Evaluation of inertia effect in finite hydrodynamic bearings with surface texturing using spectral element solver

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Abstract

Fluid inertia can have a major effect on the pressure distribution in hydrodynamic bearings with surface texturing. So far, however, the effect has been ignored in the simulation of finite bearings. In this work, we develop a spectral element solver for the Navier-Stokes equations specifically tailored to slider finite bearings textured with multiple spherical dimples. Using the solver, we studied the effect of inertia on the load-carrying capacity for 90 different bearing configurations. Our results show that the spatial arrangement of dimples has a significant impact on the inertia effect. For the Reynolds number 50 and the ratio 10 of dimple length to dimple depth, a change from full to partial texturing reduced the effect by roughly 85%.

Keywords:

Hydrodynamic, Texturing; Inertia; Spectral element method

1. Introduction

Hydrodynamic lubrication is an effective way of reducing friction and wear at the contact between two surfaces in relative sliding [1]. One way to form the lubricating film is to texture the surfaces with dimples [2–5]. For certain combinations of the ratio D of dimple length to dimple depth and the Reynolds number Re (roughly $D<50$ for $Re>30$), the lubricant flow transits from purely viscous to visco-inertial [6–8]. This renders the Reynolds equation of viscous lubrication inadequate to simulate the lubricant flow. To rectify this problem, full Navier-Stokes equations (NSE) have been employed instead. However, due to substantial computational effort required to solve the equations in three dimensions (3-D) for complex geometries, studies of the visco-inertial flow in hydrodynamic bearings have been limited to single dimple configurations [8]. In addition, periodic boundary conditions have been used; causing a dependence of results on the location and value of the reference pressure used in solving the NSE [6].

In the current work, we develop a spectral element solver for 3-D NSE that governs the lubricant flow in hydrodynamic bearings with multiple dimples. The solver decomposes the computational domain of the bearing into a small number of pre-defined elements. For each element, a discrete and independent NSE is obtained using the spectral element method (SEM) [9] and a specifically designed Schur complement of the Jacobian matrix. The solution of the NSE for the entire computational domain is obtained by combining the solutions calculated for individual elements, reducing the computational time. Using the solver we conducted a systematic study on the inertia effect in hydrodynamic bearings for 90 different combinations of dimples and the Reynolds numbers, i.e.: 90 = 15 (spatial arrangements of dimples) × 3 (ratios of dimple length to its depth 10, 50 and 100) × 2 (Reynolds numbers 1 and 50). Other effects such as mass-conserving cavitation and

temperature are not studied since they would considerably complicate the numerical simulations. For this reason, a simple Gumbel (half-Sommerfeld) cavitation boundary condition is employed while the effect of temperature on lubricant viscosity is ignored.

2. Methods

For hydrodynamic bearings with laminar and steady flow, and Newtonian, incompressible and isoviscous lubricant, the governing equations consist of the NSE and mass continuity equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)-\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\partial p}{\partial x}=0, \tag{1a}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)-\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\partial p}{\partial y}=0, \tag{1b}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)-\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\partial p}{\partial z}=0, \tag{1c}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1d}$$

where ρ and μ denote the density and dynamic viscosity of the lubricant, p is the pressure function, and u, v and w are the velocity functions in the x, y and z directions, respectively. The unknowns are the pressure and velocity. For the equations to have a unique solution, it is necessary to specify appropriate boundary conditions. In this work, the pressure is set to zero on the boundaries that do not correspond to the bearing surface and the velocity is set to zero (the sliding velocity U) at the boundary that corresponds to the stationary top (sliding bottom) bearing surface (Fig. 1).

In this study, the bearing performance is defined as the load-carrying capacity W^+:

$$W^+=\iint_A p^+ dxdy, \tag{2}$$

where $p^+ = \max(p, 0)$, A denotes the area of the sliding surface and it is assumed that the sliding is in the x direction. The reason for using the load-carrying capacity is that inertial flow mostly affects the pressure distribution [6,7]. The integration is over the positive part of the pressure distribution in Eq. (2) since the Gumbel boundary condition is used in this study.

2.1. Spectral solver

The spectral solver developed in this work first discretises the governing equations (1a-d) into a system of nonlinear algebraic equations using the SEM, and then provides the solution of the system using the Newton's method with preconditioning.

2.1.1. Discretization of the governing equations

Discretization of the governing equations is performed in four steps. First, the governing equations are discretised in a reference domain (i.e., a domain that is independent of the bearing geometry). Next, a physical domain is decomposed into a small number of pre-defined elements. Transformations from the reference domain to each element are then calculated and a system of nonlinear algebraic equations is produced.

Discretization in reference domain

3-D reference domain is defined as a cube $\Omega_*=[-1,1]^3$ with the Cartesian coordinates x_*, y_*, $z_* \in [-1,1]$. For the x_* (y_* and z_*) coordinate, a set of points $x_{*,k}=1,\ldots,K$ ($y_{*,m}=1,\ldots,M$ and $z_{*,n}=1,\ldots,N$) is chosen based on the nodes of the Gauss-Legendre-Lobatto (GLL) quadrature. Example of the domain for K=4, M=5 and N=6 is shown in Fig. 2. The points are then used to construct a set of Lagrange polynomials $l_{x,k}(x_*)$ ($l_{y,m}(y_*)$ and $l_{z,n}(z_*)$) of degree K-1 (M-1 and N-1) in a such way that the k-th (m-th and n-th) polynomial equals to one at $x_{*,k}$ ($y_{*,m}$ and $z_{*,n}$)

and to zero at the remaining points. Using the polynomials, the velocity function in the x direction is approximated as

$$u(x_*, y_*, z_*) \approx \hat{u}(x_*, y_*, z_*) = \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} u_{kmn} l_{kmn}(x_*, y_*, z_*), \tag{3}$$

where u_{kmn} denotes the unknown values to be found and $l_{kmn}(x_*,y_*,z_*)=l_{x,k}(x_*)l_{y,m}(y_*)l_{z,n}(z_*)$. The velocity functions in the y and z directions and the pressure function are approximated similarly. The approximations are then substituted in the equations (1a-d) and put into a weak form according to the Galerkin formulation [9]. For the first equation (1a) the form and the subsequent discretization steps are given by

$$\iiint_{\Omega_*}\left[\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial x_*}+\hat{v}\frac{\partial\hat{u}}{\partial y_*}+\hat{w}\frac{\partial\hat{u}}{\partial z_*}\right)-\mu\left(\frac{\partial^2\hat{u}}{\partial x_*^2}+\frac{\partial^2\hat{u}}{\partial y_*^2}+\frac{\partial^2\hat{u}}{\partial z_*^2}\right)+\frac{\partial\hat{p}}{\partial x_*}\right] l_{rst}\, d\Omega_* = 0, \tag{4}$$

where $d\Omega_*$ is the volume differential and l_{rst} denotes the same set of polynomials as l_{kmn} except that the indices r=1,…,K, s=1,…,M and t=1,…,N are independent of k, m and n. Eq. (4) must be satisfied for all values of the indices r, s and t. The equations (1b-d) have similar weak forms. The last step of the discretization in the reference domain is the reduction of the order of derivative in the diffusion term using the divergence theorem

$$\begin{aligned}
&-\iiint_{\Omega_*}\mu\left(\frac{\partial^2\hat{u}}{\partial x_*^2}+\frac{\partial^2\hat{u}}{\partial y_*^2}+\frac{\partial^2\hat{u}}{\partial z_*^2}\right) l_{rst}\, d\Omega_* = \\
&\iiint_{\Omega_*}\mu\left(\frac{\partial\hat{u}}{\partial x_*}\frac{\partial l_{rst}}{\partial x_*}+\frac{\partial\hat{u}}{\partial y_*}\frac{\partial l_{rst}}{\partial y_*}+\frac{\partial\hat{u}}{\partial z_*}\frac{\partial l_{rst}}{\partial z_*}\right) d\Omega_* - \iint_{S_*}\mu\left(n_{x_*}\frac{\partial\hat{u}}{\partial x_*}+n_{y_*}\frac{\partial\hat{u}}{\partial y_*}+n_{z_*}\frac{\partial\hat{u}}{\partial z_*}\right) l_{rst}\, dS_*,
\end{aligned} \tag{5}$$

where S_* denotes the reference domain boundary, dS_* is the differential element of area and $[n_{x*},n_{y*},n_{z*}]$ is the unit normal vector pointing outward from the reference domain. Substituting Eq. (5) into Eq. (4) gives the final form of the discretised Eq. (1a) in the reference domain.

Decomposition of physical domain into elements

The physical domain of the bearing is decomposed in two steps. First, the domain is decomposed into a grid of 5×5 cells (Fig. 3a). The cells represent the lubricant volume between the stationary top and sliding bottom bearing surfaces. For each cell, the top surface is either flat or textured with a single spherical dimple and the bottom surface is flat. Second, each cell with textured (flat) top surface is decomposed into five (one) elements (Fig. 3b). The decomposition of the textured cell ensures that the top surface of each element is smooth, increasing the accuracy of spectral approximation [9,10] and the overall number of grid/mesh points is kept as low as possible by means of a trade-off between the numbers of elements and the numbers of GLL points inside the elements. The physical domain is decomposed into multiple elements in the x and y directions, and into a single element in the z direction (i.e. across the film thickness). This has two advantages: (1) the total number of elements is approximately equal to those in a corresponding 2-D problem, and (2) each element has well-defined boundary conditions for the velocity since its top and bottom walls intersect with the top and bottom bearing surfaces, respectively. To account for the inertia effect across the film thickness, a medium-degree (6-10) Lagrange polynomial is used to approximate the solution in the z direction. The degree is high enough to capture changes in the solution as shown in the validation example in section 3.2. The exact value of the degree depends on the ratio D and the Reynolds number Re. For $D \geq 50$ and $Re=1$ (small inertia effect), we set the value to 6 whereas for other combinations of D and Re we set it to 10.

Transformations from reference domain to each element

Since the discretisation of governing equations is performed in the reference domain without taking into account the actual bearing geometry, transformations from the reference domain to each element must be identified for the partial derivatives and the volume and area

differentials. This is achieved by first expressing the physical coordinates as functions of the reference coordinates, i.e. $x=x(x_*,y_*,z_*)$, $y=y(x_*,y_*,z_*)$ and $z=z(x_*,y_*,z_*)$, and calculating the corresponding Jacobian matrix J. The transformation of the partial derivatives from the reference domain to an element (i.e., $\partial/\partial x_*$, $\partial/\partial y_*$ and $\partial/\partial z_*$ to $\partial/\partial x$, $\partial/\partial y$ and $\partial/\partial z$) is then obtained by noting that $[\partial/\partial x,\partial/\partial y,\partial/\partial z]^{\mathrm{T}}=(J^{\mathrm{T}})^{-1}[\partial/\partial x_*,\partial/\partial y_*,\partial/\partial z_*]^{\mathrm{T}}$. To transform the volume and area differentials (i.e., $d\Omega_*$ to $d\Omega$ and dS_* to dS), the determinants of their corresponding Jacobian matrices are used according to the rules of transformations of variables. In particular, the differentials are equal to $d\Omega=|J|d\Omega_*$ and $dS=|J_{\mathrm{area}}|dS_*$, where $|\ |$ denotes the matrix determinant and J_{area} is the Jacobian matrix that depends on the boundary being considered. For example, when transforming the differential dS_* of the top boundary, the two coordinates x_* and y_* can vary while the third coordinate z_* equals to 1 and hence the Jacobian matrix is

$$J_{\mathrm{area}}=\begin{bmatrix}\dfrac{\partial x}{\partial x_*} & \dfrac{\partial x}{\partial y_*}\\[2ex] \dfrac{\partial y}{\partial x_*} & \dfrac{\partial y}{\partial y_*}\end{bmatrix}. \tag{6}$$

The Jacobian matrices for the other five boundaries (i.e., bottom, left, right, near and far) are found in a similar way. Once the transformations of the partial derivatives and the differentials have been identified, the discretised equation Eq. (1a) can be expressed in physical coordinates, i.e.

$$\begin{aligned}&\iiint_{\Omega_*}\left[\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial x}+\hat{v}\frac{\partial\hat{u}}{\partial y}+\hat{w}\frac{\partial\hat{u}}{\partial z}\right)+\frac{\partial\hat{p}}{\partial x}\right]l_{rst}\,d\Omega+\iiint_{\Omega_*}\mu\left(\frac{\partial\hat{u}}{\partial x}\frac{\partial l_{rst}}{\partial x}+\frac{\partial\hat{u}}{\partial y}\frac{\partial l_{rst}}{\partial y}+\frac{\partial\hat{u}}{\partial z}\frac{\partial l_{rst}}{\partial z}\right)d\Omega\\&-\iint_{S_*}\mu\left(n_x\frac{\partial\hat{u}}{\partial x}+n_y\frac{\partial\hat{u}}{\partial y}+n_z\frac{\partial\hat{u}}{\partial z}\right)l_{rst}\,dS=0,\end{aligned} \tag{7}$$

where $[n_x,n_y,n_z]$ denotes the unit normal vector pointing outward from the physical domain. We note that all integrands and differentials in Eq. (7) are now functions of the reference coordinates and thus the use of Ω_* and its boundary S_* as regions of integration is correct.

System of nonlinear algebraic equations

To produce the system of nonlinear algebraic equations, the integrals in Eq. (7) must first be evaluated analytically or approximated numerically. In this paper, the integrals are approximated numerically using the GLL quadrature formula [9]. The formula uses the physical points $x_{kmn}=x(x_{*,k},\ y_{*,m},\ z_{*,n})$, $y_{kmn}=y(x_{*,k},\ y_{*,m},\ z_{*,n})$ and $z_{kmn}=z(x_{*,k},\ y_{*,m},\ z_{*,n})$ and replaces each integral with a weighted sum of the integrand values at these points. The weights α in the sum are taken from the GLL quadrature. For example, the approximation of the first term in Eq. (7) is given by:

$$\iiint_{\Omega_*}\left[\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial x}+\hat{v}\frac{\partial\hat{u}}{\partial y}+\hat{w}\frac{\partial\hat{u}}{\partial z}\right)+\frac{\partial\hat{p}}{\partial x}\right]l_{rst}\,d\Omega\approx\alpha_{rst}\left[\rho\left(u_{rst}\frac{\partial\hat{u}}{\partial x}+v_{rst}\frac{\partial\hat{u}}{\partial y}+w_{rst}\frac{\partial\hat{u}}{\partial w}\right)+\frac{\partial\hat{p}}{\partial x}\right], \quad (8)$$

where the partial derivatives are evaluated at the point $(x_{rst},y_{rst},z_{rst})$ and α_{rst} denotes the GLL weight at that point. The other terms are approximated similarly. As a result, an algebraic equation is produced for each combination of the indices r, s and t. Using vector notation for the approximated velocity function in the x direction $\mathbf{u}=[u_{111},\ldots,u_{kmn},\ldots,u_{KMN}]$ and similarly for other velocities $\mathbf{v}$ and $\mathbf{w}$, and pressure $\mathbf{p}$, the algebraic equation obtained for particular values of r, s and t can be written as $F_{rst}(\boldsymbol{u},\boldsymbol{v},\boldsymbol{w},\boldsymbol{p})=0$. Combining the equations for all indices yields the system of algebraic equations

$$\mathbf{F}(\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{p})=\begin{bmatrix}F_{111}(\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{p})\\ \vdots\\ F_{rst}(\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{p})\\ \vdots\\ F_{KMN}(\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{p})\end{bmatrix}=\mathbf{0}. \quad (9)$$

At this step, the values of the pressure and velocities taken from the boundary conditions are substituted in Eq. (9) and then the corresponding rows of **F** and elements of **u**, **v**, **w**, and **p** are ignored.

The system in Eq. (9) is produced from the discretization of Eq. (1a) and the transformation of the reference domain to a single element. Similar systems are produced when the reference domain is transformed to other elements. In the case where a point in physical domain is shared between two or more elements, the corresponding rows in the associated systems are replaced with their sum. For example, if the transformations of the reference points ($x_{*,N}$, $y_{*,1}$, $z_{*,1}$) in one and ($x_{*,1}$, $y_{*,1}$, $z_{*,1}$) in another element give the same point in physical domain, then the rows F_{N11} and F_{111} in the algebraic systems produced for the first and second element, respectively are replaced with $F_{N11}+F_{111}$. In this way, the total number of unknowns is equal to the total number of rows in the algebraic system.

2.1.2. Iterative solution of the system

Throughout this section, it is assumed that the algebraic systems produced from all the governing equations (1a-d) and for all the elements are assembled in a single system $\mathbf{G}(\mathbf{s})=\mathbf{0}$, where **s** denotes the combined solution of the velocity and pressure. Since the system is nonlinear, the Newton's method can be used to solve it as follows

$$\mathbf{s}^{(i+1)} = \mathbf{s}^{(i)} - (\partial\mathbf{G} / \partial\mathbf{s})^{-1}\mathbf{G}, \tag{10}$$

where *i* denotes the Newton iteration and $\partial\mathbf{G}/\partial\mathbf{s}$ and **G** are the Jacobian matrix and the system calculated at the current solution $\mathbf{s}^{(i)}$. Using the notation $\Delta\mathbf{s}^{(i)} = \mathbf{s}^{(i+1)} - \mathbf{s}^{(i)}$, $\mathbf{A} = \partial\mathbf{G} / \partial\mathbf{s}$ and $\mathbf{b} = -\mathbf{G}$, Eq. (10) can be formulated as a linear system

$$\mathbf{A}\Delta\mathbf{s}^{(i)} = \mathbf{b}. \tag{11}$$

Since the matrix **A** obtained from the SE discretization is typically dense, non-symmetric and ill-conditioned [9], the solution of the linear system in Eq. (11) is more difficult to obtain than when using, for example, the finite element or finite volume methods. This problem is often addressed using a domain-decomposition method and an iterative linear solver [9]. In this work, we use a specifically designed Schur complement of matrix **A** and the preconditioned generalized minimal residual (GMRES) method. In particular, we first decompose the solution vector as $\Delta\mathbf{s}^{(i)} = \left[\Delta\mathbf{s}_E^{(i)} \quad \Delta\mathbf{s}_I^{(i)}\right]^{\mathrm{T}}$, where the index *E* corresponds to the velocities shared between the spectral elements and the pressure, and the index *I* corresponds to the velocities inside the elements. Using the Schur complement method, the system in Eq. (11) can be decomposed into two smaller systems

$$\begin{bmatrix} \mathbf{A}_{EE} & \mathbf{A}_{EI} \\ \mathbf{A}_{IE} & \mathbf{A}_{II} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{s}_E^{(i)} \\ \Delta\mathbf{s}_I^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_E \\ \mathbf{b}_I \end{bmatrix}, \tag{12}$$

and then solved by sequentially solving the systems as follows

$$\begin{aligned} &\left[\mathbf{A}_{II} - \mathbf{A}_{IE}\mathbf{A}_{EE}^{-1}\mathbf{A}_{EI}\right]\Delta\mathbf{s}_I^{(i)} = \mathbf{b}_I - \mathbf{A}_{IE}\mathbf{A}_{EE}^{-1}\mathbf{b}_E, \\ &\mathbf{A}_{EE}\Delta\mathbf{s}_E^{(i)} = \mathbf{b}_E - \mathbf{A}_{EI}\Delta\mathbf{s}_I^{(i)}. \end{aligned} \tag{13}$$

We note that the matrix $\mathbf{A}_{EE}$ is a block matrix, where each block corresponds to a single element with well-defined boundary conditions. This allows for solving each block directly, accelerating the calculation of the Schur matrix $\mathbf{A}_{II} - \mathbf{A}_{IE}\mathbf{A}_{EE}^{-1}\mathbf{A}_{EI}$ in the top system and the solution of bottom system in Eq. (13). The top system is then solved iteratively using the GMRES method. To accelerate convergence of the GMRES, a coarse preconditioner is employed [9]. In this way, instead of solving one large linear system for the entire domain, the pressure and velocities are found by solving a number of smaller systems. The system **G** is considered to be solved at the *i*-th iteration if the relative residual is $\mathbf{G}(\mathbf{0})/\mathbf{G}(\mathbf{s})<10^{-8}$. Once the solution is found, the negative pressure values are set to zero and the load-carrying capacity W^+ in Eq. (2) is numerically approximated using the GLL quadrature formula.

3. Numerical simulations

The spectral solver was first validated against fluid dynamics software ANSYS CFX 14. The solver was then used to simulate finite hydrodynamic bearings with all possible combinations of 15 dimple arrangements, two Reynolds numbers and three ratios of dimple length to dimple depth. For all the simulations, the degrees of the Lagrange polynomials used in the spectral solver were set to 8 in the x and y directions and to 6-10 in the z direction. For the 5×5 grid of cells, this resulted in the number of nodes ranging from 43680 (one textured cell) to 114720 (25 textured cells).

3.1. Simulations of finite hydrodynamic bearings

For the simulations, 15 dimple arrangements were used (Fig. 4), i.e. all possible combinations of the numbers of textured cells T_x=1, 2, 3, 4, 5 and T_y=1, 3, 5. The arrangement of dimples in the y direction was always symmetrical with respect to the mid-line of the bearing. The Reynolds number $Re=\rho U h_{\min}/\mu$ and the ratio of the dimple length and depth $D=L_{\text{dimple}}/h_{\text{dimple}}$ took values of 10, 50 or 100 and 1 or 50 respectively. The values of Re and D were obtained by adjusting the bearing length and width L, sliding speed U and viscosity μ. The fluid density, minimum film thickness, and the ratio of dimple height and the minimum film thickness were fixed at ρ=850 kg/m^3, $h_{\min}$=10 μm and $h_{\text{dimple}}/h_{\min}$=1. Similar values were used in other studies [6].

3.2. *Validation*

For the validation, we compared the load-carrying capacities calculated for a textured bearing and inertial flow using the spectral solver and ANSYS CFX. A single cell textured with a spherical dimple defined in section 2.1.1 and the highest Reynolds number Re=50 and the

lowest ratio D=10 were used. Previous studies showed that the changes in pressure and velocity due to inertia are located at the dimple and they are the strongest for high Reynolds numbers and low ratios of the dimple length to its depth [6,7,8,11]. Thus, the use of a single cell configuration with Re=50 and D=10 was deemed sufficient to validate the spectral solver. A mesh convergence was conducted for the ANSYS CFX until the difference in the load-carrying capacity between two consecutive solutions was less than 0.5%. Figure 5a shows that this was achieved for 41,996 nodes. Pressure distributions at the dimple mid-line in the y direction calculated using the spectral solver (5,920 nodes) and ANSYS CFX (41,996) are shown in Fig. 5b. A good match between the distributions was obtained with less than 4% difference in the load-carrying capacity. It can be seen from Fig. 5a that the gain in using the spectral solver is in the reduced number of nodes. ANSYS CFX would require roughly 15,000 nodes to achieve the same accuracy as the spectral solver (~6,000 nodes).

4. Results and discussion

The effects of inertia were measured by relative differences $\Delta W^+ = (W^+_{\mathrm{NSE}} - W^+_{\mathrm{SE}}) / W^+_{\mathrm{SE}}$ between the load-carrying capacities W^+ calculated from the solutions of the NSE and SE. For each combination of the spatial arrangement of dimples, the ratio D and the Reynolds number Re, a bar plot was constructed (Fig. 6).

Spatial arrangement of dimples

As can be seen in Fig. 6, the spatial arrangement of dimples affected ΔW^+, except when $D \geq 50$ and Re=1. This effect was particularly pronounced between the full (T_x=5) and partial ($T_x \leq 4$) texturing in the x direction. For example, for D=10 and Re=50, the value of ΔW^+ decreased by ~85% when T_x changed from 5 to 4 or less. Once partial texturing was employed ($T_x \leq 4$), the inertia effect was almost independent of T_x. This could be explained by the fact that the

increase in hydrodynamic effect, observed for partial texturing as compared to full texturing [11], can overshadow the inertia effect. For the full texturing (T_x=5), the values of ΔW^+ were in a good agreement with those published for single-dimple configurations [6]. The fact that for partial texturing ($T_x \leq 4$) they were substantially greater indicates that the results obtained for 3-D bearings textured with a single dimple should not be generalized to realistic finite bearings with multiple dimples and partial texturing.

For most of the combinations, an increase in ΔW^+ was observed for a decreasing number T_y of textured cells in the *y* direction. For T_x=5 and $T_y \leq 3$, however, the increase in ΔW^+ was small. These results show that the change from the full (T_x=5) to partial texturing ($T_x \leq 4$) in the sliding has the strongest impact on the inertia effect.

Ratio of dimple length and dimple depth

Previous studies showed that for bearings textured with a single dimple the value of ΔW^+ depends on *D* [6]. Our results indicate that this dependence holds for finite bearings and the difference ΔW^+ decreases approximately proportionately with the increase of *D*. For the typical value of *D*=50 [6] and partially textured bearings, ΔW^+ is less than 5% (Fig. 6, middle row). For the fully textured bearings, the difference ΔW^+ reached 10% and 16% for T_y=3, 5 and T_y=1 respectively. This demonstrates that the partial texturing with high values of the ratio *D* can significantly reduce the effect of inertia.

Reynolds number

The value of ΔW^+ increases with the Reynolds number; up to 50 times between *Re*=1 and *Re*=50 for $D \leq 50$ and T_x=5 (Fig. 6, top and middle rows). However, the effect was substantially reduced when the partial texturing was employed. For example, for *D*=50 an

increase in *Re* resulted in 10–15% higher values of ΔW^+ for T_x=5 but only 3% higher for $T_x \leq 4$. This result indicates that for high Reynolds numbers (≥50), the spatial arrangement of dimples has to be included in the study of inertial flows in finite bearings with surface texturing.

Physical explanation

The above results show that the spatial arrangement of dimples is a major factor determining the effect of inertia on the load-carrying capacity in hydrodynamic bearings. The effect was most pronounced for bearings with full texturing. Previous study on purely viscous flow showed that in fully textured bearings with parallel surfaces the hydrodynamic effect is virtually non-existent [11]. Consequently, it can be expected that changes in the pressure distribution due to inertia and perhaps also other effects would be clearly manifested. On the contrary, in partially textured bearings where the dimples essentially imitate the geometry of a step bearing, the hydrodynamic effect can be substantial [11], thus overshadowing the inertia effect.

5. Conclusions

In this work, a spectral element solver has been developed for solving the NSE in finite hydrodynamic bearings textured with multiple spherical dimples. The solver decomposes the bearing domain into a number of pre-defined elements and solves the discretised NSE in each element using the Schur and GMRES methods, significantly accelerating the calculations. This allowed for studying the inertia effect in finite bearings with complex geometries.

The study was carried out for 90 different combinations of the spatial arrangement of dimples, the shapes of individual dimple and the Reynolds number. The results obtained

show that the inertia effect increases with increasing Reynolds number and decreasing ratio of dimple length to dimple depth. The results also revealed that the spatial arrangement of dimples has a major impact on the inertia effect. Specifically, the effect decreases with a change from full to partial texturing. For example, an 85% decrease was observed for the Reynolds number 50 and the ratio of dimple length to dimple depth equal to 10. Based on the results obtained we conclude that the error in using the Reynolds equation for inertial flows remains small as long as the ratio of dimple length to dimple depth is large (D>10) and partial texturing is employed.

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