Bandwidth allocation strategy for traffic systems of scale-free network

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Abstract

In this paper, the bandwidth resource allocation strategy is considered for traffic systems of complex networks. with a finite resource of bandwidth, an allocation strategy with preference parameter α is proposed considering the links importance. The performance of bandwidth allocation strategy is studied for the local routing protocol and the shortest path protocol. When important links are slightly favored in the bandwidth allocation, the system can achieve the optimal traffic performance for the two routing protocols. For the shortest path protocol, we also give a method to estimate the network traffic capacity theoretically.

Key words: bandwidth allocation, traffic flow, complex network
PACS: 89.75.Hc, 45.70.Vn, 05.70.Fh

1 Introduction

Dynamical properties of network traffic have attracted much devotion from physical and engineering communities. The prototypes studied include the transfer of packets in the Internet, the flying of airplanes between airports, the motion of vehicles in urban network, the migration of carbon in bio-systems, and so on. Since the discovery of Small-World phenomenon [1], and Scale-Free

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property [2], it is widely proved that the topology and degree distribution of networks have profound effects on the processes taking place on these networks, including traffic flow [3–6].

In the past few years, the phase transition phenomena [7–11], the scaling of traffic fluctuations [12–15] and the routing strategies [16–36] of traffic dynamics have been widely studied. However, the link's bandwidth are usually assumed to be infinity in most studies of network traffic properties and routing strategies. On the contrary, many real transportation and communication systems have different bandwidth for links. Meanwhile, with the same routing protocol, the network capacity can be very different when the links are with different bandwidth [37]. To alleviate traffic congestion, people usually increase links' bandwidth or router's capacity. Therefore, the following question should be important: with a finite total resource of bandwidth, how to allocate the bandwidth so that the network capacity can be maximal?

In this paper, we will investigate this question for traffic flow of complex networks. We consider two types of routing protocols: the local routing protocol and the shortest path routing protocol. For a given resource of bandwidth, we introduce an allocation strategy based on the product of end nodes' degree of links: $p_{ij} = k_i \times k_j$. By simulation and analytical estimation, we show that the network's traffic capacity can be optimized when the links with bigger p_ij are with slightly more bandwidth.

The paper is organized as follows. In the following section, the network model, traffic model and bandwidth allocation strategy are introduced. Section 3 gives the simulation results and some theoretical estimation of network capacity using the proposed bandwidth allocation strategy. The paper is concluded in the last section.

2 The Models

Recent studies indicate that many communication systems such as the Internet and the World Wide Web are not homogeneous as random or regular networks, but heterogeneous with a degree distribution following the power-law distribution $P(k) = k^{-\gamma}$. The Barabási-Albert (BA) model [2] is a well-known model which can generate networks with a power-law degree distribution. Without loss of generality, we construct the underlying network structure with the BA network model. In this model, starting from m_0 fully connected nodes, a new node with m edges $m \leq m_0$ is added to the existing graph at each time step according to preferential attachment, i.e., the probability $\prod_i$ of being connected to the existing node i is proportional to the degree k_i of the node.

The bandwidth D_{ij} of link l_{ij} is the maximal number of packets that can pass from node i to node j through the link per time step. For the convenience of theoretical estimation, this paper considers traffic flow on directed networks so that D_{ij} may be different from D_{ji}. Previous studies usually consider that the link's bandwidth is infinite [16–35]. This is not in consistence with real situations. In fact, link bandwidth often restricts the performance of the networked system. The packets can not be forwarded smoothly through the links so that they accumulate in the node's queue. This mechanism causes the network system to congest. With a finite resource of bandwidth, a good allocation strategy for the resource is important.

Here we propose a conceptual bandwidth resource allocation strategy and study its effects on the local routing protocol and the shortest path protocol. The bandwidth allocation strategy can be formulated as:

$$D_{ij} = \frac{(k_i k_j)^\alpha}{\sum_{i=1,j=1,i\neq j}^{N} A_{ij}(k_i k_j)^\alpha} D, \tag{1}$$

where A_{ij} is adjacency matrix of network, k_i and k_j are the degrees of node i and node j respectively, α is a tunable parameter, N is the size of network, and D is the total resource of bandwidth. We use the product of end nodes' degrees of a given link ($p_{ij} = k_i \times k_j$) to denote the importance of the link. When $\alpha > 0$, the links with bigger p_{ij} will have larger bandwidth, and vise versa. When $\alpha = 0$, Eq.(1) reproduces the model that all links have the same bandwidth. The total resource of bandwidth should be calculated as $D = \sum_{i=1,j=1,i\neq j}^{N} A_{ij} D_{ij}$. If all links have the same bandwidth of 1.0, the total resource will be $D = \langle k \rangle N$, with $\langle k \rangle$ as the network's average degree of nodes. Without loss of generality, we set $D = \langle k \rangle N$ in the following study.

The traffic model is described as follows. At each time step, there are R packets generated in the system, with randomly chosen sources and destinations. One node i can deliver at most C_i packets to its neighboring nodes, and C_i depends on the sum of link bandwidths connecting to its neighbors: $C_i = \Sigma_{j=1}^{N} A_{ij} D_{ij}$. Once a packet arrives at its destination, the packet will be removed from the system. The queue length of each node is assumed to be unlimited and the first in first out rule (FIFO) is applied to all queues. In this paper, we investigated two routing protocols: the shortest path protocol and the local routing protocol. In the local routing protocol, the packets are delivered by the local information of link bandwidth. In each time step, each node performs a local search among its neighbors. If a packet's destination is found within the searched area, it is delivered directly to its target. Otherwise, the packet is delivered to a neighboring node j with probability depending on the corresponding link's bandwidth:

$$P_{i\to j} = \frac{D_{ij}}{\Sigma_{j=1}^{N} A_{ij} D_{ij}} \tag{2}$$

where D_{ij} is the bandwidth of the link pointing from node i to node j, the sum runs over all neighbors of node i. This local routing protocol reflects that the packets are forwarded according to the links' bandwidth.

3 Simulation and analysis

The simulations are carried out on Barabási-Albert (BA) scale-free networks with size $N = 1000$ and parameters $m_0 = m = 4$. In order to describe the phase transition of traffic flow in the network, we use the order parameter [8]:

$$\eta(R) = \lim_{t\to\infty} \frac{\langle \Delta N_p \rangle}{R\Delta t} \tag{3}$$

where $\Delta N_p = N_p(t+\Delta t) - N_p(t)$, $\langle ... \rangle$ indicates the average over time windows of width Δt, and $N_p(t)$ is the number of data packets within the network at time t. With increasing packet generation rate R, there will be a critical value of R_c which characterizes the phase transition from free flow ($\eta = 0$) to congestion ($\eta > 0$). In the free flow state ($R < R_c$), due to the balance of created and removed packets, $\langle \Delta N_p \rangle = 0$ and $\eta = 0$. When $R > R_c$, the packets accumulate continuously in the network, and η becomes a constant larger than zero. Because the packets are congested, they will pile up in the network and the system will collapse ultimately. Therefore R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. The network's overall capacity can be measured by the maximal generating rate R_c at the phase transition point.

We first investigate the bandwidth allocation strategy Equ.(1) for the local routing protocol. Figure 1(a) shows the evolution of total packet number N_p for different packet generating rate R with network size $N = 1000$, average degree $\langle k \rangle = 8$ and routing preference $\alpha = 0$. When $R \leq R_c = 67$, the evolution of N_p will reach a balance state, while N_p will increase without bound when $R = 68 > R_c$. Figure 1(b) shows the order parameter η vs R with $\alpha = 0$. One can see that $\eta = 0$ for $R \leq 67$ and $\eta > 0$ for $R > 67$. So the network capacity is $R_c = 67$ under this bandwidth allocation strategy of $\alpha = 0$. We also show the scaling of R_c with network size N in fig.1(c).

In Fig.2, we investigate the network's capacity R_c for different values of α. The network's capacity R_c reaches maximum when $\alpha = 0.5$. This indicates that one should allocate more bandwidth for the links with bigger value of p_{ij}. In Fig.3(a), the evolution of hub node's traffic load is investigated with $\alpha = 0.1$, 0.5 and 0.7 respectively. The traffic load of node i is defined as L_i/C_i, with L_i denoting the queue length in node i and C_i denoting the delivery capacity of node i. Here, the hub node is defined as the node having the largest traffic load. One can see that, when $\alpha = 0.1$ and 0.7, the hub node is congested that

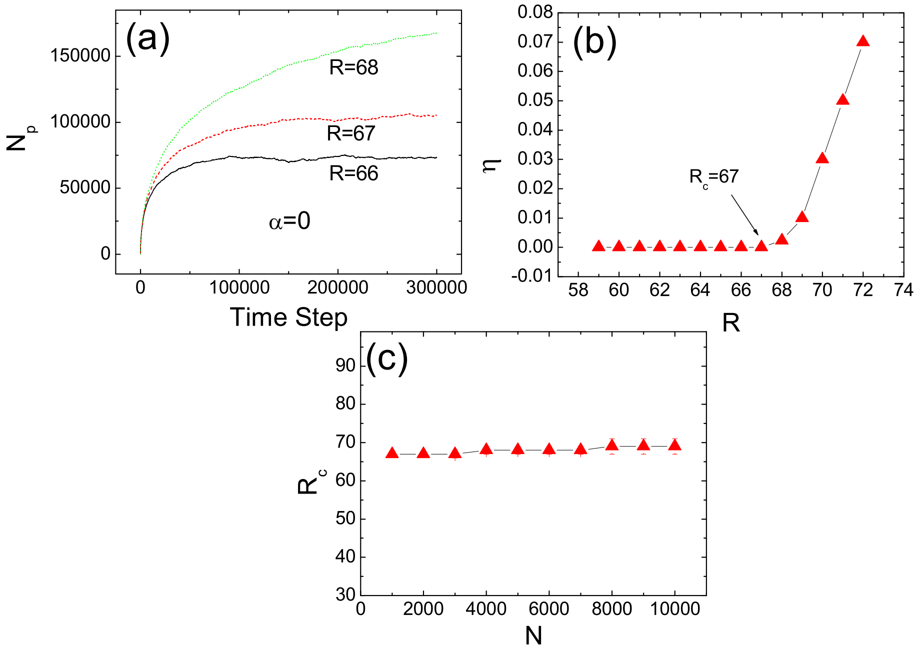

Fig. 1. (Color online.) (a) Evolution of packet number N_p with different packet generation rate R; (b) Order parameter η vs R; (c) R_c vs N. Other parameters are network size $N = 1000$, average degree $\langle k \rangle = 8$, bandwidth resource $D = 8000$, and routing preference $\alpha = 0$.

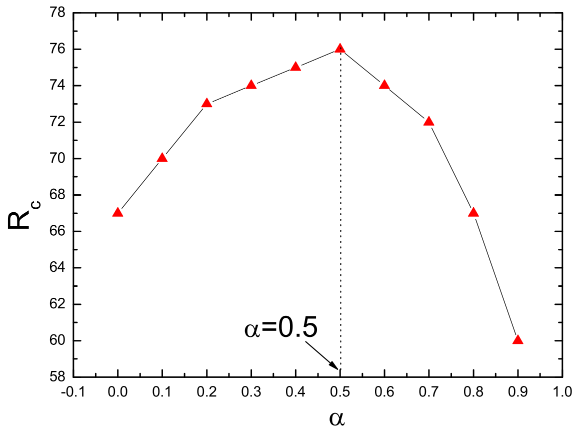

Fig. 2. (Color online.) The network capacity R_c vs the tunable parameter α. As is shown, the network capacity R_c can reach the optimum when $\alpha = 0.5$. Network size is $N = 1000$, average degree is $\langle k \rangle = 8$, and total bandwidth resource is $D = 8000$.

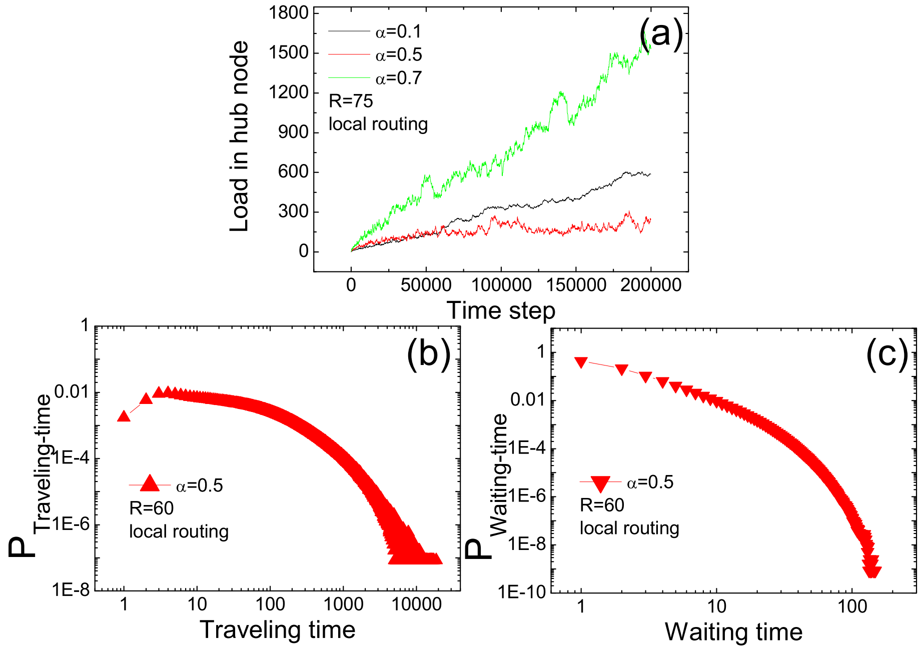

Fig. 3. (Color online.) (a) Evolution of hub node's traffic load for $\alpha = 0.1$, 0.5 and 0.7 with $R = 75$; (b) The probability distribution of packet's travelling time in free-flow state ($R = 60 < R_c$) with $\alpha = 0.5$; (c) The probability distribution of packets' waiting time in free-flow state ($R = 60 < R_c$) with $\alpha = 0.5$. Other parameters are network size $N = 1000$, average degree $\langle k \rangle = 8$, and total resource of bandwidth $D = 8000$.

traffic load increases without bound. However, when $\alpha = 0.5$ the traffic load of hub node can reach a balance state, indicating that it is still in free-flow state. Thus the bandwidth allocation with $\alpha = 0.5$ can achieve a larger network capacity.

Then we investigate the probability distribution of the travelling time and the waiting time in the free-flow state. Packet's travelling time and waiting time are important factors for characterizing the network's behavior. The travelling time is the time that a packet spends travelling from source to destination, and the waiting time is the time that a packet waits in the queue of a node. Figure 3(b) shows that the distribution of travelling time roughly follows a power law. Most packets can arrive at their destinations in a short time while some packets need to spend very long time. Figure 3(c) shows that the waiting time also roughly follows a power law.

Next, we investigate the bandwidth allocation strategy Eq.(1) for the shortest path routing protocol. For the shortest path protocol, we can calculate the node's or link's betweenness (B_i or B_{ij}), which is the number of shortest paths passing through node i or link l_{ij}. If every node in the system has the same delivering capacity and every link has an infinite bandwidth, traffic congestion will firstly appear on the node with largest betweenness. So the network's traffic capacity R_c can be estimated as [22]:

$$R_c = \frac{N(N-1)}{B_{max}}, \tag{4}$$

where B_{max} is the maximum node's betweenness in the system, and $N(N-1)$ is the total number of paths between nodes in the system. In the following, we propose a similar method to estimate the network's capacity with variation of link bandwidth. Considering the bandwidth allocation strategy of Equ.(1), we introduce an efficient betweenness B_{ij}^{eff} for link l_{ij}:

$$B_{ij}^{eff} = \frac{B_{ij}}{D_{ij}} \tag{5}$$

For the shortest path routing strategy, traffic congestion will firstly occur on the link with the largest efficient betweenness. Therefore, following Eq.(4), R_c can be estimated as:

$$R_c = \frac{N(N-1)}{B_{max}^{eff}} \tag{6}$$

where B_{max}^{eff} is the largest efficient betweenness of links.

Figure 4 shows the simulation results and theoretical estimation of network capacity R_c vs the bandwidth allocation parameter α under the shortest path protocol. The theoretical estimation results are calculated by Eq.(6). One can see that the theoretical results are in good agreement with simulation. When $\alpha = 0.35$, the network capacity R_c reaches the optimum.

For the shortest path protocol, if the efficient betweenness of link l_{ij} (pointing from node i towards node j) is the largest in the system, the packets will firstly accumulate in node i and node i should be considered as the hub node. In Fig.5(a), the evolution of traffic load on hub node is shown with different values of α. With packet generating rate $R = 900$ and total bandwidth $D = 8000$, the traffic load can reach a balance state when $\alpha = 0.35$, while it increases without bound with other parameters. So the bandwidth allocation strategy with $\alpha = 0.35$ can reduce the traffic load on the hub node and thus increase the network capacity. Figure 5(b) and (c) shows the the probability distribution of packets' travelling time and waiting time respectively in a free-flow state ($R = 600 < Rc$) with $\alpha = 0.35$. One can see that the probability of travelling time approximately follows a Poisson distribution. This behavior is quite different from that of local routing protocol. The probability of packets' waiting time

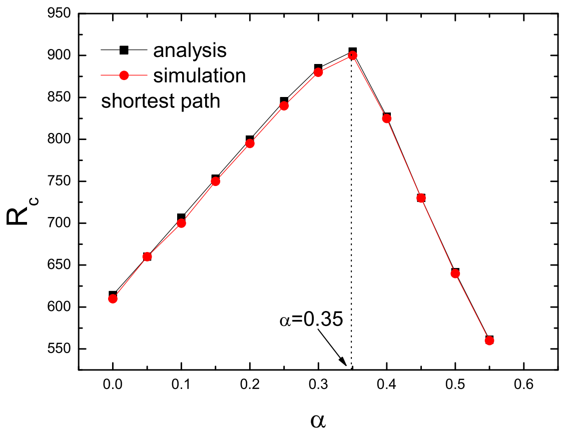

Fig. 4. (Color online.) Simulation and theoretical estimation of network capacity R_c vs tunable parameter α for the shortest path routing protocol. Network capacity R_c reaches optimum when $\alpha = 0.35$. Network size $N = 1000$, average degree $\langle k \rangle = 8$, and total resource of bandwidth is $D = 8000$.

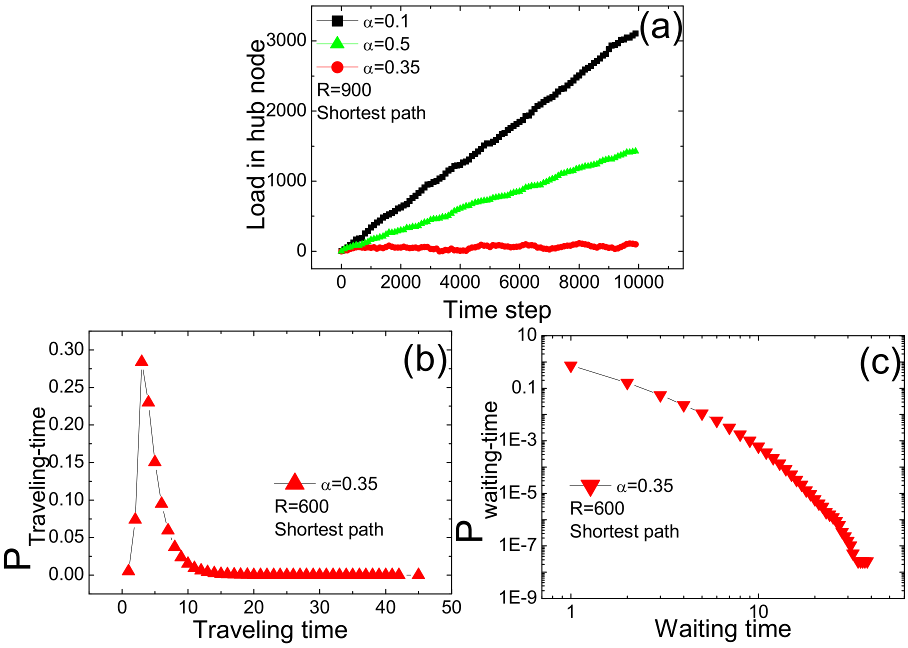

Fig. 5. (Color online.) (a) The evolution of traffic load in the hub node with different α with the shortest path protocol; (b) The probability distribution of travelling time in the free state with $\alpha = 0.35$; (c) The probability distribution of waiting time in free state with $\alpha = 0.35$. Other parameters are total resource of bandwidth $D = 8000$, network size $N = 1000$ and average degree $\langle k \rangle = 8$.

follows a pow-law distribution. But the maximum values of both travelling time and waiting time are much less than those of local routing protocol. Although the local routing protocol can forward the packets without knowing the whole system's topological information, its routing cost is much higher than that of shortest path protocol.

4 Conclusion

In summary, we have investigated the link's bandwidth allocation strategy for the local routing and the shortest path protocols. It is found that with a finite resource of link's bandwidth, one should slightly allocate more bandwidth resource to the links with bigger value of $k_i \times k_j$. By simulation, the optimal value of bandwidth allocation preference parameter α is sought out: $\alpha = 0.5$ for the local routing protocol, and $\alpha = 0.35$ for the shortest path protocol. For the shortest path protocol, we introduce a value of efficient betweenness for the links. Considering that traffic congestion will appear on the link with the largest efficient betweenness, we can estimate the network's capacity theoretically. The estimation results are in good agreement with the simulations.

This investigation may be helpful for designing realistic communication network like the Internet, the urban transportation systems, the wireless sensor network, the airway network, and so on.

Acknowledgements

This work is funded by National Basic Research Program of China (No.2006CB705500), the NNSFC Project No. 10872194, the Fundamental Research Funds for the Central Universities, the Australian Endeavour Cheung Kung Fellowship, and the Innovation Foundation for Graduate Students of University of Science and Technology of China (No.W21103010012).

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