State Variable Implementation of Stationary Reference Frame Filters for Active Filter Systems

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Abstract

This paper presents a novel and elegant implementation of a synchronous reference frame filter for power system harmonic extraction. This filter does not require the sine and cosine terms normally required of SRF filters. The novelty of this approach is the use of a state variable representation to develop the filter structure. Filters based in the stationary reference frame have been presented based on classical control approaches, but these filters tend to be inelegant in their implementation. Another beauty of this approach is that a structural relationship exists between the SRF filter and its stationary frame state variable implementation, which allows the approach to be easily extended to higher order filters or controller structures.

1. INTRODUCTION

The development of active power filters has followed a path of incremental advancement since the first installation of a passive tuned filter in the mid 1940's, [1]. The signal processing required to generate reference signals has also developed in an incremental way. The most common mechanisms presented in current literature involve the use of either Instantaneous Reactive Power (IRP), [2-4], or Synchronous Reference Frame (SRF), [5-8], based filters. These filters use mathematical transformations to convert the sinusoidal quantities found in three phase systems into DC quantities, which are much more easily filtered and processed (eg PI controllers work well with DC quantities). Quantities in the three phase system are referred to as stationary frame quantities, and the transformed DC quantities are referred to as synchronous frame. An example of this transformation process for an all-pass harmonic controller is illustrated in Figure 1.

The synchronous rotations require sine and cosine terms synchronous with the supply frequency and this increases computational complexity. More recent work has explored the relationship between stationary frame and synchronous frame, [9,10]. This allows filters to be implemented directly in the stationary frame, which mimic exactly the SRF filters. Classical control techniques are used to develop these filters from the SRF implementations, but the process is laborious.

In this paper a state variable approach is implemented. This produces state variable based stationary reference frame filters which are again identical to their synchronous reference frame counterparts. The state variable approach offers a more elegant implementation and simple design than the classical approach. This approach is also more generally applicable to the variety of signal processing requirements found in active filter applications.

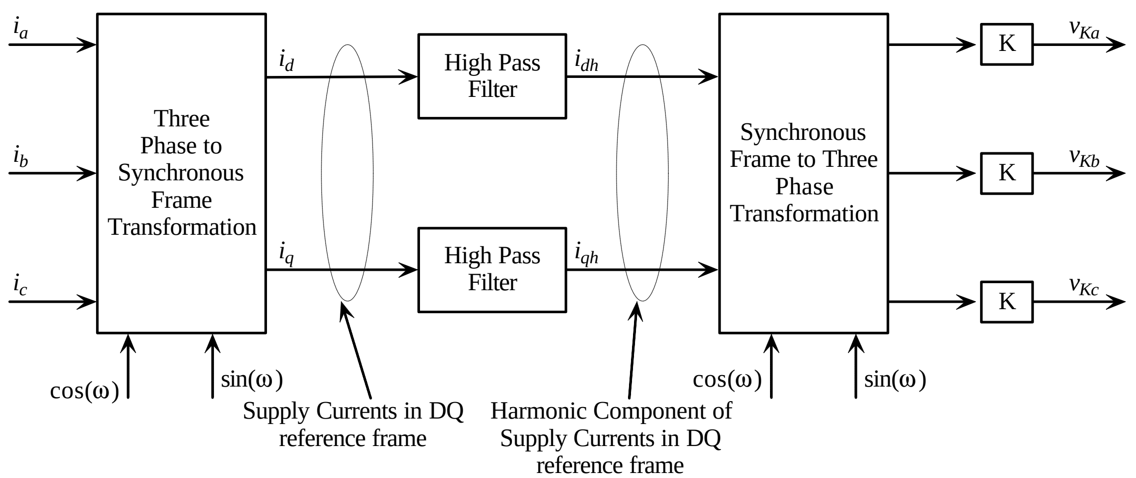


Figure 1. All pass harmonic controller in synchronous frame.

2. CLASSICAL CONTROL APPROACH

The filter systems presented in Figure 1 are effectively multi-input, multi-output systems and these may be modeled in the Laplace domain using classical techniques. The classical control approach outlined by several authors produces a set of cross-coupled transfer functions, dependent on the nature of the filters used after the synchronous transformation, [9,10]. A typical block diagram is shown in Figure 2.

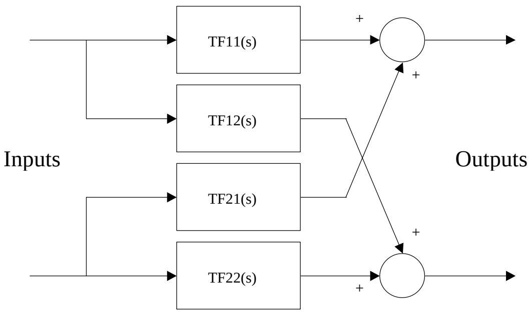


Figure 2. Classical representation of Muti-input, Multi-output system.

The transfer function shown in Figure 2 may be directly implemented in digital hardware or software to produce a system which does not require the synchronous sine-cosine terms for the rotations. Some work has been presented on discrete implementations of these filters that overcome problems of finite word size and rounding, which plague this type of implementation, [11].

3. STATE VARIABLE APPROACH

The main work presented in this paper is a state variable model, which is equivalent to the synchronous reference frame filters. This model may be used to replace the cumbersome implementations of both the synchronous frame filters and the classical transfer function approach. The resulting state models are simple and elegant and easily extended to higher order systems.

An example will be given to demonstrate the development of state models from the filters of Figure 1. For the development of this model the system shown in Figure 3 will be used. The signal flow representation indicates a low pass, first order transfer function also shown in the figure. This system implements a narrow band-pass filter designed to extract a specific frequency component. The process can be easily extended to higher order systems and this will be demonstrated later in the paper.

The state equations relating v_d,v_q to v_{fd},v_{fq} can be written directly from the figure and are given in equation (1).

$$\begin{bmatrix} \dot{x}_d \\ \dot{x}_q \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$
$$\begin{bmatrix} v_{fd} \\ v_{fq} \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \qquad (1)$$

The transformation matrices to convert from Stationary frame to synchronous frame and vice versa are given in Equation (2).

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} = R1 \begin{bmatrix} x_a \\ x_b \end{bmatrix}$$
$$\begin{bmatrix} x_a \\ x_b \end{bmatrix} = \begin{bmatrix} \cos(wt) & \sin(wt) \\ -\sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} = R1^{-1} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \qquad (2)$$

where: w is the frequency of the rotation

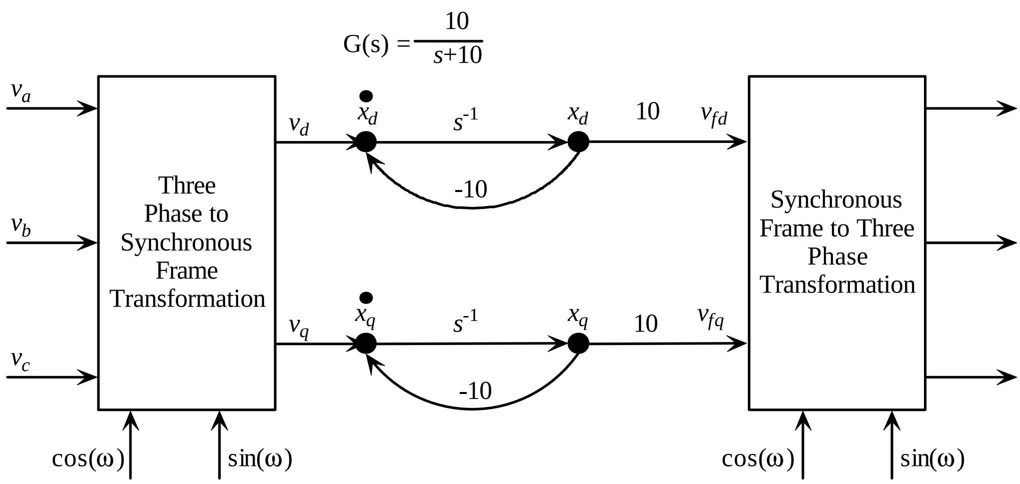


Figure 3. First order filters in the synchronous reference frame.

The first step is to define the state derivatives in the dq domain. The sine-cosine transformations are not linear and therefore the derivatives must be calculated. The process of calculation is now given.

$$\begin{bmatrix}\dot{x}_d\\ \dot{x}_q\end{bmatrix}=\frac{d}{dt}\begin{bmatrix}x_d\\ x_q\end{bmatrix}$$
$$\begin{bmatrix}\dot{x}_d\\ \dot{x}_q\end{bmatrix}=\frac{d}{dt}\left([R1]\begin{bmatrix}x_a\\ x_b\end{bmatrix}\right)$$
$$\begin{bmatrix}\dot{x}_d\\ \dot{x}_q\end{bmatrix}=\left(\frac{d}{dt}[R1]\right)\begin{bmatrix}x_a\\ x_b\end{bmatrix}+[R1]\frac{d}{dt}\begin{bmatrix}x_a\\ x_b\end{bmatrix}$$
$$\begin{bmatrix}\dot{x}_d\\ \dot{x}_q\end{bmatrix}=\left(\frac{d}{dt}[R1]\right)\begin{bmatrix}x_a\\ x_b\end{bmatrix}+[R1]\begin{bmatrix}\dot{x}_a\\ \dot{x}_b\end{bmatrix} \qquad (3)$$

The derivative of *R1* can be easily calculated and is given in Equation (4).

$$\frac{d}{dt}[R1]=w\begin{bmatrix}-\sin(wt) & -\cos(wt)\\ \cos(wt) & -\sin(wt)\end{bmatrix} \qquad (4)$$

Equations (3) and (4) can now be substituted into equation (2), yielding a representation which does not require the transformation matrix R1.

$$\left(\frac{d}{dt}[R1]\right)\begin{bmatrix}x_a\\ x_b\end{bmatrix}+[R1]\begin{bmatrix}\dot{x}_a\\ \dot{x}_b\end{bmatrix}=\begin{bmatrix}-10 & 0\\ 0 & -10\end{bmatrix}[R1]\begin{bmatrix}x_a\\ x_b\end{bmatrix}\dots$$
$$+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}[R1]\begin{bmatrix}v_a\\ v_b\end{bmatrix} \qquad (5)$$

Equation (5) may be solved to yield only state derivatives on the LHS of the equals sign.

$$\begin{bmatrix}\dot{x}_a\\ \dot{x}_b\end{bmatrix}=-[R1]^{-1}\left(\frac{d}{dt}[R1]\right)\begin{bmatrix}x_a\\ x_b\end{bmatrix}+\dots$$
$$[R1]^{-1}\begin{bmatrix}-10 & 0\\ 0 & -10\end{bmatrix}[R1]\begin{bmatrix}x_a\\ x_b\end{bmatrix}+[R1]^{-1}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}[R1]\begin{bmatrix}v_a\\ v_b\end{bmatrix} \qquad (6)$$

Further simplification may be realized using the relationship in equation (7).

$$[R1]^{-1}[A][R1]=[A] \qquad (7)$$
where: *A* is any matrix

Applying this simplification to equation (6) yields:

$$\begin{bmatrix}\dot{x}_a\\ \dot{x}_b\end{bmatrix}=-w\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_a\\ x_b\end{bmatrix}+\begin{bmatrix}-10 & 0\\ 0 & -10\end{bmatrix}\begin{bmatrix}x_a\\ x_b\end{bmatrix}+\dots$$
$$+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_a\\ v_b\end{bmatrix} \qquad (8)$$

The final state equations in the αβ reference frame are given in equation (9).

$$\begin{bmatrix}\dot{x}_a\\ \dot{x}_b\end{bmatrix}=\begin{bmatrix}-10 & w\\ -w & -10\end{bmatrix}\begin{bmatrix}x_a\\ x_b\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_a\\ v_b\end{bmatrix}$$
$$\begin{bmatrix}v_{fa}\\ v_{fb}\end{bmatrix}=\begin{bmatrix}-10 & 0\\ 0 & -10\end{bmatrix}\begin{bmatrix}x_a\\ x_b\end{bmatrix} \qquad (9)$$

Close inspection of equations (1) and (9) will reveal that the only difference is the addition of a cross-coupling term *w* in the state matrix. In implementing Equation (9) the sine and cosine transformations are not required. This simplifies the calculations required significantly, compared to the traditional approach in Figure 1. Equation (9) also retains the sequence sensitivity of the synchronous frame filter. The outputs of the two approaches are identical in every respect.

Another advantage of this approach is that a structural relationship can be found between the state matrices describing the synchronous reference frame and the stationary reference frame systems. This allows the stationary form to be derived directly from any existing filter structure of any order.

4. HIGHER ORDER EXTENSION

If the low pass filters of system shown in Figure 3 are modified to be second order with the same cut-off frequency and critical damping then one possible set of state equations for the filters in the DQ frame are given in equation 10.

$$\begin{bmatrix}\dot{x}_{d1}\\ \dot{x}_{d2}\\ \dot{x}_{q1}\\ \dot{x}_{q2}\end{bmatrix}=\begin{bmatrix}-10 & 10 & 0 & 0\\ 0 & -10 & 0 & 0\\ 0 & 0 & -10 & 10\\ 0 & 0 & 0 & -10\end{bmatrix}\begin{bmatrix}x_{d1}\\ x_{d2}\\ x_{q1}\\ x_{q2}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_d\\ v_q\end{bmatrix}$$
$$\begin{bmatrix}v_{fd}\\ v_{fq}\end{bmatrix}=\begin{bmatrix}10 & 0 & 0 & 0\\ 0 & 0 & 10 & 0\end{bmatrix}\begin{bmatrix}x_{d1}\\ x_{d2}\\ x_{q1}\\ x_{q2}\end{bmatrix} \qquad (10)$$

Applying the transformations to these equations yields the αβ frame representation in equation 11.

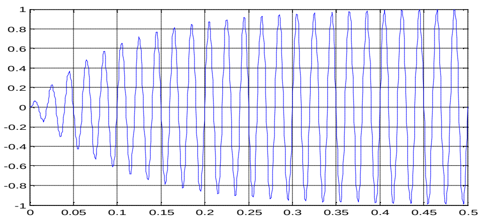
$$\begin{bmatrix} \dot{x}_{d1} \\ \dot{x}_{d2} \\ \dot{x}_{q1} \\ \dot{x}_{q2} \end{bmatrix} = \begin{bmatrix} -10 & 10 & w & 0 \\ 0 & -10 & 0 & w \\ -w & 0 & -10 & 10 \\ 0 & -w & 0 & -10 \end{bmatrix} \begin{bmatrix} x_{d1} \\ x_{d2} \\ x_{q1} \\ x_{q2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

$$\begin{bmatrix} v_{fd} \\ v_{fq} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} x_{d1} \\ x_{d2} \\ x_{q1} \\ x_{q2} \end{bmatrix} \qquad (11)$$

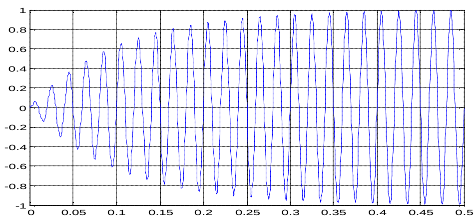
The structural relationship is evident and transformations of any filter structure may now be realised by writing the equations in the DQ frame and simply adding the *w* terms into the DQ frame equations to give the αβ frame equations. The decoupled nature of the DQ frame equation ensures that it is always possible to write the equations as a decoupled set similar to Equation (10).

5. VERIFICATION

Figure 4 shows the results from a simulation using the different first-order filter structures. Figure 4(a) is the results for Phase A using the traditional filter of Figure 3. This filter requires synchronized sine and cosine rotations. Figure 4(b) is the results for Phase A using the state variable representation of equation 9. Figure 4 shows generally the same response and general characteristics in both approaches.



(a)



(b)

Figure 4. Comparison results using (a) traditional and (b) state variable filter structures.

Closer inspections reveal that the filters are identical in every respect, including the sequence sensitivity. Figure 5 shows the results if the input to the state variable filter is negative sequence.

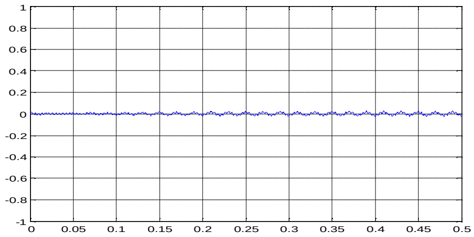
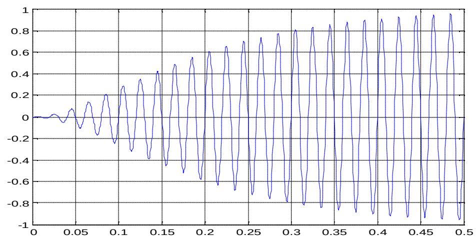


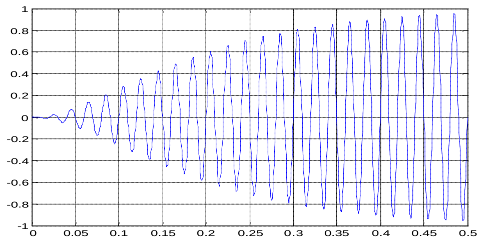
Figure 5. Output of state variable filter if input is negative sequence.

A further advantage of the state variable filters is the ease with which the sequence sensitivity may be changed. If the polarity of the *w* terms in Equation (9) is changed the sequence shifts from positive to negative.

Figure 6 shows the Phase A output for both filter types with second order DQ filters as described in Equation (10) and (11). Once again the similarity of response is evident.



(a)



(b)

Figure 6. Second order system comparison results using (a) traditional and (b) state variable filter structures.

6. DISCRETISATION

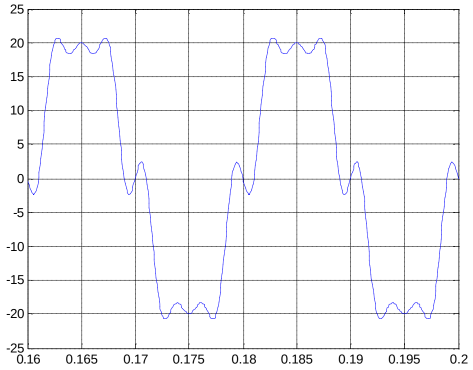
State variable equations are always first order and may easily be converted to a discrete form by approximating the derivative. Although several algorithms exist, the least complex approach is to use a first order approximation. This allows the filters to be directly implemented in a micro-controller based control system. As well as the obvious advantages of not requiring complex sine and cosine operations, it is likely that the first order nature of state equations also offers benefits in terms of rounding and overflow errors in the discrete form. Future work will look into this and determine if this is always true.

7. SYSTEM SIMULATION

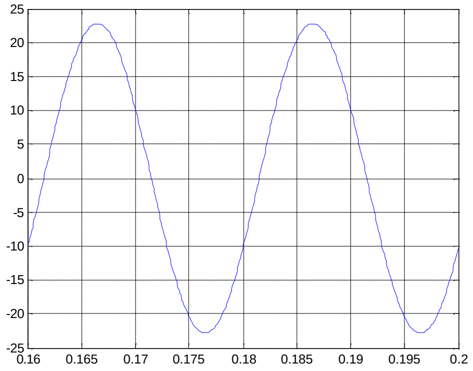
Figure 7 shows the single line diagram of a typical system with harmonic load and hybrid active/passive filter. Control of the active element to remove additional harmonic is normally achieved using multiple feedback loops, each targeting a specific harmonic as shown, [12]. Each loop requires a set of transformations at the harmonic frequency, a filter and PI controller and then an inverse transformation.

The approach presented in this paper was used to convert the filter structures used in previous work to a state space form. Figure 8 shows the simulation results for this system. For clarity in the results a simplified load, consisting of fundamental, fifth and seventh harmonic, was used. Filter loops to remove fifth and seventh are implemented, allowing the results to show a harmonic free line current.

The PI controllers used in traditional DQ controller implementations have zero steady state error at the harmonic frequencies. The results from this simulation demonstrate that the state variable controllers maintain all of the characteristics of their traditional counterparts, including the zero steady state error at the harmonic frequencies.



(a)



(b)

Figure 8. Results for complete system simulation.
(a) Phase A Load Current.
(b) Phase A Line Current.

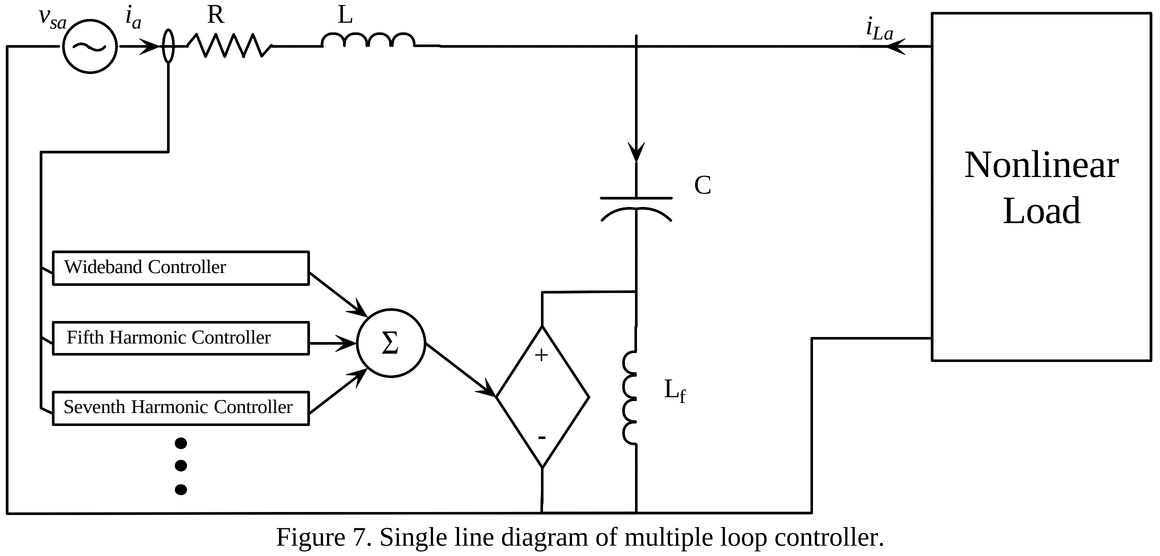


Figure 7. Single line diagram of multiple loop controller.

8. CONCLUSIONS

This paper has presented an alternate implementation of synchronous reference frame filters in the stationary reference frame. Other authors have presented a classical approach to implementing the multi-input, multi-output systems, whilst this paper has taken a modern control approach. The state variable approach allows implementation of the synchronous reference frame filters without the sine and cosine operations required by traditional DQ filters. The approach is also more elegant than the classical control approach, which requires cross-coupled transfer functions.

This paper also presents a very simple structural relationship between the DQ reference frame and the stationary reference frame. This allows ease of modification and expansion, without having to recalculate the equations each time a change is made.

The state equations also allow easy discretisation and implementation in a micro-controller based system. It is expected that this implementation will reduce problems due to rounding and overflow, which are inherent in fixed-point micro-controllers. Further work must be done to verify this.

9. REFERENCES

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