A Retail Category Management Model Integrating Shelf Space and Inventory Levels

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ABSTRACT

A retail category management model that considers the interplay of optimal product assortment decisions, space allocation and inventory quantities is presented in this paper. Specifically, the proposed model maximizes the total net profit in terms of decision variables expressing product assortment, shelf space allocation and common review period. The model takes into consideration several constraints such as the available shelf space, backroom inventory space, retailer's financial resources, and estimates of rate of demand for products based on shelf space allocation and competing products. The review period can take any values greater than zero. Results of the proposed model were compared with the results of the current industry practice for randomly generated product assortments of size six, ten and fourteen. The model also outperformed the literature benchmark. The paper demonstrates that the optimal common review period is flexible enough to accommodate the administrative restrictions of delivery schedules for products, without significantly deviating from the optimal solution.

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INTRODUCTION

Many business organizations are forced to continually seek improvements in productivity due to the increasing costs in labor, space resources, volatile sales, and intense competition. Cachon (2001) emphasized the need for retailers to constantly strive for excellence in operations due to extremely narrow profit margins. The major contributing factors to retailers' costs are rental cost (Bultez and Naert 1988) and the opportunity cost of shelf space allocation to a product (Brown and Lee 1996). An efficient use of available shelf space and effective choice of product assortment would lead to increased retail productivity.

Recent developments in category management have focused on product categories rather than individual products/brands to investigate the performance of the retail industry. Campo and Gijsbrechts (2005) discuss the issues that interplay between retail assortment, shelf and inventory management and further emphasize the need to integrate these three decision areas of category management. The management of an independent franchise retailer requires recurring decisions including which products to stock (assortment decisions), how much shelf space to allocate for each product, the inventory order size of each product, and the frequency of assortment evaluation. The existing models in the literature do not address this important problem in an integrated manner from the perspective of an independent franchise retailer. The problems facing an independent retailer differ from those of other retailers, such as chain stores, in many ways. An independent retailer owns only one retail outlet, allowing him to make independent decisions and not tying up capital in other outlets. There is also the flexibility in choosing suppliers, retail formats and devising strategy. Customer segments can be selected instead of the mass market, so that product assortments, prices, and store hours are set consistent with the market. The main contributions of this paper are as follows:

(i)   The development of a non-linear mixed integer programming model that integrates product assortment, demand, shelf space, back room inventory space and inventory value restrictions in the context of independent franchise retailing.

(ii)   The development of a heuristic method based on the Excel Solver add-in.

(iii)   Illustration of the proposed methodology using examples with real life data drawn from an independent franchise retailer.

(iv)   Comparison of the proposed method with the commonly used industry practice and also the simulated annealing method proposed by Borin et al. (1994).

The next section provides a brief background and review of the literature available in this area. This is followed by a presentation of the mathematical model for the category management problem introducing all the necessary notations and assumptions. In the next section, the methodology is illustrated through several examples generated from real life data arising from an independent franchise retailer. The conclusions are presented in the last section.

BACKGROUND

The existing category management models have evolved from the 1960's literature on the optimization of shelf space allocation (Brown and Tucker 1961; Lee 1961). Lee (1961) proposed a simple allocation model using shelf space elasticity as a demand effect. Curhan (1973) formalized the shelf space elasticity and provided a valuable synthesis of shelf space conceptual models and experiments exploring the relationship between the space allocated to a product and the unit sales of the product. A similar approach was adopted by Anderson and Amato (1974). Campo and Gijsbrechts (2005) provided a recent review of the literature in this area. Table 1 provides a brief outline of the subsequent literature in the area of shelf space allocation models and their main features.

Table 1: List of some important contributions to shelf space allocation models

Author(s)	The Model	Main features and method used
Hansen and Heinsbroek (1979)	Shelf space allocation model as a constrained optimization problem.	It is an extension of earlier models. Models the demand using the shelf space elasticity, past sales and shelf space allocation. Develops near optimal solution, using Lagrangian technique.
Corstjens and Doyle (1981)	Shelf space allocation model.	Extends Hansen and Heinsbroek (1979) model by incorporating cross elasticities within the demand formulation. Develops branch and bound method.
Corstjens and Doyle (1983)	Dynamic shelf space allocation model.	Extends their previous model to a dynamic model incorporating product life cycle and consumer preference. Develops near optimal solution.
Zufryden (1986)	Dynamic programming formulation of shelf space allocation model.	Extends the model by Corstjens and Doyle (1981) incorporating space elasticity and non-space factors such as price, advertising, and promotion.
Bultez and Naert (1988)	SH.A.R.P. (Shelf Allocation for Retailers Profit) model.	Extended the work of Corstjens and Doyle (1981, 1983) and its application. They estimated space elasticities using a symmetric attraction model.
Bultez et al. (1989)	SH.A.R.P. II model.	This is an extension of SH.A.R.P. model integrating it with an asymmetric variant attraction model for the cross elasticities.
Bookbinder and Zarour (2001)	Shelf space allocation model.	This model incorporates Direct Product Profitability (DPP) considerations within the model by Corstjens and Doyle (1981).

Yang (2001) considered a simple shelf space allocation model to maximize the store profit constrained only by the length of the shelf, the length of the product, the product profit per facing and bounds on number of facings. The author proposed a heuristic algorithm to solve this as a multi constrained knapsack problem, but optimal solutions were only found to simplified versions of the original model. Lim, Rodrigues, and Zhang (2004) developed metaheuristic methods such as Tabu search and a Squeaky Wheel Optimization to solve the simplistic model proposed by Yang (2001), as well as a simplified version of the Corstjens and Doyle (1981) model. Recently, Chen and Lin (2007) explored the data mining approach to product assortment and shelf space allocation. The first category management model that integrates the shelf space allocation and the inventory decisions was developed by Borin, Farris, and Freeland (1994). Table 2 provides a brief account of the category management models available in the literature.

Table 2: Integrated category management models

Author(s)	The Model	Main features and method used
Borin et al. (1994).	Category management.	This constrained optimization model integrates shelf space allocation and its influence on demand. They developed a heuristic method based on simulated annealing to maximize the category return on inventory.
Urban (1998)	Integrated category management.	This model improves the model of Borin et al. (1994) by separating the consequences of backroom and displayed inventories and by keeping track of the competitive and complementary product considerations. They provided a gradient reduction algorithm and a genetic algorithm to find near optimal solutions while maximizing the average net profit.
Hwang et al. (2005)	Integrated category management.	The model distinguishes the product display levels in the shelf. A gradient search heuristic and a genetic algorithm were proposed to solve the model while maximizing the average net profit.

Apart from the academic literature, commercial software programs such as Apollo (Information Resources) and Spaceman (ACNielsen) have contributed to the area of allocation of shelf space through non optimal methods. The types of factors relevant to allocation of space considered by these programs are turnover, gross profit margins and constraints such as inventory costs. Although such programs assist in investigating alternate shelf space allocations through simulation (Dreze et al. 1994), these software packages can not be considered as optimization tools (Desmet and Renaudin 1998).

Over the years the emphasis of shelf space and product assortment models research has shifted towards an integrated category management optimization model. Retail managers are required to make a number of vital decisions that include what products to stock on the shelf (assortment decisions), the number of facings to allocate to each product (shelf space allocation decisions), the ordering levels, and the frequency of category assortment evaluation or review. Such decisions are all dependent upon the accurate estimation of demand and available inventory space.

In order to understand the current industry practice and the usage of the existing models, detailed interviews were conducted with small and medium size independent franchise retailers. It was noted that the current decision processes (regarding product assortment and space allocations) used by retailers are ad hoc. In fact, the retailers lack an understanding of the alternative decision strategies that could improve their profit. Further, it was found that the space decisions were made through either a share-of-sales-equals-share-of-space rule, or planograms designed by high market share manufacturers such as Kellogs and Coca-Cola, or those designed by the franchiser. In general, these planograms do not take note of the individual independent franchisee's interests such as demand or contribution to net profit that is relevant to the specific geographic location of the retailer. Some of the retailers expressed a keen interest in exploring alternative methods of determining optimal review periods.

The model presented in this paper specifically addresses the issue of product assortment, shelf space allocation, back room inventory, monetary restriction on inventory and the optimal period of review (evaluation) by considering demand as a function of allocated shelf space and the cross-elasticities of competing products.

At first glance the proposed model may be viewed as an extension of one of the models proposed by Borin et al. (1994), Urban (1998), or Hwang et al. (2005). The following observations reiterate the benefits of our proposed model and the suggested methodology:

(i)   The category management model proposed by Borin et al. (1994) maximizes the category return on inventory. The integrated category management models of Urban (1998) and Hwang et al. (2005) maximize the average net profit per unit time. But the independent franchise retailer would rather maximize the expected total net profit accumulated for a group of competing products of a category during a common review period for them. The proposed model uses this objective function which is relevant to the independent franchise retailer.

(ii)   The model proposed by Borin et al. (1994) is not explicit in its formulation and hence its applicability may not be satisfactorily validated. In fact their formulation does not include restrictions on back room inventory and monetary levels. The integrated category management models of Urban (1998) and Hwang et al. (2005) also exclude monetary restrictions that are vital to the independent franchise retailer. The model proposed by Hwang et al. (2005) does not include product assortment. The model proposed in this paper is in an explicit form and direcly addresses the needs of independent franchise retailers.

(iii)   The proposed simulated annealing heuristic of Borin et al. (1994) does not use any of the constraints of their model in an explicit form. The gradient search heuristics proposed by Urban (1998) and Hwang et al. (2005) are partially based on their models, but not implemented in the commonly available Excel environment. Our proposed methodology is

explicit and permits modeling of many of the commonly occurring constraints in the Excel environment.

PROPOSED INTEGRATED CATEGORY MANAGEMENT MODEL

In this section we present a category management model that maximizes the expected total net profit accumulated during a common review period for an independent franchise retailer. Our model differs from previous approaches on two levels. First, the model incorporates the considerations of space allocation, product assortment determination, inventory quantities, retailer's financial resources, demand and the determination of the common review period. Second, the planning horizon for the model is flexible, depending on the needs of the retailer.

The assumptions and notations required for the proposed model are:

1. Let $N = \{1..n\}$ denote the set of products that are included in the model from a specified category (for example, say from 'Hot Beverages or Soft Drinks') such that they are competing with each other for the limited shelf space and backroom inventory space.

2. The time period between two consecutive reviews of stocks (for ordering and replenishing the backroom inventory of all the products in the category) is called the review period. The proposed model assumes a common review period for all the products of the category. Furthermore, this review period is a decision variable of the model and is denoted by t.

3. The objective is to maximize the expected total net profit during the review period of length t, considering that all the related monitory transactions are often carried out exactly once in every review period.

4. The decision variable s_j denotes the number of shelf facings allocated to product j, for j in N. Let $\ell_j \ (\geq 0)$ and u_j respectively denote the lower and upper bounds on s_j. The variable z_j is defined through a constraint $s_j(1 - z_j) = 0$ so that a positive allocation of shelf space for product

j forces z_j to be 1.   Furthermore, for an optimal solution, the maximizing objective function

forces the model to choose z_j at zero level when $s_j = 0$.   Next, the variable r_j (used in modeling

the demand) is defined using the constraint $r_j = (1 - z_j) + s_j z_j$ such that $r_j = s_j$ or 1 according

to whether s_j is positive or zero respectively. Let $C_{s,j}$ denote the fixed shelf space cost per

facing for product j.

5.   The average (expected) demand rate (that is demand per unit time) of product j is denoted by d_j.

The expected demand d_j per unit time is modeled in the typical polynomial form as a function of

α_j, the potential expected sales; s_j, the number of shelf facings allocated to product j; β_j, the

space elasticity for product j corresponding to one shelf facing; δ_{jk}, the cross elasticity

between product j and product k which can be either negative (substitutable competitive

relationship) or positive (complementary relationship); and λ_i, the resistance of a customer to

compromise in the situation where the desired product i is not included within the assortment.

More precisely, the expected demand rate d_j is modeled (Borin et al. 1994) as follows: Let $N^+ =$

$\{ j : j$ is in N and $s_j > 0 \}$, that is the set of products included in the product assortment. Let $N^- =$

N - N^+, that is the set of products excluded from the product assortment.   Further, in the

following model for the expected demand d_j we take $s_j = 1$ whenever j is an element of N^-.

$$d_j = \alpha_j s_j^{\beta_j} \left[\prod_{k \in N^+} s_k^{\delta_{jk}} \right] \left[1 + \sum_{i \in N^-} \left(-\lambda_i \left\{ \frac{\delta_{ij} \alpha_j s_j^{\beta_j} \left[\prod_{k \in N^+} s_k^{\delta_{jk}} \right]}{\sum_{m \in N^+} \delta_{im} \alpha_m s_m^{\beta_m} \left[\prod_{k \in N^+} s_k^{\delta_{mk}} \right]} \right\} \right) \right].$$

The term $\alpha_j s_j^{\beta_j} \left[\prod_{k \in N^+} s_k^{\delta_{jk}} \right]$ represents the modified demand for a product j and incorporates the

effect of shelf space elasticities and cross elasticities given a product assortment set N^+. The

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second term $\sum_{i \in N^-} \left(-\lambda_i \right) \left(\dfrac{\delta_{ij} \alpha_j s_j^{\beta_j} \left[\prod_{k \in N^+} s_k^{\delta_{jk}} \right]}{\sum_{m \in N^+} \delta_{im} \alpha_m s_m^{\beta_m} \left[\prod_{k \in N^+} s_k^{\delta_{mk}} \right]} \right)$ represents the total demand acquired by

a product j from the set of products excluded from the product assortment, that is in N^-. The

acquired demand is dependent upon the resistance of consumers of product i to compromise

purchase intention and switch to product j, and is measured by λ_i. The volume of acquired

demand is also moderated by the relative share of shelf space of product j to the total space

allocated to the set of products N^+ included in the assortment. Using the variables z_j and r_j

one can remodel the expected demand rate in terms of s_j, r_j, z_j and N as follows:

$$d_j = \alpha_j s_j^{\beta_j} \left[\prod_{k \in N} r_k^{\delta_{jk}} \right] \left[1 + \sum_{i \in N} \left(-\lambda_i \right)\left(1 - z_i \right) \left(\dfrac{\delta_{ij} \alpha_j r_j^{\beta_j} \left[\prod_{k \in N} r_k^{\delta_{jk}} \right]}{\sum_{m \in N} \delta_{im} \alpha_m s_m^{\beta_m} \left[\prod_{k \in N} r_k^{\delta_{mk}} \right]} \right) \right].$$

Note that in this modeling of d_j whenever allocated shelf space s_j is zero, the model forces
d_j to be zero, though the basic expected sales α_j is positive.

6. The parameters used in the modeling of d_j are estimated either from the past data or from
   specially conducted experiments.

7. Note that the per unit time potential sales is a random variable and its mean is estimated as the
   potential expected sales α_j from the past sales data for product j.

8. Note that the model uses the proportionality assumption and thus the order quantity for the review
   period of length t is equal to the expected potential demand during the review period and that is
   equal to td_j for product j. The managers would prefer to set the order quantity for the review
   period of length t as $td_j + SS_j$ where SS_j is the safety stock determined on the basis of the
   stochastic behavior of the potential demand, the cost of idle inventory, and the cost and risk of out

of stock situation for product j. The present model assumes that SS_j is zero. Let $Q_j^{\min}$ and $Q_j^{\max}$ respectively denote the lower and upper bounds on the number of units ordered of product j.

9. The shelf space is continuously replenished from the backroom inventory.

10. Let p_j and c_j denote respectively the retail price and the purchasing cost of one unit of the product j. Let $C_{o,j}$ denote the fixed procurement (ordering and receiving) cost of product j for every delivery. Note that $C_{o,j}$ does not depend on the delivered quantity. Let $C_{h,j}$ denote the fixed backroom storage cost per unit of product j irrespective of the duration of storage.

11. Since the model uses a short time planning horizon, it is reasonable to assume that all the cost parameters are constant and do not change with time during the planning horizon. Furthermore, the model can be resolved with appropriate changes to the cost parameters while planning a sales promotion with the supplier for some of the products.

12. Let B_{SL}, B_{SW} and B_{SH} respectively denote the length, depth and height of the available shelf space. The volume of the available shelf space is equal to $B_s = (B_{SL})(B_{SW})(B_{SH})$.

13. Let $b_{s\ell j}$, b_{swj} and b_{shj} denote respectively the length, depth and height of one unit of product j. The volume of one unit of product j is $b_j = (b_{s\ell j})(b_{swj})(b_{shj})$. Note that the product j can be displayed in the shelf with either depth facing or length facing and it is given by

$$F_j = \begin{cases} W \text{ - if depth facing required.} \\ L \text{ - if length facing required.} \end{cases}$$. Furthermore, for some products its height b_{shj} compared to shelf's height B_{SH} permits vertical stacking. The stack-ability of product j is given by SP_j which assumes value 1 or 0 according as the product j is stackable or not.

14. Let k_j denote the maximum number of items of product j that can be stored on the shelf given an allocation of one facing, and the facing requirement F_j of product j.

Note that,
$$
k_j = \begin{cases} \left\lfloor \dfrac{B_{SH}}{b_{shj}} \right\rfloor \left\lfloor \dfrac{B_{SW}}{b_{s\ell j}} \right\rfloor, & \text{if } F_j = W, \\[2em] \left\lfloor \dfrac{B_{SH}}{b_{shj}} \right\rfloor \left\lfloor \dfrac{B_{SW}}{b_{swj}} \right\rfloor, & \text{if } F_j = L, \end{cases} \quad \text{or} \quad \begin{cases} \left\lfloor \dfrac{B_{SW}}{b_{s\ell j}} \right\rfloor, & \text{if } F_j = W, \\[2em] \left\lfloor \dfrac{B_{SW}}{b_{swj}} \right\rfloor, & \text{if } F_j = L \end{cases} \quad \text{according to}
$$

whether SP_j is equal to 1 or 0 respectively. Thus if product j is allocated s_j shelf facings then the number of items of product j that will be accommodated on the shelf is $s_j k_j$.

15. Let B_b denote the volume of available backroom inventory space. Similarly let M_I denote the upper bound on the maximum money value of the idle capital in the form of the total inventory on hand at any point of time.

16. It is also assumed that no stock is currently on hand by the retailer which can be used to cater for expected sales.

Mathematical formulation

$$
Maximize \prod_n = \sum_{j \in N} \left((p_j - c_j) td_j - C_{o,j} z_j - C_{s,j} s_j - (d_j - s_j k_j) C_{h,j} \right), \tag{1}
$$

subject to

$$
\sum_{\substack{F_j = W \\ j \in N}} b_{swj} s_j + \sum_{\substack{F_j = L \\ j \in N}} b_{s\ell j} s_j \le B_{SL}, \tag{2}
$$

$$
\sum_{j \in N} b_j s_j k_j \le B_s, \tag{3}
$$

$$
\sum_{j \in N} b_j (td_j - s_j k_j) \le B_b, \tag{4}
$$

$$
\sum_{j \in N} c_j (td_j) \le M_I, \tag{5}
$$

$$
td_j \le Q_j^{\max}, \quad j \in N, \tag{6}
$$

$$
td_j \ge Q_j^{\min} z_j, \quad j \in N, \tag{7}
$$

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$$s_j k_j \leq t d_j, \quad j \in N, \tag{8}$$

$$\ell_j \leq s_j \leq u_j, \quad j \in N, \tag{9}$$

$$s_j (1 - z_j) = 0, \quad j \in N, \tag{10}$$

$$r_j = (1 - z_j) + s_j z_j, \quad j \in N, \tag{11}$$

$$d_j = \alpha_j s_j^{\beta_j} \left[\prod_{k \in N} r_k^{\delta_{jk}} \right] \left[1 + \sum_{i \in N} (-\lambda_i)(1 - z_i) \left(\frac{\delta_{ij} \alpha_j r_j^{\beta_j} \left[\prod_{k \in N} r_k^{\delta_{jk}} \right]}{\sum_{m \in N} \delta_{im} \alpha_m s_m^{\beta_m} \left[\prod_{k \in N} r_k^{\delta_{mk}} \right]} \right) \right], \quad j \in N, \tag{12}$$

(Note that the expected demand d_j is modeled as detailed in Assumption 5.)

$$z_j \geq 0 ; \quad z_j \leq 1; \quad z_j \leq s_j ; \quad s_j \text{ integer} \quad j \in N. \tag{13}$$

The objective function (1) maximizes the expected net profit accumulated during the review period and it is equal to the gross revenue subtracted by procurement costs, shelf space costs and backroom inventory costs. Note that the contributions to the objective function by a product j in N during a review period of duration t are (i) the gross revenue: $(p_j - c_j) t d_j$; (ii) the procurement cost: $- C_{o,j} z_j$; (iii) the shelf space cost: $- C_{s,j} s_j$; and, (iv) the backroom inventory cost: $- C_{h,j}(td_j - s_j k_j)$.

Constraint (2) restricts the sum of the allocated linear metric space of the shelf allocation to be less than or equal to the available shelf space length, B_{SL} with the given values of F_j. Constraint (3) ensures that the total volume of products allocated to the shelf space is less than or equal to the available shelf space volume B_s. Next, constraint (4) restricts the total volume of the backroom inventory space of the products in the assortment to be less than or equal to the total volume of the backroom inventory space, B_b. Similarly, the constraint (5) assures that the total idle money locked up in inventory (both backroom and shelf) will not exceed M_I at any point of time.

In order to satisfy the expected sales during period t we take the order quantity as td_j given that the rate of demand is d_j. Constraints (6) and (7) place an upper (Q_j^{max}) and lower (Q_j^{min}) order limit on the quantity ordered for each product j. These upper and lower limits permit the retailer to strategically model situations that arise due to demand during product life cycle or unexpected events or constraints imposed by supplier (Corstjens and Doyle 1983). Constraint (8) states that the quantity ordered must be greater than the number of units required to be stored on the shelf given the allocation of s_j facings.

The constraint (9) places lower and upper limits on the number of facings (s_j) to be allocated to a product j. Often such constraints arise from administrative convenience, contractual obligations between supplier and retailer.

A product is excluded from the category assortment if it is not allocated a shelf facing. The constraints (10) and (11) define respectively the variables z_j and r_j. The next constraint (12) recollects the modeling of the expected demand d_j. Finally, the constraint (13) ensures that z_j is non-negative and at most equal to s_j. Note that in our model the product assortment is decided through the shelf space allocation s_j by virtue of selecting s_j to be zero. On the other hand Urban (1998) considered them as distinct problems.

PRACTICAL EVALUATION OF THE PROPOSED MODEL

From the interviews conducted with independent franchise retailers, it was found that the **current practice** is to allocate shelf space for a product approximately equal to its market share. This is the most commonly used allocation method by practitioners (Borin and Farris 1995). The aim of this section is to carry out extensive comparisons of our model with the current practice.

A number of studies (Sandgren (1981), Haggag (1981), Kao (1988) on a variety of problems, including applications in optimizing the inventory and shelf space allocation model (Urban, 1998; Kar et al., 2001), have been undertaken showing that the GRG is a robust and reliable approach to solving nonlinear programming problems. Colville (1968) conducted a comprehensive comparison of fifteen codes from industrial firms and universities and Generalized Reduced Gradient (GRG) was ranked the best method among them. In particular, the method was found to be consistent in programs used to solve real-life industrial problems.

We note that neither GRG nor any other nonlinear optimization package can guarantee finding the global optimum in cases where there are distinct local optima. The only exception is if the problem is convex, in which case any local optimum is also global. The proposed search method overcomes some of these drawbacks by generating several initial solutions that are used as input for the GRG method to develop a set of local optimum solutions. Furthermore, the search method uses a rounding heuristic to search around the generated local optimum solutions for the feasible solutions of our model. In the process the method generates a number of feasible solutions and selects the best in terms of the objective function value.

Ramaseshan, Achuthan and Collinson (2008) developed a Category Management Decision Support Tool (*CMDST*) in the environment of Microsoft Excel using its spreadsheet capabilities, Visual Basic for Applications and an add-in facility called *Solver* (Frontline Systems). This was done keeping in view the inexpensive and easy accessibility of Microsoft Excel software to independent retailers. The CMDST has user driven menus to enter input data and choose optimizing criteria. The CMDST is a search method developed for the underlying model with a user specified limit on the number of iterations. It uses the following call routines: *Init_Solution*, *Random_Init_Solution*, *Solver* and *Round*. At each iteration i, the best solution S^*_i, from a set of feasible solutions is compared with the best known solution S_{best} and updated if necessary.

The pseudo code of CMDST is shown in Figure 1.

Set *Iter* $\leftarrow$ 0, S_{best} $\leftarrow$ ϕ, $\Pi_n^{(best)}$ $\leftarrow$ 0

S_{init} $\leftarrow$ Init_Solution

Repeat
    Iter $\leftarrow$ *Iter* + 1

    $i \leftarrow Iter$

    S_i $\leftarrow$ Random_Init_Solution(S_{init})   [*Select Random Solution*]

    S'_i $\leftarrow$ Solver(S_i)   [*Call Excel's Solver*]

    S^*_i $\leftarrow$ Round(S'_i)   [*Call Rounding Heuristic*]

    $\Pi_n^{(i)}$ $\leftarrow$ Π_n (S^*_i)

    If $\Pi_n^{(i)} > \Pi_n^{(best)}$ **Then**

        S_{best} $\leftarrow$ S^*_i

    End If

Until *Iter* = *MaxIter*

Output S_{best}

Figure 1. Pseudo Code of the proposed Search Method.

In the following we briefly outline the important features of the call routines used in our search method.

Init_Solution :

1.   Fix the variable $d_j = \alpha_j$, the estimated average per unit time demand of the product *j* and consider

       the reduced problem (1) – (9) and (13).

2.   Solve the reduced relaxed problem (1)- (9) and (13) relaxing the integer restrictions on s_j and z_j.

Note that this relaxed problem is a Linear Programming Problem and can be solved by the Solver.

Let the solution obtained by the Solver be denoted by $S_{init} = (t, s_j, z_j, d_j)$. Note that if $S_{init} = (t, s_j, z_j, d_j)$ satisfies (10) to (12) then it is a feasible solution to the relaxed problem (1) to (13), relaxing the integer restrictions.

3.   When $S_{init} = (t, s_j, z_j, d_j)$ does not satisfy (10) – (12) we perform the following steps:

  (a)   If $s_j \leq 0.1$ and $s_j(1 - z_j) > 0$ then fix $s_j = 0$ and $d_j = 0$.

  (b)   If $s_j \geq 0.9$ and $s_j(1 - z_j) > 0$, then fix $z_j = 1$.

  (c)   If $0.1 < s_j < 0.9$, $s_j(1 - z_j) > 0$ and $(1-z_j) \leq 0.1$, then fix $z_j = 1$.

  (d)   Revise the non zero d_j by the constraint (12) using $r_k = \max\{1-z_k, s_k\}$.

  (e)   Consider the reduced problem (1) – (9) and (13) eliminating the variables

         that are fixed by steps (a) to (c).

4.   Repeat steps (2) and (3) until $S_{init} = (t, s_j, z_j, d_j)$ satisfies (2) – (13), relaxing the integer restrictions.

Random_Init_Solution(S_{init}):

  1. Using the initial solution $S_{init} = (t, s_j, z_j, d_j)$, determine for each product j, the integer interval $I_j = [0, \lfloor s_j \rfloor]$. Select a random integer k between $\left\lfloor \dfrac{n}{4} \right\rfloor$ and $\left\lfloor \dfrac{n}{2} \right\rfloor$. The choice of this interval was

made after investigating several alternatives. Then choose k products at random from the set of n products.

2. Define a new reduced problem on $(n\text{-}k)$ products by fixing the levels of the k chosen products in the formulation (1) to (13). If a product j is one among the k chosen products, then fix the variable s_j as a random integer from the Interval I_j. If s_j is fixed as zero, then revise the variables z_j and d_j also as zero. If s_j is fixed as a non zero value, then fix z_j as 1 and d_j by (12). Next, construct the new reduced problem with $(n\text{-}k)$ products eliminating the k products from the model (1) to (13).

3. Apply the procedure Init_Solution to the reduced problem on $(n\text{-}k)$ products. Let S_i denote the solution obtained by the application of Init_solution to the reduced problem. For notational convenience, the solution S_i will include the details for all the n products of the original problem.

Solver (S_i):

The Solver add-in program consists of two computer programs. The first is an Excel Visual Basic program used to convert the spreadsheet model to an internal representation used by the second program. While the second program, an independent software module, optimizes the model using a variant of the Generalized Reduced Gradient (GRG) method, GRG2, and returns the solution to the first. GRG is a generalization of the reduced gradient method by allowing nonlinear constraints and arbitrary bounds on the variables. If the model and constraints are linear this procedure reduces to the Simplex Method, and if no constraints are present it is a gradient search. In the ith iteration of our method the solution S_i (generated by Random_Init_Solution) is used as an initial solution to the Solver. The output of Solver is dented by $S_i{}'$.

Round($S_i{}'$):

The output $S_i{}'$ of Solver may not satisfy all the integer restrictions. Under such situations, the routine Round ($S_i{}'$) constructs the smallest closed and bounded convex set C containing the

solution (S_i') such that the vertices of the convex set C satisfy all the integer restrictions. A vertex of the convex set C is called a feasible vertex if the corresponding solution satisfies the constraints (2) to (13). Furthermore, this routine chooses the best solution S^*_i from a set of n^2 feasible solutions corresponding to randomly chosen n^2 feasible vertices of the convex set C. Since the number of vertices of convex set C increases exponentially with n, the number of feasible vertices considered was limited to n^2.

We illustrate our model through an example and compare it with current practice for an assortment of ten products. This data represents an actual (reduced) product assortment of a medium size national grocery retailer obtained by Flynn (2004). The retailer provided all the relevant data including the monthly demand pertaining to 'Baked Beans and Noodles'. The retailer noted that the current practice is to order every month. Table 3 gives the product name, product dimensions (L – length, H – height and D – depth), retail price (RP), wholesale price (WP) and potential demand (PD) that are used in the model consisting of a product assortment of ten products.

Table 3: Product assortment details

Name	L	H	D	RP	WP	PD
Heinz Baked Beans 420G	110	75	75	$1.22	$1.10	152
HNZ Spaghetti 420 G	110	75	75	$1.22	$1.10	44
Maggi Two Minute Noodle Chicken 85G	95	140	38	$0.69	$0.56	34
HNZ Baked Beans 220G	80	74	74	$0.98	$0.78	27
HNZ Spaghetti EXT Cheesy 420G	110	75	75	$1.22	$1.10	27
Maggi Two Minute Noodle Beef	95	140	38	$0.69	$0.56	25
Watties Baked Beans 420G	110	75	75	$1.16	$0.93	21
HNZ Baked Beans EXT CHEESY 420G	110	75	75	$1.22	$1.10	21
HNZ Baked Beans S/RED 420G	110	75	75	$1.22	$1.10	19
SPC Baked Beans 425G	110	75	75	$1.30	$1.08	18
						388

Table 4 presents the direct space elasticities (β_j) and cross elasticities (δ_{jk}) generated by Flynn (2004) using the method proposed by Borin et al. (1994).

Table 4: Product assortment direct space and cross elasticities

	1	2	3	4	5	6	7	8	9	10
1	0.10005	-0.01568	-0.01630	-0.00251	-0.00402	-0.01029	-0.01699	-0.01422	-0.01881	-0.01458
2	-0.00942	0.10302	-0.01287	-0.00781	-0.00392	-0.00331	-0.01927	-0.01361	-0.01768	-0.01619
3	-0.01560	-0.00891	0.10637	-0.00237	-0.00378	-0.01463	-0.00529	-0.00199	-0.01809	-0.01687
4	-0.01770	-0.01533	-0.00829	0.22687	-0.00364	-0.01890	-0.00809	-0.00206	-0.00661	-0.01464
5	-0.01519	-0.00150	-0.00294	-0.00350	0.24393	-0.01516	-0.01247	-0.01001	-0.01248	-0.00871
6	-0.01663	-0.01943	-0.01369	-0.01995	-0.01501	0.26379	-0.01342	-0.00419	-0.01719	-0.01495
7	-0.01647	-0.01675	-0.01691	-0.00187	-0.01028	-0.01933	0.27934	-0.00825	-0.00509	-0.00486
8	-0.00344	-0.00207	-0.00189	-0.01076	-0.01912	-0.01439	-0.00201	0.28448	-0.01165	-0.00184
9	-0.01761	-0.00790	-0.01999	-0.00321	-0.01426	-0.00398	-0.00961	-0.00359	0.29577	-0.01857
10	-0.00145	-0.00285	-0.00146	-0.00836	-0.01720	-0.01463	-0.00231	-0.00422	-0.00275	0.29817

For each of the products listed in Table 3 the facing constraints (F_j), maximum facings (u_j) and ordering constraints ($Q_j^{\min}, Q_j^{\max}$) are respectively taken as L, 12, 0 and 1000. The available shelf space dimensions (length B_{SL}, depth B_{SW} and height B_{SH}, in meters) are respectively taken as 3, 0.45 and 0.45. The volume of the available backroom inventory space B_b was taken to be three times that of the shelf space. The unit time considered for the model is one month. Accordingly, the shelf costs ($C_{s,j}$), inventory costs ($C_{h,j}$) and ordering costs ($C_{o,j}$) are taken as 0.5, 0.05 and 0.03 (in \$'s) respectively for all the products. All parameters were supplied by the retailer.

The expected net profit was evaluated corresponding to the allocation strategies given by: (i) the current practice of the retailer (Borin and Farris 1995); and (ii) the solution given by our model. The expected net profit derived through an allocation of shelf space based upon the current practice within one time period (one month) is given in Table 5.

Table 5: Space allocation and net profit from 'current practice' method

Product Name	Shelf Allocation	Backroom Inventory	Net Profit
Heinz Baked Beans 420G	12	0	$16.24
HNZ Spaghetti 420 G	4	0	$3.74
Maggi Two Minute Noodle Chicken 85G	1	0	$3.59
HNZ Baked Beans 220G	2	0	$4.84
HNZ Spaghetti EXT Cheesy 420G	2	0	$2.59
Maggi Two Minute Noodle Beef	1	0	$2.39
Watties Baked Beans 420G	1	0	$3.92
HNZ Baked Beans EXT CHEESY 420G	2	0	$1.92
HNZ Baked Beans S/RED 420G	2	0	$1.58
SPC Baked Beans 425G	1	0	$3.31
			$44.13

Table 6 provides the optimal solution given by our model where the optimal review period works out to be 2.01 (approximately representing eight weeks).

Table 6: Space allocation and net profit determined by the model

Product Name	Shelf Allocation	Backroom Inventory	Net Profit
Heinz Baked Beans 420G	10	17	$39.36
HNZ Spaghetti 420 G	2	20	$9.05
Maggi Two Minute Noodle Chicken 85G	2	5	$7.91
HNZ Baked Beans 220G	1	15	$8.99
HNZ Spaghetti EXT Cheesy 420G	1	16	$4.95
Maggi Two Minute Noodle Beef	1	14	$4.93
Watties Baked Beans 420G	1	4	$8.42
HNZ Baked Beans EXT CHEESY 420G	1	6	$4.19
HNZ Baked Beans S/RED 420G	1	0	$3.79
SPC Baked Beans 425G	1	0	$7.39
			$98.98

Suppose that the retailer uses the shelf space allocation provided by Table 6, with zero backroom inventory and one month review period, then the total net profit for a period of length 2.01 is $92.70 as compared to $98.98 (refer to Table 6) given by our model. Furthermore, the current practice evaluated for a period of 2.01 yields a net profit of $88.70. Thus the percentage increase in net profit by our model over the allocation methods (i) and (ii) above are respectively 11.59% and 6.77%. This increase is of

significant value to a retailer considering the thin margins under which they operate in the competitive retailing environment.

The above results highlight the significant contribution the optimal review period makes in increasing the net profit. In other words, the current practice does not ensure the best profit in the absence of the optimal review period. If the retailer notices significant changes in his operations, then at the end of the optimal review time, the retailer has the option to change the parameters and resolve the model.

In practice, the optimal review period given by the model may not satisfy the delivery restrictions. We use the above example to illustrate how this situation can be overcome easily. Suppose that the last delivery is made on 20 February, 2006, then according to the above solution the next delivery should be on 17 April, 2006. This date, however, falls on Easter Monday – a public holiday when deliveries are not made. The retailer must adjust the review to occur on either week seven or week nine, whichever is feasible and provides the best net profit. In this case a review in week nine (24 April) provides the best profit of $107.79. This translates to an increase in net profit of 8.07% over the current practice, and is not far off from the near-optimal solution. This demonstrates that the model is flexible enough to accommodate the administrative restrictions of delivery schedules.

Model assessment

The following discussions provide an assessment of the performance of our model through simulation of three trials of product categories of size 6, 10 and 14. Each trial of product category was solved 10 times. The data for product assortment categories was randomly generated from a category of size 300 provided by an independent retailer. For each set of trial data of assortment categories, the problem was solved by the following three methods.

1. Method 1: current practice solution;

2. Method 2: our model restricted to the current practice assortment; and

3. Method 3: our model without any assortment restrictions (i.e. products are allowed to be excluded from the assortment).

Of the three methods used, Method 3 produced the best solution yielding the maximum total net profit. The percentage increase in the net profit provides a good comparison of Methods 2 and 3 with the current practice. Results of the simulation are summarized in Table 7, giving the mean and standard deviation of the percentage increase in net profit.

Table 7: Product assortment model: comparison with current practice

Number of Products in the Product Assortment	Allocation Method	Number of Runs	Percentage Increase in Net Profit from the Solution Using Current Practice			
			Mean	Standard Deviation	Min.	Max.
Six	Method 2	10	10.74	6.81	0.14	20.35
	Method 3	10	10.77	7.63	1.41	22.13
Ten	Method 2	10	3.32	0.48	2.65	3.67
	Method 3	10	22.01	12.92	9.60	49.86
Fourteen	Method 2	10	11.74	6.73	3.38	23.73
	Method 3	10	23.71	10.41	3.38	34.20

As can be seen form Table 7, our model consistently outperformed the current practice allocation. The significant variation in the percentage increase of the net profit is observed from the entries in the minimum and maximum columns of Table 7. This illustrates the inconsistent performance of the current practice.

Table 8: Product assortment model: comparison with Borin et al. (1994)

Number of Products in the Assortment	Percentage Increase in ROI from the Solution Using The Method of Borin et al. (1994)			
	Mean	Standard Deviation	Min.	Max.
Six	3.12	2.43	-0.19	5.95
Ten	8.29	2.13	4.33	10.65
Fourteen	4.75	2.21	2.06	8.68

We have also compared our model with the benchmark literature model of Borin et al. (1994). We have adapted our model by utilizing all the available parameters and used the values given in their paper. Table 8 shows the percentage increase in the return on inventory (ROI) of our model over that used by Borin et al. It can be seen that our model is outperformed only once. In all other instances our method proved to be superior with the mean percentage increase in ROI ranging from 3.12% to 8.29% and a maximum increase of up to 10.65%.

Summary

Our model provides a significant improvement in the net profit compared to that of the current practice and the literature benchmark. Combining the three trials we note that the average percentage increase in the net profit was 13.72. Also, we observe that in at least 30% of instances the increase in net profit is more than 20%. This result highlights the benefit of using our model.

Figures 2 and 3 present the percentage improvement of the net profit obtained by our method compared to the solution obtained by the current practice, across thirty runs. As can be seen from Figures 2 and 3, the performance of the current practice allocation is highly fluctuating, with seventy percent of

the allocations generated by Method 3 resulting in at least a 10% increase in net profit over the current practice (thirty-three percent for Method 2).
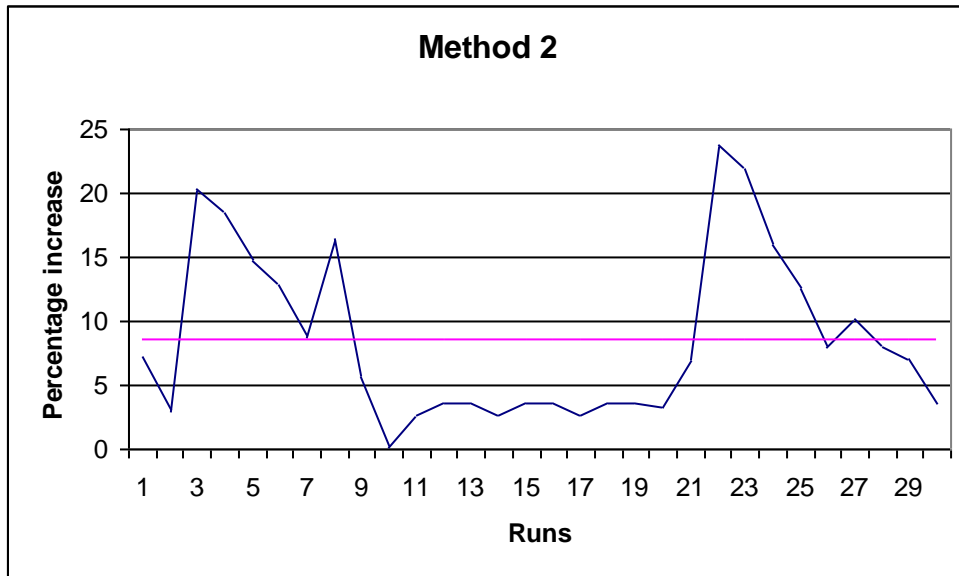
Method 2

Figure 2: Increase in net profit in comparison with the current practice
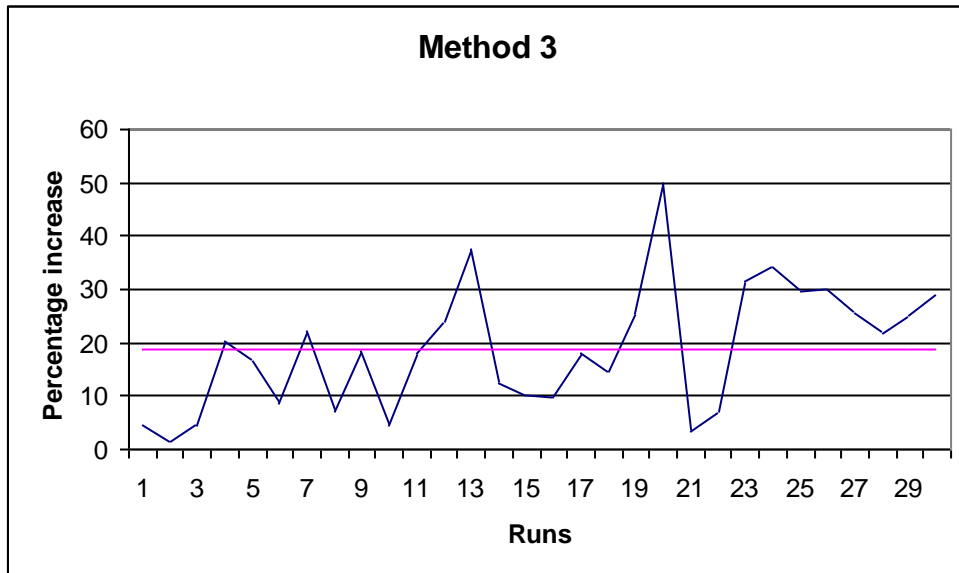
Method 3

Figure 3: Increase in net profit in comparison with the current practice

The volatile nature of performance of the current practice raises questions regarding its usage. One of the common features of the retail industries is narrow profit margins that call for a consistently well performing allocation method. Thus our method would be of great benefit to an independent retailer providing an opportunity to change the current practice and increase the net profit considerably. Note that Method 3 gives an average improvement of 18.83% (indicated by the horizontal line in Figure 3) over the current practice. For an independent retailer, Method 3 would be a natural choice to determine shelf space allocations.

Parameter sensitivity analysis

The parameters of direct shelf space elasticity, cross elasticity and consumer resistance are estimated by the process described by Borin et al. (1994) and they are assumed to be the 'true' values. To understand the robustness of the model we perturb the true values of the input parameters by specified amount, resolve the model and perform sensitivity analysis of the results obtained. The process suggested by (Borin and Ferris 1995) was used to measure the sensitivity of the model.

All the parameters were perturbed uniformly in the same direction, either positive or negative from the true value. The corresponding sensitivity analysis of the results showed that 50% deviation in either direction of the true value of the parameters resulted in approximately 15% variation from the net profit yielded by the true value of the parameters. A uniform 10% parameter estimation error yielded around 4% variation in the expected net profit using the true values.

Next, each parameter was perturbed randomly in any direction, positive or negative, from its true value. The corresponding model was solved and its solution was compared with the solution obtained for the true value. A mean absolute difference of 17.17% (from the true parameter values) yielded approximately 7.69% variation in the net profit from that of the solution obtained from true values of the parameters. A higher mean absolute difference of 66.47% in the parameter values produced around

28.16% variation in the expected net profit. Thus the model can be considered as a robust model since in real life the error in parameter estimation may not be very large. Our results are similar to that of Borin and Farris (1994, 1995).

CONCLUSIONS

In the current competitive environment with shrinking margins retailers look for every opportunity to reduce the costs and improve performance. Retail space in general and shelf space in particular, account for a significant element of retailers' costs. Traditionally, these operational factors were not integrated with the market factors while resolving the retailers' management problems. This can be attributed to the weakness in the current shelf space models which solely depend on marketing variables. The enormous benefit derived from the proposed integrated model was clearly demonstrated through model assessment and sensitivity analysis. In particular our model produced an average of 10 to 20 percent improvement over the current industry practice and an improvement of 3 to 8 percent over the model of Borin et al. (1994). Our model could be extended further by distinguishing between the display features of the shelf such as eye level, middle level and top level. The implementation of our model is limited by the capacity of the basic Excel *Solver*.

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