp of constructivism (both academic and epistemological) during the coursework stage of the graduate studies in this course.
- Analysis of changes of teaching practices due to the influence of constructivism.
- Compatibility between prevailing education practices and constructivist aspirations.
- The impact of social forces in the classroom and similar situations from other fields.

### *Stage Three*

In the Third Stage of my inquiry, to understand the extent to which the other philosophies and academic approaches are similar to or different from epistemological and academic constructivism, I reviewed several other epistemological philosophies and academic approaches such as [objectivism](#), [behaviourism](#) and [cognitivism](#), respectively. In doing so, I focussed on the following questions:

- How do these differences contrast and complement each other?
- How do these differences manifest in teachers' practices?
- Is it possible to synthesise these opposing worldviews and academic approaches to construct a coherent, internally consistent and externally valid academic philosophy and an academic approach that enriches my practice as a learner-educator and which has some relevance for other teachers?

Also, in this discourse I reflected on what effective teaching means. To gain some valuable insights I first ask the following questions.

- Were all teachers in the past ineffective?
- Are all present constructivist teachers effective?

To aid this inquiry, I considered the practices of some of the greatest teachers of humankind and paid attention to the following aspects:

- What were the teaching approaches of teachers such as Buddha, Christ, and Socrates?

- How did their strategies contribute to their success as teachers?
- Did their approaches contain elements of objectivism or relativism?

I would like to remark that when I discussed the teaching methods of Buddha and Christ, I restricted my attention to their teachings on worldly life. Also, to enrich my analysis, during my everyday teaching practice I informally surveyed students' views on effective teaching.

- Are these compatible with the teachers' views of effective teaching?
- Do students and teachers have common views on these questions?
- If students and teachers differ on what effective teaching is, why do they differ?

Education is a social process. The society at large and its powerful forces impact on educators and students both. To look at the role of social forces in an interpretive manner, I asked the following questions:

- How does the mega-culture of the society at large, affect, influence and impact students, parents, teachers and education systems worldwide?
- How does this influence affect mainstream views on effective teaching and learning?

To supplement and complement discourse on the abovementioned aspects, I:

- Conducted a literature review and an analysis of various epistemologies and pedagogies within the context of the practices of the teachers known to me, including myself.
- Inquired how current teaching practices measure up with different epistemologies and pedagogies.
- Reviewed examiner's reports on several Victorian Certificate of Education Mathematics Exams.

#### ***Stage Four***

In the Stage 4 of my inquiry I developed techniques and strategies that help me to improve my professional practice and enable me to help my students to possibly

enhance their learning capabilities (I also expect this program to be worthwhile of other teachers' consideration.)

I accomplished my goal by reflecting on and critically examining observations and experiences with my students (One of these students is myself.) and fellow teachers (One of these teachers is myself.). This stage comprised development of lesson approaches immersed in and bonded with the spirit of constructivist aspirations and based on a plurality of epistemologies and various pedagogical approaches. Furthermore, I endeavoured to analyse how my belief system contributes to the ideas presented in this work, how my pedagogical approaches are compatible with constructivist objectives and how this discourse can be of value to others.

To achieve this goal, I:

- Presented a set of guidelines and principles on which I base my pedagogical approaches. These approaches may not subscribe wholly to epistemological constructivism and academic constructivism. Yet, I hope these principles are elegant, simple, productive and constructivist in spirit. Moreover, I endeavoured to make these ideas attractive to other practitioners.
- Investigated whether it is possible to develop an integrated approach of constructivism and instructivism that can be used to promote better thinking skills and promote intellectual entertainment.
- Examined whether it is possible to incorporate a multitude of epistemologies and pedagogies into a coherent pedagogical approach that is interwoven with constructivist aspirations.
- Generated a set of mathematics, science and physics lessons, ranging from primary to senior secondary, based on the ideas generated in this inquiry.

### **Significance**

The progress and survival of our species largely depends on the success of our education system. For other species, if they have to teach at all, what has to be passed over to the next generation is exactly the same. For them most of the teaching and learning techniques are governed by reflexive mechanisms developed through evolution. This is not the case in our species. Our knowledge, skills and concept

bases are continuously expanding with ever increasing speed. Throughout the centuries, not only the teaching and learning techniques but also teaching and learning equipment have changed. It is not prudent to resist this change. It is also not wise to just become a floating straw in the flow of change. We need to understand and *manage* change.

There is a saying about change; it is impossible to enter into the same river twice. This expression magnificently captures the nature of change, but also points to the no-change aspect of change. For instance, if I say I cannot get into the Murray Valley River twice, I am

**Same Name?**

Why do we  
Still call this,  
The Murray Valley River?

Even an instant ago  
We said the very same name

emphasising the change and no-change both at the same time; the river has changed, nevertheless it is still the Murray Valley River. To manage change, we need to understand both change and no-change aspects. If we see only change, or if we see only no-change, we are not seeing it well, perhaps we are not seeing at all. There may be people who emphasise just the change aspect or no-change aspect merely because that suits their interests, or simply because that is the only way they can see.

The only way to properly manage change to our advantage is to stand on several vantage points, consider many different points of view and test a multitude of hypotheses. In this endeavour, abrupt judgements that these views and hypotheses are ‘right’ or ‘wrong’ and new or old will not serve our purpose. The concern needs to be whether we have strived to see the multidimensional and multilayered nature of the problem at hand and paid attention to both change and no-change aspects. We need to test whether we have really advanced our cause or just chosen to be a straw in the flow.

I believe that the present inquiry could provide an alternative view about education that deserves serious consideration.

**Research Methods**

This research could not have occurred if I have not worn mind lenses. In a sense, the mind lenses I am wearing are particular to me. At the same time they cannot be that

peculiar. Otherwise there is no point in writing this report. To explain this let me present the following example. Eschers'<sup>2</sup> drawings attract me because the way he sketched the world was different to the way I sketch the world. Then again his drawings mesmerise me because, underneath the obvious differences, there is a commonality to our ways of looking at the world. Therefore, to appeal to my audience to lend me a sympathetic ear I first need to explain the epistemological foundation of my inquiry.

### ***Epistemological Foundation of this Inquiry***

Epistemology consists of ways and means that we judge our knowledge claims. That is, our views and beliefs are justified by the epistemology we choose. For instance, to a Christian the existence of God is true and justified knowledge due to the theological epistemology he has chosen. It is my belief that whenever I express a view I base it on an epistemological axiom. These epistemological axioms may lie underneath my awareness at times or may take the centre-throne of my awareness at other times. What comes first; is it the epistemological axioms or the knowledge claims? At the moment, I will not discuss this chicken-egg conundrum.

In Chapters 2 and 3, through the narrative of my life experiences I have presented my epistemological evolution. It is only during the last stages of this inquiry I realised that even when I was an ardent [logical positivist](#) I had been an idealist too. Just because one professes to be an objectivist and steadfastly refuses to ask or hope for divine intervention even during an immensely troublesome time one cannot completely evade idealism. One may avoid being an idealist for some of the times of the day but not for all of the moments of the day. Similarly, every idealist is compelled to think as an objectivist at least in some moments of the day. This plurality of one's operational epistemological standing has been discussed in Chapter 4.

Since I have chosen to present this report and submit it for your value judgement it is self-evident that I employ objectivist epistemology as one of epistemological operational bases. When I say 'your value judgement' then it is clearly transparent

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<sup>2</sup> M.C. Escher (M.C. Escher Official Web Site) is a well-known graphics artist.

that I cherish relativistic epistemology as another epistemological operational base. My invitation, to you to judge my work, demonstrably indicates that I believe in some objectivity in your subjective value judgements. If I do not then there is no point in this exercise.

### ***Research Paradigms***

As cited in Ernest (1995, p. 463), Gage and Schubert have recognised three research paradigms:

- the scientific or neo-positivistic paradigm based on quantitative methods
- the interpretative or constructivist paradigm based on qualitative methods
- the critical paradigm based on critical theory

In the same paper, Ernest suggests that even if we hold strong disagreements with some of the foundational premises of each of these research paradigms we need to realise and cherish the significance, legitimacy and validity of all three research paradigms.

Another research mode which has recently gained wide acceptance is the practitioner /researcher mode. The significance and effectiveness of the practitioner/researcher mode is discussed in detail in Polkinghorne (1992) and McWilliam (2004). In the former, it is noted that the practitioner/researcher mode has helped psychologists to improve their individual practice and the practice of the profession. Polkinghorne's observations are generally valid for educational research. The practitioner/researcher mode has triumphantly gained legitimacy and now is on an equal footing with other research alternatives. Because of these new developments, education researchers are able to employ different modes of research methods or combinations of them. Being a reflective practitioner is an essential requirement in any profession. A practitioner reflects methodically on his or her practice to gain valuable insights that can raise the standards of the practitioner and the practice both.

The research paradigms and the methods I chose for this inquiry were the interpretive paradigm combined with both reflective practitioner mode and critical autobiography/ ethnography methods. These alternative research paradigms and

methods are accepted by the research community at large and are widely discussed in the literature. See Taylor (2013), Bryman (2001), Ellis and Bochner (2000) and Pereira, Settelmaier and Taylor (2005). Next I will discuss another way to look at the relationship between quantitative and qualitative research modes.

The quantitative and qualitative research modes in the social sciences can be understood by examining the parallel situation in the field of physical sciences. Consider experimental physics and theoretical physics. What 'physics' is is ultimately determined by the laboratory results of experimental physics. Once, Einstein himself said that just one single experiment can disprove his theories. Although this is valid to a large extent, this single experiment will be debated on theoretical points and experimental methodology and repeated several times before it becomes the ultimatum. Although experimental research has the edge over theoretical physics, they both complement and interplay with each other. Experimental research can motivate new conceptual awakening and theoretical research can motivate new experiments.

Another much closer parallel can be observed in medical science. Through quantitative medical research, it is possible to draw wrong conclusions unless there is proper qualitative scrutiny. This means that quantitative research alone cannot determine medical treatment. Also, just qualitatively or theoretically derived treatment options cannot be accepted until it is verified by properly controlled and qualitatively scrutinised quantitative research. Given that the proper qualitative examination has occurred, one may safely say that quantitative research rules over qualitative research.

Similarly in the social sciences there cannot be any doubt about the significance and essentiality of both qualitative research and quantitative research; they complement each other, but education policies and strategies need to be ultimately determined by properly controlled and qualitatively scrutinised quantitative research. Qualitative research on its own plays a significant role in the social sciences as theoretical physics research does in physics and qualitative research in medical science.

Choice of research paradigm is determined by the nature of the research questions that are being investigated. If we intend to examine which education policy works better, then most probably we need to adopt a quantitative framework, but here also we cannot escape from the need of qualitative inquiry. That is, whenever a quantitative research is in the offing, we are required to identify and control the relevant variables to ensure validity and reliability. The community will discard quantitative research results if there is no proper control mechanisms and qualitative analysis.

On the other hand, if our research question is why a particular method works better than another then it is a research question based on the interpretive research paradigm. In this case, it requires that the community comes to consensus on what is meant by "better" and how to measure this "better-ness". These questions require value judgements. For instance, if we examine the morality of capital punishment we need to make value judgements, and if we examine the deterrence of capital punishment then we need to make quantitative judgements. On the basis of newly acquired quantified data, to change one's position on capital punishment again requires further value judgments.

Let me further illustrate this with an example relevant to education. Suppose that I want to investigate the relationship between students' ability to perform well on multiple choice mathematics questions and to write mathematics well. Based on the results of this quantitative research I also intend to draw conclusions on whether or not we need to provide specific instructions on mathematical writing. I select a random sample of students and give them a test paper that has a section of multiple choice questions and a section that involves assessment of correct mathematical writing. Assume that I find a high statistical correlation that if a student performs well in multiple choice questions then she/he also does well in the writing section. From this result, I conclude that if the students can answer multiple choice questions in mathematics well then they will automatically know how to write mathematics well. Also further assume that on these experimental and quantitative findings, I advance the notion that specific instructions in and assessment of mathematical writing are not required even for beginners since people who develop mathematical skills automatically learn the writing skills.

One who reviews the research above in a casual manner, merely assessing the statistical techniques and the sample selection method, may be convinced that it is a sound conclusion, but is it? It is clear that students who score highly in the multiple choice section would probably also score highly in the writing skill section. Moreover, both test papers can be designed to give the false impression of providing useful information. For instance, by choosing questions from a narrow range of writing issues, scores in the writing skill section can be made to look reasonable. From this research, therefore, it is invalid to conclude that acquirement of multiple choice question skills leads to acquirement of writing skills. Here, we need the writing skill assessment paper to cover a much larger and an appropriate range of skills and use a criterion reference to measure success in both sections, such as a minimum cut-off mark. Now we will look for correlation between satisfactory performances in each section.

The most probable conclusion of such an investigation would be that even though a small number of learners assimilate mathematical writing skills without specific instructions many need structured form of instructions to properly write mathematical arguments. If we further extend this research, then we find that after the initial phase of mathematical writing instructions many more students can assimilate mathematical writing skills even without further specific instructions. This is because many have learned to pay attention to details of mathematical writing which they encounter due to the specific instructions given before. That is, the instructions given before have changed the way that they read mathematics.

In the research on the correlation between multiple choice question skills in mathematics and good mathematics writing skills it may be found that only some students who have performed satisfactorily in the multiple choice section have also performed well in the writing skill section. Even though this suggests that good performance in the multiple choice section does not necessarily indicate good performance in the writing skills section, it does not answer why some students performed well bucking the general trend. Now this is a qualitative research question, and needs to be based on the qualitative research paradigm and methods; For example, we may interview the students. Then the information gathered from the interviews may have to be assessed using the quantitative paradigm.

In the discussion above I argued that the choice of research paradigm depends on the scope of the research. The following well-known example illustrates the essentiality of the qualitative paradigm even if the research mode employed is quantitative in nature. We can show that the number of near drownings has a high statistical correlation to the amount of ice-cream sold in a beach on a hot summer day. In the absence of a qualitative analysis we may conclude that eating ice cream causes drowning. However, a qualitative analysis will show that the amount of ice-cream sold is correlated to the number of people on the beach. The number of people on the beach is correlated to the number of people in the water. Then the number of people in the water is correlated to the number of near drownings. This concludes my discussion on the interplay of qualitative and quantitative research paradigms.

Note that, to argue about the intensive and intricate interplay between qualitative and quantitative research, I have employed qualitative research techniques. The complementary and interactive nature of experimental research/quantitative research and theoretical research/qualitative research reminds us that, to deal with experiential reality, a single methodology based on a single epistemology is inadequate.

For my inquiry I have chosen the interpretive research paradigm and the critical paradigm based on critical theory. This choice is motivated by the research questions that I have posed earlier. Then the choice of the research paradigms radiates implications for the choice of the research methods that I can employ in the inquiry. I have elected the following research methods.

- Examination of many discourses, including curriculum documents.
- Reflective analysis of my life experiences using critical autobiography.
- Ethnography.
- Informal interviews with students, teachers, parents and principals.

By informal interviews I mean casual discussions on the issues at consideration with students, teachers and parents. This allowed me to assess their genuine thoughts rather than politically considered and formed opinions.

My research goal was to investigate the inter-relations between various academic approaches and epistemologies and to develop a coherent approach based on a plurality of epistemologies and methodologies. Moreover, evaluating whether one approach works better than another was not the focus of the current work. Therefore, I believe that my choice of the combined research mode of the interpretive, and critical research paradigms was the most appropriate choice available to me. This multiple research paradigm and methods has helped me to enhance my own practice as a teacher and may provide useful clues for other teachers. I do not claim that the learning approaches proposed in this report would necessarily work better than other approaches. A judgement of that nature can be made only by subsequent properly controlled long-term quantitative research. Hence, in this current research, the burden is on me to convince the community that my ideas are noteworthy of attention. I take the view that valuable research does not merely constitute the works that seek to enhance efficiency, but also includes the works that seek to promote debate on relevant issues. Eventually, these continuous discourses will lead to further research on better policies and practices. Ultimately, this will pave the way to the overall betterment of the education system.

### **Research Paradigms Employed in this Inquiry**

In a recent paper, Taylor, Settelmaier and Luitel (2012) advocate multi-paradigmatic transformative research methods. The dominant research paradigm on the twentieth century, which was based on [logical positivism](#), can be too restrictive and narrow at times. The authors articulate the rationale for the new research paradigm:

We believe that professional development of science teachers, especially via graduate research studies, should enable them to develop personally the transformative learning skills that they now are being called upon to develop in their own students, whether in school science or in college science teacher preparation courses. A pedagogy of transformative learning aims to raise students' critical awareness of the historic impact of science (and technology) on society, enabling them to develop ethical decision-making skills and a sense of personal agency for committing to make a difference, and fostering their empathic appreciation of alternative (ecological)

knowledge systems embedded in other cultures (Settelmaier 2009). These transformative learning skills constitute essential components of the higher consciousness called for by Laszlo (2008) for combating the chronic crises threatening the planet's eco-cultural systems. (Taylor, Settelmaier & Luitel, 2012, p. 377)

As the authors claim there are issues that cannot be addressed merely by quantitative research. Previously I argued that quantitative research by itself cannot exist; even to initiate a quantitative research task there needs to be a preceding qualitative inquiry. Then at the conclusion of any quantitative research there needs to be a follow up qualitative inquiry. Then again qualitative research by itself is impotent in determining which policies work better. The awareness of the significance of qualitative research paradigms is a welcome development as long as we still cherish the essential role of quantitative research.

To articulate my research findings well, I need to express my views on the research paradigms employed in this research work, and present expert views to support my choice of research paradigms.

### ***The Interpretive Paradigm***

The interpretive paradigm is based on social constructivist epistemology. An account of social constructivism is presented in Chapter 4. The noun, 'interpretation', encompasses the notion of subjectivity. Interpretations, to some significant extent, are the products of the interpreter's subjectivity. The researcher looks at the social sphere of the world through his value and belief systems. By doing so, he constructs his perceptions and views of the world. These new perceptions and views may engender a different set of mind lenses. The power of his experience compels the researcher to articulate his views and perceptions motivated by the hope of finding a sympathetic audience.

Then again if these interpretations are merely the product of the interpreter's mind then there is no transferability or confirmability, and thus the research has no relevance to the external world outside the researcher. For this reason, as positivist research treasures validity and reliability as its crown of legitimacy, the interpretive

research paradigm cherishes credibility, dependability, confirmability and transferability. See Taylor et al. (2012).

Taylor (2013) states that the interpretive research paradigm allows researchers to develop deep contextual understanding of different cultural situations and simultaneously be aware of and cherish their own cultural orientation and heritage. Because of this flexibility they can utilise ethnographic, autobiographical and narrative methods to explore their own life experience. This excavation of one's own cultural background, while appreciating other cultures, can help the researchers to emancipate themselves from cultural shackles of various forms. This view expressed in Taylor (2013) reinforces my choice of the research paradigms. As Taylor observed, emancipation from cultural shackles is a feasible goal that I experienced during this research.

Even though I will never become a Western or Eastern ideology<sup>3</sup> basher, through this inquiry I have realised that to some extent my thought processes were shackled by the Sri Lankan colonial history as I was born just seven years after independence. I also have developed an understanding that all thought processes are human in essence. For instance, when I see the drawings of M.C. Escher I can see an 'Eastern' kind of thinking radiating from his work. Nevertheless, to the best of my knowledge he did not have any significant Eastern influence. Another example is that Zeno's paradoxes resemble an 'Eastern' kind of world view. Instantaneous rate of change<sup>4</sup> is the synthesis of two complete opposites and resembles an 'Eastern' type of profound 'truth'. The Buddhist notion of right-mindfulness is somewhat similar to

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<sup>3</sup> I do not identify Western Ideology with colonialism. Any nation, Eastern or Western, due to materialistic greed, is capable of becoming colonists if the opportunity exists. Some people, for instance, identify exploitation of patients by the medical industry with Western ideology. For me, this exploitation happens due to human greed and not because medical science is a product of pure Western ideology. An example is that there are groups of people who condemn Euclidian geometry as a Western ideology based on Judaism and Christianity. In contrast, I value Euclidian geometry as a magnificent example of the power of human cognition. This power of cognition is simply human and has no ethnicity.

<sup>4</sup> How can the dependent variable have a rate of change if the independent variable does not change? See p. 194 in Chapter 6.

the notion of pedagogical awareness raised in ‘Western’ education research literature. This is because, regardless of Western or Eastern, we share a common humanity. There are differences but similarities are greater.

### ***Paradigm of Criticalism***

Taylor et al. (2012) observed that science education researchers started to welcome and employ the critical paradigm in the 1990s. The Critical paradigm can be regarded as an extension of the interpretive paradigm. While the interpretive paradigm seeks only to understand world affairs in a contextual manner the paradigm of criticalism seeks to intervene and shape world affairs. In this sense, Marxism is also an example of the critical paradigm even though later the mode of intervention became violence. Researchers who employ the critical paradigm endeavour to engage their audience in reflective and critical thinking in order to engender social change. Critical researchers invite their audience to examine their belief systems and seek to provide alternatives for existing and uncritically accepted notions. The critical paradigm has been a significant aspect of my practice as a learner and an educator. The views expressed in the literature on research paradigms reinforced my choice of research paradigms.

The Critical paradigm, to be of any value to others, needs to start from the researcher. Then it becomes the self-critical paradigm. The self-critical paradigm is essentially criticism of social affairs using self as a medium. When we criticise ourselves we criticise our interactions with our world. Therefore, self-criticism cannot be void of social content. Throughout this inquiry I employ the interpretive paradigm and critical autobiography as my research methods.

### **Quality Standards**

The quality standards of qualitative research need to be compatible with the nature of the research focus and the particular qualitative research paradigm methods employed in the study. The major research focus of this inquiry is my life experience that academic constructivism can enhance the quality of the learner and the contention that it cannot and it has not. This is an interesting situation that needed to be carefully scrutinised. At times, I felt that I was pulled both ways simultaneously by these opposing views. What I desired to do was to understand these tensions,

clarify the conceptual nature of the problems and develop a clearer view. There may be many who are subjected to the dynamics of these opposite views. They might find this discourse is useful or motivates them to examine their own struggle in a scholarly manner. There may be others who are convinced one way or the other. Since I experienced both of these sides, this account of the critical examination of my own struggle with constructivism might help them to better understand their own standpoints.

Any research, regardless of its epistemological foundation, deals with data. A major part of the data of this research came from my life experiences, observations and my reflections on them. I was duty bound to look at this data in a critical manner along with other discourses. I needed to be alert that my inability to deal with my life data in an open and critical manner could have compromised the relevance, utility and the quality of my research.

I paid special attention to the standards of wide ranging research methods, including various interpretive research methods discussed in Denzin and Lincoln (2000). Anderson (1998) provided valuable insights on how to think about the fundamentals of education research. Every interpretive/critical ethno autobiography research can have a unique character. For this reason, quality standards can be unique for the study itself, at least to some extent. Nevertheless, I would particularly appeal to my audience to judge my work on credibility, transferability and confirmability (Guba & Lincoln, 1989, p. 241-243). Triangulation (Anderson, 2004) is considered as an effective tool of evaluation of research. Most of all, I would like this work to be judged by the extent to which it raises pedagogical awareness and thoughtfulness in myself and others.

### ***In Search of Resonance with My Audience***

I write this report since I believe in a form of objectivity. I expect my audience to judge this inquiry in an 'objective' manner. As immediately as I finish the two sentences above, I find that I am in acknowledgement of subjectivity. Whose objectivity am I seeking? When I receive the judgment of my audience, I cannot help but judge the judgement I receive. Therefore, by objectivity what I mean is an agreement between my audience and myself about this work. At the end if I fail to

agree with the judgement of my audience it is my failure not that of my audience. It is my duty to seek resonance with my audience. Then again, if I seek mere resonance with my audience neither I nor my audience gains much from this inquiry. Therefore, it is clear to me that I need to construct resonance captivated with tension. In summary, if I expect my audience to totally agree or disagree with me then I am not making any contribution to the field. That is, I hope for attention to and due consideration of my views. The onus is on me to strive to gain both resonance-‘discordance’. In search of resonance, next I present an account of quality standard issues.

### *Confirmability*

I have expressed the view that notions presented in this research exposition are mind images taken by using paradigmatic lenses. If there are no lenses then there are no images. What does confirmability mean within the context of this research? For instance, if this were quantitative research based on logical positivism then there are many means to affirm the objectivity of the research. Checking data for accuracy, calibrating the equipment used and repeating the experiment many times to see whether these repeated experimental results agree with each other are some of these modes of verification. These modes of verification of objectivity may not be feasible to research of a qualitative nature.

Nevertheless, there needs to be criteria to judge and assess the ‘objectivity’ of research of a qualitative nature. Shenton (2004) acknowledged the difficulty of ascertaining confirmability in qualitative research and suggested some guidelines for achieving such confirmability. In this article he suggested:

- The research findings need to rise above the personal preferences of the researcher.
- The researcher needs to declare his or her predispositions. This means that there needs to be a discussion of research epistemologies and methods used in the research report.
- Triangulation has a significant role to play in establishing confirmability.

I appeal to my audience to evaluate my work on the first two of these suggestions. With regard to the second of these suggestions, the onus is on me to draw on the self-critical paradigm and declare my own predispositions and demonstrate how these predispositions transformed into more considered views. Then again, whether I have achieved this goal is the prerogative judgment of my audience.

### ***Triangulation***

In a discussion on establishing the validity of qualitative research, Guion, Diehl and McDonald (2011) list modes of triangulation:

- Data triangulation
- Investigator triangulation
- Theory triangulation
- Methodological triangulation
- Environmental triangulation

None of these modes seemed to be directly applicable to the current research work. Fenech and Kiger (2005, pp. 21-22) discussed the concerns expressed in the literature that triangulation just fulfils the completeness requirement rather than the confirmability aspect. Then, citing Denzin (1989), they mention the ‘person’ triangulation method involves the collection of data from different levels of persons. This has occurred in my research since the data comes from my interactions with many different people and institutions. And, as a modified version of the ‘Person’ Triangulation method, we may also opt to evaluate the responses to my current work from different persons, who happen to share similar professional aspirations to mine.

### ***Transferability***

Transferability is the extent that the research findings can be applied to different but similar situations. This requires identification of aspects of similarity and differentness. The current research is based on my life experiences as a learner and an educator. The people involved here come from different countries, as I have resided and practised in four different countries. Nevertheless, this is a small group of people. The data used here comprise the perceptions of one person, me. To what extent is this research relevant on a larger scale?

Have I critically reflected on my life experiences with a wide angle view or have I superficially looked at my life experience to bask in the indulgence of self-worth? Have I tried to see the human-commonness in my life-experiences as a human being, a learner and educator? If I have looked into my personal life experiences and perceptions in an encompassing manner then this research may be able to offer others some valuable insights and ‘aha’ moments. For instance, since every one of us is unique, the teacher Mrs Kiribandhane and the student, me, are unique. The journey we travelled on together was also unique. If it is just unique why do I need to waste the time of my audience? Even if it is distinctively unique when I narrate my story my goal is to see the story as a human story. A story is a good story to the extent that it encompasses uniqueness and commonness with equal vigour. Consider a story about a unique friendship. Why are we interested in the story? It is only because friendship is common to all of us. By reading the story we cherish our own friendships and we learn about our own weaknesses and strengths. We may be motivated to address our weakness due to this raised awareness. However, if the story has not been developed along a thin rope, balancing uniqueness and commonness, we will put down the storybook after reading just a few pages. Have I done this act of balancing well? I can only say that I strived to achieve just that, but the judgment is in the hands of my audience.

### ***Pedagogical Thoughtfulness/Awareness***

Ever since I initiated this research work, my ambition and goal has been to raise pedagogical awareness and thoughtfulness within myself and to provide some insights to learner-educators to develop such awareness. One of my best learning-teaching times was the second trimester of Year 9. During that time I enjoyed pedagogical thoughtfulness and awareness to a heightened level. Since then my ambition has been to pursue pedagogical awareness and thoughtfulness and instil such pursuit in others.

Luitel (2009, p. 54) gives a vivid account of pedagogical thoughtfulness as a quality standard measure for his doctoral thesis. There he pays attention to the following goals. Pedagogical thoughtfulness:

- is based on phenomenological-hermeneutical traditions and expects to motivate the reader to critically and reflectively interact with the information presented;
- requires educators be aware of their operational and professional core belief systems.

I would like to add that pedagogical thoughtfulness should also seek to:

- raise awareness among learners about their own belief systems;
- encourage students to question the epistemological foundation of any knowledge claims.

Liutel (2009, p. 54) appeals to his readers, to ask:

Is the research text engaging? Does the researcher invite readers to reflect upon their perspectives on issues being discussed? Does the research text offer perspectival envisioning about addressing the problem being investigated? (Liutel, 2009, p. 54)

In addition to the aspects above, I appeal to my readers to inquire critically, analytically and reflectively: Are the lesson ideas presented later in this work likely to fulfil constructivist aspirations and right-mindedness?

### ***Reflection and Emergent Praxis***

Taylor and Wallace (1996, p.1) state: “Praxis concerns the way in which the researcher attempts to stimulate the reader to take deliberate action towards changing practice.” I hope that my audience will see the lesson ideas presented in Chapter 8 as emergent praxis. During this inquiry, many times, I felt that writing was an emancipatory mind cleanser and an immensely powerful professional development tool. The literature supports this view. For instance, see (Bain, Ballantyne, Mills, & Lester, 2002, p. 10). I hope that my audience will be able to judge me on the extent to which I have used writing as a professional development tool and how well I have listened and responded to the inner voice of my narrative.

To end this section on quality standards, I would like to point to a nagging and lingering shortcoming. That is, even though the research questions are qualitative in

nature, the quality of the current inquiry could have been further raised if I had had the luxury to use appropriate quantitative tools. The use of quantitative tools in this inquiry could have enriched the multi-paradigmatic nature and the quality of this inquiry, but this thought only occurred in hindsight.

### **Thesis Structure**

The Foreword is the real first entry into this account of my journey or pilgrimage, and it signifies the underlying theme of the current research. That is, it is nearly impossible to hold a single sided view on the issues of education. Perhaps this may be true for all issues of life, but I feel that it is especially true for issues of education. The proposition that a particular approach or methodology is unconditionally superior or inferior to another approach or methodology may not have any validity. For example, a monolithic belief that the calculator is or is not a good tool in mathematics education can be detrimental to teaching and learning expectations. In the Foreword, I presented teaching as a profession of contradictions.

In this chapter (Chapter 1), *Introduction*, I have explained my motivations for carrying out this research. Also, in this chapter, I have outlined the scope of the research and the research methods chosen.

In Chapter 2, *Journey of My Education*, I have discussed the journey or pilgrimage of my education. This pilgrimage is perceived as simultaneous wandering and journeying. In writing this chapter, I strived to penetrate into my memories of a very young age of my life, to understand a significant part of my psychological makeup. Also the role of cultural traditions and rituals in forming attitudes towards education has been looked at in this chapter. I have taken the view that life itself is a school and everything that happens in it is a lesson. My world outlook had been reigned by logical positivism for the first two and half decades of my life. While logical positivism had been my epistemological framework, when I was about 14 years of age I began to develop my own notions of academic constructivism due to the influence of my Year 9 Physics teacher.

In Chapter 3, *From Logical Positivism to Constructivist Epistemology*, I have dealt with my journey from logical positivism to constructivism. This is a journey that I

had never intended to take. Because of the wide range of reading available to me during my graduate studies in mathematics in the USA, I was compelled to rethink my logical positivist epistemological world outlook. Even though I had been a proclaimed logical positivist, I could see that some hidden buds of constructivist epistemology were there even in my early version of academic constructivism and my poems. When an American Professor of English told me that my poems carried a spirit of Eastern philosophy, rather than feeling proud I felt dejected since I was thinking that my poems were manifestations of deeply held logical positivist beliefs. Even though the major strike against my logical positivist thinking came later from Quantum Physics, I realised that for any careful reader the history of science is a record of the chasms, cracks and patches of the logical positivist worldview. At the end of this chapter, I present the rainbow of epistemologies as my world view and I reject worldviews rooted in a single epistemology.

In Chapter 4, *Constructivist Views*, I have focused on various forms of epistemological constructivism and have endeavoured to understand the similarities and differences between epistemological constructivism as presented in literature and my own notions of it. In this chapter, I have reviewed the major aspects and questions of epistemological constructivism. One of these questions asks whether knowledge is transferrable. The various forms of constructivism that are discussed in this chapter include Radical Constructivism, Trivial Constructivism and Social Constructivism. I perceive that these different forms arise from emphases placed on different aspects by different authors. Nevertheless, I have developed the notion that these different views need to be taken as a coherent union of contradictions. In a salt solution, even though we cannot see salt or water separately we cannot say that there is no salt or water and it is just the salt solution. Also we cannot say that there is just the salt solution and there is no water or salt. A solution of salt is an example of coherent union of contradictions.

To insightfully look at the intricate and interlinked nature of the various forms of constructivism, two fables, one ancient (The Blind Man and the Elephant) and one modern (Two Fish View) are used. In this chapter I have also opined that no human being (including scientists, lay people or religious authorities) can operate without a belief system. Constructivism is the belief system I have chosen after a painstaking

journey of wandering. Finally, at the end of this chapter, I have presented reasons for my own version of constructivism, why I believe in it and why it works better for me, and why it makes more sense to me than any other epistemological model available to me.

In Chapter 5, *The Futility of Single Flavoured Slogans, The Actual Messages Sent out and Sunk In*, I have chosen to reflect on the subtle issue of subliminal messages. This issue has been brought to my awareness by my experiences with and perceptions of them as a classroom teacher at various institutes. The message whispered in silence throughout the day may sink into our young students' minds more than the loud outspoken message once a month or so. It is not our words that sink into our students; it is the message we propagate by our deeds that is immensely powerful. Like an electric screwdriver that goes effortlessly into a piece of metal, messages penetrate into the minds of students and implant long lasting and cancerous attitudes. We, as educators at all levels, need to be mindful of this silent issue. In this chapter, I examine Victorian Certificate of Education Specialist Mathematics Examiners' reports. If the most capable secondary school mathematics students suffer from deficiencies in basic mathematics notions, as the Examiners themselves point out, then things need to be drastically changed.

In Chapter 6, *Teachers and How They Teach*, I have strived to scrutinise the practices of many teachers. History has recorded many outstanding teachers. Among them are Buddha, Jesus and Socrates (whom I regard as the father of modern academic constructivism). What are the implications arising from the practices of these celebrated teachers? I pondered this question on the belief that their practices have a strong implication even for today's education despite the fact that we are technologically so advanced, since our basic human attributes still remain the same. The challenge is to use the technology effectively and appropriately based on a carefully chosen constructivist pedagogical framework. To initiate the discussion on these issues, I first surveyed Wave Particle Duality and other Paradoxes of Duality. Here I opined that it is inevitable that we need to find a more effective way of dealing with dualities. Still we need to keep in mind that these dualities are manifestations of our perceptions of [ontological reality](#), not the dualities of

ontological reality. That is, in a sense these dualities are creations and or discoveries of our mind and may not be ontological.

In Chapter 7, *Instructivist Constructivism or Constructivist Instructivism*, I have focused on developing a conceptual framework of constructivist aspirations. This is, in fact, the culmination of the inquiry I document in this thesis. My passionate argument here is that academic constructivism is not a recipe of what needs to be done or not done. If we take the view that academic constructivism is just a methodology then we will fail the spirit of academic constructivism. My appeal to my fellow educators is to consider academic constructivism as a set of aspirations based on a constructivist world outlook. And also I would like my audience to consider the constructivist world outlook based on the concept of a rainbow of epistemologies presented in Chapter 3. Also I have discussed the closeness between the two terms: the notion of pedagogical awareness in the constructivist literature and the concept of right-mindfulness in the Buddhist literature.

Also in this chapter, I present an example of the failure of osmosis and assimilation as a method of teaching and learning. Now I can give another episode regarding this observation. Recently, I had an opportunity to work with a retired school principal. He was visiting our classrooms to observe learning and teaching environments. His task was to help us develop better learning approaches. While he was working with a Year 7 student, I remarked that the unit of length should be written with a one letter gap from the number. That is, it should be 5 m (This is a distance measured in meters.) rather than 5m (This is the product of 5 and the pro-numeral m.). He responded that throughout all these years this had not occurred to him even though he should have seen it a million times. In Chapter 7, I gave a story of the assimilation and osmosis of cultural nature which was one of my life experiences. It was remarkable that until I wrote this story, I had not been aware of how I assimilated this learning from my Buddhist background.

In Chapter 8, *Enhancement of Learning Capabilities of Students of All Ability Levels, through a Constructivist Approach*, I have presented a small sample of lessons that I have been conducting with my students, as a component of my ongoing action research. I believe that these lessons are based on constructivist aspirations and are

able to develop pedagogical awareness (right-mindfulness). I also believe that these lessons can be good springboards or stepping-stones to constructivist learning for many students of different ability levels. As a constructivist, even if I could, I would not develop lessons that can raise learning capabilities of students of all effort levels. The success of my lessons should be measured on the following criteria: whether they are based on my set of constructivist aspirations, whether these aspirations make sense to other educators and learners, and whether they can help students to develop better learning skills and raise their right-mindfulness.

In Chapter 9, *Conclusion: Stepping-stones to Stepping-stones*, I have presented a brief overview of my inquiry. Nothing in this inquiry can be taken to be definite and complete answers. I only hope that they can at least serve as stepping-stones to a much better way of thinking.

## Chapter 2

### **Journey of My Education, Journeying by Wandering or Wandering by Journeying**

#### **Beginning**

As stated in the introduction to this thesis, a significant part of the data of this research is my life data. In this chapter, I narrate the story of my education. In doing so my expectations are to employ writing as a research tool and gain insights into my own learning and teaching that may be relevant for others. I believe that just writing this chapter mainly as a narrative story will facilitate me with a powerful way of reflecting on my own teaching practice. In later chapters, I will incorporate this data in a more analytical manner.

The following is the narrative of my education. In this chapter, I take a much broader view that education is what happens in one's life rather than what happens in just classrooms. In this view, every moment is an opportunity to learn. What happens in every moment, how we react and respond to those events, shapes who we are and who we are not. My health, wanderings, failures, frustrations, associations and friendships all were part of my journey of education. In the following chapters I will focus on education as we formally know it. Even then I would intermittently draw insights from this chapter.

Even though I have reflected on my journey innumerable times, I have been only vaguely aware that my journey consisted of many wanderings, “wrong doings and muddled thinking”. I was suffering from the illusion that I was a model student, even though many described me as such. Others evaluated me as a model student because they did not have access to my internal struggle. I did the same because I conveniently ignored looking at the root causes of my internal struggle and almost unconsciously chose not to examine them. At times, perhaps I might have been reflecting on my journey in specific periods and thus missed how much wandering had occurred. Only when I started to write this chapter did I come to realise this defect in my reflection.

In Sinhalese culture there is formal initiation into education. This occurs between the first year and the second year of life. This is called “Reading Letters”. I do not remember anything about my “Reading Letters”. My Mother told me many times that my performance at “Letter Reading” was dismal, and a cousin several months younger than me performed excellently, even though his “Letter Reading” was to come a few months later. According to my Mother, this had provided ammunition for my paternal Grandmother to hurt my Mother.

For “Letter Reading” there should be an auspicious time. This auspicious time needs to be obtained from an astrologer. Since my Dad was an astrologer (self-learned by reading books) he himself might have calculated this auspicious time, and perhaps might have checked with another professional astrologer for reassurance. As in any Sri Lankan celebration, there might have been a table full of milk rice (rice cooked in coconut milk), bananas and Sinhalese delicacies such as kaums and kokis. Mangoes should have been on the table as mangoes are called ‘amba’ which starts with the first Sinhala letter Ayanna (a). Several coconut oil lamps might have been burning beside the table or the mat where the letter reading had to occur. Kaums are oil cakes made by frying (one at a time) small quantities of mixture of rice flour and treacle. Kokis are also made in the same way, but do not include treacle as an ingredient; instead they may contain some spices for flavour. Treacle is kitul (*Caryota urens*) or palm syrup.

At the auspicious time, a learned person or a monk would point to the first letter of the Sinhalese alphabet and pronounce the letter inviting the child to repeat the sound. If the child refuses to say the first letter, then the learned person would ask pointing out to the mangoes “What are these?” When the child says ayanna or amba, the first part of the initiation is over. Next the learned person would place a pencil in the child’s right hand. Then he or she would move the child’s hand to write Ayanna. At the end of this initiation, the child will pay homage to the learned person, parents and elders in a particular Sri Lankan way of going down on the knees and putting the forehead at the feet of the person who is being paid respect and love. Next the adults will bless the child and wish her well in her education. Finally they will sit at the meal table and eat. At my “Letter Reading” I had said “Anga” instead of “Ayanna”. At that time I had a speech problem and I had been several months late to walk

compared with my younger cousin and the average age of walking. I am not certain whether the negative connotations arising from my letter reading continued to affect me in any way. For example, hearing statements such as “Ranjith cannot do anything right” directed at Mother could have sunk into my subconscious. My Mother continued to be resentful until she passed away in 2010 that my paternal Grandmother prohibited my Mother from breastfeeding me on the pretence that Mother had a lot of phlegm in her chest. My Grandmother used to breastfeed me even though there was no milk at all.

I was raised on a baby formula. My mother had been suffering both physically (breasts swollen with milk) and emotionally. Physical pain should have receded after two weeks or so, but the anguish of mental scars was tormenting her up to the very last moment of her life. Maybe for that reason I began to be extremely critical of Sinhalese medical and cultural views. Therefore I was positively biased towards modern medicine but later I found out that even though my Grandmother was wrong on the issue of breastfeeding, she was right on many other occasions and modern medical science needed more time to catch up.

During the time of my “Letter Reading” we all lived together, in Trincomalee in the East of the country. Then my Father decided to buy a house in the South very far from Trincomalee. My father still continued to work as a vendor in Trincomalee. For this reason, Father could visit us only several times a year. Initially I had been attached to my grandmother as my Mother Figure. For a long time I had been calling Mother ‘auntie’ and Grandmother ‘Mother’. My Mother, out of love, chose not to fight for my love. I vaguely remember (this could have happened when I was at the age of about four years), that whenever my Mother noticed that events were hurting me she gracefully tolerated my detachment to my own mother and my attachment to Grandmother. I began to realise that I was just a tool in my Grandmother’s hand in her war against my Mother. Before I was five years old Grandmother left us when she realised that I was no longer prepared to be in her bosom. Rather I began to defend my Mother.

After that my day to day education and discipline fell on the hands of Mother. When our Grandmother returned eight or nine years later, I had grown enough to refuse to

submit myself to her influence. I loved Grandmother immensely, but I was extremely resentful for her cruelty towards my Mother.

Even though I do not remember whether I had gone through any mental torments due to this struggle that lasted until I was four to five years old, I remember Mother painfully withdrew from me whenever there was a fight for my love. Those events should somehow have affected my development. Maybe I became more reserved, cautious, skilful about evaluating other people and somewhat reluctant of accepting love from other people. Later, particularly in my adolescent years, I was harshly critical of my Father for not protecting Mother and me. True to his self, he was genuinely apologetic. I kept saying to my parents that I was saved from being completely destroyed just because of my Mother's thoughtfulness. And, also, Mother gave me the greatest gift of love by not fighting to win my love. She let me assess the situation then choose. My mother acted so elegantly due to the wisdom springing from her unselfish love. This could have had a strong positive effect on my persona.

One of the memories still being played in my mind epitomises her demeanour towards me when I was young. It was before we came to South and I could have been about three years old. I needed to be hospitalised. Instead of my Mother, Grandmother stayed with me in the hospital. When I returned home, Mother did not come to me, hug or kiss me. I do not remember whether I was longing to see her. Then she bought me a cup of milk. It is the manner of her offering me the cup of milk in the memory flash that still hurts my soul for the pain she went through. What is remarkable in this flash of memory is that she behaved as though she was my servant. There might have been some effect from these episodes that later made me attracted to feminism.

### **My First Bold Step**

The next big event of my education was the first school day. My Father had given instructions in a letter to my Mother about the auspicious time that I should put my right foot forward. I should have been just five years old. The auspicious time was early in the morning, about 5:30 AM. I still vividly remember that I was sitting on a chair in front of the house just under the roof extension while torrential rain was beating on the ground and the small kerosene lamp was struggling to dilute the thick

darkness around me and having a blanket tightly woven around me to defend against the penetrating cold wind. That was because the auspicious time came much earlier than the school opening time, 7:30 AM. My mother, even though she was a believer in Astrology, was resentful for having to put me in that torturous circumstances due to my Father's strict adherence to auspicious time. My Mother used to remind me every now and then this horrific experience. This experience and Mother's expression of some disapproval might have planted the seeds of my rebellion against such irrational beliefs.

I remember initially I had a great deal of difficulty in writing the number two. Later I learned that this could be regarded as a symptom of dyslexia. Now looking back, I can realise that I had some other dyslexic symptoms too. For example, I took a couple of hours to make a copy of my timetable when I was in Graduate School. This defect has now vanished. My mother worked very hard with me about the number two and finally we succeeded. Before I went to school I was more than ready for school due to Mother's humongous efforts. Even though I joined school in the middle of the year I achieved First place in the class in the following Term Exam.

My favourite subject was Mathematics. From the very start, I exhibited a talent in Mathematics and my teachers were very optimistic about my academic future. In my later school years sometimes it appeared that they were wrong.

I had differences with my parents. Some of these differences were sheerly due to my ignorance. I still stand by the other differences I had with them. I rebelled against their beliefs. Nonetheless, if I have gained any success in my life and if I have nurtured a decent strong value system including my appreciation for education, that feat is solely due to their influence, love and respect. If I have had any failures in my life they are of my own making. I know this contradicts my basic belief that I should take the credit or blame for my own successes or failures. This is the nature of teaching and learning. It is impossible for me to overestimate the greatness of my first two teachers, my parents. No matter how rebellious I looked at times, I was the boy that they dearly wanted me to be. They always knew it. Only later did I gain that insight. Most of my life I was thinking that I was following a path that was

completely against their wishes. In essence, contradictorily to superficial appearance, I was nurturing the values that they really wanted me to have.

### **My First Two Teachers**

When Father came home he used to play with us as well as have long discussions of his life experiences. A discussion that occurred when I was about ten years old, can be summarised by a statement he quoted “Eat to live, do not live to eat”. Those words penetrated my mind, heart and soul. This sermon came from a man who had to abandon his education when he was in Year Three (at the age of eight years) to look after his family due to the sudden death of his own father. Once, he told me (noticing my uncombed hair), “Dress well to demand respect from others and to express that you respect them, not for fashion.” I did not comb my hair, at that time, to express my displeasure at fashions.

Knowing that I was strong in Mathematics, he used to tease with me challenging mathematical problems that he memorised from reading newspapers. He was proud that he himself had worked out the solutions for some of these problems. Buying many story books, he directed us to reading. A significant part of his gifts were books, and sometimes he sent us books by mail. Until she passed away, my Mother used to be critical about one of the books he gifted me when I was in Year 3. My Dad was misled by the name of the book, “Daddy of the Baby”. When Mother got an opportunity to read the book, I had already finished reading it. Even though it was a decent book, it was a collection of short stories of adult themes. My Mother thought that I should not have read about such complex issues in such a tender age. I did not agree with Mother and defended Dad.

When I was in Year Five or Six, he bought me a book, “Science and You.” This book was a collection of trivial scientific facts and their explanations. I still remember that first page of the book explained how carbohydrate was transformed into simpler sugars in the mouth itself due to the enzyme ptyalin. I was fascinated with the book to the extent that I began to explain those facts to my classmates.

I impatiently waited for the second part of the book to be published. By the time it was printed, I had outgrown it and my wait became a huge disappointment.

However, due to the sensational effect that the first exerted on me, I began to tune into radio programs to gain scientific knowledge. Access to books or any information was really limited. I used to pick up pieces of newspapers from the road and read them while I was walking to and from the town.

My mother's love and care set me on the right path. Only occasionally did I feel harshness from either of my parents. I did not feel that my freedom was restricted or they were overprotective of me. For instance, when my mother caught us watching people gambling in a corrupted place on the other side of our fence, she took us (me and my brother) home and gave us a sermon. At the end of the sermon, she asked us to go and watch if we still so wished. The only restriction was to keep her words in our mind while watching. We gladly went back and returned just after five minutes. After that, my mother did not have to worry about us sneaking into that corrupted place. Even in those five minutes, she somehow would have kept her watch on us. As a result of these interactions, I was developing into a person who can influence others rather than who can be led by others.

In contrast, there was another occasion she chose to act very differently in disciplining me. The effect of my mother's choice still keeps me in the right place like an untouchable, invisible giant shadowy protective and soft blanket around me. I was walking home by myself. About three hundred metres away from our home there was a house. There were two kids and one was about my age and the other was one year younger. They began to call me names. They felt powerful because I was walking in front of their house. There was a rusty tin can attached by a string lying very close to the footpath. I knew that it was one of their play things. Knowing that their verbal assault soon would change into crying, without having a need to own it I took the thing and carried it with me. They began to cry loudly providing me some entertainment.

My mother noticed that I had brought something home with me. She asked me what it was. She had to know because when I left home a few minutes earlier I did not have it. I told her the story. Mother became so furious. What was clearly evident was the strong disapproval of her beloved son's action and the disappointment with it. She took a couple of slim coconut leaf stems (these are the central fibre of a coconut

leaf.). She asked me to take the tin and lead the way to the other house. On the way she was lightly beating me with those coconut leaf stems below my knees. I felt humiliation. She asked me to talk to the boys and apologise and return the tin. I was telling her that they started abusing me first. She replied, “It is not my business to discipline those boys, it is their mother’s responsibility. My job is to discipline you.” Even though I was not crying, I was providing entertainment for my adversaries. Later I realised that the humiliation I felt was not caused by her harshness but the guilt that I had done something to make Mother so perturbed.

On one occasion of disciplining she chose to be democratic, give us a choice and preach a gentle sermon. On the other, she chose quite the opposite; to affirm her strong disapproval and demonstrate how upset and angry she was. How did my Mother know how to choose the correct way to respond? My only answer is that she acted on pure love. Love told her how to respond. I took a lead from her. When my children walked on someone else’s garden playfully disturbing nicely arranged pebbles, I went down on my knees and rearranged them. Seeing how sad and upset and how seriously I attended to rearranging the pebbles, they learned to avoid such behaviour.

Sometimes when I look back I cannot understand how my uneducated parents behaved much more brilliantly than many other learned psychologists. When my mother failed to take a step backward, my father did. When my father failed, my mother influenced him. The net result was that we were able to save our pride and innocence in front of our parents. This saved our respect for them. I certainly do not say that they were permissive. I definitely did not feel that they were authoritarian, except on a few special occasions but they were always authoritative.

Even though my mother had attended school up to Year 7 and Father only attended school up to Year 3, I felt a great deal of depth in my Father. For my Mother to attend school up to Year 7 was a remarkable achievement for a woman in that era. While my mother was motivated by unselfish love, my Dad took the lead from his depths. The difference in depths I perceived between the two, I believe, was just due to the restricted or enhanced opportunities due to the cultural expectations of gender

difference. In that era independence of a woman's mind was not something that was openly cherished.

When he came home we engaged in debating putting our points of view in the open on various issues. They might have strongly disagreed with our views, but they never used their parental authority to put down our views. My parents were not meek. I still wonder for what reason they did not stand in my way, when I was later (still at a tender age) giving up my birth religion and refusing to follow the religious line. They were certainly critical of me, but never forced me to do something I did not want to do.

I continued to excel in education. I began to study Algebra and Euclidian Deductive Geometry in Year 6 as it was required for everyone in Year 6. More than twenty five years later since she had left school, without having any form of revision or continued education, Mother was able to help me in Year six Algebra, including directed numbers and solving linear equations. Even my current students are speechless when they find their parents still remember the mathematics they learned a couple of decades earlier, though the students themselves cannot remember the chapter material right after the topic test.

In my world, there was a hierarchy of subjects in which Mathematics was held in the crown position. Drawing held the bottom position, and according to my wisdom it did not deserve my attention. This could have been a psychological trick I played on myself to avoid the pain that my inability to draw caused me. I do not know whether my drawing teachers were qualified to teach the subject. My guess is that they would not have been. My inability to write in line, unless the sheets were ruled (this persisted even when I was an undergraduate), caused me a great deal of anxiety. Another problematic flaw was my perspiring palms (I am not aware when this flaw vanished.) and the drawing became so dirty so quick. Well into my adulthood, I realised that I had very appealing visual images in my mind. I wanted to put them into drawing, but I did not/do not have the necessary skills. Four decades later I was fortunate to watch a Television program on drawing instruction. That lesson was a reawakening and realisation of my own foolishness. The thinking behind every line, dot or colour was based on the logical reasoning of how we make perceptions of

vision. These ideas are related to Mathematics and Physics. To put the picture on a paper using various kinds of lines, dots and colours, it needs patience in one's mind. Why did not I see this? I think lack of quality instruction could have been one reason, also it could be that even if the quality instruction was there I had simply tuned off. Also it could be both.

I remember when I was in Year Six I refused to complete the set drawing task for a Term Test. The teachers were very sympathetic towards my torments. (This sympathy was partly due to the fact that I was their best student). They sent a message to my mother. Mother came to the school and she made me take the test, for which now I am grateful. If I had paid attention, I could have learned that the task was not as difficult as I made it out to be. If I had learned that lesson in that occasion, it could have alleviated the pain I was about to go through. Only later did I realise that things are not as hard as we perceive them to be. If I had chosen to develop my drawing skills, it could have enhanced my mathematical skills.

Even though this hatred of drawing has affected my development in many ways, ironically the foundation for my next level of intellectual reawakening in Year 9 was also initiated by this hatred.

The book, "Science and You", set a clear learning path for my life. I wanted to become a scientist. To become a scientist I needed to go to a better school. Our Education System then consisted of four stages; Primary from Preparation Year to Year 5, Secondary from Year 6 to 8, General Certificate of Education (Ordinary) from Year 9 to 10, and General Certificate of Education (Advanced): Year 11 to 12. At the end of Year 10 and Year 12 the GCE O Level and GCE A Level Exams were conducted by the Government. Universities based their annual enrolments on the results of General Certificate of Education (A Level) Exam. The two certificates were also called GCE O Level and GCE A Level, respectively.

Since the village school that I was attending would not provide me the opportunity to study science or mathematics subjects, I began to pester my parents. I told them I needed go to a big school. Getting a place in such a school required good connections and good academic standards.

Feeling his inability to send me to a better school, Father acted rather angrily to a letter I sent to him. I do not remember whether I had provoked him by the manner I put the matter to him. The letter he sent me I did not see until about eight years later. Maybe he did not send it but kept it with him, or maybe Mother did not give it to me and hid it. I am glad that I did not see it at such a tender age. I came across the letters (the letter I sent to my dad and his reply to it) at a very crucial stage of my life when I was a throned king of despair. To refrain from going out of the chronology of my life events, I will deal with this funny episode of haunting letters when it is more appropriate.

At this point my parents would have felt that I had somewhat moved away from the sphere of their influence. They almost allowed me to make my own decisions. They might have felt that because of my debating skills and knowledge in several areas I had reached a point at which I was almost capable of handling my own affairs. My request that I should be enrolled in a better school created a qualitative change in our relationship. They might have thought that I had started to take charge of my own life. They chose to allow my independence (maybe cautiously). This independence was not a free-and-no-responsibility-attached ticket to my freedom; rather it empowered me to be cautious and righteous. Wherever I was, I always felt their gigantic presence in my small heart in a soothing and guiding manner.

### **Wandering**

They began to work on getting me into a better school. I took two qualifying exams (for two different schools) and I was offered a place from the school that was our second preference. The other school said that it would be another couple of months before the selection process was finalised. So we decided to take the first offer.

A couple of months later, I got an offer from the other school, Dharmasoka College, which still enjoys a strong reputation in Sri Lanka. At my first new school, I performed well as I was used to doing in my old school. At the second school, my academic performance fell to a very low level. It was not that I developed apathy towards learning. More than ever, I chased after learning. Yet certainly I was suffering from apathy towards school learning, memorisation, tests and exams. Even in previous years I needed to memorise many things, though I did not feel the pain of

mental drudgery. It was many years earlier that I was required to memorise my times tables. During that time my Mother was in charge of that task. This hatred of memory dominated learning was a new attitude I was developing. I was very proud of that attitude believing it was a sign of my superior intelligence. Only when I became a school teacher in Australia did I begin to properly appreciate and understand the role of memory in thinking.

Although these attitudes I was developing were not totally irrational, now I feel they were largely psychological trickery that I used to convince myself and others that my weaknesses were really strengths. I was saying to myself and others “I do not do as well in school as was expected of me. So what? I can debate. I can understand. You do not have the answers for the points that I raise”. I remember that I was arguing against much older kids. Sometimes they tried to enlist support from a second year University student. I was ready to take him on, but he agreed with my points. I was rather arrogant that I was able to stand up to a University student, even though I was a Year 7 student.

I want to make some remarks about my Year 8 Science Teacher. Even though he carried a big reputation as a gifted teacher, and his private tuition classes were full of students from nearby schools, I did not evaluate him as a great teacher, but definitely he was a good teacher. He would be my second choice when compared with Mrs Kiribandhane and Mr Mahindarathne. I have no doubts that many students in his class would have gained good marks and felt comfortable with the subject. He made it too easy for students by offering lucid explanations. I would say this was an extreme instructivist form of teaching. I believe that this form of teaching is much more harmless and helpful than the extreme form of constructivism. Then there are judgment calls. One’s left is another’s right and vice versa. Another’s middle may be the extreme right or left for some others.

Despite the fact that my favourite subject had been Mathematics and I demanded that my parents send me to a better school so that I could study science subjects my performance in both subjects was poor. Even though I was boasting about my ability to understand things, in later years I became aware of my lack of understanding due to sheer negligence.

The new school was located three and half kilometres away from our hometown, in a much larger town. I joined the library. That gave me much better access to many books. Even though there were not enough science and mathematics books, there were plenty of translations from ancient and modern world literature. I began to read many books, including Greek Philosophy. However, I was largely influenced by Russian literature, especially by the work of Fyodor Dostoevsky. Even today I feel that having had the opportunity to read his work made my life a worthwhile adventure.

Why I was performing so badly in those years is a question that I am still grappling with. My eye-sight could have been a remote factor and it was corrected only when I was in Year 11. Regardless of the fact that I was getting poor results and neglecting my homework, I was still burning with a desire to learn and understand. It was not that I became disengaged because I had other non-academic pursuits. Always my major pursuit was directly associated with understanding, learning and knowing. Was I just hapless and a powerless peg of the machine of fate? Was I just driven by some invisible chains of genes or other forms of inheritance?

I was thinking about going for a PhD in Science when I was in Year 7. Why did I not try harder? My aversion to Exams and regular schoolwork I justified to myself thinking that exam performance or regular schoolwork did not matter. What mattered was one's ability to think critically. Peers in my class respected me even though I did not bend my values to be compatible with them. I stood my ground firmly without even acknowledging their pressure. At the end of Year 7, my performance was at the lowest segment of all 275 students who were to proceed to study in Year 8.

In those years the list of “not-so-worthwhile-subjects” grew even further. It contained Mechanical Drawing, Music, History, Geography and English. Only good subjects were Mathematics, Science, Buddhism and Sinhalese. I liked the subject Sinhalese because I was an avid reader. In Year 8 there was another reason why I became fond of Sinhalese. That was the teacher. The teacher wanted us to write an essay on the origin of the Sinhala people. To motivate our ideas he first wanted us to engage in a debate, one side taking the widely acclaimed view and the other side taking the opposite view. He chose me as the leader of the opposite view. I refused

saying that I did not have the skills. He replied that he was certain that I was the best choice. Even though in a way it came as a surprise, he was verifying what I felt about myself. Then again how did he make that judgement? It could not have been based on my test results, homework, or exam performance. He most likely had based his judgement on my answers to his questions in class.

In the first half of Year 8, I was almost giving up on my dream. I began to think that if I could not study science then I would become a writer. This becoming a writer as a second option occurred to me because of the influence of the Sinhala Teacher. Year 8 was a very important year in our system. At the end of Year 8 students who were qualified to study further were divided into three streams, Science, Commerce and Art. Science was the most preferred stream and Arts was the least. I performed badly on the first two term tests. The teachers were encouraging us to study harder. I realised that to be selected in the Science Stream I needed to get to the top third of the student population.

I talked to my Mother. I asked her to intervene on my behalf and talk to the Principal if I was not selected into the Science Stream. She refused and said “You wanted to do your studies at Dahramasoka since you wanted to study science. You have not worked hard. I put you into a tuition class. You went there only two or three times. Now you need to pay the consequences. We did what we could; you did not do what you should.” I knew that she was perfectly justified and even if she wanted to make an effort on my behalf, I had not provided her any reason why she should do so. Also I was aware that she did not have the courage to go to the Principal and ask for a second chance for me. At that time, I decided to study harder and see the Principle myself, if it came to that.

I began to work harder. At the end of Year 8, from about 280 students I got a place between 70 and 80. It could have been that I was seventy second or seventy eighth. That meant that I had been selected to do science. In science there were three streams. Each stream had eight subjects. The First Stream was the Mathematics Stream with subjects Pure Mathematics, Applied Mathematics, Advanced Mathematics. The second stream had the subjects Pure Mathematics, Applied Mathematics and Biology. The third stream had the subjects Pure Mathematics,

Mechanical Drawing and Biology. All streams had the subjects Sinhalese, Buddhism, English, Physics and Chemistry. Unfortunately I was selected for the third stream with the subject Mechanical Drawing.

As a vendor my father had to walk miles and miles in a day. He felt that he was ageing. My parents decided to leave me and my youngest brother under the care of my Mother's sister's care at our own home. My Mother moved to the town where my Dad was working (where I was born.). My other brother (second son of the family) and baby sister would live with them. My parents already knew that Auntie's authority was very weak with us. My brother later passed National Scholarship Exam in Year 8. That was a feat I could not achieve, though all the teachers hoped for me to achieve the feat at the Year 5 National Scholarship Exam.

My hopes were high, since I had gained what I had wanted. But to find success, it is not sufficient to get what one wants. One needs to do the right thing with what she gets. In two years' time, I would be required to take the GCE O Level Exam. In the meantime, the school was trialling a reform program in Education. Instead of Term Exams (there were three terms in an academic year) the College endeavoured to try a Monthly Test Format. Since my weakest subject was Mechanical Drawing I paid more attention to Mechanical Drawing. Again in both Monthly Tests I was near the bottom end in my class of about thirty students. My class was the lowest class of the three science classes. However, I was rewarded in my dreaded subject Mechanical Drawing. I scored 50% and 80%.

For this class there was a particular teacher, Mrs Kiribandhane, who taught Physics. I did not know that this teacher was soon going to make a big change in my life shortly since I was still tuned out. I noticed that the teacher strived to cognitively engage us, but at that time it did not matter to me much.

### **My Journey with [Academic Constructivism](#)**

One month into the second trimester, I wanted to change class and drop Mechanical Drawing and add Applied Mathematics. I had heard from the students from the next class that Applied Mathematics had four branches: dynamics, statics, hydro dynamics and hydrostatics. I thought the solution to my problem would be to replace

Mechanical Drawing with Applied Mathematics. To do so, I needed to change my class. I cannot say what was I driven by: the need to escape from Mechanical Drawing or desire to do another mathematical subject? Maybe both of those forces were working together to embolden me. Being such a failure why did I take such a risk with even much harder subject of Applied Mathematics?

At that time there was no concept of electives. In any case it was almost the middle of the academic year. Also, because I was performing in the bottom of the class, no teacher would have supported my request to study in a higher-level class. I informed the class teacher who was also my English teacher. She said that she would talk to the Principal and ask for the approval. Her agreeing even to talk to the Principal was remarkable and encouraged me. My class teacher would have just dismissed me if she did not see any merit in it. She might have perceived that I had the ability or she might have just fallen for my boldness.

I was impatient. Contradictorily to my behaviour, I took my books and just went to the other class and arranged a desk and a chair for myself and sat there. This was extremely unusual for me or any other student for that matter. Where did I get that dauntlessness and intrepidity? In retrospect it looks like I was possessed by some divine force. It was the most decisive point in my life. Whatever had led me to behave “irrationally”, I am glad that I did. I might have feared that my request would be denied. Then if I acted the way I did it would amount to mutiny.

The news went to the Principal, Mr Wimalasuriya. (Even the staff were scared of him.) He furiously stormed into the class and began to berate me. He asked me “Are you running this college?” He pointed out that I had very low marks in all the other subjects except in Mechanical Drawing, which I claimed to be my weakest area and the single reason for my poor performance in all the other subjects. He wanted to remove me from the class immediately and he ordered me to do so. I had only one way to stop him. I told him “Sir, I would be the top of this class by the end of this term.” He stopped, looked at me, and then left. To his credit, he might have perceived the only way for him to win is to give the victory to me. He would have thought even though the might of his position and law were all on his side, crushing me with that power was not a win. This was one of the defining moments of my life.

I was so fortunate that he allowed me to continue in this class. If he did not it would have drastically changed my life. He could have seen the intense resolution in my eyes.

After his retirement (several years after I left school) Mr Wimalasuriya became closer to me in association and we discussed about the good old days. When I left Sri Lanka for the USA for my doctoral studies in Mathematics on a Graduate Teaching Assistantship I went to him to say Good-Bye. Gleaming with enormous gratitude, I told him that he was one of the chief architects of my success. That was our last meeting. He passed away three years later.

Since Mr Wimalasuriya allowed me to continue in my new class I was blessed and privileged to start my wonderful journey with Mrs Kiribandhane. She was simply amazing for the reason that she did not teach us. Even though her qualification might not have included a degree from a university (just teacher training after high school), she was very quick in thinking and was disciplined enough not to give the answers to the students. To respond to an answer of a student she would ask another question without making any comments of her own. One day, I still remember, I was able to design lab equipment. All she did was to ask question after question and then discussed my answers with the class<sup>5</sup>. Then I knew that I had found my salvation. Understanding was easy. Learning just came to me. Studying became my hobby. This particular period of three months always stands out as one of the happiest times of my life. I would trade any part of my life with that period, if I could, just for the pleasure of thinking and learning.

### **Effect of the New Teaching Approach**

Soon I began to apply the method with other subjects. Then there was no Mrs Kiribandhane to ask probing questions. I had to play the role by myself. In a way it was better that all teachers were not like Mrs Kiribandhane. If they were then it

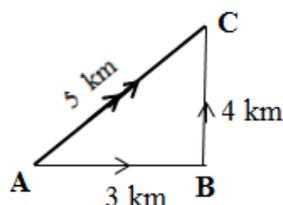
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<sup>5</sup> After such a lesson, I would ask my students, both seriously and mockingly, “Did I teach you anything today?” Then I point out to them that all I had done was to ask questions. They had come up with all the answers. That is how they should learn. They should learn to ask questions and ask questions from themselves to learn.

would have robbed me of the opportunity to apply the cognitive strategies and approaches to learning in my own space and time. If they were then this retrospect would not have generated the joy of my intellectual entertainment to the extent that it does now. Again, I have touched a contradiction. On the other hand, if all the teachers were like that then all the students would be more cognitively oriented. My task of learning would have been much easier for me. With Mrs Kiribandhane's teaching, when I understood some concept in a way I owned the understanding and the knowledge. Because other teachers were not like Mrs Kiribandhane, I had the opportunity go even further into the domain of cognition and be my own King. That was my gift from Mrs Kiribandhane; and the other teachers also (because they did not take the same approach), for that matter.

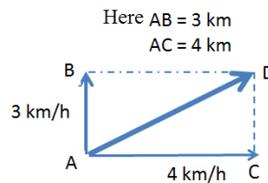
Some of the questions I asked myself were the questions usually put down as silly questions. Why are there 180 degrees on a straight line? One can take this question in a historical context. That is also important but at that time I did not have access to that kind of information. Therefore I did not raise such questions of historical context until later. Any way during the days of my naive constructivism I would not have allowed any link between Mathematics and History. Then again, such historical context cannot be derived by just reasoning. In that period of time a vast majority of my questions were mostly of conceptual nature and they probed the link between the concepts and the nature of the world.

For a long time to come, questions of historical nature eluded me. For instance, I pondered why do protons have positive charge, not negative charge? This question could be raised in a historical context but the answer I came up with was that the scientists could have taken protons to be of negative charge and the electrons to be of positive charge. Thus my answer was void of historical development but it encompassed the construction component of the concept. The following example from Applied Mathematics illustrates how I thought about concepts.



In Newtonian Dynamics, the representation of force by a line segment did not go well with me. I did not ask the teacher how it was possible; instead I asked myself. I was able to develop a nice way to look at this, which justifies the use of the cosine rule to find the resultant of two forces. First I looked at velocity. I failed. How come velocity (movement) can be represented by a fixed line segment? Therefore I decided to look at displacement. That seemed to be straightforward. If I walk 3 km in the direction of AB and then another 4 km in the direction of BC, then I will have arrived at C. Then the length of AC and its direction can be calculated by using geometry (or trigonometry). So far what I thought was not novel. Any textbook on the subject contains this explanation, one way or the other, even if the explanation is not so explicit.

Velocity still posed a problem. Consider the following diagram.



How do I add the two velocity vectors depicted in the sketch above? If I travelled from A to B with the speed of 3 km/h and, and then B to D with the speed of 4 km/h then my resultant velocity should have been  $5/2$  km/h = 2.5 km/h in the direction of AD. This did not match the vector sum answer 5 km/h in the direction of AD. Using the second sketch, I decided to look again at displacement. If I walk from A to D directly, my displacement from A to D can be regarded as the sum of the displacements, first from A to B and then from B to D (It does not really matter which movement is first.). However, in that explanation I moved in one direction at a time. To get the expected answer in the case of the velocities, I needed to move simultaneously in both AB and AC directions with the respective speeds. I could not imagine how I could move in both directions simultaneously. Then I reflected over charge of an electron.

Why does an electron have a negative charge? It does not have negative charge. We give the 'charge' of the electron the label, 'negative'. This led me to accept walking simultaneously in both AB and AC directions as a conceptual device that was not so far from reality: moving in the AD direction is equivalent to the sum of simultaneous

movements in both AB and AC directions; whether it is possible to do this physically is another matter. In doing so, I do not split myself into two pieces; one to take the path A to B and the other to take the path A to C.

I am moving in both directions not only at A, but also on all points on AD keeping myself intact. This assumption was further justified by my experience with the movement of an object when pulled in two directions. This resolved the difficulty with representation of velocity vector by a line segment and the vector sum formula of two velocity vectors since velocity is the displacement per unit time. The situation was quickly generalised to acceleration vectors. Since force is mass times acceleration the concept of representing force by a line segment made sense. It consumed my time, but the joy it engendered was worth the effort. I was making models using pencils, rulers, strings and books to understand Static problems. Also I began to be fond of Biology. After all, Biology was not so much of a “memory only” subject, as I had previously thought.

I called this method ‘the method of learning with a philosophy’. Without feeling that I was working hard, at the end of the term I came the top of my class. Two terms and one month into the last term (That meant there was only two months to the end the academic year), I decided to replace Biology with Advanced Mathematics, so I moved to another class. I missed my physics teacher, yet I did not miss her approach. It was with me.

For Year 11 and 12, our physics teacher was Mr Mahindarathne. He was soft-spoken, not as dynamic as Mrs Kiribandhane. But he had his own brand of teaching with philosophy approach, and it was effective. Its effectiveness was even beyond Mrs Kiribandhane’s approach in the sense that he was able to attract almost the whole class. He generally did not write notes for us on the board, apart from the diagrams. We needed to write our own notes. At times, if the issues were too complicated then he would dictate us to copy. I thought that they were demonstrations of how notes should be taken. The presentation of questions and their probing nature stimulated our minds. He also frequently used a Socratic questioning. Yet there was a difference between his approach and Mrs Kiribandhane’s approach. He used to say “Ranjith is the only one in this class who can think. Ranjith studies to understand and think.

Others are cramming for a grade.” Then he admonished me that I should overcome my difficulties with tests and exams.

This change in my academic performance did not change my attitude to school. I was named as a comet by Mr Mahindaratne since which days I would attend school were predictable. Those were the days when we had physics lessons with Mr Mahindaratne. In retrospect, the funniest thing was that I took criticisms against me as expressions of my greatness and I did not take any serious attempt to rectify my faults. Even if I did not attend school, I did not waste time by wagging. I stayed at home and did something academically worthwhile. Also, my criticisms over the education system grew more strongly. Again these criticisms had a sincere component and a psychological component. They were sincere because I wanted to see other teachers as intellectually challenging and stimulating as my two beloved teachers. They were psychological because I wanted to conceal my weaknesses under the clothes of pseudo intellectualism. To date I still hold these criticisms, but now they have different forms.

Another noteworthy contribution to my education came from one of my friends, Nandana, who was studying Sinhalese and Literature in Year 11 and 12 in my previous village school. Through him I was introduced to the critique of literature. Even though the paradigm he was based on leaned towards Marxism it stimulated my mind. Because of this influence I started to look at Marxism. Later on I was developing my own ideas to evaluate an artistic creation with help from many thinkers. Also, I began to write poems and short stories. A few of these poems will appear in this work when the themes of both the chapters and poems match.

### **My First Success as a Teacher**

I passed my exams at the end of Year 10 well enough to qualify for Advanced Level studies. However, these results were also a disaster as they were incompatible with the expectations of my teachers, parents and me. A year later, my brother failed his exams. My parents had high hopes for his achievement since he was a National Scholarship holder. This failure was partly due to his negligence of education. I had the confidence to promise my parents that I could help my brother to pass his exams. I promised them he would pass even at a much better level than I did.

I began to look at the concepts with him. I asked him to construct small models, using pencils, strings and other things. I posed questions so that he would make connections and associations. I tutored him for just about two months and he progressed well. He began to enjoy learning. He told me that it was time for him to study by himself and promised me that he would seek my assistance whenever needed it. At the end of the year he passed. For both Mathematics subjects he got distinctions. I was happy and sad. I was happy for two reasons; my brother passed his exams, and it was a triumph for my own intellectual prowess. I was sad because I did not achieve the same success. Why could I not get those results myself? What is wrong with me? I understood and knew everything but I almost failed. My brother's triumph helped me to console my own soul therefore I was extremely grateful to my brother.

My brother was later qualified as a Civil Engineer with a BSc degree. Now he is serving as a Mathematics lecturer at the Open University. He has built a quite good reputation as a teacher. He frequently acknowledges my role in his life. He was saved by the approach of teaching and learning with a philosophy. He is also teaching with a similar approach.

### **Debunking Myths**

I started my life as a devout Buddhist. Buddhism as practiced in Sri Lanka has many irreconcilable contradictions<sup>6</sup>. At the moment I do not intend to deal with these contradictory practices since it lies out of the scope of the current work. Buddhism is considered to be a philosophy that does not acknowledge soul. It is also a philosophy of [causality](#)<sup>7</sup>. I was walking away from my religion little by little. At the end of Year

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<sup>6</sup> Some contradictions can be reconciled to arrive at much more effective notions or magnificent concepts. This will be discussed in later chapters and the Foreword of this thesis characterised teaching as a synthesis of contradictions.

<sup>7</sup> [Causality](#) is a version of [scientific determinism](#), and therefore it leads to objective idealism. To avoid this, we need to take causality as a useful conceptual device that has some objective validity to our local environment. The standpoint that every effect has its cause is different from the standpoint that most of our experiences can be usefully explained by adopting a conceptual framework of cause and effect. Note that scientific determinism is a derivative of [determinism](#). Absolute determinism is another variant.

9, that process was complete. Even though I still rate this as a positive development the naivety of my then thinking about religion amazes me. This has taught me a lesson. When I develop an opinion, now I know that it is transitional by its very nature. I am aware of the possibility that either I would abandon it or develop it into a different form and content, or at least I will see it in a much deeper sense, encompassing many contradictory views and similar views. Still I remain a non-Buddhist. For that matter, Buddha himself was not a Buddhist.

Also, at about the same time I began to rebel over other cultural beliefs. While studying in Year 11 I was able to successfully debunk some of the village myths. I was able to gather ten or so village friends to carry out these activities. This [rationalist philosophy](#) engulfed my life for a long time to come. In Sinhalese there were only a few books to read. English books were not available. Even if they were available I would not have been able to read them. So the information I could gather was very limited.

I still believe that rationalist philosophy made a positive contribution to my life. It gave me sight, yet it took away my vision. For instance, for a long time I was thinking that there was a definite scientific argument that denied the existence of reincarnation, spirits and ghosts. That was not accurate. Why did the rationalist thinkers, who inspired me, fail to mention the inability of science to disprove these matters? After debunking stories of ghosts and reincarnation stories, after stating their scientific arguments, they should have mentioned that still it had not proved that reincarnation and ghosts did not exist. I expect this honesty from the authors. Perhaps they were writing for a different audience.

Writing on women's rights, these authors put forward a convincing argument about why the rhythmic method of contraception sanctioned by religious authorities against the contraceptive pill amounted to a measure of oppression against women. They rightfully argued that these religious authorities sanctioned sexual intercourse when women were in the lowest ebb of their sexual desires. They questioned: Do women's bodies exist only for the pleasures of men? I was very much impressed by this argument. But about sixteen years later, when I was able to read English (while I was living in the USA), I learned that here too they had failed to reveal the full

information. The contraceptive pill also can reduce the libido in women. In fact, it can do more than that. It could give blood clots. My idols fell from grace.

### **Harvest of Intellectual Arrogance**

Even though I passed GCE A Level Exams, I was not able to get University admission. I was devastated by this but now I know that I had reaped what I had planted. It came as a surprise to my parents, teachers and friends. Whether I had developed an excellent understanding or properly learned the content was unquestionable, but throughout the years I suffered from lack of cognitive discipline. My mind was everywhere wandering from one issue to another. Even though I was eager for academic success I pursued many other interests of intellectual nature and cherished lack of cognitive discipline as a sign of my greatness. A teacher also praised lack of neatness in my work as a sign of high intelligence. I was proud to hear that. Now I think otherwise.

For example, I did not even number my answer scripts before I began to write on each paper. At the end of the exam, I was not able to put the pages in order. Even though it vaguely occurred to me that this would come back to haunt me, I handed over the papers without making a real effort to number them. I was aware, albeit vaguely, that I did not display my sharp thinking skills when I wrote for an exam. Rather than striving to overcome the weakness I covered it with pseudo intellectualism. I was not simply patient. I am not sure whether it was nervousness since many saw me as a person of steel nerves in other testing life situations.

Even though I am almost certain that I might not have been described as arrogant by anyone, anytime in life, now I see that I was intellectually arrogant in my high school years and immediately after to the detriment of my own intellectual development. A decade and a half later, my Master's Thesis advisor described me as an inquisitive, bright and intellectually aggressive person. Very recently one of my students told me that at first she thought that I was intellectually arrogant. According to her, she later understood that it came from my strong desire for my students do well.

At one time in Graduate School I badly failed a test in a subject that I was making strong positive impressions on my teacher. He was very angry with me after marking

my paper. For two or three days whenever he saw me in the corridor he was expressing his disappointment, saying things like “You and your philosophy”. Then he said that he gave me an A since he was certain about my skills, understanding and hard work. Only then I decided to handle my exam taking difficulties.

In the oral segment of my preliminary exam one of my supervisors made a comment, “This may probably be your last exam of your life. Even though you have performed in the exam more than satisfactorily, your performance is much below than what we expect from a student like you. This will affect your life and I suggest you get hold of this aspect.” The other supervisors also agreed.

That was not my last exam as was predicted at the time of that meeting. Later I was required to take a couple of exams for my Graduate Diplomas in Education. I believe that I had largely resolved my problems with exams by that time. Since I did both diplomas by correspondence if I did not perform well in the exams I would have suffered negative consequences. Sometimes it surprised me that when it was crucial, I performed well in exams matching my skill levels. For example, I can cite TOEFL Exam, GRE Exams in Mathematics, Physics, and General Aptitude I took to gain graduate teaching assistantship at a University in the USA.

In those high school years and later the only thing I did to ‘resolve’ the difficulty was to become a harsh critic of the Education System. I emphatically claim that my criticisms were based on legitimate issues. On some of these issues it is true that I had taken extreme views and they have changed. When I was still in high school, reviewing my second brother’s “New Maths” curriculum, I confidently predicted that Ministry of Education in Sri Lanka would abandon it. It came true after a few years. That prediction was not an accident. I was able to make that prediction since through my own education I had developed astute judgment of educational processes. I predicted this outcome on the basis that teaching more abstract algebra to students who were robbed of the opportunity to master regular school algebra would not facilitate proper conceptual or/and skill development in students. Universities in Sri Lanka noticed a sharp drop of skill levels necessary for tertiary education. Universities demanded that school mathematics curriculum should return to the previous one. At that time I did not know that Sri Lanka was following the modern

education concepts of the developed world. Now I know that even the curriculum I grew up and experienced some success with also came from developed countries.

### **My Fate**

By the time I was in University Entrance Class my goal to gain a PhD in Physics was strengthened. If that was to fail then I wanted to become a writer. After failing to gain university admission, immediately after I left school, I obtained a teaching post. The appointment was in a rural area. Part of my work was to teach Advanced Level Mathematics. In the same month I was informed that I had won a scholarship for Chartered Accountancy. I participated in the course without resigning from my teaching post. I listened to one lecture of the day and then decided to go back to teaching. My decision dreadfully disappointed my parents, relatives and friends. I just knew that I had no life there. Even with a meagre salary I can enjoy life in teaching, I thought. Why did you kick the ladder that could take you to a golden future, was the question everybody asked?

Another significant event in my education came in a very different form. This occurred about one month after I started my teaching career immediately after leaving school. I was diagnosed as suffering from heart problems and hypertension. For a few months I had been noticing that I was suffering from breathlessness. After a month of hospitalised investigation the doctors determined that I had chronic pyelonephritis and essential hypertension. The cardiologist made a remark that if I were two years late in coming for treatment I would have lost my life. I was put on a drastically restrictive diet and several medications. These medications gave me many other health problems including kidney stones. To treat one side effect, I was prescribed another medication. That new medication gave me another health problem. At that time being a [rationalist](#), I was a pious follower of modern medicine (I do not like to use the term Western Medicine). Even in the hospital, when one of my Catholic friends offered me to pay for my health I told him in a stern voice, “Please do not, if you do so, just to see whether your prayers work, I might not take my medications”. Now I regret this event. If I know where he is now then I will keenly apologise to him since whatever my beliefs or disbelief in the effectiveness of prayers were I should have accepted his gesture of human compassion.

I just waited to die without taking care of myself. After two years or so, I recognised that I had already died since I was waiting for my death to occur. During these days I was a living dead, a zombie. Now I will discuss the two haunting letter episode that was mentioned earlier.

The immediate two and half year long period of my life after leaving school was reigned by despair. On one hand my dream of a higher education had been burnt to ashes. On the other hand, because of my illness, I was waiting to die. It was the only period of my life in which I hid my soul in a cocoon of misery that I myself created out of self-pity. For instance, I had even written a note in my diary to my parents how my funeral was to be conducted in a simple and brief manner. My first brother was entrusted to carry out my wishes. At that time I thought that I was so brave and courageous to embrace the reality of my illness and ‘almost immediate’ death but now I know that it was self-pity dressed in grandiose arrogance and pseudo braveness. Because of then my psychological need to escape from life I was regarding my doctors’ dire warnings and prognosis as a blessing. So rather than thinking of how to overcome my illness and embrace life I was embracing the illness to escape life. During those years I was receiving a great deal of sympathy from everybody. Even though I was not enjoying this sympathy outwardly it was subtly influencing me to form a dark outlook on my life and cherishing the predicament that I was in.

During that time my parents were so careful not to admonish me fearing that I would react to such sermons negatively. That is when I surprisingly found the letters ‘almost by accident’ while I was searching for something in a cupboard. That was a cupboard which we all used to go through almost in a daily basis. The sudden appearance of the letters in the cupboard was a message from my parents. The message was clear. My parents wanted to remind me about my educational aspirations. I felt a great shock reading both my letter and my Dad’s reply to it. Reading the letters, I realised that I was already dead. The task was not to bury the mind and the body but to resurrect myself. This episode presents another piece of evidence that how my parents were subtly clever in handling me. Even though they were sometimes clueless of how to handle my siblings and needed my help to do so,

they were grandmasters and extremely resourceful in guiding me to the right path with minimum interference.

The letters suddenly gave me the will to live to the fullest until my death would occur. I began to campaign against this sympathy and throw out my own self-pity and I decided to work untiringly to achieve my education dream. I began to study. My friends commented that the old merry Ranjith was back. Consequently my health issues settled down. I was able to get university admission for the most prestigious course (Engineering) at the most prestigious University (Peradeniya University) in Sri Lanka. I was happy again.

I studied eight months at Peradeniya University. I did not find intellectual stimulation in studies. There was too much work to do and a very little time to think. Instead of Mathematics there was Engineering Mathematics. Instead of Physics there was Engineering Physics. Rather than conceptual understanding the emphasis was on skill development. Again I was tormented by unhappiness. One Friday evening, without informing anybody, I just packed my bags and went home with the hope that I would be able to transfer from the engineering course to a science course at University of Colombo. Colombo is the capital of Sri Lanka. Transferring into a science course almost meant that I would become a teacher.

I met the Dean of Science and I lied to him that I wanted to change course so that I can better address my health needs because by being in Colombo I could reside with a relative of mine. I need to mention here that I did not have any intention of residing with a relative. If I had not cited my reason for the change of course as poor health I would have had to return to the engineering course. With my positive outlook on life and good care I was free of major health problems for a while and I was only taking two or three medications. The Dean was very sympathetic to my health problems after reading my health documents. I still did not break the news to my family. My parents got concerned and my Dad went to University of Peradeniya to see me. He got the news from my friends. He came to my boarding place. Seeing my father's face my heart sank; seeing me my Dad's face blossomed. He was just happy to see that I was safe.

Nevertheless he was disappointed. My father was getting old and his income as a vendor was decreasing for various reasons. He told me “Son I had the courage to walk thousand miles a day to help you to get through with your Engineering course, but I am not sure any more. I told my father that as University Education was free in Sri Lanka I would live on my education loan. Everybody was again angry with me for throwing out a great opportunity for a brighter life. Some of my friends at University of Colombo were telling me, “If you had applied for the science course here at the first instance I would have got the opportunity to do Engineering there.” On the very first day at the new University, I found what was being taught was to my liking. What attracted me was not necessarily the brilliance of teaching, even though our Mathematics Professor was really an excellent one. The depth of the conceptual thinking captivated me. I had to cover one year of work in two and half months. In one subject I tutored one of a Physics Units to a friend of mine. While he passed, I failed the unit.

My wandering still continued not because of lack of intellectual challenge but because of my own sentiments. However, I was able to pass my undergraduate degree with a second class (Lower Division) by working hard toward the last three months of the third year (final year). Two days before one of my final exams I passed a kidney stone (This was the first). That gave me great pain but I kept studying. This happened at night time. Next morning I went to the University medical clinic with the stone. The doctor tried to send me to the hospital for further investigation. I told the doctor that I had an exam the following day. Still he gave me a letter for admission to the Colombo General Hospital. I walked away with the letter and continued to study for my exams. I just took Panadol for the pain.

Immediately after I passed out from University, I obtained a graduate teaching position at a very prestigious school. My health was alright and I was taking a normal diet. When I was studying at University of Colombo, due to financial difficulties, many days I had only a glass of milk for food and I failed to take my medication, sometimes for months. I decided to go to the USA for my graduate studies. To do so I needed to learn English and take several GRE exams so that I could get a graduate teaching assistantship for my tuition and cost of living. As

always when it was so crucial I have performed well in exams, these few times also I performed sufficiently matching my skill levels.

While I was waiting for a letter of offer from a University in the USA, one night my Father noticed that the left side of my chest was prominent. He got alarmed and extremely furious. It seemed that my parents were already beginning to weep for the loss of my life. I promised them that I would go to the clinic; it was something that I had neglected to do for a long time. The doctor listened to my chest. I felt that he was upset. He ordered two X-rays, one of the chest and the other of the kidneys.

At the next clinic day the doctor looked at the x-rays and their reports. He said I needed to have a kidney operation because there was a kidney stone that could obstruct the urinary tract causing my death. I had about four months until the operation. About the chest X-ray I felt that his reaction was somewhat perturbed. I asked him. He told me that my heart has enlarged a great deal. I met a prominent medical professor at University of Colombo with my chest X-ray. I asked him to give me the diagnosis honestly. His answer was “five years at the most”.

I decided not to keep this from my parents, family and friends. I owed it to them to make them ready for my departure. It was also possible that I had a secret wish to die from my health problems, if I was not living my life in a fruitful manner.

### **The Dream Which Came True and Then Vanished**

One month later I got a letter of offer of a graduate teaching assistantship from University of Wyoming. At last my dream came true. I remember people who knew me well saying “If not for your health problems, now nothing can stop you from achieving your dream”. Despite my health problems, I decided to go to the USA for my studies. Two days before my departure Dad asked, “If you are going to die within five years, then please stay with us and die among us.” I felt sad. “Dad, if I do not go that is my death itself. If one does not follow his dream then that is a death worse than physical death. Until I die I want to keep doing what I like doing.” “Alright Son, you have my blessings.” “What are your ambitions?” he asked me. “To live and to contribute to other peoples’ lives, and at the end what matters is what you have done for others, not for yourself. For example, that Alexander Fleming won a Noble Price

for his discovery of penicillin has no meaning to him now, but for thousands of mothers, fathers, husbands, wives, sons daughters, brothers and sisters, still that discovery is a gift of life.” My Father replied, “That is a noble ambition, but remember giving a son or daughter who is well principled and thoughtful is also a great contribution to mankind”. I do not remember whether I was wise enough to recognise the wisdom of my Dad’s statement.

Thinking that it was the end of my wandering and the great beginning of my dream education, I left for the USA. It was not going to be. This time, it was for an entirely different reason. I was required to get a chest X-ray within a week of my arrival. This was required of anybody from my part of the world. I was waiting with fear expecting my doctor to call me about my X-ray. No call came. I went to him and asked about my chest X-ray. He said it was perfectly normal. Then I showed him the X-ray I carried from Sri Lanka. His conclusion was that I had not held my breath when the X-ray was taken. For me this looked unlikely. My chest was then markedly prominent. In Sri Lanka, a chest X-ray would not have been ordered as a routine matter. The doctor was alarmed and suspected something by listening to my chest. That was why I was asked to take a chest X-ray. If the X-ray was not done properly, the X-ray technician, the radiologist, and the doctors including the medical professor would have noticed it. Also, since a number was inserted in the X-ray itself it was unlikely that the X-ray was someone else’s. The only other explanation was a medical miracle due to the changed circumstances. That was also improbable.

I had to be cautious to take care of my kidney stone. Pre-existing conditions are not covered by medical insurance. I landed in the USA in the middle of January, 1984. I waited until the second week of April to discuss the matter with the University doctor. He took a plain X-ray of me and then arranged for me to go through an IVP X-ray immediately. While I was walking from the hospital to the University the doctors at the hospital had already contacted the medical doctor at the University. I was called to his office. He said that there was a medical emergency situation that I needed to go through an open kidney operation urgently. I said that I did not have any means to pay for the surgery, and the medical insurance would not pay for pre-existing conditions. He assured me that everything would be taken care of. I was instructed not to disclose that I knew that I was carrying a kidney stone. The time

frame that the Sri Lankan doctor said that I had to have the surgery was closely matched with the exact surgery time.

I was able to make a strong impression on my professors. At that time I was really into deep mathematical thinking. I began to notice the difference between mathematical thinking and “physics” thinking. Therefore I decided to finish my PhD in mathematics as soon as possible and then to go for a PhD in physics. Then the Head of the Department Mathematics, who was my Master’s thesis advisor, asked me whether I wanted to go to Princeton University to do Physics. If so, he said that he could arrange it. I told him I would first finish my PhD in Mathematics in another two to three years and then I would go to Princeton. The Head of the Department left for another University for the Chair position of the Mathematics Department. When he left he helped me to choose the supervisor for my PhD thesis.

### **Learning to Heal From Ill-Health**

Sometime after the surgery my health again deteriorated. In fact, the situation was much worse than what I had experienced in Sri Lanka. I had to make frequent hospital visits. Despite this, I was performing well in my studies. I went for a kidney biopsy because my test results indicated glomerular nephritis. The biopsy ruled out chronic pyelo-nephritis which was the diagnosis of the Sri Lankan doctors without a biopsy. I was put on a new antihypertensive medication. I needed go through medical testing for my kidney functions for twice a year. As my English was improving well, I began to read various medical books to understand my health problems. This knowledge saved my life at least two times and once saved me from condemnation of lying. One time there was another stone in the other kidney that could become dangerous at any moment. We decided to take it out through the urinary tract. I was put to sleep. As I was just coming out of my sleep, the very doctor who operated on me last time was accusing me that I had concealed the fact that I had passed the stone naturally. I was not in a situation to refute him due to the effect of general anaesthesia and immediately fell asleep but it hurt a great deal. I was discharged almost immediately. He asked me, “Do you have any exams or tests that you want to avoid”?

I was taken home by a friend. I was living alone at that time. I slept until the effect of general anaesthesia waned. Then I called the same University doctor and I asked to see him. I told him that there were some accusations and I was deeply hurt. He explained that the doctors could not find the stone when I was in the theatre. Then I asked him to explain the procedure. He said that before the doctors were searching for the stone through pictures on a monitor and they had injected a dye to make the stone visible.

Since I did not pass the stone, as it was impossible for the stone to pass through without my awareness, I reasoned that the stone was still in my body. I laughed and told the doctor, “The stone is still inside me. Your conclusion that I could not have passed the stone without my awareness because of its magnitude is absolutely correct. Furthermore, if I had passed the stone when I was with the doctors they would have noticed it. So the stone must be still inside me. Then why isn’t it visible, the doctor quipped. I told him that “Because the doctors made it invisible to the monitor”. How, roared the doctor. When you put a dye into my system you are hoping to increase the difference between the optical densities of the stone and the surrounding tissues in the hope that the monitor image of the stone would be much clearer. It is also possible that the dye could wipe out any existing optical density difference making the stone invisible on the monitor. The doctor agreed with me about the argument, but he was not convinced. He asked me, “What do you want me to do.” I asked him to take a plain X-ray. That he did. I can still vividly visualise how the doctor, with the X-ray in hand, walked towards me with bewilderment. The stone was there but it had been moved more into the kidney because of the manoeuvring of the doctors. It was visible on the new X-ray. The change of placement of the stone removed the immediate danger to my life. Later the stone was removed using ultrasound technology. These recurring kidney stones were a side effect of a medication that was given to me several years ago to counteract Calcium imbalance caused by another medication. Now I do not carry or make any of these gems inside me.

During the following years there were times that I felt that I had forgotten a great deal of Mathematics. Sometimes I felt that there were large gaps in my mathematical knowledge and skills. Once I had forgotten the quadratic formula that I had been

using since Year 8 almost on a daily basis. My car stopped several times since I forgot to refill the tank. Now I can see that there were signs that something was terribly wrong. Once when I was sitting inside my car on a street, Police surrounded my car aiming machine guns at me because neighbours had complained that I was there for a long time. The Police realised that I was harmless and I was in some form of trouble. They kindly advised me that if there was anything troubling me to seek help. I promised them but never acted on it because I was not aware of any problems. Even though alarm bells were ringing frequently and loudly I was not aware what was happening. Fortunately, my grades were not suffering. I was going to my teaching classes on time. I was returning their marked assessments on time.

I ran a stop sign and my car was totalled. Luckily nobody was seriously hurt. I stopped driving and began to use a bicycle. At one time for weeks violent thoughts were occurring in my mind to the extent that I feared that I would act on them. I threatened myself that I would go to the Police and report those violent thoughts. Fortunately the demonic thoughts went away without a trace.

December holidays came. I was sleeping all the time and refusing to answer the door or the telephone calls. One night I took the bottle full of medication and decided to end my life. At that moment it struck me that if I committed suicide the life insurance would not pay my family. That would be disastrous for them. I stopped. Then suddenly it occurred to me, “You do not commit suicide because your family will not get insurance money since it is not a natural death. The philanthropic thought is nice. But wait a minute. What is going on here?” I realised that I was losing my mind and had become almost catatonic. Why? It came to my mind that it could have been due to one of my medications. Immediately I felt some relief because this explained what was happening in my life.

The University Medical Clinic was still open. I rode my bicycle to the University medical clinic and I took my two medications with me. I first met the pharmacist. I gave him the two medications and asked him to tell me all the side effects that they could cause. One medication did not have any of the side effects I was looking for. The other medication was a beta blocker. He listed some side effects and said that they were the main ones. I asked him to go on. One of the side-effect was “catatonic

depression”. I felt so happy. I explained to the pharmacist that I was about going to commit suicide. He strongly requested me to see the doctor. The previous doctor was no longer there. I explained the situation. He wanted to put me in the observation room or admit me into the hospital. I was confident I was not going to harm myself or any living body. I told him because I knew what the situation was I was capable of handling it. The doctor advised me to stop the medication immediately.

Two days later, when I saw the sun shining in the morning I felt so happy. I had experienced the difference between hell and heaven. How beautiful and joyful and relaxing are the so many little noises and sights we just ignore. I cannot explain the happiness I felt. My condition began to improve for a while. Then I realised and doctors also agreed that I needed the help of medication for deep damage caused to my mind by the long term use of the medication. New medication meant new side effects, more hospitalisation. Also I began to develop allergies to so many medications.

Many of my friends and some of my teachers tried to persuade me to sue the doctors. I did not agree because I did not want to put the previous University doctor in difficulties. Even though it was another doctor who put me on that draconian beta blocker, I was under the care of the University doctor on a day to day basis. They would have warned me about possibilities and should have been on watch for any dreadful signs.

I quickly recovered my lost mathematical memory. Still my Sinhalese and wit in both languages suffered a great deal. To completely recover both language skills took another ten years. Before I completed my PhD, I had to go through two more surgeries. This time if not for my familiarity with some medical literature, doctors would not have advised me to have these surgeries. Those two surgeries to correct my nasal septum deformity helped me to recover from many of the health problems I was suffering. Since then for about twenty two years, my health has been satisfactory. Even though from time to time I suffer from conditions associated with my major health problem and I need to go for some medical procedures on yearly or half yearly basis now I am not a regular hospital visitor or clinic visitor. It is nothing like when I was in the USA.

When I recovered I had passed the time for chasing my dream of PhD in Physics. I could have stayed in the USA and enrolled at a University like Princeton for a PhD in Physics but I owed it to my parents to go back to Sri Lanka to visit them. Since I had been paying the insurance gaps I had not had money to visit Sri Lanka for more than five years. The little money I could save I had sent home so that my brothers and sister could continue their university education.

So I decided not to stay in the USA, instead I accepted the offer of permanent residence in Australia and decided to visit my parents. If I stayed in the USA for further studies I would have been unable to see my family for at least another five years. Even though I had been preaching about contributing to other peoples' lives that was the first significant occasion in which I put someone else's happiness ahead of my dream. I think that I had matured to the extent that I would no longer live just for my dreams. I began to live for others. That meant that I had qualified to be a family person for the first time in my life. This is also contradictory. Wouldn't I make myself unhappy by giving up one of my dreams for someone else's happiness? So maybe I was selfishly motivated for unselfish motives or vice versa or both? Ultimately, didn't I make that decision because I wanted to be happy?

Once, my students and I drifted into discussing this issue. My students told me, "You are one of the most unselfish persons we have ever seen. You give so much of your time for us." I told them one cannot be really unselfish if he is not really selfish. They looked at me with open mouths. I explained to them whatever I did for them I did it to make myself happy. Then they said, "Sir you are not religious in any sense then how does it make you happy?" I responded, "Of course I am not that selfish to use you to buy a ticket to heaven."

Think, I asked my students, "What really matters in your life? The number of pizzas you eat? The model and make of the car you drive or the size of the mansion you live in?" At the end all of us die. Therefore what matters in life is to find happiness by helping others. For example, you are helping me right now to be happy by seeking my help. You may argue that I am unselfish because I want to add to your lives and happiness and I do not want to harm you in anyway. Then again you may argue that I am selfish for the reason that I am helping you to enjoy my life and feel important.

Both statements are true simultaneously. To see this, let me ask you this question. If you feel that my helping you does not make me happy, would you come to me for help next time?" The answer was "No". "So you are also aware of my selfish motives". I told them triumphantly.

As it can be seen from the above paragraphs, in those years of my ill-health, what I learned was invaluable. It is very difficult for me to visualise myself without the experiences gained from my ill-health. If not for that beta blocker, I would have gained a PhD in Physics from a University like Princeton and I would have gained a professorship at a prestigious American University, but, would I have been the same? What would have made me a better person? Would I have acquired the similar depths that I have gained from going through enormous pain and suffering if I happened to traverse a much smoother different path? If reincarnation is a fact, then all of these experiences and learning would help me have a better life in my next life, but I do not believe in reincarnation. (Then again who knows?) Nevertheless, every bit of mental anguish I was tormented with and every drop of pain I was suffocated by taught me a valuable lesson and helped me to become a much better person. Therefore I am happy that I have a good life that suits my values and expectations.

I think that the victories that I achieved through my pain are more important to me than any other success that would have occurred. Even when I was in great pain, I was able to think that there were many others who were in far worse situations. I was not happy that I was in a better situation than some others. Also, I did not wish to be in their situation, but I could think about the enormous pain surrounding living beings. That was my victory. It was like the pain and torments I went through when I read Dostoevsky in my adolescent years. The pain and torment Dostoevsky creates in a responsive mind is almost impossible to bear, yet it is educational, informative, spiritualising, soul cleaning and peace generating. I think that Dostoevsky's novels had nurtured me immensely so that I could bear the mental and physical anguish. That is the task of true literature. I think that the most valuable lessons that can be learnt are taught by one's life itself rather than in any subject. Right mindfulness can turn our life experiences into projects of learning. What is most important is not to hurt any other person regardless of the excruciating pain we are subjected to suffer from time to time.

I left the USA and visited my family, spent some time with them and got married. In the middle of one of the worst recessions I began my residence in Australia with my wife. Only after a long time did I become aware that my PhD advisor was not writing reference letters for my job applications because he was suffering from a brain tumour. So I missed out on many job opportunities. When I knew what was happening it was too late. I decided to get a Diploma in Education. I ended up doing two of them. This was the only study I did for my survival. I have two children, a daughter and a son. All of these aspects are undoubtedly various facets of my education.

Here, I end my narrative.

I owe my life to those two teachers. I may have not been able to live up to their expectations of me but their influence on my life permeates through my approach to teaching and learning and life itself. My journey was not really a well set out journey. I doubt that anybody's life can be such a smooth and painless adventure. I journeyed through wandering and I wandered through journeying. Journey and wandering were inseparable. Which was which, it is still hard to pinpoint. Nevertheless, it did not have to be that way. I think that my wandering did not end then or ever. It only took a different form. I believe that making a journey out of our wanderings is the journey itself.

### **Aspects of My Adolescent Constructivism**

Since this work is about [Academic Constructivism](#) I end this chapter with a discussion of my philosophy on learning and teaching that I had developed up to the time that I left high school. Note that the word 'constructivism' was not known to me or I was not familiar with any form of constructivism informally or formally until I began to study for my Graduate Diplomas in Education more than two decades later.

When I was in Year 9, I needed a term to label the set of opinions that I had been forming. For that I used the phrase "Teaching and Learning with a philosophy".

The following summarises my naive constructivism.

## **Year 7-Year 8**

### Hierarchy of subjects

Thinking Subjects: Mathematics and Science

Memory Subjects: History, Geography, Health Science

In between Thinking and Memory Subjects: Sinhalese, Buddhism

Boring Subjects: English, Drawing, Music, Physical Education.

### Major Aspects

- Memorisation is useless in education and always counterproductive.
- Routine work is boring and unnecessary.
- Exams, term tests cannot measure one's understanding and therefore they are useless and unnecessary.

## **Year 9: First half of the year**

### Hierarchy of subjects

Thinking Subjects: Mathematics and Physics

Memory Subjects: Buddhism, Biology

In between Thinking and Memory Subjects: Chemistry

Boring Subjects: English, Mechanical Drawing

## **Year 9: Second half of the year and after**

### Hierarchy of subjects

Thinking Subjects: Mathematics, Physics, Chemistry and Biology

Memory Subjects: Buddhism

Boring Subjects: English.

The following are major aspects my then academic constructivist thinking.

- Concepts are partly creations and partly discovered.  
(I doubt that I would have expressed this idea clearly at that time, but when I analyse my approach I can see this as a hidden premise in my philosophy.)
- Learning is about being able to create these concepts in one's mind rather than taking them for granted.
- Understanding of a concept is paramount and learning skills based on a concept is secondary and can be at least somewhat neglected.
- Some concepts are labels. (Example: Electron has negative charge. Protons have positive charge. Here the concept of charge is not the label that I am referring to: it is the sign which happens to be a label.)

### **Summing Up**

The real purpose of this chapter was not merely to describe my background in relation to my academic philosophy and practice. The major theme of this chapter has been to uncover who and what influenced me to be who I am today. Ranging from my parents and authors to strangers there is an uncountable number of people who have influenced me to think the way I am thinking. Some contributed to my way of thinking because I was compelled to criticise their view points. This is also learning from them. The theme of this thesis is academic constructivism and how it can influence better learning and understanding in students of all abilities. Through my own learning and life experiences, I came to the view that academic constructivism can help students to enhance their learning abilities. All life events are learning events. My cultural and academic background as well as my life events discussed in this chapter can help the readers to understand my development as a social event as well as an individual event. In the next chapter I will outline how my growing academic constructivist view toppled my logical positivistic views.

## **ARC TWO-THREE: THE REVOLVING AXIS OF PARADIGM ROTATION; THE JOURNEY THROUGH MANY ISMS**

This section of my work comprises Chapter 3 and Chapter 4. In Chapter 3, I present an analytical scrutiny of my gradual transformation from logical positivism and related isms to an epistemology of a constructivist nature. Chapter 4 documents some of my reflections and views on many isms of constructivism.

Almost every ism which has come to existence has later been divided into many other branch-isms. This is true for almost all religions and other isms. To see this, consider Buddhism and Marxism. These chasms spring in the wombs of the original isms themselves. A subtle disturbance of the paradigmatic orientation gives birth to a new section of the ism. None of these isms is ontological reality. They contain perceptions, beliefs and theories which are based on axiological orientation of the viewers.

When I was a logical positivist there was only one correct view in my world. So I felt being brave, courageous and enlightened. To my then naïve mind, there was absolute proof that there was no life after death or there are no gods or ghosts. Now I have painfully learned that there are no absolute ‘scientific’ proofs even for mundane matters such as the Earth goes around the Sun. There was comfort since my world was either black or white and I did not have to acknowledge or deal with greys.

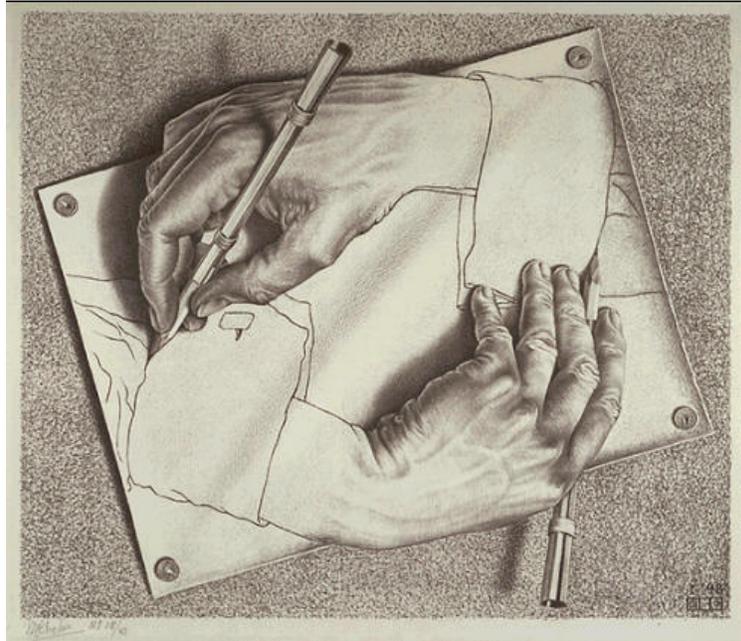
To leave this ‘comfort’ behind and embrace constructivist epistemology was a rather intellectually traumatic episode of my life. This transformation has robbed me of my ‘braveness’ and ‘courage’ to deny anything that I cannot see or directly experience, with utmost audacity. Consequently, I have become a sceptic who doubts scepticism too. Yet, this new way of thinking of reality has given me peace and ability to appreciate other points of views.

Before, I felt brave by venomously rejecting other peoples’ points of views. Now I feel brave by appreciating the views of others and incorporating them into my views in a coherent manner. This immense metamorphosis of my epistemological views has enabled me to view my Year 9-transformation from being an unengaged student

to a cognitively active student in a much more insightful manner, and to enrich my teaching practice.

Thinking of reality brings the following image, created by Escher, and the following poem, scribed by me.

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### **Game of Dice**

Is the Universe a die?

When I think  
Does my mind play  
Dice with the Universe!

Is my mind the die?

When I think of the Universe  
Does the Universe play  
Dice with my Mind!

**(2010)**

## Chapter 3

### From Logical Positivism to Constructivist Epistemology, The Journey I Never Expected To Be In

#### A Logical Positivist in Metamorphosis

Development of my academic constructivism and how this process of change was shaped by my life events were outlined in the previous chapter. Initially my mind gave residence to two quarrelling siblings, academic constructivism and [logical positivism](#). They became siblings by the blood bond of their hatred towards each other. In this chapter, I present how my academic constructivist views kept nudging my logical positivist epistemological view until it is toppled from its dominant position. This change of epistemological view has influenced my academic constructivist view. This is a cyclic process and is central to the theme of this thesis.

As two unfriendly siblings plot against each other while evading the radar view of their parents these two residents had been quarrelling for dominance in my mind. There were three possible outcomes:

- Academic constructivism was suppressed and a logical positivist framework of learning and understanding (Live-In-Partner of logical positivism) was formed.
- Logical positivism was suppressed and a constructivist epistemological worldview (Live-In-Partner of academic constructivism) is formed.
- Both academic constructivism and logical positivism and their live in partners learned to live in harmony.

As I described in the previous chapter, the formation of my academic views can be linked to the academic constructivism which was happening since the early times of my education. These views were strengthened during and after Year 9. However, for another ten to fifteen years my worldview was predominantly logical positivism. Even when my worldview had been predominantly logical positivism, the seeds of epistemological constructivism were inwardly springing due to the pressure of outwardly growing academic constructivist views. In this chapter, firstly I look at my

path to epistemological constructivism. Then, as a prelude to the discussion of academic constructivism, I explore epistemological constructivism.

I would like to remark that the academic constructivism and epistemological constructivism I espouse here may not be identical with such views in the literature but they are connected in spirit. Since my conversion from a logical positivist point of view to a form of constructivism was motivated by the reflections on scientific knowledge and nature of science- galvanized by my life experiences- later in this chapter I will discuss the issue of the nature of science.

### **My Path to Constructivist Epistemology**

When I contemplate my past, I see that constructivist epistemology was developing over the other belief systems. For instance, consider my acceptance of the equivalence of the vector sum of two simultaneous movements in two different directions and the single displacement in the direction of the third side of the triangle constructed by joining the ends of the two displacement vectors, and recognition of the sign of the charge of an electron as a label. However, this discrepancy was not noticed until much later when I started graduate studies in Constructivism, and consequently I remained an avowed objectivist for a long time to come.

As I stated in the previous chapter I started my life as a Buddhist. The [rationalist](#) paradigm was slowly snatching me from Buddhism. My rebellion against cultural beliefs and practices was a part of this process. By the time I left school I had developed an international perspective. Two years later, I embraced Marxism (Trotskyism). I believe that this was the culmination of my logical positivist belief system. Ten years later being able to read English, I came across a book that discussed the theories of Ptolemy and Copernicus. It devastated me. Since I was eight years old, I had been lied to! I began to read Quantum Physics. My world began to shatter. I began to see the relationship between [Marxism](#) and objective idealism. This saved me from Marxist ideology. [Objectivism](#) began to pose a problem, since [absolute objectivism](#) is in fact [absolute idealism](#). Note that absolute idealism is a derivative of [idealism](#) and [materialism](#) is its diametrical opposite. At that time, in my conscious level, I was developing a philosophy that I was not accustomed to. I was noticing that this developing philosophy was entertaining notions of [subjectivity](#). Only a couple of years ago, I would never have thought that

someday I would incorporate [subjectivism](#) into my worldview. In order to avoid [solipsism](#) and [objective idealism](#), I needed to incorporate objectivism and subjectivism together.

In 1980<sup>8</sup> while I was a student at University of Colombo, Sri Lanka, a collection of my poems was discussed at a conference organised (as a part of learning activity for one of the Sinhala Units) by the Sinhala department at University of Kelaniya.

This discussion was participated in by students of the unit, some other young poets and professors and lecturers of Sinhala. Some of the audience was strongly critical of the lack of rhythm and poetical words. They were also strongly critical of the simplicity. In answering, I told them: “If you really see no value in them, perhaps we ought to close this forum and go home.” The organising lecturer then interfered and said if there were no such interest then they would not have come (meaning other visitors, not the students from the unit).

They repeatedly asked me to explain the meaning of some poems. What came out of my mouth surprises me even today. “At the very moment you read them, I relinquish my authorship of them.” The audience was stunned. They asked me to explain. I said “It is clear that when you read them you recreate them in your mind, according to your life experiences, views and expectations. You expect poems to have a rhythm and nice sounding poetical words. When you do not see them you react to it. I have no problem, if you do not label them as poems because they were not written to satisfy your criteria of poem. I myself do not necessarily call them poems. For me the question is not the label. Did they tickle you? Obviously they have. Why did they tickle you? If they have tickled you, those feelings were a recreation of my work but it is not my exact work itself. If they did not tickle you, my work has failed to motivate you to recreate well. That could be my failure or yours.” These cannot be the words of an ardent objectivist as I was then but until much later I did not pay attention to the discrepancy.

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<sup>8</sup> I began postgraduate studies in mathematics in USA, January 1984.

Later while I was in the USA I took these poems to an English professor. He immediately liked them. I asked “What kind of a philosophy do they present to you?” His answer surprised and disappointed me. Now I understand his answer. He said “They come from your culture. They carry a form of Eastern philosophy and I like that philosophy.” This answer was clearly contradictory to the answer I was expecting because all the time I was thinking that the images created in the poems were fixed items of objective reality. I believe that still they could be viewed as such, yet many other interpretations are possible.

These two narratives show that on the conscious level I was behaving as an absolute objectivist, yet at the subconscious level, at least sometimes, I was thinking as a relativist.

By the time I migrated to Australia in 1992, I had ceased being a Marxist, or an objectivist. I could not accept [absolute idealism](#) or [solipsism](#). In 1997, when I was studying for my Graduate Diploma in Education, one of my teachers posed me a strange question: Is knowledge discovery or creation? That assignment led me to constructivism. Also during this Graduate Diploma Course, I realised that learning and teaching with a philosophy is in fact a form of academic constructivism.

I was thinking of academic constructivism discussed in literature as open discovery. This wrong notion has been removed during this doctoral research. Even though today I am very much closer to constructivism, I still believe many teachers have misunderstood academic constructivism. Consider this example. Piaget put forward the idea of developmental stages. Has it been interpreted appropriately? For example, up to a certain age an infant is not able to recognise colours. How can we interpret this? If this is an absolute truth (instead of some useful guiding principle), then we are advocating a genetic [determinism](#). The world is not a dream of the mind, but it is the dream of the genes. What learning and teaching approaches follow from this notion? Suppose we take away all colours from the baby’s room and stop referring to colours, until it comes to the prescribed time. Do we assume exactly at the time prescribed by Piaget’s research that the child will automatically know colours? Recall the experiment in which a kitten that was blindfolded over one eye became ‘blind’ in that eye. Not just genes but also experience plays a role in vision

development in kittens. The right approach would be to put colours into the baby's room, make reference to the colours frequently, and not get disappointed if the infant takes longer to recognise the colours.

### **To Epistemological Constructivism in the Vehicle of Academic Constructivism**

Constructivist epistemology as I experienced in my journey of education deals with the nature of knowledge itself. As I argue in this chapter, it was necessary for me to embrace a form of epistemological constructivism as a compromise between objectivity and subjectivity. I had a scornful view of subjectivism and still I do and then do not. I held a high esteem for objectivity and now I do not and then still I do. I call the resolution of this contradiction Epistemological Constructivism.

Some authors consider constructivism to be a post-epistemological paradigm. In a later section, I will argue that this is a defeatist attitude. If objectivity of knowledge or reality itself is not problematic, who needs constructivism? The very genesis of constructivism is the debatable and murky nature of reality<sup>9</sup>. On the one hand, it is impossible to deny the existence of absolute or objective reality. On the other, it is equally impossible to admit the existence of absolute or objective reality. To see this, suppose that I claim:

**There is no objective reality.**

This claim contradicts itself. Is the claim itself a piece of objective reality? If the answer is yes, then the claim itself is a falsehood. If the answer is no, then there are some objective or absolute realities. Therefore we are compelled to reword our claim: The only objective reality is that there are no other objective realities. Later I will opine that this claim does not hold either.

Also consider the question:

**Is all knowledge subjective?**

If the answer to this question is yes, then this particular knowledge itself must be objective, because all knowledge being subjective is not subjective knowledge itself (That is, all knowledge being subjective does not depend on an individual point of

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<sup>9</sup> Later in this chapter, I will make another remark on this matter.

view, time or place.) Then again, if the answer is no, that itself says that there is some objective knowledge.

This brings up another dilemma. The argument used to establish the impossibility of denying the existence of an absolute reality was of a logical and linguistic nature. This condition itself has implications for our dealing with reality. The problem does not stop here. The combination of the two words ‘absolute reality’ or ‘objective reality’ is itself contradictory, unless we define reality to be an unknowable attribute. If we take reality to be something unknowable or that we cannot perceive it, then it does not matter whether such reality is objective or subjective, absolute or relative. On the other hand, if reality is something that we can deal with, we can know and perceive, then any knowledge of such reality depends on some cognition and conscience. This implies that the term objective reality is self-contradictory.

A similar analysis is applicable to the words “subjective reality” even in the case we assume reality to mean some external and precise attribute that exists outside our mind. Then, in this case, it does not need the adjective ‘subjective’. To see this, consider that I claim “That book is black.” By this statement I acknowledge that there are other books that are not black. Therefore if we take reality to be subjective then we acknowledge the existence of an objective reality. Also, we may ask whether the claim that reality is subjective itself is a subjective judgment or not. Either way we have an inescapable contradiction.

For the sake of the argument, let us agree not to see any difficulty with the word combination, objective reality (or subjective reality), and assume that reality is an unknowable attribute. In this case, how do we know whether such unknowable reality is absolute or relative? Say, even though reality itself is unknowable it is possible for us to know whether it is objective or subjective. Then this knowledge itself depends on cognition and conscience. That is, our perceptions (pictures) of reality (external objects) are images taken by a camera (our mind), then these pictures are certainly a product of the external objects (reality), the hardware of the camera (brain functions, neuronal network and activity) and the film role sitting in the camera (the software in our mind- belief systems, value systems, etc.).

To overcome this difficulty, let us assume that there exists some supreme conscience<sup>10</sup>. We need to assume that this supreme conscience does not mirror reality as we do; rather we assume that what this supreme conscience conceives is pure [ontological reality](#). Unfortunately, this does not resolve any of our original difficulties. Knowledge or belief of such a supreme conscience clearly depends on human conscience and cognition. Even if the supreme conscience appears in person before me and proclaims the objectivity of reality, that knowledge depends on my conscience and cognition.

To complement the discussion above, now I will consider some examples from mathematics and science. Even in natural sciences, and the most pure science, mathematics, this uncertainty of objectivity lingers to be seen by anybody who wants to see it. For instance, some results obtained by Euler, one of the greatest mathematicians ever, were regarded as accurate until the concept of convergence of infinite series was constructed. After this concept was constructed, some of the results Euler obtained were considered to be wrong. Therefore, at least in principle, there is a possibility that some or all of the most ‘objectivistic’ (most objectivistic because they are derived from a set of axioms) mathematical truths we know later could be proved to be wrong.

To illustrate this point further, I will consider two examples from one of the most exact sciences, physics:

- **Ptolemy:** The Sun goes around the Earth.
- **Copernicus:** The Earth goes around the Sun.

Both of these theories are scientific theories and each of them can be used to accurately predict lunar eclipses and solar eclipses. Because of mathematical elegance and simplicity, Copernicus Theory superseded Ptolemy Theory. If we believe our scientific theories to be purely objective, it is impossible to explain how the Sun that happened to move around the Earth until the 16<sup>th</sup> century, suddenly

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<sup>10</sup> This supreme conscience does not have to be the creator of everything, but it can be readily elevated to such status. Then it follows that if such a supreme conscience exists, it must be unique.

decided to let the Earth go around it. The truth is that we do not know what goes around what. We know only that the latter theory is mathematically simpler than the former theory, and it can be applied to many more astronomical situations to make accurate predictions. There is no guarantee that another future theory cannot be much simpler and cannot be applied to even more situations.

The Second Example:

- **Newtonian Physics:** Speed of light depends on the observer's motion. Space and time are independent of the observer's motion.
- **Relativistic Physics:** Speed of light is independent of the observer's motion<sup>2</sup>. Space and time are relative to the observer's motion.

Note that, as with Ptolemy Theory and Copernicus Theory which are complete opposites of each other, these two theories are also complete opposites of each other. Still, both theories accurately predict phenomena involving low speeds, such as motion of planets, rockets and bullets. But only relativistic theory will do so if the speed under concern is close to that of light. History of science and mathematics is full of such examples. For this reason, it is impossible to maintain pure objectivity of physical theories.

Next I consider the two philosophical standpoints: Knowledge is discovery versus knowledge is creation. If knowledge is pure discovery, then all knowledge exists independent of our existence, activities and consciousness: On the other hand, if knowledge is creation then all knowledge results from our existence, activities and 'consciousness'. However, there is no way of knowing whether knowledge exists independently of our consciousness, because every effort to know it itself is an act of consciousness. Then again, to take knowledge as pure creation is too [solipsistic](#).

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<sup>3</sup> Einstein postulated that the speed of light is the maximum possible physical speed for any particle or information. Already there are evidences that this is not true. To overcome this difficulty, David Bohm suggested that the condition should be changed: For a congregate of particles average speed cannot exceed the speed of light, but individual particles can travel faster than light (See Bohm, D. (1980) p. 162). It is interesting to see how this anomaly is going to be played out, as more evidence comes out.

Then such solipsistic knowledge should exist in our minds alone. To discuss such knowledge while maintaining this solipsist point of view, we should elevate it to a collective form of solipsism. This elevation results in a common knowledge that we can share, or at least discuss, with each other. That is, eventually we are compelled to acknowledge (implicitly or explicitly) some form of objectivity of our knowledge.

The word ‘knowledge’ itself requires a knower by the very nature of the word itself. If there should be a knower to know then what is known by the knower depends on the conceptual and physiological tools used by the knower. The following discussion will illustrate this.

### **Aristotle and Newton on Motion**

Let us play a mental game in which Newton and Aristotle are both flicking books on a table. After several hours we interview them. We need to keep several things in our mind before we conduct this interview.

- They are both intellectual giants of humankind. For our purpose, we may assume that they are equally gifted in their cognitive abilities.
- They are both honest. Not like in modern times, their opinions and ideas are not treacherously manipulated by the forces of profit seekers.
- To a large extent their experience with motion is identical. This is largely true because both might have travelled in horse drawn carts or some other forms of vehicles.
- They both put a high value on their integrity and not on publicity or famousness.

**Journalist:** *Aristotle, what can you say about motion?*

**Aristotle:** *There needs to be a continuous force operating on an object for it to keep moving.*

**Journalist:** *How did you come to that conclusion with so much confidence?*

**Aristotle:** *See for yourself.*

Aristotle pushes a book on the table. The book keeps moving as long as the finger pushes it. Then Aristotle takes the finger away from the book.

*Didn't you see that?*

As the book stops, Aristotle asks triumphantly and explains:

*When the force is taken away the book cannot keep moving.*

**Journalist:** *Yes I am convinced. My car always needs petrol to run.*

Now let us interview Newton.

**Journalist:** *Newton, what can you say about motion?*

**Newton:** *If there is no force operating on an object then the object will keep moving in a straight line with uniform speed or stays at rest for ever.*

**Journalist:** *What? Are you crazy Newton? How did you come to that nonsensical conclusion?*

**Newton:** *See for yourself.*

Newton pushes a book on the table. The book keeps moving as long as the finger pushes the book. Then Newton takes the finger away from the book. Newton asks triumphantly:

*Didn't you see that?*

**Journalist:** *I saw that the book stops when you took the finger away. Aristotle is correct.*

**Newton:** *You did not really see it.*

Newton again pushes a book on the table. The book keeps moving. Then he takes the finger away from the book. Then he explains  
*Don't you see that? The book kept moving briefly even after I took my finger away from the book. During that time no force was acting on the book.*

In his quest to understand motion, Aristotle asked a single question. To explain the same observation, Newton asked two questions and attached varied significance to the questions. Aristotle asked:

- Why did the book stop?

Newton asked:

- Why didn't the book stop immediately after the force was withdrawn?
- Why did the book stop eventually?

Newton's second question was almost exactly the same as Aristotle's single question. In Newton's case, the difference in the number of questions and their order led to so many other questions and then to an elegant theory. The only explanation I have why Aristotle did not pay attention to the second question and why Newton did is the different psyches of the time they lived in.

Aristotle's knowledge of motion depended on the questions he asked. This is the same for Newton.

Now let us consider the following issue.

**Observations cannot exist independently from the observer.**

It is clear that observations cannot exist without an observer. This is because the word 'observation' itself demands the existence of an observer. The statement that observations cannot exist independently from the observer is hard to fathom since all of us deal with our day to day lives with the explicit or implicit assumption that observations are independent on the observer. Partially it is so and then again not.

To see this, consider the saltiness of salt. We discuss saltiness of salt as an absolute property of salt. Then again its subjectivity is also evident if we pay closer attention. The saltiness of salt is not just a property of salt and it cannot be just a property of the taster, either. The saltiness of salt is a property of both salt and taster, for if the taster has lost tasting ability, the taster will not feel any saltiness in salt. For the same food, different people react differently; some would say the food is too salty, and some others would say it is not salty enough.

This can be objected to by saying, one should not generalise on special examples. But, is not this the whole point? If we happen to have different brains and senses, we would not have observed the things that we observe now. To further clarify this, I note that the colour of a car being red is a property of both the seer and the car, for if the seer is a dog it will see the car as being black. Even if a human sees the car at night time, or under different light, the colour will not be red.

Previously I argued that there should be at least one absolute reality. And also I claimed that the terminologies, objective reality and subjective reality, are self-

contradictory. It is impossible to escape from this impasse. In fact, it can be shown that there exists more than one absolute reality. For instance, consider gravity. Let us agree that neither of Newton's Universal Gravitational Theory nor Einstein's General Relativity is absolute nor objective. Now consider the following claim.

*Sufficiently intelligent cognitive beings are able to construct useful and insightful models (useful and insightful to the constructors) to understand the phenomena experienced by small objects near a massive object.*

I believe that this claim represents an absolute reality. This does not refer to any particular theory. It just says that we are able to construct such models. It also does not admit that there is a grand scheme of 'gravity' at work that we do not know and cannot know. To acknowledge such a hidden grand scheme will amount to admitting the existence of a supreme conscience. (This assumption that a supreme conscience exists can be maintained logically, but it has no consequence.)

Notice that even this absolute or objective reality is not independent of conscience and cognition, because a cognitive being has derived it, and its objectivity can only be tested by some other cognitive beings. Then why is it considered to be objective? It is objective in the sense that most of us can agree on that. It can be agreeable to any sufficiently sophisticated alien conscience. This assertion will be still true even when we throw away all existing theories of gravitation in favour of a new theory. *Furthermore, there are ample examples to show that Newton's Gravitation model is an absolute gem of human cognition, even though it is not an absolute reality.*

I have argued that it is equally impossible to maintain that reality is purely objective or purely subjective. This somewhat resembles the wave-particle duality of material objects such as electrons and energy radiation such as light. Reality is simultaneously objective and subjective. To clarify my standpoint even further, I explore the following two questions:

1. *Does the moon exist only when we look at it? (Einstein posed this question. I will discuss this more in the section of nature of science.)*
2. *If a tree falls in a jungle, when there is no one around, would there be a noise?*

My ontological paradigm assumes that the moon exists even when nobody is looking at it. And it answers the latter question, “absolutely not”, because noise is a sensory perception that is generated by some physical phenomena. For a sound to exist there should be a perceiver. But if we ask, does a retina image of the moon exist, when no one is looking at it, then the answer is, “certainly not”.

### **Is Knowledge Discovery or Creation?**

If knowledge is discovery then knowledge needs to be pre-existing in fixed forms. Also, they should be lying in some space in waiting to be discovered. If this is the case then why are there different kinds of knowledge systems which depend on culture, time, expectations and belief systems? If we argue that these different knowledge forms are approximations to and or various facets of the same knowledge then we may ask why Newtonian Physics and Relativistic Physics are conceptually opposite to each other even though the formulas in the former are approximations to those in the latter? To reconcile this difficulty, we need to assume that knowledge exists in different forms, and different knowledge seekers from different backgrounds find different pre-existing knowledge systems. This amounts to refuting the notion of discovery of knowledge to some extent and embracing creation aspects to a similar extent.

The standpoint that knowledge is creation implies that seekers of knowledge create knowledge. This is compatible with the multitude of knowledge systems but then we need to account for the limitedness of the number of multitudes. Also, if we acknowledge the existence of an innumerable number of knowledge systems why do we strive to find common ground in our knowledge systems? This creation standpoint also allows for arbitrary knowledge systems but it is clear that this is not the situation. For instance, I cannot create a knowledge system in which when I drop a pen it ‘drops’ to the ceiling. This is a crude example. Nevertheless, even that crudity is enough to refute the creation standpoint. Therefore knowledge cannot be taken to be a pure creation or pure discovery.

If knowledge cannot be taken to be purely objective or purely subjective, purely discovery or purely creation, knowledge must be constructive. We construct our knowledge. To construct something, raw materials should be used. This raw material

(let us say building blocks) is provided by Nature and the mental constructs that we have already constructed.

Then again, which raw material is used or not used is not purely an objective matter. What raw material we see and use depends on subjective factors. Then we mould our raw material into a building. How we build this building depends on the way we choose to put the raw material together. Again this is not purely subjective, because some of the ways that we can put them together will not fit together. That is, the objectivity and subjectivity of reality penetrate very much deeper into each other's domain.

Sometimes, the new raw material gained via our sensory perceptions may not match the building blocks that we have already constructed. No longer does it make any sense to add to the existing castle. That is when we throw out these ideas and build new mansions. When this happens, we say that a revolution has occurred. Examples of such revolutions are Copernicus' revolution, Galileo's revolution, Newton's revolution, Einstein's revolution and quantum mechanics revolution. There may be very many to come, one after the other, without end. This belief in the nature of reality and knowledge can be named constructivism.

During the years of my graduate studies in the USA, my access to literature was enhanced, due to improved English skills and availability of books. Because of this, my view of science and reality began to alter, albeit slowly. Sometimes I summarily dismissed the very thoughts that were springing in my mind. Since my high school years we (my friends and I) were liberally using the word subjectivist as an insult and to cut down arguments against our point of view, even within the group or with outsiders. In some circles of Sri Lanka, the Sinhalese equivalent of [logical positivist](#) was considered as a compliment. To receive such a compliment was an achievement. When I realised that thought dictatorships was extended with the use of harsh name calling I decided to leave such organisations and asserted my independence. This happened three or four years after I left school.

My epistemological transformation happening was still subliminal. I did not even want to admit it to myself let alone discuss this 'diabolical' change with my friends

in Sri Lanka. In retrospect, by looking at my poems and understanding the sign of a charge being positive or negative or a magnet having south and north poles (not their behaviour, just the names in italics) as just labels now I believe that this process was happening even then beneath my conscious mind.

This reluctance to accept the developing emperor's new worldview<sup>11</sup> collapsed when I began to read philosophical debates between scientists about quantum mechanics. I would like to note that the undergraduate or graduate units in Quantum Physics I took in Sri Lanka and the USA, respectively, did not initiate or motivate or act as a catalyst for this metamorphism.

Even though I have already touched upon the nature of science in previous sections, to explain my transformation it is necessary that I discuss the nature of scientific knowledge. To achieve this feat I will resort to using one of the most beautiful stories of science, the story of light and wave particle duality. I will relate this story to my learning and teaching practice in later chapters. Also, I would like to note that after I accepted my philosophical metamorphism, during the next visit to Sri Lanka I visited many friends as many as I could, and explained to them my new way of thinking. They were flabbergasted.

### **Nature of Scientific Knowledge and the Dark Story of Light**

I have already discussed the two diagonally opposite theories of planetary motion, classical and relativistic theories of motion and Aristotelian and Newtonian theories of motion. Even though I have already discussed the nature of scientific knowledge previously through a review of those ideas to some extent, now I intend to utilise the most beautiful story of science, the rivalry of wave and particle theories of light, to further this exposition. Apart from the fact that the wave-particle duality is considered to be the single most baffling phenomenon that whirlwinded the scientific community, it is also the single greatest contributor to the transformation of my foundational and operational framework in thinking about the world I am in. And, at last it has provided me a sound philosophical foundation for the learning and

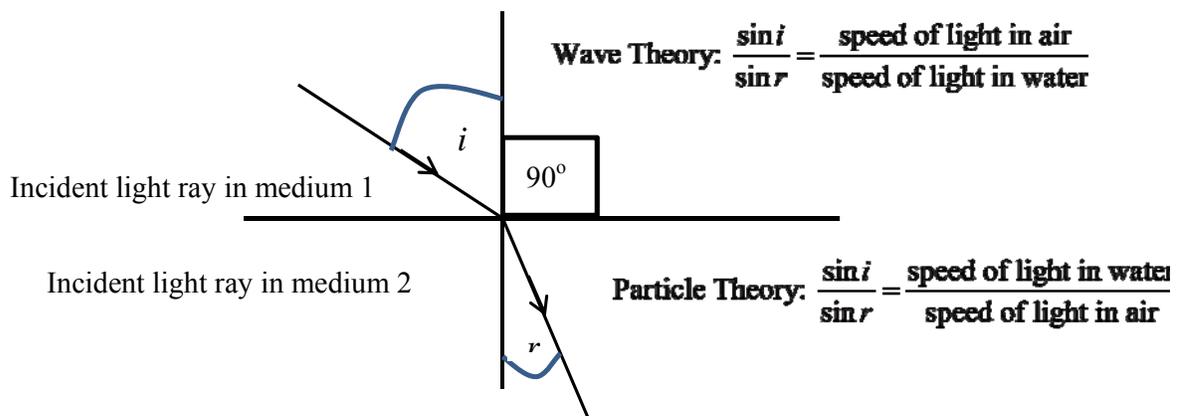
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<sup>11</sup> I am borrowing this metaphor from the title of Roger Penrose's book (Penrose, 1991).

teaching practices that I have been developing since my school years. Eventually, this paved my path to constructivist epistemology.

We may think of waves in the visual image of sea waves or waves on a string and the particles as the visual image of marbles. This imagery will suffice for the discussion here. We may also associate wave nature with the holistic nature of reality and particles with the discrete nature of reality. Waves are a means of energy transfer via oscillation of a medium such as water, rope or electric and magnetic fields. In the wave mode of energy transfer, medium does not undergo any net transfer. Electric field and magnetic field may not be considered as mediums. Number of oscillations per second is called frequency and the distance travelled by energy during one cycle (or period-usually measured in seconds) is called wavelength. The distance travelled by energy (or oscillation) per second is called the wave speed.

The rivalry between the wave theory and particle theory has been very long. During Newton's time the crown of acceptance was held by the particle theory of light. Newton's particle theory and Huygens' wave theory both predicted the ratio  $\frac{\sin i}{\sin r}$  (see the diagram below) to be a constant and this was verified by experiment. Both theories stipulated that the ratio  $\frac{\sin i}{\sin r}$  was the ratio between the speed of light in air and water, respectively; the difference was the order of the ratio. The following diagram illustrates this difference.



Notice that this again epitomises the nature of scientific theories, as discussed early in this chapter. According to the experimental results, the ratio  $\frac{\sin i}{\sin r}$  is greater than one. So, it required that the speed of light in water to be bigger than the speed of light in air; in Newton's theory the speed of light in air needed to be greater than the speed of light in water. During the era of Newton and Huygens, it was not possible to measure the speed of light in water. For this reason, it was difficult to test which ratio was accurate; was it the speed of light in air divided by the speed of light in water or was it the other way around? The scientists unwittingly used the only 'test' available; the reputation of Newton. This robbed Huygens the opportunity of producing the 'right' idea of light. So, the throne of prestige was held by Newton's particle theory until the speed of light in water was experimentally verified. With the surprising result that the speed of light in water is less than the speed of light in air, the particle theory fell from grace. By that time neither Newton nor Huygens was alive. Since then and until the photo-electricity experiment and consequent birth of the new particle theory, which was laboured on and delivered by Einstein, the indisputable theory of light was the wave theory.

In the photoelectric effect light is shone on a thin metal plate and electrons fly off from the metal producing an electric current due to the impact of light. The wave theory can explain this phenomenon in the sense that we know a tsunami can dislodge even huge buildings. However, the predictions arising from the wave theory are contradictory to the experimental results. Whether any electron would be dislodged or not, according to the wave theory of light, should depend on whether or not the intensity of light shining on the metal plate is sufficient; even then electron flow would commence only after a time lag. The experimental results verified that dislodgment of even a single electron depended only on the condition of light having or exceeding a threshold of frequency, not on intensity at all. In the case of light having or exceeding this threshold frequency, production of electric current was as immediate as light falling on the metal. Furthermore, the only right thing for the wave theory was that the number of electrons dislodged (magnitude of the photoelectric current) depended on the intensity of light, provided that the threshold frequency requirement had already been met.

To understand Einstein's explanation, imagine a tall mango tree full of fruits. A group of kids throw pebbles at mangos to fall them. The stems that attach the mangos to the tree have a special property. Each mango stem has the same strength. And when a pebble hits the stem either it breaks and falls or there is no weakening of the stem at all. In this latter case the pebbles fly away without damaging the mango or the stem. We may further assume that each pebble has the same mass and shape. Even if a mango stem is hit many times and the stem has not been broken then it does not make it any easier for the consequent hit to break the stem. Now regardless of how many pebbles per second (intensity of pebble attack) are thrown at the mangos if any of the pebbles does not possess the required energy (kinetic energy that is- determined by the speed- since all pebbles are of identical mass), no mango is going to fall, but as soon as the pebbles are thrown above a certain threshold of speed then mangos will begin to fall even if the pebbles are thrown at an extremely low intensity (say one pebble per day). Below this threshold speed, even if a billion pebbles are thrown per second no mango will fall. If the pebbles are thrown at the threshold speed then the mangoes will fall vertically. If the speed is higher than the required threshold frequency then mangos will fly away from the tree in a parabolic path.

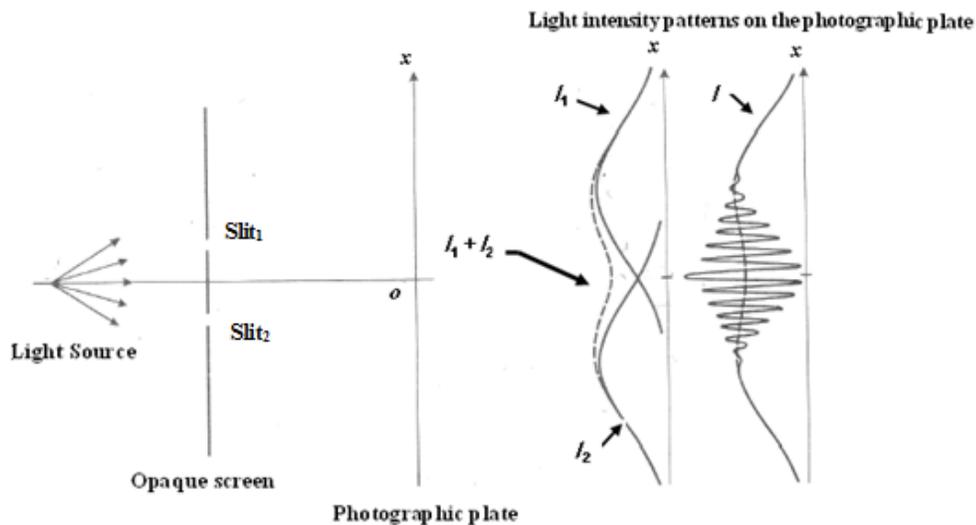
To explain the photoelectric effect, Einstein considered light as lumps of energy. The energy of the lump depends on frequency (Can you see where this linking to frequency is coming from? This assumption was motivated by the condition of threshold frequency.) In our metaphor of looking at Einstein's explanation, the pebbles play the role of particles of light (or photons) and the speed of the pebbles plays the role of frequency. When the pebbles exceed the required speed, then the larger the number of pebbles which are thrown per second, the larger is the number of mangoes that fall per second.

In summary, according to the wave theory, the intensity of light should play a decisive role in the production of the photoelectric current. That is, wave theory predicts that light should exceed a minimum intensity for electrons to be dislodged from the atoms of the metal, and that more intensity will produce a larger current while the frequency has no role in the dislodgement of electrons or in the magnitude of the current. Experimental results showed quite the opposite. Intensity does not

play any role in the dislodgement of the electrons whereas if the frequency exceeds a minimal threshold then more intensity results in more current. This result dethroned the wave theory from its kingship of being the preferred theory of light.

It looked like the wave theory was doomed forever. However, Young's double slit interference experiment was considered to be a major piece of evidence for the wave theory. See the following diagram. This diagram is a little modified version of the diagram in (Claude Cohen-Tannoudji, 1977, p. 12). In this experiment, there is an opaque screen that has two close slits. In front of this opaque screen lies a light (preferably monochromatic) source. These slits can be opened or closed individually. Light is directed upon the slits. Behind the board, there is a screen on which the light passing through the slits makes an intensity pattern. Three settings can be considered:

- The upper slit (Slit<sub>1</sub>) is open and the right slit is closed
- The lower slit (Slit<sub>2</sub>) is closed and the right slit is open
- Both slits are open



In the right corner of the diagram there are two sketches of intensity patterns, light intensity versus the distance from the point  $O$ . In the left sketch of these two, there are three graphs. The solid line graph in the left sketch, named  $I_1$ , represents the intensity of light versus the coordinate  $x$  when only Slit<sub>1</sub> is open. In all four graphical representations the larger the height at a point  $x$  the more the light arrives at that point. As expected, the intensity curve is centred at the point directly in front of Slit<sub>1</sub>. Details are identically similar for the intensity curve  $I_2$ , except that it centres at the

point which is directly in front of Slit<sub>2</sub>. The dotted line curve  $I_1 + I_2$  in the left sketch represents the numerical sum of the intensities  $I_1$  and  $I_2$  at each point  $x$ . The wavier graph to the right is the intensity curve experimentally obtained by letting light pass through both slits. This intensity pattern depicted graphically above actually looks like an alternative series of dark and bright bands of light on the photographic plate.

When I looked at these graphs for the first time I felt a sharp shivering current going through my spinal cord twisting my whole conscience. There are points on the photographic plate which light has more chance of reaching if only one of the slits is open. There are some other points which light reaches only when one slit is open, and somehow light is prevented reaching these very same points if both slits are open. More paths or opportunities (instead of one open slit, there are two slits that light can creep through) mean less chance of light reaching those points. Then there are also some other points for which more opportunities mean more success. And, the most far away ends have not been significantly affected by the number of open slits.

The wavier pattern can be easily explained by wave interference. When two waves meet interference occurs. That is, some waves get cancelled out and some others get enhanced. For instance, if a sound of single frequency (Single frequency allows this phenomenon to be easily observable.) is played by two speakers one can experience that there are some spots at which the sound is markedly enhanced and some other spots at which sound is markedly diminished (similar to dark bands and bright bands). This happens because two sound waves coming from the speakers meet at a place where they are reinforced or cancelled by each other. This interference pattern was not a problem during the time that light was considered to be a wave. In fact, it was one of the most supporting evidences for the wave theory.

Nevertheless, this interference pattern could still be explainable by particle theory if another experiment was not performed. To see this, imagine that a machine is throwing marbles in random directions. In front of this machine is a wall on which there are two small windows. The left window is closed and you stand in the other side of the wall catching the marbles coming to you through the right window. Say you catch ten marbles per minute. Then you close the right window, open the left

window and stand on the same point as before and collect the marbles coming to you through the left window. Say, you collect seven marbles per minute. After this you open both windows, stand on the same spot and collect the marbles coming to you through both windows. If the number of marbles coming to you through both windows is twelve per minute (sum of the marbles coming through individual slits- This is the result depicted by the dotted curve.) then there is no surprise. But, you find that the marbles that are likely to come to you when only one of the windows is open are not coming to you, and the marbles that are not likely to come to you when only one of the windows is open now somehow more eagerly reach you.

Even though this may impress us as a mystery at first sight there is a satisfactory explanation. If the machine throws a large number of marbles per minute then you may be able to convince yourself that the number of marbles reaching a point depends on the spot and whether both windows are open or not since when the machine throws a large number of marbles per minute then it is likely that several marbles cross the two windows simultaneously and on the way out they may hit each other and change their paths. So some marbles that were coming to you through the left window are now knocked off course by another marble coming through the right window. Also it is possible that a marble going away from you suddenly changes its direction towards you since it is hit by another marble coming through the other window.

This could be valid for Einstein's photon picture of light if they are crossing the double slits in large numbers. At this stage, scientists would have ventured to tinker with the particle theory of light to account for the above intensity graph but it was not to be. In the Year 1905, Einstein presented his paper on the photoelectric effect. G. I. Taylor wanted to test whether the reason for the strange intensity graph was due to simultaneous crossing of many photons. To achieve this, in 1909 he used a filter that minimised the intensity of light that fell on the slits. The intensity was so low that it could be safely assumed that on average no more than one photon at a given moment was present at either of the slits. This means that when a photon is crossing one slit, the probability that another photon is crossing any one of the two slits (both were left open in this experiment.) is virtually zero. The first few photographs of the light falling on the photographic plate after passing through the slits were a clear

demonstration of the particle picture of light because there were only random dots on the photographic plates. When the photographic plate was left for months and then developed the dots which hit the photographic plate at random spots one photon at a time (one dot at a time) were clearly completing the same light interference pattern as in the experiment in which the intensity was high.

This is the experiment that resurrected the wave theory to its former glory and endowed it with equal share of acceptance with its arch-rival, the particle theory. In the first glance the results of this experiment seem to indicate that individual photons behave in such a manner that they form a certain pattern ‘knowing’ that there are two alternative paths and if there were other photons then their paths would have been affected by each other (even though there are only individual photons at the slits at any given moment). So acting ‘decently and lawfully’, even when there are no ‘policemen’ to book them, they obey the traffic laws set in advance. Although photons hit the photographic plate at random spots, collectively they form the pattern. It looks like there is a communication network among the photons. “I will hit a place in random and you will do the same, but collectively we should fool our observers by creating the same pattern as we are compelled to make when crossing en masse.” To enhance visualisation of this situation, I will present another example.

Consider two streams of people coming out of a stadium through the only two (nearby) gates. The people in the two streams, as soon as they come out-noticing the people in the other stream, are compelled to adjust their paths voluntarily. Now imagine people in the stadium are only allowed to leave the stadium through one of the two open gates one person per week. If these people act like there are other people coming out of the gates and change their paths accordingly then we would send them to a psychiatrist. But that is how these individual photons behave. They ‘think’ that there are other photons coming out and ‘courteously change’ their paths to accommodate them. Later it was established that this ‘erratic’ behaviour is also a property of material particles such as electrons and protons.

The scientists tried to determine which slit the photons or electrons were going through. This new experiment reduced the double slit interference pattern to a single slit interference pattern that belongs to the slit that is open. This was interpreted as

the observer determining the path of the microscopic particles. This means that to check through which path the photon or electron is going a modification of the experiment—such as placing an electron or photon detector at one of the chosen slits—is required. The placement of this extra device produced just a single slit interference pattern ( $I_1$  or  $I_2$  depending on which slit is not provided with the detector). It looks like our conscious choice of looking (to see which slit the microscopic particle is going through) makes the particle produce the interference pattern relevant to the particular slit at which the detector is not placed. A new physics was born and it was named ‘quantum physics’.

Einstein did not like this pessimistic, defeatist and [solipsistic view](#) that our conscious mind determines the path of microscopic particles. For decades most of his energy was almost directed to finding a loophole in quantum mechanics formalism. Einstein, to put this ironical debate into context, once asked ‘Does the Moon exist only when we look at it.’”

The world of physics has never been the same since this experiment. Einstein and Bohr debated against each other’s position on interpretations of this experiment. Einstein preferred a more objective interpretation and Bohr resorted to a more subjective interpretation. Bohr almost always outsmarted Einstein in this debate. Had I known that Ptolemy was able to predict lunar and solar eclipses with his planetary motion model that the Sun goes around Earth and the deep philosophical questions arising from Taylor’s experiment and its various scientific interpretations when I was in Year 9, it could have made a tremendous difference to my worldview. I regret that I had to wait more than a decade to become aware of these aspects of scientific knowledge. It could have animated my mind and made a tremendous positive influence on me as a thinker. I hope that this story is available to our youngsters in simple language as early as possible. I am certain that this can change the lives of at least a number of kids. The world could benefit from it. The one who is affected positively by this story may be the kid who we think of as lazy and or ‘dumb’. We really do not know. We just have to offer it to them, for their sake and ours. Taking is theirs, offering is ours.

I would like to state that even though awareness of these developments snatched me from the camp of [logical positivism](#) I have been closer to the position of Einstein than that of Bohr. I am still closer to Einstein's view in the context of my constructivist epistemological views explained before. The journey I started away from logical positivism due to the impulse I received through consideration of the nature of scientific knowledge is a journey of no-return. Later in this work, I will argue that still logical positivism has a strong role to play in our affairs and it will be always like that.

Bohr thought that it was meaningless to talk about the path of an individual photon (or electron) and seemed to be happy that each individual photon or electron was crossing both slits if both of the slits were open. On the other hand Einstein needed a more objective frame of thinking. To highlight his objections, Einstein famously asked the question, as it has already been mentioned, "Does the moon exist only when we look at it?" However, just because of this quote Einstein cannot be considered as an ardent logical positivist. This following quote of Einstein demonstrates this.

He [the scientist] must appear to the systematic epistemologist as a type of unscrupulous opportunist: he appears as *realist* in so far as he seeks to describe a world independent of the acts of perception; an *idealist* in so far as he looks upon the concepts and theories as the free inventions of the human spirit (not logically derivable from what is empirically given); as *positivist* in so far as he considers his concepts and theories justified *only* to the extent to which they furnish a logical representation of relations among sensory experiences. He may even appear as a *Platonist* or *Pythagorean* in so far as he considers the viewpoint of logical simplicity as an indispensable and effective tool of his research. (As cited in Pais, 1982, p. 13)

This quote is evidence that Einstein believed in the necessity of a plurality of epistemologies for scientists to engage in their research. My Rainbow of

Epistemologies (see p. 110) appears to be an extension of this notion to a general view of the world.

Let us look again at the experimental results:

- There is randomness for individual photons or other microscopic particles with regard to the spots that the photons hit the photographic plate.
- Also there is an absolutely well-defined pattern on the collective basis. (That is how we get television pictures on our TV screens. Even though electrons hit the TV screen at random spots they collectively carry the picture and recreate it on the TV screen).
- Seemingly or interpretively there is subjectivity in the movement of microscopic particles through slits. According to the conventional view there are no objective paths of movement for microscopic particles through the slits. Paths of the particle depend on the observer.)
- Then there is clear objectivity. The interference pattern is independent of the observer or the place or the time. Also, if it is true that the paths of microscopic particles depend on the observer then that condition is independent of the observer. That is, whether the paths depend on an observer does not change from one observer to the other.
- The dots clearly and discretely imprinted on the photographic plate depict the particle aspect or locality of photons or electrons.
- The ultimate overall pattern depicts the wave nature or holistic nature of microscopic particles.

Does this mean that the wave aspect of microscopic particles goes through both slits? In that case what can we say about the 'dot' aspect? Does this 'dot' aspect go through only one slit? Which one? When we place a detector behind one of the slits, to determine which slit particles go through, it shows both the 'dot' aspect and the single slit dot pattern, that is, random dots create the single interference pattern over time. When we place the detector how does the photon or its wave-holistic aspect know where to hit the screen to create just the single interference pattern?

What does the interference experiment result say? The probability that a microscopic particle reaches a given point on the photographic plate when both venues are open is

not the same as the probability when only one slit is open. This is unequivocal and is not open to any interpretation. The explanation is the problem. Why does it happen? For me it is not surprising that the probability changes when both slits are open because by opening the other slit we are changing the material setting of the experiment. In retrospect, I would have been inclined to be surprised if opening both slits did not change the single slit dot result; but this is only in retrospect (otherwise, the question wouldn't have even occurred.). When both slits are open and a detector is placed at one of the slits does the wave aspect go only through the slit that is not attached to the detector?

To interpret the result of the experiment that was designed to look at which slit the photon went through, the mainstream scientific community take the view that there are no paths for microscopic particles or somehow they go through both slits at once. Then why do not tennis balls thrown through two windows (one per each hour) create such a pattern? If paths are not important, then why does availability of alternative paths dramatically change the result? Every effort so far made to answer these questions has failed; nevertheless computational accuracy of this new physics marvels the very best of all successful theories so far.

Our visual images, such as people streams, tennis balls or marbles, are only helpful up to the limit that we understand that these are not valid metaphors. It can be easily noticed that when a train moves it creates wind. If two trains are going through two short tunnels (assume that they move in the same direction) then we know that the wind created by trains affects the movement of each other but not up to the level at which the trains derail each other. In the sea two ships travelling on nearby parallel paths could dramatically change each other's' paths even causing a collision. This is called the Bernoulli Effect. A similar thing may happen when microscopic particles cross nearby slits in large numbers. If the slits are too closed or too far away then this spooky behaviour vanishes. This means that the spookiness is not a result created in our mind and it is the consequence of the material condition that there are two slits not too nearby or not too far away. In this paragraph I have been arguing as a logical positivist.

I wonder and speculate whether the microscopic particles are making ripples in some medium and these ripples are strong enough to disturb the paths of microscopic particles but are too weak to affect the behaviour of macroscopic particles. In that case even if a microscopic particle crosses a particular slit the ripple created by it would cross the other slit. This ripple may not be strong enough to disturb the paths of macroscopic particles but significant enough to alter the paths of microscopic particles. For a car travelling in one of two nearby short tunnels this ripple travelling through the other tunnel is not strong enough to create any noticeable alteration of path of the car. Also, other factors such as wind can be more significant than this ripple. Even the wind effect can be neglected in most situations. This ripple is computationally non-existent for the car. Then still there is a problem. The ripple has to travel through the slit at least with the speed of the microscopic particle.

David Bohm produced a theory that takes out this mystical nature and produces the same accuracy and predictions as the conventional quantum mechanics does; but this theory involved the scientific “taboo” of non-locality. Involvement of non-local variable would have been furiously resisted by Einstein. One would wonder, as Einstein was a revolutionary has his reputation become reactionary? This judgement is not up to me as I have not studied this particular work of Bohm. I have published a couple of papers on the Aharonv-Bohm effect with co-authorship of Associate Professor WP Healy. See (Healy and Samandra, 1997; Samandra and Healy, 1998). Even though a continuous debate is going on about this ‘mystery’ the majority of the scientific community, at the moment, is inclined to frown at hidden variables of any kind (such as I stipulated in the above paragraphs). For this non-acceptance, they might seemingly have valid reasons dictated by current paradigms. However, ruling out of a paradigm shift at some time in the future could be against the very nature of science since science is the only belief system one of whose major attributes is continuously striving to prove itself wrong.

Even though one cannot predict where an individual photon/electron hits on the screen the overall pattern is predictable. As previously mentioned, even though the designer of the TV circuitry cannot predict where an individual electron will land on the TV screen, he can guarantee that you will get the accurate picture (overall pattern). This is valid for the behaviour of some macroscopic particles. For instance,

traffic authorities can predict how many people are going to die on Christmas Eve on our roads but it cannot say which individual will be included in the statistics. In this case some people may say this is due to lack of information. At the microscopic level physicists hypothesise this to be a fundamental property.

With the wave-particle duality, a strange thing happened in science for the first time. This is the first time in the history of science (at least to my knowledge) that two rival theories are complementing each other and cooperatively (albeit dualistically) ruling a particular region of the scientific kingdom. Either of the Kings cannot rule the kingdom without the other King. In Chapter 6, I will briefly discuss this wave particle duality again.

Since the day of inception of quantum physics its mystery has not eased. If anything, it has only deepened. Mainstream physicists are quite happy with the status quo but there were and there are dominant physicists and philosophers who question the philosophical validity of quantum physics. The difficulty for the scientists here is to find a proper metaphor. Some aspects of microscopic particles can be explained by the marble metaphor and the other aspects by the sea wave metaphor. It is still difficult to find a simple metaphor that binds both the marble and the sea wave metaphors, and therefore the wave particle duality takes the centre stage of modern physics. Many physicists seem to believe that quantum physics has the last say on this matter. That is really unlikely. So far there is no challenge to the computational accuracy in quantum physics for any widely known experiments, but this is not a reason for solace. We had computational accuracy with Ptolemy's theory. Newtonian theory is accurate enough in its domain of validity. With time humankind will find a proper metaphor. On this I am on Einstein's side. Not because of his reputation. In fact, in this arena he was considered to be a big loser.

This is the nature of science. As it has been said many times since the inception of quantum mechanics, the task of science is not to explain the reality of nature but it is to organise what we can say about the reality of nature<sup>12</sup>. What we can say/explain about reality is subject to many constraints. One of these constraints is consensus. This consensus is built mostly by democratic debate.

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<sup>12</sup> This is quote of Neil Bohr and it appears on p. 108, in its exact form.

What does this story say to us?

- Science is not *just* a collection of indisputable facts.

Here, we need to be careful. Facts play an essential role in science. This role can never be downplayed. In one professional development program, the presenter began her exposition attacking ‘facts’? I needed clarification and asked, “To support your views on science and constructivism, do you stand on falsehoods? If, as you say, facts are useless, is the assertion, facts are useless, itself a fact? If it is a fact, then would it be a useless fact? In that case, why did you even bother to express it? Then again, if the assertion that facts are useless, is it itself a fact? If not, then why did you express that falsehood? Then again if your assertion is a fact is it a useful fact or useless fact?”

Any claim that is made into a slogan may catch the eye but may certainly elude the mind. It is not the ‘facts’ that we have a problem with; it is how we think of and how we use them create problems.

Even after the scientific community as a whole accepts a scientific theory there will still be dissidents holding for different views. Some of these scientists may not express their dissident views for fear of ridicule.

- Science is not *reality* itself.

Light is light regardless of how we choose to explain it. Light did not turn into wave overnight when the experiment verified that the speed of light in water is slower than that of light in air. And, light waves did not change into particles immediately after Einstein published his photoelectric theory. Also, light did not assume a dual nature of being waves and particles as soon as data of Taylor’s experiment was released.

- Science does not necessarily progress in *a straight line path* as depicted in many or most textbooks.

Much more correctly, zigzag is how the path of progress of science actually looks like. Reductively the multitude of zigzag paths is summed up to a straight line like path. To some extent this reductionism has some validity. Again what is wrong here is not the

approach itself but our inattentiveness to the possible illusion which can be created in students.

Kuhn (1996), in his analytical study of history of science, *The Structure of Scientific Revolutions*, has put forward a proposition that the progress of science is a transition from one paradigm to another. A paradigm is a tightly held system of beliefs that enables scientists to conduct their inquiry. Anomalies that come to light by experimental evidence compel scientists to replace their existing paradigm and form a different set of beliefs. In this analysis, the endeavour of science is indicated to be a social activity as well as a cognitive activity. Kuhn's arguments seem to support the constructivist view of the nature of scientific endeavour. Kuhn's view has been an enormous advancement in philosophy of science. Nevertheless, there are many critical views on his thesis.

Even though I happen to be a strong critic of modern education paradigms it is heartening to see now some science textbooks trying to deal with historical development. This helps to prevent teaching science as just a collection of 'facts' and encourage students to develop more rigorous thinking skills. A more appropriate way to think of science may be to think of it as a way of thinking and an approach based on 'facts'. With regards to the presentation of historical development, I advocate a well-considered middle path since it is ill-conceivable to deal with the historical development of science in minute detail. Hardly is there any reason to teach historical development in great detail. In fact, there are compelling reasons not to do so.

For instance, to a present thousand year old zigzag history of science reflecting on every idea that was ever conceived by scientists we need months or years. Even then we would make a choice about what to leave out. What is essential is to present a really brief survey of historical development (with great admiration to those thrown out ideas (that means no putdowns) of the ideas and their presenters) of many of the topics covered, and then to do somewhat more detailed presentation on one or two clearly selected topics. Some VCE physics tests have done a wonderful job of telling the story of atomic theory and wave particle duality.

The concept of ether was once embraced by general scientific community with great enthusiasm. Then it was abandoned. It is imperative to remark that there is no “proof” that ether does not exist. Hardly are there any proofs in science. This largely contrasts with the field of Mathematics<sup>13</sup>. What has been established at the moment is that the concept of ether is superfluous at best, according to the current paradigms. It may be possible for ether to make a comeback sometime in the future. At present there is a great deal of resistance even to rethink the concept.

I remember the tremendous shock and horrendous pain I felt when I first read the following statement by Neil Bohr as quoted in (Patterson, 1985, p. 12):

It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can *say* about nature.

I angrily thought “Who is speaking here, a scientist or a deluded [solipsist](#)?” That shock later turned out to be a therapy for my world outlook. Hesitantly I began to think on those lines and explored the evidence for this statement, and I was enthralled and inspired by it. That was a major event in my life similar to the life events I experienced in Year 9. How did this information elude me? I think one of the major reasons was that I could only read Sinhalese. And, even if I could have been able to read English I doubt that the relevant books would have been freely available to me in Sri Lanka at that time.

There are only four words in the phrase above “what we can say” but it speaks volumes. Here ‘we’ embodies three aspects; subjectivity (we cognise), objectivity (we debate and find consensus- as opposed to what one individual can say), sociality (since we operate in a society we are subject to its societal winds and currents.). Then the term ‘can say’ on the one hand may represent the authority of objectivity. It is not what we dream or wish or a figment of our imaginations. And also “can say” may mean what we are allowed to say by the material conditions or the authorities. These authorities could come from economic, political, military or professional bodies. Generally science strives to transcend these aspects of influence to stand on its own

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<sup>13</sup> Even some mathematics proofs may be challenged on the basis of new concepts. I have given a such example on p. 84.

feet. In the long run science can succeed only to succumb to another form of prejudice; that is the honour of the old theory or the grand old master. One reason that Huygens' wave theory was initially ridiculed could be that the honour invested in the wave theory of Newton. Then again, the prejudice vanished as soon as it was clearly shown that the speed of light in water was much slower than in air.

Before I present my position on logical positivism and objectivism I would like to slightly modify Bohr's statement above.

Science is not study of nature. Science is the study of what we can say about nature. What we can say about nature, at least to an extent, changes our experiential reality<sup>14</sup>. Also, our experiential reality changes what we can say about nature<sup>15</sup>. At times, which comes first (change in experiential reality or change in what we say about nature) is a chicken-egg conundrum. Also, at some other times, we may be able to say which comes first or second to a consensual degree of certainty. I underlined the word 'can' in the first sentence of this paragraph to emphasise that what we 'can' say about nature is not a matter of arbitrary choice.

Regardless of my firm criticisms against logical positivism I have not completely abandoned logical positivism or the [rationalist view](#). It is impossible to do so. If I do then a major part of my life activities becomes a farce. For example, I will not be able to justify why I am writing this. If someone claims to have seen a ghost my first thought is based on rationalist view of the world. That's how I operated when my community was gripped by milk drinking god statues and light radiating from Buddha statues or when there were news reports of bleeding statues of Christ. To put all this into context, I would like to remark that believing in the existence of supernatural phenomena contradicts itself. If something exists then it is part of nature and therefore it cannot be above nature, but it is in-nature. That does not mean that

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<sup>14</sup> For instance, Einstein's theory of general relativity suggests the existence of gravitational waves, and the scientists are striving to verify this. If there is no general relativity then the scientists would not be striving to observe such waves.

<sup>15</sup> The Michelson and Morley experimental result on the speed of light motivated Einstein to rethink then existing paradigm of space and time.

hitherto unexplainable/unknown phenomena do not exist. They do exist. For example, the biggest unexplainable mystery of all is my existence, or the existence of the universe, cognitive beings and cognition. I term them as a ‘miracle’ without wrapping it in any supernatural or religious aura.

It is an axiom of my belief system that I exist. I do not see any reason to engage in a debate to justify/verify or prove that I exist since if someone asks *me* to prove that I exist then the instigator has already acknowledged that I do exist. Then this existence can also be argued to be murky. I know that I exist because there is memory. For instance, if I cannot remember anything beyond the last second then still I will remain to exist but ‘myself’ will vanish into thin air. Then I take it for granted that memory exists. I may explore what memory is but I will not question or try to justify its existence. The table in front of me exists even when I do not look at it. That is also an axiom of my epistemology which I do not see any reason to debate in justifying. Many people can choose many axioms. As I do, they can also take their beliefs as axioms. As long as other peoples’ belief systems do not contravene with general human expectations of decent behaviour I respect their faiths even when I do not subscribe to them. My inability to do so is frailty of my own philosophy.

The sentiment expressed in the paragraphs above is somewhat mirrored in both Einstein’s comment that a scientist needs to be an epistemological opportunist (see p. 100.) and Niels Bohr’s comment which is quoted in Hans Bohr’s article.

Two sorts of truth: profound truths are recognized by the fact that the opposite is also a profound truth, in contrast to trivialities where opposites are obviously absurd. (Bohr, 1967, p. 328)

When I read this statement of Bohr, I always wonder “Did he think of the experimental results of Quantum Physics in the way he did because he was influenced by Eastern philosophies or was he attracted to Eastern philosophy because of the way he thought about the experimental results of Quantum Physics? Next I would like to simplify Bohr’s comment:

A naive truth is a truth whose negation is false; a profound truth is a truth whose negation is also a profound truth.

It appears that Bohr was compelled to walk away from the Aristotelian two-dimensional logical paradigm and embrace the Buddhist fourfold truth paradigm. This will be again discussed on p. 155.

### **A Rainbow of Epistemologies**

I have argued that constructivism arises from epistemological and [ontological](#) considerations of reality. To deny this is to reduce constructivism to a set of methods which blindfolds us against the precarious nature of perceivable reality. I reject any form of solipsism, or even a hint of it. Also, I reject that there exists a supreme conscience, or that any grand scheme is at work. Nevertheless, I acknowledge that, as heat energy was unified with mechanical energy, as matter was unified with energy, it may be possible to unify thought and matter. After all, if there were no conscience in the early Universe, then it is generated out of matter. Somehow there existed the potentiality that matter could be developed into living matter and then thinking matter. Was that potentiality generated later? If so, the potentiality of that potentiality was already there in the early Universe. So, maybe the dichotomy between thought and matter is just a convenient construct built by us. Maybe, just maybe. If the potentiality of genesis of conscience did not exist at the origin then the potentiality of genesis of conscience must have come to exist at a later time. From where? Then we may argue that the potentiality of lately occurred potentiality should have been there at the origin of time itself. Someone can argue that the mode of questioning in this paragraph is erroneous due to the assumption that the Universe has a beginning. These arguments take us on circular paths generated by a kitten playing with a ball of string. We do not have any escape from this maze of lexicons.

When we make deeper inroads into perceivable reality, it may be difficult for us to maintain (at philosophical level) some of the dichotomies we have created or constructed and taken for granted. For instance, it can be argued that there is no male-female gender dichotomy, only a gender spectrum.

Some may be disturbed by the state of affairs that we cannot pinpoint any absolute reality. The situation that the observer and observation are inseparable (not only at the microscopic level, i.e., quantum physics, but also at the macroscopic level) may be quite frightening. But, this is an essential predicament and an unavoidable

condition of our existence (cognition). Only way to escape this precarious nature is not to exist at all or to lose our conscience and cognition. And, this is not a nice escape.

This precariousness is also freedom. I cherish this freedom. The world, to an extent, is 'ours' to "construct". Things fall. They always did. They always will. It will not be affected, even an iota, by the flavour of the gravitational theory favoured on any given day. But we can fly too; if we cannot then we can be in a flying plane. I can be right and/or I can be wrong. I can be right and wrong at the same time too. Then again, I may not be right and I may not be wrong at the same time. Yet we will continue to ponder deeper into our wondrous world with eyes widely opened with undying excitement. Cognition is guaranteed never to be boring.

To conclude this segment on my development of epistemological views I would like to focus attention on the earlier contradiction that existed between my conscious philosophy and subconscious philosophy. While my proclaimed life philosophy was logically positivistic my subconscious thinking had been constructivist. This internal rift might have developed due to the fact that I embraced academic constructivism with great enthusiasm early in my life. When I was writing poems or in a heated debate I might have suppressed my conscious thinking, the thinking happening beneath the conscious philosophy might have popped into the open.

I hope that I have been able to demonstrate that to be sane we need to step on a very fine string hanging between two poles above a hazardous epistemological abyss. Epistemological constructivism in the form that I have discussed above provides me the fine string and the balancing rod. As a whole I did not and do not claim that absolute reality exists or reality is merely subjective or vice versa. What I attempted to argue is that any such claims lead to difficulties, at least in my mind. To deal with this situation there are three choices.

- Accept the existence of a supreme conscience and ignore the contradictions arising from that position. This choice may be useful to many people but it is difficult for me to reconcile. Contradictions arise only if we want to endow the supreme conscience with both perfect benevolence and all might. Theists have arguments against this restriction but they lead to further

contradictions. In contrast, the existence of free human will and the ability of the supreme conscience to foresee what a particular human will do or think even before he is born can be logically maintained by assuming that time and space are just a single moment and a single point to God, respectively and what humans perceive about time and space is an illusion<sup>16</sup>. As it is impossible to refute this argument, it has some merit but mere logicity may not have any relevance to ontological reality.

- Another alternative is to accept solipsism and use a jargon of entangled notions to ignore the contradictions arising within that belief system. It could lead to lawless dangerous anarchy. Any crime exists only in the mind of the doer or the perceiver and therefore it does not harm anybody. Logically it is impossible to refute solipsism. Nevertheless, by solipsism, for me, a meaningful practice cannot arise. Here I believe that I have beaten solipsists in their own game for they cannot have a counter argument against the views that I express since I do not exist. That is, my views only occur in their mind. Therefore my doubts in solipsism are the doubts of solipsists themselves. If they are true to their philosophy they need not worry about refuting other people's arguments. The solipsists would have to resort to weird and distorted answers if I ask them why they have bank accounts, last wills or refer to bus timetables. For me, this choice is even more repulsive than the first.
- Since it is also impossible to completely reject subjectivity and objectivity the third choice is to tie both views in a meaningful way and accept that we can perceive reality only as a duality of subjectivism and objectivism. This is the choice that I can live with.

A major benefit of my philosophical metamorphosis, I hope, is the cure of my intellectual arrogance that was well hidden from others and even from myself. However, occasionally and momentarily the cobra of intellectual arrogance swings its head in my mind in a rhythmic and threatening manner. At those moments, hearing my sarcastic laugh, the cobra dies, to rise again at a more opportunistic moment.

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<sup>16</sup> This comes very closer to an 'Eastern' philosophical standpoint.

No longer is there unchallengeable knowledge or wisdom. The following saying of Buddha in Alara Kalama Suthra became lucidly clear to me. “Do not accept anything just because your elders, parents, teachers, books or the whole society says so. Do not accept anything merely for its logicality. Do not accept anything because Buddha myself says so. Only accept something after careful thought”. Many Buddhists believe Buddha to be the knower of all and for this reason many monks fail to interpret the above statement appropriately. Since the very beginning of institution of Buddhism (Here I am contrasting Buddhism as an institution and Buddhism as a life philosophy espoused by Buddha), monks might have promoted the idea of Buddha being the knower of all just out of piety or to manipulate the faithful.

Even though this attribute of Buddha being the knower of all has been ingrained in Buddhist conscience there are some episodes, real or literal, that point out limitations of Buddha as a knower. For example, Buddha was initially reluctant to share his enlightenment with the masses on the basis that it might be cognitively inaccessible to them. Only after the invitation of Brahma (a living being from another world) did Buddha agree to preach to people. As theists and solipsists counter argue to circumvent the contradictions arising from their views and practices Buddhists also use arguments to escape from such difficulties.

At another instance, Buddha made his seven year old son, Rahula, a monk without the permission his guardian, King Suddhodhana, the father of Buddha, This dismayed King Suddhodhana and brought his grievances to Buddha and Buddha made a decree that no under age child can be offered monkhood without parents’ or guardians’ blessing. If Buddha was a knower of all why did he have to learn the inappropriateness of his actions from another? Whether or not these stories are actual or metaphorical is not the question. The question is whether the Buddhists heed the message that Buddha was an enlightened one but not a knower of all.

### **Colours of the Rainbow**

My worldview is a rainbow of a particular variety; a rainbow that does not exist if even one of the colours is taken out. Taken together with all the colours, the plethora of views illuminates the horizon of my epistemology as a beautiful rainbow. With

these colours I can paint beautiful images on the canvas of life and world, which is useful and soothing for me.

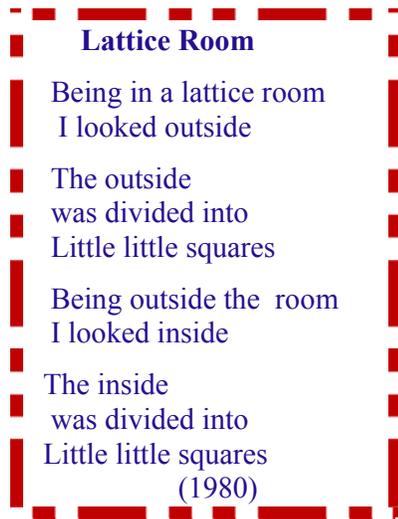
I am not even concerned which colour is dominant or significant. As a teacher, I point out to students many times the absolutes and absoluteness of knowledge. For instance consider the statement, “It does not come from that mango tree, or the sky, or the air or my brain. It comes from the statement on the board. Look very carefully.” Then in the same class I emphasise subjectivity of knowledge. Which view is dominant or considered depends on the task and aim. This may be construed or misconstrued as opportunism. In a way it is a form of opportunism and then in many ways it is not. For instance, to travel seventy kilometres one does not choose to walk unless the walk is about fund raising. It will be ludicrous to call this opportunism.

I would like to point to the similarity among Einstein’s notion of research scientist’s epistemology (see p. 100), Feyerabend’s notion that no single methodology can adequately deal with the complexities and multiplicities of scientific endeavour (Feyerabend, 1975, p. 30), and the concept of rainbow epistemology presented in this work.

I started my life as a Buddhist faithful. Due to the influence of science I became a logical positivist. In the above exposition I have outlined my transformation from logical positivism to constructivist epistemology. Ironically, this new transformation was also significantly influenced by science education. Through my life experiences, I realised how academic constructivism can help me to gain better understanding of the concepts. Academic constructivism gave me a better and novel way for me to learn. Epistemological constructivism endowed me with a superior and fresher way to think. I believe that a combination of these views can empower teachers and learners to be better teachers and better learners. In Chapter 4, I will explore various forms of constructivist views. Understanding of various forms of constructivist views can help us to better deal with the issues associated with learning and teaching.

## Chapter 4

### Constructivist Views, Many Facets of a Single Face or a Single Face Made of Many Facets



#### An Overview of Constructivism

In the previous three chapters, I have considered academic and epistemological aspects of my life voyage. In this chapter, I intend to discuss some of the main issues and contentions regarding constructivism that have been presented in the literature. By doing so, my intention is to link my experiences and reflections with the literature on the subject. My reflections and reflections over my previous reflections presented in this chapter have implications for the practice and foundational philosophy of my classroom teaching.

First we look at the three major camps of epistemology. To differentiate the positions of solipsism, objectivism and constructivism we can use their claims regarding [ontological reality](#) and [experiential reality](#):

- Solipsists: The world and the knowledge regarding it are both my creations and lie within myself.
- Objectivists: We have nothing to do with the creation of the world or the knowledge of it. Both entities exist outside our minds.

- Constructivists: We do not construct our world but we construct our knowledge of it.

The foremost implication arising from these epistemological paradigms is that, to a large extent, they determine and define how a teacher practices his profession. For instance, the constructivist epistemology invites students to participate in learning and teaching activities in a cognitively more active and creative manner.

Human beings are multidimensional beings. Therefore, it is inevitable that any theory dealing with human processes needs to be multidimensional. For this reason, it is no accident that constructivism happens to be of multiple facets. My purpose in this chapter is to discuss and understand influences of various flavours of constructivism on my practice as a learner-educator, in particular on learning and teaching in general. Specifically, I inquire:

- What are the different forms of epistemological constructivism?
- How are these different forms different?
- How are these different forms related?
- What do these different forms mean for learning and teaching processes?
- How can we use the awareness of different forms of constructivist epistemology to enhance teaching and learning practices?

The main theme of constructivism is that knowledge is basically cognition based and for this reason knowledge is a product of both external reality and our cognition. It can be noted that constructivism deals with:

- the nature of reality,
- the nature of knowledge,
- the nature of learning and teaching, and
- the methodology of learning and teaching.

Constructivism will be impotent in dealing with the latter two aspects if it cannot establish its virtue in dealing with the former two aspects. For this reason, it is not credible to put aside the former two aspects and then just claim constructivism is

'post-epistemological'. This matter will be discussed in detail in a later section of this work.

Noddings (as cited in Noddings, 1990) characterised constructivism as both a cognitive position and a methodological perspective. Constructivism can be regarded as a cognitive position because it theorises the nature of our cognition. Then it becomes a methodological perspective since this cognitive position suggests approaches to our dealing with reality and to improving our teaching and learning methods. Magoon (as cited in Noddings, 1990) also added that as a methodological perspective, constructivism assumes that human beings are knowing subjects and they mainly act or behave purposefully. These assumptions have implication for methods for studying social and other phenomena.

### **Is Knowledge Non-Transferable?**

The central theme of constructivism, that "all knowledge is constructed", has impelled some researchers to examine the nature of the claim itself. Is this claim of psychological or epistemological nature? Piaget and his followers argued that epistemology and psychology are tightly involved in each other's domains, and therefore it is not possible to separate and give them distinctive identities. According to this point of view, the genesis of this question is a philosophical error.

I may tend to agree with the followers of Piaget but the interwovenness of psychology and epistemology can be diluted to some extent by clearly defining the terms epistemology and psychology differently to the manner that the terms have been defined now. Psychology is widely defined as the scientific study of the human mind and its functions while epistemology is considered as theory of knowledge. One of the functions of the human mind is knowing. This is the basis of this entanglement of the two. Regardless of this strong basis of entanglement, one can argue that psychology and epistemology are not synonyms for or of each other, even in a remote sense since, the study of the nature of the mental functions of knowing (psychology) and the nature of the knowledge that a mind gains (epistemology) can be regarded as two distinct entities. For convenience we grant separate kingdoms to the two as we separate the study of 'Nature' into many domains, such as Physics, Chemistry and Physical Chemistry. The device of convenience can become a tool of

connivance if we forget to remember what it is and why the separation was constructed.

Is non-transferability of knowledge an underlying implicit assumption in constructivism? Does the notion of non-transferability follow from the assertion that “all knowledge is constructed”?

Suppose that I tell you that the book in my hand contains two hundred pages. If someone asks you how many pages are there in this particular book you will be able to answer that question even if you have never laid a hand on the book. Now isn't it valid for you to say that this piece of information is your knowledge? Then we may ask which aspect of this knowledge is constructed? The conceptual aspects of counting you have had constructed a long time ago. Therefore if we are to take that even this particular piece of knowledge is being constructed we face a difficulty. It is still possible to argue that just knowing the number of pages in the book (which you yourself did not count) constitutes a construction. To advance this opinion, one may argue that you need to associate the number two hundred and the number of pages in the book. Is it worthwhile to argue on such trivial matters or shall we just acknowledge that transfer of knowledge is possible but they are not always equivalent or similar? Even a trivial form of knowledge transfer is not equivalent to transferring a book from one shelf to another shelf.

If we view this question from the point of view of generations, knowledge is certainly transferrable from one generation to the other. This transfer happens at a collective level only because the transfer happens on an individual basis. If knowledge is essentially non-transferable then reading, writing, listening or speaking does not have much intellectual value. This means that the only way to construct knowledge is to engage in physical activities and thinking alone without making any connection to any of the already established knowledge. Moreover when the knowledge which is being 'transferred' is not trivial then the component of construction in this 'transfer' takes more prominence and significance.

If knowledge is not transferrable at all then why do we read books, listen to others and write books? Then one might answer that reading books and listening to lectures

are tools of facilitation of the process of construction. Then another might respond with the claim that books and lectures being tools of facilitation of the process of construction essentially amounts to some form of "transfer".

As I argued for transferability of knowledge, now I reflect on the opposite assertion of non-transferability of knowledge.

### **Is Knowledge Transferrable?**

In the previous paragraphs, it has been demonstrated that the notion of non-transferability of knowledge creates difficulties, but does this imply knowledge is merely transferrable? Borrowing a mathematics book from a library on a new mathematics field which is hitherto unknown to me certainly does not amount to transfer of knowledge even though the book has been physically transferred from a library to my room. To claim any ownership of the knowledge in the book, is it sufficient for me to read the book and merely remember some results or even the whole book? Suppose that I have a photographic memory. Now upon request I can produce any page or chapter of the book word for word. Does this amount to transfer of knowledge from the book to my mind? What has been transferred? Is it the knowledge or just the content of the book? This photographic memory is equivalent to the act of scanning the book electronically and storing an electronic copy in a computer. If I say my photographic memory is knowledge then it is quite valid to claim that the computer also has knowledge. This does not seem right.

A photographic memory of reading the book cannot be taken to be a transfer of knowledge. Then what is transfer of knowledge? Is it reflecting on the concepts and new results in the book simultaneously linking those ideas within the book itself and with external (external to the book) mathematical knowledge that I possess? What if we assume that, without having the gift of photographic memory, I study the content in the book to the extent that I can navigate from one idea to another idea as easily as following a map? Then can I claim ownership of the knowledge presented in the book? This understanding could be just momentary or at the most could be short lived. One week later, I might feel that I have never understood the concepts. Striving and paying attention to what I am thinking of I again study the book. This time I will be able to understand the book in a shorter time. Eventually when I just

open the pages of the book I will be almost immediately able to feel familiarity and friendship with the ideas. Can I claim this to be learning? Certainly this is a much better learning than mere memorisation and even perfect photographic memory.

For learning to be proper and much useful, I need to be able to apply new ideas in novel situations. Then I can confidently claim that I have ownership of the knowledge but over time I might repeatedly feel that previous understanding of the book has been inadequate and partial. (For instance, I did not understand the concept of number one properly until I started to learn Graduate School Algebra.) If we were to say that this is transfer of knowledge it does not signify the internal processes that one has to endure and entertain. For this reason, we cannot say this is just transfer of knowledge. Then again I cannot claim that I have constructed this knowledge. If I say so then I will be accused of plagiarism. So in a way the knowledge I gained resulted from simultaneously holding both characteristics of transfer and construction. Further down the track, for this field I might be able to add some new knowledge.

To summarise, the knowledge has been transferred from the mathematician to me via my interactions with the book. These interactions resulted in forming some new constructs in my mind. The process of making such constructs in my mind was motivated, guided and organised by the knowledge in the book. In the next few paragraphs I will further illustrate my ideas on this issue.

### **Transfer of Knowledge**

We may give identical copies of a Rubik cube to a group of monkeys, children and a mathematician and let them play with it for many hours. The outcomes will be different. The monkeys would not know what to do with it and it is possible that some monkeys will pull out the parts of the cube. Some may figure out that they can twist the cube. Those monkeys may start to enjoy just twisting. The child, if she has been given instructions on how to play with it, may come up with a few strategies to solve simple situations. A mathematician may develop a whole new theory.

Even though all beings are playing with identical copies of the same Rubik, the act of playing happens in different dimensions due to the different levels of cognitive

involvement. What is significant here? Is it turning the Rubik in various directions or is it the cognition involved with each turn? In the mathematician's case and to some extent in the child's case, each subsequent turn could be a result of reflections on the previous turns. Moreover, in the mathematician's case she will bring her vast knowledge of mathematics into twisting of the Rubik. She will strive to connect and link mathematical concepts she has understood and learned before. This could give rise to new concepts of mathematics or new application of existing concepts.

Now assume that I have understood and learned the ideas of the mathematician's book on the Rubik puzzle to the level that I can give a lecture based on its theoretical work. Is this construction a transfer of knowledge? If I call it a "construction" then I am committing plagiarism. If I call it "transfer" then I am denying my role in understanding and learning the concepts. It suffices to say that just having the book in my hand or a cursory reading of it will not enable me to give lectures on the subject matter of this book.

To make peace with this difficulty, rather than using a single flavour descriptor I am much more comfortable in thinking that some knowledge can be constructively transferrable or transferrably constructible in most situations. This qualification "in most situations" is required so that simply transferrable knowledge - bus time tables and number of pages of a book and the capital city of a country- can be considered as exceptions. This way of thinking about knowledge has implications for classroom practice as it signifies the roles of the teacher and the student both in learning and teaching.

Note that even though I can claim ownership of the knowledge I constructed in my mind I do not own the knowledge in the book and I do not deserve any credit for the knowledge in the book. Ownership of the transfer component of the knowledge belongs implicitly to the author of the book. The paradigm that knowledge is not transferrable is neither completely false nor completely true. Also this non-transferability of knowledge is not a modern notion. For instance, I have heard many times our teachers saying "I cannot pour knowledge into your brains". Paradoxically they made such statements while tirelessly trying to transfer knowledge to their students.

A teacher told me that he had trained a parrot to say the 'two times' table. Did the parrot acquire any knowledge? Was it constructed? It is certain that the bird engaged in activity but does this activity connect with any meaningful cognition? Can this be regarded as learning? It is clear that the parrot constructed memory of 'two times' table in its brain. Even though the cognitive involvement in the bird's achievement is minimal still it certainly resembles learning. Could this bird's knowledge become useful to cognising humans? Imagine that there is a compact disk containing the 'two times' table. The compact disk does not know the 'two times' table even though it has the memory of the 'two times' table. The compact disk player can keep playing the 'two times' table but it cannot answer the question "What is two times five?" whereas a well-trained parrot can answer "What two times five or six?". Here we may say that the bird constructed the memory of the two times table but not the concept of it.

Now we may get a CD player to keep playing the 'two times' table in front of an adult who knows only up to two times five. This adult is selling apples at \$2 each. At the precise moment he is about to sell eleven apples and that he wants to know what two times eleven is, the CD player sings that two times eleven is twenty-two. Hearing this adult is able to complete the transaction. Does the adult gain any knowledge? Has the CD player passed any knowledge to the adult? Many, including I, would think that the CD has passed a piece of knowledge to the adult even though the CD itself does not hold this particular knowledge.

How does the CD player's singing transfer meaningful knowledge to the adult? Is that because the adult was merely engaged in some activity? If the answer for this question is affirmative then to teach times tables what is required is to make students engage in some sort of activity like "selling apples". With this, pedagogical awareness can be ruled out as unnecessary. Did this piece of information become useful knowledge to the adult because the adult was reflectively interacting with the piece of information given to him by the non-cognising CD player? If this is the case we need to create activities which are pedagogically and cognitively streamlined to the concepts being taught and learned. And these activities may or may not contain physical activities. Over emphasis on physical activities or complete negligence of the role of physical activities in concept development or deterministic claims such as

knowledge is non-transferable can be thought of as slogans of one flavour. To conclude the ideas above we may opine that it could be harmful to bear a paradigm of a single flavour even though it may essentially possess some validity since the emphasis on a single flavour can produce inedible dishes.

### **Cognising as Activity**

Cognition is also an activity.

Let us consider again the example of the mathematician with the Rubik cube. Is the mathematician involved in passive learning when she does not hold the Rubik in her hand but she keeps merely playing with the mental image? Is the mathematician engaged in even more passive research when she just plays with the abstract mathematical image of the Rubik? What if she is just juggling the abstract concepts in her mind without any reference to the Rubik, its mental image or mathematical image?

Are the only meaningful learning activities 'hands on' activities? Conversely is cognition detached from the physical world and hands on activities? The famous mathematician and physicist Roger Penrose recognised Euclidian Geometry as a physical theory of space. The concepts such as points and lines in Euclidian Geometry cannot be produced in any physical way. Nevertheless these concepts are motivated by the nature of our experiential physical space. As points do not have any size and straight lines do not have any thickness, it is impossible to draw a point or a line on any paper as exactly as they are cognised in Euclidian Geometry. Nevertheless, town planning people or architects will vow to the success of Euclidian Geometry. The outcome of the monkeys' mere hand on activities with the Rubik cube warns us about improper use of hands on activities just for the sake of using them. When hands-on activities are used merely as a manipulative tool to make a restless class quiet, true learning runs away from classrooms. As a result, many meaningful hands on activities can turn out to be just time passing fun activities that waste class time and destroy the learning teaching environment.

I have no doubt that many educators take the provision of ample opportunities for students to become more fluent abstract thinkers as a primary aim of education. One

of the content areas that provide extremely rich hands on activities and cognitive challenges simultaneously is Euclidian Geometry. Given the universally accepted importance of transforming concrete thinkers to abstract thinkers, I am compelled to ask why Euclidian Geometry has been taken out of the syllabus. Modern school geometry presents Euclidian Geometry as a list of results that are true for some mysterious reasons. I remember when my mathematics teacher in Year 6 chalked and talked the concept of a point and a straight line how fascinated I became. I cannot remember how much I understood at that time about a point and a straight line but the fascination that was created still lives. That was an understanding and learning itself. The major reason of taking Euclidian Geometry from the mathematics syllabus is its perceived difficulties. However, the task of the educators is not to prune the syllabus but to construct meaningful and rich activities to make the content more accessible.

### **Constructivism as Post-Epistemological Theory**

According to Noddings (1990), constructivism is post-epistemological and the constructivist assumption should be followed by a break with epistemology. In some sense this may be an accurate account of epistemology and constructivism. In Chapter 3 I remarked that the need for constructivism arises from the precarious nature of our [experiential reality](#). One of these difficulties is that we never have direct access to ontological reality. Therefore acceptance of the existence of ontological reality amounts to a belief or a foundational axiom.

In a later section I will argue that taking constructivism as a post-epistemological theory actually weakens the foundation of constructivism. In fact, in Chapter 3 I strived to establish that constructivism is a basic epistemological paradigm. This is because constructivists (not including the pedagogical-methodological constructivists who adapt logical positivism as their epistemological worldview but use pedagogical constructivism or its variants in their instructional approach) do not believe in the existence of an ‘objective’ knowledge. This is an assumption of the nature of knowledge. Without this basic assumption there is no constructivist epistemology. Later it will be argued that this basic assumption itself essentially demands some modification in the position on the existence of objective reality. Hopefully this will

explain why constructivism needs to deal with all four aspects regarding the nature of reality and knowledge mentioned at the start of this chapter.

It is my opinion that the existence of ontological reality should be postulated as a prelude to the constructivist paradigm. I readily acknowledge that the existence of ontological reality could be questioned by simply pointing out that the existence of ontological reality has been cognised by a mind. Therefore one may argue that the postulate does not correspond to an ontological reality. As discussed earlier, there is no escape from this impasse<sup>17</sup>. I acknowledge this impasse but I choose not to surrender to this impasse. The task is to construct a belief system that does not contradict our practice and which promotes peace, harmony, serenity and prosperity. It should be noted that for the constructivist view that I hold, the existence of ontological reality does not imply a grand scheme at work. Otherwise, it can be argued that the belief of ontological reality could lead to the concept of grand scheme and then to an [absolute idealism](#). I note that all other belief systems also have their own impasses.

As the abandonment of ontological reality positions ourselves on [solipsism](#), the admission of the existence of ontological reality allows us to match the dictates of our paradigm with our practice. That is, we can comfortably speak about matters such as bus timetables (The concept of ‘timetable’ is a construct but its operations should be regarded as an ontological reality. For instance, we believe that if all regular conditions are met, the bus will arrive at a certain place around a certain time. This certain place or the certain time does not exist in our mind<sup>18</sup>), dinosaurs, the time before I was born, the time after my death<sup>19</sup>, the time before written history and the existence of the object moon before any cognitive beings walked on the Earth.

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<sup>17</sup> The difficulty discussed here may be the reason why some authors consider that constructivism is being post-epistemological. My argument here is that this is our epistemological view. Only after deep elaboration of epistemological difficulties, one can come to constructivism.

<sup>18</sup> There is a possibility that time and space can be mental illusions. Even then these mental constructs of time and space are reflections of some aspects of ontological reality and what we know about this ontological reality depends on the nature of the mind. Timetables are not about ontological time and space. They are about experiential time and space.

<sup>19</sup> Consider the document of last will or the life insurance policy.

What we speak about them may amount to mental constructs but even to start talking about them we need to accept the validity of such endeavours.

As there are many different versions of constructivism, Noddings (1990) provided a list of common characteristics of these different views. At the core of these generally agreed upon assumptions is the notion that all knowledge is constructed. This basic assumption about the nature of knowledge inevitably directs us to methodological constructivism. Consequently, in the field of education it leads to pedagogical constructivism. Now we are ready to discuss the implications of constructivism (and its different faces) to society, culture and education.

### **Radical Constructivism**

Radical Constructivism is founded on two main axioms (Von Glasersfeld 1989: 162):

- Knowledge is not passively received but actively constructed by the cognising subject;
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of [ontological reality](#).

For me the statement that knowledge is not passively received does not mean that knowledge is essentially non-transferrable and it simply means that if knowledge is transferrable then this transfer does not occur in a passive manner. When even a trivial piece of information, such as a name of a person or telephone number of a person, is given to a human being the reception of this information is not passive. When we write this piece of information on a sheet of paper, the paper receives the ink of the scribe passively. Even if the paper is interacting with the tip of the pen and the ink, it does not interact with the information in any sense. When someone gives me a simple piece of information by voice then my brain and mind interact with the sound. These connections made by the interaction of my mind with the sound are not a result of just passive action. Nevertheless, I grant that some students may be 'clever' enough to pass exams with mostly passive learning. For instance, I have encountered students who had just finished many exercises on graphing sine functions but who were unable to recognise the condition on  $a$  in the equation  $\sin x = a$ . Then I directed them to graph of the function,  $y = \sin x$ . They were able to do it quickly and

accurately, but still they were unable to see that the condition on  $a$  which was clearly depicted in the very graphs they sketched. Then I asked them, “Can you explain why you were unable to see that  $-1 \leq a \leq 1$  as you have clearly and accurately depicted this in your graphs?”

Let us look at the second statement Von Glasersfeld: The function of cognition is adaptive and serves the organisation of the experiential world. Certainly the cognition helps us to adaptively organise the experiential world. This brings the question: Is the only function of cognition to organise the physically experiential world? Does our cognition only deal with our experiential world?

Even though it is largely true that almost all of our concepts are adaptive constructs of what we can experience through our senses, the human mind has exceeded the luxurious limit of being able to cognise just the physically experiential world. Now we cognise about our mental constructs which we developed through our physical experiences. That is, we cognise without direct physically experiential sensory inputs. However there is no denying that it is possible to argue that ultimately this cognition about cognition is also tightly linked to experiential reality. These links may be short or long, curvy, cyclic or spiral.

For instance, Euclidian Geometry is an abstraction of the properties of our directly experiential physical space but not the properties of the physical space itself. If we want to check whether our experiential physical space follows Euclidian theorems or not then we should be able to draw straight lines, but this is impossible in the strictest sense since every pencil line consists of uncountably infinitely many straight lines. In Euclidian geometry there is a unique parallel line to a given line through a given point. In Riemannian geometry there are infinitely many parallel lines to a given line through a given point but we can only see the Euclidian line. We cannot experience any of the other infinitely many lines but the Euclidian parallel line. If we can directly experience Euclidian geometry through roads, building and drawing, at least to some good extent, Riemannian geometry can be only experienced through the mind.

Therefore the standpoint that cognition can deal only with the experiential world no longer seems to be absolutely correct as we can cognise about cognising and manipulate already existing concepts. If we still desire to hold onto the notion that cognition can deal only with the experiential world then we need to take our experiential world not to be limited to the world that we can immediately and directly experience with our senses. That is, we may extend our experiential world to include the world we experience through cognition. For instance, we can insist that Riemannian geometry was developed from more user friendly Euclidian geometry. However, I believe that it is much simpler to acknowledge that cognition has developed to such an extent that we are able and compelled to cognise over the things that we do not have direct experience with.

If we believe that the human mind can only deal with physical experiential reality then we will entangle ourselves in a conundrum when we try to explain Newton's thought experiment of perfectly smooth marble and the perfectly smooth floor. However, implicit in this very example is the fact that even the world we experience through mere cognition may also be ultimately tied to the world we physically experience through our senses. Even though it comes as a tautology it must be said that it is impossible to experience a pure view of our world. That is, the world we experience is necessarily an image made by our senses and our sixth sense organ, the eye of cognition.

Radical Constructivism also claims that any of our notions of reality are not discoveries. In Chapter 3, when I espoused that our knowledge is mental constructs I took the word construct to be indicative of both the cognitive (mental imaging) component and the ontological (objective) component. I believe that when we say that knowledge is not discovery it does not mean that it cannot have a 'discovery' component. To see this, let us consider the Euclidian system. The axioms, definitions and basic notions in the system are certainly much closer to creations which are ultimately motivated by the nature of physical space. After the setup of the set of notions, definitions and axioms, the theorems are hidden in the system to be discovered. These theorems are not pure creations in the strictest since the theorem has been determined by the established notions, definitions and axioms. However, we may add a new definition or a notion or even an axiom. This may result in a new

theorem. This new theorem may have both creation component and discovery component or it can be positioned in between discovery and creation.

To be specific, let me give this example. When all the necessary axioms, notions, definitions have been established then the result- that the angle sum of a triangle is a straight line angle- is already determined even if we are unaware of it. This follows from my belief that ontological reality exists whether or not I am aware of it. Then finding the angle sum result is certainly a discovery. Some of the research will fall into the category of constructive discoveries or discovery-constructs since during most research we need to construct new concepts to arrive at a new result. Some research may be pure constructs and some others may be pure discoveries and many in between. Here I need to emphasise that I have refused to entertain the question that if the result has not yet been derived how do I know that it exists? I also refuse to entertain the question how do I know whether the Moon existed before I was born? I can argue that (but I do not) even a photo of the Moon taken before I was born may not be permissible as evidence since I see the photo only now.

Scientific knowledge always has both components of discovery and creation to various degrees since, by nature, the task of scientific knowledge is to 'explain' or conceptually image our experiential reality.

### **Social Constructivism**

Humans are social beings. Therefore the knowledge that humanity has acquired must essentially be of social nature and is, at least to some extent, socially negotiated. For instance, a new revolutionary scientific theory will be negotiated through peer reviewed journals. And also, the previous planet Pluto was recently expelled from the club of planets by a popular vote of astronomers. In this example, the social nature of the knowledge is explicit. In many other examples it may be obscured due to many other parameters at work. Therefore it was inevitable that someone would have paid more attention to this social nature of knowledge. Lev Vygotsky, a cognitive psychologist by profession, shared much of the paradigm of Piaget's constructivism while highlighting the social context of knowledge, learning and teaching. This new paradigm enhances the role of teachers, elders and more experienced learners in the process of learning.

In social constructivism, mental constructs do not occur just in minds but they occur in minds which are sitting or floating in a culture. For instance, while we may recognise two sets of parallel lines sketched together as a picture of a staircase, some tribal people may see the same picture as just a set of lines. Thus cultural conditioning plays a vital role in our perceptions, hence in our knowledge systems. The core of academic social constructivism is nicely captured in the following quote as given in (Chen, n.d.):

A constructivist teacher creates a context for learning in which students can become engaged in interesting activities that encourages and facilitates learning. The teacher does not simply stand by and watch children explore and discover. Instead, the teacher may often guide students as they approach problems, may encourage them to work in groups to think about issues and questions, and support them with encouragement and advice as they tackle problems, adventures, and challenges that are rooted in real life situations that are both interesting to the students and satisfying in terms of the result of their work. Teachers thus facilitate cognitive growth and learning as do peers and other members of the child's community.

It appears that, according to the quotation above, academic social constructivism rejects the constructivist notion that teacher is merely a guide on the side. The teacher has a central role more or less equally shared with the students. Then again, I might add the teacher's involvement can be varied to suit many other parameters. Personally, I appreciate this vast range of the enhanced role for student teacher interaction offered by social constructivism. This aspect of social constructivism resonates well with my own practice of teaching.

The major assumptions of academic social constructivism are given as:

**Reality:** Social constructivists believe that reality is constructed through human activity. Members of a society together invent the properties of the world (Kukla, 2000). For the social constructivist, reality cannot be discovered: it does not exist prior to its social invention.

**Knowledge:** To social constructivists, knowledge is also a human product, and is socially and culturally constructed (Ernest, 1999; Gredler, 1997; Prat and Floden, 1994). Individuals create meaning through their interactions with each other and with the environment they live in.

**Learning:** Social constructivists view learning as a social process. It does not take place only within an individual, nor is it a passive development of behaviours that are shaped by external forces (McMahon, 1997). Meaningful learning occurs when individuals are engaged in social activities.(Kim, 2001)

As it has been claimed here, is reality constructed by human activity or is it that what we can say, perceive or image about reality is constructed by human activity? On the assumption that there is an [ontological](#) reality, what we can know and say is created by human activity. Even though the assertion that the knowledge is being socially and culturally constructed has validity and legitimacy to some extent, taking it too far can give rise to doctrines such as National Socialism and other extreme nationalistic movements. This can happen because ruthless power hungry politicians can argue that the countries should be governed according to the national realities and values in a void of wider human values. Then again, powerful countries can use these wider and more universal human values as a pretext for aggression towards countries that they want to intimidate, invade or dominate.

The notion that meaningful learning can occur *only* when individuals are engaged in social activity requires a new definition for social activity. Otherwise, the students will define the nature of the social activities that should occur in their classrooms, including team play of violent video games. Reading a book or thinking quietly in a closed room should also be defined as a social activity since many students use these modes to learn.

Social critique has been a minor but significant component in my teaching program. This is because, as social constructivists have recognised, education does not exist in a social vacuum. Even if we assume that the education content is to be somehow socially neutral the teachers who teach the content and the students who learn the content are social beings. They both are results of the mega social culture driven by big media and entertainment. However, the presence of social critique or the lack of

it can both indoctrinate the students. Hence, from the discussion above, it follows that it is necessary for the teachers to be cautious when they incorporate social constructivism and critique with academic constructivism. Even if the teacher's intentions are benevolent, reconstructions of these views in students' minds can be different in spirit to the original views expressed. It is also important that both genuine national and international perspectives are offered to the students for them to compare with the narrow-minded nationalistic and antinationalistic views<sup>20</sup>. Also, students need to be exposed to different teaching styles and instructional perspectives and learning and teaching strategies based on different epistemological paradigms to minimise possible indoctrination.

### **Implications of Social Constructivism for Education**

The main theme of social constructivism is that construction of knowledge occurs in social and cultural contexts. It is not easy to dispute this notion because, among other things, human beings are social beings. There are many notions, such as solipsism, which cannot be logically disputed. Hence, logical indisputability does not automatically qualify a notion as an appropriate view of the world. Therefore, to evaluate the claims of social constructivism, we need to ask: on what basis is the framework of principles of social constructivism compatible with our practices, in what ways can these notions be applied to improve our practices and how does it enlighten our understanding of learning and teaching process?

Firstly, since we are social beings, slighting the social and cultural nature of our knowing would amount to deluding ourselves. This delusion will lead to axiological and methodological errors in our dealing with reality. More than two decades ago, even though my teaching philosophy and practice was based on instructional constructivism, I had not conceptualised the social and cultural nature of the learning and teaching practices. However, in my subconscious, I had been influenced by the “Eastern Cultural Values” attributed to a teacher. In Eastern cultures, the teacher is almost a surrogate parent. These values have been always influential in defining my

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<sup>20</sup> It is possible that some forms of international perspectives can also be narrow-minded. My definition of genuineness of nationalist and internationalist perspectives is that they cannot be void of each other and one does not undermine the other but complement and supplement each other.

teaching practice. Therefore, regardless of the fact that I did not appreciate the social aspects in knowledge, my then teaching practice had an unconscious social component.

On the other hand, a culture is not a fossilised artefact. A culture is a breathing and perspiring organism. An individual's culture is also a living organism. My personal culture was certainly nurtured in multi-flavoured culture of Sri Lanka. Sri Lankan culture has been always shaped by the influence of the Indian culture. Buddhism is one of the most significant aspects of Sri Lankan culture. In some positive ways and negative ways Sri Lankan culture has been shaped by three colonial powers. One way or the other, my personal culture is shaped by all of these factors. Then reading of foreign literature is also a major factor in the formation of my individual culture. Moreover, I have lived in the USA, the Kingdom of Saudi Arabia and Australia. So my culture is multifaceted and international. My culture consists of Buddha, Buddhist Literature, Christ, Prophet Mohammed, Einstein, Newton, Socrates, Euclid, Dostoevsky, Beethoven, Mozart, Sri Lankan Artists Amaradeva, Mahgamasekara, Sarathchandra, Martin Wickramasinghe, and many others. Therefore it will be wrong to view my ideas as ideas of just a Sri Lankan. It will also be inaccurate to claim my culture has no Sri Lankan component. Evidently, it is doubtful that any culture is isolated in this global village. My appeal to the audience is not to view my ideas through isolationistic glasses.

Certainly my teaching has been modelled according to Socrates' view of learning and teaching. My ideas need to be evaluated on the relevance of my ideas to human learning and teaching. Even if these ideas are partly motivated by my Sri Lankan heritage that does not need to be a factor in evaluating the relevance or irrelevance. Here I am not denying my Sri Lankan heritage. Even though I do not deliberately strive to retain or shed my Sri Lankanness, I have deep emotional attachment to Sri Lanka and its culture. I am proud about this bond only because it signifies my human nature. I reiterate, while I am not ashamed or proud of my Sri Lankan heritage, I am proud of my attachment to it. Nevertheless, my basic philosophical tenet is that I am human first and then Sri Lankan second. For this reason, I believe that my ideas are human ideas first and only then they can be considered to be Sri Lankan.

As it is not prudent or productive to divorce social and cultural context from the learning process it will also be idiotic and harmful to put too much emphasis on them. Consider the notion that all learning happens through social interactions. Now, imagine a learning process in which only group learning activities (including whole-class discussions, debates and students helping one another) are used. Now imagine a class in which only individual learning activities are taking place. As we look at these two extremes, it is easier to see that such extremes do not augur well for the learning and teaching. Moreover, ultimately all learning happens in one's mind. Mind is also definitely influenced by the social aura around.

Certainly, cultural, political and social factors influence or control the content of learning. To see this, we only have to look at the learning content in Nazi Germany, Communist Russia, or any other country for that matter. To a larger or a meagre extent, even in democratic countries, teaching is social and political. In democratic countries, this control and influence may be more gentle and subtle. For example, a survey reported in Solomon (1992) has revealed that some students think scientists do experiments “to make discoveries”. This is an example of subtle thought control by democratic means. While some of these subtle thought controls may be unintentional, some others may be meticulously designed. Trivia questions such as “Who discovered this?” and “When was this discovered?” lead many people to believe science to be a field of discoveries. Though it could be unintentional, it whispers to the audience about the absoluteness of knowledge. The emotionally charged atmosphere in some TV trivia shows could have been meticulously designed for the optimum level of thought control. Instead of giving proper explanations and or inquiry activities, textbooks ask students to use calculators to convince themselves of the fact  $5 - 3 = 8$ . This approach indoctrinates students about the superiority of technology over the human mind and conceptualisation. In a way those authors have already been indoctrinated about the superiority of technology by the technology businesses. Political and social forces play a significant role in determining our education policies and even the classroom teaching practices. This is to be expected. However, problems arise when education reform is motivated by politics and finances rather than true education aspirations.

It is inevitable that the content of learning is controlled by the needs of businesses. To some extent, we need to accept and welcome this inevitability since social existence and economic prosperity are inseparable. If these business people act on the best interests of the society then we can grant the total control of learning to business organisations. Yet the businesses' first priority is their profit. All of their actions and decisions are mostly geared to profit making. Due to this profit motive, business people can be short sighted. There are many stakeholders in the process of the social endeavour of education: the government, the business, higher education authorities, the students, the educators and the parents. So ideally we need all of these stakeholders to have control of the content and the policies of Education. This means that content and policies of learning are socially negotiated.

Who controls the methodology of learning and teaching? To look at this question I first ask you to consider the question: when a patient goes to a doctor can we be certain that the sole reason for a particular treatment is benefit to the patient? I have a firsthand account that an expensive treatment was prescribed on the assumption that the treatment was covered by the patient's private health insurance. When the doctor became aware that the patient was paying from his own pocket, the treatment was cancelled (to free the patient from the financial burden of paying for the treatment). Necessity of a treatment cannot depend on who pays for it. Consider also: what brand name drug is most likely to have been prescribed by a doctor, a drug produced by a company which markets its products more aggressively or a company which markets its products less aggressively?

For the sake of the argument, let us agree not to argue about advantages and disadvantages of using calculators in our classrooms. Let us ask who determines whether to use or not use calculators in our classrooms? To answer this question, we need only to ask the question, "Who makes calculators?" The answer to the second question implies the answer to the first.

It is imperative that society give a hard look at the following question. Why do we need schools? Is that because, if there are no schools, parents cannot go to work and therefore the economy will suffer? In that case, let us build more effective child-minding centres; or do we require schools because we need our future generation to

be competent and confident thinkers and contributory participators in democracy and the economy? In that case we need better schools. If we need childcare centres let us build excellent child care centres. If we need better schools let us build effective schools. Let us not build schools to replace childcare centres and childcare centres to replace schools. By all means let us not confuse which is which. If we confuse ourselves we will confuse our future generations.

One Prime Minister wanted to extend school hours so that parents can contribute to the economy by working longer hours. This sends a subliminal message that schools are not learning and teaching places but childcare centres. Also, what was highlighted in TV news about an industrial action by teachers was that the economy suffered a loss of several millions of dollars because the parents were forced to stay at home. Again this transmits a subliminal message that schools are child care centres. Whether this was the intended message or not is a different matter. If something is directly said it can be directly opposed. Most of us could only succumb to designed or accidental subliminal messages.

People might say, on TV news, that there is no time for a clear analysis of issues. That is my point too. When you can speak only for a second you would say the most important thing, wouldn't you? Then that is what happens in TV presentations. The sound and vision bites coming from TV are just epitomes of social conscience, which again cyclically reinforces them.

### **Society and Education**

When these issues are discussed by prominent personalities and powerful media without any forethought in a casual and sensationalised manner, the damage that occurs may be far reaching and long lasting. Cumulative effect of such damage over time is that society can form a wrong operational basis. What is sadder is that, a few years later, the society will be depending on these new cultural values as national values. For example, helping parents to stay at work for longer hours by converting schools into child minding centres can become a significant national value that no one dares to question a few years later. The notions established in students' minds by social discourses may be difficult to be removed even with excellent instruction. This sentiment is captured in the following quote from Solomon (1992, p. 153).

Notions in the public domain which are reinforced through everyday discourse are not to be obliterated however good the instruction.

The term instruction can be taken to mean whatever the efforts made to undo these socially reinforced distorted notions. The social discourse sets the norms, tones and forms of values of society. These will directly influence how society looks upon the process of education. The reader will see that the quote above goes further than this. In fact, according to Solomon (1992, p. 153) everyday discourse is needed for the very understanding of speech and meaning. Also, it adds another dimension to knowledge and, perhaps, even to the student's way of thinking.

Let us look at an example of this dictation of social discourse. Suppose that a teacher happens to live and work in a gun and bullet culture (Gun and bullet cultures have a strong hold in some parts of our world.). Will it be acceptable to use gun and bullet terminology to promote concepts of addition and multiplication<sup>21</sup>. This example establishes that the strategy of taking socially prevailing speech and meaning to promote learning of new concepts is not by itself always a positive approach.

Taylor (1996) explored and reviewed other forms of social myths which are brought into classrooms, not only by students but also by teachers and other educators. These 'myths' could be based on educators' beliefs in nature of the subjects and the purpose of education. These beliefs form a dominating culture in the field of education. Building on this premise, in the article cited above, Taylor equates epistemological reform of the traditional mathematics classrooms with cultural reconstruction. This epistemological or cultural reform cannot occur if the teacher is not endowed with a remarkable degree of freedom. Because of the dominating culture this freedom does not exist in academic institutions. Therefore it is not prudent for a teacher to act as a heroic individual who rides against the tide. It needs to be a collaborative effort.

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<sup>21</sup> There was a news report that an American primary teacher used guns and bullet culture terminology to promote student engagement in numeracy. This teacher was later sacked. My sympathy lay with this teacher even though I did not and do not agree with his approach. I simply believed he misunderstood or was misled by "social constructivism".

As cited in Cobb, Wood, Yackel and McNeal (1992, p. 599), Solomon and Walkerdine premised that learning mathematics is a social and discursive practice rather than a cognitive process. As documented in Cobb et al. (1992), both Solomon and Walkerdine believe that understanding is essentially social. In the same article, they state their disagreement with Solomon's idea that students just learn how to follow already existing mathematical rules. In my long experience as a teacher I have been observing that following rules without conceptual understanding is the norm rather than the exception. Even though at times I tolerate and even promote "just learning rules", I believe that my task is to help students to construct the relevant concepts. For instance, the concept of Number 1 was constructed by our ancestors, long time ago. When children learn how to count they also construct the concept Number 1 in their minds individually. This is both a social and cognitive process. At the beginning, the concept may not be vivid for them and the skill of counting seems to be just procedural. Then years later some of these children may realise that Number 1 as a concept and it can exist only in minds. That is, one pencil or one book is not the number 1. Some other children may also develop this understanding if we ask them to explain what Number 1 is.

I can think of some arguments to support any of these competing views of the social and cognitive natures of learning as I can do the same to support [solipsism](#) and [absolute idealism](#). But, it suffices to say that even when I take learning as primarily to be a cognitive process I cannot slight the social nature of learning, and vice versa. The claim that understanding is essentially a social matter cannot be regarded as purely wrong since most of the time our understanding needs to be regulated and assessed according to set norms. Certificates test scores and awards are just manifestations of this social nature. I believe that we cannot promote, on a permanent basis, one face of learning and teaching at the expense of another.

However, there may be a need for the teachers to temporarily promote or ignore one face on the basis of pedagogical need. To instantiate this claim, please consider the surd  $\sqrt{2}$ . It is true that construction of the number  $\sqrt{2}$  was partly a social process. It is ideal if we can bring this social nature into our class discussions of the number but slighting the conceptual nature of this construction is a grave error. I believe that the

construction of  $\sqrt{2}$  is predominantly a cognitive process which is immersed in a social process. When the social nature of the construction of the concept  $\sqrt{2}$  is highlighted, the students might not feel the number as a magical creation. Also, if highlighting is done carefully and properly, then it might itself add to the cognitive nature of the endeavour.

In consideration of all these issues, my teaching consists of a component of discursive nature since I incorporate the history of the subject and may survey a topic and its many related areas. However, a well-structured component with the aim to develop conceptual and procedural skills is also a significant and major component of my teaching. What happens if all the time mathematics is taught as a survey of the field without structurally developing the concepts and skills necessary?

### ***Procedural Skills or Conceptual Mastery***

Methods of teaching of place value to very young children are focussed on procedural aspects and skill development. I have thought of a constructivist way of teaching the place value system in a much deeper conceptual way but it requires much more mature thinking skills. Just because of this restriction we cannot wait to teach the place value system until children become more mature. Therefore one alternative is to teach the place value as a set of rules until they are ready to think of them at a higher conceptual level. I also believe that learning procedural aspects of the place value system properly is by itself a cognitive process and it could lead to a deeper conceptual understanding of the place value system. To reinforce this, I look at the following question. Should we discourage toddlers from walking until they understand scientific thinking about bipedal walking?

Nevertheless, teaching mathematics as just a set of rules, not as a way of thinking, must be restricted to a bare minimum as much as possible. Even in this era of constructivism, this practice still continues unabated. For instance, rather than challenging the students to visualise the graph of  $y(x) = x^2$  from its equation itself in a conceptual manner, some teachers may just present a graphics calculator output. Then this will determine how students think about the graph of  $y(x) = x^2$  as demonstrated in the following observation.

A VCE Mathematical Method student, who was performing well in school, just two months prior to the VCE Examinations, was asked what he knew about  $x^2$ . Since the student was unable to answer I asked him to sketch the graph of  $y(x) = x^2$ . He was able to produce the graph instantaneously. Then I repeated the question. It took a few minutes and a few prompts to get him to see that  $x^2 \geq 0$  even though he has used this fact to sketch his graph. Then I asked what he meant by the statement that the curve he produced was the graph of  $y(x) = x^2$ . As expected (sadly though) he could not explain why his sketch can be regarded as the graph of  $y(x) = x^2$ . This is just one example among many. The question "what do you mean by" surprises many students and this is clearly unacceptable in this era of academic constructivism. The most striking example of converting mathematics into authoritarian sets of rules is the conversion of Euclidian Geometry into the modern textbook version of geometry. I wonder, did this happen in the name of constructivism?

Cobb et al. (1999), discuss in detail one teacher's work on the place value system with her students. Earlier I claimed that teaching of the place value system is largely centred on teaching of the procedural skills required. There is nothing wrong with that when considering the tender age at which the concept is first introduced to the students. If we want our students to investigate the place value system at a much deeper conceptual level we need to pose the following questions when they are at an appropriate cognitive skill level.

- What could have motivated the ancient people to construct the place value system?

[To answer this question, they need to think what would have been the consequences of not having the place value system. They need to make the following observations, with or without the guidance of the teacher.]

- ❖ To talk about 5000 we need 5000 thousand different symbols. The number of different symbols required is limitless.
- ❖ The symbol of a number will not give any clue to the size of the number unless we have memorised many of them.

- ❖ There will be no algorithms of addition, subtraction, multiplication and division. For instance, to add 2457 and 789, we have to advance one by one beginning from the symbol that stands for 2457 until we advance 789 places.

[The questions above address the issue of the more cognitive nature rather than the social nature.]

- Where did the magic number “10” come from?

[Consideration of this question facilitates the social nature of the construction of the concept of place value system.]

Remember that the questions above address some deeper issues of a cognitive nature associated with the place value system. Some other issues of cognitive nature related to the development of procedural skills are addressed by the work done by many teachers. This is also important.

### **Independence of Mathematics Traditions of Teachers and Students**

If school mathematics traditions exist independently of students and teachers then I cannot understand why different teachers experience different traditions with the same classes or why the same teachers experience different traditions (learning environments) with different classes? Above all, why different learning environments exist at all? Then again, can I teach  $x^2 = 4 \Rightarrow x = 2$ ? (My supervising teacher and supervising lecturer harshly criticised me for asking my students not to ignore the solution  $x = -2$  unless they state the reason for ignoring the negative solution.) This happened while I was doing my second round of teaching practice for my diploma in Australia.

I do not think that the teacher or the students have any say in the solutions of  $x^2 = 4$ . There is an 'absolutely objective' knowledge about this (ever since the concept of negative numbers has been constructed). Even though this absolutely objective knowledge exists, the teaching was subjective as my two senior supervisors contradicted my teaching. The question here is which tradition I should follow; the tradition of this objective knowledge or the subjective practices of these educators. I chose the former and risked my employability to a large extent.

A principal advised a teacher about his methods.

In this particular region, there were many building projects. Therefore a student could always find a job. Thus they were not very well motivated to think deeply. This requires an adjustment of the content and the methodology

Again tradition exists outside the teacher and the subject both; it depends on students. Is this an example of locally based curriculum?

The above is an example of the social influence on the learning content and the methodology. As a teacher, I want my students to be able to easily find jobs. One of the primary goals of education is to prepare students for their future livelihoods but it is not the sole purpose. When I teach mathematics, physics, science or any other subject (even though I talk about it explicitly only occasionally), I teach about life and society. And learning is learning to think. This is even more important on the basis of the nature of reality. If reality is a black and white issue then every one of us may not need to become thinkers since someone can just tell us what is white and what is not.

So the teaching traditions of mathematics or science are governed by the historical development of the subject, students, teachers, education administrators and education experts and businesses. For instance, consider the mathematical and physics notations which are governed by historical development. Another example of this historical aspect is manifested in the notions of conventional electric current direction and real current direction in Physics. A teacher of the subject, for pedagogical reasons, may ask students to follow different notations or guidelines that may differ from the regular notations and guidelines. A point is case that the regular notation  $\sin \theta$  has been replaced by  $\sin(\theta)$  by the Victorian Curriculum and Assessment Authority for reasons of clarity since it is difficult for students to maintain the proper distance between  $\sin$  and  $\theta$  when they write. Then many textbooks also have adopted this new notation.

In spite of mathematical sloppiness, many textbooks regard negative sign and minus sign to be the same. Consider the following example: Recall that  $a^2 = a \times a$  for any

number  $a$ . Then since  $^{-}2$  is a number,  $^{-}2^2$  should be equal to  $^{-}2 \times ^{-}2 = 4$ . Moreover, in the strictest sense of the meaning there is no number that can be written as  $^{-}2$ . This is because minus is a binary operator which means that minus can only be used with two numbers and negative is a unitary operator (here it means that it can be used only on a single number). However, many teachers will mark  $^{-}2^2 = 4$  as wrong. Even though I reason with my students that  $^{-}(2^2) = ^{-}4$  and  $^{-}2^2 = 4$ , when I mark papers I ignore what mathematics dictates to me and grudgingly comply with the dictates of modern textbooks. This discussion above again points out that dependence, independence and interdependence of many factors simultaneously contribute to the practice of teaching. Slogans of one flavour distort what exists.

Another example is Inverse Trigonometric functions. The trigonometric functions with their respective restricted domains that give rise to inverse trigonometric functions are written with upper case letter at the beginning with the same name. This upper case letter indicates the difference of the domains. Recently I have seen some books in which both functions  $\sin \theta$  with  $-\infty < \theta < \infty$  and  $\text{Sin } \theta$  with  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  are denoted by  $\sin \theta$ . Even if mathematicians have adopted this new notation I prefer the old notation since it takes the difference between two sin functions into consideration.

In one school, the Head of the Mathematics Department participated in a professional development program. Due to the influence of this program, the Department Head advised us not to use the vinculum in fractions (For instance the number  $\frac{3}{4}$  must be written as  $3 \div 4$  since research has shown that students did not understand the meaning of the vinculum.) I believe that educators' task is not to prune concepts; rather their task is to make more challenging concepts to be more accessible. The number  $\frac{3}{4}$  is a number in its own right as well as being the result of the operation 3 divided by 4 or the product of 3 and  $\frac{1}{4}$ . Replacing  $\frac{3}{4}$  with  $3 \div 4$  is similar to replacing 7 with  $4+3$  or  $9 - 2$ . I think this practice of pruning what students dislike is a culmination of a mistaken version of a student centred approach. This may be the exact thing that happened to Euclidian Geometry.

### **Parochialism and Internationalism**

I claim that all culture is international to some extent. The vast majority of us, regardless of where we live, share almost the same awareness and similar thought patterns. There are differences among us that manifest in our rituals but what lies underneath our differences is common humanity. Through my reflections in this chapter I had become aware that I possess a core of Sri-Lankan Nationalistic thinking even though I have been calling myself an Internationalist since my late teen years. Then again I am convinced that my bond to my birth country is just humanistic in nature rather than exclusively being of Sri Lankan nature. I am no longer scared to acknowledge my Nationalistic thinking since I know that all culture by itself is international. Sri Lankan culture has been nurtured by all other cultures and Sri Lanka has contributed to all the other cultures throughout history. For instance, it is said that Sri Lanka had the first known hospital.

In the social sphere, both parochialism and Internationalism could nurture prejudices and could even provoke violence. It does not have to be that way but if parochialism and Internationalism are not balanced by and with each other it could lead to such drastic situations.

### **Commonality between Various Forms of Constructivism**

What binds the various forms of constructivism, according to Spivey (as cited in Ernest, 1995), is the metaphor of construction. That is, all forms of constructivism are bound by a common thread. This common thread is that humans cannot experience the world as it is but only through constructs facilitated by our senses and cognition. Here Ernest explains:

This is about the building up of structures from pre-existing pieces, possibly specially shaped for this task. The metaphor describes understanding as the building of mental structures, and the term restructuring often used as a synonym for accommodation or conceptual change, contains this metaphor. (Ernest, 1995)

Further he (Ernest, 1995, p. 461) adds, “[some of<sup>22</sup>] previously built structures become the content in subsequent constructions.”

As Riemannian geometry is motivated by Euclidian geometry it is essentially true that some of our constructs themselves become raw material for further constructions. However, if the statement above is taken to mean that we only construct with the pre-existing constructs then that can make us land on the bank of a form of [solipsism](#). If the genesis of all building material of construction of knowledge is merely our pre-existing pieces of constructions then the world we see is merely a result of paradigms that we hold. If this is the case, then we cannot explain the replacement of a paradigm by a new one. Here it is necessary to emphasise the recursive nature of our mental construction and to take into account that these recursive structures are always tested with both sensory and cognitive information. That is, this process is a cyclic one. Our sensory data may challenge the existing paradigms and demand a new paradigm and the new paradigm can demand new sensory data.

A solipsist argument supporting a purely mental nature of recursive structuring and attacking my position on the relationship between sensory data and paradigms can be based on the argument that the sensory information that motivates a restructuring is also mental constructs and largely associated with pre-existing constructs. It is necessary to note that a solipsistic view cannot be logically disputed. For example, a hungry solipsist can go to her fridge and open it and eat something. When someone questions her, “Has she really eaten from the fridge and whether the fridge existed outside her mind?”, she may reply that the sensation of hunger, eating something from the fridge, and someone’s observation that she ate something from the fridge and the questioning of it were all mere figments of her own imagination. (It may be very well argued that, in fact, her hunger just existed in her mind because the sensation of hunger was perceived by her mind.) As a basic principle I readily acknowledge this difficulty and I refuse to engage in such futile debates. Though this is the nature of the solipsist's arguments on the nature of [ontological](#) reality, I have

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<sup>22</sup> This question mark and the words in the brackets are added by the writer.

never seen a solipsist who walked in front of an express train just because he thought the train existed only in his mind. As our hungry solipsist did, the solipsist who is convinced that the train exists only in his mind but chooses not to cross the railway track until the train passes could also argue that his waiting for the train to cross and my observation of it all are figments of imagination in his mind.

### **Two Major Forms of Constructivism: Radical and Trivial**

Radical constructivism, one of the two major versions of constructivism, has been presented earlier in this chapter. The other major version is trivial constructivism. In trivial constructivism, it is assumed that all human knowledge is constructed by each individual. Not only knowledge but also all the data that are featured in the constructions are also constructed in the brain at the very basic level of electrochemical neurological process. Unless we accept that external stimuli have a role triggering the above electrochemical neurological processes this position borders on the realm of solipsism. Maybe, the role of external stimuli is implicitly acknowledged because all constructivists acknowledge the existence of an objective reality.

Even though we can escape the trap of solipsism simply by acknowledging the role of external agents outside our mind still there is a difficulty to overcome. If all knowledge is constructed individually why should anybody bother to communicate this knowledge to anybody else? How can this knowledge be useful to anybody else? According to Ernest (1995), this problem can be addressed by assuming that individuals can construct knowledge to know existing (in other human beings' minds) objective knowledge. At best this looks like a mystical process. For instance, how can the electrochemical neurological processes in one's brain be associated with those of someone else's brain?

Radical constructivism is somewhat better positioned to explain the association between reality and human knowledge. Knowledge is an evolutionary process. This sentiment is captured in the following quotation from von Glasersfeld (as cited in Ernest, 1995).

The function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality.

Also as mentioned in Ernest (1995), many authors are critical of too much emphasis put on internal cognitive processes while neglecting the role of social interactions in the construction of knowledge in Piagetian constructivism. These authors argue that it is necessary that constructivism integrates the role of social interaction into this Piagetian framework of constructivism. This leads to social constructivism. I would like to remark that the involvement of evolutionary process to explain how we deal with external reality inevitably brings social interactions into constructivism. This is because evolution assumes the ability of surviving long enough to procreate and beyond. These long surviving people are the people who have the characteristics matching one's environment. This involves the ability to share and interpret in a more or less common manner their experiences within the social group.

According to Ernest (1995), social constructivism assumes the existence of an external world. We can share this world with each other but we cannot gain certain knowledge of it. The knowledge is socially constructed and must be socially acceptable. This, in essence, attributes some objectivity to knowledge. That is, objective knowledge is socially acceptable and accepted knowledge. A very convincing example of this assertion is the order of operations, BOMDAS or BODMAS in mathematics. The order of operations is socially agreed upon by the mathematicians of the past. These mathematicians trained the next generation; then they trained the next generation, and this goes on. Thus, the order of operation has become objective knowledge.

Ernest (1995, p. 464), citing writers such as Walkerdine, Lewin, Wood, Cobb and Yackel, warns about a danger that can arise for constructivism if constructivism is pushed down the avenue of overly child-centred romantic progressivism. He further remarks that the position that could be taken on the basis of naive and emotive interpretations of constructivism- that whatever a child does is an expression of his or her own creativity- in the end, is not productive for the learners. Experienced teachers are witness to this statement.

The unification of social constructivism and radical constructivism, according to some authors, is a welcome development. In some sense, this can be regarded as a coming of age of constructivism.

Then again if the socio-cultural role is pressed too far it may push constructivism towards a social mode of solipsism. Even though the role of society in constructing knowledge is significant and should be appropriately acknowledged, this cannot be a hindrance to the struggle of constructing conventional and consensual knowledge which is free as possible of social norms. This sentence seems to be contradictory. How can knowledge be consensual and conventional and free of social norms? It is necessary that we strive to understand different foundational paradigms in the various fields from different eras and cultures. One example is that it is hard to imagine a society or culture in which murder is acceptable. Then there had been eras and cultures in which duels were a valid form of protecting honour. For another instance, it is counterproductive to put down Chinese acupuncture or Ayurvedic paradigms just because they are counter-intuitive or ancient. It is necessary that we adapt a broad and holistic approach. Similarly, we should not embrace norms merely because they are ancient or closer to our culture or modern. This social constructivism should not lead us to national brands of realisms, socialisms or capitalisms.

This means that even though our knowledge and concepts are inevitably the products of our time and society we should not allow this condition to dictate us not to be involved in meaningful and peaceful negotiations and communications in search of better consensus. An example in this case is that oppression cannot be justified by cultural differences or social conditions. Unfortunately, there is an unintended danger contained in the example above. The powerful nations can use this principle and their might to impose their values on other nations. This mirrors the other danger. Dictators can justify their totalitarian measures by invoking national values or some distorted version of social constructivism.

It may not be difficult for someone to argue that even the counting principle,  $1+1=2$ , is socially constructed; yet it may be more appropriate to consider this as a construct which is 'forced' on us by the nature of the physical world. This does not imply that

we could not have had a different counting principle. For example, we may switch into the form  $1+1=10$ . In fact, if human beings had two digits (which is a physical condition), instead of 'ten', the number system in use could have been the binary system. *Still there is some objectivity; two apples in the former system are the same as ten apples in the latter system.* One can argue that counting was necessitated by the need to keep sheep numbers accurate, which was demanded by the social needs at the time. In modern times, we need to keep our bank accounts accurate; therefore we still keep the counting system. In a similar fashion it can be argued that almost any concept has a social motivation. This is because humans are social beings. It will be difficult to argue against the social nature of our knowledge. Does this difficulty compel us to embrace the social role of knowledge without any questioning, or demarcations? If so, we should embrace solipsism with similar enthusiasm.

The understanding of the social nature of our knowledge is a great leap forward. Nevertheless, as putting too much emphasis on the social nature of knowledge leads to a form of social solipsism, putting too much emphasis on the cognitive processes of knowledge leads to a form of subjective solipsism. The line we have to walk on is a thin line. Even in more objective research fields, such as mathematics and physics, group politics plays a vital role. Turning a blind eye to this will only make human knowledge endeavour more like 'political' dramas played in our national parliaments.

### **Correspondence between Different Versions**

To understand correspondence between different versions of constructivist views, consider the following two fables, one is age old and the other is modern: 'the blind man and the elephant' and Dr David Bohm's 'two views of the same fish' (Bohm, 1980, p. 237).

The blind man touches the same elephant at various sites of the elephant at various times and comes to different conclusions about the elephant. If the blind man can 'see' then he will see that various 'touch sensations' can be 'combined' together. This is the regular story. Instead of one blind man we can send several blind men to touch the elephant. Each goes to the elephant and touches it at only one place, each site is different for each person. Then they try to build consensus using their touch sensations obtained from different sites.

To explain ‘weird’ quantum phenomenon involving subatomic particles<sup>23</sup>, Bohm used the following fable. Imagine a set up containing an aquarium containing a fish and several (Bohm's story contained only two) television cameras anchored at different vantage points. You are only privy to the images exhibited on the different TV sets that are located around you. As you gaze at the various television monitors you might feel that the image on each TV monitor belongs to a different entity<sup>24</sup>.

Before we discuss the two fables above, let me categorically state that we are all blind in some sense. As a case in point, I cannot see the chair that I am sitting on. The image of the chair when I look at it exists only in my mind. When I sit on it, I sit outside my mind on the chair itself, not on my mental image of the chair. Similarly, the touch sensation I feel on the chair exists only in my mind.

Even though what I think of different versions of constructivism is not exactly represented by the first fable above, it captures its essence. The different touch sensations experienced by the blind men correspond to the different sites of the same elephant, but the different versions of constructivism are not different sites of the same reality or a single view of it. Bohm’s story captures the core of my view much better. As the television pictures of the fish are the images of the same fish projected

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<sup>23</sup> This is related to the EPR (Einstein, Podolsky and Rosen) quantum paradox. The paradox was constructed to show the incompleteness of quantum mechanical description of physical reality. This theoretical argument showed that quantum mechanics predicted that under certain conditions, two particles that are together, when separated ‘know’ what each other is doing, regardless how far they are from each other. This requires non-local (instantaneous) transmission of information between the two particles. However, this was an anathema to Einstein, his theory of relativity and also to a vast majority of the physicist community. This still remains the attitude of a vast majority. Through this paradox, at that time Einstein might have thought that he successfully refuted the theory of quantum mechanics. However, Alan Aspect experiment performed at University of Paris in 1982, verified this theoretical prediction.

<sup>24</sup> If you keep gazing, because of the correspondence between the different movements displayed on each monitor, you might even think that they are communicating with each other instantaneously. (This comment is not relevant for our purpose in this report.)

from different angles on the different TV monitors, the different constructivist views are the different images of the same reality.

These different versions are, in part, the products of the different sets of beliefs employed. This view may be open to attack on the ground of objectivism. It suffices to say that to see our world each one of us needs a set of beliefs. Without a belief system we cannot even start to look at our world. For instance, the idea that ‘murder is wrong’ cannot be constructed without invoking some belief system or without taking the idea itself as a fundamental belief. These belief systems do not exist independently from our [experiential reality](#). Our belief systems form the nature of our perceptions and our perceptions associated with the sensory information we gather form our belief system. This is a cyclical process.

The evolutionary process, the environment and learning (our cognition) construct the hardware component of our viewing system. One who has learned to appreciate the taste of full cream milk may initially dislike the taste of skim milk and then later he can learn to feel that skim milk is tastier than full cream milk. If the person switches back again from skim milk to full cream milk, initially he may dislike the taste of full cream milk.

The belief system itself can be a product of our environment, life experiences and the way we look at our physical experiences, at least to some extent. When the gravity of physical evidence warrants we will be forced to construct new belief systems. Again, this process is not entirely objective or subjective.

The paragraphs above explain my view about the different versions of constructivism. Now it is time for me to explain what my view of constructivism is. My view is some form of a vector sum of all these different views. As a numerical sum, three Newtons and four Newtons yield the answer seven Newtons. As a vector sum three Newtons and four Newtons can yield five Newtons plus orientation.

Now the question comes back again. Which view is more beneficial? I believe it is mine. They believe it is theirs. This sentence captures only some essence of my response. Then why do not other people convert to my view? First of all, my view is

not a static view. It changes. In fact, in a way it changes almost from one moment to another; then again it does not. These changes occur due to life experiences. These life experiences include my exposure to other people's views. Definitely these views influence my view. At least to some degree, everyone's view influences everyone else's view. In summary, even though I hold strong opinions, they are never being put away in a shut cabinet for safe keeping. They are bound to be evolved and even to be revolutionised.

### **Constructivism as a Method**

Tobin et al. (1993) began their paper by mentioning the logical impossibility of establishing the truth of any particular piece of knowledge. For example, it is impossible to establish that just right now I am writing this report (I assure you that no one is writing this paper for me.). Any evidence presented to prove that I, not anybody else, am writing this paper can be questioned. Similarly, any evidence put forward to prove that someone else is writing it on behalf of me, can be questioned. This is the essence of [scepticism](#). A healthy dose of scepticism is vital in any branch of philosophy or science and social interactions. What this healthy dose is may be open for debate.

Once again, just because it is impossible to logically refute scepticism, it is not sensible to accept scepticism as our [ontological](#) or epistemological paradigm of reality. (The irony here is also that it is also not sensible or possible to completely throw out scepticism.) That is why we have built rules, regulations, conventions and theories to determine under what conditions we should believe that something is true or is false. It is necessary that constructivism acknowledges and addresses this difficulty. If there is no such difficulty then there is no need for constructivism; [logical positivism](#), [subjective idealism](#), or [objective idealism](#) will do.

For many constructivists the point of initiation is that there is an absolute reality (later this claim will be examined in detail.) and there is no way of knowing this absolute reality. Still constructivists can use the word reality but what is meant by reality for them is different from what it could mean for a logical positivist. For a constructivist, knowledge of reality consists of a network of relationships and conventions that are built by a community of cognising beings.

We may tend to ignore addressing the nature of reality; instead opting to regard constructivism as a method of teaching that can be used to maximise student learning. However, Tobin et al. (1993) argued against the use of a constructivist framework to designate lecturing an inappropriate way of teaching. This framework notion reduces constructivism to a set of methods and constricts its power of intellectual examination of educational processes. The consequence of this view is that some methods of teaching (working in small groups, involving in activities) are thoughtlessly hailed as intrinsically constructivist and better teaching approaches. Also lecturing is denounced as a poor form of teaching. It avoids the question of improving the lecturing mode of teaching.

It must also be noted that small-group work, physical involvement or any other so designated naturally better teaching methods may fail to achieve the objective of constructivist pedagogical aspirations if they are not appropriately used. An example is that using too many concrete situations and physical activities can hinder the development of abstract thinking skills. Firstly, this can happen because there are simply no opportunities for students to develop abstract reasoning; secondly, because activities are not focussed on facilitating the construction of concepts.

Even though I consider myself to be a constructivist I also believe that constructivism or its misinterpretation has resulted in the rise of a subtle form of anti-intellectualism in education. A lesson is good only when it contains some physical activities and it does not matter how cognitively inspiring are the lively discussions engendered in classes that do not use physical activities but use 'chalk and talk'. (Here 'chalk and talk' is used to facilitate Socrates' inquiry approach.) This is a result of a blatant reduction of constructivism to a set of methods. Instead, constructivism needs to be considered as a way of thinking about reality, learning and teaching but not as a set of designated methods. Nevertheless, it is important to emphasise that the way of thinking about constructivism has implications for the learning-teaching methods we use, and how and why we use them.

## **Systems of Beliefs**

First consider the following quote from Von Glasersfeld .

Today, those constructivists who are “radical” because they take their theory of knowing seriously frequently meet the same objection—except that it is sometimes expressed less politely than at the beginning of the 18<sup>th</sup> century. Now, no less than then, it is difficult to show the critics what they demand is the very thing that constructivism must do without. To claim that one’s theory of knowing is true, in the traditional sense of representing a state of feature of an experiencer-independent world, would be perjury for a radical constructivist. One of the central points of theory is precisely that this kind of “truth” can never be claimed for the knowledge (or any piece of it) that human reason produces. (Von Glasersfeld 1990).

The central theme of the quote above is the nature (trueness) of a constructivist view of reality. Our understanding or interpretation of the nature of constructivism has significant implications for different versions of constructivism, other belief systems and their relevance for education and other social discourses and interactions.

Are the constructivist views of reality true? If I answer “no”, one can legitimately ask, “Then why do you still hold that view?” If the answer is “yes” then he may point out to me that there are many versions of constructivism, and each individual’s constructivist view is bound to be different from everyone else’s view (at least to some degree). Thus, it can be asked, how do I know that the constructivist view I hold is the right one and that other views are wrong? If I answer that someone else’s view is true then I will still have to explain why I hold onto my view. If I answer that only my own view is true then I am compelled to justify why other constructivists do not convert to my view. On the other hand, if I respond that it is not clear which constructivist description of reality is true then our opponent may triumphantly claim that if constructivism cannot be sure about itself then how it can be clear about its view of reality.

The content of the paragraph above is relevant to many other situations. We habitually expect definite answers for all of our questions. Dichotomies are the norm and the basis of our deep rooted thinking paradigms. This may not be appropriate, at

least, at times. There may be situations when we should expect that there are no definite answers for some questions. Scientific thinking, until the encounters with quantum phenomena, admitted only two dimensions for Truth; A statement is either true or false.

This is the Aristotelian two-dimensional truth paradigm. This mode of thinking is not only valid in many situations but also is essential. For instance, whether I have taken breakfast this morning has either a true or a false answer in a court room. Nevertheless, in many other situations, the Aristotelian two-dimensional truth paradigm may be simply untenable. To illustrate this untenability an example will be discussed shortly.

In contrast with the Aristotelian two-dimensional truth paradigm, Buddhist philosophy consists of a four-dimensional truth paradigm. In the Buddhist four dimensional truth paradigm, a statement can be

- true,
- false,
- true and false simultaneously, or
- neither true nor false simultaneously.

Zeno, the Greek Philosopher, asked when a pebble was moving on a path whether the pebble is at rest or on the move at any given point of its path (See Zeno's Pebble on p. 197) The dichotomy mode of Aristotle is clearly incapable of answering this question since if the pebble is at rest at any point how does it complete its journey? On the other hand, if we say the pebble is continually on the move then we may ask how we can say that the pebble is at any particular point, at any given moment. However, answers based on the four-dimensional truth paradigm will bring the wrath of the Judge in a court room.

Being cognitive beings, to view our world we wear lenses of cognitive frameworks. If there are no lenses there is no viewing. These frameworks are called belief systems. In our school days we were taught that science has nothing to do with

beliefs and is about true knowledge. First of all, this itself is a belief. Secondly, science operates on a belief system. For instance, it operates on the beliefs:

- We can understand or explain nature.
- If an experiment generates repeatedly similar results then the experimental result can be generalised for all similar experiments that may be done or not done in the future. For instance, we throw a pen vertically up ten times. It falls in all ten times. Then we generalise that the pen will fall every time when we toss it up in the air.

To contrast Physics and Mathematics, consider the following example. For a mathematician, the known condition,

$$a_n = n, \text{ for } n = 1, 2, 3, \dots, 10,000,000,$$

does not guarantee that

$$a_{10,000,001} = 10,000,001.$$

Many physicists tend to ignore the underlying belief systems of Physics and claim that there is no place for philosophy in Physics. This claim itself is of a philosophical nature. To his credit, Einstein emphasised the role of philosophy in Physics.

To view our world there are several belief systems available to choose from. We may choose one or a combination these systems, namely, logical positivism, [absolute idealism](#), [solipsism](#), constructivism, [scepticism](#), etc. Logical positivism can be challenged by analysing its knowledge claims; yet it can escape this difficulty by moving into the camp of absolute idealism. It is difficult to logically refute absolute idealism (of the existence of God) unless it is simultaneously tied with omnipotence and absolute benevolence. Solipsism cannot be attacked logically but it can be seriously challenged on the basis of the practices of solipsists. Even then they have an easy escape route. Their escape route will be mapped on the lines similar to: You and your questioning and your perceptions that my practices are incompatible with my philosophy are all part of my dream. Opponents of constructivism can attack constructivism as a form of solipsism. For constructivists, this is a grave misunderstanding and an unjustified claim.

Let us look at the question: Is the constructivist view a true view of reality? The answer is that it is a system of beliefs. As a computer cannot work without an operating system a cognitive being cannot operate without a belief system. Constructivism is a way of thinking and looking at reality. It is an operational belief system of human cognition which encompasses objectivity to some extent. Therefore the question of whether it is true is invalid. Likewise a similar question regarding [objectivism](#) or [absolute idealism](#) is invalid if a single word answer is expected.

### **Why Constructivism?**

Still, certainly there should be a way to evaluate our belief systems. To do so, we may ask: is there any advantage of a constructivist belief system over others? Even though the answer to this question partly depends on the individual, I maintain that constructivism is a better view of reality<sup>25</sup>. Its being a better view partly depends on the individual for the reason that a belief system makes sense to its holder according to what he expects from it. If an individual can be comfortable with the implications of absolute idealism- everything happens for a reason, everything is the will of a supreme conscience- then this individual may find his or her peace and calmness within absolute idealism. However, there is a drawback. The individual may have to resort to twisted logic to justify her practice. For instance, when a person gets sick instead of waiting for the will of absolute idealism to manifest itself the individual may decide to go to hospital; then she will have to justify her decision by saying that going to hospital is also the will of absolute idealism otherwise it would not have occurred. Nevertheless, it is still possible to maintain even the notion of free will within the framework of absolute idealism by using twisted logic.

The major advantage of constructivism for one's practice is the empowerment it offers to deal with all aspects of reality. For instance, if we begin to have doubts that our practice and theory are not compatible with each other then we can change the theory or the practice or both. The change of theory will result in a new form of constructivist thinking. On the other hand, we may only change our practice. In this

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<sup>25</sup> Epistemological advantages of constructivism, from a personal point of view, were discussed earlier (See p. 110). This personal view may be valid to other people too, to various degrees.

case we have developed a better understanding of applications and implications of constructivism. This is what happens in natural sciences, at least to some extent.<sup>26</sup>

Our understanding and knowledge (what we can know) of reality depends on both ontological reality and the conceptual framework we employ. This conceptual framework or paradigm is not entirely a product of our mind. And certainly it is not just an automatic manifestation of ontological reality either. The constructivist view gives us some freedom to build a framework that suits the investigation. When we study Nature what we come to understand is what we can say about Nature but not about Nature itself<sup>27</sup>. Therefore research is simultaneously discovery and creation. The aspect of creation gives a sense of empowerment. It also empowers our communication and dealing with other people. Communication is twice constructed. When an utterance is made it is constructed within the mental framework of the utterer and then it is reconstructed within the framework of the listener. While we maintain some objectivity of what we say and hear this perspective tames us to be more cautious when we interpret other people's statements and when we make statements for other people's attention.

### **Mirrors of Isms (Isms in Isms)**

Frequently I wonder about the following observation. There are many millions of people with sincerity, integrity, wisdom, intelligence and sound minds. Some of these people are Buddhists, Hindus, Christians, Catholics, Islamists, some are Marxists, objective idealists, solipsists, subjective idealists, constructivists, scientific determinists, etc. Our basic mode of thinking says that if one group of these people is

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<sup>26</sup> Companies can influence researchers by financial or other kinds of rewards and tempt the researchers to somehow mislead the public. This again points to the social nature of knowledge. Then one may argue that the knowledge is not what has been expressed due to influence. The true knowledge was the knowledge that the scientist arrived at before he altered the 'true' results due to the influences exerted by an external power. This particular social aspect applies power over scientists directly by calculating and manipulating agents. There are other social forces that are naturally arising which do not need a willing agent to apply direct influence. Even if a scientist says that her methodology involves filtering out these direct or indirect forces it only justifies the claim that our knowledge is always under the radiation of the social sphere.

<sup>27</sup> Here I am paraphrasing the quote from Neils Bohr which has been previously mentioned on p. 104 in Chapter 3.

right, then all the other groups must be wrong. I have met people who converted from other religions to Buddhism and claimed that they had found emancipation. I also have met people who were converted from Buddhism to other religions and claimed that they had found emancipation. (Some of these conversions may be due to personal, financial gain or emotional manipulation but it is clearly reasonable to assume that some of these conversions are genuinely pure.) How can I explain this discrepancy and the contradiction?

For many, Buddhism is a form of subjective idealism while Christianity is a form of [absolute idealism](#). Even though Marxists believe that they are [dialectical materialists](#) who despise absolute idealism and subjective idealism, their deterministic theory of history [Marxism](#), at its deep core, is a form of objective idealism. Science largely needs to be based on positivism since the task of science is to explain nature in an objective manner. Einstein lamented the failure of Quantum Mechanics in this regard. I share, to some extent, this concern of Einstein.

The principle of [causality](#) (There will be consequences for good and bad deeds) in Buddhism are notions of objective idealism. For instance, Buddhists believe that killing an animal is a sin, regardless of the status of the doer if he has the intention of killing it. This "if he has the intention of killing an animal" can be regarded as a subjective aspect. Nevertheless, the natural law (Killing an animal is a sin regardless of the status of the doer if he has the intention of killing it.) is clearly a form of objective idealism. So is the law of causality which claims that every result is preceded by an action or a reason. This is locally true and every one of us abides by this as much as possible. For instance, we know where we will end up if we cross the road in a wrong manner. Even the notion that there is a locally true natural law that every action bears its corresponding result is itself an element of objective idealism. When we apply this principle to the whole Universe and for all time it can soon lead into a much stronger form of objective idealism, namely theological objective idealism.

A solipsist, even though he does not believe in the existence of a world outside his mind cannot deny the existence of his own mind. This is because it is the very fundamental basis of his belief system. Then we perceive that this existence of his

mind resembles an objective condition. Then again he might argue that the existence of his mind is conceived by his mind alone. Thus his mind exists merely relative to his mind. However, in their daily lives he behaves as though he lives in a mundane world as he follows bus timetables, writes his last will and does not jump in front of cars which, according to him, exist only in his mind.

Marxists hold on to their objective idealism while contemplating nature and history with their individual minds. So Marxists cannot deny that objective idealism is tied to [subjective idealism](#) even though they call themselves dialectical materialists. Though scientists need to base their practice on a positivistic paradigm, they collect data, interpret and evaluate them as individuals and members of the scientific community. Therefore science cannot escape [subjectivism](#). Sceptics are not sceptical about their scepticism. This makes their scepticism an objectivistic belief. Again sceptics in their practice do not doubt the existence of gravity as evidenced by the fact that there are not many sceptics who jump from big heights. In summary, every ism, to a larger or meagre extent, mirrors and shadows every other ism. There is no escape from this.

Many sincere and wise people with sound minds believe in different paradigms because there is some validity in every form of paradigm that we construct; otherwise it wouldn't have occurred to humankind. This validity is partly objective and partly subjective. This 'validity' of different beliefs is mainly due to the fact that even to think about our world we need a mind. At the same time, we hold to the paradigm of the mind and matter dichotomy. I wonder whether this dichotomy is a basic and fundamental error of our viewing. Once, science believed mass and energy to be two different entities, just as we steadfastly hold on to the view that mind and matter are different entities. Time and space were separate prior to relativity. Later, time and space and mass and energy were united but now there are suggestions in the scientific literature that time and space should be again separated. Even though I take it as an axiom that matter exists that conclusion has been made possible by a mind. Therefore, matter and mind are intricately laced. Perhaps thoughts (mind) and matter can be unified too. Whether this is reality or not is a different matter. What matters here is whether the unification of matter and mind helps us to mirror reality in a more congruent manner.

We know that the Universe consists of cognitive beings. If we believe that cognition existed before any matter or Universe existed then we can ask what the cause of that cognition was. On the other hand, if we assume that cognition did not exist before the Universe or the early Universe did not have cognitive beings then we need to think that cognitive beings evolved later. Then we can ask did the early Universe contain the potential for generating cognitive beings in the future? If we say no then we should assume that the development of cognitive beings is a more or less random event. Then we can ask did the early universe contain the potentiality of randomly evolving cognitive beings? From this we have to conclude that there were cognitive beings in the early universe or the potential of generating cognitive beings or the potential of randomly producing cognitive beings was there in the baby Universe. This means that we need to tie cognition to the early universe one way or the other. Again I wonder is this difficulty arising from our belief in the dichotomy of mind and matter.

In no uncertain terms I have claimed that I reject [solipsism](#), objective and [subjective idealism](#), scepticism and positivism. Nevertheless, I also find that my daily living requires me to base my practice on each one of these paradigms as it is required by the task. For instance, I am a solipsist when I talk about the book I am going to write. I am a sceptic when someone speaks about a miraculous faith healer such as Sai Baba even though I do not categorically deny the existence of faith healing at least in some form as evidenced by the placebo effect. I am a positivist when I say that I have a copy of a particular book at home even though at the moment I am miles away from it. I am a [subjective idealist](#) when I speak or think about my personal feelings. I am an objective idealist when I use BODMAS. My form of objective idealism may not be equivalent to the theological [objective idealism](#). In short, I cannot exercise my living without acting on one or more of the paradigms available to me. Therefore, my epistemology is the rainbow of epistemologies (see p. 113) which I have outlined in Chapter 3.

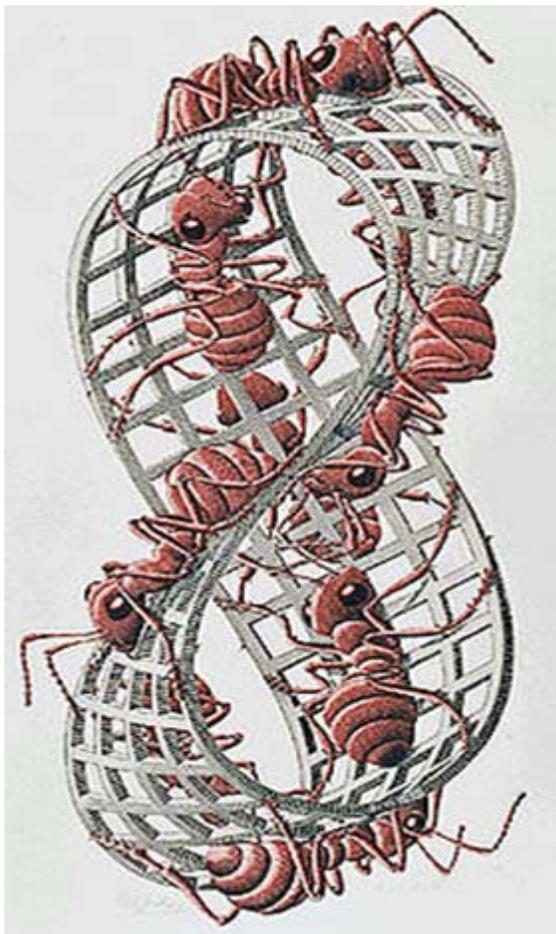
### **Summing Up**

As I have already pointed out, our epistemology influences our practice as educators. Since reality is multi-faced our pedagogical principles need to be multi flavoured. This means that our epistemology cannot be an organism of a single cell; it needs to

be multi-celled with interactions between cells. The rainbow of epistemology presented in Chapter 3 again comes to the fore here.

Implicated in the discussion above is that regardless of how ear catching, single flavoured, single sided slogans could be more harmful than beneficial and could poison constructivist aspirations. The genesis of such simple slogans is the very precarious nature of reality. If we look only from one vantage point we can only see one facet. Due to this limited vision we embrace simple slogans with great vigour. In the next chapter I will discuss how these simple slogans affect our dealing with learning and teaching issues.

Whenever I think of isms the following image created by Escher comes to my mind.



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Am I in or Out?

## **ARC THREE-FOUR: REFLECTING THROUGH PARADIGMS AND CONSTRUCTING PARADIGMS THROUGH REFLECTIONS**

Due to my epistemological metamorphosis from being a logical positivist to a constructivist epistemologist, I cannot claim that the views that I express are independent of my mind. It is a prerequisite that I straight forwardly acknowledge that my views and reflections are not objective elements of ontological existence. This open admission of subjectivity has not lessened my burden of being relevant and valuable. In fact, it requires even more efforts to make my subjective views and reflections to have any validity to the external world other than myself. My self-professed inability to ascertain the purity of thoughts is not a licence for me to be irrelevant and worthless. With this open and self-critical inquiry, I may be able to produce insights more valuable to my audience than in the case where I am simply unaware of the subjectivity of my views.

My mind operates on my own paradigms. These paradigms and values may have a shared commonality to other peoples' paradigms and values. Nevertheless, these paradigms depend and feed on my self-interests, at least to some extent. Granted, a paradigmatic shift can change my personal interests. Then still it is true that my paradigmatic framework cannot exist externally to the sphere of my personal interests.

Therefore, for my views to be of any use to others and possess any degree of legitimacy and relevance I need to directly and truthfully state what my interests are. Certainly these interests

**Right and Wrong**

Passing the lamp post  
I saw  
Light quivering on the  
ground

Looking up  
I saw  
Moths circling the lamp

**(1980)**

need to share a commonality with my audience. It is my duty to nurture this rapport with my readers. To generate resonance with my listeners, I need to be intensively

self-critical with myself, to the best of my ability. Through this critical self-reflection, I hope that I will be able to weed out petty views of narcissistic self-indulgence. As a result, I hope that the well-considered subjective notions I present in this work can generate better consensus.

Through the previous chapters, it is my hope that I have successfully built my rapport with my audience by clearly and faithfully explaining my personal and professional interests. Building rapport with my audience is one of the intended goals of using critical ethno-autobiography of narrating as one of my research modes.

This segment of the inquiry contains how I see and look at my world. Because of my 'self-interest' of being an effective educator, my world views can be easily scrutinised through my views of teaching and education.

In Chapter 5, I present my views on some observations of learning and teaching environments. These views are based on my self-interests that I nurture and the paradigms that I have formed. In Chapter 6, I reflect on my own teaching practice in a critical manner while drawing many valuable insights from the practices of some great teachers in history.

## Chapter 5

### The Futility of Single Flavoured Slogans, the Actual Messages Sent Out and Sunk In

#### Purpose

My purpose in this chapter is threefold as I intend to illustrate how

- ineffective simple slogans are,
- naïve applications of social constructivism could mislead our young students, and
- misplaced constructivist notions could obstruct development of cognitive and conceptual skills of our students.

To accomplish my purpose in this chapter, first I start with a review of my personal standing with constructivism which I have already outlined in the previous chapters. I present my views and standpoints as a genuine, passionate and an experienced learner-teacher. I do so not only with the hope that my views could contribute to the epistemological views, learning and teaching practices of others but also with the hope that any criticism could help me to develop much better views and practices.

Throughout this thesis, my thematic stance has been that our views of the world cannot be simply single sided. This condition arises from the epistemological and [ontological](#) attributes of [experiential reality](#). Reality has neither duality nor non-duality, but our views and experiences of reality are multifaceted. As a correction to a previous statement in Chapter 3, p. 81, I need to add the following.

*The murkiness is not in the nature of reality; the murkiness is in our perceptions of reality. This is because to perceive reality we must wear mind lenses. The murkiness is the taint of the lenses. Unfortunately, if there are no lenses then there is no viewing. Therefore, the onus is on us to remember that we are always wearing mind lenses and our views are always tainted in one way or the other.*

This seemingly contradictory or multi-fold nature of experiential reality certainly spills into our life practices. Even though this predicament has plagued humanity for eons, the Physics Community has only begun to endure and confront this condition

for a little more than a century since the inception of Quantum Mechanics. To deal with this predicament, I base my analysis on the following axioms:

- There exists a reality that is external to my mind.  
This view can be contested but I am immune to this contestation since it is an axiom of my choice based on my practice as a human being.
- We endeavour to conceptualise what we can know and what we can say about external reality so that a majority of us can interact with nature and among ourselves in a meaningful manner. This resembles the quotation from Bohr on p. 107.
- The task of science is to explain and describe experiential reality in an objective manner. This resembles Einstein's remark on p. 88.
- Experiential reality is multifaceted, multi-dimensional and multi flavoured. Therefore, single flavoured slogans may be more harmful than useful. (This will not compromise the power and the beauty of reductionism or hail holism as the only right world view. Holism without reductionism or reductionism without holism is equally impotent and un-aesthetic.)

### **Inadequacy of Simple Slogans**

First I would like to consider one of the Buddhist virtues: one must be satisfied with what he already has. Let us call this virtue "Self-Satisfaction". Consider the negation of this statement; one must not be satisfied with what he already has. Depressive are both the statement and its negation.

When I was a logical positivist I used to attack Buddhism on this statement. Monks preach repeatedly about the virtues of being satisfied with one already has. When they preach these 'virtues' they fail to point out that Siddhartha became Buddha because he was not happy with what he already had; a ridiculously lavish, luxurious and debauched life style. He persevered to attain enlightenment only because he was not satisfied with what he had. Moreover, Buddha has mentioned perseverance as a noble virtue. One statement of Buddha seems to advocate inactivity and another to extol perseverance. Even though I have raised some objections to the idea of being satisfied with what one already has, being unsatisfied with one already has clearly brings misery and distress. This is a situation where we need to apply the Buddhist

four-dimensional truth paradigm (see p. 107). And, also this is a truth of profound nature of Bohr's type (see p. 109).

Consider one of the most celebrated virtues in education: achieving one's own personal best. Let us call this virtue, Personal Best.

Is this virtue similar to the Buddhist virtue, Self-Satisfaction? On the other hand, is denunciation of "achieving one's personal best" equivalent to advocating unending competitiveness which most probably could bring unhappiness to one's life? Does Personal Best Virtue advocate Self-Satisfaction Virtue? If humans were happy with the status quo then we could have still been living in jungles. If Buddha was happy with the status quo then he would have still remained in the cycle of births and rebirths without attaining enlightenment. If I am not happy with what I am, who I am, what I have and what I do not have then it will lead to a life of misery and despondence. In the same token, if I am happy with the status quo I would have not enrolled in this course to further my understanding on the issues that I raise in this thesis. Again, let us look at the self-satisfaction statement.

Any religion that advocates people to be inactive and unendingly contemplate on the abject nature of life, in order for them to find serenity in life, cannot survive for long. Despite elucidative eloquence, if the content of sermons is in conflict with peoples' life aspirations, they will not heed such religions. Since that has not happened with regard to Buddhism it seems that people have taken the self-satisfaction statement in corporation with the vast metaphorical and literal Buddhist literature stories that extol perseverance. The duality of self-contentment and perseverance will remain for ever even if the quantum mysteries are resolved someday. Now I turn my attention to Personal Best Virtue.

I ponder: Is the virtue of Personal Best based on the anti-competitiveness stance of some segments of society? Here, I argue that this Personal Best slogan is too simplistic and single sided.

Is achieving personal best good enough? If not how can one achieve better than one's personal best? When does one know that he has achieved his personal best? When does a teacher know that a student has achieved the student's personal best? What

does one have to do after achieving her personal best? Let us consider an example. The shortest time taken by a particular swimmer to swim a fixed distance in the past can be taken to be an objective criterion for the swimmer's personal best. If the goal is to achieve one's personal best then should the swimmer aim at only maintaining this shortest swimming time? If not, do we not advocate that the personal best is not sufficient? On the other hand, suppose that a swimmer has taken a longer time in a particular attempt than his recorded shortest swimming time. Even though the swimmer has taken a longer time, the coach may still assess that the swimmer has done his best in this particular occasion too.

Now let us apply this into our classrooms. A teacher says to his class (say Year 8), "I want you to achieve your personal best". Hearing this, a student might think "I have already achieved my personal best. I already know two-times table. Therefore no more effort is necessary". In this case how can we put it to the students that what the student considers as his personal best is not actually good according to the dictates of the curriculum and other external realities? Let us apply this now to a workplace. A company worker will not retain his job even if he is doing his very personal best if his best does not meet the company's criteria of effective performance. Also, at any moment, are not we operating at our personal best. Consider the following statements:

- I did not pass the examination because I did not study much but I worked at my personal best during the exam. Before the examination I did not study sufficiently since I wanted to play video games. Still I reckon that I did my personal best to study hard given that I wanted to play video games.
- I did not see the red light because I was distracted due to my ongoing divorce saga. Nevertheless, I was doing my personal best to pay attention to the road conditions.

Even though the statements, "do your personal best" or "do your best" may appear as a positive trigger they could be interpreted as an endorsement of the status quo, the lack of effort and wilful submission to hopelessness.

### **Doing Personal Best or Being Perseverant and Resilient**

Even though earlier I have been frequently using the phrase "do your best" now I avoid using it. I believe that teachers need to be aware that what they think of as a positive message may subliminally transfer a negative message to the listener. Even if a statement is not designed to carry a subliminal negative message the listener might create a different meaning so that the listener can stay happily inside his narrow circle of comfort. Thus we need to remember that there are some statements for which simply the two dimensional truth paradigm is inadequate.

To convey a positive message we may need to complement the statement of "do your best" with suggestions of

- perseverance,
- excellence,
- meeting the relevant external criteria, and
- joyfully striving to do better than what one does at any moment.

This last suggestion needs to be considered as Living Life to the Fullest. Also, it is noteworthy that the above suggestions are multi-paradigmatic. The links between the suggestions and epistemologies are:

- do your best: solipsism
- excellence: criticalism: objectivism and idealism,
- meeting the relevant external criteria: positivism and objectivism, and
- joyfully striving to do better in the next moment: self-criticalism, idealism, objectivism and subjectivism.

Other people find many other links and interlinks, in additions to links I have suggested.

The dangers of simple slogans have been mentioned in (Airasian & Walsh, 1997, p. 449):

In particular, we have argued that the consequences of implementing constructivism in the classroom will be considerably more challenging than might be anticipated by simple slogans that advocates repeat.

Some such simplistic slogans are ‘Sage on the Stage’ and ‘Guide at the Side’ or ‘Chalk and Talk’.

Next I look at an example of simplistic interpretations of constructivist notions.

### **Irony of the Irony**

Newspapers and other communication outlets are full of subliminal messages that devalue and debase education. At the first sight, these news items may appear to be carrying positive information. Nevertheless, the messages sneak into the subconscious of unsuspecting young minds (even adult minds, for that matter) is quite vicious and damaging. For instance, while the newspapers carry dedicated Sports Section seven days a week, the Education Section appears only once a week. Youngsters, even though they might not be consciously aware, receive the message of supremacy of Sports over Education. The example above does not have any direct link to constructivism, but there are such circumstances that might be based on constructivist notions. To examine this aspect, I would like you to consider the following passage.

*FOOTY players are swapping brawn for brains to help primary school pupils reach goals inside the classroom. For the first time in Australia, specially trained Bombers players are using their smarts to help Grade 3 and 4 students improve their reading...*

Herald Sun, 15-05-2009, Ed: 2, Page 13, NEWS

It is clear that the passage above supposedly carries the positive message that innovative and creative approaches are employed in education. Notice the proud statement: "This happens in Australia for the first time". Regardless of whether it is designed or inadvertent, the message that sneaks into the gullible and unsuspecting public mind is the dominance of sports over education. It appears to me that the societal conscience accepts the notion that, to enhance education, use of greedy and rowdy sports is essential. This societal conscience is brought to society by the enormous revenues earned by the players, the massive profits earned by the sponsoring companies and the frenzy generated by media and company sponsored

sport events. Also these simplistic notions may have been reinforced by naïve interpretations of the constructivist tenet of child centeredness.

In the passage quoted above, there is also a hidden but (not so well hidden) indictment of the current teacher training programs. It may be true that the author of this news item or the sponsor of this novel program never consciously thought about it the way I see it. This differentiates my conscience and the social conscience. How people feel about issues depend on their subliminal value systems rather than openly expressed aspirations. Let me explain this further. Internalised beliefs create attitudes and behaviours sanctioned by those beliefs. Then people feel that these behaviours and attitudes are natural and normal. For instance, if I were to live near muddy water for years I would not sense the offensive smell of the muddy air. If one is brought up in an environment in which violent attitudes to women are normal then he will not recognise any faults with his abhorrent behaviour towards women. In fact, he may sincerely believe that gentle behaviour and attitudes towards women to be un-masculine.

For me, humanity depends on the attention we pay and the care we take to improve the quality of our cognition. Since human beings are physical and cognitive beings, both physical and cognitive exercise is necessary for proper health. When society displays attitudes of unconditional piety towards commercial sports (not physical sports that you and I play for fun and exercise), it will be unable to distinguish between debasement of education and celebration of sports.

Now let us consider again the Herald Sun article. In this article, there was an eye catching picture of an adolescent boy standing on a stack of books admiring the athletic. He is treading on the books to reach the height of the giant player. The subliminal message is extremely lucid. To get near the stardom of the athletic one needs to tread on the books.

Furthermore, the Herald Sun article claimed that the athletes had a two week training before they participated in this innovative education program of reaching primary school kids. The article is subliminally suggesting that the program is an excellent solution for education problems. We first need to train the people in their sports.

Then we train some of these athletes in education just for two weeks. This will solve all problems in Education. What an indictment is this of the education system and teacher training programs? Is this not saying that the teaching qualification obtained after four years of university learning is nowhere as good as just two weeks training of a sports person? Imagine the message that a little girl or boy infers from this program?

This, one may think, is an accidental error of a not-so-thoughtful journalist. I differ. This journalist is only expressing the attitudes and approaches of a segment of education experts and society. Here, what is at work is social conscience. Let me illustrate this with another example. During the years that I was in high school and University (in Sri Lanka) both young and older males used a popular term to refer to a female. This term can be translated into English as "thing." Their defence was that they did not mean to degrade women, and it was just a convenient slang.

I argued that, in their minds, a woman was a thing. Therefore, for them, calling a thing a thing cannot be degrading. Likewise, when social conscience overshadows our own conscience, after a while, we do not recognise racism, sexism, parochialism and anti-intellectualism in our thoughts and deeds; instead we justify them. That is, we embrace, without any forethought, criticism or assessment, the social frame of subliminal values as our own value system.

A person with a right awareness continuously strives to reflect, compare and contrast mega social conscience manipulated by various agencies and one's own conscience. Then one's own conscience may win over the mega social conscience. Perhaps such an enlightened person may even become a transformer of such subliminal or clearly expressed mega social attitudes and values.

I wonder: Whether the mode of thinking manifested in the idea of utilising sports persons as teachers or tutors is encouraged and engendered by a naive interpretation of child centeredness advocated by some constructivists? This is because the underlying foundational doctrine of this approach seems to be: to motivate children to connect and learn we need to provide what the child cherishes. How true is this tenet and how untrue is it, at the same time?

Now I turn my attention to the damage that could be caused by misplaced constructivist beliefs. It is well known that in the name of conceptual understanding, the role of memory in education has been somewhat denounced and degraded. As a naïve constructivist, when I was in Year 9-12, I also dismissed the positive and essential role of memory in education (see p. 73).

Next I look at the inadequacy of naive interpretations of constructivism in dealing with some aspects of learning and teaching. Also, I will present my views on some other issues related to education. To do so, I review some Specialist Mathematics Examination reports issued by Victorian Curriculum and Assessment Authority. I feel that these reports amply indicate the inadequacy of our current education paradigm. When I comment on these reports I assume that the examiners were careful enough not to exaggerate just slip-ups or random errors, which can be made by any person at any level of expertise. That is, the large frequency of similar errors in the cohort would have caused the examiners to deduce that the errors were not just slip-ups but were indicative of systematic errors of underlying misconceptions and lack of skills.

Moreover, I have selected the examiners' reports for Specialist Mathematics since mathematically most proficient students choose Specialist Mathematics in Victoria. For this reason, their mistakes associated with basic notions, not with Specialist Mathematics content, will be strong indicators of the ills of our present system.

### **Food for Thought**

From the Year 2008 Specialist Mathematics Exam 1 Report (Victorian Curriculum And Assessment Authority, 2008):

**Question 4:** Given that  $\sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5}-1}{4}$ , find  $\sec\left(\frac{\pi}{5}\right)$  in the form  $a\sqrt{5}+b$ ,

where  $a, b \in R$ .

**Remark:** I will add underlining to the examiners' reports to highlight the issues that I want to discuss.

**Examiners Comment:** This question was not done as well as expected, with several students not recognising the need for a double angle formula. Many attempts were let down by poor algebraic/arithmetical simplification. Many errors were seen in the expansion of  $(\sqrt{5}-1)^2$ . A number of students selected a double angle formula which complicated the question. Several sign errors were seen as were errors with fractions. Some students made mistakes in stating a double angle formula despite them being on the formula sheet.

**My Comment:** Let us ignore some students' inability to choose an appropriate double angle formula since it is a Specialist Mathematics content issue. The most alarming fact is that many attempts were let down by poor algebraic and arithmetic simplification. The examiners have observed many errors with the expansion of  $(\sqrt{5}-1)^2$ .

Let us examine this more closely. What are the concepts and notions associated with this expansion? One of the notions/skills required is the square of a difference<sup>28</sup>. In Australia, students learn the formula for the square of a difference or a sum in Year 9. Because of the presence of  $\sqrt{5}$ , another concept involved here is the square root of a non-negative number. Students learn this concept in Year 8. Therefore, students' difficulties with the expansion of the expression,  $(\sqrt{5}-1)^2$  must be related to one or more of the following aspects:

- square of a sum or a difference
- Directed Numbers
- square root of a number, its multiples and its addition to another whole number.

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<sup>28</sup> The quantity,  $(\sqrt{5}-1)^2$ , can be equally considered to be a square of a sum since  $(\sqrt{5}-1)^2 = (\sqrt{5}+^{-}1)^2$ . As "Directed Numbers" is taught in Year 8, Year 12 students must be aware of this.

Supposedly they should have learned all of these concepts and skills in Year 9 or earlier. These concepts are not the content of Specialist Mathematics. If the most mathematically able Year 12 students have difficulties with the skills and notions covered in or before Year 9, then we need to have a hard look at the content of the curriculum and the method of its delivery.

Square of a sum or difference: Some students may write

$$(a-b)^2 = a^2 + b^2 \text{ or } (a-b)^2 = a^2 - b^2$$

or some other variations. Even if these students do not really understand that these results are due to the law of distribution and the properties of directed numbers, they should be well aware of the following facts:

- the product of two negative numbers is positive,
- $(a+b)^2 = a^2 + 2ab + b^2$  or  $(a-b)^2 = a^2 - 2ab + b^2$ .

The product of two negative numbers being positive is a magical and mysterious rule for them. Nevertheless, they should be aware of these facts.

We need to acknowledge that even the original "creators" of these formulas can make occasional mistakes with the formulas. When that happens, that cannot be due to any lack of understanding or awareness. It can be just misfiring of neuronal pathways or random human mental errors. For students also this can happen. This was one of my major objections to examinations and tests when I was a secondary school student. Now, I have withdrawn my objections to examinations and tests while maintaining that they are imperfect assessment tools and a necessary evil. This means that not having tests and examinations can be more detrimental to education than having them. My failure with examinations and tests, while I was a secondary school student, was due to my intellectual arrogance and my then thinking that only understanding was sufficient. Even though understanding is of primary importance, an ability to perform and think under stress is also of significant importance. This requires understanding, learning, internalisation and the ability to remain calm under stress. We need to admire these as essential aspects of education.

These mistakes must be manifestations of memory or conceptual problems. I think that they are both. Since if they have the proper conceptual understanding then it is nearly impossible for them to use wrong versions of these formulas except in the case of sleep deprivation. Therefore, the difficulty is not nearly a memory problem. My long association with students has convinced me that their main objective is to get the right answer; not the development of proper thinking and conceptual skills. This means that the vast majority of the students who do well, even though they study conscientiously, they do not pay attention to the underlying principles of mathematics. For instance, they are not aware of the fact that  $(a+b)^2 = a^2 + 2ab + b^2$  is a direct result of

$$ma = \underbrace{a + a + \cdots + a}_{m \text{ times}} \Rightarrow m(a+b) = \underbrace{(a+b) + (a+b) + \cdots + (a+b)}_{m \text{ times}}.$$

The first of the two statements above is a notational definition and the second statement is a result following from the notational definition.

It seems that some of the educators believe that formulas are just memory things. This may be the reason why VCE Mathematics Exam papers contain so many basic formulas. From this presence of many basic formulas in the exam papers it can be inferred that many educators believe that the difficulty with the formulas is just a memory problem. Also, they seem to believe that there is no harm in not remembering the formulas. Then these educators may argue that not providing these formulas is an injustice to students, education and learning and teaching. When I was a high school student my views were similar (see p. 73).

Later I will demonstrate that some students do not remember (I explain this word choice below.) that  $\frac{2}{4} = \frac{1}{2}$  does not mean that  $2 = 1$  and  $4 = 2$  (see “Missing Equivalent Fractions” on p. 188). I used “do not remember” because some educators seem to think that many problems of understanding are just memory problems. If we go with ‘this mere memory argument’ then we need to put the following basic notion of fractions also on the formula sheet:

$$\frac{p}{q} = \frac{r}{s} \text{ does not imply that } p = r \text{ and } q = s.$$

I leave it to the authorities to make a well-considered decision on putting more and more such basic facts on the formula sheet.

Here is another observation with respect to the concept of the square root of a number. To find the value of  $(\sqrt{7})^2$ , some students go through the following calculations:  $(\sqrt{7})^2 = \sqrt{7} \times \sqrt{7} = \sqrt{7 \times 7} = \sqrt{49} = 7$ . This is a correct way of working out but is it an acceptable way? If we believe that we should accept that students can use any method they see fit then this method is acceptable. The notion that students should be allowed to use any method they see fit sometimes can lead to compromising the relevant conceptual understanding. With this acceptability, we relinquish the emphasis on conceptual understanding. I believe that we need to accentuate to the students that  $(\sqrt{7})^2 = 7$  simply because  $\sqrt{7}$  is the number whose square is 7. They need to internalise that the ancient mathematicians created the number  $\sqrt{7}$  so that there is a number whose square is 7.

This tendency of calculating the square of a square root of a number, without employing the concept of the square root of a number, indicates that the students do not grasp quite well the notion of the square root.

Some educators seem to have advocated invalidating the vinculum and replacing it with the operation of division, on the basis that many students do not understand the vinculum (see p. 143). Should we now also invalidate the notion of square roots? On the other hand if one does not want to invalidate the notion of the square root then this basic notion also must be put in the formula sheet:

For  $a \geq 0$ ,  $\sqrt{a}$  is the non-negative number whose square is  $a$ .

Maybe, to ensure that students perform well we also put some examples of square roots in the formula sheet.

Now we look at the examiner's remark: "some students made mistakes in stating a double angle formula despite them being on the formula sheet". We pay attention to

this comment because of its relevance to the roles of memorisation (remembering) and thinking in education.

In relation to the double angle formula, from the examiner's report it can be inferred that:

- It did not occur to some students the need of a double angle formula.
- Many students chose a double angle formula that complicated the question.
- Some students chose the right double angle formula but stated it wrongly even though all the double angle formulas were on the provided formula sheet.

The fact that some students did not choose a double angle formula even though the formulas were on the examination formula sheet does not surprise me. In fact, I would have been surprised if it did not happen. My experience tells me that this should happen to a reasonable fraction of the students who do not remember the formula. To demonstrate this situation to my students, I give them a question that can be solved by applying Pythagoras' Theorem or some other well-known mathematical facts such as Similar Triangle Properties. During this exercise, their note books and text books lie in front of them. Also, the theorems they need to apply may be prominently written on the whiteboard.

Unless it occurs to them that the question is about Pythagoras' Theorem they will not refer to the textbook, notes or the whiteboard even if they have done similar work just few minutes earlier. Unless the theorem is in their mind, they cannot connect the problem with the theorem. On the other hand, if the theorem is in their mind then there is no need for them to refer to a textbook.

A well experienced mathematics teacher has put this to my daughter in another effective manner. The regulations of Victorian Certificate of Education allow students to bring a textbook of students' choice and another set of written notes to the Exam II of both Mathematical Methods and Specialist Mathematics. According to my daughter, all the students in her class were ecstatic about the availability of a textbook and their own set of notes. The teacher addressed the class about this matter

and told them: during the many years of her exam supervision she has only noticed one or two students using the textbook during the exam. She told them that just the use of the textbook most probably indicated that those students would have done extremely poorly. Referring to this practice, I ask my students to imagine that Road Authorities allow people to drive motor vehicles while referring to the driver's manual. For this comment, some students might respond that driving is a real life task and Mathematics is not. This is exactly the response that I anticipate. This attitude is the reason that they find it is difficult for them to internalise even basic concepts. Mathematics not 'being real life' can only be true for some ancient tribes who lived eons ago in thick forests. Even then they were creating mathematics or pre-mathematics.

Memory skills are a basis and a part of cognition. If there were no memory skills we cannot develop cognitive skills. That is, while memory skills are not equivalent to thinking skills, without memory skills one cannot even begin to think. To hail cognition, denouncing memory skills is to denounce cognition itself which we think of as being supreme. Consider this example. The Physics formula sheet provided to the VCE students contains the formula,  $F = ma$  (The Specialist Mathematics formulas sheet also contains this formula as  $R = ma$ ). This sends a powerful subliminal message to the students that students do not need to understand Newton's Second Law since if one conceptually understands this law then this understanding of the relationship between mass, force and acceleration is itself the formula. It will be an amazing learning experience for me if I can understand Newton's Second Law without remembering the formula,  $F = ma$ .

Performances of this nature remind me of trained animal performances. In Sri Lanka, there is a species of small monkeys, 'toque monkeys'. Monkey Masters train toque monkeys to play many tricks. Then the Masters direct the toque monkeys to dance to the Masters' tune at market places. The Masters' livelihood is the money collected from the audience. In one of these dances, the master asks the monkey to show people how it carries gifts to its Mother in Law. The monkey performs. People laugh and the Master gets money. In this dance, even though the monkey performs to perfection each time, it does not understand the concept of Mother in Law or gifts.

To answer a Physics or Mathematics Exam question, if one needs to refer to the formula  $F = ma$ , and to an example of its application, then this performance is no different from the toque monkey dance. Also, performances of this nature are equivalent to sorting mail for their respective pigeonholes: John's mail goes to John's pigeon hole; Jane's mail goes to Jane's pigeonhole. I term performances of this kind 'pigeonhole mathematics'.

When I was a secondary school student, being a naive constructivist, I occasionally yearned for a formula sheet and thought that not providing a formula sheet with exam papers was a manifestation of anti-intellectualism. Now this has gone too far to the other end; exams provide even the very basic formulas to the students. As I previously mentioned, this sends an extremely powerful subliminal message of anti-intellectualism to the students. Even if educators do not send this message by design, this practice creates fertile grounds in students' minds for them to cultivate such anti-intellectual views. If we continue a practice which creates breeding grounds in students' minds inviting them to construct such anti-cognitive messages then we are the messengers; the students are just receivers.

In my days of naive constructivism of a distant past, I never thought that one day I would argue in favour of strong memory skills or that I would tie conceptual understanding to memory skills. It is difficult to understand why many educators fail to see that, for many formulas, understanding of a formula is itself the formula. Some exceptions to this assertion are long formulas. For a long formula, understanding may not be sufficient to remember it even though we may be able to derive it in two or three minutes. In such situations, a formula sheet is a good thing to students and mathematicians.

As an addendum to my strong criticism against formula sheets, I give my students the following advice on how to prepare their own information sheet or the 'cheat sheet' as they call it. I ask them to choose a minimum number of formulas to put into their sheets. To utilise this facility fully, they need to be aware when they have to use each one of these particularly troublesome formulas even before they leave home for the exam (This condition is true even for easy formulas.). That is, they

must know all the formulas which are on the sheet as conceptual relations expressed in symbolic form. Also, they need to understand the few troublesome formulas thoroughly (which they put on the sheet) at the conceptual level. Furthermore, they need to know which situations require any of these relationships (which may or may not be on the sheet) to be applied, even before they leave home for the exam. For instance, what formulas (conceptual relationships) are available to calculate a length? The answers are many: Trigonometric Ratio, Cosine Rule, Sine Rule, Pythagoras' Theorem and Similar Triangles, etc. Then they need to ask: which one of these conceptual relations is relevant to the question at hand?

I have seen many of these information sheets that students take to the tests and examinations. Say, the test is about percentages. Then these information sheets contain examples of textbook problems done in class. Inspection of these information sheets has convinced me that students have developed one or more of the following four skills:

- the ability to put a colossal amount of information in a small area,
- the ability not to contemplate under stress how they will be able to fish the right piece of information from the sea of information,
- the ability to fish the right information occasionally from the sea of information,
- Pigeonhole mathematics. [That is, students only need to select which number goes to which place. There is no need to reflect, connect, link or conceptualise. They just have to follow the pigeonhole scheme which they have practised well. In promoting cognitive skills it appears that we have created anti-cognitive practices.]

Policies developed while looking through a tube angled in a fixed direction are not well informed. Even if the policy makers have the best intentions, the policies they develop may end up being antithetical to their own noble aspirations. We need to look through all the angles. For instance, clean water is an essential nutrient to sustain life. The same clean water drunk in excess can bring death to humans as has happened on several occasions. Likewise, it is impossible to argue that the

information sheet is just bad. Also, it is impossible to assume that the information sheet is simply good. Dissent on issues needs to be encouraged and considered.

At the end of the day we need to make a ruling whether to allow or not to allow the use of an information sheet. When we make that ruling, we need to establish a mechanism to minimise the negative aspects and enhance the positive aspects. For instance, we may not allow students to bring their own information sheets but provide with exams a few particularly troublesome formulas. The ruling itself needs to be repeatedly evaluated. Also, the students need to be educated about the proper use of the formula sheet facility.

Here is another question from the same VCE Specialist Mathematics Exam 1, 2008 (Victorian Curriculum and Assessment Authority, 2008) and the examiner's comments on the question and students' work.

**Question 10: a.** Let  $w = 1 + ai$  where  $a$  is a real constant. Show that  $|w^3| = (1 + a^2)^{\frac{3}{2}}$ .

**b.** Find the values of  $a$  for which  $w^3 = 8$ .

**Examiner's comment on (10) b:** This was a straightforward question but far too many students were unable to simplify expressions involving indices. Of those who correctly obtained  $a^2 = 3$ , several carelessly gave the answer as  $a = 3$  or  $\pm 3$ .

Two general remarks on this report were:

- Poor algebraic skills: This was evident in several questions, and the inability to simplify expressions often prevented students from completing a question.
- As students were not allowed to bring calculators into the examination, there was an expectation that students would be able to simplify simple arithmetic expressions. Many students were unable to do this and lost marks as a consequence.

**My Comments:** If the best of Year 12 Mathematics students cannot deal with indices and cannot solve the equation  $a^2 = 3$  then it is an indication of a grave underlying

problem. In most of these situations what is lacking is pedagogical awareness or right mindfulness (see p. 238 and 225).

Frequent are the comments on poor algebraic skills and arithmetic skills in the Examiner's reports on students' performance in both Specialist Mathematics and Mathematical Methods. Why are poor arithmetic skills prevalent? Is it because children are born with some mysterious DNA deficiency or is it because there is something wrong in the modern education paradigm? For instance, assume that as soon as babies are born we tie their legs. Then after several years we find that they cannot walk. Is this a genetic condition? To solve this new problem of walking disability, suppose that we provide robotic assistance to children. Centuries later we will have a new species that cannot walk. Then it will become genetic. Why are even the best mathematics students poor in arithmetic skills? Why do shop assistants, even to find the double of \$2.50, use calculators? Is this a sign of progress and highly developed thinking skills, or is this a genetic disorder. It will be beneficial to look through all the angles, to make better judgments.

Let us examine more comments from examiners.

The following question is from the VCE Specialist Mathematics Exam 1 Assessment Report, 2007 (Victorian Curriculum and Assessment Authority, 2007).

**Question 1** : Express  $\frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i}$  in polar form.

**Examiner's Comments:** This question was quite well done. Two methods were widely used; rationalising using the complex conjugate and converting the numerator and denominator to polar form before proceeding. In both cases it was clear that several students had not learned the relevant exact values for circular functions.

Most students used a rationalisation approach. The typical errors for this approach included converting  $2i$  to  $2\text{cis}(0)$   $\sqrt{2}\text{cis}\left(\frac{\pi}{2}\right)$  or similar and simplifying the denominator as  $1 - 3 = -2$  . A small number of students did strange things with the fraction, such as using incorrect cancellation and writing the complex number in the

form  $\frac{2\sqrt{3}}{1+\sqrt{3}i} + \frac{2}{1+\sqrt{3}i}i$  and then attempting to treat the fractions as real coefficients.

Writing the fraction as  $2\sqrt{3} - \frac{2}{\sqrt{3}i}i$  was also seen. Several students correctly found  $z = 2i$  and then stopped. Presumably most of these had not read the question carefully enough. Those who converted the numerator and denominator to polar form before proceeding often had difficulty with the quadrant for the denominator. Students should be encouraged to draw small diagrams to indicate in which quadrant the complex number lies. The typical errors for the polar approach included: finding the arguments to be  $\frac{\pi}{3}$  in the numerator and  $\frac{-\pi}{6}$  in the denominator and hence

fortuitously getting the correct answer; and simplifying incorrectly, such as

$$\frac{\pi}{6} - \frac{-\pi}{3} = \frac{2\pi}{3} + \frac{\pi}{3} = \pi.$$

**My Comments:** It is nearly impossible that the statement,

$$\frac{\pi}{6} - \frac{-\pi}{3} = \frac{\pi}{6} + \frac{\pi}{3} = \frac{2\pi}{3} + \frac{\pi}{3} = \pi,$$

is just a random error or a slip-up. If some of the best mathematics students make such errors of a conceptual nature regarding equivalent fractions we need to trace these students to find out the reasons behind this performance. The case studies of such mistakes need to be carefully documented, explored and analysed so that educators can genuinely assess the education system. Notice that the incorrect statement

$$\frac{\pi}{6} + \frac{\pi}{3} = \frac{2\pi}{3} + \frac{\pi}{3} = \pi$$

is similar to the incorrect statements

$$\frac{5}{6} + \frac{5}{3} = \frac{2 \times 5}{3} + \frac{5}{3} = 5 \text{ and } \frac{1}{6} + \frac{1}{3} = \frac{2}{3} + \frac{1}{3} = 1.$$

To obtain an equivalent fraction, what these students have done was to multiply the numerator by 2 while dividing the denominator by 2. Also, these students are not simply aware of the magnitudes of the fractions involved. Explicitly expressed, the students who wrote the statement  $\frac{\pi}{6} + \frac{\pi}{3} = \frac{2\pi}{3} + \frac{\pi}{3} = \pi$  were not aware that

$\frac{1}{6} + \frac{1}{3} \neq 1$ , as each of the two fractions is less than  $\frac{1}{2}$ . Therefore the sum cannot be equal to 1. This clearly indicates a lack of understanding of common fractions among a considerable number of Specialist Mathematics students. This situation is simply unacceptable.

Also the examiner has highlighted the error,

$$\frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i} = \frac{2\sqrt{3}}{1} + \frac{2}{-\sqrt{3}i}i = 2\sqrt{3} - \frac{2}{\sqrt{3}}.$$

This cannot be anything but systematic conceptual error. Notice that the statement above is equivalent to any of the following statements:

$$\frac{a+b}{p+q} = \frac{a}{p} + \frac{b}{q} \quad \text{and} \quad \underbrace{\left(\frac{1+1}{1+1}\right)}_{\frac{2}{2}=1} = \frac{1}{1} + \frac{1}{1} \Rightarrow 1 = \frac{1}{1} + \frac{1}{1} \Rightarrow 1 = 2.$$

Since Year 3, they have been working with problems such as  $\frac{4}{11} + \frac{3}{11} = \frac{4+3}{11}$ .

Therefore they should be well aware of the law of distribution:

$$(4+3) \div 11 = 4 \div 11 + 3 \div 11 \quad \text{or} \\ \frac{4+3}{11} = \frac{4}{11} + \frac{3}{11}.$$

Then

$$\frac{7}{11} = \frac{4+3}{6+5} = \frac{4}{6+5} + \frac{3}{6+5}.$$

Therefore

$$\frac{a+b}{p+q} = \frac{a}{p+q} + \frac{b}{p+q}$$

and

$$\frac{a+b}{p+q} \neq \frac{a}{p} + \frac{b}{q}.$$

Specialist Mathematics students need to develop much more general understanding of the law of distribution than these counter examples provide.

Here is Question 10 from the same examination (Victorian Curriculum and Assessment Authority, 2007). To the examiner's comments, I have added numbered underlines for the convenience of reference.

**Question 10:** Given that  $\tan(2x) = \frac{4\sqrt{2}}{7}$  where  $x \in \left[0, \frac{\pi}{4}\right)$ , find the exact value of  $\sin(x)$ .

Examiner's Comments:  $\sin(x) = \frac{1}{3}$ .

There were many approaches used for this question, some of which led to (1) extremely complicated algebra which almost always resulted in errors. Some students arrived at the correct answer by circuitous and convoluted pathways but (2) most of these got lost in the algebra. The most successful method involved first finding,  $\cos(2x)$  either by using the  $\sec^2(\theta)$  formula or, more commonly, by drawing a right-angled triangle and using Pythagoras' theorem, then using the appropriate double angle formula. Another common approach, though not as successful, was to use the double angle formula for  $\tan(2x)$  to try to find  $\tan(x)$  first. (3) Many of these students were unable to solve the resulting quadratic in  $\tan(x)$ . Those who solved for  $\tan(x)$  then used either a right-angled triangle together with Pythagoras' theorem or, more rarely, the  $\csc^2(\theta)$  formula. (4) Some of these students successfully found  $\tan(x) = \frac{1}{2\sqrt{2}}$  and then stopped, either having forgotten what the question asked, not knowing what to do next, or running out of time. A not uncommon error was to initially write

$$\tan(2x) = \frac{4\sqrt{2}}{7} \Rightarrow \tan(x) = \frac{2\sqrt{2}}{7}.$$

Some students tried to use

$$\frac{\sin(2x)}{\cos(2x)} = \frac{4\sqrt{2}}{7},$$

but this was rarely successful. (5) Several of these attempts included equating the numerators and denominators, thereby stating the impossible equations

$$\underline{\sin(2x) = 4\sqrt{2}} \text{ and } \underline{\cos(2x) = 7}.$$

**My Comments:**

As usual, the Examiner has highlighted algebra errors. The first three underlined phrases refer to algebraic deficiencies in relation to Year 9 and Year 10 content. The fourth underlined phrase contains three aspects:

- (i) The students could not remember what the question has asked. (This again verifies that there cannot be any thinking without any memory skills.)
- (ii) To find what  $\sin(x)$  is, the students did not know what to do after obtaining the condition,  $\tan(x) = \frac{1}{2\sqrt{2}}$ . (This is Year 9 Trigonometry and Pythagoras' Theorem work.)
- (iii) Running out of time. (fair enough)

The fifth underlined phrase refers to the errors related to the Year 10 and/or 11 content. That is,  $-1 \leq \sin(x) \leq 1$  and  $-1 \leq \cos(x) \leq 1$  for all  $x$ . The exam formula sheet has given this fact indirectly by indicating the domain and the range of the inverse trigonometric functions. See below.

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$R$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Even if the formula sheet contains this fact explicitly, it would not have mattered to many of them. This is a basic conceptual fact which students should do well to remember. This is not about cramming or memorisation. In fact, this is the well celebrated conceptual understanding in constructivism. Students just had to visualise the unit circle and recall the definition of sine and cosine functions. The unit circle embeds the range and domain of sine and cosine functions in a clearly visible manner. I wonder how it is possible to understand the concept of the sine function without internalising its range. According to my experience, many of these students who failed to recognise the range of the sine and cosine functions would have been able to graph the functions in few seconds. Again this reminds me of the torque monkey dance and pigeonhole mathematics. Many students do routine things without any pedagogical awareness (see p. 225 and 238).

Here, it is necessary that we understand that this objection to memory came as a 'correctional' reaction (not response) to some educational approaches which prevailed decades ago. If we swing a pendulum to a large height in one direction it also swings in the other direction to an equally large height. Extreme rejection of memory skills has resulted in a tight embrace of pigeonhole mathematics and a strong denouncement of cognitive learning.

As a secondary school student, I suffered due to memory driven learning and my reaction (not response) to it was to denounce memory skills altogether. Nevertheless, there were brilliant teachers and brilliant teaching. In Chapter 1, I have discussed how I was inspired by the two great teachers who were a product of pre-constructivist constructivism.

Educators need to inquire whether

- the current curriculum approaches promote this kind of non-conceptual learning,
- we equate constructivist learning, thinking about thinking (meta cognition) and all other aspirations, which we describe with many other superfluous adjectives and phrases, to this kind of cursory learning.

We need to ask these questions with sincere intentions to find honest answers, without holding onto any dogmas. We owe this to the future generations. We owe this to ourselves. While engaged in this inquiry, we need to remember that thinking, learning and teaching are not new phenomena. Constructivist teaching is also not merely a modern teaching practice. For instance, consider Socrates and other revered teachers in history.

### **Missing Equivalent Fractions**

The Examiner missed another significant issue that some students did not understand fractions at all. For instance, the inference that

$$\frac{\sin(2x)}{\cos(2x)} = \frac{4\sqrt{2}}{7} \Rightarrow \sin(2x) = 4\sqrt{2} \text{ and } \cos(2x) = 7$$

is equivalent to the inference that

$$\frac{1}{2} = \frac{2}{4} \Rightarrow 1 = 2 \text{ and } 2 = 4.$$

Again, we need to investigate why even some of the best mathematics students have these misconceptions of an elementary nature. If the examiners frequently comment about Year 11, 12 material and deficiencies in higher order thinking skills then it is an indication of the efficiency of the current education system, but the examiners frequently comment about the lack of conceptual understanding of elementary content such as equivalent fractions. The solution to this crisis cannot be achieved by thinking that students need more than thirteen years of schooling to understand that

$$\frac{1}{2} = \frac{2}{4} \text{ does not imply that } 1 = 2 \text{ and } 2 = 4.$$

To solve these problems, if we just prune the curriculums or simply accept that only at the University level students need to understand equivalent fractions clearly then we simply dumb down future generations.

Are these deficiencies manifestations of lack of practice? A properly designed drill can be conceptually illuminating while simultaneously providing proper practice. If the drill does not contain varieties and twists then it cannot be an effective drill. As cited in Osberg (1997), the Apple Classroom of Tomorrow Project publication “The Imperative to Change Our Schools” has designated drill and practice as a tool of traditional instruction. In contrast, according to this report, constructivist learning replaces drill and practice with communication, collaboration, information access and retrieval and expression. This is too simplistic. A method cannot exclusively belong to any learning and teaching paradigm. Only how and when a method is used may exclusively belong to a particular paradigm.

To end this chapter, let us look at some formulas given with the VCE Specialist Mathematics Examination 1, 2012 (Victorian Curriculum and Assessment Authority, 2012). The choice of these formulas is not an indication that I believe that the other formulas should be in the examination formula sheet.

### Circular Trigonometric Functions

$$\cos^2(x) + \sin^2(x) = 1$$

### Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2}, \quad -\pi < \operatorname{Arg} z \leq \pi$$

### Calculus

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

### Constant (Uniform) Acceleration

$$v = u + at$$

### Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2}$$

### Mechanics

$$\underline{p} = m\underline{v}$$

$$\underline{R} = m\underline{a}$$

$$F \leq \mu N$$

Inclusion of these formulas in the formula sheet is a clear and loud statement of the curriculum philosophy which is seemingly based on "Do not learn, Do not understand". This practice of providing basic formulas may have been motivated by the simplistic, naïve and dogmatic promotion of cognitive skills. In this endeavour, our tube vision has blinded us to the multi fold nature of cognition. When we deal with one side, we lose sight of many other sides. It is vividly clear that if one understands the concept of the absolute value of a complex number then this understanding itself is the formula. The motion equation  $v = u + at$  is also just the

concept of uniform acceleration. I am challenged to understand how it is simultaneously possible to understand the concept of uniform acceleration and not to know the formula  $v = u + at$ . Therefore if we cherish conceptual understanding why do we send subliminal messages to students denouncing relational understanding and thinking?

The formula  $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$  is just the concept of a three-dimensional vector. I wonder how it is possible to understand the concept of a three dimensional vector without remembering or internalising the notation,  $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ . Here is another classic example:  $\underline{p} = m\underline{v}$ . I hope to learn how to understand the concept of momentum without knowing that  $\underline{p} = m\underline{v}$ . Thus the subliminal message that we send to the students is no longer subliminal. The message is a loud and clear demand, "Please do not cognise".

Fortunately, teachers who are passionate about the subject teach it in a proper spirit despite the dictates radiating from authorities. It seems that the current education practices promote pedagogical unawareness. Later in this work, I will incorporate the notion of pedagogical awareness with the Buddhist Concept of Samma Sathi (right mindfulness or right awareness (see p. 225 and 238). I do not know whether it is entirely accurate to think of Samma Sathi as merely a Buddhist concept. All humans need it; for this reason it is a human concept. The term, pedagogical awareness, was introduced to me by Peter Taylor while I was thinking of incorporating Samma Sathi into my teaching practice and philosophy. When Peter talked to me about pedagogical awareness I immediately realised that all concepts are human notions.

In the previous chapter, I reviewed the multiple facets of constructivism. I believe that this multiplicity arises from the very nature of [experiential reality](#). To deal with this complexity, I base my life practice on a rainbow of epistemologies (see p. 110, 113 and 158). Every slogan carries a vision, but mono angled-monochrome tube visions are insufficient for meaningful interaction with nature. In this chapter, I have discussed some simplistic slogans of various forms arising from these tube visions.

In the next chapter, I will discuss how teachers teach. My intention is to link how teachers teach with the multi fold nature of experiential reality. How we think about reality determines, to a large extent, how we teach.

I would like to finish this chapter with the following quip:

*Blindness comes in many forms. Unfortunately, vision could be one of them.*

## Chapter 6

### Teachers and How They Teach

**I think that experiential reality is a multidimensional  
rainbow.**

**Therefore, how I teach is also a multidimensional  
rainbow.**

#### **Reductionism**

In the human endeavour of knowledge seeking, compartmentalisation has played an unequivocal role. Owing to this magnificent device of reductionism, many intellectual giants, such as Newton, were able to produce absolute gems of human cognition. Ironically, the wonders of this device of compartmentalisation have also engendered a defect in the human view of reality. We have become addicts to the device: we let the device rule us, rather than we rule the device. Both the power and limitations of the human mind in dealing with experiential reality have implications for our personal and professional practices. For this reason, before I venture into the theme of this chapter, I will have another closer look at dualities and dichotomies.

We need to keep in mind that compartmentalisation of the elephant by the blind men did not result in a proper view (see p. 149). Before venturing into the duality of instructivism and constructivism, I would like to discuss several other dualities briefly which may help us to appreciate the dualistic or multi fold nature of learning and teaching.

There are many examples that slice-by-slice views of the world could result in distortions, and unification of opposite aspects could marvel even the powerful and useful device of reductionism. To illustrate this, we consider the mathematical notion of “*Instantaneous Rate of Change*”. How can there be an instantaneous rate of change? If the time or spatial variable  $x$  is instantaneous then the dependent variable

y cannot change. That is, if something (say, the primary quantity) is instantaneous then there cannot be any change in any quantity which depends on the primary quantity. Hence, how can there be a rate of change? Nevertheless, the seemingly non-sensible unification of the utterly contradictory terms has paved the path to an absolute gem of human cognition. In the process, we also construct a meaning for zero divided by zero. These values of zero divided by zero help us to understand behaviour of mathematical functions and allow us to model innumerable number of physical phenomena.

### **Wave-Particle Duality**

In Chapter 3, I presented a detailed account of the wave particle duality. At the moment, to my knowledge, there is no single metaphor or imagery that holds the wave-particle duality together. In the current unification scheme of the wave and particle aspects, their distinct identities are still clearly visible. One cannot see light through a monocular lens; if we use the wave lens, we cannot see the particle picture. If we use the particle lens, we cannot see the particle picture<sup>29</sup>. Even though this duality has not been unified in a single metaphor until now, I am hopeful (though this is not the mainstream view) that one day a single metaphor could be constructed. Even if this happens, after a while, another crisis having much more strange characteristics could arise. Also, we need to remember that even though this duality of light relates to how we experience ontological reality the duality itself may or may not be ontological.

However, there are other fundamental dualities that we have coined together, to get a single coin. For instance, consider the table in front of me. There is no doubt in my mind that the table is not a figment of my imagination, and it exists externally to my mind. (Please do not ask me how I know. That is an axiom. As soon as I attempt to justify the assertion above, arguments will start to fly back and forth generating a time wasting futile exercise.) The image I see is constructed in my mind using the light emanating from the table itself. These wave-particles of light make neuronal electrical pulses in my brain that creates a visual image in my mind.

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<sup>29</sup> There seems to some form of objectivity here. The lens used matches the picture we obtain.

Clearly this visual sensation is not the table itself, but it certainly provides me an enormous amount of information about the table; yet, this information may not be ontological. For instance, the colour of the experiential table may not have an ontological existence since it is a perception created in my mind due to ontological qualities of the table and the light source. If my mind is in a 'proper state' then other people will also agree with my perceptions, at least to some extent. This means that socially we have constructed an 'objective' view of the table.

I am aware that my assertion that the existence of the table is an ontological reality can be questioned. As we have already mentioned before, this assertion is merely an expression of my experiential reality. For this reason, some people deny any ontological existence of tables, trees or even the world. I assert that there exists an ontological table in front of me. By this statement, I mean that my sense image of the table is associated with an ontological 'object' that exists in front of me. This sense image I create out of the table is partly forced on me and also is voluntarily made. For instance, you and I both will agree that the table has four legs but you and I may disagree whether or not it is a pretty table.

Also we may ask, does this ontological table exist as it is? Consider the question:

Is this ontological table which I see now, the same ontological table I saw one second earlier?

If the answer is 'Yes', then will this ontological table exist as it is a thousand years later? Our perceptions demand that the most sensible answer to this question is 'No'. Then we can ask, a thousand years later will this ontological table vanish or change as an instantaneous act? The answer is "No". This means that the change is happening all the time. This guides us to revisit the question and change our answer from 'Yes' to 'No'. The time interval of one second we chose in our question was an arbitrary one. We could make the time interval as infinitesimal as we wish and repeat the question. The 'Zeno's paradox' (see p. 197) again confronts us.

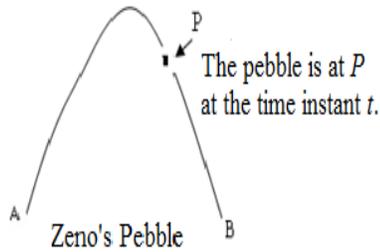
Then this confronts us with a contradiction. The table ontologically exists and it does not exist. How do we deal with this conundrum? To make peace with this difficulty, we need to assume that the ontological reality is a flow or a process endowed with

impermanence. Buddhism centres about the idea of impermanence, but this impermanence of our world is valid not only to Buddhists but also to the world itself. Again this leads to the four-dimensional truth paradigm of Buddhism.

In a normal conversation, I do not say “I cannot see the chair. I can only see its image”. It is evident that my mental image is not the chair itself but again it is not the image of something else; therefore ‘the image’ is nothing else but ‘the chair’. In other words, the image is not the chair itself, and it is not not-the-chair-itself. Here, ‘image’ is a metaphor. In our daily lives, it is natural to ‘equate’ a photo of a tree with the tree itself. This is no different from equating the sensed image of a chair with the chair itself. In most situations, this is a safe practice.

This practice of equating the object with the sensed image has spilled into the arena of philosophical deliberations. The reason why we customarily equate the image of the chair with the [ontological](#) chair is that we cannot experience directly the ontological chair. Therefore, we cannot communicate about the ontological chair. As cognitive beings, we have found an escape; we equate the summative result of what we can commonly share about the chair ([conventional reality](#)) and our own experiential reality with the ontological reality. In most of the situations, this is convenient and has no consequence, but at times this can create problems.

If I am being questioned in a courtroom whether I have seen a particular tree then I will not resort to give dual answers “yes and no”. If I insist on giving dual answers then the Judge will incarcerate me for contempt of court. Ironically, when I give a single answer- it may be straightforward yes or no- then the Judge and the lawyers will evaluate and question my answer. They will look into my sanity, whether I was on a hallucinatory drug, my credibility and ulterior motives before they accept my answer. They themselves will be thinking in dualistic terms even though if I do the same thing, the judge may choose to incarcerate me. In the following example we look further into this matter. I have discussed this paradox earlier. I repeat it here again since I want to look at a variant of it.



Consider the path of a pebble which is projected in air. There is a clear trajectory. In particular, we can see that there is a time instant  $t$  at which the pebble is at Point  $P$ . At this Point  $P$  is the pebble at rest or is it moving? As Zeno, the ancient Greek scholar,

pointed out it is impossible to answer this question.

If we say that the pebble is at rest then how does the particle stop and then start again to keep moving? On the other hand, if the stone is not at rest at the point  $P$  then what does this path mean? Why do we even say that the stone is at the point  $P$  at the time instance,  $t$ ? This duality or conundrum (say, the Stone Conundrum), is something that we have to live with at the moment. Future theories of space and time may be able to explain this.

From this well-known conundrum, arising is another conundrum. Let us name this new conundrum the Second Stone Conundrum. If one throws a stone at a person then the stone cannot hurt him since the stone is momentarily at rest at the very moment of contact. Then again since the stone is also moving at the moment of contact, the person gets hurt. This situation resembles the wave particle duality and also is similar to Shcrödinger's Cat Conundrum. The stone simultaneously hurts and does not hurt the person. This can be applied to vehicle accidents. Vehicle accidents simultaneously kill and do not kill their victims. In reality, we know that the vast majority of the vehicle accidents hurt or kill the victims. This conundrum demonstrates the incompleteness of our views of reality.

### **Paradoxes of Duality**

In the Schrodinger's Cat paradox, a cat is placed in a closed cubicle. In this cubicle, there is a single atom of a radioactive element of which the atom can decay within an hour with a probability of 50% and emit a photon (energy pulse). This photon or energy pulse activates a switch that will release a poisonous gas into the chamber. Recall that according to the wave particle duality, as we discussed in Chapter 3, it is the observation that determines whether a particle moves on a definite path.

According to the same principle, whether the cat is killed or remains alive is eventuated by an observer's looking. That is, only an act of looking into the chamber determines the fate of the cat. Until we look, the cat is in a superposition of Death and Life. This paradox was constructed to demonstrate the incompleteness of the Copenhagen interpretation of wave particle duality or Quantum Mechanics.

To resolve the Schrodinger's Cat paradox, as cited in (Penrose 91, p. 295) in his PhD thesis, Hugh Everett suggested the following hypothesis: at the event of decaying or non-decaying of the radioactive particle, the Universe splits into two Universes; in one of them the cat lives, and in the other cat dies. We realise only one of these universes. This is the many-world-interpretation of Quantum Mechanics. In the case of the Second Stone Conundrum also, we may say that the world splits into two worlds; in one of the worlds, the victim gets hurt and in the other the victim escapes any injury. We cannot experience both of these worlds simultaneously. Among all the accidents that have occurred in the world that I reside in, I have never heard of an accident in which the victim has experienced zero momentum. Maybe the victim does not get killed or injured but surely the victim experiences the momentum change, impulse.

It may be possible to resolve the Second Stone Conundrum by assuming that there is the smallest amount of time and space there can be. For my knowledge, such theories have not yet been fully developed. Detailed discussions on the paradox of Schrodinger's appear in literature yet there is no accepted resolution. It is difficult for me to fathom how Physics considered such a metaphysical idea of many-world-interpretation. Then again, what is metaphysical today can become a hotly pursued scientific research topic tomorrow.

Even though we have trained ourselves to think customarily in terms of mere dichotomies it is evident that we require a much more holistic model. For example, it is not exactly correct if we say zero cannot be divided by zero. Also, it is not true that if we say zero can be divided by zero. What is correct is that both the statements are true at the same time. Then in the synthesis of the total opposites there springs up the marvel of the derivative.

This chapter is about teachers and how they teach. In the introduction above I discussed about the dualistic, paradoxical and conundrum nature of our views of [experiential reality](#). Now we will discuss how these considerations feature in our learning and teaching practices.

### **Teaching: Instructivism or Constructivism?**

The reductionist and dichotomous views of reality have produced marvellous human cognitive gems. Denial of this will amputate our philosophical legs barring the journeys of inquiries. At the same time, if we do not acknowledge the limitations of reductionism, then we are carrying heavy baggage that hinders the journeys of inquiries. Likewise, I argue that the enforcement of mutual exclusivity of frameworks of teaching, instructivism and constructivism is nonsensical. I contend that proper and coherent incorporation of these opposites can give results of a much more inclusive learning- teaching paradigm and much more effective methodologies. Recall that we cannot synthesise the two opposites, non-divisibility of zero by zero and divisibility of zero by zero, in an arbitrary manner. They need to be synthesised in a coherent and a natural manner.

Then again, I think incorporation of instructivism and constructivism is mere reinvention of the wheel; I know, by my own personal experience and going by the historical accounts of illustrious teachers, this synthesis existed and still exists. Nevertheless, this reinvention is what should happen for betterment of education. To justify my point of view that effective teachers existed throughout history, I review the practices of a few exceptional teachers in human history. I will strive to show that good teaching has always been a nutritious soup of instructivism and constructivism. If a soup contains only one ingredient, then it cannot be that nourishing.

An observer, at times, may see that the soup contains just the constructivist ingredient; at other times he may recognise just the instructivist ingredient; on most of the other occasions he may not be able to see what isms are in the soup. To a non-compartmentalising eye, the soup is nutritious and delicious. Given that the soup is healthy and appealing, then in a way it does not matter what ingredients are in the soup unless there is a cultural or dogmatic aversion to one of the nutrients.

Nevertheless, if one wants to make such a soup it essentially matters to know how to make the soup. We need to remember that this is not a soup like a coca cola bottle; different sizes but the same soup. The nutritious education soup which I cognise is not a soup that is going to be made over and over again with the same recipe. Therefore, a cookie type recipe does not measure up. We need to construct a conceptual framework of how to think about making nutritious educational soups. To make such a soup of teaching and learning we can learn from master teachers. I intend to understand how the masters thought about their soups rather than how and with what they made their soups.

### **Buddha as a Teacher**

One of the greatest teachers of all time, Buddha, is known to have employed various teaching techniques. Again, I do not examine these techniques with the intention of learning how to teach. My intention is much broader; I review them to understand how to think about the process of teaching. As remarked in (Block, 2010), whatever the method or technique he used, Buddha's teaching was solidly based on the primary principle that teaching should make sense to the student, not merely to the teacher. The Kalama people in an Indian village were confused by the contradictory stances of many visiting religious personalities. The various personalities visiting them disdained other religions and glorified their own. When Buddha visited them the Kalamas wanted Buddha's help to resolve their confusion. Buddha said to them that it is the inquiry that should guide them not the trust one puts on religious or other authorities or ancestral beliefs. This can be summarised as "come and see for yourself". This self-reliance is the most basic principle of his teaching. Even though I was fascinated by the freedom of thought Buddha acknowledged, being a student of science, for a long time I did not understand his utterance, "Do not accept an assertion merely because it seems to be logical". In my secondary school years, I used to question this statement. Only many years later, I understood that logical reasoning is only as good as the set of initial premises even if the logical reasoning is impeccable.

Buddha insisted on self-reliance as the foundation of understanding, yet he tirelessly made many journeys of preaching around India, even up to a ripe old age. This may appear to be self-contradictory. If self-reliance had been the sole basis of his teaching

why did he choose to preach? This question arises only if we believe that the principle of self-reliance of the student excludes preaching of the teacher. This demands re-examination of our beliefs in teaching. This re-examination needs to seek integration of 'preaching' and self-reliance into a more effective way of teaching.

***Kisa Gothami: A Marvel of Constructivist Teaching***

This is a story related to Buddha's time. Kisa Gothami was married was childless for some time. She was ridiculed by her in laws for not being able to bear a child. Several years later, a son was born to her and with that good fortune returned to her. When the child reached walking age he suddenly fell ill and died. Kisa Gothami, refusing to accept the death of the child, carried her child from one house to another seeking for a cure for her son. No amount of reasoning made any difference to her belief that her son was merely sick. Finally, a gentleman directed her to Buddha for a 'cure'. She visited Buddha and requested him to treat her son. In this occasion, the preacher chose not to preach. Instead, he promised to cure the child if she could bring just a handful of mustard seeds from a house in which none of the relatives of its occupants had ever died.

Delighted Kisa Gothami walked from house to house asking for mustard seed. People willingly offered her mustard seed. Then she would ask the people at the house "Has anyone related to you died ever?" she continued her search for many hours. Every time when she was just about to receive mustard seeds with joyful heart she was repeatedly told many stories of death. Hearing about many deaths from different people she was able to come to terms with the death of her son. After burying her son, she returned to Buddha to learn Dhamma.

When Kisa Gothami pleaded with Buddha to cure her dead son the illustrious preacher chose silence. Whether it was silence or preaching he chose, his teaching was based on the tenet of self-reliance and self-realisation. Whatever he chose was merely an instrument that enabled his disciples to achieve understanding. It was not the method that was primary; it was the understanding. For instance, Buddha who preached to whole villages and towns, in this case, chose to send the lady in search of handful of mustard seeds. Why? The reason was the Kisa Gothami's dire

psychological state. She did not have any wish or will to contemplate on matters of life and death while she was holding the dead body of her son.

Kisa Gothami's situation was more complex than mere maternal grief. With the death of her son she lost her status among her in-laws. Kisa Gothami was grief stricken to the extent that she was unable to think clearly. Therefore, Buddha created a situation which compelled Kisa Gothami to go through a cycle of hope (almost getting mustard seeds) and despair (having to refuse mustard seed). Eventually, it wore out her psychological resistance to the message which she was receiving. I remember that when I heard this story as a child I disbelieved it thinking that how could Kisa Gothami be that naive. Whether or not Kisa Gothami's story was a creation, relatively recently I myself realised the deep psychological insight contained in this story.

I have gone through a great deal of physical, psychological and emotional torments throughout my life. From a very young age I have been preparing myself for death of any of my loved ones and even my death. My close friends and loved ones consider me to be an extremely tough cookie. In year 2010, I spent some time at my mother's death bed. Before she passed away I returned to Australia. I frequently called my sister who was looking after Mother. I had been calling my sister frequently. A week after my return to Australia I talked to my sister again. Ten minutes after this conversation my sister rang me. The news was sad. The words came out of my mouth were: Are you sure? Go and see again. After this conversation I was waiting to hear that they had made a mistake.

In these few minutes of agony the story of Kisa Gothami came to my mind. It surprised me that decades of contemplation of my own death and others had not prepared me for the event. It might have been even worse without this preparation than otherwise. Nevertheless, for a moment, I totally lost my rationality. I was able to recover quickly, and I continued with teaching of my tuition class that I had been conducting at our home as if nothing had happened. I consider this was the best tribute I pay to my Mother as she always wanted me to carry out my duties faithfully. Now I know that the story of Kisa Gothami was not an exaggeration.

Even though Buddha himself did not preach to Kisa Gothami, he created a situation in which many different people would talk to her about the naturalness and inevitability of death. Did not Buddha also, to create a constructivist learning environment, use talk? Consider the instructions: bring me some mustard seeds from a house whose occupants have no close association with death. Also, Buddha created a cycle of hope and despair to wear out Kisa Gothami's psychological resistance. The teaching approach taken by Buddha in this story exemplifies the modern and ancient crux of true constructivist teaching. In this teaching episode, walking (from house to house) talking to many people and contemplation were all significant aspects. Each of these aspects fed into the other.

Another occasion, while Buddha was preaching to a large group of people, several men hurriedly came and sat down among the crowd to listen. Perceiving that the guests were starving, he immediately ceased the sermon and asked his disciples to provide the guests before continuing with his sermon. Then Buddha explained that for understanding and learning to occur, firstly physical needs should be satisfied. The audience needs to be in a proper state of mind. Again, looking back at Kisa Gothami's Story, we can see that Buddha first helped her to gain proper mental state before starting with further discourse.

### ***Story of a Student Turned to a Mass Murderer***

In Buddha's time there was a brilliant student. His teacher was obnoxiously jealous of the student. At the end of learning the teacher asked for a gratuity. To create a situation in which the student would be in peril he asked for a necklace of thousand right index fingers of newly dead people. This turned the student into a mass killer. He began to kill people who were going through the nearby forest. Then he would cut off the index finger of the right hand of the victims. He would create a necklace of index fingers to offer to his teacher. Wearing this necklace he continued to kill. The word Anguli means fingers and Mala means necklace. So this murderer student received the name, Angulimala. Buddha hearing the story of Angulimala decided to save him. Buddha entered the forest.

Assuming that the monk would be an easy prey, Angulimala began to chase Buddha and asked him to stop. While walking Buddha replied, "I have already stopped, you

stop.” The obvious lie, which came from a monk, surprised the violent Angulimala. The fearless, compassionate yet authoritative manner of Buddha made his intriguing utterance “you stop, I have already stopped” might have shocked Angulimala. He asked: “You are a monk. Why do you lie?” Buddha answered, “I have stopped because I do not sin and you have not stopped since you continue to sin.” Angulimala recognised that the person in front of him was the famous preacher, Buddha. This recognition and the effect of Buddha’s thought provoking statement caused Angulimala to calm down and engage Buddha in a peaceful manner. At the end, Angulimala became a disciple. He became a monk by the name Ahinsaka, which means a person of nonviolence.

In this narrative, knowing the intellectual potential of Angulimala, Buddha used a seemingly contradictory statement to engage Angulimala. The technique was different from his regular teaching mode of preaching and the approach he took with Kisa Gothami. Would it have worked if Buddha told Angulimala to go away and first survey the people in the village to see whether killing was acceptable and then return to kill Buddha? Even though Buddha did not use preaching mode at the initial stage of his engagement with Angulimala, he used ‘talk’ to say "You stop. I have stopped." Talking can be a powerful tool of cognitive engagement.

Now, let us look at another story.

### ***Teaching with Fables***

A monk was resentful since Buddha was silent about questions such as the origin of life and the Universe. One day he threatened to leave the order unless Buddha answered his questions. Buddha asked him to think of a physician and his child. The child was wounded by an arrow. When the physician was trying to take the arrow out and treat the wound the son objected asking questions such as “From where did this arrow come? Why did someone shoot me with the arrow? Of what material is the arrow made? What kind of bow was used to shoot me? I will not accept any treatment until you answer all of these questions." Then the Buddha told the monk that he was acting like the wounded child. Buddha’s use of this parable as a tool of teaching is universal for all religions and all teaching.

### ***Taming a Demon***

In another story, the Illustrious One happened to be confronted by an extremely callous and obnoxious person named Alawaka. According to the literature, he was a demon. Buddha decided to help him and visited his residence. Alawaka invited Buddha in and offered a seat. As soon as Buddha occupied the seat, in rude language, he asked Buddha to leave. Without uttering a word the Illustrious One began to leave. As soon as he reached the door the demon boorishly commanded Buddha to come back and sit. As soon as the Illustrious One did so the demon asked in filth, him to leave. Up to this point Alawaka would have glowed with increased self-esteem that the famous preacher who had won the reverence of many was under his thumb and followed his rude orders without any protest.

The demon ordered the Illustrious One out for the third time but he did not move. This unanticipated resistance shocked the demon. He asked why Buddha refused his orders as Buddha meekly followed his dictates the first two times. The Illustrious One replied: whatever I did, I did on my own accord. In my mind I did not heed to anybody's commands. Here, Buddha was pointing out to Alawaka the difference between their perceptions of the same event. Alawaka tried to dent Buddha's composure by abusing him. Buddha's meek obedience (as perceived by Alawaka) on the first two occasions and then the gentle but steadfast refusal the third time might have surprised and put the demon out of balance. Alawaka was thinking that Buddha was following his orders but the demon suddenly understood that Buddha was acting according to Buddha's own wishes. With that exchange Alawaka came to his composure and was ready to listen.

Once again Buddha, instead of preaching, chose to create a situation in which Alawaka would become more receptive and thoughtful. At the end Alawaka also became a disciple of Buddha. Buddha's constructivist teaching was multi-folded and based on creating situations for his disciples to gain understanding. His approaches included preaching, storytelling, debating and activities of other forms.

As I was reflecting on the Alawaka story suddenly I realised that there is a link between this story and one of my university experiences. As a child from a Buddhist background I have heard and read the Alawaka story numerous times. Nevertheless,

until now I have not realised how its psychological principles had deeply sunk into my mind. The following story will exemplify this process of learning by assimilation.

Sri Lankan Universities were (and still are) plagued with ragging. According to senior students, during the first few weeks they have a 'natural' right to violate the human rights of newcomers. Before I enrolled I wrongly believed that ragging was innocent, and only the Government Authorities made a fuss about it. The general feeling among the students about ragging was that anybody who could not tolerate ragging was an unsociable and weak hearted coward. These attitudes may be still prevailing even though a few students campaign against ragging. A number of deaths that occurred in Sri Lankan Universities due to ragging in the last four decades has highlighted the magnitude of human rights violations.

For the first couple of days, I followed the orders given to me by seniors because I believed ragging was just innocent fun. However, soon my resentment began to grow. The commands given to me were mild and funny in nature except for the rudeness, authoritarianism and abusive language of the ragging students. However, following these commands began to bruise and bury my dignity. It became clear to me how young girls and boys could be molested and violated by a few sadistic ones. The acceptance by the mainstream majority that 'mild' forms of ragging are humane provides protection for these gross abuses of human rights. This is the saddest thing about ragging. I began to feel that even the milder forms of ragging were inhumane and humiliating and that any person with any dignity cannot obey these commands.

By the third day my resentment turned into strong resistance. After another couple of days seniors noted me for refusing and humouring their commands. To counter and frustrate their aggression and efforts I used wit, sarcastic smiles, loud laughs and humour. When they were just about to be violent I pretended to obey them. When their anger was gone once again I began to humour them. All of these confrontations were in open places.

This attracted the attention of more rowdy and leading elements of the raggers. By this time new students began to avoid walking with me due to the fear of reprisals. A

known senior student informed me that a gang of students were to visit my university dormitory room in the night to teach me a lesson. The friend tried to act as a peacemaker. To make peace I had to give up my dignity. I refused and was adamant that they should shed their inhumane attitude and violent behaviour.

When the gang came I asked my roommate to leave so that his presence would not distract me. The gang threatened (bluffed) to kill me and put my body in the nearby river. I listened. They ordered me to creep under the bed. I obeyed but as soon as I went in I came out. They furiously asked me “why did you come out”. I went in just because I wanted to and I came out also because I just wanted to.” This answer flabbergasted them. They again ordered me to creep under the bed. I played the same game. Third time I refused their order out rightly. In a few minutes they abandoned their mission. After that all senior students left me alone. To avoid being ragged new students preferred to walk with me.

Only now I realised that I had acted in the way I did due to the influence of the Alawaka story. Beneath my awareness and without any analysis I had learned a great deal from the story. This was due to the frequent reading and hearing of it many times. This supports my belief that a great deal of learning and understanding can occur at a subconscious level, at least to some extent. This is an example of learning by assimilation or cultural immersion. If we incorporate these subconscious processes with our awareness then certainly we can become more powerful thinkers, learners and teachers. Some people might equate learning by assimilation to learning by osmosis. Even though I agree that assimilation and osmosis need to be aspects of learning and teaching programs, to achieve significant success they need to be incorporated with pedagogical awareness. After writing the Alawaka story now I feel that I have achieved a great deal more than what I did achieve by frustrating my aggressors several decades ago. This is the power of awareness and cognition.

### **Jesus as a Teacher: Stoning the Sinner**

Recall the story of “Stoning the Sinner”. When a crowd of people was just about to throw stones at a sinner Jesus did not ask the perpetrators to stop. If he had asked then they would have stoned the preacher and the woman both. Instead, Jesus asked anyone who has never sinned to come forward and cast the first stone. These were

simple words but the power they impacted was enormous. Jesus created a situation in which his audience reflected on their lives and human fallibility. This is much stronger than asking them to stop. To construct such a conducive learning environment he used speech: cast your stone if you have never sinned. This is not plain speech made to inform. It is powerful speech made to engage. It is also a well-known fact that Jesus also chose instructivism as a powerful teaching mode.

The use of parables by Jesus in his teaching speaks volumes about the universality of teaching and learning. Instead of directly informing his listeners he used parables enriched with visual and mental images to motivate his audience to work out the message. This is constructivism. It is easy to see a parallel between the teaching approaches of Buddha and Jesus. The famous parable of Prodigal Son in the Bible is an excellent example of use of parables and metaphors in teaching.

Did Jesus use only parables? He gave direct instructions too: Love thy neighbour.

### **Socrates: The Celebrated Teacher**

Effective teachers of any time share a common thread with Socrates' teaching approach. Socrates' idea of teaching is to get students to think for themselves. Buddha's notion of self-reliance and Socrates' notion that students should see it for themselves are essentially the same. When students went to Socrates seeking knowledge, he tried to avoid answering directly the questions asked of him. Instead, he would rephrase or ask another question, making the question and the 'answer' clearer to the mind of the student. If this did not work then he would ask another question. He derived this new question from the student's answer to the previous question. This eventually enables the student to reach an answer to his original question. By the end, Socrates had never answered; he had only questioned.

This approach empowered his students to understand the issues they discussed. Socrates, the master teacher, to achieve this feat did not even use chalk; he only used talk. At the end of a discussion Socrates used to mock his students by asking "Did I really teach you anything today?"

The following is an imagined exchange between a student and Socrates.

*Student: Is murder wrong?*

*Socrates: According to the state law?*

*Student: According to the state law it is a crime. I know that, but why murder is wrong regardless of what the state law says?*

*Socrates: Well, What do you think? Is murder wrong?*

*Student: Yes, it is wrong.*

*Socrates: Then why do you ask me. You already know. Please enlighten me why murder is wrong?*

*Student: Because it is wrong to kill someone.*

*Socrates: You are merely reiterating that murder is wrong. On what ground, is murder wrong?*

*Student: Because it is wrong to take someone's life.*

*Socrates: You are just repeating that murder is wrong. Why is murder wrong?*

*Student: Murder is wrong because religious authorities have said so.*

*Socrates: Why do religions say so?*

*Student: Murder is wrong because everyone wants to live.*

*Socrates: Why is murder wrong merely because none of us wants to die?*

*Student: If murder is right we will keep killing each other; consequently our species will not survive for long.*

*Socrates: Why do you think murder is wrong merely because our species might become extinct if we accept murder is right?*

*Student: Because that is the will of God for us.*

*Socrates: Why do you think murder is wrong merely because it is God's will?*

*Student: Because He is our creator.*

*Socrates: Again, why is murder wrong merely because God is our creator?*

*Student: I do not know sir because whatever the answers I come up with you have another question to ask. So I am confused.*

*Socrates: What are you confused about, whether murder is wrong or why murder is wrong?*

*Student: It is clear to me that murder is wrong but I am not certain why it is wrong?*

*Socrates: You cannot pinpoint to a definite reason why murder is wrong but still you strongly feel that murder is wrong. Am I right?*

*Student: Yes.*

*Socrates: Can you give me a name for that kind of a thing? You strongly feel for it but there is no clear reason why?*

*Student: A belief?*

*Socrates: Now you are clear that “murder is wrong” is a belief. In your responses above, you have expressed some reasons why the vast majority holds on to this belief. Now, is murder always wrong?*

*Student: Not in the case of self-defence.*

*Socrates: How do you know that in such circumstances murder is not wrong?*

*Student: Please, not again, Sir.*

Some may wonder whether this imagined exchange between Socrates and the student is an example of constructivism or instructivism. Socrates used only ‘talk’ here. He did not even use ‘chalk’. There is no fancy breath-taking showpiece ‘activity’ in this learning task. The teacher is at the centre of the stage asking questions. The student merely answers the questions. According to the tube vision (looking through a narrow tube) from this particular angle this activity cannot meet the criteria of a constructivist lesson. On the basis of this vision, the Father of Constructivism, Socrates had never delivered a constructivist lesson.

From another point of view, we can argue that the student is at the centre of the stage and the teacher is only a moderator. From this angle, this lesson can be labelled as a constructivist lesson.

I see that the exchange above is an example of a deep teacher-student engagement in which both constructivist and instructivist aspirations are intermingled and interlaced. For instance, when the student asks the question Socrates does not ask the student to figure it out for himself. Instead, Socrates strives to motivate the student to think about his own thinking. Looking from this angle, it is not possible to say who is at the centre stage; both are at the centre stage. Each of them mirrors the other’s thinking. The teacher needs to mirror the student’s thinking since he needs to design the next question. The student has to mirror the teacher’s thinking since he needs to answer the teacher’s question. They are both in a journey together.

At the end of the conversation Socrates tried to focus the student's attention on the claim that murder in self-defence is not wrong. The student refused to take the bait. He may have understood that the belief that killing in self-defence is not wrong on the basis of the belief of the right to live.

### **My Teachers and the Teacher Me**

Teachers' influence on their students can be life-long even if it fails to manifest during the time that the interaction is taking place. Also, the same teacher can have different effects on the same student at different times. For instance, I have experienced both apathy and inspiration with the same Physics teacher. I regard my Year 9 Physics teacher, Mrs Kiribandhane, to be the most talented teacher I have ever met. During the four months I was in Year 9B, her magic did not enthral me. Later her teaching inspired me in a most enchanting way (Please see p. 51).

Even though the teacher and her approach were the same, the learning, understanding, intellectual entertainment that occurred were different during the two different intervals. What was the difference? The difference was me. When I was in 9B I did not pay close attention to the teacher. Then in the same year, about in September, I changed the class again to do a more Advanced Maths course. This time I had a different Physics Teacher. The difference between the two teachers was immense. In that Physics Class, I did not achieve much learning, understanding or intellectual entertainment, but the influence of Mrs Kiribandhane saved my day, not only in Physics but also in other subjects as well. Even though now I am aware of Mrs Kiribandhane's contribution to my life, I might not have realised the real magnitude of her influence during the time I was her student.

This experience convinces me that teachers are immensely powerful and helplessly weak in influencing students. If the student is not ready then the magic goes to the waste bin. I consider this as the First Principle of Teaching and also the Foundation of empowerment of students. In short, as the old cliché goes, we can bring water to a horse but we can never make it drink. While admitting that there are many things teachers can do to motivate discouraged and unengaged students, ultimate responsibility lies with the student. This statement could be very dangerous if it were to be interpreted as teachers cannot do anything to change a student's life. The

essence of the paragraph above is that any endeavour to influence a student's life needs to start with the admission that this change can happen only if the student is willing.

As soon as a teacher believes in his or her own magic the magic of the student can begin to fade away. A significant part of Mrs Kiribandhane's magic was to believe in my own magic. To produce her own magic and bring out the magic in me she did not take just a laid back approach or a mere spoon-feeding approach.

To be an effective teacher our teaching practice needs to be based on awareness. This awareness encompasses many dimensions. Personal, epistemological, pedagogical, subject awareness and practice awareness (awareness of how one practices as a teacher) are some examples. I want to relate an anecdote from my practice. This episode was a rude awakening for me. Our prejudices and prejudgments may blind us to our own practice. Some time ago I came across an article which hailed storytelling as a magnificent tool of teaching. This view simply generated intense disdain in me.

### **Storytelling**

One day a colleague told me that she has heard that I was a good story teller in the classroom. She wanted me to help her to develop storytelling aspect in her teaching. To this comment my response was surprise and shock. How can something I overtly disdain be a part of my own practice? How can I even be good at it? I asked her how she got that idea. Some of my students were her source. This surprised me even more. I was silently thinking that I had erred in teaching. Since I was not aware of this trait of mine I was not able to help my colleague on this aspect. I was almost defensive and dismissive. Rather than focussing on how students perceive my performance I just focussed and analysed my classroom teaching as I saw it. Not surprisingly, I could not detect any storytelling in my teaching.

Am I a storyteller? The question kept me languishing for a few days. The answer came a couple of weeks later from another student of mine. This student had no any association with the students that my colleague was referring to. I did nothing to prompt the discussion. The student voluntarily told me that my teaching of both

Physics and Mathematics were stories that invite and require students to construct the story themselves. My dismissive and defensive thoughts began to rise but I listened and asked the student to explain for his reasons for this perception. “Connectedness; one thought leads to another. This connectedness encourages students to make predictions of the next line of the story,” the student illuminated me. Also, his information relieved me since what he said was in line with my wilful practice. Still in my mind the idea of storytelling was merely their perception. Though I did not openly dismiss my student’s opinion, I was silently defensive rather than openly inquisitive.

How was it possible for two independent groups to bear the same perception? This intrigued me and strengthened my resolve to keep a watchful eye on my teaching. Because the confirmation came from an independent source of the first informer, I decided to have a closer look at my practice along the lines of my student.

Within a week I felt ashamed of myself. *I have been sleep walking in my own teaching practice. I was walking but I did not know that I was walking.* The two sentences in italics can also be interpreted as telling a story because I am attempting to stir my readers’ mind with visual and mental images. Because of the use of tunnels and mango trees and pebbles, my presentation on Quantum Physics (see p. 91 in Chapter 3.) can be thought of as a story. Since I was not aware of my own practice, I missed an opportunity to help one of my colleagues. Consequently, I determined to wilfully develop this story telling approach. *Then come to think of it, that is what all teachers do.* They all tell stories to students with a conceptual and cognitive content, inviting, encouraging and motivating students to interact, interpret, reflect, fill in the missing information and entertain with that content.

Then I faced a big dilemma. How was it possible for me to use storytelling effectively in my teaching while I was condemning storytelling as a modern gimmick? This, I say, is prejudice.

After I began to pay attention to my practice the story telling trait became more purposeful and penetrative. Then I linked the role of storytelling in our species to our teaching practices. As a species we have evolved with storytelling. Our religions

have been using techniques of allegories, metaphors and storytelling for centuries. My students and I, regardless of our different ethnic and cultural backgrounds we come from are creatures bonded with this common thread of humanity.

How does a good storybook glue us to its story? The storyteller tells the story in such a manner that the readers themselves can construct the next page of the story. The readers long to see whether their predictions come true. This attaches them to the book. If the readers are always right then the glue becomes un-stickier; the interest of the reader disappears. On the other hand, if the readers' predictions are always wrong then the reader begins to feel the story to be unbelievable. To keep the reader interested the story needs to be told in a manner that sometimes the reader is right and at other times wrong. This is true even for a story like Harry Potter. This unbelievable story is true to the readers within the pages of the book.

A good lesson can use this storytelling approach to enhance students' cognitive involvement. For instance, while telling the dark story of light (see p. 91), the story teller needs to pause and ask questions. To answer questions or to contribute to the story the only information that the listener can use is the information given in the story up to that point.

Then again, if we push this storytelling aspect too far without reflecting on the intended pedagogical outcomes the story becomes a cheap paper back filled with sentimental and adrenalin overladen mental junk food. On the other hand, in my own practice I had focused only on pedagogical expectations without paying attention to the methodology that I employed. That was the reason why I felt that I was being negatively criticised by the positive comments of my colleague and the student. If I had been aware of my own practice then I could have engaged my student and the colleague in a fruitful conversation about storytelling.

### **Storytelling of My Teacher**

Human knowledge is also an unending and evolving story with many twists, tricks and chapters. Mathematicians and scientists often wait in suspense about what would happen next, thinking how to write the next lines in the story. Consequently, in essence, all good teachers are good storytellers. Even though until this moment I

never thought of it like that, Mrs Kiribandhane was telling me stories. I felt so enchanted by her stories since I was a part writer and a part listener. Then again that part was a whole. If the next few lines I added to the story were not consistent with the overall story constructed so far then she used one of the three approaches as it was demanded by the situation:

- Guiding (not telling) me to pay attention to the contradictions between my new story lines and the story lines in the story so far developed.
- Revealing the experimental evidences that she intentionally withheld up to the point.
- Asking questions to enable me to see the logical errors I had made.

At the end it was not my story; it was not her story. It was a great human story that has been retold and rewritten many, many times. Still, she and I both have an un-copyrightable (in the commercial and in some intellectual sense) authorship of the story.

Her hope was for me to go on this path and someday create an intellectually copyrightable grand new story. That did not happen in the level that she had hoped. Nevertheless, when I teach what I want to achieve is to create feelings of:

- wonder,
- naturalness, and
- cognitive entertainment

as I had experienced as the student of my beloved two teachers.

In the next chapter, I will present a set of aspirations that guides me in my teaching. I believe that all effective teachers I happen to know, wittingly or unwittingly, must have subscribed for a similar set of aspirations. I will not think of this set of principles as a set of what to do and what not to do. Instead, I regard them as a set of principles of how to think. "How to think about teaching" may invariably result in implications for what to do and what not to do. However, these 'what to do's' and 'what not to do's' must be dictated by our aspirations and expectations. Otherwise, our aspirations and expectations will be determined by our clueless practices. There are no designated constructivist or non-constructivist methods of teaching. There are

only constructive ways of teaching. Any method or tool, in the hand of a constructivist teacher with pedagogical awareness and aspiration, are effective constructivist methods. If any so-called designated genuine constructivist teaching method is used without binding it to intended pedagogical outcomes then it could harm the learners and the learning environments.

## **ARC FOUR-ONE: CULMINATION; A BRIEF MOMENT ON A DWARF SUMMIT**

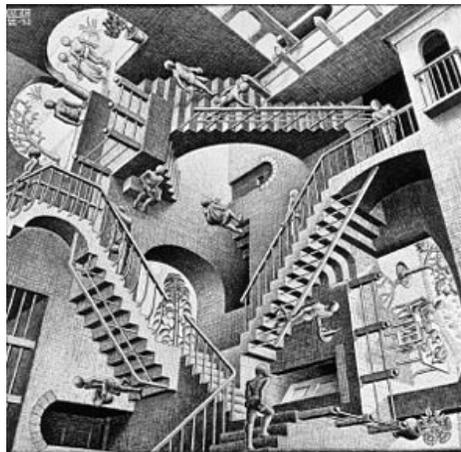
Naming of this section of my work as ‘culmination’ has some merit because of the effort I have put in and the insights I have gained. These insights tell me that this is not a summit of achievement or a permanent rest place.

Nevertheless, the three chapters in this section are a culmination of the previous chapters which bind them to the previous chapters and complete the arc. In a way, the beginning is at the end and the end is at the beginning. It has been a journey of a wanderer. I feel that I am still at the beginning since I am still inquisitively wandering. Starting from inquisitiveness, I have reached inquisitiveness on a higher plane. Therefore, the end and the beginning are the same while they are distinctively different.

In Chapter 7, I present the main theme that constructivism is not about what to do or what not to do but that it is about how to think about our world and learning and teaching of our world. This how to think about the world and learning and teaching about the world is illustrated in some lesson ideas presented in Chapter 8.

In Chapter 9, I look over my shoulder and realise that my journey has been a spiral arc which binds the end and the beginning in a strange manner as the Mobius strip does or a drawing of M. C. Escher depicts.

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## Chapter 7

### Instructivist Constructivism or Constructivist Instructivism? Coining Many Facets into One or Splitting One into Many

#### The Coin

The coin I had  
Beautiful and smart  
To see the inner beauty,  
The coin, I did split  
Seeing no interior  
I split the split  
(2011)

In this chapter, my motive is to develop a set of aspirations to guide my learning and teaching practice. I earnestly hope that these aspirations can help other learners and teachers. I intend to develop these guidelines on the basis of my rainbow epistemology that I outlined before (see p. 110 and 113). Before I do this, I will endeavour to discuss the pitfalls lying on my path to take me down:

- addiction and reverence to past or present
- addiction and aversion to technology (I will discuss only about addiction because aversion is not common)
- naive notions of self-reliance

After this discussion, I will discuss about the notion of pedagogical awareness which is of primary importance for both the teacher and the learner. Then I will build on the ideas discussed so far to construct a set of guidelines and aspirations for my learning and teaching practice.

In the previous chapters, we discussed that dichotomies and compartmentalisation can help us understand nature. Based on this understanding, to deal with challenges offered by nature we can develop effective methodologies. Also, dichotomies and compartmentalisation can distort our view and understanding of experiential reality. Consequently, this can hinder our dealings with nature. Since experiential reality is

multilayered and multisided, single angled tube vision is impotent. To overcome this inadequacy, I presented a constructivist epistemology in previous chapters, which is a plethora of epistemologies woven together. In the previous chapter, I contended that good teaching in any era must be of a constructivist nature. Such good teaching should have based on a plethora of epistemologies and methodologies.

The statement that effective teaching existed long before our time does not mean that we cannot advance the effectiveness of teaching any further. Essentially, to improve our teaching effectiveness further we need to cherish rather than disdain our past. As time progresses many paradigms are constructed. Therefore, the net result of time progress may be an improvement of the practices in any domain if the parties with vested interests do not manipulate the social conscience to their advantage and allow the natural zigzag evolution of knowledge to take place. That is, for a relatively short duration things can deteriorate due to sincere and honest errors. This innate zigzag nature of knowledge evolution can be derailed by groups with vested interests. Usually the natural sciences, such as Physics, Chemistry, Biology and Mathematics, etc. are mostly immune to such distortions but the social sciences such as education theory and practices and medical science (which is not exactly a social or a natural science), are significantly affected by these vested interests. For instance, deliberately distorted Mathematics or Physics cannot produce profit but wrong medical practices and wrong education policies can make money.

In any field, there can be short periods of 'decay'. This 'decay' itself is a 'progress'. Consider the zigzag story of light (see p. 91). Experimental results can be misinterpreted and/or experimental data can be misreported for various reasons, intentionally or accidentally. As I have earlier mentioned, this fallibility is relevant for social sciences more than it is for the natural sciences. Moreover, even in natural sciences, some scientific knowledge may be inferior to traditional knowledge. Here is an example. While many old medicinal paradigms 'knew' that there was a door to the body through the skin, until about fifteen years ago Medical Science and Biology steadfastly clinched into the paradigm that un-punctured human skin acts as an impenetrable barrier between the body and the environment. While traditional medical practitioners used ointments modern medical practitioners ridiculed them. Now modern medical science itself uses nicotine, contraceptive and insulin patches

on the skin to reach the body. Cultures such as Sri Lanka knew this perhaps for more than a thousand years.

Is there something that we can certainly call Past? Is there something that we can definitely call Present? Is there something that we can unquestionably call Future? Conventionally yes, but every Past once had been a Future and then a Present. Each moment of Present once was a Future and soon will be Past. Every moment of Future soon will be Present and then Past. Once we understand this illusion of the flow of time there will be no glorification of Past or Present and there will be no adherence to the Past or reverence to the Present. With this liberation from time, we can find better and more effective solutions to the challenges we need to confront.

Progress of time marks real progress only if we respect and learn from the past and then properly manage the present. Learning from the past does not mean adherence and reverence to the past. Progressively looking forward to changing for better does not mean condemnation of everything in the past.

There would not have been an Einstein if there were no Newton.

There would not have been a Newton if there were no Galileo.

There would not have been a Galileo if there were no Aristotle.

As we know

the progress from Ptolemy to Copernicus was achieved by turning the Solar system model upside down.

The progress from Newton to Einstein was achieved by turning the Newton's model upside down. (See, p. 84 in Chapter 3.)

This does not mean that progress is always achieved by turning an existing model upside down. Otherwise, progress from the Einstein's model should be back to the Newton's model. Nevertheless, in relation to the scientific theories of the nature of light, the progress happened in back and forth movements. Finally, it settled in a duality.

Even the notion of 'progress' sometimes can be used for oppression. For instance, to liberate women, some Marxists suggested establishing mass scaled baby care centres.

This program would have brought 'liberation' from parenthood, to both men and women. According to these Marxists, the society would have benefited from this scheme. Here, the Marxists were looking at a human situation from a tunnel vision while forgetting, ignoring, neglecting and condemning a multitude of other angles. Had this program been actuated, it would have surpassed the horrendous crime of the Stolen Generation on a horrendous scale. It would have resulted in many mothers (and fathers) suffering from the grief of forced separation in horrendous camps of Auschwitz type. In short, it would have buried humanity.

Our technology is developing in quantum leaps. This progress is enslaving humanity to technology. The addiction to technology is enslaving us on a much more colossal scale than the magnificent reductionism has enslaved our cognition. However, it is not the technology and reductionism that enslave us. We enslave ourselves to them because we choose to sleep walk. Then big profit greedy companies work tirelessly and generously, day and night, to help us enslave ourselves to their products, as tobacco companies work hard to help us addict ourselves to Nicotine. These addictions have bearing to our education practices. To discuss these ideas, I present the following four examples.

#### **First Example: An Autistic Child**

An autistic child was in a special development program which maintained a close and an optimum level of human interaction with the child. According to the father of the child, she improved considerably in her awareness, focus and concentration. Unfortunately, the father had to return to Australia so that his elder daughter can go to school while their mother can go to work. Upon return, they enrolled the child in a special education program. The program required the child to use an I-Pad loaded with a multitude of educational software. Her dad told me that this has significantly deteriorated the condition of the child. With the I-Pad now she is in a world of her own. I saw the child while we were in conversation about his elder daughter's education. It immensely saddened me to see the child who had become entirely lost in the I-Pad. Enormously anguishing, her dad confessed that he was powerless to take away the I-Pad. If he did, it would have created many problems for all of the family.

**My comment:** Autistic children run a risk of developing obsessions. Authorities have ignored this possibility when treating the child. That was why they carelessly give her a toy which could completely take away her awareness. I have seen even normal children who are addicted to video and computer games behave as autistic children. Furthermore, these addicted normal children lose the ability to concentrate and focus on other tasks. Since autistic children are deficient in social skills, I believe that structured human interaction can better help the child. That was the reason why the child was improving in the first program. This is an example of wrong use of technology. This misplacement of the role of technology may not be merely accidental and could be powered by the technology companies, like the cigarette companies power the campaign that promotes the glamorousness of smoking.

### **Second Example: Vanishing Fractions**

A lecturer of Mathematics Education from a well renowned University wrote in her lecture notes that she sent me: because of calculators fractions were vanishing. They give way to decimals. The lecturer has forgotten that many decimals are also fractions. Now, had we extended this argument to irrational numbers then we would have abandoned them too as calculators had only decimal approximations to irrational numbers until the inception of Computer Algebra System (CAS) calculators. Then, in the name of progress and modernity, we would have gone centuries back into the past.

Fortunately, the technology has revived the 'dying' fractions and irrational numbers with CAS calculators. If our educators bury their pedagogical awareness due to wrong conceptualisation of technology, imagine what it can do to our students. They will say that due to the invention of the calculator, the four operations have vanished. To get the answer to  $5+7$  who needs to know what '+' means. Just hit the buttons; the answer comes out.

For the record, I would like to mention that I am a good user of Mathematica, the renowned Computer Algebra System, in my teaching and learning. It is the way how I think of Mathematica and how I use Mathematica that make the difference.

### **Third Example: Lost Giftedness**

This is a firsthand experience. This involves a daughter of one of our family friends. Since the birth of this girl, I observed that she was a gifted child. She passed her all milestones in extremely quick time. At four and half years of age she was able to read the analogue clock to five minutes consistently. She was enrolled in Prep. Her teacher had asked her parents to come for an interview. They were hoping to hear confirmation of their daughter's giftedness, but the teacher told them that the child lacked thinking and processing skills. When the teacher gave her a task she performed the task in quick time in an impulsive and totally wrong manner.

This assessment devastated the parents. They contacted me. I visited them. I performed some tests with the child. The teacher was absolutely right. I was shocked as I expected the child to finish her PhD in at the age I finished my high school or even before. This simply could not be. Then I asked the parents whether the child had any sickness or whether she had any head injury. The answers were negative.

Then I asked what kind of changes they introduced to the child since I had seen her last. They finally said that, to make her learn about shapes, they gave her a computer program. Just out of curiosity I wanted to check the program. My inspection of the program did not alarm me of any faults. Then I asked the child to play the game.

There were several shapes on the screen and a depressed area. The child has to choose the right shape to go into the depressed region. She moved a shape to the depressed area without any thinking or processing of information. The computer yelled. Then she moved another piece. The computer again yelled. Finally, when she got the right piece, the computer gave her a huge applause. She had been playing the game for months spending a great deal of time with it. Immediately, I realised the problem and asked parents not to let her play this game ever and not to give her any video games without consulting me first. Also, when she was playing a permitted video game, an adult should supervise how she played the game. Also, I asked the parents to engage her in some tasks while demonstrating how to process visual information.

One week later, the teacher was delighted as her student began to perform much better, but it is sad to say that she never recovered to the level where she was at before. She is turning nineteen, and she is in the second year of her university education. Unfortunately, even though she is improving she is still somewhat impulsive. If not for that computer program, she would have finished her PhD in a science subject when she was just eighteen. Whenever I see her, I become sad. I could have prevented her fate if I visited her more frequently during that time.

#### **Fourth Example: A Driving Lesson of a Strange Kind**

This episode is about my daughter's driving lessons. I came across with a new Driver education program, which was sanctioned by authorities. I decided to give it a go since it appeared to be based on self-reliance. I explained to the instructor that my daughter was essentially a new learner, and she was not competent or confident even in steering. After the lesson, my daughter came back with tears in her eyes. It was a lesson in which self-reliance had gone too far. The instructor's argument was that since she was to drive on her own, she had to learn it on her own.

During the lesson the instructor ordered my daughter to drive to a huge shopping centre going through extremely busy streets. He gave only a minimum amount of instruction while he was controlling the car. My daughter had to find her way to the shopping centre. If she had lost her way, she had to find her way back by herself. She came to a corner where it was extremely difficult to make a U turn. The instructor demanded my daughter to think on her own and make a U turn. A very long time elapsed negotiating the U turn. During this turn, the instructor's only help was to control the car when it was just about to hit something. Needless to say, my daughter was so confused and about to experience a nervous breakdown.

When he explained his methods prior to the lesson, his approach appeared to be based on self-reliance. Instead, the approach appeared to base on destroying self-confidence. It took a long time for my daughter to have another go at driving. During this 'self-reliance driving' lesson, my daughter had totally depended on the extra set of controlling apparatus of the driving instructor. This reminds me some modern "figure out for yourself" lessons.

The examples discussed above show us the pitfalls awaiting and hidden in educational approaches that are driven by tunnel vision doctrines. Now, before I look into the set of guidelines and aspirations on which I base my teaching practice I would like to consider coining instructivism and constructivism to avoid self-reliance lessons becoming confidence destroying lessons, as in the case of my daughter's driving lessons.

### **Teaching as a Coin with Many Sides**

I understand that the progress of time and technological advances have clearly given us a plenitude of opportunities to enhance quality of teaching and learning programs. This enhancement cannot happen if one starts with the idea that there was no good teaching at all any time before the last fifty years. We need to coin the coins of different times and types. The following is an ancient coin.

When we walk we need to be aware that we walk and where we move. When we talk we need to be aware that we talk and what we say. When we think we need to be aware that we think and what we think about.

The above is the Buddhist principle, Samma Sathi (samma = well, sathi = awareness) or right mindfulness. This is also pedagogical awareness when we apply this principle to learning and teaching. No amount of technological advances or progress of time can take this absolutism out of our practices. Any teaching methodology, which is not based on this absolutism is bound to fail. The failure of the I-Pad approach with the autistic child exemplifies this. Instead of raising the awareness of the child, it drowned the child in an I-Pad world. It is true that the child had learned some skills in operating the I-Pad but at the same time it has diminished her awareness.

In the last chapter, citing Buddhist folklore and other sources, I suggested that constructivism (as a teaching practice) was as old as Buddha, Jesus and Socrates. Also, I argued that their constructivism was multi-fold rather than being dogmatically mono flavoured. Socrates' teaching methodology was very much of an inseparable dualistic combination of constructivism and instructivism. I would like to contrast Socrates' constructivist approach and the driving instructor's

constructivist driving lesson. The driving instructor's lesson did not help my daughter gain self-reliance; instead, throughout the whole lesson, my daughter was desperately under the mercy of the instructor.

Socrates did not leave it merely up to the student as he was interacting with the student; also he did not impose his knowledge on students. It appeared as Socrates was holding the student's hand throughout their journey together.

We cannot improve teaching if we forget that humankind already has a rich culture of teaching. In this rich culture, teaching has never been a coin of one side. To try to make it otherwise is naive, destructive and distorting. To improve our teaching practices, we need to start from these successful practices of the past.

A single flavoured teaching approach cannot be effective in general; instructivism or constructivism. For instance, imagine that the driving instructor has now embraced a crude form of instructivist approach in place of his previous rude form of constructivism. Even in the fifth lesson he maintains full control of the vehicle and never surrenders the control of the vehicle to the student.

Even in the crudest form of constructivist teaching, a teacher must give at least some instruction such as "figure out yourself, or work out this sheet or submit this project". Even in the crudest form of instructivist teaching, a teacher would acknowledge that he cannot open the mind of a student and pour the knowledge into it. What matters is the degree of instructivism and constructivism that one employs. By the term degree I do not mean a designated amount. For instance, I am not dictating to myself that a lesson needs to be 25% instructivist and 75% constructivist. By the term, degree, I mean an approach which is grounded on a set of well-considered pedagogical aspirations and multiple epistemologies, which consists of a multitude of interwoven and interlaced methodologies.

### ***Interdependent-Independent Thinking***

No parents in the world would aspire for their newborn to be dependent on them forever as they know that their offspring most probably will outlive them. Then why do many parents attend each and every single need of their babies without

admonishing them to be independent? Should not they leave new babies in caves, in a faraway forest, so that they can be independent? Instead, insightful parents develop and nurture a trust between them and their offspring by giving care and love. Within this trusted relationship the independence of the child grows. Truly insightful parents celebrate this growing independence and slowly withdraw their assistance. Imagine parents follow the example of the driving instructor's lesson.

How do we get our students to become independent thinkers? Should we abandon them in empty cognitive 'caves' in faraway 'jungles', or shall we motivate them by modelling and demonstrating what is thinking? How do we demonstrate 'thinking' without talk, chalk or print? Then why is there such a celebrated animosity towards 'chalk and talk'? Could this be a subtle form of anti-intellectualism; can this be an animosity against cognition itself? Please consider the example of Rubik (see p. 120).

To promote independent thinking, first we need to know "what independent thinking is". Am I thinking independently while I am writing this manuscript? When I finish writing a portion of this manuscript, my first task is to ask Peter, my supervisor, to have a look. Rather than my ideas being totally independent, I see many threads interweaving my ideas to many other people's. One of these threads is my discussions with Peter and his assessment of my work. Furthermore, I cannot entirely separate my ideas from my parents even though they have never uttered the word constructivism in any language. Thereby, truly independent thinking can exist only if this world exists merely in my mind. In that case, I exist only in my mind.

Then what do we mean by independent thinking? Here, also we are faced with duality and the fourfold truth paradigm of Buddhism. This has implications for my teaching practice. I do not and cannot promote independent thinking. Only I can do is to promote interdependent-Independent dualistic thinking. To achieve this feat, I need a well thought out framework of principles and aspirations as a foundation for my practice. The following are the foundation cornerstones of my practice as a learner and a teacher.

## **My Pedagogical Commandments**

The following commandments are a result of the ongoing analysis of my experiences as a learner and a teacher. My enrolment in this doctoral course largely made these retrospections, introspections and propections possible. This framework of principles always existed as a flickering faint flame in an obscured corner of my mind. Only now do I microscope and telescope at this obscured corner with enhanced awareness. Consequently, this framework of principles comes to the frontline of my practice. I do not claim this to be a new program. This existed in the practices of passionate, dedicated and thoughtful teachers all throughout history. Still it is worthwhile to have a relook.

First, here are the Foundational Aphorisms of My Teaching:

- Education consists of Understanding, Learning and Application.
- Much of the occasions when education is occurring at an optimum level, it is nearly indistinguishable which aspect is at work.
- Understanding without Learning is impotent.
- Learning without Understanding is monkey training. (This applies to much of the learning content. There are exceptions to this due to various reasons.)

I keep in mind that this last claim is just a general statement which applies to the overall character of education, not to each and every single moment of a lesson or to even a whole lesson. At times, learning without understanding is unavoidable and therefore is somewhat tolerable. Still, both the learner and the teacher must take the ultimate goal of education as understanding, learning and application. That is, if there is to be a phase of learning without understanding it must be a step of a long journey since Education is much more than toque monkey training (see p. 179 ).

I believe that *understanding and learning without application is futile*. Here, we need to take the term application to have a much broader meaning than a physical task such as designing a bridge. It can be just reflecting over the learned content, to gain intellectual pleasure and/or better understanding. Looking at a concept in a new angle, constructing a better example, illustrating a concept and applying the concept in a novel situation are some valid forms of applications.

I take that *application without learning and understanding is non-existent*. I say to myself, for all meaningful purposes, this is merely a rough approximation to an 'absolute' truth. For instance, when I climb a hill I apply the gross motor skills I have learned, yet I have only a meagre understanding about my walking. There are many scientists who study human walking. They understand much better than I do, but they do not have a comprehensive understanding either.

Thinking is also a form of learning and teaching. I do not even know what a thought is. For instance, where does a thought exist? Is it a neuro-electro chemical imprint in the brain? What role do I play in creating them? How do I play that role? I do not have answers to any of these questions, yet still I keep thinking. When I think these thoughts, how does it happen? Where do these thoughts 'reside'? How do they reside there? I cannot wait to think until I understand the answers for those questions, to think. Therefore, it is clear that even to have a distant chance of understanding what a thought is, I need to keep thinking.

Some educators disdain 'chalk and talk' since they do not signify activities. Chalk and talk can be an extremely thought provoking activity in the hands of a talented teacher. Consider Socrates. His just 'talk' lessons stand as marvels of constructivist teaching. Consider, Richard Feynman. I have read Feynman's lecture notes. When I read his notes, I heard his authoritative voice in my mind; felt like that I was in his lecture theatre. Though, I have never listened to him in person.

I believe that even though *teaching may happen in classrooms, understanding and learning happen in the minds of the students*. Some, most or 'all' of understanding may happen in the presence of a teacher. In contrast, learning happens, only when the teacher is out of 'sight'. 'Out of sight' needs to be taken metaphorically, not literally.

### ***Independent-Interdependent Learners***

On page 227 I have mentioned that absolutely independent thinking can exist if there is only one mind. Even in the case of a single mind, the later ideas can be the offsprings of the earlier ideas. If this is not the case then single mind must be generating a fragmented set of incoherent thoughts.

If our ideas are absolutely independent then they are largely irrelevant to the world. On the other hand, if our ideas are absolutely dependent then they have no value to the world. Our ideas are most effective and most potent when they are both equally independent and interdependent. For instance Riemannian geometry is an independent-interdependent offspring of Euclidian geometry. Therefore, I take, among many other things, one of the most primary and significant purpose of education is to produce powerful interdependent-independent thinkers and learners. This notion of independent and interdependent thinking and learning encompass the social nature and continuity of cognition. For instance, relativity is both continuation and revolution of Newtonian Physics.

Who are interdependent-interdependent learners? Independent-Interdependent learners

- can reflect on the content on their own
- are able to form new connections with different knowledge claims
- strive to come to terms with knowledge ,
- feel 'ownership' for learning, of knowledge
- may be able to construct 'original' knowledge claims

Perhaps, for the majority of people, constructing new knowledge claims may not happen until they enrol in postgraduate studies or reach mature age. Even then this accomplishment is a culmination of long drawn clusters of tiny interdependent-independent thinking and learning activities. However, at any level, there is interdependent-independent learning. One reason for this is that learning and thinking happens in one's mind.

How can we motivate a much larger fraction of the student population to develop interdependent-independent learning and thinking skills? This ability is not merely a genetically determined one. Relevant pedagogical approaches based on proper pedagogical aspirations and principles can accomplish this noble ambition. Motivating pedagogical awareness is at the centre of this endeavour.

### **Foundational Aspirations of My Teaching**

I lay down a set of aspirations for my performance as a teacher. A well-considered set of principles and aspirations, I hope, can inspire my students to become aspiring and endeavouring. This set of pedagogical aspirations is the basis of assessment of my own performance.

As I discussed earlier in this work, the precarious nature of experiential reality compels any normal human living being to base his or her practice of living on many forms of epistemologies. This is true even in the cases of solipsists, absolute idealists or absolute objectivists. Each of these distinct forms of epistemologies contains the shadows of the other isms. Therefore beneath the life practices of any 'ist' there lie the fragments of other isms. These fragments influence one's life practices to some extent regardless of whether the person is aware of it or not. Even though I am a harsh opponent of solipsism I happen to think as a solipsist frequently (see p. 161 ).

- ❖ I endeavour, to base my teaching on many epistemologies knowingly and assiduously. This is not the epistemology of the opportunist. It is the epistemologist of the opportunity. I presented this notion of the rainbow of epistemologies (see p. 113).
- ❖ I exert myself to be a much sharper interdependent-independent thinker. Also, as a teacher it is my mission to exhibit qualities of an interdependent-independent thinker and promote such thinking. I expect my exhibited behaviour to be an initial model to my students. Hopefully this may motivate my students to pursue interdependent-independent thinking skills.
- ❖ As a teacher, I model myself as an inquirer and inquisitor instead of forcing inquiry and inquisition on my students. The subjects of my inquiry and inquisition are nature, my knowledge claims, and how I can inspire my students to feel the wonder and beauty about their knowledge claims as I feel about my knowledge claims and how I can motivate my students to interact with the subject matter in a thought provoking, intellectually entertaining manner.
- ❖ I endeavour to enhance and maintain my pedagogical awareness (right mindfulness). When I fail I ensure to be critical of myself in front of my

students. My occasional (I hope) lack of such awareness was discussed in the storytelling episode in the last chapter (see p. 212). This episode could be only one representative of many such situations that have not yet entered into the view of my cognitive radar. I will keep diligently looking for such failures.

- ❖ I aspire to help my students raise their pedagogical awareness by pointing out the instances when they demonstrate a lack of such awareness. I will provide some instances that can arise in learning and teaching environment later in this chapter as well as in the next chapter. I take it as my supreme duty to present to the students Physics and Mathematics as a way of thinking about the world rather than prescriptive recipes of how to do things or simply to answer exam questions.

I do not want the statement above to be misconstrued as condemnation and devaluation of exams. I was this naive only when I was a secondary school student. Now I admire the essential role of exams and acknowledge, despite being a necessary evil, that exams can play a constructivist role in education. The task of constructivism is not to condemn exams, but to enhance their standards. Also, I do not totally condemn the use of recipes. Instead, I admire their positive role in organising ideas and concepts and how to think about a process. These recipes need to be based on the concept of a way of thinking; not on the way of assembling a piece of furniture unless it is truly about assembling a piece of furniture.

One may argue that ‘how to think about’ is also an act of doing. I certainly agree. Also, another may point out that any ‘how to think about’ has implications for ‘how to do things’. With this also I agree. Simultaneously I still hold the belief ‘how to think physically or mathematically’ and ‘how to do Physics or Mathematics work’ have distinctive characters. Though subtle, having awareness of this difference could make a tremendous impact on one's conceptual understanding of Mathematics and Physics. Also, the awareness of this subtlety can result in a magical synthesis.

### **Do's and Do not's**

There is no set of absolute do not's or do's. That is, I should operate by the firm policy of employing only well-considered methodologies or strategies of learning that measure up with the pedagogical aspirations and aphorisms documented before and the curriculum dictates. By the qualification, 'well –considered', I do not mean exclusion of any methodologies; I mean that any methodology is shaped and formed to engender the right pedagogical expectations.

I do not live by dictums such as 'never use calculator', or 'use calculator frequently', or 'never talk in front of the class for more than fifteen minutes'. Any dictum is always subject to the scrutiny of the above pedagogical aspirations and aphorisms. With these principles, I accommodate curriculum dictates, such as the use of CAS calculators in the best possible pedagogically prudent manner. As a long-time observer of classroom practices, it has come to my attention that more frequently both teachers and students use CAS calculators as a tool to avoid cognition; this does not need to be so. Calculators are marvellous tools to motivate thinking skills. Also, they are wonderful tools to blunt cognition.

Also, it is my policy not to believe in my magical powers on students; instead I perform my duties on the belief in the magical powers of my students.

Teachers' belief in their own magical powers on students robs students of their own achievement. Teachers are magical creatures only when the students are aware of their own magical powers. Teachers are magical creatures only to the extent that students want and need their teachers to be. In my own story, three months ago Mrs Kiribandhane was an ordinary teacher; in the next three months she was a magician who shaped my life. I do not relate this change to my sudden maturity. Instead, it was the fact that I changed my class to suit my academic skills with an enormous aspiration to learn. This change of attitude enabled me to be receptive to Mrs Kiribandhane's magic. In that Mrs Kiribandhane and I were equally potent magicians. By thinking so and saying so I do not diminish her role in my transformation, in any form or amount. In my mind, the restrictive qualification of her magic only enhances her role in my life. In fact, if Mr Mahindarathne or Mrs Kiribandhane had monopolised their role in my learning then they would not have

contributed to my life even an iota. Teaching in a way that students can own their learning is the best of all pedagogical magic.

Now let me finish this section with a modified version of the verse I presented at the start of the chapter.

#### **The Coin**

The coin I had  
Could've fixed my tank  
If I had two of them, or  
If there were  
Only one hole.

Stopped was the leaking  
With the coin I did split

Didn't I save good  
It was only one coin  
Not the whole tank  
I had to mend

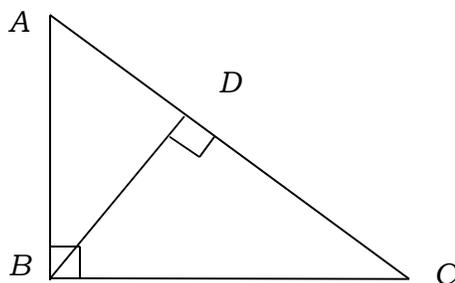
With no leaking,  
Up was the water level.  
Collapsed was the tank!  
Lost was the coin.

**(2011)**

#### **A way of Thinking Versus a Way of Doing**

I have been using the terms 'a way of thinking' and 'a way of doing' frequently. Before considering other things, now I would like to demonstrate the subtle but significant difference between 'a way of thinking' and 'a way of doing'. At the outset, I readily admit that each contains a bit of the other, yet I steadfastly maintain their essential difference. The following real-life story is merely a metaphor for many general situations.

A Year 8 student (say Namal) was struggling to copy the following diagram in to his notebook. He was extremely frustrated with his inability to copy the diagram.



First, I asked how he climbs a mountain. He would do it step by step. I instructed the student to think, how to think step by step about this challenging situation. He did not have to draw the whole picture at once. All he needed to do was to draw line by line. At the end, the diagram would be completed. To achieve this, what he needed to do was to think of the relationships between the lines.

I asked Namal to look at the diagram carefully. Here looking is doing. To look carefully, is to think.

I asked Namal to determine whether there was just one line which he can copy. Namal picked the line  $BC$ . Then I asked him to copy the line into his notebook.

The choice of the line is due to thinking. Selecting and drawing are both acts of doing. Notice that the act of thinking motivated the act of drawing.

Then I instructed Namal to think of another line that had some relationship to the Line  $BC$ , which he could copy easily. Namal picked the line  $BA$  saying that it was perpendicular to the line  $BC$ . Then he drew the Line  $AC$  saying that it was easy for him to copy since he knew how to complete a triangle. As soon as he

completed the triangle  $ABC$ , he was able to complete the remaining line.

I hope that my reader can see the inseparability of instructivism and constructivism in this exchange. For the purpose of convenience, we may label some parts as constructivist and other parts as instructivist, but in this interaction one aspect cannot exist without the other. Giving clear instructions to the student can be regarded as instructivism, yet the purpose of instructions were to motivate the student to think and come to his own conclusions; not to act on the dictates of the teacher. I am confident that when Namal has to draw such a diagram again, he will be able to use this 'way of thinking' interwoven with the 'way of doing'.

Also, it was likely that Namal had some learning difficulties (similar to the learning difficulties that I had experienced) since he was the only student who complained about the diagram. The activity visibly exhilarated him because he was confident that he could meet such future challenges. This is an example of how an instructivist and constructivist approach can inspire a student with low ability.

For the purpose of contrasting, I would like to discuss two other approaches that I could have taken:

- Approach I: This is the above approach that I used with Namal.
- Approach II: I would have given him a worksheet and kept walking around, asking him just to figure out it for himself. This is similar to the research based modern driving lesson approach my daughter was unfortunate to suffer (see p. 224).
- Approach III: I would have given him step by step instructions on how to sketch the diagram. For instance, in this approach rather than aspiring to inspire the student I would have wanted the student just complete the task in quick time. In this case, I would have asked the student to draw the line  $BC$ ; then to draw  $AB$  perpendicularly to  $BC$ , etc.

In Approach II, Namal would have approached another student, an older sibling or a parent for further instructions. In this case, the student would have received step by

step instructions (Occasionally there could be a parent, a student or an older sibling who would take Approach I but this is unlikely.) Eventually, the student would have drawn the sketch without gaining much on how to think about it. If he had practiced a number of similar sketches, he himself would have gained a vague awareness of the way to think about it but the efficiency of the method is minimal since a tiny fraction of students with similar learning difficulties would have achieved this awareness only after a relatively long time of practice. Many would have learned the copycat robotic method from their peers. I have seen it many times, the students who have experienced Approach I also instruct their peers in an interactive manner rather than in a prescriptive manner. One example is me. This was the reason why I became an inspiring teacher to my brother when I was just sixteen years of age. Then he also became a sought after teacher.

On the other hand, I could have prepared the worksheet with great care; giving a sequence of activities that could inspire students to complete the task in at increasingly more conceptual levels. Then this could have engendered success for many such students. In this case, the worksheet is just a tool of Approach I.

There is a spectrum of possibilities with regard to any learning task. One of the wavelengths in this spectrum consists of the students who require intensive interactions with a teacher. Then there is another wavelength. Even if the worksheet is not that inspiring, in a rare case among thousands of students there are a few students who are able to develop a proper approach even if the further assistance is lacking. This is due to their previous exposure to similar thinking. They have already internalised the process, and they cannot feel that, in fact, they are doing it step by step. It is like when we add two numbers such as 112 and 234. The answer, 346 comes to teachers' minds instantly. If this is not clear try 100 and 105. The answer comes instantly and we are not aware of employing the place value system. In contrast, a Year three student will have to apply the place value system painstakingly. Does the answer being instantaneous for us mean that we do not employ the place value system? Then there are many other wavelengths in between the two extremes.

Even though many may hail Approach II as a naturally constructivist approach I mostly favour and employ Approach I. I do not rule out Approach II since it might just suit my pedagogical expectations for a particular group of students at a particular moment. For instance, they may just need skill practice to develop conceptual understanding further. Skill practice cannot be separated from conceptual understanding. Sometimes, to enhance fluency, a student may need to practice a drill. Performance fluency and conceptual fluency are both important; also, one can feed the other. If students practice a carefully developed drill, it can simultaneously assist both conceptual understanding and performance fluency.

If one practices a mathematical task repeatedly with pedagogical awareness it improves conceptual understanding. This is the reason why many teachers claim that they understand many concepts after they became teachers. After they become teachers they practice the task with pedagogical awareness.

In the long run, Approach III may be somewhat more successful than Approach II in achieving the required pedagogical awareness. This is because when the students need to do tasks of Approach II type they find help in the form of Approach III, from parents, friends or tutors. This is likely to promote toque monkey type performance, in the short run. Even then, sometimes this may trigger pedagogical awareness in the performers, as in the case of teachers.

### **Pedagogical Awareness**

Pedagogical awareness or right mindfulness is simply paying attention to what one does. It amazes me how I fail in this right awareness in my day to day living. If one develops pedagogical awareness it certainly improves general right awareness. For instance, after I began to pay attention to pedagogical awareness I experience an enhancement of general awareness. Still I forget where I park my car in a large shopping mall park. Right mindfulness is an essential life skill therefore it needs to be at the centre of our learning teaching programs. At times, for various reasons, I fail to maintain my pedagogical awareness while I am teaching. These are precious educational moments for me to engage in self-criticism and promote pedagogical awareness in me and among my students.

A significant portion of my lessons is dedicated to this issue. I ask questions of the following nature. Interactions of this mode help the students to recognise their lack of awareness. I use the following example with Year 10, 11 and 12 mathematics students, who have been working with negative and positive numbers for years. On the whiteboard, I write: let  $a$  be any real number. Then I ask:

**Teacher:** Can the quantity,  $a$ , be negative?

**Students:** No. (This is the most common answer. Without commenting on the previous answer, I put another question on the whiteboard.)

**Teacher:** Is it positive?

**Students:** Yes. (Again this is the most common answer.)

**Teacher:** What about  $-a$ ? Is it always a negative value, or can it have a positive value?

They will give similar answers. Without commenting on their answers, I further ask:

**Teacher:** What about  $+a$ ? Is it always a positive value, or can it have a negative value?

To answer my questions, students consider only the sign in front of the pro-numeral. That is, for them the quantity  $+a$  or  $a$  is always a positive value and the quantity  $-a$  is always a negative value. To motivate them to reflect on their answers, I ask what any real number means. At this moment I point to my original statement on the board. Then I ask: can the value of  $+a$  and  $-a$  be zero, or positive or negative? Can we take  $a = 0$ ,  $-5$  or  $5$ ? On what basis, can we substitute, negative, positive or zero values for  $a$ ? Instead of reading the statement on the board, "let  $a$  be any real number" with awareness, they interpret the statement according to their prejudices. At the end of this discussion, I ask: Is this new knowledge? For many years they have been substituting many different values for  $+a$  or  $-a$  but they have not been aware that it is because they can have negative, zero or positive value.

This prejudice arises from their internalisation of the wrong idea that positive signed pro-numerals are always positive and negative signed pro-numerals are always negative. When they substitute  $-5$  for  $a$ , they are not simply paying attention that they are doing something against one of their internalised beliefs. If they do, they would have inquired and corrected it. This is lack of pedagogical awareness.

### An Example of Dream Walking

This example of lack of pedagogical awareness is based on absolute values. I start with a heuristic definition of absolute values with the pedagogical expectation that the students will later develop the formal definition with my help. I put this tentative definition on the board:

The absolute (modulus) value of a number is the magnitude of the number. Then I ask the students to give examples. Next I use the instructivist mode and demonstrate absolute values of numbers such as:

$$\underbrace{|7|}_{\text{The absolute value of 7}} = 7 \quad \text{and} \quad \underbrace{|-7|}_{\text{The absolute value of -7}} = 7.$$

Then I repeatedly ask them to give absolute values of the numbers I call out. My plan begins to work from the moment that they can give the absolute value of a given number instantaneously. I forewarn them that, in the next few minutes, it will reveal whether they are awake or asleep. I plead with them to pay complete attention to their thought process.

As they become comfortable with the absolute value of any negative or positive numbers, as a prelude, I ask the question, what is  $|0|$ ? This question can baffle many students. A few will answer the question. I will ignore the answer with an invitation for them to keep thinking. Then I go for my main target.

$$|x| = ?$$

Their instantaneous answer is,

$$|x| = x.$$

At this point, I write these on the board:

If  $|x| = x$ , then

$$|5| = ? \quad \text{and} \quad |-5| = ?$$

They know that  $|5| = 5$  and  $|-5| = 5$ . If  $|x| = x$  then by interactive questioning I invite them to see that  $|5| = 5$  and  $|-5| = -5$ . The last statement must be wrong. Then they change the answer:  $|x| = -x$ . In this case, by interactive questioning, I invite them to see that  $|5|$  must be  $-5$  and  $|-5|$  must be  $5$ . Now, the first statement is wrong.

The students realise that both claims  $|x| = x$  and  $|x| = -x$  are wrong. This leads them to redefine their rule:

$$|x| = \pm x.$$

The new rule does not work with any of  $x = -5$  and  $x = 5$  since according to the new rule  $|-5| = \pm 5$  and  $|5| = \pm 5$ . When they realise this predicament, to answer the question what  $|x|$  is, they go back and forth, without stopping to think. This is the moment for which I have been patiently waiting.

To motivate them to pay attention to their own thinking, I instruct them to count the number of steps they go through to figure out the answers for  $|5|$  or  $|-5|$ . Most frequently their unanimous answer is just one step. I plea with them not to concentrate on the answer for  $|8|$  or  $|-8|$ , but to pay attention to the number of steps required. For each number I write on the board, I request them to tell me the number of steps and the absolute value (in that particular order). They are adamant that they go through just a single step. After some time, realising that they require a little more explicitness, I mockingly tell them that I know about their minds better than they do and inform them that they go through two steps. They still do not see this. Then I ask them to work out slowly the absolute value of a number, while paying attention to the number of steps they go through.

At this point, one or two students acknowledge that there are two steps. After more repeated trials, I know that I need to be even more explicit with the other students. I spell it out that

- Step 1: Is the number negative or positive?
- Step 2: If the number is positive then return the same number, otherwise, remove negativity.

The majority of the class understands that there have always been these two steps underneath their awareness. Still there are a few who still do not recognise this. It is also refreshing to notice that some students are red faced since they clearly understand that they have not been aware of their thought processes. At this point I

ask them to construct the formal definition of  $|x|$ . From the two steps above, the majority of the class can construct the definition:

$$|x| = \begin{cases} x & \text{for } x \geq 0, \\ -x & \text{for } x < 0. \end{cases}$$

In education it is of primary importance to present the subject matter with the mission of developing pedagogical awareness. This mission goes beyond the goal of helping students to just get the answers. This can be effectively achieved by lessons based on a high level of interaction between the teacher and students and among students. Yet this cannot be the only single aspect of effective teaching. Even in this lesson, at times, I was explicit in my teaching. One of the most primary missions of education is to present the subject matter with the aim of developing pedagogical awareness among students. This issue is as significant as excellence in exam performance.

Here is another example.

$$\sin(x) = \frac{1}{2}.$$

Then I put the equation, on the board.

$$\sin(x) = \cos(x),$$

At this moment, I do not expect them to solve it. I just want them to develop the awareness that there is a challenge in the new equation. My expectation of them is to be able to recognise what this challenge is. Of course, if they can figure out what this challenge is then some of them may be able to solve the equation. I ask them whether the question is challenging. They agree. I tell them that they have already made progress since they recognise that it is challenging. The next step is to recognise what this challenge is. Why is it different or challenging? Even though the equation is difficult they cannot pinpoint to what the difficulty is. Then I ask why it is easier to solve the equation,  $\sin(x) = \frac{1}{2}$ ? Their answer is that while the first equation is simple the second equation is complicated.

This leads to the question; why is the first equation easy? If they cannot figure this out, I ask them to read the two equations,  $\sin(x) = \frac{1}{2}$  and  $\sin(x) = \cos(x)$ , out loud in their mind. At this point, many can see that the first equation is simple and easy to solve since it has only one trigonometric function. Also, now they can see that the second equation is complicated since there are two trigonometric functions. If having two trigonometric functions is the difficulty what can be the solution, I ask. Some students may suggest removing one of the trigonometric functions. How can we achieve this, by subtraction, addition, multiplication, division or just by deletion? I ask them to perform all of the above to consider their viability. They will realise that combining the two trigonometric functions into tan function by division is the only viable solution. The settlement of the difficulty issue of the question surprises some students that something under their nose can go unnoticed for such a long time.

Previously I stated a set of constructivist aspirations on which I base my practice as a teacher. To focus quickly, at times, one needs just one or two such principles. For this purpose I rely on the following shorter version of the set of aspirations:

My mission as a teacher is to help my students to:

- believe that they can develop self-reliance to gain knowledge
- enhance their pedagogical awareness
- believe that they can own their knowledge
- realise their mind as really a beautiful thing.

### **Failures of Assimilation and Osmosis**

With the Alawaka story in Chapter 6 (see p. 206), I gave an example from my life of how osmosis and assimilation can be effective learning processes. Just because of this possibility we should not leave teaching and learning to chance assimilation. To take the real advantage of this possibility we need to enrich the processes of osmosis and assimilation with awareness. For instance, the students can stick difficult formulas on the walls of their rooms. The students need to look at these formula posters frequently with conceptual awareness. The process of osmosis can fail. Here is an example.

Year 10-12 students have been reading books containing measurements for more than a decade, but many of these students are unaware of separation between the numerical value of a measurement and the unit of a measurement. That is, they do not recognise the differences among the expressions,  $5m^2m$ ,  $5m^2 m$ ,  $5m^3$  and  $5m^3 m$ . They may recognise  $5m^2m = 5m^3$ , but they do not recognise that  $5m^2 m$  stands for  $5m^2$  meters and  $5m^3 m$  for  $5m^3$  meters. I point out to them that they have been reading mathematics textbooks for decades without paying much attention. Also I direct my students to think why this convention (rule) is necessary. To answer this question then they need to inquire what happens if we do not leave such a space. Once, a retired principal visited my class. He heard that I have instructed my students about this conventional gap between the magnitude and the unit of a measurement. He told me that he was surprised that he did not notice such a simple thing for such a long time.

I also have internalised wrong notations despite decades of reading mathematics. Until a decade ago I have been writing the percentage symbol as  $\text{o}/\text{o}$  instead of  $\%$ . Similarly, I have written the symbol  $\leq$  as  $\ll$ . Some of my handwritten letters are peculiar to me, but I am aware of it. However, my writing of these mathematical symbols was not an example of a personal style since I was not simply aware how the real symbols are meant to be written. When I noticed this defect it shocked me. Awareness practice can make a huge difference in the quality of our cognition in all areas, particularly if we start early. If osmosis and assimilation are reliable learning tools I would have noticed how to write the symbols a long time ago. Also, my students would have noticed the separation between the unit label and the numerical value of a measurement.

### **Answers, Answers, Nothing but the Answers**

Frequently, I come across the situation that students are simply unaware of their thinking processes. Consider the following Grade One question. A basket contains five mangos, and I put two more mangos into the basket. How many mangoes are there? They can answer this question quickly and correctly. If I change the numbers to 1078 and 5478 either they will seek the calculator or try to add the numbers in their mind (this mental effort is praiseworthy), but it does not occur to them that the

answer is  $1078 + 5478$ , which they can evaluate using the addition algorithm or calculator. They are simply not aware of their thinking processes. They are looking for the answer and do not think of the operation involved. This hinders their ability to generalise and do unfamiliar questions based on the same operations.

Last year there was a Year 10 student who hated algebra and claimed algebra was nonsense. I asked him to come and visit me during lunch break. I gave him the mango question: A basket contains four mangos. I put another three mangos. How many mangos are in the basket? He gave me the answer eight. Then I asked him to show me the working out. He began to argue; showing working out for such a basic question was ridiculous. I agreed with him totally and said it was even more ridiculous to argue against a teacher's instruction too hastily. He finally agreed and wrote  $3 + 5 = 8$ . Then I asked him to pretend that he could not count beyond three and do the question again. For this, he wrote  $3 + 5$ . At this point, I claimed that what he had done was the algebra part; the answer 8 was the numerical part or the arithmetic part. Then I changed the numbers to pro-numerals. He completed the question. All these years he had been simply refusing to do anything with algebra, all the while he was doing algebra since Year 1 or even Prep.

Several years ago, one of my Year 9 students, Harin, took a chair to beat me with on the first day I met him as I had asked him not to rock the chair. Harin had no hopes and he appeared to be completely lost at that time. About four years later, a week before his VCE examinations, this student gifted me a copy of the movie, *A Beautiful Mind*. This movie tells the story of the Nobel laureate Mathematician, John Nash. Within the gift, there was a card. It read:

Mind is really a beautiful thing. You showed me how beautiful my mind is.

In the following year, he enrolled to study Engineering at Monash University.

I want to remind my reader that this is not my victory. This is my student's victory. My only victory is to know that it is not my victory, at all.

## Chapter 8

### Enhancement of Learning Capabilities of Students of All Ability Levels, Through a Constructivist Approach

#### Background

#### The Thesis Title

My thesis title is '*Enhancement of Learning Capabilities of Students of All Ability Levels, Through a Constructivist Approach*'. Notice that it is not '*Students of All Effort Levels*'. Even if there is a magical method which can enhance learning outcomes and capabilities of non-striving students, I do not want to do anything with such enslaving magic. This is because I firmly base my teaching practice on respect for and self-reliance of my students.

#### My Passion

I hope that, by now, the reader has understood my passion for concept based learning and teaching. This passion is not a genetic trait. My parents, teachers and I nurtured this value to be a part of my living. My passion for cognition-based learning and teaching arises from my heart's desire to help students to achieve the same enjoyment, fulfilment and success I experienced while I was on my learning journey with Mrs Kiribandhane. Through this mission I reap an enormous amount of pleasure. Also, because of this mission, at times I suffer heaps of torments.

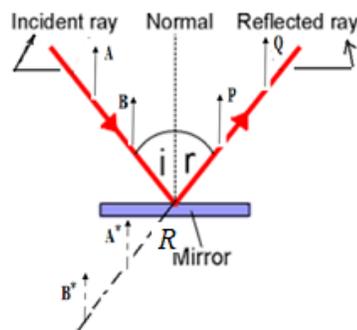
The journey of my learning has convinced me that thinking is a learnable skill; therefore it is a teachable skill. Even though my academic constructivism can be dated back to Year 9, only decades later I gained this insight by reflecting through my teachers' role, my performance as a student and a teacher. An opinion expressed in an article that thinking is a learnable and teachable skill first motivated me to rethink about thinking as a learnable and teachable skill. I do not remember when it happened, but I remember that initially I dismissed the idea completely.

I need to state that these lessons may not suit all behaviour levels. These lessons require normal human concentration and ability to focus. Simply these lessons are

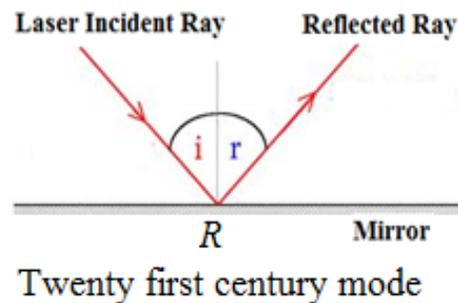
not gimmicks to make a noisy class a quiet one. Nevertheless, some misbehaving students could engage with the cognitive pleasures offered in these lessons, yet it will take time for them to develop the taste. These lessons demand intense concentration and focus from the teacher too. This is because the teacher needs to evaluate students' responses and then design the next question so that the students are somewhat more illuminated than before. Also, the students need to consider other students' responses critically and incorporate those opinions into their views. In summary, it is a journey for both the students and the teacher. In this journey, there are no gimmicks. There are only genuine cognitive exchanges and activities. These activities can be of any form and they come in many varieties. There is a commonality. Each of these activities strictly links with the learning expectations of the lesson. Through these exchanges and activities, at least to some extent, students are acting as mathematicians and scientists.

### An Example of Choice

Before I present my lesson ideas, I would like to consider two versions of the same experiment. One is of the twentieth century mode, and the other is of the twenty first century mode, respectively. This is about reflection of light.



Twentieth century mode



Twenty first century mode

The experiment depicted on the left hand side (The twentieth century mode) uses a plain mirror, paper, pins, a ruler and a pencil to investigate three laws of reflection. The reflecting surface of the plain mirror is placed on the paper. The two pins, A and B (two vertical arrows) situate on an angled line to the mirror. The two vertical arrows represent the pins standing on the paper, perpendicularly to the paper. *The starred pins A and B are the mirror images of the pins, A and B, respectively.* Then the students plant the pin P in front of the mirror on the paper so that the images A\*

and  $B^*$  and the pin  $B$  appear to be on a straight line. Finally, the students plant pin  $Q$  on the paper so that the images  $A^*$  and  $B^*$  and the pins  $P$  and  $Q$  all appear to be on a single straight line. The experiment depicted on the right hand side uses a laser light beam instead of pins.

In the first experiment, students are to remove the pins, mark the mirror line and join the holes produced by the pins  $A$  and  $B$  and  $P$  and  $Q$ . Then they need to extend these two lines  $AB$  and  $PQ$  until they meet at the point  $R$  on the mirror line. At this point, students need to construct a perpendicular to the mirror. In the second experiment also, students draw the mirror line, the incident ray, the reflected ray and the perpendicular line to the mirror at the point  $R$ .

Which one of these two experiments is better? The question is ill posed if we do not elaborate on ‘in what context better is better’. If we need the students to see the laws of reflection just visually then the laser beam experiment is clear, more effective and time saving. As soon as one turns on the beam it will be clear that  $i = r$ . On the other hand, if we need the students to feel the pain and joy of cognition, the laser beam experiment is too explicit. This explicitness could numb the minds of the students. Also, the cognitive pleasure derived from it, is minimal. However, a teacher could somewhat negate the numbing effect by motivating students to reflectively interact with the information gained.

The first experiment demands for more interpretive mental constructions. Thereby it provides more cognitive pleasure. For instance, with regard to the first experiment, the teacher can ask the students:

- What does the line  $AB$  represent?
- Why do we align the pins  $P$  and  $Q$  with the straight line created by the mirror images of the pins  $A$  and  $B$ ?
- What does the line  $PQ$  represent?
- Why are these representations valid? (I have assumed that the students are aware of the ray model of light.)

The discussion might have clearly demonstrated where my loyalty lies with respect to the two modes of the same experiment. Nevertheless, is any of these experiments absolutely better than the other? To answer this question, let us assess the many constraints that a teacher needs to consider. Think about the preparation time. For instance, there may be a video clip of the second experiment. This can save an enormous amount of time. If the students had already done the pin experiment in the last year, then the laser beam experiment or the clip can be a better alternative. Even then the current teacher may decide to repeat the pin experiment so that he can pose the questions listed in the last paragraph. In this case, the teacher may demonstrate the pin experiment as a chalk-talk-think experiment.

There is also another clear advantage with the laser beam experiment. It is easier to show that the incident ray, the reflected ray and the normal to the mirror at the intersection point of the rays are coplanar. Therefore a teacher might choose this approach:

- A virtual chalk-talk-think pin experiment (The teacher just demonstrates the experiment on the board while interactively engaging the students.) This saves time spent on some insignificant things.
- Then he asks the students to perform the laser beam experiment. It is easier to control the plane of the incident beam in the laser beam experiment (with the pin version, the plane of the incident beam is always the plane of the table surface.). Therefore students can easily investigate the coplanarity (of the incident beam, reflected beam and the normal).

The teacher may decide that the class is too advanced to engage in these trivial matters. Therefore, he may choose not to do any experiments and briefly mention the laws of reflection. This allows the teacher to focus on other priority areas with this particular group of students.

The discussion above shows that choosing a learning activity cannot be made on modernity or technology alone. Also, a simple list of do's and do not's cannot help.

## **My Mathematics Coins**

In this section, I am going to present some records of my ongoing action research. The classrooms I taught in were my universities. The students I taught were my teachers. The lessons I taught were my lessons. At times, I will endeavour to reproduce some of the actual responses of the students if it significantly contributes to the point that I strive to make. At other times, for the necessity of brevity, I will paraphrase the students' exchanges or omit them.

## **Algorithm of Long Division**

We learned the Algorithm of Long Division as a mysterious process. We just needed to develop competency and fluency in the process. Until I needed to teach long division to my daughter, I did not think of why the algorithm worked. Even in this era of celebrated academic constructivism this is the case but there is a difference. Now, students need to develop just a familiarity with the Long Division Algorithm; competence and fluency are supposedly unnecessary luxuries. I believe that practicing of very long long-division questions provides us opportunities to develop short term memory skills. These short term memory skills are essential in further learning. It is true that there are other ways to develop short term memory skills, yet for a student of mathematics it may be beneficial to use a mathematical context to develop short term memory skills.

The following is a presentation to a Year 7 class. The lesson needs to be done in chunk by chunk. At this point it is beneficial to list the learning intentions.

**Learning Intention:** At the end of this lesson, you will have learned what multiplication is (This is revision.).

Here, I have listed the learning intention in an incomplete manner. For instance, I did not write 'Repeated addition of the same number is multiplication'. Such a statement makes much of the following discussion meaningless...Even an explicit intention statement will not have mattered to many of the students due to their lack of attention. Some students may read the intention statement several times. Even then they would not grasp it. They will interpret this failure as dumbness. Therefore, I make this lack of awareness as one of the major points of my lessons. In this case, I

write such explicit revision or intention statements and assess how students interact with them. This allows me to demonstrate that their failure is not due to an inherited dumbness. Even if a teacher initially chooses not to declare learning intentions explicitly, for pedagogical reasons, during the lesson the learning intention needs to be made explicit.

*Teacher: What is multiplication?*

*Students: Students give many answers, but it is unlikely that they will recall that multiplication is repeated addition.*

*Teacher: What is  $5 \times 3$ ?*

Here, I choose a simple example so that the numerical difficulty does not hinder students' conceptual thinking or make the teacher's presentation cumbersome. This does not mean that students should do only simple stuff.

*Students: All answer it correctly.*

*Teacher: Why?*

This question may surprise them. Some might even say, "Times table says so."

*Teacher: Why does, the times table have it*

$$5 \times 3 = 15?$$

Initially some students may think that the question is weird, and this may shock them. Teachers' task is to help students to be comfortable with and appreciative of questions of this nature. Some other students may maintain quietness and think about an answer. After some time the teacher may ask students to draw a picture using balls to depict  $5 \times 3 = 15$ . Students draw this correctly as three groups of five balls.

*Teacher: Now look at your picture and think, why  $5 \times 3$  is 15? How do you interact with the picture, to get the answer? What do you do with the picture, to get the answer?*

At this point, many will see

$$5 \times 3 = 3 + 3 + 3 + 3 + 3 = 15,$$

or the teacher might write the statement on the board.

$$\underbrace{3+3+3+3+3}_{\text{Repeated addition of 3 five times}} = \underbrace{5 \text{ of } 3}_{\text{A short way of saying 5 lots of three or repeated addition of 3 five times.}} = 5 \times 3 = 15$$

A symbolic notation for repeated addition

Also,  $\underbrace{5 \times 3 = 3 \times 5}_{\text{Commutativity of multiplication}}$  (Five lots of three has the same value as three lots of five.)

At this point, teacher will write, "so far we have learned". Then students will have to complete the sentence. Then the teacher will also put a summary on the board:

**Multiplication is repeated addition of the same number.**

**Repeated addition of the same number is Multiplication.**

- ❖ Show them that Multiplication is commutative. Practice simple products such as

$$7 \times 3 = 3 \times 7$$

- ❖ Then put a large product on the board such as

$$113 \times 217 \quad ? \quad 217 \times 113.$$

It is crucial for students to understand that they do not need to calculate either side of this equality explicitly, to know both sides are equal

- ❖ Ask students to multiply  $5 \times 4 \times 3$  in any order. Ask them to make their own conclusions.

**Multiplication is associative.**

$$\text{Ex } (7 \times 3) \times 8 = 7 \times (3 \times 8).$$

Now students will do relevant questions from their textbook or a worksheet.

## Division

**Learning Intentions:** At the end of this lesson, you will have learned:

- What division is,
- Division is not commutative and
- The relationship between multiplication and division.

*Teacher: If multiplication is repeated addition what is division?*

*Students: Some may answer that it is repeated subtraction. In this case, acknowledge the answer without confirming it.*

Teacher: What is  $15 \div 3$ ?

Then, by questioning interactively the teacher will establish with the class that division is the repeated subtraction of the same number. After these exchanges, the teacher asks students to subtract 3 from 15, and subsequent results repeatedly until no longer can it be done.

$$\underbrace{15-3}_{\text{First subtraction}} = 12 \rightarrow \underbrace{12-3}_{\text{Second subtraction}} = 9 \dots \underbrace{3-3}_{\text{Fifth subtraction}} = 0.$$

This means that 3 can be subtracted from 15 five times. For the number of times that 3 can be subtracted from 15, for brevity we write 15 divided by 5. We denote this by

$$15 \div 5. \text{ Therefore } 15 \div 3 = \frac{15}{3} = 5 \quad \text{therefore} \quad 3 \times 5 = 15.$$

Another Notation for division (We omit how this is so.)

This comes from the repeated subtraction of 3, 5 times.

Instead of using repeated subtraction as our definition, we may use

$$\frac{15}{3} = 5 \quad \text{since} \quad 15 = 5 \times 3.$$

15 contains five groups of three. Therefore, we can subtract three five times.

What about  $\frac{17}{3}$ ?

$$\frac{17}{3} = 5 \text{ and a remainder } 2 \quad \text{since} \quad 15 = 5 \times 3 + 2.$$

15 contains 5 groups of 3 and another 2

The teacher asks the students to copy the board work and write a summary in their own words. Then the teacher also puts a summary and remarks that division is not commutative. For a while, they will be repeatedly doing examples on basic division, always thinking of the relationship between multiplication and division.

### Long Division

**Learning Intentions:** At the end of this lesson you will have

- Understood the Long Division Algorithm and
- Learned how to do long division.

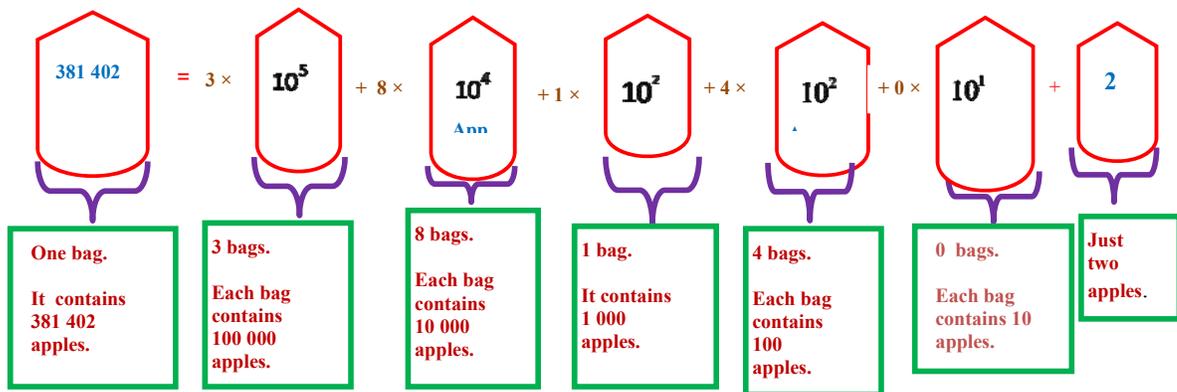
Teacher: Now, how do we divide 384 402 by 378? Either we have to keep subtracting 378 repeatedly until we get zero or a number less than 378, or keep multiplying 378 by another number until we get closest to the 384 402 without exceeding it. These are long and time consuming processes.

If we are to divide an even larger number by using one of these processes, we may need a whiteboard larger than Victoria. Now, consider the following scenario.

There are 381 402 apples. We need to divide these apples among 378 people. To do this, we can ask people to line up and give each one of them an apple at a time until we run out of apples. This involves a huge amount of walking. We need an effective way to give these apples to the people.

*Students:* Students may suggest giving them 10 apples or 100 apples at a time. This is an effective method if we already have bundles of 10's or 100's. The Long Division Algorithm is similar to this suggested approach. Note that

$$10^2 = 10 \times 10 = 100, 10^3 = 10 \times 10 \times 10 = 1000, 10^4 = 10 \times 10 \times 10 \times 10, \text{ etc.}$$



Pointing to this picture ask students what we have done here. How is this arrangement of bags related to a well-known mathematical scheme? They will see that this arrangement resembles the place value system. Now how do we start to give each one of 378 people an equal number of apples? Rather than giving apple by apple or hundred apples by hundred apples, we can start giving each a hundred thousand apples.

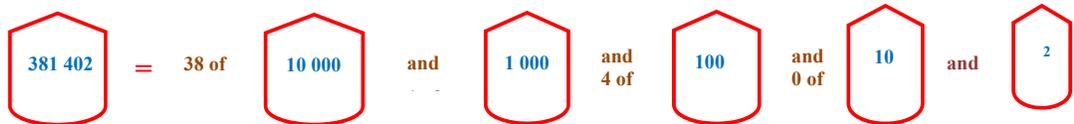
*Teacher:* Is there an enough number of bags of hundred thousand apples, to give each person a bag?

*Students:* No (Verify, by interaction, that they have a solid reason for their answer.) There are only 3 bags of hundred thousand apples, but there

are 378 people. Thus, to give each one a bag of hundred thousand apples, we need 378 bags of hundred thousand apples.

Teacher: What do we do next? [Wait. Interact. Then Guide.] Since we need to finish this division as quickly as possible, we try to give each person a bag of ten thousand apples. To do this, we need to repackage the apples. What do you suggest? [Wait, interact and guide.] Help the class to see that each bag of 100 000 apples now needs to be repackaged into bags that contain ten thousand apples. How many bags of 10 000 apples, can we make from one bag of hundred thousand apples? Consequently, we come to the situation depicted in the following diagram.

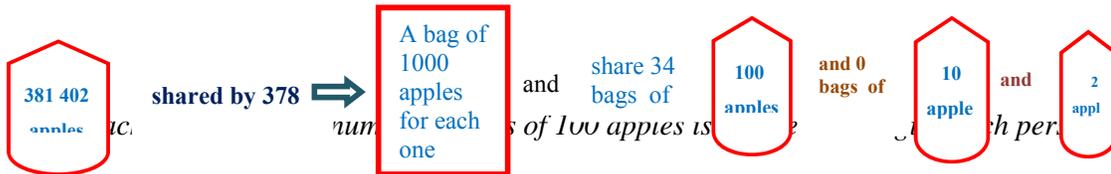
[We have dropped the word, apple, in the following diagrams.]



Teacher: Even after repackaging, we do not have a sufficient number of bags of 10 000 apples. We need 378 bags of 10 000 apples, but we have only 38 bags of them. This leads to another repackaging; this time by thousands. The following picture shows the new situation.

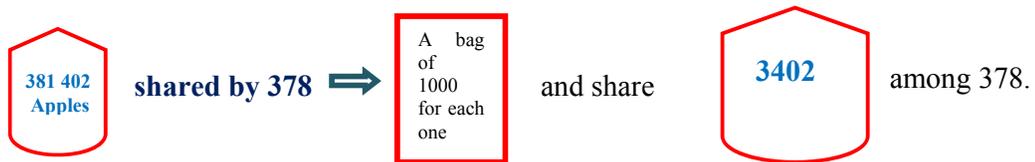
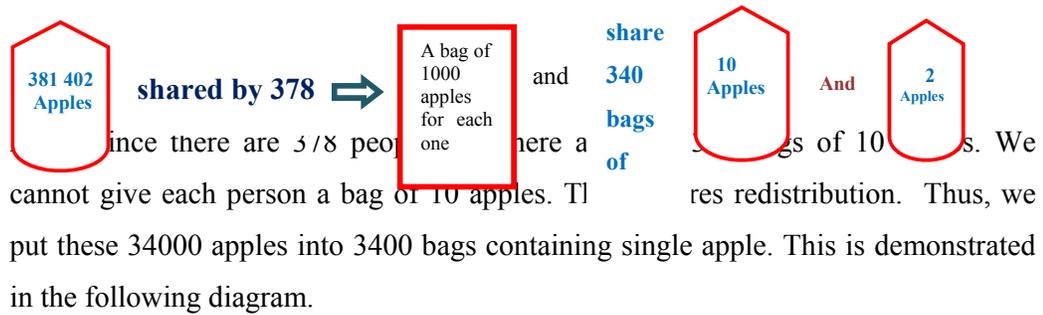


Teacher: Since there are 381 bags of 1000 apples we can give each person one bag of thousand apples. Then three bags of 1 000 apples and 4 bags of 100 and two apples remain to be divided among 378 people. Now we repackage the apples in the three bags of thousand apples into bags of hundred apples. This is depicted in the following diagram.



a bag of 100 apples. Therefore we make bags of 10 apples from these 34 bags of 100 apples. Previously we had zero bags of 10 apples. Now we

have 340 bags of 10 apples. This situation is illustrated in the following picture.



*Teacher:* With these 3402 apples, we can give each one 9 apples. So finally, everyone has received a bag of 1000 apples, zero bags of 100 apples, zero bags of 10 apples and 9 apples. Therefore,

$$\frac{381402}{378} = 1009.$$

Now redo the question using the usual algorithm and explain every step in relation to the procedure above.

$$\begin{array}{r}
 1009 \\
 \hline
 378 \overline{) 381402} \\
 \underline{378} \phantom{00} \\
 3402 \\
 \underline{3402} \\
 0
 \end{array}$$

**Comments:** How can the work of this nature enhance learning capabilities of students of all levels? The reactions from students vary:

- Some competent students may not initially see why they need to change their old behaviours and thought patterns. These competent students will complain that this is a lengthy and confusing method. Some may even become vociferous. According to them, the method they have been using is

much easier. Even if the teacher assures them that the method is the same as their preferred method they may not still be ready to pay attention. Even if the teacher guarantees that they will be continuing with their preferred method they may not be still willing to listen. Even if the teacher explains that the story is just the rationale of their preferred method still they may not budge.

I was a competent master of the division algorithm even when I was a primary student, long before I understood the algorithm at a conceptual level. When I do the long division now I still do it in the 'auto pilot' mode, but every step that I work out makes complete sense to me. That makes a tremendous difference to me.

After some time, some of these resisting students may begin to see that their thought patterns are changing, and they are learning better.

- There are also students, even though they have learned the method well enough, who feel somewhat uneasy with the methods because they lack a rationale. These students may immediately tune into lessons of this approach.
- The students who are somewhat competent could find that their competencies are enhanced because now they have a better way to think and refer.
- This rationale could also help some of the struggling students because they find an effective way to relate to the algorithm.
- The students who do not strive are not affected in either way.
- Initially students, except the students in the Rational Group, could complain that the process is lengthy and difficult but eventually persistent efforts of the teacher could win over the students.

Some schools may think of their students as clients. Receiving complaints from vociferous students against these approaches, the schools may reprimand the teacher to abandon work of this nature.

## Division of 1 by 7

When we need to obtain the decimal representation of  $\frac{1}{7}$  we need to divide. To carry out this division we need to keep adding zero. Some students have named these zeros as magic zeros.

### Learning Intentions:

At the end of this lesson, you will understand

- how to do divisions such as  $1 \div 7$ ,
- how to relate to the process of such divisions.

The teacher puts the following question on the board:

- *Can you tell me the difference between sharing an apple between seven loving siblings and seven good friends?*

Students may give several answers including the answer that there is no difference. According to the teacher, the friends do not mind if their portion is a little smaller or bigger, but siblings demand that each share to be exactly the same, particularly when the parents do the partitioning of the apple.

Then the teacher puts the following story on the board.

*A couple has seven children. They need to divide an apple among their seven children and each share must be exact. The parents have a magic knife that cuts anything, big or tiny, into ten pieces of exactly equal size. Now suggest a way to divide the apple into exactly seven portions using this magic knife.*

*Students: Cut the apple into 10 pieces and give each sibling a piece.*

*Teacher: Good.*

*Question 1: What is the size of each of the piece?*

*Question 2: Why is this process not exactly satisfying to the siblings?*

With interactive guidance, or direct information, after a while students understand that the each piece given to any of the siblings is one tenth of the apple. The remaining three pieces again have to be exactly divided among the seven siblings.

*Teacher: What are we to do now with the remaining three tenths?*

*Students: Divide each of the tenths of the apple into tenths again.*

*Teacher: When you divide 3 tenths into tenths again, how many slices do you get? What is the size of each of the new slices?*

*Students: New slices are hundredths and there are 30 slices.*

The teacher explicitly points out that since there are seven siblings and only three pieces, as in the case of sharing 381 402 apples with 378 people, we need to repackage the dividend. Similarly we repackage 3 of tenths into 30 of hundredths. This makes it is possible to give each sibling 4 of hundredths. This repackaging continues on, as always there is a remainder. This is why we add ‘mystical’ zeros to the end of the dividend. It is the very same long division algorithm which bases on the concept of repackaging when there is not enough to share.

This discussion will continue until it reaches the recurring point. While this process goes on, students may comment that the accuracy we try to achieve has no practical meaning. For this, I point out that the apple is shared, not by seven good friends but by seven loving siblings. Even after reaching the recurring point the division continues. Only difference is that, even without further division, we just know the numbers coming after the recurring point. I finish this discussion with the question, “Do the siblings get to eat the apple”? They are still in the process of dividing, to get the exact share. It has been a thousand years since the process began.

### **The Number Half**

The lesson is an introduction to fractions at a much deeper conceptual level. I assume that the students are of Year 7 or above. Therefore, they already have a good exposure to fractions. Moreover, they are willing and attentive learners yet they can be of various ability levels.

**Learning Intentions:** After the end of this lesson segment, you will understand

- that fractions are created by people out of necessity and
- what fractions are

*Teacher: What do you know about fractions?*

*Students: A part of a whole, numbers less than one, decimal numbers are some of their possible answers.*

The teacher lists their answers that have relevance to fractions. For instance, the idea that 'the fractions are less than one' is not correct.

*Teacher: What is half?*

*Students: Again students could give answers such as half of an apple, half of one, part of one, etc.*

The answer, half is a half of an apple, is clearly invalid. Some other answers may be vaguely related to the concept of half.

*Teacher: Forget all the things that you know about fractions and consider that the only numbers you know are the numbers in the set,  $N_0 = \{0,1,2,3,\dots\}$ .*

To make sure that the students understand this statement, ask the following questions:

- Can you 'see' 2 in the set,  $N_0$ ? (pointing to the set,  $N_0$ )
- Can you 'see' 10 in the set,  $N_0$ ? (pointing to the set,  $N_0$ )
- Can you 'see' million and 5 in the set,  $N_0$ ? (pointing to the set,  $N_0$ )
- Can you see 2.7 in the set,  $N_0$ ? (pointing to the set,  $N_0$ )

For the first question, they may immediately answer "yes". For the second question, many may answer "No". Ask them to think again. Then more students may say that they see ten inside the set,  $N_0$ . If that is the case, ask how they know. Next, ask from the class what does ' $\dots$ ', which appears at the end of the list,  $1,2,3,\dots$ , mean?

Subsequently, they will see that zero and all positive whole numbers are in the set,  $N_0$ . The answers for the last question may vary. The teacher again emphasises that only zero and the counting numbers are in the set,  $N_0$ . Therefore 2.7 does not belong in the set.

*Teacher: Solve the following equations. Each square has the same value, which is particular to each equation.*

$$\square + \square = 4$$

$$\square + \square = 10$$

$$\square + \square = 100$$

$$\square + \square = 1$$

*Students: Students will solve first three equations correctly and the last one wrongly*

*by giving the answer  $\square = \frac{1}{2}$ .*

This answer is wrong since  $\frac{1}{2}$  does not exist in the set,  $N_0$ . The correct solution is that there are no solutions for the quantity,  $\square$ , in the equation  $\square + \square = 1$ .

*Teacher: There are no solutions for the equation  $\square + \square = 1$ . Consider the question,*

**Is it desirable for us to be able to solve such equations? Why?**

*Students: Some students may say that it is desirable, and some may say not. The students need to provide a reason for their opinions.*

Answers that are not supported by reasoning cannot be regarded as meaningful answers. In particular, the ‘correct’ responses must be justified with reason.

To demonstrate why people needed such solutions, do the following two activities:

- Take a piece of paper. Fold it until it looks like a ruler. Use it to measure a book or another piece of paper. Also measure something which is approximately one and half of the made up ruler. (A real ruler cannot be used here since it already has halfway marks.)

- Ask a student to "sell" two pencils for one dollar. This is the set price. Now just ask him to sell one pencil for an appropriate price.

These situations show that why our ancestors had to think about equations similar to the equation,  $\square + \square = 1$  ]

*Teacher: So, we would like to have a number that can make the equation,  $\square + \square = 1$ , true. If we need and desire to have such a number, then what do we need next? What do we do when we need or desire something that does not exist? Did people need chairs? Did they always have chairs? If the ancient people did not have chairs, how did they get chairs later?*

*Students: Almost invariably, many students will suggest making up the number.*

*Teacher: What is the property that we need for this number?*

After this discussion, help them to realise that we need a number when added twice that gives one. Could there be more than one such number? Discuss this question in detail. For instance, would we like to have two distinct numbers,  $\diamond$  and  $\square$ , with the property that  $\diamond + \diamond = 4$  and  $\square + \square = 4$ ?

$\frac{1}{2}$  is the unique number when added twice that gives 1  $\Rightarrow \frac{1}{2} + \frac{1}{2} = 1$

This means that  $\frac{1}{2}$  is created to do the job, when added twice, to give 1. There are no other numbers satisfying the same property.

*Teacher: How many halves are there in one? [To answer this question, point out to the property written in the box.]*

*Teacher: Now create similar numbers and ask them to complete sentences such as:*

(a)  $\frac{1}{3}$  is the number when added ....

Therefore, there are -----  $\frac{1}{3}$  in one. [How many  $\frac{1}{3}$  are in 1?]

(b)  $\frac{1}{4}$  is the number when added....

Therefore, there are -----  $\frac{1}{4}$  in one.

(c)  $\frac{1}{7}$  is the number when added....

Therefore, there are -----  $\frac{1}{7}$  in one.

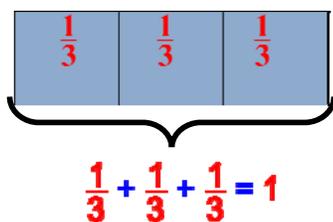
*Teacher: Very good! You have completed the sentences.*

Then the teacher repeatedly asks questions such as what is  $\frac{1}{2}$ ? What is  $\frac{1}{8}$ ? How many  $\frac{1}{12}$  are in 1 and why? If the students fail to answer the questions, in the line of the discussion above, then the teacher will briefly review the material. Wait until they answer confidently to such questions.

*Teacher: Who created these numbers?*

*Students: We created the numbers. [Some will instantaneously answer.]*

*Teacher: That is exactly what our past mathematicians did. They created these numbers. We also re-created these numbers today. Can you pictorially illustrate the idea of the sentences in (a), (b) and (c).*



Here students need to understand that the rectangle is not 1. This rectangle serves as a visual (concrete) image for us to abstractly think about one. A loaf of bread cut into three equal pieces when added together gives the whole loaf. Since each part is equal, any one of these three parts is one third of the loaf. Similarly, the rectangle has been cut into three equal parts. Therefore, any single piece is one third of the rectangle.

Now the students complete relevant exercises from a work sheet or their textbook.

More lesson ideas on fractions can be found in the Appendix. See p. 311. The lesson ideas may not be completely novel but I have strived to present them on the foundation of the constructivist pedagogy I have outlined in the previous chapter.

Next we look at Directed Numbers. Another treatment of negative numbers via the number zero is included in the Appendix. See p.329, *The Number Zero and Negative Numbers*.

### **Directed Numbers**

Imagine that you have a magic jar and magic marbles of Red and Black colour.

**Magic Rule 1:** If we put an equal number of Red and Black marbles into the jar then they cancel each other out and no marbles will be present in the jar.

**Magic Rule 2:** If we put more Red marbles than Black marbles into the jar then only the excess number of Red marbles will be present in the jar. If we put more Black marbles than Red marbles into the jar then only the excess number of Black marbles will be present.

Answer the following questions and then develop general rules for operations between magic marbles in the magic jar.

For instance, your rules should look like:

- If we put a number of red marbles and another number of red marbles into the magic jar, then the colour of the marbles appear in the jar is red/ black.
- If we put a number of black marbles and another number of black marbles into the magic jar, then the colour of the marbles appear in the jar is red/ black.
- In either of the two cases above, the number of the marbles appear in the jar is the sum of the numbers of the marbles we put into the jar.

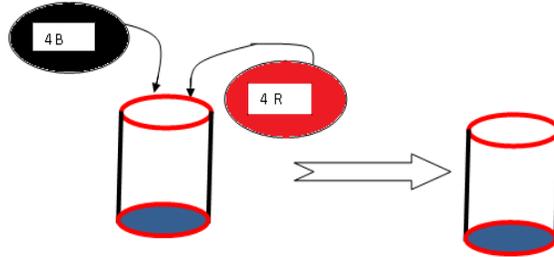
There are many other rules you can derive from the following exercises.

This worksheet needs to base on a high level of interactions between students and students and students and the teacher.

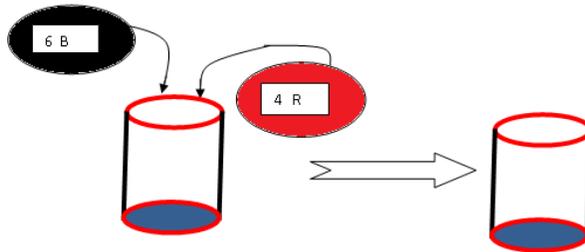
**Questions:**

Your task is to indicate the correct number of Black and/or Red marbles in the right hand jar to depict the result of the operation shown in the left hand jar.

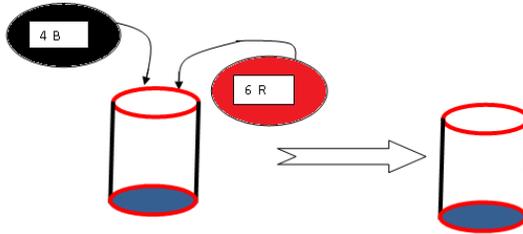
Q1 (a) You put 4 Black marbles and 4 Red marbles into the empty magic jar.



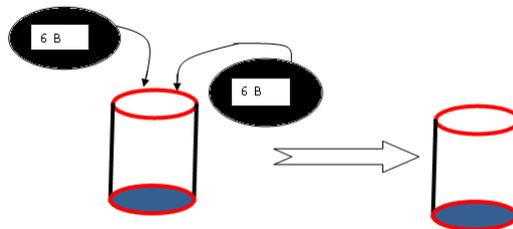
(b) You put 6 Black marbles and 4 Red marbles into the empty magic jar



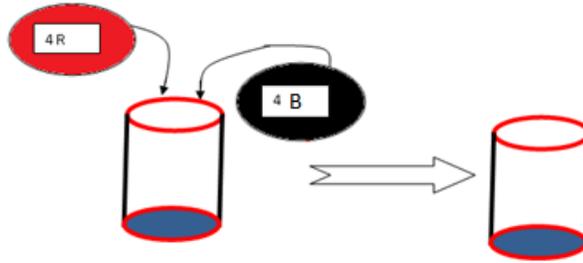
(c) You put 4 Black marbles and 6 Red marbles into the empty magic jar.



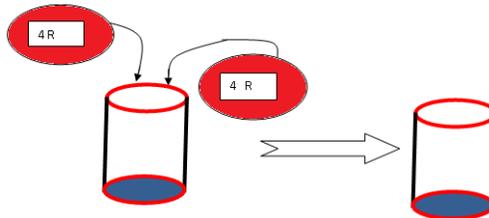
(d) You put 6 Black marbles and 6 Black marbles into the empty magic jar.



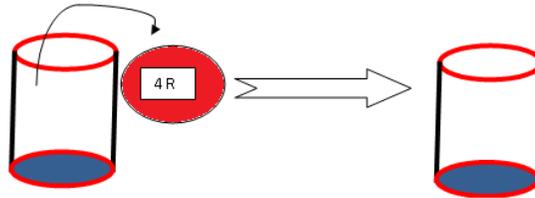
(e) You put 4 Black marbles and 4 Red marbles into the empty magic jar.



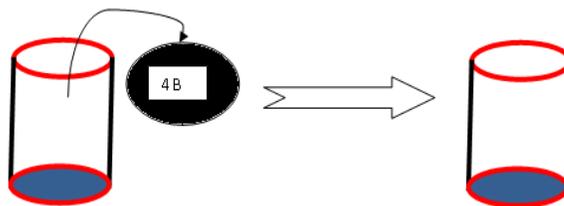
(f) You put 4 Red marbles and 4 Red marbles into the empty magic jar



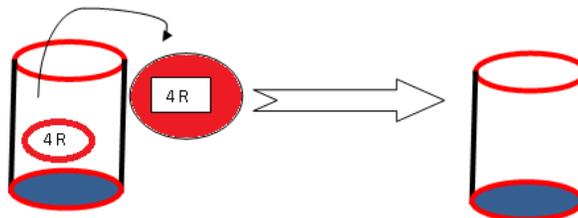
(g) You take out 4 Red marbles from the empty magic jar.



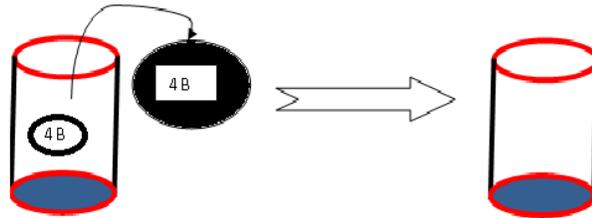
(h) You take out 4 Black marbles from the empty magic jar.



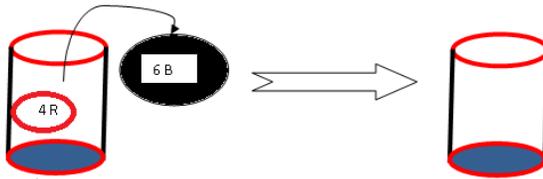
(i) You take out 4 Red marbles from the magic jar which contains 4 Red marbles



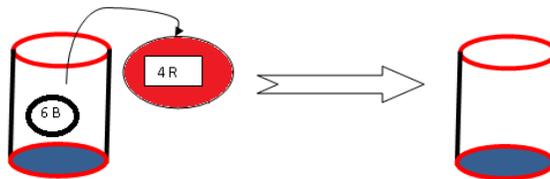
(j) You take out 4 Black marbles from the magic jar which contains 4 Black marbles.



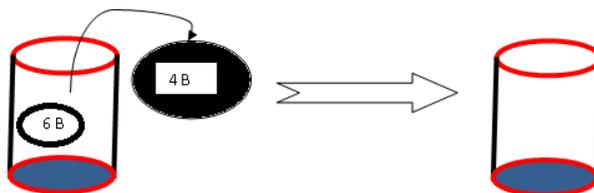
(l) You take out 6 Black marbles from the magic jar which contains 4 Red marbles.



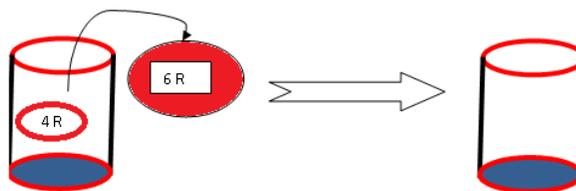
(m) You take out 4 Red marbles from the magic jar which contains 6 Black marbles.



(n) You take out 4 Black marbles from the magic jar which contains 6 Black marbles.



(o) You take out 6 Red marbles from the magic jar which contains 4 Red marbles.



Q2 Develop general rules for these operations. Then summarise them as:

**Magic Jar and Magic Marble Rules:**

Red Marbles and Red Marbles  $\implies$  ?

Black Marbles and Black Marbles  $\implies$  ?

Q 3 Red Marbles and Black Marbles

the operations of addition and subtraction

If the number of Black's = the number of Red's  $\implies$  ??

If the number of Black's > the number of Red's  $\implies$  ?

If the number of Black's < the number of Red's  $\implies$  ?

Q 4 Name the colours that represent the negative and numbers, respectively. Is there are definite colour that represent negative and positive numbers?

Q 5 Develop general rules for the addition and subtraction operations of positive negative numbers. Then summarise them as:

Negative and Negative  $\implies$  ?

Positive and Positive  $\implies$  ?

Positive and Negative

If the Positive number and Negative Number have the same size  $\implies$

If the size of the Positive number > the size of the Negative Number h  $\implies$

If the Positive number and Negative Number has the same size  $\implies$

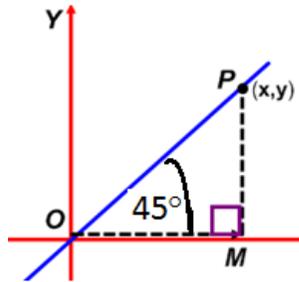
Teacher Remark: Going with the bank notation, we take the colour black to represent positive numbers and red to represent negative numbers. (Teacher may first need to introduce the bank terminology.)

An alternative treatment of negative numbers, which can be used as a supplementary and or complementary component, is presented on p. 329

Next we look at the graphs of straight lines. First we start with a special line.

### A Special Line

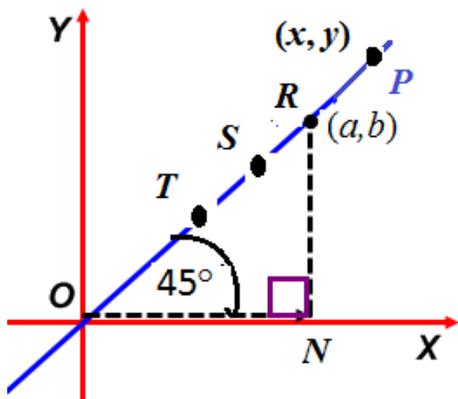
This activity is suitable for students who have an exposure to Cartesian Coordinate Systems and elementary geometry. The line  $OP$  makes a  $45^\circ$  degree angle with the positive  $X$  axis. That is,  $\angle POM = 45^\circ$ . Let  $P = (x, y)$  be any point (except the origin) on this line  $OP$ . The line,  $PM$  is vertical to the  $Y$  axis and the point  $M$  lies on the  $X$  axis.



**Questions: Discuss with your neighbours in a quiet voice.**

- (a) What do you mean by 'any point on the line'?
- (b) What do you know about  $\angle OPM$ ? Why?
- (c) Is the triangle  $POM$  of any special type? Why?
- (d) From this what can you deduce about  $OM$  and  $PM$ ?
  
- (e) What is the relationship between the  $x$  and  $y$  of the point  $P$ ?
- (f) What can you say about  $b$  and  $a$  (Refer to the point  $R$  in the diagram below.)?
- (g) Taking the answers to (e) and (f) into account, using pro-numerals write a relationship between the coordinates  $x$  and  $y$  of the points  $S$  and  $T$ .
- (h) Can  $(3, 5)$  exist on this line  $OP$ ? Why
- (i) If  $(p, q)$  is on the line what can you say about  $p$  and  $q$ ?
- (j) If  $(2p + 1, 3p - 1)$  is on the line find the value of  $p$ . Before you try to find the value of  $p$ , ask yourself what is this about?

[If the students are not familiar with linear equations then instead of  $(2p + 1, 3p - 1)$  use  $(5, p + 1)$ .]



- (j) Now give an appropriate name for this line, taking into account of the relationship between the coordinates  $x$  and  $y$  of any point on the line  $OP$ .

The following lesson develops the idea of the gradient of a straight line.

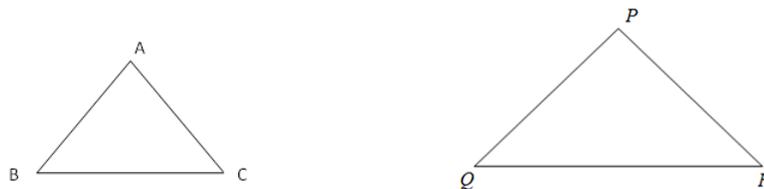
### Why Are Straight Lines Straight? I

#### Learning Intentions:

In this lesson, you will learn

- the concept of the gradient of a straight line and
- how the gradient relates to the straightness of a straight line.

Recall the properties of similar triangles. Particularly put this on the board.



If  $\angle A = \angle P$ ,  $\angle B = \angle Q$  and  $\angle C = \angle R$  then  $\frac{BC}{QR} = \frac{AC}{PR} = \frac{AB}{PQ}$ .

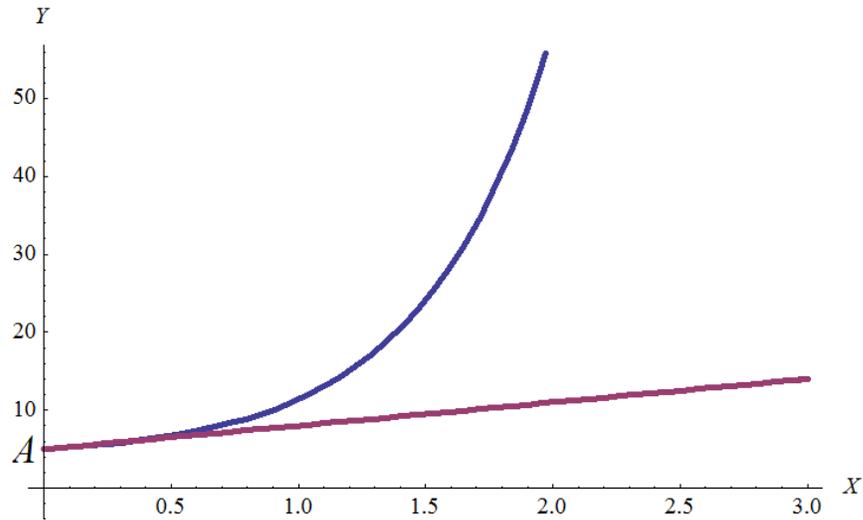
Draw a straight line on the board.

*Teacher: Why straight lines are straight?*

*Students: [They look bewildered and probably they might think that the question is weird.] Some will answer straight lines are straight since they are not curvy.*

*Teacher: Why curvy lines are curvy?*

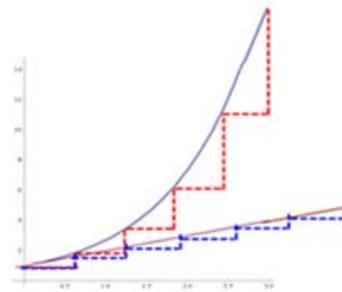
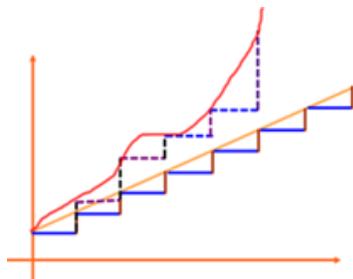
After a while, put the following graphs on the board and instruct them to copy them and say:

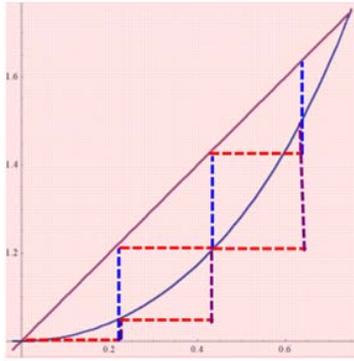


- (i) Now put your two index fingers on the point A. Move your left hand index finger slowly on the curve line. Simultaneously, move your right hand index finger slowly on the straight line. Ensure that both index fingers move with the same  $x$  coordinate. That is, the line through the index fingers is always a vertical line.

Now, observe that even though both index fingers move with the same  $x$  coordinate, they do something different. What is it? Do this many times until you see what goes on.

- (ii) See, from your activity above, whether you have grasped the variation of  $y$  which is similar to the variations depicted in the following graphs

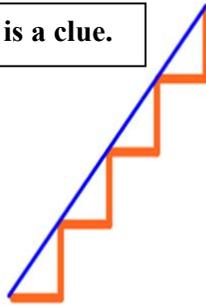




(iii) While  $y$  changes at a same rate throughout the straight line, for the curve line,  $y$  changes at different rates in different intervals.

There is something that you are very familiar in your day to day life that reminds you the variation of  $y$  for straight lines. What is it?

Here is a clue.



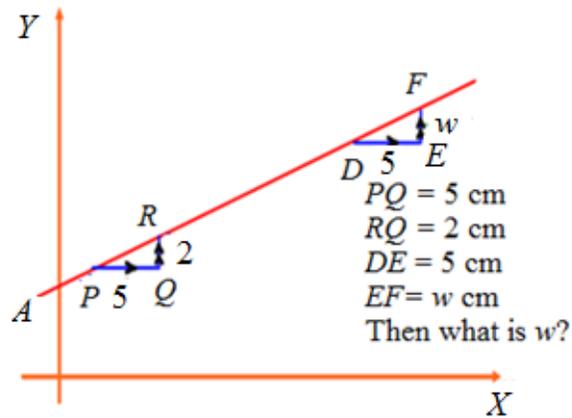
(iv) Now summarize your finding.

- Curve lines: For the same amount of change of  $x$ , the resulting change of  $y$  is different in different intervals. Thus, the curve line has changing step height.
- Straight line: For the same amount of change of  $x$ , the resulting change of  $y$  is the same in any interval.  
So, Straight lines resemble stair case of constant step height.

(v) **Staircase Principle**

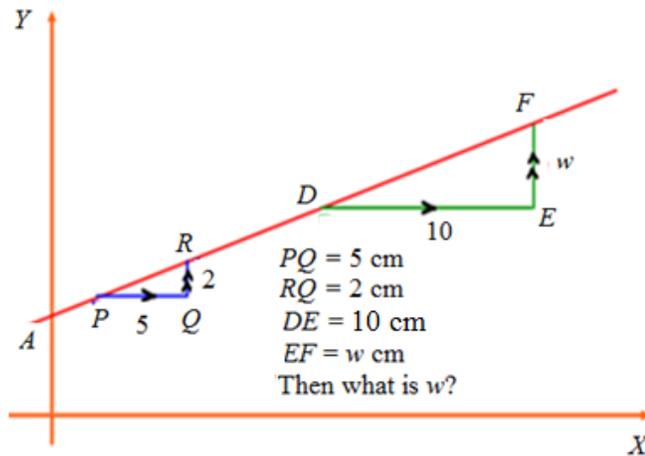
Notice, in the following diagram,  $PQ$  and  $DE$  are parallel to the  $X$ -axis while  $QR$  and  $EF$  are parallel to the  $Y$ -axis. To answer the following question, do not use any ideas learned previously. You should use, only the ideas discussed in this session.

To calculate the length,  $w$  in the diagram below, use the idea that straight lines resemble stair cases.



For the same amount of change of  $x$ , the change of  $y$  is the same we know that  $w = 2$  cm.

(vi) Look at the two triangles  $PQR$  and  $DEF$ . Then use the same "Staircase Principle", to find the distance,  $w$ , in the following situation. Notice that a stair is missing in the second triangle.



The difficulty here is that the changes in  $x$  in the two intervals are not the same. This may be thought of as missing of a stair step. In the first interval, the change of  $x$  equals 5 units; in the second interval, the change of  $x$  is 10 units. Therefore we cannot say  $EF = 2$  cm. Notice that the triangles  $PQR$  and  $DEF$  are similar triangles. (Why?)

Because the triangles are similar, their corresponding sides must have the same ratio. Thus,

$$\frac{EF}{DE} = \frac{QR}{PQ}$$

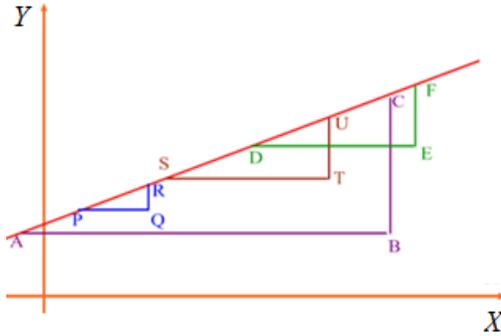
Thus,

$$\frac{w}{10} = \frac{2}{5},$$

and

$$w = 4 \text{ cm.}$$

(Vii)  $\frac{\text{Rise}}{\text{Run}}$



Take 'Run' to be the length of the horizontal base of any triangle shown. 'Rise' is the height from the base to the line. For instance, for the Triangle  $ABC$ , the Run is the distance  $AB$  and the Rise is the Distance  $BC$ . For each of the triangle, consider the ratios:

$$\frac{\text{Rise}}{\text{Run}}$$

**What do you think of these ratios of  $\frac{\text{Rise}}{\text{Run}}$  for different triangles?**

Looking at the many similar triangles drawn in the picture above, we can see that for each of the different intervals the ratio, of the step rise or height (change in  $y$ ) to step width or run (change in  $x$ ), is the same. Teacher gives the following information, after some discussion.

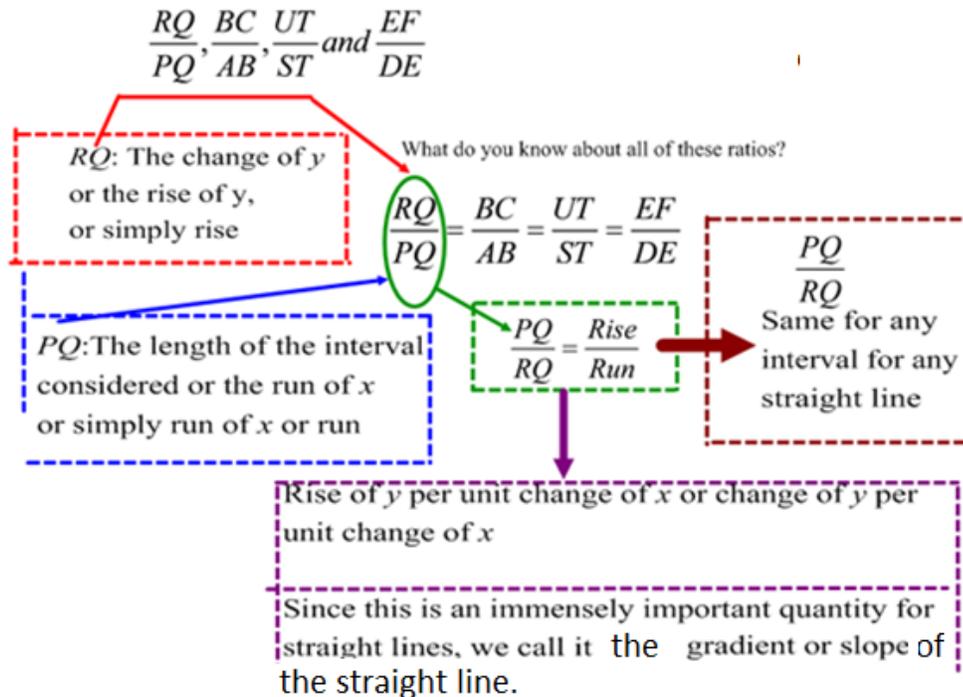
For a straight line, the ratio of step rise (simply rise) to step run (simply run) is a constant (same) for any long or short interval, far away or much closer interval. For a curvy line, the ratio of 'rise:run' keeps changing from one interval to another (in fact from one point to another). We can summarise that

In any interval for any non-vertical straight line,

$$\frac{\text{Rise}}{\text{Run}} = \text{Constant.}$$

This ratio is called the slope or the gradient of the straight line. Any given line, except for vertical lines, has its own gradient.

Next put the following summary diagram on the board.



Further treatment on straight lines is continued in the Appendix on p. 337

**Relations and Functions:** For Year Ten or Eleven

### Learning Intentions

- In this lesson, you will learn what we mean by a mathematical relation.

Define: A Relation is a set of ordered pairs.

Ask students to read the definition carefully and construct an example. Instruct them to concentrate on the keywords, to accomplish their mission. Discuss the following notions.

Set: A set is a collection of objects or numbers.

Ordered pairs: What is a pair? What do we mean by the word, order?

Question them, to see whether the students understand the concept of ordered pairs.

A pair is a couple together such as  $(a,b)$ . If we consider  $(a,b)$  to be an ordered pair, what do you think about the pairs  $(a,b)$  and  $(b,a)$ ? Are they the same?

For instance, consider the mother-daughter relation. A female called Shanti occupies the first place then Shanti is a mother. When the same person, Shanti occupies the second place then Shanti is a daughter.

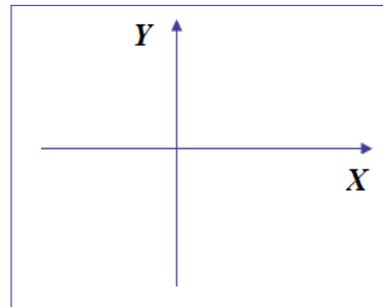
Ask them whether they have seen such ordered pairs before? Tell them that they may have seen them even when they were in Grade 5.

Put a set of axes on the board and put a dot somewhere in it? Ask, do they see an ordered pair?

Whatever the answer is, put any arbitrary point such as (2,3) on the set of axes, according to a rough scale repeat the same question. Now ask them to mark (3,2). Then ask whether they have understood the meaning of an ordered pair.

Put a set of axes on the board and ask students what they see, apart from the set of axes.

The students will be bewildered by this question and will claim except for the axes and their labels there is nothing. First, repeatedly point out what a relation is. Instruct the students to write down if they see something and not to say it out loudly. This helps to keep student thinking.



This also prevents students from adopting the answers of the other students without any thinking. Teacher is to read the comments and decide when to open these comments to the whole class. This scheme could optimise the cognitive efforts of all students.

Keep repeating the definition of a relation, while asking, “Why am I repeating the same thing again and again?” If this does not work, show any point in side the set of axes and ask, what they see there. Any point inside the set of axes is a pair of

coordinates. The immediate effect of putting a set of axes is to make any point on board, a pair of coordinates. Moreover, these are, by definition, ordered. The pair (2,3) is not the same as the pair (3,2). Therefore, as immediately we draw a set of axes, it defines a relation, but this relation is a mundane relation.

- Now draw any shape inside the set of axes.

Even if some people may be hesitant, to see just the set of axes as an expression of a relation, everyone will agree that this picture is a relation since it is a special collection of points.



A relation is a set of ordered pairs. The pairing may or may not be subjected to further restrictions.

$$f = \{ (x, y) \mid y \text{ is related to } x \text{ by the rule } y = f(x) \}$$

This reads as "The Relation,  $f$ , is the set which contains the ordered pairs of  $(x, y)$  where  $y$  relates to  $x$  by the rule  $y = f(x)$ . Even though the real notation is  $f = \{ (x, y) \mid y = f(x) \}$  for simplicity, we write  $y = f(x)$ .

Give several examples, including some examples of social relations examples. Consider the following sets. Are they relations? Why or why not.

- (i)  $\{ (M, C) \mid C = \text{The child, } C, \text{ of the mother, } M \}$
- (ii)  $\{ (C, M) \mid C = \text{The mother, } M, \text{ of the child, } C \}$
- (iii)  $\{ (x, y) \mid y = x \}$
- (iv)  $\{ (x, y) \mid y = 2x - 3 \text{ and } x \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} \}$
- (v)  $\{ (x, y) \mid y = 2x - 3 \}$
- (vi)  $\{ (1, 2), (2, 3), (3, 4), (4, 5) \}$
- (vii)  $\{ (1, 2), (2, 3), (3, 4), (4, 5), (4, 4), (2, 5) \}$

An extension of this lesson, 'Functions' can be found in the Appendix. See p.340.

## Division by Zero

Non-divisibility of a number by zero is not clearly understood by even some Year 11 and 12 students. For some of them, it is merely a fact. Even the students who know and understand this property may not have it in their awareness, with the same level of awareness that a driver has about red light signal.

### Activity Worksheet

#### Learning Intention:

If you engage in this activity faithfully it can help you to understand why division by zero is problematic.

However, this exercise, by itself, will not keep the fact in your awareness. Nevertheless, this fact needs to be in your awareness as a driver is aware that red light means "stop." This means that when you see the quantity  $\frac{1}{x}$  it should flash in your mind immediately that  $x$  cannot equal zero.

#### I

Recall that

$$a \div b = c \Leftrightarrow a = b \times c.$$

The same statement can be written in a slightly different notation:

$$\frac{a}{b} = c \Leftrightarrow a = b \times c.$$

An example of this is,

$$12 \div 3 = 4 \Leftrightarrow 12 = 3 \times 4 \text{ or } \frac{12}{3} = 4 \Leftrightarrow 12 = 3 \times 4.$$

Note that from now on we use only the second version.

Write six similar examples; 3 using just numbers, another two using only pro-numerals and the sixth example, using at least one pro-numeral and one number.

Now assume that 5 can be divided by zero. This means that there should be some real number, say  $N$  such that

$$\text{If } \frac{5}{0} = N \text{ then } \dots$$

Is there a number that can take the place of  $N$ ? Try various numbers for  $N$  and complete the sentence,

$$\text{If } \frac{5}{0} = N \text{ then ...}$$

Do you accept the implication of this completed statement? Appropriately relate the following scenario to the implication of this statement: I borrow \$5 from you and to pay back I give you \$0. In this case, do you accept that I have paid my debt? Since

$$\frac{5}{0} = N \Rightarrow 5 = 0 \times N. \text{ That is, } 5 = 0.$$

According to the statement above when I give you \$0, I have settled my debt to you. Therefore, 5 cannot be the answer for this division. Can any number  $N$  satisfy the equation

$$\frac{5}{0} = N?$$

In other words, is there any number which equals  $\frac{5}{0}$ ? If we do not want to accept the implication that zero is equal to a nonzero number, what should we think of the division of any nonzero number by zero?

## II

To investigate about the division of 0 by 0, assume that the division gives the result,

$$\frac{0}{0} = N,$$

where  $N$  is a real number. Can we say  $N = \frac{0}{0} = 0$ ? To check this, we need to test the division rule:

$$\frac{a}{b} = c \Leftrightarrow a = bc.$$

Can 5 take the place of  $N$  in  $\frac{0}{0} = N$ ? How about, -1000, 100 000 etc.? Is there a number that cannot take the place of the result  $N$ ?

Do you like the implication arising from this calculation? To answer this question, relate to the following scenario. If 5 kg of apples cost \$20, how much is one kilogram? Would you like, if the answer to this question depends on the time of the

day, location of the deal or the mood of the vendor? What happens if different answers are possible for this division?

Complete the following sentences by deleting the inappropriate terms.

- Division of a nonzero number by zero is ruled in / ruled out since there is no answer at all / there are too many [infinitely many] answers.
- Division of zero by zero is ruled in / ruled out since there is no answer at all / there are too many [infinitely many] answers.

Finding an appropriate value of  $\frac{0}{0}$ , in a contextually appropriate manner, is the core of the future topic, differentiation.

The focus of our next lesson idea is how to recognise the ‘shape’ of a function from its rule.

### **Pictures of Mathematical Functions through the Conceptual Camera of the Mind**

Sketching of the Graphs of  $y(x) = x$ ,  $y(x) = x^2$  and  $y(x) = \sqrt{x}$  on a same set of axes

**This lesson deals with:**

- the concept of the graph of a function
- ability to picture mathematical relations from their formulas
- the set of values of  $x$  for which the individual graphs exist
- the set of values of  $y$  that each function take

### **The Quadrants of existence for each graph**

How the growth rates of each function gives a different shape to their graphs

Note: For simplicity and convenience, many authors write

$$y(x) = x, y(x) = x^2 \text{ and } y(x) = \sqrt{x},$$

respectively.

## Activity Sheet

### I

Recall that  $y = x$  means that the variable  $x$  owns ( $x$  pairs itself with) a  $y$  value, which equals  $x$ . Similarly,  $y(x) = x^2$  means that  $x$  owns a  $y$ , which equals the square of  $x$  and  $y(x) = \sqrt{x}$  means that  $x$  owns a  $y$  which equals  $\sqrt{x}$ .

**Answer the following questions.**

(a) What values of 'x' can own a 'y' value which is equal to  $x$  itself?

This question will not be clear to some students. Trying to make it clearer will result in giving them the answer directly.

So, ask them another question. What does it mean by  $y(x) = \frac{1}{x}$ ?

They should be able to answer this question.

Next, ask what values of  $x$  can own a  $y$  value which is equal to  $\frac{1}{x}$ ?

This question should be clear but many will not have even a clue? Ask: is there a number, which we cannot use as a divider?

Some could give a number such as 3. They might reason out that the number 3 does not go into 1. Point it out to them that one was not divisible by three only before the construction of the rational numbers. The function  $y(x) = \frac{1}{x}$  cannot be defined when

$x = 0$ . simply because  $y$  cannot own  $\frac{1}{x}$  at  $x = 0$  since a number cannot be divided by 0.

Now go back to  $y(x) = x$ . What values of 'x' can own a 'y' value which is equal to  $x$ ? Explain this even if some students have answered it correctly.

Now ask them to graph this equation. The graph of a function is the collection of all the ordered pairs  $(x, x)$  satisfying the condition that  $y = x$ .

### II

(b) Now consider  $y(x) = x^2$  and  $y(x) = \sqrt{x}$ .

Answer the following questions.

Is there any number  $x$  for which  $x$  cannot be squared?

For what values of  $x$ , can we pair  $x^2$  with  $x$ ?

Is there any number  $x$  for which  $x$  cannot be square-rooted? Can the graph exist for these values of  $x$ ?

For what values of  $x$ , we can pair  $x$  with  $x^2$ ?

Put this on the board after they have engaged in their inquiry: Only square-root-able  $x$  can own  $\sqrt{x}$  as its  $y$  partner.

(c) Now again consider  $y(x) = x$ ,  $y(x) = x^2$  and  $y(x) = \sqrt{x}$ .

If  $y(x) = x$  then what kind of values  $x$  can own?

If  $x = 0$  then its  $y$  value is zero. If  $x$  is positive then  $x$  owns a positive valued  $y$ .

If  $x$  is negative then  $x$  owns a negative valued  $y$ .

On the basis of the answers above, in what Quadrants does the graph exist in?

If  $y(x) = x^2$  then what kind of values  $y$  can own?

If  $x = 0$  then  $y$  owns ---. If  $x$  is positive then  $x$  owns a ----- valued  $y$ . If  $x$  is negative then  $x$  owns a ----- valued  $y$ .

On the basis of the answers above, in what Quadrants does the graph exist?

If  $y(x) = \sqrt{x}$  then what kind of values  $x$  can own as its 'y' partner?

If  $x = 0$  then  $y = 0$ . If  $x$  is positive then  $x$  owns a ----- valued  $y$ . If  $x$  is negative then  $y$  is positive/negative/zero/undefined.

On the basis of the answers above, in what Quadrants does the graph exist in?

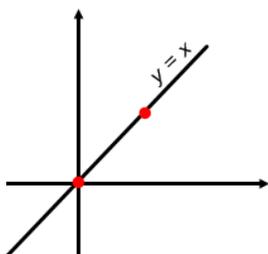
**III** Answer the following questions

(d) Are there any points that are common properties of all of these three graphs:

$y(x) = x$ ,  $y(x) = x^2$  and  $y(x) = \sqrt{x}$ ? Graph only  $y(x) = x$ . Also, on this graph

Mark these commonly owned points.

Since these points,  $(0, 0)$  and  $(1, 1)$  are on all three graphs, can all of the three graphs be the same?



Now think: What is bigger?  $x, x^2$  or  $\sqrt{x}$ ? For this question, almost all ways, almost all students answer that  $\sqrt{x}$  is the smallest (since  $x$  has been square-rooted, and  $x$  is the largest since it has been squared).

- (e) Use the following values to determine the order of the magnitudes of  $x, x^2$  and  $\sqrt{x}$  when  $x$  is between zero and 1 and when  $x > 1$ . Then think of other values, to see whether your observations hold in general.

$x$	0	$\frac{1}{4}$	1	4
$\sqrt{x}$	0	$\frac{1}{2}$	1	2
$x^2$	0	$\frac{1}{16}$	1	16

Convince yourself that if  $0 < x < 1$  then  $x^2 < x < \sqrt{x}$  and if  $x > 1$  then  $\sqrt{x} < x < x^2$ .

Use this information to determine which of the graphs,  $y(x) = x^2$  and  $y(x) = \sqrt{x}$ , goes above or below the graph of  $y(x) = x$  in the intervals,  $0 < x < 1$  and  $x > 1$ , respectively.

#### IV

- (f) Find the average rate of change (AROC) of all three functions  $y(x) = x$ ,  $y(x) = x^2$  and  $y(x) = \sqrt{x}$ , in the intervals,  $[0, 0.0001]$  and  $[10\ 000, 10,0001]$ . Use this information to gauge how fast the functions changes for small  $x$  closer to 0 and then for very large  $x$ . Note that the average gradient or average rate of change of a function in a given interval is the quotient of the rise over run in the interval.

Interval	Average Rate Of Change Of		
	$y(x) = \sqrt{x}$	$y(x) = x$	$y(x) = x^2$
$[0, 0.0001]$	100	1	0.0001
$[10\ 000, 10\ 001]$ Use the calculator here.	0.0045 (four decimal places)	1	20\ 001

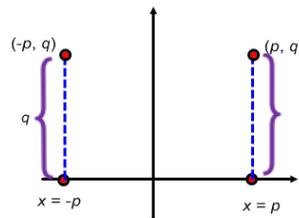
Notice that still we have to complete the graph of  $y(x) = x^2$ . The graph of the function,  $y(x) = x$  is already completed and the graph of  $y(x) = \sqrt{x}$  exists only in Quadrant I.

## V

(g) Consider the  $x$ -coordinate  $x = p$ , which is located  $p$  units to the right of the origin and  $x$ -coordinate  $x = -p$ , which locates  $p$  units to the left of the origin.

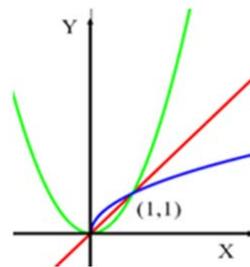
For any value  $p$ , both  $x = p$  and  $x = -p$  share the same  $y$  partner.

Depict this information in a set of axes.



Consider the following table and complete the graph of  $y(x) = x^2$ .

$x$	-5	0	5	$p$	$-p$
$x^2$	25	0	25	$p^2$	$p^2$



## VI

(h) Now go through the whole activity again and see how we generated each piece of information. Then see how you can incorporate the information into sketching of the graphs. These sketches are photos of the function taken by the conceptual camera of the mind.

See whether you can recreate all this information in your mind and picture the graphs just by looking at the functions without resorting to your memory. That is, you should not see the graph through the previous memory of the sketches; rather you need to see the graph through the work you did. For instance, as soon as you see the formula  $y = x^2$ , you should be able to process quickly and mentally that it cannot go below the  $X$  axis, and it possesses symmetry over  $Y$  axis, etc. This should be done with quick mental processing of the mathematical concepts.

## **My Science Coins**

### **How Do We See**

Ancient Greek scholars had hypothesised that people see things by emanating rays from their eyes. These rays fall upon objects. For convenience, let us call this the Greek Hypothesis. For simplicity, we consider only the objects that do not produce their own light.

### **Questions:**

Being scholars, they could not have stated any idea without long deliberation.

- (a) Assuming that you are one of these prestigious and wise scholars, design an experiment which can be done even now in class, to justify your hypothesis (the Greek Hypothesis).
- (b) Now assume that you are a modern scholar. To discredit the Greek Hypothesis, suggest an easy experiment.
- (c) Which aspects of vision, did the Greek scholars fail to consider?
- (d) Suggest a new hypothesis to explain how we see.
- (e) Suggest a simple experiment, which you may have already mentioned, to justify your new hypothesis. This hypothesis should incorporate both aspects; the aspect that the Greek scholars acknowledged and the aspect that they overlooked.
- (f) Modify the Greek Hypothesis to counter-argue the experimental results in (b) and (e).
- (g) Suggest a new experiment to defend the modified hypothesis.
- (h) Is your hypothesis in (d), the final say about vision?

- (i) Do you still think that these scholars were wise and intellectual giants of mankind?

Without presenting the extended and intensive Socratic interactive discussion, I summarise the discussion which can follow from and through these questions.

- (a) The Greek hypothesis gives rise to a prediction that if we close our eyes we cannot see. Therefore, a simple experiment is to close one's eyes; still the vision is possible. By closing our eyes, we lose vision. So this can be regarded as a tentative 'proof' or verification of the Greek hypothesis.
- (b) In a dark room, even with wide open eyes, one cannot see. This falsifies the Greek hypothesis.
- (c) Greek scholars only paid attention to the role of eyes in vision, and they overlooked the role of an external light source.
- (d) For vision to be possible, light from an external source needs to fall on the object. The light bounced back from the object should reach the eye.
- (e) The hypothesis in (d) gives rise to predictions that, to see,
  - (i) if there is no light source, regardless our eyes are close or not we cannot see,
  - (ii) if we close our eyes, regardless there is an external light source or not, we cannot see.
- (ii) even if our eyes are open and there is an external light source whose light falls on the object, if the light bouncing back from the object does not reach our eyes, we cannot see.
- (f) **Greek Hypothesis:** Rays emanates from the eye. These rays fall on the object. That makes the vision possible.  
**Modified Greek Hypothesis:** Light emanates from the external light source. This light falls on the eyes. Our eyes redirect this light on the object. That makes the vision possible.
- (g) Arrange a situation in which the light from the external source falls on the eyes, but the light from the external source does not fall on the object. Furthermore, the object is in line of sight. That is, it is possible the rays emanating from the eye, to reach the object, but the light from the external source do not fall on the object.

- (h) Nothing we say about Nature can be taken as the final say as the history of science amply demonstrates. The idea that there can or cannot be a final say is just a belief. All scientific ideas are open to modification and revolutions. For instance, Einstein once said that one single contradictory experiment result would be sufficient to falsify his theories even if there are zillions of affirmative experimental. (This is very true but Einstein was exaggerating it a little bit.)
- (i) I do not know what you think about them, but I believe that they were intellectual giants.

Also, there is a twist to this story. According to modern Physics, in fact, our eyes emanate light rays. Any object above the absolute zero temperature emanates electromagnetic radiation, which are photons (light). The frequencies of these waves tend to be higher as the temperature rises. Our eyeballs are not at the absolute zero temperature. Consequently, the eyes also emanate million times more photons than the number of photons entering into them from any normal light globe. The light rays emanate from our own eyeballs is infrared. Therefore, this light cannot create images in our brains. See Purcell (1984).

### **Atoms**

The concept of atoms is not a modern idea. The Greek philosophers, Leucippus and his pupil, Democritus introduced the concept of atoms around 450 B.C. They hypothesised that the building bricks of all matter were atoms. The word ‘atom’ originated from a Greek word, ‘atomos.’ The meaning is un-cut-able. According to Leucippus, atoms are the building blocks of all matter that exist therefore cannot be broken down any further. In other words, atoms are the smallest entity that can exist as an element. For instance, Carbon atom is the smallest amount of Carbon that can exist as Carbon. If we further divide the Carbon atom then the resulting particles are not Carbon any more. The particles obtained by ‘dividing’ an atom further are subatomic particles.

Aristotle had different ideas. According to Aristotle, all matter consists of five elements

- Water (Liquid element)
- Fire (Heat element)

- Air (Gas element)
- Earth (Solid Element)
- Ether (Heavenly Stuff)

Here, at this moment, I will not instruct my students to research on the internet about atomic theory and Aristotle's Five Elements Theory of Matter. The purpose of this is to control the amount of information they get. Certainly, one element of education is to gather information and be aware of them. The awareness of the information we have obtained is much more important than just storing information in our brains.

**Questions:**

- (1) In the ancient times, the Five Element Theory of Matter won over the Atomic Theory. This was partly due to the reputation of Aristotle. Assume that you are Aristotle. Design experiments to defend your theory against atomists.
- (2) Now assume that you were the atomists during the philosophical rein of Aristotle. Explain the difficulties of designing an experiment to back your theory. You cannot use any modern technology or ideas to do so.
- (3) Is the Five Element Theory necessarily wrong?
- (4) On what basis, do you think that the modern Atomic Theory is a better view of matter than the ancient Five Element Theory? (You may revisit this question after you complete this unit. I encourage you to do internet research and visit the school or the public library.)

Encourage students to discuss these ideas with each other while monitoring the quality of the debate. Also, ask questions and give ideas to ignite their inquiry. For instance, burning a piece of a tree can produce water, smoke, heat and ash. This verifies the existence of four of the five elements. We can 'see' that the sky and heavenly bodies exist. There were many experiments that Aristotle could have performed to verify his theory, but the atomist could perform only the thought experiment.

## Activity

Consider the following hypothesis

Atoms consist of positive charges and negative charges. (These words, positive and negative, are just labels and the essential concept here is that these charges have opposite charge of each other as we can perceive.)

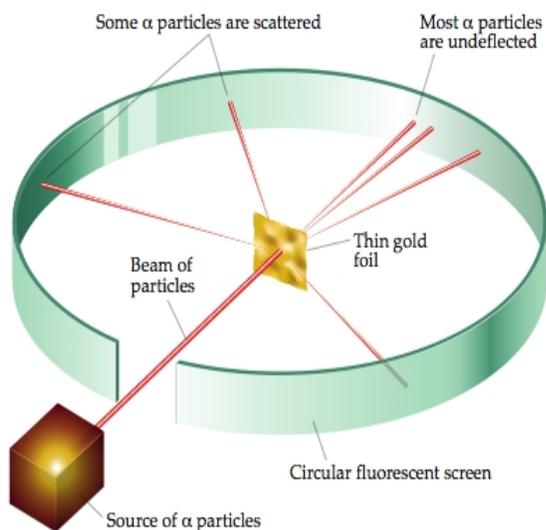
- On the basis of this assumption explain why a book is electrically neutral.
- Why can rubbing of an object against another object sometimes produce electricity?
- Why does this happen for some material and not for other material?

## The Rutherford Experiment

Consider the following information.

**$\alpha$ -particles:** Scientists knew that some elements decayed naturally into other elements by spontaneously evicting micro particles. This act of decaying of matter into different elements is radioactivity. The substances that possess can decay spontaneously are radioactive elements. Some radioactive elements emit  $\alpha$ -particles. Scientist later understood that  $\alpha$ -particles are Helium nuclei.

Rutherford bombarded a thin foil of metal and a thick foil of a metal with a stream of  $\alpha$ -particles, obtained from a radioactive source. The picture below presents a sketch of this experiment



This is the experiment. A stream of  $\alpha$ -particles is moving towards thin and thick foils of gold. The following observations were made.

[This is a Google image.]

- When the stream of  $\alpha$ -particles hits a thick sheet of gold foil all of these particles bounce back.
- When the stream of  $\alpha$ -particles hits a sufficiently thin foil of gold metal,
  - All most all  $\alpha$ -particles go through the foil to the other side.
  - A few  $\alpha$ -particles deflect.
  - Even a fewer number of  $\alpha$ -particles bounce back.

### Questions

Assume that you were the scientists who made these observations for the first time in the history. What hypothesis would you have put forward, to explain the observations above? Please do not quote from any previous knowledge. Interact only with the given information and think creatively. Try to make up a reasonable and intelligent story. There are no right or wrong answers. There exist only reasonable or unreasonable answers.

To encourage students to construct appropriate visual images, ask the following questions:

What happens if a stream of tennis balls hits a wall? In that situation, why do all the tennis balls bounce back?

What happens, if a stream of sand hits a tennis racket? Why would almost all sand goes to the other side of the racket and only a little amount of sand bounce back?

How do we relate this to Rutherford's gold foil experiment?

Now watch the animation "Nature of Atom". Make sure to observe carefully thoughtfully.

## Activity

### Background

As you have previously learned, the magnitude  $F$  of the magnetic force  $F$  on a charged particle, which is moving in a uniform magnetic field with velocity,  $v$  (speed,  $v$ ) satisfies the relationship,

$$F = qvB \sin \theta.$$

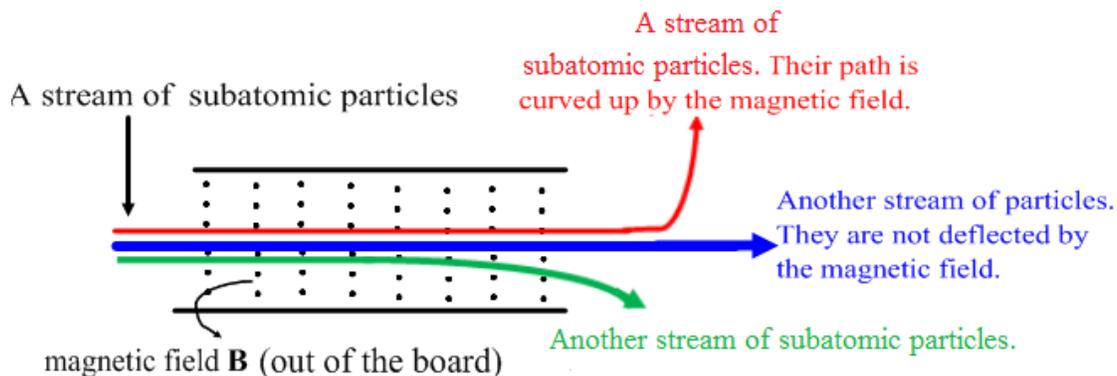
Here,  $q$  is the magnitude of the charge of the particle and  $\theta$  is the angle between the magnetic field vector,  $B$ , and the velocity vector,  $v$ . When the magnetic field vector  $B$  is perpendicular to the velocity vector  $v$  then

$$F = qvB.$$

The magnetic force  $F$  has the direction determined by Fleming's Right Hand Slap Rule and the sign of the charge. This means that the opposite charges moving in the same magnetic field with the same velocity deflect in the opposite directions.

### The Experiment of Moving Electrons in a Magnetic Field Experiment

Consider the following information. A stream of subatomic particles (particles evicted by bombarding atoms of some element with  $\alpha$ -particles) goes through a uniform magnetic field as depicted in the following diagram.



Assume that you have performed this experiment and now your task is to analyse the information and explain them. To do so, consider the following questions.

- (i) What does this experiment say about the constituents of atoms?
- (ii) Why do some particles curve up from their horizontal path?
- (iii) Why do some other particles curve down from their horizontal path?
- (iv) What can you say about the particles that go without any deflection?

To answer these questions think about the following questions.

- (a) What causes some particles to deflect from their paths?
- (b) Why one particle is deflected upward and another downward?
- (c) Why do one group of particles go through the magnetic field without any deflection?
- (d) Why do some particles display a massive deflection and another set of particles undergo only a tiny deflection? On what situation, a tennis ball suddenly deflects from its path?
- (e) Prior to this experiment you held the hypothesis that atoms consisted of positively and negatively charged micro particles. After the experimental results, can you still hold the same hypothesis?
- (f) How do you modify the hypothesis?

Through interactive questioning, help the students to arrive at following conclusions. Particles change their velocity only when a net force acts on them. Thus, some of the moving particles deflect because they experience a magnetic force. This means that the deflected particles are charged particles. Opposite directions of the deflections indicate the directions of the net forces operating on the particles. That is, some of the particles should be positively charged, and others negatively charged. Application of Fleming's right hand rule shows that the particles that deflect downward have negative charges. These particles that deflect upward have positive charges. Larger the deflection is smaller the ratio of mass to charge ( $m/q$ ) for the particle. The smaller deflection indicates a larger mass to charge ratio ( $m/q$ ) for the particular particle. The particles which did not show any deflection did not experience any net force; therefore they did not have any charge. Consequently this falsifies our initial hypothesis that the atoms had only positively and negatively charged micro particles.

We can summarise the new Atomic arising from these experimental results:

Atoms consist of

- negatively charged particles (electrons),
- positively charged particles (protons) and
- neutral particles that have no charge.

Imagine that you are the proud scientists who have discovered the new particle which has no charge. Also remember that you have just dropped the hypothesis that atoms consist of just electrons and protons only.

#### More Background Information

Suppose that through scientific journals, you gather the following information:

- Electrons and protons have the same magnitude of charge. That is, their charges differ only by the sign. The sign indicates that their charges are opposite of each other.
- A proton is 1840 times more massive than an electron.

#### Questions:

- If atoms have charged subatomic particle constituents, why objects, such as books, pencils and pens, are neutral in charge?
- On account of the answer to the question above, what could be the composition of the neutrons?

Some students by considering the answer to the first question may suggest that the neutron is neutral because it is a combination of an electron and a proton. If they suggest this hypothesis tell them that, this was what scientists suggested after the experiment. Ask, as they are the scientists who have made the hypothesis that

$$\text{Neutron} = \text{Electron} + \text{Proton},$$

what will be their next step?

Consider the information below and check your hypothesis.

$$\text{electron mass} = 9.10938188 \times 10^{-31} \text{ kg}$$

$$\text{proton mass} = 1.6726 \times 10^{-27} \text{ kg}$$

$$\text{neutron mass} = 1.6749 \times 10^{-27} \text{ kg}$$

Some students will check and verify that

$$\text{Mass}_{\text{Neutron}} > \text{Mass}_{\text{Proton}} + \text{Mass}_{\text{electron}}.$$

Then they may conclude that Neutron cannot be a combination of electron and a proton. Therefore, the new charge-less particle, neutron must be a particle in its own

right. This was what exactly scientists concluded. This means that students themselves can think as scientists.

However, later there was a twist to this story:

According to later experimental results scientists concluded that a Neutron can be decayed into an electron, proton and an anti-neutrino (another subatomic particle).

### **Magnets**

Suppose that you were ancient scholars in the area of Magnesia where magnetic rocks were abundant. Suggest experiments to establish that

- magnets have two types of poles
- similar poles repel each other
- opposite poles attract each other

A magnetic rock, if free to move, always aligns in a particular direction, with a particular type of pole always turning into a specific direction.

Use the discussion above to suggest that the Earth itself is acting as a magnet.

Magnetic rocks do not have marked poles. The students cannot just use the repulsion or attraction of poles as a clear indication of distinct types of poles. Of course, repulsion and attraction vaguely suggests the existence of two different kinds of poles, but the task is to establish this in a much more convincing manner. They may hang ten or twenty magnetic rocks and let them aligned in their preferred direction. Then mark the ends in two different colours. Repulsion of magnets occurs when we move the poles marked with the same colour closer. Attraction happens when we move the poles marked with different colours closer. This establishes the two pole theory.

The preferred direction of the rocks suggests that (assuming that there are no other known magnets in the area of the experiment) there is another invisible magnet somewhere. By repeating the experiment in various places, we can establish that this magnetic effect is not particular to buildings or their contents or any particular

geographical area. This leads to the only viable conclusion that the Earth acts as a magnet. This Earth Magnet influences the alignment of hanging magnetic rock, which is free to move.

### **Summing Up**

In this chapter, we presented lessons that could help students to see that mathematical and scientific thinking are not different from normal human thinking.

Many of these lesson ideas have roots in my Year 9 work; the way I taught myself when there was no Mrs Kiribandhane to inspire me. Even though some of these lesson ideas may not be novel I strived to present them in a novel way to create opportunities for students to think as scientists and mathematicians. I hope that my reader will see in these lessons, how constructivism and instructivism go in hand in hand tied in the knot of the rainbow epistemological philosophy.

I based these lessons on the belief that within every student there lies a great scientist or a mathematician. After all, every renowned scientist or a mathematician once has been a primary school or secondary school student. Scientific and mathematical thinking are 'natural' to humans and human thinking. In the kitchen, at home, in the garden and in our lives we think as scientists and mathematicians. For instance, if one cannot see the tool one laid on the ground just a minute ago, he would look for it under the leaves. That is scientific thinking. Mathematical and Scientific thinking are just a little more intensive and much more persevering only because such thinking is practiced with great pedagogical awareness, pride and commitment.

In the next chapter, in the conclusion, I will present a summary of this work as a total.

## Chapter 9

### Conclusion; Stepping-Stones to Stepping-Stones, Looking Back Over My Shoulder

**Journey**  
Steps  
:  
:  
Steps  
:  
:  
Some cyclic  
Some spiral  
  
True to themselves  
They never end  
  
Still  
Some cyclic  
Some spiral  
  
**(2012)**

I write this chapter with an enormous amount of satisfaction and a magnificent sense of achievement. Nevertheless, when I wrote my Mathematics PhD thesis (Samandra, 1991, 1994) I claimed; in Chapter X, I proved and established this; in Chapter Y, I proved this and that, so on and so forth. However, in this current work, I cannot pinpoint proofs or think of such things, even in a remote sense. Therefore, there is an irritating sense of non-achievement. But if I do not have this lingering and nagging sense of non-achievement, then it would be a clear sign of significant failure. Floating above this ground level sense of non-achievement, there is an immense sense of accomplishment; I have looked at my journey of life and education in an ‘objective’ and scholastic manner.

I come to know myself better. I now better understand my practice as a living being and a teacher-learner, better. I understand my strengths; I realise my debilities. I see my humility; I sense my arrogance. I feel my wisdom; I comprehend my stupidity. I am a pilgrim; I am a wanderer. Still, I am. I have a vision; therefore, sometimes I am

blind. I have learned; still I am ignorant. .... This list can continue. I am a synthesis of contradictions, yet I am not a profound truth of the Bohr type (see p. 109 ).

I wish that I could start my journey again as a much more powerful learner, but I cannot. Instead, I need to continue the journey I am already travelling. If the story of my journey helps someone to journey their journeys in a much more thoughtful manner then my effort is not in vain.

In Chapter 2, I presented a partial narrative of my journey with a considerable amount of detail. While writing this chapter I understood, to a much deeper level than before, why I tick the way I do.

### **Themes of the Inquiry**

One of the thematic dispositions of this inquiry has been that we all act on a plethora of epistemologies even if we are devotees of a single epistemological view. For instance, a sceptic who doubts the existence of things could look for the book in his bookshelf which he wrote to espouse his sceptical views on ontological reality. If he cannot find the book then he would think that someone has taken the book rather than thinking that the book never existed. The sceptic in this situation is acting as a logical positivist.

We can take both logical positivism and scepticism to be constructs of the mind. Similarly, whatever isms we stand on can be argued to be the constructs of the mind. Also, it can be counter argued that the mind derives its isms due to the compulsions radiating from the external world. In this case, the question arises: why do all minds not subscribe to a single ism with the same intensity? The net result is that our perceivable worlds and our minds are inextricably linked together.

Furthermore, regardless of what ism we cherish, in most of our daily lives we act as logical positivists. For instance, even the most ardent solipsist will not walk in the direction of the River Gomit in order to reach the River Ganga as if he believes that the rivers exist external to his mind.

As I have argued before, it appears to me that every ism is cyclic to the other isms. Even constructivism is not immune from this cyclic nature. The notion that we construct our world view is also a mental construct and, therefore, it forms cyclic reflections over mind. This is the predicament that we need to bear for the sake of owning a mind. To deal rationally with this challenge during this inquiry, I developed the concept of the rainbow of epistemology which I discussed in Chapter 3 in detail. Furthermore, I outrightly took it as an axiom that there is an external world which is external to my mind. This I cannot prove, and I am not ready to argue on this issue. Many people can point out that this axiom is a construct of my mind. I agree completely; then I do not. Some people take Buddha's or Jesus's claims to be axiomatically true. Therefore, in this regard, I am not doing anything different from what other ism holders do. As I see it:

- Objectivists think as solipsists more frequently than they acknowledge. Whatever they perceive about their ontological and objective world it is always a product of the intercourse of their mind and the external world.
- Solipsists almost always act as though they live and believe in an external world.

In summary, in Chapter 3, I looked into my metamorphosis from logical positivism to multi-layered and multi-dimensional relativism.

While solipsists believe that the world exists only in their mind I believe that there is an external world. In some sense, solipsists and I are not far apart. They appear to behave contrary to their beliefs while I cannot prove my belief. If there is a photograph of a tree then there should be a mind, a tree and a camera. Even if a camera and a photographer do not exist a tree can exist. If there is neither camera nor photographer then photographs of the tree cannot exist. Likewise, I would like to believe that the world can exist independently from my existence, but I cannot know the properties of this world without having a mind but through the mind only I can see are the images of the world, not the world itself.

This rainbow of constructivism can be seen as the epistemology of the opportunist. Certainly, it runs this risk. For instance, the theory of evolution can be used either to

justify Nazi type elimination of weaker genes to better the gene pool of humankind or to promote utmost kindness to all animals. Likewise, the risk of degeneration of the rainbow epistemology into an opportunistic epistemology is real. For me, it is not the epistemology of the opportunist; it is the epistemology of the opportunity. It suffices to say that the rainbow epistemology is not about choosing an epistemology to justify our deeds and thoughts. It is choosing an epistemology to lead and guide our deeds and thoughts. It is to act in a more sincere and straightforward thoughtful manner while keeping in mind that a single worldview cannot account for how we interact with our world; we require a multitude of epistemologies. This rainbow epistemology has vital implications for our daily lives; and therefore for education and learning and teaching. That is, our learning and teaching cannot be based on a single worldview.

As I see now, even when I was an ardent logical positivist my learning and teaching contained some aspects of relativism. Even though now I am a relativist, I know that my teaching contains elements of logical positivism and other isms. This I have explained in Chapter 3.

In Chapter 4, I reviewed many forms of constructivism. There, I argued that this multitude of constructivist forms arises from the very precarious nature of our experiential reality. Thus, I have formed my standing on the issue of the multitude of forms of constructivism on the basis of the rainbow constructivism. Placing an emphasis on any particular view generates slogans of tunnel vision. This can impair the effectiveness of our dealing with reality.

Another significant thematic disposition of this inquiry has been that educators and learners are not immune from the impact of social forces. This is not by itself a drawback. Moreover, it is a natural proviso since education itself is a social process; yet, since social forces are dictatorial and could become distorted because of the profit motive or other agendas they can impart negative influences on education. This influence can be exerted through subliminal and explicit messages. Profit driven economic forces can see education as an industry to make a profit in an easier way. Therefore, they can spread their tentacles into the field of education invasively and conqueringly. We reviewed these issues in Chapters 4, 5 and 6.

In Chapter 5, I tied simple and single sided views to some approaches and opinions that prevail in the arena of education. These approaches may exert influences directly or subliminally. Most of the time, subliminal messages can be more powerful and dangerous than straight messages. For instance, in some schools students are allowed to play just outside the classrooms while learning and teaching (even at Year 12 level) is continuing. There are two scenarios:

- Open the windows; the noise comes in and concentration elopes.
- Close the windows, the temperature rises and concentration collapses.

The noise outside perturbs the learning and teaching environment in the classroom. The players outside are not aware of the learning activities taking place inside the classroom. Open or close the windows, the learning and teaching environment is the loser. When students play aggressive sports during lunch intervals their adrenalin levels go up. After such play, the students are unable to concentrate on subjects like English, Mathematics and Science. Principals and teachers can preach about the importance of education, but the subliminal messages they send out by keeping a blind eye to the practices I discussed above is clear, loud and penetrative.

I dedicated Chapter 6 to reviewing how teachers teach. There, I put my contention forward that effective teachers use a multitude of approaches of constructivism and instructivism. Also, I examined my own teaching practice. I became aware that, at times, I have suffered from a lack of awareness of my own practices. Also, I realised that I have based my teaching practice on the rainbow epistemology even when I was a logical positivist.

Through this inquiry of my journey of education, which consists of successes and failures, I was able to recognise the system of beliefs on which I had been operating as a learner and an educator. I presented a refined version of this belief system in Chapter 7.

In Chapter 8, I presented a collection of lessons that I have been conducting with my students. I believe that these lessons are based on constructivist academic principles and aspirations of raising pedagogical awareness and right mindfulness. These

lessons are not magic tricks that can enhance learning abilities of students of all effort levels. These lessons cannot help students who are in emotional distress due to many social factors or students who fall asleep during the class due to nightlong Facebook activities, internet chatting and video games. However, some of these students may become much better thinkers due to the effect of these lessons.

To help students become sharp independent thinkers appears to be the most universally accepted purpose of education. In searching for how to achieve this goal, teachers need to model themselves as thinkers rather than presenters of knowledge. Educators cannot achieve this aspiration by demanding beginning students to think for themselves. The students need to experience and think what thinking is. They need to learn how to think about thinking. Moreover, they need to have opportunities to think rather than merely to get answers. They need to have something to think about. In short, thinking needs to be modelled.

The lessons I presented in Chapter 8 and Appendix illustrate these aspects. In Mathematics, it means that we need to take our primary attention away from the answers and place it on the mathematical processes. This includes using proper notations and writing mathematical statements elegantly. However, this does not exclude rote learning. Rote learning has a role to play in learning and teaching.

Nonetheless, the lessons presented here are not magical elements, although they can become magical elements to the motivated. They also could motivate some of the unmotivated. These lesson items are not magical elements for all, for the simple reason that they demand a high level of focus and concentration.

### **Educators as Change Agents**

Teachers are change agents in the lives of students. Let us consider this metaphor. The shape of rocks which lie in the seawater changes because seawater is a change agent. To change the shape of the rock seawater takes a longer time. Every time the seawater hits the rock the waves created reflect and interfere. The seawater, without being subject to these impacts, reflections and interferences, cannot change the rock. To make an impact on the rock seawater itself also undergoes an impact. At the end, even though the seawater retains its identity in a large scale, when the seawater

changes the shape of the rock the shape of the seawater itself around the rock also changes.

Likewise, every time a teacher ‘changes’ a student it changes the teacher. I do not like to consider this change process of the teacher as an adaptation of the teacher to a group of students or an individual student. Rather prefer to see this as the teacher’s response to the needs of the group of students or the individual student. The teacher chooses or develops the resources to suit the pedagogical needs of the students.

Suppose that the parents of an intellectually challenged student entrust me with the student’s mathematics education. Then it is obvious that many of the lessons I presented in Chapter 8 and the Appendix may not help this student unless I modify the lessons. Even then I will base myself on the same spirit of the very same lessons. In this case, even if the subject content and some learning aspirations are different, my constructivist aspiration would remain the same: to help the student develop awareness, a way of thinking and useful skills. This means that regardless of the level of intelligence of the student, my aspiration is to inspire him to make better sense out of this world through learning. Therefore, this approach is flexible enough to help all ability levels.

Also, if this student is one among many other students in a class then I will not be able to respond to the needs of the student properly. To think otherwise is outright deception. I may be able to occupy the child somehow, but I will not be able to occupy the child in a cognitively effective manner. Occupying the child ‘somehow’ is to change my role from being a teacher to being a nanny. That is, whatever the intelligence level of my students the learning activities need to inspire them to think better.

### **What is Next?**

Since it is outside the scope of this inquiry, the effectiveness of these lessons has not been quantitatively examined, but it is imperative (see Chapter 1) that properly designed quantitative research with qualitative scrutiny is conducted to measure their effectiveness. This proposed research initiative is one of the most significant which

springs from the current inquiry. There are other quantitative and qualitative research issues arising from this current inquiry:

- Are there means to improve the attention span of students rather than taking fifteen minutes, as suggested by some research, to be an absolute limit?
- The best way to learn how to speak a language is the immersion of students in an environment of the language. How well can this work for other linguistic learning such as reading, writing, thinking and interpretation?
- How effective is the method of osmosis as a tool of learning and teaching? Does it always work in all areas of learning and teaching? Could a combination of osmosis and explicit instruction be better than osmosis? For instance, do students absorb mathematical notations if teachers just use them correctly without explicitly referring to them? See the example about writing of the measurement unit far away from the numerical value of a measurement.
- We know that mathematical skills depend on mathematical understanding. Do the mathematical skills depend also on mathematical memory? Does understanding exclude the need of mathematical memory or awareness? Does not understanding of a formula itself generate the memory of the formula, at least most of the time? For instance, consider the formula,  $\text{distance} = \text{speed} \times \text{time}$ .
- How does the working memory affect learning? How can we incorporate the development of working memory skills to our regular teaching and learning programs in an integrated manner?
- Can the methods of learning and teaching a first language impact on learning and teaching of mathematics and science?
- On several occasions throughout this work I have mentioned the classroom discipline as an issue. It seems that all over the world, this concern is gaining more and more attention of educators. If we, as educators, believe that our lessons need to be cognitively challenging at an appropriate level for each of our students then the issue of discipline is crucial. Do we need to change our discipline paradigm? If so, how?

The issues above need to be examined by using both qualitative and quantitative research methodologies. Possibly there are many other research issues arising from this current work.

As a concluding remark, I would like to note that the best success I can hope from my inquiry is that it helps other learners and educators to reflect on their learning and teaching experiences in a more organised way. Then they will be able to come to their own versions of constructivism which share common threads.

If my inquiry becomes a stepping-stone to more stepping-stones, it is more than I can hope. I hate to think that somebody will see a commercial value in this work, producing commercial lessons of constructivism. Ultimately, this could turn my notion of 'a way of thinking' to the approach of 'a way of doing'. In that case, the ideas presented here will be distorted for marketing purposes and turned into slogans of tunnel vision. Consequently, the true spirit of this work will be buried alive. This is what has happened to many magnificent ideas in education. Living and breathing notions have been turned into dead dogmatic fossils.

Knowledge in any form (say a book) is a non-breathing and non-pulsing dead fossil unless a human nose is breathing in front of it. Unless this breathing nose is attached to an active, pulsing and probing brain, knowledge does not come to life.

Teachers need to inspire epistemological reform within their learning and teaching environments. I earnestly hope that this current work can provide teachers a way to think about such reforms. However, Taylor warns teachers not to act on individual heroism in order to reform school education. Rather, teachers need to work collaboratively with their colleagues in the pursuit of educational and epistemological reform.

To expect experienced teachers of mathematics to undertake epistemological reform as a solo endeavour is to assume, therefore, a remarkable degree of freedom from the coercive influence of technical accountability. The overwhelming majority of secondary

school mathematics teachers are subject to the enculturating influence of their immediate school communities, including administrators, peers and parents. It is important to avoid the danger of perpetuating the myth of the teacher as an heroic individual and creating irreconcilable ethical dilemmas for reform minded teachers as their technical and moral accountabilities clash. (Taylor 1996, p.169

This is an end which never ends.

I will end this inquiry with the following verse which I have published in Sinhalese, and the following image.



<http://joshspear.com/blog/tag/exhibition>

#### **Searching For Path**

Where does this path lead to  
Inquired one who has lost his path

This also leads to another path  
Replied another path searcher

**(1981)**

## References

- Airasian, P. W., & Walsh, M. E. (1997). Constructivist cautions. *Phi Kappan Delta*, 78(6), 444-449.
- Anderson, G. (1998). *Fundamentals of educational research* (2<sup>nd</sup> ed.). London: Falmer Press.
- Bain, J. D., Ballantyne, R., Mills, C., & Lester, N. C. (2002). *Reflecting on practice: Student teachers' perspectives*. Flaxton, Queensland: Post Pressed.
- Block, J. (Bhikkhu Bodhi) (2010). *The Buddha and his dhamma*. Retrieved on 25/12/2012 from Access to Insight:  
<http://www.accesstoinsight.org/lib/authors/bodhi/wheel433.html>
- Bohm, D. (1980). *Wholeness and the implicate order*. New York: Routledge & Kegan Paul.
- Bohr, H. (1967). My Father. In S. Rozental (Ed.), *Neils Bohr: His life and works as seen by his friends and colleagues*. (pp. 328). Amsterdam, Holland: John Willey.
- Bryman, A. (2001). *Social research methods*. London: Oxford University Press.
- Chen, I. (n.d.). *An electronic textbook on instructional technology*. Retrieved on 10/04/2012 from Social Constructivist Theories:  
<http://viking.coe.uh.edu/~ichen/ebook/et-it/social.htm>
- Claude Cohen-Tannoudji, B. D. (1977). *Quantum mechanics* (Vol. I). New York: A Wiley-Interscience Publication, John Wiley & Sons, Hermann, Paris.
- Cob, P., Wood, T., Yackel, E. & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal*, 29 (3), 573-604.
- Confrey, J. (1992). What constructivism implies for teaching. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics*. *Journal for Research in Mathematics Education Monograph 4*, 107-122.
- Denzin, N., (1989). *The research act: A theoretical introduction to sociological methods* (3<sup>rd</sup> ed.). Englewood Cliffs, NJ : Prentice Hall.
- Denzin, N., & Lincoln, Y. (Eds.). (2000). *The handbook of qualitative research* (2<sup>nd</sup> ed.). Thousand Oaks, CA: Sage Publications.
- Driver, R.A., Leach, J., Mortimer, E., & Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher*, 23(7) 5-12.

- Elkind, D. (2004). The problem with constructivism. *The Educational Forum*, 68(4): 306–312.
- Ellis, C., & Bochner, A. (2000). Autoethnography, personal narrative, reflexivity: Researcher as subject. In N. Denzin & Y. Lincoln (Eds.), *The handbook of qualitative research* (2<sup>nd</sup> ed., pp. 733-768). Thousand Oaks, CA: Sage.
- Ernest, P. (1995). The one and the many. In L.P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 459-486). Hillsdale, NJ: Lawrence Erlbaum.
- Fenech M. A., & Kiger A. (2005). The use of triangulation for completeness purposes. *Nurseresearcher*, 12 (4), 19-29.
- Geelan, D.R. (1997). Epistemological anarchy and the many forms of constructivism. *Science & Education*, 6, 15-28.
- Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: Sage.
- Guion, L. A., Diehl, D. C., & McDonald, D. (2011). *Triangulation: Establishing the validity of qualitative studies*. Retrieved on 05/04/2013 from University of Florida IFAS Extension: <https://edis.ifas.ufl.edu/fy394>
- Healy, W. P., & Samandra, R. A. (1997). Polarisation and magnetization fields in the Aharonov-Bohm Effect. *Physical Review A*, 55(2), pp. 890-899.
- Hirsch, E. D. Jr. (2001). Romancing the child. *Education Matters*, 1(1), 34-39.
- Kim, B. (2001). Social Constructivism. In M. Orey (Ed.), *Emerging perspectives on learning, teaching, and technology*. Retrieved on 09/04/2013 from <http://projects.coe.uga.edu/epltt/>
- Kruckberg, R. (2006). A Deweyan perspective on science education: Constructivism, experience, and why we learn science, *Science and Education*, 15(1), 1-30.
- Kuhn, T. S. (1996). *The structure of scientific revolutions* (3<sup>rd</sup> ed.). Chicago: University of Chicago Press.
- Luitel, B. C. (2009). *Culture, worldview and transformative philosophy of mathematics education in Nepal: A cultural-philosophical inquiry*. Unpublished PhD Thesis, Curtin University of Technology, Perth, W.A.
- McWilliam, E. (2004). Whither practitioner research? *The Australian Education Researcher*, 31(2), 113-126.
- M. C. Escher *The official web site*. (n.d.). Retrieved on 04/04/2013 from <http://www.mcescher.com/>

- Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics, Journal for Research in Mathematics Education Monograph* (4), (pp. 7-18). Reston, VA
- Osberg, K. M. (n.d.). *Chapter 2: conceptual frameworks*. Retrieved on 24/12/2012 from Constructivism in practice: The case for meaning-making in the virtual world: <http://www.hitl.washington.edu/publications/r-97-47/index.html>
- Pais, A. (1982). *Subtle is the Lord*. New York: Oxford University.
- Paterson, A. (1963). The Philosophy of Neil Bohr. *The Bulletin of the Atomic Scientists*, 19 (7), 8-14.
- Penrose, R. (1991). *The Emperor's new mind: Concerning computers, minds, and the laws of physics*. New York: Penguin Books.
- Pereira L., Settelmaier, E., & Taylor P. C. (2005). Fictive imagining and moral purpose: Autobiographical research as/for transformative development. In W. M. Roth (Ed.), *Auto/biography and auto/ethnography: Praxis of research method* (pp. 49-74). Rotterdam, The Netherlands: Sense Publishers.
- Polkinghorne, D. E. (1992). A postmodern epistemology of practice. In S. Kvale (Ed.), *Psychology and postmodernism* (pp. 146-165). London: Sage.
- Purcell, E. M. (1984). Electromagnetic energy in your eyeball. Retrieved on 04/ 01/ 2013 from *Back of the envelope problems*, American journal of physics: <http://ajp.dickinson.edu/Readers/Purcell/February1984-Problem1.pdf>
- Richardson, L. (2000). Writing: A method of inquiry. In N. Denzin and Y. Lincoln (Eds.), *Handbook of qualitative research* (2<sup>nd</sup> ed.) (pp. 923-949). Thousand Oaks, CA: Sage.
- Richardson, V. (2003). Constructivist pedagogy, *Teachers College Record*, 105 (9), 1623-1640.
- Samandra R.A. (1991). *Inverse Eigenvalue Problems for Sturm-Liouville Differential Equations*. Unpublished PhD Thesis, University of Wyoming, Laramie, USA.
- Samandra, R.A. (1994). *Inverse Eigenvalue problems for a Sturm-Liouville differential operator with a weight function part I,II,III*. Research Report Series, Department of Mathematics, Monash University, Clayton.
- Samandra, R. A., & Healy, W. P. (1998). An asymptotic formula for energy eigenvalues in the bound-state Aharonov-Bohm effect. *J. Phys. A: Math. Gen.*, 31, pp. 9547-9554.

- Seeskin, K. (1987). *Dialogue and discovery: A study in Socratic method.* (R. C. Neville, Ed.) Albany, New York, USA: State University of New York Press.
- Shenton, A. (2004). Strategies for ensuring trustworthiness in qualitative research. *Education for Information*, 22(2), 63-75.
- Soma Thera. (1994). *Kalama Sutta*. Retrieved on 26/12/2012 from Access to Insight: <http://www.accesstoinsight.org/lib/authors/soma/wheel008.html>
- Solomon, J. (1992). Images of physics: How students are influenced by social aspects of science. In R. Duit, F. Goldberg, & H. Niedderer (Eds.), *Research in physics learning: Theoretical issues & empirical studies* (pp. 141-154). IPN, University of Kiel, Germany.
- Taylor, P.C. (1996). Mythmaking and mythbreaking in the mathematics classroom. *Educational Studies in Mathematics*, 31, 151-173.
- Taylor, P. C., & Wallace, J. (1996). *Doing qualitative research in science and mathematics education*. London: Sage.
- Taylor, P. C. (1998). Constructivism: Value added. In B.J. Fraser & K.G. Tobin (Eds.), *The international handbook of science education* (pp. 1111-1123). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Taylor, P. C., & Settelmaier, E. (2003). Critical autobiographical research for science educators. *Journal of Science Education in Japan*, 27(4), 233-244.
- Taylor, P.C, & Willison, J. W. (2006). Complementary epistemologies of science teaching: Towards an integral perspective. In P. Aubusson, A. Harrison & S. Ritchie (Eds.), *Metaphor and analogy in science education* (pp. 25-38). The Netherlands: Springer.
- Taylor, P.C., Settelmaier, E., & Luitel, B.C. (2012). Multi-paradigmatic transformative research as/for teacher education: An integral perspective. In K. G. Tobin, B. J. Fraser & C. McRobbie (Eds.), *Second international handbook of science education* (pp. 373-387). Dordrecht, The Netherlands: Springer.
- Taylor, P. (in press/2013). Transformative science education. In R. Gunstone (Ed.), *Encyclopaedia of science education*. Dordrecht, The Netherlands: Springer.
- Tobin, K. & Tippins, D. (1993). Constructivism as a referent for teaching and learning. In K. Tobin (Ed.), *The practice of constructivism in science education* (pp. 3-21). Hillsdale, NJ: Lawrence Erlbaum.
- Von Glasersfeld, E. (1989). Constructivism in education. In T. Husen and T. Neville Postlethwaite (Eds.), *The international encyclopaedia of education* (pp. 162-163). Supplementary Volume 1. Oxford: Pergamon Press.

- Von Glasersfeld, E. (1990). An exposition of constructivism: why some like it radical. In R. B. Davis, C.A. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 19-29). Reston, VA: NCTM.
- Von Glasersfeld, E. (1995). A constructivist approach to teaching. In L. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 3-15). Hillsdale, NJ: Lawrence Erlbaum.
- Ziffer, D. (2006, Oct). Educators ignore proven method of improving students' learning, *School Reform News*,15. Retrieved on 14/02/2012 from <http://vikings.coe.uh.edu/~ichen/ebook/et-it/social.htm>
- Victorian Curriculum and Assessment Authority. (2007). Retrieved on 20/ 12/ 2012 from [http://www.vcaa.vic.edu.au/Documents/exams/mathematics/specialist\\_maths\\_1\\_assessrep\\_07](http://www.vcaa.vic.edu.au/Documents/exams/mathematics/specialist_maths_1_assessrep_07).
- Victorian Curriculum and Assessment Authority. (2008). Retrieved on 20/ 12/ 2012 from [http://www.vcaa.vic.edu.au/Documents/exams/mathematics/specialist\\_maths\\_1\\_assessrep\\_08](http://www.vcaa.vic.edu.au/Documents/exams/mathematics/specialist_maths_1_assessrep_08).
- Victorian Curriculum and Assessment Authority. (2012). Retrieved on 04/ 04/ 2013 from [http://www.vcaa.vic.edu.au/Documents/exams/mathematics/2012/2012\\_specmath1-w.pdf](http://www.vcaa.vic.edu.au/Documents/exams/mathematics/2012/2012_specmath1-w.pdf)

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## APPENDIX

Here we present some lesson ideas that can be taken as natural extensions, supplementary or complementary components for the lessons provided in Chapter 8.

### Fractions I

This lesson on Fractions links with the lesson ended on p. 264.

The teacher reviews what the class has learned in the previous lesson. Teacher writes on the board what they have learned so far:

- People created  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{8}$  etc. We also re-create these fractions.

**The number,  $\frac{1}{2}$  is the number when added twice that gives one.**

**There are two  $\frac{1}{2}$  s in one.**

[The class practice similar definitions for other similar numbers.]

**Learning Intentions for This Lesson:** You will learn

- how to create fractions such as  $\frac{5}{7}$ , which are quotients of two integers,
- How to multiply a fraction with a whole number,

*Teacher:* What is  $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ ?

*Students:* 1

*Teacher:* Why?

Some students quickly say this is because  $\frac{1}{3}$  is the number when added three times

that gives 1. Some could say that it is by adding  $\frac{1}{3}$  three times that they get the

answer. Then the teacher asks on what basis we can add  $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ . At the

moment, we know that  $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$  only because we created the number  $\frac{1}{3}$  with

this condition. General addition of fractions we have not yet developed. Teacher points out this to students, and asks what is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}?$$

Repeated addition: 8 of  $\frac{1}{4} = 8 \times \frac{1}{4}$

Some students quickly say 2. Help and guide them to illustrate their answer:

$$\underbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}}_1 + \underbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}}_1 = 2. \text{ Therefore, } 8 \times \frac{1}{4} = 2.$$

*Teacher:* Let us extend this idea of numbers such as  $\frac{1}{2}$  and  $\frac{1}{3}$  a little bit further. If there are a mango, a mango, a mango and a mango, how many mangos are there?

*Teacher:* Likewise

$$2 \text{ mangos and } 3 \text{ mangos} = (2+3) \text{ mangos} = 5 \text{ mangos.}$$

We call this **Mango Principle** so that we can refer to it conveniently.

*Teacher:* Likewise, if I add three and three and three and three how many threes are there?

*Students:* Some will say 4 and some others will say 12.

[The answer to the question asked is 4. Instruct the students to pay attention to the question closely and not to answer their own questions.]

*Teacher:* We depict this in the following equation.

$$3 + 3 + 3 + 3 = 4 \text{ of } 3 = 4 \times 3.$$

*Teacher:* This is repeated addition. Is there any other name for repeated addition?

[When the students answer, write 4 of 3 as  $4 \times 3$  and mention that this is a notation that helps us to write things quickly.]

*Teacher:* What is  $\frac{1}{3} + \frac{1}{3}$ ?

$$\frac{1}{3} + \frac{1}{3} = \underbrace{2 \text{ of } \frac{1}{3}}_{\text{Students complete. Teacher helps.}} = 2 \times \frac{1}{3}.$$

[Again point out that we have agreed to write ‘of’ as multiplication. That is we denote “of” by  $\times$ .]

Teacher: Look at the number,  $2 \times \frac{1}{3}$ . Can we write this number,  $2 \times \frac{1}{3}$ , in a much more elegant manner? To see this, ask the students to add the number,  $2 \times \frac{1}{3}$ , 3 times. [Students using the approach depicted in (P), find that]

$$\begin{aligned}
 3 \text{ of } 2 \times \frac{1}{3} &= 2 \times \frac{1}{3} + 2 \times \frac{1}{3} + 2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\
 &= \underbrace{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}_1 + \underbrace{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}_1 = 2.
 \end{aligned}$$

Teacher: This tells us that  $2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3}$  can be denoted by  $\frac{2}{3}$  since when we add the number,  $2 \times \frac{1}{3}$ , three times that makes 2. Consequently, this gives a way to multiply a number such as  $\frac{1}{3}$  with 2. Now consider the product

4 of  $\frac{2}{3} = 4 \times \frac{2}{3}$ . It can be shown that adding the number,  $4 \times \frac{2}{3}$ , three

times gives the result 8. Therefore,  $4 \times \frac{2}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$ . Therefore, now we know how to multiply any fraction with any whole number. Note that the top number of the fraction is called the numerator and the bottom number is called the denominator.

Also recall that we previously obtained that  $8 \times \frac{1}{4} = 2$ . With this new

notation, we can write  $8 \times \frac{1}{4} = \frac{8 \times 1}{4} = \frac{8}{4} = 2$ . This last equality,  $\frac{8}{4} = 2$

contains another principle. We will look into this a little while later.

We may think of the numerator (the top number) of the fraction as the number of pieces and the denominator of the fraction (the bottom number) as the size of the each piece. Larger the bottom number, smaller is the size. For instance, in  $\frac{3}{4}$ , the

numerator, 3, is the number of pieces, and the denominator, 4, indicates the size of each piece. The bottom number, 4, indicates that four pieces together make one.

Now students work on relevant questions from their textbook or a worksheet.

## Fractions II

Revise what the class has learned so far. Then put the learning intentions for this segment of the lesson.

### Learning Intentions:

- proper, improper, and mixed fractions and

Next we consider the types of fractions: First recall the mango principle (see p. 312).

$$\left. \begin{array}{l} \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \\ 1 + \frac{3}{4} \Rightarrow \text{This is because } \underbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}}_1 + \underbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}}_{\frac{3}{4}} \end{array} \right\} \begin{array}{l} \frac{7}{4} \text{ (This is obtained by using the mango principle or the principle which} \\ \text{we have discovered (not created) that 7 of } \frac{1}{4} = 7 \times \frac{1}{4} = \frac{7}{4}.) \\ \end{array}$$

*Teacher:* Here we agree to drop the operation symbol "+" in  $1 + \frac{3}{4}$ . That

is, we write  $1\frac{3}{4}$  and still conceptually see  $1 + \frac{3}{4}$ . For example:

$$2\frac{3}{7} = 2 + \frac{3}{7} \text{ (see the invisible '+' in the left hand side.)}$$

$$-2\frac{3}{7} = -2 - \frac{3}{7} \text{ (see the invisible '-' in the right hand side.)}$$

[This is similar to one of our old agreements: We write 257 (two five seven) but read it as  $2 \times 100 + 5 \times 10 + 7$ .]

*Teacher:* Then we observe that there are three classes of fractions.

Fraction {
 

- A fraction whose numerator is smaller than the denominator  $\Rightarrow$  **Proper Fractions**, Ex :  $\frac{3}{4}$
- A fraction whose numerator is bigger than the denominator  $\Rightarrow$  **Improper Fractions**, Ex :  $\frac{7}{4}$
- A fractions which is the sum of a whole number and a proper fraction  $\Rightarrow$  **mixed fraction**, Ex :  $1\frac{3}{4}$

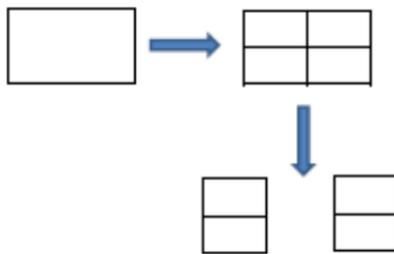
Make sure that the students have grasped the concepts of three types of fractions and developed the skills of converting from to one another. Then the teacher turns his attention to equivalent fractions.

### Equivalent Fractions

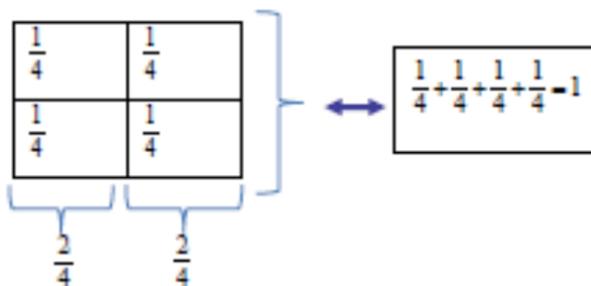
*Teacher: We cut a loaf of bread into four equal pieces. From these four pieces, we create two groups of two pieces. Answer the following questions.*

- i Ask students to draw this in a picture.
- ii How much of the loaf is each of the four pieces?
- iii How much of the loaf is each of the two groups of two pieces?

*Teacher: After a while, the teacher draws these diagrams on the board. Ask students to reflect how these diagrams relate to the questions of cutting the loaf.*



or



One loaf of bread is not 1, but we may take the bread of loaf as a visual aid to think about the numbers we work with at the moment. The visual diagram says that 2 pieces of  $\frac{1}{4} = 2 \times \frac{1}{4} = \frac{2}{4}$  is the same as  $\frac{1}{2}$  of the loaf. Even without this visual aid, we can see what this diagram says from the statement that  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ .

$$\begin{array}{c} \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1 \\ \underbrace{\qquad\qquad\qquad}_{\downarrow} \quad \underbrace{\qquad\qquad\qquad}_{\downarrow} \\ \frac{2}{4} + \frac{2}{4} = 1 \end{array}$$

*Teacher: Remember that  $\frac{1}{2}$  is the number when added twice that gives one. We want to take that the number  $\frac{1}{2}$  to be unique as we know that a double of a number is unique. Then look at the following statements. What do these statements say?*

$$\begin{array}{c} \frac{1}{2} + \frac{1}{2} = 1 \\ \frac{2}{4} + \frac{2}{4} = 1 \end{array}$$

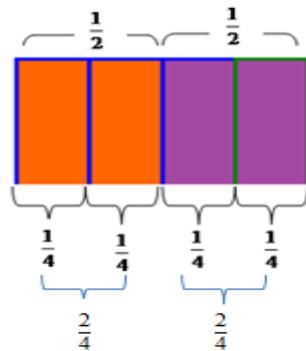
*Teacher: Besides  $\frac{1}{2}$ , is there any other number on the whiteboard when added twice that gives one?*

*Students: Some students might see the equivalence of  $\frac{1}{2}$ , and  $\frac{2}{4}$  and explain the equivalence saying that  $\frac{2}{4}$  does the same job as  $\frac{1}{2}$ . Therefore, they should be the same.*

Teacher: There seem to be two cooks,  $\frac{1}{2}$  and  $\frac{2}{4}$ , in the same kitchen doing the same job. That is two of  $\frac{1}{2}$  is one and 2 of  $\frac{2}{4}$  is also 1. This compels us impose that

$$\frac{2}{4} = \frac{1}{2}.$$

That is, when we multiply the numerator and the denominator of a fraction by a nonzero number, the resulting fraction is equivalent to the initial fraction. This is called the Principle of Equivalence. Also, if we divide the numerator and the denominator by a same non-zero number then the resulting fraction is equivalent to the original number. To depict the Principle of Equivalence, we redraw the following picture. Also, note that we have not proved this equivalence for all the fractions. We just gave an example and took the generalised result for granted.



The picture itself says that  $\frac{2}{4} = \frac{1}{2}$ .

Now the teacher shows many examples with equivalent fractions. The students do work from a worksheet or their textbook. The teacher may assign some additional questions for students to work. For instance:

Q 1. What equivalent fractions do the following diagrams denote?



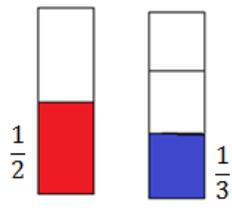
Q 2. What is the number that cannot be used, as a multiplier or a divider to generate an equivalent fraction? Why?

Q 3. To generate an equivalent fraction, which number do not we use even though we can use it as a divider or multiplier? Why ?

Advice to students: Fractions should be expressed in the most reduced form by applying the Principle of Equivalence. Example  $\frac{10}{15} = \frac{2}{3}$ .

**Special Remark:** By defining  $\frac{1}{n}$  as the number when added  $n$  times that gives 1, and requiring these newly created numbers,  $\frac{1}{n}$  to follow the distributive law, the commutative law and associative law of multiplication and addition, one can construct rational numbers in a rigorous manner. Instead of doing this, here we have given some justification why rational numbers behave in the manner they do.

From now on, we omit the learning intention statement, to save space.



Which number is bigger,  $\frac{1}{3}$  or  $\frac{1}{2}$ ?

Which gives a bigger piece of bread, cutting it into two equal pieces or three equal pieces?

Which number is bigger,  $\frac{2}{5}$  or  $\frac{4}{5}$ ? Consider:

- A loaf of bread is cut into 5 equal pieces.
- Which gives you a larger amount of bread, two pieces or four pieces?

Some students need concrete visual imagery to work out these comparisons. The teachers need to guide students to move from the concrete mode of thinking to the abstract mode of thinking, as parents help children to move from drinking from a bottle to drinking from a cup. To achieve this goal with respect to comparing of fractions we can use the following approach.

What is bigger,  $\frac{3}{7}$  or  $\frac{5}{7}$ ?

We need 7 of  $\frac{1}{7}$ . Therefore, 5 of  $\frac{1}{7}$ , which is the same as  $\frac{5}{7}$  must be bigger than 3 of  $\frac{1}{7}$ , which is the same as  $\frac{3}{7}$ .

To make number 1, it takes three of  $\frac{1}{3}$ . Similarly, it takes 2 of  $\frac{1}{2}$  to make 1, respectively. Therefore,  $\frac{1}{3} < \frac{1}{2}$ .

We may automate the process of comparing two fractions if they have either the same denominator or the same numerator. In these two special situations, just inspection is sufficient.

- If the denominators are the same, larger the numerator, the bigger the fraction is. Here, we compare the number of pieces of equal size.
- If the numerators are the same, then smaller the denominator, the bigger the fraction is. Here, we compare the size of the pieces since we have the same number of pieces.

### **Question:**

How do you compare two fractions if both the denominators and numerators are different? For instance, consider  $\frac{5}{6}$  and  $\frac{7}{9}$ .

If the students do not see the way out, repeatedly ask when it is easier to compare fractions. Then students may suggest making the numerator or the denominator the same for both the fractions.

### **Addition and Subtraction of Fractions**

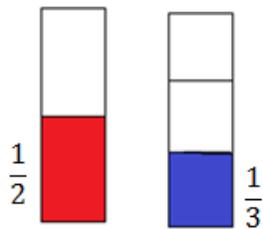
It is easier to add fractions with the same denominator using the Mango Principle.

Let students practice the Mango Principle with several examples, such as

$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} \Rightarrow$  One of seventh plus 2 of seventh plus 3 of seventh is (1+2+3) of seventh.

$$\text{Therefore, } \frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{1+2+3}{7} = \frac{6}{7}.$$

After practicing the Mango Principle many times, put  $\frac{1}{3} + \frac{1}{2}$  on the board. Ask students, whether we can use the Mango Principle to add these two fractions. Listen to their answers. Some of them will say that since the denominators are different, the Mango Principle is inapplicable. Ask: why the Mango Principle is inapplicable when the denominators are different. This can be illustrated using a picture:



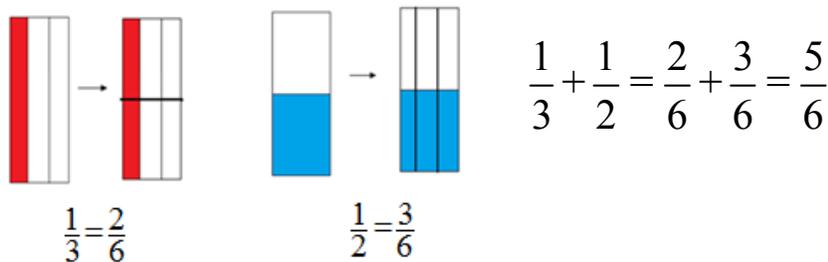
To make one, we need three of  $\frac{1}{3}$  or two of  $\frac{1}{2}$ . Thus, the two numbers are not of equal size. Therefore, 4 of  $\frac{1}{3}$  and 3 of  $\frac{1}{2}$  do not make 7 of  $\frac{1}{3}$  or 7 of  $\frac{1}{2}$ .

Teacher: Suggest a way to add these **unequal** parts,  $\frac{1}{3}$  and  $\frac{1}{2}$ .

**Questions:**

1. Why it is easier to add fractions with an equal denominator?
2. Then how do you suggest adding fractions with different denominators?

If it is easier to add fractions with equal denominators, then to add fractions with different denominators, we should make the denominators of the fractions equal. This equalisation of denominators can be achieved by using the principle of equivalent fractions. See the picture below.



The students need to understand that the fractions  $\frac{1}{3}$  and  $\frac{2}{6}$  and the fractions  $\frac{1}{2}$  and  $\frac{3}{6}$  are the very same fractions, respectively. Now addition and subtraction of many forms of fractions can be practiced. We omit this here.

## Multiplication of Fractions

Multiplication is repeated addition. Repeated addition of any fraction can be easily done using the Mango Principle (see p. 312). What is  $3 \times \frac{2}{5}$ ? Ask students to explain how they work it out. Remind them that

$$3 \times 7 = 7 + 7 + 7.$$

$$\begin{aligned} 3 \times \frac{2}{5} &= \frac{2}{5} + \frac{2}{5} + \frac{2}{5} \\ &= \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 6 \text{ of } \frac{1}{5} \\ &= 6 \times \frac{1}{5} = \frac{6}{5} \end{aligned}$$

Emphasise with the students again and again that we are trying to understand the magic behind the method of multiplication.

Can we do this in a much more elegant and a quicker way? Then ask: Is 6 random? Is there any connection between 6 and the numbers present in our initial multiplication? Another example may help. Then guide them to see  $3 \times \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$ .

Then practice some more multiplication problems.

We can easily multiply  $3 \times \frac{2}{5}$  by using the Mango Principle as we have already

demonstrated. Then how do we multiply  $\frac{2}{5} \times 3$ ? we can add  $\frac{2}{5}$  repeatedly 3 times

but how can we add 3, repeatedly  $\frac{2}{5}$  times? To do so, we require our newly created

fraction numbers to be 'family' with the family that they have been born into, the whole numbers. That is, these new fraction numbers must obey the laws of commutativity and associativity of both addition and multiplication, as our old friends, whole numbers, do. In addition, they need to obey the law of distribution.

Then, invoking the commutative law of multiplication, we get

$$\frac{2}{5} \times 3 = 3 \times \frac{2}{5} = \frac{3 \times 2}{5}.$$

Practice a few more examples. When they are comfortable with this process, then put

a new multiplication question, such as  $\frac{2}{5} \times \frac{3}{7}$ .

For this question, there is no escape route through the law of commutativity. First, we notice that

$$\frac{2}{5} \times \frac{3}{7} = 2 \times \frac{1}{5} \times 3 \times \frac{1}{7}.$$

In the statement above, we have used:  $\frac{2}{5} = 2 \times \frac{1}{5}$  and  $\frac{3}{7} = 3 \times \frac{1}{7}$ . Now we use the Associative Law of Multiplication (Numbers can be multiplied in any order.) and obtain.

$$\frac{2}{5} \times \frac{3}{7} = 2 \times 3 \times \frac{1}{5} \times \frac{1}{7} = (2 \times 3) \times \frac{1}{5} \times \frac{1}{7} = 6 \times \frac{1}{5} \times \frac{1}{7}.$$

Now let us look at the product,  $\frac{1}{3} \times \frac{1}{2}$  as an easier example which can help us to

figure out how to deal with the product,  $\frac{1}{5} \times \frac{1}{7}$ . Consider six of  $\frac{1}{3} \times \frac{1}{2}$ .

$$\begin{aligned} 6 \times \frac{1}{3} \times \frac{1}{2} &= \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \\ &= \left( \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \right) + \left( \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \right) + \left( \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \right) \\ &= \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right). \end{aligned}$$

To get the third line from the second line, we have used the converse of the distributive law which is the common factor law.

Now using the facts that  $\frac{1}{2} + \frac{1}{2} = 1$  and  $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ , we get

$$6 \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

What does this statement say? It says that when the number,  $\frac{1}{3} \times \frac{1}{2}$  is repeatedly added six times, the result is 1. Therefore

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6} = \frac{1}{3 \times 2}.$$

Thus,

$$\frac{1}{5} \times \frac{1}{7} = \frac{1}{5 \times 7} = \frac{1}{35}.$$

Then, as we have already seen that

$$\frac{2}{5} \times \frac{3}{7} = 2 \times 3 \times \frac{1}{5} \times \frac{1}{7} = (2 \times 3) \times \frac{1}{5} \times \frac{1}{7}.$$

Hence, we conclude that

$$\frac{2}{5} \times \frac{3}{7} = 2 \times 3 \times \frac{1}{5 \times 7} = \frac{2 \times 3}{5 \times 7}.$$

Therefore, to multiply two fractions, we multiply their numerators and denominators.

This is shown by the example above.

Similarly,

$$\frac{12}{5} \times \frac{25}{48} = \frac{12 \times 25}{5 \times 48}.$$

However, we should reduce the fractions (by using the Principle of Equivalence) before we multiply. That is,

$$\frac{12}{5} \times \frac{25}{48} = \frac{\cancel{12}^1}{5} \times \frac{\cancel{25}^5}{\cancel{48}^4} = \frac{1 \times 5}{\underbrace{1 \times 4}} = \frac{5}{4}.$$

This step  
is not necessary

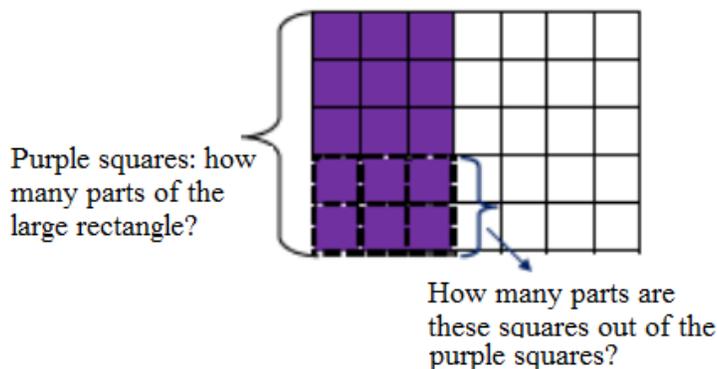
If we choose to multiply, without simplifying, then

$$\frac{12}{5} \times \frac{25}{48} = \frac{12 \times 25}{5 \times 48} = \frac{30\cancel{0}}{24\cancel{0}} = \frac{\cancel{30}^5}{\cancel{24}^4} = \frac{5}{4}.$$

This is a waste of time and effort. Therefore, we always simplify the fractions before we multiply their numerators and denominators.

Some students benefit from concrete approaches. Therefore, let us again consider the product,  $\frac{2}{5} \times \frac{3}{7}$  in a pictorial way.

$$\frac{2}{5} \times \frac{3}{7} = \frac{2}{5} \text{ of } \frac{3}{7}.$$



**Questions:**

1. How many squares are there in the whole rectangle?
2. What fraction of the whole rectangle is coloured with purple?
3. What fraction of the purple squares is marked with broken lines?
4. How is this related to our multiplication problem?

Construct your own examples for several other fractions.

**Division of Fractions**

Consider the division of  $\frac{2}{5}$  by  $\frac{3}{7}$ . Division is repeated subtraction of the Divisor; initially from the Dividend and then from each subsequent result, until the result is zero or less than the Divisor. In this question, the Dividend is  $\frac{2}{5}$  and the Divisor is

$\frac{3}{7}$ . How can we repeatedly subtract  $\frac{3}{7}$ , starting from  $\frac{2}{5}$  and from each subsequent result? Notice that, to divide  $\frac{1}{2}$  by  $\frac{1}{4}$ , we can apply the repeated subtraction method

directly but if we attempt to do the same for the division of  $\frac{1}{4}$  by  $\frac{1}{2}$ , it will give us a spurious result. To avoid this difficulty, instead of directly performing repeated subtraction, we use the relationship between multiplication and division. This link is illustrated in

$$12 \div 3 = 4 \Leftrightarrow 12 = 3 \times 4.$$

Instead of doing so, first we look at the operation of division in a less general way.

**Exercises:****Question 1:**

$$\frac{\left(\frac{4}{7}\right)}{\left(\frac{4}{7}\right)} = \frac{4}{7} \div \frac{4}{7} = ? \quad \text{WHY?}$$

When a nonzero number is divided by itself, the result must be 1.

**Question 2:**

$$\frac{4}{7} \times \frac{7}{4} = ? \quad \text{WHY?}$$

The answer is 1. We get this answer by multiplying the two fractions.

**Question 3:** By looking at your answers to the two questions above, can you conjecture (hypothesise) a way to divide fractions?

It appears that

$$\frac{4}{7} \div \frac{4}{7} = \frac{4}{7} \times \frac{7}{4}$$

In other words, it looks like division by a fraction is the multiplication by its reciprocal (upside down). To verify this, do similar examples. So we may generalise that

$$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} = \frac{2 \times 7}{5 \times 3} = \frac{14}{15}$$

$\times$   
 change the operation of division to multiplication  
 $\frac{7}{3}$   
 change the divisor to its reciprocal

That is,

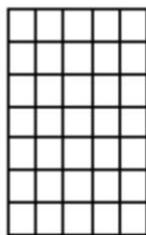
$$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$$

Noting that  $\frac{7}{3}$  is the multiplicative inverse (reciprocal) of  $\frac{3}{7}$ , we generalise the result (without proof) that

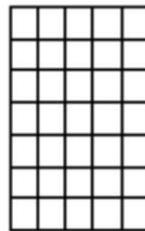
**Division Principle** Division of a fraction (**Dividend**) by another fraction (Divisor) is multiplication of the **Dividend** by **the multiplicative inverse (reciprocal) of the Divisor**.

The following exercise will help the students who need concrete visual imagery.

**Exercise:** Illustration of  $\frac{2}{5} \div \frac{3}{7}$ . Consider the following two rectangles. Each one of these rectangles is of size 5 by 7. Complete the following steps.



Mark  $\frac{2}{5}$  of the squares in red.



Mark  $\frac{3}{7}$  of the squares in blue.

How many squares are there in each of the two rectangles?

Why do we consider a rectangle of 5 by 7?

Count the number of red squares.

Count the number of blue squares.

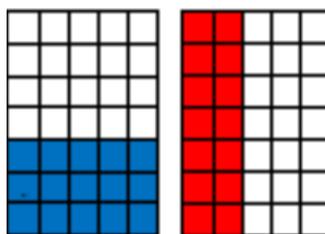
Now consider the following facts:

$6 \div 3$  means that how many threes go into 6, and this is denoted as  $\frac{6}{3}$ .

$11 \div 8$  means that how many eights go into 11 and is denoted as  $\frac{11}{8}$ .

Also note that  $140 \div 20 = (14 \text{ of } 10) \div (2 \text{ of } 10) = 14 \div 2$

Students will produce the following shaded rectangles.



$\frac{2}{5} \div \frac{3}{7} = \frac{\text{the number of squares in ----- of the rectangle in the left}}{\text{the number of squares in ----- of the rectangle in the right}}$  (Fill in each of the blanks with the relevant fraction.)

$= \frac{\text{the number of ----- squares in the rectangle in the left}}{\text{the number of ----- squares in of the rectangle in the right}}$  (Fill in each of the blanks with the relevant colour.)

Now interpret your answer with respect to the Division Rule.

Using the pictorial method, check this procedure with some other fractions.

Finally, show that the Division rule works with even whole numbers.

## A Little More Formal Method of Looking At the Operation of Division

Assume that

$$\frac{2}{5} \div \frac{3}{7} = *.$$

*Teacher:* What kind of a thing is \*?

*Students:* Students could give many irrelevant answers. Ask them to concentrate. Encourage them to answer the question teacher asks, not the questions they make up.

*Teacher:* Is this \*, an apple, a car or a table? If  $\# = 8 \div 2$ , then # is a number. We know this number explicitly;  $\# = 8 \div 2 = 4$ . In general, if a number is divided by a nonzero number then the answer must be a number. Also, we know that

$$8 \div 2 = 4 \Leftrightarrow 8 = 2 \times 4.$$

Likewise we require that (We need divisions of fractions to follow the same properties as divisions of whole numbers.)

$$\frac{2}{5} \div \frac{3}{7} = * \Rightarrow \frac{2}{5} = * \times \frac{3}{7}.$$

Multiply both sides of  $\frac{2}{5} = * \times \frac{3}{7}$  by 7. Then, by using the principles so far we have developed, we get

$$\frac{2}{5} \times 7 = * \times \frac{3}{\cancel{7}} \times \cancel{7} \Rightarrow \frac{2}{5} \times 7 = * \times 3.$$

Now we multiply this last statement,  $\frac{2}{5} \times 7 = * \times 3$ , by the fraction by,  $\frac{1}{3}$  to get

$$\left(\frac{2}{5} \times 7\right) \times \frac{1}{3} = (* \times 3) \times \frac{1}{3} \Rightarrow \frac{2}{5} \times \underbrace{\left(7 \times \frac{1}{3}\right)}_{\frac{7}{3}} = * \times \left(3 \times \frac{1}{3}\right)$$

Thus,

$$\frac{2}{5} \times \underbrace{\left(7 \times \frac{1}{3}\right)}_{\frac{7}{3}} = * \times \underbrace{\left(3 \times \frac{1}{3}\right)}_1 \Rightarrow \frac{2}{5} \times \frac{7}{3} = *$$

This verifies that

$$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}.$$

**Comments:** There could be many criticisms on the ideas presented above.

- Are not these ideas too deep?
- Do the students possess necessary physical and mental discipline, to follow through and engage with these ideas?

**The First Question:** With this same argument, Euclid’s Deductive Reasoning Geometry was removed from the Mathematics Curriculums all over the world. Strangely enough this removal was made in the era of academic constructivism and in the name of thinking skill development. Several decades ago, the Euclidean Geometry was taught from Year 6 onwards. It was definitely a challenging subject. The solution to this problem was not to remove the content but to develop more effective learning and teaching approaches. If academic contents need to be removed on the basis of their depths of abstractness then we should also remove counting or place value system. The Counting and the Place Value System are two of the most cognitively challenging concepts that have ever been developed by mankind.

**The Second Question:** To an extent, the content and methods need to be chosen to match the students but we should not choose content and methods to match students’ behaviour levels. For instance, apart from behaviour problems, students could have developed quick fix attitudes. The students could demand “Give the steps we have to do”. Should a teacher comply with these demands? Consider the following example. To calculate 5% of 250, they could prefer instructions such as

1. Divide 250 by hundred.
2. Then multiply the answer by 5.

This scheme is totally unacceptable to me, unless it is required by a learning difficulty of a particular student. I would like my students to develop this calculation skill through a conceptual approach. For this to happen, each and every one of them needs to ask and answer the question,

What do I mean by 5% of a quantity?

This thinking scheme may be appealing to even some of many capable students, due to their prevailing attitudes but the educators’ task is to win over such students.

## **The Number Zero and Negative Numbers**

This lesson and provides an alternative way to present negative numbers. In Chapter 8, I have presented negative numbers with a different approach. See p.264.

In this lesson, I intend to help my students to construct the number zero and the negative numbers. Again, I start with the learning intentions: Even though the proposed work needs to be a sequence of lessons, I present it here as a single lesson, to save space.

At the end of these learning activities, it is expected that the students will:

- understand the number zero as a constructed (made up) number,
- understand the properties of zero,
- learn that why zero is the identity of addition.

*Teacher: What is number zero?*

*Students: The number before 1.*

*Teacher: Do we use the number zero to count?*

*Students: No.*

If they answer “yes,” then ask them to count some pencils. This will help them to see that zero is not a counting number. Note that there are some counting situations which start with the count zero. We ignore this.

*Teacher: Then how do we know that there is a number before one?*

*Students: ??*

*Teacher: Can you think of the difficulties we face because of the absence of zero?*

*Students: ??*

*Teacher: Think about number 305. What does the notation 305 stands for? Can you think of a way, to write the number 305 without using zero?*

After some discussion, the teacher gives some information directly.

*Teacher: People initially invented number zero as a place value holder. At the beginning, it did not have the status of a number. If you are interested in learning the history of the number zero, put the words "The history of the*

number zero" into the Google search bar. This history will be a extraordinarily interesting story to some of you. I invite you to discuss with me if any such information puzzles you.

This segment partly establishes that people constructed the number zero.

*Teacher:* Now you need to forget all about zero. It does not exist for us in the next few minutes. Consider that the only known numbers are  $\{1,2,3,\dots\}$ . Find the numbers that satisfy the following number sentences.

$$\# + 5 = 8,$$

$$6 + \# = 11,$$

$$5 + \# = 5.$$

*Students:* Many students solve the first two of these equations without any difficulty. Some students give the number zero as the solution of the third equation. Tell them that we do not know of such a number, therefore the solution,  $\# = 0$ , is unacceptable. A few students may say that there are no numbers satisfying the third equation. Again the conclusion that there is no solution is a valid solution in some situations.

As in the case of fractions, after some interactive exchanges, the class agrees to construct a number to satisfy the condition,  $5 + \# = 5$ .

*Teacher:* Let us denote this new number by  $\tilde{0}$ . Remember that this is not the number zero since we agreed to forget its existence. Then we know

$$5 + \tilde{0} = 5.$$

*On the basis that  $5 + \tilde{0} = 5$ , what can we say about  $\tilde{0} + 5 = 5$ ?*

Since numbers have the property of commutativity of addition we demand that this new number  $\tilde{0}$  also has this property. Therefore we know that

$$* \quad 5 + \tilde{0} = \tilde{0} + 5 = 5.$$

*Teacher:* Then what is  $8 + \tilde{0} = ?$  Remember, that we can only use the property in (\*).

*Students:* Students may answer that  $8 + \tilde{0} = 8$ , on the basis of the properties of zero. Otherwise, many of them do not have a clue. If they give the answer, 8,

*ask them to justify their answer without using the properties of the number 0.*

*Teacher: The teacher writes  $8 + \tilde{0} =$  in huge letters. Since we only know the property given in (\*), the only way to find out the answer for  $8 + \tilde{0} =$ , is to use ---? (Pause to invite the students to complete the sentence.) Repeat this statement many times.*

Are they clearly listening? Even if they are listening, are they hearing? Ask these questions and invite them to listen with awareness paying attention to every word. Write the incomplete sentence on the board. Ask the students to read the question silently, still hearing their voices inside their minds. Some students will see the connection that the statement, (\*) needs to be used to evaluate the value of  $8 + \tilde{0}$ . Still they may not see how the statement, (\*), can be used. After sufficient interactive exchanges, explicitly instruct the students to make sure that all students see the necessity of linking the statement, (\*), with  $8 + \tilde{0}$ .]

*Teacher: Referring to the huge writing of  $8 + \tilde{0} =$  on the whiteboard ask whether students can see five in the left hand side.*

*Students: Perplexed by the question, they will say NO!*

*Teacher: If that is the case, then we can never know what  $8 + \tilde{0}$  is. Maybe I am seeing things. I see the number five there. Do you think that I need to see a psychiatrist? [The students will gladly agree that the teacher needs to see a doctor.] To be able to see 5 there, you need to use something you learned in Year 1 or 2. Look very carefully.*

If they still do not see five in the left hand side of  $8 + \tilde{0} =$ , then ask whether they can see 8.

*Students: Yes*

*Teacher: I was really worried that you could not see 8. Then you must see five there.*

*Students: Some may say that  $8 = 5 + 3$ .*

*Teacher: Alright, then  $8 = 3 + 5$ , also. Now, does anyone see the value of  $8 + \tilde{0}$ ? Remember, you must use (\*). That is the only way.*

$$8 + \tilde{0} = (3 + 5) + \tilde{0} = 3 + (5 + \tilde{0}) = 3 + 5 = 8 \text{ and}$$

$$\tilde{0} + 8 = \tilde{0} + (5 + 3) = (\tilde{0} + 5) + 3 = 5 + 3 = 8.$$

How do we know that

$$(3 + 5) + \tilde{O} = 3 + (5 + \tilde{O}) ?$$

Similarly

$$\tilde{O} + 2 = \tilde{O} + (5 - 3) = (\tilde{O} + 5) - 3 = 5 - 3 = 2.$$

We can show that if we add the number  $\tilde{O}$  to or subtract it from any other number it does not change the number. That is why we call the number  $\tilde{O}$  the identity of addition. That is, for any number  $\square$  :

$$(*1) \quad \square + \tilde{O} = \tilde{O} + \square \text{ and } \square - \tilde{O} = \square.$$

It should be clearly explained to the student what "any number,  $\square$ " means.

Suppose that there is another number  $s$ , satisfying,

$$(*2) \quad \square + s = s + \square, \text{ for any number, } \square.$$

Then

$$\tilde{O} + s = ?$$

Here, repeatedly point out to students the statements, (\*1) and (\*2).

$$\tilde{O} + s = s \text{ by } (*1) \text{ and } \tilde{O} + s = \tilde{O} \text{ by } (*2).$$

If we add 3 and 2 in Sydney and do the same in Melbourne, can we get two different answers? Likewise, if we add  $\tilde{O} + s$  in Sydney and Melbourne should we get two answers? Therefore, we must conclude that

$$\tilde{O} + s = s = \tilde{O}. \text{ That is, } s = \tilde{O}.$$

Thus, the number we constructed to satisfy (\*) is unique. We call this number zero, 0.

You can also see that subtraction or addition of 0 from or to another number does not change this second number. For this reason of preserving identity under addition, we call it the additive identity.

This segment of the lesson has established that people have constructed the number zero. The students have constructed the number zero. Now summarise the discussion above and show that multiplication by zero results in zero.

*Teacher: What is  $0 + 0$ ?*

*Students: Many will answer 0.*

*Teacher: Why?*

*Teacher: What happens if we assume that  $0 + 0 \neq 0$ ? Say  $0 + 0 = 2$ . Consider*

$$5 + \underbrace{(0 + 0)}_2 = 5 + 2 = 7, \quad \text{but on the other hand}$$
$$5 + (0 + 0) = (5 + 0) + 0 = 5 + 0 = 5.$$

So there are two different answers for  $5 + (0 + 0)$ . They are 5 and 7. This leads to the absurd result,  $5 = 7$ . Since this is an absurdity, we need to conclude that  $0 + 0 \neq 0$ , is wrong. That is,

$$0 + 0 = 0.$$

Now any multiple of zero and the zero multiple of any number is zero. Thus, in particular,

$$\underbrace{0 \times 5}_{\substack{\text{This should} \\ \text{equal to } 5 \times 0 \text{ due to} \\ \text{commutativity.}}} = \underbrace{5 \times 0}_{\substack{\text{If it is challenging} \\ \text{to add 5 zero times} \\ \text{we can add zero} \\ \text{five times}}} = 0 + 0 + 0 + 0 + 0 = 0. \text{ Negative Numbers}$$

**Learning Intentions:** At the end of these activities, you will understand that

- why we need negative numbers,
- People have constructed the negative numbers,
- negative numbers are additive inverses of positive numbers and vice versa,
- how negative numbers and positive numbers relate to the additive identity, zero, and
- subtracting a number is the addition of the additive inverse of the number.

To demonstrate why we need negative numbers we may consider a bank account scenario.

A bank account allows the account holders to borrow up to thousand dollars more than the amount of money you have in the account.

*Teacher: If I just have \$100 and withdraw \$200, what will the bank say or what will the bank expect me to do?*

*Students: Pay the excess money back.*

*Teacher: How much do I have to pay them?*

*Students: \$100.*

*Teacher: Tell me several different ways that the bank can say this?*

*Students:*

- *You owe the bank \$100.*
- *You have withdrawn \$100 in excess.*
- *Your debt is \$100.*
- *You are \$ 100 into the red.*

Up to this moment the negative numbers do not exist. Therefore, negative number answers are not valid.

*Teacher: Try to express this in several different symbolical ways.*

After a while, the teacher suggests these symbols.  $^*100$ ,  $^+100$ ,  $100^*$ ,  $100^-$ ,  $-100$ ,  $^{\square}100$ , etc.

Some students will object to the use of the symbol “+” to mean lack of something. However, if we had chosen “+” to mean "lack of something" and kept using it for a sufficiently long enough time, then we would have objected if someone suggested using “+” for abundance of thing. So all these choices of symbolical expressions could be valid as long as we agree to use the symbol in the intended meaning and they do not contradict our previous notations.

*Teacher: Later, we will agree, on an appropriate notation and terminology to deal with notions such as a number below 0 or a debt of \$ 100.*

*Teacher: Previously, we believed that we cannot subtract a larger number from a smaller number. For instance, you cannot take away 5 pencils from 3 pencils. Nevertheless, the banks need to do such transactions in a meaningful way. There are many other everyday life tasks which need such notions.*

Ask students to suggest such situations. After a while, ask them to consider the next activity.

Exercise: See how the following activity relates to the situations presented in the previous activity. Assume that the number zero exists, but the negative do not yet

exist. The following equations can be solved by using the numbers we already know:  $\{0,1,2,\dots\}$ .

$$5 + \# = 8,$$

$$5 + \# = 5.$$

What about the equation,

$$5 + \# = \# + 5 = 2?$$

We know that  $5 - 3 = 2$ . Therefore it appears that the addition of  $\#$  to five has the effect of subtracting three from 5. Initially, the teacher will strive to get this out of students by interactive questioning. There will be some students who will be able to realise this situation. Otherwise, after a reasonable time, the teacher needs to explain it. Suggest students to inquire about this further.

*Teacher: In the equation,  $5 + \# = 2$ , can you see 3 somewhere?*

*Students: No!*

*Teacher: What can you see?*

*Students: We can see the number 5, the black triangle, the number 2, the equal sign and the plus sign.*

*Teacher: That's a relief. You have seen the number 3 but just fumbled over it.*

Give some time and then repeat "5 is..." several times. This invites the students to complete the sentence.

*Teacher: Can you walk 5 meters without walking 3 meters first?*

*Students: No.*

*Teacher: Then do you see 3 somewhere in  $5 + \# = 2$ ?*

*Students: Yes,  $5 = 2 + 3$ .*

*Teacher: Therefore we can write*

$$5 + \# = (2 + 3) + \# = 2 + (3 + \#) = 2 + 0 = =$$

*Teacher: By simply looking at the equation above, what can we say?  $3 + \#$  is equal to?*

*The quantity,  $3 + \#$  behaves like ...?*

Give some time to students to see that is acting like the number zero. Since we have only one zero,

$$3 + \# = 0.$$

What are the properties of this new number# ?

- When we add # to 3, both get annihilated and results in zero.
- The only number, # annihilates, by addition, is 3. The only number that can annihilate 3, by addition, is# .
- When we add #, to any other number, the result will not be zero.

For these reasons, this number#, can be denoted by adding a marker to the number 3. As we did before, we can choose many symbols. Here, we agree to term the numbers we already have as positive numbers and their annihilators as negative numbers. Clearly, this terminology of positive and negative are just labels. We could use the label positive numbers for negative numbers and vice versa. Since the people before us have already determined a notation we do not have the liberty to choose our own notation. Therefore we accept the existing notation. So note that

$$5 = \underbrace{+5}_{\substack{\text{We agree to drop the} \\ \text{positive sign and still} \\ \text{mean the } +5 \text{ by } 5.}} \quad \text{and} \quad ^-5 \text{ has the property that } 5 + ^-5 = 0 = ^-5 + 5.$$

Similarly, we create negative numbers with the annihilating property:

$$10 + ^-10 = 0, 25 + ^-25 = 0, \text{ etc.}$$

Notice that, as we have remarked previously, addition of  $^-10$  to any other number (say 17) has the effect of subtracting 10 from 17.

$$17 + ^-10 = ^-10 + (10 + 7) = (^-10 + 10) + 7 = 0 + 7 = 7$$

That is,

$$17 + ^-10 = 17 - 10 = 7.$$

Also, subtracting 10 from 17 is equivalent to adding its additive annihilator to 17. From now on we take these rules for granted. Furthermore, since 10 and  $^-10$  annihilate (produce zero) each other through addition, we call them additive inverses of each other.

### Questions:

- (1) One withdraws \$200 from an account holding just \$100. This creates a debt of \$100. How do you relate this to the negative numbers?
- (2) What is  $0 - 5$ ?

- (3) What is bigger, 0 or  $-5$ ? Why?
- (4) Arrange the three numbers, 0, 5 and  $-5$ , in the ascending order of their magnitudes.
- (5) (i) What do you mean by the additive inverse of a number?  
(ii) Why do you say that  $-25$  is the additive inverse of 25?  
(iii) What is the additive inverse of  $-25$ ? Why?  
(iv) What is  $0 + 0$ ? Thus, what is the additive inverse of 0?

Answer For Question (2)

$$0 - 5 = \underbrace{0}_{\text{Additive Identity}} +^{-}5 =^{-}5.$$

The fact that  $0 - 5 =^{-}5$  tells us that subtraction of five from zero produces the number  $-5$ . Therefore, negative numbers must be smaller than zero.

### Why Straight Lines Are Straight? II

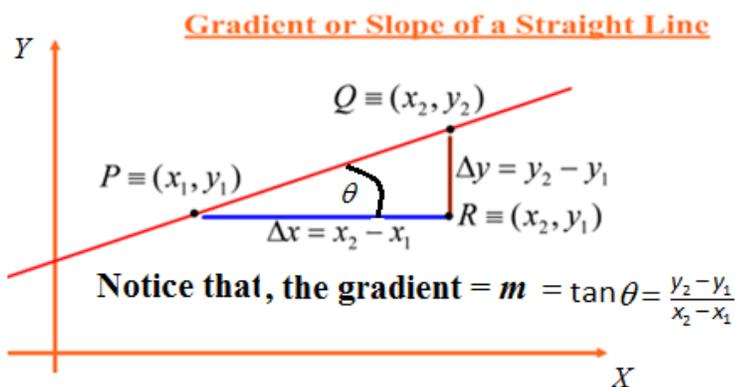
This lesson extends the activities presented in *Why Straight Lines Are Straight I?* See p. 270. We start with the following questions.

#### Questions

- (a) Draw a coordinate system in your book. Draw 5 straight lines going through a common point. One of these lines has to be a vertical line, and another has to be horizontal. The other three lines are neither horizontal nor vertical. Say their common point is  $P$ . Name the lines as  $APB$ ,  $CPD$ ,  $EPF$  etc.
- (i) Do they have the same gradient?  
(ii) How do you associate the steepness of a line with its gradient?  
(iii) Are there lines with zero gradients? Explain. Can you draw such a line?  
(iv) Are there lines with undefined gradients? Explain. Can you draw such a line?  
(v) Are there lines with gradient one? Can you draw such a line?
- (b) Can you draw a line for which when  $x$  increases
- $y$  also increases,
  - $y$  decreases,
  - $y$  remains the same?
- (c) Can you draw a line for which  $x$  remains the same but  $y$  changes.  
(d) Comment about the gradients of the lines drawn in (b), (c) and (d).

**Teacher summary:**

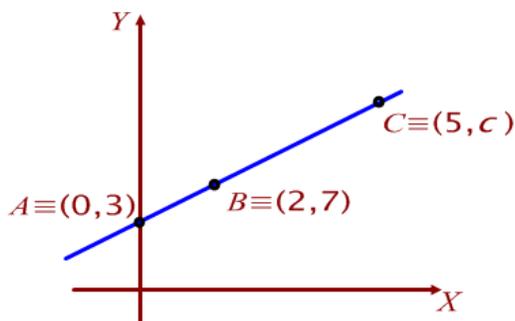
The gradient of a straight line is a measure of the inclination of the straight line. The larger the ratio,  $\frac{\text{Rise}}{\text{Run}}$  the steeper the line is. The symbol  $\Delta y$  and  $\Delta x$  denote change of  $y$  and change of  $x$ , respectively. The gradient of a straight line is the trigonometric ratio of  $\theta$ , where  $\theta$  is the angle between the line and the positive  $X$  Axis. The following diagram depicts this information.



**Slope or Gradient,**  $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Through the discussion above, we come to the understanding that the straight lines are straight because they have the same gradient in each and every interval of their existence.

**Problems:**



**Now consider the diagram shown. Find the value of  $c$ .**

Solution: The line is a straight line. Therefore each line segment of the line must have the same gradient. In particular,

$$\text{Gradient}_{AC} = \text{Gradient}_{AB}$$

$$\frac{c-3}{5-0} = \frac{7-3}{2-0} = 2$$

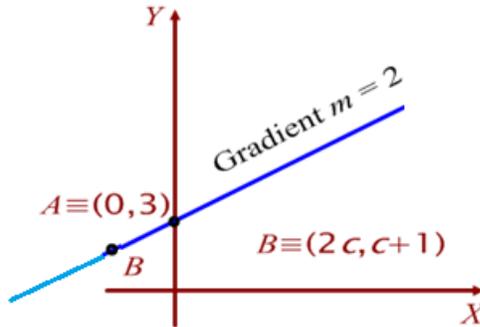
Therefore,

$$c-3 = 5 \times 2 = 10$$

Thus

$$c = 13.$$

Consider the information given in the following graph. Find the value of  $c$ . Has the point  $B$  marked in the correct Quadrant?



Solution: The line is a straight line. Therefore each line segment must have the same gradient. In particular,

$$\text{Gradient}_{AB} = \text{Gradient Given}$$

$$\frac{c+1-3}{2c-0} = 2$$

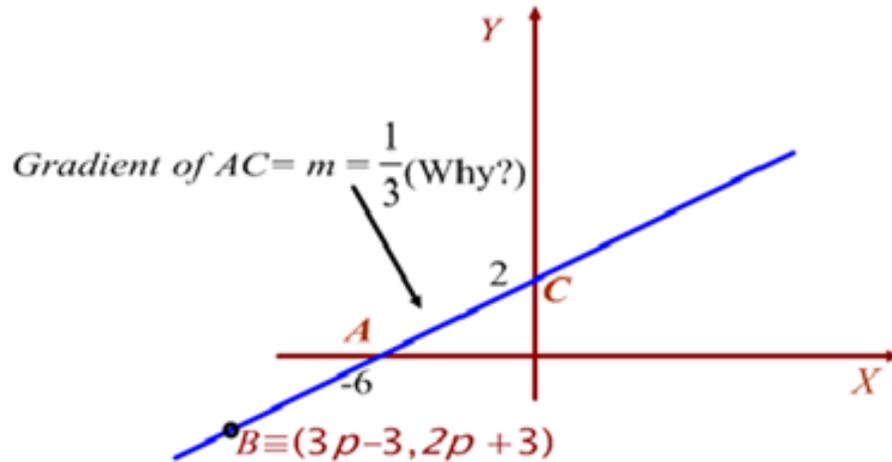
$$\text{So, } c-2 = 4c$$

$$3c = -2$$

$$\text{Hence, } c = \frac{-2}{3}.$$

Therefore,  $B \equiv (2c, 2c-1) = \left(\frac{-4}{3}, \frac{1}{3}\right)$ . Therefore, the point  $B$  is in the right Quadrant.

Consider the line and the information given in the following graph. Find the value of  $c$ .



Solution: The line is a straight line. Therefore each line segment must have the same gradient. In particular,

$$\begin{aligned} \text{Gradient}_{CB} &= \text{Gradient}_{AC} \\ \frac{2p+3-2}{3p-3} &= \frac{1}{3} \\ \frac{2p+1}{3p-3} &= \frac{1}{3} \\ \text{So, } 6p+3 &= 3p-3 \\ 3p &= -6 \\ p &= -2. \end{aligned}$$

## Functions

This lesson is an extension of *Functions and Relations*. See p.275.

### Learning Intention:

- In this lesson, we will learn what we mean by a mathematical function.

Consider the examples (questions) given in the last session.

- The relations (ii) and (vi) are of the same nature.
- The relations (i) and (vii) are of the same nature.

Elaborate on the statement (a) and (b). On what basis they could be similar? Understand the question. They are relations. Thus they are similar. Beyond this obvious similarity, why are they similar?

How does the relationship in (a), differ from the relationship in (b)? On what characteristic they are similar or not similar?

Following the same line of thinking, (a) and (b), categorise the other relations into the type (a) or type (b).

Think:

If we ask a mother, “who are you children” does the question always have a single answer?

If we ask a child, “who is your mother?” can we ever have two different answers?

From the relations (i) to (vii), which relations have the characteristic that the first member pairs with one and only one second member?

**A function** is a special relation with the restriction that each first member of the relation has a one and only one (unique) partner related with it.

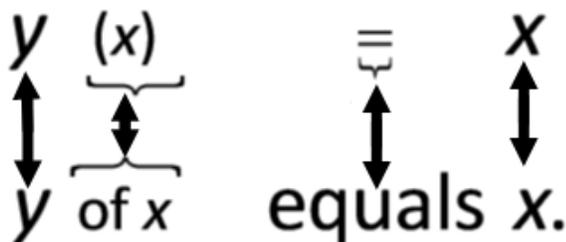
$$f = \{(x, y) \mid y = f(x) \text{ where } f(x) \text{ is unique for each } x.\}$$

There are some other notions to be developed about functions and relations, but we omit dealing with those here.

**Another way to look at a function:**

Ask yourself what you mean by  $y(x) = x$ ?

All students know how to graph this relation but even after learning the concepts of relations and functions, for many of them the question, what  $y(x) = x$  means, may appear to be a weird one. Therefore persist and repeatedly ask the question, while encouraging them to read the function,  $y(x) = x$  with full awareness. They should hear their internal voice. Write the equation,  $y(x) = x$ , in large letters.

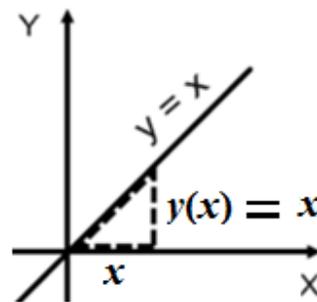


Does this enhance their perceptibility? Give a couple of minutes and make some prompts. Then say, in mathematics "of" can mean two different things and ask what those two different things are? Multiplication is one. What is the other? When we say the colour of the wall is blue what does the word "of" express? An ownership. Then, here does this  $y$  of  $x$  mean an ownership or multiplication of  $y$  by  $x$ ?

Here, mathematicians need to think about changing their notations since  $y(x+h)$  could mean both  $yx + yh$  and the value of  $y$  evaluated at  $(x+h)$ .

Look at the function,  $y(x) = x$ . The function,  $y(x) = x$ , means that 'x' owns a 'y' which is the same value as  $x$ .

- Now, ask them to think the values of  $x$  for which the relation,  $y(x) = x$  cannot work. That is, for what values of  $x$ , it cannot be paired with  $x$  itself. Now on a set of axes, roughly mark the points for which  $y = x$ . This, means marking all the points  $(x, y)$  for which  $x$  and  $y$  coordinates are equal. Since for any value of  $x$  it can be paired with  $x$ , the domain of  $y(x) = x$  is the whole set of real numbers, unless we choose to restrict the domain.



We did not develop the all the concepts related to

functions and relations. Through individual work, class discussions and teacher demonstrations, it needs to discuss many examples of functions and many other concepts in detail. The concept of domain needs to be discussed using many examples. Here, we have just mentioned it.

### Evaluating a Function at a Given $x$ Value

Many students know how to evaluate a function at a given point. Even so, many of them do not have a clear understanding why the value of  $x$  needs to be substituted into the rule of the function. To deal with this difficulty, I use a function with a simple rule. To achieve better success, these exercises needed to be done with complex rules.

#### Learning Intention:

- Through these exercises, students will understand why they need to substitute to evaluate values of a function.

**Questions:** If  $f(x) = 2x^2 + 3x + 4$ , what is

- (i)  $f(0)$ ?
- (ii)  $f(1)$ ?
- (iii)  $f(3)$ ?
- (iv)  $f(3123)$ ?

Many students concentrate on getting numerical answers and do not link their work to the concept of the rule of a function. Therefore, some students feel dumbfounded about the question (iv) unless they have a calculator in their hand. This exercise is not about the numerical answer. Therefore, students should not use calculators.

Up to this point, the teacher does not correct the students or provide any feedback.

*Teacher:* What is it meant by  $f(x) = 2x^2 + 3x + 4$  ?

It is more probable that there will be no proper answers to this. First warn the students that the repetitive aspect of the following questions is a design to generate a message. Ask them to pay attention to this message and their inner voice.

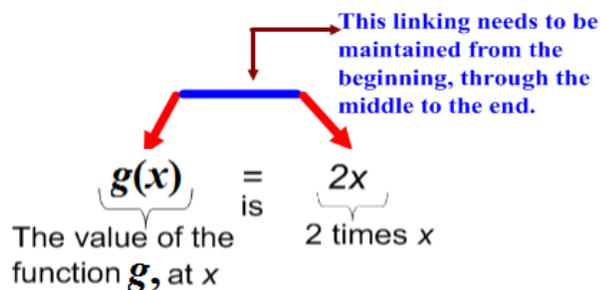
*Teacher:* Given that  $f(x) = 2x^2 + 3x + 4$ , what is

- (i)  $f(p)$
- (ii)  $f(q)$
- (iii)  $f(r)$
- (iv)  $f(a)$

$f(*)$   
 $f(**)$   
 $f(\#)$   
 $f(ocean)$

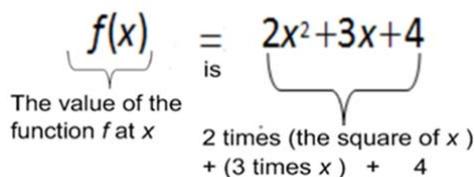
$f(\text{Newton}) = f$  of Newton

Do not comment about their answers. If someone's answer is wrong, ask them again what is  $f(1)$ ,  $f(2)$ ...? If they can get the answers for  $f(1)$ ,  $f(2)$ ..., then they should be able to get the answers for any one of them. Now, to explain why they need to substitute the value of the variable  $x$ , put this on the board.



The idea is here, not just to get the student to learn the technique of substitution. The task is to get the student to see how the technique of substitution embeds itself in the notation itself. When the students read the notation "g of  $x$  equals 2 times  $x$ " they read it without hearing what they read. That is, they read without attaching, linking or connecting any concepts. By the time, they cross over the equal sign, to read "2 times  $x$ " they do not remember that this "new"  $x$  is the same "old"  $x$ . For this reason, the substitution becomes a meaningless and an automated activity. In other words, they need to look at the notation,  $g(x) = 2x$ , as a whole. When they start to read/look at  $g(x)$ , they need to have in mind that there is an expression in the other side, that depends on  $x$ . This is also an issue of pedagogical awareness.

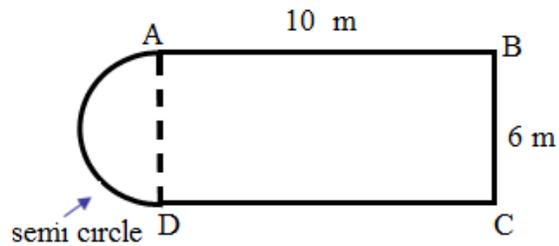
Ask them to evaluate the following items and many others, while keeping the ideas above in their minds.



If  $f(x) = 2x^2 + 3x + 4$ , then find  $f(x+h)$ ,  $f\left(x + \frac{1}{x}\right)$  and  $f(2x^2 + 3x + 4)$ .

### Some Other Miscellaneous Examples

#### A Perimeter Problem



Consider the following perimeter problem. Many students who answer this question do so without thinking what perimeter is. They vaguely remember that they need to add some numbers. So, they immediately begin to add. It is true that some of them get the answer. Nevertheless, Mathematics is not about just getting answers.

The students who get wrong answers will be willingly participate in this exchange because they know that they have something to gain. The students who get correct numerical answers will resist this exchange because they think they have nothing to gain; some of them may regretfully miss an opportunity to develop pedagogical awareness. The following is a report of an exchange between some of my students and me.

*Student: How do you do this question?*

*Teacher: Can you show me how did you do it?*

*Student: Don't you add 10 and 6, and then double the result?*

*Teacher: Before you made any calculation, have you asked yourself what perimeter is?*

*Student: No?*

*Teacher: Please do so and tell me what perimeter is?*

*Student: The length around it.*

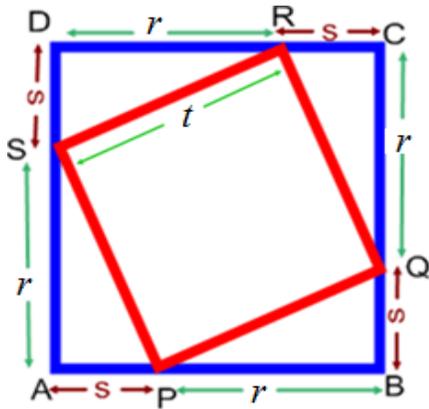
Teacher: Perimeter of a 2-D shape is the length around it. Can you show me the perimeter of the shape by running your finger around the shape? While doing so, add up the distances that your finger walks on the edges of the shape.

Student: While running her fingers on the edges of the rectangle (missing the semicircular arc) the student adds up the lengths and gets the perimeter of the rectangle.

Teacher: Now do the same thing again and run your fingers on the shape keep thinking about the meaning of the perimeter of a 2D shape.

This time students do it right.

### Some Area Problems



#### Area Problem 1

Consider the square within the square in this diagram.

- What is the area of the larger square?
- Again, what is the area of the larger Square?(Same question as Part (a), different vantage point)
- What do you know about  $(r + s)^2$  and  $t^2 + 2rs$  ?
- What can you conclude from (c)?

**Clue:** A famous theorem on a particular type of triangles.

**Answers:** (a)  $(r + s)^2$  (b)  $t^2 + 2rs$   $t^2 + 2rs$

#### Comments

Some of the students may not know how to think of the part (a) since they lack pedagogical awareness. For instance, to find the area of a square of side length of 5

m, they just multiply 5 m and 5 m without thinking that they are multiplying the lengths. To help these particular students to recognise this lack of awareness, the teacher may need to ask the following questions:

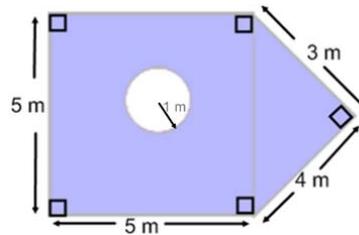
How do you find the area of a square?

What is the side length of the larger square?

Then they will be able to answer the question (a). To answer the question (b), ask them to interact with the diagram and ask them to visualise, making of the larger square by assembling smaller shapes. This is the principle learned in primary school that the whole quantity is the sum of the part quantities.

### Area Problem 2

Consider the following shape.



Your task is to find the area of the shaded region. Do you see an Addition and subtraction?

Build up the discussion around this question of addition and subtraction rather than the answer itself.

### Calculus

#### The Chain Rule

Problem: Differentiate, with respect to  $x$ , the following functions. Use the Product Rule as indicated.

(i)  $(x^3 + 2x + 3)$

(ii)  $(x^3 + 2x + 3)^2 = (x^3 + 2x + 3)(x^3 + 2x + 3)$

(iii)  $(x^3 + 2x + 3)^3 = (x^3 + 2x + 3)(x^3 + 2x + 3)^2$  (Use the answer in (ii).)

(iv)  $(x^3 + 2x + 3)^4 = (x^3 + 2x + 3)(x^3 + 2x + 3)^3$  (Use the answer in (iii).)

(v)  $(x^3 + 2x + 3)^5 = (x^3 + 2x + 3)(x^3 + 2x + 3)^4$  (Use the answer in (iv).)

(vi)  $(x^3 + 2x + 3)^n$  (Use the pattern above to guess the answer. This result can be more rigorously derived by using the method of induction, but we omit this here.)

Now various functions of similar type can be differentiated using this particular form of the Chain Rule. This exercise can be taken to be a prelude to the most general form of the chain rule of differentiation. Here is another precursor to the development of the concepts and skills related with the Chain Rule.

**Problem:** Differentiate the following functions, with respect to indicated variable.

$$(i) \quad \frac{d}{dx} x, \quad \frac{d}{d(x^2)}(x^2), \quad \frac{d}{d(x^3)}(x^3), \quad \frac{d}{d(x+3)}(x+3),$$

$$\frac{d}{d(x+3)^2}(x+3)^2 \quad \text{and} \quad \frac{d}{d(x+3)^3}(x+3)^3 :$$

Quickly reason out why all these derivatives should yield the result 1, without referring to variable transformation directly.

$$(ii) \quad \frac{d}{d[\sin(x)]}[\sin(x)], \quad \frac{d}{d[\sin(x^2)]}[\sin(x^2)], \quad \frac{d}{d[\sin^2(x)]}[\sin^2(x)],$$

$$\frac{d}{d[\sin(3x^2 + 7x + 5)]}[\sin(3x^2 + 7x + 5)]$$

Quickly reason out why all these derivatives should yield the result 1. (You cannot refer to variable transformation directly.)

$$(iii) \quad \frac{d}{d(e^x)}(e^x), \quad \frac{d}{de^x}e^{2x} = \frac{d}{d(e^x)}(e^x)^2, \quad \frac{d}{de^x}e^{kx} = \frac{d}{d(e^x)}(e^x)^k$$

Quickly reason out why all these derivatives should yield the result 1. (You cannot refer to variable transformation directly.)

$$(iv) \quad \frac{d}{dx}e^x, \quad \frac{d}{d(2x)}e^{(2x)}, \quad \frac{d}{d(kx)}e^{(kx)}, \quad \frac{d}{d(x^2)}e^{(x^2)}, \quad \frac{d}{d(x^2+x+7)}e^{(x^2+x+7)}$$

Get the answers for these derivatives above, by using the fact that they all look like

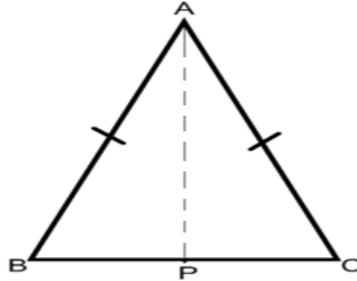
$$\frac{d}{dx}e^x.$$

## Integrals Which Are a Prelude to Change of Variable Transformation

$$(i) \int x dx, \int x^2 d(x^2), \int \sqrt{x} d(\sqrt{x}), \int e^x d(e^x) \text{ and } \int e^{x^2} d(e^{x^2})$$

### Triangle Properties:

Year 8:



(i) This is a triangle of a special type.

- Why it is special?
- What is this type?

A triangle in which two of the sides have equal lengths is an isosceles triangle.

(ii) Imagine a straight line going from  $A$  to  $P$  which bisects the  $\angle BAC$  where  $P$  is on the line segment  $BC$ . Thus,  $\angle BAP = \angle CAP$ .

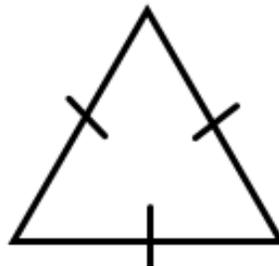
Now imagine that you fold the triangle  $ABC$  along the line  $AP$  so that the line  $AC$  folds towards the left.

(iii) Where does the line  $AC$  fall on; to the left of  $AB$ , to the right of  $AB$  or exactly on the line  $AB$ ? Why?

(iii) What is the final position of the point  $C$ ; to the right of  $C$ , to the left of  $C$  or exactly on  $B$ ? Why?

(iv) What do you see about  $\angle B$  and  $\angle C$ ?

Consider the following triangle.



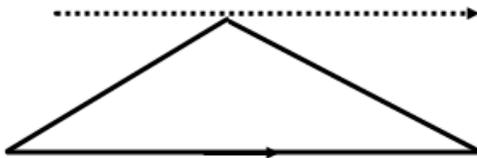
(i) What kind of a triangle is this? Why?

- (ii) Do you see that all three angles of the triangle have equal measure? Can you provide reasons for your answer? Clue: Apply the result for the part (iv) in the previous problem.

I emphasise that the question is not that do you know. The question is “Do you see”?.

If it is something that you just know but do not see, you should not answer until you see it.) Knowledge (even contained in books) is dead fossils until someone interacts with it but thinking is always alive.

Carefully look at the following diagram. (The students know the parallel line theorems.)



Do you see (not that do you know) that the angle sum of a triangle is  $180^\circ$ ? Why?

## GLOSSARY OF TERMS

**Remark I:** These definitions are not complete and they are mere references to the basic ideas of defining terms. To define these terms thoroughly, requires longer expositions. Detailed discussions of some of these terms are in Chapter 3 and Chapter 4 and perhaps also in other chapters.

**Absolute Idealism:** Absolute idealism assumes the existence of a grand conscience, absolute mind or a spiritual unity which embraces the whole Universe.

Even if one initially takes absolute idealism to be distinct from objective idealism, a careful inquiry will show that it is untenable to maintain this distinction. It is easier to see that absolute idealism entertains the philosophies of objective idealism.

Since for an objective idealist there exists a Grand Conscience independently from any other conscience, there is no point in maintaining any distinction between this Grand Conscience and the Absolute Idea.

Also, see the entries on subjective idealism, objectivism, and materialism.

**Behaviourism:** The major tenet of behaviourism is that psychology should focus only on observable behaviours of people and animals. This means that behaviourism should not concern itself with mental processes inside minds since they are unobservable.

If educators apply this principle to learning, then they must pay attention only to the behaviour changes that happen due to learning. Behaviourists seem to consider the learner's ability

to add two numbers is of primary importance, not their understanding of the addition and the addition algorithm.

Then one can define thinking as a form of behaviour. Also, one can regard understanding as a change of behaviour. Then educators can assess these behaviours by asking appropriate questions. Therefore, it is legitimate to consider thinking and understanding as observable.

**Causality:** Causality is the belief that each event is a result of a previous action. If one puts his hand into a fire, then his hand gets burnt. The cause is 'putting the hand into the fire' and the effect is the burning of the hand. Causality affirms that if there is no cause then there is no effect; if there is an effect there must be a cause.

**Cognitivism:** Cognitivism affirms that the observable behaviours are the manifestations of internal cognitive behaviours. Therefore, the educator's primary concern needs to be the cognitive processes of the learner, not the behaviours of the learner.

Cognitivism and behaviourism are two sides of a thin coin. Both theories are valid to some extent and wrong to another extent. Even though a Synthesis of both theories is still imperfect it may be better than either of the two.

**Constructivism:** Many regard constructivism as a theory of learning. A major tenet of constructivism is that people make sense out of their world through a series of mental constructs. People build these constructs individually in their mind. Some authors may think of constructivism as a theory of knowledge (See (von Glasersfeld, 1989, P. 162). Constructivism has many variants. When dealing with issues of learning we name it 'academic constructivism.' Also, for some (including me) constructivism

is regarded as an epistemology. In this case, we simply name it 'constructivism.' In this current work the focus has been on both forms of constructivism and its many variants.

**Conventional  
Reality:**

Conventional reality is what we agree upon to be true. An example: One must walk on the right hand side of the street.

**Determinism:**

Determinism is the belief that what happens in the universe at a given moment is inevitable due to the events preceding the moment.

Some differentiate determinism from absolute determinism or pre-determinism which affirms that each single event is programmed at the very inception of time and space.

If one assumes that the Universe has an inception, then determinism itself leads to the philosophy of absolute determinism. In this case, it is redundant to maintain two versions of determinism. Marxism is a philosophy of historical determinism.

**Dialectical**

**Materialism:**

Dialectical Materialism is the foundational philosophy of Marxism.

Matter is primary. All changes occur because of movement of matter. Conflict of opposites is common for any object, event, or idea. These conflicts lead to a synthesis.

For instance, society consists of two opposite classes, capitalists and workers. They are always in conflict. This conflict is resolved in the revolution that annihilates the classes and produces the synthesis (unity), communism. Unity is impermanent and conflict is permanent. In the classless

utopian society, conflict continues in the form of the struggle between man and nature.

Accrue ment of small quantitative changes leads to a qualitative change (a revolutionary change) of matter or society. When water is heated, the kinetic energy of the water molecules slowly rises. Then there comes a critical point where water liquid becomes water steam.

**Epistemology:** Epistemology is the theory of knowledge. Some of its focus points are the nature of knowledge, how knowledge is acquired, the limits and the extent of the validity of knowledge claims.

**Experiential Reality:** Experiential reality is the images of reality that we make through our senses. That is, what we experience is experiential reality.

According to this definition, hallucinations experienced by a person on a hallucinatory drug may also be considered as the experiential reality of that person.

Also, Neil Armstrong's experiential reality of the Moon is different from my own experience of the Moon. Notwithstanding, I consider my reading of Neil Armstrong's description also as my own experience. Therefore I have incorporated Neil Armstrong's experiential Moon with my own experiential Moon.

**Idealism:** The belief that the universe is merely thought or mind stuff; reality is the thoughts of ours or God. The genesis of the matter is Grand Conscience Being or Grand Conscience. Therefore, conscience is primary, and matter is secondary. See the entries on materialism, objective idealism, subjective idealism and objectivism.

**Logical Positivism:** Knowledge claims are valid only if they can be examined by the rigorous scientific method. Any knowledge claim that cannot be examined by the scientific method must be ignored.

**Marxism:** Marxism is a socio-economic philosophy. Also, it is a philosophy of historical determinism. It affirms that the society has a pre-determined fate of attaining a Utopian classless society in which there is no government, injustices or inequalities. Also, see the entry on dialectical materialism.

**Materialism:** The Universe is merely matter. All reality is of materialistic nature. The genesis of thoughts and conscience is matter; Matter is primary and conscience is secondary. See the entries on idealism and objectivism.

**Objectivism:** Objectivism claims that there is an external reality; humans can access or approximate this reality through sensory information and cognition. What we can know about external reality is more or less the reality that exists.

Objectivists can quickly degenerate into absolute objectivists unless they maintain epistemological awareness. Examples of this possibility are scientific determinism, determinism, historical determinism and Marxism.

If the philosophy of objectivity is extended to encompass the whole Universe then this extension stops at Grand Conscience. For instance, by objectivity if we assume the existence of a set of laws operating governing the Universe then this set of laws is the Grand Conscience.

**Objective Idealism:** Objective Idealism states that reality is spiritual and like thought. In other words, reality is Grand Conscience.

Moreover, this Grand Conscience exists externally to the conscience of any observer other than the Grand Conscience itself. See the entry on absolute idealism.

**Ontology:** Ontology is the study of the nature of the existence of reality. For instance, consider this statement. I know that my sensory perception of the Moon exists, but is there really a Moon. If so how do I know that it exists? What is the nature of its existence?

Ontological reality is the reality that exists independently of our minds. For instance, my concept of the Moon and my perception of it exist only in my mind. This is my experiential reality of the Moon.

Then there exists the true Moon which existed even before I was born and that will exist even after I die. This is the ontological Moon. (Do not ask me how I know that the Moon existed in Year 1875 or it will exist in Year 2075. This is an axiom.) My experiential Moon is derived from the ontological Moon. The two Moons are not the same. My experiential Moon is an image of the ontological Moon, taken with the camera, my mind.

**Rationalism:** Rationalism is a system of beliefs in which reason is paramount in understanding our experiences. For instance, if we see a stone moving inside the room then the stone must have experienced a Newtonian force and there must have been a thrower of the stone. Rationalism denies that the stones can fly by themselves without being subject to a physical force. Therefore if a stone flies, either there should be a physical thrower (not ghosts) or another physical process (like an earth tremor) that generates the force.

Rationalism guides much of our everyday tasks. However, I have discussed the failures of rationalism briefly on p. 57.

**Scepticism:** Scepticism espouses the significance of evidence of supporting knowledge claims. Sceptics question the validity of evidence of knowledge claims. Even though scepticism is generally a sincere guard of trueness of our knowledge claims, scepticism can be extended to deny the validity of any affirmations. For instance, some sceptics can question whether there are any books in the world. They can question any form of evidence we throw at them, to show that books exist again and again, quite logically but not ‘sensibly.’

**Scientific Determinism:** Scientific determinism is a form of determinism where the laws of determination are scientific laws. According to this view, the processes of the Universe are determined by the scientific laws. This means that if we have all the data and if we know all the scientific laws there are, we can, at least in principle, predict what will happen in any future time. See the entries on absolute idealism, objectivism and Marxism.

**Solipsism:** The only certain thing to exist is one’s own mind. Everything else is a figment of imagination of the mind.

**Subjectivism:** There is no truth claims independent of the subject. Therefore, there is no external reality. The reality of any subject is merely how and what any subject perceives. Consequently, there are no objective moral or ethical values. Sensory perceptions are just the creations of one’s mind.

**Subjective Idealism:** Subjective Idealism claims that only mind stuff exists, and all material stuff is the creation or construct of the individual.

**Remark:** All isms concerning reality, except constructivism, can be categorised into two main camps.

- Idealism Camp: Mind stuff is primary. Matter is created by mind stuff.
- Materialism Camp: Matter is primary. All mind stuff is based on matter.