Department of Mining Engineering and Metallurgical Engineering
Western Australian School of Mines
Faculty of Science and Engineering

Real Options Valuation for Mining Projects Under Dual Economic Uncertainty

Md. Aminul Haque

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgments have been made. This thesis contains no material which has been accepted for any other degree or diploma in any University.

Signature: ______________

Date: *March 14, 2016*

This thesis is dedicated to my departed parents, my wife Tamanna and my daughter Abdia, for their collective great sacrifice, love, endurance and understanding.

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PUBLICATIONS INCORPORATED INTO THIS THESIS

This thesis incorporates papers that have been published during the period of this thesis preparation, the details of these publications being listed below.

1. Haque, M. A., Topal, E. and Lilford, E. (2014). A numerical study for a mining project using real options valuation under commodity price uncertainty. Resources Policy, 39:115-123.

2. Haque, M. A., Topal, E. and Lilford, E. (2015). Relationship between the gold price and the Australian dollar-US dollar exchange rate. Mineral Economics. 28: 65-78.

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5. Haque, M. A., Topal, E. and Lilford, E. (2015). Evaluation of a mining project under the joint effect of commodity price and exchange rate uncertainties using real option valuation. Journal of the Engineering Economist (publication under review process).

ABSTRACT

Cash flows arising from mining projects are typically volatile and are largely influenced by a number of exogenous variables including commodity price and exchange rate uncertainties. In addition, investments made into mining projects tend to be irreversible. Therefore, with minerals and mining being a complex industry sector, investing into a mining project is both a challenging and risky undertaking. Volatility associated with commodity prices and exchange rates often carries significant income risk for a mining company, and often creates negative sentiment around the minerals industry at large. Due to the dramatic decline in mineral commodity prices, some companies and operations have break-even prices above the *de facto* commodity spot prices. As with commodity price uncertainty, exchange rate uncertainty is an equally important economic factor for investment decision-making in the mining sector. Mining companies and operations that have revenues dependent on the US dollar exchange rate commensurate with expenses incurred in local currencies, face significant uncertainty with foreign exchange rates that can be regarded just as important as commodity price uncertainty. Therefore, mine management should address these economic risk exposures before making and acting on investment decisions.

When there is an opportunity to acquire either a new or an operating mine, the owners of the mining project need to know what value or return the project can generate over its life. However, determining the value of a mining project is not a simple task due to numerous associated uncertainties inherent in the project. Consequently, any valuation assessment demands an appropriate evaluation methodology that closely reflects real life circumstances, responding to global market dynamics and providing managers and stakeholders with relevant and appropriate information before acting on a decision to invest or divest in or from the project, respectively.

In this economically competitive world, as with many other industries, mining companies need to adopt more risk mitigation strategies including commodity and currency hedging strategies to respond to economic uncertainties, typically obviating risks such as commodity price volatility, exchange rate fluctuations and other economic attributes, in order to reduce downside risk and avoid or at least minimise

operating losses. Through the use of hedging strategies and futures contracts, mine management can reduce the potential for generating losses. Moreover, mine management often has some flexibility and, with embedded optionality to carry out mining projects, may exploit new information as well as new market conditions to improve the economics of a project. Regrettably, until now, many mining companies have deployed traditional Discounted Cash Flow (DCF) methods for mining project evaluations, which fail to capture these economic uncertainties and typically do not incorporate hedging strategies.

Over the last few years, many studies have presented the real options valuation (ROV) methodology as a promising technique in evaluating natural resource investments under conditions of uncertainty. However, due to its complexity, mining companies that use ROV methods with continuous time option based modelling incorporating a hedging strategy are rare. Whether using a modification of the Black-Scholes pricing equation or partial differential equations (PDEs), continuous time models always demand a reasonably sophisticated knowledge of a variety of advanced mathematical techniques which is habitually limited in mine managers and associated industry practitioners. Stated differently, continuous time models are poorly understood by practitioners, and are deemed too complex to implement in a mining company. However, to incorporate a meaningful hedging strategy and futures contracts in a mining valuation model, continuous time stochastic models and resulting PDEs are beneficial for approximating project values.

This thesis primarily focuses on the dual economic uncertainties in mining projects, and incorporates them into ROV methods for the evaluation of mining projects using continuous time modelling and hedging strategies. It provides the fundamentals of a novel mine valuation framework in real options valuation methods through developing new PDEs which incorporate commodity price and exchange rate uncertainties in the evaluation process. In this study, stochastic models are considered to address the economic uncertainties around commodity price and exchange rate fluctuations. Thereafter, resulting new PDEs are developed using these stochastic models, hedging strategies and futures contracts, incorporating a variety of financial and mining parameters. These proposed novel PDEs are utilised in ROV methods to approximate the mining project values through numerical solutions.

Since commodity prices and exchange rates are the key economic uncertainties associated with accurate mining project evaluations, the relationship between mineral commodity prices and corresponding exchanges rates have a significant impact on the evaluation of individual mining projects and effective hedging strategies, and these exogenous factors must be addressed in the modelling process to estimate mining project values. Consequently, this thesis also offers an outline and investigates the relationship between mineral commodity prices and corresponding exchange rates, and has utilized the results in ROV methods for approximating mining project values. This new ROV method assesses the value of a mining project by providing the advantage of mitigating financial losses and enhancing financial gains.

Keywords: Mining project evaluation, discounted cash flow, real options, hedging strategy, futures contracts, stochastic differential equations, partial differential equations, Geometric Brownian Motion, mean-reverting model, finite difference method, unit roots, cointegration, vector auto regression, vector error correction model, structural vector auto regression.

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LIST OF ACRONYMS

AGDFAT: Australian Government Department of Foreign Affairs and Trade

AIMR : Australian Identified Mineral Resources.

AMHA: Australian Mining History Association

CAPEX: Capital Expenditure

DCF: Discounted Cash Flow

DT: Decision Tree

FDM: Finite Difference Method

FEVD: Forecast Error Variance Decomposition

FOREX: Foreign Exchange

GBM: Geometric Brownian Motion

IRF: Impulse–Response Function

MCS: Monte Carlo Simulation

MRM: Mean-Reverting Model

NPV: Net Present Value

OPEX: Operational Expenditure

PDE: Partial Differential Equation

RBA: Reserve Bank of Australia

ROV: Real Option Valuation

SDE: Stochastic Differential Equation

SFEVD: Structural Forecast Error Variance Decomposition

SVAR: Structural Vector Auto Regressive

VAR: Vector Auto Regressive

VEC: Vector Error Correction

WACC: Weighted Average Cost of Capital

CHAPTER 1

INTRODUCTION AND OBJECTIVE OF THIS RESEARCH

1.1 Background

Cash flows emanating from mining projects are highly volatile as a direct consequence of being notably influenced by a number of exogenous variables. In addition to this, mining projects are complex businesses and many of their executed investment decisions are irreversible. Commodity prices and exchange rates are continuously fluctuating and at times they exacerbate the risks associated with the mining industry. Collectively, mine management needs to address these potential risk exposures before making or acting on an investment decision. Trench and Sykes (2014) stated that the most common way of mitigating commodity price risk for a mining operation is through hedging that commodity, and the eventual goal of the commodity price hedging strategy is to bring certainty to the revenue generating component of the operation through the introduction of a protection mechanism against a possible fall in commodity prices. FitzGerald (2014) reported that, the director of Surbition, Sandra Close, explained the benefits of gold price hedging by the miners in that several gold producers achieved over $100 per ounce more than the average spot price through delivering some of their production into a hedging strategy. Close also stated that, wisely, hedging mechanisms provide certainty and reduce risk, and added that it was curious and quite inconsistent for a company to insure its plant and equipment, but not to secure the minimum price it can receive for its product. However, it is noted that hedging needs be done cautiously. Therefore, due to the deterioration and fluctuation of mineral commodity prices and exchange rates, for a successful mining project acquisition or development, an important and appropriate investment strategy should include hedging out much of the economic risk i.e. to reduce any potential substantial losses suffered by a company. Another essential strategy is to incorporate management and operational flexibilities in a mining operation, which flexibilities evolve during the project’s life.

Hence, when there is a possibility of acquiring either a new or an operating mine, owners and stakeholders of the mining project need to know the value of the project that can be generated over the operating life of mine. Therefore, it demands appropriate evaluation methods that can consider real life circumstances, respond to global market dynamics, and provide managers and stakeholders with relevant information before taking the decision to invest in or develop the project. These are the fundamental bases for making the final irreversible decisions about going ahead with the natural resource project's investment. The choice over such a vital decision for the investment is formally referred to as an option.

The estimation of a mining project's value is a complicated task, as several uncertainties are attached during the life of a project. This is because, in general, values of mining projects are influenced by many underlying economic and physical uncertainties, such as commodity price, metal grade, foreign exchange rates, operating costs, size of the deposit, environmental and political issues, and more. Consequently, mining projects are facing considerable challenges and risks, most notably due to the uncertainty surrounding commodity prices, exchange rates and operating costs, along with geological uncertainties.

The valuations of mining projects are developed on the basis of the project acquisition, investment and financing decisions by both the investors and the financial institutions in the mineral sector. In general, in this competitive world, as with other industries, a mining company needs to be more rehearsed and knowledgeable in hedging strategies to respond to commodity price and exchange rate uncertainties for optimising profits and minimizing or protecting mining losses. The traditional discounted cash flow (DCF) valuation methods fail to respond to these economic uncertainties and useful hedging strategies as the methods rely on the risk adjusted discount rate to address these uncertainties.

1.2 Statement of problem

Mining projects have the unique characteristic of being location specific, typically with longevity and exposed to technical and significant economic uncertainties i.e. commodity price, exchange rate and cost uncertainties, as well as reserve risk. The fluctuation of commodity prices is a major uncertainty for the mining industry. A weakened commodity price generally creates a negative sentiment around the mineral and mining industry. Brazil's Vale, the world's biggest producer of iron ore and associated pellets (Vale, 2015), reported its second quarter earnings results on

Thursday July 30, 2015, and stated that, due to the sharp decline in iron ore prices, the result was a significantly negative impact on the company's financial outcome. The company's net operational revenues for the second quarter in 2015 stood at US$6.97 billion, which is significantly lower than the figure of US$9.90 billion in the corresponding period of the previous year, 2014, and with the main reason being attributed to the dramatic decline in iron ore prices. Gold mining industries have also been hit hard due to the tremendous fall in gold prices. The gold price has continued to fall since 2011, and this is creating severe pain for the gold mining industry. Mark Bristow, the chief executive officer of Randgold Resources, expressed his view that the gold mining industry is evidently "stuffed" at US$1,140/oz, and it will be a bloodbath if the price reaches below US$1,000/oz (Bristow, 2015). Jennifer Hewett stated on July 28, 2015, that miners are trying to drastically reduce their costs, with Fortescue Metals Group (FMG) being in a more vulnerable position than other iron ore producers due to low iron ore prices and its required break-even price being above current iron ore spot prices, commensurate with the company still carrying around US$9 billion worth of debt (Hewett, 2015).

In a report dated July 27, 2015, Williams mentioned that due to gold's ongoing price decline, one can expect an accelerated rate of gold mine shutdowns around the world (William 2015). Along with commodity price uncertainty, exchange rate uncertainty is an equally important economic factor to be considered in the investment decision making process in the mining sector. The recent depreciation of the US dollar against the currencies of major mineral-exporting countries has shown that the foreign exchange rate (FOREX) can be a major contributor to the economic risk of mining investments. For mining companies that have revenues pegged to the US dollar exchange rate and expenses incurred in local currencies, the uncertainty over FOREX can be as important as commodity prices. For instance, in the six month period ended June 30, 2013, Jaguar Mining in Brazil (Jaguar Mining, 2013) reported on August 7, 2013 that it had incurred a net loss of US$4.7 million due mainly to changes in the foreign exchange rate. The Australian natural resources group BHP Billiton reported that positive exchange rate movements added AUD$190 million to its net profit compared with the March quarter of the previous year (BHP, 2001). However, until now, foreign exchange rate uncertainty in the mining sector has largely been ignored. Furthermore, as the commodity price and exchange rate uncertainties are crucial components for mining investments, it is important to know the relationship between specific mining commodity prices and exchange rate movements before evaluating a mining project or mining investment opportunity.

Although there are sophisticated tools and valuation methods currently available, including stochastic modelling processes to determine and quantify mineral resources and reserves and to minimise project risk, there is still insignificant literature available that addresses exchange rate uncertainty in the evaluation of a mining project. Furthermore, there is no literature available regarding the modelling process necessary to incorporate the joint effect of these dual economic (commodity price and exchange rate) uncertainties for approximating mining project values. Therefore, this research aims to develop an outline for approximating explicit mining project values numerically through ROV methods considering continuous time stochastic modelling, to address these dual economic uncertainties. Due to the variability and volatility of commodity prices and exchange rates, hedging becomes a key strategy for mining companies in their response to these economic uncertainties. Using financial hedging strategies and forward and futures contracts, mine management can reduce potential operational financial losses i.e. economic risk management. However, hedging should be approached with caution.

The discounted cash flow (DCF) valuation method has commonly been used in the mining industry for a long time. Furthermore, Binomial Option Pricing is generally used as a methodology to evaluate a mining project using the ROV method. However, previous research around DCF and ROV methods has typically encountered the following pitfalls:

1. Other than through simple sensitivity analyses, DCF evaluation methods ignore integrating various types of uncertainties, such as commodity price, costs, the exchange rate, reserves and others, as these uncertainties are generally considered only by using a single parameter risk adjusted discounted rate.

2. In general, in a DCF method, it is difficult to accurately determine an appropriate single dynamic risk adjusted rate of return.

3. Prior studies have often ignored the uncertainty associated with exchange rates, even though exchange rate assumptions have a significant impact on the estimation of project values.

4. Predominantly, in the past, the Binomial Option Pricing (BOP) model was used to calculate ROV results. One of the limitations of the BOP method is that the methodology excludes the use or incorporation of futures contracts and hedging strategies for reducing risk and optimising cash flow results. Another major shortcoming is that binomial models are trees and consequently, when used for modelling and valuing investment opportunities in conjunction with real options, can quickly become large and cumbersome. An additional problem in using BOP methods is that it may be difficult to combine more than one uncertainty in an evaluation process using hedging strategies and, in practice, it becomes computationally intensive as the number of scenarios demanding evaluation increases exponentially.

5. A number of previous studies have developed complicated theoretical models in continuous time processes, but there is no up-to-date solution to find mining project values using these models, especially in the case when modelling more than one uncertainty. Consequently, even though prior studies claim that the ROV model is better than the DCF method, there is still no noticeable replacement-value in preferentially using the DCF method over the ROV method in industry. Furthermore, for investment opportunities with non-standard features (such as project specific characteristics, multiple uncertainties, etc.), the general Black-Scholes equation i.e. the general partial differential equation (PDE), may not always be sufficient. In these circumstances, the best alternative is to undertake a given stochastic process for the underlying asset, typically a Geometric Brownian Motion (GBM), and then derive an appropriate resulting new PDE which can capture the project specific characteristics and parameters.

6. Whether using a modified version of the Black-Scholes pricing equation or PDEs, continuous time models always require a reasonably sophisticated

knowledge of a variety of advanced mathematical techniques, which is in general not common amongst mine managers and valuation practitioners. Therefore, continuous time models are not popular, are less well understood, and are generally deemed too complex to implement in the mining industry.

Therefore, in this study, through the incorporation of financial hedging strategies and stochastic models, new partial differential equations have been derived through incorporating different financial, economic and mining parameters, and these PDEs have been solved numerically to find approximate project values.

1.3 Objective of research

This research focuses mainly on economic uncertainties in mining projects and incorporates these uncertainties into ROV methods for the evaluation of mining projects using continuous time models. Mine management often has some flexibility and inherent optionality associated with carrying out mining projects, and can incorporate new information and revised market conditions to improve the economics of a project. Commodity prices and exchange rates are the key economic uncertainties in mining project evaluations. As a consequence, the relationship between the mineral commodity price and the corresponding exchanges rates have a significant impact on the evaluation of individual mining projects, and these exogenous factors need to be addressed in the modelling process to estimate mining project values. This research has addressed many of these issues and has used them in ROV methods for approximating mining project values. Hence, the objectives of this research are sequentially as follows:

1. Review the existing methodologies and literature for mining project evaluations.

2. Review the existing literature regarding the relationship between mineral commodity prices and exchange rates.

3. Introduce new PDEs using continuous time stochastic models and hedging strategies, and implement these in ROV methods for mining project evaluations considering price uncertainty.

4. Develop methodologies and investigate the relationship between mineral commodity prices and exchange rate movements using econometric models. As a part of this investigation, two case studies are considered:
   - (i) The relationship between the gold price and the AUD/USD exchange rate, and
   - (ii) The relationship between the iron ore price and the AUD/USD exchange rate movement.

5. Propose new PDEs using continuous time stochastic models and hedging strategies and futures contracts, and utilize these PDEs in ROV methods for valuing mining projects under the dual effect of commodity price and exchange rate uncertainties.

1.4 Scope and limitation of research

This study focuses on dual economic uncertainties being commodity prices and exchange rates for evaluating mining projects through continuous time stochastic processes/models and using the resulting new partial differential equations (PDEs). Although some economic parameters are determined to approximate the project values by solving the new PDE numerically, the limitation of this research is that some parameter values are assumed, due to the time constraints in collecting real data and calculating the necessary parameter values. As a consequence, hypothetical mining projects are considered, as proxies, to approximate the project value. Furthermore, there is still scope for future research to develop the models and resulting PDEs based on ongoing mining situations and market conditions, and also to develop an interface program to approximate the project values.

1.5 Significance of the research

The key significance of this PhD research is to develop new partial differential equations (PDEs) and ROV methods addressing economic uncertainties, specifically the commodity price and exchange rate. The ultimate goal of this research is to develop and use these new PDEs for project evaluations which are constructed by incorporating hedging strategies through futures contracts, instead of using traditional DCF methods. Mining companies should incorporate a hedging strategy and futures contract program to minimise economic risks and optimise profits. Another significant consideration is to develop the methodologies in order to understand the relationship between mining commodity prices and exchange rates. To use the hedging strategy in a mining company, management needs to know the relationship, if any, held between these economic indicators before investing in a mining project. Furthermore, to estimate mining project values considering more than one uncertainty through continuous time stochastic models, the correlation parameter between these two economic variables is required. The significance of this research can be summarised as follows:

1. A new reserve dependent PDE using continuous time stochastic GBM models and a hedging strategy has been developed, and applied it to ROV methods in mining projects evaluation.

2. A new time dependent PDE using a continuous time stochastic mean reverting model (MRM) and hedging strategy has been developed, and employed the result in ROV methods for mining projects evaluation.

3. Investigate the relationship between mineral commodity prices and exchange rates considering two case studies.

4. Develop new PDEs and estimate project values under the joint effect of commodity price uncertainty and exchange rate uncertainty using the ROV method. This is the first ROV method that integrates exchange rate uncertainty with price uncertainty for mining project evaluations.

5. A comparison study has been shown between the project values considering single uncertainty commodity price and the joint uncertainties commodity price and exchange rates. In this study, the key finding is that project values are over

estimated when the singe uncertainty commodity price is considered rather than the joint uncertainties of commodity prices and exchange rate.

1.6 Thesis organization

This thesis is prepared into six chapters. The chapters are organized as follows:

Chapter 1 provides a general overview of the background of this research. The statement of the problem and the objectives of the research are stated, with the scope and limitation of this research also being described here. The significance of this research and the organization of the whole research work are also presented in this chapter.

Chapter 2 provides an introduction and describes different types of financial options. The basic idea of discounted cash flow (DCF), hedging and futures contracts, and real options (RO) techniques are also presented in this chapter. Different types of methodologies for project evaluations are also described in this chapter. Lastly, a detailed review of most published literature in the main focus area of the research is presented.

Chapter 3 develops new PDEs and implements ROV methods for mining projects evaluation considering price uncertainty through continuous time stochastic models. In addition, the gold price volatility is determined from historical data through the log return method of analysis, and different available real options for mining projects are also described. In this chapter, two different techniques are developed to approximate project values: firstly from a reserve dependent PDE which considers a GBM stochastic model, and secondly, from a time dependent PDE which considers a mean reverting stochastic model. In this chapter, new PDEs are derived incorporating mining parameters and taxes directly within the PDEs to approximate project values using single uncertainty.

Chapter 4 develops and applies different econometrics and statistical methodologies for investigating the relationship between the commodity price and exchange rate. As a case study, the long term relationship between gold prices and the AUD/USD exchange rate is shown here. In this chapter the relationship of iron ore prices and the value of the Australian dollar (i.e. AUD/USD exchange rate) is also presented.

Chapter 5 proposes a novel PDE considering two stochastic models under the joint effects of commodity price and exchange rate uncertainties. This new PDE is utilized in ROV methods for approximating mining project values and this is the first model which has incorporated the exchange rate uncertainty with the price uncertainty for the evaluation of mining projects. In this chapter, price paths of the commodity have also been simulated and shown (as an example two case studies being iron ore and gold are considered here). This simulated average price is used to approximate the project values. New PDEs are implemented in iron ore and gold case studies to estimate project values and the results are presented.

Chapter 6 concludes the thesis by presenting an introduction, summary and original contribution of the thesis, as well as recommendations for future research work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In the real world, to make the right decision around choosing the most profitable investment strategy for a company is critical. Unexpected price fluctuations, economic recessions, political changes, improvements in technology, changes in consumer tastes, competitive markets, choices concerning innovation and other aspects are all risk factors affecting investment decision making (Botteron, 2001). The company shareholders expect high returns. Therefore, it is necessary to ensure the most efficient use of resources. What is the decision for initiating a project? Invest now, wait, or abandon the project altogether? To answer these questions, it is necessary to determine the optimal assessment method and then to apply it for each project evaluation.

Mining is a challenging business, so before initiating or investing in a mining project, an evaluation of the project is necessary and the resultant values must be considered in order to devise effective strategies which maximize profits and minimize losses. Investment in a mining project is a difficult task, as mining projects are associated with internal and externals risks. Furthermore, investments in mining projects are generally irreversible. In addition, there is a strong relationship between the price of the mined commodity and the exchange rate. Consequently, mining projects are looking to be assessed by a new version of evaluation techniques that can adequately capture financial market conditions and the decision-flexibility inherent in the project combined with the association between economic variables, rather than depending on the traditional project evaluation techniques. This chapter is organized as follows: for the evaluation of project values, the first part of the reviews are concerned about the project evaluation methods, and for the investigation of the relationship between commodity price and exchange rates, the second part of the reviews are concerned about the existing literature of the association between mineral commodity prices and exchange rates (as some information regarding these relationships is crucial for hedging strategy, and some parameters of this relationship will be used in continuous time stochastic modelling and associated partial differential equations (PDEs) for evaluating project values).

2.2 Important definitions on project evaluation

Some important definitions on project evaluation are discussed here.

2.2.1 Discounted Cash Flow (DCF) technique

The present value (PV) derived from a discounted cash flow (DCF) is a valuation technique which is used to estimate the project/firm value. The mathematical formula to calculate PV is typically as follows:

$$PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \cdots\cdots\cdots\cdots + \frac{CF_n}{(1+r)^n}$$

$$PV = \sum_{i=1}^{n} \frac{CF_i}{(1+r)^i} \qquad (2\text{-}1)$$

Where

PV = present value

CF_i = cash flow in year i

r = discount rate

n = the number of periods in the valuation model

2.2.2 Financial Options

Financial options are types of derivatives contracts where the underlying assets are financial instruments such as stocks, bonds or an interest rate. The options on financial instruments give a buyer the right to either buy or sell the underlying financial instruments at a specified price on a specified future date. Even though the buyer has the rights to buy or sell the underlying instruments, there is no obligation for the buyer to exercise this option. However, on the counterpart, the seller of the contract is under an obligation to buy or sell the underlying instruments if the option is exercised by the buyer.

There are two main types of options: call options and put options. Call options give the buyer the right to buy the financial instrument at the specified price at a future date, whereas, under a put option, the buyer of the contract acquires the right to sell the financial instrument at the specified price at the specified date**.**

2.2.2.1 European Call and Put options

A European call option gives the holder the right, but not the obligation, to purchase an asset or the financial instrument at the strike price on the maturity date. Whereas, a European put option gives the investor the right, but not the obligation, to sell an asset or the financial instrument at the strike price on the maturity date.

2.2.2.2 American call and Put options

An American call option gives the holder the right, but not the obligation, to purchase (exercise option) an asset or the financial instrument at any time prior to, and including on the maturity date. An American put option gives the holder the right, but not the obligation, to sell an asset or the financial instrument at any time prior to, and including on the maturity date.

As an American option there is an opportunity of early exercise during the life of contract, therefore, an American option is more valuable than a European option under certain conditions.

2.2.3 Hedging for commodity price and exchange rate

Hedging is an investment position which is used to offset potential losses or gains that may be experienced by an investment. In simple terms, hedging is used to diminish any potential extensive losses or gains experienced by an individual or a company. Portfolio managers, individual investors and companies usually use the hedging techniques to reduce their exposures to various risks. Futures contracts are one of the most common derivatives used to hedge risk. The main reason that companies or corporations use futures contracts is to offset their risk exposures and thus limit potential negative financial impacts from fluctuations in a commodity price or in exchange rate.

Forwards and swaps are also used for the purpose of hedging for mitigating risk. A forward contract is a non-standardized contract between two parties to buy or sell an asset at a specified future time at a price agreed upon today. A forward contract can be used for hedging or speculation, although its non-standardized nature makes it particularly apt for hedging. Forward contracts do not trade on a centralized exchange and are therefore regarded as over-the-counter (OTC) market instrument. In addition, swaps are also used to hedge for mitigating risk, and commodity swaps are usually enable producers and consumers to hedge commodity prices risk.

2.3 The real options (RO) technique

An option which is embedded in, or attached to, physical assets or real assets is termed a real option. A real option is not a financial derivative instrument, rather it is an alternative or choice that becomes available with a business investment opportunity. Real options can include opportunities or strategic decisions such as delaying an investment, temporary closure, expanding, downsizing or abandoning projects. The real option valuation technique, which is based on modern asset pricing (MAP) theory, usually uses the concept of futures contracts to evaluate projects by understanding and controlling the effect of uncertainty, managerial flexibility and inherent risk in the project. This is possible due to the major advances in asset pricing theory. A real option is similar to a financial option, and the real option analysis is a valuation and strategic decision that applies financial option theory to real assets (Rogers, 2002). In real options, the underlying assets are usually tangible rather than that in financial options. Some similarities and differences between financial options and real options are given in Table 2-1.

Table 2-1 Similarities and differences between financial and real options (Source: Martinez, 2010).

Financial options	**Real options**
Short maturity often quoted in months	Longer maturity usually in years
Underlying variable driving the value which is the equity price or price of a financial product.	Underlying assets or exposures are usually tangible.
Values are usually small	Values are very large, usually in the millions or billions of dollar decisions
Competition and market effects are irrelevant to its exercise price	Competition and market drive the value of a strategic option.
Usually solved using partial differential equations and binomial lattices with simulation of the underlying variables.	Usually solved using partial differential equations and binomial lattices with simulation of the underlying variables
Management assumptions and actions have no bearing on valuation.	Management actions drive the value of a real option.
Marketable and traded security with comparable and pricing information	Not traded and propriety in nature with no market comparable.

2.4 Project evaluation methodologies

Based on cash flows, there are five major economic evaluation methodologies for determining firm/project values being Multiples, Discounted Cash Flow (DCF), Monte Carlo Simulation (MCS), Decision Tree (DT) and Real Options Valuation (ROV). The Multiples evaluation method is based on the idea that similar assets sell at similar prices, and is usually applied at the beginning of a new project (Brealey et al., 2006, Dixit and Pindyck., 1994). One of the most well-known traditional methods in project evaluation is DCF. This standard valuation technique estimates the present value of expected future cash flows. That means it uses future free cash flow projections and discounts them to arrive at a present value, which is used to evaluate the potential for investment. The DCF technique evaluates the whole project by adjusting or discounting the project's periodic cash flows for the effects of risk and time. Net Present Value (NPV) is the most frequently cited result of applying DCF analysis and is widely used for evaluating an investment project.

The MCS method is a simulation technique and it estimates the outcome of the project by using the marginal distribution of all the parameters appearing in the NPV equation. Monte-Carlo simulation is mainly a risk assessment tool. This method simply uses statistical distributions, such as normal, lognormal, triangular, and uniform, to evaluate the uncertainty in the parameters within the project (Topal, 2008). Due to the development of computer technology, this technique has been used increasingly as a beneficial tool for analysing projects under uncertainty. In every simulation, the values are selected randomly from each parameter distribution for every time period and substituted into the NPV equation in order to generate one possible outcome of the project (Galli et al., 1999). This process is repeated hundreds or thousands of times for calculating an average or expected NPV of the project. The more simulations done, the more accurate the approximation of the outcome of the project will be (Walls, 1999). The MCS method focuses on modelling the uncertainty with the input parameters and still uses in many mining companies for valuing projects.

The other method is DT which estimates the probability of possible outcomes of a project by generating appropriate decision branches that have probabilities of their likelihood of occurrence. This method allows for the decision maker to resolve a large problem easily by breaking it down into a series of smaller ones. The disadvantage of this method is that when the number of possibilities increases, the tree grows exponentially (Topal, 2008). This method is generally utilised in the probabilistic analysis of mining projects.

Finally, the ROV method has been developed for project evaluations. ROV is a process by which a real or tangible asset with uncertainties can be valued when there is flexibility or a potential option is available. The ROV method includes the application of the mathematical skills found in financial options in the field of business to assist a project manager in assessing the best option for a company when faced with a real-life decision. In 1985, Brennan and Schwartz firstly used the ROV method to capture the value of flexibility of a project in a sound mathematical model using the concept of futures markets, and gradually ROV emerged to exploit uncertainty in projects evaluation (Cortazar and Schwartz, 1993, 1994; Cortazar et al., 1998b, 1999, 2001; Brennan and Trigeorgis, 2000; Copeland and Antikarov, 2001; Colwell et al., 2003). The standard DCF approach to valuation in corporate finance practice is gradually being supplemented with real options valuation. ROV is usually employed in mining projects, and the researchers and evaluators are now increasingly explaining the advantages of ROV over the more traditional DCF methods.

Figure 2-1 demonstrates briefly the historical trend of project evaluation methodologies starting from Multiples to ROV. During the period 1960-1970, methods that were used for project evaluations included Multiples and DCF which ignored uncertainties inherent in projects. In 1980, the MCS method was introduced and it assessed uncertainty during the project evaluation stage. However, around 1990 the DT method attempted to manage this uncertainty. Finally the ROV method appeared to exploit uncertainty and operational flexibility in mining projects.

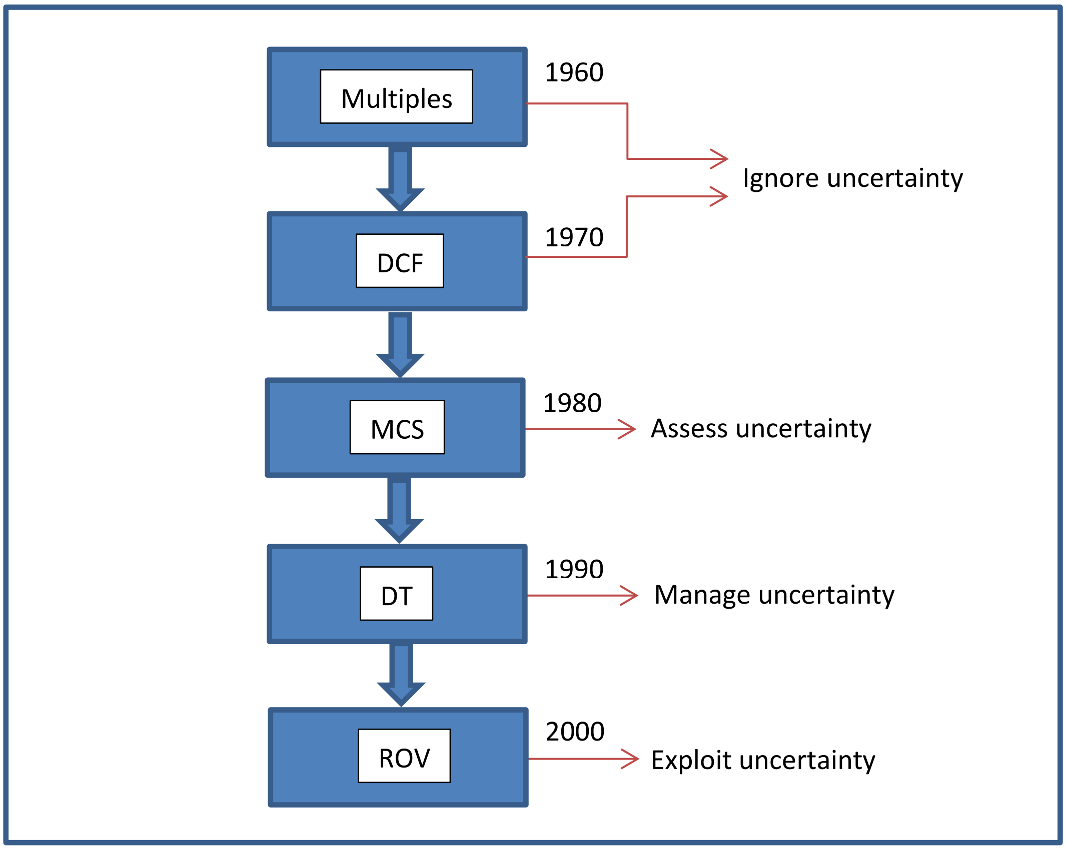

Figure 2-1 The historical trend of project evaluation (Source: Faiz, 2001).

2.5 Limitations of DCF method

Due to its simplicity, the traditional DCF analysis is the most widely accepted way of making practical investment decisions. Certainly for resource projects with healthy NPVs and stable cash flows, the DCF analysis will remain the dominant investment decision-making tool for the mining industry. However, the validity of this approach is undermined where there is a high degree of uncertainty in future cash flows and where management has the flexibility to respond to these uncertainties (Martinez and McKibben, 2010). Since various options are ignored, DCF analysis undervalues a project. While conceptually very simple, the DCF method has some major shortcomings.

2.5.1 Dealing with uncertainty

In DCF analysis, a manager has to predict a cash flow outcome for some period in the future. Under this scenario, a disproportionate amount of time and effort has to be spent on assessing the likely impact of a loss and how to alleviate this outcome. A further limitation of the DCF method involves the use of a risk adjusted discount rate which is not able to capture several sources of uncertainty in the project. Therefore, this method has the effect of ignoring the real sources of uncertainty, as there are many factors which can influence the value of a mining project including technical, financing, commodity price, exchange rate, geopolitical, geological and market elements (Martinez and McKibben, 2010). In a practical sense, it is also very difficult to determine an appropriate single dynamic WACC risk adjusted rate of return. Furthermore, the riskiness of the project may change over time depending on how uncertainties unfold and management has to react to these uncertainties (Smith and McCardle, 1998, 1999).

2.5.2 Dealing with flexibility

The traditional DCF method ignores the flexibility of management for valuing a project and, consequently, there may be some hidden or even invisible possibilities of additional value associated with the mining project (McKnight, 2000, Shafiee and Topal, 2008a). In reality, if external or internal factors changes in a mining project, mine managers have some flexibility such as delaying the project, accelerating or decelerating the mining rate, temporarily closing or abandoning the operation etc.

However, due to regulations or local issues, sometimes these options such as temporary closure of mine or changes in production rate may not be possible.

2.6 Real option valuation in natural resources investment

Options valuation was launched in 1973, when three famous economists Fischer Black, Myron Scholes and Robert Merton established a model, known as the Black-Scholes-Merton option pricing model (Black and Scholes, 1973; Merton, 1973). Cox and Ross (1976), and Cox et al. (1979) presented the fundamental economic principle for option pricing using arbitrage methods and provided a numerical procedure for valuing financial options. About four decades ago, Myers (1977) coined the term "real options" in a well-known paper observing that corporate investment opportunities can be viewed as call options on real assets. Since then, numerous researchers addressed investment opportunities under uncertainty by using a real options approach. In a seminal paper, Brennan and Schwartz (1985) solved the value and optimal production policy of a natural resource investment, contingent on the price of a commodity with a futures market. These authors presumed that there are no arbitrage opportunities available in trading in the real (the natural resource) and the financial (the futures market) assets. The production policy is defined by the schedules of prices at which production is delayed, closed or abandoned.

McDonald and Siegel (1985) demonstrated an essential lesson from microeconomics theory, which is that a project should be shut down if operating revenues are less than the total operating costs. This straightforward theory introduced the initial question of whether and when “a project should be opened or closed”. These authors also demonstrated the value of a project through waiting or holding off in the actual investment (McDonald and Siegel, 1986). Trigeorgis (1993) studied real options valuation and interactions with financial flexibility. In this study, the author provided an overview of the evaluation of real options, and illustrated the value of several types of operating options embedded in capital investments through simple examples.

Trigeorgis (1996) assisted to give a form in the field of real options by introducing new flexibilities in corporate resource allocation and in the evaluation of alternative investments. Trigeorgis and Mason (1987), and Trigeorgis (2000) also provided some examples of the use of real options valuation to several projects. These authors explained options to defer, to temporarily close and to expand several natural resource investment project decisions. In this study, it was also pointed out that decision making for natural resources projects significantly depends on the fluctuation of commodity prices. Paddock et al. (1988) extended the theory of financial options to the evaluation of an offshore petroleum lease. Ekern (1988) used a binomial option pricing method to evaluate a petroleum project. In this study, the author discussed several real options in the context of a petroleum project which include the flexibility value with compound development, the development and operations of satellite fields, and the incremental capacity of break-even values. Cortazar and Schwartz (1997) studied a real option model for valuing an undeveloped oil field. The study showed that a significant fraction of the oil field value could be provided due to the flexibility of delaying the investment and this flexibility value decreases when the oil price increases.

Real Option theory was presented gradually as a new method of appraisal for a variety of industries. Mardones (1993) employed the financial option theory as an application of Contingent Claims Analysis (CCA) for finding the value added by managerial flexibility in a copper project in Chile. Berger et al. (1996) studied the abandonment option of a firm through real options, and claimed that the abandonment option was valuable. Armstrong and Galli (1997) also reviewed the Brennan and Schwarz model and confirmed that option pricing is a promising approach in valuing mineral properties.

Modern Asset Pricing (MAP) is a valuation technique which specifies the effects of uncertainty on value at an earlier stage of the analysis. In this method, the economic analysis of risk is applied to the uncertain inputs to the project cash flow model rather than to model net cash flow. Laughton (1996) utilized the modern asset pricing (MAP) techniques to analyse the mining financial outlines. The author compared the MAP valuation techniques with the DCF valuation techniques and showed that the DCF method inadequately valued a long life mine compared to the valuation which is provided through MAP techniques. The MAP valuation techniques extended the valuation by examining the project under different combinations of management flexibility (Samis and Poulin, 1998). These authors evaluated copper and gold mines in two different papers and calculated the project value by DCF and ROV techniques and concluded that ROV was more flexible and better suited for mining projects in comparison with DCF. Salahor (1998) has described how MAP may be used for oil and gas project evaluations. Kelly (1998) considered a binomial lattice approach for valuing a mining property initial public offering (IPO) and provided an insight into the suitability of this method as an alternative to the DCF valuation technique. Zhu et al. (2012) evaluated an overseas oil investment project using a real options based simulation model. In this study, the MCS technique was used for evaluating different sized oil fields.

Researchers have also applied the principle of flexibility to a range of real mining operations. Slade (2001), Moel and Tufano. (2002), Colwell et al. (2003), and Kelley (2004), have individually valued managerial flexibility for 21 copper mines in Canada, 285 gold mines in the USA and 27 gold companies in Australia, respectively. These studies have concluded that flexibility in mine projects is significant, and it will lead mining project managers to open, to defer, or to shut down projects under different circumstances. Blais et al. (2005) integrated price risk for the evaluation of a multi-mineral mine through real option valuation. This study extended the application of real options in the minerals industry by developing a model through the Monte Carlo valuation method.

Davis and Samis (2006) provided evidence to the benefit of using real options to value and manage exploration exposures to a mining project. In this study, the authors demonstrated that traditional DCF analysis overlooks the nonlinearity of payoffs to exploration and thus undervalues exploration activity. However, the real options framework that captures the nonlinearity of payoffs, discloses that the exploration activity is positive value creating, and the project should proceed, contrary to the initial valuation assessment using a DCF analysis wherein a negative value was determined. Guj and Garzon (2007) presented evidence that in mining projects the MAP valuation technique is a valuable real option complement to the DCF modelling analysis. These two authors determined that MAP gives a minimum risk adjusted ‘floor’ value which represents a valuable supplement compared to DCF valuation alone.

Evidence in assessing mining and oil projects using ROV methods in comparison with DCF methods have also shown that the ‘shut down’ or temporary closure of projects can increase profitability (Dessureault et al., 2007; Wu et al., 2007). Hall and Nicholls (2007) also examined the difference between real options valuation and a DCF valuation of mining projects taking simplified examples through Binomial option pricing techniques, and concluded that ROV is a more appropriate method than DCF. Moreover, some research using hypothetical examples has determined that ROV is a suitable method for mining companies since operational flexibilities are thought to be an essential component of mining and oil project values (Dogbe et al., 2007; Shafiee and Topal, 2008a). Huang et al. (2014) integrated overconfidence into a real option based decision making model for metal resources mining projects. Haque et al. (2014) approximated the numerical value of a mining project using real option valuation (ROV) approaches under commodity price unpredictability, considering a hypothetical gold mine as a case study. The authors also estimated gold price volatility from the historical data, and established that price volatility has a significant impact on mining project values.

Zhang et al. (2014) studied the real option value of a mining operation using mean–reverting commodity prices. These authors determined the optimal prices for exercising the mining options, which translates to being the price at which mining production commences. However, these authors did not determine the numerically explicit project values through continuous time modelling. The ROV method is also applied in mine planning and production scheduling, and it has been shown to be a significant benefit in the implementation of real options in the short-term mine planning phase (Li and Knights, 2009). Ajak and Topal (2015) implemented real options in the case of flexible decision making at a mine operation level through the binomial decision tree. These authors showed that project values increased when operational flexibility was included in the mine design.

Furthermore, technical and geological risks, and managerial operational flexibilities have influence on the value of a mining project (e.g. Musingwini et al. 2007, Dimitrakopoulos et al., 2007, Sabour and Wood 2009, Dimitrakopoulos and Grieco 2009, Groeneveld and Topal, 2011, Ajak and Topal, 2015, Salama et al., 2015). However, there are more sophisticated techniques available including stochastic ore body modelling to address several types of technical and geological risks. The ROV and MCS are also applied in integrating uncertainty and operation flexibility into open pit mine design selection (Sabour and Dimitrakopoulos, 2011). Groeneveld et al. (2012) compared the robust, flexible and operational mine design strategies using a combination of MCS and MIP (mixed integer programming) techniques, and extended the application of real options in design theory to mining. These authors revealed that a fully flexible design approach produces the best project value whereas the traditional method which does not consider the flexibility and the uncertainty was the worst performing approach. As a result, mining companies are increasingly interested in flexibility in their projects, and have changed their methods accordingly in evaluating projects. However, based on the survey report of R2 Mining/CostMine (R2 Mining/CostMine, September 11, 2012), it has been shown that more than 50% of mining projects are still evaluated using the simple NPV technique which is used in DCF method as central tool, and just less than 5% of mining companies have utilised real options valuation methods to evaluate Australian mining projects.

Although some research has been done to implement real options theory in mining projects through continuous time stochastic models, most of the studies were limited to being academic and theoretical work (Brenann and Schwartz, 1985; Cortazar and Casassus, 1998; Colwell et al., 2003; Cortazar et al., 2001). Some authors formulated complicated models, but due to the degree of difficulty there is no simplistic solution for the numerical approximation or valuation of the mining projects. Mathematical complexity attached to project uncertainties as well as the difficulty of solving the complex models do not make the real options valuation methods more useful to practitioners or to the mining industry at large.

Furthermore, there is still limited literature available discussing the valuation of mining projects considering more than one uncertainty using continuous time stochastic models, and the commensurate numerical solution of the resulting partial differential equations (PDEs). Through continuous time stochastic modelling, it is conveniently possible to use a hedging strategy for minimizing risk and maximizing profits, and a mining company should use this for mitigating its risk by locking in the commodity price and exchange rate. Cortazar et al. (2001) explored optimal investment under price and geological uncertainties. These authors assumed a zero drift stochastic process in the case of geological uncertainty. Lima and Suslick (2006) estimated the project volatility only, considering price and operating cost uncertainties but did not determine the value of the project. Castillo and Dimitrakopoulos (2014) studied the joint effect of commodity price and geological uncertainty. The authors analysed the efficiency of traditional evaluation methods in assessing the potential performance of a mining operation under uncertain geological and commodity price scenarios, and provided an alternative real options based method that included the option of expanding or contracting the initial ultimate pit limit, subject to these uncertainties.

So far there is no study which considers multiple uncertainties with solutions in ROV through continuous time modelling using a hedging strategy. Therefore, in this thesis, the ROV method will be studied for approximating project values incorporating a hedging strategy and futures contract through continuous time stochastic models. The aim of this research is focussed on the investigation of the long-term relationship between the mineral commodity prices and exchange rates, and thereafter to develop a technique to approximate the project values considering commodity price and exchange rate uncertainties as the main stochastic variables. No research that undertakes a mining project evaluation considering the joint effects of commodity price and exchange rate uncertainties has been found, even though these are two vital factors for evaluating mining project values. The fluctuation of the commodity price and its associated volatility is a crucial factor for the evaluation of mining projects. The exchange rate uncertainty is also an imperative valuation factor for mining companies, as the exchange rate not only reflects the local currency sales of a commodity that is priced in an offshore currency, but also impacts and associates with mining costs such as capital expenditure (CAPEX), operational expenses (OPEX) etc. Therefore, in order to evaluate the economic feasibility of a mining project correctly, the exchange rate volatility should be modelled appropriately and should be included in the evaluation process.

2.7 Long-term relationship between mineral commodity prices and exchange rates

The breakdown of the Bretton Woods system on 15 August, 1971, and the commensurate adoption of freely floating exchange rates, has motivated the growing curiosity to examine the relationship between exchange rate movements and economic parameters. Commodity price is one of the vital economic parameters that directly impacts on currency exchange rate movements in addition to a country's overall economy. Most notably, countries including Australia, Canada, New Zealand, Chile and South Africa are dependent on commodity exports for significant levels of foreign income. The world prices of commodities seem to have a robust and systematic relationship with a number of currencies which are known as commodity currencies. Chen and Rogoff (2003) studied these commodity currencies using Newey-west heteroscedasticity, linear regression, and Chow and Hansen tests. The study noted that there is a close association between the price of specific primary commodities such as gold, oil and agricultural products and the commodity currencies. The value of a commodity currency typically increases or decreases depending on the value of the country's primary commodity exports.

According to the Australian Government's Department of Foreign Affairs and Trade (AGDFAT) in 2013-2014, Australia earned approximately AU$151.18 billion through exporting minerals, and the main contributors were iron ore, gold and coal. Commodity prices affect the strength and weakness of the AUD including the AUD/USD exchange rate as a direct consequence of Australia's dependency on minerals and, to a lesser extent, farming exports. As a consequence, when there is a mining boom, Australia's economy improves and, conversely, when a downturn in the resources sector occurs, Australia's economy is negatively impacted. Therefore, the typical return on investment (ROI) in the mining and minerals industry is heavily biased towards commodity prices and commensurate exchange rate volatilities. Hence, it is imperative to analyse whether evidence exists to prove that an empirical relationship between mining commodity prices and exchange rate movements holds and, if so, to find the strength of their association i.e. the coefficient of correlation. However, when the mining industry booms, some other industries may suffer such as Australian tourism industry due to currency appreciation of AUD. Furthermore, the commodities boom can drain significant skills away from other sectors and other States in Australia.

Cashin et al. (2004) studied commodity currencies and the real exchange rate and found evidence in favour of the long–run comovement of national real exchange rates and real commodity-export prices. Zhou (1995) examined the response of real exchange rates due to different economic shocks and found that the variation of oil prices plays a key role in real exchange rate movements. Ihrig et al. (2006) examined the exchange rate pass-through in G-7 countries. These authors assessed that 10 percent depreciation in the local currency could increase import prices nearly 7 percent on average across these countries in the late 1970s and 1980s. They also found that a decline in import-price and consumer- price pass–through occurred for almost all of the G-7 countries. Sekine (2006) studied the exchange rate pass-through using a time varying parameter with a stochastic volatility model for some industrial countries. This study categorized the exchange rate pass-through in two stages, and revealed that in both stages the exchange rate pass-through has declined over time for the sample countries. Otani et al. (2006) re-examined the decline in exchange rate pass-through evidence from Japan’s import prices. This study revealed that the decline in the exchange rate pass-through to Japan’s import prices excluding primary commodities is mostly attributable to the decrease in the exchange rate pass-through in each product. Chen and Chen (2007) investigated oil prices and real exchange rates. These authors found that real oil prices may have been the dominant source for the movement of real exchange rates, and there exists a cointegrating relationship between real oil prices and real exchange rates.

Sari et al. (2010) studied the dynamics of oil and precious metals prices, and the US dollar/euro exchange rate. The study did not apparently find long-run equilibrium relationships between those spot price returns and changes in the exchange rate. In the short-run, however, there is evidence that spot precious metal prices and the exchange rate may be closely linked when shocks occur. Lizardo and Mollick (2010) studied the oil price fluctuations and the US dollar exchange rates by adding oil prices to the monetary model of exchange rates. It was found that an increase in real oil prices lead to a remarkable depreciation of the US dollar against the net oil exporter currencies, such as Canada, Mexico and Russia. Han et al. (2012) studied an interval method for exploring the relationship between the gold price and Australian dollar against the US dollar exchange rate taking data over the period 2002 to 2008. They compared the interval and point methods and revealed that the difference between the ordinary least square (OLS) and indirect least square (ILS) estimates becomes larger while moving from weekly to quarterly data.

Beckman and Czudaj (2013) investigated the relationship between oil prices and effective dollar exchange rates through a multivariate Markov-switching vector error correction model. The findings show that causality runs from nominal exchange rates to nominal oil prices, but not nominal oil prices to nominal exchange rates. Apergis and Papoulakos (2013) investigated the relationship in terms of means and conditional volatilities between the Australian dollar and gold prices considering data from 2000-2011.These authors found the evidence in favour of a cointegration relationship between the Australian dollar and gold prices. Pustov et al. (2013) studied and applied the approaches to forecast the long-term iron ore price modelling considering marginal costs versus investment price. The study revealed that this price was crucial for the valuation of investments in greenfield iron ore projects with an horizon of more than five years. Ma (2013) examined the impact of changes in the forward pricing mechanism on the volatility of iron ore spot prices considering prices from October 2008 to September 2012. The results of this study showed that the implementation of the quarterly pricing mechanism alleviated the volatility of the iron ore spot price.

Ndlovu et al. (2014) investigated the link between the nominal exchange rate of the South African Rand and commodity prices. Their findings confirm that there is a direct relationship between commodity price changes and exchange rate changes in South Africa. However, the strength of the relationship is significantly weaker than in other commodity exporting countries. Reboredo and Rivera-Castro (2014) investigated gold’s hedging and value-preserving properties when the US dollar depreciates. The study investigated gold’s ability to reduce downside risk and to hedge currency risk for different kinds of portfolios. Warell (2014) studied the effect of a change in pricing regime on iron ore prices considering monthly data over the period January 2003 to August 2012. The study showed that the change in pricing regime did not have a significant impact on iron ore prices, but gross domestic product (GDP) growth in China, iron ore prices and freight rates are found to be cointegrated when regressed with a market dummy variable. Reboredo and Rivera-Castro (2014a) studied the hedging and downside risk benefits using gold for currency risk management at different investment horizons. In this study, it was observed that a positive dependence between gold and US dollar devaluation against a wide set of currencies existed.

Regrettably, limited literature is available for the investigation of long term relationship surrounding the mineral commodity prices and the exchange rate and their strength of association or correlation. Therefore, in this research the relationship between mineral commodity prices and exchange rates have been investigated, and as a case study, the gold prices and AUD/USD exchange rates, and iron ore prices and AUD/USD exchange rates have been considered. The volatility of commodity prices and the AUD/USD exchange rates and their strength of association may have a significant effect on the evaluation of mining project value using real option valuation method.

2.8 Summary and discussion

This chapter provided a detailed review of the relevant literature in the study area. Firstly, it gives the overview of some preliminary aspects such as the discounted cash flow (DCF) technique, history of option pricing theory, and several types of options. It also includes similarities and differences between financial and real options, hedging strategy and futures contracts. Thereafter, project evaluations methodologies are discussed. Then, the literature of ROV methods in relevant natural resources investments are reviewed, and the relationship between mining commodity price and exchange rate are studied. Based on the literature, earlier research studies that used discounted cash flow (DCF) and ROV methods for mining project evaluations have some drawbacks. For instance: DCF evaluation methods overlook several sources of uncertainties, such as commodity price, exchange rate, costs, reserves etc. In these methods managerial operational flexibilities are also ignored. In general, it is also very difficult to determine an appropriate single dynamic risk adjusted rate of return for evaluating project values using the DCF method. For implementing ROV methods, past studies mainly used the Binomial Option Pricing (BOP) technique. However, one of the limitations of the BOP method is the lack of using futures contracts and the ever-useful hedging strategy for mining project evaluation. The other main problem with using BOP is that in practice, it becomes computationally intensive as the number of scenarios increases exponentially. Finally, sometimes it is very problematic to combine more than one uncertainty in the evaluation processes.

Although prior studies developed some complicated theoretical models in continuous time processes, there is no up to date solution to find the project values of these models, especially in the case of more than one uncertainty. Therefore; even though prior studies claim that the ROV model is better than DCF methods, there is still no noticeable replacement to using the DCF method in industry. One of the major limitations of prior studies was ignoring the uncertainty of exchange rates in ROV frameworks, even though exchange rates have significant impacts on the estimation of project values, and there is also a relationship between mining commodity prices and exchange rates. As a consequence, this study aims to address those problems and implement a new ROV framework through developing new PDEs by incorporating new parameters for approximating mining project values.

In addition, there was limited literature available for the investigation of the relationship surrounding mining commodity prices and the exchange rate and their strength of association or correlation. Although prior studies investigated the association between gold prices and the Australian dollar, this was limited to shorter periods of data which reduced the accuracy of results. Furthermore, other than gold, there is no literature on mining commodities including iron ore, coal etc. However, to determine/approximate mining project values considering multiple uncertainties, it is necessary to know the relationship between economic variables. As some information surrounding these relationships is useful for choosing effective hedging strategies. Therefore, this research will also investigate the relationship between the mining commodity and the exchange rate, and incorporate this correlation parameter and hedging strategy with a new ROV framework through developing the new PDE.

CHAPTER 3

PROJECT EVALUATION UNDER SINGLE UNCERTAINTY-COMMODITY PRICE

3.1 Introduction

As a value-influencing factor, the locally-stated commodity price is vital for mining companies. The price volatility is a key parameter for mining project evaluation and investment decision making. The conventional Discounted Cash Flow (DCF) methods of valuation are generally used for mining project evaluations, however, due to the commodity price uncertainty and operational flexibilities, it is difficult and often inappropriate, to determine mining project values through traditional DCF methods alone. Declining commodity prices have an adverse effect on the mining industry. Therefore, in order to more precisely assess the economic viability of a mining project, the commodity price and its inherent volatility should be modelled appropriately and incorporated into the evaluation process. As a consequence, researchers and practitioners continue to develop and introduce Real Options Valuation (ROV) methods for mining project evaluations under commodity price uncertainty, incorporating continuous time stochastic models. However, most of the models that have been developed to-date are generally limited to theoretical research and academia and consequently, the application of ROV methods is often not used in project evaluations. Analytical and numerical solutions derived through the applications of ROV methods are rarely found in practice due to the complexity associated with solving the partial differential equations (PDEs) which depend on several conditions and parameters. Therefore, the greatest challenge to ROV modelling is to find numerically explicit project values. This chapter contributes towards the further development of known theoretical work, and enhances an approach to approximating explicit numerical project values considering a continuous time stochastic model.

To ensure the project is profitable and to reduce commodity price uncertainty, delta hedging and futures contracts have been used as options for deriving the PDE. Delta hedging is an options strategy that aims to reduce (hedge) the risk associated with price movements in the underlying asset by offsetting a long and a short position. For instance, a long call position can be delta hedged by shorting the underlying stock. Furthermore, as a part of the development, new parameters have been incorporated within the PDE. This new PDE has been utilised to approximate the numerical values of mining projects considering hypothetical gold mines as case studies. The explicit finite difference method (FDM) and MatLab software have been used and implemented to solve this PDE, and to determine the numerical project values considering the available options associated with a mining project. In addition, commodity price volatility has been determined from the historical data to approximate the project values. This chapter has shown two different methods for approximating project values, firstly for reserve dependency i.e. finite reserve, and secondly for time dependency i.e. infinite reserve.

3.2. Real options and continuous time stochastic model for mining projects evaluation

The application of real options valuation concepts for valuing real assets is a growing area in the theory and practice of finance and economics analysis. Over the last few years, many studies have presented the ROV method as a promising technique for valuing natural resources investments under conditions of uncertainty. Although DCF methods are broadly used for project evaluations, several studies have found that managers often do not necessarily follow the forecasts reflected in a DCF method (Clowel et. al., 2003; Hayes and Garvin, 1982; Hayes and Abernathy, 1980). Almost one–and-a-half decades later, researchers are beginning to understand why the DCF method and its variants do not properly explore the explicit value of an investment under uncertainty as they typically fail to take into account management flexibilities and financial options associated with reducing risks and optimizing profits and investment returns.

Due to the extensive growth of financial markets and embedded managerial options, the flexibilities around taking decisions for investments largely depend on market conditions. The literature on real options in the mining project investment arena is not new. The idea of using real options frameworks in the mining industry was first introduced in a seminal paper by Brennan and Schwartz (Brennan and Schwartz, 1985). Although gradually considerable work has been done to implement this real options theory in mining projects using continuous time stochastic models, most of the studies were limited to being academic and theoretical work due mainly to the mathematical complexity for the attachment of project uncertainties and the difficulty of solving the complex models. As a consequence, the numerical simulation concept for determining project values through the ROV method is not readily available to practitioners and the numerical results of mining project values have typically been overlooked and probably understated. An additional reason for the method's limited use can be the mathematical challenge in solving higher order and dimensional PDEs due to the inclusion of several parameters and conditions. Moreover, to find a mining project's value is a challenging task compared to other industries due to extensive uncertainties and the finite project life attached to that mining project. Consequently, in this chapter, ROV methods will be utilised to investigate the numerically explicit mining project value under commodity price uncertainty which is one of the most important factors for mining investments.

3.3 Project evaluation considering reserve dependence

Mining project values are typically determined while considering the total reserve being a determinant of its value i.e. project values depend on the total reserve. As the commodity price moves randomly, a Geometric Brownian Motion (GBM) model was considered to represent the price movement of the commodity. GBM is commonly assumed to be an appropriate choice to describe the behaviour of stock prices and commodity prices. (Black and Scholes, 1973; Cox et al., 1979, Brennan, and Schwartz, 1985). Furthermore, GBM has also been largely used in corporates and natural resources project valuation using real options analysis (Trigeorgis, 1993, Brennan and Schwartz, 1985). Consequently a GBM model had been considered here to represent the price movement of the commodity. To reduce mining project risk and to make investment decisions for the mining project, several real and financial options have been discussed in this chapter and implemented, which is not generally possible in traditional DCF methods or in Binomial option pricing (Binomial lattice/tree model) methods. In order to calculate numerical values for a gold mining project, a higher ordered PDE (Black- Scholes equation type-PDE*)* will be constructed using the GBM model incorporating financial market tools such as hedging and futures contracts. This PDE will then be solved numerically through FDM and MatLab tools to obtain values for that gold project.

3.3.1 The model for commodity price movements

Mining project developments and investments have the inherent managerial flexibility of selecting options for opening, temporarily closing, postponing or abandoning the project arising mainly from the uncertainty around commodity prices. While a mine is temporary closed, it will typically incur maintenance costs during the closing period. It is assumed that the dynamics of the commodity price (i.e. output price), P will be according to the following stochastic differential equation (Cortazar et. al., 2001; Lima and Suslick, 2006):

$$\frac{dP}{P} = (r - \delta)dt + \sigma\, dw \tag{3-1}$$

where,
P is the spot unit price of the commodity,
r is the risk free rate of interest,
δ is the mean convenience yield on holding one unit of that commodity,
σ is the volatility of returns of P,
dw is the Wiener increments of the Geometric Brownian Motion of P.

The value of the mine is defined as $V = V(P, Q, i, \phi)$. Here, Q is the total reserve, i is the indicator variable which takes the value one if the mine is open and zero if it is closed, ϕ describes the operating policy such as opening, temporarily closing, postponing and abandoning the mine.

3.3.2 Derivation of the partial differential equation (PDE) using the stochastic differential equation (SDE)

To find the numerical project value, a PDE will be derived through equation (3-1), with the help of commodity futures market and hedging strategies. If the commodity futures market is arbitrage free, being that is there is no arbitrage opportunity, then management has the flexibility to enter a long position for investment in the mining project and a short position in the futures contracts for hedging commodity price risk

(Brennan and Schwartz, 1985; Bellalah 2001; Colwell et al., 2003; Cortazar et al., 1997; Dixit and Pindyck,1994).

The value of the mine can be defined as: $V = V\left(P,Q\right)$. Now, consider the mining company adopting the options as a long position for investment in the mining project $V = V\left(P,Q\right)$, and a short position in $\frac{\partial V}{\partial P}$ units of output (commodity). In dt interval of time, this project leads to a cash flow, $q\left(P-C\right)\left(1-G\right)dt - \delta\frac{\partial V}{\partial P}P\,dt$, where q is the production rate , C is the total cost (CAPEX, OPEX, working capital, etc.) per unit of commodity and G is the total tax.

Therefore, the total return on the portfolio is:

$$dV \; - \frac{\partial V}{\partial P}dP + q\left(P-C\right)(1-G)\,dt - \delta\frac{\partial V}{\partial P}P\,dt \tag{3-2}$$

Applying Ito's Lemma in, $V \; = V\left(P,Q\right)$

$$dV \; = \frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial Q}dQ + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 \tag{3-3}$$

Substituting the value of dV from equation (3-3) into equation (3-2), the total return on this portfolio becomes

$$\frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial Q}dQ + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 - \frac{\partial V}{\partial P}dP + q\,\left(P-C\right)(1-G)dt - \delta\frac{\partial V}{\partial P}P\,dt \tag{3-4}$$

According to Bellalah (2001) and Cortazar et al. (1997), a mining company with a long position in an investment in a mining project and a short position in commodity futures contracts can hedge its risk and should earn a return equal to at least the risk free interest rate plus the country risk premium associated with the country where the mining project is situated. The country risk premium is denoted by λ_C. Therefore;

$$\frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial Q}dQ + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 - \frac{\partial V}{\partial P}dP + q\left(P-C\right)\left(1-G\right)dt - \delta\frac{\partial V}{\partial P}P\,dt =$$
$$\left(r+\lambda_C\right)V\,dt - r\frac{\partial V}{\partial P}P\,dt \tag{3-5}$$

When the mine is operated at a rate q, then the reserve will change accordingly, $dQ = -q\,dt$, and equation (3-5) becomes the following PDE:

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2} - q\frac{\partial V}{\partial Q} + (r-\delta)P\frac{\partial V}{\partial P} - (r+\lambda_C)V + q(P-C)(1-G) = 0 \qquad (3\text{-}6)$$

Subject to the boundary conditions:

$$V\,(P,\,0) = 0$$

This means that when the reserves are exhausted, the value of the mine is zero, and

$$V\,(0,\,Q) = 0$$

This means that if the price of the commodity is zero, then the value of the mine is zero. Furthermore, also assume that $\lim_{P\to\infty}\frac{V(P,Q)}{P} \to 1$ i.e. $\frac{\partial^2 V}{\partial P^2}(\infty, Q) = 0$.

Brennan and Schwartz (1985) mentioned that there is no analytical solution for this type of problem i.e. PDE (3-6). Therefore, the numerical solutions of the PDE (3-6) will be approximated through the finite difference method (FDM) using MatLab software, which will commensurately provide the numerical values of the mining project.

3.4. Determination of commodity price historical volatility

To find the numerical values of a mining project, commodity price volatility is required, as it is one of the important input parameters to solve the PDE (3-6). Therefore, the commodity price volatility is needed to be determined from the historical data.

3.4.1 Historical volatility

In economics and finance, the concept of standard deviation is comparatively diverse and is generally applied to the annual rate of return of an investment to compute the investment's volatility. Standard deviation is also known as historical volatility which is a statistical quantity that highlights the unpredictability of the asset value or price over time. It is the standard deviation of an asset’s historical returns. Commodity price volatility can be determined from historical data through the log return method. Let P_t denote the price (closed price/adjusted price) of a commodity at the end of day t and P_{t-1} denote the price (closed price/adjusted price) of a commodity at the end of day $t-1$. Assuming no dividend is paid, then the log return on an investment in a commodity between days $t-1$ and t is defined as:

$$r_t = \ln\left(\frac{P_t}{p_{t-1}}\right)$$

3.4.2 Historical volatility estimation

Black and Scholes (1973) indicated that the parameter σ^2 is the variance rate of return on the stock prices. Black and Scholes considered this as a known parameter which is constant throughout the life of an option. In their paper prior to a seminal one, Black and Scholes provided additional perception into the variance rate of return and assessed the instantaneous variance from the historical series of stock prices. Thereafter, volatility was defined by them as the amount of irregularity in the returns of the underlying assets. Black and Scholes determined what today is known as the historical volatility, which is used as a proxy for the expected volatility in the future.

3.4.3 Gold prices historical volatility

As an example, gold is considered here in the analysis for determining the gold mining project values, so the estimation of gold's historical volatility is based on the calculation of the standard deviation of gold's return over time. The steps and the formulae for the estimation of the gold price's historical volatility follow:

1. Taking the log returns of gold prices, relating today's gold price (P_t) to yesterday's gold price (P_{t-1}). This is $r_t = \ln\left(\frac{P_t}{p_{t-1}}\right)$, the continuously compounded return.
2. Calculating the variance of the log returns based on the formula $\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(r_t - \bar{r}\right)^2$

   Where $\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$
3. Taking the square root of the variance to get the standard deviation, and converting it to an annualised volatility. The daily historical volatility is given by

$$\sigma = \sqrt{\left(\frac{1}{n-1}\sum_{t=1}^{n}\left(r_t - \bar{r}\right)^2\right)}.$$

To compare volatilities for different interval lengths, it is generally articulated in annual terms by the formula, $\sigma_{an} = \sigma * \sqrt{h}$, where h is the number of intervals per annum. If daily data are used, the interval is one trading day and h=252, if the interval is a week, h=52 and for monthly data, h=12.

Figure 3-1 demonstrates gold prices in nominal money terms (monthly average price data have been used) from October 1996 to December 2014. It is apparent that the gold price has randomly changed and has escalated between year 2007 and 2014. In January, 1997 the monthly average gold price was US$354.11/oz, since then, it has slightly decreased in each month until November, 2003. After that the average gold price has increased gradually until May 2006. Due to the global financial crisis (GFC) in 2008, the gold price fluctuated significantly from January 2008 to December 2008.

In January 2008, the average gold price was US$889.59/oz, in April 2008 the price was US$909.70/oz, and in November 2008 the price was US$760.86/oz. However, the figure shows that the maximum average gold price was US$1,755.81/oz in August 2011 during the period of 1996-2014. In October 2012, the average gold price was US$1,747.01/oz.

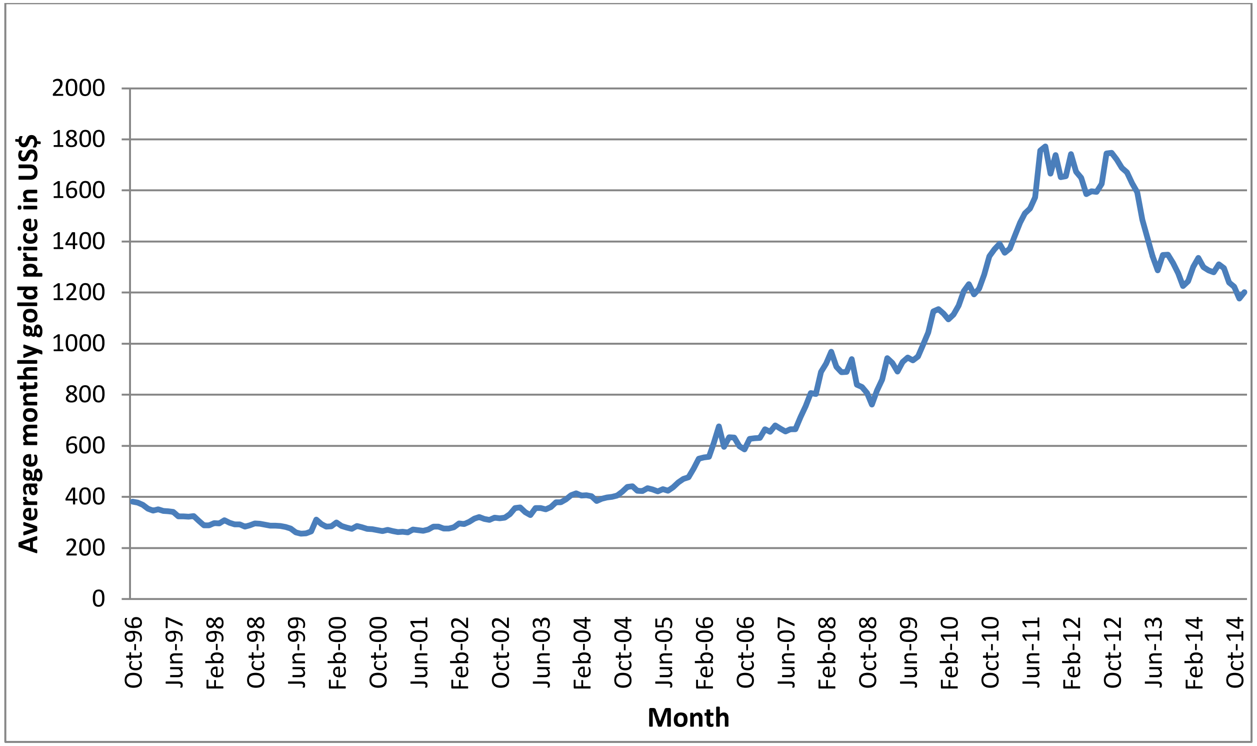

Figure 3-1 Gold price in US dollar per ounce from October 1996 to December 2014 (monthly average data have been used).Data source: kitcogold.com. http://www.kitco.com/

Therefore, the past observation shows that over the last 15 years, the gold price has increased noticeably before early 2013. Thereafter, the gold price retraced noticeably, being during the period beyond May 01, 2013. As a consequence, in May 2014 the average gold price was US$1,287.52/oz and in December 2014 the average gold price was only US$1202.29/oz. At present, the average gold price continues to decrease.

The gold price (in nominal money terms) volatility has been determined from historical data using Microsoft Excel through the log return method of analysis. Figure 3-2 represents the gold price volatilities over the period from 1997 to 2014. Gold price data have been collected over that period, in order to use the immediate past 16 to 17 years historical data to get an overview of the historical gold price volatility. Moreover, during this period and specifically between the year 2007 and mid - 2013, gold prices have been fluctuating significantly. Furthermore, there was a significant global recession in 2008, a consequence of the global financial crisis, which impacted the gold market and almost every other market. In 1997, the annual historical gold price volatility was 10.99%. This volatility gradually increased until the year 1999, reaching 15.46%.

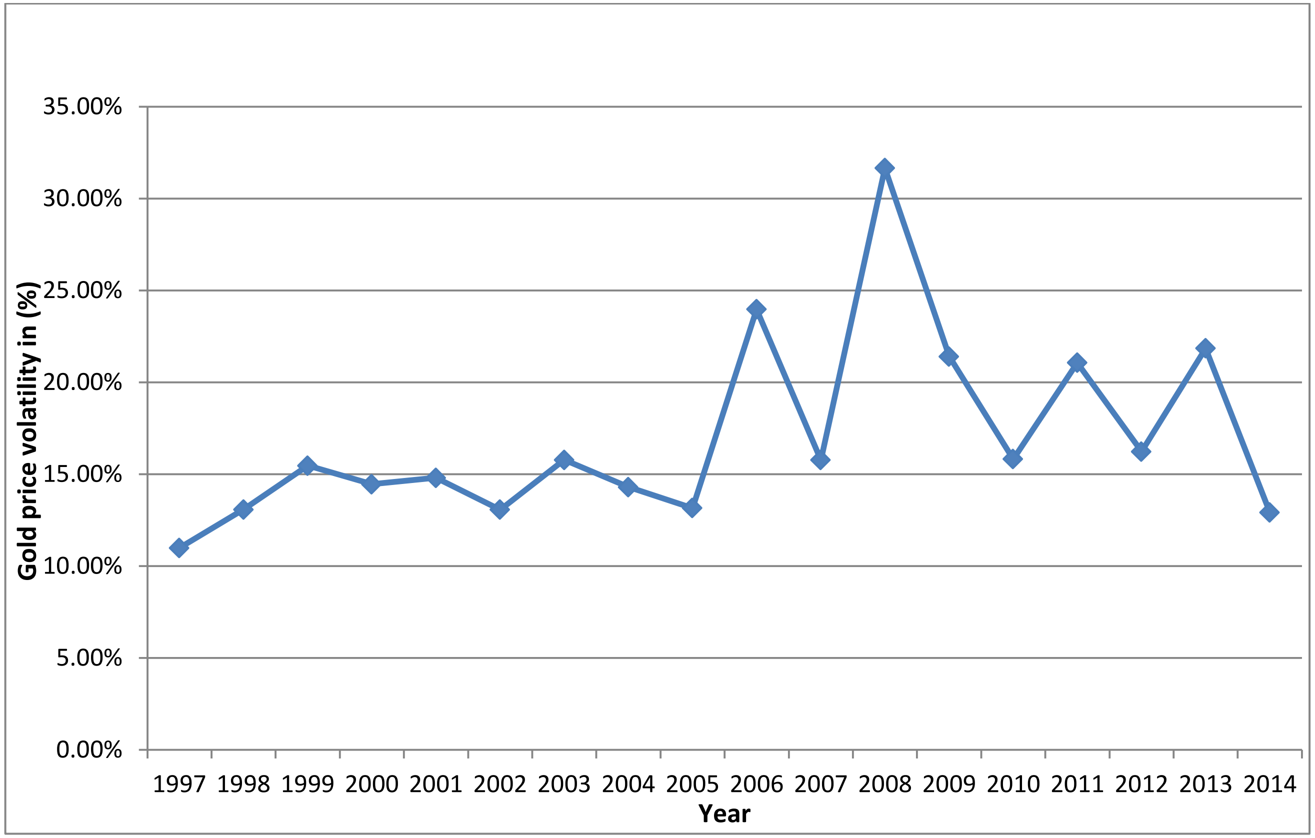

Figure 3-2 Annual historical gold price volatility from January 1997 to December 2014 (daily data have been used). Data source: kitcogold.com. http://www.kitco.com/

The volatility fluctuated until 2005. Thereafter, there was a dramatic increase in volatility in 2006 to 23.97%, and the next year in 2007 the volatility declined rapidly again. During the period 2005 to 2014, there were remarkable fluctuations in the volatility of the gold price. The gold price volatility can be explained by the underlying drivers of gold prices during the period January 1997 to December 2014. The average gold price volatility was 16.99% during this period and the standard deviation was 5.13% over the mean gold price volatility. Over the same period, the minimum gold price volatility is shown as 10.99% and the maximum gold price volatility is shown as 31.65%. Therefore, the maximum peak volatility is around 31.65% in 2008, and it occurred as a direct consequence of the turmoil in most international markets due to the GFC and commensurate global recession during this period (Shafiee and Topal, 2010).

3.5 Available options for a mining project

In real option valuation methods, management can choose to exercise different real options during the life of the mine for different scenarios, as the mining project involves many uncertainties while an investment in a mining project is mostly irreversible. Therefore, management should choose the following real options to invest in a mining project for maximizing profits or alleviating losses. Several flexibilities and options have been discussed and numerical results are highlighted for different scenarios.

3.5.1 Delay or Deferral option

Due to the commodity price depreciation, management can defer any initiation of the project which has not started up yet or delay the commencement of the mine operation if it is a developed mine. In this option, management can observe whether the commodity price goes up or down, the currency depreciates or appreciates, etc. A delay/deferral option gives the opportunity to management to wait until circumstances become more advantageous. When undertaking the project immediately has a negative cash flow or extremely low values, then sometimes the opportunity to wait and invest later would generate a positive worth (cash flow). The option to defer (wait) is analogous to exercising a call option.

3.5.2 Temporary closure option

If the mine is already operational and the commodity price declines noticeably, then management can discontinue mine production temporarily and incur maintaining costs during the closure period. At a future point, when the commodity price improves, management can then reopen the mine. For this, maintaining and reopening costs need to be calculated and added to the other costs notably, CAPEX and OPEX for determining the total cost per unit of commodity.

3.5.3 Accelerate or decelerate and expansion options for mine operation

Acceleration or deceleration of mine production rates has an impact on mining project values. If the commodity price increases markedly (or the currency depreciates against the US$), then management can accelerate the production rate which helps to recognize a higher project value and investment returns. To increase the production rate, management may need to invest additional capital during this time for obtaining the necessary surplus capacity, but the overall mining costs may be decreased due to economies of scale and a commensurate reduction in the life of the mine. Conversely, when the commodity price decreases strikingly (or the currency appreciates) then management can decelerate the production rate which will give lower project values and returns at that moment but it may increase the life of mine for future production. If the commodity price is higher than it is determined beforehand, then management can choose the expansion option and increase the mine production rate to lock in higher project values and returns. In this option, some part of the mineral resource which was not economical to exploit previously will now be economical and the life of mine may be increased exploiting that newly economic portion before final mine closure occurs. In addition, this option can also be managed through change in cut-off grades. The relationship between production rate and cut-off grade strategy can be determined by the more rigorous method presented by Lane (1998, 1997). Lane’s approach considers the constraints placed on the operation by the mine, mill and refinery or market.

3.5.4 Abandon option

For the downside protection, the management has the option to abandon the project for its salvage value or the value inherent in its best alternative (switch) use. When the maximum project value is extremely low or approaches 0, or the total cash flows of the project are negative, then management can wait to start operating of the project or temporarily close the mining operation if the mine operation is continuing, and observe the scenario whether the commodity price goes up or down, currency depreciates or appreciates, etc. After waiting, if the entire perceived outcome remains unfavorable and the total cash flow of this project is still negative, then management can adopt the abandon option and leave the project permanently. When the commodity price decreases over a protracted period, management should take the abandon option and sell the capital equipment (salvage assets) or the whole project. The abandon option is tantamount to exercising an American put option.

3.6. Numerical Analysis and results

The procedure of discretizing the PDE (3-6) has been demonstrated through the finite difference method (FDM), and considered a case study of a hypothetical gold mine for the determination of numerical project values.

3.6.1 Discretization of PDE with Finite Difference Method (FDM)

The explicit finite difference method (FDM) has been employed and discretized the PDE (3-6) as follows:

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2} - q\frac{\partial V}{\partial Q} + (r-\delta)P\frac{\partial V}{\partial P} - (r+\lambda_C)V + q(P-C)(1-G) = 0$$

$$\Rightarrow \frac{\partial V}{\partial Q} - \frac{\sigma^2}{2q}P^2\frac{\partial^2 V}{\partial P^2} - \frac{1}{q}(r-\delta)P\frac{\partial V}{\partial P} + \frac{1}{q}(r+\lambda_C)V - (P-C)(1-G) = 0$$

$$\Rightarrow \frac{V_{n,\,j+1} - V_{n,\,j}}{\Delta Q} - \frac{\sigma^2}{2q}(n\Delta P)^2\left(\frac{V_{n+1,\,j} - 2\,V_{n,\,j} + V_{n-1,\,j}}{(\Delta P)^2}\right) - \frac{1}{q}(r-\delta)\,n\Delta P\left(\frac{V_{n+1,\,j} - V_{n-1,\,j}}{2\Delta P}\right)$$

$$+\frac{1}{q}(r+\lambda_C)\,V_{n,\,j} - (n\Delta P - C)(1-G) = 0$$

$$\Rightarrow V_{n,\,j+1} = V_{n,\,j} + \frac{\sigma^2}{2q} n^2 \left(V_{n+1,\,j} - 2\,V_{n,\,j} + V_{n-1,\,j}\right) \Delta Q + \frac{n}{2q}(r-\delta)\left(V_{n+1,\,j} - V_{n-1,\,j}\right)\Delta Q$$

$$-\frac{1}{q}(r+\lambda_C)\,\Delta Q\; V_{n,\,j} + (n\Delta P - C)(1-G)\Delta Q$$

$$\Rightarrow V_{n,\,j+1} = \left(\frac{1}{2}\frac{\sigma^2 n^2}{q}\Delta Q - \frac{n}{2q}(r-\delta)\,\Delta Q\right)V_{n-1,\,j} + \left(1 - \frac{\sigma^2 n^2}{q}\Delta Q - \frac{(r+\lambda_C)}{q}\Delta Q\right)V_{n,\,j} +$$

$$\left(\frac{1}{2}\frac{\sigma^2 n^2}{q}\Delta Q + \frac{n}{2q}(r-\delta)\Delta Q\right)V_{n+1,\,j} + (n\Delta P - C)(1-G)\Delta Q$$

We find the solution:

$$V_{n,\,j+1} = a_n\, V_{n-1,\,j} + b_n\, V_{n,\,j} + c_n\, V_{n+1,\,j} + d_n \qquad (3\text{-}7)$$

Where

$$a_n = \frac{1}{2}\frac{\Delta Q}{q}\left(n^2\sigma^2 - n\,(r-\delta)\right)$$

$$b_n = 1 - \frac{\Delta Q}{q}\left(n^2\sigma^2 + (r+\lambda_C)\right)$$

$$c_n = \frac{1}{2}\frac{\Delta Q}{q}\left(n^2\sigma^2 + n\,(r-\delta)\right)$$

$$d_n = \Delta Q\left(n\,\Delta P - C\right)(1-G)$$

Here, $P = n\Delta P\,,\quad n = 0\,,1,\cdots\cdots N$

$Q = j\,\Delta Q\,,\quad j = 0\,,1,\cdots\cdots J$

$V_{n,\,j} = V(n\Delta P,\, j\Delta Q)$

3.6.2 Case study of a hypothetical underground gold mine

The following case study has been considered to show the approximate numerical values of a mining project under different scenarios and some available real options. Consider a hypothetical underground gold mine which extracted 425,560 ounces of gold at an average grade of 6.52 g/t, over the years 2006 to 2009.

Table 3-1** Characteristics/data sets of the hypothetical gold mine are as follows (all financial figures expressed in real money terms).

Total reserve	425,560 ounces of gold
Average ore production rate	616,790 tonnes per year (approximately)
Average gold production rate	141,850 ounces per year (approximately)
Average grade of gold	6.52 g/t
Gold price	US$1,606.55 per ounce
Total milling and sales costs	US$655.70 per ounce
The fixed cost and pre-tax operating costs (CAPEX, OPEX etc.)	US$141.71 per ounce
*Gold price mean volatility	16.99%
*Standard deviation of volatility from the mean volatility	5.13%
Risk free interest rate	6.00%
Country risk premium	3.00%
The depreciation and amortization charges	0
Convenience yield for holding gold	2.00%
Corporate taxes	30%

* Parameter values in Table 3-1 to Table 3-2 were determined from historical data.

The management continued the mine operation for 3 years, and later on due to the decreased gold price the mine was closed in 2010. The management closed the mine temporarily and incurred the maintaining costs during closure period. In 2011, the gold price increased markedly and the management of the mine decided to reopen the mine and exploit the lower grade material which had an average grade of 4.54 g/t. The gold mine has a possible remaining reserve of approximately 335,620 ounces of gold (2.095 million tonnes of ore).

Table 3-2 Characteristics/data sets of the hypothetical gold mine are as follows (the mine is reopened after being temporarily closed; all financial figures expressed in real money terms).

Total remaining reserve	335,620 ounces of gold
Average ore production rate	577,320 tonnes per year(approximately)
Average gold production rate	92,454 ounces per year (approximately)
Average grade of gold	4.54 g/t
Gold price	US$1,627.35 per ounce
Total milling and sales costs	US$635.10 per ounce
Fixed costs and pre-tax operating costs (CAPEX, OPEX, etc.)	US$170.68 per ounce
Total mine closure costs	US$4,550,000
Total cost for reopening the mine	US$5,350,000
*Gold price mean volatility	16.99%
*Standard deviation of volatility from the mean volatility	5.13%
Risk free interest rate	6.00%
Country risk premium	3.00%
Depreciation and amortization charges	0
Convenience yield for holding gold	2.00%
Corporate taxes	30%

****Note:** Data in Table 3-1 to Table 3-2 have been considered based on US dollars, therefore to determine project values in other currencies, the data and currency must be converted from their local currency.

In this case study from Table 3-1, the fixed cost and pre-tax operating cost (CAPEX, OPEX, etc.) per ounce of gold is US$141.71. Therefore, the total production and sales cost of each ounce of gold is US$797.41 (US$655.70+US$141.71)

Gold price volatility has been calculated from the past historical data over the period from January, 1997 to December, 2014. The average gold price volatility during this period was 16.99%, whereas the maximum and minimum gold price volatilities were 31.65% and 10.99% respectively, and the standard deviation was 5.13%. Therefore, to approximate the gold mining project's values, future gold price volatility may be forecast to $\pm 5.13\%$ with the mean volatility 16.99% as a proxy for future gold price volatility. Numerical project values have been shown using those parameter values with MatLab software (MatLab, 2013a).

Typically mine management and industry practitioners only need to know the approximate (as it is difficult to estimate an exact value) numeric values or value ranges before investing in a project or its expansion, contraction, suspension, etc. Consequently, in each case, the approximate maximum project value is calculated which will help mine management and practitioners make an appropriate investment decision associated with that mining project. In addition, graphical representations are given (though graphical figures might not be necessary for mine managers and industry practitioners for making the decision) to provide just an overview of the project values.

Using the parameter values from Table 3-1, the approximate maximum project value is determined as US$78,213,000. In addition, graphically, Figure 3-3 displays the approximate maximum project value of the mine in linear scale when the reserve is 425,560 ounces of gold and volatility 16.99%, with the approximate maximum project value being US$78,213,000. To approximate project values in each case (each figure), 60 by 1,800 matrixes (60x1, 800) have been created, and each grid point project value has been calculated which has then generated the figures.

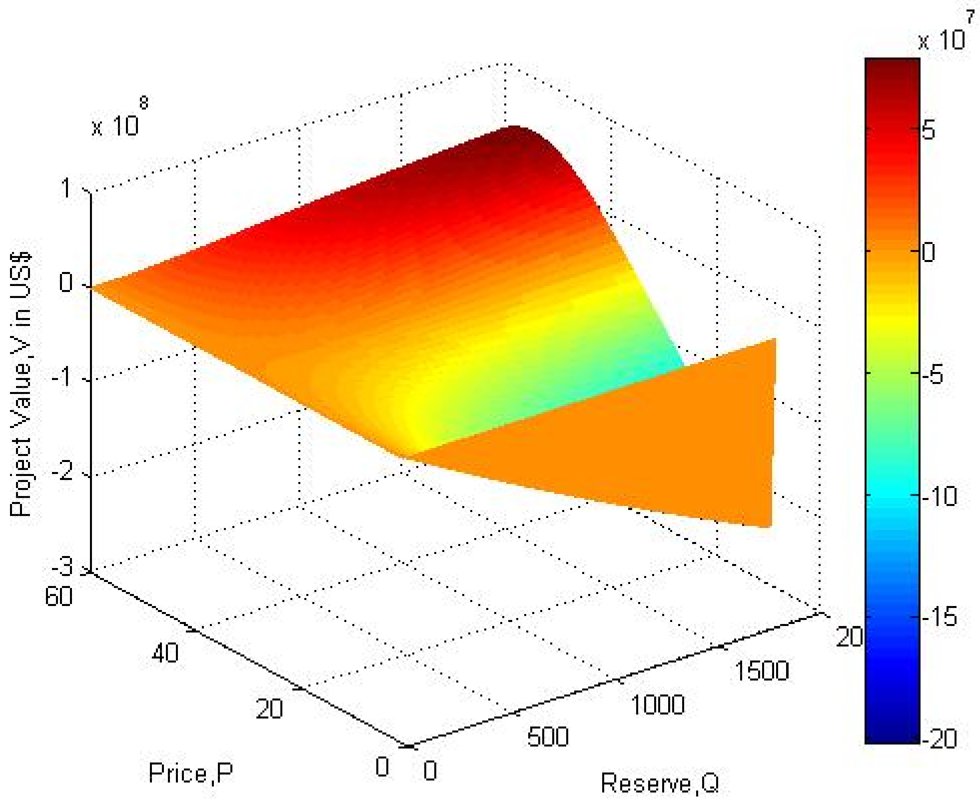

Figure 3-3 Project values of the mine in linear scale (input data are used from Table 3-1, and q=141,850 oz, P=1606.55 US$/oz, C=797.41 US$/oz).

We have also shown project values considering different commodity price volatilities to see the impact of price volatility on the project value. From the analysis, it has been observed that when historical volatility goes up or down 1% from 5.13%, the project value changes -22.57% and 28.33% respectively (for details refer to appendix of Chapter 3). Therefore, price volatility has a great impact on mining project values.

The option to defer investing

Due to commodity price depreciation, management can defer initiation of the project (the mine operation) for a year by incurring extra costs (maintenance costs), and management can monitor whether the commodity price goes up or down over that period. Suppose the above project described in Table 3-1 defers commencement of operation for a year, and the average gold price after one year increases to US$1,632.40 per ounce. The total production and sales cost of each ounce of gold increases to US$802.35. In this case, when the reserve is 425,560 oz. of gold and the average gold price is US$1,632.40 per ounce after deferring the project (mine operation) for a year, the approximate maximum project value is calculated as US$81,086,000.

Figure 3-4 shows the approximate maximum project value is US$81,086,000 when the average gold price is US$1,632.40 per ounce after deferring the project's start-up for a year.

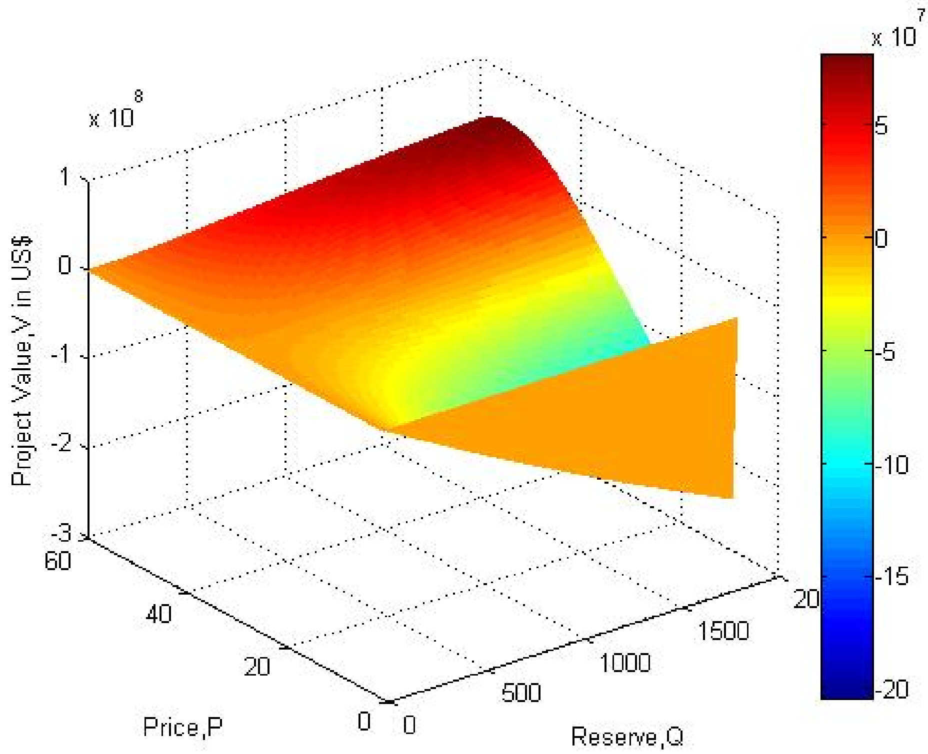

Figure 3-4 Project values of the mine in linear scale (input data are used from Table 3-1, and q=141,850 oz, P=1632.40 US$/oz, C=802.35 US$/oz).

In this scenario, the approximate maximum project value is US$81,086,000, so the deferral option added value to the project.

Accelerate or decelerate options for mine operation

If the commodity price goes up markedly then management can accelerate the production rate (q) which recognises a higher project value and associated returns (Figure 3-5). However, if the commodity price decreases significantly, then management can reduce the production rate (q) which will generate a lower project value and commensurate returns at that time (Figure 3-3), but may increase the life of mine and offer an opportunity to wait until favorable conditions return for realizing a higher project value.

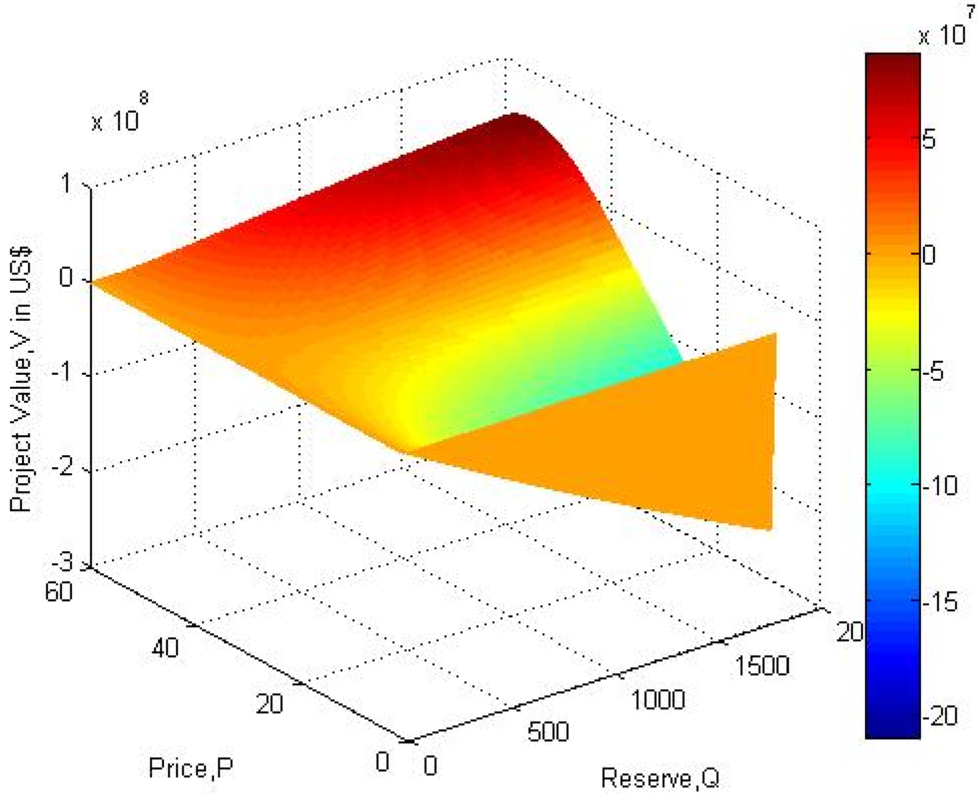

Figure 3-5 Project values of mine in linear scale (input data are used from Table 3-1, and q=168,850 oz, P=1625.60 US$/oz, C=810.50 US$/oz).

From the simulation results it has been observed that when the annual mine production rate is 141,850 oz. of gold the maximum numerical project value is US$78,213,000. However, when the production rate is 168,850 oz. of gold, the average gold price is US$1625.60 per ounce (usually the mine production capacity accelerates when commodity prices increase) and the total cost is US$810.50 per ounce (as the option to accelerate the mining production rate or expand the scale of production may need an extra investment outlay) the maximum project value is US$85,736,000 (from Table 3-1, other parameters remain the same in these two cases). This shows that when the production capacity of the mine (i.e. acceleration of production rate) increases, the project value increases, so the expansion option added value to the project. Figure 3-3 and Figure 3-5 display these results graphically.

Resultant values largely depend on total costs too because numerical simulations show that if the total operating cost is lower, then the project's value increases (Figure 3-6).

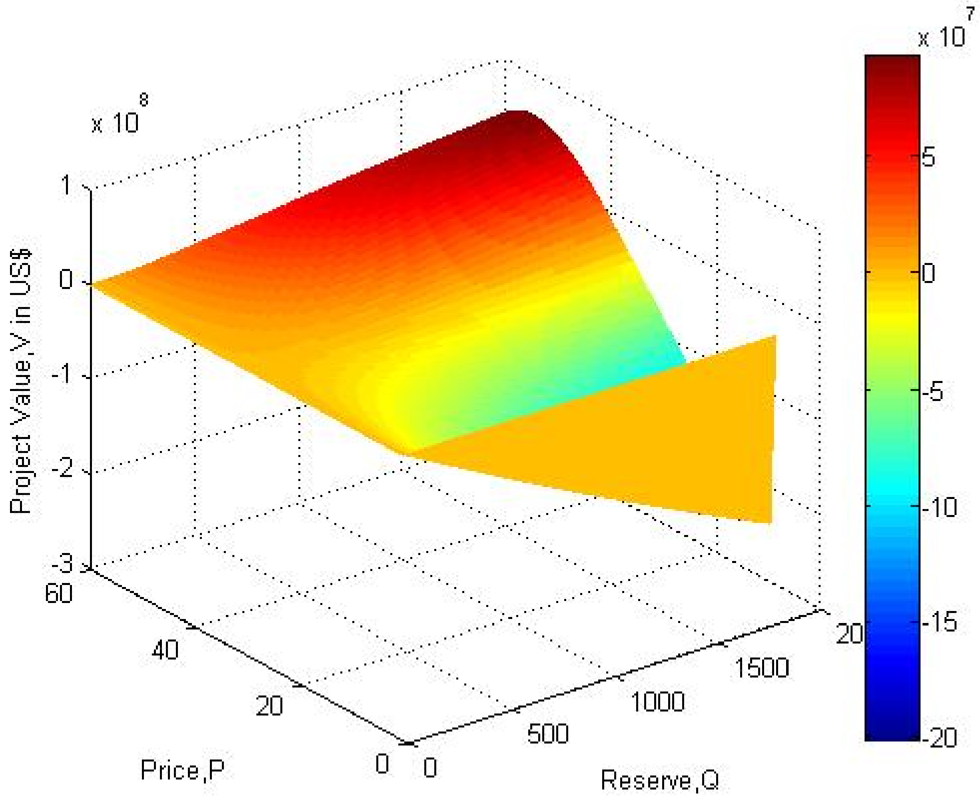

Figure 3-6 Project values of the mine in linear scale (input data are used from Table 3-1, and q=168,850 oz, P=1625.60 US$/oz, C=777.41 US$/oz).

From the simulation (input parameters are considered from Table 3-1) results, it has been observed that when the total cost per ounce of gold is US$810.50, the maximum project value is US$85,736,000 and if the total cost is US$777.41 per ounce of gold the maximum project value is US$92,696,000 (other parameters remain the same in these two cases).

Temporary closure and reopening option

After the temporary closing of the mine, when the commodity price rises notably, management can reopen the mine. In this case management could approximate the project value through this same PDE (3-6). To achieve this, maintaining and reopening costs need to be calculated and added to the other operating costs. Consequently, when management reopens the mine for extracting the remaining reserves of 335,620 oz. of gold, from the case study in Table 3-2, the fixed and pre-tax operating cost per ounce of gold is US$170.68, with the closing and reopening mine costs i.e. the maintenance costs, adding US$29.50 per ounce of gold. Hence, the total cost per ounce of gold is now US$835.28 (US$635.10+US$170.68+ US$29.50). In this case, the maximum value of this hypothetical gold mine is US$52,185,000 (using data from Table 3-2).

Figure 3-7 displays the approximate maximum project value of the mine in linear scale when the hypothetical gold mine has a remaining reserve of 335,620 oz. and the production rate is 92,454 oz of gold.

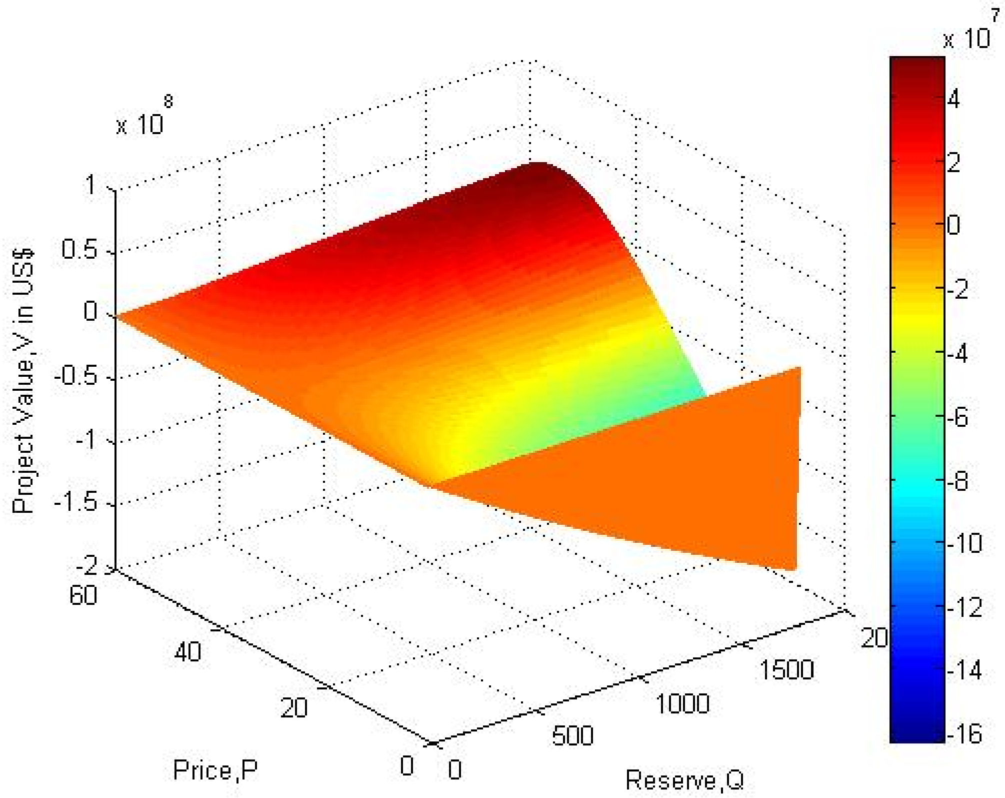

Figure 3-7 Project values of the mine in linear scale (input data are used from Table 3-2).

Abandon option for salvage value

If the maximum project value is determined to be extremely low or approaches 0, or the total cash flows of the project are negative, management can delay the project or temporarily close the mining operation if the mine is open, until advantageous conditions return. If, after a period, the total cash flows of this project are calculated to still be negative, then management can adopt the abandon option and leave the project permanently and sell the whole project for its salvage value or its best alternative use. This scenario usually happens when the total operating cost per unit of commodity is closest to or higher than the unit price of the commodity.

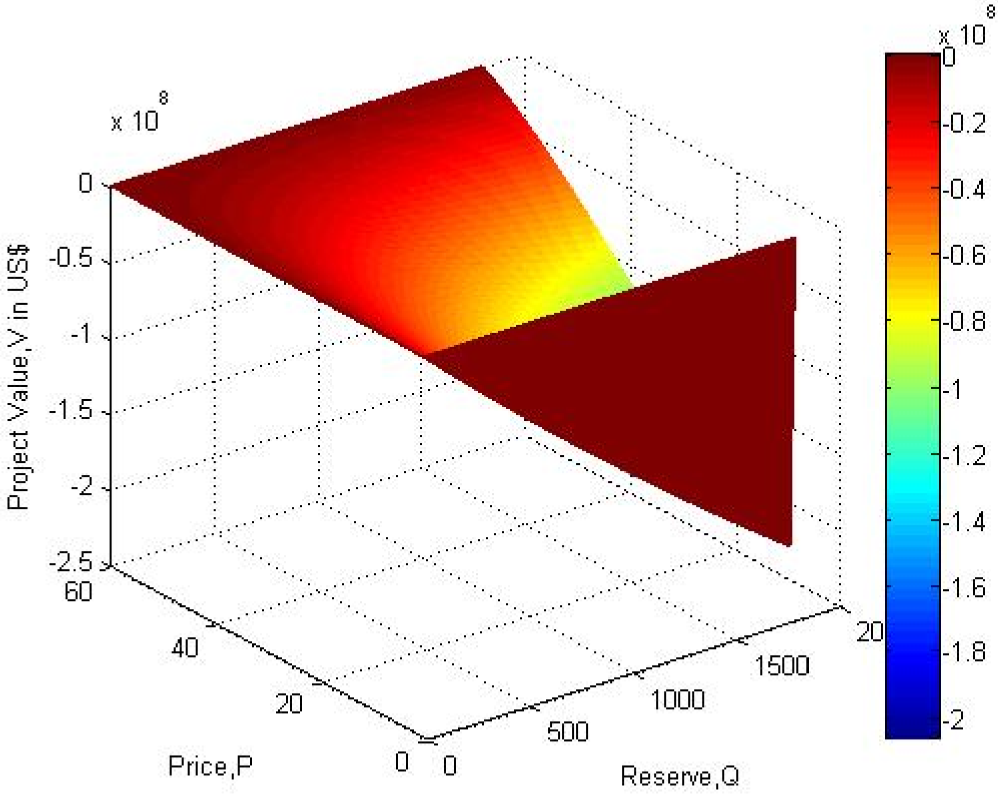

Figure 3-8 Project values of the mine in linear scale (input data are used from Table 3-1, and P=824.90 US$/oz).

Figure 3-8 reflects the value of the project as negative when the reserve is 425,560 oz of gold and the volatility is 16.99%. Therefore, management can delay the project, temporarily close or adopt the abandon option and leave the project. In this case the lowest gold price is US$824.90 per ounce for which the project value is negative and there is no positive cash flow. This might be considered as the critical gold price for this project at which the temporary closure of the mine is triggered.

Figure 3-9 represents the value of the project being negative when the remaining reserve is 335,620 oz of gold and the volatility is 16.99%.

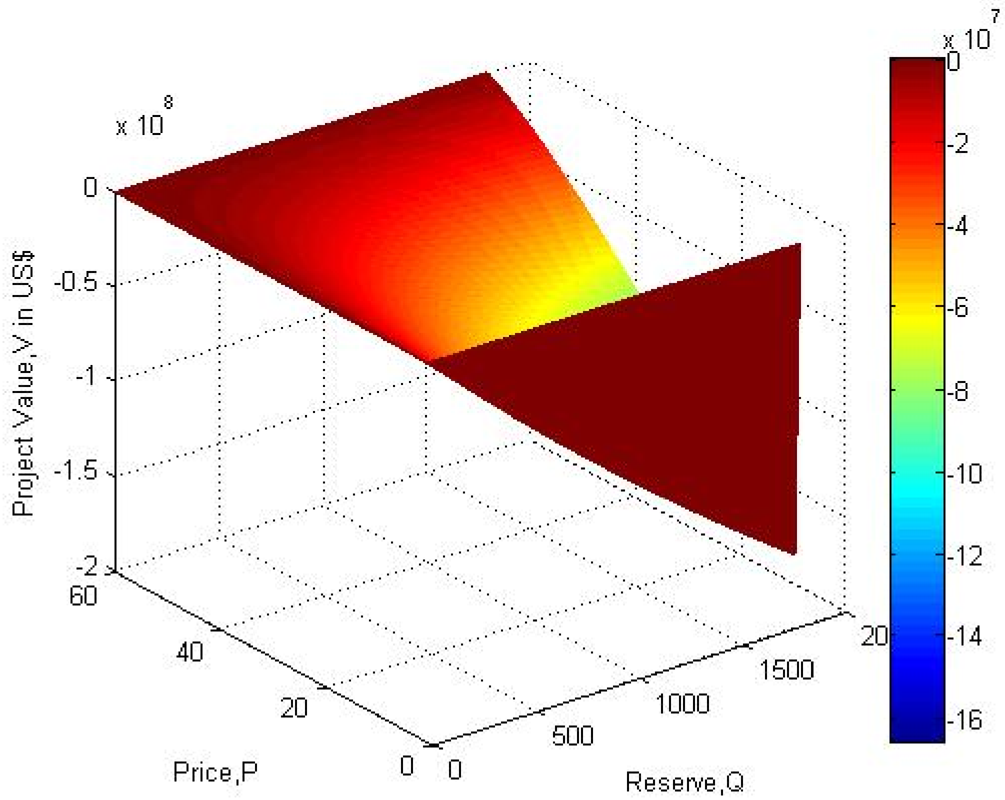

Figure 3-9 Project values of the mine in linear scale (input data are used from Table 3-2, and P=864.05 US$/oz).

Therefore, management should adopt the abandon option and leave the project permanently. In this case the lowest gold price is US$864.05 per ounce for which the project value for the remaining reserve is negative and there is no positive cash flow. This may be reflected as the critical gold price for this project above which point reopening the mine may be considered. Therefore, mining project values are greatly influenced by commodity price as well as volatility.

3.7 Project evaluation considering time as a variable of mine value (time dependent and mean reverting commodity price)

In the preceding section, the project value has been approximated considering reserve dependency, however, in a real life project, to determine the appropriate total reserve is not an easy task, and sometimes the uncertainty of economic reserves is considered as a direct result of flexible operational cut-off grades in response to price fluctuations. Due to any commodity price change there is a huge influence on the ore cut-off grade (Asad and Topal, 2011; Ajak and Topal, 2015). In general, the reserve is determined under a feasibility study at the beginning of the project, but it could be changed (economic reserve) due to economic circumstances i.e. market conditions and mining total costs. Moreover, in a logical sense, reserves may not be diminished in a linear manner, but the true fact is that the economic reserve might be diminished due to the commodity price and mining cost. Hence, it would be better to approximate mining project values for time dependence; i.e., time is considered as an independent variable for the function of mine value instead of total reserve. Therefore, the project value will be approximated now considering time dependence, instead of reserve dependence.

Another aim is to approximate the project value in the case of a mean reverting mining commodity price model. That means before going to evaluate a specific mining project, it might be better to investigate whether this specific commodity has a mean reverting behaviour or not, and if it exists, what is the speed of reversion, as based on this idea some authors used mean reverting models for commodity prices (Dixit and Pindyck, 1994; Cortazar and Schwartz ,1997; Cortazar and Casassus, 1998; Hall and Nicolas, 2007). Cortazar and Schwartz (1997), and Cortazar and Casassus (1998) used the mean reversion stochastic price model for the evaluation of project values considering a reserve dependent continuous time PDE. Hall and Nicolas (2007) also utilized the mean reversion stochastic price model for mining projects. However, these authors did not determine the numerically explicit project values through continuous time modeling. However, the price of all commodities may not strongly follow the mean reverting criteria, and based on this terminology some authors used continuous time geometric Brownian motion (GBM) processes (Brennan and Schwartz, 1985; Cortazar et al., 2001, Colwell et.al. 2003, Lima and Suslick 2006, Haque et al., 2014; McDonald and Siegel, 1986; Zhu et al., 2012). However, to replicate the mean-reversion of commodity prices, a mean reversion stochastic model is considered here to represent the commodity price fluctuations for approximating the project values.

3.7.1 Model for commodity price fluctuations

In this section, a mean reversion stochastic model is reflected on the fluctuation of the commodity price, and thereafter, a PDE is constructed for approximating the value of the mining projects. The mean reversion is the price or returns of an asset, especially commodities or stocks, which eventually move back towards the mean or average. This mean or average might be the historical average of the price or return of the respective commodities or stocks. According to Cortazar and Schwartz (1997) and Cortazar and Casassus (1998), it is assumed that the dynamics of the commodity price, P will follow the mean reversion stochastic process described by the model (3-8) below. The existing behaviour of mean reversion in prices is accommodated here with drift term, since if the speed of reversion is zero then the price of the commodity follows the Geometric Brownian motion (GBM) model.

$$\frac{dP}{P} = (r - \delta + \mu(\overline{P} - P))dt + \sigma\, dw \qquad (3\text{-}8)$$

where,

P is the spot unit price of the commodity,

$\overline{P}$ is mean or average unit price of the commodity,

r is the risk free rate of interest,

δ is the mean convenience yield on holding one unit of that commodity,

σ is the volatility of returns of P,

μ is the speed of the reversion,

dw is the Wiener increments of the mean reversion process.

The advantage of this stochastic model is that, if the specific mineral commodity price does not follow the mean reversion behavior, then the value of μ can be assigned in a value of 0, and eventually the model (3-8) will turn into a Geometric Brownian motion model (3-1).

Now consider the value of the mine to be time dependent i.e. mine value is also a function of the operating time rather than the reserve. During the mine operation, the value of the mine at time t $(0 \le t \le T)$ can be defined as $V = V(P, t, i, \phi) \cong V(P, t)$. Here i is the indicator variable which takes the value one if the mine is open and zero if it is closed, ϕ describes the operating policy such as opening, temporarily closing, postponing and abandoning the mine, and T is the total time of operation.

3.7.2 Derivation of the partial differential equation (PDE) using the SDE (3-8) and time is considered as variable of mine value

A PDE will be constructed using equation (3-8), with the help of commodity futures markets and hedging strategies. If the commodity futures market is arbitrage free, that is there is no arbitrage opportunity, then management has the flexibility to enter a long position for investment in the mining project and a short position in the futures contracts for hedging commodity price risk (Brennan and Schwartz, 1985; Bellalah 2001; Colwell et al., 2003; Cortazar and Schwartz, 1997; Dixit and Pindyck, 1994).

Suppose the commodity spot price is P, and its delivery date under the futures contract is t years, so the value of the futures contract is $F(P,t)$.

Applying Ito's Lemma to the value of the commodity future gives:

$$dF(P,t) = \frac{\partial F}{\partial P}dp - \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial p^2}P^2\sigma_P^2 dt \tag{3-9}$$

If there is no arbitrage opportunity in commodity futures markets, then an investor through a portfolio of long one unit of commodity and short in $\left(\frac{\partial F}{\partial P}\right)^{-1}$ units of the futures contract has hedged the price risk and should earn a risk free interest rate (Cortazar and Schwartz, 1997). Therefore, in time interval dt, this portfolio becomes

$$dP + \left(\delta - \mu\left(\overline{P} - P\right)\right)P\,dt - \frac{dF}{F_P} = r\,p\,dt \tag{3-10}$$

In real life, a mining company can adopt the hedging opportunity to lock up the commodity price for minimizing risk in its project. That means a mining company with a long position for an investment in a mining project $V = V(P,t)$, and short position in $\frac{V_P}{F_P}$ units of commodity could hedge its risk and should earn a return equal to the risk free interest rate plus the country risk premium associated with the country where the mining project is situated (Cortazar and Schwartz, 1997; Colwell et al., 2003; Bellalah 2001), and the portfolio becomes

$$dV + q\,g_a\left(P - C\right)(1-G)dt - \frac{V_P}{F_P}dF \tag{3-11}$$

Where, q is the ore production rate in tonnes per unit time, g_a is the average grade of the commodity, C is the total cost (CAPEX, OPEX, working capital, etc.) per unit of commodity, and G is the total tax.

Applying Ito's Lemma, $V = V\left(P, t\right)$

$$dV = \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 \qquad \text{(3-12)}$$

Substituting the value of dV from the equation (3-12) into the equation (3-11),

the total return on this portfolio becomes

$$\frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 + qg_a\left(P - C\right)(1 - G)dt - \frac{V_P}{F_P} dF$$

If there is no arbitrage opportunity, then a mining investment with a long position for investment in the mining project and short position in the futures contracts hedges its risk and should earn a return equal to the risk free interest rate plus the country risk premium associated with the country where the mining project is situated. Therefore, it becomes,

$$\frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 + qg_a\left(P - C\right)\left(1 - G\right)dt + \frac{\partial V}{\partial P}\left(r\,P\,dt - dP - \delta\,Pdt + \mu\left(\overline{P} - P\right)P\,dt\right) = \left(r + \lambda_C\right)Vdt$$

$$\Rightarrow \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}\left(dP\right)^2 + qg_a\left(P - C\right)\left(1 - G\right)dt + \frac{\partial V}{\partial P}\left(r - \delta + \mu\left(\overline{P} - P\right)\right)P\,dt = \left(r + \lambda_C\right)Vdt \qquad (3-13)$$

As production in the mine continues, the overall mine life logically decreases i.e. the total mine operational time will decrease. Therefore, the relationship of the change in time can be written as

$$h(t) = T - t \quad , \qquad 0 \le t \le T \qquad \text{(3-14)}$$

Using (3-14), the following relation can be derived

$$\frac{\partial V(P,t)}{\partial t} = \frac{\partial V(P,h(t))}{\partial h(t)} h'(t) = -\frac{\partial V(P,h)}{\partial h} \qquad \text{(3-15)}$$

Using the relation (3-15), equation (3-13) can be written as

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2} - \frac{\partial V}{\partial t} + \left(r - \delta + \mu\left(\overline{P} - P\right)\right)P\frac{\partial V}{\partial P} - \left(r + \lambda_C\right)V + qg_a\left(P - C\right)\left(1 - G\right) = 0 \qquad \text{(3-16)}$$

Numerical solutions of the PDE (3-16) can be approximated through the finite difference method (FDM) using MatLab software, which will commensurately provide the numerical values of the mining project. For solving the PDE (3-16) numerically, some conditions need to be imposed. Since a real option is similar to a financial option, one of the boundary conditions is considered as $V\left(0,t\right)=0$, that is when the output price is zero at any time in the mine's life, the value of the mine is zero. The other condition is $V\left(P,0\right)=0$, implying that when the time is zero or time is exhausted for the mine operation, the value of the mine is zero. Furthermore, it is also assumed that $\lim_{P\to\infty}\frac{V(P,t)}{P}\to 1$ i.e. $\frac{\partial^2 V}{\partial P^2}\left(\infty,t\right)=0$.

3.8. Numerical results

In this section, the PDE (3-16) is discretized, and thereafter, a case study will be considered to estimate project values numerically.

3.8.1 Discretization of PDE with FDM

The explicit finite difference method has been employed to discretize the PDE (3-16):

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2}-\frac{\partial V}{\partial t}+\left(r-\delta+\mu(\overline{P}-P)\right)P\frac{\partial V}{\partial P}-\left(r+\lambda_C\right)V+q\,g_a\left(P-C\right)\left(1-G\right)=0$$

The discretized form of this PDE:

$$\Rightarrow\frac{V_{n,\,j+1}-V_{n,\,j}}{\Delta t}-\frac{\sigma^2}{2}\left(n\Delta P\right)^2\left(\frac{V_{n+1,\,j}-2\,V_{n,\,j}+V_{n-1,\,j}}{\left(\Delta P\right)^2}\right)-\left(r-\delta+\mu(\overline{P}-P)\right)n\Delta P\left(\frac{V_{n+1,\,j}-V_{n-1,\,j}}{2\Delta P}\right)$$
$$+\left(r+\lambda_C\right)V_{n,\,j}-qg_a\left(n\Delta P-C\right)(1-G)=0$$

The solution can be found as the form:

$$V_{n,\,j+1}=A_n\,V_{n-1,\,j}+B_n\,V_{n,\,j}+C_n\,V_{n+1,\,j}+D_n \tag{3-17}$$

where

$$A_n=\frac{1}{2}\Delta t\left(n^2\sigma^2-n\left(r-\delta+\mu\left(\overline{P}-n\Delta P\right)\right)\right)$$

$$B_n = 1 - \Delta t\left(n^2\sigma^2 + \left(r + \lambda_C\right)\right) ,$$

$$C_n = \frac{1}{2}\Delta t\left(n^2\sigma^2 + n\left(r - \delta + \mu\left(\overline{P} - n\Delta P\right)\right)\right) ,$$

$$D_n = qg_a\Delta t\left(n\,\Delta P - C\right)(1 - G) .$$

Here, $P = n\Delta P\,, \quad n = 0\,, 1, \cdots\cdots M$

$T = j\,\Delta t\,, \quad j = 0\,, 1, \cdots\cdots N$

$V_{n,\,j} = V\left(n\Delta P,\, j\Delta t\right)$

3.8.2 Case study of a conceptual open pit gold mine

To find the approximate numerical values for a mining project under different scenarios and with a number of available real options, consider the following case study.

Scenario 1

Consider a conceptual open pit gold mine that extracted ore at a rate of 616,790 tonnes per year (approximately) as presented in Table 3-3, and continued operating for 1 year, starting from January 2012 to December 2012. It is just assumed that gold prices follow the mean reversion and the speed of reversion is 0.02, however, after the investigation/analysis, if gold prices do not follow the mean reverting criteria, then the speed of reversion, μ can be assigned a zero value.

For scenario 1, the total production and sales cost of each ounce of gold is US$834.41(US$685.70+US$148.71). Using the data from Table 3-3, the approximate maximum project value for one year of operation under the effect of commodity price uncertainty is calculated as US$49,006,000.

Table 3-3** Characteristics/data sets of a conceptual underground gold mine representing scenario 1 (all financial figures expressed in real money terms).

Total operation	1 year
Average ore production rate	616,790 tonnes per year (approximately)
Average grade of gold	0.1816 ounce per tonne
Gold price	US$1,565.25 per ounce
Total milling and sales costs	US$685.70 per ounce
The fixed cost and pre-tax operating costs (CAPEX, OPEX etc.)	US$148.71 per ounce
*Gold price mean volatility	16.99%
*Standard deviation of volatility from the mean volatility	5.13%
Risk free interest rate	6.00%
Average/mean gold price	US$1250.10 per ounce
Speed of mean reversion	0.02
Country risk premium	2.00%
The depreciation and amortization charges	0
Convenience yield for holding gold	2.00%
Corporate taxes	30%

Figure 3-10 shows the project values of the mine when the average gold price is US$ 1,565.25 per ounce, and the approximate maximum project value under the effect of commodity price uncertainty is US$49,006,000.

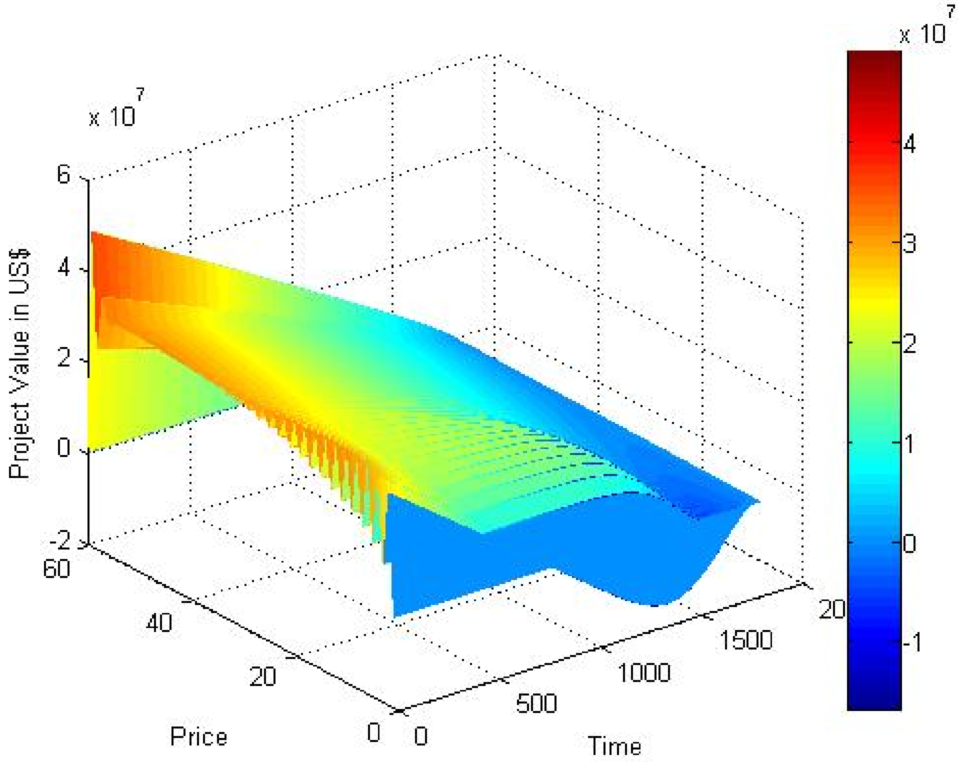

Figure 3-10 Project values of the mine in linear scale (data sets are used from Table 3-3).

Option to defer investment

If the company has a one year lease which providing it is a proprietary right to defer undertaking the project or if the company can hold off production by incurring a cost, then due to commodity price depreciation, management can defer the start- up (initiation) of the project (the mine operation) for a year. Suppose the above project described in Table 3-3, defers initiation of mine operation for a year, and the average gold price for next year increases to US$1599.40 per ounce. The total production and sales cost of each ounce of gold increases to US$836.35.

In this situation, the approximate maximum numeric project value is US$50,234,000. Therefore, the defer option added value to the project. Figure 3-11 shows the project values of the mine when the average gold price is US$ 1,599.40/oz, after deferring production at the mine for year.

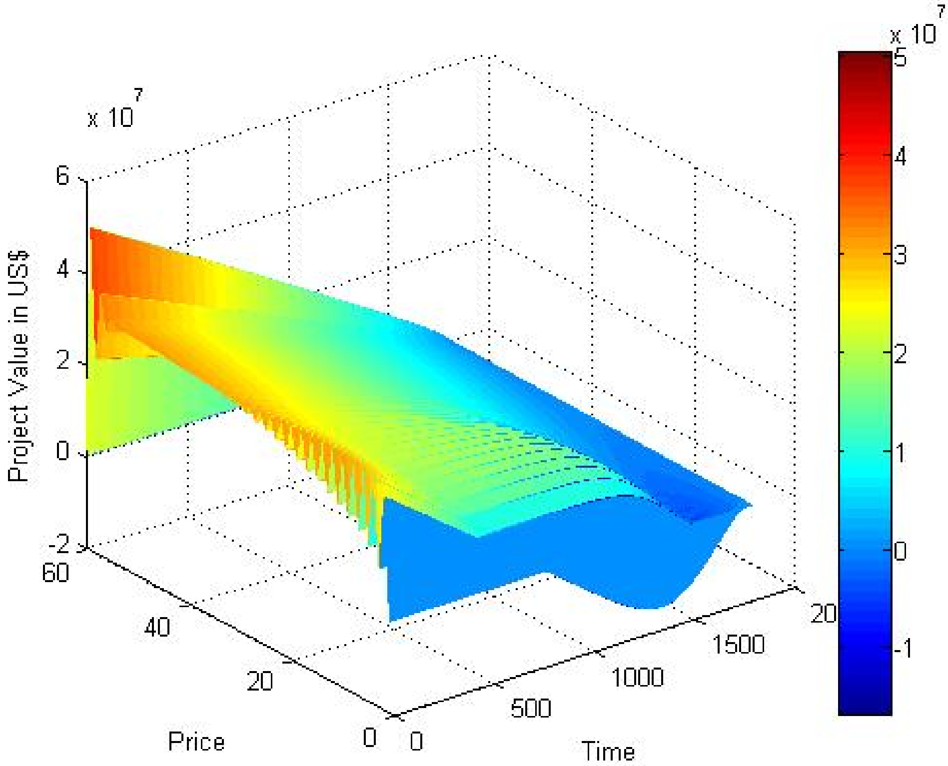

Figure 3-11 Project values of the mine in linear scale (data sets are used from Table 3-3 and, P=1599.40US$/oz, C=836.35 US$/oz).

Scenario 2

Management continued the mine operation over 2012 and, thereafter, due to the reduced gold price, mining operations were suspended in 2013. Management closed the mine temporarily and incurred the maintaining costs during the closure period. In 2014, the gold price increased and the management of the mine decided to reopen the mine and exploit from the remaining reserves at an extraction rate of 799,890 tonnes per year as presented in Table 3-4. The gold deposit has a possible remaining reserve of approximately 6 years of extraction. After one year of extraction in 2014 (from January 2014 to December 2014), management again stopped mining production due to depressed gold prices.

Table 3-4 Characteristics/data sets of a conceptual underground gold mine representing scenario 2 (the mine is reopened after being temporarily closed. All financial figures expressed in real money terms).

Total operation	1 year
Average ore production rate	799,890 tonnes per year (approximately)
Average grade of gold	0.1416 ounce per tonne
Gold price	US$1,598.35 per ounce
Total milling and sales costs	US$640.10 per ounce
The fixed cost and pre-tax operating cost (CAPEX, OPEX, etc.)	US$173.68 per ounce
Total mine closure costs	US$1,422,600
Total cost for reopening the mine	US$1,918,600
*Gold price mean volatility	16.99%
*Standard deviation of volatility from the mean volatility	5.13%
Risk free interest rate	6.00%
Average/mean gold price	US$1,250.10 per ounce
Speed of mean reversion	0.02
Country risk premium	2.00%
Depreciation and amortization charges	0
Convenience yield for holding gold	2.00%
Corporate taxes	30%

* Parameter values in Table 3-3 to Table 3-4 were determined from historical data.

****Note:** Data in Table 3-3 to Table 3-4 have been considered based on US dollars, therefore to determine project values in other currencies, the data and currency must be converted from their local currency.

Temporary closure option

After the temporary closure of the mine, when the commodity price rises significantly, management can reopen the mine. In this case, management could approximate the project value through the same PDE (3-16). To achieve this, maintaining and reopening costs need to be calculated and added to the other operating costs. Temporary closures may carry a benefit but most delays incur a cost (Eschenbach et al. 2009). Intuitively, in mining projects, a significant amount of maintaining and reopening costs are involved due to a temporary closure option.

From Scenario 2, for reopening the mine after a temporary closure, the total production and sales cost of each ounce of gold is US$843.28 (US$640.10 +US$173.68+US$12.56+US$16.94). Using the data from Table3-4, the approximate maximum project values for a one year operation under the effect of commodity price uncertainty is calculated as US$49,930,000.

Figure 3-12 shows the project values of the mine when the mine is reopened after temporary closure, and the approximate maximum project value is US$49,930,000.

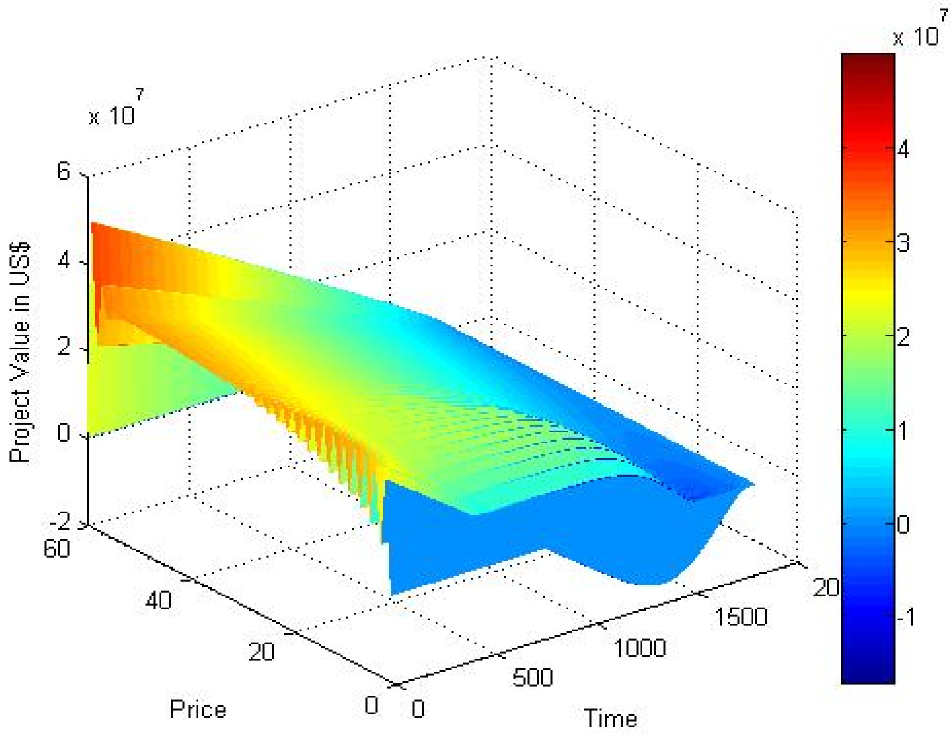

Figure 3-12 Project values of the mine in linear scale (data sets are used from Table 3-4, average ore production rate 799,890 tonnes per year and speed of mean reversion 0.02).

Temporary closure option and accelerate or decelerate options for mine operation

If the commodity price rises considerably, management can reopen the mine after a temporary closure. Furthermore, if the commodity price increases noticeably then management can accelerate the production rate which helps to realize a higher project value and associated returns (Figure 3-13). Alternatively, when the commodity price decreases significantly, then management can reduce the production rate which will generate a lower project value and return at that time, but may increase the life of the mine and provide an opportunity to wait until more favorable conditions return for realizing a higher project value (retaining the option value).

Hence, if the mine is again reopened after a temporary closure and management adopted the expansion option (increased production rate) by incurring an extra cost, the total cost of per ounce gold will be increased to approximately US$845.10 per ounce, and ore production rate 838,820 tonnes per year (other parameter values are remain same). Therefore, in this scenario, the approximate maximum project value is US$52,133,000 and it has been observed that the expansion option added value to the project.

Figure 3-13 displays the project values of the mine when the mine is reopened after a temporary closure and management adopts the expansion option (increased production rate) by incurring an extra cost.

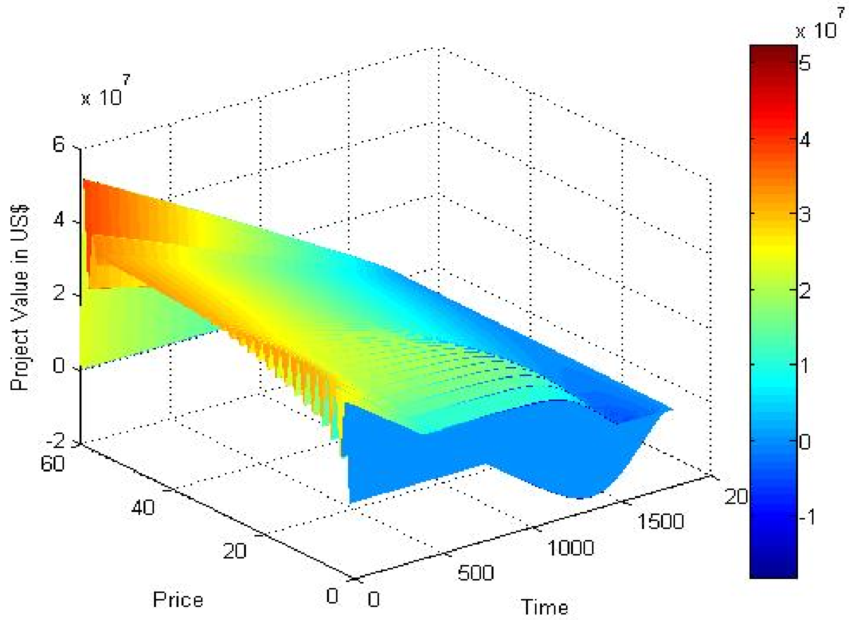

Figure 3-13 Project values of the mine in linear scale (data sets are used from Table 3-4, average ore production rate 838,820 tonnes per year).

Figure 3-14 shows the project values of the mine when the country risk premium is considered as zero (other parameters remain same). That is, when the mine is reopened after a temporary closure and management adopts the expansion option. In this case the approximate maximum project value is US$52,638,000, and it was revealed that when we considered the country risk premium is zero the project value increases US$505,000 (comparing the maximum project values US$52,133,000 and

US$52,638,000). Therefore, country risk premium has an impact on the value of a mining project.

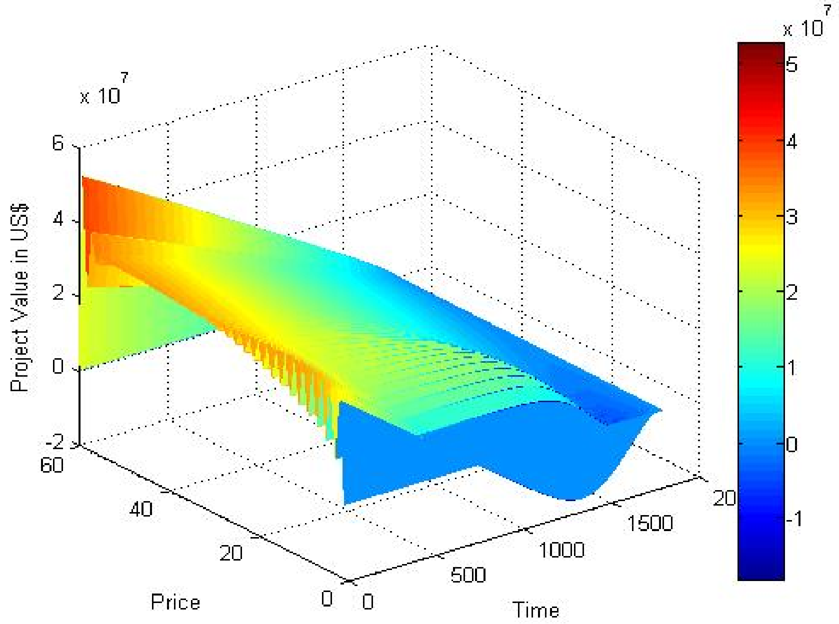

Figure 3-14 Project values of the mine in linear scale (data sets are used from Table 3-4, average ore production rate 838,820 tonnes per year, and the country risk premium is zero)

When we considered GBM model and associated PDE (3.2a) the approximate project values shows US$29,148,000 (For details see Figure A3-4, in Chapter 3, Appendix 3.2A). Hence from our simulation results it has been observed that mean reverting commodity price give higher project values compare to Geometric Brownian motion model. If we considered the speed of reversion, mu is 0.01 then the maximum project values shows US$41,928,000 (calculated from PDE (3-16), see Figure A3-5, in Chapter 3, Appendix 3.2A). Hence we observed that when the speed of reversion decrease project value also decreases. Therefore, the speed of reversion has an impact on project values

3.9. Summary and discussion

In this chapter, two different new techniques have been presented for approximating project values, firstly for reserve dependence i.e. finite reserve, and secondly for time dependence i.e. infinite reserve. For reducing mining project risk and capturing commodity price uncertainty, new PDEs have been derived using hedging strategies and futures contracts through commodity futures markets, and new mining parameters are directly incorporated with these PDEs. This will help mining companies to approximate project values through optimizing profits and minimizing mining losses. In traditional DCF methods as well as Binomial lattice/tree models, it might not be possible to adequately consider these types of financial and economic fundamentals. Besides this, real options and several management flexibility options are considered to approximate mining project values which seemingly assist the mining company and its associated management to take the appropriate decisions for investment under different scenarios. Moreover, the procedures are shown to approximate the numerical values of the mining project by solving the PDE numerically through the use of FDM and MatLab software. Additionally, to find the numerical values of a mining project, as a case study historical gold price volatility has been determined from the historical data as, it is one of the more important input parameters to solve the PDE and revealed that price volatility has great impact on mining project values.

It has also been shown that project values depend on production rates, commodity prices, total costs (CAPEX, OPEX, taxes, etc.), and country risk. When the commodity price increases (or the currency depreciates), then management can adopt the option to accelerate the mine operating rate influencing the parameter q, which apparently benefits in obtaining higher project values and simultaneously decreases unit mining costs. However, when the commodity price decreases (or the currency appreciates) management may adopt the option to decrease the mine production rate which may increase the life of mine and provide the opportunity to wait until favorable conditions return for recognizing a higher project value.

Furthermore, due to commodity price depreciation and large mining costs, management can defer the project, take the temporary closure option or adopt the abandon option. If the maximum project value becomes extremely low or approaches 0, or the total cash flows of the project are negative, then management can choose the temporary closure option and wait until a favourable commodity price returns or adopt the abandon option and leave the project permanently when the overall outlook becomes unfavourable. Although a hypothetical gold project is considered as a case study to determine the numerical values of a mining project, using these PDEs, it is possible to evaluate any mining project values for any other commodities, both for underground and open pit mines too. In the case study of mean reversion commodity prices for finding numerical results, it is presumed that gold prices follow the mean reversion and the speed of reversion is 0.02 (as an assumption). However, after investigation, if it is found that gold prices or any specific mineral commodity prices do not follow the mean reverting criteria, then the speed of reversion, μ can be assigned a zero value and using the same time dependent PDE it is possible to estimate the project's values. In this chapter, only commodity price uncertainty is considered, but exchange rate uncertainty has an impact on a mining project value as well. Therefore, the project values will be approximated considering the joint effect of commodity price and exchange rate uncertainties, and will be discussed further in Chapter 5.

CHAPTER 4

RELATIONSHIP BETWEEN THE COMMODITY PRICE AND THE EXCHANGE RATE

4.1 Introduction

Commodity prices and exchange rates have a great impact on the economics of several industries including mining, oil and gas, agriculture, textiles and others that are exposed to international trade. As an example, the US dollar-based gold price and the exchange rate between the Australian dollar against the US dollar (AUD/USD) have a combined and significant impact on both the trend of the Australian minerals industry and the overall Australian economy. The trend in this case refers to the economic sustainability and growth of the minerals industry as well as the physical extraction rates of the collective industry. For approximating mining project values, mine managers and decision makers need to be cautious about different uncertainties including financial and economic uncertainties, and the inter-correlation and long-run relationships among these variables. Therefore, to estimate mining project values, it is necessary to determine the relationship between the commodity price, stated in US dollars, and the relevant exchange rate against the US dollar. The aim of this chapter is to explore the relationship between the US dollar quoted commodity price and the relevant periodic exchange rates, and to determine the strength of association between them which then can be used to approximate the value of mining projects.

Commodity prices are collectively one of the most important economic parameters that directly impacts currency exchange rate movements. Most notably, countries including Australia, Canada, New Zealand, Chile and South Africa are dependent on significant levels of foreign income through exporting commodities. The Canadian dollar, the Australian dollar and the New Zealand dollar are the most frequently traded commodity currencies, and these countries are inextricably linked to global capital markets and active participants in international trade. Chen and Rogoff (2003) studied these commodity currencies and observed that there is a close association between the price of specific primary commodities such as gold, oil and agricultural products and specific currencies. Ma (2013) examined the impact of changes in the forward pricing mechanism on the volatility of iron ore spot prices considering prices from October 2008 to September 2012. The results of the study showed that the implementation of the quarterly pricing mechanism alleviated the volatility of the iron ore spot price. Warell (2014) studied the effect of a change in pricing regime on iron ore prices considering monthly data over the period January 2003 to August 2012. In the study, it has been demonstrated that the change in pricing regime does not have a significant impact on iron ore prices, but gross domestic product (GDP) growth in China, iron ore prices and freight rates are found to be co-integrated when regressed with a market dummy variable.

The combined impact of large volume and high value exports, notably coal, gold and iron ore, strengthened the Australian dollar against a basket of important global currencies, including the US dollar. Specifically, the Australian dollar recorded its strongest level, being its highest value (highest exchange rate), against the US dollar of 110.56 cents on July 28, 2011 (Ali and Rahman 2012), attaining this level due mainly to the export volumes of Australian minerals supported by the deemed insatiable demand of its largest commodities' trading partner, China. The value of a commodity currency typically increases or decreases depending on the value of the country's primary commodity exports. Commodity prices affect the strength and weakness of the AUD including the AUD/USD exchange rate as a direct consequence of Australia's dependency on mining and, to a lesser extent, farming exports. Therefore, when there is a mining boom, Australia's economy improves and, conversely, when a downturn in the resources sector occurs, Australia's economy is negatively impacted. Therefore, the typical return on investment (ROI) in the mining and minerals industry is heavily biased towards commodity prices and commensurate exchange rate volatilities. During 1996-2014, the lowest or weakest AUD/USD exchange rate was 0.4857, recorded on April 2, 2001, and the highest or strongest exchange rate was 1.1056, recorded on 28 July 2011. On December 24, 2014, the Australian dollar depreciated to a low of 81.12 US cents, and it has fallen almost 14.23 per cent from its July 02, 2014 peak of 94.58 US cents. On February 3, 2015, Jamie Smyth reported that the Australian dollar depreciated to a five and half year low of 76.67 US cents (Smyth, 2015). Since then, the Australian dollar has continued to depreciate, and on January 11, 2016 it was trading at 69.76 US cents (Source: RBA).

However, Rob Brierley, the head of research at Paterson's Securities, explained that since commodities are sold in US dollar prices, the weak Australian dollar sometimes is a boom and gives relief to Australian miners (Babs, 2013). Gilroy (2014a) also described that weakening currencies provide some relief to iron ore producers, and added that Australia's two main export-revenue earners are iron ore and coal, and they have been negatively impacted by the sharp price drops this year. Australia's currency has not typically depreciated at the same rate of weakening as the basket of commodity prices, but it does continue to depreciate. This is somewhat positive for domestic producers including Rio Tinto (RIO), BHP Billiton (BHP) and Fortescue Metals Group (FMG). For commodity countries such as Australia and Brazil, when commodity prices decline, export revenues also usually decline. Therefore, applying the same logic to Brazil, it is noted that its currency has also depreciated against the US dollar. This is positive for domestic producer, Vale.

On December 01, 2014, Barry FitzGerald, a resource editor, reported that "Weaker dollar cushions gold producers from slump". In his report, it was cited that Surbition (a Melbourne-based consultant) director Sandra Close released a survey that in the face of negative sentiment on the US dollar gold price, the industry's capability to maintain production was due basically to the weakening of the Australian dollar, the impact of gold hedging activities and the high grade of gold operations. Close added that while the US dollar gold price continued to trend downwards, the Australian dollar gold price was more stable, particularly over the last few months, since it benefitted from the lower Australian dollar exchange rate (FitzGerald, 2014). The reason behind this is that the Australian mining industry relies on its Australian labour force and commensurately an Australian cost base. Commodities are priced in US dollars and incomes from sales are converted to Australian dollars at the weaker exchange rate, therefore effectively receiving a higher income in Australian dollars when converting to the local currency. This may be good for mineral commodity exporters, but it may not be so good for Australia's overall economy and the importers of mining equipment, as equipment prices are often paid in US dollars. Reciprocally, as costs approach levels closer to revenue levels due to real domestic cost increases, an exchange rate appreciation may place the miner into a loss-making position.

Therefore, it is crucial for the mining and relevant industry participants to understand the relationship between economic variables, such as mineral commodity prices and exchange rates, so that better assessments or more accurate methodologies in calculating the project revenue as well as capital and operating costs can be performed. As a consequence, the mine management team should acknowledge the relationship between mineral commodity prices and the exchange rate, and thereafter determine the numerical value (correlation coefficients) for evaluating mining projects before making and acting on an investment decision or decisions around other investment commitments. This is required in order to estimate the mining project values considering more than one uncertainty through a continuous time stochastic model. That is, the correlation parameter is required. Furthermore, for minimizing risk and maximizing profits due to several uncertainties, mining companies need to incorporate a useful hedging strategy. Regrettably, limited literature is available surrounding the long term relationship of a mineral commodity price and the exchange rate and their strength of association or correlation. Therefore, the relationship between mineral commodity prices and exchange rates has been investigated in this chapter. Some parameters of this relationship will be used in the ROV method in chapter 5 for approximating the numerical values of a mining project.

For this investigation, two case studies are considered:

1. Relationship between gold prices in US dollars and the Australian dollar-US dollar (AUD/USD) exchange rate.

2. Relationship between iron ore prices in US dollars and the value of the Australian dollar i.e. AUD/USD exchange rate.

In exploring these relationships between mineral commodity prices and AUD/USD exchange rates, the concept of Dutch Disease needs to be discussed as it relates to Australia.

4.2 Dutch Disease concept in terms of Australia

The impact associated with Dutch Disease, a term that originated in the Netherlands during the 1960s when high income levels were noted as being generated from the country's natural gas discoveries, led to a notable decline in the competitiveness of the country’s other, non-booming, non-gas sector. What eventuated was that the country's skills generally diverted to the booming energy and gas sectors, but in the longer term, at a cost. The cost was the arising reduced focus on other sectors, which culminated in a significant skills-improvement (including specialisation) in the gas and energy sectors, but simultaneously with a notable decline in the acquired skills in the non-gas and non-energy sectors. In spite of the revenue windfall associated with the new gas discoveries, over time the Netherlands experienced a drastic decline in overall economic growth. This seems counter-intuitive but is explained by the removal from industry of skills not aligned to gas and energy. This paradox has been recognized as the situation in which a booming sector adversely affects the performance of other sectors of an economy. Over the past two decades, voluminous literature covering the Dutch Disease has examined the commodity booms experienced by some countries. Globally, the petroleum boom from 1973 to 1979 produced the most generally significant consequences.

In Australia, the commodities boom drained significant skills away from other sectors and other States wherein these skills were applied, and placed them in the iron ore-rich region of the Pilbara in Western Australia. However, with the recent decline in iron ore prices, these skills are again available to other sectors, but the Dutch Disease concept had already occurred with the pull or migration away from other sectors with these skills taken up in the iron ore sector.

Taking this outline to Australia's currency and its relative strength against other currencies, Chinese demand for iron ore and metallurgical coal meant that, while the world suffered at the hands of the Global Financial Crisis (GFC), Australia’s economy and hence currency strength was somewhat protected due to the demand for the country’s commodities. However, as China's demand for raw products reduced amid a slowing economy, the iron ore and coal prices have declined and, commensurately, Australia's commodity-dependent currency, referred to as a commodity currency, has been exposed and the country’s currency has weakened against its trading partners due to the lower demand for Australia’s natural resources from China.

4.3 Necessary tests for the investigation of the relationship between commodity price and exchange rate

In time series data analysis in econometrics, it is very important to ascertain whether the data are in a stationary or non-stationary condition. Non-stationary time-series data can be regarded as potentially a major problem for empirical econometrics analysis. It is known that trends, either stochastic or deterministic, may generate spurious regressions, uninterpretable student-t values and other statistical results that render regression results rather problematic to evaluate. Therefore, before further consideration is given to the bivariate model for commodity prices and exchange rates, the unit root test will be performed to examine whether the data series is stationary or non-stationary.

To investigate the long-term relationship between the commodity price and the exchange rate, cointegration tests will be performed. Standard regression approaches, such as the ordinary least squares (OLS) method, require that the variables be covariance stationary (or stationary). Cointegration investigation offers an outline for estimation and interpretation when the variables are not covariance stationary.

Vector Error Correction Model (VECM) analysis is performed to estimate the speed at which a dependent variable returns to equilibrium after a change in an independent variable. In addition, VECM and vector auto regression (VAR) tests have been employed for testing the stability of this relationship. To test the causality relationship between commodity price and exchange rate, a Granger Causality test is performed, as it is a statistical hypothesis test for determining whether a one time series variable is useful in forecasting another time series variable. Granger Causality effects are revealed through the VAR model.

The structural vector auto regression (SVAR) model is used as a new powerful tool for analysing the impulse-response function (IRFs) to track the responses of a system’s variables due to contemporaneous shocks on the systems. It is also very useful to analyse the forecast error variance decomposition (FEVD). Finally, correlation analyses have been shown between the mineral commodity price and the exchange rate through the Spearman rank correlation coefficient method. The above methodologies are described as follows.

4.3.1. Unit root tests for commodity price and exchange rate

It is often visually difficult to forecast the nature of the variables i.e. time series data, and to ascertain whether they are stationary or not. However, taking the first difference may result in a stationary process (Box and Jenkins 1970). Therefore the formal test for unit root of the data must be undertaken. There are several tests available to determine a stationary state including Dickey-Fuller and DF-GLS tests for unit root. However, the standard and most popular test for unit root was pioneered by Dickey and Fuller (Dickey-Fuller unit root test).

For this test, consider the autoregressive AR (1) model:

$$X_t = \delta X_{t-1} + \varepsilon_t \quad (4\text{-}1)$$

Here X_t represents the commodity price at time t or exchange rate at time t, and the ε_t represents independently and identically distributed (i.i.d) term with mean zero and a finite variance σ^2.

If $\delta = 1$, then the model (4-1) follows a random walk without a drift and it has a unit root. Running the regression, it is not possible to estimate the model regressing the series on its lagged value because the t-statistics for the δ coefficient are severely biased due to the presence of a unit root. Therefore, it is necessary to manipulate the equation (4-1) and express it differently.

Subtracting the lagged value from both sides, the following is obtained:

$$X_t - X_{t-1} = (\delta - 1)X_{t-1} + \varepsilon_t \Rightarrow \Delta X_t = (\delta - 1)X_{t-1} + \varepsilon_t$$

Define: $\rho = (\delta - 1)$, then $\Delta X_t = \rho X_{t-1} + \varepsilon_t$ (4-2)

Now estimate this model (4-2) and obtain ρ and test the hypothesis:

$H_0 : \rho = 0$ (implies that there is a unit root and the series is non-stationary), and the against hypothesis

$H_A : \rho < 0$ (implies that there is no unit root and the series is stationary).

The Dickey-Fuller test is conducted here in the following cases depending on whether a series displays a trend or not.

$$\Delta X_t = \rho X_{t-1} + \varepsilon_t \quad (4\text{-}3)$$

Equation (4-3) represents a random walk with no drift, no time trend. However, equation (4-4) represents a random walk with drift, no time trend.

$$\Delta X_t = \alpha + \rho X_{t-1} + \varepsilon_t \tag{4-4}$$

To develop this unit root test, repeat the model in equation (4-4) and include a time trend along with a constant term α and a coefficient ρ, which are important in the test statistics to be developed.

This extended model is shown in equation (4-5) below:

$$\Delta X_t = \alpha + \rho X_{t-1} + \partial t + \varepsilon_t \tag{4-5}$$

For the above cases (4-3), (4-4) and (4-5), the test hypotheses are:

$H_0 : \rho = 0$ (means that there is a unit root, and the series is non-stationary or it has a stochastic trend).

Against $H_A : \rho < 0$ (means that there is no unit root and the series is stationary).

To eliminate the possibilities of serial correlation in the lagged variables, the regression in equation (4-5) can be extended by measuring k lagged differences and fitting a model as shown in equation (4-6) below:

$$\Delta X_t = \alpha + \beta X_{t-1} + \partial t + \sum_{i=1}^{k} \phi_i \, \Delta X_{t-i} + w_t \tag{4-6}$$

Equation (4-6) represents the Augmented Dickey-Fuller (ADF) regression with k lags, where Δ is the difference operator, x_t is the variable whose time series properties needed to be examined and w is the white noise error term. In this study, different methodologies (ADF and DF-GLS) for unit root tests have been employed for econometric analysis through STATA software to obtain the more accurate decision around whether the time series has unit root or not.

4.3.2 Cointegration test for long-run relationship between commodity price and exchange rate

The concept of cointegration was first introduced by Granger (1981). Granger (1981) and Granger and Weiss (1983) pointed out that a vector of variables, all of which achieve a stationary state after differencing, may have linear combinations which are stationary without differencing. Thereafter, Engle and Granger (1987), in their seminal paper, delivered a firm theoretical base for the representation, testing, estimating and modelling of cointegrated non-stationary time-series variables.

For the cointegration, in a bivariate model for the variables Y_t and X_t, there exists a (cointegrating vector) β such that $Y_t - \beta X_t$ is I(0) i.e. stationary even though Y_t and X_t are non-stationary processes. This indicates that two variables are cointegrated or have a long-run equilibrium relationship with Y_t and X_t being individually stochastic. The cointegration analysis offers applied econometricians an effective formal framework for testing models and estimating long-run relationships among cointegrated variables from the actual time-series data. For the cointegration test, the Johansen cointegration methodology has been employed.

4.3.2.1 The Johansen cointegration test for commodity price and exchange rate

A test for cointegration will be realistic when the variables i.e. data sets, satisfy the fundamental condition that they contain unit roots. The Johansen methodology (Johansen, 1991, 1995, 1988; Johansen and Juselius, 1990) is a maximum likelihood approach for testing cointegration in multivariate autoregressive models. The Johansen test relies on the relationship between the rank of the matrix and its characteristic roots. This method is used here to examine the cointegrating relationship between the commodity price and the exchange rate. The Johansen trace statistics method (Johansen, 1991) has been conducted in this research for determining r which is widely used for testing cointegration. The null hypothesis for the trace test is that there is at most r $(0 \le r < p)$ cointegrating vectors among p variables. Therefore, the trace test i.e. the likelihood ratio (LR) test, statistics for the hypothesis that there are at most r distinct cointegrating vectors against a general alternative of n cointegrating vectors is given by

$$\lambda_{trace} = -T \sum_{j=r+1}^{n} \ln(1-\hat{\lambda}_j) \qquad (4\text{-}7)$$

where $j = r+1, \cdots\cdots, n$, are the $(n-r)$ smallest squared canonical correlations, $r = 0,1,2,\ldots\ldots, n-1$, and $\lambda_{trace}(r) = 0$, when $\lambda_j = 0$, and T is the sample size.

The test statistics do not follow a chi-square distribution in general. However, asymptotic critical values can be found in Johansen and Juselius (1990), MacKinnon et al. (1999), and are also specified by most econometric software packages.

4.3.2.2 Vector error correction (VEC) model for commodity prices and exchange rates

If the variables are cointegrated under the Johansen cointegartion test, it is essential to estimate the vector error-correction model (VECM). For the estimation of the VECM for the commodity price and the exchange rate, the following vector error-correction model has been considered:

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots\ldots\ldots + \Gamma_{k-1} \Delta X_{t-k+1} + v + \delta t + \varepsilon_t \qquad (4\text{-}8)$$

Equation (4-8) can be written as

$$\Delta X_t = \Pi X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + v + \delta t + \varepsilon_t \qquad (4\text{-}9)$$

Now substituting $\prod = \alpha \beta'$ in equation (4-9), it becomes

$$\Delta X_t = \alpha \beta' X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + v + \delta t + \varepsilon_t \qquad (4\text{-}10)$$

where, α represents the coefficient matrix of the error correction term and adjusted long-run disequilibrium of the variables, whereas β represents the coefficient matrix for the cointegrating vectors. Economic importance is placed on coefficients α and β .For the existence of cointegration, matrices α and β need to be reduced in rank. Γ_i are coefficients that estimate short–run shock effects on ΔX_t. υ represents the constant term and δt represents the linear time trend term, whereas ε_t represents the identically and normally distributed error term.

4.3.2.3 Vector auto regressive (VAR) model for commodity prices and exchange rates

There is an issue of whether the variables in a vector auto regression (VAR) need to be stationary or non- stationary. Sims (1980) and Sims et al. (1990) suggested against differencing even if the variables contain a unit root. These authors argued that the goal of a VAR analysis is to determine the interrelationships among the variables, not to just determine the parameter estimates (Enders 2004, pp-270). Hence, according to the recommendation of these authors VAR can be used both for the stationary and the non-stationary variables. Another issue is whether VAR models will be estimated for variables at levels or variables at first differences, that is I (1) variables. When the variables are I (1) stationary and cointegrated, VAR needs to be estimated at levels, because differencing discards information contained in the cointegrating relationships. If the variables are I (1) and not cointegrated, it would be better to estimate VAR at first differences (Enders 2004, pp-358).

For the estimation, the following 2×2 VAR (k) models for commodity prices and exchange rates are considered:

$$\ln CP_t = \alpha_{10} + \sum_{j=1}^{k} \alpha_{1j} \ln CP_{t-j} + \sum_{j=1}^{k} \alpha_{2j} \ln EX_{t-j} + e_{1t} \tag{4-11}$$

$$\ln EX_t = \beta_{10} + \sum_{j=1}^{k} \beta_{1j} \ln CP_{t-j} + \sum_{j=1}^{k} \beta_{2j} \ln EX_{t-j} + e_{2t} \tag{4-12}$$

Where $\ln CP_t$ represents the natural logarithmic price of commodity and $\ln EX_t$ represents the natural logarithmic value of exchange rate, and e_{1t} and e_{1t} are stochastic disturbances i.e. error terms.

4.3.3 Correlation Analysis

For any investigation of the relationship between two logically linked variables, the correlation analysis is proven to be a useful tool for determining the strength of that relationship (Ali and Rahman 2012; Robert, 2006). Robert (2006) stated that correlation is primarily concerned with finding out whether a relationship exists and with determining its direction and strength. If the correlation between two variables X and Y is positive, then they are directly related. That is, when X increases, Y tends to increase. However, if the correlation is negative, then X and Y are inversely related which means when X increases, Y will tend to decrease, and vice versa. The Pearson correlation coefficient method measures the strength of the linear relationship between variables X and Y. However, in the case of nonlinear, but monotonic relationships, a suitable measure is the Spearman's rank correlation coefficient method. Spearman's rank correlation is a non-parametric test which is used to measure the degree of association between two variables. It is usually defined by the symbol ρ, and can be computed by the following mathematical formula:

$$\rho = 1 - \frac{6\sum d_i^2}{n\left(n^2 - 1\right)} \tag{4-13}$$

where

ρ = Spearman’s rank correlation coefficient

d_i = the difference between the ranks of corresponding values X_i and Y_i

n = number of value in each data set.

The Spearman’s rank correlation coefficient method has been used to investigate the strength of association i.e. correlation, between commodity prices and exchange rates.

4.3.4 Granger causality test

Granger (1969) came out with a decision for testing a causality relationship in econometrics. According to Granger, in a bivariate model, a variable X_t is said to be Granger-caused for another time series variable Y_t, if X_t helps to predict Y_t. Using models (4-11) and (4-12) the Granger Causality effects between commodity price and exchange rates have been presented.

4.3.5 The impulse–response functions (IRFs) analysis in structural VAR (SVAR)

According to Kilian (2013) structural VAR models are continually used as workhorses of macroeconomics and finance, and they have four main applications. Two of the main applications are described as, firstly, to study the average response of the model variables to a given one time structural shock, and secondly, they are applied in the construction of the FEVDs which compute the average contribution of structural shock due to the variability of data. As a consequence, the structural VAR model has been considered here for the analysis of the IRFs, and the FEVDs between commodity prices and exchange rates.

The following SVAR model was considered here without exogenous variables.

$$A\left(I_K - A_1 L - A_2 L^2 - \dots\dots\dots - A_P L^P\right) y_t = A u_t = B w_t \qquad (4\text{-}14)$$

where A, B, and A_i, $i = 1, \dots\dots, P$ are $K \times K$ matrices of parameters, L is the lag operator, u_t is $K \times 1$ vector of a mean zero serially uncorrelated error term, also referred to as pure structural innovations or structural shocks, and w_t is a $K \times 1$ vector of orthogonalized disturbances. The variance–covariance matrix of the structural error term is generally normalized as $E(u_t\, u_t') \equiv \sum_u = I_K$.

The matrices A and B are assumed to be non-singular, and restrictions are placed on their elements. The Cholesky restrictions have been imposed on this system by applying equality constraints with the constraint matrices $A = \begin{bmatrix} 1 & 0 \\ . & 1 \end{bmatrix}$ and $B = \begin{bmatrix} . & 0 \\ 0 & . \end{bmatrix}$, where . denotes a free parameter, and $y_t = \begin{bmatrix} fdLNCP \\ fdLNEX \end{bmatrix}$. Here *fdLNCP* denotes first difference of natural logarithm of commodity prices and *fdLNEX* denotes first difference of the natural logarithm of exchange rates.

4.4 Case study 1: Relationship between gold prices in US dollars and the Australian dollar-US dollar (AUD/USD) exchange rate

Historically, gold was first discovered in Australia by James Mcbrien in 1823 in New South Wales. Additional traces of gold were found in the following decades in New South Wales and in Victoria. Gradually, the Australian golden age started, and by the 1850s, almost 40 per cent of the world’s gold was produced in this country (Source: Australia’s Mining History, AMH, 2015). However, the Western Australian (WA) gold rush began with the first discovery of gold only in the late 1890s. The gold was first officially discovered in WA in 1892 by William Ford and Arthur Bailey in the Coolgardie region which was previously named Fly Flat. Coolgardie was believed by many to be the mother of the Western Australian goldfields. The second gold rush occurred in the Kalgoorlie area (WA), with the name Kalgoorlie being said to be derived from an Aboriginal word meaning “place of the silky pears”. A trio of Irish gold prospectors, Paddy Hannan, Tom Flannagan and Dan Shea, found gold near Mount Charlotte in 1893. Thereafter, news of the gold discoveries spread as fast as the region's wildfires and soon gold prospectors were arriving to seek their fortunes and set up gold rush towns in the dusty landscapes of the Kalgoorlie, Goldfields and Murchison regions (Source: Australia's Golden Outback, Gold rush history). According to an AIMR 2014 report, Australia continued to hold the largest gold resource of about 18% of the world total, whereas South Africa and Russia maintained their places as hosting the second and third largest gold resource inventories, respectively, in the world rankings (AIMR, 2014).

The price of gold is considered as one of the major factors which affect the AUD/USD nominal exchange rate (spot rate) because the commodity is deemed fundamental to the Australian economy (Han et al., 2012). The uncertainty of commodity prices in the international market is an important factor for the mining industry. Babs (2013) stated that all commodity prices have declined - iron ore, coal, copper, lead, silver, phosphate, and zinc, but none of them more intensely than gold, which lost US$200 per ounce in April 2013 alone. At that time gold traded at around US$1,300 per ounce which was down from US$1,700 per ounce three months prior to that. In Australian dollar terms, this translates to around AU$1,400 per ounce. Currently, gold is trading at around US$1,081/oz and this has created negative sentiment around the gold mining industry. In 2015, the price slump of iron ore to 6 year lows was worse than gold's price slump. Hewett (2015) mentioned that iron ore was trading below US$50 per tonne which was about half of its average price of the previous year, and it is anticipated to drop further.

Australia is facing its greatest challenge with the collapse in the price of mineral commodity exports including iron ore, gold, coal and liquefied natural gas. A drop in mining investment and fears over the economic growth of China are casting gloominess over the Australian economy which has not experienced a recession in almost a quarter of a century. The volatility of the gold price is a vital factor for the Australian gold mining industry and, therefore, the overall Australian economy. When the price of gold in international markets rises, it encourages the production of additional gold in Australia which evidently provides stronger support than expected to the Australian economy and, consequently, leads to the appreciation of the AUD against the USD. Furthermore, if the demand for gold increases, demand for AUD will increase to buy the gold which will cause an appreciation (strengthening) of the AUD against the USD.

It is therefore crucial for gold mining and other relevant industry participants to understand the relationship between gold prices and AUD/USD exchange rates, so that improved and more accurate assessments can be made when calculating the project revenue in conjunction with capital and operating costs for investment decision making in a gold mine. The information derived from this relationship is used for new ROV techniques utilizing hedging strategies and by solving the resulting PDE to approximate the mining project values (in Chapter 5). Other economic and noneconomic parameters such as other commodities, inflation, countries' exports, demand and supply may have influences on the AUD/USD exchange rate, but it is beyond the scope of the present study to contemplate these additional parameters.

Unfortunately, very little literature covering the relationship of gold prices and the exchange rate movements between Australian dollars against the US dollar exists. As a consequence, this study of the relationship between the AUD/USD exchange rate and the gold price will clearly provide thought provoking results to the mining and associated investment industry, and more broadly to the general economy in Australia. There is no other industrial economic endeavour that has such a profound influence on the Australian economy (Huleatt and Jaques 2005; Mudd 2007).

4.4.1 Sources of data for the investigation of the relationship between gold prices and AUD/USD exchange rates

In order to perform this empirical study, gold price and AUD/USD exchange rate data have been collected from different sources. While the spot gold price (cash price) data have been collected from kitcogold.com, the AUD/USD exchange rate spot data have been collected from the Reserve Bank of Australia (RBA). Since both these assets have forward prices, only the commonly reported headline rate is necessary, which is the spot rate. Gold prices based on the US dollar and the historical AUD/USD exchange rate data have been collected from the trading period between 1996 and 2014. Both the gold price and exchange rate data are stated in nominal money terms for the entire test. For this analysis, weekly average data in gold prices and AUD/USD exchange rates have been used. The weekly averages for both series have been calculated. For the purposes of empirical analysis, the natural logarithm of the gold price and the natural logarithm of the AUD/USD exchange rates have been calculated and used for the entire analysis. Moreover, percentage log returns for gold prices and AUD/USD exchange rates have been calculated by taking the daily difference of natural logarithms and multiplying by 100. For all the econometrical and statistical tests, STATA version 13c (STATA, 2013) software has been used.

4.4.2 Unit root tests for gold prices and AUD/USD exchange rate

ADF and DF-GLS tests have been performed through STATA software (STATA, 2013) for achieving a more appropriate decision around whether the time series has unit root or not, to explore the relationship between gold prices and exchange rates (AUD/USD). In Table 4-1 to 4-6, ADF and DF-GLS tests results are only reported at 1%, 5% and 10% significance levels with critical values. For the entire unit root test, the null hypothesis shows the variable has a unit root i.e. non stationary. If the test statistics value (i.e. absolute value) is less than the critical value (i.e. absolute value) or the probability value is greater than 5%, the null hypothesis cannot be rejected.

ADF test results are presented in Table 4-1 and Table 4-2. From the ADF test results in Table 4-1, the null hypothesis cannot be rejected. Therefore, both time series have unit roots i.e. both series are non-stationary at level (without taking first difference).

Table** 4-1 Results of ADF test for unit roots at level (gold prices and AUD/USD exchange rates).

Variables at level	Specification	Lags	Test Statistics Z(t)	1% critical value	5% critical value	10% critical value	McKinnon approximate p value for Z(t)	Decisions
Gold price	Trend	3	-2.745	-3.960	-3.410	-3.120	0.2180	Non Stationary
	Drift	6	0.121	-2.331	-1.647	-1.283	0.5481	Non Stationary
	Regress with no constant	1	1.523	-2.580	-1.950	-1.620		Non Stationary
Exchange rate	Trend	3	-2.722	-3.960	-3.410	-3.120	0.2269	Non Stationary
	Drift	6	-1.204	-2.331	-1.647	-1.283	0.1144	Non Stationary
	Regress with no constant	1	-0.895	-2.580	-1.950	-1.620		Non Stationary

However, when first differences are taken, the null hypothesis is rejected at a 1% significance level (Table 4-2) for both the variable's gold prices and the exchange rate (AUD/USD), and therefore both series are stationary at first differences.

Table 4-2 Results of ADF test for unit roots at first difference (gold prices and AUD/USD exchange rates).

Variables at first difference	**Specification**	**Lags**	**Test Statistics Z(t)**	**1% critical value**	**5% critical value**	**10% critical value**	**McKinnon approximate p value for Z(t)**	**Decisions**
Gold Price	Trend	3	-15.939	-3.960	-3.410	-3.120	0.0000	
	Drift	6	-12.086	-2.331	-1.647	-1.283	0.0000	Stationary
	Regress with no constant	4	-14.733	-2.580	-1.950	-1.620		Stationary
Exchange rate	Trend	3	-13.281	-3.960	-3.410	-3.120	0.0000	Stationary
	Drift	5	-10.529	-2.331	-1.647	-1.283	0.0000	Stationary
	Regress with no constant	2	-15.216	-2.580	-1.950	-1.620		Stationary

DF-GLS test results are provided through Tables 4-3 to Table 4-6 (to reduce the table size, only a few values are shown in each table). It has been observed that the null hypothesis cannot be rejected for the series gold prices for all lags 1 to 20 both in no trend specification and in Elliott et al. (1996) proposed Elliott, Rothenberg and Stock (ERS) specification (Table 4-3). Therefore, the time series has unit roots i.e. it is non–stationary.

From Table 4-4, it has been observed that the null hypothesis has been rejected for time series gold prices at a 1% significance level for all lags 1 to 20 both in no trend specification and ERS proposed specification. Therefore, the time series has no unit roots i.e. stationary when first differences are taken.

Table 4-5 presents the DF-GLS test result, showing that the null hypothesis can be accepted for AUD/USD exchange rate both in no trend specification and ERS specification. Therefore, this test has confirmed that the time series has unit roots i.e. non stationary in levels.

From Table 4-6 it has been observed that the null hypothesis has been rejected for time series AUD/USD exchange rates at a 1% significance level for all lags 1 to 20 both in no trend specification and ERS specification. Therefore, the time series has no unit roots i.e. both series are stationary when first differences are taken.

Table 4-3 Results of DF-GLS test for unit roots in gold prices (variables in level).

Variable in level	**Specification**	**Test Statistics**		**Lags**	**1% critical value**	**5% critical value**	**10% critical value**	**Decision**
		Tau	**mu**					
Gold price	No trend		0.784	20	-2.580	-1.947	-1.628	Non Stationary
			…….	….	….	….	…..	……
			1.148	10	-2.580	-1.959	-1.639	Non Stationary
			……	….	……	…..	……	……
			0.651	1	-2.580	-1.969	-1.647	Non Stationary
	*ERS	-0.883		20	-3.480	-2.890	-2.570	Non Stationary
		…..		…	…..	…..	…..	……
		-0.756		10	-3.480	-2.890	-2.570	Non Stationary
		……		….	…	….	…..	……
		-1.029		1	-3.480	-2.890	-2.570	Non Stationary

**Note: ERS means Elliott, Rothenberg and Stock presented interpolated critical values which were proven in 1996. They proposed a modified version of the Dickey Fuller t test which has substantially improved its applicability when an unknown mean or trend is present.*

Table 4-4 Results of DF-GLS test for unit roots in gold prices (variable in first difference).

Variable in first difference	Specification	Test Statistics		Lags	1% critical value	5% critical value	10% critical value	Decision
		Tau	mu					
Gold price	No trend		-4.978	20	-2.580	-1.947	-1.628	Stationary
			….	….	….	….	…..	…..
			-8.148	10	-2.580	-1.959	-1.639	Stationary
			…..		…..	…..	…..	….
			-19.395	… 1	-2.580	-1.969	-1.647	Stationary
	ERS	-5.415		20	-3.480	-2.890	-2.570	Stationary
		….		….	….	….	….	…..
		-8.588		10	-3.480	-2.890	-2.570	Stationary
		….		….	….	….	….	….
		-19.629		1	-3.480	-2.890	-2.570	Stationary

Table 4-5 Results of DF-GLS test for unit roots in exchange rates (variable in level).

Variable in level	**Specification**	**Test Statistics**		**Lags**	**1% critical value**	**5% critical value**	**10% critical value**	**Decision**
		Tau	**mu**					
Exchange rate	No trend		-1.282	20	-2.580	-1.947	-1.628	Non stationary
			…..	….	….	…..	…..	
			-1.102	10	-2.580	-1.959	-1.639	Non stationary
			…..	…..	……	….	….	……
			-1.115	1	-2.580	-1.969	-1.647	Non stationary
	ERS	-1.429		20	-3.480	-2.890	-2.570	Non stationary
		…..		…	….	….	….	…..
		-1.253		10	-3.480	-2.890	-2.570	Non stationary
		….		…..	….	….	…..	…..
		-1.267		1	-3.480	-2.890	-2.570	Non stationary

Table 4-6 Results of DF-GLS test for unit roots in exchange rates (variable in first difference).

Variable in first difference	Specification	Test Statistics		Lags	1% critical value	5% critical value	10% critical value	Decision
		Tau	mu					
Exchange rate	No trend		-4.437	20	-2.580	-1.947	-1.628	Stationary
			….	….	….	….	….	….
			-7.140	10	-2.580	-1.959	-1.639	Stationary
			….	….	….	….	….	….
			-17.534	1	-2.580	-1.969	-1.647	Stationary
	ERS	-4.946		20	-3.480	-2.890	-2.570	Stationary
		….		….	….	….	….	….
		-7.682		10	-3.480	-2.890	-2.570	Stationary
		….		….	….	….	….	….
		-17.973		1	-3.480	-2.890	-2.570	Stationary

***** Note :*** *1. For the entire unit root testing (Table 4-1 to Table 4-6) the null hypothesis: The variable has a unit root (i.e. non stationary)*

2. For all the figures and tests both the gold prices and exchange rates data have been used in nominal terms.

From the tests, ADF and DF-GLS, it has been confirmed that both the gold price and exchange rate time series are I(1) stationary. Therefore, to investigate the long-run equilibrium relationship of the gold price and the Australian dollar exchange rate against the US dollar (AUD/USD), the Johansen cointegration test will be employed since it is a commonly used test for determining the behaviour of long-run relationships. Non-stationary time series may generate spurious regressions, uninterpretable student-t values and other statistical results that render regression results rather problematic to evaluate giving rise to non-stationary time-series data, being the main problem for empirical econometrics. Therefore, if the time series is not I(1) stationary or integrated of order 1, then it can be checked for I (2) stationary or whether it is integrated of order 2. Finally, if the time series data are not integrated at the same order, the analysis would be complicated and it may justify the use of the ARDL model for the analysis.

4.4.3 Long term relationship between the gold price and AUD/USD exchange rate

The Johansen trace test has been implemented to determine the cointegrating relationship between gold prices and AUD/USD exchange rates. In Table 4-7, the trace statistics at $r = 0$ exceeded its critical values at 5% significance levels. Hence, the null hypothesis has been rejected for no cointegrating equation which implies that a cointegrating equation exists. From the case $r = 1$, it has been seen that one cointegrating equation exists in the bivariate model.

Table 4-7 Results of Johansen cointegration trace test for gold prices and AUD/USD exchange rate.

Lags	Rank	Maximum likelihood (LL)	Eigenvalue	Trace statistics	5% critical value
3	r= 0	4369.6671	.	18.8736	18.17
	r=1	4375.3439	0.01359	3.5201	3.74
	r=2	4379.1039	0.00902		

Therefore, from the test results it can be concluded that there exists a long-run relationship between gold prices and AUD/USD exchange rates, since a cointegrating equation exists in the bivariate model. In Table 4-7, there is no trace statistics and critical values when r =2 because, in the Johansen cointegration test approach, the consistently maximum rank's eigenvalue (last row) computes the trace statistics and critical value for the previous line at r =1. Similarly the eigenvalue for r=1 computes the trace statistics and critical value for the previous line that reflects r = 0.

4.4.4 Estimation of the VECM parameters for gold prices and AUD/USD exchange rate

From the equation (4-10), the VECM parameters were estimated which have economic importance. The main interests of estimates are the matrix β that contains the cointegrating parameters, α , which is the speed of adjustment. The stability test for the VEC model will also be reported showing that the specified model is stable, and that the specified number of cointegrating equations for the cointegration relationship between gold prices and AUD/USD exchange rates has been determined.

From the VEC model estimation, the long-run disequilibrium adjustment matrix was obtained $\hat{\alpha} = (-0.0196405, 0.0042265)$, and the normalizing cointegrating vector i.e. the β parameters, $\hat{\beta} = (1, -0.50)$. The speed of adjustment parameters -0.0196405 (p-value 0.001) and 0.0042265 (p-value 0.026), and the normalizing beta value -0.50 (p-value 0.000) are significant and meaningful.

The speed of adjustment coefficient -0.0196405 suggests that approximately 1.9 percent of exchange rates per week can be attributed to the disequilibrium between actual and equilibrium levels. The adjustment coefficient 0.0042265 shows that the variability of gold prices induces a positive change in the exchange rate. The normalizing beta value -0.50 shows that a one percent increase in the gold price leads to an appreciation of the AUD/USD nominal exchange rate by approximately 0.5%.

4.4.5 Stability test for VEC model between gold prices and AUD/USD exchange rates

According to Lutkephol (2005) and Hamilton (1994), the stability condition of the VECM is that if the companion matrix (see Appendix 4.4A in Chapter 4) of a

VECM has k endogenous variables and r cointegrating equations, then there will be k-r unit eigenvalues and the remaining eigenvalues lie inside the unit circle.

From Table 4-8 it is observed that all the eigenvalues are less than 1, except one.

Table 4-8 Stability test for cointegration relationship between gold prices and AUD/USD exchange rates (Eigenvalue stability condition).

Eigenvalue	**Modulus**
1	1
0.9730517	0.9730517
0.2670259 + 0 .294281i	0.397372
0.2670259 - 0.294281i	0.397372
-0.03038044 + 0.2577965i	0.25958
-0.03038044 -0 .2577965i	0.25958

Figure 4-1 presents roots of the companion matrix of the VECM between gold prices and AUD/USD exchange rates.

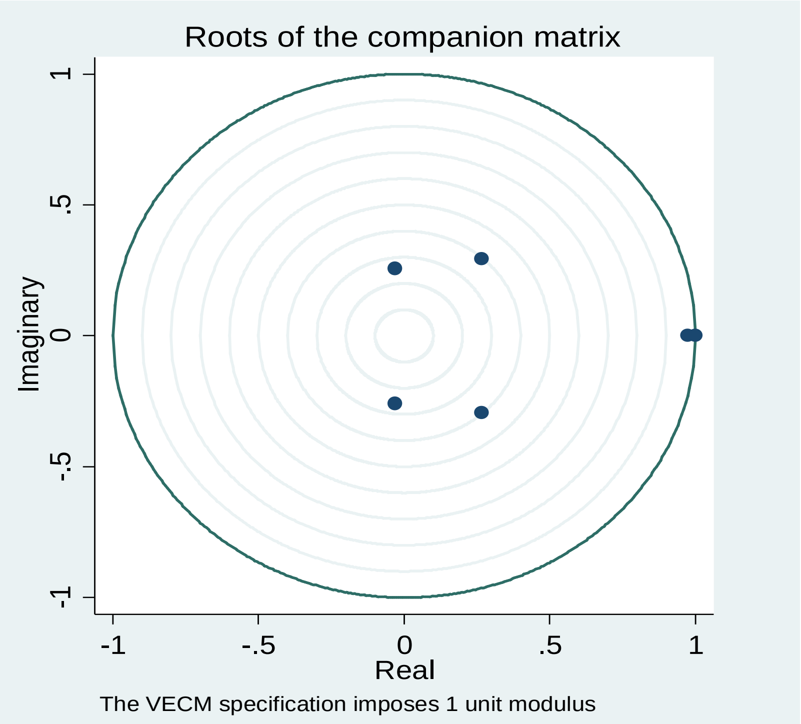

Figure 4-1 The stability of the cointegration relationship between gold prices and exchange rates in the VEC Test.

From Figure 4-1, it can be seen that three eigenvalues lie inside the unit circle and one eigenvalue lies on the circle, because the VECM specification imposes 1 unit modulus. Therefore, from Table 4-8 and Figure 4-1, it is concluded that the VECM satisfies the stability condition for a cointegration relationship between gold prices and AUD/USD exchange rates, and the number of cointegrating equations have been correctly specified.

4.4.6 Granger causality test for gold prices and AUD/USD exchange rates

The results of the Granger causality test are reported in Table 4-9. Here, LNGP means natural logarithmic of gold prices, and LNER means natural logarithmic of AUD/USD exchange rates

Table 4-9 Granger causality Wald tests for gold prices and AUD/USD exchange rates.

Equation	Excluded	*chi2	df	Prob>chi2
LNER	LNGP	26.769	3	0.000
LNER	ALL	26.769	3	0.000
LNGP	LNER	10.471	3	0.015
LNGP	ALL	10.471	3	0.015

** Note: Chi2 means chi-square; df: degrees of freedom*

As can be seen from Table 4-9, the Granger causality test confirmed that there is a bi-directional causality between the gold price and the AUD/USD exchange rate.

4.4.7 Stability test for VAR model

According to Lutkephol (2005) and Hamilton (1994), if the modulus of each eigenvalue of the companion matrix is strictly less than one, then the estimated VAR is stable.

Table 4-10 Eigenvalue stability condition for gold prices and AUD/USD exchange rates

Eigenvalue	**Modulus**
0.9997985	0.9997985
0.9801911	0.9801911
0.2616431 + 0.2947875i	0.394153
0.2616431 - 0.2947875i	0.394153
-0.02758123 + 0.2626929i	0.264137
-0.02758123 - 0 .2626929i	0.264137

From Table 4-10, it is observed that all eigenvalues are less than 1. Moreover, Figure 4-2 shows all the roots of the companion matrix lie inside the unit circle. Therefore, it can be concluded that VAR satisfies the stability condition.

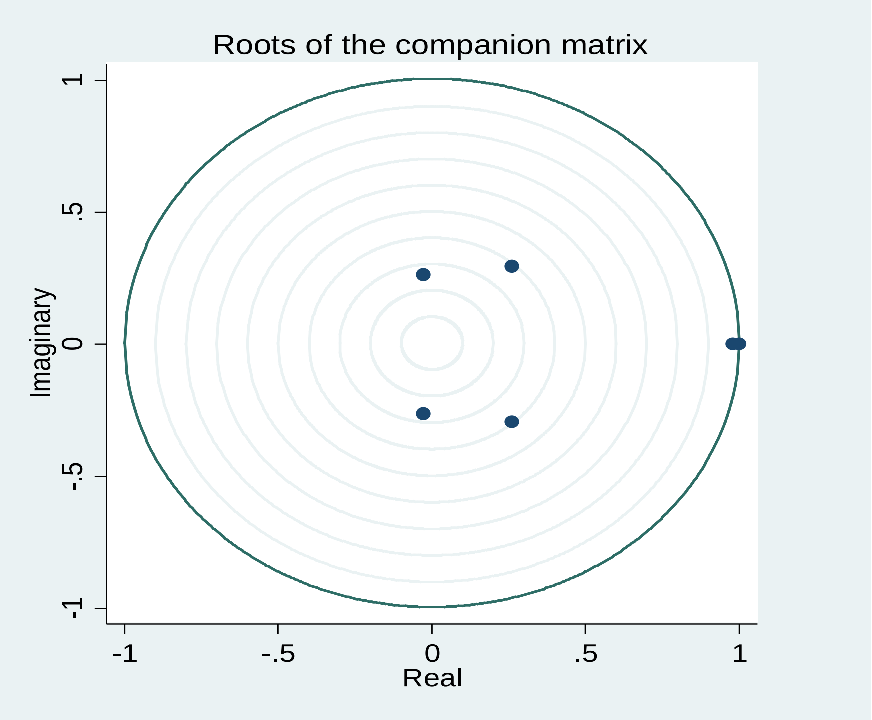

Figure 4-2 The stability of VAR test for gold prices and AUD/USD exchange rates.

4.4.8 The impulse–response functions (IRFs) analysis

The VAR methodology also offers a new powerful tool in the impulse-response function (IRF) analysis to track the responses of a system's variables due to impulses of the system's shocks. However, to show the effect through the impulse–response functions (IRFs) analysis, the structural vector auto regression (SVAR) would be more helpful, so it has been considered here for the IRF analysis.

Figure 4-3 shows the significant response of the fdLNER due to shocks of fdLNGP for some periods and then it dies out after roughly 6 periods. Here, fdLNER means the first differences of natural logarithmic of AUD/USD exchange rates, and fdLNGP means the first differences of natural logarithmic of gold prices.

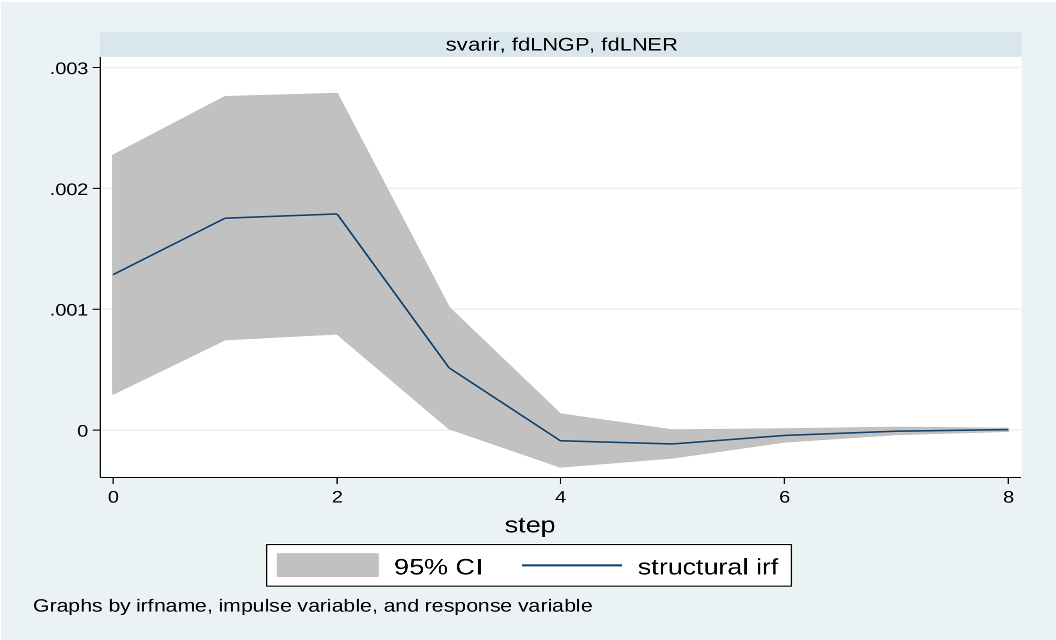

Figure 4-3 The structural impulse (fdLNGP) –response (fdLNER) functions for gold prices and AUD/USD exchange rates.

Figure 4-4 demonstrates the significant response of the fdLNGP due to the impulse of fdLNER for some periods and then it dies out after roughly 5 periods.

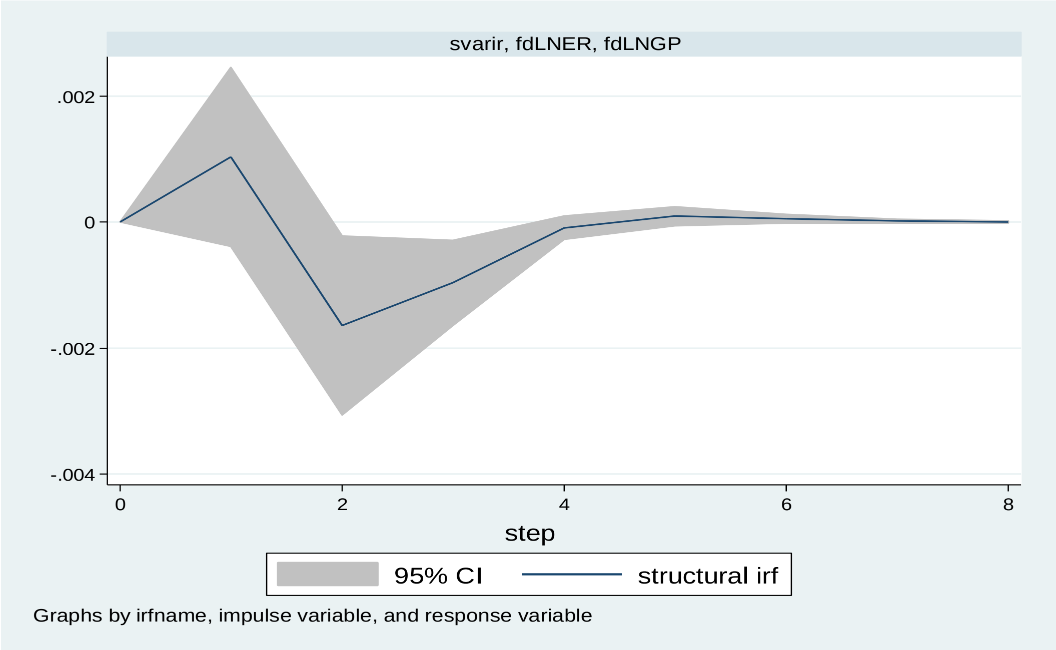

Figure 4-4 The structural impulse (fdLNER) –response (fdLNGP) functions for gold prices and AUD/USD exchange rates.

Hence, the impulse–response functions (IRFs) analysis also supports the idea that gold prices generate significant effects on the AUD/USD exchange rate, and vice versa.

4.4.9 Correlation Analysis between gold prices and AUD/USD exchange rates

The value of the correlation coefficient was determined as 0.91. When time lag differences were considered and the correlation coefficients were determined, the results did not change noticeably. These results can be seen in Table 4-11.

Table 4-11 The value of correlation coefficients between gold prices and AUD/USD exchange rates.

Variables	**Lags**	**Correlation coefficients**
Gold prices and AUD/USD exchange rates	0	0.9117
	3	0.9109
	5	0.9104

4.4.10. Granger causality for gold return and AUD/USD exchange rate return

If the time series is non-stationary at levels, being I (1) stationary, then the log return series should be I (0) stationary. The return series of gold prices and exchange rates at I(0) stationary (as returns series are mainly differencing series) has been examined. Since VAR can be used both for stationary and non-stationary variables, the following 2×2 unrestricted VAR (p) models for the gold returns and the AUD/USD exchange rate returns have been estimated here.

$$\Delta X_t = \alpha_{10} + \sum_{k=1}^{p} \alpha_{1k} \Delta X_{t-k} + \sum_{k=1}^{p} \alpha_{2k} \Delta Y_{t-k} + e_{1t} \quad (4\text{-}15)$$

$$\Delta Y_t = \beta_{10} + \sum_{k=1}^{p} \beta_{1k} \Delta X_{t-k} + \sum_{k=1}^{p} \beta_{2k} \Delta Y_{t-k} + e_{2t} \quad (4\text{-}16)$$

To examine the Granger causality and to test the stability, the tests have been performed and the results are reported in Table 4-12 and Table 4-13, respectively.

Table 4-12 Granger causality Wald tests for returns of gold prices and AUD/USD exchange rates.

Equation	Excluded	chi2	df	Prob>chi2
LNRGP	LNRER	23.71	3	0.000
LNRGP	ALL	23.71	3	0.000
LNRGP	LNRER	11.895	3	0.008
LNRGP	ALL	11.895	3	0.008

From Table 4-12, it has been observed that there exists a bi-directional causality between the continuously compounded percentage return of gold prices and the returns of AUD/USD exchange rates. Here LNRGP denotes the natural logarithmic return of gold prices, and LNRER denotes the natural logarithmic return of AUD/USD. From Table 4-13, it can be seen that all eigenvalues are less than 1. Moreover, Figure 4-5 shows that all the roots of the companion matrix lie inside the unit circle. Therefore, it is concluded that VAR satisfies the stability condition.

Table 4-13 Eigenvalue stability condition for returns of gold prices and AUD/USD exchange rates.

Eigenvalue	**Modulus**
-0.1002284 + 0.4668501i	0.477488
-0.1002284 - 0.4668501i	0.477488
0.4131378	0.4131378
0.2378172 + 0.277409i	0.365394
0.2378172 - 0.277409i	0.365394
-0.22094	0.22094

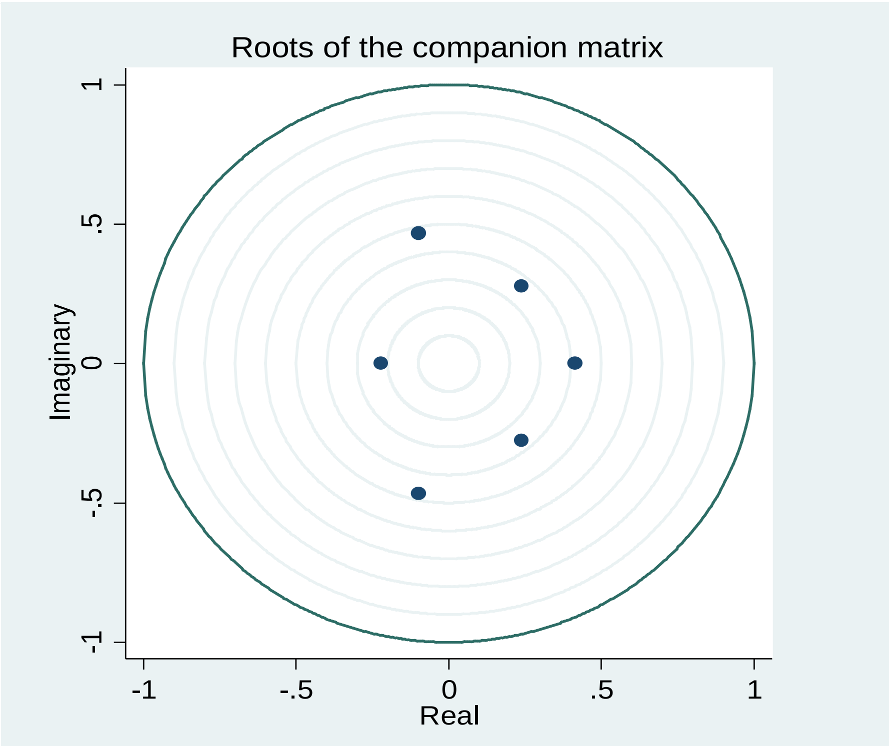

Figure 4-5 The stability of VAR test for gold and AUD/USD exchange rate returns.

4.4.11 The impulse–response functions (IRFs) analysis for gold return and AUD/USD exchange rate return

The structural vector auto regression (SVAR) is also considered here for the IRF analysis between gold and the AUD/USD exchange rate return series.

Figure 4-6 shows the response of fdLNRER due to shocks on fdLNRGP through both the orthogonalized impulse –response function (IRF) and the structural impulse –response function (IRF). Here fdLNRER denotes first differences of the natural logarithmic return of AUD/USD, and fdLNRGP denotes first differences of the natural logarithmic return of gold prices.

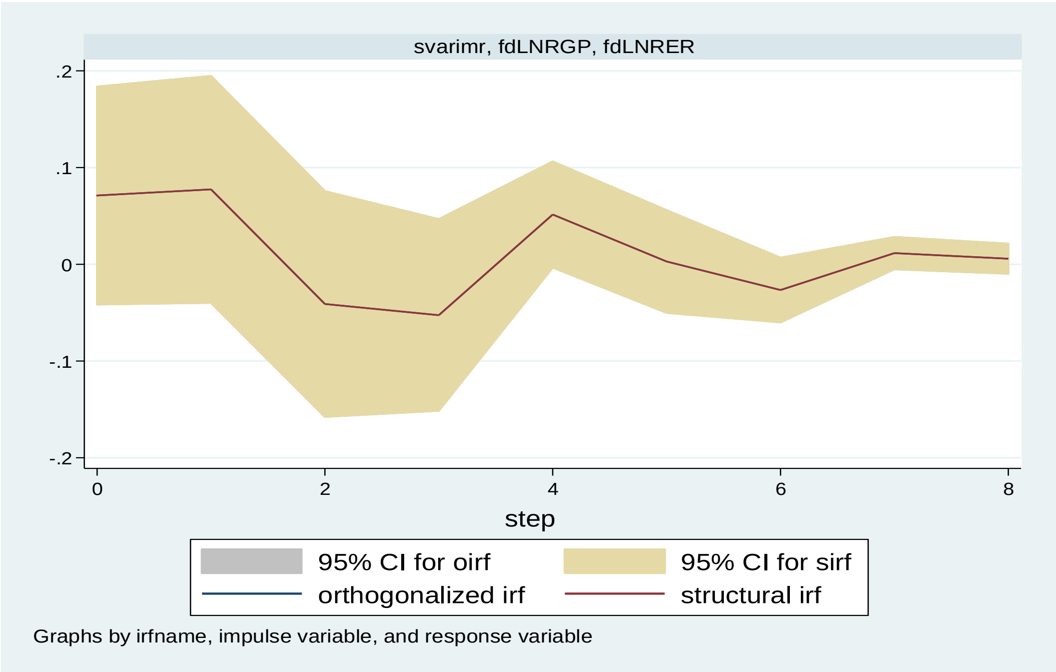

Figure 4-6 The response of fdLNRER due to the impulse on fdLNRGP through both orthogonalized IRF and structural IRF.

Figure 4-7 shows the response of fdLNRGP due to shocks on fdLNRER through both the orthogonalized impulse–response function (IRF) and the structural impulse–response function (IRF).

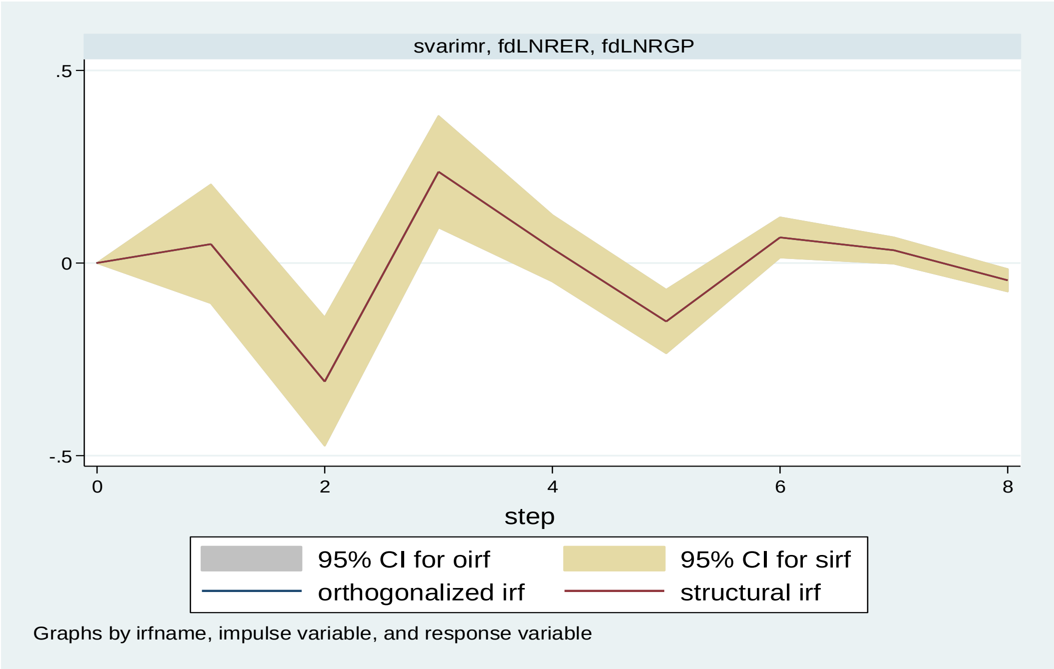

Figure 4-7 The response of fdLNRGP due to the impulse on fdLNRER through both orthogonalized IRF and structural IRF.

From Figure 4-6 and Figure 4-7, interestingly, it is observed that the return of gold prices generates significant effects to the return of AUD/USD exchange rates due to shocks on the return of gold prices, and vice versa.

4.5 Case study 2: Relationship between iron ore prices in US dollars and the value of the Australian dollar

The history of Australia's iron ore pricing began with the development of iron ore mines in the Pilbara Region in the mid-1960s, and the lifting of an iron ore export embargo from Australia. Prior to this period, the Australian government thought that iron ore was not an abundant resources and it needed to be maintained as a strategic resource for future national security. However, after the discovery of iron ore in the Pilbara region by Norman and Hilditch and Lang Hancock, it became clear that, in fact, Australia had abundant iron ore resources (Sukagawa 2010). Historically, trade in iron ore was based on long-term contracts. However, in recent years, there has been a change towards shorter-term pricing, including spot market trades, and subsequently iron ore prices now replicate market developments more rapidly (Caputo et al. 2013).

Due to the continued strong demand of iron ore in China, production of Australia's iron ore has ensured that this commodity is singularly the highest value-ranking, exported mineral commodity for this country. As a consequence, the iron ore price and the value of the Australian dollar have substantial influences on the Australian economy. Throughout more recent years, iron ore has been the top value-ranking mineral exporting commodity in Australia due mainly to growing markets in China. China accounts for one-third of the world's total iron ore consumption and also ranks first in the world in iron ore imports (Ma et al. 2013). As a result, to meet its iron ore demand, China increasingly depends on imports from countries including Australia, Brazil and India. Australia is now one of the world's major iron ore producers and exporters. According to Australian Identified Mineral Resources (AIMR 2014) reports, it hosts some of the largest iron ore deposits globally and has a significant share of the world's economically demonstrated resources (EDR) of iron ore at 28%, followed by Brazil at 17%. Australia's EDR of iron ore increased by 18% to 52,578 million metric tonnes (Mt) during 2013 with the EDR of contained iron estimated as 23,035 Mt. The total production of iron ore resources in Australia increased in 2013 to 609 Mt and the main contributing state was Western Australia (WA). In Australia, WA has the largest share of iron ore which accounts for approximately 89% of Australia's EDR and the majority of it is in the Pilbara region. In 2013, WA State produced 593 Mt or 97% of Australia's total production of iron ore, up from 507 Mt in 2012.

According to the Australian Government's Department of Foreign Affairs and Trade, in 2014 Australia earned approximately AU$74.67 billion through exporting iron ore with the growth of iron ore exports up by around 9.5% over the previous year. It is apparent that there is an increasing trend in exporting iron ore and accumulating associated earnings derived from this material. While iron ore is recognised as one of the major export earning commodities in Australia's economy, less cognisance is given to the relationship between the price of iron ore and the Australian dollar exchange value. The exchange rate of the Australian dollar against the US dollar is numerically escalating (weakening Australian dollar against the US dollar) due to the reduction in commodity prices. The volatility of the iron ore price is important for the Australian iron ore industry and therefore the overall Australian economy. Iron ore is generally sold internationally in over-the-counter (OTC) markets and, until November 2008, its trade was mainly based on longer-term contract prices. However, from December 2008, the trade of this commodity moved to shorter-term pricing mechanisms such as monthly contracts and spot sales. The importance of short-term pricing in iron ore is very noticeable in the market. Monthly based contract prices and spot transactions account for a large share of global trade (Caputo et al. 2013). Before 2005, the price of iron ore did not change significantly, and over the period 2008-2014, the prices varied remarkably. From January 2014, iron ore prices started to decline, and in early March 2014, the iron ore price had recorded its most remarkable decline over any prior 18 month period. Another round of collapsing stock prices, preceded by "sneezes" in the Chinese economy, took place and marked the bottoming of iron ore prices. In response to this decline in iron ore prices, listed share prices of BHP Billiton (BHP**)** fell 4.14%, Rio Tinto (RIO) dropped 5.76%, Fortescue Metals (FMG**)** slipped 9.39%, Atlas Iron (AGO**)** shares fell 10.14%, and shareholders of BC Iron (BCI**)** lost 8.18% (Source: The BULL.COM. AU, April 2014). Furthermore, in July 2015, the iron ore price was below US$50 per metric tonne, and at present the price continues to decline or keep at least flat which creates negative sentiment in mining companies. Therefore, the variation in the iron ore price has directly and indirectly led to the appreciation or depreciation of the AUD/USD exchange rate. Arising from this, it is necessary to derive the relationship between the iron ore price and the Australian dollar exchange value and to determine their strength of association before investing in the iron ore sector or while evaluating iron ore mining projects, as the combined volatility of commodity prices and AUD/USD exchange rates and their nature of association may have a significant impact on the valuation of mining projects. Unfortunately, until now, scarce literature exists regarding the relationship between iron ore prices and exchange rate movements and their commensurate strength of association. Therefore, the objective of this study is to investigate the long-term relationship and the strength of association between iron ore prices and the AUD/USD exchange rate, by considering historical monthly data (which will be used to approximate the mining project values in Chapter-5). While other economic and non-economic variables and parameters may have an influence on these two variables and how they behave, especially on AUD/USD exchange rates, these additional factors lie beyond the scope of this study and will therefore not be considered in any detail, if at all.

4.5.1 Sources of data for the investigation of the relationship between iron ore prices and AUD/USD exchange rates

Historical trade in iron ore was based on long-term and short-term contracts, and therefore the iron ore price did not vary month-on-month for a specific year. That is, each month's iron ore price was identical to every other month in each year. Therefore, for this study, monthly iron ore price data have been considered over the period from 2008 to 2014 (i.e. the period when iron ore prices started to change monthly), and collected from Indexmundi. As iron ore price data are available on a monthly basis, the monthly Australian dollar against the US dollar (AUD/USD) exchange rate data have been considered and sourced from the Reserve Bank of Australia (RBA). Both the iron ore price and the exchange rate data are expressed in nominal terms for the entire test. For the purposes of empirical analysis, the natural logarithm of iron ore prices (LNIP) and the natural logarithm of AUD/USD exchange rates (LNEX) have been calculated and used for the entire test. For all the econometrical and statistical tests, STATA software (STATA version 13c) has been used.

4.5.2 Brief statistics of iron ore prices

An overview of iron ore prices has been provided. Figure 4-8 shows the iron ore prices in US dollars per dry metric tonne over the period 1999 to 2014. From this figure, it can be observed that iron ore prices did not vary significantly between 1999 and 2005.

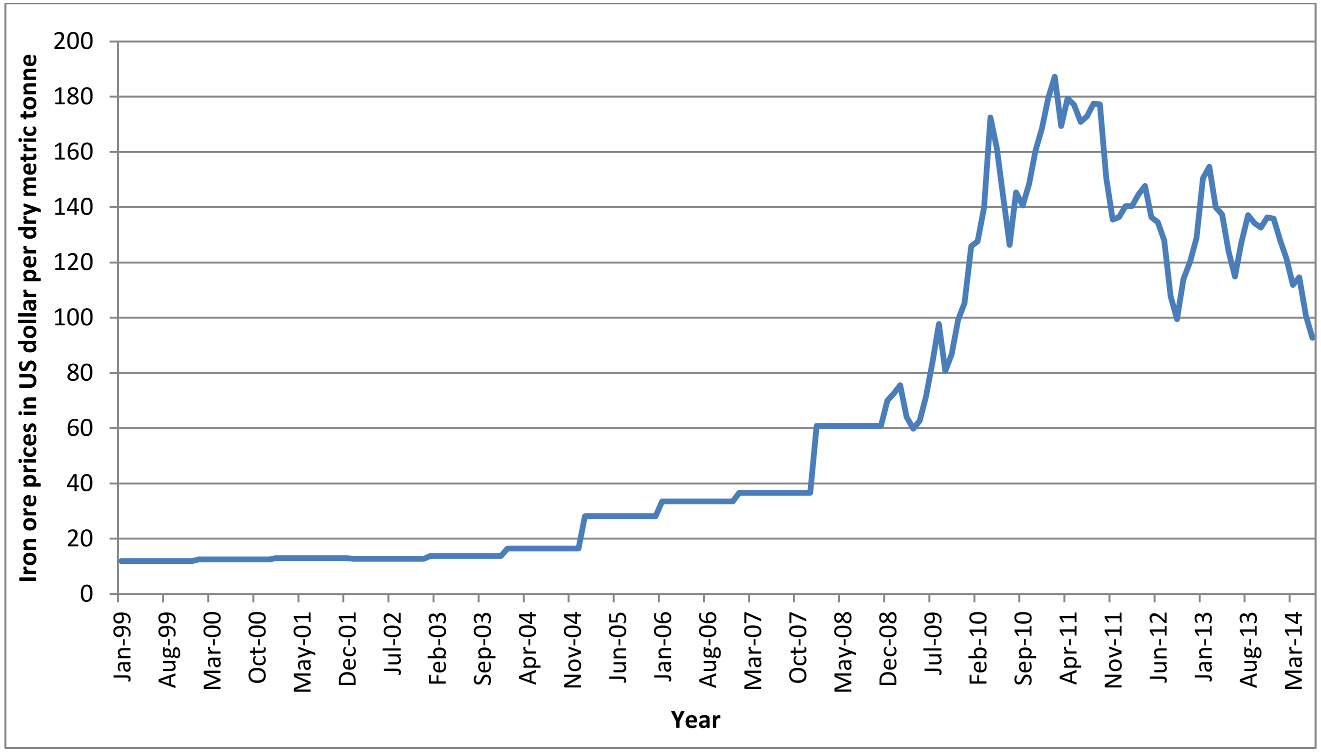

Figure 4-8 Iron ore prices in US dollars per dry metric tonne over the period 1999 to 2014 (monthly data). Data source: http://www.indexmundi.com

In January 1999, the price of iron ore was only US$11.93 per dry metric tonne, and in January 2004, it was US$16.39 per dry metric tonne. However, in January 2011, the price had risen to US$179.63 per dry metric tonne. After that, the iron ore price dropped markedly in 2012 and thereafter there were notable fluctuations in the price until December 2013. Iron ore prices started to decline again from January 2014, and in early March 2014, the iron ore price was recorded at US$104.70 per dry metric tonne. Furthermore, in July 2015, the iron ore price was only US$51.50 per dry metric tonne, and at present the price continues to decline or at least remain flat at these low levels. The primary reason for the price drop may be due to the decreased demand and the simultaneous contagion effects of China's economy, compounded by increasing iron ore production rates.

4.5.3 Unit root tests for the iron ore price and AUD/USD exchange rate

Table 4-14 presents ADF test results and from these results, it has been ascertained that the null hypothesis has been accepted at 1% significance for both iron ore prices and exchange rates (AUD/USD) when variables are considered in levels. Therefore, both time series have unit roots i.e. non-stationary in levels. However, the null hypothesis has been rejected at 1% significance for both time series when variables are considered in first differences. Therefore, both the variables are I(0) stationary in first differences. Hence the ADF test has confirmed that both variables are I(1) stationary at 1% significance.

Table 4-14 Results of ADF test for unit roots (iron ore prices and AUD/USD exchange rates).

Variables		Specifications	Lags	Test Statistics Z(t)	1% critical value	5% critical value	10% critical value	McKinnon approximate p value for Z(t)	Decisions
In levels	Iron ore price	Trend	1	-1.563	-4.117	-3.485	-3.171	0.8066	Non Stationary
		Regress with no constant	3	0.327	-2.615	-1.950	-1.610		Non Stationary
	*FX rate	Trend	2	-3.326	-4.119	-3.486	-3.172	0.0620	Non Stationary
		Regress with no constant	3	-1.738	-2.617	-1.950	-1.610		Non Stationary
In first differ-ences	Iron ore price	Trend	1	-5.860	-4.119	-3.486	-3.172	0.0000	Stationary
		Regress with no constant	3	-4.211	-2.615	-1.950	-1.610		Stationary
	FX rate	Trend	2	-4.920	-4.121	-3.487	-3.172	0.0000	Stationary
		Regress with no constant	3	-3.818	-2.615	-1.950	-1.610		Stationary

**Note: FX means foreign exchange*

Furthermore, from the DF-GLS test results (see details in Appendix Chapter 4.1A, Tables, A4-1 to A4-4), it has been observed that the null hypothesis has been accepted both for iron ore prices and exchange rates (AUD/USD) both in no trend specification, and in Elliott, Rothenberg and Stock, 1996 (ERS) proposed specification when variables are in levels. Therefore, the time series has unit roots i.e. non-stationary in levels. However, the null hypothesis has been rejected for both time series when variables are considered in first differences. Therefore, both the variables are I(0) stationary in first differences. Hence the DF-GLS test has also confirmed that both variables are I(1) stationary. Therefore, from these two tests, being ADF and DF-GLS tests, it has been confirmed that both iron ore prices and the AUD/USD exchange rate time series are non-stationary in levels, and stationary in first differences (I(1) stationary).

4.5.4 The Johansen cointegration test for the iron ore price and AUD/USD exchange rate

For investigating the cointegrating relationship of the iron ore price and the Australian dollar exchange rate against the US dollar (AUD/USD), the Johansen cointegration test was performed. As in a cointegration test, it is expected that the combination of cointegrated variables or series gives a stationary process, with one of the commonly used testing methods to verify this being the Johansen's trace statistics. Therefore, Johansen trace tests have been completed here to determine the cointegrating relationship between iron ore prices and AUD/USD exchange rates. In Table 4-15, the trace statistics at $r = 0$ exceeded its critical values at 5% significance levels. Therefore, the null hypothesis has been rejected for no cointegrating equations. However, in the case $r = 1$, the null hypothesis has been accepted for at most one cointegrating equation. Hence, there exists one cointegrating equation when $r = 1$.

Table 4-15 Results of Johansen cointegration trace tests for iron ore prices and AUD/USD exchange rates.

Lags	Rank	Maximum likelihood (LL)	Eigenvalue	Trace Statistics	5% critical value
3	r=0	218.55488	.	25.8832	19.96
	r=1	229.2462	0.28402	4.5006	9.42
	r=2	231.4965	0.06791		
5	r=0	221.0351	.	26.1524	25.32
	r=1	231.43612	0.28503	5.3504	12.25
	r=2	234.11132	0.08268		

Therefore, the test results have revealed that there exists a long-run relationship between iron ore prices and AUD/USD exchange rates, since a cointegrating equation exists in the bivariate model.

4.5.5 Estimation of the vector error correction model parameters

From the equation (4-10), the VECM parameters for iron ore prices and AUD/USD exchange rates have been estimated which have economic importance. Therefore, the main interest of estimates lies in the matrix β that contains the cointegrating parameters, α, being the speed of adjustment and the short-run coefficient matrix, Γ. The stability test for the VEC model will then be reported and it will be deduced that the specified model is stable to examine the relationship. Furthermore, the autocorrelation test in lag order will be shown as will the normality test.

From the VEC model estimation, the following is obtained:

The long-run disequilibrium adjustment matrix, $\hat{\alpha} = (-0.09198, 0.37046)$

The normalizing cointegrating vector i.e. the β parameters, $\hat{\beta} = (1, -0.31)$

The short–run coefficient matrix, $\hat{\Gamma} = \begin{pmatrix} 0.51444 & -0.09187 \\ 0.52389 & 0.21297 \end{pmatrix}$

The speed of adjustment parameters -0.09198 (p- value 0.043) and 0.37046 (p-value 0.024), the normalizing beta value -0.31 (p-value 0.000), and the short-run coefficients 0.51444 (p-value 0.000) and -0.09187 (p-value 0.006) are statistically significant and meaningful.

The short-term variability of the AUD/USD exchange rate is attributed to two parts, the error correction terms which ensure the exchange rate returns towards the long-term equilibrium, and the contemporaneous variability of iron ore prices. The speed of adjustment coefficient -0.09198 suggests that an approximately 9.2 percent change in the exchange rate per month can be attributed to the disequilibrium between actual and equilibrium levels. The adjustment coefficient 0.37046 shows that the variability of iron ore prices induces a positive change in the exchange rate. The normalizing beta value -0.31 indicates that a one percent increase in iron ore prices leads to an appreciation of the AUD/USD exchange rate by approximately 0.31 percent. The value of the R-square is 0.32, which suggests that there is a positive association between them.

4.5.5.1 Stability test for VEC model

According to Lutkephol (2005) and Hamilton (1994), the stability condition of the VECM is that if the companion matrix of a VECM has k endogenous variables and r cointegrating equations, then there will be k-r unit eigenvalues, and the remaining eigenvalues lie inside the unit circle. Table 4-16 shows all but one of the eigenvalues are less than 1.

Table 4-16 Stability test for cointegration relationship between iron ore prices and AUD/USD exchange rates (Eigenvalue stability condition).

Eigenvalue	**Modulus**
1	1
0.5113213 + 0 .2330609i	0.561931
0.5113213 - 0 .2330609i	0.561931
0.4994143	0.499414

From Figure 4-9, it is noted that three eigenvalues lie inside the unit circle and one eigenvalue is on the circle, because VECM specification imposes 1 unit modulus. Therefore, from Table 4-16 and Figure 4-9, it can be concluded that the VECM satisfies the stability condition for a cointegration relationship between iron ore prices and AUD/USD exchange rates.

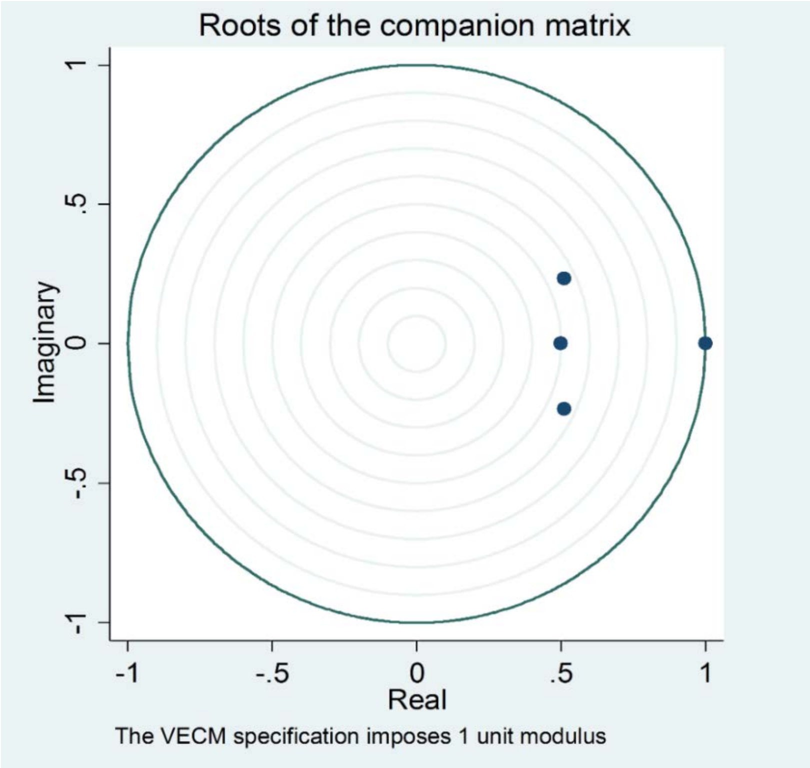

Figure 4-9.The stability of the cointegration relationship between iron ore prices and exchange rates in the VECM Test.

4.5.5.2 Autocorrelation test

To see the autocorrelation between iron ore prices and AUD/USD exchange rates, the Lagrange-multiplier test has been performed. The results are reported in Table 4-17. It has been seen from Table 4-17 that there is no autocorrelation at lag order.

Table 4-17 Output of autocorrelation test.

Lags	**chi2**	**df**	**Prob>chi2**
1	3.0498	4	0.54952
2	5.9244	4	0.20486
3	0.8514	4	0.93142
4	7.8736	4	0.09632
5	2.2188	4	0.69558
6	2.7975	4	0.59226
7	4.8399	4	0.30412

Note: H_0 (null hypothesis): no autocorrelation at lag order

4.5.5.3 Test for normally distributed disturbances

For the distribution of the error terms of the model, the Jarque-Bera test, Skewness test and Kurtosis tests have been performed to examine the null hypothesis that the errors are normally distributed. The assumption of the model was that the error terms are normally distributed with zero mean and finite variance. However, if the error terms do not follow the normal distribution but are merely independently and identically distributed with zero mean and finite variance, then the parameters' estimation are still consistent but they are deemed inefficient.

From Table 4-18, it can be observed that all three tests have confirmed the errors are normally distributed.

Table 4-18 Test for distribution of the error terms.

Jarque-Bera test

Equation	chi2	df	Prob>chi2
D_LNEX	2.036	2	0.36132
D_LNIP	2.617	2	0.27027
ALL	4.653	4	0.32483

Skewness test

Equation	Skewness	chi2	df	Prob>chi2
D_LNEX	-0.42904	1.994	1	0.15791
D_LNIP	-0.29464	0.940	1	0.33215
ALL		2.935	2	0.23054

Kurtosis test

Equation	Kurtosis	chi2	df	Prob>chi2
D_LNEX	3.1243	0.042	1	0.83796
D_LNIP	3.7867	1.676	1	0.19543
ALL		1.718	2	0.42359

4.5.6 Vector auto regressive (VAR) model for iron ore prices and AUD/USD exchange rates

As, under the unit root tests process in this investigation, both series are I(1) stationary and also cointegrated, then to examine the Granger causality, a VAR model in levels has been estimated. The results are reported in Table 4-19.

From this VAR parameter estimation, it can be concluded that, to explain AUD/USD exchange rates, the lag prices of iron ore and the lag AUD/USD exchange rates are statistically significant. On the other hand, to explain iron ore prices, the lag prices of iron ore are statistically significant, whereas the coefficients of lag of AUD/USD exchange rates are not zero but are also not statistically significant.

Table 4-19 VAR model parameters estimation.

Parameter values	**P- value**
α_{10} =0.826	0.008*
α_{11} = 1.084	0.000*
α_{12} = -0.252	0.031*
α_{21} = 0.665	0.102
α_{22} = -0.374	0.359
β_{10} = -0.044	0.601
β_{11} = 1.335	0.000*
β_{12} = -0.426	0.014*
β_{21} = -0.079	0.014*
β_{22} = 0.088	0.005*

*Note: Here * denotes statistically significant values.*

4.5.6.1 Granger causality test

The results of the Granger causality test are reported in Table 4-20. From the Granger causality test results, it is observed that that there is one-directional causality between iron ore prices and AUD/USD exchange rates. That means iron ore prices incur Granger-causes to AUD/USD exchange rates whereas exchange rates do not seem to Granger-cause impacts to iron ore prices. In this case the probability value is exactly 5% for rejecting the null hypothesis. Here, LNIP denotes

Table 4-20 Granger causality Wald tests, variables in levels.

Equation	**Excluded**	**chi2**	**df**	**Prob>chi2**
LNEX	LNIP	7.945	2	0.019
LNEX	ALL	7.945	2	0.019
LNIP	LNEX	5.9491	2	0.051
LNIP	ALL	5.9491	2	0.051

the natural logarithm of iron ore prices, and LNEX denotes the natural logarithm of AUD/USD exchange rates.

However, when Granger causality tests were performed considering variables in first differences, the results were obtained suggesting that only iron ore prices have Granger-causes to AUD/USD exchanges rates. The test results are reported in Table 4-21. Here, fdLNEX denotes first difference of the natural logarithm of AUD/USD exchange rates and fdLNIP denotes first difference of natural logarithm of iron ore prices.

Table 4-21 Granger causality Wald tests, variables in first differences.

Equation	Excluded	chi2	df	Prob>chi2
fdLNEX	fdLNIP	9.7901	2	0.007
fdLNEX	ALL	9.7901	2	0.007
fdLNIP	fdLNEX	5.3064	2	0.070
fdLNIP	ALL	5.3064	2	0.070

4.5.6.2 Stability test for VAR model

According to Lutkephol (2005) and Hamilton (1994), if the modulus of each eigenvalue of the companion matrix is strictly less than one, then the estimated VAR is stable. From Table 4-22, it is observed that all eigenvalues are less than 1.

Table 4-22 Eigenvalue stability condition (for VAR model**).**

Eigenvalue	Modulus
0.9097894	0 .909789
0.5795964	0.5795964
0.4654647 + 0.2227076i	0.516
0.4654647 - 0.2227076i	0.516

Moreover, Figure 4-10 shows all the roots of the companion matrix strictly lie inside the unit circle. Therefore, it can be deduced that VAR satisfies the stability condition.

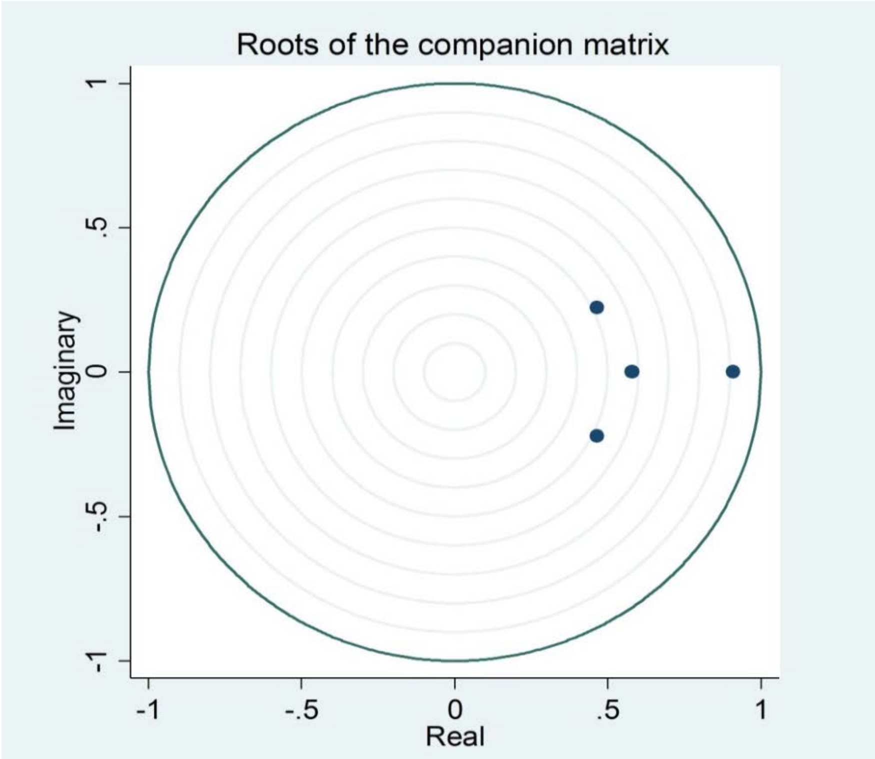

Figure 4-10 The stability of VAR test for iron ore prices and AUD/USD exchange rates.

4.5.6.3 Autocorrelation test for VAR model

For the autocorrelation test, the Lagrange-multiplier test (LM) has been used. The results are reported below in Table 4-23. As can be seen from Table 4-23, there is no autocorrelation at lag order.

Table 4-23 Output of autocorrelation (LM) test (VAR model).

Lags	chi2	df	Prob>chi2
1	1.9803	4	0.73939
2	6.8628	4	0.14332
3	2.1546	4	0.70735
4	8.1060	4	0.08777
5	1.7149	4	0.78800
6	2.5419	4	0.63715
7	4.6476	4	0.32540

Note: H_0: no autocorrelation at lag order

4.5.6.4 Test for normally distributed disturbances

For the distribution of the error terms of the VAR model, the Jarque-Bera test, Skewness test and Kurtosis test have been performed to examine the null hypothesis that the errors are normally distributed.

Table 4-24 demonstrates that all three tests have confirmed the errors are normally distributed.

Table 4-24 Test for distribution of the error terms (VAR model).

Jarque-Bera test

Equation	**chi2**	**df**	**Prob>chi2**
LNEX	0.951	2	0.62142
LNIP	2.983	2	0.22501
ALL	3.935	4	0.41491

Skewness test

Equation	**Skewness**	**chi2**	**df**	**Prob>chi2**
LNEX	-0.26543	0.751	1	0.38601
LNIP	-0.10293	0.113	1	0.33215
ALL		0.864	2	0.64905

Kurtosis test

Equation	**Kurtosis**	**chi2**	**df**	**Prob>chi2**
LNEX	3.2739	0.200	1	0.65470
LNIP	4.0375	2.870	1	0.09023
ALL		3.070	2	0.21543

4.5.7 The impulse–response functions (IRFs) analysis in structural VAR (SVAR)

The structural vector auto regression (SVAR) has been considered here for the analysis of the impulse–response functions and the forecast error variance decompositions between iron ore prices and AUD/USD exchange rates.

Figure 4-11 shows the response of fdLNEX due to the impulse/shocks on fdLNIP through the orthogonalized impulse–response function (IRF).

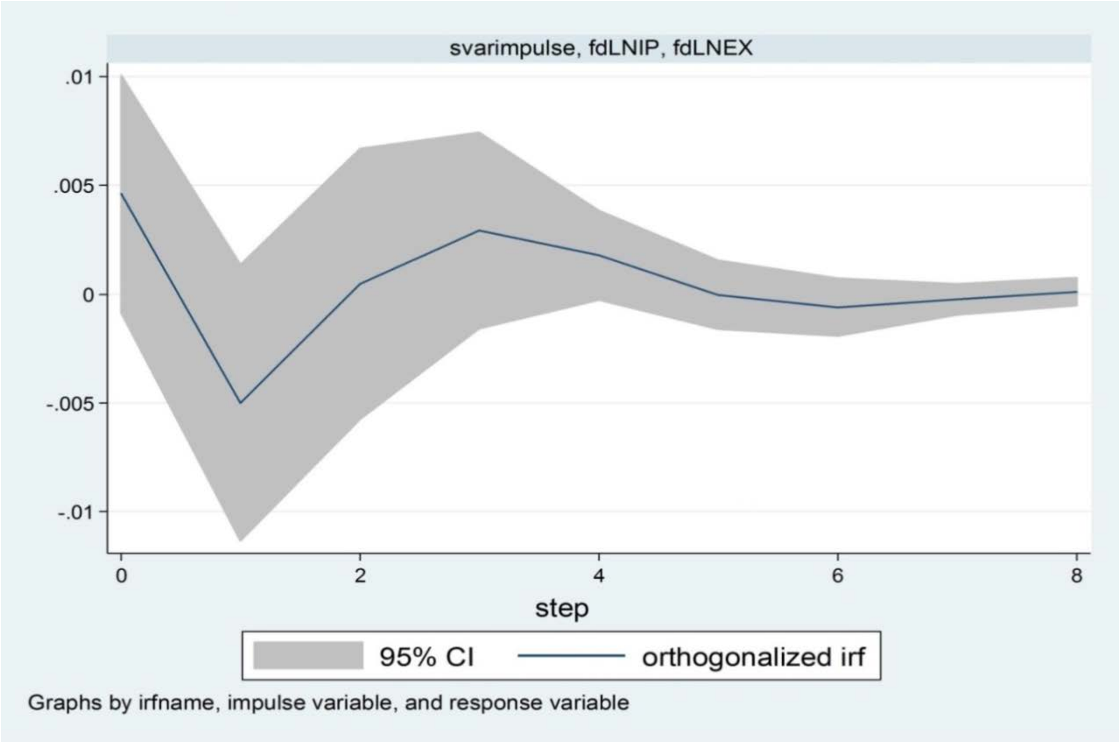

Figure 4-11 .The response of fdLNEX due to the impulse on fdLNIP through orthogonalized IRF

Figure 4-12 shows the response of fdLNEX due to shocks on fdLNIP through both the orthogonalized impulse–response function and the structural impulse–response function.

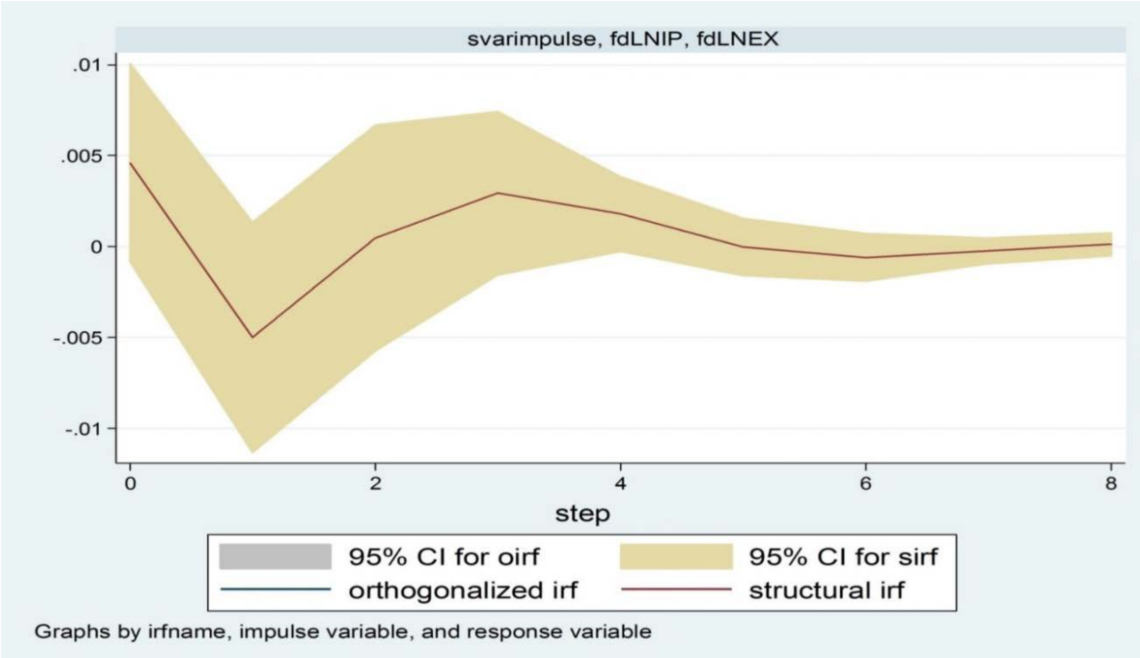

Figure 4-12 The response of fdLNEX due to the impulse on fdLNIP through both orthogonalized IRF and structural IRF.

As can be seen from Figure 4-11 and Figure 4-12, AUD/USD exchange rates have significant responses due to shocks on iron ore prices through orthogonalized IRF and the structural IRF analysis. Iron ore prices initially have a small positive impact on AUD/USD exchange rates, but within 1-2 periods it becomes negative, and interestingly, after 2 periods it has regained positive effects on AUD/USD exchange rates, which continues before diminishing after roughly 6 periods.

Figure 4-13 displays the response of fdLNIP due to the shocks on fdLNEX through the orthogonalized impulse–response function.

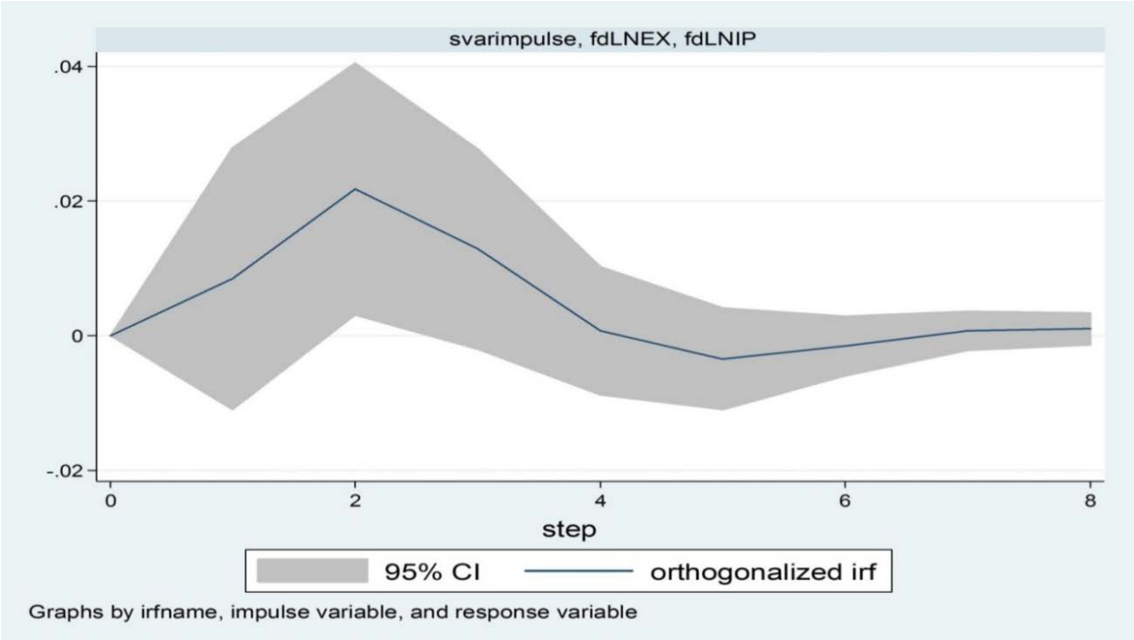

Figure 4-13 The response of fdLNIP due to impulse on fdLNEX through orthogonalized IRF.

Figure 4-14 shows the response of fdLNIP due to shocks on fdLNEX through both the orthogonalized impulse–response function and the structural impulse–response function.

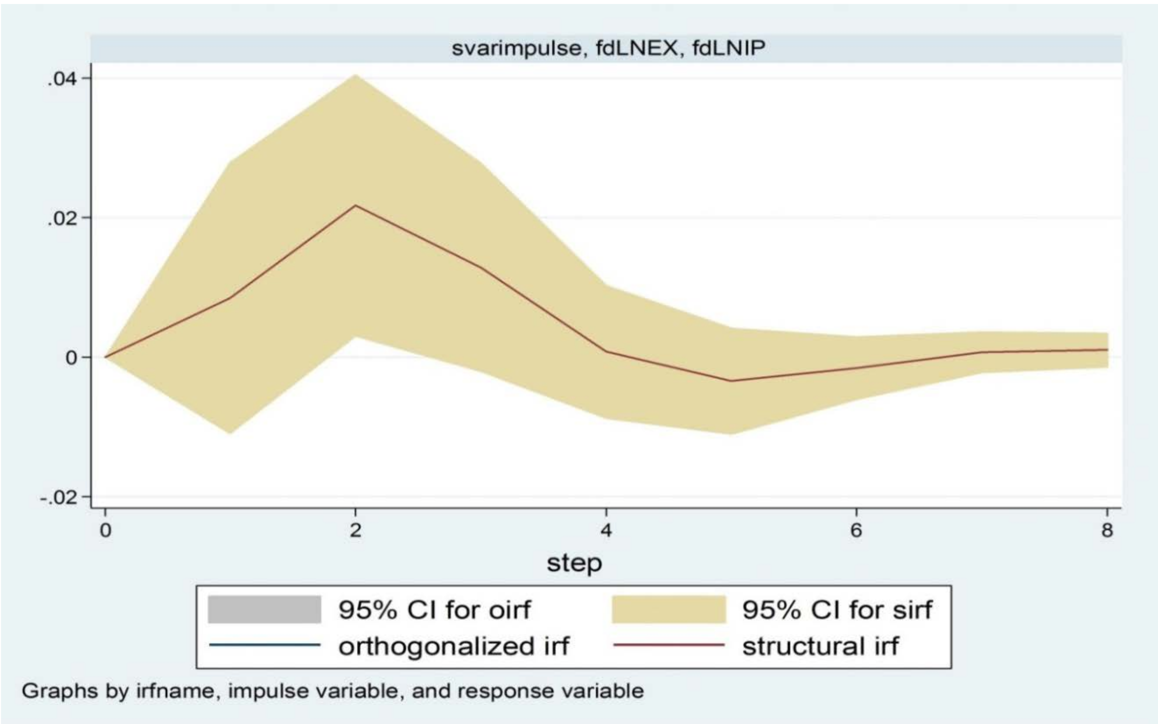

Figure 4-14 The response of fdLNIP due to impulse on fdLNEX through both orthogonalized IRF and structural IRF.

However, from Figure 4-13 and Figure 4-14, it has been observed that AUD/USD exchange rates generate significant causes to iron ore prices due to shocks on the AUD/USD exchange rates. AUD/USD exchanges rates have a significant positive effect on iron ore prices until period 4, but after that it becomes negative before diminishing after around 7 periods. Even though a diverse lag order is considered in the model and performed in the tests, the results (Appendix 4.2A, Figure A4-1 to A4-4) confirmed that due to the shocks on AUD/USD exchange rates, iron ore prices have responses, and vice versa. Furthermore, the forecast error variance decomposition and the structural forecast error variance decomposition tests have been performed (Appendix 4.3A, Table A4-5 to A4-6, and Figure A4-5 to A4-6). They also supported the results implied by impulse-response analysis.

4.5.8 Correlation analysis between iron ore prices and AUD/USD exchange rates

In the analysis, a positive correlation between iron ore prices and the AUD/USD exchange rate has been obtained, with the value of the correlation coefficient being determined as 0.78. When lag differences are considered and the correlation coefficients determined, the results did not change significantly. This can be seen in Table 4-25.

Table 4-25 The value of correlation coefficients between iron ore prices and AUD/USD exchange rates.

Variables	lags	Correlation coefficients
Iron ore prices and AUD/USD exchange rates	0	0.7800
	2	0.7872
	3	0.7876

4.6 Summary and concluding remarks

To estimate mining project values, mine management need to be vigilant about various uncertainties including financial and economic uncertainties, and the inter correlation and long-run relationships among these variables. Consequently, it is imperative for the mining and relevant industry participants to analyse whether evidence exists to prove that an empirical relationship between mineral commodity prices and exchange rate movements hold and, if so, to find the strength of their relationship i.e. the coefficient of correlation. This parameter is needed to estimate the project values considering the dual uncertainty of commodity prices and exchange rates.

In this chapter methodologies are presented to investigate the relationship between mining commodity prices and exchange rates. As an application for this investigation, two case studies have been considered for mineral commodities and the Australian dollar against the US dollar exchange rate. In the first case study, almost 17 years of weekly average data were considered for both gold prices and AUD/USD exchange rates over the period 1996 to 2014, and different tests were performed i.e. Augmented Dickey-Fuller (ADF) and modified Dickey-Fuller methods using generalised least squares (DF-GLS) to identify whether the series is stationary or not. All of the tests have confirmed that both series are I(1) stationary at a 1% significance level. The Johansen cointegration test and VECM test revealed the evidence in favour of a long-term stable relationship between gold prices and the AUD/USD exchange rate during this period. In the VECM test, the normalizing beta value shows that a one percent increase in the nominal gold price leads to an appreciation of the AUD/USD nominal exchange rate by approximately 0.50%. In the test, the speed of adjustment coefficient -0.0196405 proposes that approximately 1.9 percent of an exchange rate per week can be attributed to the disequilibrium between actual and equilibrium levels. The adjustment coefficient 0.0042265 shows that the variability of the gold price induces a positive change in the exchange rate.

Moreover, this study revealed that the AUD/USD exchange rate has a strong positive correlation with gold prices. The value of the correlation coefficient between the gold price and the AUD/USD exchange rate was determined as 0.91, even though when time lag differences are considered and determined, the corresponding correlation coefficient results did not change significantly. Furthermore, this study has also shown the Granger causality through the VAR test and has obtained interesting results. This test revealed that there is bi-directional causality between gold prices and the AUD/USD exchange rate, which is also supported by the structural vector auto regression (SVAR) impulse–response functions (IRFs) analysis.

In addition, the continuously compounded percentage return series for gold prices and AUD/USD exchange rates has been considered, which revealed an interesting finding in that there exists a bi-directional causality between the continuously compounded percentage return of gold prices and the returns of AUD/USD exchange rates. This result is also supported by the structural vector auto regression (SVAR) impulse–response functions (IRFs) analysis. However, other economic and noneconomic factors such as the countries' exports, demand and supply, inflation, other commodities and other markets might have influences on the AUD/USD exchange rate and gold prices, and their returns too, but it is beyond the scope of the present study to evaluate the effects of these parameters.

According to these results, it has been revealed that it would be more accurate to consider prolonged period data for taking better-informed decisions to find the relationships and their consequent strength of association. This is due to the limitation with shorter periods, wherein data loss inhibits the information variability and consequently compromises improved accuracy of results. Therefore, in the present study, approximately 17 years of data was considered. As a direct result, more accurate results in their relationships were determined. These results and associated information are crucial for the gold industry especially for gold mining project evaluations in Australia, and for notable gold and exchange rate stock markets. This study has clearly delivered an insight into the strong relationship between the gold price movement and the AUD/USD exchange rate. While the price of gold in international markets rises, it encourages the production of extra gold in Australia which apparently delivers strong support to the Australian economy and subsequently leads to the appreciation of the AUD against the USD. However, if the price of gold decreases in international markets, the production of gold apparently decreases and leads to the depreciation of the AUD against the USD.

In the second case study, the relationship between iron ore prices and the value of the Australian dollar have been investigated. For this study, monthly data have been considered over the period from 2008 to 2014 both in iron ore prices and the AUD/USD exchange rate, and it was examined, under different tests, whether these series are stationary or not. All the tests have confirmed that both series are I(1) stationary under the study. The Johansen cointegration test and VECM test revealed the evidence in favour of a long term stable relationship between iron ore prices and the AUD/USD exchange rate. In the VECM test, the normalizing beta value shows that a one percent increase in iron ore prices leads to an appreciation of the AUD/USD exchange rate by approximately 0.31 percent (other economic or non-economic variables and parameters may influence these two variables, but this was beyond the scope of this study). The value of the correlation coefficient between the iron price and the AUD/USD exchange rate was determined as 0.78, even though when time lag differences are considered, the corresponding correlation coefficient results did not change considerably.

Furthermore, this study has also shown the Granger causality through the VAR test. From the Granger causality test, it has been observed that there is one-directional causality between iron ore prices and the AUD/USD exchange rate, which means that iron ore prices have significant Granger-causes on the AUD/USD exchange rates whereas exchange rates do not have significant Granger-causes on iron ore prices. The VAR has been estimated both in levels and in first differences to examine Granger causality. When Granger causality tests were performed considering variables in levels, it was observed that iron ore prices seem to have Granger-causes effects on AUD/USD exchange rates whereas exchange rates did not seem to incur Granger-causes on iron ore prices, with the probability value being exactly 5% in this case for rejecting the null hypothesis.

Nevertheless, when Granger causality tests were performed considering variables in first differences, the results were obtained suggesting that only iron ore prices have Granger-causes to AUD/USD exchanges rates. However, while the structural vector auto regression (SVAR) is considered, interestingly, the impulse-response functions (IRFs) analysis shows that the impulses/shocks of AUD/USD exchange rates have a significant effect on iron ore prices, and due to shocks of iron ore prices, AUD/USD exchange rates have a significant response. This investigation has also revealed that the Granger causality test may not always provide the correct information of bi-directional causality. These results and the associated information might be crucial for the iron ore industry especially for iron ore mining project evaluations, and for iron ore and exchange rate markets.

This study also clearly delivered an insight into the relationship between the iron ore price and the AUD/USD exchange rate which may be helpful for several industries including, importantly, the mining industry. It is important for mining industry participants to understand the movement and correlation between economic variables, such as a mining commodity price and exchange rates before making any investment decisions in mining projects. To estimate the mining project values considering more than one uncertainty through a continuous time stochastic model, the correlation parameter is essential. This study achieved an important outcome, as the value of the parameter will be included in the new ROV technique in Chapter 5 to approximate the project values. Furthermore, to use the suitable hedging strategy in a mining company, management needs to know and be able to interpret specific information regarding the long-term relationship between these economic indicators.

CHAPTER 5

EVALUATION OF MINING PROJECTS UNDER THE JOINT EFFECT OF COMMODITY PRICE AND EXCHANGE RATE UNCERTAINTIES

5.1 Introduction

The value of a mining project is extensively influenced by exogenous variables, notably commodity prices and exchange rates. The traditional discounted cash flow (DCF) method, which is usually used for economic analysis of feasibility studies and mining project evaluations, presents deficiencies since the method fails to adequately address uncertainties and operational flexibilities and often ignores certain specific market conditions. Numerous studies have been carried out for mining project evaluations using the real option valuation (ROV) methods critically assessing commodity price uncertainty, but there is no research available on the combined effects of price and exchange rate uncertainties. Therefore, the objective of this chapter is that in order to assess the economic feasibility of a mining project more accurately, the commodity price and its inherent volatility, the exchange rate and its inherent volatility and the correlation parameters between them have been incorporated into the model and used in the ROV based evaluation process. This new ROV technique will explore the opportunity to utilise an alternative methodology for approximating project values and to identify valuation opportunities to enhance economic gains or to mitigate economic losses, where the DCF valuation method does not.

Mining projects host inherent uncertainties and risks and, consequently, the evaluation of these projects must give due consideration to these numerous risks and uncertainties or risk yielding inaccuracies and inappropriate results. As a consequence, mine management and stakeholders associated with the industry make less than optimal decisions based on this inaccurate information. Economic uncertainty represents one of the most crucial sources of uncertainty and, if unchecked, may have a critical impact on the evaluation of the mining project. The fluctuation of commodity prices and the exchange rate are, singularly and combined, the most important external uncertainties facing mining companies. Over recent years, the fluctuation of the price of mining commodities (e.g. iron ore, coal, gold) has had a tremendous effect on mining and its associated industries.

Depressed commodity prices generally create a negative sentiment around the minerals and mining industry. Gilroy (2014) reported that the dramatic price downfall of iron ore prices created a negative impact on all iron ore companies. Validakis (2015) reported that due to the extremely low iron ore prices, Australia's Fortescue Metals Group (FMG) generated a loss of around US$1.37 billion in 2015. FMG had a first half profit of US$331 million, which was down from a profit of US$1.7 billion from the previous year. Global rating agency, Moody stated in July 2015, that due to the continuing deterioration of gold prices the impact is credit negative for South African gold mining companies, including AngloGold Ashanti Limited and Gold Fields Limited. Gold Fields, and probably others too, had to readdress its reserve statement at the lower gold price of US$1,300/oz. At a gold price below this level, the company's leverage will rise, and consequently the net debt will increase (Moody, 2015).

Williams (2015) stated that the continuing fall in the gold price, will lead to an acceleration in the rates of closure of a number of gold mining operations around the world, and bring an unprecedented halt to many new gold mine developments, affecting gold supply fundamentals quicker than previously predicted. In addition, Mason (2015) stated that the fall in the iron ore price is a fresh six- year low, and added that most companies have a break-even price above the current iron ore spot price despite having reduced their costs. A further example, BC Iron, has a breakeven price ofUS$61 per tonne of iron ore, whereas Atlas Iron's break-even price is US$64 per tonne and Mount Gibson's is US$54 per tonne. Exchange rates have an enormous impact on the evaluation of a mining project too. For instance, Jaguar Mining reported that over the second quarter of 2013, it incurred a foreign exchange loss of US$4.7 million due to the change in the exchange rate (Jaguar Mining, 2013). Anglo American (2002) reported that due to the favourable exchange rate changes, largely relating to the South African rand and to a lesser extent the Chilean peso and the Brazilian real added US$147 million to their headline earnings over that financial reporting period.

The traditional DCF method of valuation is readily used for mining project evaluations. However, based on economic and non-economic uncertainties and the lack of managerial flexibilities, the DCF method often fails to determine the appropriate mining project value. After almost one and a half decades, researchers are beginning to understand why the DCF valuation method cannot determine implicit mining project values.

It has been determined that the DCF valuation method:

- fails to capture financial options;
- ignores economic uncertainties;
- fails to adequately incorporate strategies to reduce risk and maximize profit; and
- ignores management flexibilities to handle the uncertainties.

Although some researchers and authors have attempted to implement the evaluation methodology in mining projects through ROV methods using continuous time stochastic models, in most of the cases the analyses were limited to only price uncertainty. However, Cortazar et al. (2001) explored optimal investments under-price and geological uncertainties. These authors assumed a zero drift stochastic process in the case of geological uncertainty. Lima and Suslick (2006) estimated the project volatility only, considering price and operating cost uncertainties but did not determine the value of the project. Castillo and Dimitrakopoulos (2014) studied the joint effect of commodity price and geological uncertainty. They analyzed the efficiency of traditional valuation methods in assessing the potential performance of a mining operation under uncertain geological and commodity price scenarios, and provided alternative real options based method that includes the option of expanding or contracting the initial ultimate pit limit, subject to these uncertainties. Zhang et al. (2014) studied the real option value of a mining operation using mean reverting commodity prices. However, these authors did not determine the explicit value of the project.

There is still limited literature available discussing the valuation of mining projects considering more than one uncertainty using continuous time stochastic models and the commensurate numerical solution of the resulting PDEs. Therefore, in this chapter, the ROV method has been studied under joint commodity price and exchange rate uncertainties as the main stochastic variables for a mining project evaluation. This is the first research that undertakes a mining project evaluation considering the mutual effect of commodity price and exchange rate uncertainties. This chapter contributes the development of theoretical work and enhances an approach to approximate explicit numerical project values under the joint commodity price and exchange rate uncertainties. The fluctuation of a commodity price and its associated volatility is a crucial factor for the evaluation of mining projects. The exchange rate uncertainty is also an imperative valuation factor for mining companies, as the exchange rate does not only reflect the local currency sales of a commodity that is priced in an offshore currency, it also impacts and associates with mining costs (CAPEX, OPEX, etc.). Therefore, in order to evaluate the economic feasibility of a mining project correctly, the exchange rate volatility should be modelled appropriately and included in the evaluation process.

Furthermore, commodity price paths have been simulated through the joint effect of commodity prices and the exchange rate volatilities, using the correlation between commodity prices and exchange rates, to obtain an average realized price of the commodity over the period. As case studies, iron ore price paths have been simulated through the joint effect of iron ore prices and the AUD/USD exchange rate volatilities, using the correlation between iron ore prices and AUD/USD exchange rates (determined in Chapter 4, Haque et al., 2015a), to obtain an average realized price of iron ore over the period. This simulated average price realization of iron ore, the calculated iron ore and exchange rate volatilities, as well as their correlation have been used in the approximation of the project's values. Additionally, gold price paths have also been simulated through the joint effect of gold prices and the exchange rate volatility, and the correlation between gold prices and exchange rate (AUD/USD) as determined in Chapter 4, to obtain an average realized price of gold over the period. This simulated average price realization of gold, the calculated gold and exchange rate volatilities, as well as the correlation between gold prices and exchange rate (AUD/USD) have been used in the approximation of the gold project's values

Even though the idea regarding ROV methods arose only a few decades ago, the models that have been developed to-date are usually limited to theoretical research and academia. Therefore, the applications of ROV methods are not well understood and remain uncommon, and hence are often not used in mining project evaluations. Analytical and numerical solutions derived through the application of ROV methods are rarely found in practice due mainly to the complexity associated with solving the PDEs, which are dependent on several conditions and parameters (Haque, et al., 2014). As a consequence, it may not generally be applicable to evaluate mining projects under all project-specific circumstances. Therefore, the greatest challenge to ROV modelling is in finding numerically explicit project values considering multiple uncertainties. The stochastic models have been transformed to higher ordered PDEs with the help of financial options and hedging strategies and the PDEs have been solved numerically utilising the finite difference method (FDM) and Matlab software to estimate the project values.

5.2. Modelling uncertainty factors for commodity price and exchange rate movements

Changes in commodity prices will directly affect the value and associated returns of a mining investment and, therefore, this variable alone has a significant impact on mining project decision making. Since both commodity prices and the overseas investment itself are determined in US dollars, changes in exchange rates will, to some extent, affect the local commodity price and therefore the mining project value. Consequently, a two factor model will be considered here for valuing mining projects under both commodity price and exchange rate uncertainties. It is assumed that the dynamics of the commodity price, P and exchange rate, E will be according to the following stochastic differential equations (SDEs):

$$\frac{dP}{P} = (r - \delta)dt + \sigma_P \, dW_P \tag{5-1}$$

$$\frac{dE}{E} = \alpha \, dt + \sigma_E \, dW_E \tag{5-2}$$

where P is the spot unit price of the commodity. r and σ_P represent the drift (expected rate of return) and volatility parameters of the commodity price, respectively. δ is the mean convenience yield/dividend yield on holding one unit of commodity (for mining industry cases, it might be considered either for the stock's dividend for the specific mining commodity or as a storage cost for inventory). E represents the spot exchange rate (local currency against US dollar), and α and σ_E represent the drift (expected rate of return) and volatility parameters for the exchange rate, respectively. dW_P and dW_E are the Wiener increments of Geometric Brownian Motion (GBM) of P and E respectively.

Note: α can be defined as $\alpha = r - \delta_E$, where δ_E is the dividend yield for holding one unit of exchange rate.

To approximate the numerical value of the project, according to Cortazar et al. (2001) and Qiu and Wang (2014), it is possible to collapse the joint uncertainties of price, P, and exchange rate, E, into a single state variable, Z, which can then be defined as follows:

$$Z \equiv F(P,E) = PE \tag{5-3}$$

This makes the model simpler to implement, while providing a reasonable approximation to the project value. The new state variable, Z, can be seen as a modified or shadow commodity price with a different drift and volatility, and the two stochastic processes (5-1) and (5-2) can be written as follows:

$$\frac{dZ}{Z} = (r - (\delta - \alpha - \rho\,\sigma_P\,\sigma_E))\,dt + \sigma_P dW_P + \sigma_E dW_E \tag{5-4}$$

where ρ represents the correlation coefficient between the commodity price and the exchange rate. Here the modified drift rate depends on the parameters r, α, ρ, δ, σ_P and σ_E.

For the purpose of the simulation of the commodity prices paths, according to Hull (2012) in the discrete form, the Equation (5-4) can be written as follows:

$$Z_{t+1} = Z_t + (r - (\delta - \alpha - \rho\,\sigma_P\,\sigma_E))\,Z_t\Delta t + \sigma_P Z_t \varepsilon\sqrt{\Delta t} + \sigma_E Z_t \varepsilon\sqrt{\Delta t} \tag{5-5}$$

This equation (5-5) will be used to carry out the simulation of the commodity price of a project, which will assist in finding the approximation of the project's value.

Following Cortazar et al. (2001) and Qiu and Wang (2014), the increased volatility, σ_Z (due to the joint uncertainties price, P and exchange rate, E) can be calculated as $\sigma_Z = \sqrt{\sigma_P^2 + \sigma_E^2 + 2\rho\sigma_P\sigma_E}$, and equation (5-4) can be approximated by equation (5-6) as follows:

$$\frac{dZ}{Z} = (r - (\delta - \alpha - \rho\,\sigma_P\,\sigma_E))\,dt + \sigma_P dW_P + \sigma_E dW_E$$

$$\cong (r - (\delta - \alpha - \rho\,\sigma_P\,\sigma_E))\,dt + \sigma_z dW_z$$

$$\frac{dZ}{Z} = (r - (\delta - \alpha - \rho\,\sigma_P\,\sigma_E))\,dt + \sigma_z dW_z \tag{5-6}$$

Each dW is an independent random variable of the form $\sqrt{\Delta t}\ N(0,1)$, and $N(0,1)$ denotes a normally distributed random number with zero mean and unit variance.

The mine's management has the flexibility of choosing the option for opening or temporarily closing the mine for investment due to the uncertainty of the commodity price and the exchange rate. While the operation is closed or suspended, it will incur a maintenance cost. Therefore, the value of the mine can be defined as:

$V = V(P, E, T, \phi) \cong V(Z, T, \phi) = V(Z, T)$. Here, ϕ describes managerial flexibility alone i.e. the operating policy such as opening, temporarily closing, and abandoning the mine, and T is the time (duration) of mine operations.

5.3 Derivation of partial differential equation (PDE) using futures contracts and SDE (5-6)

With the help of the futures markets it is possible to hedge the price (output) risk of a commodity. With this it is possible to derive the PDE using the SDE (5-6) and incorporate hedging strategies.

Suppose the commodity price is Z, and its delivery in the futures contract is in time t. The value of the futures contact is $F(Z,t)$.

Applying Ito's Lemma to the value of commodity futures the following is obtained:

$$dF(Z,t) = \frac{\partial F}{\partial Z} dZ - \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial Z^2} Z^2 \sigma_Z^2 dt \tag{5-7}$$

If there is no arbitrage opportunity in commodity futures markets, then an investor, through a portfolio of being long (buyer) in one unit of output (commodity) and short (seller) in $\left(\frac{\partial F}{\partial Z}\right)^{-1}$ units of the futures contract, can hedge this price risk and this should earn a risk free interest rate (Cortazar and Schwartz., 1997, Bellalah, 1999, 2001). Therefore:

$$dZ + (\delta - \alpha - \rho \sigma_P \sigma_E)\, Z\, dt - \frac{dF}{F_Z} = r\, Z\, dt \tag{5-8}$$

In real life, a mining company can implement a hedging strategy to lock up the output price to minimize economic risk in the project. That means a mining company with a long position for an investment in a mining project, $V = V(Z, T)$ and $\frac{V_Z}{F_Z}$ a short position in a futures contract could hedge its risk and should earn a return equal to the risk free interest rate plus the country risk premium associated with the country where the mining project is situated (Cortazar et al., 1997; Bellalah 2001). The portfolio then becomes:

$$dV + q(Z - C)(1 - G)dt - \frac{V_Z}{F_Z} dF = (r + \lambda_C)V\, dt \tag{5-9}$$

Here C is the total cost (CAPEX, OPEX, working capital, etc.) per unit of commodity and q is the operating rate and G is the total tax.

Applying Ito's Lemma in $V = V(Z, T)$ the following is obtained:

$$dV = \frac{\partial V}{\partial Z} dZ + \frac{\partial V}{\partial T} dT + \frac{1}{2}\frac{\partial^2 V}{\partial Z^2}(dZ)^2 \tag{5-10}$$

Using (5-8) and (5-10), we obtain from (5-9)

$$\frac{\partial V}{\partial Z} dZ + \frac{\partial V}{\partial T} dT + \frac{1}{2}\frac{\partial^2 V}{\partial Z^2}(dZ)^2 + q(Z - C)(1 - G)dt + \frac{\partial V}{\partial Z}(r\, Z\, dt - dZ - (\delta - \alpha - \rho\, \sigma_P \sigma_E) Z\, dt)$$
$$= (r + \lambda_C)V\, dt$$

When the production of the mine is ongoing, the overall mine life will logically decrease over time and will change accordingly, $dT = -dt$.

$$\Rightarrow -\frac{\partial V}{\partial T} dt + \frac{1}{2} Z^Z \sigma_Z^2 \frac{\partial^2 V}{\partial Z^2} dt + (r - (\delta - \alpha - \rho\sigma_P\sigma_E))\frac{\partial V}{\partial Z} dt + q(Z - C)(1 - G)dt$$
$$= (r + \lambda_C)V\, dt$$

$$\Rightarrow \frac{1}{2} Z^Z \sigma_Z^2 \frac{\partial^2 V}{\partial Z^2} - \frac{\partial V}{\partial T} + (r - (\delta - \alpha - \rho\sigma_P\sigma_E))Z\frac{\partial V}{\partial Z} - (r + \lambda_C)V = -q(Z - C)(1 - G)$$

(5-11)

This is a realistic PDE for approximating the project's value that incorporates several financial, economic and mining project related parameters. Brennan and Schwartz (1985) mentioned that there is no analytical solution for this type of problem i.e. PDE (5-11). Therefore the numerical solutions of the PDE (5-11) will be approximated through the finite difference method (FDM) using MatLab software, which will commensurately provide the numerical values of the mining project. For solving the PDE (5-11) numerically, a number of conditions need to be imposed. Since a real option is similar to a financial option, one of the boundary conditions is considered as $V(0, T) = 0$, being that when the output price is zero at any time of the mine's life, the value of the mine is zero. The other condition is $V(Z, 0) = 0$, implying that when the time is zero or the time is exhausted for the mine operation, the value of the mine is zero.

Furthermore, we also assume that $\lim_{Z\to\infty} \frac{V(Z,T)}{Z} \to 1$, i.e. $\lim_{Z\to\infty} \frac{\partial^2 V}{\partial Z^2}(Z,T) = 0$.

The early exercise option has also been checked, as mining projects may present an opportunity to exercise the option early, similar to an American call option.

Note: If the the mine operation is considered at time, $t \quad t \in [0,T]$ *instead of* T *where* T *is the total duration of mine operation, then it is possible to rewrite the PDE (5-11) into the familiar form, as follows:*

$$\frac{1}{2} Z^Z \sigma_Z^2 \frac{\partial^2 V}{\partial Z^2} - \frac{\partial V}{\partial t} + \left(r - \left(\delta - \alpha - \rho\sigma_P\sigma_E\right)\right) Z \frac{\partial V}{\partial Z} - \left(r + \lambda_C\right)V = -q(Z - C)(1 - G)$$

5.4 Discretization of PDE

For a numerical solution, PDE (5-11) has been discretized using the finite difference method (FDM) and solved numerically through MatLab software. The discretized form of the PDE (5-11) is as follows:

$$\Rightarrow \frac{V_{n,\,j+1} - V_{n,\,j}}{\Delta T} - \frac{\sigma^2}{2}(n\Delta Z)^2 \left(\frac{V_{n+1,\,j} - 2\,V_{n,\,j} + V_{n-1,\,j}}{(\Delta Z)^2}\right) - \left(r - \left(\delta - \alpha - \rho\sigma_P\sigma_E\right)\right) n\Delta Z \left(\frac{V_{n+1,\,j} - V_{n-1,\,j}}{2\Delta Z}\right) + \left(r + \lambda_C\right) V_{n,\,j} - q(n\Delta Z - C)(1 - G) = 0$$

$$\Rightarrow V_{n,\,j+1} = V_{n,\,j} + \frac{\sigma^2}{2}(n\Delta Z)^2 \left(\frac{V_{n+1,\,j} - 2\,V_{n,\,j} + V_{n-1,\,j}}{(\Delta Z)^2}\right)\Delta T + \left(r - \left(\delta - \alpha - \rho\sigma_P\sigma_E\right)\right) n\Delta Z \left(\frac{V_{n+1,\,j} - V_{n-1,\,j}}{2\Delta Z}\right)\Delta T - \left(r + \lambda_C\right) V_{n,\,j}\Delta T + q(n\Delta Z - C)(1 - G)\Delta T$$

Now the solution can be found as the form:

$$V_{n,\,j+1} = a_n\, V_{n-1,\,j} \; + b_n\, V_{n,\,j} + c_n \; V_{n+1,\,j} + d_n \tag{5-12}$$

where

$$a_n = \frac{\Delta T}{2}\left(n^2\sigma_Z^2 - n\,(r - \delta + \alpha + \rho\sigma_P\sigma_E\,)\,\right)$$

$$b_n = 1 - \Delta T\left(n^2\sigma_Z^2 + \left(r + \lambda_C\right)\,\right)$$

$$c_n = \frac{\Delta T}{2}\left(n^2\sigma_Z^2 + n\,(r - \delta + \alpha + \rho\sigma_P\sigma_E\,)\,\right)$$

$$d_n = q\,\Delta T\,(n\;\Delta Z - C\;)\,(1 - G)$$

Here, $Z \; = n\Delta Z\;, \quad n = 0\;,1,\cdots\cdots M$

$$T = j\;\Delta T\;, \quad j = 0\;,1,\cdots\cdots N$$

$$V_{n,\,j} = V\left(n\Delta Z,\, j\Delta t\right)$$

5.5 Determination of exchange rate and commodity price volatility

In this section, exchange rate and commodity price volatilities have been estimated based on historical data. These volatilities are needed to find the simulated commodity price paths and to approximate the mining project values numerically. As a case study, AUD/USD exchange rate and iron ore prices are considered here.

5.5.1 Determination of AUD/USD exchange rate volatility

The selected period over which the data have been collected is the period between 1996-2014, chosen because the data period reflects a long-enough meaningful period and simultaneously the immediate past, being the last 17 to 18 years. This historical data will provide an overview of the historical AUD/USD exchange rate volatility. Moreover, during this period, there was a significant global recession in 2008, a consequence of the global financial crisis (GFC) which impacted the foreign exchange markets and almost every other financial market. The AUD/USD nominal exchange rate volatility has been determined from historical data using Microsoft Excel and by employing the log return method of analysis (for details refer to Chapter 3)

As can be seen from *Figure 5-1, the annual historical AUD/USD exchange rate volatility was only 6.47% in 1996. This volatility fluctuated gradually year-on-year and, in 2008, it dramatically reached its highest peak at 24.54%. Thereafter, the volatility gradually reduced and reached 9.04% in 2012. The average volatility of the AUD/USD exchange rate was 11.88% during the period 1996-2014.

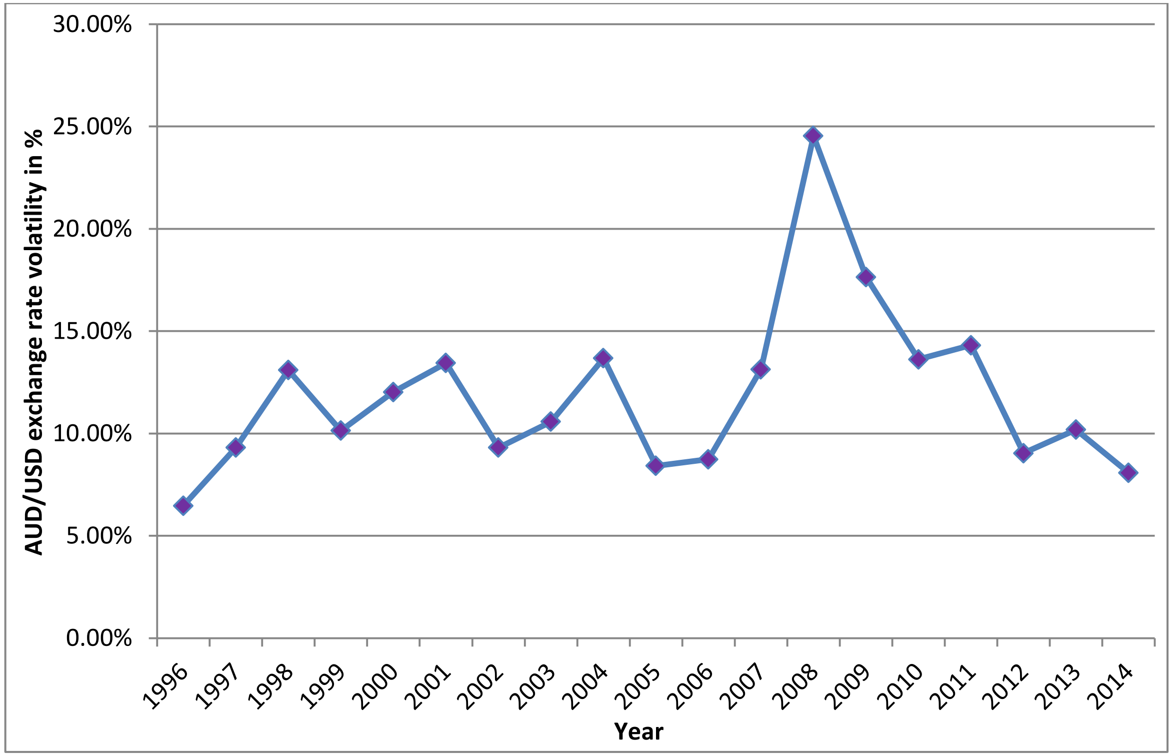

Figure 5-1 AUD/USD exchange rate volatility over the period 1996-2014. Data source: Reserve Bank of Australia. http://www.rba.gov.au/statistics/frequency/exchange-rates.html

5.5.2 Determination of iron ore price volatility

Historical trade in iron ore was based on long-term and short-term contracts, and therefore the iron ore price did not vary at all on a monthly basis over any specific year. That is, each month's iron ore price was identical for every other month in each year (for details refer to Haque et al., 2015a). Therefore, for this study, iron ore price data have been considered over the period from 2009 to 2014 being the period over which iron ore prices started to fluctuate from month to month.

Figure 5-2 shows that there was considerable fluctuation in the volatility of iron ore prices from the year 2009 to 2014. In 2009 the volatility was 40.78%, after which the volatility declined dramatically and reached 22.90% in 2011. Thereafter, the iron ore price volatility increased and reached 27.98% in 2012. The average iron ore price volatility was 27.93% during the period between 2009-2014.

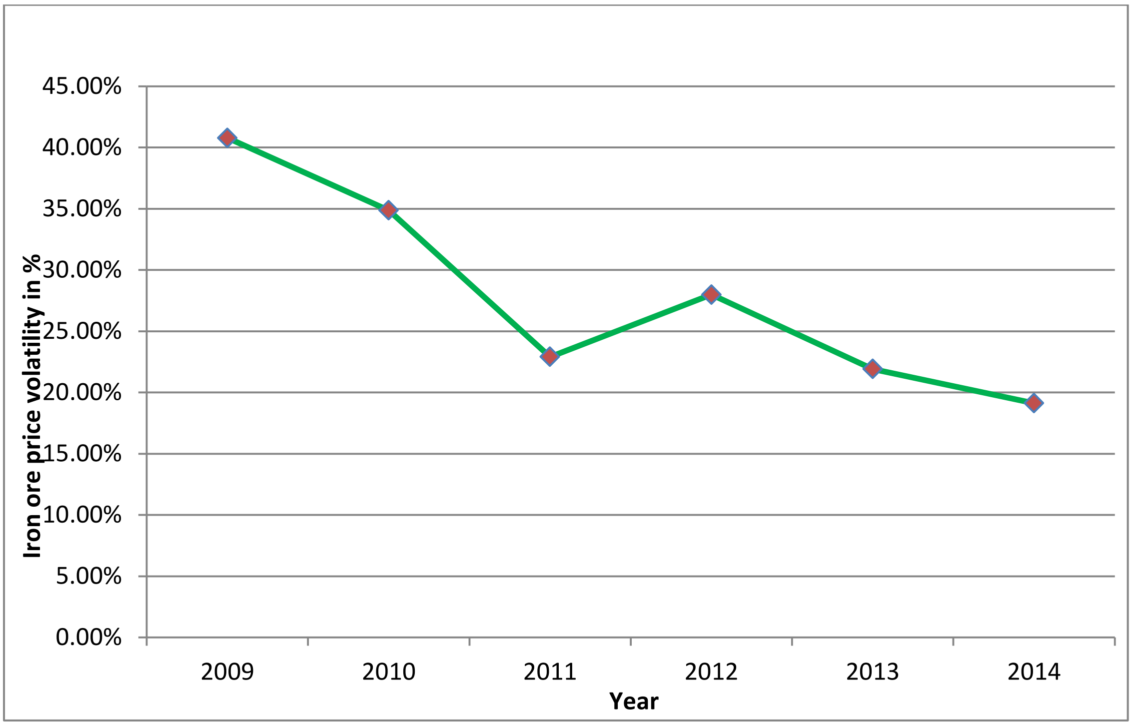

Figure 5-2 Iron ore price volatility over the period 1996-2014. Data source: http://www.indexmundi.com

*Figure 5-3 presents the comparison of volatilities of iron ore prices and AUD/USD exchange rates between 2009-2014. From the figure it is apparent that, in 2009, both the iron ore price and the AUD/USD exchange rate volatilities are determined at their highest points, following which their respective volatilities declined gradually with substantial short-term fluctuations. Between the period 2009-2014, the average volatility of the iron ore price was 27.93% whereas the average volatility of the AUD/USD exchange rate was 12.15%.

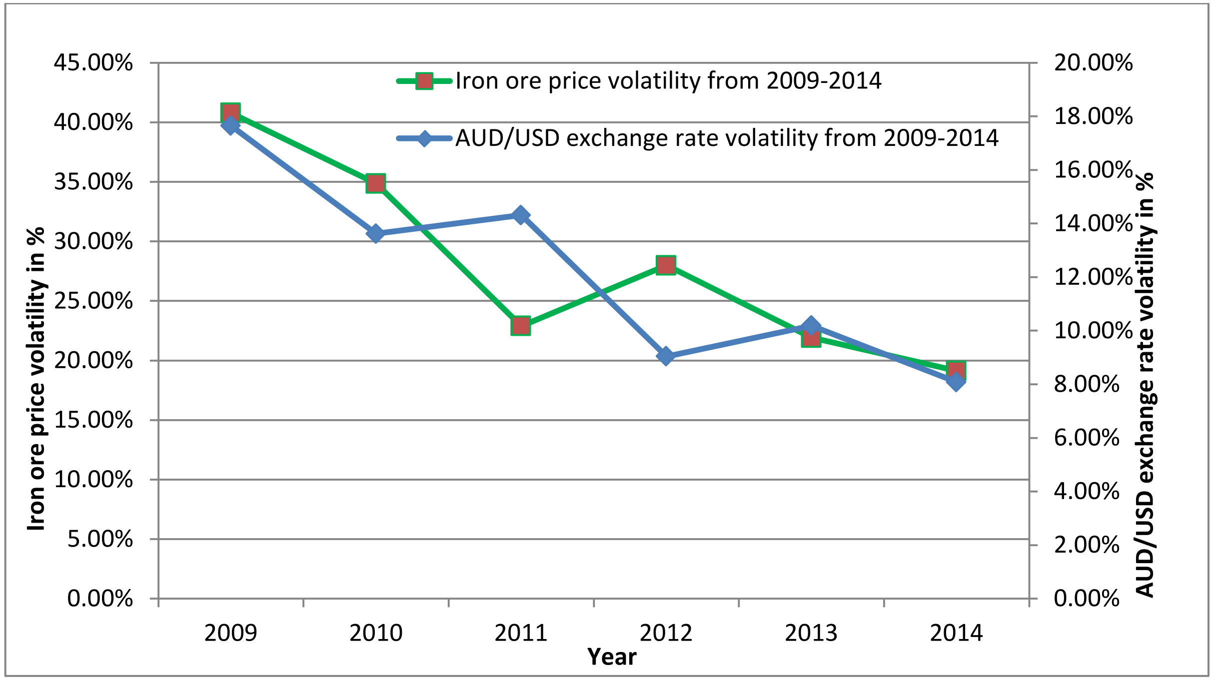

Figure 5- 3 Comparison of iron ore price and AUD/USD exchange rate volatilities between 2009-2014.

5.6. Simulated commodity price paths

To find the simulated commodity price paths under the combined volatility of commodity prices and exchange rates, and the correlation between commodity price and exchange rate, the equation (5-5) was used, and as a case study two simulated commodity price paths are shown.

5.6.1 Simulated iron ore price paths

Four illustrative simulated iron ore price paths for a period from 2015 to 2027 have been shown in Figure 5-4 using MatLab software. The iron ore price of US$52.74 per dry metric tonne (dmt) is used as the initial price (spot price), and the calculated mean iron ore price volatility and the mean exchange rate volatility have been used in the simulation.

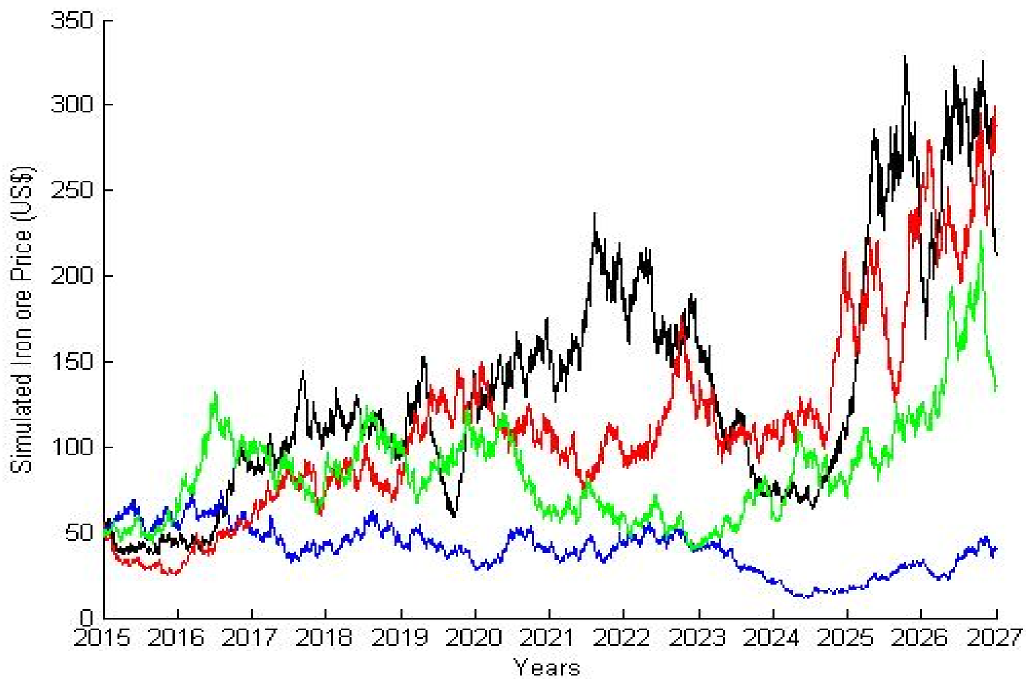

Figure 5-4 Four illustrative paths of simulated dry iron ore prices during 2015-2027.

The average realized simulated iron ore prices, per dry metric tonne (dmt), of the four paths are US$86.18, US$112.10, US$40.45, andUS$134.75, respectively. The simulated realized average iron ore price has been used to approximate the iron ore mining project values.

5.6.2 Simulated gold price paths

Four simulated gold price paths are shown in Figure 5-5 over the period 2016 to 2028. The gold spot price of US$1,085.40 per ounce is used as the initial price, and the correlation between gold prices and AUD/USD exchange rates (refer to Haque et al., 2015, chapter 4), the calculated mean gold price volatility (determined in chapter 3), and the mean exchange rate volatility are used in the simulation.

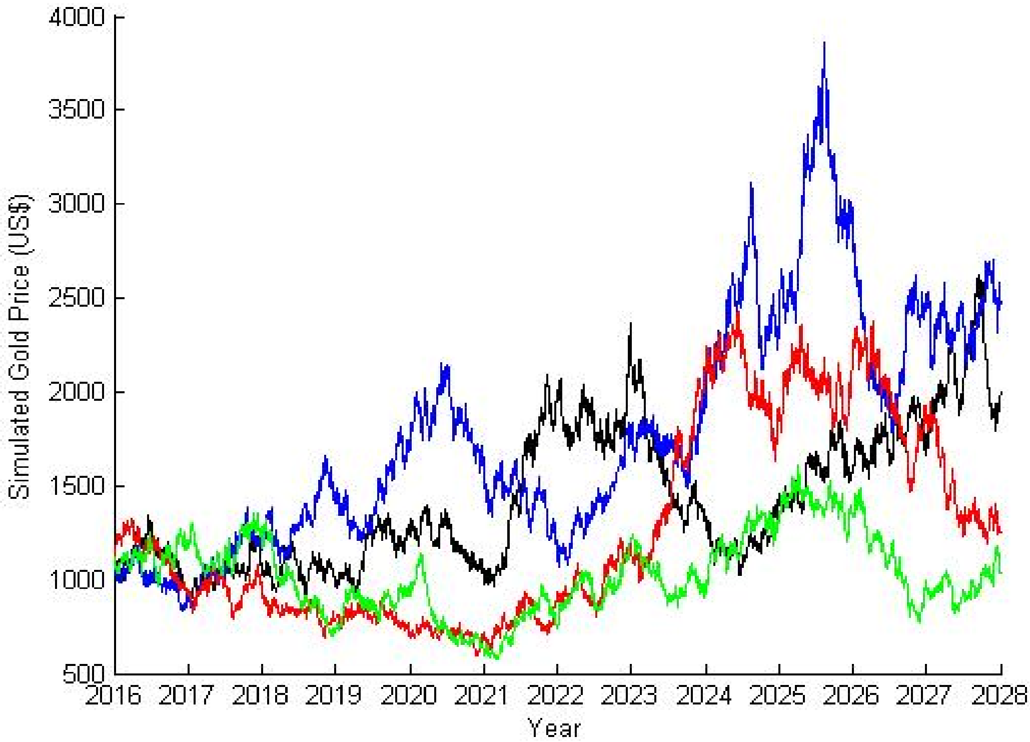

Figure 5-5 Four illustrative paths of simulated gold prices over the period 2016-2028.

The average realized simulated gold prices, per ounce, of the four paths are shown as US$1029.20, US$1,269.10, US$1,428.20, and US$1,779.50 respectively. The simulated realized average gold price is used to estimate the gold mining project values.

5.7 Case studies

To estimate/approximate mining project values by solving the PDE numerically, as case studies, we have considered two conceptual mining projects, an iron ore mine and an open pit gold mine.

5.7.1 Case study 1: A conceptual iron ore mine

To find the approximate numerical values for a mining project under different scenarios and with a number of available real options, consider the following case study.

Scenario 1

Consider a conceptual open pit iron ore mine in Western Australia that extracted iron ore at a rate of 1,340,987 dry metric tonnes per year (approximately) as presented in Table 5-1, and the operating period was 1 year, starting from January 2011 to December 2011.

For scenario 1, the total production and sales cost of each dry metric tonne of iron ore is US$36.31 (US$23.60 + US$11.31+US$1.40). To estimate project values in each case, 60 by 1,800 matrixes (60x1, 800) have been created, and each grid point project value has been calculated. Negative project values are excluded to generate figures which are presented in this chapter.

Table 5-1** Characteristics/data sets of the conceptual iron ore mine are as follows (prior to production before the mine is temporarily closed. All financial figures expressed in real money terms).

Total operation	1 year
Average iron ore production rate	1,340,987dmt per year (approximately)
Iron ore price	US$86.18 per dry metric tonne (average realization price from simulation path)
Total milling (refining, treatment, etc.) and sales costs	US$23.60 per dry metric tonne
Fixed pre-tax operating costs (OPEX, CAPEX etc.)	US$11.31 per dry metric tonne
* Iron ore price mean volatility	27.93%
Risk free interest rate	5.00%
Country risk premium	2.00%
Depreciation and amortization charges	US$1.40 per dry metric tonne
*Exchange rate (AUD/USD) mean volatility	12.15% (During 2009-2014)
Exchange rate (AUD/USD) drift(expected rate of return)	0.03
*Correlation between iron ore prices and AUD/USD exchange rates	0.78(Determined in Chapter 4)
Convenience yield for holding iron ore	2.00%
Corporate taxes	25%

* Parameter values in Tables 5-1, 5-2, 5-3 were determined from historical data.

Figure 5-6 displays the project values graphically before the mine is temporarily closed (scenario 1, and using the data from Table 5-1), and the approximate maximum project value under the joint effect of commodity price and exchange rate uncertainties is US$16,501,000.

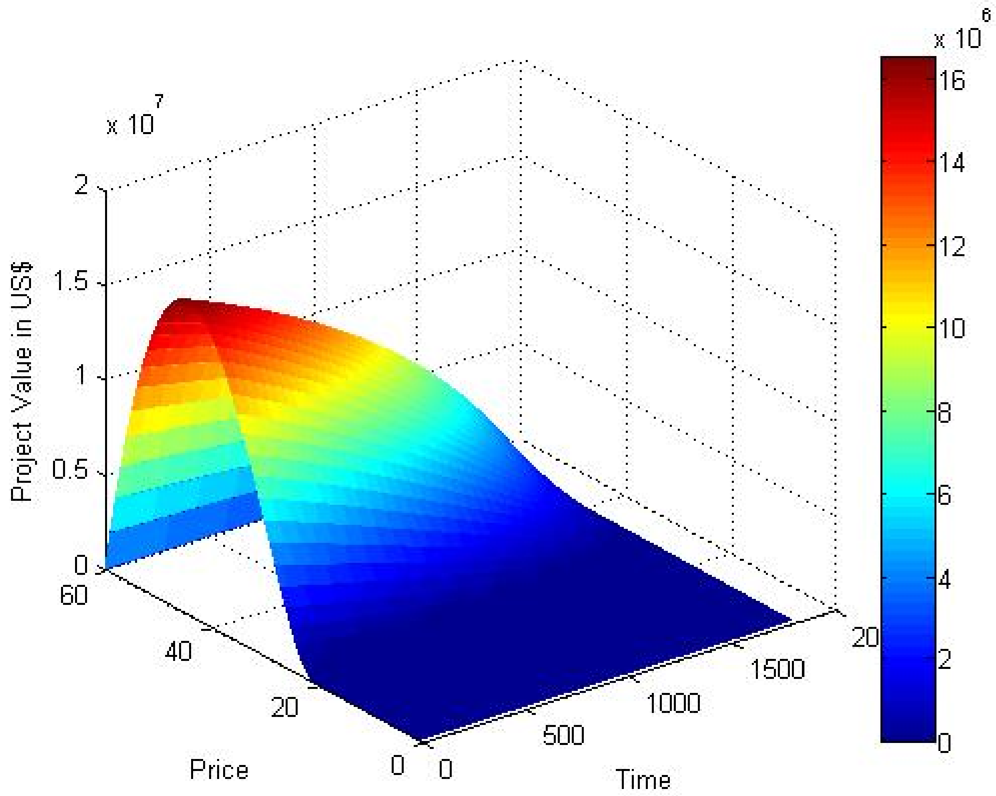

Figure 5-6 Project values of the mine in linear scale (data are used from Table 5-1).

However, when only commodity price uncertainty is considered, the approximate maximum project value for one year of operation is shown to be US$21,114,000 (calculated from the PDE (5-8a), Figure A5-1. For details, please refer to Chapter 5, Appendix 5.1A and 5.2A). When exchange rate uncertainty is added to the commodity price uncertainty in the model, the overall volatility is increased and consequently, the return is decreased i.e. the project value decreases (for details refer to Chapter 5, Appendix 5.4A).

The option to defer investment

Due to mineral commodity price depreciation, management can defer the initiation of the project (the mine operation) for a year by incurring an extra cost (holding cost), and then management can monitor whether the commodity price improves or falls further. Suppose the above project described in Table 5-1 defers mining start-up for a year and the average iron ore price after one year increases to US$100.10 per dry metric tonne. The total production and sales cost of each dry metric tonne of iron ore increases to US$38.65. In this situation, the approximate maximum project value of the project is US$21,776,000 under the commodity price and exchange rates as presented in Figure 5-7. Therefore, the deferral option enhanced the value of the project.

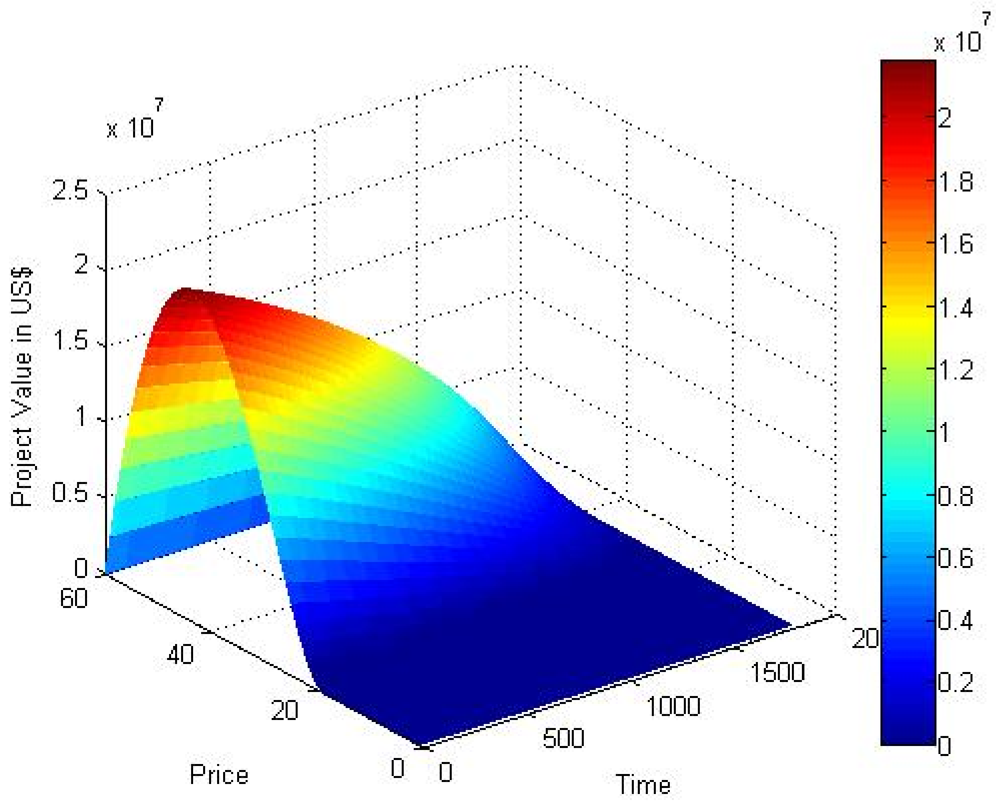

Figure 5-7 Project values of the mine in linear scale (data are used from Table 5-1, and, P=100.10 US$/dmt, C =38.65US$/dmt).

Scenario 2

Management continued the mine operation over the period from January 2011 to December 2011 and, thereafter, due to the reduced iron ore price, mining operations were suspended in early 2012. Management closed the mine temporarily and incurred the maintaining costs during the closure period. In 2013, the iron ore price increased markedly and the management of the mine decided to reopen the mine and exploit from the remaining iron ore reserves at an extraction rate of 1,662,986 dry metric tonnes per year as presented in Table 5-2. The iron ore deposit has a possible remaining reserve of approximately 6 years of extraction. After one year of extraction in 2013, management again stopped mining production due to depressed iron ore prices.

Table 5-2 Characteristics/data sets of the conceptual iron ore mine are as follows (the mine is reopened after being temporarily closed. All financial figures expressed in real money terms).

Total operation	1 years
Average iron ore production rate	1,662,986dmt per year (approximately)
Iron ore price	US$112.10 per dry metric tonne (average realization price from simulation path)
Total milling (refining, treatment etc.) and sales costs	US$24.70 per dry metric tonne
Fixed pre-tax operating costs (OPEX, CAPEX etc.)	US$11.75 per dry metric tonne
Total mine closure costs	US$7,250,600
Total cost for reopening the mine	US$6,768,400
* Iron ore price mean volatility	27.93%
Risk free interest rate	5.00%
Country risk premium	2.00%
Depreciation and amortization charges	US$1.40 per dry metric tonne
*Exchange rate (AUD/USD) mean volatility	12.15% (During 2009-2014)
Exchange rate (AUD/USD) drift	0.03
*Correlation between iron ore prices and AUD/USD exchange rates	0.78
Convenience yield for holding iron ore	2.00%
Corporate taxes	25%

Temporary closure option

After the temporary closure of the mine, when the commodity price rises significantly, management can reopen the mine. In this case, management can approximate the project value through the same PDE. To achieve this, maintaining and reopening costs need to be calculated and added to the other operating costs. Temporary closure or project delays may carry a benefit but most delays also involve a cost (Eschenbach et al. 2009). Of course, in mining projects a significant amount of maintaining and reopening costs are involved due to temporary closures or deferral real options.

From scenario 2 and Table 5-2, for reopening the mine after temporary closure, the total production and sales cost of each dry metric tonne of iron ore is US$46.28 (US$24.70 +US$11.75+US$4.36+US$4.07+US$1.40). Using the data from Table 5-2, the approximate maximum project values for a one year operation under the joint effect of commodity price and exchange rate uncertainties is US$27,487,000.

Figure 5-8 shows the project values of the mine when the mine is reopened after temporary closure, and the approximate maximum project value is US$27,487,000 under the joint effect of commodity price and exchange rate uncertainty.

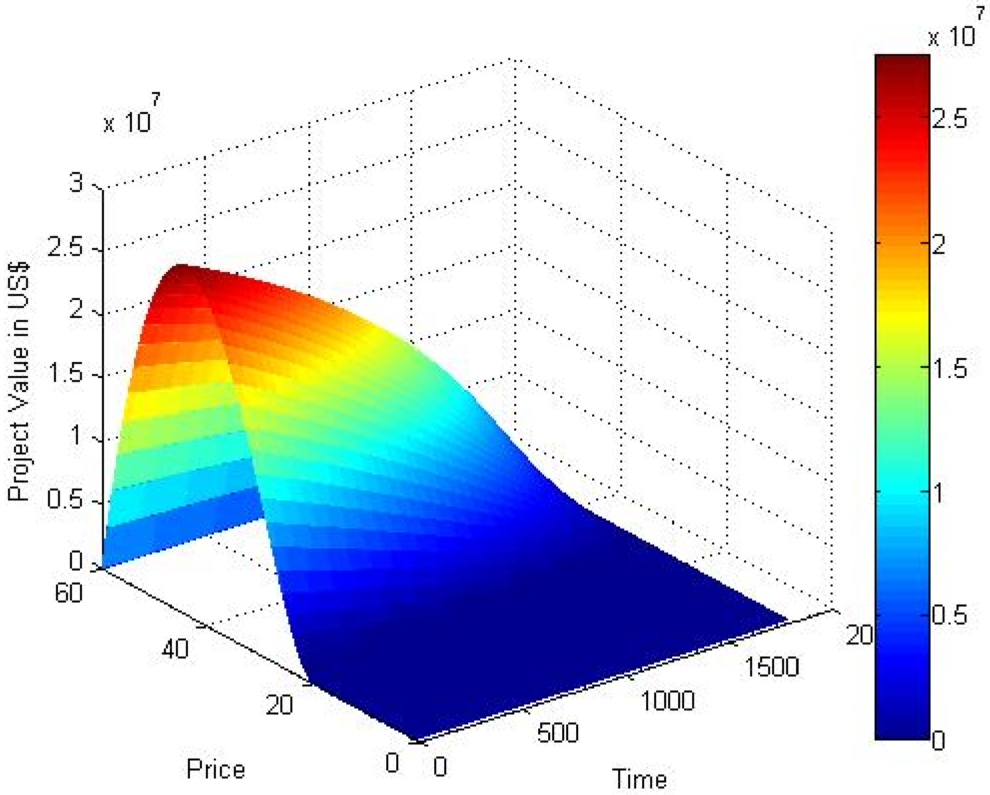

Figure 5-8 Project values of the mine in linear scale (data are used from Table 5-2).

However, when only the commodity price uncertainty is considered, the approximate maximum project value for a one year operation is calculated as US$35,003,000 (calculated from the PDE (5-8a), refer to Figure A5-2 in Chapter 5, Appendix 5.2A).

Scenario 3

After one and a half years, the iron ore price again increases dramatically and the increased price continues to hold over the full year. Management can reopen the mine and also adopt the expansion option for mine production. That means management can increase the operating rate/production rate during this year, as presented in Table 5-3.

Temporary closure option and accelerate or decelerate options for mine operation

If the commodity price rises considerably, management can reopen the mine after a temporary closure. Furthermore, if the commodity price increases noticeably then management can accelerate the production rate which helps to realize a higher project value and associated returns (Figure 5-9). Alternatively, if the commodity price decreases significantly, then management can reduce the production rate which will generate a lower project value and return at that time, but may increase the life of the mine and provide an opportunity to wait until more favorable conditions return for realizing a higher project value (retaining the option value).

For scenario 3, management can exercise the real option and again reopen the mine after the second temporary closure. As the iron ore price is high, management can also adopt the accelerate option and increase the production or operating rate.

Now the total production and sales cost of each dry metric tonne of iron ore is US$53.83(US$29.60+US$13.65+US$4.83+US$4.35+US$1.40) which is presented in Table 5-3.

Table 5-3 Characteristics/data sets of the conceptual iron ore mine are as follows (the mine is again reopened after being temporarily closed and has adopted the acceleration option i.e. increased operating rate. All financial figures expressed in real money terms).

Total operation	1 year
Average iron ore production rate	1,862,986 dmt per year (approximately)
Iron ore price	US$134.75 per dry metric tonne (average realization price from simulation path)
Total milling (refining, treatment etc.) and sales costs	US$29.60 per dry metric tonne
Fixed pre-tax operating costs (OPEX, CAPEX etc.)	US$13.65 per dry metric tonne
Total mine closure costs	US$8,995,700
Total cost for reopening the mine	US$8,105,300
*Iron ore price mean volatility	27.93%
Risk free interest rate	5.00%
Country risk premium	2.00%
Depreciation and amortization charges	US$1.40 per dry metric tonne
*Exchange rate (AUD/USD) mean volatility	12.15% (During 2009-2014)
Exchange rate (AUD/USD) drift	0.03
*Correlation between iron ore prices and AUD/USD exchange rates	0.78
Convenience yield for holding iron ore	2.00%
Corporate taxes	25%

****Note:** Data in Tables 5-1, 5-2, 5-3 have been considered based on US dollars, therefore to determine project values in other currencies, the data and currency must be converted from their local currency at the prevailing and forecast future currency exchange rates.

Figure 5-9 displays the project values of the mine when the mine is again reopened after a temporary closure and management adopts the expansion option (increased production rate).

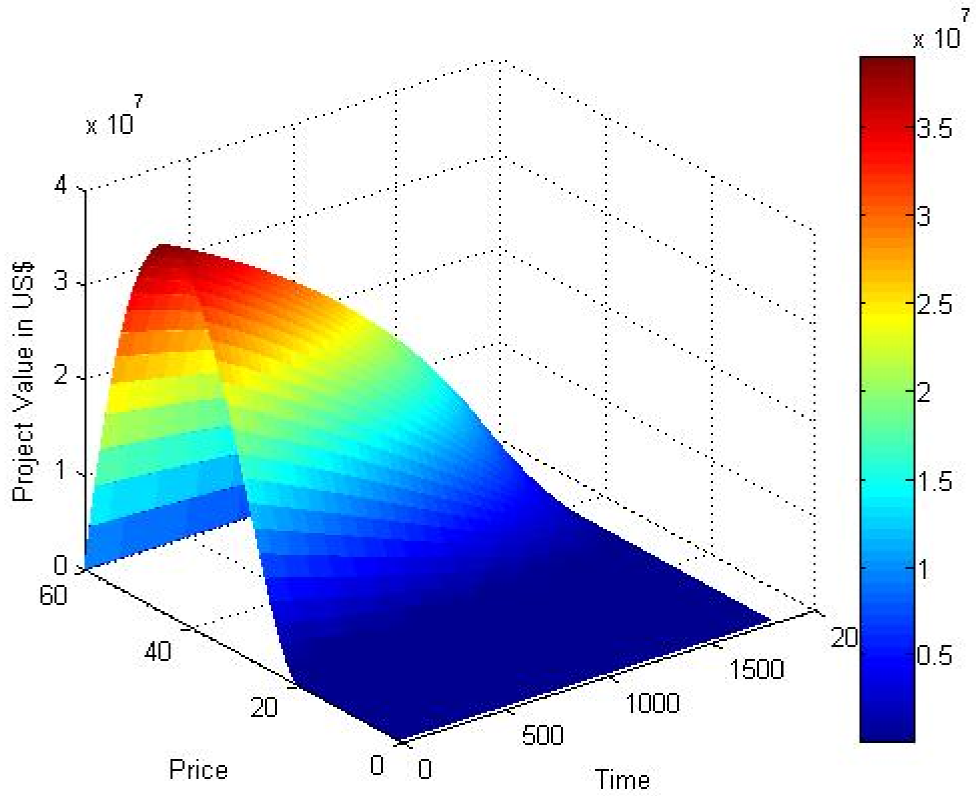

Figure 5-9 Project values of the mine in linear scale (data are used from Table 5-3).

In this case the approximate maximum project value under the joint effect of commodity price and exchange rate uncertainties is US$38,865,000, and it has been observed that the expansion option added value to the project.

However, if only the commodity price uncertainty is considered, the approximate maximum project value for a one year operation is calculated as US$49,184,000 (see Figure A5-3, in Chapter 5, Appendix 5.2A).

5.7.2 Case study 2: A conceptual open pit gold mine

To find the approximate numerical values or value ranges for a gold mining project, consider the following scenarios.

Scenario 1

Consider a conceptual small open pit gold mine in Australia that extracted gold at a rate of 856,890 tonnes per year (approximately) as presented in Table 5-4, and continued operating for 1 year, starting from January 2014 to December 2014.

For scenario 1, the total production and sales cost of each ounce of gold is US$585.10(US$450.10+US$135.00).Using the data from Table 5-4, the approximate maximum project value for one year of operation under the joint effect of commodity price uncertainty and exchange rate uncertainty is calculated as US$16,197,000.

Table 5-4 ** Characteristics/data sets of the conceptual open pit gold mine are as follows (all financial figures expressed in real money terms).

Total operation	1 year
Average ore production rate	856,890 tonnes per year(approximately)
Average grade of gold	5.25 g/t
Average gold production rate	158,690 ounces per year(approximately)
Gold price	US$1029.20 per ounce (average realization price from simulation path)
Total milling and sales costs	US$450.10 per ounce
The fixed cost and pre-tax operating costs (CAPEX, OPEX etc.)	US$135.00 per ounce
*Gold price mean volatility	16.99% (Determined in Chapter 3)
*Standard deviation of volatility from the mean volatility	5.13%
*Exchange rate (AUD/USD) mean volatility	11.88% (During 1996-2014)
Exchange rate (AUD/USD) drift	0.03
Correlation between gold prices and AUD/USD exchange rates	0.91(Determined in Chapter 4)
Risk free interest rate	5.00%
Country risk premium	2.00%
The depreciation and amortization charges	0
Convenience/dividend yield for holding gold	2.2%
Corporate taxes	25%

Figure 5-10 shows the project values of the mine when the average gold price is US$1029.20/oz, and the approximate maximum project value under the joint effect of commodity price uncertainty and exchange rate uncertainty is calculated as US$16,197,000.

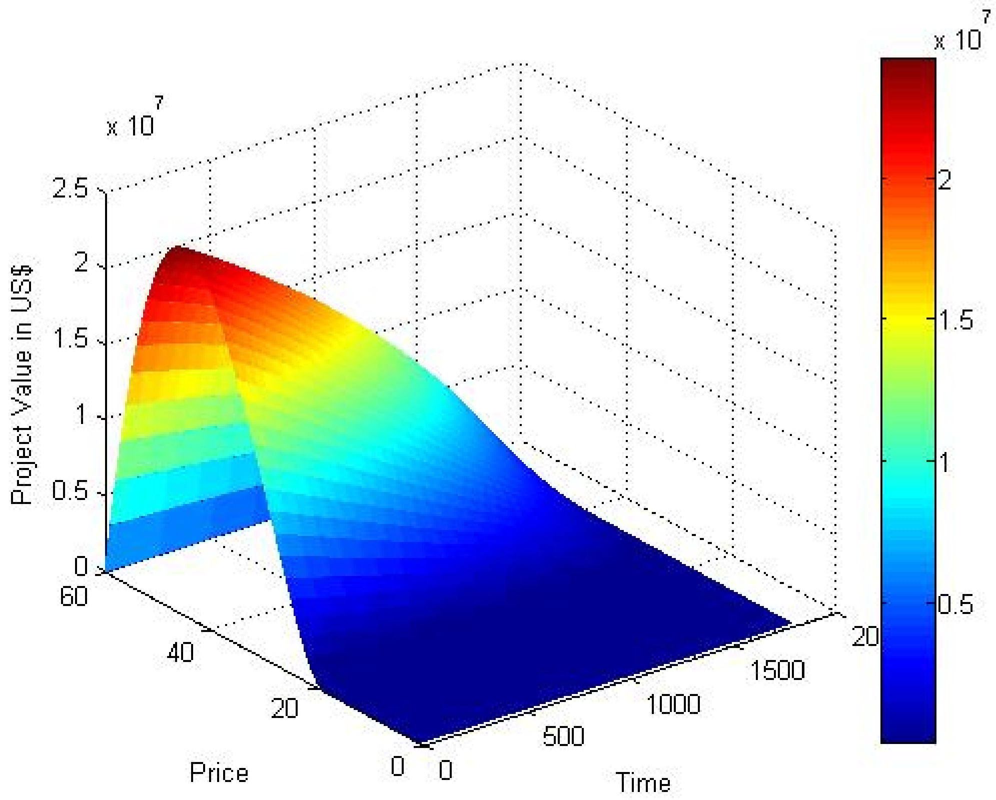

Figure 5-10 Project values of the mine in linear scale (data are used from Table 5-4).

Nevertheless, when only the commodity price uncertainty is considered, the approximate maximum project value for one year of operation is shown to be US$24,190,000 (calculated from the PDE (5-8a), Figure A5-4, for details, refer to Chapter 5, Appendix 5.3A).

If the company has a one year lease which, providing it is a proprietary right to defer commencing the project or if the company can wait just by incurring a holding cost, then, due to commodity price depreciation, management can defer to initiate the project (the mine operation) for a year. Suppose the above project described in Table 5-4 is deferred for a year, and the average gold price for the following year increases to US$1140.50 per ounce. The total production and sales cost of each ounce of gold increases to US$605.25. In this situation, the approximate maximum numeric project value is US$21,540,000. Therefore, the defer option added value to the project.

Figure 5-11 shows the project values of the mine when the average gold price is US$ 1,140.50/oz, after deferring the mine operation for one year.

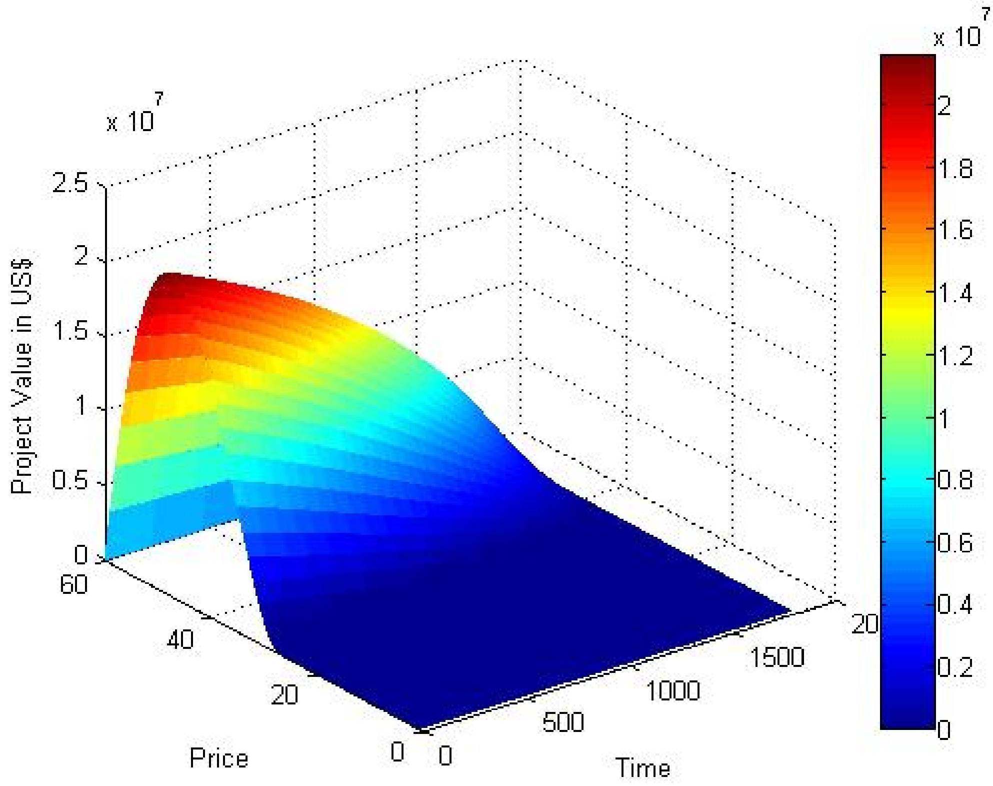

Figure 5-11 Project values of the mine in linear scale (data are used from Table 5-4, and, P=1140.50 US$/oz, C=605.25 US$/oz)

However, the approximate maximum project value for a one year operation is calculated as US$31,035,000, when a single uncertainty i.e. the commodity price uncertainty, is considered (calculated from the PDE (5-8a), see Figure A5-5 in Chapter 5, Appendix 5.3A).

Scenario 2

Consider management continuing the mine operation over 2014 and, thereafter, due to the reduced gold price, mining operations were suspended in 2015. Management closed the mine temporarily and incurred maintaining costs during the closure period. In 2016, the gold price increases and the management of the mine decided to reopen the mine and exploit from the remaining reserves at an extraction rate of 1,099,690 tonnes per year as presented in Table 5-5. The gold deposit has a possible remaining reserve of approximately 5 years of extraction. After one year of extraction in 2016 (from January 2016 to December 2016), management again stopped mining production due to depressed gold prices.

Table 5-5 Characteristics/data sets of the conceptual open pit gold mine are as follows (the mine is reopened after being temporarily closed. All financial figures expressed in real money terms).

Total operation	1 year
Average ore production rate	1,099,690 tonnes per year approximately)
Average grade of gold	4.70 g/t
Average gold production rate	182,320 ounces per year (approximately)
Gold price	US$1,428.20 per ounce (average realization price from simulation path)
Total milling and sales costs	US$490.15 per ounce
The fixed cost and pre-tax operating cost (CAPEX, OPEX, etc.)	US$150.28 per ounce
Total mine closure costs	US$2,096,600
Total cost for reopening the mine	US$2,670,988
Risk free interest rate	5.00%
*Gold price mean volatility	16.99%
*Standard deviation of volatility from the mean volatility	5.13%
Exchange rate (AUD/USD) drift	0.03
*Exchange rate (AUD/USD) mean volatility	11.88% (During 1996-2014)
Correlation between gold prices and AUD/USD exchange rates	0.91
Country risk premium	2.00%
Convenience/dividend yield for holding gold	2.2%
Depreciation and amortization charges	0
Corporate taxes	25%

* Parameter values in Table 5-4 to Table 5-5 were determined from historical data.

****Note:** Data in Table 5-4 to Table 5-5 have been considered based on US dollars, therefore to determine project values in other currencies, the data and currency must be converted from their local currency.

After the temporary closure of the mine, when the commodity price rises significantly, management can reopen the mine. In this case, management could approximate the project value through the same PDE (5-11). To acquire this, maintaining and reopening costs need to be calculated and added to the other operating costs.

From Scenario 2, for reopening the mine after temporary closure, the total production and sales cost for each ounce of gold is US$666.58 (US$490.15+US$150.28 +US$11.50+US$14.65). Here, total milling and sales costs is US$490.15 per ounce, the fixed cost and pre-tax operating cost (CAPEX, OPEX, etc.) is US$150.28 per ounce, total mine closure costs is US$11.50 per ounce and total cost for reopening the mine is US$14.65 per ounce. Hence, using the data from Table 5-5, the approximate maximum project values for a one year operation under the joint effect of commodity price uncertainty and exchange rate uncertainty is calculated as US$40,283,000.

Figure 5-12 shows the project values of the mine when the mine is reopened after temporary closure, and the approximate maximum project value is US$40,283,000.

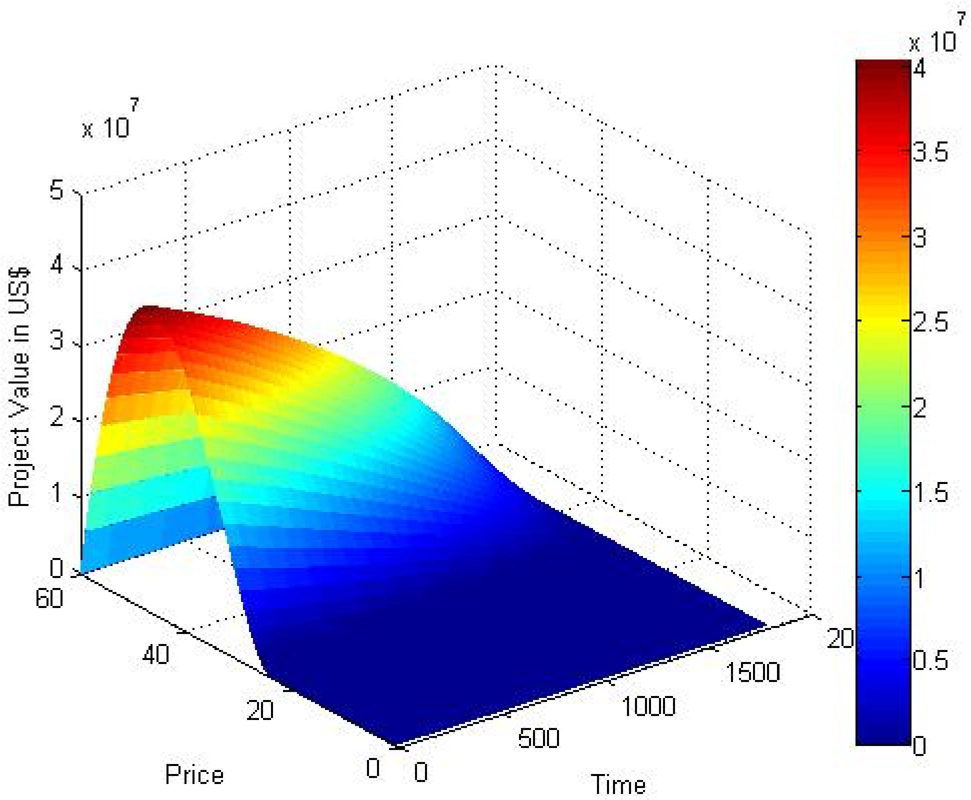

Figure 5-12 Project values of the mine in linear scale (data are used from Table 5-5, average gold production rate is 182,320 ounces per year).

On the other hand, while only the commodity price uncertainty is considered, the approximate maximum project value for one year operation is calculated as US$55,121,000 (calculated from the PDE (5-8a), see Figure A5-6 in Chapter 5, Appendix 5.3A).

When the commodity price rises considerably, management can reopen the mine after a temporary closure period. Furthermore, if the commodity price increases noticeably then management can accelerate the production rate which helps to realize a higher project value and associated returns (Figure 5-13). Alternatively, when the commodity price decreases significantly, then management can reduce the production rate which will generate a lower project value and return at that time, but may increase the life of the mine and provide an opportunity to wait until more favorable conditions return for realizing a higher project value.

Hence, if the mine is again reopened after a temporary closure and management adopted the expansion option (increased production rate) by incurring an extra cost, the total cost per ounce of gold will be increased and it is assumed at US$672.10 per ounce. The increased gold production rate is 229,860 ounces per year (other parameters are remain same). Therefore, in this scenario, the approximate maximum project value is US$50,072,000 and it has been observed that the expansion option added value to the project.

Figure 5-13 displays the project values of the mine when the mine is reopened after a temporary closure and management adopts the expansion option (increased production rate) by incurring an extra cost.

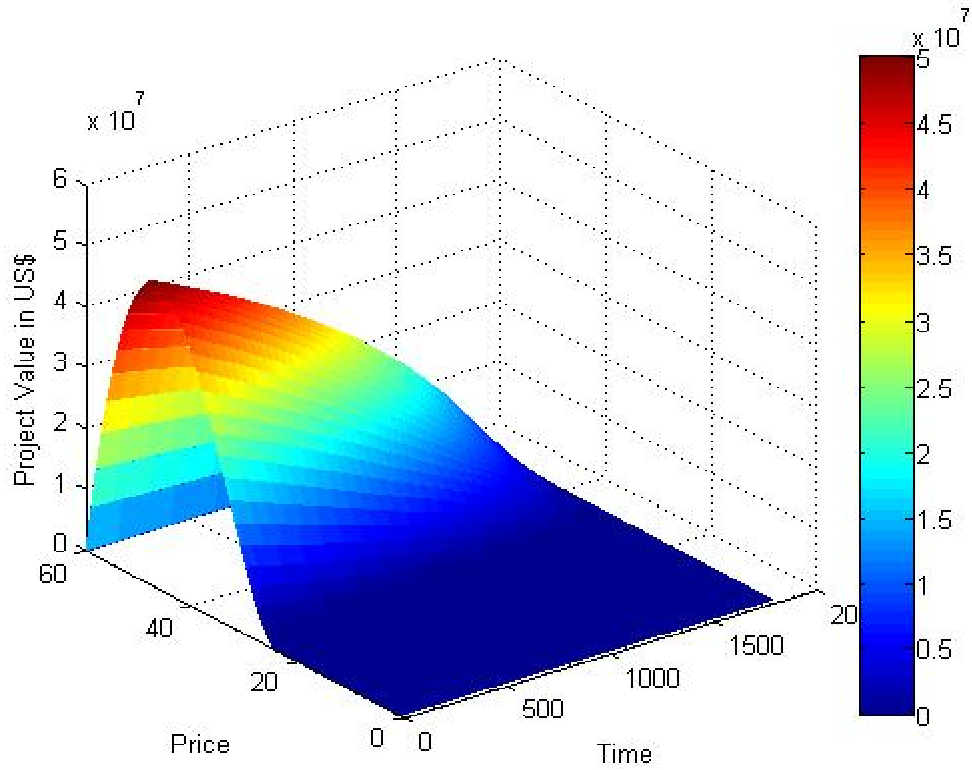

Figure 5-13. Project values of the mine in linear scale (data are used from Table 5-5, average gold production rate is 229,860 ounces per year).

5.8. Summary and concluding remarks

In this chapter, consideration has been given to both commodity price and exchange rate uncertainties to approximate mining project values, numerically, through the use of real options incorporating management flexibilities. This new methodology will help mining companies to approximate project values through optimizing profits and minimizing mining losses. In traditional DCF valuation methods, in addition to Binomial lattice/tree models, it may be difficult and even in some cases impossible to adequately consider these types of financial and economic fundamentals. In the study, managerial flexibilities were evaluated through several real options for approximating project values. These seemingly assisted the mining company and its associated management to take appropriate decisions for investment under different scenarios.

As with commodity price uncertainty, exchange rate uncertainty is also an imperative factor for mining companies. For that reason, in order to more accurately evaluate the economic feasibility of a mining project and its associated value, the exchange rate volatilities were modelled appropriately and included into the evaluation process. Furthermore, the iron ore price paths were simulated during the period 2015-2027 using the joint effect of iron ore price volatility and exchange rate volatility, and the correlation between them was determined to obtain an average realization price of iron ore. This simulated average realization price of iron ore, and the calculated iron ore and exchange rate volatilities, along with their correlation, have been used to approximate the project values. In addition, gold price paths were simulated and shown over the periods from 2016 to 2028. The calculated correlation values between gold prices and AUD/USD exchange rates, the calculated mean gold price volatility (determined in Chapter 3), and the mean exchange rate volatility are used in the simulation. The simulated realized average gold price is used to estimate the gold mining project values. One of the key findings revealed in the study is that project values are over estimated if only commodity price uncertainty is considered in evaluating the project value instead of the joint effect of commodity price and exchange rate uncertainties. Furthermore, it has been observed that the option to defer and option to acceleration of production enhanced value to the projects.

Analytical and numerical solutions derived through the application of ROV methods are rarely found in practice due to the complexity associated with solving the partial differential equations, which are dependent on several conditions and parameters. As a result, it may not generally be applicable to evaluate mining projects under all project-specific circumstances. Consequently, the greatest challenge to ROV modelling is in finding numerically explicit project values. Therefore, the stochastic models have been transformed to higher ordered PDEs with the help of financial options and hedging strategies, and the resulting new PDEs have been solved numerically through the finite difference method and using MatLab software to estimate project values. These tools are useful for the valuation of mining projects under project specific circumstances and remain relevant with the change in financial market conditions.

Furthermore, due to commodity price depreciation and high mining costs, management can delay the project, take the temporary closure option or adopt the abandon option. When the maximum project value is extremely low or approaches zero or the total cash flows of the project are negative, then management can choose the temporary closure option and wait until a favourable commodity price returns or they can adopt the abandon option and leave the project permanently when the overall outlook becomes or remains unfavourable. Although a conceptual iron ore and a conceptual gold project were considered as case studies to determine the numerical values or value ranges of mining projects, using the PDE in this new ROV technique may make it possible to evaluate any mining project value for any commodity, both for underground and open pit mining methods.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

Mining and the development of a mining project are complex and exhibit the sole characteristic of being location–specific. Over longer operational periods and development times, significant economic uncertainties arise commensurate with other geological-technical uncertainties. Cash flows in mining projects are generally volatile and are fundamentally influenced by a number of exogenous variables including commodity price and exchange rate uncertainties. The volatilities related to commodity prices and exchange rates often carry substantial income risk for a mining company, and often create negative sentiment around the minerals industry. Beside commodity price uncertainty, exchange rate uncertainty is an equally crucial economic factor for investment decision-making in the mining sector. As a consequence, mine management needs to address these economic risk exposures before taking an investment decision in a project.

The DCF method is generally used to evaluate project values in the mining industry and has been used for a long time. Beside this, in ROV methods, typically the Binomial Option Pricing (BOP) technique is used as a methodology to evaluate a mining project. However, DCF and BOP methods have several shortcomings (which have been discussed in the problem statement in Chapter 1). Hence, explicit valuations demand appropriate evaluation methods that can address economic uncertainties, consider real life circumstances, and provide mine managers with relevant information before taking a decision of investment in the project. Commodity price uncertainty and exchange rate uncertainty need to be prioritized and considered as the most influential economic uncertainties for decision-making in potentially investing in the mining sector. Due to the dramatic decline of mineral commodity prices, some companies and operations have break-even prices above the ruling commodity spot prices. The recent depreciation of the US dollar against the currencies of major mineral-exporting countries has demonstrated that the foreign exchange rate (FOREX) is also a key contributor to the economic risk surrounding mineral investments. Unfortunately, in many cases, foreign exchange rate uncertainty in the mining sector is ignored. Therefore, in a period of falling commodity prices, it is important to note that there are other economic risk-mitigating options available, with the challenge to mine managers and associated executives being to identify these options and to evaluate them in the face of potential price and currency uncertainties. Additionally, as the mining commodity price and exchange rate uncertainties are crucial factors for mining investment decision-making, it is beneficial to know the relationship between a specific mining commodity price and exchange rate movements before evaluating the mining project or acting on a mining investment decision.

Even though there are sophisticated tools and methods available to gauge mining and mining investment opportunities and their associated risks, including stochastic modelling processes, there is minimal research available that addresses the exchange rate uncertainty for the evaluation of a mining project. Moreover, there is no study considering the modelling process which integrates the combined effect of the dual economic uncertainties of commodity price and exchange rate for approximating mining project values. Therefore, the research aim contemplated in this thesis was to develop the outline for approximating explicit mining project values numerically through a ROV framework considering continuous time stochastic modelling addressing these dual economic uncertainties. Due to the fluctuation and depression of commodity price and exchange rates, hedging should be the main goal and strategy for mining companies in future to respond to these economic uncertainties. Using a financial hedging strategy, mine management can optimise profits and minimize losses at an operation to manage economic risk in a mining company.

As an outcome, this thesis has provided the fundamentals of a novel mine valuation framework in a real options valuation methodology through developing new PDEs which incorporate commodity price and exchange rate uncertainties in the evaluation process. In this study, stochastic models were considered to address the economic uncertainties of commodity prices and exchange rates. Thereafter, new PDEs were developed using the stochastic models and hedging strategies, incorporating a variety of financial and mining parameters. These novel PDEs were utilised to approximate the mining project values through numerical simulation. Furthermore, this thesis provided an outline on how to investigate the relationship between mining commodity prices and exchange rates considering two case studies. This information is crucial to mine management in choosing an appropriate hedging strategy and in taking an informed investment decision for a mining project. To incorporate the ROV method in the valuation of a mining project, previous researchers have typically used only one uncertainty being the commodity price as the main stochastic variable. The reason is that if the degree of uncertainty is higher than one, it raises greater complexity and it may become difficult to manipulate the continuous time stochastic model. Therefore, in the present study, the ROV methods considered more than one uncertainty as the main stochastic variable for approximating project values. These new ROV techniques estimate the value of a mining project by offering opportunities to intensify gains and to mitigate losses.

6.2 Summary and original contributions

This thesis has been structured into six chapters. The first chapter provides a general overview of the background of this research. This chapter elucidates the statement and objectives, the scope and limitations, and the significance of the research. Chapter 1 clarifies the statement of problems and the way that the thesis (overview of this thesis) addresses them. The discounted cash flow (DCF) methods are generally used in the evaluation of mining projects. There are several shortcomings in DCF methods, as DCF evaluation methods ignore integrating various types of uncertainties, such as commodity price, exchange rate, costs, ore grade, etc., as these uncertainties are generally considered only by using a single parameter risk adjusted discounted rate. Previous research that has conducted ROV methods also has some pitfalls. Prior studies have often ignored the uncertainty associated with exchange rates, even though exchange rate assumptions have a significant impact on the estimation of project values. The previous studies introduced to calculate ROVs have predominantly used the Binomial Option Pricing model (BOP). One of the limitations of the BOP method is that the methodology excludes the use or incorporation of futures contracts and hedging strategies for reducing risk and optimising cash flow results. Another major shortcoming is that binomial models are trees and consequently, when used for modelling and valuing investment opportunities in conjunction with real options, can quickly become large and cumbersome. An additional problem in using BOP methods is that it may be difficult to combine more than one uncertainty in an evaluation process using hedging strategies and, in practice, it becomes computationally intensive as the number of scenarios demanding evaluation increases exponentially. A number of previous studies have developed complicated theoretical models in continuous time processes, but there is no up-to-date solution to determine mining project values using these models, especially in the case when modelling more than one uncertainty. Furthermore, for investment opportunities with non-standard features, the general Black-Scholes equation i.e. the general partial differential equation (PDE), may not always be sufficient. In these circumstances, the best alternative is to undertake a given stochastic process for the underlying asset, typically a Geometric Brownian Motion (GBM), and then derive an appropriate resulting new PDE which can capture the project specific characteristics and parameters. These pitfalls are attempted to be addressed and resolved by adding new contributions individually, in chapters' three to five.

Chapter 2 reviews the ROV method and how it applies to mining projects. This chapter demonstrates the advantages of applying new valuation techniques versus traditional techniques for the evaluation of mining projects. Chapter 2 describes different types of financial options, including hedging strategies and futures contracts. Different types of methodologies for project evaluations are described in this chapter. The basic premise of real options and real options techniques are also presented in this chapter. Lastly, a detailed review of most published literature in the body of the research is presented.

Chapter 3 outlines the link between financial markets, the use of hedging strategies, futures contracts and the general mineral/mining commodity market using the example of a gold opportunity. This chapter develops new PDEs and implements ROV methods for mining project evaluations considering price uncertainty through continuous time stochastic models. Furthermore, the gold price volatility is determined from historical data using the log return method of analysis, and different available real options for mining projects are also discussed. In this chapter, two different new techniques are developed to approximate project values, firstly from a novel reserve dependent PDE considering a GBM stochastic model, and secondly, from a novel time dependent PDE based on a stochastic mean reverting model (MRM). In this chapter, it was revealed that commodity price volatility and country risk premium have great impact on mining project values. From the sensitivity study, it has been observed that when historical volatility goes up or down 1% from 5.13%, the project value changes -22.57% and 28.33%, respectively. In this study, numerical simulations results also suggest that it would be profitable to run a mining project, if the commodity price volatility lies either below the average volatility or at the average volatility. When considering the country risk premium as zero in the simulation, the project value was increased. Furthermore, the deferral option and expansion option increased to the maximum values of the project which are US$1,228,000 (from results US$49,006,000 and US$50,234,000) and US$2,203,000 (from results US$49,930,000 and US$52,133,000), respectively, compared to the base case.

Chapter 4 contributes to the investigation of mineral/mining commodity prices and exchange rates. Since the commodity price and exchange rate uncertainties are crucial components for the investment decision-making process in mining projects, it is essential to know the relationship between specific mining commodity prices and exchange rate movements before evaluating the mining project or mining investment. This chapter initially reviews the existing literature on commodity prices and the exchange rate, and the history of the Australian gold and iron ore minerals/mining sector. This chapter develops and applies different new econometrics and statistical methodologies to investigate the relationship between the commodity price and the exchange rate. As a case study, the long term relationship between gold prices and the AUD/USD exchange rate is shown here. The speed of adjustment coefficient suggests that approximately 1.9 percent of exchange rates per week can be attributed to the disequilibrium between actual and equilibrium levels. The adjustment coefficient 0.00423 shows that the variability of gold prices induces a positive change in the exchange rate and the normalizing beta shows that a one percent increases in the gold price leads to an appreciation of the AUD/USD nominal exchange rate by approximately 0.5%.

In this chapter the relationship of iron ore prices and the value of the Australian dollar (i.e. AUD/USD exchange rate) are also presented. This relationship was utilized in the project evaluation using the novel ROV method considering multiple uncertainties in Chapter 5. The adjustment coefficient 0.37046 shows that the variability of iron ore prices induces a positive change in the AUD/USD exchange rate. One of the key findings is that a one percent increase in iron ore prices leads to an ap preciation of the AUD/USD exchange rate by approximately 0.31 percent. Consequently, to use a suitable hedging strategy in a mining company, management needs to know specific information regarding the long-term relationship between these economic indicators.

Chapter 5 proposes the new version of the ROV method for the estimation of mining project values. In this chapter, the AUD/USD exchange rate volatility and commodity price volatility have been calculated from historical data. This chapter presents a novel ROV technique in which was developed a new PDE through futures contracts and a hedging strategy under the joint effect of commodity price uncertainty and exchange rate uncertainty, and utilizes this PDE for the evaluation of mining projects. In this chapter, a new stochastic differential equation has been derived incorporating several parameters including commodity price volatility, exchange rate volatility and the correlation between commodity price and exchange rates. Using this new stochastic differential equation, price paths of the commodity have also been simulated and shown graphically (as a case study iron ore and gold are considered here). This simulated average price is used to approximate the project values. This is the first ROV method that integrates exchange rate uncertainty together with commodity price uncertainty, and the correlation between commodity price and exchange rate for mining projects evaluation which would help mine management to estimate project values through earning profits and minimizing potential mining losses by considering different financial and real options and operational flexibilities. New PDEs are implemented to iron ore and gold case studies to approximate project values or value ranges. One of the important findings revealed in the study is that project values are over estimated if only commodity price uncertainty is considered instead of under the joint effect of commodity prices and exchange rate uncertainties. In addition, it has also been observed that option to defer and option to expansion enhanced values to the projects.

6.3 Recommendations

As future work, further improvement can be achieved in the following areas:

In this thesis, project values are estimated considering the single uncertainty, commodity price through two different techniques using a Geometric Brownian motion model and a mean reverting commodity price model. For future research, it can be investigated whether the specific mineral commodity price follows the mean reversion behavior or not.

In this study, project values are approximated under the dual economic uncertainties, commodity price and exchange rates through continuous time modeling. For future research, it can be extended under the effects of commodity price, exchange rate and cost uncertainties.

In addition to price and exchange rate uncertainties, risk free interest rate is also an important uncertain financial parameter to estimate project values. In the proposed new PDEs, the parameter risk free interest rate has also been incorporated. For future research, it can be hedged through forward contracts, futures, swaps or options

This thesis have investigated the relationship between the mineral commodity price and the exchange rate considering two case studies, gold prices and AUD/USD exchange rates, and then iron ore prices and AUD/USD exchange rates. In future research, the relationship between iron ore prices and the value of the Australian dollar including the variable of aggregate demand from high consumer countries such as China, can be examined and non-linear association can also be checked.

In this research, ADF and DF-GLS tests have been used to investigate the unit root tests of commodity prices and exchange rates. However, artificial neural network (ANN) techniques can also be utilised in the place of ADF and DF-GLS tests to investigate the unit root tests of commodity prices and exchange rates and this presents an opportunity for further research.

In this study for approximating project values, it was necessary to calculate and use historical volatilities. However, for future research, implied volatility can also be examined and utilised to approximate project values. In addition, for future research, the effects of resource degradation and technological innovation can be considered, as these two parameters may lead to the non-linearity in operation costs.

In this research for solving PDEs numerically, the finite difference method (FDM) and MatLab software/programs have been used to approximate project values. However, an interface program can be developed to solve this PDE numerically for estimating project values as a future work.

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APPENDIX TO CHAPTER 3

Appendix 3.1A   Sensitivity study for commodity price volatility

To see the impact of commodity price fluctuations on project values, consider here the PDE (3-6), and Table 3-1 from the case study 3.6.2. To estimate the gold mining project values, gold price volatility can be forecasted to $\pm 5.13\%$ with the average volatility 16.99 % as a proxy for future gold price volatility. Below are project values considering different volatilities (parameter values are used from Table 3-1).

Figure A3-1 shows the maximum project value is US$60,563,000 when the volatility is 22.12% (the average volatility +5.13%, standard deviation), and the reserve is 425,560 oz. of gold (before the mine is temporarily closed).

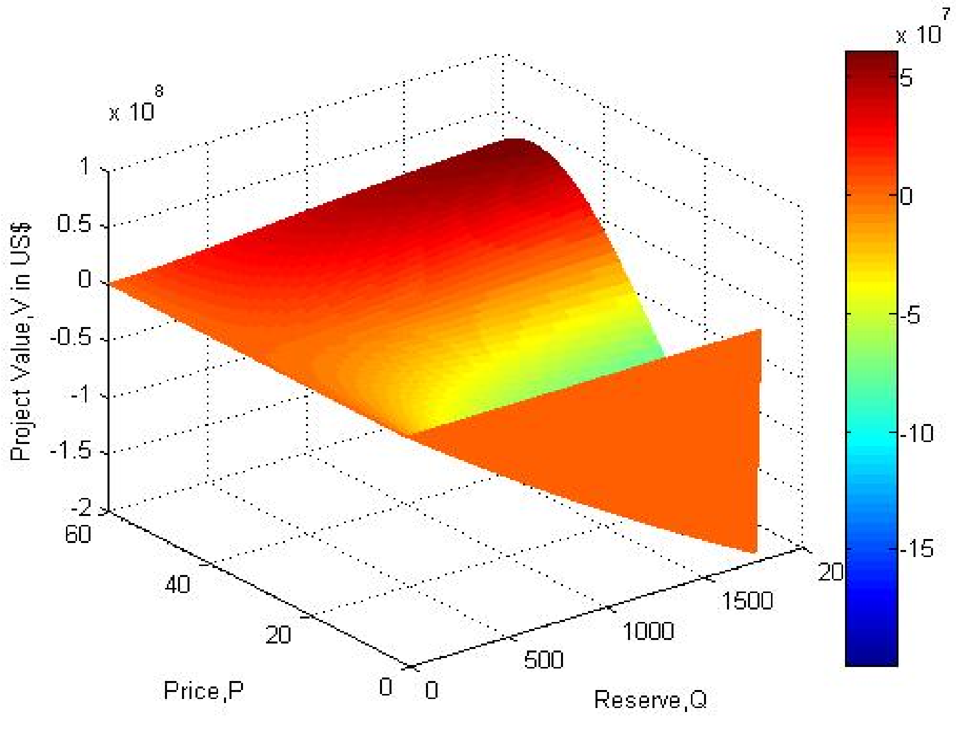

Figure A 3-1 Project values of the mine in linear scale (input data are used from Table 3-1) when reserve $Q = 425{,}560$ oz, $r = 6.00\%$, $\sigma = 22.12\%$, $\delta = 2.00\%$, $q = 141{,}850$ oz, $\lambda_C = 3.00\%$.

Figure A 3-2 demonstrates the maximum project value is US$100,370,000 when the volatility is 11.86% (the average volatility - 5.13%, standard deviation).

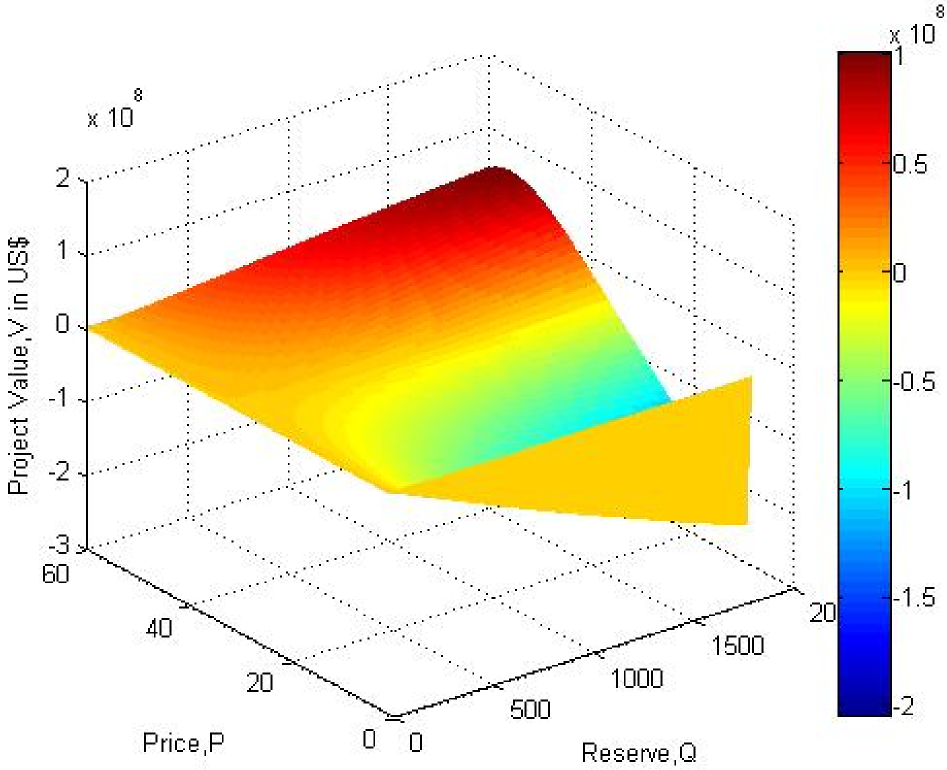

Figure A3-2 Project values of the mine in linear scale (input data are used from Table 3-1) when reserve $Q = 425{,}560$ oz, $r = 6.00\%$, $\sigma = 11.86\%$, $\delta = 2.00\%$, $q = 141{,}850$ oz, $\lambda_C = 3.00\%$.

Figure A3-3 exhibits the maximum project value is US$35,910,000 when the gold price volatility is 31.65% (the maximum volatility during this period).

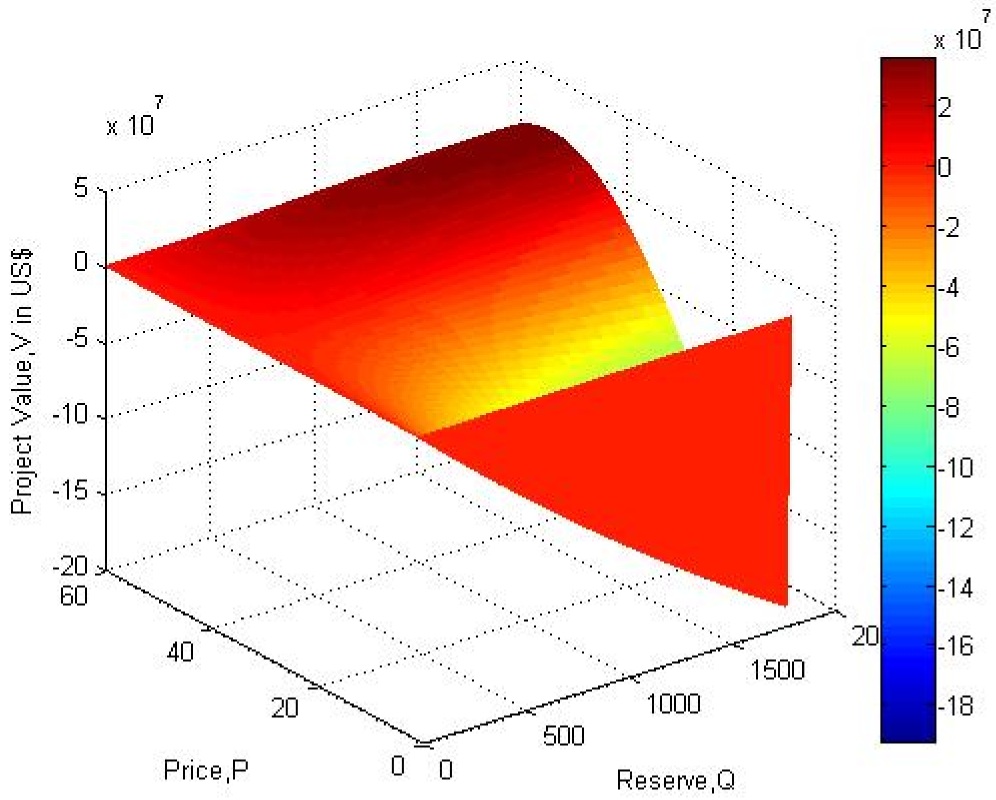

Figure A3-3 Project values of the mine in linear scale (input data are used from Table 3-1) when reserve $Q = 425{,}560$ oz, $r = 6.00\%$, $\sigma = 31.65\%$, $\delta = 2.00\%$, $q = 141{,}850$ oz, $\lambda_C = 3.00\%$.

Hence, from the simulation results (Figure 3-3 in Chapter 3, and Figure A3-1 to Figure A3-3) before the mine is temporarily closed, it can be observed that commodity price volatility is a significant issue for the evaluation of a mining project. When the gold price volatility is 16.99% (the average volatility), the maximum project value is US$78,213,000, and if the gold price volatility increases to 5.13% from the average volatility i.e. 22.12%, the maximum project value becomes US$60,563,000. Conversely, if the gold price volatility decreases to 5.13% from the average volatility i.e.11.86%, the maximum project value becomes US$100,370,000. Furthermore, when the maximum volatility is 31.65%, the maximum project value becomes US$35,910,000 (other parameters values are remain same). This study shows that project value is dependent (sensitive) on price volatility.

From the analysis, it shows that when historical volatility goes up or down 1% from 5.13%, the project value changes -22.57% and 28.33% respectively. Therefore, mining project values can be significantly influenced by commodity price volatility. The numerical simulation results suggest that it might be more profitable to run a mining project when the commodity price volatility lies either below the average volatility or at the average volatility.

Appendix 3.2A Comparison of results among different speed of reversion

If the speed of reversion, mu is considered as zero, then ultimately the model (3-8) in chapter 3 will turn in to a Geometric Brownian motion model.

$\frac{dP}{P} = (r-\delta)dt + \sigma\, dw$ (3-1a), and the resulting PDE will be

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2} - \frac{\partial V}{\partial t} + (r-\delta)P\frac{\partial V}{\partial P} - (r+\lambda_C)V + qg_a(P-C)(1-G) = 0 \qquad \text{(3-2a)}$$

In this case using the data from scenario 2 (Table 3-4), and if GBM model and associated PDE (3.2a) have been considered, the approximate maximum project value for a one year operation under the effect of commodity price uncertainty shows US$29,148,000. Hence from the simulation results, it has been observed the maximum project value will be higher if mean reverting commodity price model is considered instead of Geometric Brownian motion model.

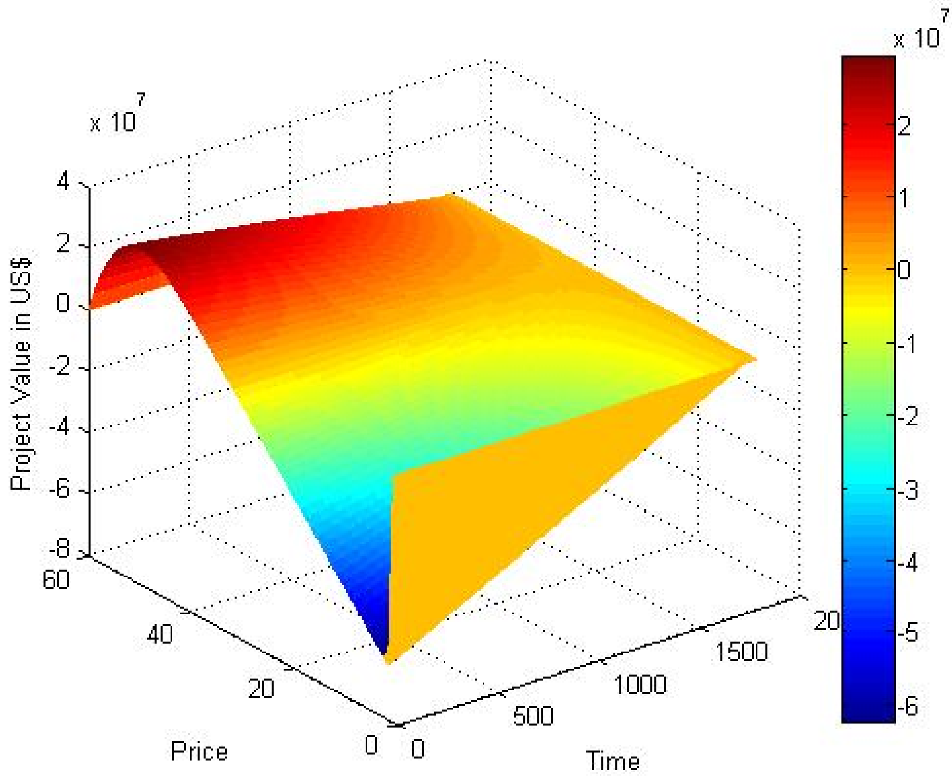

Figure A3-4 Project values of the mine in linear scale (data sets are used from Table 3-4, average ore production rate 799,890 tonnes per year).

When the speed of reversion is considered as 0.01, the approximate maximum project value shows US$41,928,000 (calculated from PDE (3-16), and using data from Table 3-4).

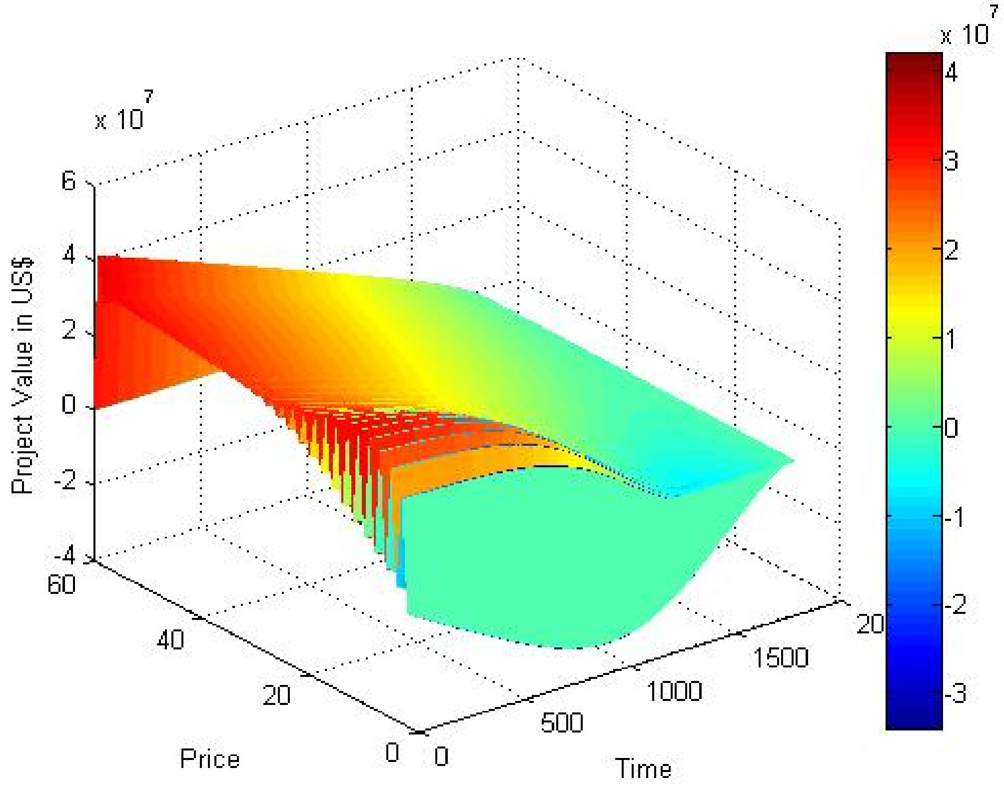

Figure A3-5 Project values of the mine in linear scale (data sets are used from Table 3-4, average ore production rate 799,890 tons per year and speed of reversion is 0.01).

Hence, it has been observed that when the speed of reversion decrease project value also decreases. Therefore, the speed of reversion has an impact on project values.

APPENDIX TO CHAPTER 4

Appendix 4.1A

From the DF-GLS test results in Table A4-1, it has been observed that the null hypothesis has been accepted for iron ore prices at 1% significance levels both in no trend specification, and in Elliott, Rothenberg and Stock, 1996 (ERS) proposed specification. Therefore, this test has confirmed that the time series has unit roots i.e. non stationary in levels.

It can be seen from the DF-GLS test results in Table A4-2 that when the variable is in first differences, the null hypothesis is rejected for the iron ore prices at 1% (lags 1 to 5) and 5% (lag 6) significance, both in no trend specification and ERS specification. Therefore, this test has confirmed that the series has no unit roots i.e. I(0) stationary.

Table A4-3 presents the DF-GLS test result, showing that the null hypothesis can be accepted for iron ore prices at a 1% significance level both in no trend specification and ERS specification. Therefore, this test has confirmed that the time series has unit roots i.e. non stationary in levels.

From the DF-GLS test results in Table A4-4, it has been observed that when taking the first difference of variables, the null hypothesis has been rejected for the AUD/USD exchange rate both in no trend specification and ERS specification. Through 1 to 6 lags, the null hypothesis is rejected at a 1% significance level, and for lag 7 at a 5% significance level. Therefore, this test has confirmed that the time series has no unit roots i.e. I(0) stationary.

Table A4-1 Results of DF-GLS test for unit roots in iron ore prices (variables in levels).

Variable at level	**Specification**	**Test Statistics**		**Lags**	**1% critical value**	**5% critical value**	**10% critical value**	**Decision**
		Tau	**mu**					
Iron ore price	No trend		-1.215	9	-2.613	-2.037	-1.741	Non stationary
			…	…	…	…	…	Non stationary
			-0.774	5	-2.613	-2.130	-1.834	Non stationary
			…	…	…	…	…	Non stationary
			-0.944	1	-2.613	-2.217	-1.915	Non stationary
	*ERS	-1.306		9	-3.705	-3.136	-2.839	Non stationary
		…		…	…	…	…	Non stationary
		-0.810		5	-3.705	-3.136	-2.839	Non stationary
		…		….	…	…	…	Non stationary
		-0.998		1	-3.705	-3.136	-2.839	Non stationary

**Note: ERS means Elliott, Rothenberg and Stock presented interpolated critical values which were proven in 1996. They proposed a modified version of the Dickey Fuller t test which has substantially improved its applicability when an unknown mean or trend is present.*

Table A4-2 Results of DF-GLS test for unit roots in iron ore prices (variable in first differences).

Variable in first difference	**Specification**	**Test Statistics**		**Lags**	**1% critical value**	**5% critical value**	**10% critical value**	**Decision**
		Tau	**mu**					
Iron ore price	No trend		-2.508	6	-2.614	-2.108	-1.813	Stationary
			…	…	…	…	…	Stationary
			-3.561	3	-2.614	-2.181	-1.882	Stationary
			…	…	…	…	…	Stationary
			-4.566	1	-2.614	-2.222	-1.920	Stationary
	ERS	-3.542		6	-3.709	-3.139	-2.842	Stationary
		…		…	…	…	…	Stationary
		-4.345		3	-3.709	-3.139	-2.842	Stationary
		…		…	…	…	…	Stationary
		-5.183		1	-3.709	-3.139	-2.842	Stationary

Table A4-3 Results of DF-GLS test for unit roots in exchange rates (variable in levels).

Variable in level	Specification	Test Statistics		Lags	1% critical value	5% critical value	10% critical value	Decision
		Tau	mu					
Exchange rate	No trend		-0.374	9	-2.613	-2.037	-1.741	Non stationary
			…	…	…	…	…	Non stationary
			-0.287	5	-2.613	-2.130	-1.834	Non stationary
			…	…	…	…	…	Non stationary
			-0.513	1	-2.613	-2.217	-1.915	Non stationary
	ERS	-1.291		9	-3.705	-3.136	-2.839	Non stationary
		…		…	…	…	…	Non stationary
		-1.027		5	-3.705	-3.136	-2.839	Non stationary
		…		…	…	…	…	Non stationary
		-1.199		1	-3.705	-3.136	-2.839	Non stationary

Table A4-4 Results of DF-GLS test for unit roots in exchange rates (variable in first differences).

Variable in first difference	Level	Test Statistics		Lags	1% critical value	5% critical value	10% critical value	Decision
		Tau	mu					
Exchange rate	No trend		-2.329	7	-2.614	-2.084	-1.788	Stationary
			…	…	…	…	…	Stationary
			2.422	4	-2.614	-2.157	-1.860	Stationary
			…	…	…	…	…	Stationary
			-4.822	1	-2.614	-2.222	-1.920	Stationary
	ERS	-3.318		9	-3.709	-3.139	-2.842	Stationary
		…		…	…	…	…	Stationary
		-3.196		5	-3.709	-3.139	-2.842	Stationary
		…		…	….	…	…	Stationary
		-5.595		1	-3.709	-3.139	-2.842	Stationary

Note: To reduce the table sizes, only a few lag values are reported in Tables A4-1, A4-2, A4-3 & A4-4.
For all the unit root tests (Tables 4-14, and A4-1 to A4-4), the null hypotheses were: the series has unit root i.e. non stationary.

Appendix 4.2A

When diverse lag order is considered in the model and performed in the tests, the results confirmed that, due to the shocks on AUD/USD exchange rates, iron ore prices have responses, and vice versa.

Figure A4-1 shows the response of fdLNEX due to the shocks on fdLNIP through the orthogonalized impulse–response function (IRF).

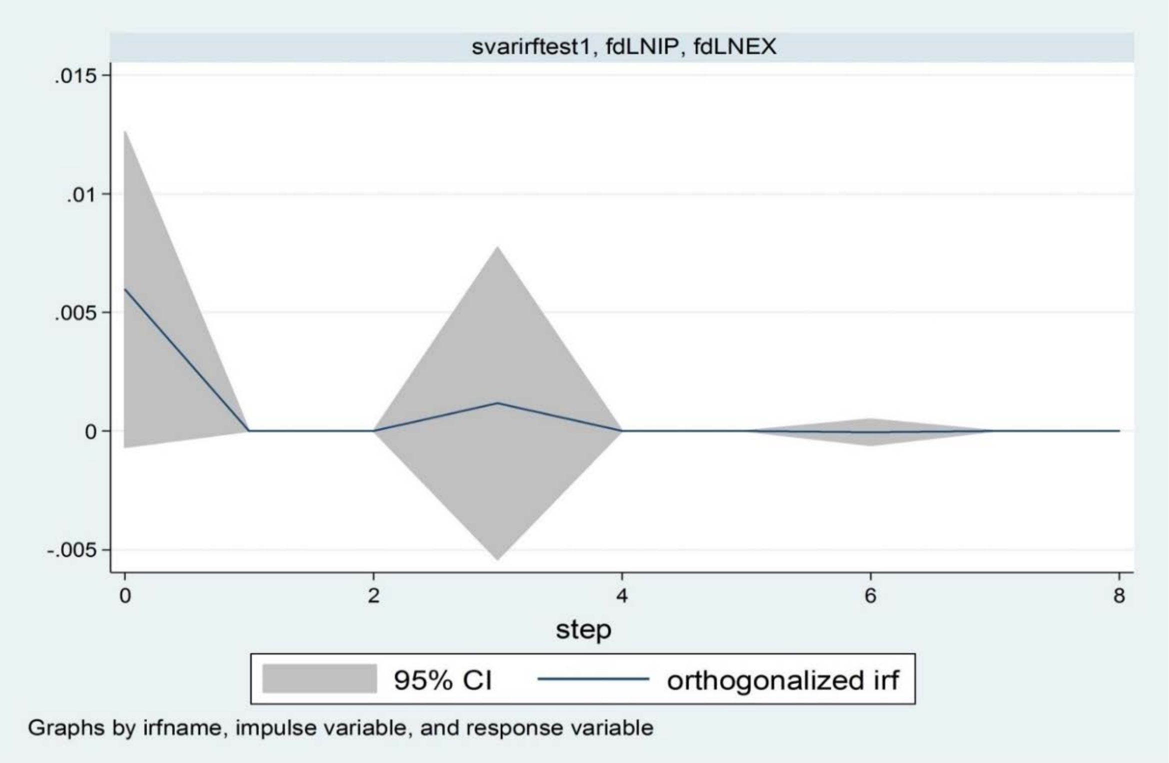

Figure A4-1 The response of fdLNEX due to shocks on fdLNIP through orthogonalized IRF.

Figure A4-2 shows the response of fdLNEX due to shocks on fdLNIP through the structural impulse–response function (IRF).

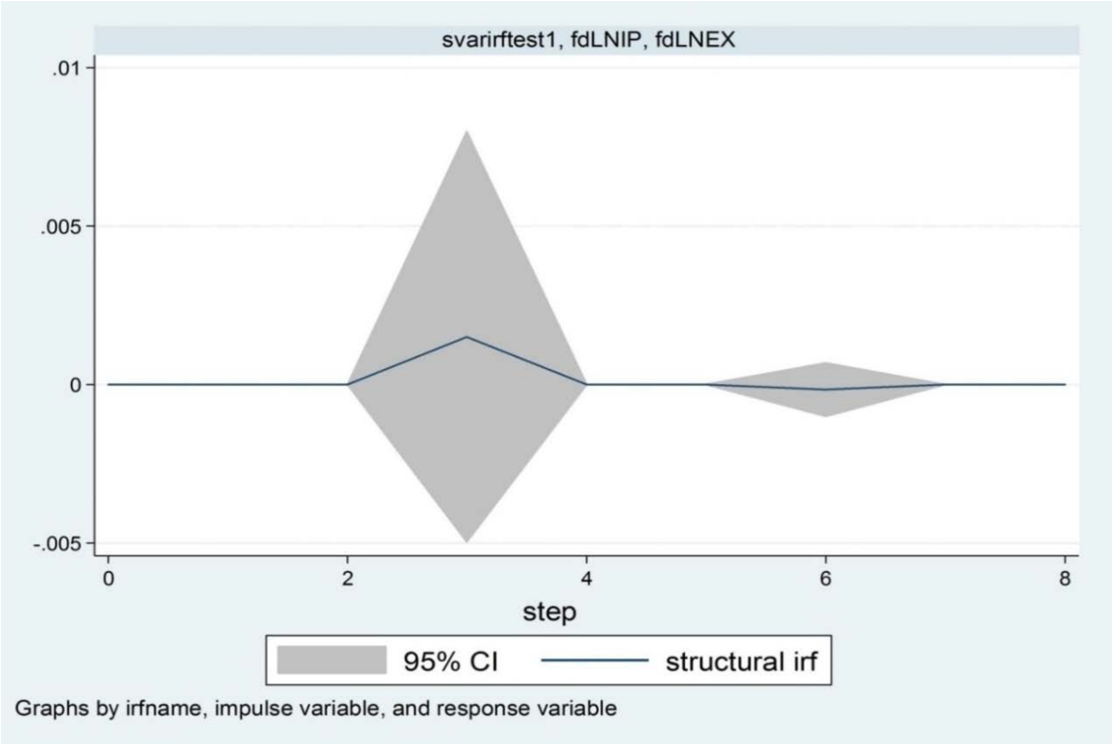

Figure A4-2 The response of fdLNEX due to impulse on fdLNIP through structural IRF.

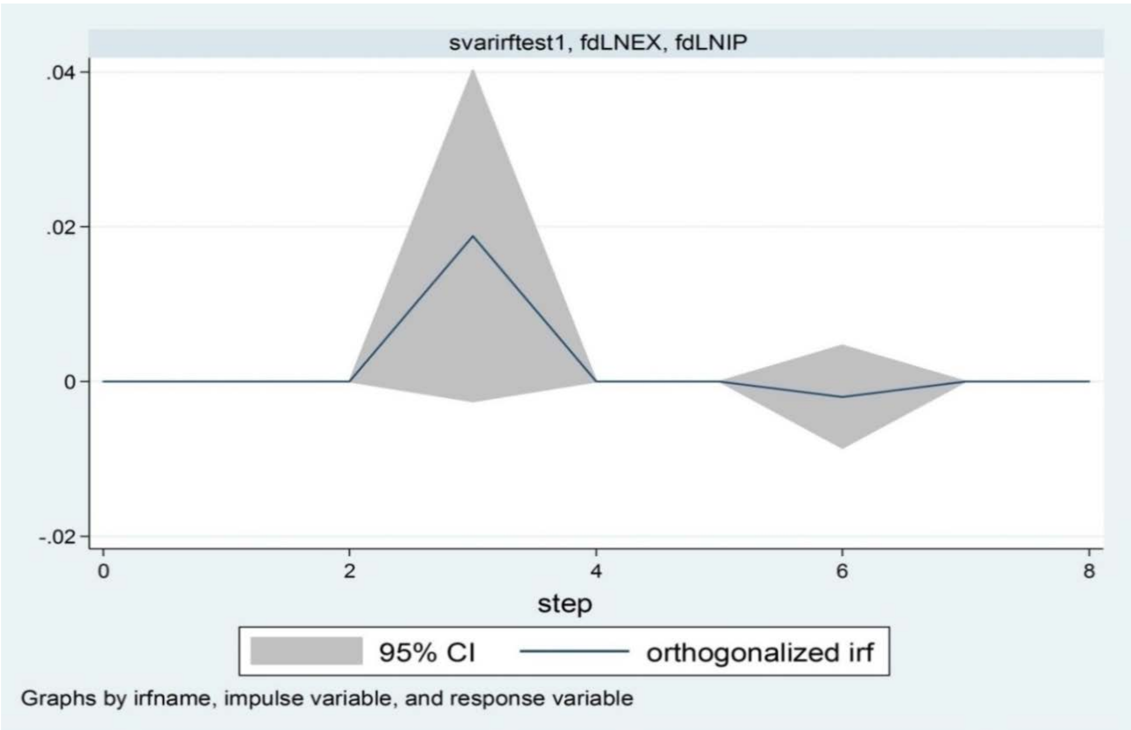

Figure A4-3 The response of fdLNIP due to shocks on fdLNEX through the orthogonalized IRF.

Figure A4-3 displays the response of fdLNIP due to the shocks on fdLNEX through orthogonalized impulse–response function.

Figure A4-4 shows the response of fdLNIP due to impulse on fdLNEX through the structural impulse–response function.

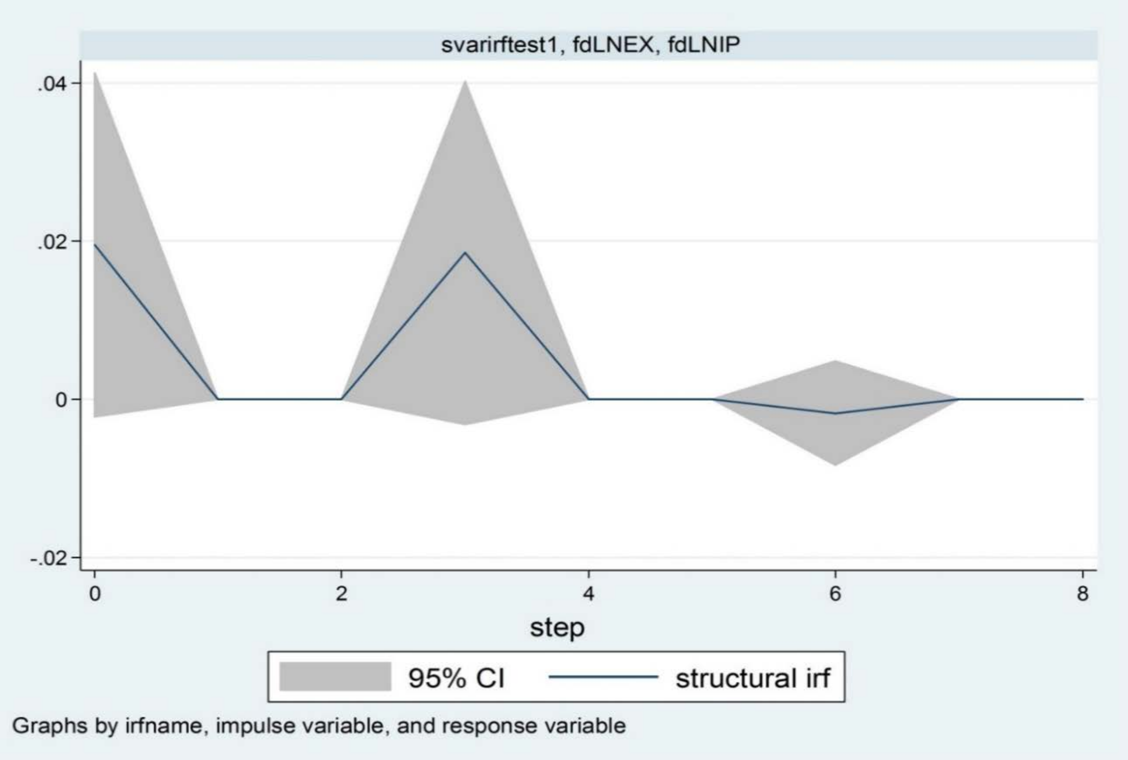

Figure A4-4 The response of fdLNIP due to impulse on fdLNEX through structural IRF.

From Figures A4-1 to A4-4, it has been observed that due to shocks on iron ore prices, AUD/USD exchange rates have responses. On the other hand, due to shocks on AUD/USD exchange rates, iron ore prices have responses too. But these shocks are not persistent and die out after number of periods.

Appendix 4.3A

Forecast error variance decomposition (FEVD): The forecast error variance decomposition method is also referred to as the innovation accounting method which gives information about the dynamic relationships among jointly analysed VAR and SVAR system variables. When the shocks are uncorrelated, it is often useful to calculate the forecast error variance decomposition.

The forecast error variance decomposition and the structural forecast error variance decomposition tests have been performed and the results are reported in Table A4-5 and Table A4-6, and also shown in Figure A4-5 and Figure A4-6.

Table A4-5 Results of forecast error variance decomposition (FEVD) of iron ore prices and AUD/USD exchange rates.

Step	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)
	FEVD	**S. E.**	**FEVD**	**S. E.**	**FEVD**	**S. E.**	**FEVD**	**S. E.**
0	0	0	0	0	0	0	0	0
1	1	0	0.041746	0.048947	0	0	0.958254	0.048947
2	0.990824	0.021306	0.065914	0.042668	0.009176	0.021306	0.934086	0.042668
3	0.934661	0.057529	0.06545	0.041033	0.065339	0.057529	0.93455	0.041033
4	0.916849	0.069213	0.076252	0.043334	0.083151	0.069213	0.923748	0.043334
5	0.916803	0.069279	0.080106	0.045455	0.083197	0.069279	0.919894	0.045455
6	0.91566	0.069991	0.080095	0.045435	0.08434	0.069991	0.919905	0.045435
7	0.915431	0.070101	0.08048	0.045775	0.084569	0.070101	0.91952	0.045775
8	0.915379	0.070189	0.080549	0.045816	0.084621	0.070189	0.919451	0.045816

(1) irfname = svarimpulse, impulse = fdLNIP, and response = fdLNIP

(2) irfname = svarimpulse, impulse = fdLNIP, and response = fdLNEX

(3) irfname = svarimpulse, impulse = fdLNEX, and response = fdLNIP

(4) irfname = svarimpulse, impulse = fdLNEX, and response = fdLNEX

Table A4-6 Results of structural forecast error variance decomposition (SFEVD) of iron ore prices and AUD/USD exchange rates.

Step	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)
	FEVD	**S. E.**	**FEVD**	**S. E.**	**FEVD**	**S. E.**	**FEVD**	**S. E.**
0	0	0	0	0	0	0	0	0
1	1	0	0.041746	0.048947	0	0	0.958254	0.048947
2	0.990824	0.021306	0.065914	0.042668	0.009176	0.021306	0.934086	0.042668
3	0.934661	0.057529	0.06545	0.041033	0.065339	0.057529	0.93455	0.041033
4	0.916849	0.069213	0.076252	0.043334	0.083151	0.069213	0.923748	0.043334
5	0.916803	0.069279	0.080106	0.045455	0.083197	0.069279	0.919894	0.045455
6	0.91566	0.069991	0.080095	0.045435	0.08434	0.069991	0.919905	0.045435
7	0.915431	0.070101	0.08048	0.045775	0.084569	0.070101	0.91952	0.045775
8	0.915379	0.070189	0.080549	0.045816	0.084621	0.070189	0.919451	0.045816

(1) irfname = svarimpulse, impulse = fdLNIP, and response = fdLNIP

(2) irfname = svarimpulse, impulse = fdLNIP, and response = fdLNEX

(3) irfname = svarimpulse, impulse = fdLNEX, and response = fdLNIP

(4) irfname = svarimpulse, impulse = fdLNEX, and response = fdLNEX

Figure A4-5 shows the forecast error variance decomposition (FEVD) of iron ore prices and AUD/USD exchange rates and Figure A4.6 shows the structural forecast error variance decomposition (SFVD) of iron ore prices and AUD/USD exchange rates.

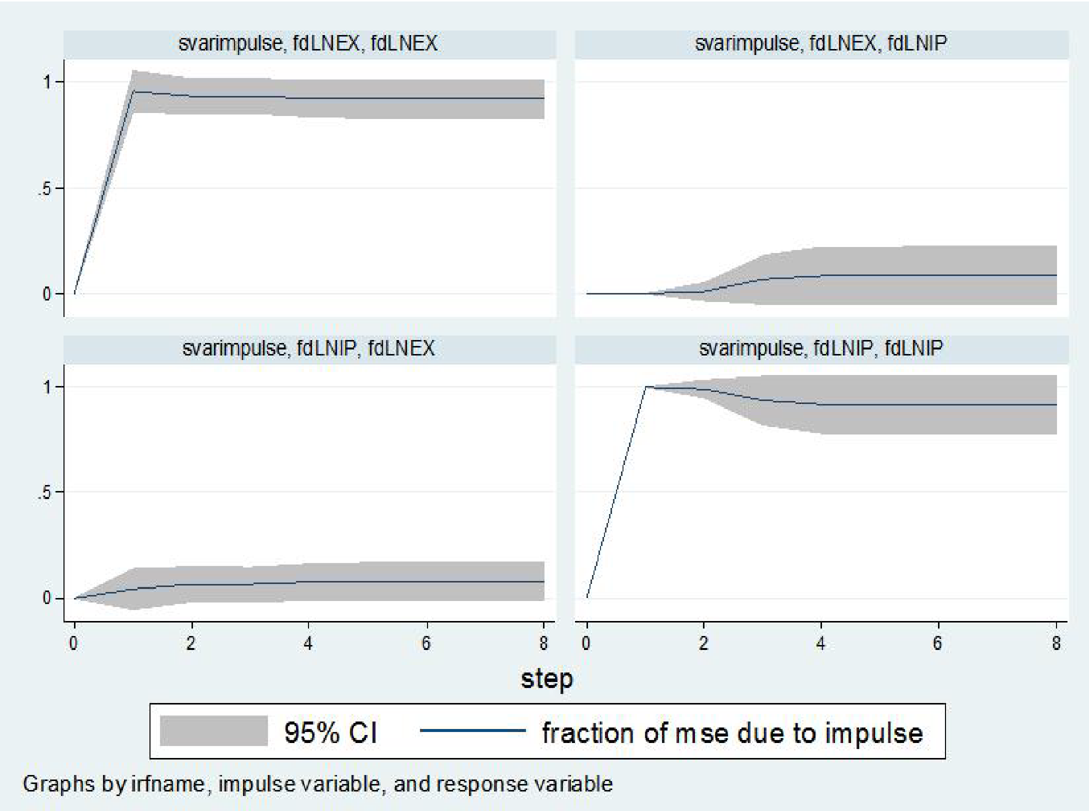

Figure A4-5 Forecast error variance decomposition (FEVD) of iron ore prices and AUD/USD exchange rates.

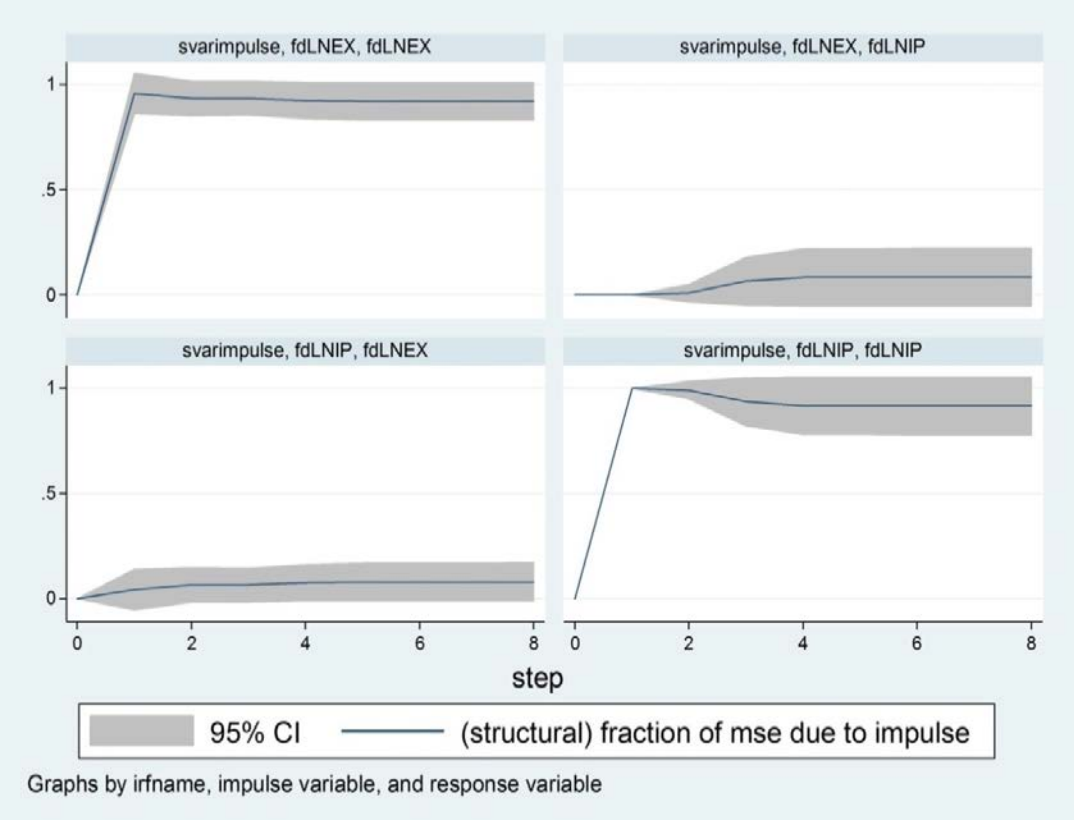

Figure A4-6. Structural forecast error variance decomposition (SFEVD) of iron ore prices and AUD/USD exchange rates.

From Table A4-5 and Table A4-6, and Figure A4-5 and Figure A4-6, it has been observed that the forecast error variance decomposition and the structural forecast error variance decomposition results are similar, and they support the results implied by impulse-response analysis.

Appendix 4.4A

Companion matrix and its application to test the stability of VAR.

Consider a polynomial of degree n,

$$f(y) = y^n + \lambda_1 y^{n-1} + \lambda_2 y^{n-2} + \cdots\cdots + \lambda_{n-1} y + \lambda_n \qquad (4\text{-}1a)$$

As it is well known, the matrix

$$A = \begin{pmatrix} -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{n-1} & -\lambda_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad (4\text{-}2a)$$

has the property that

$$\det(yI - A) = f(y).$$

So the matrix A, or some of its modification, is called a companion matrix (Fiedler, 2003) of the polynomial $f(y)$.

A VAR model of order k can be written as a VAR (1) representation (this is also known as the companion form of VAR (k) model) with the help of the following companion matrix (4-3a):

$$A = \begin{pmatrix} A_1 & A_2 & \cdots & A_{k-1} & \lambda_k \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & I & 0 \end{pmatrix} \qquad (4\text{-}3a)$$

If the modulus of each eigenvalue of the companion matrix is strictly less than one, then the estimated VAR will be stable (Lutkephol, 2005; Hamilton, 1994).

APPENDIX TO CHAPTER 5

Appendix 5.1A

A PDE will be derived considering only the commodity price uncertainty using the model (5-1) in Chapter 5. During the mine operation, the value of the mine at time t $(0 \le t \le T)$ can be defined as $V = V(P,t,\phi) \cong V(P,t)$. Here, ϕ describes managerial flexibility alone i.e. the operating policy such as opening, temporarily closing, and abandoning the mine, and T is the total duration of the mine operation.

Derivation of partial differential equation (PDE) using futures contracts and only SDE (5-1) in chapter 5

Suppose the commodity price is P, and its delivery in the futures contract is in t years' time. The value of the futures contract is $F(P,t)$.

Applying Ito's Lemma to the value of commodity futures:

$$dF(P,t) = \frac{\partial F}{\partial P} dP - \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 P}{\partial P^2} P^2 \sigma_P^2 dt \tag{5-1a}$$

If there is no arbitrage opportunity in commodity futures markets, then an investor through a portfolio of long one unit of commodity and short in $\left(\frac{\partial F}{\partial P}\right)^{-1}$ units of the futures contract has hedged the price risk and should earn a risk free interest rate (Cortazar and Schwartz, 1997, Bellalah 2001). Therefore, in time interval dt, this portfolio becomes

$$dP + \delta P\, dt - \frac{dF}{F_P} = r\, p\, dt \tag{5-2a}$$

In real life, a mining company can adopt the hedging opportunity to lock up the commodity price in order to minimize risk in its project. That means a mining company with a long position for an investment in a mining project $V = V(P,t)$, and short position in $\frac{V_P}{F_P}$ units of commodity could hedge its risk and should earn a return equal to the risk free interest rate plus the country risk premium associated with the country where the mining project is situated (Cortazar and Schwartz, 1997; Colwell et al., 2003; Bellalah 2001), and the portfolio becomes

$$dV + q(P-C)(1-G)dt - \frac{V_P}{F_P} dF \tag{5-3a}$$

Where, q is the mine production rate in tonnes/ounces, C is the total cost (CAPEX, OPEX, working capital, etc.) per unit of commodity, and G is the total tax.

Applying Ito's Lemma in $V = V\,(P, t)$

$$dV = \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(dP)^2 \tag{5-4a}$$

Substituting the value of dV from the equation (5-4a) into the equation (5-3a).

The total return on this portfolio becomes

$$\frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(dP)^2 + q(P-C)(1-G)dt - \frac{V_P}{F_P} dF$$

If there is no arbitrage opportunity, then a mining investment with a long position for investment in the mining project and short position in the futures contracts hedges its risk and should earn a return equal to the risk free interest rate plus the country risk premium associated with the country where the mining project is situated. Therefore, it becomes

$$\frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(dP)^2 + q(P-C)(1-G)dt + \frac{\partial V}{\partial P}(r\,P\,dt - dP - \delta\,Pdt) = (r + \lambda_C)Vdt$$

$$\Rightarrow \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(dP)^2 + q(P-C)(1-G)dt + \frac{\partial V}{\partial P}(r-\delta)Pdt = (r + \lambda_C)Vdt$$

(5-5a)

As production from the mine continues, the overall mine life logically decreases i.e. the total life of mine will decrease. Therefore, the relation of the change in time can be written as

$$h(t) = T - t\,, \qquad 0 \le t \le T \tag{5-6a}$$

Using (5-6a), the following relation can be derived

$$\frac{\partial V(P,t)}{\partial t} = \frac{\partial V(P,h(t))}{\partial h(t)} h'(t) = -\frac{\partial V(P,h)}{\partial h} \tag{5-7a}$$

Using the relation (5-7a), equation (5-5a) can be written as

$$\Rightarrow \frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2} - \frac{\partial V}{\partial t} + (r-\delta)P\frac{\partial V}{\partial P} - (r+\lambda_C)V + q(P-C)(1-G) = 0 \quad \text{(5-8a)}$$

There is no analytical solution for this type of problem i.e. PDE (5-8a). Hence the numerical solutions of the PDE (5-8a) will be approximated through the finite difference method (FDM) using MatLab software, which will commensurately provide the numerical values of the mining project. For solving the PDE (5-8a) numerically, some boundary conditions are needed. As a real option is similar to a financial option, one boundary condition is similar to that prescribed by the call option $V(0,t)=0$. The other boundary condition is $V(P,0)=0$, implying that when the time is zero or the time is exhausted for the mine operation, the value of the mine is zero. Furthermore, it is also necessary to assume $\lim_{P\to\infty}\frac{V(P,t)}{P}\to 1$ i.e. $\frac{\partial^2 V}{\partial P^2}(\infty,t)=0$.

The explicit finite difference method has been employed to discretize the PDE (5-8a):

The discretized form of this PDE is:

$$V_{n,\,j+1} = A_n^1\ V_{n-1,\,j} + B_n^1\ V_{n,\,j} + C_n^1\ V_{n+1,\,j} + D_n^1 \quad \text{(5-9a)}$$

where

$$A_n^1 = \frac{1}{2}\Delta t\left(n^2\sigma^2 - n\,(r-\delta)\right),$$

$$B_n^1 = 1 - \Delta t\left(n^2\sigma^2 + (r+\lambda_C)\right),$$

$$C_n^1 = \frac{1}{2}\Delta t\left(n^2\sigma^2 + n\,(r-\delta)\right),$$

$$D_n^1 = q\Delta t\,(n\,\Delta P - C)(1-G).$$

Here, $P = n\Delta P,\quad n = 0,1,\cdots\cdots M$

$T = j\,\Delta t,\quad j = 0,1,\cdots\cdots N$

$V_{n,\,j} = V(n\Delta P, j\Delta t)$

Stability conditions are as follows:

$\Delta t \leq \frac{1}{N^2 \sigma^2}$, and $\Delta t \leq \frac{\sigma^2}{(r-\delta)^2}$ (for details, see Appendix 5.5A)

The approximate numerical values or value ranges are shown considering only the effect of commodity price uncertainty by solving the PDE (5-8a) numerically.

Appendix 5.2A Case study 1: A conceptual iron ore mine under single uncertainty

Figure A5-1 shows the project values of the mine before the mine is temporarily closed (using data from Table 5-1, in chapter 5), and the approximate maximum project value under the effect of commodity price uncertainty is US$21,114,000.

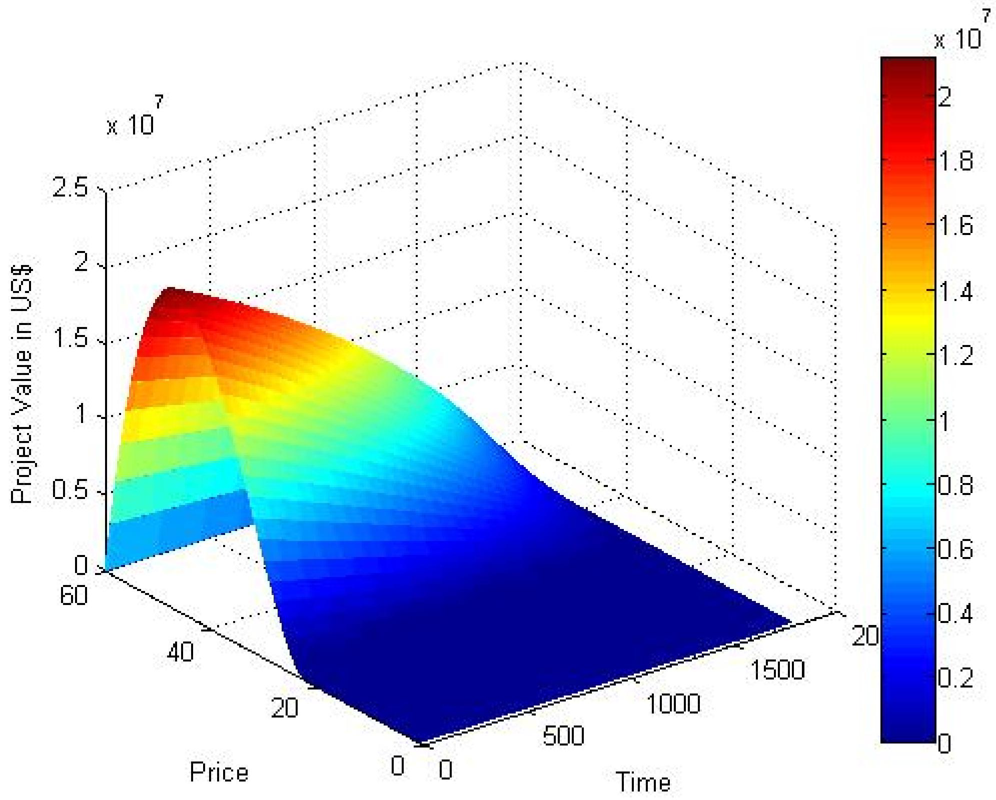

Figure A5-1 Project values of the iron ore mine in linear scale under only the commodity price uncertainty (data are used from Table 5-1).

Figure A5-2 shows the project values of the mine when the mine is reopened after temporary closure, and the approximate maximum project value is US$35,003,000 under the effect of commodity price uncertainty (using data from Table 5-2, in chapter 5).

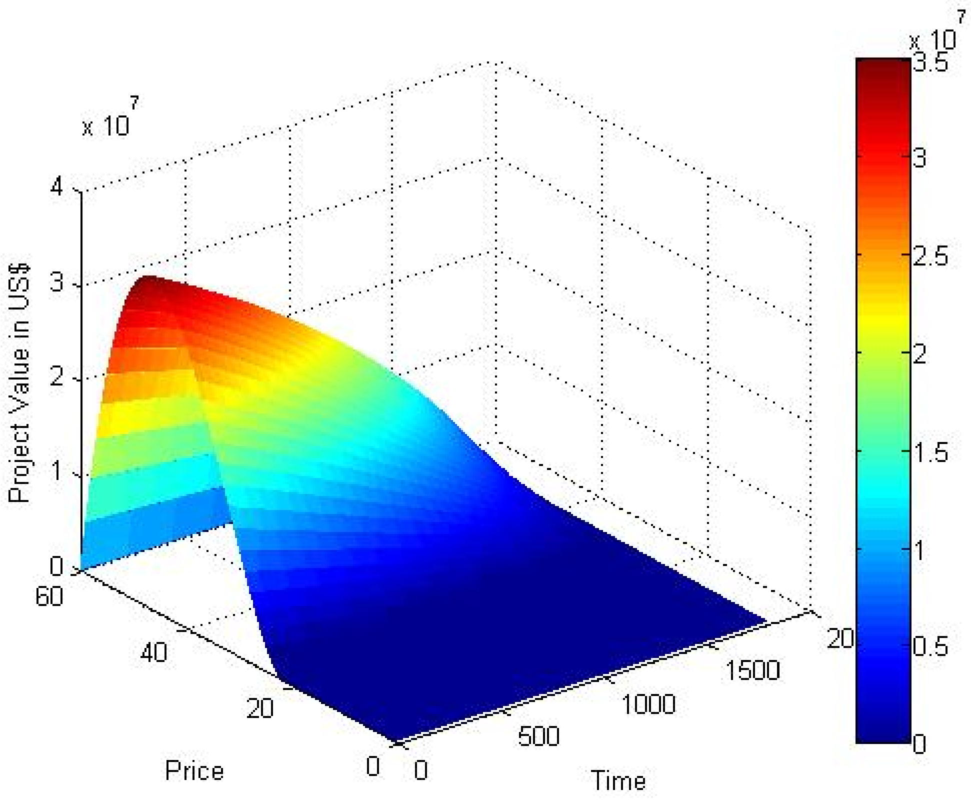

Figure A5-2 Project values of the iron ore mine in linear scale considering the single price uncertainty (data are used from Table 5-2).

Figure A5-3 displays the approximate maximum project value for a one year operation, being, US$49,184,000 when only the commodity price uncertainty is considered (using data from Table 5-3, in Chapter 5).

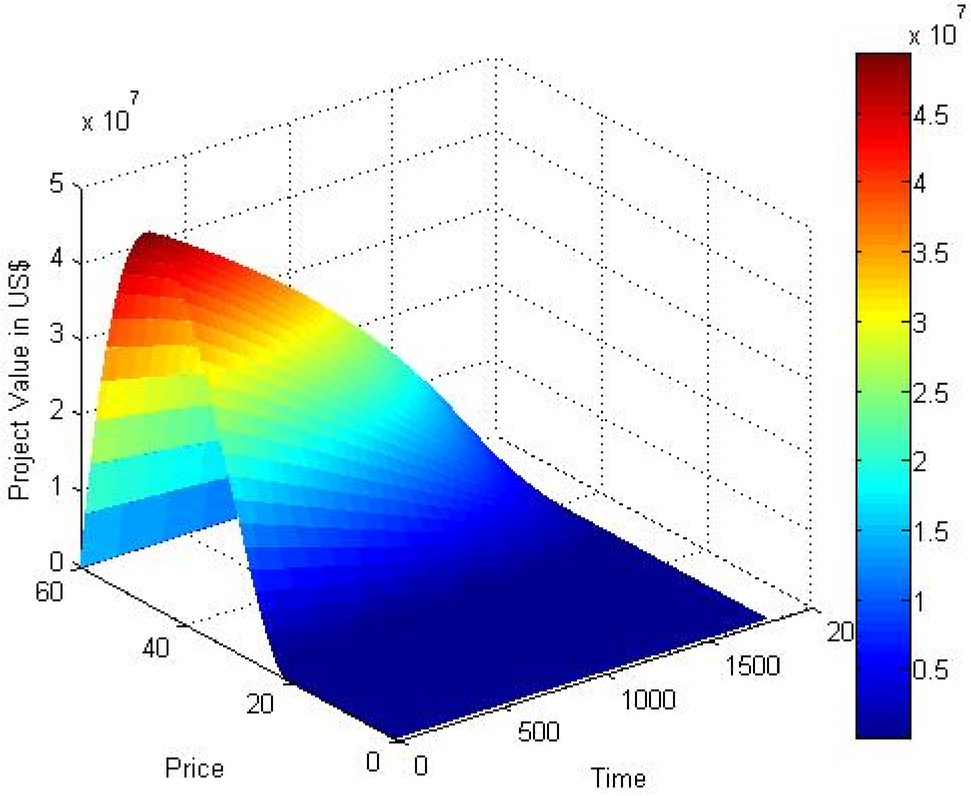

Figure A5-3 Project values of the iron ore mine in linear scale under only the price uncertainty (data are used from Table 5-3).

Appendix 5.3A Case study 2: A conceptual gold mine under single uncertainty

Figure A5-4 shows the project values of the mine before the mine is temporarily closed (using data from Table 5-4, in Chapter 5), and the approximate maximum project value under the effect of commodity price uncertainty is US$24,190,000.

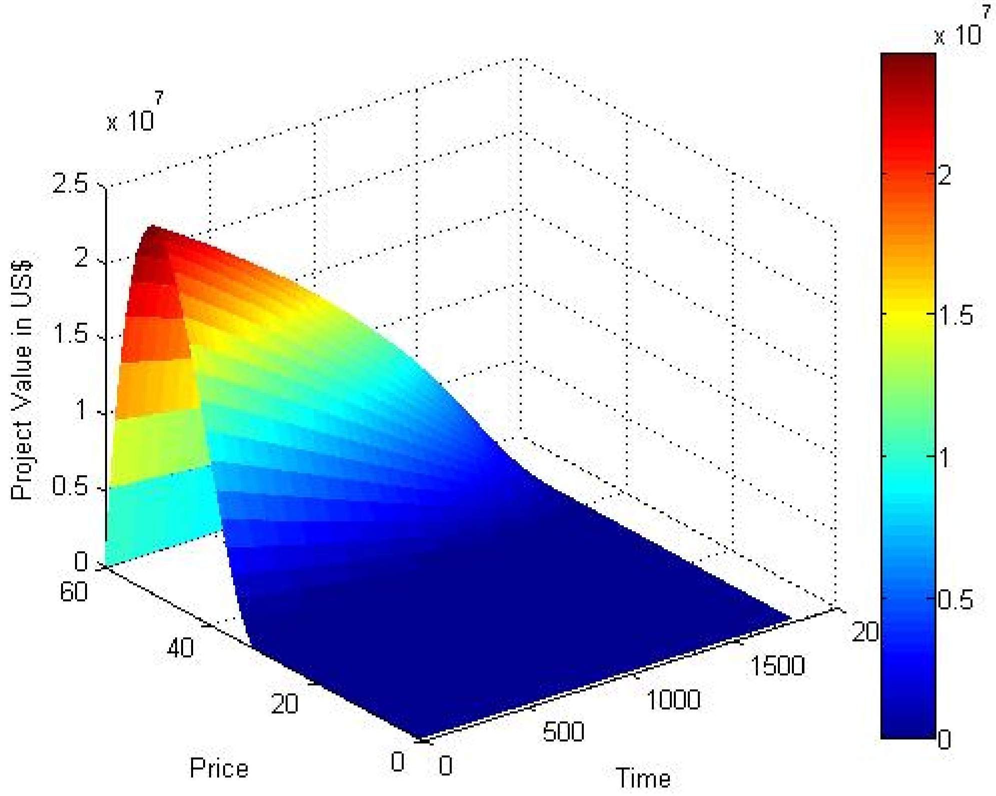

Figure A5-4 Project values of the gold mine in linear scale under the single price uncertainty (data are used from Table 5-4).

Assume that the above project described in Table 5-4, defers the commencement of mining for a year, and the average gold price for next year increases to US$1140.50 per ounce. The total production and sales cost of each ounce of gold increases to US$605.25. In this case, Figure A5-5 shows the maximum approximate project value of the mine is US$31,035,000, under the effect of single commodity price uncertainty.

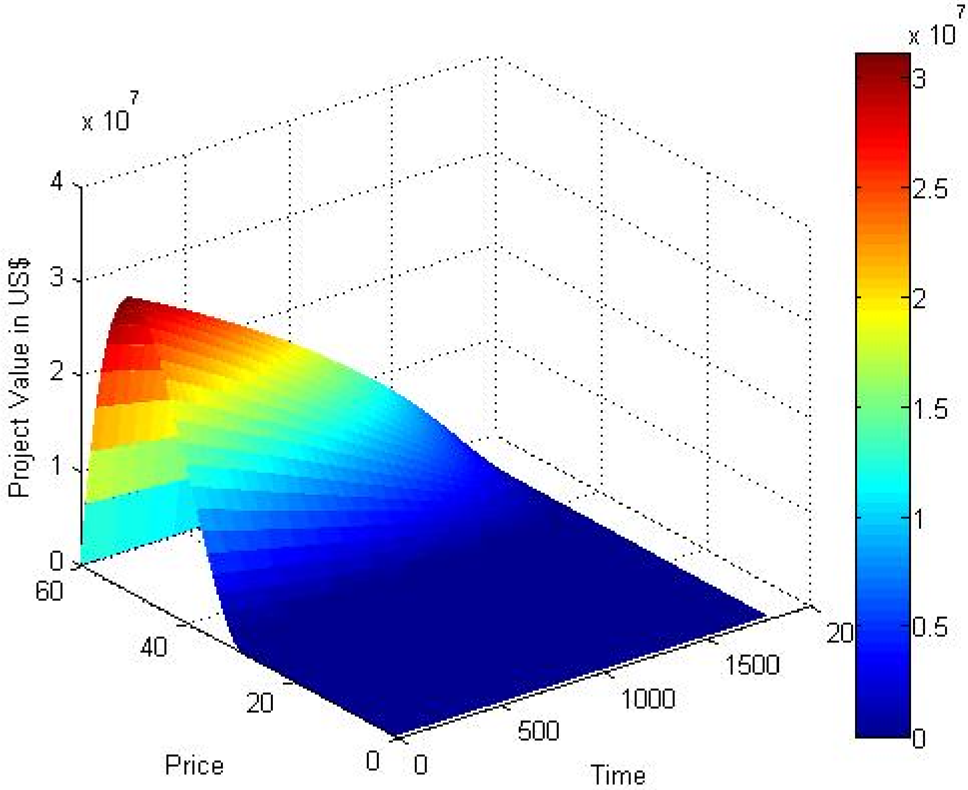

Figure A5-5 Project values of the gold mine in linear scale considering only the price uncertainty (data are used from Table 5-4)

Figure A5-6 displays the approximate maximum project value for a one year operation, being US$55,121,000**,** when only the commodity price uncertainty is considered (using data from Table 5-5 in Chapter 5).

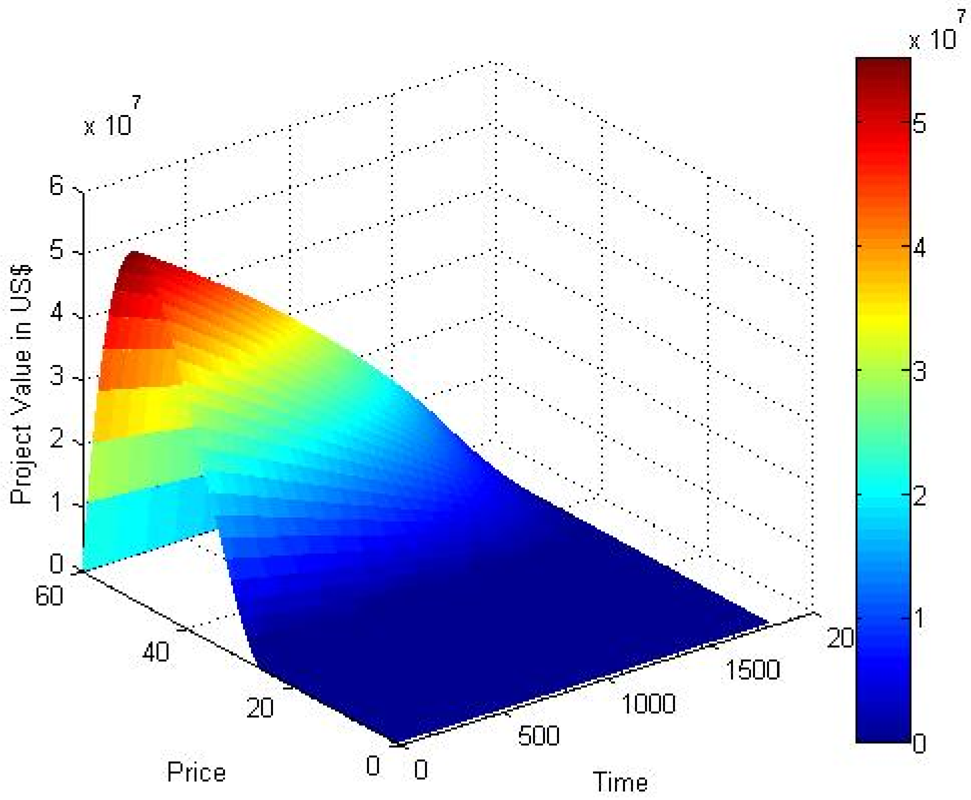

Figure A5-6 Project values of the gold mine in linear scale under the single price uncertainty (data are used from Table 5-5)

Appendix 5.4A Volatility's impact on market returns

For securities, the higher the standard deviation, the greater the dispersion of returns and the higher the risk associated with the investment. Therefore, there is a strong relationship between volatility and market performance (Wagner, 2007). When the volatility increases the stock markets fall, and when the volatility declines the stock markets rise. This is because, when volatility increases, the risk increases and commensurately returns decrease. There are some relevant literature regarding the relationship between stock market returns and volatility. The literature documents that low stock returns are associated with increased volatility (Bae et al., 2007). In this study the investigation was why stock returns and volatility are negatively correlated. Bekaert and Wu (2000) studied asymmetric volatility and risk in equity markets and claimed that returns are negatively correlated with volatility. Whitelaw (2000) investigated stock market risk and return and observed that stock market returns are negatively correlated with stock market volatility. Li et al., (2005) investigated the relationship between expected stock returns and volatility in international stock markets. They found evidence of a significant negative relationship between expected returns and volatility. Vliet et al., (2011) studied the relationship between volatility and expected returns during the period 1963-2009. They found that the empirical relationship between historical volatility and expected returns is negative. Crestmont (2011) examined the historical relationship between stock market performance and the volatility of the market. This research shows that higher volatility corresponds to a higher probability of a declining market (diminishing returns), and lower volatility corresponds to higher probability of a rising market (improving returns). Haque et al., (2014) studied the numerical value of a mining project through real option valuation (ROV) approaches under the commodity price uncertainty considering a hypothetical gold mine as a case study. In this study, they revealed that the price volatility has a significant impact on mining project values. When the commodity price volatility increases, returns decrease and consequently project value decreases, and vice versa.

Appendix 5.5A Stability analysis/condition

Consider again the PDE (5-8a).

$$\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2}-\frac{\partial V}{\partial t}+(r-\delta)P\frac{\partial V}{\partial P}-(r+\lambda_C)V+q(P-C)(1-G)=0 \qquad (5\text{-}8a)$$

The PDE (5-8a) can be written as follows

$$\frac{\partial V}{\partial t}-\frac{1}{2}P^2\sigma^2\frac{\partial^2 V}{\partial P^2}-(r-\delta)P\frac{\partial V}{\partial P}+(r+\lambda_C)V-q(P-C)(1-G)=0 \qquad (5\text{-}10a)$$

$$\Rightarrow \frac{\partial V}{\partial t}+a(P,t)\frac{\partial^2 V}{\partial P^2}+b(P,t)\frac{\partial V}{\partial P}+c(P,t)V+d(P,t)=0 \qquad (5\text{-}11a)$$

where

$$a(P,t)=-\frac{1}{2}\sigma^2P^2$$

$$b(P,t)=-(r-\delta)P$$

$$c(P,t)=r+\lambda_C$$

$$d(P,t)=-q(P-C)(1-G)$$

Now taking the explicit approximation to the derivatives, which gives

$$\Rightarrow \frac{V_{n,\,j+1} - V_{n,\,j}}{\Delta t} + a_{n\,,\,j}\left(\frac{V_{n+1,\,j} - 2\,V_{n,\,j} + V_{n-1,\,j}}{\Delta P^2}\right) + b_{n\,,\,j}\left(\frac{V_{n+1,\,j} - V_{n-1,\,j}}{2\Delta P}\right) +$$

$$c_{n\,,\,j}V_{n,\,j} + d_{n\,,\,j} = O\left(\Delta t, \Delta P^2\right) \tag{5-12a}$$

Now define: $u_1 = \frac{\Delta t}{\Delta P^2}$ and $u_2 = \frac{\Delta t}{\Delta P}$, and rearrange this difference equation

(5-12a) to put all of the $j+1$ term on the left side.

$$V_{n,\,j+1} = V_{n\,,\,j} - u_1 a_{n\,,\,j}\left(V_{n+1,\,j} - 2V_{n\,,\,j} + V_{n-1,}\,j\right) - \frac{1}{2}\,u_2 b_{n\,,\,j}\left(V_{n+1,\,j} - V_{n-1,\,j}\right) - c_{n\,,\,j}\Delta t\,V_{n,\,j}$$

$$-\Delta t\,d_{n\,,\,j} + O\left(\Delta t^2, \Delta t\Delta P^2\right)$$

$$\Rightarrow V_{n,\,j+1} = \left(\frac{1}{2}u_2\,b_{n,j} - u_1 a_{n,j}\right)V_{n-1,\,j} + \left(1 + 2\,u_1 a_{n\,,\,j} - c_{n\,,\,j}\Delta t\right)V_{n,\,j} -$$

$$\left(\frac{1}{2}u_2 b_{n,\,j} + u_1 a_{n,\,j}\right)V_{n+1,\,j} - \Delta t\,d_{n\,,\,j} + O\left(\Delta t^2, \Delta t\Delta P^2\right) \tag{5-13a}$$

Now equation (5-13a) can also be written as

$$V_{n,\,j+1} = \frac{1}{2}\left(\sigma^2 n^2 - n\left(r - \delta\right)\right)\Delta t\,V_{n-1,\,j} + \left(1 - \left(\sigma^2 n^2 + \left(r + \lambda_C\right)\right)\Delta t\right)V_{n,\,j} +$$

$$\frac{1}{2}\left(\sigma^2 n^2 + n\left(r - \delta\right)\right)\Delta t\,V_{n+1,\,j} - \Delta t\,d_{n\,,\,j} + O\left(\Delta t^2, \Delta t\Delta P^2\right) \tag{5-14a}$$

For determining the stability conditions, using the equation (5-13a), and it can be written as follows

$$V_{n,\,j+1} = \left(\frac{1}{2}u_2\,b_{n,j} - u_1 a_{n,j}\right)V_{n-1,\,j} + \left(1 + 2\,u_1 a_{n\,,\,j} - c_{n\,,\,j}\Delta t\right)V_{n,\,j} - \left(\frac{1}{2}u_2 b_{n,\,j} + u_1 a_{n,\,j}\right)V_{n+1,\,j} - \Delta t\,d_{n\,,\,j} \tag{5-15a}$$

For the stability, consider finding the solution of the Von Neumann type of the form

$$V_{n\,,\,j} = e^{at}\;e^{i\,n\,k\,\Delta P} \tag{5-16a}$$

Since in the difference equation the error is linear, it is sufficient to consider the growth of error of a typical exponential term. Using equation (5-16a) the following can be obtained:

$$V_{n-1,\,j} = e^{at}\;e^{i\,(n-1)k\,\Delta P}$$

$$V_{n+1,\,j} = e^{at}\;e^{i\,(n+1)\,k\,\Delta P}$$

$$V_{n,\,j+1} = e^{a(t+\Delta t)}\ e^{i\,n\,k\,\Delta P}$$

Now substituting these above values in equation (5-15a), we obtain

$$e^{at}e^{a\Delta t}e^{i\,n\,k\,\Delta P} = \left(\frac{1}{2}u_2\,b_{n,\,j} - u_1 a_{n,\,j}\right)e^{at}e^{i(n-1)k\,\Delta P} + \left(1 + 2\,u_1 a_{n,\,j} - c_{n,\,j}\Delta t\right)e^{at}e^{i\,n\,k\,\Delta P} - \left(\frac{1}{2}u_2 b_{n,\,j} + u_1 a_{n,\,j}\right)e^{at}e^{i(n+1)k\,\Delta P} - \Delta t\,d_{n\,,\,j}$$

$$\Rightarrow e^{a\Delta t} = \left(\frac{1}{2}u_2\,b_{n,\,j} - u_1 a_{n,\,j}\right)e^{-i\,n\,k\,\Delta P} + \left(1 + 2\,u_1 a_{n,\,j} - c_{n,\,j}\Delta t\right) - \left(\frac{1}{2}u_2 b_{n,\,j} + u_1 a_{n,\,j}\right)e^{i\,n\,k\,\Delta P} - \Delta t\,d_{n\,,\,j}\ e^{-at}e^{-i\,n\,k\Delta P}$$

(5-17a)

To get the stability, it is necessary that $\left| e^{a\Delta t} \right| \le 1$, and for this assume that all the coefficients are slowly varying over the ΔP scales.

$$c_{n,j} \ge 0$$

$$a_{n,j} \le 0$$

$$2u_1 a_{n,\,j} - c_{n,\,j}\Delta t \le -1$$

$$\left(u_2 \left|b_{n,\,j}\right|\right)^2 \le -2u_1 a_{n,\,j}$$

First constraint is satisfied, as in finance r is usually positive, second condition is also satisfied as σ^2 and P^2 are positive. Typically, choose u_1 to be $O(1)$ so that the third constraint is approximately, $u_1 \le -\dfrac{1}{2\,a_{n,\,j}}$.

$$\text{i.e. } \Delta t \le -\frac{\Delta P^2}{2\,a_{n,\,j}} = -\frac{\Delta P^2}{2\left(-\frac{1}{2}\sigma^2 P^2\right)} = \frac{1}{N^2\sigma^2}$$

and the fourth constraint becomes

$$u_2^2 \le -\frac{2\,u_1 a_{n,\,j}}{\left|b_{n,\,j}\right|^2}$$

$$\text{i.e. } \Delta t \le -\frac{2\left(-\frac{1}{2}\sigma^2 P^2\right)}{(r-\delta)^2 P^2} = \frac{\sigma^2}{(r-\delta)^2}$$

Note: $d_{n,j} \to 0$, as for large t, $e^{-at} \to 0$, and $\left|e^{-ink\Delta P}\right| = 1$, $\left|e^{ink\Delta P}\right| = 1$

Therefore, the stability conditions are as follows:

$$\Delta t \le \frac{1}{N^2\sigma^2}, \text{ and } \Delta t \le \frac{\sigma^2}{(r-\delta)^2}$$

Stability condition for the numerical solution of the PDE (5-11)

We have considered the PDE (5-11) from the body of Chapter 5.

$$\frac{1}{2}Z^Z\sigma_Z^2\frac{\partial^2 V}{\partial Z^2} - \frac{\partial V}{\partial T} + (r - (\delta - \alpha - \rho\sigma_P\sigma_E))Z\frac{\partial V}{\partial Z} - (r + \lambda_C)V = -q(Z - C)(1 - G)$$

The PDE (5-11) can be written as

$$\frac{\partial V}{\partial T} - \frac{1}{2}Z^2\sigma_Z^2\frac{\partial^2 V}{\partial Z^2} - (r - (\delta - \alpha - \rho\sigma_P\sigma_E))Z\frac{\partial V}{\partial Z} + (r + \lambda_C)V \ - q(Z - C)(1 - G) = 0$$

(5-18a)

$$\Rightarrow \frac{\partial V}{\partial T} + a(Z, T)\frac{\partial^2 V}{\partial Z^2} + b(Z, T)\frac{\partial V}{\partial Z} + c(Z, T)V \ + d(Z, T) = 0$$

(5-19a)

Here we define,

$$a(Z, T) = -\frac{1}{2}\sigma_Z^2 Z^2$$

$$b(Z, T) = -(r - (\delta - \alpha - \rho\sigma_P\sigma_E))Z$$

$$c(Z, T) = r + \lambda_C$$

$$d(Z, T) = -q(Z - C)(1 - G)$$

In similar procedures as described above, the stability conditions for the numerical solution of the PDE (5-11), as follows:

$$\Delta T \le \frac{1}{N^2\sigma_Z^2}, \text{ and } \Delta T \le \frac{\sigma_Z^2}{(r - (\delta - \alpha - \rho\sigma_P\sigma_E))^2}$$