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# A Simple Spatial Model for Edgeworth Cycles

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## *Abstract*

Maskin & Tirole (1998) formalise Edgeworth's (1925) model of a dynamic equilibrium between two players where prices increase sharply and decrease slowly; the Edgeworth Cycle. Here, we present an application of the model, showing how Edgeworth Cycles might arise in a marketplace where spatial competition is important. We illustrate the approach using the example of retail gasoline markets where Edgeworth Cycles have been widely observed.

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## A Simple Spatial Model for Edgeworth Cycles

Maskin & Tirole (1988) formalise the suggestion of Edgeworth (1925) of a regular cycle of prices in an oligopoly. Many of the assumptions made by Maskin & Tirole (1988) are relaxed by in subsequent work. Noel (2007a), shows that such cycles still occur when marginal costs, elasticities, product and strategies differ and when there are three firms. Eckert (2003) shows that they obtain when the firms are different sizes and Lau (2001), shows how cycles can arise even when players move simultaneously. There has also been considerable empirical work exploring the nature of Edgeworth Cycles in retail gasoline markets in Canada (Eckert & West, 2004a,b, 2005, Atkinson, Eckert & West, 2009, Noel, 2007a, b), the US (Lewis, 2008 and Doyle, Muehlegger & Samphantharak, 2008) and Australia (Wang, 2009, ACCC, 2007, Wills-Johnson & Bloch, 2010a, b).

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*Figure One about here*

We further extend the analysis by deriving Edgeworth Cycles in a spatial model. This is appropriate given the importance of spatial competition in gasoline markets where they are commonly observed. Our analysis is based on a simple framework in order to emphasise its intuitive appeal.

### *The Model*

The model is based upon the work of Hoover (1937) and MacBride (1983), who show how firms price discriminate between customers, charging closer customers higher prices than those more distant as closer customers face higher costs in accessing the firm in a spatially differentiated market. Our point of departure from their models is to assume that customers come to the firm (rather than having goods delivered to them), which means that the firm cannot ascertain from whence its customers have come and must thus charge a single price to all of them.

Consider the situation of two firms, A and B, located along a section of road, and selling an homogenous product to consumers who purchase one unit each of the product. The uniformly distributed consumers each have a travel plan that takes them past one of the retailers but they would have to deviate to frequent the premises of the other retailer. They would only choose to deviate if the second retailer had prices lower than the retailer they plan to pass (and can thus patronise at zero cost) by a margin greater than the extra travel costs of deviating. Each retailer maximises profit by trading off the extra per-unit profits that can be made by charging higher prices to those consumers for whom deviating to the competing retailer is costly against the extra gross profits which can be made by selling to more customers if a retailer undercuts its rival.

The situation faced by the firms is illustrated in Figure 2, where the cross-hatched shaded area indicates the profit Firm A gains from expanding market share through its lower price and the diagonally marked shaded area indicates the profit it loses on its most captive customers by not pricing at the same level as Firm B.

*Figure Two about here*

Assuming that consumers' planned departure points from the highway are uniformly distributed along the road between the two firms, the profit function of each firm can be expressed as

$$\pi_i = \begin{cases} q_i(p_i - c_i)d - q_i(p_i - c_i)\left(\dfrac{p_i - p_j}{d\tan\alpha}\right) & if \quad p_i > p_j \\ q_i(p_i - c_i)d & if \quad p_i = p_j \\ q_i(p_i - c_i)d + q_j(p_i - c_i)\left(\dfrac{p_j - p_i}{d\tan\alpha}\right) & if \quad p_i < p_j \end{cases} \tag{1}$$

where:

$p_i$ = price charged by firm *i*.

$q_i$ = proportion of overall customers that pass firm *i* first.

$c_i$ = marginal cost of firm *i*.

$d$ = distance between firm *i* and firm *j*.

tan$\alpha$ = the per-unit cost of travel (cost/distance).

Consider a sequential game where Firm A moves first and sets a price $p^*$, which is above an as yet undefined minimum. If *m* and *q* are, respectively, the travel cost and number of customers whose travel plans take them past Firm A and *n* and *r* represent the same variables for Firm B, then the best response of Firm B to $p^*$ is:

$$p = \min\left( mp,\ {}^{1}\!/_{2q}\left(nq + d^2 \tan\alpha r + qp^*\right)\right) \tag{2}$$

The best response to this price, when Firm A responds in turn, will be:

$$p = \min\left( mp,\ \frac{1}{4qr}\left(d^2 \tan\alpha\left(2q^2 + r^2\right) + +qr\left(2m + n + p^*\right)\right)\right) \tag{3}$$

The process continues until each firm reaches a minimum, below which it will not go. This minimum is:

$$p = \min\left( mp,\ \frac{1}{3qr}\left(\left(2q^2 + r^2\right)d^2 \tan\alpha + (2m + n)qr\right)\right) \text{ for Firm A,}$$

and (4)

$$p = \min\left( mp,\ \frac{1}{3qr}\left(\left(q^2 + 2r^2\right)d^2 \tan\alpha + (m + 2n)qr\right)\right) \text{ for Firm B.}$$

The question is whether the firms will move from this minimum. Raising price, as Maskin & Tirole (1988) suggest is a “public good” in that one firm doing so provides benefits both firms by raising the price that must be beaten to capture market share. A firm might decide to be the first mover if it believes that its rival will do so the next time they both reach a minimum, or if its own marginal costs and share of customers whose travel plans take them past it are such that it can gain profits over the course of the cycle (compared to both firms remaining at the minimum) even if it moves first.

However, the incentive to raise price need not be related to having the lowest costs, the most customers or an expectation of future moves. Consider the case where $m=n$ and $q=r$, and Firm A again increases its price to $p^*$. The best response of Firm B is now:

$$p = \frac{1}{2}\left(d^2 \tan\alpha + p^*\right) \tag{5}$$

The best response of Firm A to B's best response is:

$$p = \frac{1}{4}\left(3d^2 \tan\alpha + p^*\right) \tag{6}$$

This continues until each firm reaches its minimum price:

$$p = \left(d^2 \tan\alpha\right) \tag{7}$$

In this case, the profit for each firm if both remain at this minimum price is:

$$\pi_i = qd^3 \tan\alpha \tag{8}$$

The profits for each firm if Firm B relents first are shown in Table 1 overleaf, where the shaded cells indicate who is moving in which round. Note that $p^*$ is now $\overline{p}$; the maximum price that is feasible for this particular outlet. Subsequent responses to Firm B's increase in price are all a function of the distance between the two firms, the cost of travel and the price to which Firm B raises, not the minimum price. Thus, to maximise its returns over the cycle, Firm B will raise its price as high as it can.

*Table One about here*

Table One shows that there will clearly be cases where cycling gives higher profits than both firms remaining at the minimum price. However, the profit functions are complex. To simplify matters still further, we consider the results over a range of prices $\bar{p} = \delta \times d^2 \tan \alpha$, where $\delta$ is a simple scalar mark-up over the minimum price. The results are shown in Table Two, over the same five rounds as in Table One.

*Table Two about here*

For Firm A, which does not initiate the price hike, net profits are an increasing function of the size of the price hike. This is because it can capture a large profit in Round One, by pricing just below Firm B, but it does not have a compensating loss in Round Zero from a price hike which loses market share. Firm B, however, is different. When the price hike reaches roughly twice the minimum, its net profits turn negative. It thus seems unlikely that firms will raise prices above this level, even if the optimal response of a market-wide monopolist would be to do so.

## Conclusions

This paper introduces a very simple spatial model that is intuitively reasonable in the context of a market like that of retail gasoline. Whilst not proving that Edgeworth Cycles obtain in every case, like Noel (2007a), it suggests that they are highly likely over a wide range of plausible scenarios. Moreover, it shows that one does not need differences in costs or in market share (measured by customers passing an outlet first) in order to provide a unilateral incentive to increase price.  This may help in explaining why Edgeworth Cycles are widely observed in retail gasoline markets.

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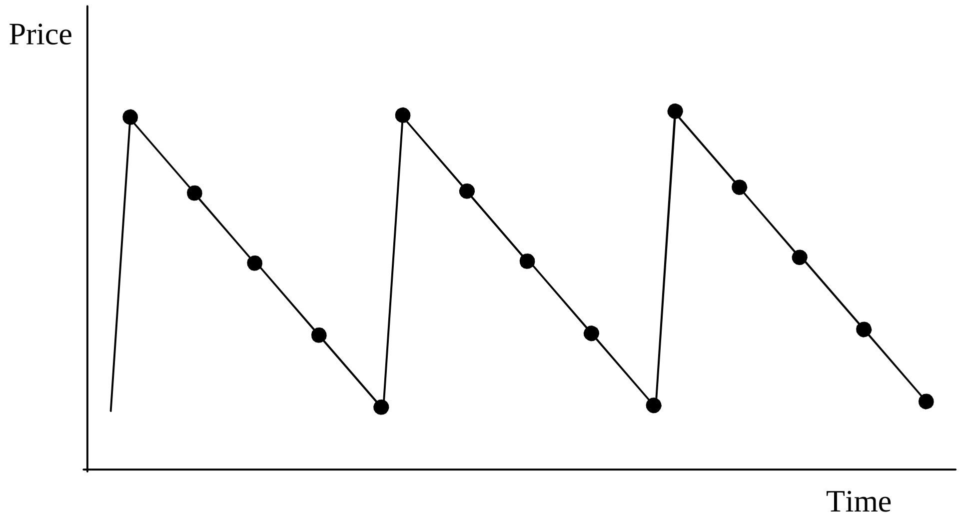


**Fig. 1.** A diagrammatic representation of an Edgeworth cycle

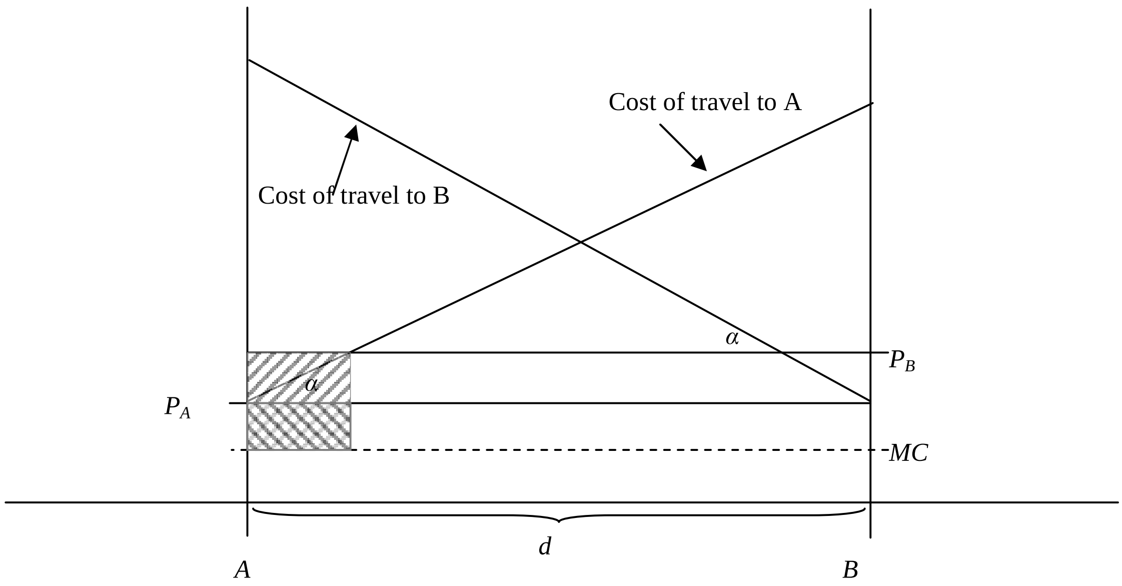


**Fig. 2.** Pricing game for two retail petroleum outlets.

**Table 1**

Four-round game results

| | ***Best Response Price*** | | ***Profits*** | |
|---|---|---|---|---|
| | *Firm A* | *Firm B* | *Firm A* | *Firm B* |
| Round 0 | $d^2 \tan\alpha$ | $\overline{p}$ | $dq\overline{p}$ | $2dq\overline{p} - \dfrac{q\overline{p}^2}{d\tan\alpha}$ |
| Round 1 | $\frac{1}{2}\left(d^2\tan\alpha + \overline{p}\right)$ | $\overline{p}$ | $\dfrac{q\left(d^2\tan\alpha + \overline{p}\right)^2}{4d\tan\alpha}$ | $\dfrac{q\left(3d^2\tan\alpha - \overline{p}\right)\overline{p}}{2d\tan\alpha}$ |
| Round 2 | $\frac{1}{2}\left(d^2\tan\alpha + \overline{p}\right)$ | $\frac{1}{4}\left(3d^2\tan\alpha + \overline{p}\right)$ | $\dfrac{q\left(5d^4\tan\alpha + 4d^2\overline{p} - \overline{p}/\tan\alpha\right)}{8d}$ | $\dfrac{q\left(3d^2\tan\alpha + \overline{p}\right)^2}{16d\tan\alpha}$ |
| Round 3 | $\frac{1}{8}\left(7d^2\tan\alpha + \overline{p}\right)$ | $\frac{1}{4}\left(3d^2\tan\alpha + \overline{p}\right)$ | $\dfrac{q\left(7d^2\tan\alpha + \overline{p}\right)^2}{64d\tan\alpha}$ | $\dfrac{q\left(27d^4\tan^2\alpha + 6d^2\tan\alpha\overline{p} - \overline{p}^2\right)}{32d\tan\alpha}$ |
| Round 4 | $\frac{1}{8}\left(7d^2\tan\alpha + \overline{p}\right)$ | $\frac{1}{16}\left(15d^2\tan\alpha + \overline{p}\right)$ | $\dfrac{q\left(119d^4\tan^2\alpha + 110d^2\tan\alpha\overline{p} - \overline{p}^2\right)}{128d\tan\alpha}$ | $\dfrac{q\left(15d^2\tan\alpha + \overline{p}\right)^2}{256d\tan\alpha}$ |
| | | ***Sum of Profits*** | $\dfrac{q\left(329d^4\tan^2\alpha + 294d^2\tan\alpha\overline{p} - 75\overline{p}^2\right)}{128d\tan\alpha}$ | $\dfrac{5q\left(117d^4\tan^2\alpha + 214d^2\tan\alpha\overline{p} - 75\overline{p}^2\right)}{256d\tan\alpha}$ |
| | | ***Net Profits*** | $\dfrac{q\left(-311d^4\tan^2\alpha + 294d^2\tan\alpha\overline{p} + 17\overline{p}^2\right)}{128d\tan\alpha}$ | $\dfrac{5q\left(-139d^4\tan^2\alpha + 214d^2\tan\alpha\overline{p} - 75\overline{p}^2\right)}{256d\tan\alpha}$ |

**Table 2**

Net profits over a range of multiples of minimum price equilibria

| $\delta$ | Net Profit for Firm A | Net Profit for Firm B |
|---|---|---|
| 1.1 | $0.2576d^3\tan\alpha$ | $0.1103d^3\tan\alpha$ |
| 1.2 | $0.2576d^3\tan\alpha$ | $0.1103d^3\tan\alpha$ |
| 1.3 | $0.5178d^3\tan\alpha$ | $0.1914d^3\tan\alpha$ |
| 1.4 | $0.7807d^3\tan\alpha$ | $0.2432d^3\tan\alpha$ |
| 1.5 | $1.0462d^3\tan\alpha$ | $0.2656d^3\tan\alpha$ |
| 1.6 | $1.3144d^3\tan\alpha$ | $0.2588d^3\tan\alpha$ |
| 1.7 | $1.5853d^3\tan\alpha$ | $0.2226d^3\tan\alpha$ |
| 1.8 | $1.8588d^3\tan\alpha$ | $0.1572d^3\tan\alpha$ |
| 1.9 | $2.1350d^3\tan\alpha$ | $0.0625d^3\tan\alpha$ |
| 2.0 | $2.4138d^3\tan\alpha$ | $-0.0615d^3\tan\alpha$ |
| 2.1 | $2.6953d^3\tan\alpha$ | $-0.2148d^3\tan\alpha$ |