An Inverse Hyperbolic Sine Heteroskedastic Latent Class Panel Tobit Model: an Application to Modelling Charitable Donations

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Abstract:
We apply a latent class tobit framework to the analysis of panel data on charitable donations at the household level where the latent class aspect of the model splits households into two groups, which we subsequently interpret as “low” donators and “high” donators. The tobit part of the model explores the determinants of the amount donated by each household conditional on being in that class. We extend the standard latent class tobit panel approach to simultaneously include random effects, to allow for heteroskedasticity and to incorporate the inverse hyperbolic sine (IHS) transformation of the dependent variable. Our findings, which are based on U.S. panel data drawn from five waves of the Panel Study of Income Dynamics, suggest two distinct classes of donators. There is a clear disparity between the probabilities of zero donations across these classes, with one class dominated by the observed zero givers and associated with relatively low levels of predicted giving. We find clear evidence of both heteroskedasticity and random effects. In addition, all IHS parameters were significantly different from zero and different across classes. In combination, these findings endorse the importance of our three modelling extensions and suggest that treating the population as a single homogeneous group of donors, as is common in the existing literature, may lead to biased parameter estimates and erroneous policy inference. Although we use this model to explain charitable donations, we note that it has wide applicability for researchers across the social sciences.

Key Words: Charity; Donations; Latent Class; Panel Data; Tobit.

JEL Classification: D19; C23; C24

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I. Introduction and Background

Recent figures from Giving USA 2011 estimate total charitable contributions in the U.S. in 2011 at $290.89 billion, which relates to total charitable contributions from U.S. individuals, corporations and foundations and includes both cash and in-kind donations. Given the economic significance of such donations, it is not surprising that an extensive empirical and theoretical literature exists exploring why individuals make contributions to charity, with much of the existing research focusing on charitable donations at the individual and household level in the U.S. (see, for example, Andreoni, 2006).¹

The statistical methodology used to analyse charitable donations has increased in sophistication since the early studies, which typically adopted a simple log-linear approach to analyse the amount of donations. Reece (1979) made an early methodological contribution by applying the tobit model to the analysis of cross-section data on the amount of household donations accounting for the fact that donations are censored at zero: a significant proportion of individuals and households do not make charitable donations.² The tobit approach has been adopted by a number of empirical studies of charitable donations including Kingma (1989), Auten and Joulfaian (1996) and, more recently, Brown et al. (2012).³ However, a fundamental problem with the tobit approach, relating to the treatment of the censored observations, lies in the possibility that the decision to donate and the decision regarding how much to donate,

¹ It should be acknowledged that the implications of charitable behaviour have also been analysed at the country level. For example, Elgin et al. (2013) analyse how religion motivates individuals to engage in charitable giving and this leads them to prefer making their contributions privately and voluntarily rather than through the state, with religiosity resulting in lower levels of taxes and hence lower levels of spending on both public goods and redistribution.
² The tobit approach has been used in a very wide range of applications characterised by truncated observations: for recent examples, see Addessi et al. (2014) in the context of innovation activity, Al-Malkawi et al. (2014) in the context of dividend smoothing and Chen et al. (2014), who analyse the intellectual capital and productivity of insurers.
³ In contrast to the current paper, the focus of Brown at al. (2012) lies in analysing cross-section data from the 2005 US Panel Study of Income Dynamics to explore the relationship between donations to the victims of the 2004 Indian Ocean tsunami disaster and other charitable donations, i.e. to further our understanding of the relationship between donations made to different causes.

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may be characterised by different influences. As a consequence, the double-hurdle model has also found favour in the existing literature (see, for example, Yen et al., 1997); this approach allows covariates to have different effects on the probability of making a donation and the level of donation. Thus, one interpretation of the double-hurdle approach is that it is based on the premise that a significant proportion of households, the “non-participants”, will never donate, which we argue here may not necessarily be the case. In a cross-section case, this is true by definition: the identified “non-participants” cannot donate; in a panel setting, such as ours however, it is possible to allow the participation decision to vary over time.

For example, a stark feature of our data reveals that once we consider households, as opposed to simply observations, the proportion that never donate drops dramatically with the number of times they are observed. Even over the relatively short period of time we observe households for (nine years in total, for which we have data for five), of those households observed over the full length of the panel, only 15% never donate, compared to 44% of households regardless of length of time in the panel. Clearly if we could observe all these households over a longer period of time, the logical conjecture is that this percentage would fall even further and start to approach zero. For example, even for habitual zero-observed donators, it is possible that a significant shock (such as a closely related traumatic event) will increase their propensity to donate.

Hence, it appears that a double hurdle approach may be inappropriate in this context. The latent class approach is an alternative modelling strategy which is arguably well-suited to the analysis of charitable donations, given the potential for very diverse
donating behaviour within a population. The latent class approach (probabilistically) splits the population into a set of homogeneous groups. Within each class, or group, an appropriate statistical model applies (in our case, this is based upon a tobit specification to take into account the censored nature of the data).

Such an approach is advantageous, as it simultaneously introduces heterogeneity into the empirical framework and \textit{ex post} allows for splitting of the population into various sub-groups of donating behaviour. Moreover, this approach, in splitting the population into different types of givers, explicitly allows the probability of zero donating to differ in each class, thereby leading to a richer layered characterisation of the “zero-donation” process. In essence, our suggested latent class approach will “push” some groups towards zero donations whilst “pulling” others away from it. In all situations, there remains a non-zero probability of a zero donation, which is likely to be higher in the groups pulled towards zero.

Building on the heterogeneity afforded by the latent class approach, we take advantage of the panel data available to us to account for unobserved heterogeneity that will undoubtedly drive household donating behaviour. That is, we explicitly allow for unobserved effects. Finally, as is well documented in the statistics literature (see Wooldridge, 2010, for example), estimation issues have arisen with respect to the tobit model including inconsistency in the face of both heteroskedasticity and non-normality. Therefore, we accommodate both non-normality, by employing the inverse hyperbolic sine (IHS) transformation, and heteroskedasticity, with an explicit parameterization of the disturbance variance(s).

In the existing literature, all of these extensions have been explored in isolation to each other. Our contribution is that we allow for all of these extensions within an

\footnote{This approach has been applied in a wide variety of areas ranging from consumer behaviour (see, for example, Reboussin et al., 2008, and Chung et al., 2011), to health economics (see, for example, Deb and Trivedi, 1997, and Bago d’Uva, 2005) to transport mode choice (see, for example, Shen, 2009).}
integrated statistical framework. Indeed, the joint consideration of each of latent classes, unobserved (random) effects, non-normality and heteroskedasticity, is extremely important, as if any of these are present and not accounted for, as is well-known in the literature, biased and inconsistent estimates would result (see, Wooldridge, 2010, for example). Our extended statistical framework thus augments the existing latent class model in a number of ways which are fundamentally important for its application to the analysis of panel data with a censored dependent variable.\(^5\)

Although we apply this model to explain the level of charitable donations, we note that this statistical model is potentially of wide interest to researchers right across the social sciences. Moreover, it is flexible enough to accommodate any specific nuances that are warranted by the particular modelling strategy (for example, the researcher may not have panel data to hand, so she would simply omit the relevant components of our suggested modelling approach).

II. **Statistical Framework: A Panel Latent Class Tobit Model**

Our basic hypothesis is that there are inherently more than one type of charity donators in the population; “high” givers and “low” givers is a natural partition. However, clearly these inherently different types of households will not be directly observed. Thus, the broad approach we follow here is that of “latent class” or “finite mixture” models (for a comprehensive survey of latent class models see McLachlan and Peel, 2000). Essentially such an approach assumes that the observed data are drawn from a mixture of underlying populations. In undertaking such an approach, care needs to be taken of the specific nature of our dependent variable: household charitable donations. As is common in the existing literature on charity (see Andreoni, 2006 for a thorough survey),

\(^5\) We note that one relevant existing study is Islam (2007), who considers a very restricted version of a latent class tobit model, but only allows intercepts to vary by class (in addition, classes do not vary by observed characteristics; neither heteroskedasticity nor non-normality are allowed for; and any unobserved effects in the tobit part of the model are ignored).
we treat this as a corner solution model, such that we need to employ censored regression (tobit) model techniques to take into account the quite significant amount of censoring at zero (Maddala, 1983). In our case the censoring amounts to some 40% of observations.

Thus, the general framework we adopt is a latent class tobit model. This approach amounts to first (probabilistically) splitting the sample into two, or more, samples (which, prior to estimation we envisage to correspond to “high” and “low” donators) and then, for each of these subpopulations, separate tobit models apply. In this way, the same explanatory variables in the tobit (or “amount of giving”) equation can have differing effects across the different classes.

The probabilistic splitting of the sample is usually based on a logit specification (Greene, 2012), which can be either a constant across households, or allowed to be a function of observed household and head of household characteristics, $z_i$. It is possible to allow for a theoretically large number of such latent classes. However, we restrict ourselves here to two, as any greater number of classes yields an overly parameterised model that is difficult to interpret.6 In practice the optimal number of classes is usually determined on the basis of (Akaike) information criteria, AIC (see, for example, Deb and Trivedi, 2002).

As Greene (2012) points out, the availability of panel data significantly aids in the identification of latent class models. Essentially this arises as, being time-invariant, we now have several observations $(T_i)$ on each household upon which to base class membership, as opposed to the single one in a cross-section. Following the existing literature, for example, Clark et al. (2005), Bago d’Uva and Jones (2009) and Greene (2012), we parameterise our model such that time-invariant head of household

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6 Indeed, convergence problems were encountered in the case of the three-class model, suggesting that this was the case: one, or more, of the three probabilistic points of support was degenerate.
characteristics \( z_i \) affect the probability of being in each class (with associated coefficients \( \eta \)) and the remaining head of household and household characteristics, along with any further economic variables (such as price), determine the amount of giving by the household within the class. In effect, specifying time invariant head of household characteristics in this way amounts to parameterising the household’s “fixed effect” of being in each class.

Let \( x_{it} \) be the vector of explanatory variables determining the level of donations by household \( i \) in period \( t \), and let there be \( j = 1, \ldots, J \) latent classes (in our case, \( J = 2 \)). There will be \( J \) parameter vectors \( (\beta_j, \sigma_j) \) associated with \( x_{it} \) in the different classes (where \( \sigma_j \) is the standard deviation of the error term within each class). Post-estimation, based on the estimated parameter vectors, it is possible to estimate (average) expected values of giving across the classes, and in this way to determine which classes are the “high” and “low” donators.

Conditional on class membership, which is constant over time by definition, the \( y_{it} \) observations on charity donations for household \( i \) \( (i = 1, \ldots, N) \) in period \( t \) \( (t = 1, \ldots, T_i) \) are independent – we reconsider this assumption below. For a group of \( T_i \) observations, the joint density of the sequence of \( y_{it} \) is

\[
f\left(y_{it} | class = j, X_i, \beta_j, \sigma_j \right) = \prod_{t=1}^{T_i} f\left(y_{it} | class = j, x_{it}, \beta_j, \sigma_j \right) \quad (1)
\]

where for household \( i \) in period \( t \), the density \( f\left(y_{it} | class = j, x_{it}, \beta_j, \sigma_j \right) \) is given by the tobit formulation (Maddala, 1983) and \((y_{it}, X_i)\) denotes the \( T_i \) periods of observed data on household \( i \).

The density for the \( it' \)th observation for the tobit model is derived from the latent regression,
\[ y_{it}^* \mid (\text{class} = j) = \beta_j' x_{it} + e_{itj}, \ e_{itj} \sim N(0, \sigma_j^2), \] with \[ y_{it} = y_{it}^* \] if \( y_{it}^* > 0 \) and \( y_{it} = 0 \) otherwise.

The implied density for the observed \( y_{it} \) is therefore

\[ f(y_{it} \mid \text{class} = j, x_{it}, \beta_j, \sigma_j) = \left[ \Phi \left( \frac{-\beta_j' x_{it}}{\sigma_j} \right) \right]^{1-D_{it}} \left[ \phi \left( \frac{y_{it} - \beta_j' x_{it}}{\sigma_j} \right) \right]^{D_{it}} \] (3)

where \( D_{it} \) equals 1 if \( y_{it} \) is greater than zero, and 0 otherwise, and \( \phi \) and \( \Phi \) represent the standard normal p.d.f. and c.d.f., respectively.

The log-likelihood for a panel of data on charitable donations will accordingly be

\[ \log L(\beta_j, \sigma_j, \eta_j), \ j = 1, ..., J \] \[ = \sum_{i=1}^{N} \log \left[ \sum_{j=1}^{J} p_{ij}(z_{it}, \eta_j) \prod_{t=1}^{T} f \left( y_{it} \mid \text{class} = j, x_{it}, \beta_j, \sigma_j \right) \right] \] (4)

where \( p_{ij}(z_{it}, \eta_j) \) are the logit probabilities of being in class \( j \):

\[ p_{ij}(z_{it}, \eta_j) = \frac{\exp(z_{it}' \eta_j)}{\sum_{j=1}^{J} \exp(z_{it}' \eta_j)}, \ j = 1, ..., J; \ \eta_j = 0 \] (5)

and \( \eta_j = 0 \) for identification. Note that all parameters of the model, that is, those in the logit model determining class membership and those in the multiple tobit equations, are jointly estimated (see, for example, Deb and Trivedi, 2002, for maximum likelihood estimation of latent class models). The latent class specification groups the population into two types (classes) of donors. Prior to estimation, we know nothing about which households will be in each class; and nothing about the donating behaviour within each class.

Within each class, donating behaviour follows a corner solution model, whereby each household, in each time period, chooses an optimal level of donation. For some households, in some time-periods, this choice will be zero. Moreover, this decision
process will (primarily) be driven by observed changes in the household’s economic and social environment, i.e., $x_{it}$. Thus, this optimisation process combined with a changing observed (economic and social) environment, $x_{it}$, means that the statistical model explicitly allows for households to move from zero to positive consumption from year-to-year; or from positive to zero; or from large donations to small; and so on.

Post-estimation, two estimates of the probability of being in each class are available. Prior probabilities can be obtained by simply evaluating the above expression for $p_{ij}(\eta, z_i)$. However, for prediction purposes it is more useful to look at the posterior, or conditional on the observed data, probabilities (Greene, 2012). Using Bayes Theorem, we obtain

$$P(\text{class} = j | \text{observation } i) = \frac{f(\text{observation } i | \text{class} = j) P(\text{class } j)}{\sum_{j=1}^{J} f(\text{observation } i | \text{class} = j) P(\text{class } j)}$$

$$= \frac{\prod_{t=1}^{T} f(y_{it} | x_{it}, \beta_j, \sigma_j) p_{ij}(z_i, \eta_j)}{\sum_{j=1}^{J} \prod_{t=1}^{T} f(y_{it} | x_{it}, \beta_j, \sigma_j) p_{ij}(z_i, \eta_j)}.$$  \hspace{1cm} (6)

The specification thus far can be considered a “standard” application of a latent class model where panel data are available (see Greene, 2012, for example), and the model can be estimated using standard software, such as NLOGIT/LIMDEP. We suggest three important extensions to this basic set-up that significantly increase the flexibility and robustness of this latent class approach, whilst fully taking advantage of the panel nature of the data.

**Heteroskedasticity**

As is well-known in the literature (see, Maddala, 1983, for example) if, as is likely with unit-level data, there is heteroskedasticity present in the data and this is ignored in estimation of nonlinear models (by maximum likelihood techniques), biased and
inconsistent parameter estimators will result (effectively as a result of maximising an incorrect likelihood function). The conventional assumption in the (latent class) tobit model, is that

$$E(e_{ij}^2|x_i) = \sigma_j^2.$$  \hspace{1cm} (7)

That is, that the error term in the model in (2) is orthogonal to the covariates and homoskedastic within each class. A common approach to allow for heteroskedasticity is Harvey’s (1976) model, in which the variance varies by observed characteristics $w$ with unknown weights $\delta$

$$\sigma_{ij}^2 = \sigma_j^2 \left[ \exp(w'\delta_j) \right]^2. \hspace{1cm} (8)$$

The exponential transformation ensures that the variance(s) under the assumption of heteroskedasticity ($\delta \neq 0$), is (are) both identified and positive. It is also convenient in that a test of $\delta_j = 0, j = 1, 2, \ldots$ provides a test of the heteroskedasticity model versus the homoskedastic one. Following the bulk of the censored regression literature (see, for example, Yen and Jones, 1997), the variables chosen to enter into $w$ are the household scale variables available to us (income and wealth). With this extension, $\sigma_{ij}$ in (8) replaces $\sigma_j$ in the tobit model in (2).

**Unobserved Heterogeneity**

Although the standard panel data latent class model (described above) allows one to identify the classes more strongly (as opposed to simple cross-sectional data), it is possible to exploit the panel nature of the data even further by using the within household variation throughout the window of the panel. Accordingly, we will also include unobserved time invariant common, or random, effects into the tobit parts of our model specification (the case of fixed effects in censored regression models is considered by Honoré, 1992). As is common in the panel data literature (see, for
example, Baltagi, 2005), we add to $\varepsilon_i$ into equation (2), a household (and class) varying error $u_{ij}$ such that the latent regression becomes

$$y_{ij}^* = \mathbf{x}'_i \mathbf{\beta}_j + \varepsilon_{ij} + u_{ij}.$$  

(9)

Again, as is common in the literature, these unobserved household specific effects are (initially) assumed to be orthogonal to the covariates in the model, and follow a normal distribution with mean zero and variance $\theta_j^2$. However, it is also straightforward to allow these unobserved effects to be arbitrarily correlated with covariates in the model following the usual Mundlak (1978) approach (which essentially entails entering group means of time-varying covariates by individual into the model). Note that as the two unobserved effects implicitly relate to two distinct different groups of the population, they are assumed to be independent. However, due to the presence of the common $u_{ij}$, observations on the particular household are no longer independent across periods.

The density for the observed $y_{ij}$ is now formed by first conditioning on the unobserved heterogeneity. It is useful to write $u_i = \theta v_i$ where $\theta^2 = \text{Var}[u_i]$ and $v_i \sim \text{N}[0,1]$. Then,

$$f(y_{ij} | \text{class} = j, \mathbf{x}_i, u_{ij}, \mathbf{\beta}_j, \sigma_j, \theta_j) = \left[ \frac{\Phi \left( -\frac{\theta y_{ij} - \mathbf{\beta}'_j \mathbf{x}_i}{\sigma_j} \right) }{\sigma_j} \right]^{1-\theta_j} \left[ \frac{1}{\sigma_j} \phi \left( \frac{y_{ij} - \mathbf{\beta}'_j \mathbf{x}_i - \theta v_{ij}}{\sigma_j} \right) \right]^{\gamma_j}$$

(10)

The density for the observed $y_{ij}$ is now formed by integrating the unobserved $v_{ij}$ out of the conditional density. We return to this point below where we obtain the log likelihood for the sample.

**Allowing for Non-Normality**

If the assumption of normality that is central in the tobit models considered thus far is invalid, the (pseudo-) maximum likelihood estimator of the parameters will be biased and inconsistent. It has been commonplace in models of charitable donations (and indeed, in related areas, such as trade flows where there is a preponderance of zero
observations; see, for example, Harris et al., 2012); to model the natural logarithm of (one plus) the actual level of donations (see, for example, Yen, 2002). Although often not explicitly stated, this is presumably so that the resulting distribution of charitable donations is more nearly normally distributed. However, it is not clear that a zero in the logarithmic scale is equivalent to the same in the untransformed scale; and moreover, the addition of one is simply an arbitrarily chosen number to ensure that the log transformation is defined for all households.\(^7\)

A recently used parametric approach to deal with this issue of non-normality that originates with Burbidge et al. (1988), is to use the inverse hyperbolic sine (IHS) transformation of the dependent variable. The IHS transformation, \(I(y; \gamma)\), of a variable \(y\), takes the form

\[
I(y, \gamma) = \frac{1}{\gamma} \sinh^{-1}(\gamma y) = \frac{1}{\gamma} \log \left[ \gamma y + \left( \gamma^2 y^2 + 1 \right)^{0.5} \right]
\]

where \(\gamma\) is a scalar parameter to be estimated, and where the transformation is symmetric around zero (so typically only nonnegative values of \(\gamma\) values are considered). The transformation is linear as \(\gamma\) approaches zero. For a wide range of values of \(\gamma\), the transformation behaves logarithmically, as it does for large values of \(y\).

A major advantage of the IHS transformation is that it renders estimation on the transformed variable robust to non-normality of the original error terms. The IHS transformation has been used before in more simple models of charitable donations by, for example, Yen et al. (1997).

We note here that the use of the IHS transformation is not just to deal with nonnormality, but also extreme values (as will be present in models related to wealth). Moreover, it has a short, but illustrious, history in the study of wealth and

\(^7\) We note that the issue of non-normality (and heteroskedasticity), within the context of modelling charitable giving at the cross-sectional level has also been considered by Wilhelm (2008).
related issues (see, for example, Friedline et al., 2015), where there is not only nonnormality, but also the presence of extreme values (see also, Burbidge et al., 1988, and Pence, 2006). The IHS transformation has the virtue that it is smooth and continuous, exists for the entire real support, not just nonnegative values, and does not make an abrupt transition at zero (the way that the Box-Cox transformation does, for example). 

With this modification, the density of $y$ becomes

$$f(y_{it} | \text{class } = j, x_{it}, u_{ij}, \beta_j, \sigma_j, \theta_j, \gamma_j) = \left[ \Phi \left( \frac{-\beta' x_{it} - \theta u_{ij}}{\sigma_j} \right) \right]^{\gamma_j - D_j} \times \left[ \frac{J(y_{it}, \gamma_j)}{\sigma_j} \phi \left( \frac{I(y_{it}, \gamma_j) - \beta' x_{it} - \theta u_{ij}}{\sigma_j} \right) \right]^{\theta_j}$$

where $J(y_{it}, \gamma_j)$ is the Jacobian of the transformation from $I(y_{it}, \gamma_j)$ to $y_{it}$,

$$J(y_{it}, \gamma) = [1 + (y_{it} \gamma_j)^2]^{1/2}.$$  

We allow $\gamma_j$ to vary across classes, as it is possible that different transformations are appropriate for the different sub-groups of the population. If the IHS parameters do vary across classes, this would suggest that using a single transformation for all households (using logs, for example) would be inappropriate.

The suggested extensions (heteroskedasticity, random effects and non-normality) are new to the literature of panel data latent class models. Importantly if any of these innovations are found to be statistically significant (which they all were in our application, as discussed below), ignoring them in estimation will lead to biased and

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8 Following the existing literature (beginning with Burbidge et al., 1988, and including MacKinnon and McGee, 1990, Jensen and Yen, 1995, Yen and Jones, 1997, Yen et al., 1997, Newman et al., 2003, Pence, 2006, Yen, 2007, and most recently Friedline et al., 2015), we apply the IHS transformation to the dependent variable. This approach ties in with the long line of literature on the Box-Cox transformation, as reviewed in Sakia (1992). An alternative approach relates to applying the IHS transformation to deviations from the conditional mean function, which we highlight as a potential area for future research, given our aim to position our analysis within the existing literature. We are grateful to an anonymous reviewer for bringing this to our attention.
inconsistent parameter estimates (Wooldridge, 2010). Allowing for unobserved heterogeneity significantly increases the complexity of the estimation. The random effects need to be integrated out of the likelihood function. The approach we take here to evaluate these integrals is to use simulation techniques, using 500 Halton draws (Train, 2003). In estimation, we note that the results based on 500 random draws were essentially identical to those using 100 Halton draws, suggesting that 500 Halton replicates were sufficient. The simulated log likelihood with all extensions in place is given by

\[
\text{LogL}_S = \sum_{i=1}^{N} \log \left( \sum_{j=1}^{J} p_{ij}(\eta_i, z_i) \left[ \frac{1}{R} \sum_{r=1}^{R} \left( \prod_{t=1}^{T} f(y_{it} \mid j, x_{it}, z_i, \nu_r, \beta_j, \sigma_j, \delta_j, \gamma_j) \right) \right] \right)
\]

(14)

where the simulation is over the \( R \) draws on \( \nu_r \). The simulated log likelihood is maximized using the BFGS (Broyden, Fletcher, Goldfarb, and Shanno) algorithm in NLOGIT 5.0.

Predictions and partial effects are complicated in this model by the presence of the IHS transformation. To assemble this, we note in general, the potentially interesting margin

\[
\text{Prob}(y_{it} > 0 \mid \text{class} = j, x_{it}, z_i, u_i) = \Phi \left( \frac{\beta'_j x_{it} + \theta_j \nu_i}{\sigma_j} \right).
\]

(15)

We will evaluate this probability at the expected value of \( \nu_i \) (i.e., zero). The expected donation given that the donation is positive is

\[
E[y_{it} \mid y_{it} > 0] = \int_{0}^{\infty} y_{it} f(y_{it} \mid y_{it} > 0) dy_{it}
\]

\[
= \left[ \Phi \left( \frac{\beta'_j x_{it}}{\sigma_j} \right) \right]^{-1} \int_{0}^{\infty} y_{it} \left[ \frac{f(y_{it} \mid \gamma_j)}{\sigma_j} \right] \phi \left( \frac{I(y_{it} \mid \gamma_j) - \beta'_j x_{it}}{\sigma_j} \right) dy_{it},
\]

(16)

the unconditional expected donation is

\[
E[y_{it}] = \text{Prob}(y_{it} = 0) \times 0 + \text{Prob}(y_{it} > 0) E[y_{it} > 0]
\]
\[ \int_0^\infty \gamma_i \left[ \frac{J(y_i, \gamma_j)}{\sigma_{ij}} \right] \phi \left( \frac{I(y_i, \gamma_j) - \beta'x_{it}}{\sigma_{ij}} \right) dy_i. \]  

There are no closed forms for these integrals, so they must be approximated. We used the Newton-Cotes method (rectangles with end-point correction). Partial effects of these conditional means also require integration. The derivatives of the integrals are simpler than it might appear at first, as in order to differentiate with respect to \( x_{it} \) and \( z_{it} \), it is only necessary to differentiate with respect to \( E_{it} = \{I(y_i, \gamma_j) - \beta'x_{it}\}/\sigma_{ij} \) and \( \sigma_{ij} \). Partial effects are then multiples of these primitive derivatives. Standard errors for the partial effects are obtained by the delta method.\(^9\)

### III. Application: Data

We use data from the U.S. Panel Study of Income Dynamics (PSID), which is a panel of individuals ongoing since 1968 conducted at the Institute for Social Research, University of Michigan. In the PSID waves 2001, 2003, 2005, 2007 and 2009, there are a series of detailed questions related to giving to charity.\(^10\) Households are asked about total donations to charity over the respective calendar years. The mean (median) total value of donations in each of the calendar years are as follows: 2001, $1,181.2 ($160); 2003, $1,170.7 ($114.2); 2005, $1,467.9 ($248.2); 2007, $1,743.9 ($251.8), and 2009 $1,589.6 ($242.2).\(^11\) The potential for recall error here, should be acknowledged given that households are asked to recollect their donating behaviour over the past year. However, Wilhelm (2006) explores the quality of the PSID data on charitable donations in terms of two dimensions: missing data and the amounts reported. He compares the PSID charitable donations data with data on charitable deductions from the Internal

\(^9\) This model is available in Version 6.0 of NLOGIT (2015, Econometric Software, Plainview, New York.) and version 10.0 of LIMDEP (same publisher). In the appendix, we also provide syntax as to how to estimate this model.

\(^10\) The definition of a charitable organization in the PSID includes 'religious or non-profit organizations that help those in need or that serve and support the public interest'. It is clearly stated that the definition used does not include political contributions.

\(^11\) Note that for estimation purposes, donations were entered as thousands of dollars.
Revenue Service and finds that the reported amounts generally compare well across the data sources except above the 90th percentile. He thus confirms that the PSID data on charitable donations are ‘high quality’.

We analyse an unbalanced panel of data, where, on average, households are in the panel for 3 waves and the minimum (maximum) number of waves is 1 (5). Following the existing literature (such as Auten et al., 2002), to avoid changes in income and in charitable donations being related to changes in household composition, households are only included in the sample if their marital status is unchanged over the period. Our findings are robust to including all households regardless of changes in marital status.

In our statistical framework, we include numerous explanatory variables, which have previously been employed (see, for example, Andreoni, 2006, and Auten and Joulfaian, 1996). In terms of those in the latent class component of the model, i.e. in \( z_i \), following Clark et al. (2005), Bago d’Uva and Jones (2009) and Greene (2012), for example, we include time invariant head of household characteristics: years of completed schooling; gender; the ethnicity of the head of household (where groups other than white form the reference category); religious denomination, that is, Catholic, Protestant or other religion (with no religious denomination as the omitted category); the natural logarithm of permanent income, which is defined as the average household income prior to the commencement of the estimation sample; and the following year of birth categories, born before 1949, 1950-59, 1960-69 and 1970-1979 (born after 1980 is our reference category).

The tobit part of the model, i.e. \( x_{it} \), is in line with much of the existing literature. Here we include the number of adults in the household, the number of children in the household, the age of the head of household, the employment status of the head of
household and their spouse (with unemployed or not currently in the labour market as the reference category), the marital status of the head of household (with all states other than married or cohabiting as the base), the natural logarithm of the income of the head of household and their spouse, and the natural logarithm of household wealth.\textsuperscript{12}

Finally, we also include the price of donating in the tobit model. Taxpayers in the U.S. can choose to report itemized deductions such as donations to charity in their federal income tax returns as eligible expenses to reduce the level of income subject to tax. The majority of taxpayers in the U.S. choose between itemized deductions and the standard deduction depending on which is the largest. For households who itemize charitable donations in their tax return, the price of the donation is defined as one minus the household’s marginal tax rate on the contribution made, whereas for households who do not itemize charitable donations, the price of the donation is one; donating one dollar means that there is one dollar less for consumption. One key advantage of the PSID is that households are asked to indicate whether they made an itemized deduction for charitable contributions. Households which itemize are assigned the relevant tax rate using the National Bureau of Economic Research (NBER) TAXSIM programme (http://www.nber.org/~taxsim/), which calculates federal state tax liabilities for survey data based on a range of factors such as earnings, marital status and children.\textsuperscript{13,14}

\textsuperscript{12} As is standard practice, we focus on head of household characteristics. We have checked however, that our results are robust to using average characteristics of the head of household and their spouse for variables such as age and education. The results are unchanged, which is not surprising given that, for example, the mean age of the head of household is 45.48 years compared to 44.62 for the average of the head and spouse, similarly with respect to years of schooling, 13.16 years compared to 12.90 years.

\textsuperscript{13} The TAXSIM programme includes both state and federal law, which is important given for example changes in federal taxes in 2001, 2003 and 2004 during this period (see Backus, 2010, for recent discussion of the effects of these changes).

\textsuperscript{14} One additional issue, which has arisen in the existing literature, is that the decision to itemise is arguably not fully exogenous: the decision to itemise may be influenced by the level of donations. To account for this, as is common in the existing literature (see Clotfelter, 1980, and Auten et al., 2002), we exclude ‘endogenous itemisers’ who are defined as those who have itemised but would not have done so in the absence of their actual charitable donations. Due to an additional source of possible endogeneity relating to the price of a charitable donation being a function of both the donation and income, see Auten et al. (2002), we calculate the price variable firstly by assuming that charitable donations equal zero (i.e. the first dollar price) and then after including a predicted amount of giving set at 1 per cent of average
Summary statistics for our estimation sample are presented in Table 1, where, on average, the head of household has 13 years of schooling; 70 per cent are male; 49 per cent of household heads are born between 1950 and 1969; and 53 per cent are married or cohabiting. All monetary variables in the analysis are deflated to 2001 prices.

IV. Application: Results

In this section, we discuss the results from estimating the panel latent class tobit model detailed above. Table 2 presents the results relating to the determinants of class membership (with Table 3 presenting the remaining results). Out of the 9,755 observations, 2,274 are predicted to be in class 1 and 7,481 in class 2, the sample proportions in each class being 0.23 and 0.77, respectively. Note that these class separations are determined by the estimated posterior probabilities (based upon the maximum probability rule). In Table 2, we also present the probability of reporting zero donations within each class (evaluated at sample means).

From Table 2, there is a clear disparity between the probabilities of zero donations across the classes, with class 1 (at 0.61) being significantly lower than class 2 (at 0.76). We can use these findings, in part, to help us identify the two classes: so class 2 is dominated by the predicted zero givers. To paint a clearer picture of our findings, consider the results presented in Table 3. This table presents the results relating to the analysis of the determinants of the amount of donations. We will return to the estimated coefficients shortly, but for now will focus on the expected values, E(V), of donations. As before, we split the sample into class 1 or 2, based upon their predicted posterior probabilities. Within each class, we then consider two expected values of charitable donations: the simple, unconditional, sample average of observed donations for these income. We then take an average of the two price variables. As stated by Auten et al. (2002), p.376, ‘this procedure yields a tax price consistent with the actual costs of giving, but not endogenous to individual donation decision’.

18
households; and the averaged predicted expected value of donations (that is, based upon observed personal and household characteristics).

From the unconditional expected values, it seems clear that class 1 contains “high” and class 2 contains “low” givers to charity. Average actual donations for classes 1 and 2, respectively, are $3,309 and $313. This ties in with the findings presented in Table 2. Households predicted to be in class 1 have a relatively low probability of making zero donations and are predicted to donate, on average, much more than those predicted to be in class 2. This finding is reinforced when we evaluate the predicted expected value of the level of donations for each class. Due to the IHS transformation, these predicted expenditure levels are computed following the approach of Yen and Jones (1997) as described above. We now find that the average predicted level of donations amongst those in class 1 is $2,063, which is again significantly higher than that of class 2, at $1,487. After summarising the results in general, we now turn our attention to the specific drivers of both class membership and donation levels.

Class Membership

As the coefficients in Table 2 correspond to class 1 membership (relative to class 2), these coefficients can be interpreted as follows: positive ones being associated with higher probabilities of being in class 1 (relative to class 2); and negative ones being associated with a higher probability of being in class 2. The results suggest that households with a male head are significantly more likely to be in class 2, the low donating group characterised by a relatively high probability of making zero donations, than households with a female head (at the 5% level), which ties in with the existing literature. Life cycle effects are also evident with the likelihood of being in class 1
(relative to class 2) found to be positively associated with the birth cohort controls born before 1949, born 1950-59, and born 1960-69.\textsuperscript{15}

The education and ethnicity of the head of household are also significant predictors of class membership, with the years of schooling of the head of household and having a white head of household being positively associated with being in class 1 (the high donators group). Interestingly, having a household head in a Protestant religious denomination is positively associated with being in class 1, whilst being in a Catholic religious denomination is positively associated with being in class 2, albeit at a lower level of statistical significance. Such findings highlight the importance of distinguishing between different religious denominations in modelling donations to charity.

\textit{Total Donations to Charity}

The results from modelling the level of total household donations are presented in Table 3, where the coefficients are reported by class. With regard to individual (and joint) parameter significance, it is apparent that, in general, the model is well-specified, with many covariates attaining statistical significance. Moreover, given the above specification tests, the overall model also appears to be well-specified. There are some interesting differences between the effects of some covariates across the two classes thereby revealing the flexibility, and appropriateness, of the latent class approach. For example, statistically significant effects from whether the head of household or his/her spouse are employed are apparent for class 1 but not for class 2. In contrast, being married or cohabiting is positively associated with the amount of donations in both classes. The price variable, on the other hand, is statistically insignificant in both classes.

\textsuperscript{15}These cohort results could be capturing generational effects.
We next consider the statistical significance of our three proposed extensions as validation of our suggested modelling approach. Specifically, in terms of our ancillary parameters (Table 3), importantly we find evidence of heteroskedasticity in class 2, with the variance decreasing in both wealth and income in class 2, both effects being statistically significant at the 1% level (and jointly so, using a likelihood ratio test). This evidence of heteroskedasticity in one of the classes highlights the importance of extending the modelling framework to deal with this issue. Random effects are also significantly present in both classes, being much larger for class 1, therefore strongly indicating the presence of unobserved heterogeneity and endorsing this novel extension to the modelling framework.

Both IHS parameters are significantly different from zero which would appear to suggest that a linear approach is inappropriate and that the standard untransformed tobit model, for example, would be mis-specified (on the assumption that the model is well-specified, as is suggested by the above specification tests). Interestingly, these parameters vary dramatically across classes suggesting a single transformation for all households (as, for example, in a simple log–transformed model) would also be mis-specified.

Individually, therefore, each of the above tests provides, in essence, a specification test for mis-specification in the case of the simpler (appropriately nested) model. Indeed, these individual ones do provide strong evidence in support of our approach. Unfortunately, there does not appear to be an appropriate specification test that can simultaneously address all three areas of possible mis-specification.

We focus our discussion of the remaining results, on two key covariates which have attracted interest in the existing literature, namely: wealth and income. It is apparent that wealth exerts a statistically significant effect in the case of both classes,
where wealth is positively associated with the amount of donations. Similarly, income exerts a positive influence on the amount of donations for both classes.

Table 4 presents partial effects, computed separately for each class, for our two covariates of particular interest (wealth and income). For each class we present partial effects relating to the probability of making a positive donation, the overall expected value of donations and the expected value of donations conditional on donating. These effects all relate to a 10% increase in both income and wealth.

Thus in class 1 (Panel A), we see that a 10% rise in wealth results in an increase in the probability of observing a positive donation, albeit of a relatively small magnitude. On the other hand, a rise in income in this class has a statistically insignificant effect. For class 2 (Panel B), a 10% increase in wealth has a larger effect than in class 1, on the probability of observing a positive donation. We also find a positive income effect for class 2, which is relatively large. Although these effects are rather small in absolute value, the difference between the classes is dramatic: for class 2 the wealth effect is almost double that of class 1; whereas that for income is nearly tenfold.

Turning now to the estimated partial effects across both classes (Panel A and B) we find statistically significant positive effects for income and wealth for both the overall expected value of donations and the expected value of donations conditional on donating. For class 1 these effects are rather similar in magnitude (across $E[y]$ and $E[y|y > 0]$). Across classes, the effects of income are noticeably greater than that of wealth. However, these effects are considerably larger in class 2. For example, whereas a 10% increase in income for class 1 results in an increase in $E[y|y > 0]$ of 0.007 ($000’s 2001), the equivalent in class 2 is 0.08 ($000’s 2001). Such differences serve to
highlight the flexibility of our latent class approach in terms of unveiling how the influences on donating behaviour vary across sub-groups of the population.

Model Comparison and Evaluation

In this sub-section we compare our results to a selection of alternative models that conceivably could also have been considered. Specifically, we estimate: a standard tobit model, a fixed effects tobit model,\(^\text{16}\) a standard panel latent class model; and a double-hurdle model. We note that as these models are not all nested in the usual parametric sense, it is not straightforward, or indeed obvious, as to how one may statistically test for a “preferred” model. However, in such a case it is common to use model selection techniques based on Akaike Information Criteria, AIC (Cameron and Trivedi, 2005).\(^\text{17}\) To this extent, our model dominates all of the others so-considered. This suggests that our latent class approach is by far the preferred one here (see Table 5).

In Table 5, we also present estimated coefficients and, for purposes of comparison across the models, partial effects for wealth and income. It is apparent that across the five models the estimated coefficients for wealth and income are all positive and statistically significant confirming that income and wealth are important drivers of donating behaviour. This is also the case for the estimated partial effects, although there are some distinct differences in terms of the magnitude of the effects across the five models. With respect to wealth, the fixed effects tobit model reveals the smallest partial effect (0.00319) and the standard tobit model the largest (0.00849). Interestingly, the difference in magnitude is similar across the partial effects estimated in our proposed latent class framework across the two classes at 0.00474 (class 1) versus 0.00822 (class

\(^{16}\) We have also estimated a random effects tobit model. These results are not reported here as due to a very low scaling factor, partial effects could not be recovered. The coefficients for income and wealth estimated for this model tie in with the findings summarised in Table 5.

\(^{17}\) Although we only present AIC measures, as these appear to be more commonly used in these types of models, the findings are robust to choice of particular information criteria (such as the Corrected AIC and the Bayesian Information Criteria).
2). One interpretation of this finding is that in estimating a single effect, the fixed effects and standard tobit results, are yielding lower and upper bounds for this effect.

In terms of income effects, our new latent class model reveals both the smallest partial effect (at 0.00705 for class 1) and the largest partial effect (at 0.05517 for class 2), encompassing the income effects for all other models. In this instance it appears that the other models are essentially estimating an average effect between the two classes. Thus, our flexible framework appropriately identifies the extent of the difference in the income effects across the two classes, which would be overlooked by the other estimation approaches.

V. Conclusion

We have extended the standard latent class tobit panel approach to simultaneously include random effects, heteroskedasticity and the IHS transformation of the dependent variable. We have applied this extended latent class framework to the modelling of donations to charity, an interesting application because of the potential for distinct groups of households in the population to have quite divergent behaviour with respect to their donating behaviour. Our findings, which are based on U.S. panel data drawn from five waves of the Panel Study of Income Dynamics, indicate that there are, indeed, two clearly defined groups of charitable donors: one which gives much more, and has an associated lower probability of zero-donation; and the other, which donates much less and has a higher probability of not donating. This suggests that treating the population as a single homogeneous group of donors, could well lead to biased parameter estimates and erroneous policy inference, as indicated by the comparison of the findings from our extended modelling framework with those from other approaches commonly used in the related literature. It is apparent that our modelling framework can potentially be applied
to analysis of other areas of household behaviour typically modelled in the existing literature via a tobit approach, where different groups potentially exist in the population.
References


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<tr>
<th></th>
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<th>STANDARD DEVIATION</th>
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</thead>
<tbody>
<tr>
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<td>$1,826</td>
</tr>
<tr>
<td></td>
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<td>0.38</td>
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<td>0.43</td>
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<tr>
<td>Born 1960-69 [0/1]</td>
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<td>0.43</td>
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<td>0.42</td>
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<tr>
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<td>20.09</td>
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<td>0.44</td>
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<td>Spouse Employee or Self Employed [0/1]</td>
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<td>0.47</td>
</tr>
<tr>
<td>Married or Cohabiting [0/1]</td>
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<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Household Characteristics</strong></td>
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<td>0.78</td>
</tr>
<tr>
<td>Number of Children [0+]</td>
<td>0.92</td>
<td>1.16</td>
</tr>
<tr>
<td>Log Income of Head and Spouse (2001 prices)</td>
<td>10.42</td>
<td>1.07</td>
</tr>
<tr>
<td>Log Permanent Income (2001 prices)</td>
<td>9.13</td>
<td>1.14</td>
</tr>
<tr>
<td>Log Wealth (2001 prices)</td>
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<td>3.05</td>
</tr>
<tr>
<td>Price</td>
<td>0.77</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>OBSERVATIONS</strong></td>
<td></td>
<td>9,755</td>
</tr>
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**TABLE 2:** Estimates of the Determinants of Class One Membership

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-6.9551</td>
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</tr>
<tr>
<td>Years of Schooling</td>
<td>0.3250</td>
<td>0.0207***</td>
</tr>
<tr>
<td>Male</td>
<td>0.2532</td>
<td>0.1172**</td>
</tr>
<tr>
<td>White</td>
<td>0.1688</td>
<td>0.0999*</td>
</tr>
<tr>
<td>Catholic</td>
<td>-0.4045</td>
<td>0.1626**</td>
</tr>
<tr>
<td>Protestant</td>
<td>0.4185</td>
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<td>Other Religion</td>
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<tr>
<td>Log Permanent Income</td>
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<td>Born =&lt; 1949</td>
<td>1.4752</td>
<td>0.1646***</td>
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<tr>
<td>Born 1950-59</td>
<td>1.3693</td>
<td>0.1491***</td>
</tr>
<tr>
<td>Born 1960-69</td>
<td>0.6479</td>
<td>0.1553***</td>
</tr>
<tr>
<td>Born 1970-79</td>
<td>-0.0784</td>
<td>0.1676</td>
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</tbody>
</table>

- Proportion predicted in Class 1 ($p_1$) 0.23
- Proportion predicted in Class 2 ($p_2$) 0.77
- Probability of Class 1 – Zero donations 0.61
- Probability of Class 2 – Zero donations 0.76

**TOTAL OBSERVATIONS** 9,755

Notes: (i) *** significant at the 1% level; ** significant at the 5% level; and * significant at the 10% level. (ii) COEF denotes estimated coefficient and S.E. denotes standard error.
<table>
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<tr>
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<th>S.E.</th>
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<th>S.E.</th>
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<td>Intercept</td>
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<td>0.3511</td>
<td>-1.5047</td>
<td>0.0950***</td>
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<tr>
<td>Married or Cohabiting</td>
<td>0.5034</td>
<td>0.0677***</td>
<td>1.1714</td>
<td>0.0128***</td>
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<td>Number of Adults</td>
<td>-0.0344</td>
<td>0.0741</td>
<td>0.0138</td>
<td>0.0205</td>
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<tr>
<td>Number of Children</td>
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<td>0.0565</td>
<td>0.0091</td>
<td>0.0141</td>
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<tr>
<td>Employed</td>
<td>0.2805</td>
<td>0.0538***</td>
<td>0.0213</td>
<td>0.0133</td>
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<td>Spouse Employed</td>
<td>-0.1185</td>
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<td>-0.0007</td>
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<td>Log Wealth</td>
<td>0.0343</td>
<td>0.0060***</td>
<td>0.0219</td>
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<td>Log Income</td>
<td>0.1007</td>
<td>0.0192***</td>
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<td>σ</td>
<td>0.6883</td>
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<td>0.0144</td>
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<td>γ (IHS)</td>
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<td>0.0501***</td>
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Notes: (i) ***significant at the 1% level; **significant at the 5% level; and *significant at the 10% level. (ii) COEF denotes estimated coefficient and S.E. denotes standard error.
**TABLE 4:** Random Effects Latent Class Tobit Model – Partial Effects – Wealth and Income

**PANEL A: CLASS 1**

|                      | Prob($y > 0$|class 1) | $E[y]$ | $E[y|y > 0]$ |
|----------------------|-------------|--------|-------------|
|                      | P.E.        | S.E.   | P.E.        | S.E.         | P.E.        | S.E.         |
| Log Wealth           | 0.00027     | 0.00009*** | 0.00474     | 0.00132***    | 0.00438     | 0.00117***   |
| Log Income           | 0.00033     | 0.00023 | 0.00705     | 0.00372*      | 0.00703     | 0.00333***   |

**OBERVATIONS**

| 2,274 |

**PANEL B: CLASS 2**

|                      | Prob($y > 0$|class 2) | $E[y]$ | $E[y|y > 0]$ |
|----------------------|-------------|--------|-------------|
|                      | P.E.        | S.E.   | P.E.        | S.E.         | P.E.        | S.E.         |
| Log Wealth           | 0.00045     | 0.00004*** | 0.00822     | 0.00060***    | 0.01186     | 0.00101***   |
| Log Income           | 0.00297     | 0.00014*** | 0.05517     | 0.00294***    | 0.08020     | 0.00477***   |

**OBERVATIONS**

| 7,481 |

Notes: (i) ***significant at the 1% level; **significant at the 5% level; and * significant at the 10% level. (ii) P.E. denotes partial effect; and S.E. denotes standard error.
**TABLE 5: Model Comparison and Evaluation**

**PANEL A: Latent Class Panel IHS Heteroskedastic Tobit Model**

<table>
<thead>
<tr>
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<th>COEF</th>
<th>S.E.</th>
<th>P.E.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Log (Wealth)</td>
<td>0.0343</td>
<td>0.0060***</td>
<td>0.00474</td>
</tr>
<tr>
<td></td>
<td>Log (Income)</td>
<td>0.1007</td>
<td>0.0192***</td>
<td>0.00705</td>
</tr>
<tr>
<td>2</td>
<td>Log (Wealth)</td>
<td>0.0219</td>
<td>0.0017***</td>
<td>0.00822</td>
</tr>
<tr>
<td></td>
<td>Log (Income)</td>
<td>0.1450</td>
<td>0.0070***</td>
<td>0.05517</td>
</tr>
</tbody>
</table>

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**PANEL B: Tobit Model**

<table>
<thead>
<tr>
<th>COEF</th>
<th>S.E.</th>
<th>P.E.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (Wealth)</td>
<td>0.1618</td>
<td>0.0085***</td>
<td>0.00849</td>
</tr>
<tr>
<td>Log (Income)</td>
<td>0.5677</td>
<td>0.0284***</td>
<td>0.02977</td>
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</tbody>
</table>

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**PANEL C: Fixed Effects Tobit Model**

<table>
<thead>
<tr>
<th>COEF</th>
<th>S.E.</th>
<th>P.E.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (Wealth)</td>
<td>0.0340</td>
<td>0.0074***</td>
<td>0.00319</td>
</tr>
<tr>
<td>Log (Income)</td>
<td>0.3235</td>
<td>0.0311***</td>
<td>0.03033</td>
</tr>
</tbody>
</table>

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**PANEL D: Latent Class Tobit Model**

<table>
<thead>
<tr>
<th>Class</th>
<th>COEF</th>
<th>S.E.</th>
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<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Log (Wealth)</td>
<td>0.1050</td>
<td>0.0134***</td>
<td>0.00440</td>
</tr>
<tr>
<td></td>
<td>Log (Income)</td>
<td>0.3776</td>
<td>0.0439***</td>
<td>0.01905</td>
</tr>
<tr>
<td>2</td>
<td>Log (Wealth)</td>
<td>0.0484</td>
<td>0.0029***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Log (Income)</td>
<td>0.2381</td>
<td>0.0096***</td>
<td></td>
</tr>
</tbody>
</table>

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**PANEL E: Double Hurdle Model**

<table>
<thead>
<tr>
<th>COEF</th>
<th>S.E.</th>
<th>P.E.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (Wealth)</td>
<td>0.1527</td>
<td>0.0080***</td>
<td>0.00786</td>
</tr>
<tr>
<td>Log (Income)</td>
<td>0.5332</td>
<td>0.0238***</td>
<td>0.02745</td>
</tr>
</tbody>
</table>

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Notes: (i) ***significant at the 1% level; **significant at the 5% level; and *significant at the 10% level. (ii) P.E. denotes partial effect; and S.E. denotes standard error. (iii) The partial effects presented in this table all relate to a 10% rise in the covariate on $E[d]$. (iv) In Panel D the partial effects are a weighted average across the two classes.
Appendix: LIMDEP/NLOGIT Syntax

Below is the syntax to estimate this model:

TOBIT ; Lhs = < dependent variable >
    ; Rhs = < list of independent variables >
    ; Model = IHS
    ; Marginal Effects $

Extension to add heteroskedasticity in the disturbance is
    ; Heteroscedasticity ; Hf1 = < list of variables >

Extensions to accommodate latent heterogeneity
I. Random Effects
    ; Pds = <panel data specification - this is optional >
    ; Draws = < number of draws for simulation > ; Halton
    ; RPM ; Fcn = one(n)
II. Latent class specification
    ; LCM o r ; LCM = <variables that enter the prior class probs.>
    ; Pts = < the number of classes >
    ; Pds = < panel data specification - this is optional >