

Western Australian School of Mines

**Stope Boundary Optimisation in Underground Mining Based on a
Heuristic Approach**

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Doctor of Philosophy
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In loving memory of my grandmother, Wasantha Sandanayake (1917-2000)

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

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ABSTRACT

A stope can be defined as an underground production zone where ore is extracted from the surrounding rock mass using underground mining methods. Employing the optimal layout of stopes maximises the discounted value of future cash flows subject to inherent physical, geotechnical and geological constraints over the life span of underground mining operations. Therefore, stope optimisation is a key requirement over the duration of an underground mining venture. Numerous approaches have been introduced to address the stope layout optimisation problem. Considering the computational complexity and size of the problem, none of these approaches guarantee true optimality in three-dimensional (3D) space. Thus, the research focus of this PhD thesis is the development of an innovative heuristic algorithm to generate a near-optimal stope layout for a given resource model. Due to the non-deterministic polynomial-time hard (NP-hard) combinatorial nature of the problem, a new heuristic algorithm is proposed to solve the stope optimisation problem. The proposed algorithm takes a 3D resource model as its input, generates a set of possible solutions and selects the solution with the maximum cash flow subject to associated costs of extraction as the optimum solution to a given resource model. Improvements to the algorithm increased the performance by applying multi-threading concepts from object-oriented programming. Case studies (referenced by A to G in the alphabetical order) are presented to demonstrate the applicability of the algorithm based on a resource model that represents a copper deposit in different mining scenarios. Among these defined cases, case D with a variable stopes size of $3 \times 3 \times 3 - 3 \times 3 \times 4$ generated the most profitable solution. Further changes to the input configuration improved the solution values of cases A and D by 3.8%. Validation studies show that the proposed algorithm generates more profitable solutions than two existing algorithms: the maximum value neighbourhood (MVN) algorithm and the Sens and Topal heuristic approach. In comparison to the MVN algorithm, the proposed algorithm generated 11.2% and 22.5% superior solutions for cases B and G. In comparison to the Sens and Topal heuristic approach, the proposed algorithm generated 7.3% and 5.6% superior solutions for cases A and D.

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CHAPTER 1 INTRODUCTION

Stope optimisation plays a pivotal role in the planning stages of underground mining projects, because the optimal stope design impacts the value of the project revenue or the discounted value of the future cash flows. In addition, the guidance of mining operations given the best stope design is of paramount importance as it ensures the best utilisation and management of the human and financial resources involved in an underground mining project. Few algorithms have been developed to optimise the stope boundary, and none guarantee true optimality in a three-dimensional (3D) space. Thus, an efficient algorithm needs further development by mine planning engineers to optimise stope layouts while satisfying physical mining constraints and maximising the profit from operations.

1.1 PROBLEM STATEMENT

The demand for mineral commodities has been increasing rapidly due to global industrial and population growth with limited resources and depletion of shallow, easily accessible deposits. As a consequence, the mining industry has been compelled to explore deeper deposits and extract minerals using underground mining methods to meet increasing market demands. To maintain a healthy balance between market demand and mine production rates, underground mine planning and design tools need to be optimised in a manner that facilitates the maximum recovery of a resource and minimum extraction costs (Kumral, 2004; Little et al., 2011). Therefore, it is essential that mine planners employ high quality underground optimisation tools that can guarantee the generation of the optimum design solutions for mining-related problems.

Over the years, underground mine planning has advanced into three major areas: stope boundary and layout definition; development and infrastructure placement; and production scheduling (Sens & Topal, 2009). Of these three areas, stope boundary optimisation is particularly significant for underground mining projects because the optimal stope design ensures optimal production rates of ore with minimum amount of waste, and it guides production scheduling over the life-term of the mine. However, developing techniques for stope optimisation is a complex task because it

requires the satisfaction of a multitude of mining constraints while maximising the overall profit.

Underground deposits are modelled using exploration data with geostatistical techniques in order to facilitate the selection of stoping methods and stoping dimensions (Asad et al., 2013; Kumral, 2013; Topal, 2008). These models are collections of 3D blocks that represent shapes, volumes, tonnages and grades of solids, which comprise ore zones, waste zones and other zones of geological interest. The models used are small-scale representations of ore deposits and their surroundings. Ore is accompanied by waste material, thus it is a difficult task to identify economic collections of production zones and satisfy objectives of maximising productivity in mining operations (Asad, 2011; Hustrulid, 2006).

The number of blocks that can be included in a stope depend on other factors such as geotechnical constraints and the sizes of the machines utilised (Alford et al., 2007; Ataee-pour, 2004). Two or more profitable stopes may share a few blocks in some cases, and decisions have to be made to select the most profitable stopes, because overlapping stopes cannot be physically mined together. In general, these requirements dictate the minimum and maximum stope dimensions, final stope layout and mining operations.

Figure 1.1 illustrates the requirement of a robust algorithm for the complex stope optimisation problem using a hypothetical 2D (xy -plane) block model consisting of 80 mining blocks, i.e. 10 blocks along x -axis and 8 blocks along y -axis.

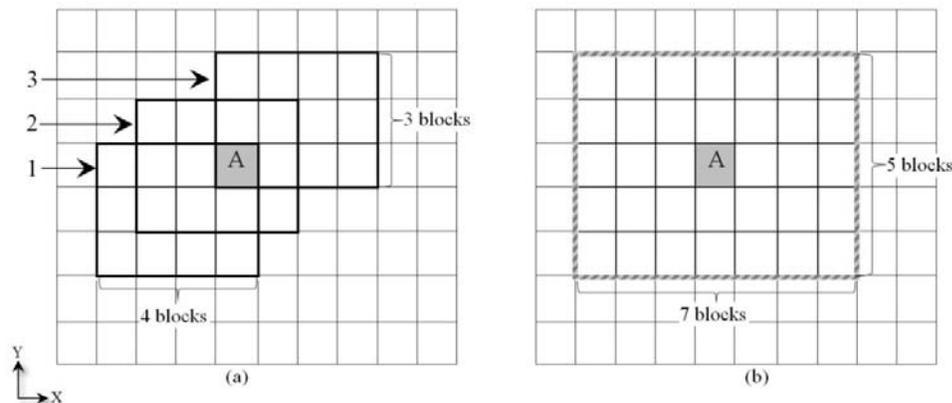


Figure 1.1 (a) Three possible stopes (b) Border of the candidate pool of blocks for stope creation.

In this hypothetical situation, the mining constraints restrict the stope size to four blocks along the x axis and three blocks along the y axis, i.e., a total of 12 mining blocks inside a stope. Consequently, Figure 1.1 (a) shows that there are three possible stopes around a given mining block A. Figure 1.1 (b) indicates the boundary of a candidate pool of 35 mining blocks, i.e., seven blocks along the x dimension and five blocks along the y axis, where 11 blocks can be aggregated with mining block A to create stopes that satisfy the stope size constraint. If the z axis is also considered, the candidate pool of blocks expands further and the stope boundary becomes much larger. However, mining block A may belong to a single stope, because it is not physically possible to mine overlapping stopes in underground mining situations.

From an economic perspective, a stope may be included in a stope layout only if it makes a positive contribution to the overall profit of a layout (Richmond & Beasley, 2004). Therefore, the economic value of a stope should be evaluated to determine its suitability for a stope layout.

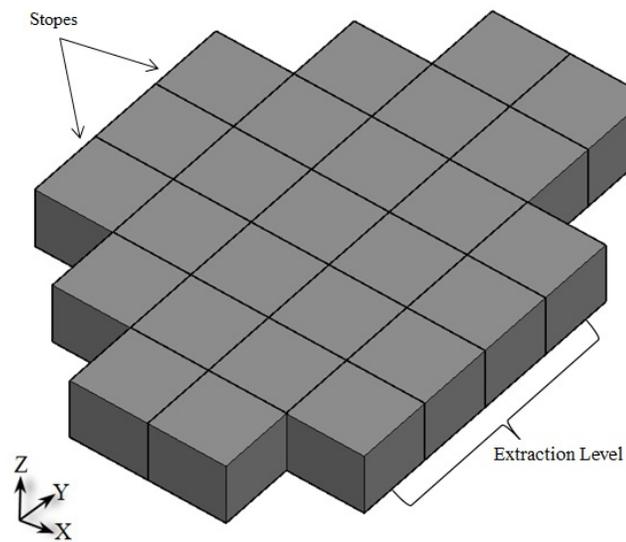


Figure 1.2 Representative layout of a stope in an underground mine.

Figure 1.2 shows a 3D view of a non-overlapping stope layout in a section of an underground mine. Given this stope layout, the profit of each stope is calculated from the quality and quantity data of mining blocks combined to form the stopes. If the total economic value of the ore blocks exceeds that of the waste blocks, a stope may

have a positive economic value. Similarly, if the total economic value of the ore blocks is below that of the waste blocks, the overall economic value of a stope may be negative. This adds to the computational complexity of the problem, because a stope optimisation algorithm may have to perform additional work to evaluate the profitability of each stope among all the possible stopes. Therefore, selecting an optimal stope layout using realistic ore body models that comprise millions of blocks in 3D (xyz plane) space is challenging.

In conclusion, generating an optimal stope layout for underground mining requires a robust algorithm, which can identify and select the best combination of profitable stopes for the mining blocks within a given resource model. Over the years, a number of algorithms have been tested and implemented to solve the optimal stope layout problem in underground ore bodies. However, they have failed to generate an optimal stope layout for a given resource model. The aim of the present study is to address this problem by developing an algorithm that finds the optimum stope layout in three dimensions. The proposed methodology will be immensely useful for mine planners and it will maximise the efficiency, and productivity of underground mining operations.

1.2 OBJECTIVES

Objectives of this research are:

- Review the existing stope optimisation algorithms and identify the areas that need further research and development.
- Develop a heuristic-based algorithm using object-oriented programming to optimise the underground stope layout and to maximise the overall profit of the layout.
- Incorporate the physical mining constraints in the structure of the algorithm to represent real mining scenarios.
- Implement the proposed algorithm on a real ore body model and validate the algorithm based on comparisons with existing algorithms.
- Develop a user-friendly interface to generate the optimal results based on different input parameters.

1.3 SCOPE

The scope of this project is limited to optimising the underground stope layout while satisfying the physical mining constraints, such as the minimum stope dimensions, stope size variations, pillar separation and level optimisation. The optimisation of underground development and infrastructure, and the optimisation of production schedules are two other key areas in underground mine optimisation, but this project does not contribute to these areas. Furthermore, the influences of geological and economic variables, such as metal prices on the outcomes of optimisation are not considered, although, these parameters may affect the outcomes. Thus, stochastic analysis is beyond the scope of the current study.

1.4 METHODOLOGY

Stope optimisation can be specified as a combinatorial optimisation problem due to its nature, as discussed in Section 1.1. Therefore, a heuristic algorithm is proposed to seek the optimal solution among a finite set of possible solutions. Both constructive and improvement characteristics related to heuristic algorithms are incorporated in the development of the proposed algorithm to ensure that the best quality layout of stopes is obtained. The algorithm incorporates fixed and variable stope sizes, pillar widths and level separations in its structure to represent realistic underground mining scenarios. Performance of the algorithm was analysed and improved by incorporating multi-threading concepts in object-oriented programming. The algorithm was implemented using the C# object-oriented computer language on the Visual Studio 2010 development platform (Albahari & Albahari, 2010; Deitel & Deitel, 2008; Hejlsberg et al., 2010). Microsoft SQL Server 2008 (Morelan et al., 2009) was used to facilitate storing and retrieving of block model data. Surpac mine design software (GEOVIA Surpac™ version 6.3.1, 2012) was employed for 3D visualisations of optimum solutions.

1.5 SIGNIFICANCE AND RELEVANCE

This research makes the following contributions:

1. Presentation of an innovative heuristic algorithm that generates the optimal stope layout for a given ore body model based on combinatorial optimisation.

2. An algorithm which overcomes the computational complexity by reducing the set of infinite solutions to a discrete set of finite solutions in order to find the best solution.
3. An algorithm that introduces an innovative method for identifying overlapping stopes and avoiding their inclusion in the optimum stope layout.
4. An implementation of the task-parallelism concept in object-oriented programming to improve the performance of the proposed stope optimisation algorithm.
5. An algorithm which incorporates important stope optimisation characteristics, including pillar separation, level separation and stope size variation, to generate solutions representing realistic underground mining scenarios.
6. Alterations to the configuration of input demonstrate further improvements to the solution values.
7. Implementation of the algorithm using Microsoft C# object-oriented programming language enables the execution of the application on windows-based computers with different architectures and/or platforms.
8. A user-friendly interface which enables rapid implementations of the stope optimisation algorithm in various mining scenarios.

1.6 THESIS OVERVIEW

Chapter 1 presents an introduction to the stope optimisation problem, including descriptions of its complexity, project objectives, scope, methodology and the significance of the project for the mining industry.

Chapter 2 presents an in-depth review of the existing stope optimisation algorithms. Furthermore, it includes a discussion about the applications of heuristics in combinatorial optimisation and their relevance to the stope optimisation problem.

Chapter 3 explains the proposed algorithm for stope boundary optimisation, including the general flow of the algorithm, improvements for different mining scenarios and modifications that address the computational complexity of the problem.

Chapter 4 presents an implementation of the proposed algorithm for an authentic resource model. Based on six cases, the applicability of the algorithm to different

mining scenarios is demonstrated. Furthermore, it includes a validation study of the proposed algorithm.

Chapter 5 concludes the outcomes, analyses the performance of the proposed algorithm, and provides recommendations for future research.

CHAPTER 2 UNDERGROUND STOPE OPTIMISATION

A variety of mine optimisation techniques have been developed since the early 1960s, and surface mine planning and design have improved remarkably as a result. For example, the current commercially available Lerchs-Grossmann algorithm has been implemented successfully to determine the ultimate pit limits for an open pit mine operation. By contrast, underground stope optimisation has received little attention. The lack of tools and appropriate computer programs to address underground mining problems remains an issue (Alford & Hall, 2009; Little & Topal, 2011).

An optimum stope layout dictates the maximisation of net present value in subsequent production scheduling, which is critical when planning an underground mining project (Hustrulid & Bullock, 2001; Kumral, 2010). Stope optimisation is far more complex than open pit mining optimisation with various mining methods being used in underground mining and their applications varying among mine sites. Further underground mining projects may require employment of more than one mining method when the ore body has a complex geometry, to improve safety and production. Furthermore, the development of underground stopes requires the consideration of physical and geotechnical constraints, for instance, the size of mining equipment, extent of mine development levels that provide access to the stopes, size and orientation of the ore body size of ore pillars which ensure stability of underground production areas (Gertch & Bullock, 1998; Krishna & Chanda, 1997). These parameters also affect the design of the dimensions (width, height and length) of stopes.

Geometrically, there are numerous possible options for the removal of a particular mining block relative to underground mining, as mentioned in Section 1.1 of Chapter 1. However, for a given slope angle constraint, there is only one possible option for removing a particular block in an open pit mine, thus generating an ultimate pit limit which is not as computationally complex as selecting the optimum stope layout for

underground mining. Figure 2.1 shows a two-dimensional (2D) hypothetical situation that illustrates the simplicity of block removal during open pit mining.

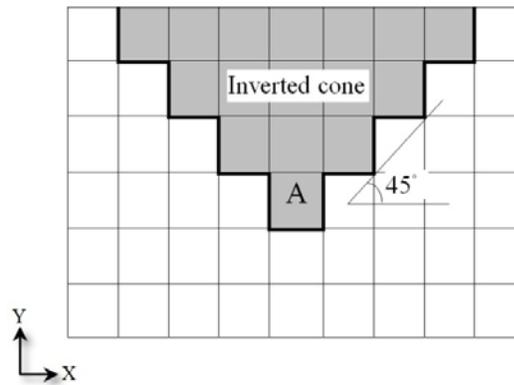


Figure 2.1 Two-dimensional view of an open pit mining geometry.

Given a slope angle of 45° , the space (shaded blocks in Figure 2.1) required to remove mining block A is a unique inverted cone (Ataee-pour, 2005). Altering the slope angle may result in a change in the size of the cone, however, for the given angle of 45° , there is only one solution.

Section 2.1 provides an overview of the stoping methods to which the proposed algorithm is applicable. Section 2.2 reviews the existing algorithms and describes their advantages and disadvantages in detail. Finally, Section 2.10 explains the role of heuristic algorithms in solving combinatorial optimisation problems.

2.1 FUNDAMENTALS OF STOPING METHODS

Stoping is a systematic engineering process that involves extracting valuable ore from an underground mine using specific mining methods. The selection of an underground stoping method is largely based on the spatial orientation of a given ore body. Operational/capital costs, production rates, labour availability, ground support and considerations for equipment also play a vital role in the selection of a stoping method. The fundamental aim is to select a method that offers the best combination of operational safety, production economics and mining recovery (Okubo & Yamatomi, n.d.). Within a given stope layout, the ore is excavated through a general cycle of drilling, blasting, loading, and haulage operations. Figure 2.2 shows a close-up of this stope production cycle within an underground stoping operation.

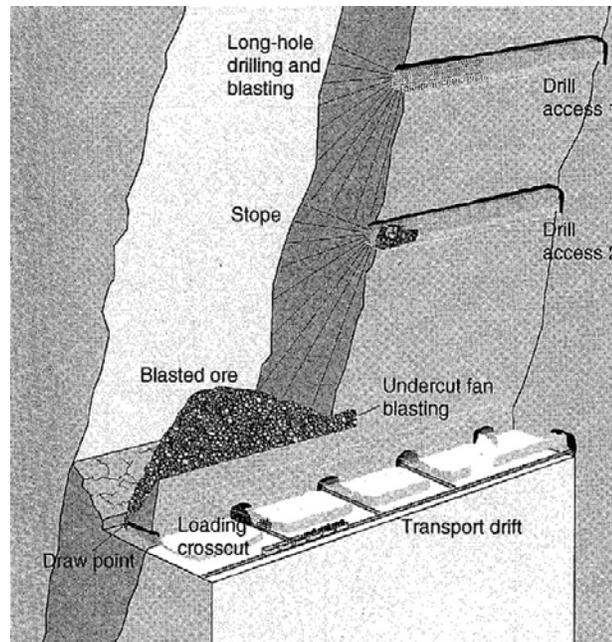


Figure 2.2 A close-up of production operations with a stope of an underground mine (Gertch and Bullock, 1998).

Underground stoping methods can be categorised in two classes based on the extent of the ground support required: naturally supported and artificially supported. Room and pillar mining and sublevel stoping are the methods available for naturally supported stoping methods. Examples for artificially supported stoping methods include cut-and-fill mining, shrinkage stoping and square-set mining. These two categories are briefly explained in Subsections 2.1.1 and 2.1.2 as follows.

2.1.1 NATURALLY SUPPORTED STOPING METHODS

The naturally supported stoping methods are applied to ore bodies requiring minimum development of an artificial system to support the stability of the underground mining areas. Within these ore bodies, the geotechnical conditions of the surrounding mass of the production areas are favourable to adequately withstand the loads and thereby assuring the safety of the underground mining operations. However, rock/roof bolts or timber/steel structures may be required in some cases to a lesser degree compared to the artificially supported stoping methods. For example, deposits with large vertical heights may require the installation of rock bolts to support roof stability where mining progresses in slices (Gertch and Bullock, 1998).

Among naturally supported stoping methods, room-and-pillar mining is used for deposits with limited thickness such as coal, limestone, and shale. The sublevel stoping mining method is applied to vertical or steeply inclined ore bodies.

2.1.2 ARTIFICIALLY SUPPORTED STOPING METHODS

The artificially supported stoping methods are employed for the extraction of ore bodies which require a higher degree of ground control systems. The load-carrying capacity of the rock mass in these ore bodies is not sufficient to sustain safety of the underground operations and openings. As such, support structures are provided in terms of timbers, backfilling, cribs, and hydraulics to improve stability of underground mining (Hustrulid & Bullock, 2001).

Among artificially supported stoping methods, Cut-and-fill mining exploits the ore body in horizontal slices generally from the bottom of a stope upwards similar to shrinkage stoping. This method is applied to steeply dipping ore bodies which are moderately competent. Waste rock may be utilised as fill material though in the modern practice, hydraulic filling methods are used. Shrinkage stoping guides ore extraction from the bottom of a stope upwards in terms of horizontal slices. This method is best utilised for steeply dipping ore bodies. Square-set mining is most suitable for small high-grade ore bodies and based on timber-support structures. Though it can be applied to any orebody shape and ground condition, it demands a steady supply of timbers and intensive labour. In some cases, variations of sublevel stoping are categorised in to artificially supported stoping methods when the pillars are recovered and the voids are filled with hydraulic fill, paste or cemented rock (Darling, 2011).

2.2 THE EXISTING STOPE OPTIMISATION ALGORITHMS

The few existing algorithms for optimising stope layouts may be classified as rigorous and heuristic (Ataee-pour, 2005). Rigorous algorithms are supported by mathematical proofs, and some examples of this category include the dynamic programming algorithm (Riddle, 1977), branch and bound technique (Ovanic & Young, 1995) and network flow method (Bai et al., 2013). Heuristic approaches solve optimality problems based on rules that are applied during the search of optimal stope layouts. Examples also include the octree division algorithm (Cheimanoff et al., 1989), floating stope algorithm (Alford, 1995), Sens and Topal

heuristic approach (Sens & Topal, 2009) and maximum value neighbourhood algorithm (Ataee-pour, 2000).

2.2.1 DYNAMIC PROGRAMMING SOLUTION FOR STOPE OPTIMISATION

Riddle (1977) proposed a dynamic programming-based solution for defining stope boundaries in block caving-related mining systems. During initialisation, the algorithm assumes that there is no footwall in the optimisation process and that the maximum profit is achieved irrespective of the footwall design. Next, it defines a footwall in operational regions and examines the profitability of all feasible solutions for mining and non-mining regions. In the subsequent steps, the process divides the defined footwall region into two sub-regions if the profit is greater than or equal to an assumed profit. The process terminates when there are no more profitable footwalls to examine, or no further feasible footwalls can be introduced into the operational region. This 2D approach is useful for the stope optimisation problem because of its simplicity, but it does not support the realistic 3D mining situations.

2.2.2 OCTREE DIVISION ALGORITHM

Cheimanoff et al. (1989) developed an algorithm known as the Octree division algorithm. During initialisation, the algorithm gathers borehole data, geostatistical analysis data, shapes of geological objects, etc. Next, the collected data are utilised to build a geometric model, which transforms the geological resources into reserves. Using the geometric model, the algorithm identifies reserves based on economic evaluations of the geological resources and determines the mining sequences. Consequently, the algorithm divides the reserves into sub-volumes for further economic evaluations. These sub-volumes are either removed from the model or stored in the model, if their mineral content or dimensions violates the constraints set by the algorithm. Figure 2.3 (a) shows the division of a block into eight sub-volumes and Figure 2.3 (b) shows sub-volume removal with the Octree division algorithm.

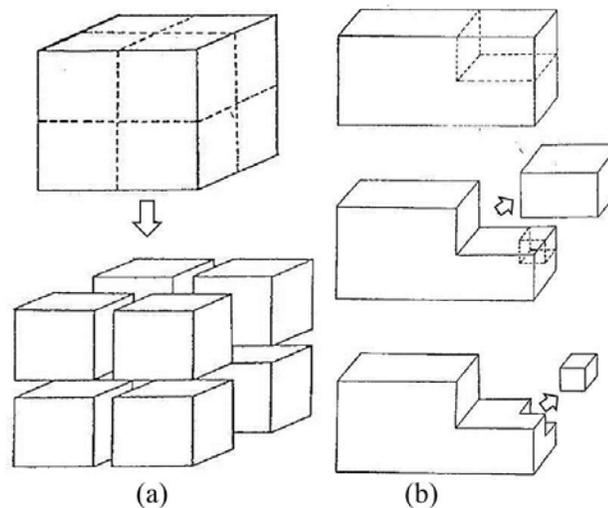


Figure 2.3 (a) Octree division. (b) Successive removal of sub-volumes (Cheimanoff et al., 1989).

The Octree division algorithm generates a 3D feasible solution for the stope geometry, but it leads to more waste being added in the final mine layout because the algorithm does not analyse the sub-volumes jointly. It includes individual sub-volumes that consider the minimum allowable dimensions in the final layout but not the amount of waste in these sub-volumes. As such, the algorithm does not guarantee the optimality of stope layouts.

2.2.3 BRANCH AND BOUND ALGORITHM

Ovanic and Young (1995, 1999) introduced a mixed integer programming technique known as “type-two special ordered sets” for the economic optimisation of stope layouts. The objective of this algorithm is to identify optimal start and finish locations for mining based on given stope dimensions. The algorithm achieves this by employing two piecewise linear cumulative functions, one to optimise the start location and the other to optimise the finish location. These two functions calculate the cumulative economic values of the blocks in each row of the model. The application of the cumulative economic evaluation is illustrated for a hypothetical situation with eight mining blocks (Figure 2.4).

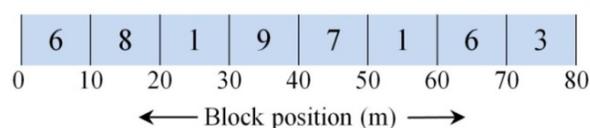


Figure 2.4 Row of economic value blocks.

The blocks are represented in blue colour cells and spaced 10 m apart. The figures within each cell represent the economic value for that block. Figure 2.5 shows the linear function of the cumulative economic values for each sequential block position.

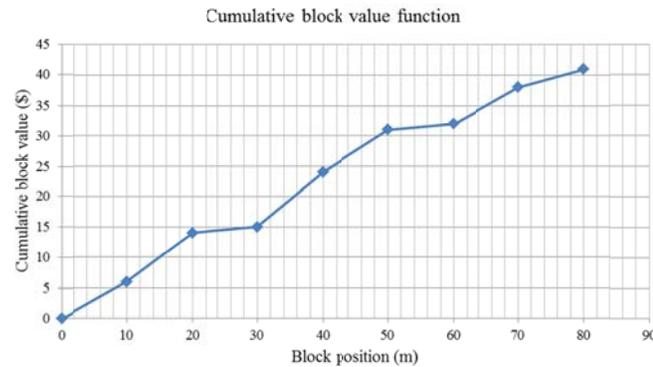


Figure 2.5 Cumulative function for the row of economic blocks.

Based on the given stope dimension values, the algorithm identifies the optimal start and finish locations that maximises the cumulative economic function of each row of mining blocks in the resource model. The main advantage of this technique compared with other methods is that the block geometry does not need to be regular or orthogonal. However, the algorithm only optimises the stope boundary in one dimension and lacks a 3D implementation.

2.2.4 FLOATING STOPE ALGORITHM

Alford (1995) introduced a stope optimisation method called “floating stope”. This algorithm is commercially available in the design software Datamine (CAE Studio 3, 2014). During initialisation, the algorithm specifies a stope with the minimum stope dimensions. The specified stope is floated in the ore body relative to an origin, with defined float increments in three orthogonal directions. During the floating process, the grade inside the floating stope is calculated by averaging the grades of the blocks that fall within this specified stope boundary. The stopes that should be selected for inclusion in the final stope layout depend on the optimisation objectives, i.e., maximising the ore tonnage, the grade, the metal contained or the accumulated (dollar) value.

This algorithm has a major disadvantage because it generates overlapping stopes, with shared mining blocks. Therefore, manual adjustments may be required to

exclude these shared blocks from the final layout, because a block cannot physically belong to multiple stopes. Consequently, the economic values of the shared blocks are summed more than once depending on their number of overlaps. Figure 2.6 illustrates this issue using a hypothetical 2D scenario. In this scenario, two stopes that measure 5×5 blocks, i.e., five blocks along the x axis and five blocks along the y axis, overlap and share six mining blocks. The two overlapping stopes are designated as *Stope 1* and *Stope 2* and the economic values of their shared mining blocks are indicated. It is assumed that *Stope 1* yields a total economic value of 6 and that *Stope 2* yields a total economic value of 3.

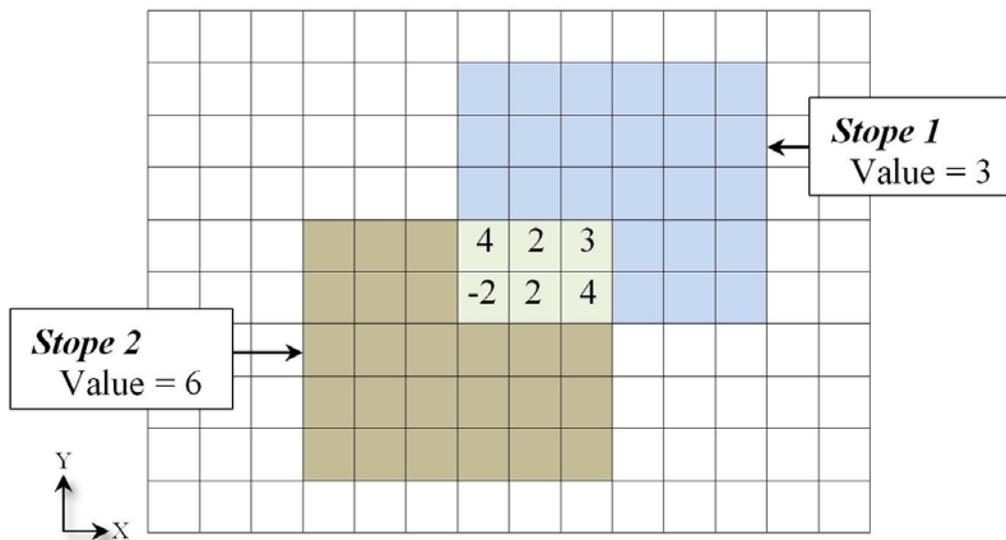


Figure 2.6 The overlapping problem with the floating stope algorithm.

1. Total economic value calculated by the algorithm = $6 + 3$
= 9
2. Actual total economic value of *Stopes 1* and *2* = $(6+3) - (4 + 2 + 3 - 2 + 2 + 4)$
= -6

The total economic value of the combination of stopes differs from the actual economic value. Therefore, the algorithm does not guarantee an optimal solution to the problem.

2.2.5 MULTIPLE PASS FLOATING STOPE PROCESS

The multiple pass floating stope process (MPFSP) was developed by Cawrse (2001), to extend and improve the functionality of Datamine software (CAE Studio 3, 2014).

The MPFSP process is divided into three steps: input parameter definition, file generation and file management. Input parameters such as the head grade, cut-off grade and maximum waste are defined by the user. During the file generation step, economic stope envelopes are created for each set of parameters. Finally, the data files or the statistical files generated in the process are converted into a Microsoft Excel compatible (CSV) format.

Subsequently, the MPFSP process provides an informative evaluation of the block model examining the stopping feasibility and identifying possible areas for the development of stopes. However, it does not eliminate the shortcomings of the “floating stope” algorithm. The method can facilitate stope boundary selection and design but it does not generate optimum stope layouts (Cawrse, 2007).

2.2.6 MAXIMUM VALUE NEIGHBOURHOOD ALGORITHM

Ataee-pour (2000) developed an algorithm based on a heuristic approach called the maximum value neighbourhood (MVN). The algorithm defines a number of mining blocks that should be mined together to satisfy the minimum stope size constraint. Next, sets of feasible neighbourhoods are identified for each block of the model. The economic value of each neighbourhood is determined to identify the MVN, i.e., the neighbourhood with the maximum economic value. Finally, all of the MVN blocks are flagged for inclusion in the final stope layout. Some checks are performed to ensure the exclusion of waste blocks, and consequently, an enhancement in the performance of the algorithm. For example, if a block is already flagged or if a block has a negative value, the algorithm terminates the neighbourhood evaluation process for that particular block.

The MVN algorithm has advantages in terms of its simple concept and computational implementation, but it also has several drawbacks. First, the algorithm identifies a collection of MVNs rather than mineable stopes. The algorithm defines the neighbourhood dimensions based on a minimum stope width requirement, but in some cases, this requirement may be violated. Figure 2.7 shows an example of this issue using a hypothetical row of six mining blocks. The six mining blocks are labelled *a-f* in alphabetical order, for reference. The values shown in each block are the hypothetical block economic values.

-20	10	30	40	15	-25
<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

Step 1 : Block *a* is negative and algorithm stops the block from further proceedings.

Step 2 : Feasible neighbourhoods for block *b* is evaluated and MVN is selected as follows.

- (i)

-20	10	30
-----	----	----

 \Rightarrow Neighbourhood Value : $-20+10+30=20$
- (ii)

10	30	40
----	----	----

 \Rightarrow Neighbourhood Value: $10+30+40=80$

Blocks *c* and *d* are flagged and neighbourhood (ii) is selected as the MVN for block *b*

Step 3: Blocks, *c* and *d* are flagged already. Thus, feasible neighbourhoods of block *e* is evaluated and MVN is selected as follows.

- (i)

30	40	15
----	----	----

 \Rightarrow Neighbourhood Value: $30+40+15=85$
- (ii)

40	15	-25
----	----	-----

 \Rightarrow Neighbourhood Value: $40+15-25=30$

Block *e* is flagged and neighbourhood (i) is selected as the MVN for block *e*.

Step 4 : Block *f* is negative. Thus, the algorithm stops them from proceeding further.

Step 5 : Final stope layout consists of blocks which are hatched as follows.

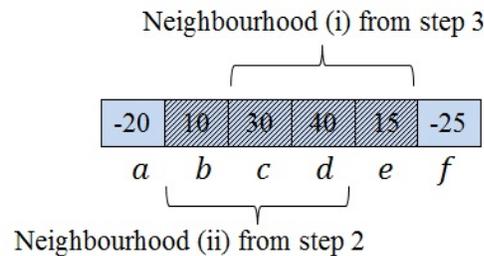


Figure 2.7 Example illustrating the main problem with the MVN algorithm.

According to this hypothetical example, a neighbourhood of three blocks is defined as the minimum stope size. The algorithm derives two MVNs, which comprise blocks *b*, *c*, *d* and *e*. However, selecting either of the neighbourhoods may result in violation of the minimum stope width constraint. For example, if neighbourhood (i) is selected from step 2, mining of block *b* may be impossible because it is a single block, thus it violates the minimum stope size constraint. Similarly, mining of block *e* may be impossible, if neighbourhood (ii) is selected from step 3. Therefore, the implementation of the algorithm does not yield a practical mining solution for the stope optimisation problem. Furthermore, the algorithm generates different outcomes depending on the selection of the start point, thereby indicating that the algorithm

cannot ensure that the best solution to the problem is obtained (Little, 2012; Little et al., 2013).

2.2.7 MIXED INTEGER PROGRAMMING-BASED ALGORITHM

Grieco and Dimitrakopoulos (2007) developed a probabilistic optimisation algorithm based on mixed integer programming. In this approach, the ore body is divided into layers initially. Each layer is then subdivided into a number of panels and each panel is subdivided into a series of rings. Each ring is assigned to a binary variable of the mixed integer model. The objective function of the algorithm works by maximising the metal content at a given time. The minimum and maximum mining rings, and the sizes of the pillars that need to be left unmined between two primary stopes, are limited by the constraints of the model. This method considers the geological uncertainty during the stope design stage, but it does not allow accurate examinations of stopes in different locations. It generates the optimum stope layout based on the rings that are defined relative to the location and size of the most profitable stopes. Further, the algorithm does not ensure precise examination of stopes in different locations of the ore body. Furthermore, the performance of the algorithm is proportional to the number of binary variables in the mixed integer model, causing the solution time to increase as the number of rings increase (Dimitrakopoulos & Grieco, 2009).

2.2.8 SENS AND TOPAL HEURISTIC APPROACH

Sens and Topal (2009) introduced a new algorithm based on a heuristic approach. During initialisation, the given mining block model is converted to a regularised block model, i.e., a block model that constitutes mining blocks with consistent sizes. Given the dimensions (height, length and width) stopes are generated from the regularised block model. Finally a heuristic stope optimisation algorithm is implemented in Matlab software based on the economic value of stopes and the results are visualised with respect to different user-defined parameters. The advantage is that it locates stope boundaries using different stope sizes and selection strategies in three dimensions.

To its disadvantage, the algorithm selects the stopes in descending order of the stope economic values while removing the overlapping stopes. To clarify, this process

discards the possibility of multiple stope combinations that can be derived from a given stope set. Amongst these multiple combinations, there may be combinations with higher total economic values. Figure 2.8 illustrates this problem based on a hypothetical situation. In this 2D diagram, the stope size is fixed at 5×4 , i.e., five blocks along the x axis and four blocks along the y axis.

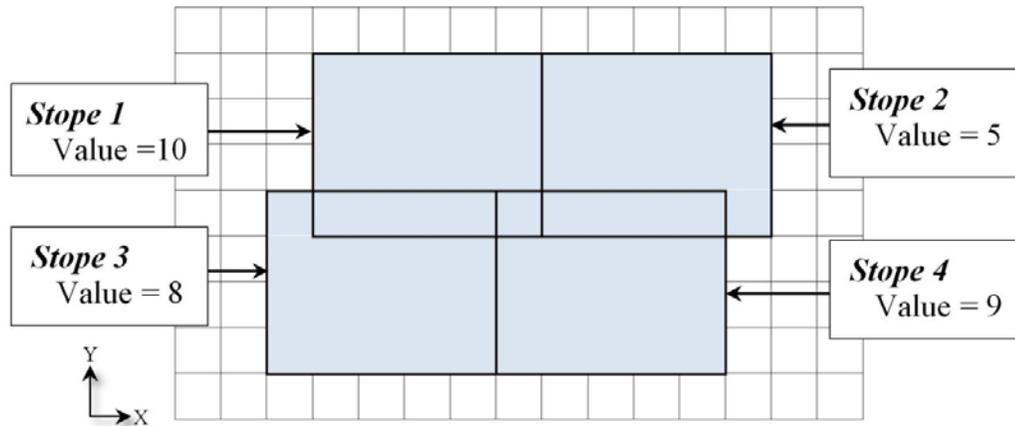


Figure 2.8 The problem with Sens and Topal heuristic approach.

1. Combination selected by the algorithm = *Stope 1 and Stope 2*
(Total economic value = $10 + 5 = 15$)
2. Most profitable combination = *Stope 3 and Stope 4*
(Total economic value = $8 + 9 = 17$)

Therefore, the final stope selection criterion employed by this algorithm fails to generate the most profitable stope combination.

2.2.9 NETWORK FLOW METHOD

Bai et al. (2013) introduced the graph theory and network flow method for underground stope optimisation. This approach was implemented specifically for underground mines that operate using the sub-level stoping mining method. The algorithm specifies an initial location and extent for development of a raise, before defining a cylindrical coordinate system around the specified raise location. The cylindrical system establishes the precedence links between mining blocks, which are required to satisfy the geotechnical constraints on the footwall and the hanging wall slopes. Figure 2.9 (a) shows a block model using the cylindrical coordinate system. Figure 2.9 (b) shows the arcs with vertical links of precedence for a given block A,

where r , Δr and Δz denote the distance to a particular block from the raise centre line, the length of a block and the vertical height of a block, respectively.

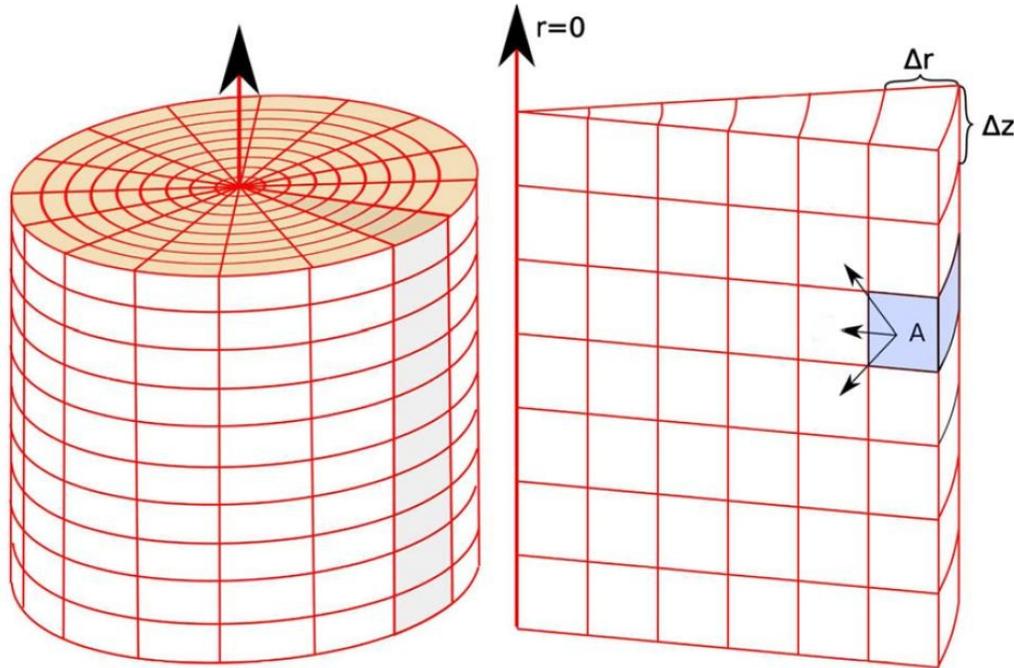


Figure 2.9 (a) Block model constructed using the cylindrical coordinate system. (b) Typical arcs with vertical links of precedence (Bai et al., 2013).

The graph is constructed using vertical arcs for the footwall and hanging wall slope constraints, with horizontal arcs for the stope width constraint. Selecting a particular mining block for a stope should aim to maximise the stope value subject to two additional constraints: the maximum distance of a block from the raise and the horizontal width required to bring the most distant mining block to the raise. The stope profit is optimised as a function of location and height of the raise and consequently, the best location and height for raises are identified. Finally, the performance of the algorithm was compared with the floating stope algorithm and obtaining better solution values. However, it is limited to sub-level stoping mining method and small mineralised ore bodies.

2.3 USE OF HEURISTICS FOR COMBINATORIAL OPTIMISATION

Combinatorial optimisation can be defined as a mathematical method that involves the search for an optimal set and arrangement of discrete objects from a finite set of objects. Like any other optimisation related problem, combinatorial optimisation problems are concerned with searching for the best values for one or several decision variables, which must satisfy the objectives without violating the defined constraints.

However, most combinatorial optimisation problems are NP-hard, as defined in computational complexity theory, meaning it is often impossible to solve the instances in reasonable time (Karapetyan, 2010). Implementation of exact optimisation algorithms may also be impossible because many modern optimisation problems tend to be complex and require analysis of large data sets. Two examples of NP-hard problems are stated as follows.

1. The travelling salesman problem (TSP): Given a list of cities and their spatial coordinates where an origin city is defined, find the shortest possible route that visits each city once and only once, and returns to the city of origin (Basel & Willemain, 2001; Lam & Newman, 2008; Punnen, 2007).

2. The subset sum problem (SSP): Given a set of integers $A = \{x_1, x_2, \dots, x_n\}$ and f is an integer number, determine whether there exists a subset J of $\{1, 2, \dots, n\}$ such that $\sum_{j \in J} x_j = f$, i.e., determine whether there exists a set $B \subset A$ where the sum of the members of B is f (Bernstein et al., 2013).

In both cases, finding the optimal solution, i.e., the shortest route in the case of TSP and a set B with sum f in the case of SSP requires search methods to select the optimal solution among a set of possible solutions. However, as the problem size increases, i.e., number of cities in TSP and the cardinality (number of members) of set A in SSP, the search for possible solutions becomes intense and exhaustive. In such cases, heuristic algorithms are essential because they may generate a near-optimal solution for a given NP-hard problem. In some cases, they may be the only practical alternative for NP-hard optimisation problems, subject to computational complexity, problem size and solution time constraints (Gonçalves & Resende, 2011; Martí et al., 2013; Rardin & Uzsoy, 2001).

2.3.1 TYPES OF HEURISTIC ALGORITHMS

Heuristic algorithms may be used to build a solution constructively (piece by piece) or by progressive improvements (Semančo & Modrák, 2011; Žerovnik & Žerovnik, 2011). Many constructive heuristics are “greedy”, which means they take the best solution without regard for the remaining solutions. For example, a constructive approach to the TSP is to take the nearest city in consecutive steps. Similarly, a constructive approach for the SSP would be to randomly select an $x_j \in A$ and to

search for any $x_{j'} \in A$ from a set $C \subset A$. The objective is to ensure that the cardinality of C , i.e., the number of members of C , remains as low as possible to address the computational complexity, which may be caused by the cardinality of A .

By contrast, improvement heuristics are “intelligent”, which means that they employ techniques to obtain better quality solutions using the given specified objects. An example of an improvement approach would be to generate a possible solution for TSP and then to swap the order of a few cities to generate a better solution. A possible improvement approach for the SSP is to randomly select any $x_j \in A$ and then to search for any $x_{j'} \in A$ such that $x_{j'} \cong f - x_j$. The objective of this approach is to find any better value for $x_{j'}$ from set A such that $f - x_j$ is as close as possible to zero.

In general, a heuristic algorithm can be compound in nature, i.e., a combination of constructive and improvement types. A compound heuristic solves an NP-hard optimisation problem in two phases: a constructive phase followed by an improvement phase (Derigs & Vogel, 2014).

2.4 SUMMARY

Numerous approaches have been developed to solve the stope optimisation problem, but none of them guarantees the generation of an optimal solution that satisfies the physical mining constraints in 3D space. Therefore, stope optimisation for underground mining requires a robust algorithm that can handle the computational complexity and satisfy the physical mining constraints to generate an optimal solution.

According to the concepts employed by combinatorial optimisation in operations research, the stope optimisation problem can be defined as an NP-hard optimisation problem because of its computational complexity and the problem size. Therefore, this problem may be addressed using a heuristic algorithm that searches for the possible solutions and finds the optimal solution among them while satisfying the physical mining constraints. The constructive and improvement characteristics discussed in Subsection 2.10.1 are incorporated into the proposed algorithm to generate better quality solutions in the least possible time.

CHAPTER 3 DEVELOPMENT OF THE PROPOSED ALGORITHM

The proposed algorithm comprises six major stages that are implemented sequentially for a given resource model. Figure 3.1 shows of this sequential process through a flow diagram.

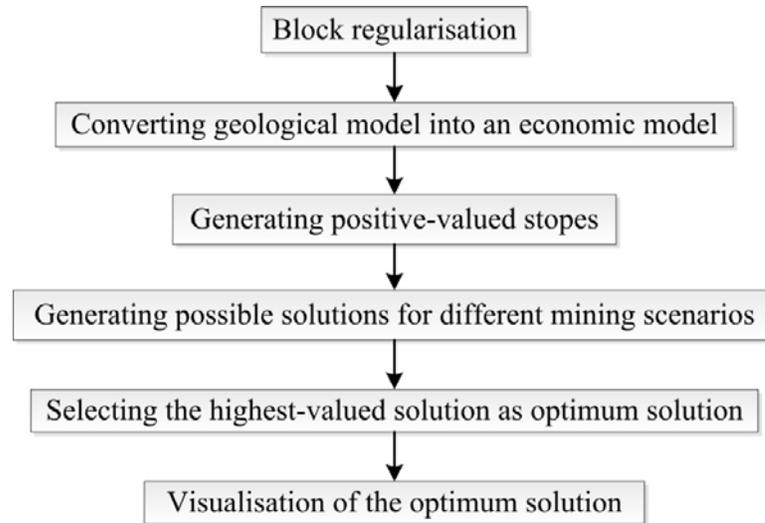


Figure 3.1 General flow of the algorithm.

The process begins by regularising a resource model, i.e., by converting the irregular block model into a model with equal-sized blocks. The algorithm then converts the regularised geological block model into an economical block model. Stopes are generated based on fixed or variable size and a set of positive-valued stopes are extracted using the economical block model as input. Consequently, the algorithm generates a set of possible solutions for given mining scenarios using the positive-valued stopes as input. Finally, the algorithm selects the solution with the highest economic value and visualises it.

3.1 BLOCK REGULARISATION

The resource models comprise 3D blocks (cubes and cuboids) of different sizes. The stope optimisation algorithm requires a resource model with equal-sized blocks, i.e., a regularised model, as the input for generating stopes. A regularised resource model facilitates the definition of the stope size using a fixed or variable number of blocks

from x , y and z axes, thus a given ore body model is regularised at the beginning of the stope optimisation process.

The dimensions (length, width, and height) of the regularised blocks are defined by the algorithm. The minimum coordinate among the set of coordinates for the original blocks becomes the minimum coordinate of the regularised block model. Similarly, the maximum coordinate among the set of coordinates for the original blocks becomes the maximum coordinate of the regularised block model. After defining the new dimensions and the two limits, the algorithm regularises the original model, generates a new regularised block model and updates their attributes including grade, density, tonnage, metal weight and economic value. Figure 3.2 shows an example of this process using a section of an ore body model.

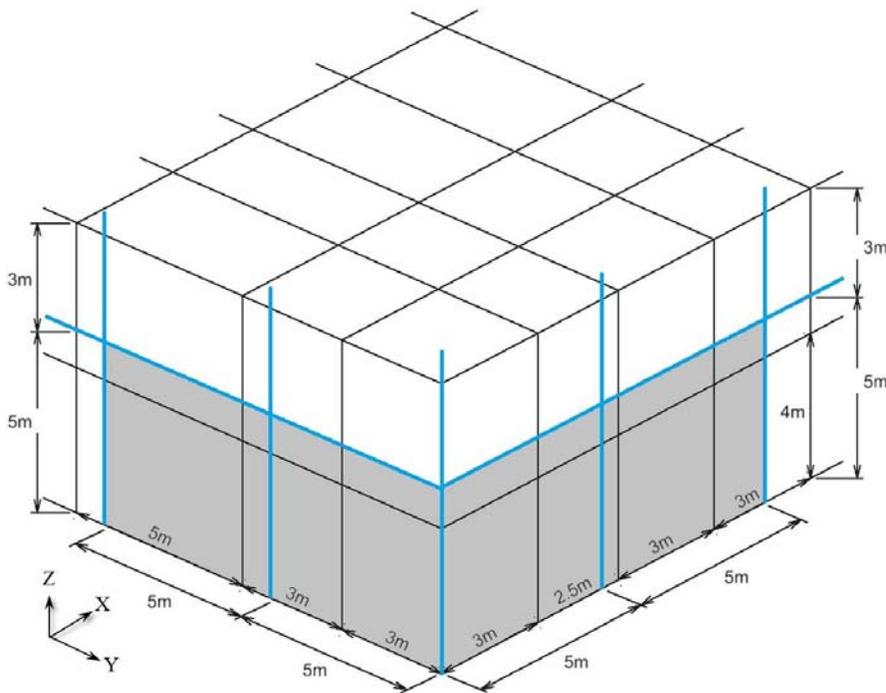


Figure 3.2 Example illustrating the block regularisation process.

The hypothetical model comprises mining blocks with variable sizes ($3 \times 3 \times 4$ m, $2.5 \times 3 \times 4$ m, $3 \times 5 \times 4$ m and $4 \times 5 \times 4$ m). In this situation, regularisation of the original blocks is performed to obtain new blocks with a fixed size of $5 \times 5 \times 5$ m. The blue borders indicate the limits of the regularised blocks and the shaded areas represent the faces of the blocks that have been regularised. It can be seen that the regularised blocks comprise a collection of partial and full original blocks. The

algorithm updates the mining parameters for a regularised block based on the attributes of the partial or complete original blocks, which are inside a regularised block. The following notations are defined to explain the mathematical formulations of the block attributes.

o = block indicator of a given original block model

r = block indicator of a regularised block model

g_o = original block grade(% or gram per tonne)

d_o = original block density(tonne per m^3)

W_r = regularised block width(meter)

H_r = regularised block height(meter)

L_r = regularised block length(meter)

W_{ro} = overlapping width of original block o with new block r (meter)

L_{ro} = overlapping length of original block o with the new block r (meter)

H_{ro} = overlapping height of original block o with the new block r (meter)

O_{ro} = overlapping volume of original block o with the new block r (m^3)

m_{ro} = metal weight of O_{ro} (gram)

t_{ro} = Tonnage of O_{ro} (tonne)

t_r = tonnage of block r (tonne)

V_r = volume of block r (m^3)

v_r = economic value of block r (\$)

d_r = density of block r (tonne per m^3)

m_r = metal weight of block r (gram)

g_r = grade of block r (% or gram per tonne)

Equations (1) – (8) represent formulations of the block attributes, where $o' \in \alpha_o$ and α_o is a set of blocks that overlap with a block r .

$$O_{ro'} = W_{ro'} \times L_{ro'} \times H_{ro'} \quad (1)$$

$$V_r = W_r \times L_r \times H_r \quad (2)$$

$$t_r = \sum_{o'} (d_{o'} \times O_{ro'}) \quad (3)$$

$$d_r = t_r/V_r \quad (4)$$

Equations (5) and (6) represent m_r and g_r formulations for metals where the grade is defined in %.

$$m_r = \sum_{o'} (d_{o'} \times g_{o'} \times O_{r_{o'}}) \times 10^6 \quad (5)$$

$$g_r = [m_r \times (t_r \times 10^6)] \times 100 \quad (6)$$

Equations (7) and (8) represent m_r and g_r formulations for metals where the grade is defined in grams/tonne.

$$m_r = \sum_{o'} (g_{o'} \times t_r) \quad (7)$$

$$g_r = (m_r/t_r) \quad (8)$$

3.2 CONVERTING THE GEOLOGICAL MODEL INTO AN ECONOMIC MODEL

In the second stage of the process, the algorithm calculates the block economic values to translate the regularised resource model into an economic model. The following notations are used to explain the mathematical formulations of the block economic values. The C# object-oriented source code for this formulation has been included in Subsection 1 in Appendix B.

$p =$ Metal price (\$/tonne or \$/ounce)

$e_r =$ Mining cost(\$/tonne)

$c_r =$ Processing cost(\$/tonne)

$r_r =$ Refining cost(\$/tonne)

$y =$ Recovery(%)

$v_r =$ Block economic value(\$)

Equation (9) is used to calculate v_r .

$$v_r = [(p - r_r) \times g_r \times y - (e_r + c_r)] \times t_r \quad (9)$$

3.3 STOPE GENERATION

Given the regularised economic model, the algorithm generates stopes in the third stage of the process. It defines the stope size in terms of the number of blocks from

the x , y and z axes, and generates a set of possible stopes. Figure 3.3 shows an example of a $2 \times 3 \times 2$ stope size definition, i.e., two blocks along the x axis, three blocks along the y axis and two blocks along the z axis, which are inside a block model that has been regularised to a block size of $5 \times 5 \times 5$ m.

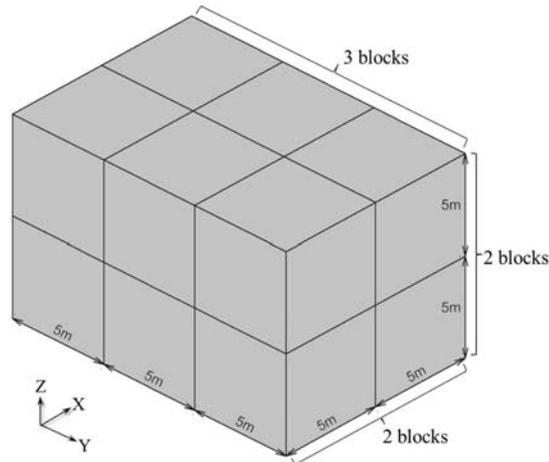


Figure 3.3 Block model with a stope size of $2 \times 3 \times 2$ blocks and a block size of $5 \times 5 \times 5$ m.

The defined stope size is floated along the x , y and z axes to identify all possible stopes within the economic model.

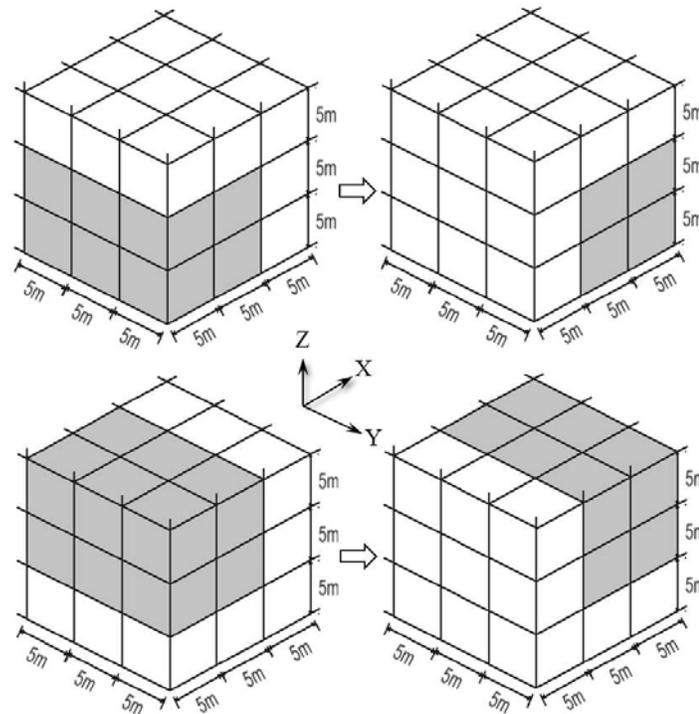


Figure 3.4 Stope generation process.

Figure 3.4 illustrates an example of the stope generation process using a regularised block model comprising of 27 mining blocks, i.e., three blocks along the x axis, three blocks along the y axis and three blocks along the z axis. The stope size defined in Figure 3.3 is floated along x , y and z axes and four possible stopes are generated within this block model.

The following notations are used to explain the mathematical formulations for the stope generating algorithm.

s = stope indicator

x_r, y_r, z_r = location (x, y, z coordinates) of block r

$\bar{x}_s, \bar{y}_s, \bar{z}_s$ = location (x, y, z) of stope s

$\bar{L}_s, \bar{W}_s, \bar{H}_s$ = length (m), width (m), and height (m) of stope s

\bar{g}_s = metal content or grade (% or grams/ tonne) of stope s

\bar{d}_s = material density (tonnes/ m^3) of stope s

\bar{m}_s = metal weight (grams) of stope s

\bar{t}_s = total weight (tonnes) of stope s

\bar{v}_s = economic value (\$) of stope s

nx_s, ny_s, nz_s = number of mining blocks inside stope s along the x, y , and z axes

sx_s, sy_s, sz_s = location (x, y, z coordinates) of the starting (origin) block of stope s

ex_s, ey_s, ez_s = location (x, y, z coordinates) of the end (last) block of stope s

i, j, k = block identifier that facilitates stope generation

Figure 3.5 illustrates some of these notations.

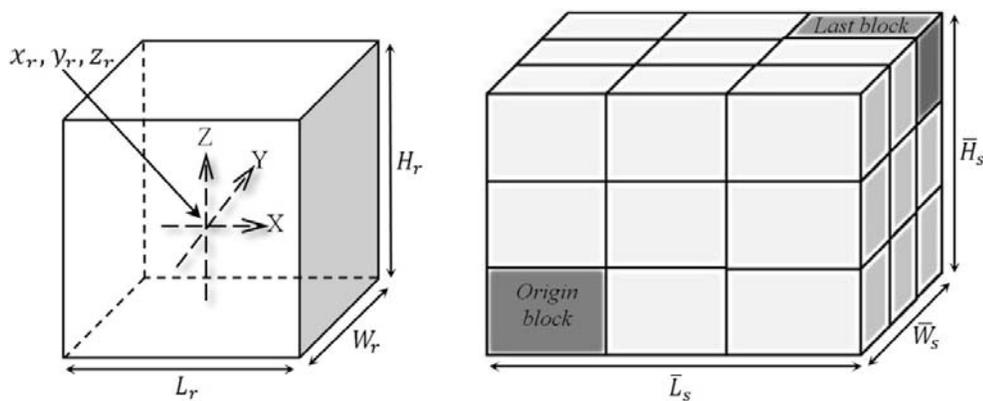


Figure 3.5 Mining block and stope notations.

3.3.1 STOPE GENERATION ALGORITHM

A unique block identifier (i, j, k) is defined to facilitate the stope generation process. The algorithm assigns $(1,1,1)$, i.e., $(i = 1; j = 1, k = 1)$, to the origin block of the block model, before increasing (i, j, k) with an increment of 1 along the x, y, z axes and updating all the blocks in the model. The increase in (i, j, k) is proportional to the ascending order of the x, y, z coordinates of the mining blocks. Figure 3.6 illustrates this process using a 2D block model comprising of 25 blocks.

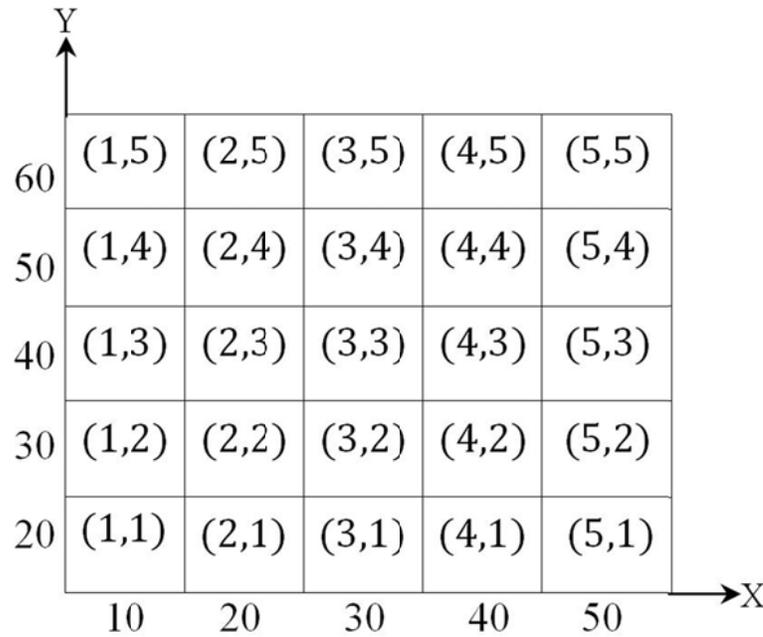


Figure 3.6 Block identifier for a two-dimensional example.

In this case, the algorithm identifies the origin of the block model, i.e., $X = 10, Y = 20$ and assigns $(1,1)$, i.e., $(i = 1; j = 1)$, to the origin block. Next, according to the ascending order of the X and Y coordinates, the algorithm advances (i, j) with an increment of 1 and updates all the blocks in the model with a unique (i, j) number.

The unique identifier (i, j, k) facilitates the identification and aggregation of the mining blocks to generate stopes. Given stope dimensions (nx_s, ny_s, nz_s) , the algorithm then establishes the (i, j, k) of the last block using the (i, j, k) of the origin block of a stope. Equation (10) shows the mathematical formulation of this process.

$$i'', j'', k'' = (i' + (nx_s - 1)), (j' + (ny_s - 1)), (k' + (nz_s - 1)) \quad (10)$$

where

$$(i', j', k'), (i'', j'', k'') \in \alpha_{ijk}$$

α_{ijk} = Set of unique integer identifiers for a regularised block model

(i', j', k') = (i, j, k) of the starting block of a stope

(i'', j'', k'') = (i, j, k) of the end block of a stope

The C# object-oriented source code for this formulation has been included in Subsection 2 in Appendix B.

Next, the algorithm identifies the block with the maximum i, j, k , i.e., $i_{max}, j_{max}, k_{max}$, to determine the candidate pool of blocks for generating stopes. Within the limits (i', j', k') and (i'', j'', k'') , and subject to $(i'' \leq i_{max}, j'' \leq j_{max}, k'' \leq k_{max})$, the set of possible stopes are generated. A block r with a unique identifier (i, j, k) is included in stope s if and only if the following condition is true.

$$(i', j', k') \leq (i, j, k) \leq (i'', j'', k'')$$

For example, given a unique identifier for the origin block = $(2,3)$, i.e., $i' = 2$ and $j' = 3$, and the stope size = 2×2 , i.e., $nx_s = 2$ and $ny_s = 2$, the formulation of (i'', j'') is as follows in the example 2D situation in Figure 3.6.

$$i'', j'' = (2 + (2 - 1)), (3 + (2 - 1))$$

$$i'', j'' = (2 + 1), (3 + 1)$$

$$i'', j'' = 3, 4$$

In this case, $i_{max} = 5$ and $j_{max} = 5$, thus $i'' \leq 5$ and $j'' \leq 5$.

Consequently, for any block r to be included in the stope s' originating from the block at $(2,3)$, the unique identifier (i, j) of r must satisfy the following condition.

$$(2,3) \leq (i, j) \leq (3,4)$$

Figure 3.7 shows the stope boundary that satisfies this condition for the example model.

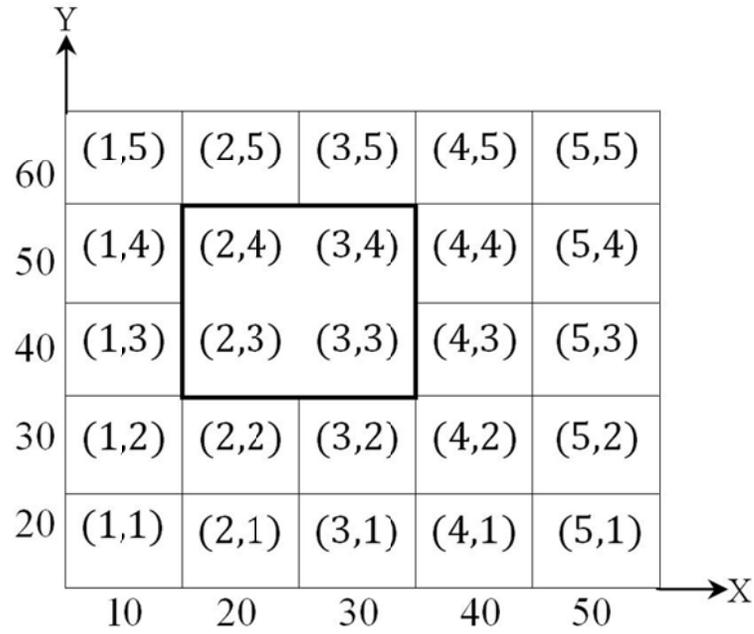


Figure 3.7 The stope that begins at the block identifier (2,3) and ends at the block identifier (3,4).

For each generated stope, the algorithm then formulates the stope attributes as follows. Equations (11)–(15) represent the stope attribute formulations.

$$\bar{L}_s, \bar{W}_s, \bar{H}_s = (L_{r'} \times nx_s), (W_{r'} \times ny_s), (H_{r'} \times nz_s) \quad (11)$$

$$\bar{d}_s = \frac{\sum_{r'} d_{r'}}{(nx_s \times ny_s \times nz_s)} \quad (12)$$

$$\bar{m}_s = \sum_{r'} m_{r'} \quad (13)$$

$$\bar{w}_s = \sum_{r'} w_{r'} \quad (14)$$

$$\bar{v}_s = \sum_{r'} v_{r'} \quad (15)$$

Equation (16) is used to calculate \bar{g}_s for metals, where the grade is defined in grams per tonne.

$$\bar{g}_s = \left(\frac{\bar{m}_s}{\bar{t}_s} \right) \quad (16)$$

Equation (17) is used to calculate \bar{g}_s for metals, where the grade is defined in %.

$$\bar{g}_s = \left(\frac{\bar{m}_s}{\bar{t}_s} \right) \times 100 \quad (17)$$

where $i' \in \alpha_s$, and α_s = the set of mining blocks inside the stope s .

Figure 3.8 explains the steps of the stope generation algorithm through a flow chart. The C# object-oriented source code for this formulation has been included in Subsection 3 in Appendix B.

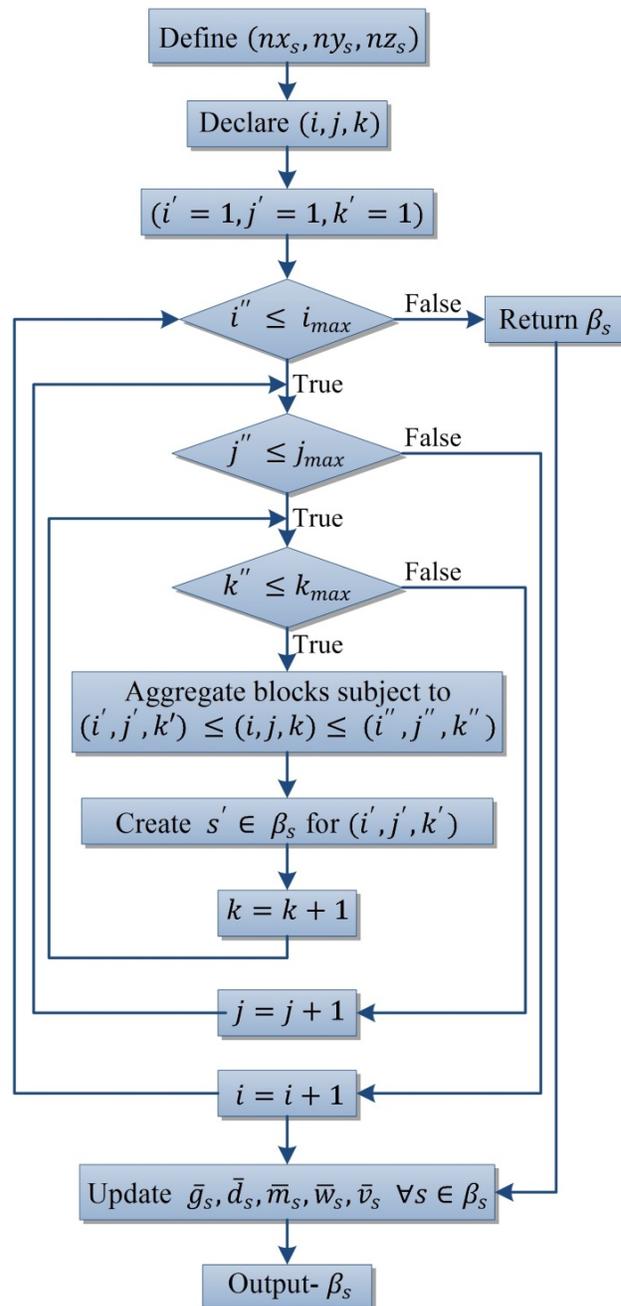


Figure 3.8 Steps of the stope generation algorithm.

3.3.2 EXTRACTING POSITIVE ECONOMIC STOPES

After generating all of the possible stopes β_s , the algorithm identifies and discards negative economic stopes to extract a set of positive stopes γ_s according to the following condition.

$$\begin{aligned} & \text{Select } \gamma_s \text{ from } \beta_s \\ & \text{Such that } \bar{v}_s > 0 \text{ \& } \gamma_s \subset \beta_s \end{aligned}$$

Figure 3.9 illustrates this process using a 2D example with a block model that comprises 25 mining blocks, i.e., five blocks along the x axis and five blocks along the y axis. The figures in the cells, i.e., the mining blocks, represent the economic values of the respective mining blocks. The algorithm defines and floats a $3 \times 3 \times 1$ stope size along the x axis and y axis to generate β_s and extract γ_s for this example. It can be seen that the floating process produces β_s , i.e., all possible nine stopes for this hypothetical situation. For reference, the stopes are indicated by (i) – (ix) in sequential order.

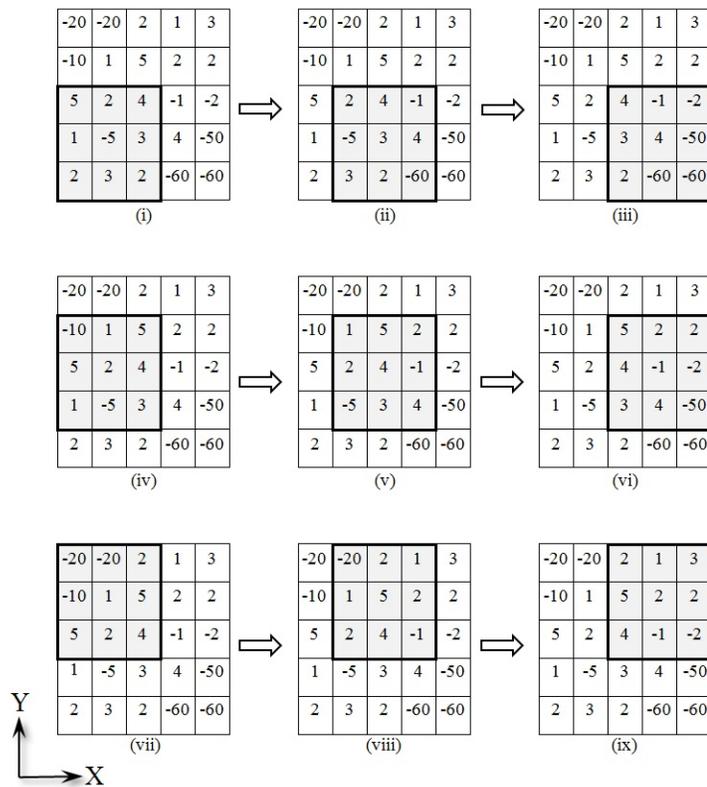


Figure 3.9 Generation of nine possible stopes for a two-dimensional example.

Table 3.1 shows the results obtained after generating β_s and the calculations of \bar{v}_s for the model. According to the calculations of \bar{v}_s , stopes (i) , (v) and (ix) yield positive economic values, i.e., $\bar{v}_s > 0$, thus the algorithm derives $\gamma_s = \{(i), (v), (ix)\}$ from β_s . The C# object-oriented source code for this formulation has been included in Subsection 4 in Appendix B.

Table 3.1 Summary of the stopes generated for the example.

Stope	Total economic value(\$)
(i)	14
(ii)	-53
(iii)	-161
(iv)	-14
(v)	13
(vi)	-33
(vii)	-51
$(viii)$	-4
(ix)	16

3.4 GENERATING THE OPTIMUM SOLUTION

Given a set of positive stopes γ_s , the algorithm generates a family of unique non-overlapping stope sets over γ_s and selects the set with the highest economic value as the optimum solution during the fourth and the fifth stages of the process.

The following notations are used to explain the mathematical formulation of the stope optimisation algorithm.

k = indicator of a set in a family of non-overlapping stope sets

u = indicator of a unique set in a family of non-overlapping stope sets

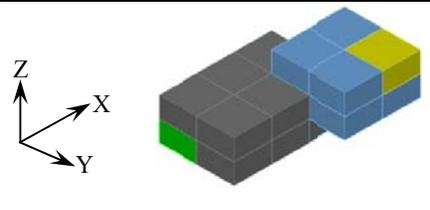
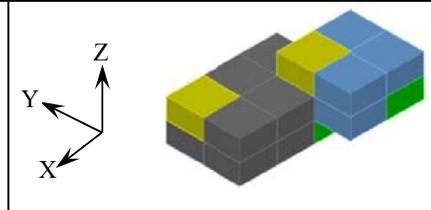
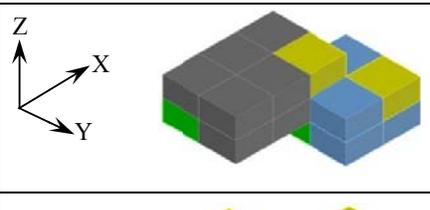
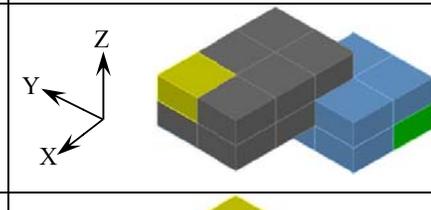
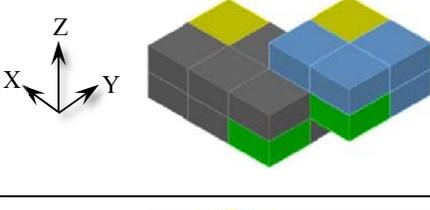
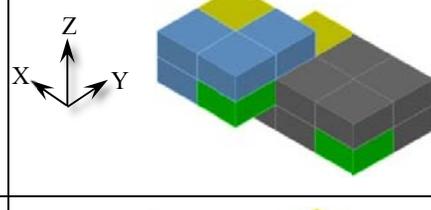
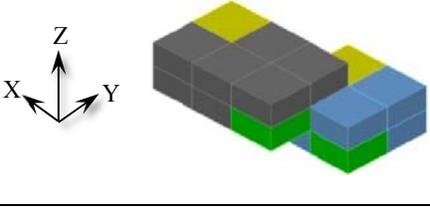
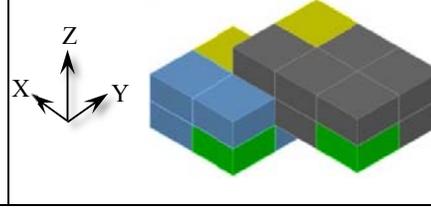
\bar{v}_u = economic value (\$) of the unique non-overlapping stopes set u

The algorithm incorporates two constraints, i.e., the non-overlapping stope constraint and the unique set constraint, to generate a family of unique non-overlapping stope sets over γ_s , as described in subsections 3.4.1 and 3.4.2.

3.4.1 DEFINITION OF THE NON-OVERLAPPING STOPE CONSTRAINT

The stope coordinates, i.e. $sx_s, sy_s, sz_s, ex_s, ey_s, ez_s$ facilitate the identification of overlapping stopes. Given (sx_s, sy_s, sz_s) and (ex_s, ey_s, ez_s) , two stopes may overlap for any of the eight scenarios shown in Table 3.2.

Table 3.2 Overlapping scenarios for two stopes.

No.	Scenario	No.	Scenario
i		v	
ii		vi	
iii		vii	
iv		viii	

The illustrations in the Table 3.2 represent the eight overlapping scenarios for any two stopes $j, j' \in \gamma_s$. Stope j is shown in grey and it comprises 12 mining blocks, i.e., three blocks in the x dimension, two blocks in the y dimension and two blocks in the z dimension. Stope j' is shown in blue and comprises of eight mining blocks, i.e., two blocks along the x axis, two blocks along the y axis and two blocks along the z axis. The green and yellow blocks represent the origin and the last blocks for stopes j and j' respectively.

The mathematical formulations of the eight conditions are as follows.

$$((sx_s \leq sx_{s'} \leq ex_s) \& (sy_s \leq sy_{s'} \leq ey_s) \& (sz_s \leq sz_{s'} \leq ez_s)) \quad (i)$$

$$((sx_s \leq sx_{s'} \leq ex_s) \& (sy_s \leq sy_{s'} \leq ey_s) \& (sz_s \leq ez_{s'} \leq ez_s)) \quad (ii)$$

$$((sx_s \leq sx_{s'} \leq ex_s) \& (sy_s \leq ey_{s'} \leq ey_s) \& (sz_s \leq ez_{s'} \leq ez_s)) \quad (iii)$$

$$((sx_s \leq sx_{s'} \leq ex_s) \& (sy_s \leq ey_{s'} \leq ey_s) \& (sz_s \leq sz_{s'} \leq ez_s)) \quad (iv)$$

$$((sx_s \leq ex_{s'} \leq ex_s) \& (sy_s \leq ey_{s'} \leq ey_s) \& (sz_s \leq sz_{s'} \leq ez_s)) \quad (v)$$

$$((sx_s \leq ex_{s'} \leq ex_s) \& (sy_s \leq ey_{s'} \leq ey_s) \& (sz_s \leq ez_{s'} \leq ez_s)) \quad (vi)$$

$$((sx_s \leq ex_{s'} \leq ex_s) \& (sy_s \leq sy_{s'} \leq ey_s) \& (sz_s \leq ez_{s'} \leq ez_s)) \quad (vii)$$

$$((sx_s \leq ex_{s'} \leq ex_s) \& (sy_s \leq sy_{s'} \leq ey_s) \& (sz_s \leq sz_{s'} \leq ez_s)) \quad (vii)$$

where $j, j' \in \gamma_j$

3.4.2 DEFINITION OF THE UNIQUE SET CONSTRAINT

The algorithm generates a set of sets or a family of sets F_k , which may comprise sets and their supersets, or duplicated sets (repeated elements). Thus, F_k have the characteristics of a multi set. The subsets, proper subsets or duplicated elements are removed from F_k to generate a family of unique non-overlapping stope sets F_u . This process improves the performance of the algorithm.

For example, $A_{k'}$, $A_{k''}$, $A_{k'''}$ and $B_{k'}$ are four sets of non-overlapping stopes a , b , c , d and e , respectively, where $A_{k'} = \{a, b, d\}$, $A_{k''} = \{a, b\}$, $A_{k'''} = \{a, c, e\}$, $B_{k'} = \{a, b, d\}$ and $A_{k'}$, $A_{k''}$, $A_{k'''}$, $B_{k'} \in F_k$. Since, $A_{k''} \subset A_{k'}$ and $A_{k''} \subset A_{k'''}$, then $A_{k''}$ should be discarded. In addition, $A_{k'}$ and $B_{k'}$ are duplicated elements, i.e., $A_{k'} \subset B_{k'}$ and $B_{k'} \subset A_{k'}$. Therefore, either $A_{k'}$ or $B_{k'}$ should be discarded. Finally, the unique set $A_{k'''}$ is combined with either $A_{k'}$ or $B_{k'}$ to create a family of unique non-overlapping stope sets F_u , i.e., $F_u = \{A_{k'''}, A_{k'}\}$ or $F_u = \{A_{k'''}, B_{k'}\}$.

3.4.3 STEPS OF THE ALGORITHM

After defining the non-overlapping stope and unique set constraints, the algorithm generates F_u over set γ_s . Among the members of F_u , the algorithm selects the non-overlapping stope set with the maximum \bar{v}_u as the optimum solution for the given ore body model. Figure 3.10 illustrates the steps of the stope optimisation algorithm using a flow chart.

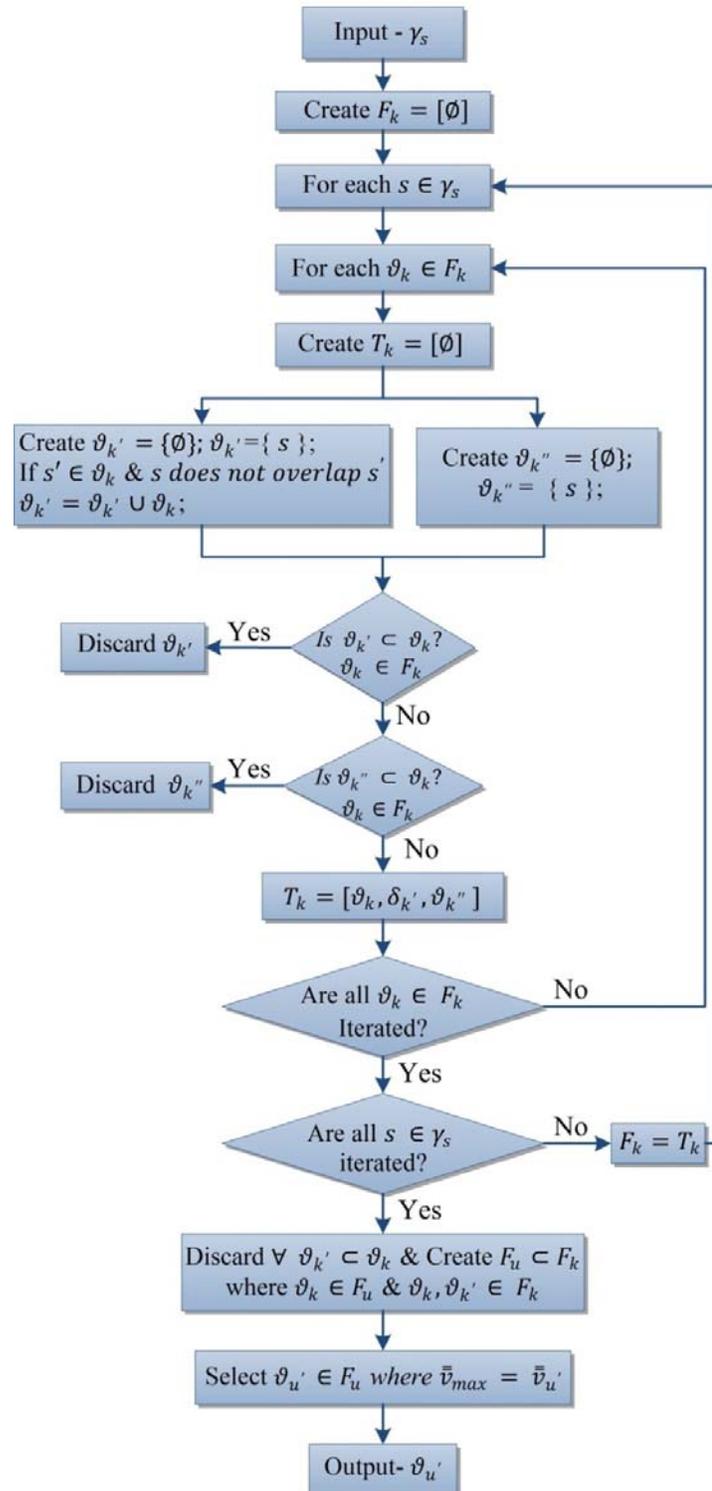


Figure 3.10 Algorithmic flow required to generate the optimum solution.

The steps of the algorithm illustrated in Figure 3.10 are as follows.

- i. Read the stope economic values, i.e., \bar{v}_s , of γ_s .
- ii. Create a null family of sets F_k and iterate through steps (iii)–(ix) $\forall s \in \gamma_s$.
- iii. Iterate through steps (iv)–(ix) $\forall \vartheta_k \in F_k$.
- iv. Create a null family of sets of stopes T_k .
- v. Create a set of stopes $\vartheta_{k'}$ and add stope s to $\vartheta_{k'}$. If any $s' \in \vartheta_k$ does not overlap with s , then combine sets $\vartheta_{k'}$ and ϑ_k .
- vi. Create a set of stopes $\vartheta_{k''}$ and add stope s to $\vartheta_{k''}$.
- vii. Discard $\vartheta_{k'}$ or $\vartheta_{k''}$ if they are subsets of any set $\vartheta_k \in F_k$.
- viii. Add $\vartheta_k, \vartheta_{k'}, \vartheta_{k''}$ to T_k .
- ix. Assign T_k to F_k and iterate through steps (iv)–(viii) $\forall \vartheta_k \in F_k$.
- x. Remove all sets $\vartheta_k \in F_k$ that are subsets of other sets in F_k to generate F_u .
- xi. Determine the maximum economic value \bar{v}_{max} among \bar{v}_u and select $\vartheta_{u'} \in F_u$ such that $\bar{v}_{max} = \bar{v}_{u'}$ is the optimum solution.

3.5 HANDLING MINING SCENARIOS

This section discusses the further developments of the stope optimisation algorithm to handle the following scenarios.

1. Application of variable stope sizes.
2. Provision of mining levels.
3. Provision of pillars.
4. Stope shaper post-optimum improvements.

3.5.1 APPLICATION OF VARIABLE STOPE SIZES

The stope generation algorithm is modified to handle variable stope sizes, i.e., generating more than one stope size during the stope generation process. The algorithm achieves this by defining a minimum stope size (ox_s, oy_s, oz_s) and an increment (p_x, p_y, p_z) that facilitates stope size variations. Then it determines the maximum number of x , y and z blocks that a stope may contain, i.e., the maximum stope size. The modified algorithm defines stope sizes between the minimum and the maximum stope sizes, thereby obtaining a range of stope sizes. The following notations are defined to mathematically formulate the stope size variation algorithm:

ox_s, oy_s, oz_s = minimum number of mining blocks inside stope s along x , y , and z axes,

p_x, p_y, p_z = maximum incremental number of mining blocks along the x , y , and z axes for stope generation.

Given ox_s, oy_s, oz_s and p_x, p_y, p_z , the variable stope size nx_s, ny_s, nz_s can be defined as follows, where $n_x, n_y, n_z \in \text{set of positive integers}$.

$$(nx_s, ny_s, nz_s) = (ox_s + n_x, oy_s + n_y, oz_s + n_z) \quad (18)$$

where

$$n_x \leq p_x, n_y \leq p_y, n_z \leq p_z$$

&

$$ox_s, oy_s, oz_s \leq (nx_s, ny_s, nz_s) \leq (ox_s + p_x, oy_s + p_y, oz_s + p_z)$$

The flow chart in Figure 3.11 shows the steps employed by the stope size variation algorithm.

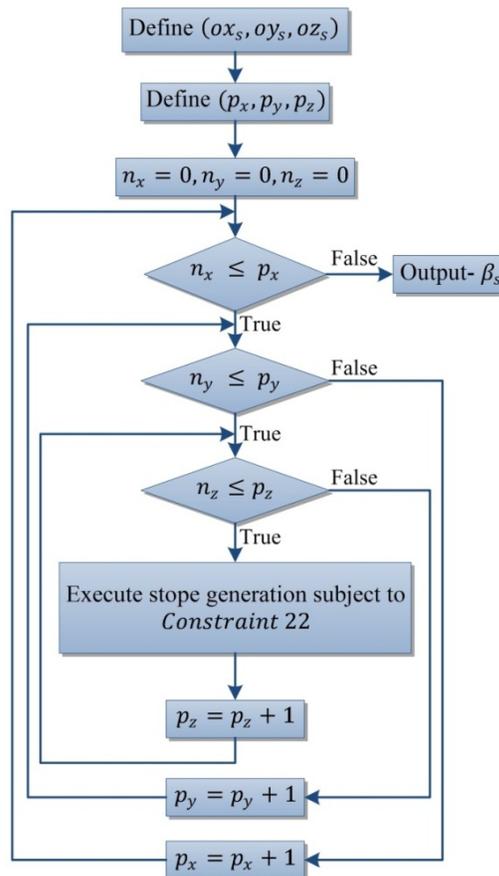


Figure 3.11 Steps of the stope size variation algorithm.

The following three examples illustrate the stope size variation algorithm.

Example 1

Given $ox_s = 3, oy_s = 3, oz_s = 3$ and $p_x = 1, p_y = 0, p_z = 0$, build the variable stope sizes. Table 3.3 shows the mathematical formulation of (nx_s, ny_s, nz_s) and the resulting stope sizes.

Table 3.3 Stope size creation for Example 1.

(nx_s, ny_s, nz_s) formulation	Stope size
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 0), (3 + 0), (3 + 0)$	$(3 \times 3 \times 3)$
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 1), (3 + 0), (3 + 0)$	$(4 \times 3 \times 3)$

Example 2

Given $ox_s = 3, oy_s = 3, oz_s = 3$ and $p_x = 1, p_y = 1, p_z = 0$, build the possible stope sizes. Table 3.4 shows the mathematical formulation of (nx_s, ny_s, nz_s) and the resulting stope sizes.

Table 3.4 Stope size creation for Example 2.

(nx_s, ny_s, nz_s) formulation	Stope size
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 0), (3 + 0), (3 + 0)$	$(3 \times 3 \times 3)$
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 1), (3 + 0), (3 + 0)$	$(4 \times 3 \times 3)$
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 0), (3 + 1), (3 + 0)$	$(3 \times 4 \times 3)$
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(3 + 1), (3 + 1), (3 + 0)$	$(4 \times 4 \times 3)$

Example 3

Given $ox_s = 2, oy_s = 3, oz_s = 2$ and $p_x = 0, p_y = 0, p_z = 1$, build the possible stope sizes. Table 3.5 shows the mathematical formulation of (nx_s, ny_s, nz_s) and the resulting stope sizes.

Table 3.5 Stope size creation for Example 3.

(nx_s, ny_s, nz_s) formulation	Stope size
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(2 + 0), (3 + 0), (2 + 0)$	$(2 \times 3 \times 2)$
$(ox_s + n_x), (oy_s + n_y), (oz_s + n_z)$ $(2 + 0), (3 + 0), (2 + 1)$	$(2 \times 3 \times 3)$

Figure 3.12 illustrates the stope generation process for Example 3 using a block model that comprises of 27 mining blocks, i.e., three blocks along x axis, three blocks along y axis and three blocks in z axis. The shaded blocks represent the blocks that belong to the stope size. In this hypothetical situation, six possible stopes (four stopes of $2 \times 3 \times 2$ and two stopes of $2 \times 3 \times 3$) are generated from floating of the two stope sizes within the block model.

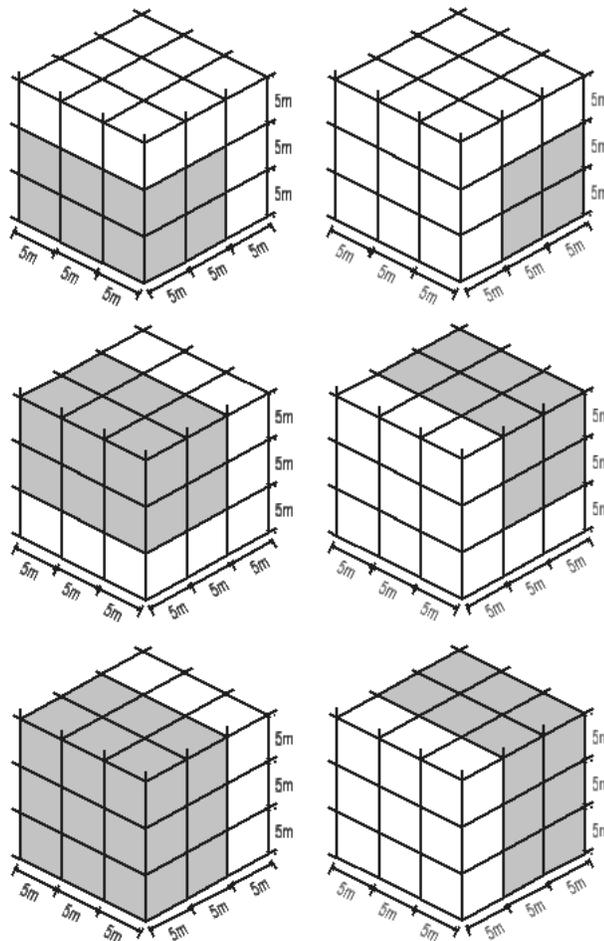


Figure 3.12 Generating variable stope sizes.

3.5.2 PROVISION OF MINING LEVELS

The stope generation algorithm is modified to allow mining level definition during the stope generation process. The modified algorithm achieves this by setting stope height = level height, thereby creating stopes with stope height intervals rather than block height intervals. Consequently, in the algorithm, $k = k + 1$ becomes $k = k + nz_s$. Figure 3.13 illustrates the provision of levels using an example block model that comprises of 125 mining blocks. Given a stope size of $5 \times 5 \times 2$, Figure 3.13 (i)–(iii) shows the typical situation where stope generation is based on block height intervals. In this situation, the algorithm generates three possible stopes based on the block height, (H_r). For the same stope size, Figure 3.13 (iv) and (v) illustrates the stope generation process when the mining level height = stope height, i.e., two mining blocks along the z axis. In this scenario, the modified algorithm generates two possible stopes based on the level height (stope height or nz_s).

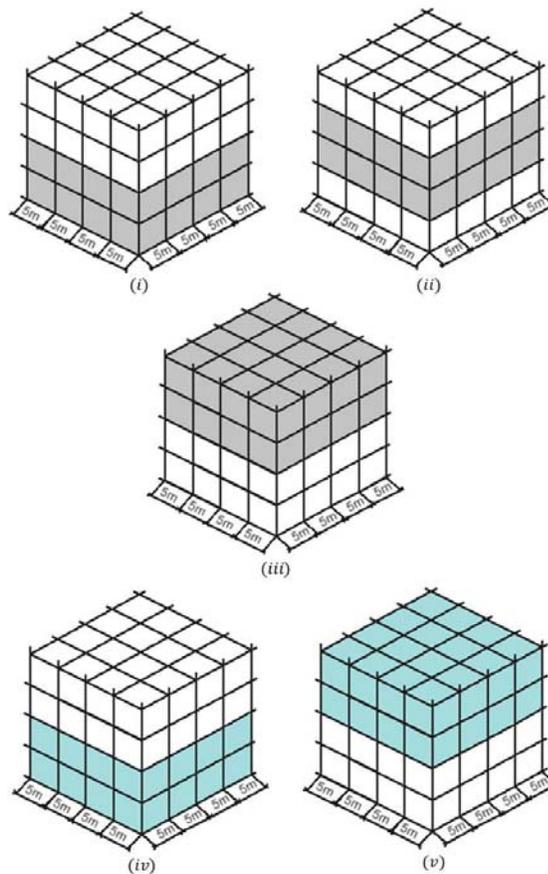


Figure 3.13 Generating stopes on a level by level basis compared with a block by block basis along the z axis.

The flow chart in Figure 3.14 shows the steps of the stope generation algorithm that incorporates level definitions.

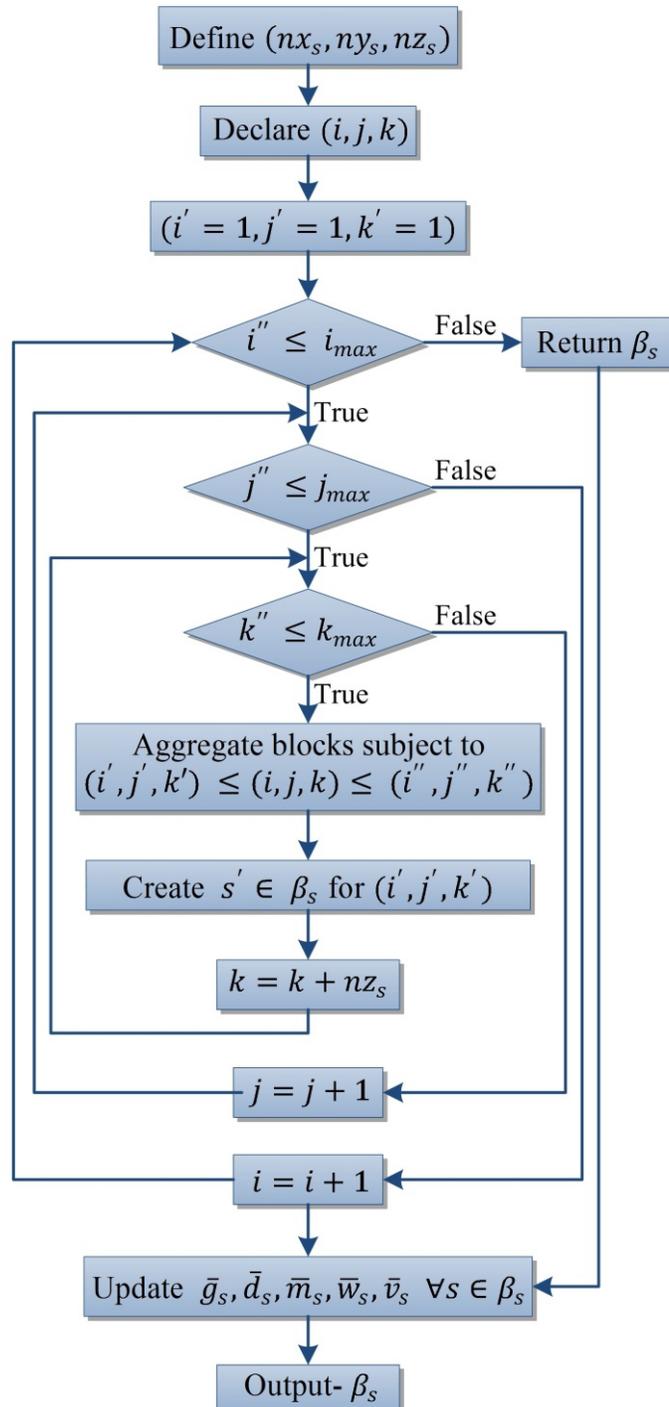


Figure 3.14 Stope generating algorithm with incorporation of levels.

3.5.3 PROVISION OF PILLARS

A modification to the structure of stope optimisation algorithm allows the allocation of space for the provision of pillars, thus it facilitates the stability improvements in pillars required for underground mining development.

The algorithm defines a condition that allows the provision of pillars during the stope optimisation process. The following notations are used to explain this condition.

i = Indicator for the level of the stopes

j = Pillar width (m)

For a given stope level i , Figure 3.15 indicates the notations used for the mathematical formulation of the condition.

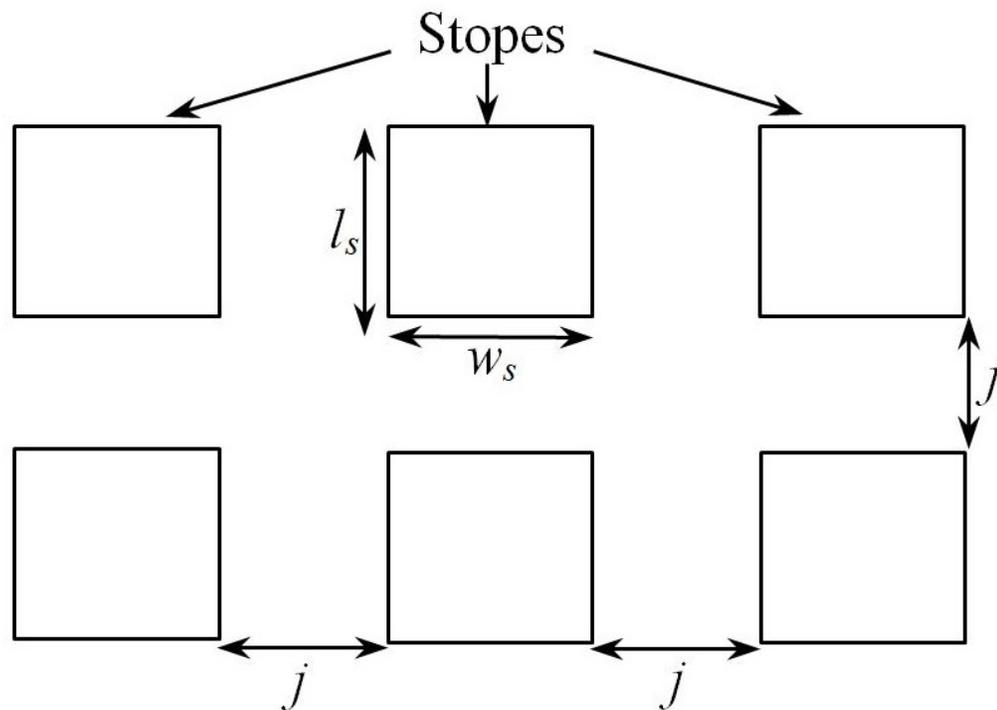


Figure 3.15 Notations for pillar separation.

Given a stope level i , the condition that facilitates identification of stopes that are located within a pillar distance of j is as follows.

$$[|(sx_s - sx_{s'})| \geq j \cup |(sy_s - sy_{s'})|] \geq j$$

$$\text{where } s, s' \in \theta_{si}: \theta_{si} \subset \delta_s$$

$$\theta_{si} = \text{set of stopes in level } i$$

Consequently, the modified stope optimisation algorithm incorporates the condition for provision of pillars, generates a family of sets F_u over set γ_s and selects the set with \bar{v}_{max} as the optimum solution.

3.5.4 STOPE SHAPER: POST-OPTIMUM IMPROVEMENTS TO THE SOLUTION

The stope generator creates regular shapes (cubic or cuboid) of stopes. Thus, the optimum stope layout comprises regular stopes, which might not follow the irregular boundaries of ore body models. Therefore, waste blocks are present inside the stopes located along the boundary of the ore body model. These waste blocks on the boundary stopes decrease the value of the stope layout. Thus, an algorithm is proposed to remove the waste blocks from the boundary stopes to overcome this issue.

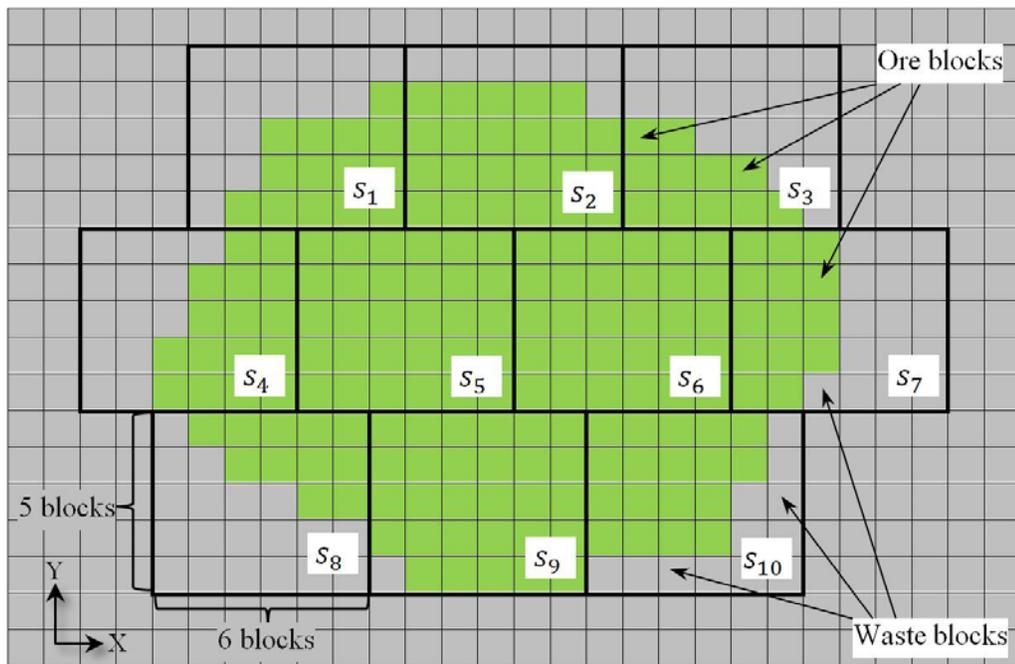


Figure 3.16 Optimum stope layout for a given level on the x,y plane.

Figure 3.16 illustrates the stope shaper algorithm using an example set of 10 stopes, for a given mining level. The ten stopes are comprised of 6 blocks along the x axis and 5 blocks along the y axis, which are labelled sequentially from s_1 to s_{10} . In this 2D example, the green blocks represent ore blocks whereas the grey blocks represent

the waste blocks. As shown in Figure 3.16, the enclosed stopes, s_5 and s_6 , contain ore blocks only, whereas, the boundary stopes, $s_1, s_2, s_3, s_4, s_7, s_8, s_9$ and s_{10} , comprise both ore blocks and waste blocks.

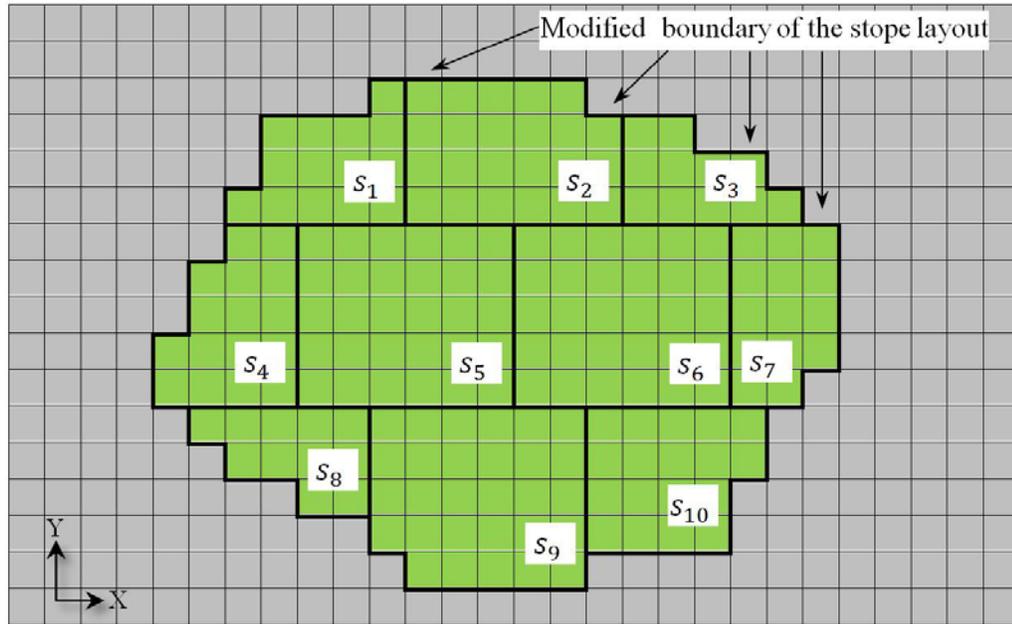


Figure 3.17 Layout of the modified stope boundary for the example.

For the example illustrated in Figure 3.16, the stope shaper algorithm removes waste blocks from the boundary stopes and alters the original boundary which is shown in Figure 3.17. Thus, the modified stope boundary becomes the ore body boundary. The following steps explain the mathematical formulations used by the stope shaper algorithm for a given optimum solution ϑ_u .

1. Derive a set of enclosed stopes τ_{si} from the set of stopes $\theta_{si} \subset \vartheta_u$ at a given level i .

A given stope s at a stope level i is an enclosed stope, if there is a minimum of four surrounding stopes, s_1, s_2, s_3 , and s_4 , and the following condition is true.

$$[(sx_s - sx_{s_1}) \geq 0 \cap (sx_{s_2} - sx_s) \geq 0 \cap (sy_s - sy_{s_3}) \geq 0 \cap (sy_{s_4} - sy_s)] \geq 0$$

$$\text{where } s_1, s_2, s_3, s_4 \in \theta_{si}; \theta_{si} \subset \vartheta_u$$

$$\theta_{si} = \text{set of stopes in level } i$$

2. Subtract τ_{si} from θ_{si} to identify a set of boundary stopes v_{si} , where $\tau_{si} \cup v_{si} = \theta_{si}$ and $\tau_{si} \cap v_{si} = \{\emptyset\}$.

3. Identify and extract the set of ore blocks q_{ri} where $v_{ri} > 0$ from the set of blocks σ_{ri} within the boundary stopes.
4. Combine τ_{si} and q_{ri} to regenerate the stope layout boundary for all the levels of ϑ_u .

3.6 PERFORMANCE IMPROVEMENTS AND LIMITATIONS

This section describes modifications that improve the performance of the stope generation and optimisation algorithms.

3.6.1 STOPE GENERATION PERFORMANCE

Using an Intel dual core processor running at 3.00 GHz and with 4.00 GB RAM, the algorithm generated 576 stopes for a synthetic model that comprised 1200 blocks within 5 s. However, it required 8 h and 10 min to generate 255,120 stopes for a realistic ore body model that comprised of 287,984 blocks. Thus, the structure of the algorithm was investigated using the EQATEC (EQATEC .NET version 1.1, 2014) code profiler to identify time consuming methods in the algorithm. Figure 3.18 shows a screenshot of EQATEC analysis for the stope generation algorithm.

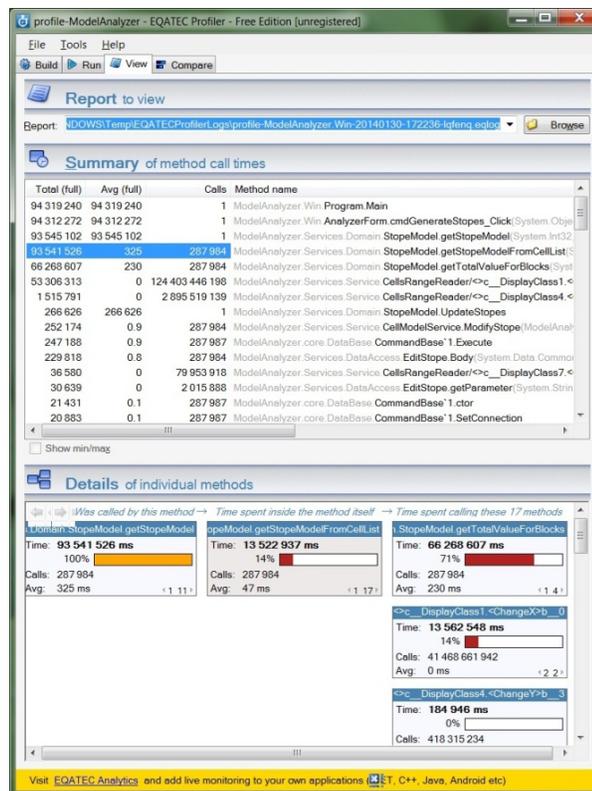


Figure 3.18 EQATEC profile showing method details of the original stope generation algorithm.

The methods indicated in red are time-consuming operations. After analysing the execution flow or the interaction between the methods in the algorithm, a new execution sequence was designed and implemented to reduce the time required to obtain a solution. The execution sequence of the original algorithm is as follows.

1. Regularise the block model and define the stope dimensions, which are the inputs for stope generation.
2. Each block may become a candidate as the origin block of a stope.
3. Each block is given an integer identifier to facilitate last block selection.
4. A last block is identified by adding the stope dimensions to the unique identifier.
5. The coordinates of an origin block become the starting coordinates of a stope.
6. The coordinates of the last block become the end coordinates of a stope.
7. Blocks within the starting coordinates and the end coordinates are selected to determine the stope attributes.
8. The stope attributes are determined using the mathematical formulations defined in Subsection 3.3.1.

According to the analysis, the method employed to identify a candidate block to generate stopes was the most time-consuming operation. For this method, the number of calls was also higher compared with the other methods in the program. Therefore, this method was restructured and the execution sequence was modified as follows.

1. Regularise the block model and define the stope dimensions, which are used as the inputs for stope generation.
2. Identify the lowest coordinates of the block model as the starting coordinates for the stope model.
3. Create a 3D object with defined stope dimensions.
4. Float the 3D object in the block model. The blocks within the 3D object are aggregated to determine the stope attributes.
5. The start coordinates and the end coordinates of a stope are determined by selecting the lowest coordinates and the highest coordinates of the selected blocks.
6. The stopes are generated and the stope attributes are determined using the mathematical formulations defined in Subsection 3.3.1.

The number of operations in the new execution sequence was lower than that in the original sequence. The method used to identify candidate blocks was also omitted in the new sequence. These two changes reduce the time from 8 hours and 10 minutes to just 4 minutes and 30 seconds, thus the performance of the algorithm improved significantly. Figure 3.19 shows a screenshot of the analysis for the modified stoppe generation algorithm.

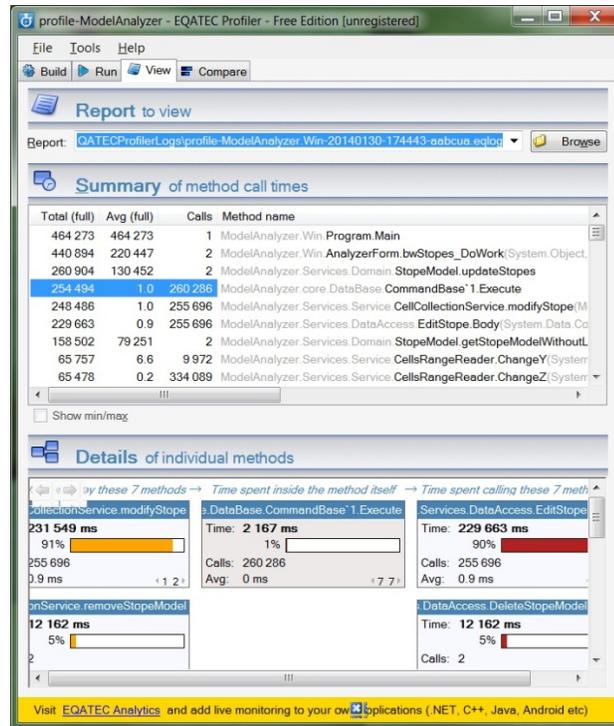


Figure 3.19 EQATEC profile showing the method calls and details for the modified stoppe generation algorithm.

3.6.2 STOPE OPTIMISATION PERFORMANCE

It was observed that the solution time increased exponentially with increases in C_{T_k} . This is demonstrated in the cases in Chapter 4. In addition, the solution time is proportional to the number of positive stopes in the input used to generate sets of non-overlapping combinations.

Therefore, step (iii) of the algorithm was modified further to improve the performance. The algorithm achieved improved performance after the implementation of the task parallelism concept from object-oriented programming (James, 2013; "C# Threads," 2013; "Parallel Class," 2013). First, the algorithm

divides set T_k , into two unique subsets $\mu_{1k}, \mu_{2k} \subset T_k$, such that $\mu_{1k} \cap \mu_{2k} = \{\emptyset\}$ and $\mu_{1k} \cup \mu_{2k} = T_k$.

Next, it performs a parallel implementation of step (iii) on each defined subset using the multiple threads available for computational processing. The multiple threads execute step (iii) asynchronously, thus the algorithm achieves a performance gain. The cases in Chapter 4 compare the performance of the original algorithm with that of the modified algorithm. Figure 3.20 shows the incorporation of parallel multi-threads within the structure of stope optimisation algorithm in step (iii).

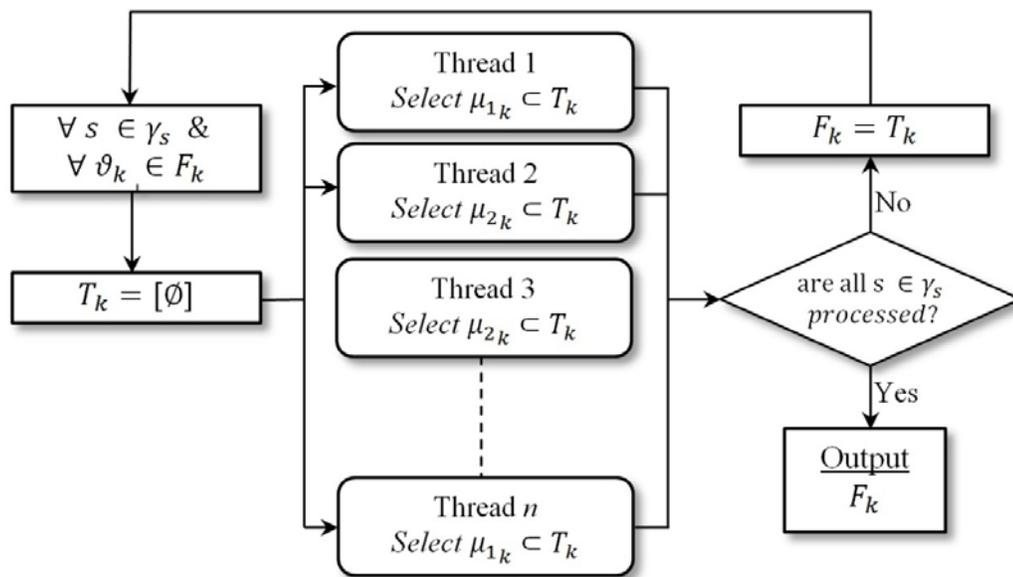


Figure 3.20 Incorporation of parallel threading into step (iii) of the algorithm.

3.6.3 LIMITATION OF THE OPTIMUM SOLUTIONS GENERATED

Given the positive-valued stopes from step (i), the possible combinations that can generate sets of non-overlapping stopes in step (viii) are infinite, thereby increasing the computational complexity of the problem. Thus, the algorithm was modified to address this issue, where the modified algorithm implements step (viii) over a number of iterations thereby incorporating an improvement characteristic to the proposed heuristic algorithm. During each iteration, the algorithm orders T_k according to the descending values of \bar{v}_k . Next, it limits the sets of non-overlapping stopes T_k to a predefined count C_{T_k} , i.e., it selects the first C_{T_k} number of T_k to create subset T'_k of T_k . Finally, it implements steps (viii) to (xi) and records the highest

value solution \bar{v}_{max} . For each subsequent iteration, it increments C_{T_k} to a new value, repeats steps (viii) to (ix), performs a comparison of the new and old solutions and continues until the convergence of the solution. In Figure 3.21, the modified steps of the stope optimisation algorithm are highlighted in green.

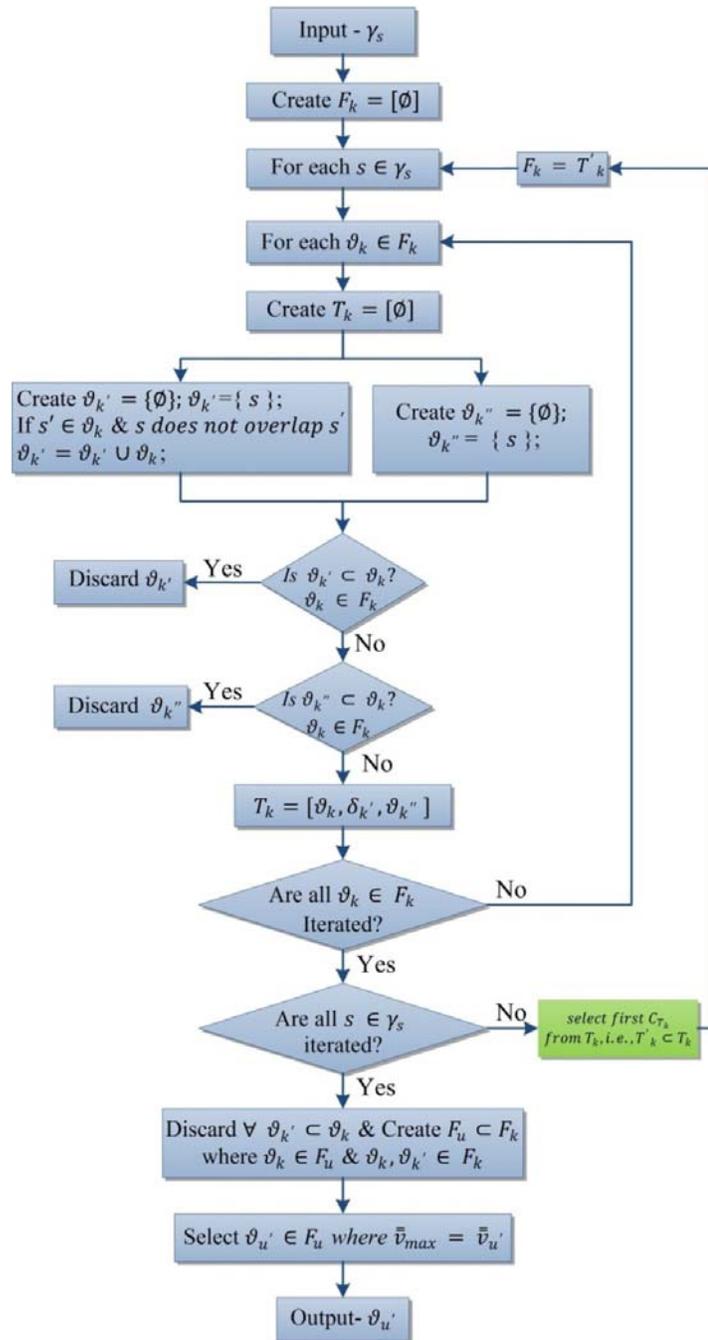


Figure 3.21 Steps of the modified stope optimisation algorithm.

CHAPTER 4 IMPLEMENTATION OF THE PROPOSED ALGORITHM

The algorithm was implemented for a real block model test case representing a copper deposit. Table 4.1 summarises details of this ore body model.

Table 4.1 Details of the original block model.

No. of blocks	47,052
Block sizes	Blocks of many sizes, with the largest being $20 \times 20 \times 20$ m
d_o variation	From 2.62 tonnes/m ³ to 4.66 tonnes/m ³
g_o variation	From 0 to 15.3197 %
Spatial extent (x,y,z)	From 5610, 4600, -100 to 6610, 5600, 245

Table 4.1 shows the grade distribution of the copper deposit.

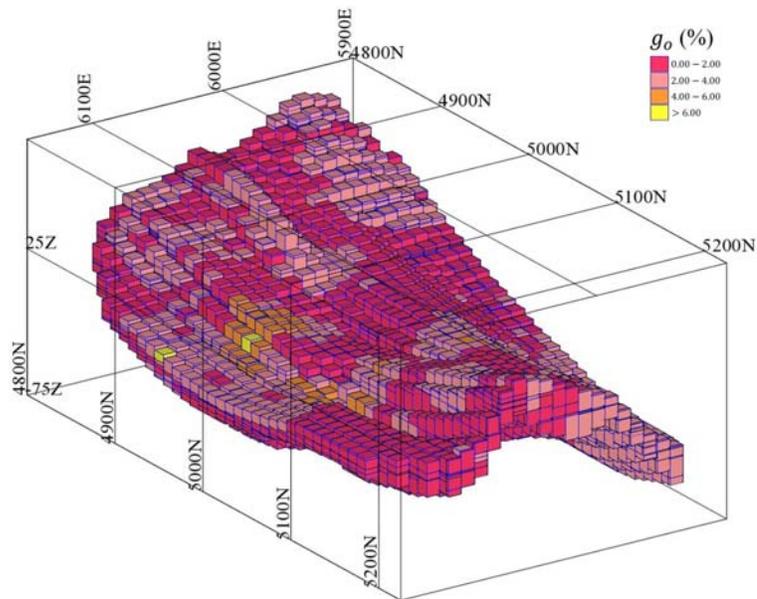


Figure 4.1 Grade distribution of the copper deposit.

4.1 BLOCK REGULARISATION FOR CASES A TO F

The algorithm used this model as its input, regularised the block model, and converted the regularised model into an economic model to facilitate implementation of the stope optimisation algorithm for Cases A-F defined in Section 4.3.

The block model was regularised to a fixed size of $10 \text{ m} \times 10 \text{ m} \times 10 \text{ m}$. Table 4.2 presents the details of the regularised block model.

Table 4.2 Details of the regularised block model.

No. of blocks	287,984
Block size	10 × 10 × 10 m fixed size
d_r variation	2.62 tonnes/m ³ to 4.51 tonnes/m ³
g_r variation	0 to 8.91%
Spatial extent (x,y,z)	5610, 4600, -100 to 6610, 5600, 245

4.2 CONVERTING THE GEOLOGICAL MODEL INTO AN ECONOMIC MODEL FOR CASES A TO F

The economic values of the new blocks were calculated using the mathematical formulae described in Section 3.2 in Chapter 3. For block economic calculations, the copper price and mining cost were considered to be \$ 8,000/tonne, and \$ 30/tonne respectively. Consequently, the calculated economic values of the blocks varied from \$ -95,405 to \$ 2,514,245. Figure 4.2 shows the ore blocks, i.e., blocks where $v_r > 0$, for the economic ore body model.

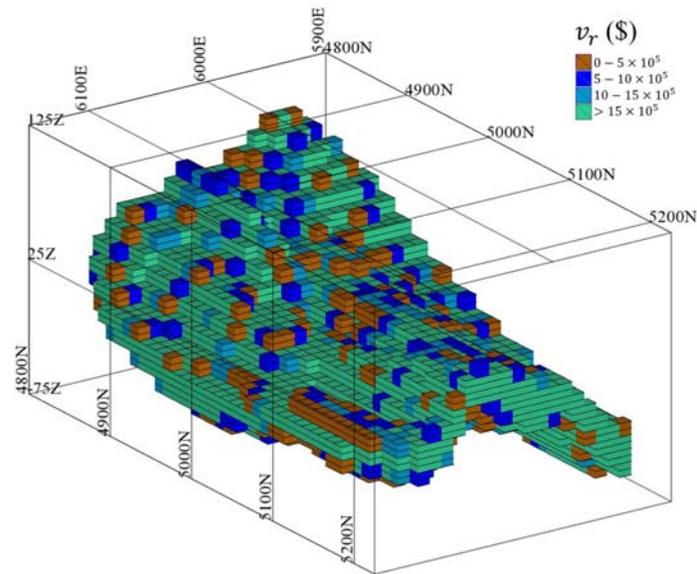


Figure 4.2 The regularised economic block model.

4.3 CASES BASED ON DIFFERENT MINING SCENARIOS

After converting the regularised geological model into an economic model, the following six cases were used to demonstrate the performance of the algorithm in different mining situations.

Case A: Fixed stope size limit of 27 ($nx_s = 3, ny_s = 3, nz_s = 3$) mining blocks, i.e., a stope size of $3 \times 3 \times 3$ but without the provision of pillars and levels.

Case B: Fixed stope size limit of 27 ($nx_s = 3, ny_s = 3, nz_s = 3$) mining blocks, i.e., a stope size of $3 \times 3 \times 3$ with the provision of levels but without the provision of pillars.

Case C: Fixed stope size limit of 27 ($nx_s = 3, ny_s = 3, nz_s = 3$) mining blocks, i.e., a stope size of $3 \times 3 \times 3$ with the provision of levels and pillars (10 m width).

Case D: Variable stope sizes of 27 ($nx = 3, ny = 3, nz = 3$) and 36 ($nx = 3, ny = 3, nz = 4$) mining blocks, i.e., stope sizes of $3 \times 3 \times 3$ and $3 \times 3 \times 4$, but without the provision of pillars and levels.

Case E: Variable stope sizes of 27 ($nx = 3, ny = 3, nz = 3$) and 36 ($nx = 4, ny = 3, nz = 3$) mining blocks, i.e., stope sizes of $3 \times 3 \times 3$ and $4 \times 3 \times 3$, with the provision of levels but without the provision of pillars.

Case F: Variable stope sizes of 27 ($nx = 3, ny = 3, nz = 3$) and 36 ($nx = 4, ny = 3, nz = 3$) mining blocks, i.e., stope sizes of $3 \times 3 \times 3$ and $4 \times 3 \times 3$ with the provision of levels and pillars (10 m width).

For each case, stopes were generated based on the defined stope sizes. Table 4.3 summaries the attribute values of the stope generation process.

Table 4.3 Summary of the stopes generated for the cases.

Case	$ \beta_s $	$ \gamma_s $	\bar{g}_s (%)		\bar{d}_s (tonne/m ³)		Solution time (h: min: s)
			Min	Max	Min	Max	
A	255,120	4,547	0.00	4.199	2.62	3.86	0: 4: 30
B & C	88,232	1,530	0.00	4.066	2.62	3.83	0: 1: 40
D	500,636	9,496	0.00	4.199	2.62	3.86	0: 8: 10
E & F	175,395	3,090	0.00	4.077	2.62	3.83	0: 3: 00

After stope generation, the algorithm used γ_s as the input and generated optimum solutions for the six cases. The set count, i.e., C_{T_k} , was incremented from 0 to 10,000

and the solutions were recorded. The solution values were plotted against C_{T_k} to identify the convergence of the solution values. The outcomes of the stope optimisation process for each case are summarised in Subsection 4.3.1 as follows.

4.3.1 SUMMARY OF OUTCOMES FOR CASES A TO F

Table 4.4 presents the parameter values for the optimal solutions to Cases A-F.

Table 4.4 Values of the parameters for the optimal solution to Cases A-F.

Case	Parameter				
	\bar{v}_{max} (\$)	Average \bar{v}_s (\$)	Average \bar{g}_s (%)	No of stopes	Solution time (h: min: s)
A	779,872,119	4,456,412	1.0120	175	7: 00: 00
B	751,953,177	3,978,588	0.9468	189	0: 26: 14
C	436,341,939	5,073,743	1.1010	86	0 : 0 : 45
D	790,524,917	4,849,846	1.0134	163	14 : 30 : 05
E	734,563,937	4,619,898	0.9122	159	2: 21:05
F	382,129,243	5,619,547	1.0146	68	2: 05: 12

Figure 4.3 shows the increase and convergence in the solution values for Cases A-F.

Figure 4.4 shows the visualised optimal stope layouts for Cases A-F.

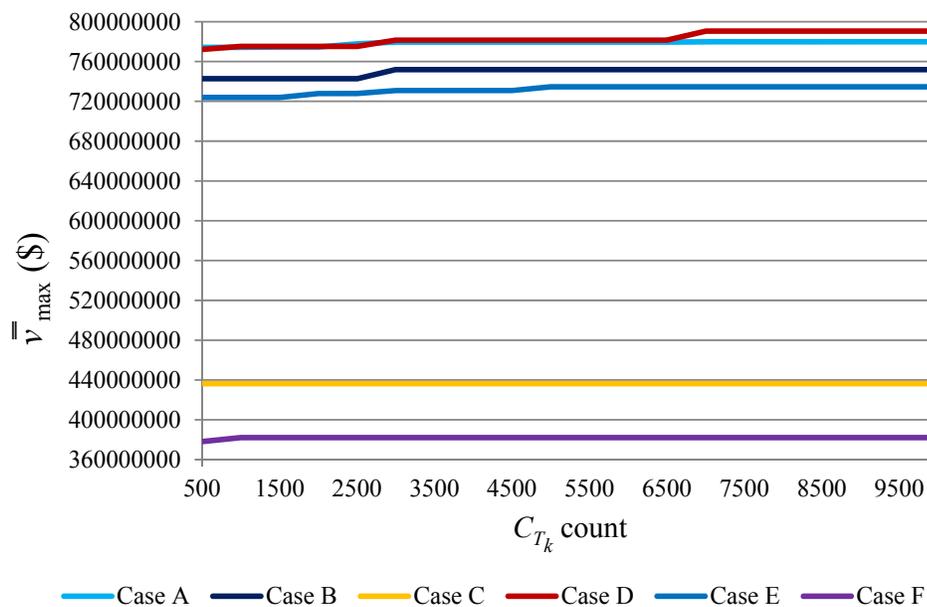


Figure 4.3 Convergence of the solution value for Cases A-F

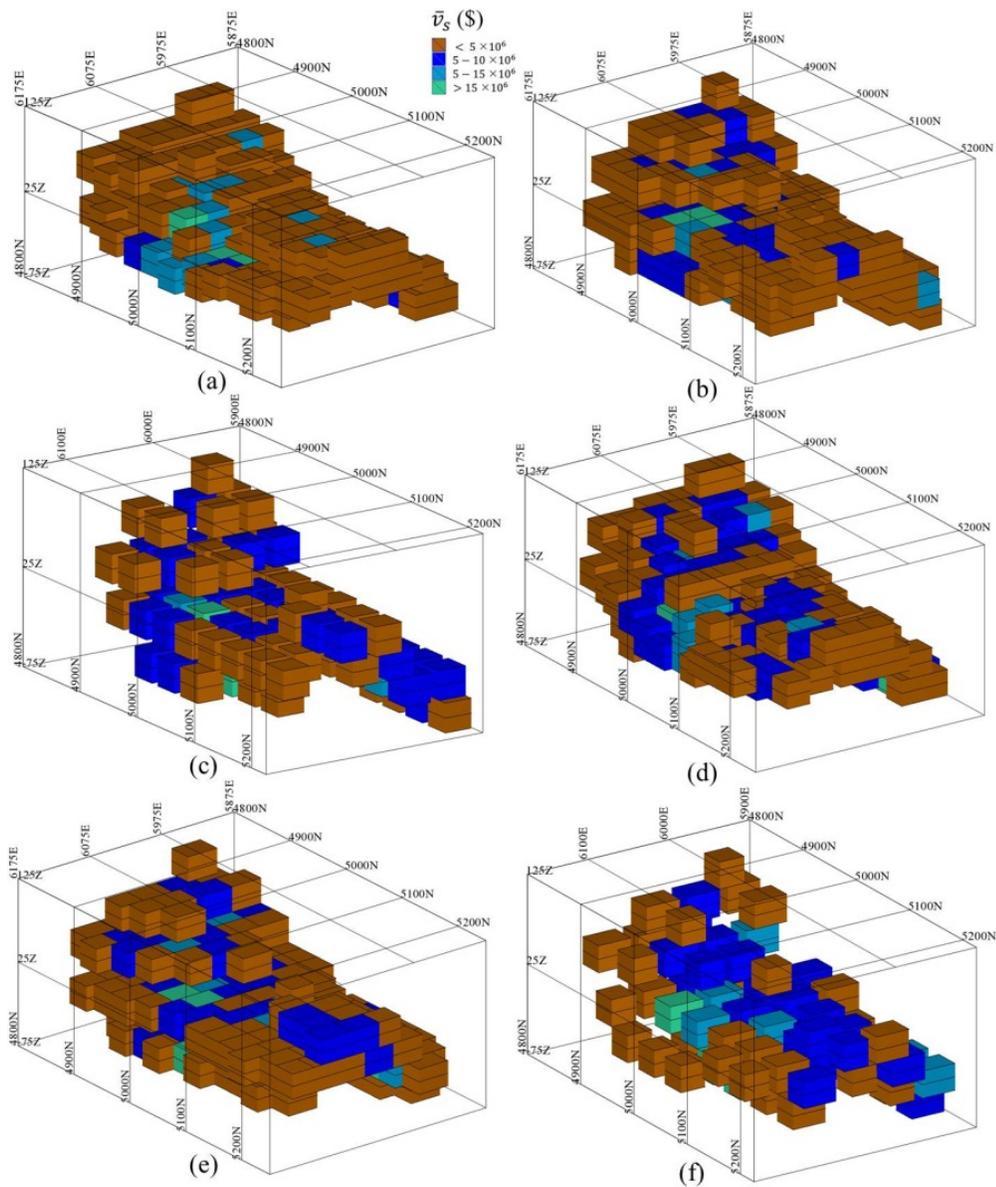


Figure 4.4 Three-dimensional view of the optimal stope layouts in Cases A-F.

4.4 APPLICATION OF PARALLEL MULTI-THREADING

This section presents an implementation of the multi-threading-based stope optimisation algorithm described in Subsection 3.6.2 in Chapter 3 for Cases A-F. Figure 4.5 shows the exponential relationship between the count C_{T_k} and the solution time when the original stope optimisation algorithm was implemented for Cases A-F.

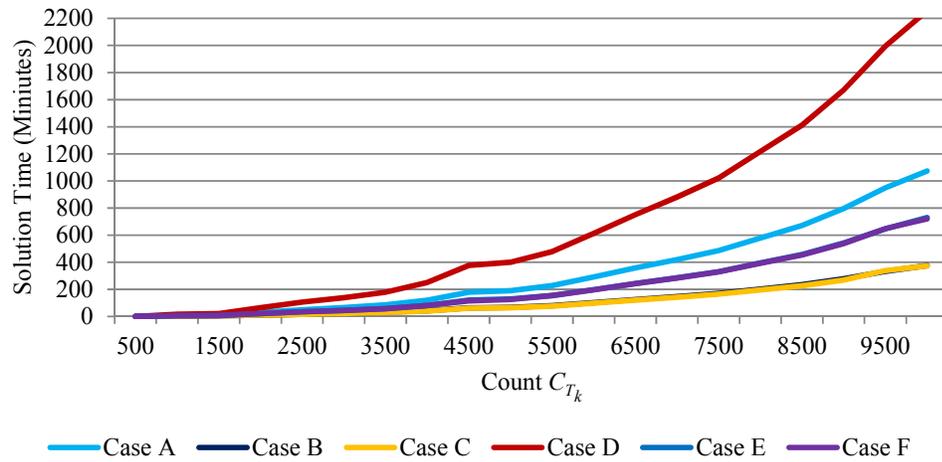


Figure 4.5 Solution times for Cases A-F prior to the implementation of multi-threading.

Cases A-F were replicated using the multi-threading-based stope optimisation algorithm and performance of the original and modified algorithms was compared for each case. Figure 4.6–4.11 illustrate the performance comparisons between the modified algorithm and the original algorithm for Cases A-F in sequence.

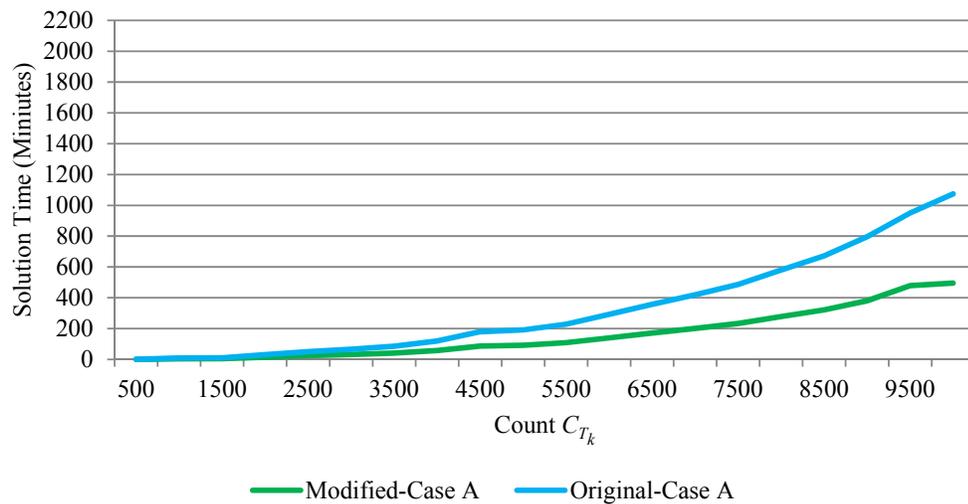


Figure 4.6 Performance comparison between the original and modified algorithms for Case A.

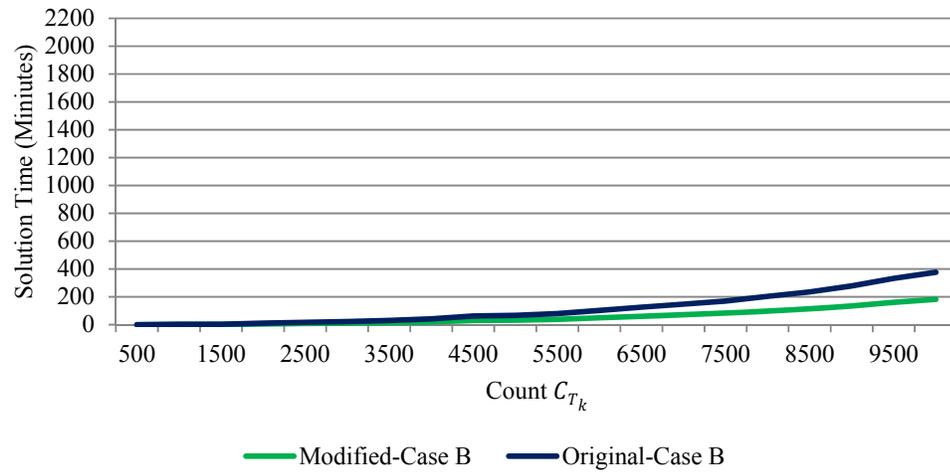


Figure 4.7 Performance comparison between the original and modified algorithms for Case B.

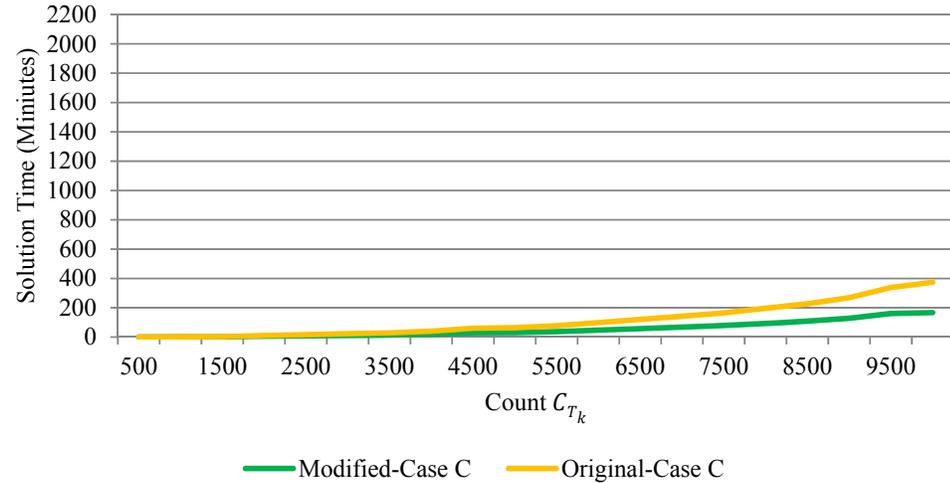


Figure 4.8 Performance comparison between the original and modified algorithms for Case C.

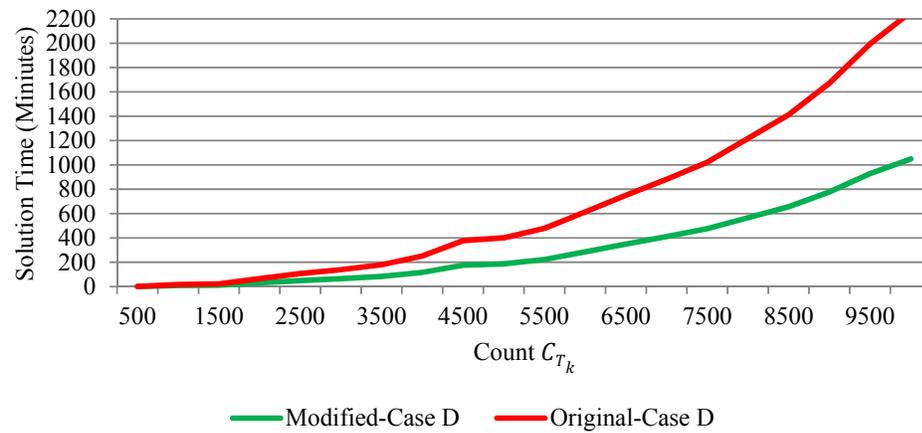


Figure 4.9 Performance comparison between the original and modified algorithms for Case D.

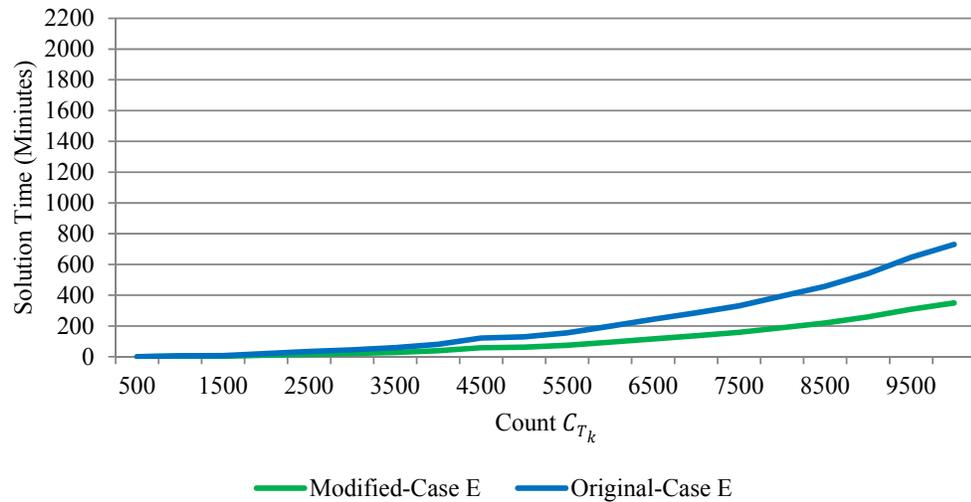


Figure 4.10 Performance comparison between the original and modified algorithms for Case E.

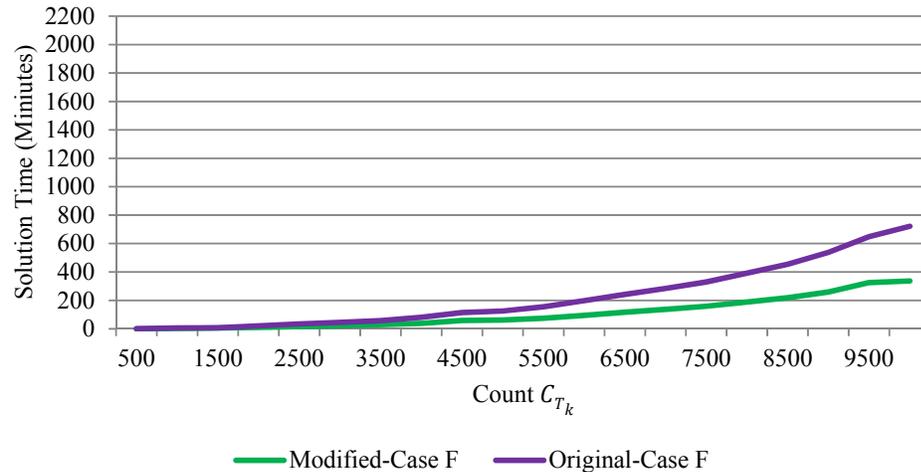


Figure 4.11 Performance comparison between the original and modified algorithms for Case F.

4.5 APPLYING THE STOPE SHAPER

This section describes an implementation of the stope shaper for the optimum stope layout in Case C. Figure 4.12 (a) shows the boundary and enclosed stopes that are identified by stope shaper. Figure 4.12 (b) shows the extracted ore blocks within the boundary stopes combined with the enclosed stopes. Figure 4.13 (a) shows the superimposition of the ore blocks in the resource model, i.e., blocks where $v_r > 0$, with Figure 4.12 (a). It shows that the stope layout does not follow the shape of the ore body model. Figure 4.13 (b) shows the superimposition of the ore blocks in the model, i.e., blocks where $v_r > 0$, with Figure 4.12 (b). In this scenario, the stope layout follows the shape of the resource model.

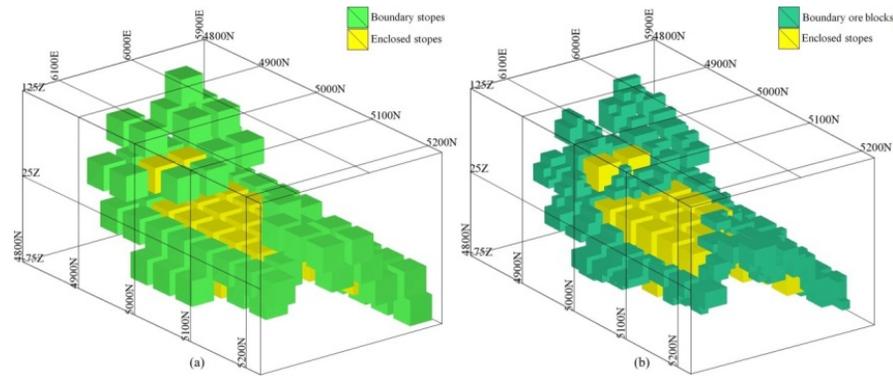


Figure 4.12 (a) Boundary and enclosed stopes, (b) enclosed stopes and boundary ore blocks.

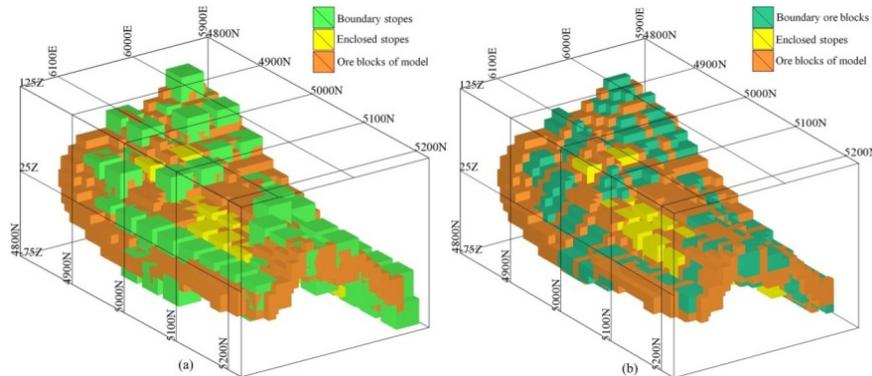


Figure 4.13 Superimposition of the ore blocks in the resource model.

4.6 IMPROVEMENTS TO THE SOLUTION BASED ON STRUCTURAL CHANGES IN THE INPUT

An input structure-based study is conducted to achieve better quality outcomes in the stope optimisation process. The structure of the input used for stope optimisation, i.e., the positive-valued stopes, was changed based on the following two parameters.

1. 3D orientation of the input
2. Percentage of the input based on economic values

After making these changes to the structure of the input, stope optimisation was performed and the optimum solutions were recorded. The outcomes were compared with the solutions of the original cases to identify improvements. The study is described in Subsections 4.6.1, 4.6.2 and 4.6.3, as follows.

4.6.1 3D ORIENTATION OF THE STOPE STRUCTURE

For the six cases described in Section 4.3, the inputs used for stope optimisation were ordered based on the ascending values of the x , y and z stope coordinates. However, the input stopes could be ordered in five more orientations as follows.

1. x,z,y
2. y,x,z
3. y,z,x
4. z,x,y
5. z,y,x

Case A was replicated using these five additional spatial orientations as inputs. Table 4.5 summarizes the outcomes of this analysis.

Table 4.5 Summary of the optimum solutions for six stope orientations.

3D orientation of input	\bar{v}_{max} (\$)	Number of stopes
x,y,z	779, 872, 119	175
x,z,y	773, 279, 060	169
y,x,z	809, 991, 068	155
y,z,x	801, 584, 416	160
z,x,y	706, 584, 946	189
z,y,x	720, 425, 272	185

As shown in Table 4.5, the 3D orientation of the input affected the profit of the outcome. Furthermore, the y,x,z orientation and y,z,x orientation yielded more profit than the x,y,z orientation employed in the case studies. By contrast, input orientations of x,z,y ; z,x,y ; and z,y,x yielded less profit than the x,y,z orientation. Thus, the y,x,z

orientation may be considered as the best 3D orientation because it generated the maximum profit among the six different orientations.

Case D was also replicated using the y,x,z slope input orientation to verify this finding. The optimal solution for Case D with the y,x,z orientation generated \$ 30,085,585 higher solution than that for the initial case with the x,y,z orientation, thereby confirming the accuracy of the result obtained for case A.

In the algorithm, the sequence of selecting stopes for inclusion in a layout is based on the input orientation of stopes. Consequently, the amount of high-valued stopes located in a given orientation impacts the profit value of the overall stope layout selected by the algorithm. Accordingly, for the given copper model, it can be concluded that the input stopes from y,x,z orientation consist of the maximum number of high-valued stopes among the six orientations. Table 4.6 presents a comparison of the results obtained using the x,y,z and y,x,z orientations for the two Cases.

Table 4.6 Comparison of solutions obtained using x,y,z and y,x,z orientations for Cases A and D.

Case	\bar{v}_{max} (\$)	
	x,y,z orientation	y,x,z orientation
A	779, 872, 119	809,991,068
D	790,524,917	820,610,425

4.6.2 PERCENTAGE OF POSITIVE STOPES BASED ON ECONOMIC VALUE

In this analysis, the input stopes were reordered in descending economic value before generating the combinations. Similar to Subsection 4.7.1, Cases A and D were replicated using these changed input stopes. The percentage was varied from 10% to 100% with 10% intervals where $C_{T_k} = 500$, and optimum solutions were generated for Cases A and D. The y,x,z orientation was used in each case because it generated the best solutions among the six possible different orientations. Table 4.7 shows the outcomes of this analysis for Cases A and D.

Table 4.7 Summary of the optimum solutions for Cases A and D based on the percentage of stopes.

% of stopes (value based)	Case A		Case D	
	\bar{v}_{max} (\$)	No. of Stopes	\bar{v}_{max} (\$)	No. of stopes
10	409,407,384	33	450,885,487	34
20	590,481,701	61	602,008,063	61
30	671,592,575	82	661,762,868	84
40	733,457,984	100	747,772,181	95
50	761,684,012	124	771,479,825	112
60	798,480,400	135	784,140,760	124
70	808,263,344	146	801,490,425	131
80	805,827,591	152	791,208,353	141
90	798,123,701	153	793,209,939	154
100	801,959,111	155	759,809,034	160

A graphical representation of the results obtained for the two scenarios is shown in Figure 4.14.

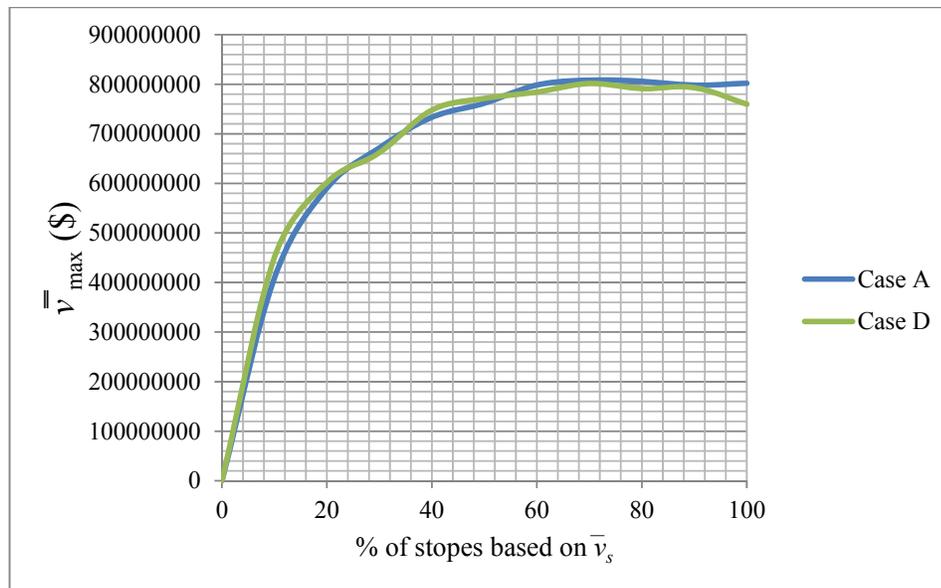


Figure 4.14 Results obtained after variation in the input based on the percentage of stopes for Cases A and D.

The results show that the optimum solution increased in the region between 10% and 70%. The increase between 10% and 70% was attributed to the increase in the number of economically high valued stopes in the input. However, when a large

proportion of stopes were considered (a combination of both economically high valued and low valued stopes) the results began to fluctuate or decrease. Therefore, it can be concluded that reducing the percentage of economically low valued stopes increases the likelihood of generating better results with the proposed algorithm.

A comparison of the results obtained using the general approach, i.e., the x,y,z 3D orientation for inputs, and the modified approach, i.e., the y,x,z orientation with 60–100% input stopes, is shown in Table 4.8 for Cases A and D. For Case A, using 60% stopes to 100% stopes based on their economic values yielded more profitable solutions than the general approach. Similarly, for Case D, using 70% stopes to 90% stopes based on their economic values yielded more profitable solutions than the general approach.

Table 4.8 Summary of the optimum solutions obtained based on 60–100% stopes after 500 iterations.

Case	\bar{v}_{max} (10^6 \$)					
	x,y,z orientation	y,x,z orientation where $C_{T_k} = 500$				
		60%	70%	80%	90%	100%
A	779	798	808	805	798	801
D	790	784	801	791	793	759

The results of this analysis show that the region between 70% and 90% comprised the optimum input percentages, where the algorithm generated significantly better solutions than the region between 10% and 50%.

4.6.3 COMBINATION OF THE OPTIMUM 3D ORIENTATION AND INPUT PERCENTAGE

In this analysis, the optimum 3D orientation, i.e., y,x,z 3D orientation, and the optimum percentage region, i.e., 70% to 90%, were combined to improve the results for Case A. The number of iterations varied from 0 to 10,000 using three scenarios, i.e., 70%, 80%, and 90%. A summary of the optimum solutions is provided in Table 4.9.

Table 4.9 Summary of the optimum solutions for Case A based on the combined input parameters.

% of stopes	\bar{v}_{max} (\$)	No. of stopes
70	808,263,344	146
80	818,766,686	145
90	809,611,742	150

As shown in Table 4.9, the algorithm achieves superior solutions to the general case. Furthermore, at 80% of stopes, the algorithm generates the best outcome with a \$ 10,503,342 higher solution value than that for the general case.

4.7 VALIDATION OF THE PROPOSED ALGORITHM

In Subsections 4.7.1 and 4.7.2, the performance of the proposed algorithm is compared with the Sens and Topal heuristic approach (Sens and Topal, 2009) and the MVN algorithm (Ataee-pour, 2000).

4.7.1 COMPARISON WITH THE SENS AND TOPAL HEURISTIC APPROACH

The Sens and Topal heuristic approach supports cases with fixed and variable stope sizes but not cases with level optimisation or the provision of pillars. Among the six cases defined in Section 4.3, Cases A and D accommodate such constraints. Therefore, Cases A and D were replicated based on Sens and Topal approach using the copper resource model as input. Figure 4.15 and Figure 4.16 provide a visual comparison of the optimum stope layouts obtained using the two algorithms for Case A and Case D respectively.

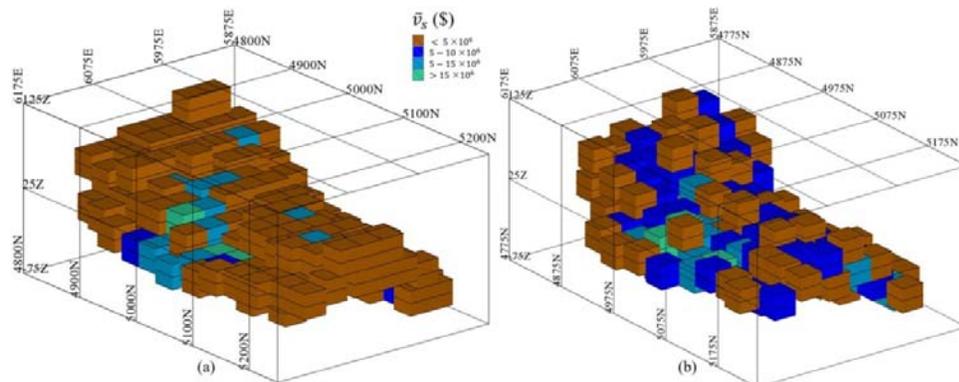


Figure 4.15 Visualisations of the optimum solutions to Case A using (a) the proposed algorithm and (b) the Sens and Topal approach.

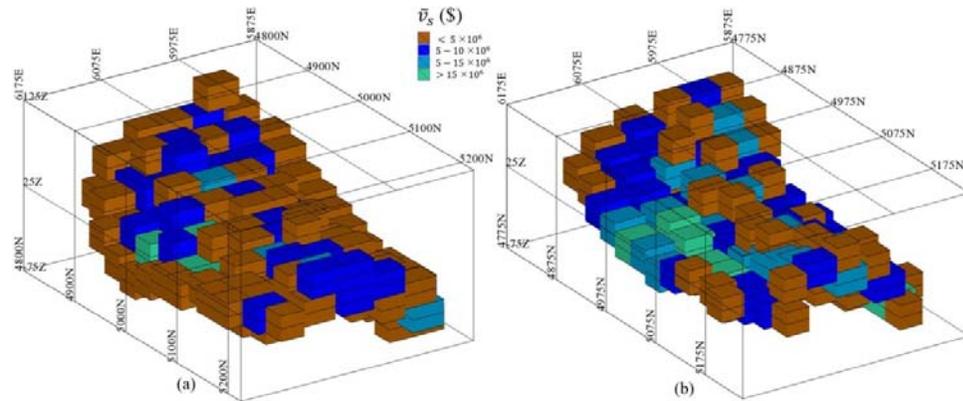


Figure 4.16 Visualisations of the optimum solutions to Case D using (a) the proposed algorithm and (b) the Sens and Topal approach.

Table 4.10 summarises the parameters of the optimal solutions that were generated using the proposed algorithm and the MVN approach for Cases A and D respectively.

Table 4.10 Comparison of solutions obtained using proposed algorithm and Sens and Topal approach.

Parameter	Proposed algorithm		Sens and Topal approach	
	Case A	Case D	Case A	Case D
\bar{v}_{max} (\$)	779,872,119	790,524,917	722,804,983	745,561,899
Average \bar{v}_s (\$)	4,456,412	4,849,846	5,736,547	6,840,017
Average \bar{g}_s (%)	1.0120	1.0134	1.1801	1.1467
No of stopes	175	163	126	109
Solution time (h: min: s)	1: 35: 00	8: 36: 00	0: 00: 09	0: 00: 55

As can be seen from Table 4.10, the proposed algorithm generates 7.3% and 5.6% more profitable solutions than the Sens and Topal approach for Cases A and D. Both the proposed algorithm and the Sens and Topal approach are heuristics. However, while the proposed algorithm needs to generate a set of discrete solutions to identify the optimum solution, the Sens and Topal approach generates only a single solution for a given resource model as described in Section 2.8 in Chapter 2. As such the

solution time for the Sens and Topal approach is less than that for the proposed algorithm.

4.7.2 COMPARISON WITH THE MVN ALGORITHM

The MVN algorithm supports cases with level optimisation and fixed stope sizes but not cases with variable stope sizes and pillar provision. Among the six cases defined in Section 4.3, only Case B accommodates such constraints. Therefore an additional case was created to allow better comparisons and show the significance of the proposed algorithm.

Additional case (Case G): Fixed stope size limit of 64 ($nx_s = 4, ny_s = 4, nz_s = 4$) mining blocks, i.e., a stope size of $4 \times 4 \times 4$ with the provision of levels but without the provision of pillars.

Upon defining Case G, Cases B and G were replicated based on the MVN algorithm using the copper resource model as input. The MVN algorithm produces a set of mining blocks as the outcome and it does not delineate mineable stopes, whereas the proposed algorithm generates a set of delineated stopes as the outcome. In order to achieve visual comparisons, mining blocks were extracted from the optimal stope layouts generated using the proposed algorithm. Figure 4.17 and Figure 4.18 provide a visual comparison of the mining blocks that were obtained using the two algorithms for Cases B and G respectively.

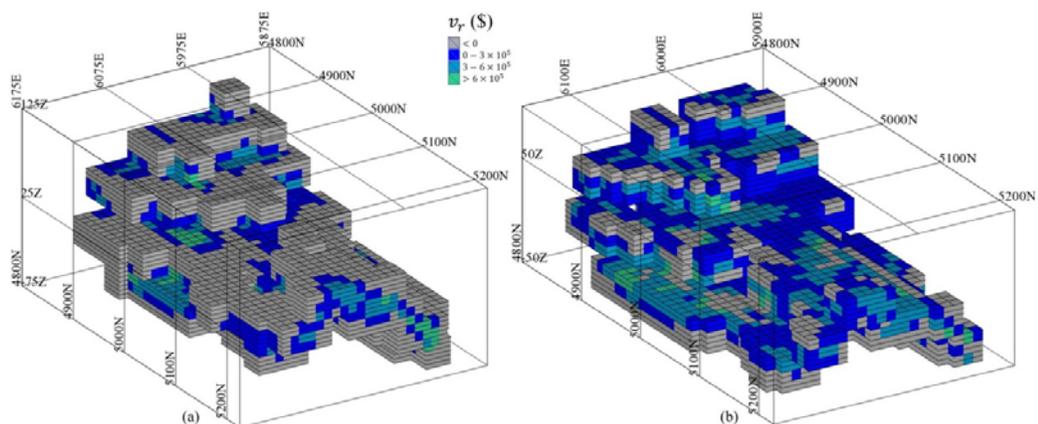


Figure 4.17 Visualisations of the mining blocks to Case B using (a) the proposed algorithm and (b) the MVN algorithm.

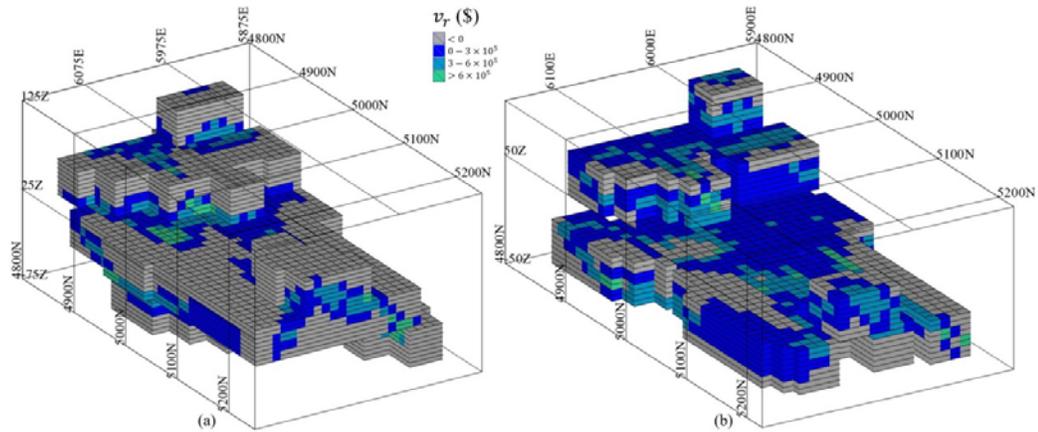


Figure 4.18 Visualisations of the mining blocks to Case G using (a) the proposed algorithm and (b) the MVN algorithm.

Table 4.11 presents a comparison of optimal solutions obtained using the proposed and the MVN algorithms in Cases B and G.

Table 4.11 Comparison of solutions obtained using the proposed algorithm and the MVN algorithm for Case B.

Parameter	Proposed algorithm		MVN algorithm	
	Case B	Case G	Case B	Case G
\bar{v}_{max} (\$)	751,953,177	701,635,964	667,468,447	543,442,075
Average \bar{v}_s (\$)	3,978,588	8,995,332	N/A	N/A
No of mining blocks	5103	4992	3312	3400
No of stopes	189	78	N/A	N/A
Solution time (h : min : s)	0: 23: 00	0: 21: 00	0: 00: 35	0: 00: 57

As can be seen from Table 4.11, the proposed algorithm generates 11.2% and 22.5% better solutions than the MVN algorithm for Cases B and G, respectively. Both the proposed and the MVN algorithms are heuristics. However, while the proposed algorithm needs to generate a set of discrete solutions to identify the optimum solution, the MVN algorithm generates a single solution based on the maximum neighbourhood concept as discussed in Section 2.6 in Chapter 2. Furthermore, the proposed algorithm incorporates the non-overlapping stope constraint requiring identification and avoidance of overlapping stopes during the optimisation process, whereas in MVN algorithm, a non-overlapping stope constraint is not applied. As

such the solution time for the MVN algorithm is less than that of the proposed algorithm.

4.8 APPLICATION INTERFACE

A user-friendly interface was designed and built using the Visual Studio interface development tools to allow implementation of the algorithm for different mining scenarios. This user-friendly application enables resolution of stope optimisation problems, but without the need to manually modify the C# source code. Figure 4.19 shows the C# Windows-based interface for the stope optimisation algorithm. Instructions about how to use the application are provided in the appendix A.

Figure 4.19 User interface for the stope optimisation algorithm.

CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

This chapter is divided into Subsections 5.1 and 5.2. Subsection 5.1 summarises the contributions of the thesis to the field of underground mining and outcomes of a case study that demonstrates the applicability of the proposed algorithm to a real case resource model representing a copper deposit. Subsection 5.2 provides recommendations for improving the proposed algorithm based on observations of its performance and applicability.

5.1 THESIS SUMMARY

This thesis presents four major contributions as follows.

1. The original contribution is the development of a robust heuristic algorithm in order to generate an optimal stope layout for a given resource model. The algorithm incorporates pillar separation, level separation and stope size variation, successfully to generate solutions subject to physical mining constraints. Previous heuristic approaches lack the incorporation of physical mining constraints. Therefore, the proposed algorithm becomes an innovative mine planning tool for generating optimum stope layouts that represent realistic mining scenarios.
2. The proposed algorithm establishes the exponential relationship between the solution time and possible solutions to the problem. The thesis presents an innovative method to handle the computational complexity and implements the method successfully to a resource model which represents a copper deposit. Further, the performance of the algorithm is improved using the task parallelism concept from object oriented programming, thereby presenting an efficient method to develop solutions to block models consisting of thousands of mining blocks.
3. The algorithm incorporates an innovative method for identifying overlapping stopes and avoiding their inclusion in the optimum stope layout, thus eliminating the problem within the industrially recognised floating stope algorithm.

4. Development of user-friendly software that implements the stope optimisation algorithm for different mining scenarios including Microsoft C# object-oriented programming language and the Microsoft SQL server. The advantage of using this combination is that it enables the execution of the software on windows-based computers with different architectures. As a result of this development, manual changes to the source code are not required to implement the algorithm in different mining scenarios.

Due to the combinatorial optimisation nature of the problem the algorithm employs a heuristic approach. The design of the heuristic algorithm has a compound nature, i.e., constructive and improvement characteristics which are incorporated to ensure that better quality solutions are achieved in the shortest possible time.

The algorithm was implemented on a resource model that represented a copper deposit, where six cases (Cases A to F) were defined based on a variety of mining scenarios to demonstrate the suitability of the algorithm for different mining situations. For the given resource model, Case D with a variable stope size of $3 \times 3 \times 3 - 3 \times 3 \times 4$ yielded the most profitable solution among the six cases. The provision of pillars and levels reduced the solution value compared with cases without the provision of pillars and levels. This was because the provision of pillars and levels prior to optimisation reduced the number of positive-valued stopes in the input, and thus the number of possible solutions.

In a further analysis that aimed to improve the solution values for Cases A and D, the structure of the input positive-valued stopes, were modified to identify the optimum spatial orientation and the optimum input percentage for the high-valued stopes. Based on the outcomes, the y,x,z orientation was selected as the optimal spatial orientation because changing the orientation from x,y,z to y,x,z increased the solution value by 3.8% in both cases. For the input stopes, a range of 70–90% was identified as the optimal percentage range. A modified input structure that comprised the y,x,z stope orientation and 80% of the high-valued stopes improved the solution value in Case A by 1.2%. This analysis proves that altering the orientation or the amount of high-valued stopes in the input impacts the sequence in which the stopes are selected to generate a solution, thereby increasing the solution value. Therefore, it can be

concluded that restructuring the input has potential to obtain better solutions when applying the proposed algorithm to resource models.

The algorithm was validated based on comparisons with two existing heuristic algorithms: the MVN algorithm and Sens and Topal's heuristic approach. The proposed algorithm obtained superior solutions than both of the existing algorithms for selected cases. The MVN algorithm was implemented only for Cases B and G because it does not support variable stope sizes and the provision of pillars. The proposed algorithm generated solutions which were 11.2% and 22.5% more profitable than the solution obtained using the MVN algorithm. The Sens and Topal heuristic approach was implemented for Cases A and D specifically because it does not support the provision of levels and pillars. The proposed algorithm generated solutions, which were 7.3% and 5.6% more profitable than the solutions obtained using the Sens and Topal heuristic approach for Cases A and D, respectively. An advantage of the proposed algorithm compared with the MVN algorithm is that it allows the provision of pillars or variable stopes sizes when developing solutions. Similarly, an advantage of the proposed algorithm compared with the Sens and Topal heuristic approach is that it can incorporate level definitions and the provision of pillars when developing solutions. This reflects the flexibility of the proposed algorithm for generating solutions to realistic underground mining scenarios.

5.2 RECOMMENDATIONS

This research has made a significant contribution to the development of an algorithm to optimise stope boundary layout in underground mining as discussed in this thesis. However, the proposed algorithm can be further improved to optimise its performance and applicability in the following areas.

Even though the scope of this research was limited to stope boundary optimisation, production scheduling must be incorporated to the stope optimisation process because they share a close relationship with one area influencing the other. Currently, it is a common mine planning practice to consider optimisation of each area separately. Consequently, the resulting revenues may be, less than a simultaneous consideration of both areas in the optimisation process. In order to handle the specific geotechnical aspects, i.e. size and orientation of the ore body, load carrying capacity of rock mass, etc., given ore body models can be divided in to

sub sections allowing the optimisation of stope layout for those sections separately. As such, further modifications to the proposed algorithm are required to satisfy the geotechnical parameters which influence the stope layout as well as production scheduling.

A versatile extension to the developed interface would be a stand-alone visualiser, which would facilitate the rapid visualisation of 3D stope layouts. The current version of the stope optimisation software relies on visualisation using external mine design software tools. This process requires time to transfer and convert the output files into compatible formats for current visualisation tools.

The application of the multi-threading concept improved the performance of the algorithm, but this needs to be investigated further and tested rigorously. In the present study, only two sets of non-overlapping stopes were defined as the input for multi-threading. Further divisions of the input file may result in greater performance improvements and shorter solution times. However, the addition of more sets to the computational process may result in proportional increases in the processing power and memory requirements.

The algorithm generates solutions for resource models that contain one mineral type, thus further improvements are required to the structure of the algorithm for multi-commodity resource models. An option to select either single-commodity or multi-commodity can be added to the developed interface to facilitate this requirement.

REFERENCES

- Albahari, J., & Albahari, B. (2010). C# 4.0 in a nutshell. 4th edition ed., U.S.A.: (O'REILLY).
- Alford, C. (1995, 9-14 July). *Optimisation in underground mine design*, Brisbane. [http://dx.doi.org:10.1016/0148-9062\(96\)80055-6](http://dx.doi.org:10.1016/0148-9062(96)80055-6)
- Alford, C., Brazil, M., & Lee, D. (2007). Optimisation in Underground Mining. In A. Weintraub, C. Romero, T. Bjørndal, R. Epstein & J. Miranda (Eds.), *Handbook Of Operations Research In Natural Resources* (Vol. 99, pp. 561-577): Springer US. http://dx.doi.org/:10.1007/978-0-387-71815-6_30
- Alford, C., & Hall, B. (2009, 21-22 April 2009). *Stope Optimisation Tools for Selection of Optimum Cut-Off grade in Underground Mine Design*. Paper presented at the Project Evaluation Conference, Melbourne, Vic 21-22 April 2009 Accessed
- Asad, M. W. A. (2011). A heuristic approach to long-range production planning of cement quarry operations. *Prod. Plan. Control*, 22(4), 353-364. <http://dx.doi.org/10.1080/09537287.2010.484819>
- Asad, M. W. A., Dimitrakopoulos, R., & Eldert, J. v. (2013). Stochastic production phase design for an open pit mining complex with multiple processing streams. *Engineering Optimization*, 46(8), 1139-1152. <http://dx.doi.org/10.1080/0305215X.2013.819094>
- Ataee-pour, M. (2000). *A heuristic algorithm to optimise stope boundaries*. Doctor of Philosophy University of Wollongong, New South Wales, Australia.
- Ataee-pour, M. (2004). Optimisation of stope limits using a heuristic approach. *Transactions of the Institution of Mining and Metallurgy Section a-Mining Technology*, 113(2), A123-A128. <http://dx.doi.org/10.1179/037178404225004959>
- Ataee-pour, M. (2005). A critical survey of the existing stope layout optimization techniques. *Journal of Mining Science*, 41(5), 447-466. <http://dx.doi.org/10.1007/s10913-006-0008-9>
- Bai, X. Y., Marcotte, D., & Simon, R. (2013). Underground stope optimization with network flow method. *Computers & Geosciences*, 52, 361-371. <http://dx.doi.org/DOI 10.1016/j.cageo.2012.10.019>
- Basel, J., III, & Willemain, T. (2001). Random Tours in the Traveling Salesman Problem: Analysis and Application. *Computational Optimization and Applications*, 20(2), 211-217. <http://dx.doi.org/10.1023/A:1011263204536>

- Bernstein, D., Jeffery, S., Lange, T., & Meurer, A. (2013). Quantum Algorithms for the Subset-Sum Problem. In P. Gaborit (Ed.), *Post-Quantum Cryptography* (Vol. 7932, pp. 16-33): Springer Berlin Heidelberg.
- C# Threads. (2013). Retrieved January 21, 2013 <http://www.dotnetperls.com/thread>
- CAE Studio 3. (2014). software available at <http://www.cae.com/mining/>
- Cawrse, I. (2001). *Multiple Pass Floating Stope Process*. Paper presented at the 4th Biennial Strategic Mine Planning Conference, Melbourne
- Cawrse, I. (2007). Multiple pass floating stope process, *held in Strategic Mine Planning Conference*, (pp. 87-94). Perth, Australia
- Cheimanoff, N. M., Deliac, E. P., & Mallet, J. L. (1989). *GEOCAD: an alternative CAD and artificial intelligence tool that helps moving from geological resources to mineable reserves*. Paper presented at the 21st International APCOM Symposium, USA
- Darling, P. (2011). *SME mining engineering handbook* (Vol. 2): SME.
- Deitel, P. J., & Deitel, H. M. (2008). *Visual C#® 2008 How to Program*. U.S.A.: Prentice Hall Press.
- Derigs, U., & Vogel, U. (2014). Experience with a framework for developing heuristics for solving rich vehicle routing problems. *Journal of Heuristics*, 20(1), 75-106. <http://dx.doi.org/10.1007/s10732-013-9232-z>
- Dimitrakopoulos, R., & Grieco, N. (2009). Stope design and geological uncertainty: quantification of risk in conventional designs and probabilistic alternative. *Journal of Mining Science*, 45(2), 152-163.
- EQATEC .NET version 1.1. (2014). Software available at <http://eqatec-profiler.eqatec-as.blueprograms.com/>
- GEOVIA Surpac™ version 6.3.1. (2012). Software available at <http://www.gemcomsoftware.com/products/surpac>
- Gertch, R. E., & Bullock, R. L. (1998). *Techniques in Underground Mining*. USA: Society for Mining, Metallurgy, and Exploration, Inc.
- Gonçalves, J., & Resende, M. C. (2011). Biased random-key genetic algorithms for combinatorial optimization. *Journal of Heuristics*, 17(5), 487-525. <http://dx.doi.org/10.1007/s10732-010-9143-1>
- Grieco, N., & Dimitrakopoulos, R. (2007). Managing grade risk in stope design optimisation: probabilistic mathematical programming model and application in sublevel stoping. *Mining Technology*, 116(2), 49-57.
- Hejlsberg, A., Torgersen, M., Wiltamuth, S., & Golde, P. (2010). *The C# programming language*. U.S.A.: Addison-Wesley.

- Hustrulid, W. A. (2006). *Open pit mine planning & design / William Hustrulid, Mark Kuchta*. London: London : Taylor and Francis.
- Hustrulid, W. A., & Bullock, R. L. (2001). *Underground Mining Methods*. Colorado, USA: Society for Mining, Metallurgy, and Exploration Inc.
- James, M. H. (2013). C#.NET Little Wonders. Retrieved January 23, 2013 <http://geekswithblogs.net/BlackRabbitCoder/archive/2012/12/20/c.net-little-wonders-the-parallel.invoke-method.aspx>
- Karapetyan, D. (2010). *Design, Evaluation and Analysis of Combinatorial Optimization Heuristic Algorithms*. Doctor of Philosophy University of London, United Kingdom.
- Krishna, R., & Chanda, M. W. (1997). Re-thinking in the optimization of stope dimensions. *Eighth International Congress on Rock Mechanics, Vol 3, Proceedings*, 1381-1384. <Go to ISI>://A1997BH39W00069
- Kumral, M. (2004). Genetic algorithms for optimization of a mine system under uncertainty. *Production Planning & Control*, 15(1), 34-41. <http://dx.doi.org/10.1080/09537280310001654844>
- Kumral, M. (2010). Robust stochastic mine production scheduling. *Eng. Optimiz.*, 42(6), 567-579. <http://dx.doi.org/10.1080/03052150903353336>
- Kumral, M. (2013). Optimizing ore-waste discrimination and block sequencing through simulated annealing. *Applied Soft Computing*, 13(8), 3737-3744. <http://dx.doi.org/http://dx.doi.org/10.1016/j.asoc.2013.03.005>
- Lam, F., & Newman, A. (2008). Traveling salesman path problems. *Mathematical Programming*, 113(1), 39-59. <http://dx.doi.org/10.1007/s10107-006-0046-8>
- Little, J. (2012). *Simultaneous optimisation of stope layouts and production schedules for long-term underground mine planning*. Doctor of Philosophy University of Queensland, Queensland, Australia.
- Little, J., Knights, P., & Topal, E. (2013). Integrated optimization of underground mine design and scheduling. *Journal of the Southern African Institute of Mining and Metallurgy*, 113(10), 775-785.
- Little, J., & Topal, E. (2011). Strategies to assist in obtaining an optimal solution for an underground mine planning problem using Mixed Integer Programming.
- Little, J., Topal, E., & Knights, P. (2011). Simultaneous optimisation of stope layouts and long term production schedules. *Mining Technology*, 120(3), 129-136. <http://dx.doi.org/doi:10.1179/1743286311Y.0000000011>
- Martí, R., Gallego, M., Duarte, A., & Pardo, E. (2013). Heuristics and metaheuristics for the maximum diversity problem. *Journal of Heuristics*, 19(4), 591-615. <http://dx.doi.org/10.1007/s10732-011-9172-4>

- Morelan, R. A., Ekberg, T., Brown, J., & Berger, I. (2009). *SQL Queries Joes 2 Pros: SQL Query Techniques For Microsoft SQL Server 2008, Volume 2*: BookSurge Publishing.
- Okubo, S., & Yamatomi, J. (n.d.). Underground Mining Methods and Equipment. *Civil Engineering*, 2 Retrieved from <http://www.eolss.net/sample-chapters/c05/e6-37-06-02.pdf>
- Ovanic, J., & Young, D. S. (1995). Economic optimization of stope geometry using separable programming with special branch and bound techniques. Paper presented at the 3rd Canadian Conference on Computer Applications in the Mineral Industry, McGill University, Montreal
- Ovanic, J., & Young, D. S. (1999). *Economic optimisation of open stope geometry*. Paper presented at the 28th International APCOM Symposium, Colorado School of Mines, Golden
- Parallel Class. (2013). Retrieved January 23, 2013 [http://msdn.microsoft.com/en-us/library/vstudio/system.threading.tasks.parallel\(v=vs.110\).aspx](http://msdn.microsoft.com/en-us/library/vstudio/system.threading.tasks.parallel(v=vs.110).aspx)
- Punnen, A. (2007). The Traveling Salesman Problem: Applications, Formulations and Variations. In G. Gutin & A. Punnen (Eds.), *The Traveling Salesman Problem and Its Variations* (Vol. 12, pp. 1-28): Springer US. http://dx.doi.org/10.1007/0-306-48213-4_1
- Rardin, R., & Uzsoy, R. (2001). Experimental Evaluation of Heuristic Optimization Algorithms: A Tutorial. *Journal of Heuristics*, 7(3), 261-304. <http://dx.doi.org/10.1023/A:1011319115230>
- Richmond, A. J., & Beasley, J. E. (2004). An Iterative Construction Heuristic for the Ore Selection Problem. *Journal of Heuristics*, 10(2), 153-167. <http://dx.doi.org/10.1023/B:HEUR.0000026265.47626.23>
- Riddle, J. M. (1977). *A dynamic programming solution of a block caving mine layout*. Paper presented at the 14th APCOM Symposium, New York
- Semančo, P., & Modrák, V. (2011). Hybrid GA-Based Improvement Heuristic with Makespan Criterion for Flow-Shop Scheduling Problems. In M. Cruz-Cunha, J. Varajão, P. Powell & R. Martinho (Eds.), *ENTERprise Information Systems* (Vol. 220, pp. 11-18): Springer Berlin Heidelberg. http://dx.doi.org/10.1007/978-3-642-24355-4_2
- Sens, J., & Topal, E. (2009). A New Algorithm for Stope Boundary Optimisation. *Ausimm New Leaders Conference 2009*, 2009(4), 25-28. <Go to ISI>://000271285400004
- Topal, E. (2008). Early and late start algorithms to improve the solution time for long-term underground mine production scheduling. *The Journal of the South African Institute of Mining and Metallurgy*, 108(2), 101-107.

Žerovnik, G., & Žerovnik, J. (2011). Constructive heuristics for the canister filling problem. *Central European Journal of Operations Research*, 19(3), 371-389.
<http://dx.doi.org/10.1007/s10100-010-0164-5>

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APPENDIX A

The appendix A provides a guide to the Windows-based stope optimisation application.

1. Data set up process

The data set up process is as follows.

1. Create a new database on the SQL server, as shown in Figure 1.

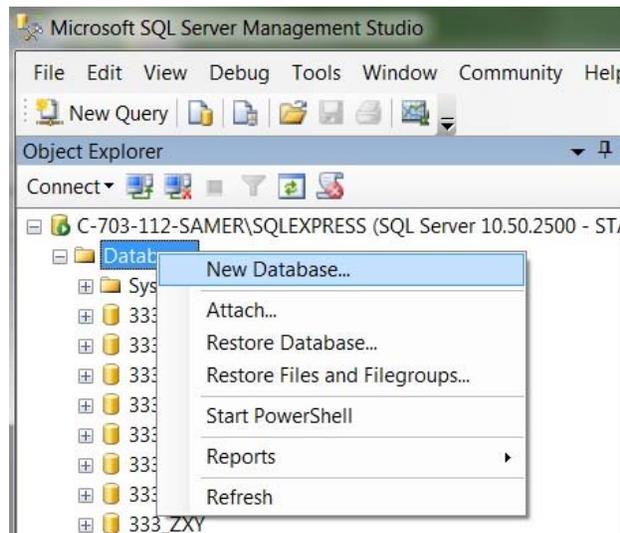


Figure 1. Creating a new database.

2. Name the database as TestModel, as shown in Figure 2.

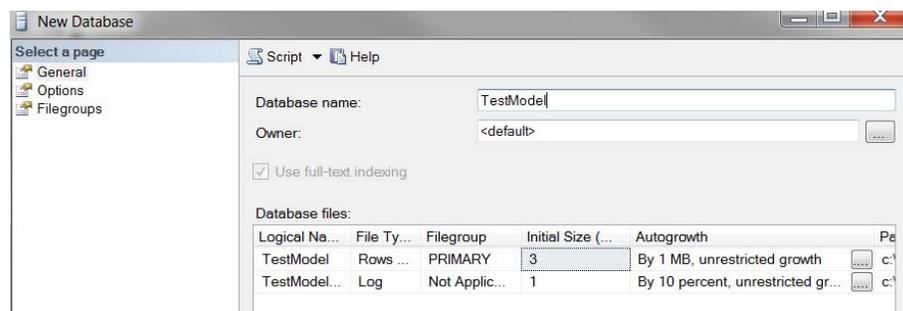


Figure 2. Naming the database.

3. Create a new SQL table and name it as OriginalCollection in the TestModel database, and then import the block model data into the table (this table will contain the block model used for stope optimisation). The data structure of the table should follow the order shown in Figure 3.

	X	Y	Z	XINC	YINC	ZINC	Density	Grade	Eval	TotalWeight	MetalWeight
1	5620	4610	-90	20	20	20	3.18	0	0	0	0
2	5620	4610	-70	20	20	20	3.18	0	0	0	0
3	5620	4610	-50	20	20	20	3.18	0	0	0	0
4	5620	4610	-30	20	20	20	3.18	0	0	0	0
5	5620	4610	-10	20	20	20	3.18	0	0	0	0
6	5620	4610	10	20	20	20	3.18	0	0	0	0
7	5620	4610	30	20	20	20	3.18	0	0	0	0
8	5620	4610	50	20	20	20	3.18	0	0	0	0
9	5620	4610	70	20	20	20	3.18	0	0	0	0

Figure 3. Data structure of the original block model.

A description of each column of this data model is given in Table 1.

Table 1- Description of the columns in the OriginalCollection data model.

Column Name	Data Type	Description
X	float	x coordinate of a block
Y	float	y coordinate of a block
Z	float	z coordinate of a block
XINC	float	Width of a block
YINC	float	Length of a block
ZINC	float	Height of a block
Density	float	Material density of a block
Grade	float	Grade of a block
Eval	float	Block economic value (initially set to zero prior to value calculation)
TotalWeight	float	Block total weight (initially set to zero prior to regularisation)
MetalWeight	float	Block metal weight (initially set to zero prior to regularisation)

Figure 4 shows the design of OriginalCollection on the SQL server.

Column Name	Data Type	Allow Nulls
X	float	<input checked="" type="checkbox"/>
Y	float	<input checked="" type="checkbox"/>
Z	float	<input checked="" type="checkbox"/>
XINC	float	<input checked="" type="checkbox"/>
YINC	float	<input checked="" type="checkbox"/>
ZINC	float	<input checked="" type="checkbox"/>
Density	float	<input checked="" type="checkbox"/>
Grade	float	<input checked="" type="checkbox"/>
Eval	float	<input checked="" type="checkbox"/>
TotalWeight	float	<input checked="" type="checkbox"/>
MetalWeight	float	<input checked="" type="checkbox"/>

Figure 4. Design of OriginalCollection on the SQL server.

Note: This data format can also be found in a sql script named as OriginalCollection_Format.sql that has been created to facilitate the application of the optimisation process.

4. Open a new query editor on the SQL server and run the CreateTables.sql script to create the following 11 SQL tables in the TestModel SQL database. (CreateTables.sql is an sql script associated with the stope optimisation application).

- i. BorderStopes
- ii. CombinationCollection
- iii. FeasibleSolution
- iv. MineableCells
- v. MineableStopes
- vi. NewCollection
- vii. OptimumSolution
- viii. PositiveStopes
- ix. Stopes
- x. ValueCalculatedCollection
- xi. ValuedCombinationCollection

The stope optimisation program requires this table structure for data access, retrieval and saving purposes.

2. Implementation

The generation of the optimum solution requires the implementation of the following major processes in the order given.

1. Set the SQL and block model unit variables.
2. Block regularisation.
3. Economic value calculation (i.e. converting the regularised model into an economic model).
4. Stope generation (for level-based and stope size-based scenarios).
5. Generate the optimum solution (for pillar-based and variable input-based scenarios).
6. Apply the stope shaper and extract the ore blocks to identify the blocks of stopes that are located at the boundaries of an ore body (see subsection 3.5.4 in Chapter 3).

7. Quick Solver: a rapid method for generating a feasible solution to the stope optimisation problem.

Figure 5 shows the developed interface for the stope optimisation process.

Figure 5. Interface of the stope optimisation application.

2.1 SETTING THE SQL AND BLOCK MODEL UNIT PARAMETERS

Setting the correct SQL parameters and block model units prior to the implementation of the stope optimiser is important for avoiding program failures and calculation errors. Thus, the three parameters shown in Figure 6 need to be set first.

Figure 6. Setting the SQL and block model parameters.

1. Server Name: The Server Name text box allows the user to select of the SQL authentication mode, i.e., Windows-based or mixed mode. In Windows-based authentication, the users do not have to log onto the SQL Server separately after they

have logged onto the computer. With mixed authentication, a user login and password are required to log onto the SQL server. For the stope optimisation application, Windows authentication is suggested because it does not require user credentials. The user types in the server address (in most cases the computer name) in the text box to log onto the SQL server.

2. SQL DB: The SQL DB combo box facilitates the selection of the ore body model that needs to be optimised. The user creates the data structure, as explained in section 1 of the appendices, and types the database name (for example, TestModel in this case) to access the required database using this combo box.

3. Set Units: The Set Units combo box facilitates the selection of a suitable grade unit for the type of metal in the ore body model. For example, metal Au requires that the grade unit is expressed in grams/tonne (g/t) and metal Cu requires that the grade unit be expressed as a percentage (%).

2.2 REGULARISER

First, the original block model is regularised. During the regularisation process, the blocks of different sizes are converted into a size that defined by the user via the interface. The size is defined by the X-block width (m), Y-block length (m) and Z-block height (m). After defining the size, the user may click the Regularise button (see Figure 7) to regularise the model.

The screenshot shows a software interface titled "Regulariser". It contains several input fields and a button. The fields are arranged in two rows. The first row has three fields: "X-Width (m)" with the value "10", "Y-Length (m)" with the value "10", and "Z-Height (m)" with the value "10". The second row is titled "Outer-Layer Parameters Assigner" and contains two fields: "Density (Tonnes/m3)" with the value "3" and "Grade (% or g/t)" with the value "0". To the right of these fields is a large button with a red downward-pointing arrow and the text "Regularise".

Figure 7. Block regulariser.

The outer-layer parameter assigner is used to adjust the Density (tonnes/m³) and Grade (%) values manually for any new volumes of the regularised block model that do not overlap with the original block model.

2.3 ECONOMIC VALUE CALCULATOR

The economic values of the regularised block model can be calculated based on the Metal Price (\$), Mining Cost (\$/tonne) and Mining Cost Factor parameters (\$/tonne). The Mining Cost defines the amount of money required to mine a particular block in the ore body model. The Mining Cost Factor defines the additional amount of money required to mine a particular block due to the depth of the block in the ore body model. After defining the parameters, the user may click on the Calculate button to determine the economic values for the regularised blocks in the model (Figure 8).

Economic Value Calculator		
Metal Price (\$/t)	Mining Cost (\$/t)	Mining Cost Factor (\$/t)
8000	30	0
Refining Cost (\$/t)	Processing Cost (\$/t)	Recovery (%)
0	0	100

Calculate

Figure 8. Economic value calculator.

2.4 STOPE GENERATOR

Stopes can be generated according to a fixed size or variable sizes. A fixed stope size is defined in terms of the number of blocks that participating from the x , y and z dimensions. Variable stope sizes are defined using additional parameters: X Cells Increment, Y Cells Increment, and Z Cells Increment. The user may define these parameters according to their specific requirements and they can then click the Generate Stopes button to create stopes with either fixed or variable sizes (Figure 9). The Stope Generator produces all the possible stopes and extracts the positive-valued stopes as inputs for the stope optimisation process.

Stope Generator		
No of X Cells	No of Y Cells	No of Z Cells
3	3	3
X Cells Increment	Y Cells Increment	Z Cells Increment
0	0	0

Create Levels
Generate Stopes

Figure 9. Stope generator.

The following three examples illustrate the stope generation process with a fixed stope size and variable stope sizes.

1. 3,3,3 Fixed size, i.e., No. of X Cells = 0, No. of Y Cells = 0 and No. of Z Cells = 0 (Figure 9), is defined for a regularised block model with a block size of $10 \times 10 \times 10$ m, where the size of a stope is set to $30 \times 30 \times 30$ m (stope width \times stope length \times stope height).
2. 3,3,3 to 4,3,3 Variable size, i.e., the user defines a stope size of 3,3,3 and defines the following: X Cells Increment = 1, Y Cells Increment = 0, Z Cells Increment = 0 (Figure 10), and stopes with the following two sizes are generated.
 - i. 3,3,3
 - ii. 4,3,3

No of X Cells	No of Y Cells	No of Z Cells
3	3	3
X Cells Increment	Y Cells Increment	Z Cells Increment
1	0	0

Figure 10. Variable stope sizes from 3,3,3 to 4,3,3.

3. 3,3,3 – 4,4,3 Variable size, i.e., the user defines a stope size of 3,3,3 and defines the following: X Cells Increment = 1, Y Cells Increment = 1, Z Cells Increment = 0 (Figure 11), and stopes with the following four sizes are generated.
 - i. 3,3,3
 - ii. 4,3,3
 - iii. 3,4,3
 - iv. 4,4,3

No of X Cells	No of Y Cells	No of Z Cells
3	3	3
X Cells Increment	Y Cells Increment	Z Cells Increment
1	1	0

Figure 11. Variable stope sizes from 3,3,3 to 4,4,3.

Another option that is available on the stope generator is the Create Levels check box: Create Levels. When a user checks this box, a suitable stope size can be defined and, after clicking the Generate Stopes button, the application divides the stopes according to the Z No. of Cells based on the Z dimension. For example, if the defined stope size is 3,3,3 and Create Levels is checked, the level height is fixed to a length of $3 \times$ the height of a regularised block.

2.5 STOPE OPTIMISER

The stope optimiser can be implemented for the different mining scenarios depicted in the tree diagram shown in Figure 12.

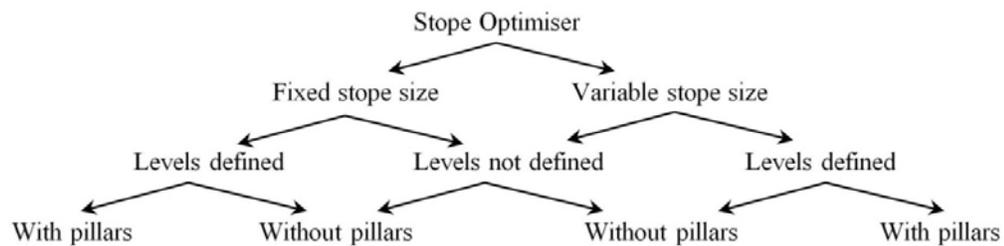


Figure 12. Tree diagram showing different stope optimiser scenarios.

In addition, the input for the stope optimiser, i.e., the positive-valued stopes, can be arranged according to two scenarios to analyse the effects of those scenarios on the optimum solution and to select the scenario that provides the best solution. The work flow of the stope optimiser is as follows.

1. Scenario-1: Select the spatial orientation of the stopes.
2. Scenario-2: Select the percentage of stopes according to their economic values.
3. Implementation of stope optimisation (Figure 13).

Figure 13. Stope Optimiser.

2.5.1 SELECTING THE SPATIAL ORDER OF STOPES

The spatial order of the input stopes can either be in ascending order or descending order, with the following six different orientations.

- i. XYZ
- ii. XZY
- iii. YXZ
- iv. YZX
- v. ZXY
- vi. ZYX

The spatial order can be changed using the **Ascending order** check box **Ascending order** (Ascending order is activated if the box is checked whereas Descending order is activated if it is unchecked). The spatial orientation of the input stopes can be changed using the **Stope Orientation** combo box (Figure 14).



Figure 14. Stope Orientation combo box.

2.5.2 SELECTING THE PERCENTAGE OF STOPES

The number of stopes used in the input can be changed based on the economic values of the stopes. To select the number of stopes used in the input, the user can define the percentage using the **% of stopes** combo box (Figure 15)

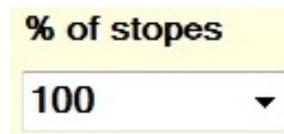


Figure 15. Percentage of stopes combo box.

For example, if 90% is selected as the percentage, the algorithm orders the input stopes according to their descending economic value before selecting the top 90% of the input stopes as the new input.

2.5.3 IMPLEMENTATION OF THE STOPE OPTIMISER

After defining the input parameters, the user can perform stope optimisation using different numbers of iterations. The iteration count limits the number of operations for the stope optimisation algorithm. With a higher number of iterations, there is a greater likelihood of the convergence of the solutions. However, increasing the iteration count may also require longer times to obtain a solution.

The iteration count can be set manually or by applying the default values given in the **Iteration count** combo box shown in Figure 16.

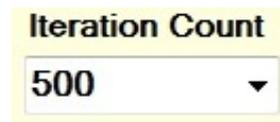


Figure 16. Setting the Iteration Count.

In summary, the user may define the stope order, stope orientation, percentage and iteration count parameters, before clicking the **Optimum Solution** button to generate the optimum solution.

2.6 QUICK SOLVER

This approach is based on the heuristic algorithm developed by Sens and Topal (Sens and Topal, 2009). The Quick Solver was embedded in the application to allow the rapid generation of a feasible solution and to allow comparisons with the proposed algorithm (Section 2.8 in Chapter 2 provides a description of the algorithm used in this approach). To generate a feasible solution, no additional parameters are required by the stope optimiser. Thus, after generating the positive-valued stopes, the user can simply click on the **Feasible Solution** button to generate a solution rapidly, as shown in Figure 17.

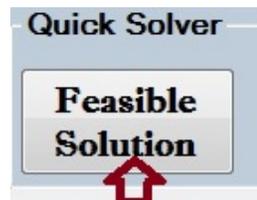


Figure 17. Generating a Feasible Solution.

2.7 STOPE SHAPER AND EXTRACTION OF ORE BLOCKS

The stope generator creates stopes with regular shapes (cubic or cuboid shapes). The optimum stope layout comprises these regular shapes, which might not follow the shape of the boundary of the ore body. Thus, dilution of the solution may occur due to the waste blocks included in the stopes along the boundary of the ore body (refer to section 4.5 in Chapter 4). The Stope Shaper shown in Figure 18 can be used to identify these stopes and extract the ore blocks from them. If the volumes of the ore blocks satisfy the production targets, they can be included in the solution, otherwise they can be discarded. Two steps are used to achieve this goal in the application.

1. First, the user clicks the **Stope Shaper** button on the interface to identify and extract the boundary stopes.
2. Second, the user clicks the **Extract Ore blocks** button on the interface to identify and extract the ore blocks in the boundary stopes.

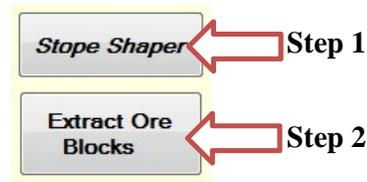


Figure 18. Stope Shaper.

2.8 PROGRESS REPORTER

The Progress Reporter presents updated progress information continuously for a particular implementation related to the application. For example, during the implementation of the stope optimiser, the Progress Reporter presents updated information about how many stopes have been evaluated during the generation of the optimum solution. Figure 19 shows a typical report provided by the Progress Reporter when the stope optimiser is implemented. According to this example, 51 out of a total of 4547 positive stopes have been evaluated to generate the optimum solution at the time considered.



Figure 19. Progress Reporter for the stope optimiser.

APPENDIX B

The appendix B shows the C# object-oriented programme source code for the following four cases.

- I. Converting the geological model into an economic model
- II. Creating a unique identifier to facilitate stope generation
- III. Generating stopes
- IV. Extracting a list of positive stopes

1. CONVERTING THE GEOLOGICAL MODEL INTO AN ECONOMIC MODEL

```

public List<Cell> CalculateEconomicValuesForPercentage(double metalprice, double miningcost,
double incrementcost, double refining, double processing, double recovery, object sender,
DoWorkEventArgs e)
{
    BackgroundWorker bWorker = (BackgroundWorker)sender;
    double z; double CollectionUpper; double CollectionLower;
    List<Cell> rest_levels_set = null;
    List<Cell> combinedset = new List<Cell>();

    CollectionLower=cellService.getLowerForCalculations();
    CollectionUpper = cellService.getUpperForCalculations();
    List<Cell> converted_collection = cellService.getConvertedCellList();
    double level_increment = converted_collection.First().Dim.CellH;
    int count = 0; // for background worker
    for (z = CollectionUpper; z >= CollectionLower; z -= level_increment)
    {
        rest_levels_set = converted_collection.Where(rest => rest.Coord.ZC == z).ToList();
        foreach (var cell in rest_levels_set)
        {
            cell.EVal=((metalprice-refining)*(recovery/100)*cell.metalweight)-
            ((miningcost+processing)*cell.totalweight);
        }
        miningcost += incrementcost;
        combinedset.AddRange(rest_levels_set);
        count++;
        bWorker.ReportProgress(count);
    }
    values_calculated_collection = combinedset;
    cellService.modifyValueCalculatedCollection(values_calculated_collection);
    e.Result = "";
    return combinedset;
}

```

2. CREATING A UNIQUE IDENTIFIER TO FACILITATE STOPE GENERATION

```

public void LableCells()
{
    if (cellList != null)
    {
        int xLabelValue = 1;
        int yLabelValue = 1;
        int zLabelValue = 1;
        double prvX = -20000;
        double prvY = -20000;
        double prvZ = -20000;
        int index = 0;
        foreach (Cell cell in cellList)
        {
            CellLabel label;
            if (index == 0)
            {
                label = new CellLabel(1, 1, 1);
            }
            else
            {
                if (cell.Coord.XC > prvX)
                {
                    xLabelValue++;
                    yLabelValue = 1;
                    zLabelValue = 1;
                }
                else if (cell.Coord.YC > prvY)
                {
                    yLabelValue++;
                    zLabelValue = 1;
                }
                else if (cell.Coord.ZC > prvZ)
                {
                    zLabelValue++;
                }
                label = new CellLabel(xLabelValue, yLabelValue, zLabelValue);
            }
            cell.Label = label;
            prvX = cell.Coord.XC;
            prvY = cell.Coord.YC;
            prvZ = cell.Coord.ZC;

            index++;
        }
    }
}

```

3. GENERATING STOPES

```

public ICollection<Stope> getStopeModelWithLevels(int cellsInX, int cellsInY, int cellsInZ,
String gradeUnit, object sender, DoWorkEventArgs e)
{
    BackgroundWorker bWorker = (BackgroundWorker)sender; int count = 0;
    cellList.OrderBy(cell => cell.Coord);

    if (this.cellList == null &&
        CollectionLowerLimit == null && this.cellList.ToList().Count == 0)
    {
        initStopeModel();
        if (this.cellList == null &&
            CollectionLowerLimit == null && this.cellList.ToList().Count == 0)
        {
            return null;
        }
    }
    var reader = new CellsRangeReader(this.CellList);

    int incrmntX = (cellsInX - 1); int incrmntY = (cellsInY - 1); int incrmntZ = (cellsInZ - 1);
    int lblMaxX = cellList.Max(i => i.Label.LabelX) - (cellsInX - 1);
    int lblMaxY = cellList.Max(i => i.Label.LabelY) - (cellsInY - 1);
    int lblMaxZ = cellList.Max(i => i.Label.LabelZ) - (cellsInZ - 1);

    Cell first = cellList.First();
    double dimX = first.Dim.CellW; double dimY = first.Dim.CellL;
    double dimZ = first.Dim.CellH;
    double stopeDimX = dimX * cellsInX; double stopeDimY = dimY * cellsInY;
    double stopeDimZ = dimZ * cellsInZ;
    int lblx = 0;

    do
    {
        int endX = lblx + cellsInX; reader.ChangeX(lblx, endX);
        double coordStartX = CollectionLowerLimit.XC + (lblx - 1) * dimX;
        double coordEndX = coordStartX + dimX * incrmntX;
        double coordCenterX = coordStartX + (dimX / 2) * incrmntX;
        lblx++;
        int lbly = 0;

        do
        {
            int endY = lbly + cellsInY; reader.ChangeY(lbly, endY);
            double coordStartY = CollectionLowerLimit.YC + (lbly - 1) * dimY;
            double coordEndY = coordStartY + dimY * incrmntY;
            double coordCenterY = coordStartY + (dimY / 2) * incrmntY;
            lbly++;
            int lblz = 1;

            do
            {

```

```

int endZ = lblz + cellsInZ; reader.ChangeZ(lblz, endZ);
double coordStartZ = CollectionLowerLimit.ZC + (lblz - 1) * dimZ;
double coordEndZ = coordStartZ + dimZ * incrmntZ;
double coordCenterZ = coordStartZ + (dimZ / 2) * incrmntZ;

lblz += cellsInZ;

double EVal = 0; double metalweight = 0; double totalweight = 0;
double density = 0;
int TotalCellsInAStope = cellsInx * cellsInY * cellsInZ;
if (reader.GetCells().Count() == TotalCellsInAStope)
{
    foreach (Cell cell in reader.GetCells())
    {
        EVal += cell.EVal; metalweight += cell.metalweight;
        totalweight += cell.totalweight;
        density += cell.density ?? 0;
    }

    Stope newStope = new Stope();
    newStope.stopeEVal = EVal; newStope.metalweight = metalweight;
    newStope.totalweight = totalweight;
    newStope.density = density / TotalCellsInAStope;

    if (gradeUnit == "Grade- %")
    {
        newStope.grade = (metalweight / totalweight) * 100;
    }

    else if (gradeUnit == "Grade- g/t")
    {
        newStope.grade = (metalweight / totalweight) * 100;
    }

    newStope.Coord = new Coord(coordStartX, coordStartY, coordStartZ);
    newStope.EndCoordinates = new Coord(coordEndX, coordEndY, coordEndZ);
    newStope.CenterCoordinates = new Coord(coordCenterX, coordCenterY, coordCenterZ);
    newStope.Dim = new Dim(stopeDimX, stopeDimY, stopeDimZ);
    this.stopeList.Add(newStope);
    count++;

    if (count % 1000 == 0)
    {
        bWorker.ReportProgress(count);
    }
}
} while (lblz <= lblMaxZ);
} while (lbly <= lblMaxY);
} while (lblx <= lblMaxX);
e.Result = "";
return this.stopeList;
}

```

4. EXTRACTING A LIST OF POSITIVE STOPEs

```
public int positvestopes_count = 0;
public Boolean UpdatePositiveStopes()
{
    cellService.removePositiveStopes();
    List<Stope> positvestopes = cellService.extractPositiveStopes();
    positvestopes_count = positvestopes.Count;
    if (positvestopes != null && positvestopes.Count > 0)
    {
        foreach (Stope stope in positvestopes)
        {
            cellService.modifyPositiveStopes(stope);
        }
        return true;
    }
    else
    {
        return false;
    }
}
```