

**Science and Mathematics Education Centre**

**Gender Differences and the Effects of Cooperative Learning in  
College Level Mathematics**

**ViAnn Esther Olson**

**This thesis is presented as part of the requirements for  
the award of the Degree of Doctor of Mathematics Education  
of the Curtin University of Technology**

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## **Declaration**

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

Signature: \_\_\_\_\_

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## ABSTRACT

The purpose of this study was to evaluate the effectiveness of cooperative learning in a liberal arts mathematics course and to examine any gender-related differences in the effects of cooperative learning in terms of achievement, composition of the cooperative groups, mathematics anxiety, attitudes toward mathematics, attendance, and retention. The quasi-experimental design compared a control section using individualized learning methods with three treatment sections using cooperative learning methods based on the Learning Together model of Johnson and Johnson (1991). The compositions of the three treatment groups varied: heterogeneous ability/heterogeneous gender, heterogeneous ability/homogeneous gender, and self-selected. The Academic Skills Assessment Program (ASAP) was used as a pre-test to show that the groups were equal. The Revised Math Attitude Scale and the Mathematics Anxiety Rating Scale (MARS) were used as pre- and post-tests to measure changes in attitudes and anxiety. Achievement was measured by individual and group course grades. Group grades were determined by taking 67% of a student's individual exam score plus 33% of the group's average on the exam.

Cooperative learning and composition of groups had no significant effect on achievement. The differences between individual and group grades were insignificant, and the group grading method benefited the grades of only five students. Attendance had a large effect on achievement, and the ASAP score and the Math Attitude Post-test were significant predictors of achievement. The MARS post-test was negatively correlated with individual course grade.

While no significant gender-related differences surfaced, some trends appeared. The ASAP, MARS, and Math Attitude pre-test scores were equal, however, females achieved slightly higher course grades than males. In each of the four research groups the individual course grades were higher for females than for males. Also, females had a larger decrease in mathematics anxiety with a drop of 22 points compared to the males' drop of 10 points. Males and females each improved their attitudes toward mathematics by only one point, however, in three of the four research groups, females had smaller attitude changes than males, and two of the cooperative learning groups had decreases in their attitude scores.

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## **CHAPTER ONE: INTRODUCTION**

I have taught mathematics for the past 26 years. The first 12 of these were at high schools (grades 7-12), and the last 14 at a community college. Like other teachers, I am continuously searching for new ideas and teaching methods in an attempt to reach more students. During the summer of 1991 I took a cooperative learning course at St. Cloud State University. It was a methods course on how to incorporate cooperative learning into your classes. The instructor had been a student of David and Roger Johnson at the University of Minnesota. The course was geared toward K-12, and I was the only postsecondary teacher in the class, but the instructor convinced me that cooperative learning was a viable method at the college level. The following semester I began using cooperative group work occasionally, and quite tentatively. If a student didn't want to work in a group, I allowed the student to work individually. At that time, many students had not experienced working cooperatively. Since then I have become a believer in the benefits of cooperative learning, not just to learn the mathematics but to learn how to work cooperatively rather than competitively or individually. Now I incorporate cooperative group work daily, and students must participate in groups. While my perception is that cooperative learning has been successful, I did have some questions that I wanted to investigate. Hence, the present research study.

This chapter begins with the statement of the problem, followed by a brief review of the literature that informed this study. Next is a list of the research questions. Following that are sections on definition of terms, the research design, significance of the study, and the research site. The final section provides an overview of the dissertation.

### **Statement of the Problem**

As a community college mathematics instructor I am keenly aware of the fact that many of my students come to class with weak backgrounds in mathematics, low mathematics self-esteem, high mathematics anxiety, low

interest in the subject matter, and poor attitudes towards mathematics. They are in the class because it is the lesser of the evils that fulfils the mathematics requirement for their programs. In an attempt to reach these students I began using cooperative group work several years ago. It seemed to help some students, but I have always questioned the overall effectiveness of it. This research study examined the effectiveness of cooperative learning in a liberal arts mathematics course at a Midwestern community college. The effects examined were achievement, anxiety, attitudes, attendance, and course retention. Gender-related differences were also examined.

## **Background**

The purpose of this brief literature review is to supply a background of some of the research that informed the present study. The first three sections review research conducted in the fields of cooperative learning, mathematics anxiety, and gender differences in mathematics, respectively. These are followed by sections on group composition and social loafers. The final section is the summary.

### Cooperative Learning

Cooperative learning is understood to occur when small groups of people work together to help each other learn academic content (Davidson & Kroll, 1991). Cooperative learning encompasses a variety of teaching methods, which are explained in Chapter Two, and the one I have chosen to use in this study is the Learning Together model (Johnson & Johnson, 1991).

The Learning Together model of Johnson and Johnson (1991), developed at the University of Minnesota, requires five essential components to be successful: positive interdependence, face-to-face verbal interaction, individual accountability, social skills, and group processing. In the Learning Together model, students are assigned to groups of two to five members.



Over the last 20 years much research has been conducted on the effectiveness of cooperative learning. The Johnsons' Learning Together models have been widely used and widely studied, particularly in comparison to individualistic and competitive models. Further details of the model and research conducted into its effectiveness are provided in Chapter Two.

Slavin (1995) updated his earlier (1990) research review with a meta-analysis of 90 studies of the practical applications of cooperative learning in elementary and secondary schools. Analyses of the data clearly indicate that achievement effects vary widely, depending on the cooperative learning method employed and other factors.

### Mathematics Anxiety

Mathematics anxiety is a phenomenon that creates difficulties for both the student and the teacher. According to Richardson and Suinn (1972, p. 551), "Mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations." They developed the Mathematics Anxiety Rating Scale (MARS), which has been widely validated and used extensively in research studies. Brush (1978) administered the MARS to university students of various academic majors. Her study validated Richardson and Suinn's results and confirmed that MARS is a reliable instrument to test for mathematics anxiety.

Mathematics anxiety seems to be related to prior achievement in mathematics (Clute, 1984; Hembree, 1990). "Mathematics anxiety is related to poor performance on mathematics achievement tests. It relates inversely to positive attitudes toward mathematics and is bound directly to avoidance of the subject (Hembree, 1990, p. 33)." Mathematics anxiety is widespread among college students and highest among college students majoring in elementary education (Hembree, 1990).

### Gender-related Differences

It is a well documented and researched fact that males outnumber females in participation in advanced mathematics courses and in mathematics-related careers. Studies of achievement in mathematics indicate that gender differences tend to be small, depending on the type of mathematical task (Baker & Jones, 1992; Battista, 1990; Ethington, 1990; Hyde, Fennema, & Lamon, 1990), that the magnitude of the differences seems to have declined over the years (Baker & Jones, 1992; Hyde et al., 1990), and that gender differences increase with age (Ethington, 1992; Frost, Hyde, & Fennema, 1994; Hyde et al., 1990; Leder, 1996).

Several studies examined affective factors that may have contributed to differences in performance, including confidence (Bohlin, 1994; Leder, 1996; Norton & Rennie, 1998; Sax, 1994), anxiety (Frost et al., 1994; Norton & Rennie, 1998), and the stereotyping of mathematics as a male domain (Ethington, 1992; FitzSimons, 1997; Norton & Rennie, 1998). Females were found to be less interested in activities and careers requiring mathematical or technological applications, preferred a less competitive learning environment, and had a greater need for structure in that environment than did males. Boaler (1997a) found that females were more interested in understanding the mathematics, while males just wanted to get it done.

There is evidence to suggest that cooperative learning is a preferred learning style for women and that women tend to be “connected knowers” (Belenky, Clinchy, Goldberger, and Tarule, 1986). Jacobs and Becker (1997) discuss how cooperative learning lends itself to those connected knowers. “There is evidence that women (and students of color) not only prefer a more collaborative, less competitive atmosphere in the classroom, but that they achieve more in that milieu as well” (AAUW, 1992, as cited in Jacobs and Becker, 1997, p. 110).

### Group Composition

Group composition is an interesting phenomenon. Studies on small group interactions (Holden, 1993; Webb, 1984, 1985, 1991) found that if the groups were equal in the number of males and females, both sexes had similar learning experiences. Females were disadvantaged if the groups were unequal. Webb's studies included ability grouping. She found that in uniformly high-ability groups, everyone assumes everyone else knows how to do the work, so no help is given, while in uniformly low-ability groups, no one feels competent to offer help. The most helping behaviours were exhibited in uniformly medium-ability groups and mixed-ability groups. However, if the mixed-ability groups contained high-medium-low-abilities, the mediums tended to be ignored while the highs helped the lows.

### Social Loafers, Free Riders, and Suckers

The use of cooperative learning may produce some social loafers or free riders, which may influence attitudes and achievement. One definition of a social loafer is someone who has reduced his or her individual effort when working with a group on an additive group task. "Additive tasks require the summing together of individual group members' inputs to maximize the group product" (Johnson & Johnson, 2000, p. 299). A free-rider, on the other hand, is someone who benefits from the work of others while contributing no effort oneself. The essential difference between social loafers and free riders is that social loafers "hide in the crowd," while free riders "get something for nothing" (Johnson & Johnson, 2000, p. 301). These definitions come from the perspective of cooperative groups in an educational setting. On the other hand, Comer's definition (1995), taken from economic theory and applied to her study of cooperative group work in the workplace, states that "a free rider is someone who derives benefits from membership in a large group (such as a community) that are disproportionately larger than his or her contributions to the group" (p. 648). And she describes a social loafer as someone who "profits from the work of other group members without working up to his or her potential" (p. 648).

## Summary

The research clearly shows that cooperative learning methods that include both individual accountability and group goals will result in positive effects on achievement. However, the majority of these studies have been conducted with grades K-12. The question is whether or not similar results are found at the university level. This gap in the research assures me that my study will be both original and significant. These studies, as well as those on anxiety, gender, and group composition have informed my research and have directed me to the following research questions.

## **Research Questions**

I wanted to know if the use of cooperative methods increases achievement, improves attitudes toward mathematics, reduces anxiety, increases attendance, and reduces the number of withdrawals from the course. I was also curious if there is any difference between genders.

The research questions are:

1. What are the effects of cooperative learning, in terms of:
  - a) Achievement in mathematics?
  - b) Mathematics anxiety?
  - c) Attitudes toward mathematics?
  - d) Attendance?
  - e) Retention (number of student withdrawals from the course)?
  
2. What are the gender-related differences in the effects of cooperative learning, in terms of:
  - a) Achievement in mathematics?
  - b) Mathematics anxiety?
  - c) Attitudes toward mathematics?
  - d) Attendance?
  - e) Retention (number of student withdrawals from the course)?

## **Definition of Terms**

### Academic Skills Assessment Program (ASAP)

The Academic Skills Assessment Program (ASAP) consists of a group of placement tests for reading, writing, and mathematics. The ASAP tests are used at all state colleges and universities in Minnesota. The ASAP tests are produced by Educational Testing Service, Princeton, New Jersey, U.S.A.

### College

A college is a Minnesota State Colleges and Universities institution authorized to offer certificates, diplomas, and associate degrees.

### Developmental Courses

Developmental courses are high school level courses taught at colleges and universities to students who are not prepared for college level work. College credit is not awarded for developmental courses. Elementary Algebra is the developmental course that is the prerequisite to the course that is the vehicle for this study.

### Free Rider

A free rider is a person who gains from the work of others without making a contribution.

### Liberal Arts Mathematics Course

A liberal arts mathematics course is a survey course that covers several diverse mathematics topics, but doesn't cover them in depth. It is a college level course designed for students majoring in education, the humanities, the social sciences, and other liberal arts fields. The liberal arts mathematics course in this study is called Contemporary Concepts in Mathematics.

### Postsecondary Enrollment Options (PSEO)

The Postsecondary Enrollment Options program allows high school students to receive both high school and college credit for college or university courses completed. This program is limited to juniors (year 11) who are in the upper one-third of their class and to seniors (year 12) in the upper one-half of their class.

### Social Loafer

A social loafer is a person who gains from the work of others while making a minimal contribution.

### Sucker

A sucker is a person who is easily cheated or taken advantage of.

### University

A university is a Minnesota State Colleges and Universities institution authorized to offer degrees at the associate level and above.

## **Research Design**

Acting as both the researcher and the teacher I conducted this research study with my own classes, and my own students were the participants. A quasi-experimental design was used to compare a control section, which used individualized learning methods, and treatment sections, which used cooperative learning methods. Based on the literature reviewed, various group compositions were tested: heterogeneous ability groups, homogeneous gender groups, and self-selected groups. The instruments used in this research study included the ASAP placement test, the Mathematics Anxiety Rating Scale (MARS), the Revised Math Attitude Scale, a researcher-constructed questionnaire, and interviews with selected students. Achievement effects were measured by final semester grades, which were determined by teacher-constructed unit examinations and in-class daily grades. Attendance was recorded by using in-class daily assignments. The design of the study is explained in more detail in Chapter Three.

## **Significance**

The review of the literature found a large body of research on the effectiveness of cooperative learning in terms of achievement gains in K-12 education and a much smaller body of similar studies at the university level, indicating a need for more university level studies. The present study will help to determine whether the recommendations resulting from previous studies by other researchers done at the high school level can be generalised to the community college and university level.

The literature on gender differences in mathematics suggests that cooperative learning is a preferred learning style for women and that gender differences increase with age. There is a need to discover which learning environments are most beneficial to both genders at the university level because mathematics has become the filter that keeps people out of the higher-paying fields of work. This study will add to the body of knowledge that currently exists on gender differences and mathematics and cooperative learning.

Mathematics anxiety and attitudes towards mathematics are two affective factors that make students avoid studying mathematics at the university level. Does lack of success in the subject cause mathematics anxiety, or does mathematics anxiety produce lack of achievement in mathematics? If cooperative learning is effective in enhancing achievement in mathematics, will it also reduce mathematics anxiety? This study will provide evidence to help answer these questions.

In addition, this study will compare various group compositions to determine if the group composition makes a difference at the university level. It will also determine if working cooperatively encourages better attendance and better course retention.

This study is significant because of its uniqueness. No other study combines the elements of cooperative learning, mathematics anxiety, attitudes

towards mathematics, gender differences, attendance, and group composition all into one study.

However, the greatest significance of this study may be to me, the teacher/researcher. This study will help me determine if the use of cooperative learning is a worthwhile instructional method and will inform me on the best group compositions for using cooperative learning. Further, it will inform me on gender differences that I may not otherwise recognize are present in my classroom.

### **Research Site**

Rochester Community and Technical College (RCTC) is a two-year college, which offers both liberal arts and technical programs, and is located in Rochester, Minnesota, U.S.A. It is part of the Minnesota State Colleges and Universities (MnSCU) system. Minnesota State Colleges and Universities are a network of 34 two- and four-year colleges and universities, consisting of 53 campuses in Minnesota, and serving about 225,000 students. The 53 campuses are scattered throughout the state, so that anyone living anywhere in Minnesota should have access to a college or university within 45 to 50 miles of their home. However, only 7 of the institutions are four-year universities. The other 27 colleges are technical colleges, community colleges, or combined community and technical colleges.

During Fall Semester, 1999, RCTC had an enrolment of 4407 students: 1749 males, and 2658 females. Of the 4407 students, 2382 (54%) were full-time students, and 2025 (46%) were part-time. Approximately 12% of the students were of non-Caucasian ethnicity.



## Overview

This study began with a thorough search of the available literature on the achievement effects of cooperative learning in mathematics, which produced evidence that suggests students achieve better in a cooperative rather than competitive or individualistic environment. However, the cooperative learning needs to follow a model that incorporates individual accountability as well as group interdependence. Group composition and social loafers are two factors that may affect achievement. The body of research on cooperative learning at the K-12 level is much larger than that at the university level, indicating a need for more research at this level.

The research also suggests that females benefit more than males in a cooperative learning environment. So I expanded my literature review to include gender-related differences in mathematics. One of the gender differences that appeared frequently was mathematics anxiety. Therefore, I expanded the literature review to include mathematics anxiety as well. Mathematics anxiety is related to attitudes towards mathematics, and mathematics attitude is another area where gender differences appear. Since research shows that gender differences tend to increase with age, these topics are appropriate to study at the university level. The literature review is found at Chapter Two.

Achievement in mathematics, mathematics anxiety, attitudes towards mathematics, attendance, and retention of students in the course, may all be affected by the use of cooperative learning. And the effects may differ between genders. Therefore, the research questions, stated on page six, were developed.

The quasi-experimental research design compared a control group of individual learners to three treatment groups using the Teaching and Learning model for cooperative learning. Group composition varied among the three treatment groups. The research participants were students in my liberal arts mathematics course at a two-year community college in the Midwestern United States. The ASAP placement test was used to determine if groups were equal.

The MARS and the Revised Math Attitude Scale were used to collect pre- and post- measurements of anxiety and attitude levels. Achievement effects were measured by final semester grades, and attendance was recorded daily. A thorough explanation of the design of the study is found at Chapter Three.

In summary, this study began with a thorough search of the available literature on the achievement effects of cooperative learning, gender-related differences in mathematics, and mathematics anxiety, particularly at the university (college) level. Chapter Two provides the review and synthesis of the literature. The study involved my students completing the ASAP placement test, pre- and post mathematics anxiety and attitude questionnaires, a researcher-constructed questionnaire, as well as completing the normal coursework and exams for a semester grade. An explanation of the methodology of the study is found at Chapter Three. Chapter Four presents the data analysis and interpretation. Chapter Five concludes and summarizes the study.

## CHAPTER TWO: LITERATURE REVIEW

Any research study is informed by research that precedes it. The literature review is a general review and synthesis of existing literature. It provides background information for the researcher and places the current study within the context of previous research in the field. Chapter Two provides an explanation of the literature search procedures followed by a review of recent research literature in three areas: (a) the achievement effects of cooperative learning, (b) gender-related differences in mathematics, and (c) mathematics anxiety. Finally, a summary ties it all together. These three areas help contextualise the framework of the current study, as is reported in the research methodology in Chapter Three.

### Literature Search Procedures

The literature search began in 1999 at Curtin University of Technology in Perth, Western Australia. A manual search of Curtin's library included browsing through stacks of journals such as *Educational Studies in Mathematics* and *Journal For Research in Mathematics Education*. An electronic search of databases allowed access to libraries at other universities in Perth: Edith Cowan University, Murdoch University, and University of Western Australia. The search continued in the United States from 1999-2002 at University Center Rochester, Rochester, Minnesota. The electronic search allowed access to libraries at any college or university in Minnesota or neighbouring states that are members of the Project for Automated Library Systems (PALS) network. PALS is a consortium of over 125 libraries interested in resource sharing, which uses a centralized library automation system (PALS) developed over the past twenty years. Databases that were searched included ERIC, Dissertation Abstracts International (DAI), and Info Trac. Info Trac contains 18,252,698 articles and the more recent ones can be downloaded.

In searching for articles on the achievement effects of cooperative learning in mathematics, the descriptors cooperative learning, collaborative learning, university, and mathematics were used alone and in combination with each other. Gender and math, girls and math, and math anxiety were the descriptors used for searching for articles on gender-related differences in mathematics and articles on mathematics anxiety. In addition to the searches for cooperative learning, gender, and anxiety in mathematics, separate searches were conducted to obtain the quantitative instruments used in this research study. MARS, mathematics anxiety, and math attitude scale were the descriptors used.

### **The Achievement Effects of Cooperative Learning**

This part of the literature review consists of three sections. The background section provides an explanation of cooperative learning methods. In the next section, a meta-analysis of research on cooperative learning conducted by Robert Slavin is reviewed. The third section is a review of some studies done specifically with mathematics at the university level. And the final section is a summary.

#### Background

Cooperative learning is understood to occur when small groups of people work together to help each other learn academic content (Davidson & Kroll, 1991). However, it is not as simple as that. Cooperative learning encompasses a variety of teaching methods, and because many of the research studies have looked at the various methods, an explanation of each method is necessary.

Slavin (1995) described the five Student Team Learning (STL) methods that were developed and researched at Johns Hopkins University. These methods include Student Teams-Achievement Division (STAD), Teams-Games-Tournaments (TGT), Jigsaw II, Team Accelerated Instruction (TAI), and Cooperative Integrated Reading and Composition (CIRC). Three concepts common to all STL methods are team rewards, individual accountability, and

equal opportunities for success. STAD teams are four-member learning teams that are set up to be heterogeneous in performance level, gender, and ethnicity. Students study together but take tests independently. Team rewards are granted on the basis of students' improvement. Teams-Games-Tournaments is similar to STAD but with the added dimension of the games. Jigsaw II is designed such that students work in groups where each student is assigned the responsibility to become an "expert" in part of a reading assignment. The experts on each topic first get together with each other to discuss and become more expert, then they return to their original groups and teach the material to the other members of their original group. Team Accelerated Instruction (TAI) is similar to STAD, but it incorporates cooperative learning with individualised instruction. It is specifically designed for grades 3-6. Cooperative Integrated Reading and Composition (CIRC) is a comprehensive reading and writing program for upper elementary and middle grades.

Other cooperative models described by Slavin (1995) include Aronson's Jigsaw, Group Investigation, and Structured Dyads. Jigsaw is similar to Jigsaw II except that Jigsaw places more emphasis on the task structure, and Jigsaw II places more emphasis on the reward structure. Group Investigation, developed by Sharan and Sharan at the University of Tel Aviv, is a method where students form groups of two to six members, select a topic from a unit being studied, investigate that topic, and report back to the entire class. Structured Dyads is a name for a variety of methods of students working in pairs to help each other learn.

The Learning Together model of Johnson and Johnson (1991), developed at the University of Minnesota, is similar to the STL models, but it requires five essential components to be successful: positive interdependence, face-to-face verbal interaction, individual accountability, social skills, and group processing. Students are assigned to groups of two to five members. Group rewards are used to create the positive interdependence. Face-to-face verbal interaction simply refers to the physical arrangement of students in groups. Individual accountability needs to be part of the assessment of the assignment. Unlike the STL methods,

the Learning Together model is designed to teach social skills while students work in groups. Group processing, the process of discussing how well the groups worked together, is also not included as part of the STL models.

#### Review of the Meta-analysis

Over the last 20 years much research, and analyses of research, has been conducted on the effectiveness of cooperative learning. Slavin's (1995) meta-analysis of 90 studies of the practical applications of cooperative learning in elementary and secondary schools included the subject areas of mathematics, science, biology, social studies, history, spelling, language arts, reading comprehension, English literature, English as a second language, and drafting. Locations ranged from schools in various U.S. cities to Israel, Nigeria, and the Netherlands. A minimum duration of four weeks was a requirement for the study to be included in the review. Other methodological criteria included the following: studies had to compare cooperative learning groups with control groups studying the same material; the studies had to show evidence of initial equality or use statistical controls for pre-test differences; and achievement assessment tools had to assess objectives taught in experimental as well as control classes. Effect size was used as the measure of the effect of cooperative learning on achievement. Some of the 90 studies compared multiple cooperative learning methods to control groups and were listed more than once, so Slavin had a total of 99 comparisons between control and cooperative learning methods. Enough data were included to calculate effect sizes for 77 of the 99 comparisons. Of those comparisons, 64% significantly favoured cooperative learning, while only 5% significantly favoured the control groups. Achievement effects varied widely, leading Slavin to further analyse the data according to type of cooperative learning method used.

Student Team Learning (STL) Methods. More than half of the studies used some form of STL, such as STAD, TGT, CIRC, or TAI. Of 29 studies involving STAD, 69% had significantly positive achievement effects and none of them was significantly negative. Of the 12 TGT studies, 75% were significantly

positive and none was significantly negative. All of the 8 CIRC studies and all of the 6 TAI studies were significantly positive. Overall, the STL studies had 77% significantly positive achievement effects, 23% no difference, and none was significantly negative.

Learning Together. Johnson and Johnson (1991) discuss three learning models: individualistic, competitive, and cooperative. The individualistic model is where the student works independently, and the individual's goals and rewards are not related to the goals and rewards of anyone else. In the competitive model the success of one student depends on the failure of others, hence by the design of this model, not everyone can succeed. The cooperative model is structured so that students work together to accomplish shared goals. The Johnsons agree that ideally, all three will be used in the classroom, as appropriate. However, it is the cooperative model that they promote. Their Learning Together models have been widely used and widely studied, particularly in comparison to individualistic and competitive models. According to Johnson and Johnson (1991), more than 375 studies over the last 90 years have looked at the achievement effects of cooperative learning:

When all of the studies were included in the analysis the average cooperator performed at about two-thirds of a standard deviation above average student learning within a competitive situation (effect size = 0.66) or an individualistic situation (effect size = 0.63). When only the high quality studies were included in the analysis, the effect sizes were 0.86 and 0.59, respectively. Cooperative learning, furthermore, resulted in more higher-level reasoning, more frequent generation of new ideas and solutions (i.e., process gain), and greater transfer of what is learned within one situation to another (i.e., group-to-individual transfer) than did competitive or individualistic learning. (p. 39)

However, many of their studies have been for a duration of less than four weeks and were not included in Slavin's (1995) review. Of the 12 such models that were included five (42%) were significantly positive, five (42%) showed no difference, and two (17%) were significantly negative.

Jigsaw. Thirteen of the studies used the Jigsaw or Jigsaw II model. The achievement effects were highly variable: 31% were significantly positive; 46% showed no difference; and 23% were significantly negative.

Group Investigation, Structured Dyads, and Other. Of the six studies modelled on group investigation three showed significantly positive results, and three showed no difference. All of the 6 structured dyad studies displayed significantly positive results. Of the seven “other” models of group work 29% showed significantly positive results and 71% showed no difference.

In attempting to determine what factors contribute to achievement effects, one must look not only at the different models used but also at the rewards used. From early on, studies have shown that the greatest effects of cooperative learning on achievement occur when groups are rewarded based on the individual learning of their members (Slavin, 1983). Individual accountability is important because it gives students the incentive to learn, and group goals or group rewards are important because they give students the incentive to help others learn. Slavin’s review showed that 64 of the 99 studies included both group goals and individual accountability. Of those, 78% had significantly positive achievement effects, while none of them was significantly negative. The median effect size was + .32 over 52 studies, whereas it was + .07 for the nine studies using group goals only, and also + .07 for the 12 studies using only individual accountability.

### Review of University Level Studies

According to Johnson, Johnson, and Smith (1998), most of the cooperative learning research prior to 1970 had been conducted at the university (college) level. Beginning in the 1970s, K-12 educators wanted to find out if cooperative learning would be as effective in their classrooms as it was demonstrated to be in college classrooms. It wasn’t until the 1990s that interest returned to researching the use of cooperative learning at the college level.

Johnson et al. (1998) conducted a meta-analysis of over 350 studies at the college level. Although the earliest dated back to 1924, 68 percent of the studies



were conducted after 1970. They found that “cooperative learning promotes higher individual achievement than do competitive approaches (effect size = 0.49) or individualistic ones (effect size = 0.53)” (p. 31). These results held for verbal, mathematical and procedural tasks. In addition to academic success the meta-analysis showed that “cooperative effort promotes greater liking among students than does competing with others (effect size = 0.68) or working on one’s own (effect size = 0.55)” (p. 31). Also, students working cooperatively perceived more social support from both peers and instructors than do students working competitively (effect size = 0.60) or individualistically (effect size = 0.51).

I reviewed several other university level studies, some of which would have been included in the meta-analysis discussed above. I have selected twelve studies to review more critically. Classroom instructors like myself who are experimenting with cooperative learning did many of the studies. All of the studies incorporated some form of cooperative learning as the treatment, and ten of the twelve studies had a control group. The study by Houston and Lazenbatt (1996) did not have a control group, but they compared the results of the cooperative learning treatment to results of previous classes that did not use cooperative learning. The study by Owens et al. (1998) also did not have a control group, but they measured changes in attitude and achievement, using pre- and post-tests. Half of the twelve studies indicated significant positive effects on achievement due to cooperative learning. Most of the others indicated no significant difference. Only one of them had the control group doing better than the treatment group. However, many of the studies have serious flaws that prevent them from being generalisable. The studies are summarized in Table 1 and a discussion of their weaknesses follows.

Table 1. Cooperative Learning Studies in University Level Mathematics.

Article/Location	Sample	Duration	Treatment	Evaluation Design/Effects
Dees 1982 Purdue University Calumet	100 university students in a remedial algebra and geometry course	1 semester	Cooperative learning lab groups with teaching assistants 6 hours per week	T-test No significant difference in achievement.
Freeman 1997 Lexington Community College Kentucky	584 Intermediate Algebra students – 253 treatment, 331 control	2 years	2-hour small-group workshop with supplemental homework problems in addition to standard lectures	Inter. Algebra: 49% treatment passed vs. 31% control. College Algebra: 82% vs. 51% (same students).
Harding & Fletcher 1994 Tennessee Technological University	154 TTU students in PreAlgebra and Beginning Algebra	2 semesters	Peer tutoring groups, STAD	ANOVA No significant difference in achievement.
Houston & Lazenbatt 1996 University of Ulster Northern Ireland	University students doing a module on mathematical modelling	1 semester	Peer tutoring groups with independent learning activity rather than teacher lecture model	Students didn't like the independent learning and achieved more or less the same as previous years.
Johnson, Johnson, Stanne, & Garibaldi 1990 Xavier University, New Orleans	48 high ability Black American entering high school seniors and college freshmen participating in Project Excel	3 hours	Cooperative learning with and without group processing	ANOVA All cooperative groups did better than the control, but highest achievement was in the group with teacher-and-student-led processing.
McCuen 1996 American River College California	3002 college students	1 semester	Collaborative study groups	86.6% success rate for treatment group, 55.2% for control.
Mears 1995 University of South Florida	14 sections of Basic College Algebra	1 semester	Modified lecture method with cooperative learning strategies	Significant positive difference in the groups that met Tuesdays and Thursdays, but not in those meeting Monday, Wed., Friday.
Mwerinde & Ebert 1995 University of Delaware	108 university students in College Algebra and Statistics	1 semester	Cooperative learning groups—heterogeneous based on algebra pre-test	Videotapes, audiotapes, analysis of written work. End of semester grades had control group do better.
Obiekwe 1991 Memphis State University Tennessee	219 Intermediate Algebra students at MSU	1 semester	STAD cooperative learning groups	ANCOVA No significant difference on achievement

Owens, Perry, Conroy, Geoghegan, & Howe 1998 University of Western Sydney, Macarthur	Students in pre-service primary teacher education course	1 semester	Problem solving in cooperative groups	Two-tailed t-test. Significant increase in attitude as well as achievement.
Rieck, Clark, Lopez 1995 University of Southwestern Louisiana	140 university students in Algebra for College Students and Integrated Collegiate Mathematics	1 semester	Integrated Collegiate Mathematics was a new course using cooperative learning. Homogeneous ability, no group reward	Two-tailed t-tests, paired and unpaired. No significant difference in achievement, but a positive change in attitude.
Wood 1991 Central Florida Community College	54 CFCC Intermediate Algebra students	1 semester	Cooperative group work and computer lab. Homogeneous achievement groups, group tests, individual final.	Experimental group had significantly more students pass and less withdraw.

Mwerinde and Ebert (1995) conducted the study in which the control group did significantly better than the treatment group. Baseline assessments had the experimental group more mathematically experienced, but the control group achieved higher grades. A weakness of this study is that the group work was only on four assignments. In addition, there's no mention of group goal or individual accountability on those assignments.

Next, the studies in which there were no significant differences are examined. In the first of these, Dees (1991) never actually assigned groups or required group work. Students were given lists of names and phone numbers of other students and were encouraged to work together. Because the students were not formally assigned to groups, there was no group interdependence.

Harding and Fletcher (1994) tried various grouping methods, but prior research has shown that heterogeneous ability grouping is better than homogeneous ability grouping. Like that of Dees, this study makes no mention of group rewards. How did they achieve group interdependence?

The Houston and Lazenbatt (1996) and Rieck, Clark, and Lopez (1995) were similar studies in that they both experimented with new courses. In the

study by Houston and Lazenbatt the attempt was to use peer tutoring to replace the teacher in an independent learning situation, but the difficulty of the content may have been a factor in the negative attitudes and the lack of achievement gains. Rieck et al. compared two different courses with different content and different tests, which could be an issue, but the biggest problem with their study was the 50% attrition, from 140 students at the start of the semester to 71 at the end, making the validity of their conclusions questionable.

Obiekwe (1992) had a 15% attrition rate, but he doesn't break this down for the treatment and control groups. If he followed the STAD principles exactly, one can assume that he had heterogeneous ability groups and group rewards, although no mention was made to that effect. Four teachers were involved in this study, but no mention was made of the training they had in cooperative methods.

Next, I examine the studies that were successful in achievement gains. Mears (1995) had interesting results in that the groups meeting for a longer period of time, fewer times a week (Tuesday, Thursday) had significant differences in achievement, but those meeting for a shorter time, more times per week (Monday, Wednesday, Friday) had no significant difference. The various groups met at seven different time periods throughout the day for the same total amount of time per week. Cooperative group work takes time, and the results suggest that the MWF groups didn't spend enough class time working together. An important factor in using cooperative learning methods is the teacher. Mears indicated that the teachers involved in his study would have preferred more extensive training in the cooperative methods. No mention is made of group goals.

Freeman (1997) followed students over two courses and continued the study for two years. His results look really great, but they may be biased by the fact that students in his treatment group chose to attend an extra two-hour workshop per week where they were given extra problem sets. Students who choose to do this extra work tend to be the more highly motivated, higher achievers. Hence, the possible bias.

Johnson, Johnson, Stanne, and Garibaldi (1990) looked at different types of group processing. Cooperative learning with no processing, with student-led processing, with teacher-led processing, and with teacher-and-student-led processing were all examined. However, Slavin's criticism of some of their earlier research would hold here as well. The situation was artificial in that it was done during the summer with elite students, not with a typical class. By the time they divided 48 students into all those groups they had a very small sample size for each group. That and the fact that it was just a three-hour experiment serve to make the study's findings ungeneralisable.

Wood (1992) included group homework assignments as well as group tests. The individual accountability came in the form of the final exam. Hence, she included both group goals and individual accountability. However, the final exam counted for only 10% of the final grade. To test for achievement gains she should have compared final exam scores instead of final course grades, since 90% of the course grade was assessed on group work.

The study by McCuen (1996) involved students in a Peer Assisted Learning (PAL) program involving collaborative study groups in selected mathematics and science classes. This is an ongoing program for which changes are made based on annual evaluations of the project. Of the 3002 students who took PAL courses, 887 participated in collaborative study groups. These 887 students had an 86.6% success rate compared to 55.2% for the other 2,115 PAL students. When students self select to participate in outside study groups, there is a bias created by the fact that students who prefer the cooperative mode of study and who are motivated and willing to put in extra study time outside of class are more likely to be in the treatment group.

The final study, by Owens, Perry, Conroy, Geoghegan, and Howe (1998), was unique in that it dealt with students studying to become primary teachers. There were no control and treatment groups to compare. The study used

cooperative groups as a method for teaching problem solving. Attitude improvement as well as achievement gains were the outcomes.

### Summary

One may agree with Slavin (1995) that the Johnsons' studies of duration less than four weeks were artificial and lacking generalisability, however, excluding those studies, there are still ample studies involving a variety of cooperative learning models. The data presented by Slavin and the Johnsons clearly show that cooperative learning when implemented according to one of the models described and including both individual accountability and group goals will result in positive effects on achievement. After 20 years of extensive research the consensus is that at the elementary and junior high level (with basic skills objectives) cooperative methods using group goals and individual accountability "accelerate student learning considerably" (Slavin, 1989, p. 54). Even when no significant gain is shown in achievement, there are often significant gains in attitude, attendance, social skills, and interpersonal relations, which warrant using this teaching method.

What these studies have not tested is the effect of group processing, which is one of the components of Learning Together, but not a component of the other models. Group processing as defined by Johnson and Johnson (1991) refers to group members reflecting on a group session, deciding which actions of the group members were productive or non-productive and which actions or behaviours to change or leave the same. Is group processing really necessary? Some of the research (Johnson et al., 1990) indicates higher achievement among groups that process. However, further research is needed on the effects of group processing.

Also, the reviews are mixed regarding the use of group work without group goals. Group goal or group reward is a technique for making the groups more interdependent. The idea is to get students to take responsibility for their learning as well as the learning of others. Are group goals necessary at the university level? Also, is cooperative learning equally effective for both genders?

And does cooperative learning have an effect on mathematics anxiety at the university level?

### **Gender-Related Differences in Mathematics**

The fact that males outnumber females in participation in advanced mathematics courses and in mathematics-related careers is well documented and well researched. Baker and Jones (1992) cited work by Benbow and Stanley in the early 1980s that ignited the controversy between biological and sociological explanations for gender differences in mathematics. According to Baker and Jones, Benbow and Stanley maintained that their findings indicated male superiority in mathematical ability, and they attributed this superiority to biological differences between the sexes. Baker and Jones refute the biological argument from two perspectives. First, if the gender differences are biological, there should be no cross-national variances, which research shows exist. And second, the biological view doesn't explain the fact that gender differences have changed over time. Fennema, Walberg, and Marrett (1985) argue that studying biological variables in sex differences is not worthwhile in the sense that it assumes nothing can be changed and is not useful in effecting change. Leder (1996) states that even those who argue that biology is an important factor in gender differences admit that biological factors have a small effect compared to the social and cultural effects. Hence, much research has been devoted to sociological factors, which are alterable. This review will focus on the research literature relating to sociological factors effecting gender-related differences in mathematics. The first section is devoted to the literature defining the gender differences in mathematics, while the second section examines teacher-student and student-student interactions. In the third section, university level studies that examine gender effects are reviewed. A summary of gender-related differences in mathematics follows in the fourth section.

### What Are the Gender-related Differences in Mathematics?

Achievement Differences. To support their sociological stance, Baker and Jones (1992) analysed data from the 1982 Second International Mathematics Study (SIMS). The SIMS data for population A (13-year olds) contained test scores from a forty-item test administered to a representative sample of middle school mathematics classrooms in twenty-four countries. The data also included qualitative information from students about their curriculum access and their perceptions of parents' encouragement to study mathematics. In addition, the researchers gathered data from published sources in each country regarding gender participation in advanced education and the labour market. To check for cross-national variances, mean differences between genders were calculated by subtracting the mean score of females from the mean score of males for each country. They found variation between the size of the differences as well as the direction. In eight national systems there were no significant gender differences in the scores; in seven systems males scored better than females; and in four systems females did better than males. To determine if gender differences have changed over time, they compared the SIMS data to that of the First International Mathematics Study (FIMS) of 1964. There was a change in seven of the nine national systems for which they had both FIMS and SIMS data. Five of those seven cases went from a male advantage in 1964 to no difference in 1982. One went from a female advantage in 1964 to a male advantage in 1982 and one reversed that advantage. Two countries maintained their male advantage. Their other data, gathered from students and published sources, correlated with gender differences on test scores. Countries that promote equal opportunities for females and males in access to higher education and the labour market had smaller differences between genders in mathematical performance. Also, the general decline in gender differences in mathematics performance corresponds to broader opportunities for females. It is important to note the grade level of the students participating in the SIMS study. They were all seventh and eighth graders. It is worth noting that, in their respective studies, Ethington (1992); Frost, Hyde, and Fennema (1994); Hyde, Fennema, and Lamon (1990); and Leder (1996) found that gender differences increase with age.



The SIMS data were also analysed by Ethington (1990). She looked at the longitudinal data file of SIMS, which included eight of the 24 countries. Data were from the Population A (eighth grade in the U.S. and other countries and seventh grade in Japan). Results indicate no significant gender differences in any of the content areas (fractions, ratio/proportion/percent, algebra, geometry, and measurement), and where there were slight effects, they favoured females more frequently than males.

Casey, Nuttall, and Pezaris (2001); Fierros, (1999); Lokan, (1999); and Wester and Henriksson (2000) analysed the Third International Mathematics and Science Study (TIMSS) data for gender differences in performance. Wester and Henriksson examined the effect of change from multiple-choice to open-ended questions on the TIMSS. Their sample involved sixth, seventh, and eighth graders in Sweden. Ten questions were selected and the students were all given identical questions, but some students had test booklets in which questions were a multiple-choice format, while others had booklets in which the choices were removed, leaving open-ended, short-answer questions. “The results showed no significant changes in gender differences when item format is altered” (p. 87). Females’ achievement was slightly better in both formats. This contradicted prior research (Hellekant, 1994; Ben-Shakarait & Sinai, 1998) cited by Wester and Henriksson that suggested the multiple-choice format favoured males and the open-ended format favoured females.

On the other hand, Fierros (1999) examined the TIMSS data for gender differences by item format (multiple choice, short answer, or extended response) and found little difference at the eighth grade, but larger differences in all three formats in the twelfth grade. He also analysed the data by gender for low-, middle-, and high-performers, and again found few gender differences in the eighth grade, but in the twelfth grade in a majority of the countries, high-performing (top 25%) males outperformed high-performing females.

Lokan (1999) attempted to examine equity issues in the TIMSS Performance Assessment tasks based on variables such as gender, language, and socioeducational background. Lokan cites prior research on equity issues (Klein, Jovanovic, Stecher, McCaffrey, Shavelson, Haertel, Solano-Flores, & Comfort, 1997) that found that “girls tended to do better than boys on the performance measures, although boys tended to do better on particular types of questions” (p. 298). However, internationally, few gender differences were found on Performance Assessment tasks of the TIMSS.

Gender differences of the Mathematics Scholastic Aptitude Test (SAT-M) for a sample of college-bound students were examined by Casey, Nuttall, and Pezaris (1997). They found that the male advantage is mediated by two factors: better mental rotation ability and higher mathematics self-confidence. Further, the mental rotation ability had almost twice the effect of self-confidence. They found similar results in their recent study of the TIMSS United States eighth-grade data (Casey et al., 2001). In this eighth-grade sample “the composite measure of spatial-mechanical skills accounted for three times the mediational gender effects of mathematics self-confidence” (p. 46).

In their study, Walsh, Hickey, and Duffy (1999) conducted experiments with Canadian students of two age groups to determine if gender labelling (male character, female character, or gender neutral) accounted for gender differences in mathematical problem solving. A sample of seventh and eighth graders were given a modified version of the Canadian Test of Basic Skills, and a sample of university students were given a modified version of a model Standardized Achievement Test. The results of this study found that gender labelling affected seventh and eighth graders’ performance, but it did not account for gender differences because both males and females were more accurate in answering male-labelled items than either female-labelled or gender neutral items. The performance of the university students was not affected by gender labelling.

Hyde et al. (1990) conducted a meta-analysis of 100 studies of gender differences in mathematics performance. They, too, found that the magnitude of gender differences in performance has declined over the years. They determined that gender differences are small and are dependent on the type of mathematics problems. The overall effect size was 0.15, and the largest effect sizes they found were 0.29 for problem solving in high school and 0.32 for problem solving in university studies. Females are superior in computation; there is no difference in understanding of mathematical concepts; and males are superior in problem solving. These differences do not appear until high school. They suggest looking at factors other than ability to explain why fewer numbers of females enrol in college-level mathematics courses and choose mathematics-related careers.

Tate (1997) reviewed studies that included elementary, middle, and secondary students. He was trying to determine national trends in mathematics achievement of various social groups, including gender. He found that achievement differences tended to favour males but that the differences were small and insignificant. On the other hand, in his study of American high school geometry students, Battista (1990) found that males performed significantly better on spatial visualization, geometry achievement, and geometric problem solving. He found no gender differences on tests of logical reasoning.

While females in her study received higher course grades, Bohlin (1994) discovered that they scored lower on standardized tests, such as the PSAT and SAT. This phenomenon may be explained by the studies mentioned earlier that examined the types of problems in which males or females perform better. Course grades are based on teacher-constructed tests, which require the students to use methods they learn in class. It's possible that standardized tests require more creative problem solving methods, or methods not taught in class.

Affective Differences. Several studies examined affective factors that may have contributed to differences in performance. Females were found to be less confident of their mathematical ability (Bohlin, 1994; Leder, 1996; Norton &

Rennie, 1998; Sax, 1994) and to be more math anxious (Frost et al., 1994; Norton & Rennie, 1998). Males and females, but mostly males, stereotyped mathematics as a male domain (Ethington, 1992; FitzSimons, 1997; Norton & Rennie, 1998). Females were less interested in activities and careers requiring mathematical or technological applications (American Association of University Women, 1992, 2002; Boaler, 1997a; Bohlin, 1994; Correll, 2001; Fennema, 2000; Frost et al., 1994; Hyde et al., 1990; Turner & Bowen, 1999; Zeldin & Pajares, 2000), preferred a less competitive learning environment (Boaler, 1997a; Sax, 1994), and had a greater need for structure in that environment (Bohlin, 1994). In addition, females were more interested in understanding the mathematics, whereas males just wanted to get it done (Boaler, 1997a). However, the meta-analysis of 113 studies by Ma and Kishor (1997) found no significant gender effect on the relationship between attitude and achievement, and the meta-analysis of 26 studies by Ma (1999) found no significant gender differences on the relationship between mathematics anxiety and achievement.

Ethington (1992) examined gender differences in psychological influences on mathematical achievement. For both males and females prior achievement was by far the main factor influencing current achievement. For males, task value was the only other significant factor, and it had a much lower effect on current achievement. Whereas, for females, perception of difficulty, stereotyping mathematics as a male domain, and family help and encouragement were all significant factors. However, Ethington notes that while expectation for success did not appear as a significant factor in her study of eighth graders, other studies have found that the influence of expectation for success on current achievement increases with age.

In their meta-analysis, Frost et al. (1994) analysed 100 studies of gender differences in mathematical performance and 70 studies relating to gender and mathematics-related attitudes and affect. The studies included ages five to over 26. They found that across all studies differences in mathematical performance were small in favour of males. Concurring with Hyde et al. (1990), the largest

gender differences occurred in problem solving, beginning in the high school years. They concluded that the small differences in attitudes and affect individually were not likely to account for differences in participation and achievement, but collectively they might have an effect. Frost et al. believed that males, in the form of peers, teachers, and family members, may be subtly discouraging females from participation and achievement. They also found that the gender differences in mathematics anxiety were too small to account for differences in participation and achievement, and concluded that

the results indicate that gender differences in mathematics performance and mathematics-related attitudes and affect are, at most, small to moderate. Consequently, it appears that we must look beyond abilities and intrapsychic factors in order to account for the substantial under representation of females in advanced mathematics classes and mathematics-related occupations (Frost et al., 1994, p. 383).

However, consistent with Ethington (1992); Hyde et al. (1990); and Leder (1996) they found significant age trends for attitudes toward mathematics, mathematics self-concept and self-confidence, and the usefulness of mathematics. Among older subjects (19–25-year olds) the gender differences were larger. Consistent with Norton and Rennie (1998), they found that males are more likely than females to stereotype mathematics as a male domain.

Both Norton and Rennie (1998) and Atweh and Cooper (1995) studied the effects of school environment in Australian schools. Norton and Rennie compared single-sex and coeducational schools, one each of private and government schools. Using five of the Fennema-Sherman Attitude Scales they found that as females move to higher grade levels in school their attitudes toward females' mathematical ability become less positive. They also found that females from single-sex schools had more positive attitudes than those from coeducational schools on all of the five scales. Atweh and Cooper conducted a case study that compared two private all-girls' schools of different socio-economic and ethnic backgrounds. The perceived needs of the students varied. Those from the low socio-economic school needed mathematics to be useful and applicable to everyday life. These needs were in conflict with the curriculum. Those from the

high socio-economic school needed mathematics that would gain them entry into university. These needs matched the curriculum design.

### Teacher-Student and Student-Student Interactions

Whole-Class Interactions. Jungwirth (1991) studied teacher-student interactions during whole-class discussions. She found that male students are more willing to participate than females. She states that studies that examined the frequency of teacher-male and teacher-female interactions found that teachers call on males more often, ask males more questions, give males more feedback, and in general, are more encouraging of males. In her study encompassing grades five through twelve, Jungwirth found that “The more ambiguous a question is, the more students have to rely on trial and error, the less girls actively participate in the interaction” (p. 269). In an attempt to explain this, she cites work done by Maltz and Borker who discuss social differences within same-sex peer groups. Females tend to develop a closeness where they treat each other as equals and criticize each other without being harsh. Males, on the other hand, have to jockey for a position within their peer group by telling stories and jokes, which are often met with criticism, mockery, or other challenges. In this sense, the boys are more prepared for a challenging interaction with a teacher. They are used to having someone challenge their responses and don’t mind it the way a girl would mind a teacher correcting her. Another study (Fennema, Peterson, Carpenter, & Lubinski, 1990) determined that teacher attitude affects gender differences in that it influences both students’ participation and motivation.

If it is true that teachers direct the interactions that take place in whole-class discussions, then cooperative group work should benefit females because the group work minimizes direct interaction from the teacher.

Small Group Interactions. Holden (1993) also found that boys dominate classroom talk in whole-class situations. However, in her study of small groups conducted with 4-11-year olds in Britain, she found that in cooperative groups the gender balance of the group affects the amount and nature of talk. Females’

abstract level of talk is depressed in groups where males outnumber females. In language tasks females talked more if the number of females was greater than or equal to the number of males, but not when males outnumbered females. In mathematical tasks, males talked more if the numbers of males and females were equal, and there was equal talk if the females outnumbered the males.

Webb (1984, 1985, 1991) conducted similar studies of gender composition of cooperative groups with mostly seventh and eighth graders, and a few ninth and eleventh graders. She used three types of groups, majority male (3M, 1F), majority female (3F, 1M), and equal males and females (2F, 2M). She found that if the groups were equal in the number of males and females, both sexes had similar learning experiences. But if the groups were unequal, females were disadvantaged. In the majority male groups, males focused their attention on the other males and the female was ignored. In the majority female groups, the females turned to the male for answers. They asked for explanations, but their questions went unanswered. In addition to studying the gender composition, she also studied the ability grouping. She found that students in uniformly high-ability groups assumed that everyone else knew how to do the work, so no help was given. In uniformly low-ability groups, no one felt adequate to help others. The uniformly medium-ability and mixed-ability groups exhibited the most helping behaviours. But if the mixed-ability groups contained high-medium-low-abilities, the mediums tended to be ignored while the highs helped the lows. It is important to note that Webb's studies were of short duration, the longest lasting three weeks. Also, these were mostly junior high school students. These results may not be generalisable to adult students in a classroom situation lasting one semester.

Another perspective for viewing small group interactions is from the status of group members. Status refers to social rankings in which it is believed that it is better to be higher rather than lower. Peterson, Johnson, and Johnson (1991) examined status within the group from the perspective of expectation states theory and social interdependence theory. "Expectation states are stable beliefs about the

performance of a person with a given status characteristic” (Peterson et al., 1991, p. 718). Expectation states theory suggests that in cooperative groups the male status is a reflection of the male’s status within society and that the status will increase the longer the group works together. “Social interdependence exists when each individual’s outcome is affected by the actions of others” (Peterson et al., 1991, p. 719). Social interdependence theory suggests that status is irrelevant within the group. The study by Peterson et al. involved 75 sixth graders in groups of four who met 55 minutes per day for 14 days. Their results support the social interdependence theory. There were no significant differences in achievement, verbal participation, perceived leadership, or status. The results were the same whether the gender composition of the group was male-majority, female-majority, or equal-sex. On the other hand, Cohen and Lotan (1995) took the perspective that the expectation states theory held, and attempted to find treatments to limit the impact of status on interactions. Not enough information was given as to how the groups were structured regarding goals, but the differences in these two studies could be that Peterson et al. (1991) built interdependence into the group structure using group goals. In their study of fifth and sixth grade science students Johnson, Johnson, Scott, and Ramolae (1985) reiterate the need for this. They found no significant difference in the interactions of mixed-sex and single-sex groups. “Since previous research indicates that in mixed-sex learning groups males will dominate, it may be concluded that clearly structuring cooperative interdependence within learning groups does increase the participation of female students” (p. 218).

A study by Perrenet and Terwel (1997), conducted with secondary students in the Netherlands, assigned the roles of captain, facilitator, recorder, provider of resources, and reporter to the group members. They found that when the captain was a boy there was no resistance, but when the captain was a girl the boys showed resistance most of the time. Girls rarely offered resistance.



### University Level Studies

Evidence suggests that cooperative learning is a preferred learning style for women (Belenky, Clinchy, Goldberger, & Tarule, 1986; Boaler, 1997a; Boaler, 1997b; Holden, 1993; Fennema et al., 1990). According to Belenky et al. (1986), women tend to be “connected knowers.” In their discussion of cooperative learning as one of the four principles of feminist pedagogy, Jacobs and Becker (1997) discuss how cooperative learning lends itself to those connected knowers. As noted in Chapter One, according to the American Association of University Women (1992), women prefer and are more successful in a collaborative, less competitive classroom. Several studies at the tertiary level on the effectiveness of cooperative learning also examined the gender-related effects. A few of them are highlighted here and some of them were mentioned previously in this review.

Joiner (1999) conducted research involving calculus reform in a university setting. He found that males achieved better in the traditional, lecture method classroom, whereas females performed better in the calculus reform models, which included cooperative learning. In another calculus study, Bryant (1998) compared the small group instruction method to the traditional lecture method and found no significant differences with respect to gender or race. Informal studies of the use of cooperative groups in calculus (Kast, 1993; Myers, 1993) did not compare gender differences, but concluded that generally, cooperative groups were successful.

Rieck et al. (1995) compared a new course, Integrated Collegiate Mathematics, which incorporated cooperative learning, to a traditional Algebra for College Students. The cooperative groups were of homogeneous ability, and there was no group goal. The results indicated no significant difference between genders. There was a significant attitude change overall, but it was not significant between genders. It is important to note that this study compared achievement results of tests where the content of the two courses was different. Also, there was

a large dropout from 140 students to 69 who actually completed the pre- and post-surveys and a large gender imbalance of 52 females and 17 males.

There have been other studies that have investigated gender differences at the tertiary level, but not involving cooperative groups. Druva-Roush (1994) conducted a study of 105 Intermediate Algebra students, 54 females and 51 males, at a large Midwest American university. She was interested in gender differences as well as differences between math-anxious and non-math-anxious students. There were no gender differences on overall comprehension scores or on overall anxiety scores. But she found that males and females perform differently on different kinds of tasks and at different anxiety levels. For females, as the anxiety level increases, the ability to comprehend problem statements of computational and analytic type problems decreases.

FitzSimons (1997) studied adult and vocational mathematics education in Australia. She argued that vocational mathematics does not serve the interests of the individual, nor does it address issues of gender equity. She also included a feminist critique of competency-based education. On the other side of the world, Sax (1994) criticized American universities for reinforcing the gender gap. The results of her study, including 8,997 women and 6,053 men at 392 four-year universities and comparing pre- and post- self-ratings, indicated that after four years of college both men and women become less confident in their mathematical abilities. The effect was stronger for women. However, those women whose curricular program is more math-oriented gain confidence while at university. I question whether college impacts mathematics self-concept so much as not studying mathematics does. Would these same people have a decline in mathematics self-concept if they were not in college and not studying mathematics? To blame college for reinforcing the gender gap is a little extreme. This study shows that college reinforces the decline in math confidence by not requiring students to study more mathematics. If students are not required to study any more mathematics, then what is the purpose of researching their mathematics self-confidence?

Norwood (1995); Walker and Plata (2000); and Thomas, Higbee, and Dwinell (1992) studied under-prepared students attending community colleges and universities. Students were placed in developmental mathematics courses based on a placement test of algebra skills. Developmental courses are remedial courses for students who are not prepared to take college level mathematics. Success rates in these courses vary along the lines of both gender and race. Some recommendations were given to improve the success rate, and group learning was advocated to increase the success rates with women, older adult students, African-American and Hispanic students.

### Summary

Many factors, biological, psychological, and sociological, contribute to gender differences in mathematics. There are achievement as well as affective effects. The importance of the factors seems to change over time, so that the effects tend to increase with age. The achievement differences tend to be small, depending on the type of mathematical tasks, and the magnitude of the differences seems to have declined over the years. We seem to be on the right track, but we still have a long way to go.

“Gender differences in achievement and participation in mathematics no longer are regarded as either natural or inevitable” (Willis, 1996, p. 41). Nor are the existing differences permanent. Teachers can change the way they interact with their students, and they can, to some degree, control student-student interactions by controlling group composition. As Gill (1997) stated, “The task now for mathematics education research is to identify those features of mathematics learning which are working for female students and to build on this experience.” I would include male students as well, because not all male students succeed with the traditional methods. Cooperative learning appears to be one of the experiences we should build on. According to Terwel, Herfs, Mertens, and Perrenet (1994, p. 229), “Cooperative learning is a resource-sharing process. Students benefit from the collective supply of knowledge, experiences, concepts, reasoning skills, and problem solving strategies in their group.” However, the

cooperative learning must be structured to include individual accountability as well as group interdependence. And an interesting phenomenon is the importance of the gender composition and ability composition of the group.

There is an abundance of research on gender and mathematics, and an abundance of cooperative learning studies. However, there is a need to discover what teaching methods and learning environments are most suitable for both genders at the university level, particularly, if it is true that gender differences increase with age. Another reason this type of research is important is because many people choose their careers based on the mathematical requirements in the program of study.

More than any other school subject, mathematics serves as a race, sex and class filter. There is power and status associated with the study of mathematics. For those who achieve in the subject, there are substantial rewards in the opportunities available for entrance into well-paying fields of study and work (Kreinberg & Lewis, 1996, p. 181).

Another affective factor in mathematics achievement is mathematics anxiety. Kast (1993) found that the use of cooperative learning helped to reduce mathematics anxiety. That is also a part of the present study, and the literature review continues with a review of mathematics anxiety research.

### **Mathematics Anxiety at Colleges and Universities**

Mathematics anxiety is a phenomenon that creates difficulties for both the student and the teacher. Not all mathematics students with negative attitudes have mathematics anxiety, nor do all students with mathematics anxiety have negative attitudes. However, in some students there exists a negative correlation between the degree of mathematics anxiety and their attitudes toward the study of mathematics. The question is whether the use of cooperative learning helps to reduce anxiety and to improve attitudes. That is a part of the current research study involving the effectiveness of cooperative learning. This review looks at

some of the literature on mathematics anxiety and attitudes toward mathematics. The first section looks at some of the research that has been done to define mathematics anxiety and its components. The second section reviews some studies done on mathematics anxiety and attitudes toward mathematics at the university level. A summary is provided in the third section.

### What Is Mathematics Anxiety?

A certain amount of anxiety is normal and benefits learning, but mathematics anxiety is often of the “panic” type that inhibits learning. The fear is out of proportion to the situation. As stated in Chapter One, Richardson and Suinn (1972) found that mathematics anxiety consists of tension and anxiety so severe that it interferes with everyday activities, such as managing money. They also found that mathematics anxiety exists among university students who generally don’t suffer from other types of anxiety. Their Mathematics Anxiety Rating Scale (MARS) has been widely validated and used extensively in research studies. Brush's (1978) administering of the MARS to university students of various academic majors validated Richardson and Suinn’s, confirming that MARS is a reliable instrument to test for mathematics anxiety.

Rounds and Hendel (1980) viewed mathematics anxiety from the clinical psychologists’ perspective and proposed that for counselling purposes, to tailor treatment to individual needs, it would be beneficial to look at the various factors of mathematics anxiety. They took the MARS instrument and isolated the factors that measure Mathematics Test Anxiety and Numerical Anxiety. Their study involved 350 women participating in a mathematics anxiety treatment program at a large Midwestern university. Ferguson (1986) took that a step further and in addition isolated Abstraction Anxiety. Bessant (1995) also used a factor analysis of the MARS to analyse the dimensions of mathematics anxiety. His conclusion was that mathematics anxiety is multi-dimensional, consisting of general evaluation anxiety, everyday numerical anxiety, passive observation anxiety, performance anxiety, mathematics test anxiety, and problem-solving anxiety.

In researching affective issues involved with problem solving, McLeod (1988) compared emotion and attitude. He said that emotion is very intense, but of short duration, whereas attitude is not very intense but of long duration. Mathematics anxiety can have peaks of intensity, but it does not seem to be of short duration. Test anxiety may be an emotion, but based on observations of her students, this researcher believes that mathematics anxiety is consistent and of long duration. Mathematics anxiety is more an attitude than an emotion. Moreover, I agree with Middleton and Spanias (1999) that mathematics anxiety is learned. In a discussion of motivation they say that motivational patterns are learned. They suggest that students learn to dislike mathematics and the dislike becomes part of their mathematics self-concept.

#### University Studies of Mathematics Anxiety

Prevalence. Betz (1978) tried to document the prevalence of mathematics anxiety in college students. She used a revised version of the Fennema-Sherman Mathematics Anxiety Scale with 652 freshmen and sophomore students at Ohio State University. Three groups of students were selected. Group one was students in a developmental mathematics course, group two was students in a pre-calculus course, and group three was students in an introductory psychology course. Group one represented students who were under-prepared for a college mathematics course, group two represented students who were majoring in fields that required calculus, and group three represented students in a variety of major fields, since this introductory psychology course was required for many degree programs. Half of the developmental group and one-fourth of the other two groups expressed feelings of mathematics anxiety. Half in all three groups indicated mathematics test anxiety. The developmental mathematics and psychology groups had significantly higher levels of anxiety among females. The pre-calculus group had no significant difference between genders. Levels of mathematics anxiety related to age of females, with older women having more anxiety. And levels of anxiety were related to number of years of high school mathematics in both males and females, indicating that under-prepared students are more mathematics anxious.

Anxiety and Achievement. Clute (1984) cites several studies (“Aiken, 1970; Callahan and Glennon, 1975; Crosswhite, 1972; Hendel, 1977; Richardson and Suinn, 1972; and Szetela, 1971”) that have shown that high achievement is related to low anxiety in mathematics. This held for primary school through university. Clute’s study investigated the relationship of the two factors, anxiety and teaching method, to achievement in mathematics. The study involved 81 students at two California universities. The MARS was administered to determine levels of mathematics anxiety, and then students were randomly assigned to treatment groups. She compared two methods of instruction, discovery method and expository method. Both methods were teacher led, but involved lots of teacher-student interaction. The difference was that the expository method had the teacher giving more directions, whereas the discovery method had the students discovering what to do. The results were that the groups with lower levels of mathematics anxiety achieved more with the discovery method and the groups with high levels of anxiety achieved more with the expository method. In her university study mentioned earlier, Brush also found a negative correlation between MARS scores and a test of mathematical knowledge. Hembree (1990) in his meta-analysis of 151 studies on mathematics anxiety concluded that, “Mathematics anxiety is related to poor performance on mathematics achievement tests. It relates inversely to positive attitudes toward mathematics and is bound directly to avoidance of the subject” (p. 33). One important construct Hembree did not link to mathematics anxiety is intelligence. Some people who are highly successful in other areas are found to be mathematics anxious, which contributes to the theory that mathematics anxiety is learned.

Cooperative Learning Studies. Several university studies involving cooperative learning effects on achievement also examined mathematics anxiety or attitudes toward mathematics. Rieck et al. (1995) designed a college mathematics course based on the NCTM standards and used reform strategies such as “real world” problem solving and cooperative learning. They compared this to a control group that was a traditional college algebra course. Pre- and post-attitude inventories showed a significant positive change for the cooperative

learning group but not for the control group. However, with two different courses being compared there was a big difference in course content as well as the difference between traditional and reform teaching strategies.

Mathematics anxiety is widespread among elementary education majors. Hembree (1990) found that college students majoring in elementary education maintain the highest levels of mathematics anxiety as compared with other college majors. Pre-service elementary teachers in a mathematics methods course were the focus of studies by Owens et al. (1998) at the University of Western Sydney, Macarthur. Their experiential learning cycle approach involved using problem solving activities and cooperative learning methods. The four phases of the learning cycle were experiencing, discussing, generalizing, and applying. The students met for 13 three-hour sessions. Each session consisted of working in small groups to solve problems. Attitude and belief questionnaires were administered at the start and end of the semester. The results indicated very significant increases in attitudes toward mathematics with this model of teaching. Sloan, Vinson, Haynes, and Gresham (1997) conducted a similar study on mathematics anxiety among pre-service teacher candidates in a mathematics methods course at Athens State College, Alabama. The MARS was administered at the beginning and again at the end of the semester. The methods course was taught using concrete manipulatives and active learning approaches. The results indicated a significant decrease in the anxiety levels when taught by this method.

### Summary.

Does a weak background or lack of knowledge and understanding in mathematics cause mathematics anxiety, or does mathematics anxiety produce a lack of knowledge in mathematics? There's no definitive answer to this question. However, much of the research tends to support the former. Students with prior poor achievement in mathematics tend to see themselves as bad at the subject, and this poor self-efficacy creates or increases the anxiety. Research does indicate that mathematics anxiety is inversely related to achievement. It also concludes that mathematics anxiety is widespread among university students and is highest



among elementary education majors. This is relevant to my study because elementary education majors are one of the larger groups enrolled in the liberal arts mathematics courses.

Be it emotion or attitude, mathematics anxiety has a powerful influence on people's lives in the sense that it causes mathematics avoidance and causes people to choose their careers based on the mathematical requirements in the program of study. There is little doubt that many students experience anxiety, be it mathematics anxiety or test anxiety. For this reason it's important to research the causes, the effects, and the treatments. Because it is a psychological factor, clinical and counselling psychologists have done much of this research. However, I think it's time that teachers examine the environments they are creating in their classrooms to determine if they influence the level of anxiety. Since achievement and anxiety seem to be inversely related, increasing the achievement may also decrease the anxiety. There is ample research (Johnson & Johnson, 1991) that clearly indicates that cooperative learning methods that incorporate both individual accountability and group goals result in positive gains in achievement. Increased achievement and communication with group members should help reduce anxiety. Another advantage of working cooperatively is that students can credit their successes to themselves and attribute failures to the group, thus avoiding the development of negative self-efficacy. According to the Johnsons (1991, p. 43), "Compared to competitive and individualistic learning experiences, cooperative ones promote more positive attitudes toward the subject area being studied." And in their discussion of motivation for achievement in mathematics, Middleton and Spanias (1999, p. 78) stated,

In short, when teachers emphasize understanding of mathematical concepts and provide facilitative classroom environments, students tend to be more receptive and less anxious with regard to mathematical activities than when teachers stress rote activities and are perceived to be authoritarian.

The research indicates that cooperative learning should show positive gains in both anxiety and achievement. The ultimate goals are to improve achievement and reduce anxiety. The two goals seem to be positively correlated,

so that success with one should bring some success with the other as well. This research study will attempt to determine if cooperative methods help to attain these goals.

### **Summary**

This review has examined some of the vast amounts of literature in three areas that may influence mathematics achievement at the college and university level: (a) the achievement effects of cooperative learning, (b) gender-related differences in mathematics, and (c) mathematics anxiety. These three areas help form the framework of the current study, as is reported in the research methodology in Chapter Three.

## **CHAPTER THREE: METHODOLOGY**

### **Overview**

I was both researcher and classroom teacher as I conducted this research study, which examined the effects of cooperative learning in mathematics courses at the community college level. The type of course researched was a liberal arts mathematics course entitled Contemporary Concepts in Mathematics. The effects that were examined were achievement, anxiety, attitudes, attendance, and course retention. Also, gender-related differences were examined for each of these effects. This chapter provides an explanation of the design of the study, the sample, instrumentation, and data collection, followed by a discussion of issues of reliability and validity, data analysis and interpretation, and limitations of the study.

### **Design of the Study**

This was a quasi-experimental study because I could not randomly assign students to treatment or control groups. Students began enrolling in May for courses that started in late August. They were allowed to enrol up until the end of the first week of the semester, depending on seat availability. Students selected their section of the course based on the time that it was offered, so that it fit their schedule of other courses. Hence, to a certain extent, students randomly placed themselves in various sections of the course.

### **Sample**

This section describes the research site, the research participants, the role of the participants, and the treatment method used in this study.

#### **Research Site**

The University Center Rochester (UCR) is made up of three partnering institutions. Rochester Community and Technical College (RCTC) is a two-year college, which offers both liberal arts courses and technical programs, and which

is located in Rochester, Minnesota, U.S.A. The community college is both a bridge between high school and university as well as a second chance for students who didn't take high school seriously. There are no entrance requirements other than a high school diploma (or the equivalent). RCTC students range from Presidential Scholars (students in the top 10% of their high school graduating class), who aren't quite ready to make the big move to the big city and the big university, to students who barely graduated from high school. Students range in age from 16-year-old high school students participating in a Postsecondary Enrollment Options (PSEO) program to middle-aged working adults returning to improve themselves on the job or looking for a complete career change. Students can earn a one-year or two-year degree in a technical field, or they can earn an associate degree in science or liberal arts and transfer to a four-year university for their last two years of study. Since there is no four-year university in Rochester, RCTC has what's called a 2 + 2 program with Winona State University (WSU), whereby RCTC offers the first two years and WSU offers the next two years on our campus for certain programs. That way, Rochester area students who participate in the 2 + 2 program can get a four-year degree without commuting the 50 miles to Winona. RCTC has a similar relationship with the University of Minnesota, which offers graduate degrees on the UCR campus.

### Research Participants

I was the instructor for all students in the sample that consisted entirely of RCTC students. Prior to enrolling at RCTC students are required to take a placement test called the Academic Skills Assessment Program (ASAP), a group of tests used at all state colleges and universities in Minnesota. The ASAP consists of three tests: reading, writing, and mathematics. Upon completion of the ASAP tests, students are each given a list of possible English and Mathematics courses for which they may enrol. This system worked fine until a few years ago when the state switched to a new software registration package that is not capable of checking prerequisites as students register for courses. Therefore, the reality is that students can and do enrol in any course they choose, regardless of their ASAP scores. The majority of the students place into developmental mathematics

courses. Developmental courses are high school level courses taught at the university level to students who are under-prepared for college level work. College credit is not awarded for developmental courses. Since students don't like paying for a course that does not give them college credit, they often disregard their ASAP recommendations and enrol in courses beyond their capabilities.

The college level course in this study is called Contemporary Concepts in Mathematics. This course is the easiest college level course that fulfils the mathematics requirement for the social science, education, humanities, and other liberal arts majors. It is a survey course that covers a wide variety of mathematics applications, but doesn't cover any of the topics in depth. Topics studied include Numeration Systems, Logic, Problem Solving, Geometry, Trigonometry, Finance Mathematics, Probability, and Statistics. The prerequisite for this course is a passing grade in Elementary Algebra or an appropriate score on the mathematics placement test. Many students come to this class with weak backgrounds in mathematics, low math self-esteem, high math anxiety, low interest in the subject matter, and poor attitudes. They are in the class because it is the lesser of the evils that fulfils the mathematics requirement for their programs. These students were the subjects of this quasi-experimental research study.

During fall semester of 1999, which ran from late August to late December, I taught four sections of 35-40 students per section of the course called Contemporary Concepts in Mathematics as described above. One section met for 50 minutes three times a week on Mondays, Wednesdays, and Fridays; two sections met for 75 minutes twice a week on Tuesdays and Thursdays; one section (an evening class) met for 150 minutes one night per week. All sections met for a total of 16 weeks. Based on a study done by Mears (1995), I chose the Monday/Wednesday/Friday group to be the control group, which did not use cooperative learning. Mears found that 50 minutes was not enough time for the cooperative activities. The Tuesday/Thursday group that met at 10:00 was arbitrarily selected to be treatment group one (T1). This class was assigned to

groups that were heterogeneous in ability and, to the extent possible, also heterogeneous in gender. The Tuesday/Thursday group that met at 12:00 was arbitrarily selected to be treatment group two (T2). This class was assigned to groups that were homogeneous in gender and, as much as possible, heterogeneous in ability. The evening class remained to be treatment group three (T3). This class self-selected the groups with whom they worked. Hence, some of the groups were made up of friends, while others were made up of total strangers.

Since many students disregard the recommendation of their placement test, I decided to use ASAP scores to determine if all treatment groups were equal as well as to place students in cooperative groups within their treatment groups. Because the students had not all taken the same ASAP test at the same time, I needed to administer the ASAP during class time. Since Elementary Algebra is the prerequisite course, all students completed the Elementary Algebra ASAP test during our first class meeting. Based on these scores the top third of the class was designated high-ability, the middle third was designated medium-ability, and the lowest third was low-ability. Using the scores and the gender of each student I formed groups that consisted of two each of high-medium, medium-low, or high-low students as well as two males and two females for T1, and all males or all females for T2. As cited in Chapter Two, Holden (1993) and Webb (1984, 1985, 1991) studied small group interactions and found that if the groups were composed of high-medium-low ability the medium students were ignored. And if the males outnumbered the females, the females were ignored. Therefore, I tried to avoid either of those grouping situations.

### Role of Participants

In addition to taking the ASAP test, students completed pre- and post-questionnaires on attitudes and anxiety. During the semester, students in the control group completed all assignments and exams individually, while those in the treatment groups worked cooperatively on assignments and took exams individually, but were graded based on how the group did (group interdependence). At the end of the semester all students completed a researcher-

constructed questionnaire about their thoughts on the teaching method used in their classroom. Six students participated in individual interviews for the purpose of validating their responses to the questionnaire.

### Treatment Method

As mentioned earlier, one section was designated the control section, which was taught using individualized learning. The other sections were taught using cooperative learning methods. The treatment method chosen was based on the Learning Together model of Johnson and Johnson (1991). I chose this method because it was the most similar to what I was already using in my classroom; therefore, I was more familiar with it than with the other models. As described in Chapter Two, the Learning Together model requires five essential components to be successful: face-to-face verbal interaction, individual accountability, positive interdependence, social skills, and group processing. Students were assigned to groups of four and kept the same groups the entire semester.

The face-to-face verbal interaction was achieved by having the students sit toe-to-toe, knee-to-knee, eye-to-eye, two facing two. That was easier in some classrooms than in others, depending on the layout of the room. In one room the chairs were three in a row with an aisle between them, so some group members had to cross the aisle to join their groups. The other classrooms had four chairs in a row; so two members of the group just turned their chairs to face the other members of the group. These rooms also had plenty of room for me to walk around, observe, and answer questions while the students worked.

To incorporate individual accountability students were required to do unit exams independently. However, the method of grading those exams was where the group goals (positive interdependence) were incorporated. While each student in the control section received 100% of his or her exam score, each student in a treatment section received 67% of his or her exam score plus 33% of the group's average. This group goal required a student to assume responsibility for his or her own learning as well as the learning of the others in the group. For comparing end

results, the scores of the individual exams were used for both the control and treatment groups. This showed whether the group goal motivated individuals to higher achievement. To prevent the experiment being detrimental to students' final course grades, each treatment section student was given the higher of the two grades, the 100% individual exam scores or the 67% individual and 33% group average.

Social skills were discussed only at the start of the semester after the groups were first formed. The expectations of group members were listed and presented to the students by means of an overhead projector. A copy of that list is included in Appendix A. Group processing was very informal if it occurred at all. Occasionally, I noticed that a group was dysfunctional and I intervened to help them get back on the track of working together. A few times students called on me to help them resolve disagreements. Mostly, the students seemed to have no trouble working together. However, comments from students on the researcher-constructed questionnaire showed me that I should have spent some time having students formally processing within their groups about how the groups were working.

### **Instrumentation**

The instruments used in this study included the ASAP placement test, the MARS anxiety scale, the Revised Math Attitude Scale, a researcher-constructed questionnaire, interviews with selected students, course grades, attendance, and retention. Each of them is described in this section.

#### ASAP

The mathematics part of the ASAP test booklet consists of four tests ranging from Arithmetic to Pre-Calculus. The Elementary Algebra Skills Test was the ASAP test used for this study. A score ranging from 11-23 placed a student in Elementary Algebra, while a score ranging from 24-31 placed the student in Contemporary Concepts in Mathematics. Results of this test served



two purposes. First, they were used to form the cooperative groups as described above in the Treatment Method section. Second, scores on this test were compared between groups to determine if the control and treatment groups were equivalent at the start. Since this is a proprietary test, the test cannot be included here, however, the sample elementary algebra questions that RCTC gives students to help them decide which test to take are found in Appendix A. These questions are very similar to those found on the elementary algebra ASAP test.

### MARS

I selected the Mathematics Anxiety Rating Scale (MARS) adult version (Richardson & Suinn, 1972) as the instrument to assess students' mathematics anxiety levels because it was cited so frequently in the literature I reviewed. Obviously, there are other instruments available, but none as widely validated and as extensively used as the MARS. It is a 98-item questionnaire where each item states a situation that may arouse anxiety. The subject is required to self rate the amount of anxiety associated with the statement, choosing from the dimensions "not at all," "a little," "a fair amount," "much," or "very much." These results are coded 1-5 with 1 being "not at all" and 5 being "very much." The MARS was administered to all groups of students during the first and last weeks of the semester to assess changes in anxiety levels from the start to the end of the semester. The MARS is a proprietary test, so just a few sample questions are included in Appendix A.

### Revised Math Attitude Scale

The Revised Math Attitude Scale (Aiken & Dreger, 1961), designed to assess attitudes toward mathematics, uses a five-point scale ranging from "strongly disagree" to "strongly agree." Some of the statements are negatively worded. The subjects are required to rate their attitudes toward statements such as: "I am happier in a math class than in any other class," and "Mathematics makes me feel uncomfortable, restless, irritable, and impatient." Results are coded 0-4 from strongly disagree to strongly agree, with negatively worded statements recoded. I chose this questionnaire because of its simplicity. Other

attitude questionnaires that I considered were too long and too complex, with several subsections. Since I had already chosen the very long MARS for the anxiety questionnaire, I wanted to keep this one simpler. I didn't want students to get so tired of the questionnaires that they wouldn't answer them sincerely. All students completed the Revised Math Attitude Scale at the start and at the end of the semester. Again, the aim was to note changes in attitudes from beginning to end of the semester. A copy of the questionnaire is found in Appendix A.

#### Researcher-Constructed Questionnaire

The researcher-constructed questionnaire collected personal information on the students, such as gender, age group, previous mathematics courses taken, as well as perceptions on how their groups worked, attitude changes, and anxiety changes. All students in both the control and treatment groups completed the questionnaire at the end of the semester, however, the control group did not complete all questions. A copy of the questionnaire is included in Appendix A.

#### Student Interviews

The researcher-constructed questionnaire was also used when the researcher interviewed a total of six students from the treatment groups – two from each ability group and three of each gender – for the purpose of validating the questionnaire and triangulating the data. During the interviews the students were asked to clarify and expound on their responses to the written questionnaire.

#### Course Grades

Course grades were collected throughout the semester and included daily grades, assignments, and examinations. Daily grades were in-class assignments based on the previous day's assignment or on the current day's lecture. They were usually one or two problems, worth four points. Assignments were bigger projects completed in class, such as the Mortgage Problem, which required students to find a house for sale, calculate the mortgage needed, monthly payment, insurance, taxes, and compare the 30-year versus 15-year options. At the conclusion of each of the seven units, a 40-point teacher-constructed examination

was given. Students completed these examinations independently. Course grades were calculated using a weighted average in which the examinations counted for 75% of the semester grade and the daily grades and assignments made up the other 25%. Copies of the examinations and samples of the daily grade problems are found in Appendix F.

### Attendance

Student attendance or absence was recorded daily using the in-class daily assignments. A student choosing not to do the assignment handed in a paper with only his or her name on it. At the end of the semester I calculated the percentage of days attended. Since attendance was not (and should not have been) normally distributed, I recoded it as a nominal variable as follows: 4 = 90-100%, 3 = 80-89%, 2 = 70-79%, and 1 = 60-69%. No student had less than 60% days attended.

### Retention

Students at RCTC are allowed to change their schedules by dropping or adding courses during the first ten days of the semester. So the official class list is the “tenth day” list. For each student on my tenth day lists I assigned a final status code of 1 = withdrew and 2 = completed, based on whether or not the student completed the course.

## **Data Collection**

On the first day of the semester, students were given information sheets describing the study as well as participant consent forms to sign. (Copies of these forms are included in Appendix A.) Also, each student was assigned a unique student identification number to use on the questionnaires in place of their names. This was to protect their data privacy as well as to prevent any bias by myself as instructor/researcher, who was recording all of the information.

During the first week of the semester students completed the ASAP, the MARS, and the Revised Math Attitude Scales. All of these were scored or coded and recorded first on paper and later on my home computer in an SPSS file. The ASAP total score was recorded, but each individual scale response was recorded

for the MARS and the attitude scales. As the course proceeded, grades were collected and entered in an Excel file on my office computer. After each examination, students in the treatment groups were given two results: their individual scores, and their scores based on group averages. Both of these scores were stored in the Excel file. After the course was finished and course grades were calculated, they were transferred to the SPSS file, along with attendance and retention codes. At the end of the semester the MARS, the Revised Math Attitude Scale, and the researcher-constructed questionnaires were completed and recorded in the SPSS file. Each response to each individual question was recorded for these three questionnaires. Since some students withdrew from the course during the semester, a complete set of data was lacking for some students. With the exception of retention analyses, only the subjects for whom there was a complete set of data were included in the data analysis.

### **Reliability and Validity**

An internal consistency reliability analysis, using Cronbach's alpha, was run on pre- and post- MARS and Revised Math Attitude Scale questionnaires. The MARS results were  $\alpha = .9743$  ( $N = 116$ , mean = 217.8448) for the pre-test and  $\alpha = .9820$  ( $N = 108$ , mean = 200.3241) for the post-test. The Revised Math Attitude Scale results were  $\alpha = .9687$  ( $N = 119$ , mean = 37.6471) for the pre-test and  $\alpha = .9642$  ( $N = 115$ , mean = 38.7478) for the post-test. This indicates very strong internal consistency for both questionnaires.

### **Data Analysis and Interpretation**

Data analyses were completed using SPSS version 10.0 and procedures recommended by Morgan, Griego, and Gloeckner (2001). Cronbach's alpha was used to test for internal consistency of the MARS and the Revised Math Attitude Scale, as indicated above.

Frequency distributions for the ASAP, pre- and post- MARS, pre- and post- Revised Math Attitude Scale, and individual and group course grades were examined for normality.

One-way ANOVA was used first on the pre-test scores of the anxiety, attitude, and ASAP instruments, hoping that no differences between treatment and control groups would emerge. This procedure was repeated on the post-tests and individual and group course grades. If differences emerged, the Tukey HSD test was used to examine where differences occurred between groups. One-way ANOVA was also used to test each of the ASAP, MARS, Revised Math Attitude Scale, and course grades for differences both by research group and by gender.

Factorial ANOVA analyses were used to compare MARS changes to gender and research groups as well as Math Attitude changes to gender and research groups.

Achievement was analysed several ways. First, a paired samples t-test was used to examine differences between individual and group course grades. Since no differences were found, individual course grade became the dependent variable for further analyses. Factorial ANOVA was used to compare individual course grade to gender, research group, attendance, and any interactions among these variables. However, this doesn't take into account the differences between the subjects at the start – their math placement scores. So, ANCOVA was used to analyse individual course grade, adjusting for ASAP (the covariate), with research group as the factor of interest. This analysis was repeated with gender as the factor of interest. Finally, a regression model was run to analyse individual course grade based on ASAP, MARS post-test, Revised Math Attitude Scale post-test, gender, and research group.

Crosstabs and correlations were used to examine associations between gender and retention, and research group and retention, as well as various pairs of items on the researcher-constructed questionnaire.

## **Limitations**

One of the limiting factors of this methodology may be sample size. Creating a control and three treatment groups will make each of the sample sizes fairly small. The fact that the sample is limited to students from one mathematics course at one community college limits the generaliseability of the results. Another limitation may be the number of questionnaires that the students must complete during the first and last weeks of the semester. Will they take them seriously or will they get tired of questionnaires and not take the time to think about their answers? A major limitation is not having one instrument to measure achievement gains. Since the topics are so diverse, and since many students would not have studied most of the topics, it is not realistic to use an achievement pre-test covering the course topics. Also, since the course does not teach algebra, it does not make sense to use the ASAP test as a post-test measure of achievement. Finally, any time the researcher is also the classroom teacher there is the possibility for bias, which would be another limitation.

## **Summary**

Three pre-test assessments, ASAP, MARS, and Revised Math Attitude Scale; three post-test assessments, MARS, Revised Math Attitude Scale, and Course Grade; achievement; attendance; and course retention were analysed for differences among research groups and between genders. In addition, a researcher-constructed questionnaire provided personal information on students' backgrounds and their perceptions of how their groups worked, attitude changes, and anxiety changes. Chapter Four examines the results of these assessments of anxiety, attitudes, and achievement as well as correlations among the items of the researcher-constructed questionnaire.

## **CHAPTER FOUR: DATA ANALYSIS AND INTERPRETATION**

This chapter outlines the results of the research conducted with students in a liberal arts mathematics course at Rochester Community and Technical College during Fall Semester, 1999. Several students withdrew from the course during the semester, so the data in this study include only the cases for which pre- and post-data exist. Also, there were instances where students missed answering one or more questions of the attitude or anxiety questionnaires, and those individual cases were excluded by SPSS when appropriate. Hence, the number of cases varies depending on the analysis. In the first section the pre-test assessments – ASAP, MARS, and Revised Math Attitude Scale – are examined for normality, differences by research group, and differences by gender. In the second section the post-test assessments – MARS and Revised Math Attitude Scale – are examined for normality, differences by research group, and differences by gender, followed by an examination of the changes between the pre- and post-tests. The third section examines achievement assessments and attempts to identify factors that effect achievement. Section four analyses associations between research group and retention and between gender and retention. Student responses to the researcher-constructed questionnaire are summarized in section five. Finally, section six provides a summary of the results of the study.

### **Pre-Test Assessments**

Three pre-test assessments were used – ASAP, MARS, and Revised Math Attitude Scale. Each of these assessments was analysed by first verifying the normality of the distribution, then looking for differences between research groups and differences between gender.

### Academic Skills Assessment Program (ASAP)

The ASAP scores were used to determine if all control and treatment groups were equal prior to the research study. First, the distribution of the ASAP scores is examined for normality, and then differences in ASAP scores are examined by research group and by gender.

Distribution. The ASAP score was recorded for each of the 121 students who finished the course. The distribution, shown at Figure 1, has a mean of 23.4 (out of 35 questions), a standard deviation of 4.63, and a skewness of -.309. Since the skewness is between -1 and +1, the distribution is considered normal (Morgan et al., 2001). Also, the normality of the ASAP score was assessed separately for each of the research groups. The results were as follows: the control group had skewness = -.561; the heterogeneous ability/heterogeneous gender group had skewness = .127; the heterogeneous ability/homogeneous gender group had skewness = .061; the self-selected group had skewness = .023. A second check for normality using the SPSS recommendation that skewness divided by standard error of skewness should be between negative two and positive two (SPSS Inc., 1999) confirmed that all groups were normal. The histograms for each group are found in Appendix B. It is important to note that an ASAP score of 24 is the prerequisite for this course. The mean is lower than 24. Of the 121 students who completed the course, 47% did not meet the prerequisite. It's possible that some students did not take the test seriously. In response to their queries as to what would happen if they didn't do well, I promised that no one would be removed from the class due to a low ASAP score.



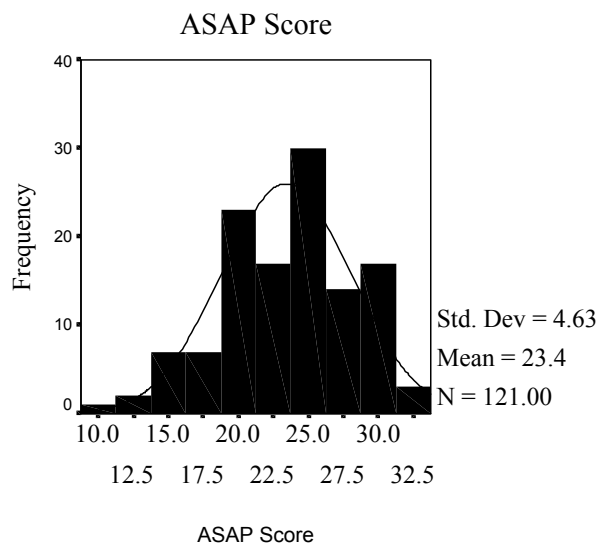


Figure1. Distribution of ASAP Score.

Differences by Research Group. One-way ANOVA was used to test the ASAP scores for differences among the four research groups. Table 2 displays the mean and standard deviation of ASAP scores for each group. The test for homogeneity of variances using the Levene statistic,  $F(3, 117) = 5.598, p = .001$ , indicates that the assumption of equal variances is violated. The one-way ANOVA (Table 3) shows that there are significant variances between groups for the ASAP scores:  $F(3, 117) = 4.675, p = .004$ . A post hoc multiple comparison implementing the Tukey HSD (honestly significant differences) test determined that the significant variances were between the self-selected and the heterogeneous ability/homogeneous gender groups (Table 4). I had expected the self-selected treatment group to be different because 25 of the original 36 students did not score high enough to meet the prerequisite for the course. If the school had enforced its prerequisites those 25 students would not have been allowed to enrol. This was the evening class, consisting mostly of non-traditional students who work by day and take courses at night. They are in a hurry to finish their degrees and are not willing to take prerequisites that are not required for their majors. Their motivation helped them finish the course, but their weak backgrounds in algebra forced me to spend more time reviewing algebra.

Table 2. Mean and Standard Deviation for ASAP Scores by Research Group.

Research Group	<u>n</u>	<u>M</u>	<u>SD</u>
Control	33	23.73	3.79
Heterogeneous Ability/ Heterogeneous Gender	27	23.67	5.39
Heterogeneous Ability/ Homogeneous Gender	32	25.16	3.21
Self-selected	29	20.97	5.23
Total	121	23.43	4.63

Table 3. One-Way ANOVA for ASAP Scores by Research Group.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	275.923	91.974	4.675	<b>.004</b>
Within Groups	117	2301.730	19.673		
Total	120	2577.653			

Table 4. Tukey HSD<sup>ab</sup> Homogeneous Subsets for ASAP Scores by Research Group.

Research Group	<u>n</u>	<u>Subset for alpha = .05</u>	
		1	2
Self-selected	29	20.97	
Heterogeneous Ability/ Heterogeneous Gender	27	23.67	23.67
Control	33	23.73	23.73
Heterogeneous Ability/ Homogeneous Gender	32		25.16
Sig.		.080	.564

Note: Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 30.059.

b. The group sizes are unequal. The harmonic mean of the group sizes is used.

Differences by Gender. One-way ANOVA was also used to test for differences in ASAP scores between genders. There were no significant gender differences in the ASAP scores (Tables 5 and 6). The Levene statistic,  $F(1, 119) = 1.393, p = .240$ , confirms that the variances are equal.

Table 5. Mean and Standard Deviation for ASAP Scores by Gender.

Gender	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
Male	51	23.55	4.35	13	33
Female	70	23.34	4.86	10	32
Total	121	23.43	4.63	10	33

Table 6. One-Way ANOVA for ASAP Scores by Gender.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	1	1.254	1.254	.058	<b>.810</b>
Within Groups	119	2576.399	21.650		
Total	120	2577.653			

Mathematics Anxiety Rating Scale (MARS)

The MARS questionnaire was used to determine the initial levels of mathematics anxiety in order to later determine any changes in anxiety among the groups. First, the distributions are examined. Then differences are examined by research group and by gender.

Distribution. The MARS questionnaire consisted of 98 statements to which a student responded on a five-point scale of strongly disagree to strongly agree. Each statement was scored on a scale of 1 to 5, with five being the most anxious. Since no statement was negatively worded, no recoding was necessary. The student's score was the sum of the numerical weights. The scores could range from 98 (no anxiety) to 490 (extremely high anxiety). The MARS Pre-test scores for 116 students were normally distributed with mean = 217.84, SD = 54.58, and skewness = .125. The histogram is shown in Figure 2. Skewness divided by its standard error is .556, confirming normality. An assessment of the normality of each research group concluded that all research groups were normal. Those histograms are found in Appendix B.

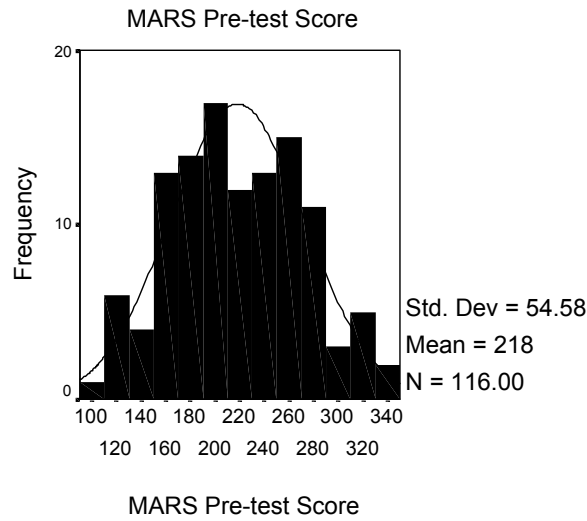


Figure 2. Distribution of MARS Pre-test.

Differences by Research Group. Tables 7 and 8 display the results of the one-way ANOVA comparing the means of the MARS pre-test by research group. The assumption of equal variances is confirmed by the Levene statistic,  $F(3, 112) = 1.506, p = .217$ . There are no significant differences between research groups.

Table 7. Mean and Standard Deviation for MARS Pre-test by Research Group.

Research Group	<u>n</u>	<u>M</u>	<u>SD</u>
Control	32	221.41	44.98
Heterogeneous ability/Heterogeneous gender	27	219.78	51.14
Heterogeneous ability/Homogeneous gender	30	222.90	61.65
Self-selected	27	206.07	60.98
Total	116	217.84	54.58

Table 8. One-Way ANOVA for MARS Pre-test by Research Group.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	5014.270	1671.423	.554	<b>.646</b>
Within Groups	112	337608.937	3014.366		
Total	115	342623.207			

Differences by Gender. The results of the one-way ANOVA comparing MARS pre-test to gender indicate no significant gender differences in the MARS pre-test scores (Tables 9 and 10). The Levene statistic,  $F(1, 114) = .506, p = .479$ , indicates that the assumption of equal variances is upheld, and the ANOVA table shows that the differences between gender are not significant.

Table 9. Mean and Standard Deviation for MARS Pre-test by Gender.

Gender	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
Male	49	219.71	52.72	120	327
Female	67	216.48	56.26	98	344
Total	116	217.84	54.58	98	344

Table 10. One-Way ANOVA for MARS Pre-test by Gender.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	1	296.490	296.490	.099	<b>.754</b>
Within Groups	114	342326.716	3002.866		
Total	115	342623.207			

### Revised Math Attitude Scale

The Revised Math Attitude Scale was used to determine the initial attitudes toward mathematics in order to later determine any changes in attitudes between and within research groups. First, the distributions are examined. Then differences are examined by research group and by gender.

Distribution. The Revised Math Attitude questionnaire consisted of 20 statements that were answered on a five-point scale ranging from strongly disagree to strongly agree. Each item was scored on a scale of 0 to 4, from strongly disagree to strongly agree, with negatively worded items recoded. The sum of the numerical weights constituted the total score for each student, so the scores could range from 0 (poor attitude) to 80 (good attitude). Scores for 119 students had a mean = 37.65, SD = 16.12, and skewness = .292, indicating a normal distribution, which was confirmed by dividing skewness by its standard error. The histogram is found at Figure 3. Also, an exploration of the individual groups confirmed that they are each normal. Those histograms are found in Appendix B.

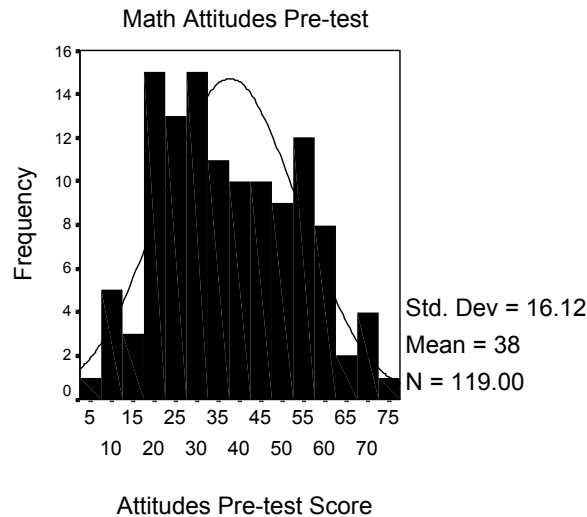


Figure 3. Distribution of Math Attitudes Pre-test.

Differences by Research Group. Once again one-way ANOVA was used to determine whether the variances were equal between research groups (Tables 11 and 12). The Levene statistic,  $F(3, 115) = .683, p = .564$ , indicates that the assumption of homogeneity of variances was upheld, and Table 12 shows that the differences between research groups were not significant.

Table 11. Mean and Standard Deviation for Math Attitudes Pre-test by Research Group.

Research Group	<u>n</u>	<u>M</u>	<u>SD</u>
Control	32	37.16	15.78
Heterogeneous Ability/ Heterogeneous Gender	27	41.52	14.35
Heterogeneous Ability/ Homogeneous Gender	32	35.53	17.48
Self-selected	28	36.89	16.72
Total	119	37.65	16.12

Table 12. One-way ANOVA for Math Attitudes Pre-test by Research Group.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	571.570	190.523	.728	<b>.537</b>
Within Groups	115	30091.607	261.666		
Total	118	30663.176			

Differences by Gender. The results of the one-way ANOVA of Math Attitudes Pre-test by Gender, displayed at Tables 13 and 14, had a Levene statistic of  $F(1, 117) = 2.590, p = .110$ , which indicates no significant gender differences in the Math Attitudes Pre-test scores. However, it is interesting to note that the



minimum score for males was higher than the minimum score for females, but the maximum score for males was lower than the maximum score for females.

Table 13. Mean and Standard Deviation for Math Attitudes Pre-test by Gender.

Gender	<u>n</u>	<u>M</u>	<u>SD</u>	Minimum	Maximum
Male	50	37.80	14.07	15	69
Female	69	37.54	17.56	7	76
Total	119	37.65	16.12	7	76

Table 14. One-way ANOVA for Math Attitudes Pre-test by Gender.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	1	2.017	2.017	.008	<b>.930</b>
Within Groups	117	30661.159	262.061		
Total	118	30663.176			

### Post-Test Assessments

Post-test assessments included the mathematics anxiety (MARS) and attitude (Revised Math Attitude Scale) questionnaires. Since the course did not teach algebra, it was inappropriate to use the ASAP as a post-test assessment. Instead, course grades were used to measure achievement. Each of these assessments is analysed for normality and for differences between research groups and differences between genders. Finally, differences between the pre- and post-tests for anxiety and attitudes are examined.

### MARS Post-Test

The MARS questionnaire was used again at the end of the semester to determine if any changes in anxiety took place. First, the distributions are examined for normality, and then differences in the post-test are examined by research group and gender. Finally, I calculated the MARS Changes by subtracting the pre-test scores from the post-test scores. A negative change indicates a decrease in the level of mathematics anxiety. These changes are also compared by research group and gender.

Distribution. Figure 4 displays the histogram for the MARS Post-test scores. It appears to be fairly normal, although slightly positively skewed. It has skewness of .778, which passes Morgan's test for normality. However, the skewness divided by its standard error is 3.339, which by the SPSS standard, is not normal. I chose to consider it normal. Examination of the histograms of the various research groups indicates that all are normal. The histograms are found in Appendix B. The pre-test having a normal distribution and the post-test having a not quite normal distribution together indicate that some changes took place. Those changes are discussed later.

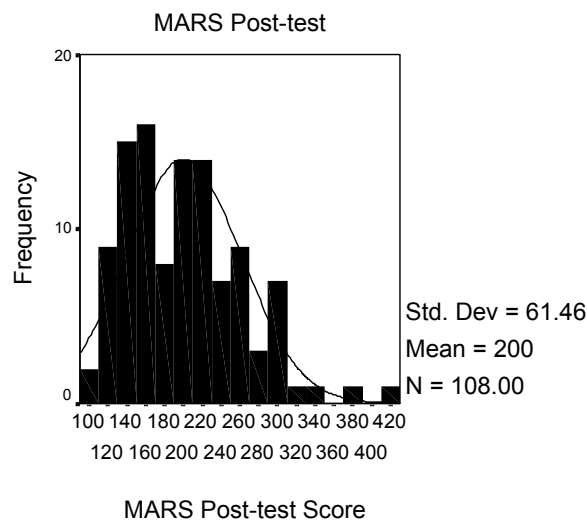


Figure 4. Distribution of MARS Post-test.

Differences by Research Group. Tables 15 and 16 display the results of the one-way ANOVA comparing the means of the MARS post-test by research group. The Levene statistic,  $F(3, 104) = .437, p = .727$ , confirms the assumption of equality of variances. There are no significant differences between research groups.

Table 15. Mean and Standard Deviation for MARS Post-test by Research Group.

Research Group	<u>n</u>	<u>M</u>	<u>SD</u>
Control	29	202.72	61.17
Heterogeneous ability/Heterogeneous gender	24	201.63	65.28
Heterogeneous ability/Homogeneous gender	30	209.33	62.11
Self-selected	25	185.48	58.23
Total	108	200.32	61.46

Table 16. One-Way ANOVA for MARS Post-test by Research Group.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	8151.333	2717.111	.713	<b>.546</b>
Within Groups	104	396068.325	3808.349		
Total	107	404219.657			

Differences by Gender. The results of the one-way ANOVA comparing MARS post-test to gender indicate no significant gender differences in the MARS post-test scores (Tables 17 and 18). The Levene statistic,  $F(1, 106) = .315, p = .576$ , indicates that the assumption of equal variances is upheld, and the ANOVA table shows that the differences between gender are not significant.

Table 17. Mean and Standard Deviation for MARS Post-test by Gender.

Gender	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
Male	43	209.98	64.64	102	414
Female	65	193.94	58.91	109	372
Total	108	200.32	61.46	102	414

Table 18. One-Way ANOVA for MARS Post-test by Gender.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	1	6656.927	6656.927	1.775	<b>.186</b>
Within Groups	106	397562.731	3750.592		
Total	107	404219.657			

MARS Changes. The MARS Changes were calculated by subtracting the MARS pre-test score from the MARS post-test score. Therefore, a negative change indicates that the anxiety has been reduced from the start to the end of the semester. Figure 5 displays the histogram for the pre- to post- changes in anxiety. The changes are normally distributed with a skewness of .059, a mean of -17.0, and a standard deviation of 46.05. The 104 students for whom pre- and post-MARS questionnaires were complete had a mean decrease in anxiety of 17.0. A factorial ANOVA analysis was done with MARS Changes as the dependent variable and gender and research group as the factors. Results are shown in Tables 19 to 21. The Levene statistic,  $F(7, 96) = 1.164, p = .331$ , affirms the hypothesis that the error variance of MARS Changes is equal across groups. Eta squared from Table 21 indicates small effect size (Cohen, 1988, as cited in Morgan et al., 2001) for gender and research group. 1.6% of the variance in MARS Changes can be predicted from gender, while 1.1% of it can be predicted

from research group. Even though gender's not significant, the graph at Figure 6 shows that in 3 of the 4 treatment groups females had a much larger decrease in mathematics anxiety than did males. The mean change for females was -21.56, whereas the mean change for males was -9.93.

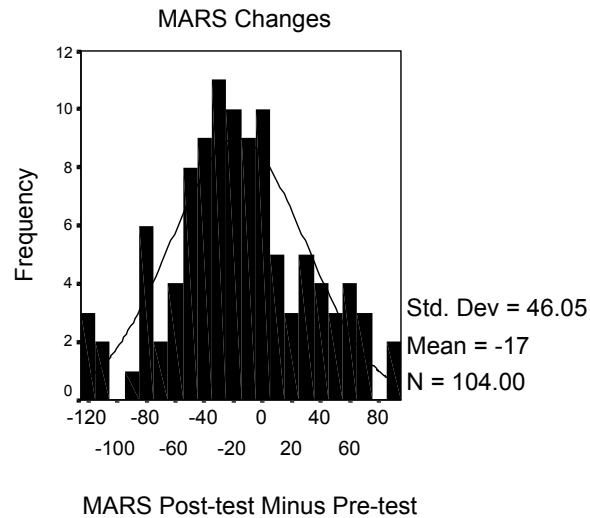


Figure 5. Distribution of MARS Changes.

Table 19. Between-Subjects Factors for MARS Changes.

	Value	Label	N
Gender	1	Male	41
	2	Female	63
Research Group	1	Control	28
	2	Heter. ability/heter. gender	24
	3	Heter. ability/homo. gender	29
	4	Self-selected	23

Table 20. Descriptive Statistics for MARS Changes.

Gender	Research Group	M	SD	N
Male	Control	-14.6667	50.7818	12
	Heter. ability/heter. gender	.8182	36.9508	11
	Heter. ability/homo. gender	-.2222	37.6124	9
	Self-selected	-26.4444	40.8293	9
	Total	-9.9268	42.2838	41
Female	Control	-25.5000	62.2672	16
	Heter. ability/heter. gender	-30.3846	36.4155	13
	Heter. ability/homo. gender	-17.4000	46.0805	20
	Self-selected	-14.7857	45.4570	14
	Total	-21.5556	48.1140	63
Total	Control	-20.8571	56.8720	28
	Heter. ability/heter. gender	-16.0833	39.2139	24
	Heter. ability/homo. gender	-12.0690	43.7092	29
	Self-selected	-19.3478	43.1400	23
	Total	-16.9712	46.0480	104

Table 21. Tests of Between-Subjects Effects for MARS Changes.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	10450.599 <sup>a</sup>	7	1492.943	.689	.681	.048
Intercept	25154.140	1	25154.140	11.612	.001	.108
GENDER	3440.497	1	3440.497	1.588	.211	.016
RESGROUP	2242.112	3	747.371	.345	.793	.011
GEND*RESGROUP	5435.967	3	1811.989	.836	.477	.025
Error	207952.315	96	2166.170			
Total	248357.000	104				
Corrected Total	218402.913	103				

a R = .219, R Squared = .048 (Adjusted R Squared = -.022)

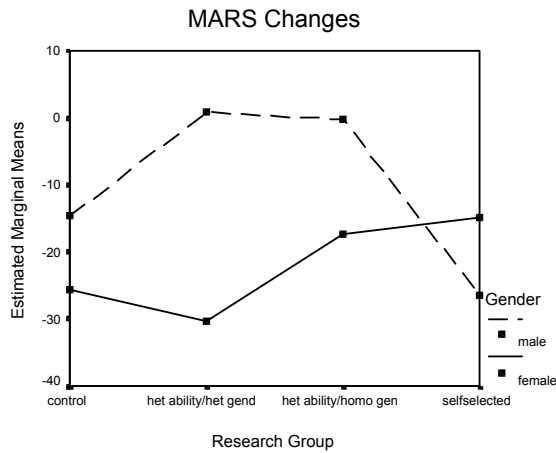


Figure 6. MARS Changes by Research Group and Gender.

Revised Math Attitude Scale Post-test

The Revised Math Attitude Scale was administered again at the end of the semester to determine if any attitudes toward mathematics had changed. First, the post-test distributions are examined; then differences are examined by research

group and by gender. Finally, I calculated the Math Attitude Changes by subtracting the pre-test score from the post-test score. A positive change indicates a more positive attitude toward mathematics. These changes are also compared by research group and by gender.

Distribution. The histogram for the Math Attitudes Post-test is displayed at Figure 7. It has skewness of .036, which indicates that the distribution is normal. The histograms of the individual research groups, found in Appendix B, indicate that all are normal.

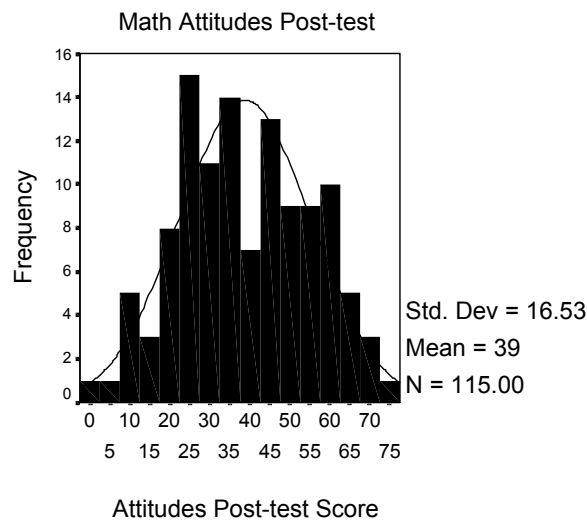


Figure 7. Distribution of Math Attitudes Post-test.

Differences by Research Group. One-way ANOVA compared the means of the Math Attitudes Post-test by research group. The Levene statistic,  $F(3, 111) = 1.104$ ,  $p = .351$ , confirms the assumption of equal variances. There are no significant differences between research groups. Tables 22 and 23 display the ANOVA results.



Table 22. Mean and Standard Deviation for Math Attitudes Post-test by Research Group.

Research Group	<u>n</u>	<u>M</u>	<u>SD</u>
Control	32	40.88	15.30
Heterogeneous ability/Heterogeneous gender	26	39.31	18.84
Heterogeneous ability/Homogeneous gender	29	33.79	17.56
Self-selected	28	40.93	14.10
Total	115	38.75	16.53

Table 23. One-Way ANOVA for Math Attitudes Post-test by Research Group.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	998.033	332.678	1.225	<b>.304</b>
Within Groups	111	30141.654	271.546		
Total	114	31139.687			

Differences by Gender. The one-way ANOVA comparing the means of the Math Attitudes Post-test by gender resulted in the Levene statistic of  $F(1, 113) = 6.388$ ,  $p = .013$ , which indicates that the assumption of equal variances is violated. Since gender has only two groups, the Tukey HSD test is not valid in this case. Therefore, I used the independent samples t-test, equal variances not assumed, which resulted in  $t(112.593) = .013$ ,  $p = .990$ , indicating no significant gender differences. An examination of the descriptive statistics at Table 24 caused me to question the minimum score of zero. I located the questionnaire and verified that it really was a score of zero. Either that student was in a negative mood when she completed the post-test or the course caused her attitude toward mathematics to drop considerably. Changes in attitudes are discussed in the next section.

Table 24. Mean and Standard Deviation for Math Attitudes Post-test by Gender.

Gender	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
Male	48	38.77	13.88	10	67
Female	67	38.73	18.29	0	76
Total	115	38.75	16.53	0	76

Math Attitude Changes. The Math Attitude Changes were calculated by subtracting the attitudes pre-test score from the post-test score. A positive change indicates that attitudes have improved from beginning to end of the semester. Figure 8 displays the histogram for the pre- to post- changes in attitudes. The changes are normally distributed with a skewness of -.061, a mean of 1.1, and a standard deviation of 12.04. A factorial ANOVA analysis was completed using Math Attitude Changes as the dependent variable and gender and research group as the factors. Results are displayed at Tables 25 to 27. The Levene statistic,  $F(1, 111) = .997, p = .320$ , affirms that the error variance is equal across groups. Eta squared from Table 27 indicates a small effect size for gender and research group. Approximately 6% of the variance in Math Attitude Changes can be predicted from the interaction of gender and research group, while 4% can be attributed to research group and almost nothing to gender. The mean attitude change for males was 1.02 and for females was 1.14, however, Figure 9 shows that in 3 of the 4 research groups females had less improvement in attitudes than did males. Two of the 4 research groups had a decline in attitudes toward mathematics for both males and females. Both females and males in the heterogeneous ability/heterogeneous gender declined, as did females in the heterogeneous ability/homogeneous gender and males in the self-selected.

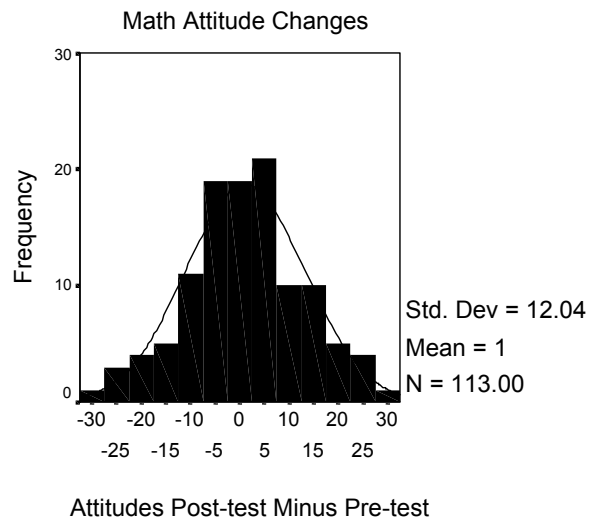


Figure 8. Distribution of Math Attitude Changes.

Table 25. Between-Subjects Factors for Math Attitude Changes.

	Value Label	N
Gender	1 Male	47
	2 Female	66
Research Group	1 Control	31
	2 Heter. ability/heter. gender	26
	3 Heter. ability/homo. gender	29
	4 Self-selected	27

Table 26. Descriptive Statistics for Math Attitude Changes.

Gender	Research Group	M	SD	N
Male	Control	5.46667	8.3312	15
	Heter. ability/heter. gender	-2.0769	8.7508	13
	Heter. ability/homo. gender	2.0000	11.0567	9
	Self-selected	-2.5000	16.9460	10
	Total	1.0213	11.4274	47
Female	Control	1.5000	8.5323	16
	Heter. ability/heter. gender	-2.9231	8.2306	13
	Heter. ability/homo. gender	-2.5500	15.4016	20
	Self-selected	8.2353	12.4123	17
	Total	1.1364	12.5403	66
Total	Control	3.4194	8.5353	31
	Heter. ability/heter. gender	-2.5000	8.3343	26
	Heter. ability/homo. gender	-1.1379	14.1591	29
	Self-selected	4.2593	14.9038	27
	Total	1.0885	12.0376	113

Table 27. Tests of Between-Subjects Effects for Math Attitude Changes.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	1899.027 <sup>a</sup>	7	271.290	1988	.064	.117
Intercept	84.834	1	84.834	.622	.432	.006
GENDER	3.124	1	3.124	.023	.880	.000
RESGROUP	637.670	3	212.557	1.557	.204	.043
GEND*RESGROUP	980.225	3	326.742	2.394	.073	.064
Error	14330.088	105	136.477			
Total	16363.000	113				
Corrected Total	16229.115	103				

a R = .342, R Squared = .117 (Adjusted R Squared = .058)

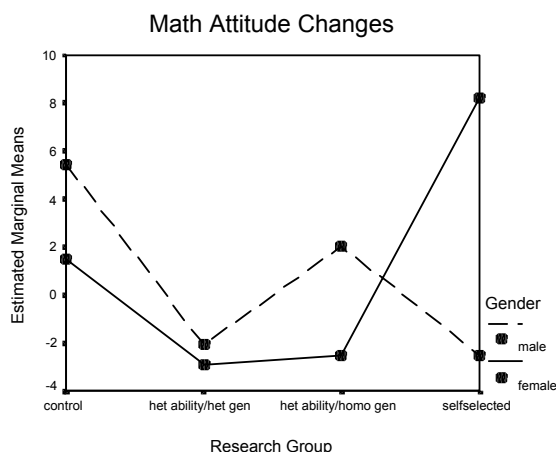


Figure 9. Math Attitude Changes by Research Group and Gender.

### Achievement Assessment

Mathematics achievement was measured by individual and group course grades. As described in Chapter 3, course grades were collected throughout the semester and included daily grades, assignments, and examinations. Students in the control group completed all daily grades, assignments, and examinations independently. Their individual grades were calculated using a weighted average in which the examinations counted for 75% of the course grade and the daily grades and assignments made up the other 25%.

Students in the three treatment groups worked cooperatively on the daily grades and assignments, but completed examinations independently. Individual group members each received the same grade on daily grades and assignments. Their individual course grades were calculated the same way as the control group's, using the weighted average of 75% for examinations and 25% for daily grades and assignments. In addition to individual course grades, I also calculated group course grades for students in the three treatment groups. In place of an individual exam score, each student received 67% of his or her exam score plus 33% of the cooperative group's average. The purpose of this was to incorporate a

group goal, requiring a student to assume responsibility for his or her own learning as well as the learning of the others in the group.

Individual and group course grades are examined individually for normality, differences by research group, and differences by gender, then the differences between the individual and group course grades are examined. Also, in my search for answers as to what factors effect achievement, I use factorial ANOVA, ANCOVA, and regression analyses to look for effects of gender, research group, anxiety, attitude, and attendance, while controlling for ASAP.

### Individual and Group Course Grade

The analysis of achievement results examined both individual course grades and group course grades together. First, distributions are examined for normality; next differences are examined by research group and by gender; finally differences between individual and group course grades are examined.

Distributions. The histogram at Figure 10 displays the distribution of the 121 Individual Course Grades. It is normal, with skewness =  $-.285$ , mean =  $80.0$ , and standard deviation =  $9.94$ . Figure 11 displays the distribution of the 88 Group Course Grades, which is also normal, with skewness =  $-.712$ , mean =  $80.4$ , and standard deviation =  $9.40$ .

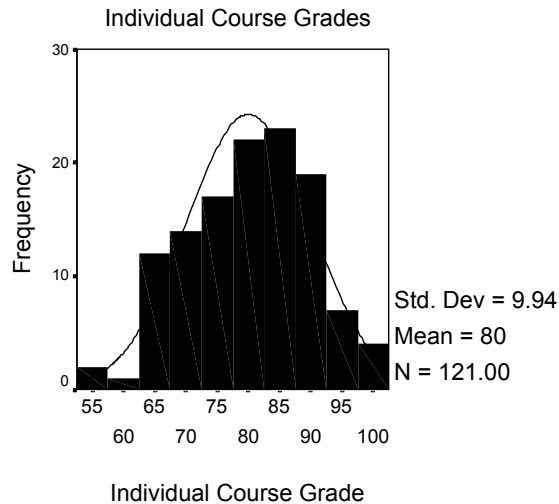


Figure 10. Distribution of Individual Course Grades.

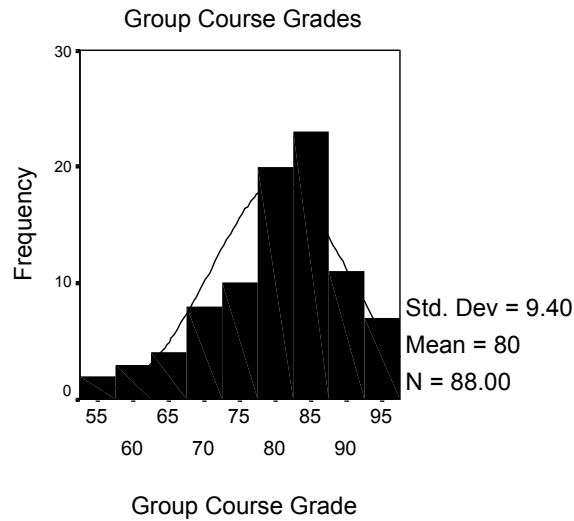


Figure 11. Distribution of Group Course Grades.

Differences by Research Group. One-way ANOVA was used to test both Individual Course Grade and Group Course Grade for differences among the four research groups. Tables 28 and 29 display the results of this ANOVA. The Levene statistic for Individual Course Grade is  $F(3, 117) = 1.405, p = .245$ , and for Group Course Grade is  $F(2, 85) = 1.624, p = .203$ , indicating no violation of assumption of equal variances in either case. The ANOVA table indicates no significant differences between research groups for either Individual or Group Course Grade.

Table 28. Mean and Standard Deviation for Individual and Group Grades by Research Group.

Research Group		<u>n</u>	<u>M</u>	<u>SD</u>
Individual Grade	Control	33	77.67	8.30
	Heter. ability/Heter. gender	27	79.52	11.61
	Heter. ability/Homo. gender	32	82.28	9.99
	Self-selected	29	80.45	9.80
	Total	121	79.97	9.94
Group Grade	Control	0		
	Heter. ability/Heter. gender	27	79.30	10.35
	Heter. ability/Homo. gender	32	81.47	9.93
	Self-selected	29	80.38	7.97
	Total	88	80.44	9.40

Table 29. One-Way ANOVA for Individual and Group Grades by Research Group.

Source		<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Individual Grades	Between Groups	3	358.153	119.384	1.215	<b>.307</b>
	Within Groups	117	11491.715	98.220		
	Total	120	11849.868			
Group Grades	Between Groups	3	69.290	23.097	.255	<b>.858</b>
	Within Groups	84	7618.426	90.696		
	Total	87	7687.716			



Differences by Gender. An independent samples t-test compared means of individual and group grades by gender. Means and standard deviations are shown at Table 30. Levene’s test for equality of variances resulted in  $F(1, 119) = .488, p = .486$ , for Individual Course Grade, and  $F(1, 86) = 2.352, p = .129$ , for Group Course Grades, verifying the assumption of equal variances in both cases. The t-test result for Individual Course Grade, equal variances assumed, was  $t(119) = -1.477, p = .142$ , and for Group Course Grade, equal variances assumed, was  $t(86) = -1.311, p = .193$ , which indicate no significant gender differences in either Individual or Group Course Grades.

Table 30. Mean and Standard Deviation for Individual and Group Course Grades by Gender.

	Gender	<u>N</u>	<u>M</u>	<u>SD</u>
Individual Course Grade	male	51	78.41	10.30
	female	70	81.10	9.58
	Total	121	79.97	9.94
Group Course Grade	male	34	78.79	10.39
	female	54	81.48	8.66
	Total	88	80.44	9.40

Differences between Individual and Group Grades. Individual and group course grades were compared for the 88 students in the three cooperative learning treatment groups. A paired samples t-test (Table 31) indicates that there were no significant differences between individual course grades and group course grades:  $t(87) = 1.448, p = .151$ . The mean individual course grades are higher than group grades by .39, but the confidence interval (-.14, .92) indicates that individual grades could be as low as .14 lower than group grades or as high as .92 above group grades. The differences are definitely small, leading me to believe that the “free rider” effect is not a concern. The free rider effect is discussed in Chapter 5.

Table 31. Paired Samples Test for Individual and Group Course Grades.

	Paired Differences			t	df	p
	M	SD	SEM			
Individual course grade – Group course grade	.39	2.50	.27	1.448	87	.151

What Factors Effect Achievement?

Based on the analyses done in the preceding section, I could conclude that neither gender nor research group alone effects achievement. The use of cooperative groups and group grading did not make a significant difference in grades. However, there are some things I have not considered. First, I haven't analysed the possible effect of attendance. Second, there could be an interaction effect from gender and research group. Third, my analyses haven't taken into account the differences between the subjects at the start – the ASAP scores. This section will address those issues. First, individual course grade and attendance are compared using a one-way ANOVA analysis. Next, I used factorial ANOVA to compare individual grade to gender, research group, and attendance. Following that, an ANCOVA analysis compares individual course grade to gender and research group, while controlling for ASAP. Finally, regression is used to determine which combination of factors predicts individual course grade. Since the differences between individual and group grades were so small, and since not all students had a group grade, only the individual course grade is used for these analyses.

Attendance. As stated in Chapter 3, attendance was first recorded as a percentage of days attended out of the total days the class met during the semester. Later it was recoded as 1 = 60-69%, 2 = 70-79%, 3 = 80-89%, and 4 = 90-100%.

No student attended less than 60% of the time. One-way ANOVA was used to compare means of Individual Course Grade among the four attendance groups. Table 32 displays the mean and standard deviation of Individual Course Grade for each group. Note that the majority of the students, 76 out of 121, attended 90-100% of the classes. This 90-100% group had a higher mean as well as higher minimums and higher maximums than the other three groups. The result of Levene's test of homogeneity of variances,  $F(3, 117) = .859, p = .464$ , affirmed the assumption of equal variances across groups. The one-way ANOVA (Table 33) shows that there are significant variances between attendance groups for the Individual Course Grade:  $F(3, 117) = 18.566, p < .001$ . A post hoc multiple comparison implementing the Tukey HSD (honestly significant differences) test determined that the significant variances were between the 90-100% group and the other three groups (Tables 34 and 35). The differences between the first three groups are not significant.

Table 32. Mean and Standard Deviation for Individual Course Grade by Attendance.

Attendance	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
60-69	5	73.20	10.62	64	90
70-79	17	71.12	9.48	53	84
80-89	23	73.83	8.07	55	88
90-100	76	84.25	7.92	68	99
Total	121	79.97	9.94	53	99

Table 33. One-Way ANOVA for Individual Course Grade by Attendance.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	3821.749	1273.916	18.566	<b>.000</b>
Within Groups	117	8028.119	68.616		
Total	120	11849.868			

Table 34. Tukey HSD Multiple Comparisons for Individual Course Grade by Attendance.

(I) Attendance	(J) Attendance	Mean Difference (I-J)	<u>SE</u>	<u>p</u>
60-69	60-69			
	70-79	2.08	4.21	.960
	80-89	-.63	4.09	.999
	90-100	-11.05	3.82	<b>.024</b>
70-79	60-69	-2.08	4.21	.960
	70-79			
	80-89	-2.71	2.65	.737
	90-100	-13.13	2.22	<b>.000</b>
80-89	60-69	.63	4.09	.999
	70-79	2.71	2.65	.737
	80-89			
	90-100	-10.42	1.97	<b>.000</b>
90-100	60-69	11.05	3.82	<b>.024</b>
	70-79	13.13	2.22	<b>.000</b>
	80-89	10.42	1.97	<b>.000</b>
	90-100			

Table 35. Tukey HSD<sup>ab</sup> Homogeneous Subsets for Individual Course Grade by Attendance.

Attendance	<u>n</u>	<u>Subset for alpha = .05</u>	
		1	2
70-79	17	71.12	
60-69	5	73.20	
80-89	23	73.83	
90-100	76		84.25
Sig.		.843	1.000

Note: Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 12.680.

b. The group sizes are unequal. The harmonic mean of the group sizes is used.

Considering how the individual course grade was determined—a weighted average of 75% for examinations and 25% for daily grades, which could not be completed if the student was not in attendance—it seems appropriate that higher attendance would beget higher grades. That’s built into the grading scheme. So, the question arises as to whether or not attendance would make a difference if it was not part of the grading scheme. To analyse this, I created a new variable for individual course grade based 100% on examination scores and called it Exams Only Individual Course Grade. I then used one-way ANOVA to compare Exams Only Individual Course Grade to attendance. The Levene statistic,  $F(3, 117) = 1.450, p = .232$ , affirms the assumption of equal variances. The ANOVA table (Table 37), with  $F(3, 117) = 8.043, p < .001$ , indicates that there are significant differences among attendance groups. The results of the Tukey HSD test (Table 38) confirm that students who attended 90-100% of the time scored significantly higher on examinations than did those who attended 70-79% or 80-89% of classes. Ironically, they did not score significantly higher than those who attended only 60-69% of their classes. However, the Tukey homogeneous subset table (Table 39), which identifies subsets that do not differ from one another, shows no differences in means for Exam Only Individual Grade by Attendance. This apparent contradiction to the information in Table 38 can be explained by the

large differences in sample sizes of the attendance groups, which range from 5 to 76. The homogeneous subset test does multiple range calculations using the harmonic mean of the sample sizes of the four groups, 12.68, whereas, the pairwise comparisons (Table 38) used the sample size of the two groups being compared. Therefore, the pairwise tests used larger sample sizes and it was easier to detect significance.

Table 36. Mean and Standard Deviation for Exam Only Individual Course Grade by Attendance.

Attendance	<u>n</u>	<u>M</u>	<u>SD</u>	<u>Min</u>	<u>Max</u>
60-69	5	77.20	10.18	64	88
70-79	17	72.94	12.42	47	88
80-89	23	73.87	8.67	57	88
90-100	76	82.96	9.72	63	100
Total	121	79.59	10.81	47	100

Table 37. One-Way ANOVA for Exam Only Individual Course Grade by Attendance.

Source	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Between Groups	3	2396.107	798.702	8.043	<b>.000</b>
Within Groups	117	11619.231	99.310		
Total	120	14015.339			

Table 38. Tukey HSD Multiple Comparisons for Exam Only Individual Course Grade by Attendance.

		Mean		
(I) Attendance	(J) Attendance	Difference (I-J)	<u>SE</u>	<u>p</u>
60-69	60-69			
	70-79	4.26	5.07	.835
	80-89	3.33	4.92	.906
	90-100	-5.76	4.60	.595
70-79	60-69	-4.26	5.07	.835
	70-79			
	80-89	-.93	3.19	.991
	90-100	-10.02	2.67	<b>.002</b>
80-89	60-69	-3.33	4.92	.906
	70-79	.93	3.19	.991
	80-89			
	90-100	-9.09	2.37	<b>.001</b>
90-100	60-69	5.76	4.60	.595
	70-79	10.02	2.67	<b>.002</b>
	80-89	9.09	2.37	<b>.001</b>
	90-100			

Table 39. Tukey HSD<sup>ab</sup> Homogeneous Subsets for Exam Only Individual Course Grade by Attendance.

Attendance	<u>n</u>	<u>Subset for alpha = .05</u>
70-79	17	1
80-89	23	1
60-69	5	1
90-100	76	1
Sig.		.060

Note: Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 12.680.

b. The group sizes are unequal. The harmonic mean of the group sizes is used.

Gender, Research Group, and Attendance. A factorial ANOVA analysis, using individual course grade as the dependent variable and gender, research group, and attendance as the factors, indicates that attendance has a large effect on individual course grade. The results are displayed at Tables 40 to 42. The significance of the corrected model verifies the significance of the multiple correlation coefficient,  $R$ .  $R = .709$ , and the adjusted  $R^2 = .372$ , indicating that 37.2% of variance in individual course grade can be predicted from the three variables, gender, research group, and attendance. This is a large effect according to Cohen (1988, as cited in Morgan et al., 2001). Attendance has the largest individual effect:  $F(3, 95) = 20.301, p < .001$ , and eta squared = .391, indicating that 39.1% of variation in individual grades can be predicted by attendance. The interaction of attendance and research group also has a large effect size.  $F(7, 95) = 3.03, p = .006$ , and eta squared = .183, which indicates that 18.3% of the variation in individual course grades can be predicted by the interaction of attendance and research group. The other factors and interactions are not significant. While gender is not a significant factor, it's interesting to note the graph at Figure 12, which compares individual course grade by research group and gender. In all four of the research groups females had higher course grades than did males.



I was interested in the differences in attendance across research groups. Since both are nominal variables, I did a crosstabs analysis, and since the crosstabs table was larger than 2 x 2, I used Cramer's V. In this case, Cramer's V = .191,  $p = .153$ , indicating no significant relationship between research group and attendance. However, I still was not convinced because I observed that my evening classes that met once a week had better attendance ( $M = 94.52$ ) than my day classes ( $M = 86.42$  for control group). It seemed that this difference should be significant. So I went back to my original recording of attendance as a percentage. I ran one-way ANOVA comparing attendance as a percentage to research group. The Levene statistic indicated that the assumption of equal variances had been violated; the ANOVA table indicated significant differences between groups; and the Tukey HSD test indicated that the differences were between the self-selected and control groups. The self-selected group met in the evening once a week and the control group met in the morning three times a week. These results confirmed my observations. But then I remembered why I had recoded the attendance data – because it was not normally distributed. Therefore, ANOVA was not the appropriate analysis to use. The nonparametric substitute for ANOVA, the Kruskal-Wallis Test, was used to compare attendance as a percentage to the four research groups. The results,  $\chi^2 = 9.732$ ,  $df = 3$ ,  $p = .021$ , indicate significant differences among the groups.

Table 40. Between-Subjects Factors for Individual Course Grade.

	Value	Label	N
Gender	1	Male	51
	2	Female	70
Research Group	1	Control	33
	2	Heter. ability/heter. gender	27
	3	Heter. ability/homo. gender	32
	4	Self-selected	29
Attendance	1	60-69	5
	2	70-79	17
	3	80-89	23
	4	90-100	76

Table 41. Descriptive Statistics for Individual Course Grade.

Gender	Research Group	Attendance	M	SD	N
Male	Control	60-69	66.00		1
		70-79	74.00	7.87	4
		80-89	74.00	1.63	4
		90-100	82.25	7.83	8
		Total	77.41	8.04	17
	Heterogeneous ability/ Heterogeneous gender	60-69	90.00		1
		70-79	64.00	4.24	2
		80-89	66.67	3.51	3
		90-100	87.29	7.93	7
		Total	79.15	12.67	13
	Heterogeneous ability/ Homogeneous gender	60-69	69.00		1
		70-79	63.50	.71	2
		80-89	86.00		1
		90-100	85.43	5.91	7
		Total	80.00	10.57	11
	Self-selected	60-69			
		70-79			
		80-89	68.75	10.31	4
		90-100	83.17	8.35	6
		Total	77.40	11.38	10
	Total	60-69	75.00	13.08	3
		70-79	68.88	7.70	8
		80-89	71.42	7.90	12
		90-100	84.50	7.40	28
		Total	78.41	10.30	51

Table 41 continued.

Gender	Research Group	Attendance	M	SD	N
Female	Control	60-69	70.50	9.19	2
		70-79	79.00	7.00	3
		80-89	69.50	9.19	2
		90-100	81.11	8.31	9
		Total	77.94	8.82	16
	Heterogeneous ability/ Heterogeneous gender	60-69			
		70-79	58.50	7.78	2
		80-89	80.33	1.53	3
		90-100	84.44	7.13	9
		Total	79.86	11.00	14
	Heterogeneous ability/ Homogeneous gender	60-69			
		70-79	76.00	8.64	4
		80-89	81.00	9.90	2
		90-100	85.80	9.44	15
		Total	83.48	9.71	21
	Self-selected	60-69			
		70-79			
		80-89	74.75	8.54	4
		90-100	84.00	7.99	15
		Total	82.05	8.77	19
	Total	60-69	70.50	9.19	2
		70-79	73.11	10.88	9
		80-89	76.45	7.75	11
		90-100	84.10	8.29	48
		Total	81.10	9.58	70

Table 41 continued.

Gender	Research Group	Attendance	M	SD	N
Total	Control	60-69	69.00	7.00	3
		70-79	76.14	7.38	7
		80-89	72.50	4.89	6
		90-100	81.65	7.86	17
		Total	77.67	8.30	33
	Heterogeneous ability/ Heterogeneous gender	60-69	90.00		1
		70-79	61.25	6.02	4
		80-89	73.50	7.87	6
		90-100	85.69	7.37	16
		Total	79.52	11.61	27
	Heterogeneous ability/ Homogeneous gender	60-69	69.00		1
		70-79	71.83	9.30	6
		80-89	82.67	7.57	3
		90-100	85.68	8.33	22
		Total	82.28	9.99	32
	Self-selected	60-69			
		70-79			
		80-89	71.75	9.33	8
		90-100	83.76	7.89	21
		Total	80.45	9.80	29
Total		60-69	73.20	10.62	5
		70-79	71.12	9.48	17
		80-89	73.83	8.07	23
		90-100	84.25	7.92	76
		Total	79.97	9.94	121

Table 42. Tests of Between-Subjects Effects for Individual Course Grade.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	5954.047 <sup>a</sup>	25	238.162	3.838	<b>.000</b>	.502
Intercept	301734.524	1	301734.524	4861.881	<b>.000</b>	.981
GENDER	90.376	1	90.376	1.456	.231	.015
RESGROUP	165.939	3	55.313	.891	.449	.027
ATTEND	3779.787	3	1259.929	20.301	<b>.000</b>	.391
GEND*RESGROUP	64.951	3	21.650	.349	.790	.011
GEND*ATTEND	133.660	3	44.553	.718	.544	.022
RESGR*ATTEND	1316.320	7	188.046	3.030	<b>.006</b>	.183
GEND*RESGROUP*						
ATTEND	460.378	5	92.076	1.484	.202	.072
Error	5895.821	95	62.061			
Total	785610.000	121				
Corrected Total	11849.868	120				

a R = .709, R Squared = .502 (Adjusted R Squared = .372)

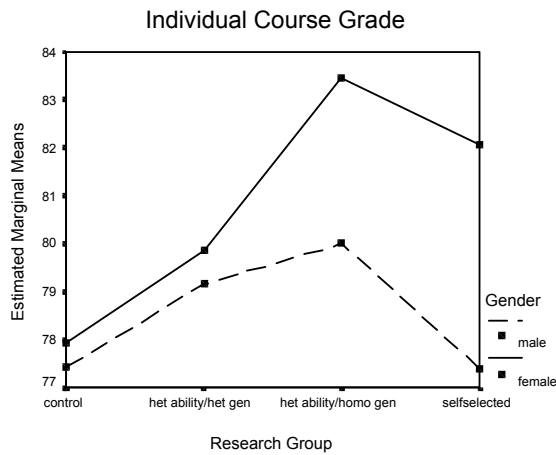


Figure 12. Individual Course Grade by Research Group and Gender.

Controlling for ASAP. Since there was variation in the ASAP scores at the start, analysis of covariance (ANCOVA), with individual course grade as the dependent variable, research group as the factor of interest, and ASAP as the covariant, was used to determine whether the research group makes a difference in the final grade for students of equal initial ASAP score. This analysis was done twice, first with the four research groups, and again with the three cooperative learning groups combined into one treatment group. With the four research groups, Levene's test of equality of error variances resulted in  $F(3, 117) = .671, p = .571$ , affirming the hypothesis that the error variance of individual course grade is equal across research groups. The ANCOVA results are displayed at Tables 43 to 45. The between-subjects effects information (Table 45) affirms that research group is not a significant factor of individual grade when controlling for ASAP scores:  $F(3, 117) = 1.769, p = .157$ . Eta squared = .044 is a small effect; however, ASAP is a medium effect: eta squared = .158. The significance of the corrected model verifies the significance of the multiple correlation coefficient,  $R$ .  $R = .428$ , and adjusted  $R^2 = .155$ , effect size = .39, a medium to large effect (Cohen, 1988, as cited in Morgan et al., 2001). The results were the same when repeating the test comparing the combined treatment group with the control group. Research group still had a small effect:  $F(1, 119) = 3.532, p = .063$ , eta squared = .029. These tables can be found at Appendix C.

Table 43. Between-Subjects Factors for Individual Course Grade by Research Group.

	Value	Label	N
Research Group	1	Control	33
	2	Heter. ability/heter. gender	27
	3	Heter. ability/homo. gender	32
	4	Self-selected	29

Table 44. Descriptive Statistics for Individual Course Grade by Research Group.

Research Group	M	SD	N
Control	77.67	8.30	33
Heter. ability/heter. gender	79.52	11.61	27
Heter. ability/homo. gender	82.28	9.99	32
Self-selected	80.45	9.80	29
Total	79.97	9.94	121

Table 45. Tests of Between-Subjects Effects for Individual Course Grade by Research Group.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	2172.764 <sup>a</sup>	4	543.191	6.511	<b>.000</b>	.183
Intercept	14269.166	1	14269.166	171.045	<b>.000</b>	.596
ASAP	1814.611	1	1814.611	21.752	<b>.000</b>	.158
RESGROUP	442.757	3	147.586	1.769	.157	.044
Error	9677.104	116	83.423			
Total	785610.000	121				
Corrected Total	11849.868	120				

a R = .428, R Squared = .183 (Adjusted R Squared = .155)

ANCOVA was also used to determine if there are gender effects on individual course grade when controlling for ASAP scores. The results are displayed at Tables 46 to 48. Levene's statistic,  $F(1, 119) = .043$ ,  $p = .836$ , affirms the hypothesis that the error variances of individual course grade are equal across genders. The ANCOVA results (Table 48),  $F(1, 119) = 2.878$ ,  $p = .092$ ,  $\eta^2 = .024$ , indicate no significant gender effect on individual course grade when controlling for ASAP score. However, the ASAP does have an effect:  $F(1, 119)$



= 20.996,  $p < .001$ ,  $\eta^2 = .151$ . The significance of R is verified by the significance of the corrected model.  $R = .407$ , and the adjusted  $R^2 = .152$ , effect size = .39, a medium to large effect (Cohen, 1988, as cited in Morgan et al., 2001).

Table 46. Between-Subjects Factors for Individual Course Grade by Gender.

	Value	Label	N
Gender	1	male	51
	2	female	70

Table 47. Descriptive Statistics for Individual Course Grade by Gender.

Gender	M	SD	N
Male	78.41	10.30	51
Female	81.10	9.58	70
Total	79.97	9.94	121

Table 48. Tests of Between-Subjects Effects for Individual Course Grade by Gender.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	1970.972 <sup>a</sup>	2	985.486	11.771	.000	.166
Intercept	16439.639	1	16439.639	196.366	.000	.625
ASAP	1757.757	1	1757.757	20.996	.000	.151
GENDER	240.965	1	240.965	2.878	.092	.024
Error	9878.895	118	83.719			
Total	785610.000	121				
Corrected Total	11849.868	120				

a R = .407, R Squared = .166 (Adjusted R Squared = .152)

Regression. Multiple regression was used to determine if individual course grade can be predicted better from a combination of ASAP, MARS Post-test, and Math Attitude Post-test than from any one of them alone. I ran this analysis twice, once using the SPSS enter method, which considers all variables simultaneously, and again using the stepwise method. The end results were the same, so I have included only the enter method here. All requested variables were entered simultaneously. The descriptive statistics are found at Table 49, while the correlations are at Table 50, the ANOVA at Table 51, and the regression coefficients at Table 52.

The correlations show that ASAP and Math Attitude Post-test are positively correlated with individual course grade, whereas MARS Post-test is negatively correlated with Individual Course Grade. To state it another way, as ASAP scores increase, individual course grades increase, but as anxiety increases, grades decrease.

The multiple correlation coefficient,  $R$ , using all predictors simultaneously, is significant with a value of .477, and the adjusted  $R^2 = .205$ , which means that 20.5% of variance in individual course grade can be predicted from the three variables, ASAP, Math Attitude Post-test, and MARS Post-test. This is a large effect according to Cohen (1988, as cited in Morgan et al., 2001).

The ANOVA table shows that there is at least one significant predictor among the independent variables, since  $F(3, 101) = 9.926, p < .001$ . This is to be expected since some of the correlations were significant.

The coefficients table shows me which variables make a significant contribution to the prediction of individual course grade over and above the contribution of all other variables. The constant and unstandardized coefficients can be used to predict a student's individual grade given the scores for ASAP, Math Attitudes Post-test and MARS Post-test. The  $t$  and  $p$  columns show me that

ASAP and Math Attitude Post-test scores make significant additions to the prediction of a student's individual course grade.

Table 49. Mean and Standard Deviation for Multiple Regression, Predicting Individual Course Grade.

	<u>n</u>	<u>M</u>	<u>SD</u>
Individual Course Grade	105	80.12	9.86
Placement Test Score (ASAP)	105	23.39	4.74
Math Attitude Post-test	105	38.67	16.92
MARS Post-test	105	201.80	61.66

Table 50. Correlations for Multiple Regression, Predicting Individual Course Grade.

Measure	1	2	3	4
1. Individual Course Grade	--			
2. ASAP Score	.380**	--		
3. Math Attitudes Post-test	.360**	.207*	--	
4. MARS Post-test	-.173*	-.131	-.520**	--

Note. Listwise N = 105.

\*\*p < .001, two tailed.

\*p < .05, two tailed.

Table 51. ANOVA for Multiple Regression, Predicting Individual Course Grade.

Model	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Regression	3	2300.159	766.720	9.926	.000 <sup>a</sup>
Residual	101	7801.232	77.240		
Total	104	10101.390			

a Predictors: (Constant), MARS Post-test, ASAP, Math Attitude Post-test

Table 52. Coefficients for Multiple Regression, Predicting Individual Course Grade.

Model	Unstandardized Coefficients		Standardized Coefficients	t	p
	B	SE	Beta		
(Constant)	56.635	6.430		8.808	.000
ASAP	.666	.186	.320	3.583	<b>.001</b>
Math Attitude Post-test	.180	.060	.308	2.972	<b>.004</b>
MARS Post-test	.005	.016	.030	.289	.773

### Retention

It has already been stated that retention of students from one semester to the next is a problem at RCTC. Many students do not even complete their current semester before dropping out. Mathematics courses are renowned for their high dropout rate. I am interested in knowing if working in cooperative groups has an effect on retention of students in this course. Does the help from others and the group's camaraderie give students the support and encouragement they need to stay in the class? This section attempts to answer that question.

A new SPSS data file was set up for the 141 students initially enrolled in this course. In addition to research group and gender, a new variable was created and called Final Status. Each student's final status indicated whether the student withdrew or completed the course and was recorded as 1 = withdrew and 2 = completed. A crosstabulation analysis was used to compare the final status first with research group and then with gender.

### Research Group

The semester began with 141 students and ended with 121 students completing the course, a retention rate of 86%. The control group lost 4 of the

original 36 students, a retention rate of 89%, whereas, the three cooperative learning groups combined lost 16 of 105 students, a retention rate of 85%. These data are displayed at Table 53. Since both variables are nominal and it is a 2 x 2 table, the appropriate test is phi. The phi statistic = -.052,  $p = .540$ , which indicates that the association between research group and retention is insignificant, effect size = .003.

Table 53. Research Group by Final Status Crosstabulation.

Research Group		Final Status		Total
		Withdrew	Completed	
Control	Count	4	32	36
	Expected Count	5.1	30.9	36.0
	% within Research Group	11.1%	88.9%	100.0%
	% within Final Status	20.0%	26.4%	25.5%
	% of Total	2.8%	22.7%	25.5%
Cooperative	Count	16	89	105
	Expected Count	14.9	90.1	105.0
	% within Research Group	15.2%	84.8%	100.0%
	% within Final Status	80.0%	73.6%	74.5%
	% of Total	11.3%	63.1%	74.5%
Total	Count	20	121	141
	Expected Count	20.0	121.0	141.0
	% within Research Group	14.2%	85.8%	100.0%
	% within Final Status	100.0%	100.0%	100.0%
	% of Total	14.2%	85.8%	100.0%

As seen at Table 54, a breakdown of the three treatment groups shows retention rates of 87% for the heterogeneous ability/ heterogeneous gender group,

87% for the heterogeneous ability/homogeneous gender group, and 81% for the self-selected group. The association was still insignificant when comparing the three treatment groups instead of one. Since this table is 4 x 2, Cramer's V is the appropriate statistic to use. Cramer's V = .091 with  $p = .760$  and effect size = .008. I expected that the self-selected group would have the lowest retention because it had such a large number of students—25 out of 36—who did not meet the prerequisite for the course, however, I was slightly surprised that the control group had the highest retention. Cooperative learning does not seem to make a difference in retention of students.

Table 54. Research Group by Final Status Crosstabulation (with Four Groups).

Research Group		Final Status		Total
		Withdrew	Completed	
Control	Count	4	32	36
	Expected Count	5.1	30.9	36.0
	% within Research Group	11.1%	88.9%	100.0%
	% within Final Status	20.0%	26.4%	25.5%
	% of Total	2.8%	22.7%	25.5%
Het.Ability/ Het. Gender	Count	4	27	31
	Expected Count	4.4	26.6	31.0
	% within Research Group	12.9%	87.1%	100.0%
	% within Final Status	20.0%	22.3%	22.0%
	% of Total	2.8%	19.1%	22.0%
Het.Ability/ Homo. Gender	Count	5	33	38
	Expected Count	5.4	32.6	38.0
	% within Research Group	13.2%	86.8%	100.0%
	% within Final Status	25.0%	27.3%	27.0%
	% of Total	3.5%	23.4%	27.0%

Table 54 continued.

Self-selected	Count	7	29	36
	Expected Count	5.1	30.9	36.0
	% within Research Group	19.4%	80.6%	100.0%
	% within Final Status	35.0%	24.0%	25.5%
	% of Total	5.0%	20.6%	25.5%
Total	Count	20	121	141
	Expected Count	20.0	121.0	141.0
	% within Research Group	14.2%	85.8%	100.0%
	% within Final Status	100.0%	100.0%	100.0%
	% of Total	14.2%	85.8%	100.0%

### Gender

Table 55 displays the results of the crosstabulation of gender and final status. Of the 141 students who started the course, 61 were male and 80 were female. Of the 61 males initially enrolled in the course, 53, or 87%, completed the course. Of the initial 80 females, 68, or 85%, completed the course. The phi statistic =  $-.027$ , with  $p = .751$ , indicating that there was not a significant association between gender and retention. Effect size =  $.0007$ , which is a small effect (Cohen, 1988, as cited in Morgan et al., 2001).

Table 55. Gender by Final Status Crosstabulation.

Gender		Final Status		Total
		Withdrew	Completed	
Male	Count	8	53	61
	Expected Count	8.7	52.3	61.0
	% within Research Group	13.1%	86.9%	100.0%
	% within Final Status	40.0%	43.8%	43.3%
	% of Total	5.7%	37.6%	43.3%
Female	Count	12	68	80
	Expected Count	11.3	68.7	80.0
	% within Research Group	15.0%	85.0%	100.0%
	% within Final Status	60.0%	56.2%	56.7%
	% of Total	8.5%	48.2%	56.7%
Total	Count	20	121	141
	Expected Count	20.0	121.0	141.0
	% within Research Group	14.2%	85.8%	100.0%
	% within Final Status	100.0%	100.0%	100.0%
	% of Total	14.2%	85.8%	100.0%

### **Researcher-Constructed Questionnaire**

The researcher-constructed questionnaire consisted of two parts. The first part, which was completed by all students in both the control and treatment groups, requested personal information, such as gender, age group, previous mathematics courses taken, and preference for cooperative or individual learning. The second part, which was completed by students in just the three treatment



groups, was a survey on the use of cooperative learning. It asked items such as students' perceptions on whether cooperative learning helped, hindered, or had no effect on their course grades, and whether cooperative learning increased, decreased, or had no effect on their mathematics anxiety and attitudes. While the questions required categorical responses, most questions also allowed students the opportunity for written comments. A copy of the questionnaire is found at Appendix A. Crosstabulations and correlations were used to examine associations between various items on the questionnaire with gender and research group. The crosstabs and correlations tables are found at Appendix D, and a listing of all student comments is found at Appendix E. This section provides the results of the associational analyses, followed by a discussion of interviews that were conducted with six students for the purpose of validating the questionnaire, and, finally, a summary of written comments of all students.

#### Age Group

Were there gender differences in the ages of the students? A crosstabulation of gender by age group indicated a significant association at the .05 level. Cramer's  $V = .237$  and  $p = .033$ , but the effect is small (.056). An inspection of the crosstabs table confirms that the largest difference is found in the high school age group where 88% were female and 12% male. In the traditional college age group (18-22 years), 48% were male and 52% female, and in the non-traditional age (over 22) 43% were male and 57% female. The overall student population at RCTC is 60% female and 40% male. However, the overall PSEO (high school) population is 63% female and 37% male. The percentage of females in this age group in this study was considerably larger than the percentage of females in this age group in the entire RCTC student body.

Significant associations occurred between research group and age group. Cramer's  $V = .368$ ,  $p < .001$ , effect size = .135. This was not surprising since 67% of the non-traditional age students were in the evening class (self-selected research group). Twenty-one of the 121 students were non-traditional age, and 16 of the 121 were high school students. Of the high school students, two were in the

control group, one was in the self-selected group, and the others were split 6 and 7 in the other two treatment groups.

Did age group have anything to do with attendance? The crosstabulation indicates that the association is insignificant: Cramer's  $V = .155$  and  $p = .445$ . However, it is interesting to note that 63% of high school students, 58% of traditional age students, and 81% of non-traditional students attended 90-100% of the time. This might be explained by the fact that high school students won't graduate from high school if they fail this course, and non-traditional students are usually paying for the course themselves. As the students I interviewed pointed out, these groups of students "have more riding on their grades" than the typical traditional student.

#### Mathematics Background

Did males and females differ on their previous mathematics courses taken? A crosstabulation of gender by mathematics background indicated no significant association: Cramer's  $V = .225$ , and  $p = .191$ , an effect size of .051. The students had stronger mathematics backgrounds than I had expected. Only two students, both male, had taken only high school Algebra I, which is usually taken by ninth grade. Fifty-three percent of the students, 26% male and 27% female, had taken all three of Algebra I, Geometry, and Algebra II. Only 23% of the students, 8% male and 15% female, took the prerequisite Elementary Algebra at RCTC. This strong mathematics background explains why so many students successfully completed the course when their ASAP scores were so low. Most of them did not take four years of high school mathematics, so they would have had at least one year of no mathematics prior to entering RCTC. That long break may be what caused students to score so low on the ASAP test.

Did research groups differ on previous mathematics courses taken? The crosstabulation shows that there was no significant association between the research group and the students' mathematics backgrounds. Cramer's  $V = .224$ ,  $p = .108$ , and effect size = .05. The backgrounds were quite strong. Only 15 of

the 121 students (12%) had taken the prerequisite elementary algebra course at RCTC. But 100% of the control group, 85% of the heterogeneous ability/heterogeneous gender group, 81% of the heterogeneous ability/homogeneous gender group, and 86% of the self-selected group either had taken at least three high school mathematics courses or had taken elementary algebra at RCTC.

Did mathematics background differ across age groups? The crosstabulation indicates a significant difference in previous mathematics courses taken across the age groups: Cramer's  $V = .303$ ,  $p = .004$ , effect size = .092. The high school students have had more mathematics courses. Seventy-five percent of high school students have had Algebra I, Geometry, and Algebra II, compared to 57% of traditional college age students and 19% of non-traditional students. However, since the PSEO program does not allow high school students to take high school level courses at the college, 0% of the high school students took elementary algebra at RCTC prior to taking this course, while 10% of the traditional and 33% of the non-traditional students took the elementary algebra as the prerequisite for this course.

#### Enjoyment of Group Work

Did females more than males like working in cooperative groups as suggested by the literature reviewed in Chapter 2? A 2 x 2 crosstabulation resulted in  $\Phi = .033$ ,  $p = .760$ , effect size = .001, indicating no significant difference between male and female opinions on this. Of the people who liked working in groups, 58% were female and 42% were male. Another way to interpret the results is that 24% of males and 21% of females liked working in groups.

Was there an association between treatment group and whether students liked working in groups? Only 26% of the heterogeneous ability/heterogeneous gender group, 19% of the heterogeneous ability/homogeneous gender group, and

21% of the self-selected group said that they like working in groups. This is not significant: Cramer's  $V = .072$ ,  $p = .800$ , effect size = .005.

Was there a correlation between individual course grade and whether a student liked working in groups? Spearman's  $\rho = -.153$  and  $p = .157$ , which indicates that while the correlation was negative, it was also insignificant.

#### Effect on Individual Grade

Were there gender-related differences associated with students' perceptions of whether working cooperatively raised, lowered, or had no effect on their grades? The results of the crosstabulation: Cramer's  $V = .168$ ,  $p = .296$ , effect size = .028, lead me to conclude that there were no gender differences. Eighteen percent of males and 17% of females felt that working cooperatively raised their grades, while 27% of males and 43% of females felt it lowered their grades, and 55% of males and 40% of females felt it had no effect on their grades.

Spearman's  $\rho$  was used to correlate individual course grade with students' perceptions of the effect of working cooperatively on their course grades. There was a significant negative correlation: Spearman's  $\rho = -.263$ ,  $p = .014$ . This means that students with higher grades were more likely to feel that working cooperatively lowered their grades.

Were there research group-related differences associated with students' perceptions of whether working cooperatively raised, lowered, or had no effect on their grades? The crosstabulation yielded: Cramer's  $V = .150$ ,  $p = .421$ , effect size = .023, indicating no significant differences.

#### Effect on Attitude Toward Mathematics

Was there an association between gender and students' perceptions of the effect of working cooperatively on their attitudes? The gender by effect on attitude crosstabulation resulted in Cramer's  $V = .130$ ,  $p = .690$ , and effect size = .017, so the association is insignificant. Thirteen percent of females and 9% of

males felt it had a positive effect, 47% of females and 44% of males felt it had a negative effect, and 38% of females and 47% of males felt it had no effect.

The research group by effect on attitude crosstabulation showed that there is no significant association between research groups and students' perceptions of whether working cooperatively had positive, negative, or no effects on their attitudes toward mathematics: Cramer's  $V = .177$ ,  $p = .486$ , and effect size =  $.031$ . Overall, 12% of the students said working cooperatively had a positive effect, 46% said it had a negative effect, and 41% said it had no effect, while 11% of the heterogeneous ability/heterogeneous gender group, 13% of the heterogeneous ability/homogeneous gender group, and 11% of the self-selected group said it had a positive effect on their attitudes toward mathematics.

Many students felt that working cooperatively had a negative effect on their attitudes toward mathematics. Spearman's correlation indicated that there was no significant correlation between students perception of the effect on attitude and the actual change from pre- to post-attitude scores: Spearman's  $\rho = -.039$  and  $p = .731$ .

#### Effect on Mathematics Anxiety

Was there an association between gender and students' perceptions of the effect of cooperative work on their mathematics anxiety? The crosstabulation indicates that the effect is insignificant, Cramer's  $V = .091$ ,  $p = .701$ , and effect size =  $.008$ . Both males and females felt that working in groups decreased their anxiety. Sixty-four percent of females and 73% of males thought it reduced their anxiety, while 13% of females and 9% of males felt it increased their anxiety, and 23% of females and 18% of males felt it had no effect on their mathematics anxiety.

The association between research group and effect on anxiety was quite different from that of attitude. The crosstabulation result: Cramer's  $V = .213$ ,  $p = .100$ , effect size =  $.045$ , indicates no significant differences among research

groups. However, overall, only 12% of the students thought that working cooperatively increased their mathematics anxiety, 67% said it decreased their anxiety, and 21% said it had no effect. Examining individual groups, I found that 59% of the heterogeneous ability/heterogeneous gender group, 81% of the heterogeneous ability/homogeneous gender group, and 61% of the self-selected group said working cooperatively decreased their mathematics anxiety.

Many students believed that working cooperatively decreased their mathematics anxiety. However, Spearman's correlation showed no significant correlation between their beliefs in the effect on anxiety and the actual change from the pre- to post-MARS scores: Spearman's  $\rho = -.189$  and  $p = .108$ .

#### Take a Similar Course

Did males and females differ on their opinions about whether they would take another course taught this way? No, both groups were overwhelmingly opposed to taking another course taught this way: 85% of males and 83% of females said they would not take another course taught this way.  $\Phi = -.056$ ,  $p = .547$ , and effect size = .003.

Did the research groups differ on whether the students would take another course taught this way? The crosstabulation results: Cramer's  $V = .228$ ,  $p = .107$ , effect size = .052, indicate no significant differences by research group. Overall, 83% of the students said they would not take another course taught this way. But that includes the control group, where 94% of the students said they would not take another course taught this way. The breakdown of the three cooperative treatment groups was 74% of the heterogeneous ability/heterogeneous gender group, 87% of the heterogeneous ability/ homogeneous gender groups, and 74% of the self-selected groups said that they would not take another course taught this way. In other words, the students in the control group were not happy working individually, and the students in the three treatment groups were not happy working cooperatively.

### Meeting Outside of Class

Did treatment groups differ on how much time they spent meeting outside of class? Each of the three groups had 74-75% of the students respond that they never met outside of class. This was very consistent throughout the groups, and the crosstabulation verified that: Cramer's  $V = .064$ ,  $p = .951$ , and effect size =  $.004$ .

### Other Correlations

Finally, Pearson's correlation was used to determine any associations between ASAP, Individual Course Grade, Attitudes Pre-test, Attitudes Post-test, MARS Pre-test, and MARS Post-test. There was a positive correlation,  $r(119) = .382$ ,  $p < .001$ , between ASAP scores and individual course grades. Likewise, there was a positive correlation,  $r(119) = .184$ ,  $p = .045$ , between ASAP scores and the attitude pre-test. However, there was a negative correlation,  $r(119) = -.381$ ,  $p < .001$ , between the MARS pre-test and the attitude pre-test. This negative correlation was upheld with the MARS and attitude post-tests,  $r(119) = -.520$ ,  $p < .001$ . Individual grade was positively correlated with the attitude post-test score:  $r(119) = .330$ ,  $p < .001$ .

### Student Interviews

Six students, three of each gender and two of each ability group, were selected from the treatment groups to be interviewed regarding their responses to the researcher-constructed survey on cooperative group work. At the time I made the selection of students they had not yet completed the questionnaire. I purposely selected students who attended regularly because I felt that they were the ones with the most experience working cooperatively. Also, I selected students from groups that appeared to work well as well as those that didn't seem to work well. I chose not to interview students from the evening class because most of them worked during the day and the evening class didn't meet during final exams week. However, I did select a non-traditional age day student. No student declined to be interviewed. Individual interviews of the students took place in my office during the final week of the semester. The interviews had to be

scheduled around both the students' and my final examination periods, so they were conducted over a period of four days. During each interview, I had the student's questionnaire in front of me. I had read it in advance and was prepared with a list of questions I wanted clarified. In some cases, the question had been misinterpreted. The interviews lasted approximately twenty minutes and were recorded on audiotape. This section includes a synopsis of each interview followed by a general summary of all student comments. The complete listing of student comments is found at Appendix E. I should remind the reader that the range of possible scores on the MARS was 98-490, and on the Revised Math Attitude Scale was 0-80.

Female 1. This student was chosen because she was in the only group composed of three males and one female. Due to unequal numbers, this was the only unbalanced group with respect to gender. Also, she had exhibited very negative body language throughout the semester. It appeared that she was not happy working cooperatively. She was a traditional age student in her first semester of college. She was one of the two high ability members of her high-low heterogeneous ability/heterogeneous gender group. She was very cooperative with me during the interview, however, I was still somewhat surprised at her extreme negativity toward group work. Her own attendance was 100%, but two of the people in her group were absent often, and when they did attend, they didn't participate much. Her stated reason for disliking groups was,

I would much rather think for myself. If I screw up I know it is my fault and I can take responsibility for my actions. I don't like to depend on others for things I hope to accomplish.

Even so, she and one other group member met outside of class twice a week for 3-4 hours total for the purpose of working together on the homework. She acknowledged that working cooperatively helped her to a certain point. Since she often had to explain things to the other person, it helped her, while at the same time held her back because she had to stop and explain things. In assessing her achievement, she felt that working cooperatively lowered her semester grade. At the time of the interview her final course grade had not been determined. In fact, her individual course grade was 88%, whereas, the group grade was 86%, but her



individual examination score, which excluded the daily group activities, was only 85%. These were insignificant differences and they were all the same letter grade, “B”. She felt that working cooperatively had a negative effect on her attitude and increased her anxiety due to the fact that she was always concerned about the test scores of other group members. She didn’t trust that the others would do the work and do well on exams. Her pre- and post- attitude and anxiety scales indicated an increase of five points from 61 to 66 in her attitude towards mathematics and a decrease of 46 points from 227 to 181 in her mathematics anxiety. Obviously, her perception of what was happening and what was actually happening were not quite the same. When I asked how she felt about being the only female in a group with three males, she responded that since this was her first semester in college she felt extremely intimidated. But it happened that the other group member who attended regularly was in two of her other classes, so they developed a good rapport and she was no longer intimidated by any of the group members. I agree that she did not get a good group to work with, but I think that her personality, one of very strong self-motivation and determination to do well and very little tolerance for slackers, was not conducive to working cooperatively. However, she was the type of person who would benefit from learning to work better with others, and I’m sure that she would have many more opportunities for group work in other college courses for her business management degree. She said that she would “absolutely not” take another course taught this way.

Male 1. This student was chosen because he always seemed positive, he was usually in class (96% attendance), and he was of non-traditional age (over 22 years). He was one of the high ability members of his high-medium heterogeneous ability/heterogeneous gender group, and he was the only non-traditional age member of his group. He did not like working in groups, “I would prefer to be responsible for my own education. Math class was enough of a worry for me let alone worrying if 3 other people were studying.” He felt that working cooperatively had no effect on his grade, but had a positive effect on his attitude in that he made acquaintances within the group, and it forced him to “come out of his shell and communicate with people.” He felt that the pressure of knowing that

his performance affected others increased his anxiety. In fact, his math anxiety increased 15 points from 251 to 266, and his attitude towards mathematics dropped from an already low 27 to 22. His individual course grade was 93%, the group grade was 91%, and his individual examination score was 93%, all of which are “A” grades. He commented that this was “the first math course that I’ve ever done well in,” and that he would take another course taught this way if it was in another subject area. His group did not meet outside of class time, due to scheduling logistics. I think if they had met to study together he would have had a sense of how his colleagues were doing, which may have reduced his anxiety. At the end of the interview he acknowledged that back when he was a traditional age college student he had flunked out after his first year. This may have biased his expectation of traditional age students.

Female 2. This student was chosen because she was a high school student in the PSEO program. She was one of the medium ability members of a medium-medium ability, all-female research group. She had 100% attendance, but her group members did not. One student never returned after the first week of the semester, and the other two had sporadic attendance. She said that she was always good at mathematics and didn’t need help from her group, so she never encouraged them to study together outside of class. She did not like working in groups, and because she felt that her group depended on her too much, she retaliated by trying to hurt their grades without hurting her own. “I feel that my group relied on me and that started to encourage me not to try so not to help them all the time but I tried not to hurt my grade.” She felt that working cooperatively lowered her course grade, had a negative effect on her attitude, but had no effect on her anxiety. In fact, her attitude increased 22 points, from 43 to 65, and her anxiety decreased from 254 to 229, a decrease of 25 points. However, she was correct on her course grade assessment. Her individual course grade was 92%, the group course grade was 87%, and her individual examination score was 91%. Her letter grade based on the group grade would have been a “B,” whereas working alone would have given her an “A”. As was stated on the participant information sheet that they received at the start of this study, nobody’s grade was lowered

because of their participation in this research study. All students were given the higher of the two grades. She was one of eight students who could have been adversely affected by this grading method. Even though the difference between her individual and group grade may not have been statistically significant, the fact that her grade was so close to the border between an “A” and a “B” made a small difference become significant to her. She would not take another course that was taught this way because she felt that she works better alone.

Male 2. This student was selected because he was a traditional college age student of medium ability who worked with a group of all males, who were also all medium ability. He had 100% attendance, and most of his group attended and participated regularly. This was his second year at RCTC and the second mathematics course taken there. He liked working in groups because it helped when another student explained things. He had taken Elementary Algebra and group work was not utilized and he didn’t do as well. His group met outside of class usually once a week to get caught up on homework and to prepare for examinations. He felt that working cooperatively raised his grade because it forced him to study “so as not to let others down.” He felt that working in groups had no effect on his anxiety, but it had a positive effect on his attitude because it made him feel more comfortable. In fact, his anxiety score increased 87 points, from an already high 327 to 414, and his attitude score decreased 5 points, from an already low 29 to 24. His individual and group grades were the same, 83%, and his examinations score was 80%, all “Bs”. His was by far the largest increase in anxiety. I can’t explain that. I didn’t have that score at the time I interviewed him, or I would have pursued that in our discussion.

Female 3. This person was selected because she was so obviously benefiting from working cooperatively in a group that worked very well together. She was a traditional college age student with a physical handicap that required her to use a crutch and required her to be absent a few times for medical appointments. This was her first semester at RCTC. Her ASAP score of 17 was well below the required 24, which made her the low ability member of her

medium-low ability group from the heterogeneous ability/heterogeneous gender research group. She should have been in a mixed gender group, but due to unequal numbers, hers was all female. She liked working in groups because if she had a problem she could count on her group members to answer her questions. Because of scheduling logistics, her group never met outside of class, but they occasionally worked together over the telephone, and twice a week she worked with a friend who was in one of the other treatment groups. She felt that working cooperatively raised her grade, had both positive and negative effects on her attitude and it increased her anxiety. The attitude and anxiety effects were because “it put more stress on me to do well for the group.” In fact, her attitude score decreased 15 points from a very low 20 to an extremely low 5, and her anxiety score increased from 334 to 336. She stated that it’s her personality to go out of her way to make others happy even if it makes her unhappy. Since she “had never been good at math,” she was very concerned that her grade would bring down the grades of her group members. She was one of the five students for whom the group grading method was an advantage. Her individual course grade was 69%, a “D”, her group grade was 71%, a “C”, and her individual examination score was 63%. She benefited from the daily grades as well as from the group grading method. I suppose one could call her a “free rider,” however, I know how hard she worked herself, so I wouldn’t label her a free rider. (Free riders are discussed in Chapter 5.) I asked her if she thought the grading was fair. She did not think the grading was fair because she should not have benefited from other people’s test scores, but at the same time, she was happy to accept the higher grade. She liked the group work and getting help from classmates and she would take another course taught this way.

Male 3. This student was a first year, traditional college age student from the heterogeneous ability/homogeneous gender treatment group. His ASAP score of 21 made him a low ability representative in his medium-low group. He had perfect attendance and enjoyed working in groups because “it gives me a sense of how other people think.” He liked hearing other people’s ideas, but he disliked it when people didn’t do their fair share. His group had one person whose

attendance was very irregular. The group did not meet outside of class time, and they all were confident that the other group members would study and “would come through on the tests.” He felt that working cooperatively had no effect on his attitude or anxiety, but that it raised his course grade. In fact, his attitude score went from 55 to 56, and his anxiety score increased from 234 to 235. His individual and group course grades were both 87%, and his individual examination score was 84%, all “B” grades. Since he was so positive about working cooperatively, I was surprised by his response that he would not take another course taught this way. When I asked for more detail on that response, he said the course went too fast for him and he had trouble with the algebra. The low ASAP score would explain that.

#### Summary of Spoken and Written Comments

The interviews were enlightening. I learned that some of my questions were poorly stated and were interpreted other than I had intended. For example, one question was, “Would you have preferred to be in one of the research groups other than the one in which you participated?” I was referring to control versus treatment groups, but some students interpreted it to be the cooperative group with which they were working and responded based on the fact that they would have preferred working with other people.

The attitudes that came through were interesting. I was surprised at Female 2’s admission that she wanted to hurt the grades of her group members as long as she didn’t hurt her own grade. I realize that six cases are not enough on which to form generalizations, but I can’t help thinking that maturity is a big factor here. The older student and students in their second year of college tended to be more concerned that their grades would damage someone else’s than that someone else’s grades would damage theirs.

These students were typical of all respondents in not meeting with their group outside of class. Seventy-five percent of all students said they did not meet their groups outside of class, and 52% of those that did meet did so for the

purpose of preparing for examinations, not on a regular basis. The logistics issue kept surfacing as the excuse for not meeting. I think that is typical of a community college. Students have overloaded schedules, working full-time and attending school part-time, or attending school full-time and working part-time. Some of them commute 45-50 miles, so they schedule all of their classes on two or three days of the week, and they don't come to campus the other days. They don't stay on campus any longer than necessary.

Based on the literature I had read, I was not surprised that some females were happy to be in all-female groups, but I was surprised that the males were happy to be in all-male groups. Overall, the students didn't seem to care about the composition of the group as long as the members participated. Those who were in a group with members who attended and participated enjoyed it and felt they benefited from working cooperatively.

The students liked the grading scheme of 75% tests and 25% daily grades. Some of them said their attendance may not have been as good had they not received daily grades. However, they definitely did not like the group grading method of 67% of their test and 33% of the group's average. Although I can't measure it, I feel very strongly that this had a negative effect on both attitudes and anxiety. On the first day of the semester each student received a participant information sheet that stated that his or her grades would not be hurt by this research study. However, I didn't tell them that they would get the higher of the two grades, because this would have biased the experiment. It would have negated the method for creating group interdependence. The students indicated during interviews that this was why they said they would not take another course taught this way. Overall, 83% of the students agreed that they would not take another course taught this way.

## Summary of Results

The purpose of this study was to evaluate the effectiveness of cooperative learning and to examine any gender-related differences in the effects of cooperative learning in terms of achievement, composition of the cooperative groups, mathematics anxiety, attitudes toward mathematics, attendance, and retention.

Cooperative learning had no significant effect on achievement. The differences between individual and group grades were insignificant, and the group grading method benefited the grades of only five students. The factors that did affect achievement were the ASAP placement test score, attendance, and the Math Attitude Post-test score. The MARS post-test was negatively correlated with individual course grade, meaning that as anxiety increased the individual course grade decreased.

The composition of the cooperative groups had no effect on course grade, mathematics anxiety, attitude toward mathematics, or retention. The significant difference between the control group and the self-selected treatment group for attendance can be attributed to the fact that the control group met three times per week and the self-selected (evening) class met once per week and the fact that the evening class was composed of more non-traditional age students, rather than to the fact that the cooperative groups were self-selected.

While no significant gender-related differences surfaced, some trends did appear. The ASAP, MARS, and Math Attitude pre-test scores were almost equal, however, females achieved slightly higher course grades than males. In each of the four research groups the mean individual course grades were higher for females than for males. Also, females had a larger decrease in mathematics anxiety with a drop of 22 points compared to the males' drop of 10 points. Males and females each improved their attitudes toward mathematics by only one point, however, in three of the four research groups, females had smaller attitude

changes than males, and two of the cooperative learning groups had decreases in their attitude scores. The female, self-selected group had positive attitude gains that were high enough to offset those of the other female groups.

It seems somewhat of an anomaly that females, having higher grades and less anxiety than males, don't also have better attitudes. This is discussed further in Chapter 5.



## CHAPTER FIVE: CONCLUSIONS AND IMPLICATIONS

Many of the results of this study were consistent with results of similar research reviewed in Chapter Two. However, this investigation raised several questions that weren't addressed in previous studies that I examined. Those questions are discussed in the summary section of this chapter, followed by a discussion of limitations of the study, recommendations for future research, and finally, the conclusion.

### Summary

#### Achievement and Anxiety

The results of this investigation are generally consistent with those of prior studies reviewed in Chapter Two. In terms of achievement, six of the twelve college level studies reviewed in Chapter Two indicated positive effects on achievement due to cooperative learning, five found no significant differences, and only one had the control group significantly better than the treatment group (Table 1). Although the present study resulted in each of the three cooperative learning groups achieving just a little higher than the control group, it is consistent with the five studies that found no significant differences.

The fact that mathematics anxiety was negatively correlated with achievement in this study is an affirmation of research cited previously (Brush, 1978; Clute, 1984; Hembree, 1990; Ma, 1999), which also found mathematics anxiety and achievement to be negatively correlated.

I was surprised that the anxiety levels decreased, even if not significantly. I had sensed the attitudes and anxiety from the students, and I had thought my 67% - 33% group grading scheme was the cause of the anxiety. I never attributed the anxiety to the attitudes they brought with them. Sometimes it's difficult to distinguish between attitude and anxiety. The results of the study showed that

they were negatively correlated. As the anxiety increased the attitudes became less positive. If the anxiety had decreased significantly, would the attitudes have increased significantly?

Although the differences were not significant, it is interesting to note that the heterogeneous ability/homogeneous gender treatment group had the highest post-test anxiety scores, the poorest attitudes, and the highest course grades. The self-selected group, which met in the evening and consisted of several non-traditional age students, had the least anxiety, the most positive attitudes, and the second highest course grades, even though they had the lowest ASAP scores. The control group had the lowest course grades, the second highest attitudes, and the second highest anxiety scores.

### Attitude

“Attitude is everything,” is a motto heard often in various settings, from sportsmen to businessmen. These words come out of my mouth for the benefit of my students at least once a semester. Attitudes towards mathematics played a big role in this study. Pre-test attitude scores were positively correlated with ASAP scores. The fact that 47% of the students’ ASAP scores were lower than the prerequisite score corresponds to similarly low attitude scores. What causes the poor attitudes towards mathematics? And what do the poor attitudes affect? When prospective students take the ASAP examinations prior to enrolling at RCTC, they are given a computer printout listing their scores and the courses for which they are qualified to enrol. I believe that some of the 47% of the students who didn’t score high enough when they took the ASAP from me probably had scored high enough to enrol in the class the first time they took the exam, because whenever a retest is given there is usually a small amount of variation between the scores. I also believe that some of the students didn’t take the exam seriously when they took it from me because they knew it didn’t count for anything, as they were already enrolled. But I also believe that many of those students knowingly enrolled in a course that would be difficult for them. They skipped the prerequisite course because they didn’t want to spend the money and time taking a

course for which college credit is not awarded. Would knowing that one is under-prepared cause one to have more anxiety, apprehension, and poorer attitudes? And would that person be more reluctant to work in a cooperative group because he or she feels inadequate? I wonder what the outcome of the study would have been if those 47% had scored high enough to meet the course prerequisite. Would the attitudes have been much better? And if the attitudes in general had been better, would the cooperative groups have worked better? Based on both the written comments and the student interviews, I concluded that much of the poor attitudes about cooperative work stemmed from the groups that contained social loafers and free riders. Comer (1995) cited Weldon and Mustari (1988), “Loafers and free riders are allowed to benefit because in each case, the outcome of group performance . . . is shared equally by all group members, regardless of their input. (p. 331)” This certainly applied to the grading scheme used in my study for daily grades. Group members were each awarded the same number of points no matter how much they contributed to the group effort. Comer concluded that the perception of loafing by one’s fellow group members might encourage one to become a loafer as well, possibly to avoid becoming a sucker.

The sucker effect is when a person is willing to reduce one’s own rewards to prevent being a sucker to free-riding partners (Kerr, 1983, as cited in Johnson & Johnson, 2000). The behaviour of Female 2, as recorded in the student interviews, is a prime example of the sucker effect.

King, Day, and Zehnder (1999) examined student passivity in small-group cooperative learning settings. They found that low achievers tended to become passive. While their study involved elementary school students, it’s possible that the results would hold true for students in the present study as well. It’s possible that other members of the group may perceive a passive student as a social loafer.

A student receiving an unearned grade due to working in groups is the main argument I hear from my colleagues who are opposed to using cooperative learning. I know that on any given day any student may be tired or “brain dead”

and will become a social loafer for that day. But the number of persistent social loafers in this study appeared to be few, and I wouldn't classify any of the students as free riders. The fact that examinations were taken independently encouraged students to participate to some degree. Only five students received their group course grade because it was higher than their individual course grade, and in four out of five instances the differences were two percentage points or less (the fifth case was a difference of 5%). It just happened that the scores were so close to the border between letter grades that their grades were changed to the next higher letter. I would describe all five of these students as occasional social loafers, but not as free riders, because they were mostly low-ability students who tried their best. I was surprised that there were only five such cases. While the grading scheme of the examinations did not encourage social loafing, that of the daily grades certainly did. In order to discourage social loafing I need to find a better method of awarding the daily grades that would give the students the perception that they are judged on their individual contributions to the group effort.

### Gender Differences

The gender differences in this study began with the enrolment – 80 females (57%) compared to 61 males (43%). There are two explanations for why females outnumbered males in this liberal arts mathematics course. First, the overall enrolment at RCTC during Fall Semester, 1999, was 60% females (2658) compared to 40% males (1749). The percentages in my sample were very similar to those of the total student population. The enrolment data also show that 51% of the females and only 38% of the males were part-time students. Why do more females attend the community and technical college? Is it because they have family and work commitments, so they cannot attend full-time? Are males not attending post-secondary schools, or are they more likely to leave home and move to where they can attend four-year universities? These are topics for future research.

The second explanation for why females outnumbered males in this liberal arts mathematics course is the fact that males outnumber females in the advanced mathematics courses required for science, mathematics, engineering, and technology-related (quantitative) careers. As stated in Chapter Two, females were less interested in activities and careers requiring mathematical or technological applications (AAUW, 1992, 2000; Boaler, 1997a; Bohlin, 1994; Fennema, 2000; Frost et al., 1994; Hyde et al., 1990; Turner & Bowen, 1999; Zeldin & Pajares, 2000).

The meta-analysis by Ma and Kishor (1997) found no significant gender differences on the relationship between attitude and achievement, and the meta-analysis by Ma (1999) found no significant gender differences on the relationship between mathematics anxiety and achievement. The results of the present study affirm no significant gender differences in either anxiety or attitude scores, as well as no significant gender differences in achievement, attendance, or retention.

#### Group Composition

The composition of groups had no effect on course grade, mathematics anxiety, attitude toward mathematics, or retention. Due to the few numbers of students of non-Caucasian ethnicity (1 to 2 per research group), ethnicity was not a factor of this study. These group composition results may have been different had there been more minority students. (The overall college enrolment had 12% non-Caucasian students during this semester. However, many of the immigrants are studying for quantitative careers, such as computer science. As a result, the Discrete Mathematics course usually has more minorities than Caucasians.) The heterogeneous-gender and heterogeneous-ability issues were controlled during the formation of the groups, based on the results of previous research, so the results were not unexpected for those groups. However, I was surprised that the self-selected groups also had no effect. I wonder, if the day classes had been self-selected, would the results have been the same? The fact that the evening class, consisting mostly of highly motivated, non-traditional age students, worked well

in their somewhat randomly self-selected groups doesn't mean that the day students would work as well in self-selected groups.

The fact that group composition had no effect is significant to me because I usually don't spend the time forming groups that I spent for this study. Usually, I ask the students if they want me to randomly assign the groups or if they want to select their own. Generally, in the first semester they request me to form the groups, since most of them don't know each other. But by second semester, the consensus is often to self-select. I assign groups by having students randomly select cards from a partial deck of cards, which I have selected so that there are the same numbers of cards as students. Then the three or four students with cards of the same rank form a group. This method of forming groups is efficient for me and deemed fair by the students.

### Summary

This study found few results that are significant and generaliseable, and it may have raised as many questions as it answered. However, the results are significant to me in that they have informed my instructional methods.

### **Limitations**

One of the limiting factors of this study was sample size. Creating four research groups made each of the sample sizes fairly small. The groups started out small and the withdrawal of some students made them even smaller. In addition, several students did not answer all questions on all questionnaires, and the SPSS analyses excluded those cases that were missing any items.

The students had to complete several questionnaires during the first and last weeks of the semester. Was it too much? Did they take them seriously, or did they get tired of questionnaires and not take the time to think about their answers?

The course taken by the students requires a fair amount of reading. One of its prerequisites is a minimum score of 29 on the reading portion of the ASAP. I did not control for that. That could have been a confounding variable in terms of achievement. Also, a student with reading difficulties may have become passive in order to avoid group members detecting the problem.

Another limitation was the inability to measure achievement gains using pre- and post- achievement test scores. Due to the nature of the survey course with its variety of topics, it was not realistic to use an achievement pre-test covering the course topics. And since algebra was not taught, it wasn't appropriate to use the ASAP as a post-test measure of achievement. Also, the measures of achievement used, teacher's grades based on teacher/researcher-designed mathematics examinations, may have been a limiting factor. Ma's meta-analysis (1999) determined that achievement instruments of this type tend to overestimate the magnitude of the relationship between mathematics anxiety and achievement compared to commercially developed achievement instruments. He theorized that this is due to lack of control of the difficulty level of the items:

The ceiling effect (items are so easy that many students perform well) or the floor effect (items are so difficult that many students perform poorly) is more likely to occur when mathematics teachers' grades or researcher-designed mathematics tests instead of commercial tests are used as achievement measures. (p. 525)

In fact, when I construct an examination, I intentionally include a few questions that are so easy that everyone should have the correct answer as well as a few items that only those well-prepared students will successfully answer.

Finally, the researcher as the instructor may have been a limiting factor in the sense that while trying to avoid bias I did not do as much as I could have to help the cooperative small groups within the treatment groups function better.

## **Recommendations**

Since the attitudes toward mathematics at the start were correlated to the ASAP score, this study should be repeated when the course prerequisites are enforced, so the study would include only students who do meet the prerequisite ASAP scores for both mathematics and reading. That way, students whose negative attitudes are attributed to the fact that they are in a course for which they are under-prepared are eliminated. The attitude pre-test scores may start higher, and the mathematics anxiety pre-test scores might start lower. If the attitudes are better and students are not feeling overwhelmed by the mathematics and their ability (or lack thereof), would they work better and be more successful in their groups?

Theoretically, this course, with its variety of topics, should have been ideal for this group grading scheme because each group member may have excelled at a different topic, thereby balancing the efforts, strengths, and weaknesses of the group. However, theory and practice are not the same. A similar study should be done without the 67% individual and 33% group grading method implemented in the current study. I believe that the group grading method caused poorer attitudes and created more anxiety. Another method should be found to incorporate group interdependence. One I have tried in another course is to assign problem sets A, B, and C. Each of the three members of the group is assigned one of the problem sets to work out and another set to mark. So each student does two problem sets – the one they were assigned to do and the one they were assigned to mark – and in the process of marking someone else’s work the students get together and discuss the errors found. Students are given group grades based on the corrected versions as well as individual grades based on their marking of the other’s work. This worked fairly well in a mathematics course for elementary education majors, but may not work as well in a liberal arts mathematics course due to the diversity and the larger number of students. Another possible method for incorporating group goals is the one used by Ruppnow and Bogenschild (1998). For their study, tests



were taken independently, but bonus points were earned based on the average improvement of everyone in the group.

More research needs to be done at the university level on the interaction among group members, particularly, regarding the nature and quality of explanations and help given. Also, I'm not convinced that group processing is necessary at the college level. However, discussion should take place on how to deal with social loafers and free riders within the groups. More research needs to be done to determine the effectiveness of group processing.

Finally, RCTC needs to do further studies of the scores used to determine cut-off points for entry into their mathematics courses, particularly, the Contemporary Concepts in Mathematics course. Only two of the 57 students who failed to meet the prerequisite actually failed the course, and another ten students received "D" grades. This raised the question, is the cut-off score too high?

### **Conclusion**

No significant differences in achievement were found between cooperative and individualised learning methods. Attendance had a large effect on achievement (Table 42). The ASAP placement test score and the Revised Math Attitude Scale post-test were significant predictors of achievement (Table 52). In addition, achievement and anxiety were negatively correlated. The composition of the cooperative groups had no effect on achievement, mathematics anxiety, attitudes toward mathematics, or retention.

The gender-related differences that surfaced were not significant, however, trends did appear. Females had slightly higher course grades and slightly larger decreases in mathematics anxiety than males, but smaller positive gains in attitudes. Since mathematics seems to be the key to open the door to the quantitative and more lucrative careers, it is important that females have every

opportunity to excel at mathematics. Cooperative learning may be a tool that helps them learn mathematics.

Even if cooperative learning does not have significant effects on achievement, anxiety, and attitudes, there are two reasons why cooperative learning should be a part of educational methods. First, knowing how to work cooperatively is an important skill that students need when they enter the workforce. Second, cooperative learning will prepare students to become better members of society. Johnson and Johnson (1996) discuss the fact that cooperation, not competition or individualism, is at the heart of a democracy. For that matter, cooperation is an underlying factor in all social systems, including the family. They stated,

Cooperation is the heart of forming a more perfect Union, establishing justice, ensuring domestic tranquillity, providing for the common defense, promoting the general welfare, and securing the blessings of liberty for ourselves and our posterity. It is time to recognize the relationship between cooperative learning and commitment to the values underlying American democracy. (p. 65)

On a personal note, after 26 years of teaching, I am more convinced than ever that there is no “perfect” teaching method that will reach all students. I admit that I was disappointed in the results of this study. I had hoped that it would confirm without a doubt that cooperative learning methods are better than individualised ones. But I was pleased that it also did not confirm that individualised learning is the better method. I was also pleased at the small number of free riders in the class. Research is not emphasized, expected, or even encouraged at MnSCU community colleges. Teaching, not research, is the emphasis for faculty, and teaching assignments are such that there is little time left to conduct research. As such, faculty try new methods and use their gut feelings, comments from students, and anecdotal observations to determine if the method was effective enough to make it worth using again. This study was significant to me in that it is more than just anecdotal proof that cooperative learning is a method worth using.

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## **APPENDIX A: FORMS AND QUESTIONNAIRES**



## **SCIENCE AND MATHEMATICS EDUCATION CENTRE**

### **Research Project: The effectiveness of cooperative learning in college level and developmental mathematics.**

#### **Participant Information Sheet**

Your instructor, ViAnn Olson, is conducting this research project as part of her Doctor of Mathematics Education program. The section you are attending is either a control section or a treatment section. In all sections the assignments and tests are the same. The differences are whether or not you use cooperative groups, how the groups are formed, and how the exams are scored. At the end of the semester ViAnn will compare final grades among the different sections. At the beginning and end of the semester you will be asked to complete attitude and anxiety questionnaires. Also, at the end of the semester you will be asked to complete a questionnaire on the teaching method used. In addition, a few students will be asked to participate in a follow-up interview. ViAnn will analyze all of the data. If you are asked to put your name or student number on the questionnaires, it is for the purpose of correlating questionnaire results with achievement results. Your name or student number will never be used in any written reports. Only ViAnn and her supervisors at Curtin University will view the data. The data will be secured electronically for a period of five years, after which they will be destroyed.

You have the right to refuse to be part of this study. In the event that you choose not to be included, you will still be required to do all of the required course work. Group work and group grading will not change. The only differences will be that you will not be required to complete the questionnaires and that your data will not be included in the study. Also, you may choose to withdraw from the study at any stage. Note that your final semester grade will not be negatively affected by this study.

If you have any questions please contact ViAnn Olson at 285-7457.



## SCIENCE AND MATHEMATICS EDUCATION CENTRE

**Research Project: The effectiveness of cooperative learning in college level and developmental mathematics.**

### **Participant Consent Form**

I agree to participate in the research project being conducted by ViAnn Olson. I understand that in addition to participating in the regular course work, I will be asked to complete some questionnaires and may be asked to participate in a follow-up interview.

It is my understanding that my name or any other identifying information will not be used in any written reports; that, if individuals are identified, pseudonyms will be employed. I am assured that the data collected will be secured electronically for a period of five years, after which they will be destroyed. I understand that my final semester grade will not be negatively affected by this study. I also understand that I may choose to withdraw from the study at any stage.

---

Date

---

Name (Please Print)

---

Signature

Note: If you are under 18 your parent's signature is needed.

**APPENDIX A: Note**

**Samples of ASAP and MARS Questionnaires  
Removed**

**Note: For copyright reasons sample ASAP and MARS questionnaires have not  
been reproduced (pp. 145-148)**

**(Co-ordinator, ADT Project, Curtin University of Technology, 24.09.03)**

## Cooperative Learning Objectives

- Students will learn to work cooperatively.
- Students will learn mathematics.
- Students will increase their oral and written communication skills.
- Students will get to know other students.

## Social Skills

- One person is the reader.
- One person is the recorder.
- At least two people are the calculators.
- Everyone is an encourager.
- Everyone should act as checker/quality controller.
- Roles should be rotated daily.
- Everyone contributes.
- No one dominates.
- All opinions are valuable.
- No put-downs.
- Sit face-to-face, rather than all in a row.
- Hand in one copy of the assignment for the group.
- Each member must sign his/her own name, acknowledging that you agree with the answer(s). Do not sign someone else's name for them.



## Research On Cooperative Group Work Survey

**Items with an asterisk are coded as follows:**

	<b>Coding</b>
1. Your student posting number assigned by Ms. Olson _____	<b>001-200</b>
2. *Gender <input type="checkbox"/> Male	<b>1</b>
<input type="checkbox"/> Female	<b>2</b>
3. *Which age group best describes you?	
<input type="checkbox"/> PSEO (high school)	<b>1</b>
<input type="checkbox"/> Traditional college age (18-22 years)	<b>2</b>
<input type="checkbox"/> Nontraditional college age (> 22 years)	<b>3</b>
4. *What was your math background prior to this course?	
<input type="checkbox"/> High school Algebra I only	<b>1</b>
<input type="checkbox"/> High school Algebra I and Geometry	<b>3</b>
<input type="checkbox"/> High school Algebra I, Geometry, Algebra II	<b>4</b>
<input type="checkbox"/> Elementary Algebra taken at RCTC	<b>2</b>
<input type="checkbox"/> Other, please list them	<b>5</b>
5. Which research group were you a part of?	
<input type="checkbox"/> Control group (met MWF at 10:00)	
<input type="checkbox"/> Cooperative groups, mixed gender (Tues./Thurs. at 10:00)	
<input type="checkbox"/> Cooperative groups, gender homogeneous (Tues./Thurs. at 12:00)	
<input type="checkbox"/> Cooperative groups, self-selected (Mon. eves.)	

6. \*Would you have preferred to be in one of the research groups other than the one in which you participated? Please explain.

Comment:

- Yes      **2**  
 No        **1**

If you were in the control group (MWF 10:00 class), skip to #17.

7. \*Do you like working in groups? Why or why not?

Comment:

- Yes      **2**  
 No        **1**

8. \*Were there times when no one in the group knew what to do because everyone assumed that someone else would listen to the directions?

Comment:

- Yes      **2**  
 No        **1**

9. \*Did you always participate fully or were there times when you sat back and let the others do the work? Explain.

Comment:

- Always participated fully    **3**  
 Usually participated fully    **2**  
 Rarely participated fully    **1**

10. \*Approximately how many hours per week did your group meet outside of class time?

Comment:

- None                            **1**  
 1-2 hours                      **2**  
 3-4 hours                      **3**  
 5-6 hours                      **4**  
 more than 6 hours          **5**

11. \*What was the purpose of those meetings? Check all that apply.
- |   |          |
|---|----------|
| <input type="checkbox"/> Work on homework on a regular basis. | 1        |
| <input type="checkbox"/> Study for tests.                     | 2        |
| <b>both</b>   | <b>3</b> |
| <input type="checkbox"/> Other _____                          | <b>4</b> |
- 

12. \*In assessing what you learned in this class, do you feel that working cooperatively helped you, hindered you, or had no effect on what you learned?

Comment:

<input type="checkbox"/> helped	3
<input type="checkbox"/> hindered	1
<input type="checkbox"/> no effect	2

13. \*In assessing your achievement in this class, do you think that working cooperatively raised, lowered, or had no effect on your semester grade?

Comment:

<input type="checkbox"/> raised	3
<input type="checkbox"/> lowered	1
<input type="checkbox"/> no effect	2

14. \*Do you think that working cooperatively affected your attitude in any way? If so, in what way?

Comment:

<input type="checkbox"/> positive effect	3
<input type="checkbox"/> negative effect	1
<input type="checkbox"/> no effect	2

15. \*If you had feelings of anxiety at the start of the semester, did working cooperatively have any effect on that? If so, in what ways?

Comment:

<input type="checkbox"/> increased anxiety	1
<input type="checkbox"/> decreased anxiety	3
<input type="checkbox"/> no effect on anxiety	2

16. Please comment on the method of grading.

17. \*Would you take another course that is taught this way?

yes      **2**  
 no        **1**

Comment:

18. Feel free to make any other relevant comments regarding the course, the instructor, suggestions for improvement, etc. (not necessarily pertaining to group work). For example, are there topics we didn't cover that you would have liked to cover? Or do you think we should cover fewer topics in more depth?

Thanks very much for your help with my research!  
Have a great holiday and a restful break!

**APPENDIX B: HISTOGRAMS FOR INDIVIDUAL  
RESEARCH GROUPS**

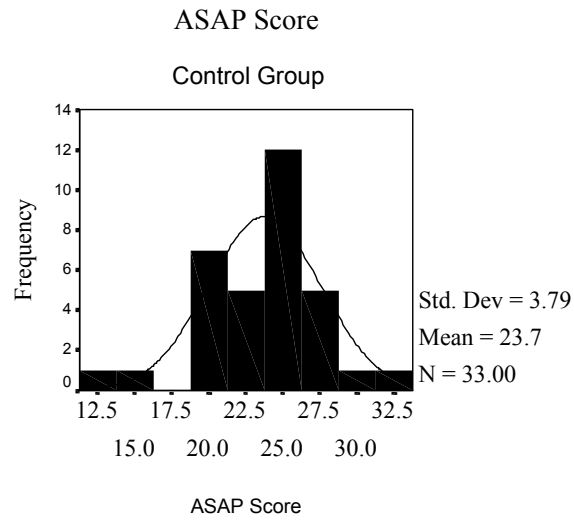


Figure B1. Distribution of ASAP Score for Control Group.

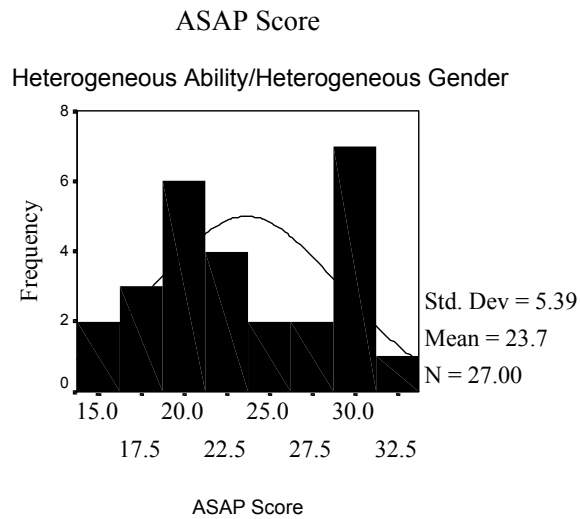


Figure B2. Distribution of ASAP Score for Heterogeneous Ability/Heterogeneous Gender Group.

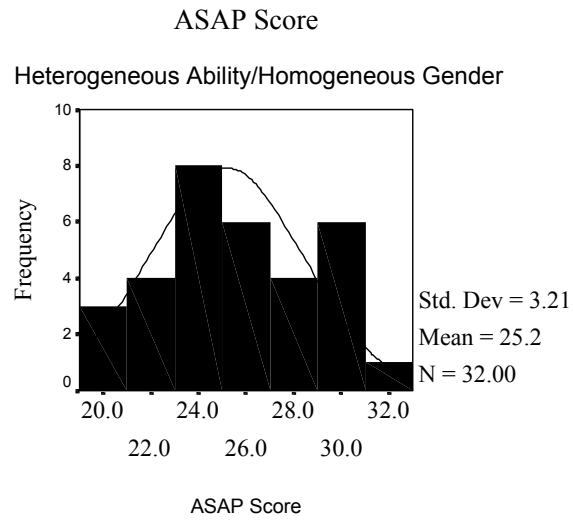


Figure B3. Distribution of ASAP Score for Heterogeneous Ability/Homogeneous Gender Group.

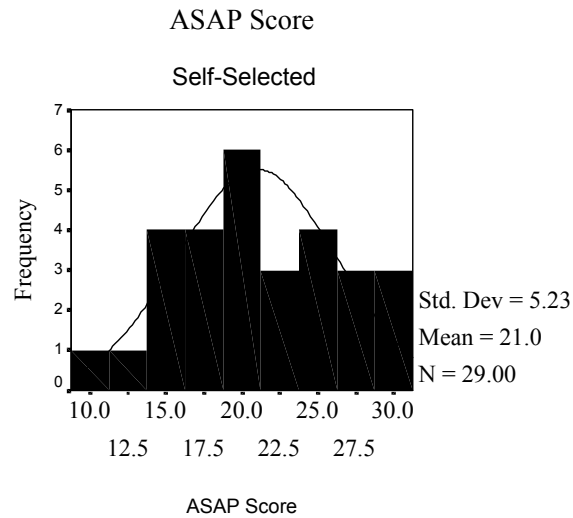


Figure B4. Distribution of ASAP Score for Self-selected Group.

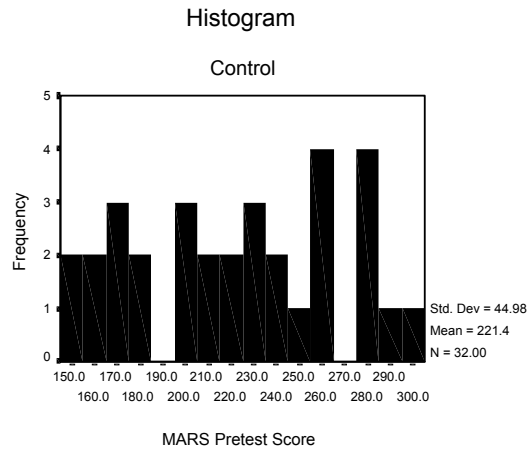


Figure B5. Distribution of MARS Pre-test for Control Group.

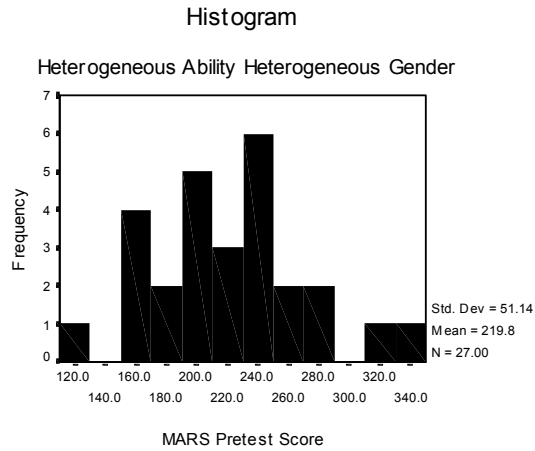


Figure B6. Distribution of MARS Pre-test for Heterogeneous Ability/Heterogeneous Gender Group.



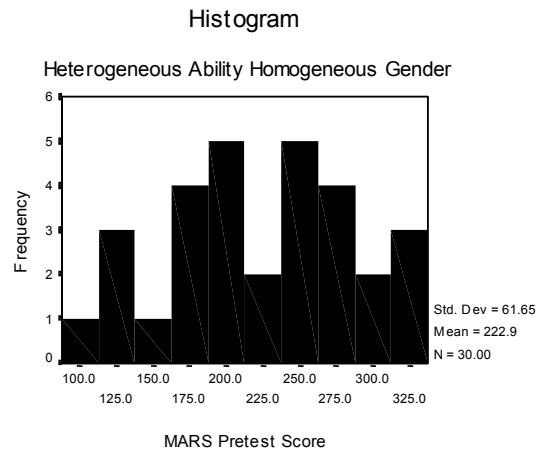


Figure B7. Distribution of MARS Pre-test for Heterogeneous Ability/Homogeneous Gender Group.

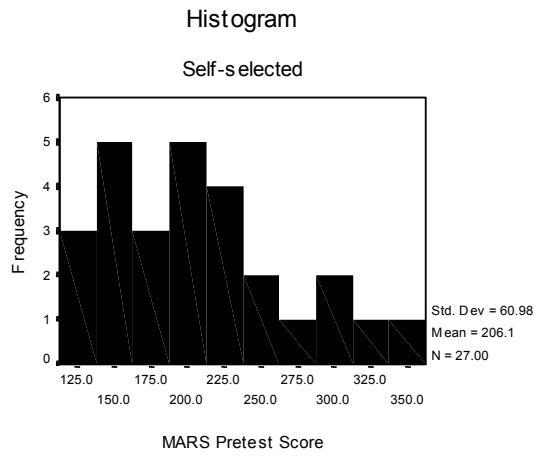


Figure B8. Distribution of MARS Pre-test for Self-selected Group.

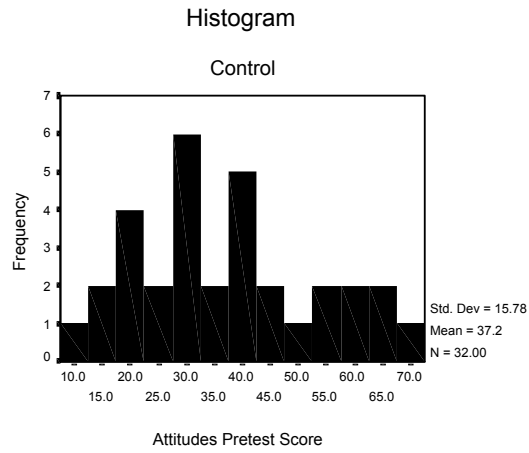


Figure B9. Distribution of Attitudes Pre-test for Control Group.

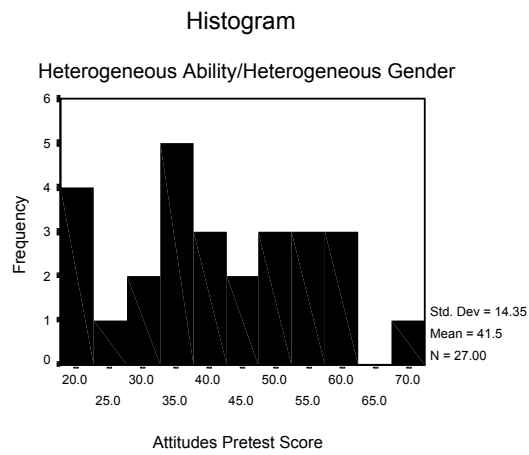


Figure B10. Distribution of Attitudes Pre-test for Heterogeneous Ability/Heterogeneous Gender.

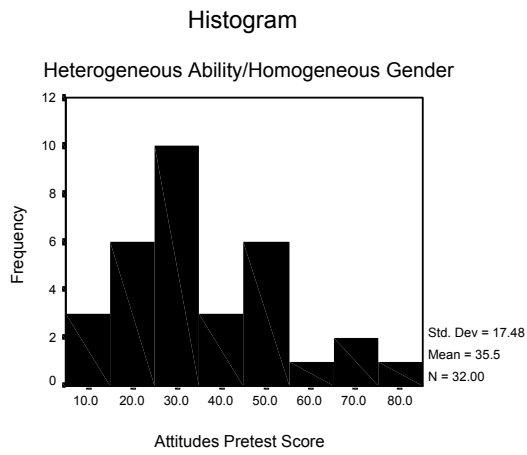


Figure B11. Distribution of Attitudes Pre-test for Heterogeneous Ability/Homogeneous Gender.

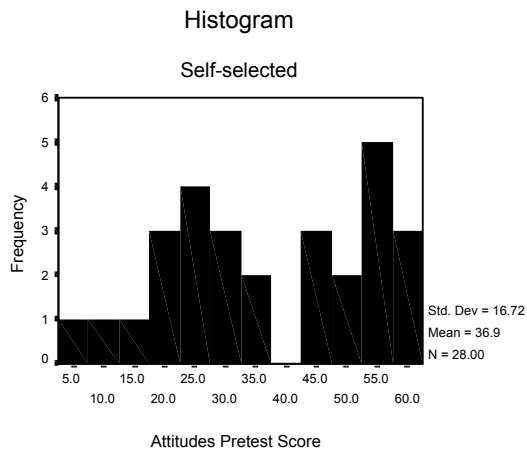


Figure B12. Distribution of Attitudes Pre-test for Self-selected Group.

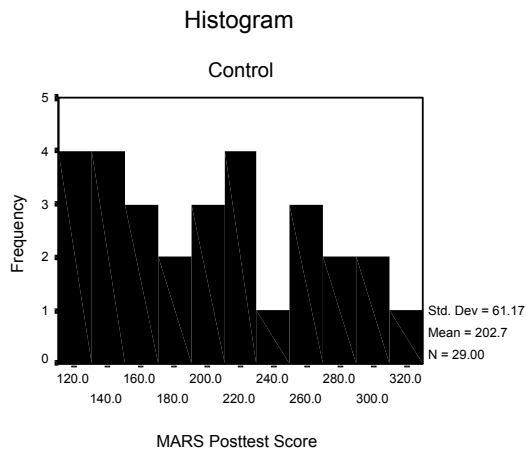


Figure B13. Distribution of MARS Post-test for Control Group.

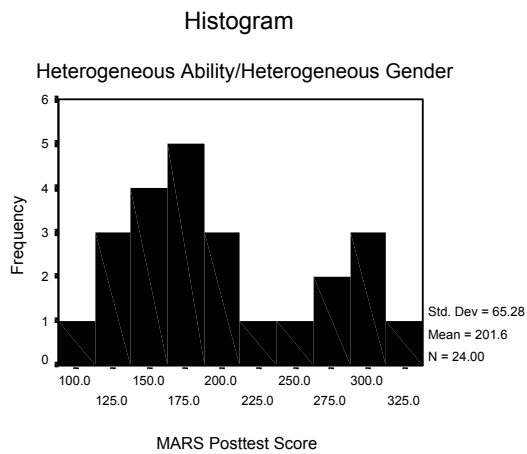


Figure B14. Distribution of MARS Post-test for Heterogeneous Ability/Heterogeneous Gender Group.

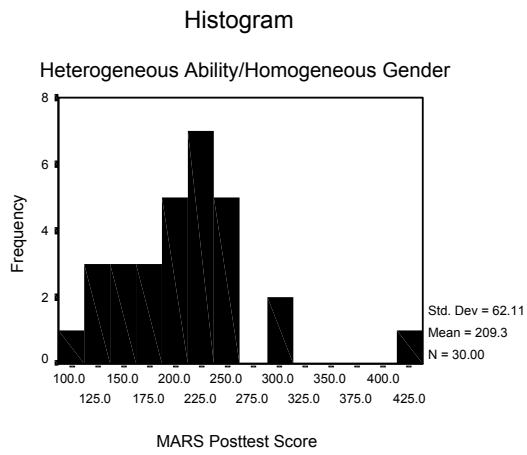


Figure B15. Distribution of MARS Post-test for Heterogeneous Ability/Homogeneous Gender.

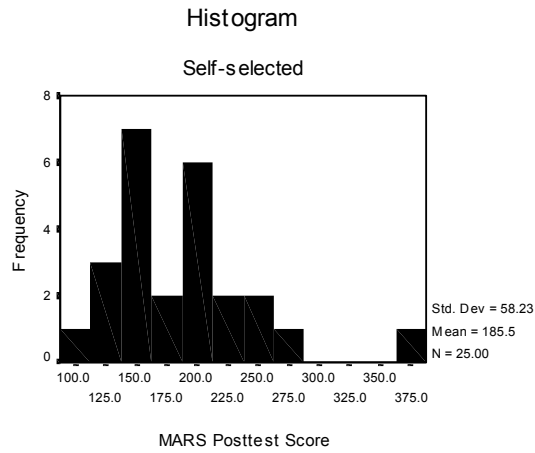


Figure B16. Distribution of MARS Post-test for Self-selected Group.

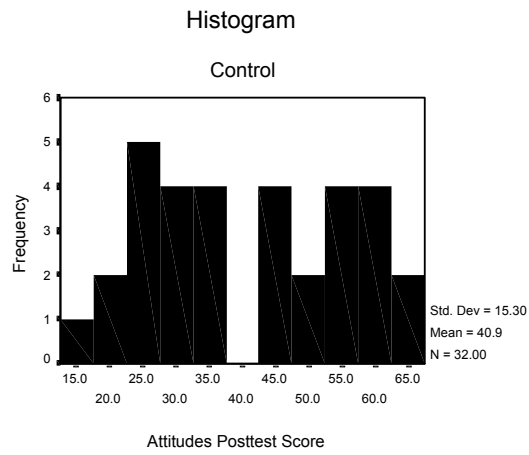


Figure B17. Distribution of Attitudes Post-test for Control Group.

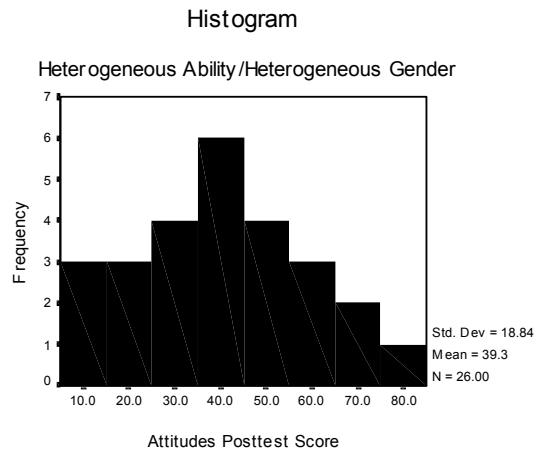


Figure B18. Distribution of Attitudes Post-test for Heterogeneous Ability/Heterogeneous Gender Group.

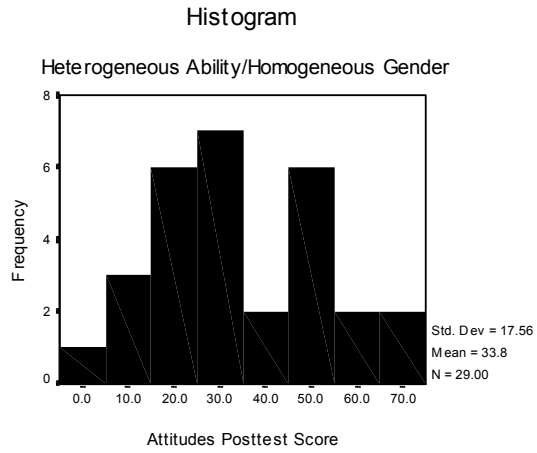


Figure B19. Distribution of Attitudes Post-test for Heterogeneous Ability/Homogeneous Gender Group.

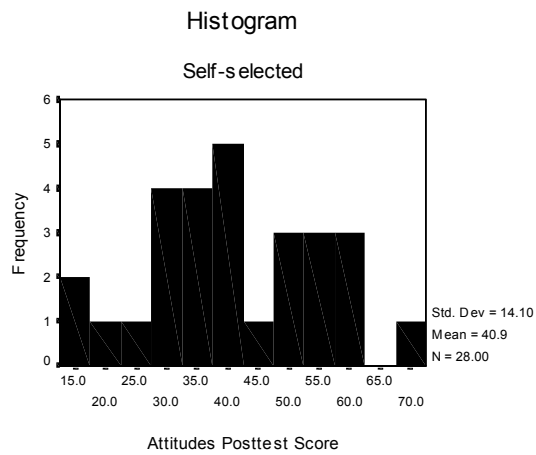


Figure B20. Distribution of Attitudes Post-test for Self-selected Group.

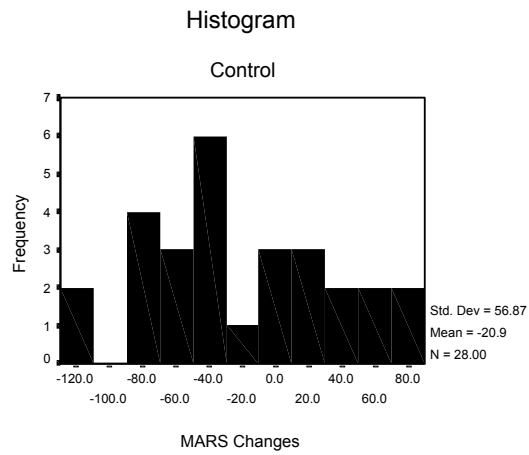


Figure B21. Distribution of MARS Changes for Control Group.

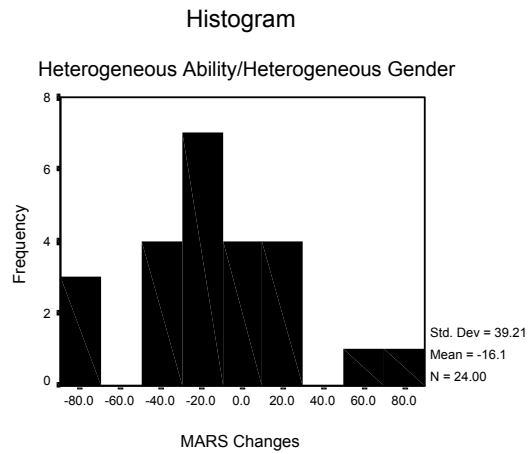


Figure B22. Distribution of MARS Changes for Heterogeneous Ability/Heterogeneous Gender Group.



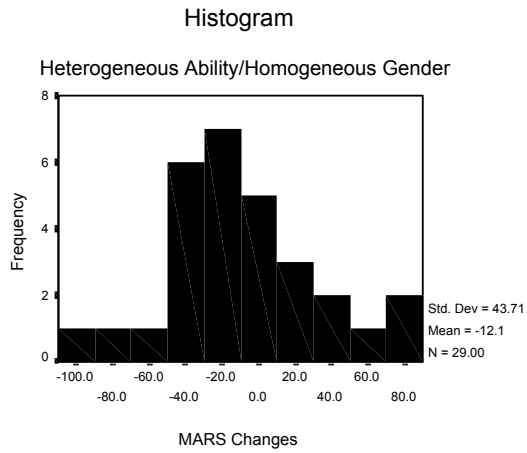


Figure B23. Distribution of MARS Changes for Heterogeneous Ability/Homogeneous Gender Group.

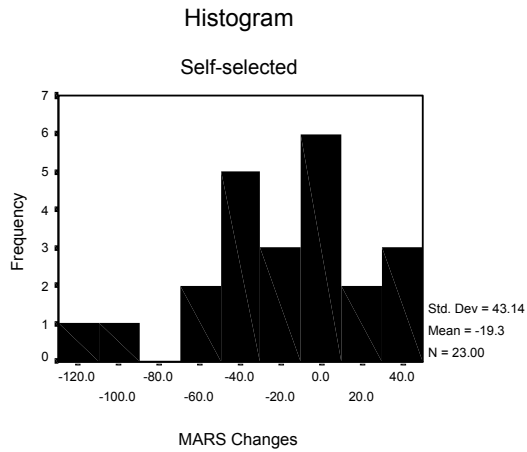


Figure B24. Distribution of MARS Changes for Self-selected Group.

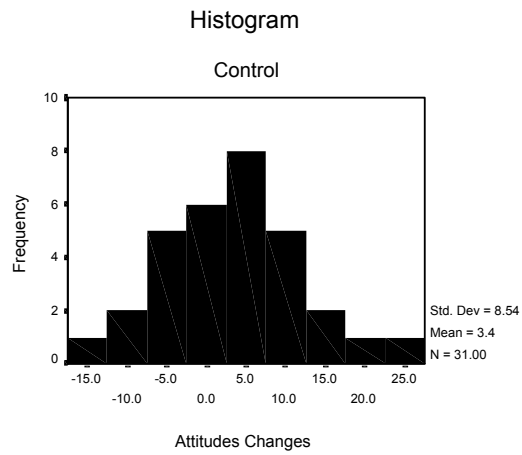


Figure B25. Distribution of Attitudes Changes for Control Group.

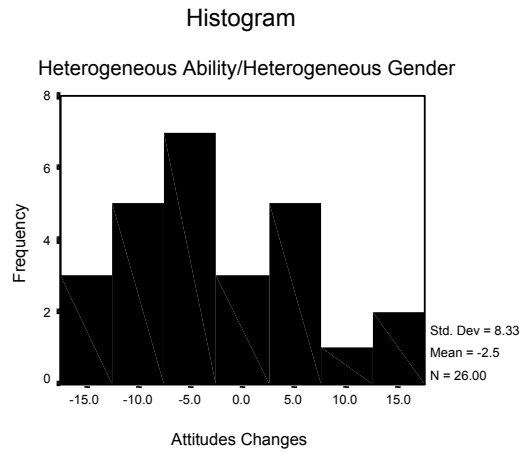


Figure B26. Distribution of Attitudes Changes for Heterogeneous Ability/Heterogeneous Gender Group.

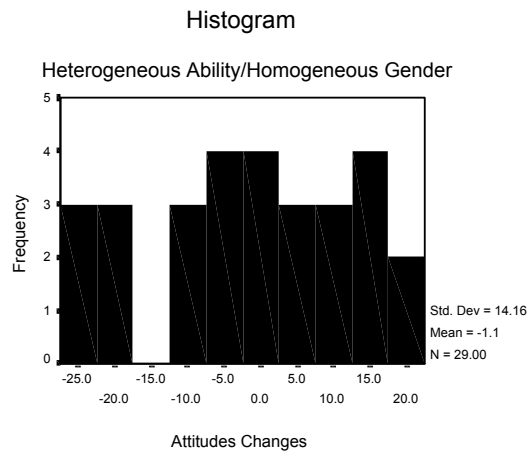


Figure B27. Distribution of Attitudes Changes for Heterogeneous Ability/Homogeneous Gender Group.

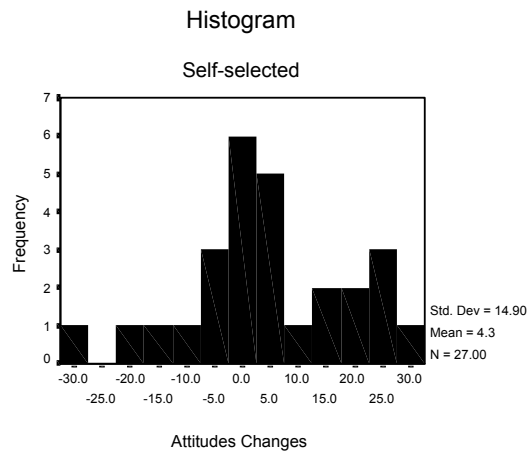


Figure B28. Distribution of Attitudes Changes for Self-selected Group.

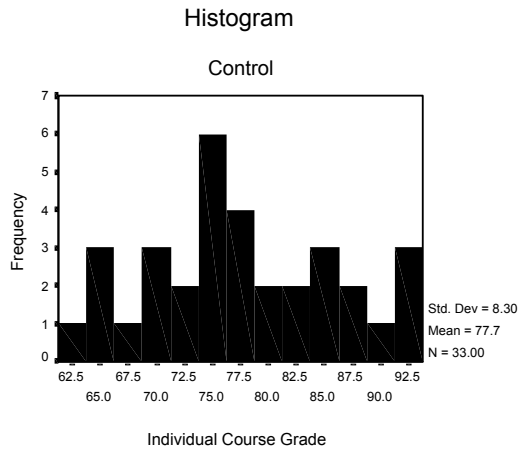


Figure B29. Distribution of Individual Course Grade for Control Group.

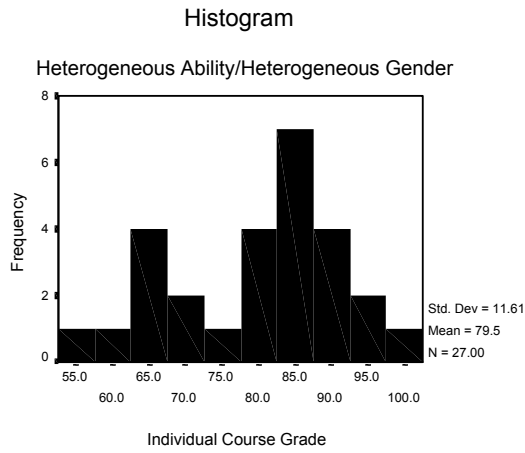


Figure B30. Distribution of Individual Course Grade for Heterogeneous Ability/Heterogeneous Gender Group.

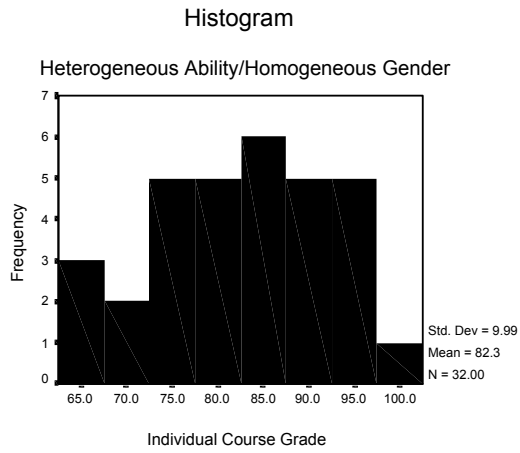


Figure B31. Distribution of Individual Course Grade for Heterogeneous Ability/Homogeneous Gender Group.

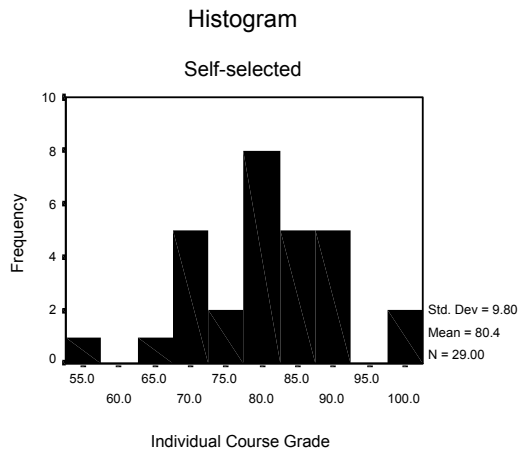


Figure B32. Distribution of Individual Course Grade for Self-selected Group.

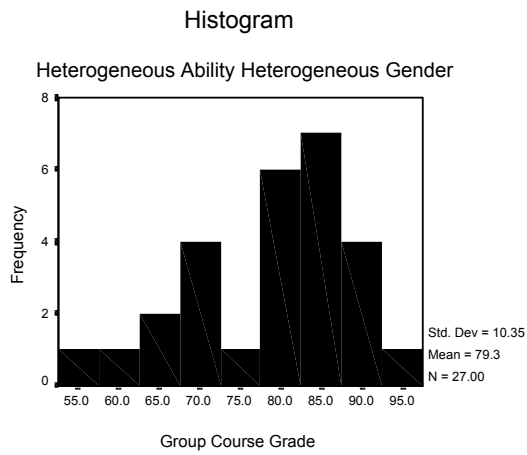


Figure B33. Distribution of Group Course Grade for Heterogeneous Ability/Heterogeneous Gender Group.

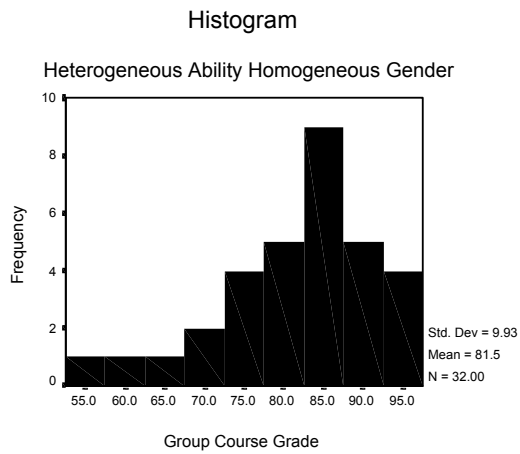


Figure B34. Distribution of Group Course Grade for Heterogeneous Ability/Homogeneous Gender Group.

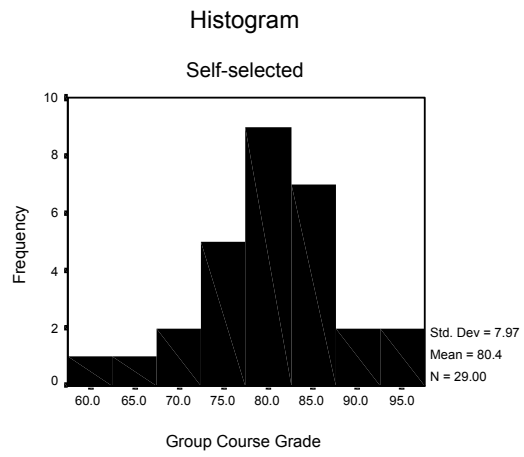


Figure B35. Distribution of Group Course Grade for Self-selected Group.

**APPENDIX C: ANCOVA FOR INDIVIDUAL COURSE  
GRADE BY RESEARCH GROUP USING TWO  
RESEARCH GROUPS**



Table C1. Between-Subjects Factors for Individual Course Grade by Research Group.

	Value	Label	N
Research Group	1	Control	33
	2	Cooperative	88

Table C2. Descriptive Statistics for Individual Course Grade by Research Group.

Research Group	M	SD	N
Control	77.67	8.30	33
Cooperative	80.83	10.40	88
Total	79.97	9.94	121

Table C3. Tests of Between-Subjects Effects for Individual Course Grade by Research Group.

Source	Type III SS	df	MS	F	Sig.	Eta <sup>2</sup>
Corrected Model	2024.133 <sup>a</sup>	2	1012.067	12.154	.000	.171
Intercept	15792.108	1	15792.108	189.652	.000	.616
ASAP	1784.042	1	1784.042	21.425	<b>.000</b>	.154
RESGROUP	294.126	1	294.126	3.532	.063	.029
Error	9825.735	118	83.269			
Total	785610.000	121				
Corrected Total	11849.868	120				

a R = .428, R Squared = .183 (Adjusted R Squared = .155)

**APPENDIX D: RESULTS OF RESEARCHER-CONSTRUCTED  
QUESTIONNAIRE – CROSSTABS AND CORRELATIONS**

Table D1. Gender by Age Group Crosstabulation.

Gender		Age Group			Total
		<18	18-22	>18	
Male	Count	2	40	9	51
	Expected Count	6.7	35.4	8.9	51.0
	% within Gender	3.9%	78.4%	17.6%	100.0%
	% within Age Group	12.5%	47.6%	42.9%	42.1%
Female	Count	14	44	12	70
	Expected Count	9.3	48.6	12.1	70.0
	% within Gender	20.0%	62.9%	17.1%	100.0%
	% within Age Group	87.5%	52.4%	57.1%	57.9%
Total	Count	16	84	21	121
	Expected Count	16.0	84.0	21.0	121.0
	% within Gender	13.2%	69.4%	17.4%	100.0%
	% within Age Group	100.0%	100.0%	100.0%	100.0%

Table D2. Research Group by Age Group Crosstabulation.

Research Group		Age Group			Total
		<18	18-22	>18	
Control	Count	2	26	5	33
	Expected Count	4.4	22.9	5.7	33.0
	% within Research Group	6.1%	78.8%	15.2%	100.0%
	% within Age Group	12.5%	31.0%	23.8%	27.3%
Het. Ability/ Het. Gender	Count	7	19	1	27
	Expected Count	3.6	18.7	4.7	27.0
	% within Research Group	25.9%	70.4%	3.7%	100.0%
	% within Age Group	43.8%	22.6%	4.8%	22.3%
Het. Ability/ Hom. Gender	Count	6	25	1	32
	Expected Count	4.2	22.2	5.6	32.0
	% within Research Group	18.8%	78.1%	3.1%	100.0%
	% within Age Group	37.5%	29.8%	4.8%	26.4%
Self-selected	Count	1	14	14	29
	Expected Count	3.8	20.1	5.0	29.0
	% within Research Group	3.4%	48.3%	48.3%	100.0%
	% within Age Group	6.3%	16.7%	66.7%	24.0%
Total	Count	16	84	21	121
	Expected Count	16.0	84.0	21.0	121.0
	% within Research Group	13.2%	69.4%	17.4%	100.0%
	% within Age Group	100.0%	100.0%	100.0%	100.0%

Table D3. Attendance by Age Group Crosstabulation.

Attendance		Age Group			Total
		<18	18-22	>18	
60-69	Count	1	3	1	5
	Expected Count	.7	3.5	.9	5.0
	% within Attendance	20.0%	60.0%	20.0%	100.0%
	% within Age Group	6.3%	3.6%	4.8%	4.1%
70-79	Count	3	12	2	17
	Expected Count	2.2	11.8	3.0	17.0
	% within Attendance	17.6%	70.6%	11.8%	100.0%
	% within Age Group	18.8%	14.3%	9.5%	14.0%
80-89	Count	2	20	1	23
	Expected Count	3.0	16.0	4.0	23.0
	% within Attendance	8.7%	87.0%	4.3%	100.0%
	% within Age Group	12.5%	23.8%	4.8%	19.0%
90-100	Count	10	49	17	76
	Expected Count	10.0	52.8	13.2	76.0
	% within Attendance	13.2%	64.5%	22.4%	100.0%
	% within Age Group	62.5%	58.3%	81.0%	62.8%
Total	Count	16	84	21	121
	Expected Count	16.0	84.0	21.0	121.0
	% within Attendance	13.2%	69.4%	17.4%	100.0%
	% within Age Group	100.0%	100.0%	100.0%	100.0%

Table D4. Math Background by Gender Crosstabulation.

Math Background		Gender		Total
		Male	Female	
HS Algebra I only	Count	2	0	2
	Expected Count	.8	1.2	2.0
	% within Math Background	100.0%	.0%	100.0%
	% within Gender	3.9%	.0%	1.7%
HS Algebra I and Geometry	Count	4	8	12
	Expected Count	5.1	6.9	12.0
	% within Math Background	33.3%	66.7%	100.0%
	% within Gender	7.8%	11.4%	9.9%
HS Algebra I, Geometry, and Algebra II	Count	31	33	64
	Expected Count	27.0	37.0	64.0
	% within Math Background	48.4%	51.6%	100.0%
	% within Gender	60.8%	47.1%	52.9%
Elem. Algebra at RCTC	Count	4	11	15
	Expected Count	6.3	8.7	15.0
	% within Math Background	26.7%	73.3%	100.0%
	% within Gender	7.8%	15.7%	12.4%
Other	Count	10	18	28
	Expected Count	11.8	16.2	28.0
	% within Math Background	35.7%	64.3%	100.0%
	% within Gender	19.6%	25.7%	23.1%
Total	Count	51	70	121
	Expected Count	51.0	70.0	121.0
	% within Math Background	42.1%	57.9%	100.0%
	% within Gender	100.0%	100.0%	100.0%

Table D5. Math Background by Research Group Crosstabulation.

Math Background		Research Group				Total
		C	T1	T2	T3	
HS Alg. I only	Count	0	0	1	1	2
	Expected Count	.5	.4	.5	.5	2.0
	% within Res. Group	.0%	.0%	3.1%	3.4%	1.7%
	% within Math Backgr.	.0%	.0%	50.0%	50.0%	100%
HS Alg. I, Geometry	Count	0	4	5	3	12
	Expected Count	3.3	2.7	3.2	2.9	12.0
	% within Res. Group	.0%	14.8%	15.6%	10.3%	9.9%
	% within Math Backgr.	.0%	33.3%	41.7%	25.0%	100.0%
HS Alg. I, Geometry, Algebra II	Count	20	17	17	10	64
	Expected Count	17.5	14.3	16.9	15.3	64.0
	% within Res. Group	60.6%	63.0%	53.1%	34.5%	52.9%
	% within Math Backgr.	31.3%	26.6%	26.6%	15.6%	100.0%
Elem. Alg. at RCTC	Count	4	1	2	8	15
	Expected Count	4.1	3.3	4.0	3.6	15.0
	% within Res. Group	12.1%	3.7%	6.3%	27.6%	12.4%
	% within Math Backgr.	26.7%	6.7%	13.3%	53.3%	100.0%
Other	Count	9	5	7	7	28
	Expected Count	7.6	6.2	7.4	6.7	28.0
	% within Res. Group	27.3%	18.5%	21.9%	24.1%	23.1%
	% within Math Backgr.	32.1%	17.9%	25.0%	25.0%	23.1%
Total	Count	33	27	32	29	121
	Expected Count	33.0	27.0	32.0	29.0	121.0
	% within Res. Group	100.0%	100.0%	100.0%	100.0%	100.0%
	% within Math Backgr.	27.3%	22.3%	26.4%	24.0%	100.0%

Table D6. Math Background by Age Group Crosstabulation.

Math Background		Age Group			Total
		<18	18-22	>22	
HS Alg. I only	Count	0	1	1	2
	Expected Count	.3	1.4	.3	2.0
	% within Math Backgr.	.0%	50.0%	50.0%	100.0%
	% within Age Group	.0%	1.2%	4.8%	1.7%
HS Alg. I, Geometry	Count	3	7	2	12
	Expected Count	1.6	8.3	2.1	12.0
	% within Math Backgr.	25.0%	58.3%	16.7%	100.0%
	% within Age Group	18.8%	8.3%	9.5%	9.9%
HS Alg. I, Geometry, Algebra II	Count	12	48	4	64
	Expected Count	8.5	44.4	11.1	64.0
	% within Math Backgr.	18.8%	75.0%	6.3%	100.0%
	% within Age Group	75.0%	57.1%	19.0%	52.9%
Elem. Alg. at RCTC	Count	0	8	7	15
	Expected Count	2.0	10.4	2.6	15.0
	% within Math Backgr.	.0%	53.3%	46.7%	100.0%
	% within Age Group	.0%	9.5%	33.3%	12.4%
Other	Count	1	20	7	28
	Expected Count	3.7	19.4	4.9	28.0
	% within Math Backgr.	3.6%	71.4%	25.0%	100.0%
	% within Age Group	6.3%	23.8%	33.3%	23.1%
Total	Count	16	84	21	121
	Expected Count	16.0	84.0	21.0	121.0
	% within Math Backgr.	13.2%	69.4%	17.4%	100.0%
	% within Age Group	100.0%	100.0%	100.0%	100.0%



Table D7. Gender by Enjoyment of Groups Crosstabulation.

Gender		<u>Enjoy Groups</u>		Total
		yes	no	
Male	Count	8	26	34
	Expected Count	7.4	26.6	34.0
	% within Gender	23.5%	76.5%	100.0%
	% within Enjoy Groups	42.1%	38.2%	39.1%
Female	Count	11	42	53
	Expected Count	11.6	41.4	53.0
	% within Gender	20.8%	79.2%	100.0%
	% within Enjoy Groups	57.9%	61.8%	60.9%
Total	Count	19	68	87
	Expected Count	19.0	68.0	87.0
	% within Gender	21.8%	78.2%	100.0%
	% within Enjoy Groups	100.0%	100.0%	100.0%

Table D8. Research Group by Enjoyment of Groups Crosstabulation.

Research Group		<u>Enjoy Groups</u>		Total
		yes	no	
Het. Ability/	Count	7	20	27
Het. Gender	Expected Count	5.9	21.1	27.0
	% within Res. Group	25.9%	74.1%	100.0%
	% within Enjoy Groups	36.8%	29.4%	31.0%
Het. Ability/	Count	6	26	32
Homo. Gender	Expected Count	7.0	25.0	32.0
	% within Res. Group	18.8%	81.3%	100.0%
	% within Enjoy Groups	31.6%	38.2%	36.8%
Self-Selected	Count	6	22	28
	Expected Count	6.1	21.9	28.0
	% within Res. Group	21.4%	78.6%	100.0%
	% within Enjoy Groups	31.6%	32.4%	32.2%
Total	Count	19	68	87
	Expected Count	19.0	68.0	87.0
	% within Res. Group	21.8%	78.2%	100.0%
	% within Enjoy Groups	100.0%	100.0%	100.0%

Table D9. Gender by Effect on Achievement Crosstabulation.

Gender		<u>Effect on Achievement</u>			Total
		Raised	Lowered	No Effect	
Male	Count	6	9	18	33
	Expected Count	5.8	12.3	15.0	33.0
	% within Gender	18.2%	27.3%	54.5%	100.0%
	% within Effect on Achievement	40.0%	28.1%	46.2%	38.4%
Female	Count	9	23	21	53
	Expected Count	9.2	19.7	24.0	53.0
	% within Gender	17.0%	43.4%	39.6%	100.0%
	% within Effect on Achievement	60.0%	71.9%	53.8%	61.6%
Total	Count	15	32	39	86
	Expected Count	15.0	32.0	39.0	86.0
	% within Gender	17.4%	37.2%	45.3%	100.0%
	% within Effect on Achievement	100.0%	100.0%	100.0%	100.0%

Table D10. Research Group by Effect on Achievement Crosstabulation.

Research Group		Effect on Achievement			Total
		Raised	Lowered	No Effect	
Het. Ability/	Count	6	8	13	27
Het. Gender	Expected Count	4.7	10.0	12.2	27.0
	% within Res. Group	22.2%	29.6%	48.1%	100.0%
	% within Effect on Achievement	40.0%	25.0%	33.3%	31.4%
Het. Ability/	Count	6	10	16	32
Hom. Gender	Expected Count	5.6	11.9	14.5	32.0
	% within Res. Group	18.8%	31.3%	50.0%	100.0%
	% within Effect on Achievement	40.0%	31.3%	41.0%	37.2%
Self-Selected	Count	3	14	10	27
	Expected Count	4.7	10.0	12.2	27.0
	% within Res. Group	11.1%	51.9%	37.0%	100.0%
	% within Effect on Achievement	20.0%	43.8%	25.6%	31.4%
Total	Count	15	32	39	86
	Expected Count	15.0	32.0	39.0	86.0
	% within Res. Group	17.4%	37.2%	45.3%	100.0%
	% within Effect on Achievement	100.0%	100.0%	100.0%	100.0%

Table D11. Gender by Effect on Attitude Crosstabulation.

Gender		Effect on Attitude				Total
		Positive	Negative	No Effect	Other	
Male	Count	3	15	16	0	34
	Expected Count	3.9	15.6	14.1	.4	34.0
	% within Gender	8.8%	44.1%	47.1%	.0%	100.0%
	% within Effect on Attitude	30.0%	37.5%	44.4%	.0%	39.1%
Female	Count	7	25	20	1	53
	Expected Count	6.1	24.4	21.9	.6	53.0
	% within Gender	13.2%	47.2%	37.7%	1.9%	100.0%
	% within Effect on Attitude	70.0%	62.5%	55.6%	100.0%	60.9%
Total	Count	10	40	36	1	87
	Expected Count	10.0	40.0	36.0	1.0	87.0
	% within Gender	11.5%	46.0%	41.4%	1.1%	100.0%
	% within Effect on Attitude	100.0%	100.0%	100.0%	100.0%	100.0%

Table D12. Research Group by Effect on Attitude Crosstabulation.

Research Group		Effect on Attitude				Total
		Positive	Negative	No Effect	Other	
Het. Ability/	Count	3	12	11	1	27
Het. Gender	Exp. Count	3.1	12.4	11.2	.3	27.0
	% within					
	Res.Group	11.1%	44.4%	40.7%	3.7%	100.0%
	% within Effect on Attitude	30.0%	30.0%	30.6%	100.0%	31.0%
Het. Ability/	Count	4	18	10	0	32
Hom. Gender	Exp. Count	3.7	14.7	13.2	.4	32.0
	% within					
	Res.Group	12.5%	56.3%	31.3%	.0%	100.0%
	% within Effect on Attitude	40.0%	45.0%	27.8%	100.0%	36.8%
Self-selected	Count	3	10	15	0	28
	Exp. Count	3.2	12.9	11.6	.3	28.0
	% within					
	Res.Group	10.7%	35.7%	53.6%	.0%	100.0%
	% within Effect on Attitude	30.0%	25.0%	41.7%	.0%	32.2%
Total	Count	10	40	36	1	87
	Exp. Count	10.0	40.0	36.0	1.0	87.0
	% within					
	Res.Group	11.5%	46.0%	41.4%	1.1%	100.0%
	% within Effect on Attitude	100.0%	100.0%	100.0%	100.0%	100.0%

Table D13. Gender by Effect on Anxiety Crosstabulation.

Gender		<u>Effect on Anxiety</u>			Total
		Increased	Decreased	No Effect	
Male	Count	3	24	6	33
	Expected Count	3.8	22.3	6.9	33.0
	% within Gender	9.1%	72.7%	18.2%	100.0%
	% within Effect on Anxiety	30.0%	41.4%	33.3%	38.4%
Female	Count	7	34	12	53
	Expected Count	6.2	35.7	11.1	53.0
	% within Gender	13.2%	64.2%	22.6%	100.0%
	% within Effect on Anxiety	70.0%	58.6%	66.7%	61.6%
Total	Count	10	58	18	86
	Expected Count	10.0	58.0	18.0	86.0
	% within Gender	11.6%	67.4%	20.9%	100.0%
	% within Effect on Anxiety	100.0%	100.0%	100.0%	100.0%

Table D14. Research Group by Effect on Anxiety Crosstabulation.

Research Group		<u>Effect on Anxiety</u>			Total
		Increased	Decreased	No Effect	
Het. Ability/	Count	6	16	5	27
Het. Gender	Expected Count	3.1	18.2	5.7	27.0
	% within Res. Group	22.2%	59.3%	18.5%	100.0%
	% within Effect on Anxiety	60.0%	27.6%	27.8%	31.4%
Het. Ability/	Count	2	25	4	31
Hom. Gender	Expected Count	3.6	20.9	6.5	31.0
	% within Res. Group	6.5%	80.6%	12.9%	100.0%
	% within Effect on Anxiety	20.0%	43.1%	22.2%	36.0%
Self-selected	Count	2	17	9	28
	Expected Count	3.3	18.9	5.9	28.0
	% within Res. Group	7.1%	60.7%	32.1%	100.0%
	% within Effect on Anxiety	20.0%	29.3%	50.0%	32.6%
Total	Count	10	58	18	28
	Expected Count	10.0	58.0	18.0	28.0
	% within Res. Group	11.6%	67.4%	20.9%	100.0%
	% within Effect on Anxiety	100.0%	100.0%	100.0%	100.0%



Table D15. Gender by Take Another Course Crosstabulation.

Gender		<u>take another course</u>		Total
		yes	no	
Male	Count	7	41	48
	Expected Count	8.2	39.8	48.0
	% within Gender	14.6%	85.4%	100.0%
	% within Take Another			
	Course	35.0%	42.3%	41.0%
Female	Count	13	56	69
	Expected Count	11.8	57.2	69.0
	% within Gender	18.8%	81.2%	100.0%
	% within Take Another			
	Course	65.0%	57.7%	59.0%
Total	Count	20	97	117
	Expected Count	20.0	97.0	117.0
	% within Gender	17.1%	82.9%	100.0%
	% within Take Another			
	Course	100.0%	100.0%	100.0%

Table D16. Research Group by Take Another Course Crosstabulation.

Research Group		<u>Enjoy Groups</u>		Total
		yes	no	
Het. Ability/	Count	2	31	33
Het. Gender	Expected Count	5.6	27.4	33.0
	% within Res. Group	6.1%	93.9%	100.0%
	% within Take Another			
	Course	10.0%	32.0%	28.2%
Het. Ability/	Count	7	20	27
Homo. Gender	Expected Count	4.6	22.4	27.0
	% within Res. Group	25.9%	74.1%	100.0%
	% within Take Another			
	Course	35.0%	20.6%	23.1%
Self-Selected	Count	4	26	30
	Expected Count	5.1	24.9	30.0
	% within Res. Group	13.3%	86.7%	100.0%
	% within Take Another			
	Course	20.0%	26.8%	25.6%
Total	Count	20	97	117
	Expected Count	20.0	97.0	117.0
	% within Res. Group	17.1%	82.9%	100.0%
	% within Take Another			
	Course	100.0%	100.0%	100.0%

Table D17. Research Group by Hours Outside of Class Crosstabulation.

Research Group		<u>Hours Outside of Class</u>			Total
		None	1-2	3-4	
Het. Ability/	Count	20	4	3	27
Het. Gender	Expected Count	20.2	4.3	2.5	27.0
	% within Res. Group	74.1%	14.8%	11.1%	100.0%
	% within Hours	30.8%	28.6%	37.5%	31.0%
Het. Ability/	Count	24	6	2	32
Hom. Gender	Expected Count	23.9	5.1	2.9	32.0
	% within Res. Group	75.0%	18.8%	6.3%	100.0%
	% within Hours	36.9%	42.9%	25.0%	36.8%
Self-selected	Count	21	4	3	28
	Expected Count	20.9	4.5	2.6	28.0
	% within Res. Group	75.0%	14.3	10.7%	100.0%
	% within Hours	32.3%	28.6%	37.5%	32.2%
Total	Count	65	14	8	87
	Expected Count	65.0	14.0	8.0	87.0
	% within Res. Group	74.7%	16.1%	9.2%	100.0%
	% within Hours	100.0%	100.0%	100.0%	100.0%

Table D18. Research Group by Attendance Crosstabulation.

Research Group		Attendance				Total
		60-69	70-79	80-89	90-100	
Control	Count	3	7	6	17	33
	Exp. Count	1.4	4.6	6.3	20.7	33.0
	% within Res.Group	9.1%	21.2%	18.2%	51.5%	100.0%
	% within Attendance	60.0%	41.2%	26.1%	22.4%	27.3%
Het. Ability/ Het. Gender	Count	1	4	6	16	27
	Exp. Count	1.1	3.8	5.1	17.0	27.0
	% within Res.Group	3.7%	14.8%	22.2%	59.3%	100.0%
	% within Attendance	20.0%	23.5%	26.1%	21.1%	22.3%
Het. Ability/ Hom. Gender	Count	1	6	3	22	32
	Exp. Count	1.3	4.5	6.1	20.1	32.0
	% within Res.Group	3.1%	18.8%	9.4%	68.8%	100.0%
	% within Attendance	20.0%	35.3%	13.0%	28.9%	26.4%
Self-selected	Count	0	0	8	21	29
	Exp. Count	1.2	4.1	5.5	18.2	29.0
	% within Res.Group	.0%	.0%	27.6%	72.4%	100.0%
	% within Attendance	.0%	.0%	34.8%	27.6%	24.0%
Total	Count	5	17	23	76	121
	Exp. Count	5.0	17.0	23.0	76.0	121.0
	% within Res.Group	4.1%	14.0%	19.0%	62.8%	100.0%
	% within Attendance	100.0%	100.0%	100.0%	100.0%	100.0%

Table D19. Spearman's Rho Correlation of Individual Course Grade and Enjoyment of Group Work

Measure	1	2
1. Individual Course Grade	--	
2. Enjoyment of Group Work	-.205	--

Note. Listwise  $N = 87$ .

\*\*  $p < .01$ , two tailed.

\*  $p < .05$ , two tailed.

Table D20. Spearman's Rho Correlation of Individual Course Grade and Students' Perceptions of Effect of Group Work on Course Grade

Measure	1	2
1. Individual Course Grade	--	
2. Effect on Course Grade	-.263*	--

Note. Listwise  $N = 86$ .

\*\*  $p < .01$ , two tailed.

\*  $p < .05$ , two tailed.

Table D21. Spearman's Rho Correlation of Attitude Change and Students' Perceptions of Effect of Group Work on Attitude Change

Measure	1	2
1. Attitude Change	--	
2. Effect on Attitude Change	-.039	--

Note. Listwise  $N = 81$ .

\*\*  $p < .01$ , two tailed.

\*  $p < .05$ , two tailed.

Table D22. Spearman's Rho Correlation of Anxiety Change and Students' Perceptions of Effect of Group Work on Anxiety Change.

Measure	1	2
1. Anxiety Change	--	
2. Effect on Anxiety Change	-.189	--

Note. Listwise  $N = 74$ .

\*\* $p < .01$ , two tailed.

\* $p < .05$ , two tailed.

Table D23. Pearson's Intercorrelations of Individual Course Grade, ASAP, Attitudes Pre-test, Attitudes Post-test, MARS Pre-test, and MARS Post-test.

Measure	1	2	3	4	5	6
1. Individual Course Grade	--					
2. ASAP Score	.382**	--				
3. Math Attitudes Pre-test	.166	.184*	--			
4. Math Attitudes Post-test	.330**	.178	.734**	--		
5. MARS Pre-test	-.111	-.143	-.381**	-.277**	--	
6. MARS Post-test	-.171	-.136	-.423**	-.520**	.696**	--

Note. Pairwise  $N = 121$  for 1 and 2,  $N = 19$  for 3,  $N = 115$  for 4,  $N = 116$  for 5, and  $N = 108$  for 6.

\*\* $p < .001$ , two tailed.

\* $p < .05$ , two tailed.

**APPENDIX E: STUDENTS' COMMENTS FROM THE  
RESEARCHER-CONSTRUCTED QUESTIONNAIRE**

The responses are grouped by research group. Multiple, similar responses are indicated by tally marks following the comment.

**6. Would you have preferred to be in one of the research groups other than the one in which you participated? Please explain.**

Group 1 Control group (MWF 10:00)

Yes

I enjoyed it in another class (Elem Algebra).

Would to be partner w/other students beside the ones I sat next to.

I like working in groups. I

I would rather be in the group with cooperative learning because I work better in a group.

No

I would rather work alone than in a group. IIIII

I do not like group work. I wish to depend upon myself for my grade.

If you were in a group you might not have learned as much and you would not have been as prepared for the test.

I wouldn't want other people bringing my grade down and I also wouldn't bring others grades down.

Because it really doesn't matter to me, you would not have had a choice in the matter anyway.

It really doesn't matter to me.

I liked the way it was done.

I thought there was nothing wrong with this group.

Worked fine just the way it was.

Being on my own, made me think for myself. In return I had to know the information.

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Yes

I felt the work wasn't fairly distributed between the four of us, I felt if I didn't do it, it might not get done.

I felt that 2 people out of the 4 participated willingly and were dedicated. The other 2 never showed up and didn't participate well in class.

Self-selected would have been nicer and easier to deal with.

Two of the people in our group never would work with us.

Self-selected one felt more comfortable with picking my own group.

The other people in group either dropped out or skipped a lot which put more stress on me from daily assignments to tests.

I'm not a teacher and I can't explain how to do things and I was often asked to explain and I couldn't.

No

It doesn't really matter to me. I

I liked the group I was in.



This was fine.  
The mixed cooperative is fine the others seem alright too.  
I felt my group for the most part, did an excellent job.  
I liked the group I was in but I don't like the fact that my grade depended on other people.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Yes

It would have been nice to choose ourselves.  
Control Group.  
Control group because I can't make someone else show up for class and it shouldn't effect my grade.  
Control group, I work better on my own.  
I would prefer to be in a mixed gender group, I think that works better.  
I didn't want to be segregated from the opposite gender.

No

I liked the group I was in. 1  
It worked well.  
Mine was fine.  
This group is just as good as the next/doesn't matter 1  
I liked working with just other females.  
I felt better working with all girls that way we helped each other better.  
I liked working in groups because I came to class more than I would have otherwise.  
Our group did really well – so I don't think I'd change – it's easier working with guys. (*male respondent*)  
With all girls I didn't feel nervous to participate, easier to work with.  
I liked the way we worked with groups on daily grades because that helps study for test. They explain if you don't understand. I feel it's good to put responsibility on this, if we don't do good brings group down etc. Makes you try harder.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Yes

Some of the group didn't really care.  
Cooperative groups mixed gender, with diverse groups it's an equal chance for everyone.  
Not a lot of cooperation  
I don't like the group thing.  
I feel I may get a B, because of lower scores from others in my group.

No

I liked my group (we worked well together). 11  
I liked being able to select my group myself. 11  
I liked working in groups.  
Night classes worked best with my work schedule.  
Every one of us knew certain parts better. We got much needed help.

I like the groups on Monday night.

**7. Do you like working in groups? Why or why not?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Yes

It was nice being able to help each other out/learn from each other IIII

I like having people around who can explain things to me if I don't understand something.

Because it helps you understand the lessons better. I

Helps you learn more and you are able to get help with homework. I

So you get help or help others and don't have to wait for teacher to come around and show you.

I think it helps with daily grades, I'm not sure that I would have known how to do all of them without people. It's hard when one of you group members is ALWAYS Absent.

Yes, because we get a variety of ideas to solve problems. We can check our answers.

Help with confusing or harder work.

More minds working on a problem/more input. I

If everyone is here it's great.

I like to hear everyone else's opinion. When I didn't understand something at least one other person was able to explain it to me. I didn't have to bother the teacher every other minute.

No

I would much rather think for myself. If I screw up I know it is my fault and I can take responsibility for my actions. I don't like to depend on others for things I hope to accomplish.

I like to work by myself.

It helped out however I could have gotten a study partner without having to go through a set group.

Don't think it was fair for our grades to reflect our own grade.

I would prefer to be responsible for my own education. Math class was enough of a worry for me let alone worrying if 3 other people were studying.

I like my grade to be mine only.

I have no patience.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Yes

It makes some things easier to learn if they are explained by someone else.

Group helps me understand/learn the lessons. IIII

It's easier to learn that way, without so much teacher attention.

It's easier to learn when you have 3 different people with 3 different skills and 3 different levels.

Group can answer questions.

1

If I don't understand something, I don't have to ask in front of the class.

They can help you if you are having problems.

Helps me figure out and get answers to questions when I didn't know how to do things.

Gives you more insights about problems.

Gives me a sense of how other people think.

You can compare answers, ask questions, and learn from each other.

I felt more open to ask questions and I would get the same answer from almost everyone, so you can be more sure of the answer.

It made each person do less work, also if we had a problem usually one of us knew the answer.

I'm not always responsible for all the work. Responsibility is split up evenly.

Pushed me to work harder.

I like to meet new people.

It makes class fun. It's boring to work alone.

No

It's nice to meet people, but it's hard to help others if they don't understand and it's easier for me to learn and work on my own.

I didn't learn any more, most of the time we just rushed though it to get done.

I don't like to be responsible for other people.

I don't really like groups because, lots of past bad experience and I still always feel like I'm the only one who wanted to be here and do well therefore I end up doing more, which I don't always mind but sometimes bugs me.

I work better by myself.

I work and think better on my own.

#### Group 4 Cooperative groups, self-selected (Mon.eves.)

Yes

It helps.

I think it helps in solving problems. You have more than one person thinking about the answer so you're more likely to get it.

You get a better chance to get it right.

More ideas and thoughts.

Shared knowledge.

Four times the knowledge was applied to studying.

Because you get more people's heads together to figure out the problem.

But preferably not in a math class.

Because I liked my group members.

Because you can ask questions of your peers and not feel so stupid.

1

Group members help each other out when they don't understand something.

III

It helps to work with others to better explain problems.

I did learn more from the group.

Some people are very good at 1 thing and can show the others. Next time, another person can show the others.

But if I have outgoing partners. They were too "self doers."

Easier to see how problems work out with multiple ideas of how to attack it. Checking each other's work was also helpful. If someone didn't understand, someone else would.

No

I prefer to struggle and work through problems and discover the answers myself. We never really had time to get together.

I only liked working with ~~~ because she cared and didn't make excuses and miss meeting times.

Prefer to work on own and go through problems at own pace, ask questions in a way that I would answer.

Not this early in the work, maybe later after we had all figured it out and not on test.

**8. Were there times when no one in the group knew what to do because everyone assumed that someone else would listen to the directions?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Yes

Yes, but all figured out what to do.

2 out of 4 were observant and paid attention.

No

A someone always knew what to do. III

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Yes

It happened quite often.

This happened once. We weren't quite sure, so we had to ask.

No

Sometimes one of us wasn't sure, but then we would tell them.

We both listened.

We all listened and participated usually.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Yes

Sometimes one person would not have read the homework.

No

Everyone seemed to have an idea as to what should be done.

All or a couple of us always seemed to have a general idea of what to do.

Some people didn't do their homework so they didn't know how.

We all brought our ideas to the group.

Unlike previous group work in other classes, my group worked not only equally, but well together.

We are all responsible.

**9. Did you always participate fully or were there times when you sat back and let the others do the work? Explain.**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Always participated fully:

There were two people in my group who were passive and the other one was shy but knew exactly what he was doing so I need to take initiative.

I always had an example way to solve the problem, and I always worked it out.

I felt if I didn't, it wouldn't get done, and the answers would be right.

However I am slower at math and found myself lost when others were ahead.

There were times when I didn't understand something and the rest of the group would just keep working ahead answering questions and I would still be back working on the second question.

Usually participated fully:

I usually participated.

We all had our strong points.

I think we all at times let others do the work for us.

It really depended on if I understood how to do it or if I needed to watch to understand what was going on. I am a very hands on learner.

Sometimes I felt too stupid to ask a question because seemed to grasp the concept fully.

I wasn't here sometimes.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Always participated fully:

If I knew what we were doing I would help others or I would do the recording.

1

There was only two of us.

I knew what was expected and worked to get it done.

Our whole group always participated fully.

Usually participated fully:

When everyone understands something, it is easy to let someone else do the work.

Tried, but sometimes didn't understand fully about what we were doing.

At times someone would understand something better than I, so I would watch and learn.

Some people knew the material better sometime.

The first times we met we got used to working with each other and didn't always put in a lot of input. After that I always participated, but another member usually sat there and let us other two do the work, because she didn't understand and we wanted to hurry and get it done instead of taking the time to explain to her.

Sometimes I didn't understand how to do the problems so I would listen to find out or have the others help me. It's hard to participate fully when sometimes you don't know how to do things.

Sometimes I wasn't sure, so I asked questions or just paid attention to what my group members were saying. I  
I really hate math. Sometimes I just couldn't understand what to do, and I let the others do the work.  
Sometimes I was gone.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Always participated fully:  
I never try to put the work on someone else. Our group seemed to participate.  
We all did our share.  
Unless I didn't know how to do problems, I would fully participate.  
There were times when I didn't actually punch the numbers in the calculator, or write the problem, but I feel that I added my input.

Usually participated fully:  
Sometimes I did the majority of the work and sometimes I just watched the others work out the problem while I learned. It just depended on our individual knowledge.  
Most of the time I stepped in and did the work. Sometimes I did sit back to see if they knew what they were doing.  
Sometimes the group knew it all and finished quickly without needing my help.  
Sometimes the group "whizzed" through things that took me longer to answer.  
All except for just a few times everyone in our group participated fully.  
Sometimes I got tired.  
Once, where I was not quick on the reading. They took over and did not care if I had understood or not.  
I felt like they wanted to do all the work and wouldn't listen to my ideas.

**10. Approximately how many hours per week did your group meet outside of class time?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

None:  
Never needed.  
Our schedules always conflicted. III  
We only met once all semester.  
Conflicting schedules, if there was an assignment, we would all do it and then compare in class, and then turn one in.  
Two of us met a couple times that was it. We tried to get everyone there but was not possible.

3-4 hours:  
Two of us would meet an hour or more before each class on Tuesday and Thursday. However, often two refused to come.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

None:  
Too busy, never had time. L

Conflicting schedules

Didn't need to, we always got our work done in class.

Our group is pretty separate and it would've been very difficult to meet, not to mention we rarely saw each other in class.

1-2 hours:

Met usually when we had a group assignment.

Before tests we would meet an hour before class.

Conflicting schedules

When we had extended assignments or tests.

Group 4 Cooperative groups, self-selected (Mon.eves.)

None:

I work during the day, and no one suggested it.

Conflicting schedules/too busy      II

1-2 hours:

Only with ~~~ no one else.

3-4 hours:

Every Monday before class.

## **11. What was the purpose of those meetings?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

None (10), with other response (11):

Work on group projects.      III

We didn't except one time for about a half hour was as long as we could have a time where everyone was available.

In class work not finished.

1-2 hours (10), other:

Do class assignments that couldn't get done in class.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

1-2 hours (10), with other response (11):

group assignments      II

extended assignments

3-4 hours (10), other:

To do work we were behind on and Daily grades.

Group 4 Cooperative groups, self-selected (Mon.eves.)

None (10), with other response (11):

We met once to study for test

**12. In assessing what you learned in this class, do you feel that working cooperatively helped you, hindered you, or had no effect on what you learned?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Helped & Hindered:

Helped to a certain point, but held me back for explaining, but I helped someone else understand what he was doing.

Helped:

If I had problems the group could help.

It really helped that my boyfriend ~~~ had the same class at the 12:00 hr, because he helped me a lot. Every Mon. and Wed. we would get together to just do math which helped too!

Hindered:

I believe that if everyone had come it would have helped.

I like my grade to be mine only.

No effect:

I think I would have done the same on all tests and daily grades.

I think there was no change. I was working hard in math before we were assigned a group and would have done the same.

I'm not sure "cooperatively" is the right word for our group work.

I don't need any help in math, but if my grade has been reduced I would say it hindered me.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Helped & Hindered:

Some of the individuals didn't care and got poor test scores.

Helped:

Made things more clear.

Sometimes if I didn't understand, someone else would explain it. 1

Working with the group probably helped me understand things better when I explained them to other group members.

My group helped me to understand things I probably wouldn't have asked the instructor. 1

I am not very good at math, so without my group I would have a bad daily grade.

It helped with the daily assignments.

It helped - to be able to ask questions.

Hindered:

I feel that my group relied on me and that started to encourage me not to try so not to help them all the time but I tried not to hurt my grade.



Group 4 Cooperative groups, self-selected (Mon.eves.)

Helped:

If I didn't get a problem, someone else would and would be able to explain it so I would understand. III

Got to do hands on in class – rather than just watching it done on the board.

I pushed myself more.

I understood material better.

Hindered:

The other two knew they wouldn't get an "A" in the class so they commented they didn't care anymore.

No effect:

You can only learn by how much effort you put in yourself.

I freeze up on math.

But met with a different group and it helped a lot.

Difficult schedules (work, childcare, etc.) made it difficult for all to meet.

**13. In assessing your achievement in this class, do you think that working cooperatively raised, lowered, or had no effect on your semester grade?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Raised:

Things I didn't understand were worked out.

It may have raised it in some cases (on certain tests) and lowered it or had no effect on it in other cases.

With group assignment

Lowered:

I think that when our test scores were based on how well others did wasn't fair.

The other kids (Adults) grades affected mine.

Some of my test scores were lower.

No effect:

The people that would have hindered it dropped out.

I don't know my grade yet.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Raised:

Not sure, it did help a lot.

If I didn't know how to do something I had someone explain it.

My group helped me a lot, taught me a lot.

Lowered:

I usually scored higher on all tests than the groups average so it brought me down. Since we didn't have to do homework (optional) I didn't do it much. I wasn't forced, so in turn, I didn't learn. That's my fault though.

No effect:

We were all about even in test scores.

At the beginning, my first couple tests scores were lowered by the group score but now I'm not sure.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Raised:

I think working in groups, especially for a daily grade, raised my grade because you're more apt to get the right answer.

A lot of times the group knew more, or understood more clearly.

We were better together I think we worked harder in our groups.

I think that actually doing problems in class made things more clear.

No effect:

However the group grade was lower than my individual grade.

Raised, lowered, no effect?

I'm not sure. I had some group members quit halfway through the semester.

**14. Do you think that working cooperatively affected your attitude in any way? If so, in what way?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Positive effect:

Made me see you have to trust others.

I was willing to except different possible answers to solve different problems.

Gained the relationship within the group (made acquaintances).

Negative effect:

I wondered what my other groups test scores were and how it was affecting mine.

I couldn't concentrate on my score alone.

When people wouldn't show I became upset.

Don't work well with others.

Positive and Negative effect:

Both because sometimes it put more stress on me to do well for the group.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Positive effect:

When you have three other people depending on you, you feel obligated to do good.

Saw how others perceived information, learned from them.

It made me feel better about myself and my grade because my group was very helpful always at least one person or more knew what they were doing.

Negative effect:

I didn't want to feel responsible for their learning.

It's just the whole group thing. I do better on my own.

I didn't like my grade being lowered.

No effect:

It is nice to have help, but sometimes I felt like I couldn't contribute as much because my group members worked faster than me.

I'm used to working with people all the time.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Positive effect:

Working with others generally makes me in a better mood as long as they don't have a negative attitude.

Nice working with people.

Wanted to do better to help my grade and my group's grade. I

knew others felt the same towards things.

It was easier to understand math concepts.

We had fun as well as work time.

Helped communicating with others and working with others.

It kept me involved in the material that we were covering.

Negative effect:

It was maddening when some people didn't care.

**15. If you had feelings of anxiety at the start of the semester, did working cooperatively have any effect on that? If so, in what ways?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Increased anxiety:

I didn't want to let my group down. 1

Worried about others grades.

Pressure of knowing performance affected others and vice-versa.

Decreased anxiety:

I knew that if I had a question I could ask anyone in my group for help.

It may helped my anxiety when it came to do assignments. I felt more confident.

No effect on anxiety:

I still hate math

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Increased anxiety:

Afraid of what kind of people would be in my group.

Decreased anxiety:

Got help from group on harder problems/when I had trouble II  
Working cooperatively with others helped decrease my anxiety by my knowing that I was more ready for the test than before speaking with them.

No effect on anxiety:

I knew I would get a good grade on daily grades because I had a good group, that worked well together.  
I still hate math.  
No anxiety.

Group 4 Cooperative groups, self-selected (Mon.eves.)

Increased anxiety:

Others grades were affected by mine.

Decreased anxiety:

Because I counted on my partners to answer, helped me if I did not understand it.  
Got to know people in the class.  
I felt more comfortable because I knew if I had any difficulty with the assignments or materials I could go to my group with those questions.

No effect on anxiety:

Groups did not effect my anxiety. The only time I was stressed was when I was treated unfairly or embarrassed in front of the class.

**16. Please comment on the method of grading.**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

It is okay I don't think we should get docked points if we were sick.  
I am not sure if it's fair that a grade should be effected by others. No matter how much we worked with one of our group members, he still didn't do as well as the rest of us. (also some have disabilities) I  
I thought it was ridiculous. I mean, a weighted group grade!?! It was not my responsibility or my peers to teach each other. We don't have degrees in teaching because we don't know how to teach. That is what the professor is paid to do.  
I don't think that grades for assignments and/or tests should be affected by the group

(group affecting individual score is unfair, etc) IIII

I don't know because it never brought my grade down but if it did I don't think I would like it.

I think the method of grading was very ingenuitive, however, I don't think it was fair to the people who scored the highest in the group.

It's alright/fine/good/very well/fair IIIIII

Tests challenged me and the daily grades just honed my skills for the test. The grading system was good.

Your grading is good, couldn't think of any other way to do it.

Don't care for it.

The method of grading was okay, it helped people that are not good test takers.

I feel that I am in college working hard for my education, my education alone. I am here to earn grades for myself. If a person decides to slack off, I shouldn't be affected.

The group grades work well as long as all members of the group work together equally.

I did not think that the rest of the group's test scores should have effected mine.

I thought it was very unfair that our homework grade wasn't worth anything.

Some people just can't take test and as a result their grade lowers to a D or an F.

Even though they did try and did their best a test based on numbers does not show the character or the skill of the individual.

It's okay, it's difficult at times when you are working with group members because whatever the other group members do it has some effect on you too. On the other hand it motivates you to do better because you don't want to bring everybody else's grade down.

### Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Grading style is good but some people that don't show up should not effect others grades.

To be honest, I really don't know the grading method to well.

I don't really know the point of group grade.

Sometimes the grading method was helpful, sometime it hurt my grade.

Good/fair grading                      lllllll

Fair. Majority based on tests = good.

I found that the grading was O.K., however I am not sure of my current grade so that feeling could change.

I thought it was fair – daily grade helped those who came regularly and was a disadvantage to those who rarely came to class.

I don't think it is fair because I earned an A on all but one test and that grade helped raise other's grades if they didn't do as well, it lowered my grade if they didn't know what they were doing on tests. (neg. effects by group members grades)      lll

Hated having combined group scores. Don't think it is fair.

It is bad for the other people because when I get a bad grade it lowers them.

Not really fair.

I think that we should get graded on group work but we shouldn't have a group test grade!                      l

I think the method of grading makes perfect sense. The daily grade can really help people who are not very good test takers.

The method of grading I like, having extra credit offered really helped. Group grades sometimes helped bring up the scores too.

It didn't affect much.

It works O.K. If you do bad on a test your group can raise you.

It made me nervous at first, but since the group can't drop us below our actual grade – It didn't really bother me.

It was good because of people who aren't good at math, like myself, it boosts our grade up – in most cases.

I think the grading was good because it made you try harder because you had an effect on everybody else.

Group 4 Cooperative groups, self-selected (Mon.eves.)

I don't believe the method of grading helped my grade.

Fine/(very)fair           lllll

Fair and straightforward

Fair, in class work helped a lot to keep grade up.

It's a fair method, I think the weighted grades are even.

Somewhat confusing when trying to figure it out on my own.

The grading was fair and it was terrific to have extra credit points to bring up our grades.

I liked the method of grading, for the fact it helped my overall grade.

More anxiety when my grade affected everyone else's – more pressure.

I don't think it was fair for the group if one did poorly (which was me). I feel bad because I didn't do very good.

I prefer grading on an individual basis.

I feel that someone else's grade should not bring the others in the group down.

This should be more of a positive experience.           L

I'd rather do homework assignments with others (group). But test I feel would be better individually.

I liked the extra credit, actually, I'm thankful for it. Computer grading was a problem. Sometimes ones I got right were marked wrong because the computer couldn't read it.

I rely on the instructor to grade as they see fit. I have no comment on the method other than feeling offended that I am viewed as a slacker because I happen to have a major downfall with this subject. When in fact, any business, social type classes I may out grade my team members.

I guess I agree with it because we cannot be hurt by working in groups. Also, it all depends on how much effort we put into it.

**17. Would you take another course that is taught this way?**

Group 2 Cooperative groups, mixed gender (T/TH 10:00)

Yes

I like the help.

I really like working with others. Two minds are better than one.

Because by working in groups, you're able to catch up if you missed and if you don't understand something.

I like working in groups and meeting other people and this was a good class to do so.

If the students could have picked their own groups.

But would prefer traditional, would be more apt to take a course in this manner in another subject area.

It doesn't bother me.

No

Absolutely not. I am in college because I want to be. I chose to be here.  
The grading was awful.

Yes and No

Only if the grade were not affected by everyone.

Group 3 Cooperative groups, gender homogeneous (T/TH at 12:00)

Yes

It helped me a lot.

It could be helpful.

I would rather work in groups than alone. It really helps me to understand things better.

I like working in groups/interacting 1

Group work was okay not the grading

Not graded this way.

If we could actually meet out of class.

It was nervous at first with the groups and all, but I think that they really helped out.

I thought it was great learning experience.

No

The speed of teaching was much too fast and blunt.

I work better alone.

It really depends on the class.

Maybe, doesn't matter. Possibly. 1

Group 4 Cooperative groups, self-selected (Mon.eves.)

Yes

I like working in groups. 1

I liked the idea of getting help from others, and sometimes showed me easier ways to do a problem compared to how it was done in class.

Hope to have a better experience.

I felt that it was a very beneficial method of learning.

No

I don't like working in groups.

Undecided 1

Group 1 Control group (MWF 10:00)

Yes

It was okay.

Because this is the way most of my classes are taught.

Fair grading, made me want to come to class.

It was a very well taught class, I learned a lot by working by myself. It made me think.

I don't always like working in groups. It's too easy for me to slack.

Because if a person (student) stays in the same group, that student will not learn anything.

I didn't see much difference in this class from others I've taken.

We have more freedom, to choose between working by ourselves or with others.

Because I liked the way you followed the syllabus and daily grades.

It was understandable.

It is kind of fun to be involved in an experiment.

It was fairly easy.

The information was thorough and well taught.

No

There was not good group work.

It made me feel uncomfortable.



**APPENDIX F: MATHEMATICS EXAMINATIONS  
AND SAMPLES OF DAILY GRADES**

**Use the scantron form for all questions except the extra credit. (1 point each).  
Note that on the scantron form A is true and B is false.**

**I. True/False: Determine whether each of the following statements is true or false.**

1. Every irrational number is a real number.
2. Every whole number is a rational number.
3. Every natural number greater than one is either prime or composite.
4. Every natural number greater than one is either abundant or deficient.
5. Every imaginary number is a complex number.
6. Every real number is a complex number.
7. The square root of a negative number is an imaginary number.
8. The cube root of a negative number is a real number.
9. Imaginary numbers have no practical purpose in the sciences.
10. Negative numbers are not real numbers.
11. Every prime number is deficient.
12. 37 is a prime number.
13. 21 is an example of an abundant number
14. A numeral is a symbol for a number.
15. Binary numbers have no purpose in real life.

**II. Multiple Choice.**

16. Recall that in the Roman numeration system I, V, X, L, C represent 1, 5, 10, 50, 100, respectively, and that the Roman system used both addition and subtraction. How is the number 239 represented?
  - a. CCXXLXI
  - b. CCXXLIX
  - c. CCXXXXI
  - d. CCXXXIX

17. Recall that the Babylonian system was base 60 and used  $\vee$  and  $<$  to represent 1 and 10 respectively. What number is represented by  $<<<< \quad <\vee$ ?
- 51
  - 2411
  - 2412
  - 2501
18. Recall that in the Attic system, I,  $\Gamma$ ,  $\Delta$ ,  $\Upsilon$ , represent 1, 5, 10, and 50, respectively, and that the Attic system used addition. How is 48 represented in this system?
- $\Delta \Delta \Delta \Delta \Gamma \text{ III}$
  - $\Gamma \Gamma \Gamma \Gamma \Delta \text{ III}$
  - $\Gamma \text{ II}$
  - $\Gamma \Delta \Delta \Delta \text{ IIII}$
19. The number 51 is . . .
- a prime number.
  - a composite number.
  - both of these.
  - none of these.
20. The number 45 is . . .
- a perfect number.
  - a deficient number.
  - an abundant number.
  - an amicable number.
21. What is the smallest abundant number?
- 2
  - 4
  - 6
  - 12
22. What is the smallest deficient composite number?
- 2
  - 4
  - 6
  - 12
23. What is the smallest number greater than 20 that is a prime number?
- 21
  - 22
  - 23
  - 31
24. Which of the following is **not** a square number?
- 1
  - 9
  - 15
  - 25

25. What is the pentagonal number that continues the sequence 1, 5, 12, 22, . . .
- a. 26
  - b. 29
  - c. 35
  - d. 37
26. What is the hexagonal number that continues the sequence 1, 6, 15, 28, 45, . .
- a. 66
  - b. 70
  - c. 64
  - d. 65
27. In the numeral 3,427,586 the digit 2 represents
- a. two hundreds
  - b. two ten-thousandths
  - c. twenty thousand
  - d. two hundredths
28. Write  $13.2_4$  as a decimal numeral.
- a. 7.2
  - b. 13.2
  - c. 7.5
  - d. 13.5
29. Write  $3AB_{16}$  as a decimal numeral.
- a. 31,011
  - b. 24
  - c. 939
  - d. None of these.
30. Which of the following cannot be a base 3 numeral?
- a.  $131_3$
  - b.  $22_3$
  - c.  $110_3$
  - d.  $212_3$
31. What are the proper factors of 20?
- a. 1, 2, 3, 4, 5, 10, 20
  - b. 1, 2, 4, 5, 10, 20
  - c. 1, 2, 4, 5, 10
  - d. 2, 4, 5, 10

32. Identify the rational numbers among the following:  $6, 0, 5/9, \sqrt{2}, \sqrt{-2}$
- $5/9$
  - $\sqrt{2}, \sqrt{-2}$
  - $6, 0, 5/9, \sqrt{2}$
  - $6, 0, 5/9$
33. Identify the integers among the following:  $6, 0, 5/9, \sqrt{2}, \sqrt{-2}$
- $6$
  - $6, 0$
  - $6, 0, 5/9$
  - all of them
34. Identify the real numbers among the following:  $6, 0, 5/9, \sqrt{2}, \sqrt{-2}$
- $6, 0$
  - $6, 0, 5/9$
  - $6, 0, 5/9, \sqrt{2}$
  - all of them.
35. Identify the imaginary numbers among the following:  $i, \pi, 0$ .
- $i$
  - $\pi$
  - $i$ , and  $\pi$
  - all of them
36. Which of the following numbers is largest?
- $123_6$
  - $321_4$
  - $213_5$
37. Solve the following:  $23_4 + 23_5 = \underline{\hspace{2cm}}_6$
- $24_6$
  - $46_6$
  - $4_6$
  - $40_6$
38. Suppose  $300_B = 48$ . Find B.
- $2$
  - $4$
  - $8$
  - $16$

39. Which of the following numerals is not equal to the other three?
- V <VV
  - Γ Δ Δ Ι Ι
  - LXXIII
  - 72

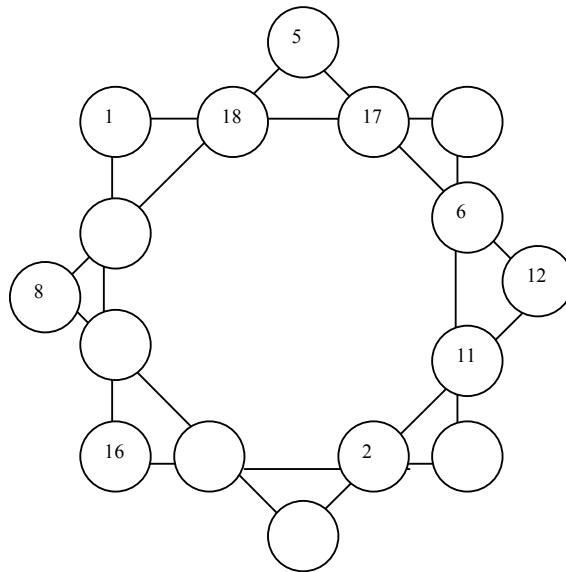
40. What number must be placed in the center in order to make the following a magic square?

2	7	6
9	1	
4	3	8

- 5
- 6
- There is no number that will make it a magic square.
- Nothing. It's a magic square already.

\*\*\*EXTRA CREDIT\*\*\* (3 points)

A figure is created by placing a number at each intersection point on the lines of a star. If the sum of the numbers along each line of the star is the same as every other such sum in that star, the star is called a magic star. Determine the missing values that will make the star below a magic star.



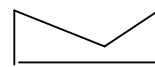
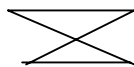
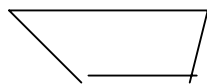
\*\*\*These formulas may be helpful:

$A = 180(n - 2)/n$  where  $A$  is the interior angle of a regular polygon and  $n$  is the number of sides of the polygon.

Dimension of a fractal:  $d = \log N/\log(l/r)$  where  $N$  is the number of new objects and  $r$  = ratio of the length of the new object to the length of the original object.

I. **MULTIPLE CHOICE:** Feel free to write on the test, but put final answers on the scantron form for 1-40. Do the extra credit on the test.

- Which of the following numbers is **not** a Fibonacci number?  
a. 1                      b. 3                      c. 13                      d. 23
- A polygon for which short extensions of some of the edges pierce the interior is called...  
a. concave.  
b. convex.  
c. regular.  
d. irregular.
- A proposition which is **proven** to be true is called...  
a. a theorem.  
b. a proof.  
c. an axiom.  
d. none of these.
- An angle which measures less than  $90^\circ$  is called...  
a. an acute angle.  
b. an obtuse angle.  
c. a right angle.  
d. a straight angle.
- Among the figures below, how many are polygons?



- only one
- exactly three

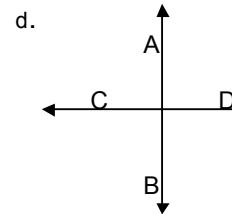
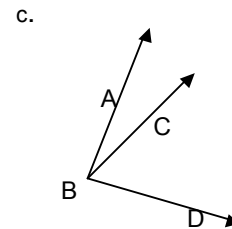
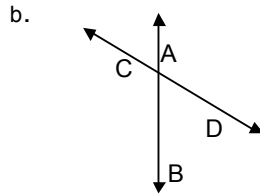
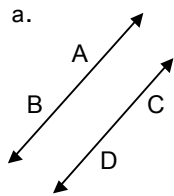
- exactly two
- all four

6. Each interior angle of a regular pentagon measures
  - a.  $180^\circ$
  - b.  $128.6^\circ$
  - c.  $108^\circ$
  - d.  $120^\circ$
  
7. If you are drawing a regular decagon you will need to use central angles that measure
  - a.  $144^\circ$
  - b.  $45^\circ$
  - c.  $36^\circ$
  - d.  $40^\circ$
  
8. Which of the following people is famous for his study of fractals?
  - a. Mandelbrodt
  - b. Escher
  - c. Euclid
  - d. Fermat
  
9. In which geometry can a triangle be formed whose interior angles sum to less than  $180^\circ$ ?
  - a. Euclidean
  - b. Lobachevskian
  - c. Riemannian
  - d. None of the above.
  
10. In which geometry can a square be formed?
  - a. Euclidean
  - b. Lobachevskian
  - c. Riemannian
  - d. All of the above.
  
11. In which geometry can two distinct parallel lines intersect?
  - a. Euclidean
  - b. Lobachevskian
  - c. Riemannian
  - d. None of the above.
  
12. In which geometry is it true that given a line and a point not on the line, there is exactly one line parallel to the given line and passing through the given point?
  - a. Euclidean
  - b. Lobachevskian
  - c. Riemannian
  - d. All of the above.
  
13. The sphere is used as a model of this geometry.
  - a. Euclidean
  - b. Lobachevskian
  - c. Riemannian
  - d. Fractal



14. You drop a few playing cards flat on a table and take a photograph of the result. Identify the type of perspective which best describes the result.
- overlapping shapes
  - diminishing sizes
  - atmospheric perspective
  - one-point perspective
15. You stand twenty domino tiles in a line, spacing them four inches apart, and take a photograph of the line, looking from an end of the line. Identify the type of perspective which best describes the result.
- overlapping shapes
  - diminishing sizes
  - atmospheric perspective
  - none of these

16. Which picture represents  $\overline{AB} \perp \overline{CD}$ ?

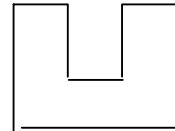


17. From the choices in #16, which picture represents  $AB \parallel CD$ ?
18. If a postcard 5" on the shorter side is in the shape of a golden rectangle, what is the length of the longer side?
- 10"
  - 8.1"
  - .324"
  - 3.09"
19. A line of equally spaced 20-foot light poles are to be part of a one-point perspective drawing. If in your drawing the first pole is 15" from the vanishing point, and the second pole is 6" high and 13" from the vanishing point, how high is the first pole?
- 8"
  - 12.92"
  - 6.92"
  - 6"
20. What is the smallest Fibonacci number greater than 40?
- 55
  - 1
  - 35
  - 34
21. Recall the snowflake fractal which was formed by dividing each line segment of an equilateral triangle into 3 equal parts, removing the middle part, and replacing it with 2 segments equal in length to the piece that was removed. Find the dimension of this fractal.
- .79
  - 1.26
  - .48
  - 1.46

22. Which of the following triples of numbers could be the dimensions of a golden box?
- 3, 4, 5
  - 7, 12, 19
  - 5, 9, 17
  - 13, 21, 34

**II. True/False: Remember that on the scantron form A is true and B false.**

23. In geometry, **point** is an undefined term.
24. Every rectangle is a quadrilateral.
25. The figure formed by two rays that share the same endpoint is called an angle.
26. The edges of a stop sign form a convex polygon.
27. The figure at the right is an octagon.
28. The figure at the right is concave.
29. The figure at the right is regular.
30. Axioms are **assumed** to be true.
31. The Sierpinski Carpet is an example of a fractal.
32. Every interior angle of **every** convex polygon has measure greater than  $90^\circ$ .
33. The technique of **atmospheric perspective** causes items far away to appear fuzzy.
34. The technique of **one point perspective** is illustrated by the effect of train tracks appearing to come together at the horizon.
35. The Chinese book K'iu-ch'ang Suan-shu shows that the Chinese used what's now called the Pythagorean theorem some 500 years before the time of Pythagoras.
36. Differential geometry is essentially the technique of applying calculus to the study of curves and surfaces.
37. Mathematics gives absolute truths about the physical world.



38. It is possible to make a sketch in the Euclidean plane of a triangle with two obtuse angles.
39. It is possible to make a sketch in the Riemannian plane (model) of a triangle with two obtuse angles.
40. It is possible to make a sketch in the Lobachevskian plane (model) of a triangle with two obtuse angles.

\*\*\*EXTRA CREDIT\*\*\* (3 pts)

Consider the fractal created by dividing a line segment into three equal pieces, replacing the middle section by three sections, each equal in length to the removed section, and replicating this process repeatedly on each subdivided segment. What is the dimension of the resulting fractal? (Round to 2 decimal places.)

Sketch the result of two iterations of this fractal.

**Use the scantron form for all questions except the extra credit. Show work for the extra credit. Feel free to write on the exam, however do NOT write on the sheet of truth tables.**

**I. True/False: A is true; B is false.**

1. The negation of the statement, "*All candy promotes tooth decay,*" is "*Some candy does not promote tooth decay.*"
2. The negation of the statement, "*This test is not easy,*" is "*This test is easy.*"
3. The following is considered a statement: "*All cows eat grass.*"
4. The following statement is considered a definition: "*All even numbers are integers.*"
5. The statement, "*All students study hard.*" is a conditional statement.
6. The Greek philosopher Aristotle is credited with being "The Father of Logic."
7. Deductive reasoning sometimes leads to incorrect conclusions.
8. The hypothesis and conclusion are parts of a conditional statement.
9. A conditional statement can be replaced by its contrapositive in a logical argument.
10. If two statements are logically equivalent, then they are either both true or both false.

**II. Multiple Choice.**

11. If the statement A is true, then . . .
  - a. the converse of A is always true.
  - b. the inverse of A is always true.
  - c. the contrapositive of A is always true.
  - d. None of the above responses are always true.
12. The contrapositive of  $A \rightarrow B$  is . . .
  - a.  $\sim A \rightarrow \sim B$
  - b.  $B \rightarrow A$
  - c.  $\sim B \rightarrow \sim A$
  - d. none of these.

13. In a logical statement the symbol  $\wedge$  means . . .
- “and.”
  - “or.”
  - “not.”
  - “therefore.”
14. Which response is a correct symbolic version of the statement: “If it is a pig, it has four legs and lies in mud,” where L = has four legs, M = lies in mud, and P = is a pig?
- $P \rightarrow (L \wedge M)$
  - $(L \wedge M) \rightarrow P$
  - $P \rightarrow L \vee M$
  - $L \wedge M \equiv P$
15. Which response is a correct symbolic version of the statement: “If I eat junk food and do not exercise, then I am not healthy,” where J = I eat junk food, E = I exercise, and H = I am healthy?
- $H \rightarrow \sim J \wedge E$
  - $J \rightarrow \sim E \wedge \sim H$
  - $(J \rightarrow \sim E) \wedge \sim H$
  - $(J \wedge \sim E) \rightarrow \sim H$
16. “Every year I gain ten pounds during December. This year, the same will occur.” This argument is an example of
- inductive reasoning
  - deductive reasoning
17. “I ate a chili dog at Joe’s and got indigestion. I ate a chili dog at Ruby’s and got indigestion. Therefore, chili dogs give me indigestion.” This argument is an example of
- inductive reasoning
  - deductive reasoning
18. “The sum of the angles of a triangle is  $180^\circ$ . Figure ABC is a triangle. Therefore, the sum of the angles of ABC is  $180^\circ$ .” This argument is an example of
- inductive reasoning
  - deductive reasoning
- 19-21. Consider the statement, “If you exercise, you will feel healthy.”**
19. The inverse of that statement is:
- If you feel healthy, you will exercise.
  - You will feel healthy if you exercise.
  - If you do not feel healthy, you will not exercise.
  - If you do not exercise, you will not feel healthy.
20. Using the same choices as in #19, find the converse of that statement.
21. Using the same choices as in #19, find the contrapositive of that statement.

**22-24. Choose a valid conclusion for each of the given arguments.**

22. If you play chess, then you are intelligent. You play chess.  
a. You are intelligent; therefore, you play chess.  
b. Therefore, you are intelligent.  
c. Therefore, you do not play chess.  
d. All chess players are not intelligent.
23. If the cat takes the rat, then the rat will take the cheese. The rat does not take the cheese.  
a. Therefore, the cat takes the cheese.  
b. Therefore, the cat takes the rat.  
c. Therefore, the cat does not take the rat.  
d. Therefore, the rat takes the cheese before the cat takes the rat.
24. If you attend class, then you will pass the course. If you pass the course, then you will graduate.  
a. Therefore, if you graduate then you passed the course.  
b. Therefore, if you don't attend class, then you won't graduate.  
c. Therefore, only those who pass the course will graduate.  
d. Therefore, if you attend class, then you will graduate.
25. **“All bankers are coin collectors. Sam is a banker. Therefore, Sam is a coin collector.”** This argument is an example of  
a. Hypothetical Syllogism  
b. Affirming the Antecedent  
c. Denying the Consequent  
d. None of them.
26. The argument in number 25 is  
a. valid  
b. not valid
27. **“All vacations are relaxing. A trip to the dentist is not relaxing. Therefore, a trip to the dentist is not a vacation.”** This argument is an example of  
a. Hypothetical Syllogism  
b. Affirming the Antecedent  
c. Denying the Consequent  
d. None of them.
28. The argument in number 27 is  
a. valid  
b. not valid
29. **“If I study every day, I will pass this class. I have not studied every day. Therefore, I will not pass this class.”** This argument is an example of  
a. Hypothetical Syllogism  
b. Affirming the Antecedent  
c. Denying the Consequent  
d. None of them.



37-39. Complete the truth table below to determine if  $[(\sim p \rightarrow \sim q) \wedge p] \rightarrow q$  is a valid argument.

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$(\sim p \rightarrow \sim q) \wedge p$	$[(\sim p \rightarrow \sim q) \wedge p] \rightarrow q$
T	T					
T	F					
F	T					
F	F					

37. The column under  $\sim p \rightarrow \sim q$  should be
- |      |      |      |      |
|------|------|------|------|
| a. F | b. T | c. T | d. T |
| T    | T    | F    | T    |
| F    | F    | T    | F    |
| F    | T    | T    | F    |

38. The column under  $(\sim p \rightarrow \sim q) \wedge p$  should be
- |      |      |      |      |
|------|------|------|------|
| a. T | b. F | c. T | d. T |
| F    | F    | T    | T    |
| T    | T    | F    | T    |
| T    | T    | F    | F    |

39. Is this a valid argument or not?
- a. valid
  - b. not valid

40. Three playing cards lie face down on the table in a row. There are a Queen, King, Ace, Diamond, Heart, and Club. You know that:
- To the right of the Queen is the Diamond.
  - To the right of the Ace is the Club.
  - To the right of the Heart is the Queen.
- Match up each card with its suit and position.
- a. Ace of hearts, Queen of clubs, King of diamonds
  - b. Queen of hearts, Ace of clubs, King of diamonds
  - c. Queen of hearts, Ace of diamonds, King of clubs
  - d. None of these.

EXTRA CREDIT (3 points)

The Turner triplets have an annoying habit--whenever a question is asked of the three of them, two tell the truth and the third lies. When I asked them which of them was born first, they replied as follows:

Tina: Trude was born first.

Trude: I am not the oldest.

Tonya: Tina is the oldest.

Which of the Turner triplets was born first?



**Use the scantron form to answer questions 1-40. Recall that A is true and B is false. Feel free to write on the test. Draw diagrams when needed. Round final answers to one decimal place, but carry at least 4 decimal places for intermediate results. DO NOT WRITE ON THE SHEET OF FORMULAS!**

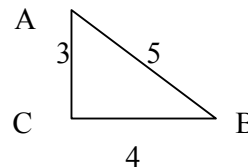
**I. True/False.**

1. The protractor is an instrument used to measure angles.
2. The shortest side of a triangle is always opposite the smallest angle.
3. The hypotenuse is always the longest side of a right triangle.
4. The numbers 12, 16, 20 form a Pythagorean triple.
5.  $\sin x = \cos(90 - x)$ .
6.  $\sin x$  is never greater than 1.
7.  $\cos x$  is sometimes greater than 1.
8.  $\tan x$  is sometimes equal to 1.
9. Bearing is a measure of direction relative to an east-west (horizontal) line.
10. The angle made between the ground and a line to the top of a tree is called an angle of depression.
11. The Greeks, whose interest in astronomy led them to study the relationship between arcs and chords of a circle, made the first significant contributions to trigonometry.

**II. Multiple Choice.**

**For questions 12-15 use the triangle ABC at right.**

12. In the triangle shown  $4/3$  is
  - a.  $\sin A$
  - b.  $\cos A$
  - c.  $\tan A$
  - d.  $\tan B$



13. Determine  $\sin B$ .
- .75
  - .8
  - 1.333...
  - .6
14. Determine  $\cos B$ .
- .75
  - .8
  - 1.333...
  - .6
15. Determine  $\tan B$ .
- .75
  - .8
  - 1.333...
  - .6
16. Which of the following is a Pythagorean triple?
- 5, 6, 7
  - 7, 9, 12
  - 5, 12, 13
  - None of these.
17. A triangle where each angle measures less than  $90^\circ$  is called ...
- a right triangle.
  - an acute triangle.
  - a Pythagorean triangle.
  - an obtuse triangle.
18. What kind of triangle has an angle greater than  $90^\circ$ ?
- right triangle.
  - an acute triangle.
  - a Pythagorean triangle.
  - an obtuse triangle.
19. Suppose you know the angles  $A$  and  $B$  and the length of side  $c$  of a **triangle ABC**. In order to determine the length of side **a**, you should use the ...
- Law of Sines.
  - Law of Cosines.
  - Pythagorean Theorem.
  - Root Spiral.

20. Suppose you know the lengths of two sides of a **right triangle**. In order to determine the length of the third side, you should use the ...
- a. Law of Sines.
  - b. Law of Cosines.
  - c. Pythagorean Theorem.
  - d. Root Spiral.
21. Which of the following **cannot** be the lengths of the sides of a triangle?
- a. 4, 5, 6
  - b. 5, 7, 9
  - c. 2, 2, 2
  - d. 2, 3, 6
22. Determine  $\tan 34^\circ$  to four decimal places.
- a. .5592
  - b. .8290
  - c. .6745
  - d. .5591
23. Determine the measure of angle A for which  $\cos A = .2567$ .
- a.  $1.0^\circ$
  - b.  $75.1^\circ$
  - c.  $75.2^\circ$
  - d.  $14.4^\circ$
24. Determine the measure of angle B for which  $\sin B = .6543$ .
- a.  $40.8^\circ$
  - b.  $49.1^\circ$
  - c.  $33.2^\circ$
  - d.  $40.9^\circ$
25. If the legs of a right triangle are 4 and 5, find the length of the hypotenuse.
- a. 9
  - b. 41
  - c. 6.4
  - d. 3
26. A square table has a diagonal distance between opposite corners of 16 ft. What is the length of each side?
- a. 2.8 ft.
  - b. 11.3 ft.
  - c. 22.6 ft.
  - d. Not enough information given to solve it.

27. Suppose a table is 7 ft. long and 4 ft. wide. What is the distance between opposite corners?
- 11 ft.
  - 8.1 ft.
  - 8.0 ft.
  - 33 ft.

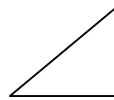
**For questions 28-30: Suppose a right triangle ABC with right angle at C has a  $35^\circ$  angle at A and the hypotenuse has length 14.**

28. To determine the angle at B you would use...
- Law of Sines
  - Law of Cosines
  - Pythagorean Theorem
  - Angle-Sum Theorem
29. Determine the length of side a.
- 11.4
  - 9.8
  - 8.0
  - None of these.
30. Determine the length of b.
- 11.4
  - 20.0
  - 11.5
  - 8.0

**For questions 31 – 34: Suppose a triangle ABC has sides of length  $a=8$ ,  $b=7$ , and  $c=9$ .**

31. To determine the measure of angle A you must use ...
- Law of Sines
  - Law of Cosines
  - Angle-Sum Theorem
  - $\tan A = 8/9$
32. Determine the measure of angle A.
- $62.7^\circ$
  - $60^\circ$
  - $58.4^\circ$
  - $48.2^\circ$

33. Determine the measure of angle B.
- $58.4^\circ$
  - $51.1^\circ$
  - $48^\circ$
  - $48.2^\circ$
34. Determine the measure of angle C.
- $73.4^\circ$
  - $48.2^\circ$
  - $78.6^\circ$
  - $58.4^\circ$
35. Suppose you have released 150 ft. of string while flying a kite and have secured the end of the string to a post in the ground. Assume that the line is taut and makes an angle of  $75^\circ$  with the ground. How high is the kite?
- 144 ft.
  - 151.0 ft.
  - 144.9 ft.
  - 144.8 ft.

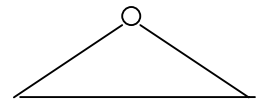


36. Suppose a 12-ft. ladder is leaning against a wall so that its base is 2 ft. from the edge of the wall. As you climb to the very top of the ladder, you find you can reach 7 ft. above the top of the ladder. To what height are you reaching?
- 18.8 ft.
  - 19.2 ft.
  - 11.8 ft.
  - 12.2 ft.



**For questions 37-38: A balloon is tethered by two ropes, one of which is 200 ft. long. The other rope is secured 100 ft. away, forming an angle of  $70^\circ$  with the ground.**

37. Find the angle formed at the balloon.
- $82.0^\circ$
  - $81.9^\circ$
  - $28.1^\circ$
  - $28.0^\circ$
38. How long is the second rope?
- 199.1 ft.
  - 210.8 ft.
  - 210.7 ft.
  - Not enough information given to solve it.



39. What angle does a 30-ft. plank make with level ground if it is placed so that it reaches 20 ft. up a vertical wall?
- $90^\circ$
  - $48.2^\circ$
  - $33.7^\circ$
  - $41.8^\circ$
40. A pilot, attempting to fly from Middletown to Westburg, a distance of 200 miles, notices after flying 180 miles that Westburg is still 35 miles away because the heading was off by a few degrees. How many degrees was the heading off of the desired course?
- 120.4
  - 50.9
  - 8.7
  - None of these

**\*\*\*BONUS\*\*\* (4 points)**

You are in a hot air balloon with your support crew on the ground. Attached to the balloon are two tether cords of length 350 ft. and 400 ft., which your crew has attached to the ground. You note that these cords form an angle of  $50^\circ$  where they meet the balloon. Assuming the lines are taut and the ground is level, with what angles do these cords meet the ground? **(Show work or no credit will be awarded. One point for the picture. Round to one decimal place.)**

**All questions except the extra credit should be answered on the scantron form. Feel free to write on the exam. Round final answers to the nearest hundredth when necessary.**

1. The code of Hammurabi, one of the first law codes in history, regulated interest.  
a. True                      b. False
2. Accelerated payments will reduce the interest rate of a loan.  
a. True                      b. False
3. For a one-year investment, an account paying a simple interest rate of 6% annually and an account which earns 6% compounded annually will earn the same amount.  
a. True                      b. False
4. A car loan is an example of an annuity.  
a. True                      b. False
5. IRA accounts are example of annuities.  
a. True                      b. False
6. In the first year of a mortgage, almost all of each payment is used to reduce the principal.  
a. True                      b. False
7. A two-year investment paying annually compounded interest will earn exactly twice the interest of a one-year investment paying the same rate.  
a. True                      b. False
8. The monthly payment of a 15-year mortgage is twice as large as the monthly payment of a 30-year mortgage.  
a. True                      b. False
9. Periodic interest rate is calculated by dividing the annual rate by the number of periods per year.  
a. True                      b. False
10. Accelerated payments will reduce the total interest paid on a loan.  
a. True                      b. False
11. Ordinary annuities are those for which the frequency of payments and the frequency of compounding are the same and payments are made at the end of each time period.  
a. True                      b. False

12. Solve the equation:  $4^x = 30$   
 a. 2.4534      b. 0.4076      c. 21.6404      d. 0.8503
13. Solve the equation:  $x^4 = 44$   
 a. 11      b. 22      c. 6.6332      d. 2.5755
14. A bank gives you two options to choose from for your investments:  
 Option A: 8% annual interest rate compounded yearly; and  
 Option B: 7.9% annual interest rate compounded quarterly.  
 Decide which is the better investment at the end of 2 years.  
 a. Option A      b. Option B
15. You are given an annual interest rate of 19.2%; find the monthly interest rate.  
 a. 2.6 %      b. 1.6 %      c. 1.8 %      d. 2.1 %
16. A savings account pays 4.25%, compounded quarterly. What is the effective rate?  
 a. 4.318%      b. 17%      c. 1.06%      d. 1.043%

**Use the formulas  $I=Prt$  and  $A=P(1 + rt)$  for problems 17-21.**

17. How much interest is earned by an investment of \$25000 for three years at an annual simple interest rate of 5%?  
 a. \$3750      b. \$2200      c. \$4000      d. \$2700
18. Mark takes a bank loan of \$3300 to start up a business at an annual interest rate of 7%. How much will he have to repay in 28 months?  
 a. \$4400      b. \$4600      c. \$3839      d. \$5456
19. Suppose you borrow \$1200 and pay back \$1550 eighteen months later. What is the annual simple interest rate for this loan?  
 a. 1.62%      b. 12.73%      c. 19.44%      d. 8.61%
20. Maya bought a car for \$15,000. To pay for it, she took a bank add-on interest loan at an annual interest rate of 4%. The loan term is 8 years. How much interest will she pay?  
 a. \$4900      b. \$19,800      c. \$48,000      d. \$4800
21. What are Maya's monthly payments (from #20)?  
 a. \$206.25      b. \$500.00      c. \$412.50      d. \$309.38



Use the formula  $A = P(1 + R/N)^{Nt}$  for problems 22-25.

22-23. Suppose you deposit \$1500 into an account which pays 6% interest compounded monthly. You want to know how much money will have accumulated in the account after 4 years.

22. How much money will be in the account after 4 years?  
 a. \$1893.72    b. \$1903.48    c. \$1592.70    d. \$1905.73
23. How much interest will have been earned?  
 a. \$393.72    b. \$405.73    c. \$92.70    d. \$403.48
24. Car insurance costs \$550 with an inflation rate of 4%. Find the cost of the car insurance in 7 years.  
 a. \$973.76    b. \$1023.76    c. \$723.76    d. \$873.76
25. Anne purchased a bond for a museum valued at \$15,000 for \$9,500. If the bond pays 6.5% annual interest compounded monthly, how long must she hold it until it reaches its full face value?  
 a. 7.05 years    b. 6.05 years    c. 5.05 years    d. 9.05 years

Use the formula  $S = PMT \left[ \frac{(1 + R/N)^{Nt} - 1}{R/N} \right]$  for problems 26-32.

Suppose you want to set up an ordinary annuity by putting \$80 each month into an account which pays 6.5% compounded monthly. You want to know the value of the account after 10 years.

26. Substitute \$80 for . . .  
 a. S    b. PMT    c. R    d. N
27. What is the **monthly** interest rate?  
 a. .005417    b. 1.005417    c. .065    d. 1.301625
28. How many total payments will be made?  
 a. 10    b. 40    c. -120    d. 120
29. How much will be in the account after 10 years?  
 a. \$4458.14    b. \$13472.55    c. \$13457.54    d. \$168.41
30. How much will you have paid into the account?  
 a. \$9600    b. \$800    c. \$960    d. \$4800
31. How much interest will have been earned?  
 a. \$3498.14    b. \$3872.55    c. \$12512.55    d. \$3857.54

32. How long must you pay into this annuity in order to have a final value of \$6000?

- a. 6 years      b. 63.11 years      c. 5.32 years      d. 5.26 years

Use the formula  $L = PMT \left[ \frac{1 - (1 + R/N)^{-Nt}}{R/N} \right]$  to do problems 33 – 40.

Suppose you buy a house and take out a mortgage for \$70,000. You agree to make equal monthly payments for thirty years, at an annual interest rate of 9.25%. You want to calculate the amount of the monthly payments.

33. \$70000 should be substituted for . . .  
a. L      b. PMT      c. R      d. N
34. Find the **monthly** interest rate.  
a. .0925      b. 1.0925      c. .007708      d. .0078
35. What is the amount of your monthly payment?  
a. \$121.56      b. \$507.57      c. \$575.85      d. \$575.37
36. What is the total amount paid on the loan after 30 years?  
a. \$43761.60      b. \$17275.50      c. \$207133.20      d. \$207306
37. How much interest is paid on the loan?  
a. \$26238.40      b. \$52724.50      c. \$137133.20      d. \$137306
38. The amount of interest paid could be reduced if you . . .  
a. refinance at a lower interest rate.  
b. pay a little extra each month to pay down the principal.  
c. set up the mortgage for 15 years instead of 30.  
d. all of the above.

Suppose you increase your payments by \$50 per month.

39. How long would it take you to pay off the mortgage?  
a. 6.75 years      b. 25.26 years      c. 1.61 years      d. 21.50 years
40. How much less interest will you pay over the life of the loan if you pay an extra \$50 per month at the start of your payments?  
a. \$161469.30      b. \$91469.30      c. \$45836.70      d. \$18000.00

**\*\*\*EXTRA CREDIT\*\*\* (3 points)**

**Paula took out a 15-year mortgage for \$130,000 for her home at an annual interest rate of 7%. She decided to refinance after 8 years. Find the unpaid balance of the loan.**

Formulas:  $P_{n,r} = \frac{n!}{(n-r)!}$        $C_{n,r} = \frac{n!}{r!(n-r)!}$

**All questions except the extra credit should be answered on the scantron form. Feel free to write on the exam.**

**I. TRUE/FALSE. A is true; B is false.**

1. The product of  $2!$  and  $3!$  is  $6!$ .
2. By their design, state lotteries are fair games.
3.  $C_{n,n}$  is equal to 1.
4.  $0!$  is equal to 0.

For questions 5-9: You are drawing one card from a deck of 52 playing cards. The probability that the card you draw is . . .

5. red is  $1/2$ .
6. a jack is  $4/52$ .
7. the jack of diamonds is  $1/52$ .
8. a diamond or a queen is  **$30/52$** .
9. The **odds in favor** of drawing a 7 is 4:52.

**II. MULTIPLE CHOICE.**

10. How many possible sets of answers are there for a true/false test of 8 questions?
  - a.  $2^8$
  - b.  $8^2$
  - c.  ${}_8C_2$
  - d.  ${}_8P_2$
11. How many ways can a kindergarten class of 15 students form a line for lunch?
  - a.  $15!$
  - b.  ${}_{15}C_2$
  - c.  ${}_{15}C_{15}$
  - d.  ${}_{15}P_2$
12. Six senators (from a group of 100) are selected to travel on a fact-finding tour in South America. How many ways can this selection be made?
  - a.  $6!$
  - b.  $100!$
  - c.  ${}_{100}C_6$
  - d.  ${}_{100}P_6$

13. If a cafe offers 4 different meats, 5 different vegetables, and 4 different salads, how many different meals containing a meat, a vegetable, and a salad can be formed?
- 13
  - 36
  - 80
  - none of these
14. Suppose that a frozen yogurt can be ordered in 3 sizes (small, medium, large), 2 flavors (vanilla, chocolate), and 4 toppings (plain, sprinkles only, fudge only, sprinkles and fudge). Find the number of possible yogurt selections.
- 9
  - 10
  - 24
  - none of these
15. A fruit bowl contains three bananas, two apples, four kiwi, and two oranges. Suppose you choose two items from the bowl for lunch. In order to determine how many possible outcomes exist, you note that this count is done ...
- with replacement, where order matters.
  - without replacement, where order matters.
  - with replacement, where order does not matter.
  - without replacement, where order does not matter.
16. According to the weather report, there is a 20% chance of snow in Olmsted county tomorrow. Which of the following statements would be appropriate?
- Out of the next five days, it will snow one of those days.
  - Out of the next 24 hours, snow will fall for 4.8 hours.
  - Of past days when conditions were similar, one out of five had some snow.
  - It will snow on 20% of the area of the county.
17. The doctor says "There is a 40% chance that your problem will get better without surgery." Which of the following statements would be appropriate?
- You expect to feel 40% better.
  - In the future you will feel better on two out of every five days.
  - Among you and the next four other patients with the same problem, two will get better without surgery.
  - Among patients with symptoms similar to yours who have participated in research studies of nonsurgical treatments, about 40% got better.

18. Calculate the value of  $9!$   
 a. 72                      b. 9                      c. 45                      d. 362,880
19. Calculate the value of  ${}_6P_4$ .  
 a. 360                      b. 15                      c. 120                      d. 30
20. Calculate the value of  ${}_6C_4$ .  
 a. 360                      b. 15                      c. 120                      d. 30
21. How many five-letter computer passwords could exist?  
 a.  $5 \cdot 26$                       b.  $5 \cdot 10$                       c.  $10 \cdot 5$                       d.  $26 \cdot 5$
22. If you're playing the Daily 3 in the Minnesota lottery you will pick 3 digits from 0 - 9. Digits may be repeated. How many sets of possible winning numbers are there?  
 a.  $9 \cdot 3$                       b.  $3 \cdot 9$                       c.  $10^3$                       d.  $3 \cdot 10$

**For questions 23-24: There are 20 horses in a race.**

23. Find the probability that you randomly pick the winner of the race.  
 a. 1:20                      b. 19:20                      c. 1/20                      d. 19/20
24. Find the probability that you do not pick the winner.  
 a. 1:20                      b. 19:20                      c. 1/20                      d. 19/20
25. The probability that a particular dog wins a greyhound race is  $3/8$ . What are the **odds in favor** of the dog winning?  
 a. 3:8                      b. 8:3                      c. 5:3                      d. 3:5
26. The **odds** for winning the \$50 prize in a particular lottery game are listed as 1:83. What is the **probability** of winning the \$50 prize?  
 a. 1/83                      b. 82/83                      c. 83:1                      d. 1/84
27. Find the **number of ways** in which exactly 3 aces and 2 kings can be drawn in a **6-card** hand from a 52-card deck.  
 a.  ${}_4C_3 \times {}_4C_2$                       b.  $12 \times {}_4C_3 \times {}_4C_2 \times {}_{47}C_1$   
 c.  ${}_4C_3 \times {}_4C_2 \times {}_{44}C_1$                       d.  $13 \times 12 \times {}_4C_3 \times {}_4C_2 \times {}_{44}C_1$
28. Find the **number of ways** in which 3 aces and a pair of any rank can be drawn in a **6-card** hand from a standard 52-card deck.  
 a.  $13 \times 12 \times {}_4C_3 \times {}_4C_2$                       b.  ${}_4C_3 \times 12 \times {}_4C_2 \times {}_{44}C_1$   
 c.  $12 \times {}_4C_3 \times {}_4C_2 \times {}_{47}C_1$                       d.  $13 \times 12 \times {}_4C_3 \times {}_4C_2 \times {}_{47}C_1$

29. Find the **probability** of drawing 3 aces and a pair of any rank in a **6-card** hand from a standard 52-card deck.

a.  $\frac{13 \times 12 \times 4C_3 \times 4C_2}{52C_6}$

b.  $\frac{12 \times 4C_3 \times 4C_2 \times 47C_1}{52C_6}$

c.  $12 \times 4C_3 \times 4C_2 \times 44C_1$

d.  $\frac{13 \times 12 \times 4C_3 \times 4C_2 \times 44C_1}{52C_6}$

**For questions 30-32: Suppose you are throwing a standard pair of six-sided dice.**

30. What is the probability that the sum will be five?  
 a. 4/12                      b. 2/12                      c. 2/36                      d. 4/36

31. What is the probability the sum will be six or ten?  
 a. 6/36                      b. 6/12                      c. 8/36                      d. 4/12

32. The probability that the sum will be eight is 5/36. If you roll the dice 360 times, what is the expected number of times the sum will be eight?  
 a. 50/360                      b. 5/36                      c. 180                      d. 50

33. By analyzing her sales records, a saleswoman has found that her weekly commissions have the following probabilities. Use the information in the chart and the expected value formula to find the saleswoman's expected commission.

<u>Commission</u>	<u>Probability</u>
0	0.05
100	0.15
200	0.25
300	0.45
400	0.10

- a. \$240.05
- b. \$240
- c. \$1000
- d. none of these

34. In a certain game, the probability of winning is .35 and the probability of losing is .65. If a player wins, the player will collect \$50 plus the \$5 paid to play. If the player loses, the player will lose the \$5 paid to play. What is the expected value of this game?  
 a. \$16                      b. \$20.75                      c. \$14.25                      d. none of these

35. If the game is played 100 times what are the expected winnings (or losses) of the player?  
 a. -\$20.75                      b. -\$1425                      c. \$1425                      d. none of these

36. In terms of expected value, when is a game considered to be fair?
- When the player comes out ahead.
  - When the expected value is positive.
  - When the expected value equals zero.
  - When the expected value is negative.

**For each situation described in problems 37-40 determine whether "mathematical" or "experimental" probability is used.**

37. Your auto insurance company sets your premium based on the probability that you'll have an accident.
- mathematical
  - experimental
38. According to a weather report, there is a 30% chance of rain tomorrow.
- mathematical
  - experimental
39. The probability of rolling a pair of dice and getting doubles is  $6/36$  or  $1/6$ .
- mathematical
  - experimental
40. After flipping a coin 500 times and getting 240 heads, you declare that the probability of getting heads is  $240/500$  or 48%.
- mathematical
  - experimental

**\*\*\*EXTRA CREDIT\*\*\* (3 points)**

Jack and Bob are roommates, and they're always arguing about whose turn it is to do the dishes. Finally, a mutual friend (a Math 1111 student) came up with a solution. Now instead of arguing, they draw marbles to see who does the dishes. Here's how it works. There are 1 red and 2 green marbles in the bag. They each draw 1 marble without replacement. If the marbles are the same color, Jack does the dishes. If the marbles are one of each color, Bob does the dishes.

Who gets stuck doing the dishes more often? Explain why.

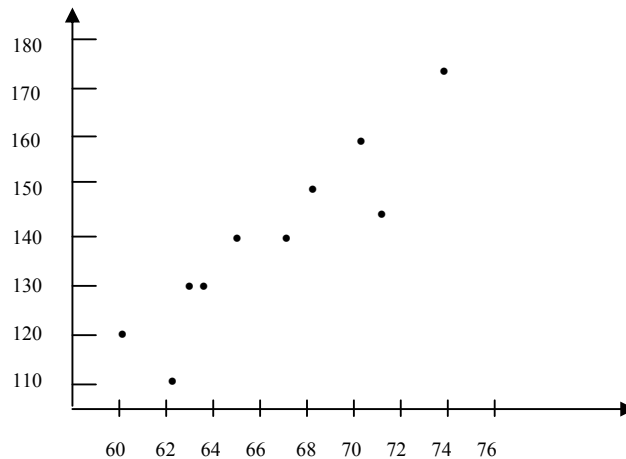
**Use the scantron form for all questions except the extra credit. Feel free to write on the exam, however, do NOT write on the normal table.**

1. Is the following statement true or false?  $\sum_{i=1}^5 i^2 = (\sum_{i=1}^5 i)^2$
2. Is the following statement true or false?  $\sum_{i=1}^5 (3i) = 3 \sum_{i=1}^5 i$
3. Is the following statement true or false?  $\sum_{i=1}^5 (4+i) = 4 + \sum_{i=1}^5 i$
4.  $\sum_{i=1}^4 i^3$  is equal to . . .  
a. 10            b. 30            c. 100            d. 1000
5. Which of the data sets below is bimodal?  
a. 1,1,2,2,3,3,3,4,5            b. 1,2,2,3,3,3,4,4,5  
c. 1,2,3,3,3,3,4            d. 1,2,2,3,4,5,5
6. The local café offers 12 different flavors of ice cream, numbered 1 to 12. A recent survey asked 200 students to choose their favorite flavor. The group's favorite flavor would be found by taking this data and determining the . . .  
a. mean            b. median            c. mode            d. range
7. Suppose you want to buy a dependable, long-lasting headlight. Comparing the lifetimes of Brand A and Brand B, you find that their means are equal, but the standard deviation of Brand A is less than that of Brand B.  
a. Brand A and Brand B are equivalent.  
b. You should buy Brand A.  
c. You should buy Brand B.  
d. The information is inconclusive.
8. Suppose you also want to buy a dependable, long-lasting taillight. The lifetime of Brand C is smaller than that of Brand D, but they have the same standard deviation.  
a. Brand C and Brand D are equivalent.  
b. You should buy Brand C.  
c. You should buy Brand D.  
d. The information is inconclusive.



9. Determine  $\sum_{i=1}^5 (4i-3)$ .
- a. 17                      b. 57                      c. 45                      d. none of the above

**10-15. Heights and weights of women volleyball players are shown on the graph below. Use the information on the graph to answer the questions.**



10. How tall is the woman who weighs 150 pounds?  
 a. 67 inches                      b. 68 inches  
 c. 69 inches                      d. 70 inches
11. What is the weight of the woman who is 70 inches tall?  
 a. 140 lbs.                      b. 150 lbs.  
 c. 160 lbs.                      d. 170 lbs.
12. Meagan is the tallest player. How tall is she?  
 a. 175 inches                      b. 170 inches  
 c. 75 inches                      d. 74 inches
13. Jessie is the lightest player, but not the shortest. Give the coordinates of the point which represents Jessie.  
 a. (110, 62)                      b. (120, 60)  
 c. (60, 120)                      d. (62, 110)
14. Julie is the tallest player in her 130-pound weight group. How tall is she?  
 a. 62 inches                      b. 63 inches  
 c. 64 inches                      d. 65 inches

15. What, if any, correlation (relationship) exists between the heights and weights shown?  
 a. positive                      b. negative                      c. no correlation

**16-20. The Truly Amazing Dudes are a group of comic acrobats. The weights (in pounds) of the ten acrobats are as follows: 155, 126, 192, 184, 146, 159, 136, 140, 168, 136. (Round answers to 2 decimal places if they don't come out even.)**

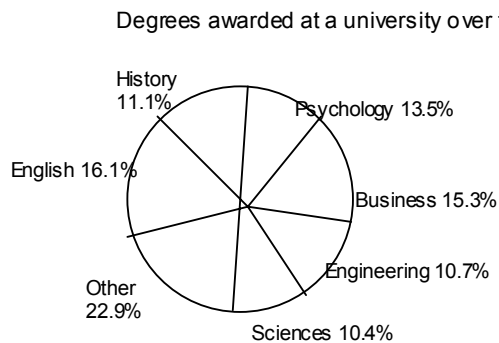
16. Compute the mean.  
 a. 136                                      b. 150.5  
 c. 154.2                                    d. 66
17. Find the median.  
 a. 136                                      b. 150.5  
 c. 154.2                                    d. 66
18. Find the mode.  
 a. 136                                      b. 150.5  
 c. 154.2                                    d. There is no mode.
19. Find the range.  
 a. 136                                      b. 150.5  
 c. 154.2                                    d. 66

20. Use this table to help you compute the standard deviation of these weights.

Weight	Difference between mean and weight	Squares of differences
155		
126		
192		
184		
146		
159		
136		
140		
168		
136		

- a. 20.63                                      b. 65.25  
 c. 425.76                                    d. 6.53

- 21-22. The circle graph below illustrates the percent of students receiving degrees in various majors at a given university over the past 10 years. The total number of degrees awarded over the past 10 years is 3600.



21. Determine the approximate number of degrees awarded in Business.  
 a. 486      b. 235      c. 267      d. 551
22. Determine the approximate number of degrees awarded in English.  
 a. 576      b. 580      c. 224      d. 310

Use the normal table and the following formula to answer questions 23-29.

$$Z = \frac{X - \mu}{\sigma} \quad \text{where } \mu = \text{mean, } \sigma = \text{standard deviation}$$

23. Determine the area under the normal curve between  $z = 0$  and  $z = 1.93$ .  
 a. .4726      b. .4738  
 c. .4732      d. .3238
24. Determine the area under the normal curve between  $z = 0$  and  $z = -1.38$ .  
 a. -.4162      b. .4162  
 c. -.4147      d. .4147
25. Determine the area under the normal curve between  $z = 1.46$  and  $z = 2.39$ .  
 a. .0637      b. .4916  
 c. .4279      d. .9195
26. Determine the area under the normal curve to the right of  $z = -1.17$ .  
 a. .1210      b. .8790  
 c. .3790      d. .8810



32. Grades in your Math 1111 course are weighted such that tests count for 75% of the semester grade and daily grades count for 25% of the semester grade. If a student has a test average of 70% and a daily grade average of 88%, what is his/her semester grade?
- a. 80%                      b. 79%  
c. 85%                      d. 75%

**The box plot below shows data from a Math 1111 class of 32 students. Use these data to answer 33-36.**

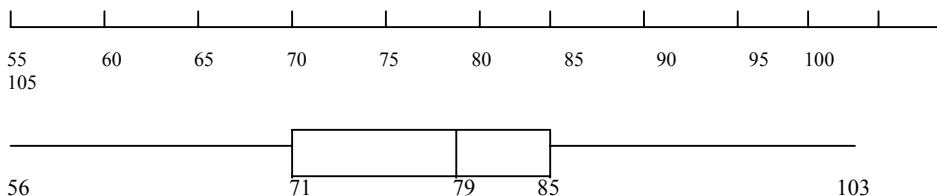
**High = 103**

**Low = 56**

**Upper Quartile = 85**

**Lower Quartile = 71**

**Median = 79**



33. What percent of the students scored above 79 (the median)?
- a. 25%  
b. 50%  
c. 75%  
d. Not enough information given.
34. What percent of the students scored below 85 (the upper quartile)?
- a. 25%  
b. 50%  
c. 75%  
d. Not enough information given.
35. How many students scored above 71 (the lower quartile)?
- a. 8  
b. 16  
c. 24  
d. Not enough information given.
36. Are there any outliers?
- a. Yes, there is at least one outlier on the right whisker, but not on the left.  
b. Yes, there is at least one outlier on the left whisker, but not on the right.  
c. Yes, there are outliers on both whiskers.  
d. There are no outliers.

**37-40. The stem-and-leaf plot below represents the resting pulse rates of a group of 15 people involved in a fitness study. Use these data to answer 37-40.**

4		7
5		48
6		1569
7		023556
8		27

37. Describe the shape of the data.
- |            |              |
|------------|--------------|
| a. skewed  | b. symmetric |
| c. bimodal | d. normal    |
38. Calculate the median.
- |       |         |
|-------|---------|
| a. 61 | b. 70   |
| c. 75 | d. 69.5 |
39. Calculate the upper quartile.
- |       |         |
|-------|---------|
| a. 61 | b. 70   |
| c. 75 | d. 69.5 |
40. Calculate the lower quartile.
- |       |         |
|-------|---------|
| a. 61 | b. 70   |
| c. 75 | d. 69.5 |

**\*\*\*EXTRA CREDIT\*\*\*(4 points)**

Suppose the class averaged 78 on the last exam, with a standard deviation of 7. The top 10% are to be As, the next 20% Bs, the next 40% Cs, the next 20% Ds, and the bottom 10% Fs.

Assume the scores are normally distributed.

- What is the score range for As?
- What is the score range for Bs?
- What is the score range for Cs?
- What is the score range for Ds?

## Samples of Daily Grades Used in the Study

Many of the daily grades were problems taken out of the textbook, or were otherwise copyrighted. This is just a sample of the types of problems that were used as daily grades.

### Daily Grade – Chapter 1

1. What number is represented by the Roman numeral DCCXXXIV if D=500, C=100, X=10, V=5, and I=1?
2. What number is represented by the Babylonian numeral  $\lll\lll \llllll$  if  $\ll = 10$ ,  $\lll = 1$ , and they used a base 60 number system?

### Math 1111

### Tessellations

There are 21 possible combinations of regular polygons that can be placed around a vertex to tessellate the plane. Find at least eight of these combinations. Use the following notation: 6-6-6 stands for three hexagons, 4-4-4-4 for four squares, and 3-3-3-3-3-3 for six triangles. The hexagon, square, and triangle, are the only polygons that will tessellate the plane by themselves. The other polygons must be used in combinations of at least two different polygons. Find eight of these combinations. Do not use the examples above. (Hint: The sum of the angles around a vertex must be  $360^\circ$ .)

### Chess Club Survey

The local chess club bulletin includes news, features, and instruction. In a survey of 114 readers, 52 liked at least the news category, 51 at least the features category, and 52 at least the instruction category. Among them, 15 readers liked at least news and features, 23 at least features and instruction, 26 at least news and instruction, and 9 liked all three. How many liked only news, only features, and only instruction? And how many liked none of them?

### Inductive Reasoning

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

1. Write the next three lines.
2. Will the pattern always continue? Explain.

### Problems Involving the Pythagorean Theorem

1. If the two legs of a right triangle have lengths of 3 and 4, find the length of its hypotenuse.
2. The diagonal of a square is 10 cm long. Find the length of a side of the square.
3. Find the length of a diagonal of a 10 cm by 10 cm square.
4. Sam leans a 30-foot ladder against a building. The base of the ladder is 8 feet from the building. How high up the building does the ladder reach?



## Trigonometry Daily Grade

Suppose a right triangle has one leg twice the length of the other leg.

1. What is the angle opposite the shorter leg?
2. What is the angle opposite the longer leg?
3. Is the hypotenuse more than 2.5 times the length of the shorter leg?

## Annuities

The local newspaper recently ran a story about a man in Ohio who has been saving pennies since he was five years old. Now at age 70, he hauled 40 large plastic garbage cans of pennies to the bank. The total was \$80,000.

By keeping the money at home all those years, the man didn't get any interest on the money. Suppose the man had deposited the \$80,000 in equal amounts each month into an account which earned 5% compounded monthly. How much would be in the account now?

## Loan Payments

A couple purchased a house by taking out a \$125,000 mortgage (loan) with monthly payments for 30 years. The interest rate on the loan was 8.25% compounded monthly.

- (a) How much would the scheduled payments on the loan be?
- (b) If they pay \$100 extra each month, how long will it take to pay off the loan?
- (c) How much does the couple save in payments over the life of the loan by paying \$100 extra each month?

## Mortgage Assignment

Using the loan formula below, answer the following questions on a separate sheet of paper. Show work when necessary. Use complete sentences for any question that requires more than a numerical answer. Do not write on this page.

$$L = P \left[ \frac{1 - \left(1 + \frac{R}{N}\right)^{-Nt}}{\frac{R}{N}} \right]$$

L = amount of the loan

P = amount of each payment

R = annual interest rate

N = number of payments per year

t = number of years

1. Find an ad in the newspaper for a house that you want to buy. Attach it to your paper.
2. Use the following interest rates for problem 6: 30-year = 7%, 15-year = 6.5%, and ARM (adjustable rate mortgage) = 5%.
3. For an estimate of annual real estate taxes on this property, multiply the sale price by 1.3%. Calculate the **monthly** payment for taxes.
4. An estimate of the annual insurance on this property is \$2.20 per \$1000 home value. Example: A \$100,000 home would be 100 x 2.20 or \$220 per year. Calculate the **monthly** insurance payment for your house.
5. Assume that you need a down payment of 10%. Calculate the amount you need to **borrow**.
6. Calculate your monthly payment for this amount for each of the 3 options.  
Option 1: 30-year loan  
Option 2: 15-year loan  
Option 3: Adjustable Rate Mortgage (ARM)
7. To this monthly payment add the monthly payments for the taxes and insurance for each option.
8. Calculate the total amount paid over the life of the loan for each of the first 2 options (not including taxes and insurance).
9. Assume a 2% increase in the interest rate for each of the next 2 years of the ARM. Calculate the monthly payments for each of the next 2 years.
10. Decide which option you would choose. Write a brief explanation justifying your decision. Include the positive and negative things about each option.

**Math 1111 Daily Grade****Methods of Counting**

1. How many different three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, and 7?
2. One telephone prefix for the college is 285. How many telephones are possible before a new prefix is needed?
3. If a coin is tossed five times, in how many different ways can the sequence of heads and tails appear?
4. Pam's Pizza House offers 3 kinds of salads, 15 kinds of pizza, and 4 kinds of desserts. How many different three-course meals can be ordered?

**Math 1111 Daily Grade****Probability**

1. What is the probability of guessing the top 3 finishers in a greyhound race of 9 dogs?
2. Dodecahedral dice have 12 sides instead of the standard 6 sides. The sides are numbered 1 through 12. What is the probability of rolling two dodecahedral dice and having the sum of the face-up sides equal 8?

**Math 1111****Odds in Favor**

Find the odds in favor of obtaining:

1. an even number in one throw of a single die.
2. an ace when drawing 1 card from an ordinary deck of 52 cards.
3. 2 tails when an ordinary coin is thrown twice.
4. a vowel when 1 letter is chosen at random from among the 26 letters of the English alphabet.

Find the odds against obtaining

1. a 4 in one throw of a single die.
2. the king of spades when drawing 1 card from an ordinary deck of 52 cards.
3. a face card (jack, queen, king) when drawing 1 card from an ordinary deck of 52 cards.
4. If the correct odds in favor of Johnny's winning a race are 3:2, what is the probability that Johnny wins?
5. The House odds on a horse race are 5:1 for a horse named Fastpacer. If you bet \$3 on Fastpacer to win, how much money will you win if you are right?

### **Expected Value or Mathematical Expectation**

1. A coin is thrown twice. If heads comes up either time, we get \$2; but if heads does not occur, we lose \$4. What is the expected value of this game?
2. Two dice are thrown. If the sum of the dots showing is even, we get \$10; otherwise, we lose \$20. What is the expected value of this game?
3. A die is thrown. A person receives the number of dollars corresponding to the dots on the face that turns up. How much should a player pay to play in order to make this game fair?

**Math 1111****Normal Distribution 1**

A survey of men aged 15-21 determined that the mean travel time to work was 18.9 min with a standard deviation of 20 min. Assume a normal distribution.

- a) What is the probability that a man from this survey, chosen at random, traveled more than 30 min to reach work?
- b) What percentage of the men traveled between 10 and 30 min to get to work?
- c) What percentage of the men traveled between 5 and 10 min to get to work?

**Math 1111****Normal Distribution 2**

A study of men aged 35-57 at increased risk of coronary disease found that the average number of cigarettes smoked per day was 20.1 with a standard deviation of 18.5. Assuming a normal distribution, determine the number of cigarettes smoked by the 10% of the group who were the heaviest smokers.

**Math 1111****Normal Distribution 3**

Suppose that an American university accepts, unconditionally, students who score in the top 10% nationally on the SAT exam. For those students in the next 15% on the SAT exam, the university accepts the students on the basis of their grades in the senior year of high school. Assume the SAT average is 940 with a Standard deviation of 275.

- a) Determine the lowest SAT score a student may have and be accepted to the university unconditionally.
- b) Determine the SAT scores that will allow a student to enter the university dependent on his or her senior-year grades.

**Math 1111****Normal distribution 4**

An Algebra class averaged 76 on the first midterm with a standard deviation of 14.2. If the top 10% received A's, the next 20% received B's, the next 40% received C's, the next 20% D's, and the final 10% F's, use the normal distribution to determine which scores receive which grades.