

Science and Mathematics Education Centre

**Mathematical Errors in Fractions Work: A Longitudinal Study of
Primary Level Pupils in Brunei**

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**This thesis is presented for the Degree
of
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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

Signature:

Date: 16 March 2004
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Abstract

This study examined the different types of mathematical errors exhibited by primary level pupils in Brunei when working with fractions. In addition, the study examined pupils' attitudes towards the learning of fractions and investigated if there were gender differences among Bruneian pupils' performances with fractions and with their attitudes towards fractions. The study was longitudinal in nature and its two phases involved a single cohort of Primary 5 pupils followed through a full year period in four government-funded primary schools in Brunei Darussalam. Pupils' mathematical errors were assessed by means of researcher-developed paper-and-pencil tests, while pupils' attitudes towards the learning of fractions were measured by means of an adapted version of attitude questionnaire that has been used previously with Bruneian pupils.

Guided by six research questions, a number of statistical analyses were carried out to ensure the validity and reliability of the instruments used. These included piloting and revising the instruments, the use of Cronbach's alpha with the items in the attitude questionnaire, and the calculation of the Pearson Product Correlation Coefficient between scales of the questionnaire. The data was analysed by calculating the percentages and means of occurrences of each type of error. Paired and independent sample *t*-tests were carried out in order to investigate gender differences in pupils' errors and the impact of further instruction on fraction at the P6 level, while the GLM test was administered in order to investigate if there were significant change in pupils' attitudes towards fractions from the pre- to the post-tests. Qualitative information obtained through pupils' interviews, field notes and lesson observations was used to support the quantitative data.

The study revealed that though pupils' achievement in the post-test improved, their performances on fraction work remained generally unsatisfactory. Many pupils in the study continued to have difficulty with the basic operations on fractions and resorted to the use of keyword strategies in dealing with word problems. Despite the pupils' unsatisfactory performance in the diagnostic tests, they generally held very positive attitudes towards the learning of fractions. No significant gender differences

were observed either in pupils' performance in working with fractions tasks nor with their attitudes towards the learning of fractions.

The findings of this study also highlight a number of issues for mathematics teachers to consider when dealing with fractions, and the findings also have implications for the quality of the instructional activities provided by the teachers, for the impact of language transfer in the medium of instruction – that is, from Bahasa Melayu to English at the pupils' Primary 4 level– and for the quality of the teacher training program in Brunei.

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CHAPTER 1

BACKGROUND AND INTRODUCTION TO THE STUDY

The principal aim of this study was to identify the patterns of errors made by a sample of Primary 5 (hereafter referred to as P5) and Primary 6 (hereafter referred to as P6) pupils in Negara Brunei Darussalam when working with fractions.

This chapter provides an introduction to the study and explains why research of this nature was undertaken, and why it is important for the education of primary school pupils in Brunei. The chapter begins with background details about the study along with a number of definitions of important terms used which will lead to the working definitions adopted for use in the study. To place the research into context, some information on the demographic background and the current education system and policy of Negara Brunei Darussalam is also provided in this chapter, including a discussion on the importance of the teaching and learning of mathematics. The chapter also briefly examines the rationale for undertaking the study; the study's aims; the six research questions that guided the study; a brief summary of the methodology employed; the significance and scope of the study as well as its limitations. The chapter concludes with an overview of the thesis.

1.1 Background to the study

The study of fractions is foundational in mathematics, yet it is among the most difficult topics for school pupils to understand (Cramer, Behr, Post, & Lesh, 1997). Despite the acknowledged difficulty in learning fractions, children encounter fractions and fraction-related concepts both in real-life and in classroom situations. A firm understanding of fractions undoubtedly helps a child make sense of a considerable number of other ideas in his or her daily life. Despite the context in which children engage fractions, it is generally agreed that this topic provides teachers with insight into general developments in children's understanding of, and relations among, numbers (Porteous, 1998). These understandings are built on children's personal experiences, intuitions, and formal knowledge gained in the classroom (Bezuk & Bieck, 1993; Brinker, 1998). Fractions are known to be

complex in character and provide important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic operations in the later years of children's school experience. The topic continues to present problems and difficulties for children in primary schools who have difficulty in making the critical conceptual link between mathematics strands such as space and measurement (Pitkethly & Hunting, 1996). Hanson (2001) wrote that one of the most frustrating areas for teachers and pupils alike is the study of fractions, specifically operations with fractions. Year after year, pupils learn and forget how to add, subtract, multiply and divide with fractions. The main reason for the difficulties, as Hanson has suggested, is that pupils seem to want to memorise formulas or algorithms instead of understanding the concepts.

The acknowledged difficulties in learning fractions are reflected and documented in a number of studies that have examined different aspects of this topic. As early as 1958, Hartung acknowledged that the fraction concepts are complex, cannot be grasped all at once, and must be acquired through a long process of sequential development. Orton (1992) supported this view by claiming that the fraction concept develops over a long period, during which time children experience the different meanings of fractions in a variety of situations. As Bezuk and Bieck (1993) have said,

“Fractions are a rich part of mathematics, but we tend to manipulate fractions by rote rather than try to make sense of the concepts and procedures. Researchers have concluded that this complex topic causes more trouble for elementary and middle school students than any other area of mathematics”. (Bezuk and Bieck, 1993 quoted in Chapin and Johnson, 2000, p.73).

Based on the acknowledged difficulties noted above, findings and recommendations from the above-mentioned researchers, it suggests that in teaching fractions, teachers should provide experiences that involve other mathematical concepts, including number, length, weight and money, and these should be set in meaningful situations to which children can relate.

More recent studies by Mack (1998); Tzur (1999) and Anderson, Anderson and Wensell (2000) have revealed that understanding and using fractions are tasks that

have traditionally been difficult for pupils. National assessment results in the United States (NCTM, 1989) showed that even older pupils have trouble in working with and understanding fractions. The National Centre for Educational Statistics in the United States (1990) reported that only 46% of twelfth graders who took the National Assessment of Educational Progress Test could consistently solve fraction problems, while Carpenter, Linquist, Brown, Kouba, Silver and Swafford (1988) in reporting on the Fourth National Assessment of Educational Progress (1988), stated that only 44% of eleventh graders could choose the correct answer for the following item:

$5\frac{1}{4}$ is the same as (a) $5 + \frac{1}{4}$, (b) $5 - \frac{1}{4}$, (c) $5 \times \frac{1}{4}$, (d) $5 \div \frac{1}{4}$

An extensive review of the literature on pupils' learning difficulties and mathematical errors in using fractions is provided in Chapter 2.

This study is also concerned with aspects of pupils' attitudes towards fractions in a classroom. In an attempt to identify the pupils' attitudes towards fractions, a search for indicators was performed. Borasi (1990) proposed that attitudes should not be viewed in isolation; rather, it should fit together into the analysis of errors and their performance in mathematics. Before errors are considered, it is necessary to define errors. According to Hawker and Cowley (1998), errors are mistakes or a condition of being wrong (p.163), and therefore "mathematical errors" are mistakes or conditions of being wrong exhibited in performing mathematical calculations and mathematical problem solving. Therefore, for the purpose of this study, the working definition of "errors" is constructed from Hawker and Cowley's definition as it is thought to be the most appropriate. A further discussion on the relationship between attitudes and pupils' performance in mathematics is provided in Chapter 2, Section 2.14, p. 60.

As Swedosh (1996) and Ashlock (2002) have demonstrated, there is a relationship between errors and misconceptions and they suggest that a number of errors exhibited by pupils in doing mathematical works were the result of misconceptions that they acquired before instruction. Even though errors are also associated with misconceptions, Bransford and Vye (1989) cautioned that research on errors should be distinguished from research on misconceptions. They noted that whereas errors

are typically associated with performance after instruction, misconceptions include inaccurate conceptions before instruction. Oliver (1989) distinguishes between slips, errors and misconceptions. According to Olivier, slips are wrong answers due to processing; they are not systematic but are sporadically and carelessly made by both experts and novices; they are easily detected and are spontaneously corrected. On the other hand, he describes errors as symptoms of the underlying conceptual structures that are the cause of errors. Accordingly, he believes that it is these underlying beliefs and principles in a pupil's cognitive structure that are the cause of the "systematic conceptual errors" that he calls misconceptions.

In addition, Bell (1984, p.58) defines a misconception as the implicit belief held by a pupil, which governs the errors that the pupil makes. From what Bransford, Vye, Oliver and Bell described, misconceptions always refer to erroneous thinking that pupils apply consistently and which can be diagnosed by listening and watching carefully when a child answers strategically designed tasks.

The current study was more concerned with the errors the pupils made in carrying out computational and word problem solving in fractions on the two diagnostic tests administered just after the pupils received formal instructions on fractions in two consecutive primary school levels and years.

Pupils' errors, if detected at an early stage, should be corrected in order to prevent them from developing further. Errors become evident at any stage and in any topics learnt by pupils. Therefore, it is the responsibility of mathematics educators to attempt to eliminate or at least to minimise this learning problem. An immediate intervention is necessary in order to avoid having those errors become deep seated in the pupils' minds. Traditionally, teachers use the re-teaching approach to overcome mathematical errors and misconceptions, but Dole, Cooper and Lyndon (1997), after a meta-analysis of earlier studies, suggest that the re-teaching approach has not always resulted in sustained conceptual change and eradication of the error. Currently, alternative programs have been described in the literature that recommends actively using pupils' error patterns/misconceptions as a focal point for intervention and conceptual change (Borasi, 1985a, 1985b). In this respect, Cox concluded from her research in 1975 that not only did children make systematic errors, but without instructional intervention they also continued with the error

patterns for long periods. Cox emphasised that teachers must look for these patterns in the work they collect from pupils having difficulty with computation as the first step in correcting the error.

The Brunei education system continues to expand rapidly, and research on primary mathematics education has gradually increased since the last half decade, especially in view of the increasing number of students enrolled in the Masters Degree in Mathematics Education program at the University of Brunei Darussalam. Comprehensive studies on Bruneian pupils' errors in mathematics, particularly on fractions, have been carried out lately but are still limited in quantity and scope. Several exceptions are those of Fatimah (1998), See and Yusof (2000), Raimah (2001) and Shamsiah and Clements (2002). The present study is one of the few on Bruneian pupils' errors in working with fractions and is in fact the first one to embark longitudinally, following the same cohort of pupils over a two-year period.

It was this knowledge of the paucity of studies in the area of pupils' mathematical errors on fractions that prompted me to embark on this study. In addition, my twenty years experience as a primary and secondary school mathematics teacher as well as my current position as a primary mathematics teacher educator also motivated me to undertake a study on the pupils' difficulties in mathematics, particularly the problems in working with fractions. It has been my interest to keep abreast with the current phenomenon concerning pupils' learning difficulties in mathematics and I have been involved in a number of small-scale studies on mathematics education conducted in Brunei schools as well as a joint comparative study with colleagues from Singapore National Institute of Education. These activities have given me the experience and opportunity to conduct the current study with full support from the Brunei Ministry of Education and the schools involved in terms of the co-operation rendered to me during the pilot and field study. It also helps me to develop a very satisfactory rapport not only with the principals of the schools involved but with all the teachers and pupils participating in this study. In addition, as a post-graduate student at an international university, I have been exposed to current issues concerning the teaching and learning of mathematics, and have been impressed with the constructivist view of learning. The main thrust of the constructivist view is that a subject such as mathematics is learnt by individuals constructing ideas, processes

and understanding for themselves rather than through the transmission of pre-formed knowledge from teacher to learner that now dominates conceptions of mathematics learning in many parts of the world – including Brunei (Glenda, 1996; Booker, Bond, Briggs & Davey, 1998). Evidence for this view began to surface as errors that children made were analysed and the misconceptions that they had formed were seen to fall into common patterns rather than reflect individual inconsistencies. This exposure to constructivism further consolidated my interest in pursuing a study concerning Bruneian pupils learning difficulties in mathematics, and particularly on fractions.

Further, I wanted to examine pupils' attitudes towards the learning of mathematics in general and fractions in particular. Attitude was considered important because it has consistently been attributed as a contributing factor in determining the pupils' performance in the learning of mathematics (Groff, 1994; Lokan & Greenwood, 2000). Although previous studies have examined associations between pupils' attitudinal outcomes and pupils' mathematical performance (Brandon, Newton & Hammond, 1987; Lokan & Greenwood, 2000; Prior, 2000), this study is distinctive in that it examined the relationship between pupils' attitude towards the learning of fractions along with their cognitive performance in this topic. As Leder (1987) pointed out, a student's attitude towards mathematics is not a one-dimensional construct. Just as there are different topics in mathematics, there are potentially a variety of attitudes towards those topics. In this study, the focus is on pupils' attitudes towards fractions.

Results from this study will have implications both for mathematics teachers and for pupils. I believe that the outcomes of the study will play an important role in allowing Bruneian mathematics teachers to develop their classroom skills, especially in identifying the types of errors exhibited by their pupils. This is because the accuracy of the diagnosis and the information obtained from it will enable teachers to plan and develop specific intervention strategies, with a greater chance of successfully helping the pupils overcome their learning difficulties and progress towards mathematical achievement (Dole *et al.*, 1997). Research on error patterns has pedagogic implications (Maurer, 1987). If teachers know the various types of errors the pupils made in relation to particular topics, they can develop programs of

instruction in an effort to prevent the perpetuation of such errors (Stefanich & Rokusek, 1992). Therefore, it is further hoped that the findings of this study can assist in improving the quality of the teaching-learning processes in primary mathematics generally, and more specifically the teaching of fractions in Brunei. This is necessary because Brunei educators have to become aware that education is continually changing and that the mathematics curriculum worldwide has seen many changes over the past ten to fifteen years regarding how fractions should be delivered and taught.

At this point it is appropriate for the readers to gain an insight into the Bruneian education system.

1.2 Brunei Darussalam in Context

Brunei Darussalam, located on the island of Borneo, was a British protected state from 1888 until it gained its independence on 1st January 1984. Following independence, a National Education System called the Bilingual System of Education was implemented in 1985. The national language of Brunei is Bahasa Melayu and it is the language spoken by the majority of people, however English is spoken and understood by a wide spectrum of the population.

1.2.1 Brunei Darussalam Education System

Negara Brunei Darussalam is located in the north-western corner of Borneo island, between the longitudes of 114 degrees 04' and 11 degrees 23', and north latitudes of 4 degrees and 5 degrees 05'. Bounded in the north by the South China Sea and by the Malaysians state of Sarawak on all the other sides, it has an area of 5,765 square kilometres.

The country's estimated population is around 350,000 of which two-thirds are Malays. The rest includes indigenous people, Chinese, Indians and expatriates. The average population growth is around 3% annually (Brunei Yearbook, 2003). Brunei Darussalam is divided into four districts; namely, Brunei/Muara, Tutong, Belait and Temburong. Bandar Seri Begawan, the capital, is the seat of the Government and centre of commerce. Muara is the country's principal port; Seria, the centre of the oil

and gas industry and Kuala Belait, Tutong and Bangar are the administrative centres for the districts of Belait, Tutong and Temburong respectively. Islam is the official religion, with His Majesty the Sultan and Yang Di-Pertuan of Brunei Darussalam as the head of the Islamic faith in the country. However, as freedom of religion is guaranteed in the Constitution, other religions practised are Christianity and Buddhism. As an Islamic nation, Brunei honours everything that embodies Islam in a moderate way (Brunei yearbook, 2003).

The Education policy of Brunei Darussalam aims to set up an effective, efficient and equitable system of education in agreement with the national policy of a Malay Islamic Monarchy, which includes the:

- Implementation of a national education system that prioritises the use of the Malay Language as the official national language and the use of major languages such as English and/or Arabic as a medium of instructions;
- Provision of 12 years of education for every student – that is, 7 years of primary education including a year of pre-school, 3 years of lower secondary and 2 years of upper secondary, or vocational or technical education;
- Provision of an integrated curriculum as well as suitable and uniform public examinations administered according to the level of education, including special needs, in all schools throughout the nations;
- Provision of Islamic Religious education in accordance with Ahli Sunnah Wal-Jamaah, through the school curriculum;
- Provision of facilities for mathematics, science, technical and information and communications technology education in order to enable students to obtain knowledge and skills that are relevant and necessary in the constantly changing world of employment;
- Provision of self-development and enrichment programs through co-curricular activities in accordance with the national philosophy;
- Provision of opportunities in higher education for those with appropriate

qualifications and experience, such opportunities to be offered based on national needs as and when they arise; and

- Preparation of the best possible educational infrastructure in order to fulfil national needs.

The above policy is reflected in the country's Aim of Education, namely to maximise the intellectual, spiritual, emotional, social and physical potential of every individual for the formation of a developed society that is strongly founded on the Malay Islamic Monarchy philosophy (Brunei Yearbook, 2003 p.78).

The formal school system adopted in Brunei Darussalam conforms to a 7-3-2-2 pattern. This pattern represents primary, lower secondary; upper secondary and pre-tertiary levels respectively. The number of school days in a year varies between 202 and 210. There are four school terms starting from January to March, April to June, July to early September and mid-September to December, with 1.5 to 2.5 weeks breaks in between the terms. The bilingual system of education was implemented in all government schools in 1985, and to all non-government schools in 1992 as the *Melayu Islam Beraja (MIB)* or Malay Islamic Monarchy and bilingual policy were implemented in response to the aim set out in the 1972 education report (Ahmad, 1989). This was to ensure the position of Malay as a National Language and to place more emphasis on Islamic studies in the school curriculum. In order to achieve these set aims, the education system in Brunei Darussalam is highly centralised and controlled by the Ministry of Education. Therefore, the primary, secondary and tertiary education schooling in government schools and institutions is free to citizens and permanent residents of Brunei Darussalam. Currently there are 126 government primary schools, 26 government secondary schools and 70 non-government primary and secondary schools. No non-government schools or their teachers are funded by the government, though they are still required to register with the Ministry of Education to conform to the Education Act, 1984 (Brunei Year Book, 2003).

Primary Education in Brunei Darussalam is divided into three stages, namely pre-school, lower primary and upper primary, which makes a total of 7 years including one year at pre-school level. Primary Education aims to give children a firm foundation in the basic skills of writing, reading and arithmetic as well as to provide

opportunities for their personal growth and character development. Pre-school education became part of the primary school system in 1979, and since then it has become compulsory for children at the age of five to enrol in pre-school classes for one year before being admitted to Primary 1. On completion of their seven years of primary level education, pupils are required to sit for the Primary Certificate of Examination (PCE), which is taken at the end of Primary 6. On completion of Primary 6, pupils enter the lower secondary level at Secondary 1 for those who take the exam once, and level 2 for those who fail the exam twice but are automatically promoted in accordance with the Brunei Darussalam Education Policy.

The lower secondary level has a duration of three years. At the end of the third year, students sit for the *Penilaian Menengah Bawah (PMB)* or Lower Secondary Assessment Examination (BJCE). Those who pass all the core subjects (English, Bahasa Melayu, Science and Mathematics) can pursue two years of upper secondary education leading to the Brunei Cambridge General Certificate of Education (GCE 'O' Level) examination, while those who fail one of the core subjects but pass other subjects such as History, Geography, Art, Commercial Studies and Religious Study, can pursue three years of upper secondary education leading to the Brunei Cambridge General Certificate of Education (GCE 'N' Level) examination. In addition, students have an option to pursue craft and basic technical level courses at technical or vocational institutions or enter the employment market after the lower secondary level.

Education at upper secondary level is general with some provision for specialisation in science, arts and technical fields. Those with adequate and relevant Ordinary level passes may go on to do a two-year pre-university course leading to the Brunei-Cambridge Advanced Level Certificate of Education examination (GCE 'A' Level). Educational and training at the post-secondary level in both academic and professional fields is provided by Universiti Brunei Darussalam, Institute of Technology, Pengiran Anak Puteri Rashidah Sa'adatul Bolkiah College of Nursing, Sultan Saiful Rizal Technical College and various vocational education institutions and training centres.

1.2.2 Teaching and Learning Mathematics in Brunei Darussalam Primary Schools

In the Brunei education system, mathematics is given considerable emphasis from the commencement of the primary school. It has always been considered a core subject together with Bahasa Melayu, English and Science. It is only during the pre-school stage that mathematics is not taught as a subject by itself but is integrated with other subjects such as language and general studies using a thematic approach. From primary 1 to primary 3, mathematics is taught as a subject by itself in Bahasa Melayu (the mother tongue) for about five hours a week, which is nearly a quarter of the total periods for all subjects taught at the primary school level. From primary 4 onwards, mathematics is taught in English (a second language), also for about five hours a week, as well as other subjects such as Geography, Science and of course English as a subject in its own right. Other subjects such as History, Art, Religious Study, and Physical Education are taught in Bahasa Melayu in addition to Bahasa Melayu itself as a subject. This system of Bilingual Education was first implemented in 1985; a year after Brunei obtained its full independence. It became evident in this study that the switch to English in the teaching of mathematics from Primary 4 onwards was a factor that had a significant impact on the pupils' performances in the diagnostic tests administered to them.

As outlined in the Mathematics Syllabus for Upper Primary Schools (Ministry of Education, 1999), the objectives of primary mathematics education in Brunei are to enable the pupils to:

- master the language of mathematics and be able to interact and communicate effectively.
- master mathematical concepts.
- comprehend and master mathematical operations together with the appropriate methodologies.
- recognise and understand the use of measuring instruments where appropriate.
- learn to solve problems connected with measurements (weight, length, volume and time) and money.
- learn to recognise, construct and use patterns in mathematics.

- learn how to collect data, as well as to construct and interpret graphs.

In line with the above list of objectives, the primary mathematics curriculum covers a wide spectrum of the number, geometry and measurement components of mathematical content. Although the mathematics curriculum emphasises pupils acquiring mathematical knowledge with understanding and the development of positive attitudes towards mathematical skills through the provision of developmentally appropriate methods of mathematics teaching and learning, local studies have shown that classroom practices do not reflect these emphases (Zaitun, 1988; Yusof, 1993). Mathematics teaching in Brunei has been viewed as to be very traditional (RoseDalilah, 1999). A number of local studies (Zaitun, 1988; Yusof, 1993; RoseDalilah, 1999) have raised several issues pertaining to the teaching and learning of mathematics. Sainah (1998) proved in her study that mathematics is a difficult subject to teach and to learn, especially from Primary 4 onwards when English becomes the medium of instruction. Word problems have been found to be the most difficult topic to teach (Veloo & Lopez-Real, 1994) while other issues in the teaching and learning of mathematics can be categorised as contextual, epistemological, pedagogical, resources-influenced and lingual.

Contextual Issues

Most teachers rely heavily on textbooks, workbooks and teachers' guides among other resources in teaching mathematics in Bruneian schools (Asmah, 2001). Teachers are highly dependent on the prescribed term scheme of work and the teachers' guides that are provided by the Curriculum Development Department of the Ministry of Education. Prior to term examinations, teachers normally use past years examination papers as revision exercises for the pupils as those exam papers normally cover the same syllabus that they are teaching. With these exercises, teachers believe that they are able to familiarise pupils with the actual examination format.

Epistemological Issues

Two epistemological issues identified by RoseDalilah (1999) in relation to mathematics teaching and learning in Brunei were that teachers are reluctant to change their beliefs about the acquisition of knowledge, and that pupils' passivity is related to cultural influences. She demonstrates that teaching in Bruneian schools is characterised by telling and explaining, demonstrating and verbal and written drill exercises. Pupils are encouraged to learn by memorising – a practice that is equivalent to rote learning. Despite the fact that during their teachers training days pre-service teachers are exposed to a number of good pedagogical practices, Asmah (2001) asserted that typically, teachers in her study asked questions that required pupils to respond with short answers. My observations of Bruneian classrooms over many years verify this view.

Pedagogical Issues

Instructional methods in the primary schools are mainly teacher-centred (Asmah, 2001). Many teachers use the traditional approach called direct instruction or the “chalk and talk” method (RoseDalilah, 1999; Asmah, 2001). Due to its highly examination-oriented system, teachers in Brunei believe that it is a priority to ensure their pupils pass examinations. In most mathematics classes, the pupils sit alone and work to solve the given exercises. If they cannot find the answers, many will become frustrated, as asking questions or explanations from the teachers is not the norm (RoseDalilah, 1999). Therefore, she concludes that this could be one of the reasons why students in the secondary schools tend to dislike mathematics. In addition to this, many students think that mathematics is only for smart or talented pupils who can solve mathematical problems easily and successfully by themselves.

Resource-related Issues

My observation in Brunei schools shows that teachers in Brunei follow a very demanding school calendar and thus many have limited time to cover the topics in a developmentally appropriate manner. Teachers tend to rush to finish the syllabi so that they have time to revise past examination papers. This is especially true for P6 teachers, as their pupils sit for the Primary Certificate Education (PCE) examination

during the first week of October, which is about two months before the actual school term ends. Many schools lack resources such as manipulating materials and well equipped mathematics resource rooms, thus teachers are unable to provide rich experiences for the children to learn mathematics in a developmentally appropriate manner so as to make teaching and learning more effective.

Teachers' reliance on textbooks is a particular issue. Many teachers are very dependent on the textbooks and workbooks provided by the schools. Consequently, many teachers of each primary level class who are provided with the "teachers' guide" do not treat it merely as a guide but adhere strictly to it and treat it as a "bible". This practice makes their teaching approach narrow and rigid. Teachers are reluctant to try alternative approaches and thus their teaching can be very monotonous (RoseDalilah, 1999).

Language Issues

The Bilingual system of Education was first implemented in all government schools in 1985 and fully implemented to all non-government schools in 1992. Although the aim of the bilingual system is to develop high level of competency in both Bahasa Melayu and the English language, local studies have shown that most pupils' mastery of English is still short of expectations (Liew, Hassan, Ramli, Jee, Baktavatchalan & Gallop, 1993) with many pupils finding that learning academic subjects in the second language is difficult. This could be one of the reasons why pupils in the primary schools in Brunei encounter learning difficulties, not only specific to mathematics, but also generally in other subject areas as well. The above issues suggest that further investigations are needed to examine the conditions of teaching and learning in mathematics classrooms in Brunei.

Though I do not have any empirical evidence to support my view, from my general observations, pupils' experiences in learning mathematics in Brunei schools are limited to learning algorithms for computation with little or no conceptual understanding. As mentioned earlier, the nature of the teaching is very traditional with an emphasis on the chalk and talk method. This was acknowledged by the Permanent Secretary in the Ministry of Education, Brunei Darussalam, in 1994 when he commented that the chalk and talk method used in teaching mathematics

would not lead to meaningful learning (Zubaidah, 1994). The Permanent secretary expressed his delight with those teachers who worked hard and used several methods of teaching designed to stimulate pupils in their learning. He stressed that the use of suitable teaching aids could help to achieve the desirable objectives.

1.2.3 Primary 4, Primary 5 and Primary 6 Syllabus Expectations with Respect to Fractions

Rationale, aims and objectives of the syllabus

The rationale for the national mathematics curriculum for upper primary schools (Curriculum Development Department, 1999) refers to the need to foster skills in calculation [with] speed and accuracy, and “to enable each pupil to become numerate” (p.1). The syllabus asks teachers to seek to develop in their pupils “a positive attitude and logical approach towards mathematics,” and “competence in basic mathematics which will also be a preparation for the study of mathematics at secondary level” (p.1). A preamble on “objectives” in the syllabus document states that by the end of primary school pupils should have “mastered the language of mathematics and be able to interact and communicate effectively.” In addition, the pupils should have “mastered the mathematical concepts” and “comprehended and mastered mathematical operations together with the appropriate methodologies” (p.2).

Content expectations for Fractions in the Primary 4,5 and 6 Syllabi.

The content of the syllabus for Primary 4 are the extension of what the pupils have already learnt in Primary 3. The content is further extended in the P5 and P6 syllabi. The focus is mainly on equivalent fractions, comparison of fractions, addition and subtraction of simple fractions, multiplication and division of fractions with denominators not greater than 100, fractions of a quantity, expressing one quantity as a fraction of another, conversion of fractions to decimals, expressing fractions and decimals as percentages as well as simple word problems on fractions. All these areas were examined in the current study in terms of pupils’ error patterns and

misconceptions encountered. The detailed syllabi of the fractions component for P4, P5 and P6 can be found in Appendix D on page 205.

1.3 Rationale for the Study

Making mistakes tends to generate feelings of frustration, not only among pupils, but among teachers as well. While errors are something we try to avoid, nevertheless they seem to be an inevitable outcome of almost any learning process. Rather than condemn errors made by pupils, teachers should capitalise on them for a better and deeper understanding of pupils' problems regardless of any tendency for the pupils to repeat them (Borasi, 1994; Swedosh, 1996).

The study of mathematical errors has significantly influenced the field of mathematics assessment and intervention, providing alternative perspectives as to what error patterns indicate (e.g. Ashlock, Johnson, Wilson & Jones, 1983; Ashlock, 1994). Traditionally, pupils who made errors in their work were regarded as suffering from some type of learning disability (Kephart, 1960), and it was considered that they made errors because they lacked the knowledge of "correct" algorithms. Such a deficit model of error production suggested that the pupils exhibiting the error had learned nothing as a result of the initial teaching effort. As such, the inadequacies of pupils' knowledge in mathematics and the resulting learning difficulties have been an important theme of research in the last decade. A number of studies in the area of concept learning have shown that pupils frequently hold ideas that are different from the accepted mathematical view (Mansfield & Happs, 1992; Fong, 1995). In some cases, those ideas were consistent and they made sense from the pupils' point of view. However in other cases, there was some evidence of pupils demonstrating a confused, inconsistent way of thinking.

The fractions component of the mathematics syllabus is not an exception. Numerous studies on learning difficulties and misconceptions on learning fractions have been carried out in the past decades (e.g. Hartung, 1958; Pitkethly & Hunting, 1996; Taber, 1999; Tirosh, 2000). This research shows that fractions are a well-identified area of difficulty for many children and even for some adults (Porteous, 1998). Various reasons have been attributed as for this difficulty, with most researchers agreeing that fractions are too abstract for children to see and understand (Hart;

1981; Saenz-Ludlow, 1995). The ways that fractions are taught in classes have also been identified as one of the reasons why children cannot grasp the concepts successfully (Hanson, 2001). This researcher believes that the direct-instruction method is unsuitable for teaching difficult topics such as fractions.

Another relevant factor is that of mathematical performance. This has often been associated with gender (Hyde, Fennema & Lamon, 1990; Entwisle, Alexander & Olson, 1994). While girls and boys commence basic schooling on an equal footing in the United States, by grade twelve girls are generally in a lower academic position than their male counterparts, particularly in the areas of mathematics (Fennema & Sherman, 1977). However, more recent researches carried out by (Leder, 1992; Forgasz, Leder & Vale, 1999) indicate that there is now a changing perspective gender differences in mathematical performance. A further discussion on gender differences in mathematical performance is presented in Chapter 2, Section 2.3 (p.31). Since this gender issue is widely reported in western literature, and therefore I considered that it would be informative to examine if such an issue is also prevalent in Brunei.

Besides the gender issue, researchers such as Odell and Schumacher (1998) and Prior (2000) have demonstrated that pupils' attitudes towards mathematics can be a causal factor in determining their performance in mathematics. Pupils with positive attitudes are said to achieve well and vice-versa. However, some researchers such Green (1994) suggest that there is little or no relationship between pupils' cognitive achievement and their perceptions of the classroom environment. One of the dimensions he studied was pupils' attitudes to learning mathematics. Therefore, with such alternative and often contradictory views in the literature, I deemed that it would be worthwhile to investigate such associations in the current study.

1.4 Aims of the Study

As a consequence of the above, the aims of the study were to:

- 1) diagnose and identify the types of mathematical errors made by a sample of Bruneian P5 pupils when learning fractions.
- 2) determine if P5 pupils' errors will decrease over time as they move to P6 and given that they had received further instruction on fractions

over the year.

- 3) investigate the pupils' general attitudes towards the learning of fractions.
- 4) investigate if pupils' attitudes towards the learning of fractions changes over time.
- 5) investigate if there is gender differences in pupils' performance in the diagnostic and attitude pre- and post-tests.

The above aims were made possible by implementing diagnostic pre- and post-tests coupled with attitude pre- and post-tests in a longitudinal study over the period of a year, following the same cohort of pupils.

1.5 Research Questions

In order to provide a focus for the six aims, the following research questions were formulated:

- 1) What are the prevalent mathematical errors on fractions held by Bruneian P5 pupils?
- 2) Will the same errors made by P5 pupils still be prevalent after they move to P6?
- 3) In which component/s of the six identified areas of the fractions unit do P5 and P6 pupils exhibit the most errors?
- 4) Are there gender differences in the errors exhibited by Bruneian P5 and P6 pupils?
- 5) What are the attitudes of Bruneian P5 and P6 pupils towards the learning of fractions?
- 6) Are there any gender differences in P5 and P6 pupils' attitudes towards the learning of fractions?

As mentioned earlier, research on error patterns has pedagogic implications (Maurer, 1987; Stefanich & Rokusek, 1992). Therefore, the answers to the above research questions resulted a number of implications for the teaching and learning of mathematics, particularly on fractions in Brunei. Those implications will be presented in detail in Chapter 5. To put the six research questions into context, it is

now appropriate for the reader to gain a brief insight into the methodology employed in this study. A detailed description of the methodology will be presented in Chapter 3 (p.67).

1.6 Methodology

A 28-item paper-and-pencil test which served as a diagnostic pre-test was administered to a sample of 396 P5 pupils during the second term of the school year in 2002. The same instrument (with two additional items), treated as a diagnostic post-test was re-administered to the same cohort of pupils a year later during the first term of their P6 year. The two extra items were added because they were relevant to the P6 syllabus. A detailed description of the instrument is provided in Chapter 3. The aim of administering the diagnostic post-test was to investigate if there were improvements in pupils' performance and a decrease in the frequencies of the different types of errors exhibited at the P5 level, after they received further instruction on fractions when they moved to P6. An analysis of pupils' written responses was carried out to collect data for the explanation on what types of error patterns were made and which were most prevalent. In addition, an attempt was made to ascertain in which of the six categories the errors were most likely to occur.

The study also adapted and modified an existing attitude questionnaire to measure the pupils' attitudes towards the learning of fractions. The aim was to support the data obtained from the two diagnostic tests along with the pupils' interviews and my lesson observations. The attitude questionnaire was administered a week after the pupils sat for the diagnostic pre-test and re-administered about a year later, that is, after the pupils sat for the diagnostic post-test. However, due to school transfers at the beginning the 2003 school term and pupils absentees, only 338 pupils sat for the diagnostic and attitude post-test compared to 396 pupils sat for the diagnostic and attitude pre-test. Data collected in the diagnostic and attitude pre- and post-tests were analysed using the SPSS program. In analysing the data, matters pertaining to quality criteria (Guba & Lincoln, 1989) were considered extensively. The matters related to quality criteria will be described further in Chapter 3 (p.103).

1.7 Significance of the Study

An intensive survey carried out by the Assessment Performance Unit (APU) into fractions between 1978 and 1982 showed that as pupils learn fractions, their use of particular algorithms and their understanding can lead to misconceptions and resulting errors (Womersley, 2000). Therefore, discovering some of the misunderstandings, difficulties in teaching the subject and misconceptions that occur within the topic is necessary in order to help the teachers in minimising pupils' errors. Data collected during the observations of pupils' on-task behaviour and semi-structured interviews conducted during the first and second phase of the study were used to inform the respective teachers of their pupils' difficulties in learning fractions so that they could develop teaching materials and teaching strategies for use during the following round of teaching.

Error patterns research has implications for teacher training programs (Dole *et al.*, 1997). Working as a primary schools mathematics educator at the University of Brunei Darussalam, I envisaged that the study would provide some guidance for future mathematics teachers by incorporating the findings and issues that arose from the study into the mathematics courses that are offered in the University's Diploma and Bachelor of Arts in Primary Education courses. Most important to me was that student-teachers should be alerted to make them aware of the types and nature of errors made by the pupils. Thipkong and Davis (1991) alerted educators to the influence of teacher errors and misconceptions upon their teaching, and thus on student learning. They shared their research findings on the pre-service teachers' misconceptions in interpreting and applying decimals, noting that the misconception "multiplication makes bigger, division makes smaller" was extremely prevalent. They suggested that if teachers were aware of their own errors in particular mathematical topics, great care would need to be taken so that such errors were not transferred to learners. It was expected that the findings on the pupils' errors in working with fractions and on their attitudes towards fractions would help teachers and parents to realise their roles in assisting the pupils to deal with those problems. Therefore, this study serves to inform mathematics teacher-training programs, particularly in Brunei.

In addition, through my presentations during in-service courses, departmental seminars, local and regional conferences, I anticipate that the information and findings of this study will reach parents, pre-service teachers, practising teachers, other teacher educators, curriculum developers and textbook writers in Brunei Darussalam. It is also the practice for officers who receive government funds for further studies to provide copies of the completed thesis to various departments of the Ministry of Education and the University of Brunei Darussalam library. With the sharing of the findings, issues and information, it should enable everyone concerned to strive for improvement in teacher education and instructional strategies and, more importantly, for mathematics teachers engaged in planning effective remedial strategies. As the paper-and-pencil diagnostic tests and the attitude questionnaires used in this study were newly developed and modified instruments, they will add to the existing available resources and it is anticipated that future researchers will use the instruments with other pupils, either within Brunei Darussalam or in other countries experiencing similar problems with the teaching and learning of fractions.

1.8 Overview of the thesis

This thesis is organised into five chapters. Chapter 1 has provided an overview of the thesis. In Chapter 1, the background of the study, current education system in Brunei Darussalam, the rationale for the study, the six research questions that guided the study, a brief summary of the methodology employed and the significance and scope of the study have been discussed. The chapter concludes with this summary of the whole thesis structure.

In Chapter 2, the literature that has been written in the areas of fractions learning, problems associated with it, error patterns and misconceptions held by pupils when learning fractions are examined. Gender issues and pupils' attitudes towards the learning of mathematics and fractions are also reviewed. The chapter provides a window into understanding the issues related to pupils' difficulties in learning fractions and endeavours to establish the significance of the study by pointing out how it extends the existing literature on such issues.

In Chapter 3, the processes involved, and instruments developed and used for gathering the data are described. The details cover the research paradigm adapted for

this study, how the sample was selected, the design of the diagnostic test and the attitude questionnaire, measurement and coding of the different error patterns as well as the coding of the responses from the attitude questionnaire and statistical analysis. Quality controls and ethical issues are also described in this chapter.

In Chapter 4, the results of the study are discussed and its outcomes are linked to the findings of other researchers. In the final chapter (Chapter 5), the conclusions of the study and its implications for the teaching and learning of fractions are presented along with a number of recommendations for future research.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

This chapter reviews the literature related to the research questions being investigated in this study. The chapter begins with a brief rationale for undertaking the study, and then a number of issues pertaining to the literature are reviewed. These include the learning difficulties and error patterns that occur when pupils learn fractions – a subject that encompasses the basic introductory concepts of fractions and the four mathematical operations on fractions as well as the application of fractional concepts and operations in solving word problems. Issues on the use of key word strategy in solving word problems are also discussed in this chapter. Gender has been considered a factor in determining pupils' performance in mathematics and fractions along with attitudes towards mathematics and fractions. Therefore, a review of literature concerning gender differences in mathematical performance particularly on fractions and attitude towards mathematics will also be provided. A brief conclusion ends the chapter.

A large number of studies both regionally and internationally have been carried out in the area of fractions (Bana, Farrel & McIntosh, 1997; Anderson *et al.*, 2000; Gabb, 2002; Shamsiah & Clements, 2002; Suffolk & Clements, 2003). However, an extensive review of the literature has revealed a paucity of studies concerning pupils' error patterns in the six areas of fractions; the basic concept and sequencing of fractions, addition and subtraction, multiplication and division procedures, determining fractions of a quantity, alternative form of fractions, and solving word problems. No longitudinal study following the same cohort of pupils over a year been carried out in a single study in Brunei either. Therefore, this study set out to contribute to the literature base by examining pupils' error patterns in the six areas of fractions and to determine if such error patterns remained as profound among pupils as they moved through their various level of schooling.

More other international research studies have been carried out to investigate pupils' difficulties in learning fractions – for example those of Mack (1990, 1998), Brinker (1998) and Alcaro, Alice, Alston and Katims (2000). One specific example is a study by Gearhart, Saxe, Seltzer, Schlackman, Ching, Nasir, Bennett, Rgine and Sloan (1999). Among other topics, Gearhart *et al.* (2000) chose to study fractions for several reasons. First, they considered that the domain of fractions is deeply related to other forms of important rational number concepts including rates, quotients, operators, measures, percents, and decimals, and therefore it is a critical curriculum target for the upper elementary grades. Second, Gearhart *et al.* (2000) believe that fractions are a domain that poses difficulties for pupils. Many upper elementary pupils do not understand what fraction symbols represent. In addition, the researchers also mentioned that another plausible source of pupils' difficulties with fractions may be teachers' difficulties in understanding fractions themselves along with the ways that pupils make sense of fractions in instructional interactions. The present study aimed to examine such pupils' difficulties in the Bruneian context.

The areas of algebra in which misconceptions and errors were most prevalent were also of interest. This is because my experience as a mathematics student before and as a mathematics teacher and teacher educator, I believe that the study of algebra is inter-related with the study of fractions especially at the pupils' secondary level of schooling. A review of this area is provided in Section 2.12 (p.59) of this chapter. It was expected that the study would fill this gap in the current literature, and thus break new ground, especially regarding findings related to pupils' difficulties and erroneous procedures in learning fractions. This information should be of particular assistance and guidance to teachers and pupils in dealing with difficulties in this area of mathematics in Brunei, as the diagnosis of errors in arithmetic followed by appropriate instruction is an essential part of evaluation in the mathematics program (Ashlock, 1990).

The role played by pupils' errors and erroneous procedures as they learn, and the knowledge of how pupils make those errors is expected to help teachers improve instruction (Borasi, 1994; Swedosh, 1996). A review of past studies on early and late elementary grade pupils' improved performance or otherwise after instructions in the learning of fractions will be one aspect of the study.

Educators such as Piaget (1973), Hiebert and Tonnessen (1978), Borasi (1994) and Sophian (2000) have shown how a developmental perspective can help to provide insights into the nature, development, treatment of mathematics learning disabilities, and their findings proved useful in this study. On the other hand, researchers such as Mullis, Dossey, Owen, Philips (1991) and Groff (1994) assert that middle grade pupils continue to do poorly on tests of fractions despite their having been given extensive and persistent instruction, and the researchers believe that fractions are exceedingly complex and difficult for children to master.

Studies of issues on gender differences pertaining to error patterns exhibited by pupils on learning fractions will also be reviewed. These include the questions of whether or not girls exhibit more errors in fractions as compared to boys. If they do, in what areas and at what level, will the differences be more significant?

The review will finally highlight a number of issues on pupils' attitudes towards the learning of mathematics and fractions. Attitude has consistently been considered a contributing factor in determining pupils' performance in the teaching and learning of mathematics (Kloosterman & Cougan, 1994; Fullarton & Lamb, 2000).

2.2 Learning Difficulties and Error Patterns When Learning Fractions.

Research in mathematics education has shown repeatedly that errors and erroneous procedures about particular concepts are widely exhibited by pupils (Ashlock 1990, 1994, 2002; Baroody & Hume 1991; Bana *et al.*, 1997). Errors in mathematics are closely associated with misconceptions that often lead pupils to make absurd conclusion and hence incorrect responses (Perso, 1991; Swedosh, 1996). Perso described the relationship between misconceptions and errors as follows: "errors are not simply failures by students but rather symptoms of the nature of the conceptions which underlie their mathematical actions" (Perso, 1991, p.349). She commented that incorrect answers could be due to guessing, low intelligence, or low mathematical aptitude, but more often they result from systematic strategies or rules that usually have sensible origins and are based on beliefs or misconceptions; they are usually distortions or misinterpretations of sound procedures. An example shown

on p. 25 shows the misconception that the smaller number is always subtracted from the larger number, results in

$$5 - 13 = 8 \quad \text{or} \quad \begin{array}{r} 23 \\ - 7 \\ \hline 24 \end{array}$$

The problems shown above are described by Perso (1991) to be based on ‘good rules’ which have been badly applied or distorted. Some pupils simply try to assimilate new procedures into familiar algorithms.

Several other researchers (Bell, 1982; Farrell, 1992; Margulies, 1993; Swedosh, 1996) have studied various aspects relating to mathematical errors and misconceptions, their frequencies of occurrence and their importance to the pupil’s future learning of mathematics. The number and range of the different types of mathematical misconceptions and errors is enormous, “and a complete list may not even be practical” (Davis, 1984, p.335).

The fractions topic has been identified as an area in which pupils tend to rely heavily on memorisation and one in which they have weak strategies for manipulating ideas (Gabb, 2002). As a result, pupils hold conceptions of fractions that discourage conceptual development and the strengthening of the mathematical process. The study of fractions is foundational in mathematics, yet it is among the most difficult topics in mathematics for pupils both at the elementary and middle school levels (Croff, 1996; Moss & Case, 1999). Hanson (2001) wrote that one of the most frustrating areas for teachers and pupils alike was the study of fractions, specifically operations involving them. Year after year, pupils learnt but forgot how to add, subtract, multiply and divide with fractions. She quoted that the main reason pupils had difficulties with fractions was that they seemed to want to memorise formulas or algorithms instead of understanding them. In addition to that, literature has shown that pupils have difficulty recognising when two fractions are equal, putting fractions in order by size and understanding that the symbol for a fraction represents a single number (Cramer *et. al.*, 1997). Therefore, it implies that pupils need to understand fractions well before they are asked to perform operations on them, such as addition or subtraction.

Research on problems of how teaching and children's construction of fractions knowledge co-emerge has been of interest to researchers for many decades (Moss & Case, 1999; Tzur, 1999). Unlike children's construction of early number knowledge that often occurs without a teacher, a great deal of children's fractions learning takes place through instruction (Clements & Del Campo, 1990). Therefore, studying the problem in fractions teaching and learning is important if one is to suggest ways to understand and improve the current state of teaching. Davis, Hunting and Pearn (1993, p.63) stated the following "The teaching and learning of fractions is not only very hard, it is, in the broader scheme of things, a dismal failure". This statement is supported by ample evidence that fractions learning is a major obstacle for children (Hiebert & Behr, 1988; Mack, 1990; Behr, Harel, Post, & Lesh, 1992; Kaput, 1994).

The fact is that fractions are not only difficult for pupils to master in the elementary grades but that many teachers find the topic hard to teach also (Barwell, 1998; Fatimah, 1998). As mentioned earlier, not only pupils at the elementary grades find it difficult, but also as Moss and Case (1999) described, the domain of fractions has traditionally been a difficult one for middle and secondary school students and also student teachers to master (Pinto & Tall, 1996; Chinnappan, 2000). These researchers commented that although most students eventually learnt the specific algorithms that they were taught, their general conceptual knowledge often remained remarkably deficient. Lopez-Real, Veloo and Maawiah (1992) made an analysis of Bruneian P6 pupils' performance in the Primary Certificate of Education (PCE) Mathematics Examination answer scripts and found that pupils had great difficulty with the symbolic manipulation of fractions. Other local researchers such as Leong, Fatimah and Sainah (1997); See and Yusof, (2000); Raimah (2001) and Shamsiah and Clements (2002) confirmed the finding. Yet the percentage of questions on fractions in the PCE examination paper has been steadily increasing – from 9% in 1996 to 13% in 2001 (Shamsiah & Clements, 2002). A similar outcome resulted earlier in a study by Ellerton and Clements (1994) of pupils in Papua New Guinea, South East Asia, Australia and New Zealand. In their interviews, Ellerton and Clements found that Grade 6 pupils were not able to apply fraction concepts to practical situations such as to "pick-up one-third of a set of twelve marbles", or to solve a fraction task involving symbol manipulation. This demonstrated that pupils could not link fraction symbols with what they were doing practically, nor could

they deal with the formal language of fractions – with expressions such as “a third of twelve”.

In a survey carried out by Barwell (1998) in Northern Pakistan it was observed that, of all areas in mathematics, fractions seemed to cause primary teachers the most problems. Over a fifth of all the mathematics teachers surveyed named fractions as the most difficult topic to teach. Moreover, almost all teachers read fractions in unusual ways. When reading $\frac{3}{4}$, they would most often say “three upon four”. This expression was different from the text in the English medium mathematics books (Barwell, 1998) where “three-quarters” was the terminology used..

There are many factors that may contribute towards primary school pupils’ poor understanding of common fractions. Based on the research results reported by Baroody & Hume (1991) and D’Ambrosio & Mewborn (1994), there appear to be three main possible causes:

- The way and sequence in which the content is initially presented to the pupils; in particular exposure to a limited variety of fractions (only halves and quarters), and the use of pre-partitioned manipulative, meaning that pupils are provided with materials which the teachers have prepared earlier but which do not provide any opportunities for the pupils to manipulate those materials;
- A classroom environment in which, through lack of opportunity, incorrect ideas and informal (everyday) conceptions of fractions are not monitored or resolved; and
- The inappropriate application of whole-number schemes, based on the interpretation of the digits of a fraction at face value, or seeing the numerator and denominator as separate whole numbers.

National assessment results in the United States (NCTM, 1989) showed that even older pupils had trouble in working with and understanding fractions. The National Centre for Educational Statistics in the United States (1990) reported that only 46% of twelfth graders who took the National Assessment of Educational Progress Test could consistently solve problems involving fractions.

Baroody and Hume (1991) suggested that pupils' errors in fractions may be caused by poor understanding of underlying concepts, as well as by an inability to recognise accurate visual representation. Educators generally agree that learning occurs when one challenges (or builds upon) a pupil's existing conceptions (Australian Education Council, 1990; National Council of Teachers of Mathematics, 1989, 1991, 2000, 2001; Tirosh, Fischbein, Graeber & Wilson, 2001). Consequently, those calling for reform in mathematics education emphasise that having knowledge of pupils' common conceptions and misconceptions about the subject matter is essential for teaching (Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989; National Council of Teachers of Mathematics, 1989, 1991, 2000, 2001; Australian Education Council, 1990; Tirosh, 2000). The current study set out to investigate if P5 pupils in Brunei also find fractions difficult and complex. If so, which particular components of fractions do they find most difficult, and how they can be helped in overcoming those difficulties?

Markovitz and Sowder (1991) investigated the types of errors made by the pupils in understanding the relationship between fractions and decimal interpretations of rational numbers where both interpretations are represented symbolically. Their study also investigated the effect of the pupils' understanding of fractions on the learning of conceptual estimation. Questionnaires, interviews and tests were used in Markovitz and Sowder's study, and the results indicated that pupils had little understanding of the meaning of the symbols used and that they found difficulty in understanding the relationship between decimals and fractions. These problems were confirmed by the responses made by pupils in a series of individual interviews that they made before and after the presentation of the fractions units. The present study incorporates a number of ideas from the 1994 study, including several items to diagnose whether the study's pupils will also exhibit similar difficulties in understanding the relationship between decimals and fractions. The diagnostic pre-test, the post-test after more formal instruction in the following year and the pupils' interviews administered throughout the study assisted me in unfolding the types of mathematical errors held by P5 pupils in Brunei and in addition helped me to investigate if mathematical errors were still prevalent at the P6 level.

As mentioned earlier, a number of studies support the notion that fractions are a difficult topic for children to learn. Neimi stated that fractions are “tough for kids” (Neimi, 1996, p.75). He quotes the results of the National Assessment of Educational Progress, which indicate that fraction problems plague some pupils throughout their high school careers. Why this is so is not clear. Neimi believes that this is because, up to the point where fractions are introduced, the principal quantitative activities pupils have engaged in have been a variety of counting operations involving units. However, understanding a fraction properly is a very complicated task, according to Niemi. He contends that understanding a fraction in mathematical terms means, among other things, seeing it simultaneously as a relation between two other numbers, each of which can represent a measured quantity and a single descriptor of quantity itself.

More recently, Sophian (2000) suggests that instruction on fractions in the middle to late elementary school years is widely considered a watershed – a transition point at which many children begin to have serious trouble with mathematics. It is often argued that fractions are difficult because they do not fit well with children’s intuitive ideas about numbers which provided a good foundation for learning about counting and related whole-number operations but not for fractions. A troubling implication of this view is that the early years of mathematics instruction, in which the focus is on whole numbers, cannot prepare children for the transition to fractions and rational number concepts.

These studies influenced me to investigate if P5 and P6 pupils in Brunei, like their contemporaries elsewhere, also find fractions as “tough to study”, and if pupils lack a sound understanding in relation to their perception of fractions. As in most other parts of the world, pupils in Brunei have been exposed to the fractions concept formally as early as primary three or at an average age of eight. Therefore, it would be interesting and informative to investigate if the pupils in this study also faced the same difficulties as reported earlier.

In addition to the above-mentioned international studies, studies carried out in Brunei by Leong *et al.* (1997), See and Yusof (2000) and Raimah (2001) have demonstrated the difficulty children experience in understanding the concepts of continuous and discrete fractions. Pupils in Leong *et al.*’s study particularly found it

difficult to complete one of the items that required pupils to indicate on the sides of an equilateral triangle $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of the lengths of the sides. In all the above three studies, pupils also found items on the subtraction of a fractional part from a whole to be difficult to deal with successfully.

Behr and Harel (1990) stated that it has been shown that pupils' errors and misconceptions in different fields of mathematics and science do not occur haphazardly, but that there is a uniformity and pattern in the pupils' erroneous behaviour. Efforts to formulate a theory to account for systematic errors in mathematics have been made by researchers such as Matz (1980) in the domain of high-school algebra, and by Brown and VanLehn (1982) in the domain of place-value subtraction. Teaching can benefit from such theory if the sources of pupils' errors are used in designing learning environments that inhibit the errors.

In addition, Fischbein, Deri, Nello, and Marino (1985), Mason (1989), Mansfield and Happs (1992) have all demonstrated that pupils' misconceptions and errors persist despite instruction in algebra, calculus, multiplication, lines, polygons, subtraction and probability. Numerous studies documenting the prevalence of errors in diverse disciplines and summarised by Confrey (1990) highlight the importance of understanding these phenomena in order to develop appropriate remedial strategies. Since the concept of fraction is known to be abstract especially to young children (Behr *et al.*, 1992), it is necessary for teachers to plan activities that have the potential to provide pupils with opportunities to build conceptual understanding rather than to teach fractions in a "traditional" way. In the present study, P5 pupils' performance in the diagnostic test and the areas in which they exhibit more errors will be shared with all the teachers concerned before the pupils move to P6 the following year. In Brunei, many schools practise a system where P4 teachers follow their pupils to P5 and subsequently to P6 – a system that allowed me to alert the teachers to areas in which they should pay more attention when they teach their pupils again in the following year.

Existing research shows that children improve in mathematics calculation when given cognitive strategy instruction. For example, Naglieri and Gottling (1997) found that those children who have weakness in a cognitive process called

‘planning’, improved by 80 per cent over baseline standards in classroom mathematics computation assignments. Children who were average in planning improved about 40 per cent over initial levels. This indicated that those who were weak in planning needed especially to receive cognitive strategy instruction followed by more math instruction. More generally, all children improve when given cognitive strategy instruction intervention (Barnes, 1983; Naglieri & Gottling, 1997). As this study supported data with lesson observations, the above-mentioned factor would be observed if the teachers concerned would incorporate the ‘planning’ process in their teachings of fractions. The information could be used to supplement data obtained from the diagnostic tests, attitude questionnaire and the pupils’ interviews.

Brown and Van Lehn, (1982) and Resnick (1987) have demonstrated that a pupil’s error in solving a problem is frequently the result of his/her invented repair of a known rule or procedure. The repaired rule is invented and applied in order to circumvent a cognitive conflict resulting from differences between the pupil’s existing knowledge and the constraints of the problem space. One can look at the invention of this rule as an attempt by the pupil to keep his/her knowledge consistent with the constraints stated in the problem, or occurring during the solution process.

2.3 The Concept of Fractions

As mentioned earlier, one of the reasons for the prevalence of errors in fractions work is the way and sequence in which the content is initially presented to pupils – in particular, exposure to a limited variety of fractions (only halves and quarters), and the use of pre-partitioned manipulative (D’Ambrosio & Mewborn, 1994). Errors arising from the initial exposure to fractions at school can encourage pupils to produce memorised ‘recipes’ for fractions – for example, many pupils visualise a fraction as made up of “numerator, line segment and denominator”.

Newstead and Murray (1998) found that there was evidence of a reproduction of pre-partitioned illustrations – that is, they found that in response to an item such as “Show $\frac{3}{4}$ in at least 3 different ways”, 12% of their fourth graders and 10% of their sixth graders responded by drawing the following three shapes (p.32):



Some pupils in Newstead and Murray's study even wrote "square, rectangle, circle", particularly in one class where such illustrations of fractions were displayed on the wall. The generalisation of the partitioning of a shape into four parts to a triangle indicates a limited concept of a fraction that does not include equal partitioning.

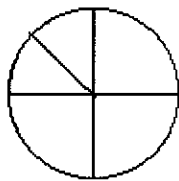
In addition to the above, Newstead and Murray (1998) explained their finding that the introduction of fractions using a mainly continuous area model, in which the fraction represents part of the whole, was also evident in their study. They stated that while 20% of Grade 4 pupils and 35% of Grade 6 pupils produced such illustrations of shaded geometric shapes, only 2% of Grade 4 pupils and 3% of Grade 6 pupils represented $\frac{3}{4}$ as a part of a collection of objects – for example:



The above "smartie" chocolate problem illustrated $\frac{3}{4}$ of a collection of discrete objects. This item addresses a particular meaning of the fractions and had a particularly poor success rate, especially at Grade 4 level (2%). The researchers concluded that it was probable that the pupils had not previously been exposed to problems that addressed the fraction as part of a collection of discrete objects. One of the current study's instruments incorporates an item that measures pupils' understanding of a similar problem. It was considered important to investigate if Bruneian pupils also experience the same problems as those pupils in Newstead and Murray's study.

A similar study was carried out by Reys, Kim and Bay (1999) with twenty Grade 5 pupils in Florida. The pupils who took part in the study had recently completed a six-week unit on fractions. One of the interview questions was to show a fraction of

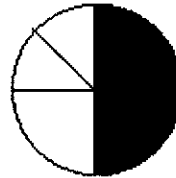
$\frac{2}{5}$ in a diagram form. Responses from seven of the twenty pupils interviewed revealed a basic misconception about fractions namely that a whole need to be partitioned into equal-sized pieces. For example, three pupils drew a circle, divided in into fourths, and then divided one of the fourths in half to form the “fifth” section. When these pupils were asked if it made any difference which two sections of the pizza were shaded to represent $\frac{2}{5}$, one pupil indicated that it did not matter whether two “large” sections or two “small” sections were shaded. The pupil was not troubled that the choice led to different representations of fractions $\frac{2}{5}$ as shown below:



$$\frac{2}{5}$$



$$\frac{2}{5}$$



$$\frac{2}{5}$$

The above two studies reveal that the above type of misconception with fractions is still prevalent with older pupils even after they receive further instructions on fractions. This misconception may lead them to make incorrect responses in comparing any given fractions. The current study set out to examine if such problems are also prevalent among P5 pupils and whether or not they would be eliminated as pupils received more instruction on fractions as they move to P6.

2.3.1 Possible reasons for misconceptions and errors acquisition in fractions learning

Why and how do the pupils acquire and develop such misconceptions and errors in fractions learning? Educators such as Niemi (1996) postulate that the initial foundation in acquiring the concept of fraction is undoubtedly the most important of all, as it is the prerequisite for doing more fraction work later, yet fractions are often considered the most difficult, mechanical and least understood topic in many primary schools (Tirosh, 2000; Raimah, 2001; Suffolk & Clements, 2003). Various

reasons have been attributed to this phenomenon. One reason is the pedagogical or instructional aspect investigated by Reys *et al.*, (1999). These researchers implied the difficulty that pupils experience with fractions was the result of the kind of instruction they were given. They added that if children naturally thought about completing wholes, but their classroom instruction focused on finding common denominators, creating improper fractions and dividing to create wholes, then children would have a difficult time incorporating that instruction into their conceptual framework. They concluded that many pupils when confronted with this instruction could not understand it at all and tried hard to learn the approach being taught by rote. From their study, Reys *et al.*, (1999) suggest that pupils must eventually develop generalised procedures to accomplish computations, such as adding fractions, and that those procedures would most likely include the traditionally taught algorithms. Therefore, they contended that pupils should not begin learning those procedures too early; rather, children should start by using their understanding of fractions to develop procedures that make sense to them (Reys *et al.*, 1999).

Studies such as those by Gearhart *et al.*, (1999) and Alcaro *et al.*, (2000) have also blamed the nature of textbooks and other instructional materials as being responsible for promoting problems, misconceptions and subsequent errors in learning fractions. These researchers commented that unfortunately, most textbooks and instructional materials present fractions as pieces of “pies” or circles, a practice that implies the part/whole relationship only. Another factor which has been attributed as the cause of the problem is the nature of assessment administered to the pupils. Neimi mentioned, “The typical achievement test doesn’t permit us to see whether or not pupils have developed an appropriate concept of fractions.” (Neimi, 1996, p.76)

To overcome this ineffective mode of assessment, Niemi (1996) developed a number of more “authentic” assessments and administered them to groups of fifth graders. For his research, fractions were selected on the basis of pupils’ identified problems in understanding the concept of fractions through middle and high school. He also hypothesised that, when asked to explain what they thought about the nature of fractions, pupils who had been instructed about fractions in terms of measurement and quantitative comparisons would demonstrate better conceptual understandings

than would pupils who had received only the traditional part/whole approach to fractions. After eight days of instructions, the fifth graders in Niemi's experiment were asked such questions as "What is a fraction? Why are there two numbers in a fraction? How can you add two fractions?" They were also asked to solve problems, for example, "Which is larger, $\frac{2}{5}$ or $\frac{2}{4}$?" and to justify their answers. Even though the instruction was relatively short, the pupils in the experimental group (who received the measurement/quantitative approach to instruction) scored higher on a scale measuring understanding of principles. Niemi considered this important because not all measurements of this type were sensitive to short-term instruction. Members of the experimental group also scored higher in problems solving and justifying answers and they demonstrated more prepositional knowledge about fractions. Niemi (1996) further added that on the downside, not many pupils showed much understanding about fractions at all. One-fourth scored at the bottom of the scale on the explanations task – which meant that their explanations contained no content knowledge. Misconceptions and errors abounded.

Many pupils in Niemi's study drew pictures indicating that they did not understand that objects had to be partitioned into equal parts in order to show a fraction. A number of pupils wrote that a fraction is a piece of something that you eat; these responses, while not scored as errors, certainly implied limitations in the pupils' thinking, most likely deriving from their classroom experiences with fractions. Ideally, the pupils should have thought that fractions did not necessarily represent parts of an area but it could also represent parts of continuous or collections of objects. Several pupils, when asked whether the uneaten part of an object could also be a fraction, said that they did not believe so (Niemi, 1996).

No doubt, such conceptions are in part responsible for the fact that 54% of the pupils in Niemi's study solved the pizza problem correctly while only 38% could handle an analogous problem involving lengths. On the other hand, some pupils displayed high levels of understanding. One pupil noted that $\frac{2}{4}$ of a pizza is bigger than $\frac{2}{5}$ of a pizza but not if the pizza cut into $\frac{2}{4}$ is a small pizza and the one cut into $\frac{2}{5}$ is a large

pizza. This indicates that an understanding that the “wholes” have to be equal is important.

Niemi expressed some concern that more than 90% of the pupils performed at a level far below anything that might remotely be considered a “world class standard”. This is an interesting judgement since no data or criteria existed for such standards at that time. Though the Third International Mathematics and Science Study (TIMSS) was carried out commencing in April 1995 in 40 countries around the world, it is unlikely that the analysis was available to Niemi at the time he reported (1996). Nevertheless, he took it as a “sign of hope” that some pupils demonstrated deep understanding after such short instruction. Niemi hoped that teachers in a classroom setting could use the assessment instrument he developed. Questions that require justification and explanation allow teachers to see where pupils might possess misconceptions and commit errors. This is much more useful information than the right/wrong data obtained from multiple-choice test or simple calculation problems. He concluded that pupils who performed well on traditional, computational tests showed a lack of understanding when asked to explain the concept. Niemi’s study reveals the importance of designing and implementing assessment strategies that identify pupils’ depth of understanding in learning about fractions.

The current study incorporated a number of items to measure pupils’ understanding regarding comparing and ordering fractions. It was important to examine if P5 and P6 pupils in Brunei exhibited similar problems to those in Niemi’s study and whether the Brunei pupils would improve after further instruction at the P6 level.

2.4 The Decimal-Fraction Link

The most basic idea about decimals is that, like fractions, they enable us to describe parts of a unit quantity. Decimals and fractions can constitute the solution to a single problem. Many young children, even those attending secondary schools, do not make the decimal-fraction link correctly (Bana *et al.*, 1997). Others exhibit the same cognitive difficulties that are encountered with fractions in their thinking about decimals. Fundamental understanding of fractions, such as dealing with equivalent

fractions, is critical to decimals as well (Resnick *et al.*, 1989; Moloney & Stacey, 1997).

In converting fractions into decimals, one misconception which caused an error identified by Irwin (2001) in his study was that pupils considered that $\frac{1}{4}$ could be written either as 0.4 or as 0.25. The misconceptions was further demonstrated when the pupils made errors by converting $93\frac{1}{4}$ to 93.04 in decimals. Irwin believed that the results of his interviews demonstrated that many pupils possessed misconceptions and errors about decimal fractions that might have affected their understanding of common fractions. Pupils seemed to have difficulty transforming their “everyday” knowledge of decimal fractions to “school” knowledge of decimal fractions. Pupils need to build their “school” knowledge upon their everyday knowledge by attaching meaning to decimal fractions. Irwin further added that the misconceptions held by pupils in his study, and others like them, suggested that they had not reflected on the scientific concepts involved in decimal fractions in any way that resolved their misconceptions, and that these misconceptions later lead them to make errors. Pupils who believed that “one hundredth” was written as 0.100 or that $\frac{1}{4}$ could be written either as 0.4 or 0.25 had not reflected on the incompatibility of these notions with principles such as place value. In light of the above learning problem, the current study sought to find out if such errors in the conversion of decimals into fractions also existed among the P5 and P6 pupils in Brunei. If they did, at what level of schooling were the errors more prevalent?

There are many contexts in which decimal fractions can be understood, including concrete models and the number line (Irwin, 2000; Kyungsoon, 2001). Problems presented in everyday settings provided the context that pupils needed for reflection on the scientific concept of decimal fractions – for example, a question on the meaning of money expressed to more than two decimal places required pupils to consider how their existing concepts related to scientific knowledge. Such problems may have provided the opportunity for reflection required to expand pupils’ knowledge of decimal fractions.

Common fractions provide more information explicitly than do decimals. The fraction $\frac{2}{5}$, for example, indicates that the reference unit (the “whole”) has been

divided into 5 equal parts and two of these parts make up this fraction. In decimal notation, the *denominator* is hidden, as is the place value of the columns in whole number numeration. Just as it is simpler to see the Roman numeral XXXII as 3 tens (XXX) and 2 ones (II) than it is to see this structure in 32, so it is easier to interpret the fraction $\frac{4}{10}$ than see the structure of the decimal 0.4 where the size of the parts (tenths) is indicated only by the place value.

Some pupils who link decimals and fractions may nevertheless fail to associate a decimal with the correct fraction. For example, a pupil may interpret 2.6 as two and one sixth or write 1.4 as the decimal for one quarter (Hiebert, 1985). Interpreting the decimal part of a number as the denominator of a fraction is referred by Hiebert as reciprocal thinking. In his research summary, he concluded that the result of his interviews demonstrated that many pupils had misconceptions about decimal and common fractions. Pupils had difficulty transforming their everyday knowledge of decimal and common fractions to school knowledge of fractions. This led to my interest in the current study to investigate if P5 and P6 pupils in Brunei also encounter such problems. I realised this by incorporating a number of items on decimals/fractions into my instrument.

2.5 Equivalence of Fractions

The concept of equivalence has been widely researched over the last 25 years. Behr and Post (1988), Ashlock (1994), Morris (1995) and Mack (1998) have found that a common error involving finding a fraction that is equivalent to another fraction involved using additive rather than multiplicative reasoning. For example, a common incorrect solution to the problem $\frac{1}{3} = \frac{?}{6}$ was 4; pupils often looked for the number they could add to 3 to get 6 ($3 + ? = 6$), and then added that same amount to the numerator ($1 + 3 = 4$), concluding that $\frac{1}{3}$ was equivalent to $\frac{4}{6}$.

Evidence from the above study suggests children can use the language of fractions even without fully understanding their nature. Cramer, Post and DelMas (2002) quoted the results of the National Assessment of Educational Progress (NAEP),

which revealed that fourth-grade pupils have limited understandings of fractions. They suggested that pupils' understanding of the fundamental concept of equivalent fractions should reflect more than just a knowledge of a procedure for generating equal fractions: it should be rich in connections among symbols, models, pictures, and context. Yet, only 42% of fourth graders in the NAEP sample could choose a picture that represented a fraction equivalent to a given fraction, and only 18% could shade a rectangular region to produce a representation of a given fraction.

Another study, one carried out by Kerslake (1986), also contains further evidence of pupils' lack of understanding of fractions, even if they succeeded in the test items based on equivalent relationships. Kerslake's report suggests that at around age 11, pupils find it easier to provide equivalent fractions if the fractions are presented as a number pattern activity rather than presented as an item on fractional equivalence. Another example of a typical question suggested by Ashlock (1994) is

$$\frac{5}{9} = \frac{10}{18} = \frac{15}{\dots} = \frac{\dots}{36}$$

Items of this type have been incorporated in the current study's tests to investigate if such problems also exist among the P5 and P6 pupils in Brunei.

Kerslake (1986) and Ashlock (1994) further suggested that the task of cancelling in a given fraction appeared easier when an example in a form of illustration was given rather than that of providing equivalent fractions without the support of an illustrated example. An illustration shown in Fig.2.1 below proved to be helpful in Kerslake's study in 1986.

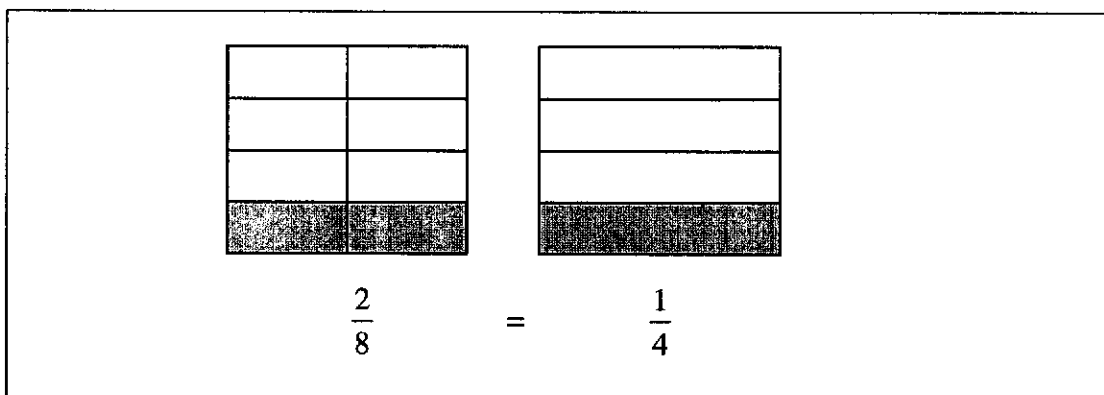


Fig.2.1: A diagrammatic example to cancel a fraction

Based on the approach used by Kerslake (1986) in his study as described on p.40, I presume that lessons on the equivalence of fractions should be based as a number pattern activity. Emphasis should be placed on whole number patterns, leaving the pupils to discover a rule or algorithm for themselves, though teachers should ensure that pupils emphasise a multiplicative rather than an additive rule. The current study examined if such an approach was employed by the teachers during their instructions on fractions.

2.6 Comparing or Ordering Fractions

In terms of ordering the magnitude of fractions, a common misconception is that the smaller the numbers in the fraction then the smaller the fraction must be. However, pupils have mixed views on fraction sizes, and often possess the vague notion that larger numbers in fractions can also imply a small fraction. Womersley (2000) quoted the Chelsea Diagnostic Research that showed 80% of pupils aged 11 were clear that $\frac{1}{1000}$ was the smallest of the 4 fractions given and that $\frac{1}{2}$ was the largest ($\frac{1}{1000}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$). However, 13% of these pupils were unclear where $\frac{1}{3}$ and $\frac{1}{4}$ should be in order of magnitude. The use of fraction wall/ fraction chart (below) can be used to clarify the problem for pupils.

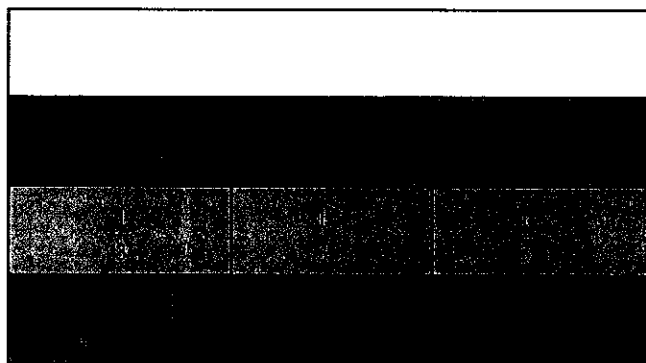


Fig 2.2: A Fraction wall / chart

Womersley (2000) further described her investigation of misconceptions in fractions involving mixed numerators and denominators and it was clear that pupils had a variety of strategies for ordering (smallest first) the following:

$$\frac{7}{\dots}, \quad 7\frac{5}{8}, \quad 7\frac{1}{8}, \quad \frac{7}{\dots}, \quad \frac{7}{\dots},$$

None of her pupils could actually put them in the correct order (either smallest or largest first).

As mentioned earlier, some pupils tend to equate decimals and fractions – a phenomenon that Hiebert (1985) and Pitkethly and Hunting (1996) referred to as reciprocal thinking, which occurs when pupils interpret decimals as fractions. They see the decimal part as the denominator of a fraction, with larger denominators creating smaller fractions. Hence, they consider 0.3 to be larger than 0.4 as $\frac{1}{3}$ is larger than $\frac{1}{4}$. Pitkethly and Hunting (1996) described this phenomenon as being quite common amongst early secondary pupils. Such pupils may interpret 2.6 as two and one sixth or as $\frac{2}{6}$. For a question which asked pupils to write a decimal to tell what part of a region was shaded, more than 25% of Grade 7 pupils in a national survey conducted in the USA wrote 1.5 for $\frac{1}{5}$ and 1.4 for $\frac{1}{4}$ (Hiebert, 1985). Many older pupils exhibited confusion between fractions and decimal notation also. Carpenter, Corbitt, Kepner, Lindquist and Reys (1981) reported the results from a study of a sample of 13 year-old children that showed that on a multiple-choice question which asked for the decimal equivalent of $\frac{1}{5}$, only 38% answered correctly, whilst just as many pupils (38%) chose 0.5.

Because decimal fractions do not explicitly show the denominator, it is likely that some pupils may assume that the numbers written represent the denominator, rather than the numerator of the decimal fraction. These are Pitkethly and Hunting's reciprocal thinkers. Two items on ordering fractions were incorporated into the current study to investigate if any P5 and P6 pupils in Brunei could be classified in the same way.

Comparing the size of fractions by considering only the size of the denominator can be considered a case of regarding the numerator and denominator as two unrelated whole numbers. For items such as:

Put these fractions in order from smallest to biggest: $\frac{2}{5}$, $\frac{2}{3}$, $\frac{2}{9}$

A number of pupils in Newstead and Murray's study (1998) responded $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{9}$ (16% of the fourth graders and 38% of the sixth graders). The percentage shows that this misconception was actually more prevalent in the case of the Grade 6 pupils than in the case of the Grade 4 pupils. The tendency to choose the larger fraction – the one with the larger denominator – has also been reported elsewhere in the literature (e.g. Baroody & Hume, 1991; Ashlock, 1994; Stepus, 1985). Despite the fact that Grade 6 pupils are more mature and more exposed to the fraction concept, they had a lower success rate in this particular item. With the current longitudinal study, it was my interest to determine if P6 pupils in Brunei who were assessed when they were in P5 would demonstrate similar error patterns to other pupils elsewhere, as reported in other studies. Alternatively, would P6 pupils show an enhanced performance after receiving more formal instruction on fractions and as they grew a year older?

2.7 Addition of Fractions

Many studies have reported that a substantial number of pupils tend to add fractions by “adding the tops and the bottoms” Ashlock (1994). Various possible causes of this behaviour have been offered:

- The pupils do not view fractions as representing quantities but see them as four separate whole numbers to be combined in one way or another. Each fraction is viewed, as two numbers separated by a line, and it seems reasonable to add the numerators to obtain the numerator of the sum and to add the denominators similarly (Oppenheimer & Hunting, 1999).
- The pupils confuse the rule of adding fractions with that of multiplying the fractions (Graeber & Baker, 1991)
- The pupils view the four numbers involved in addition of fractions as two numerators and two denominators. They believe that the adequate way to perform addition is to add “alike items”, that is, numerator to numerator, and

denominator to denominator. They view this as similar to adding the way it is done with whole numbers, in the sense that in both cases only alike terms are added – ones to ones, tens to tens, numerators to numerators, and so on (Baroody & Hume, 1991; D'Ambrosio & Mewborn, 1994; Huinker, 1998).

- There are some life situations in which such a way of operating is appropriate – such as when pupils are counting their total scores from two sections of exercises (Borasi, 1994).

The above behaviours were not only demonstrated by young pupils in a study by Tirosh *et al.*, (2001), but their results showed that even some prospective teachers who were given problems on the addition of fractions added the numerators and the denominators. Though it was only about 15% of those prospective teachers in their study who responded incorrectly to the addition problems, it could become a serious matter if this situation were to be left ignored.

The misconception of “adding” two sets of fractions by adding the numerators and adding the denominators was further evidenced in a study carried out by Newstead and Murray (1998). Their research showed that when it came to adding fractions, some pupils mechanically combined the two denominators and numerators. This illustrates again that pupils are confused with the rules for multiplication of fractions because pupils see that the application of a similar whole-number scheme can lead to success when it comes to multiplying fractions. Because of such misconceptions, pupils tend to make those errors. The interference of whole-number strategies, which results in pupils simply adding the numerators and the denominators, has been widely reported in the literature (Baroody & Hume, 1991; Mochon, 1993; D'Ambrosio & Mewborn, 1994; Alcaro *et al.*, 2000).

What happens with this misconception is that the pupils try to assimilate a new algorithm into an existing and already familiar one. Perso (1991, p.57) described the above error as “good rules but badly applied or distorted to some degree”. Such a method is incorrect, but is nevertheless systematic. It originates in what is already known to the pupils. The incorrect procedures have their roots in sensible procedures. Some pupils develop rules that work some of the time and hence cannot

be generalised. The rules have sound origins, but pupils cannot understand why they do not work all the time. Therefore, it is a challenge for teachers to ensure that pupils are aware of such problems in working with fractions.

I anticipated that similar misconceptions and mathematical errors would be exhibited by P5 pupils in carrying out the addition of fractions because this scenario seems to be universally common. I considered it interesting to examine if such misconceptions and errors would decline as pupils move to their next level of schooling and after more exposure and instruction. The second diagnostic test (post-test) given to P6 pupils was designed to resolve this matter.

2.8 Subtraction of Fractions

As in the case of the addition of fraction, many pupils are influenced by the operation of whole numbers in the subtraction of fractions. The most prevalent misconceptions reported in many studies such as those by Baroody and Hume (1991) and D'Ambrosio and Mewborn (1994) are that pupils tend to treat the numerators and denominators in fractions as distinct whole numbers, thus responding as follows:

$\frac{5}{7} - \frac{3}{4}$ as $\frac{2}{3}$ (i.e. subtracting the smaller numbers from the larger numbers).

Morris (1995) reported her research finding on subtraction of fractions involving mixed numbers. After a treatment of the topic in which she offered pictorial and symbolic solutions to the pupils, she noticed an improved performance. Children who had been unable to convert mixed numbers to improper fractions (and vice versa) on the pre-test independently began to make those conversions. For example, her pupils demonstrated the following process:

Teacher: $4\frac{2}{9} - 1\frac{7}{9} =$

Pupil 1: (on black board): $4\frac{2}{9} - 1\frac{7}{9} = \frac{38}{9} - \frac{16}{9} = \frac{22}{9} = 2\frac{4}{9}$

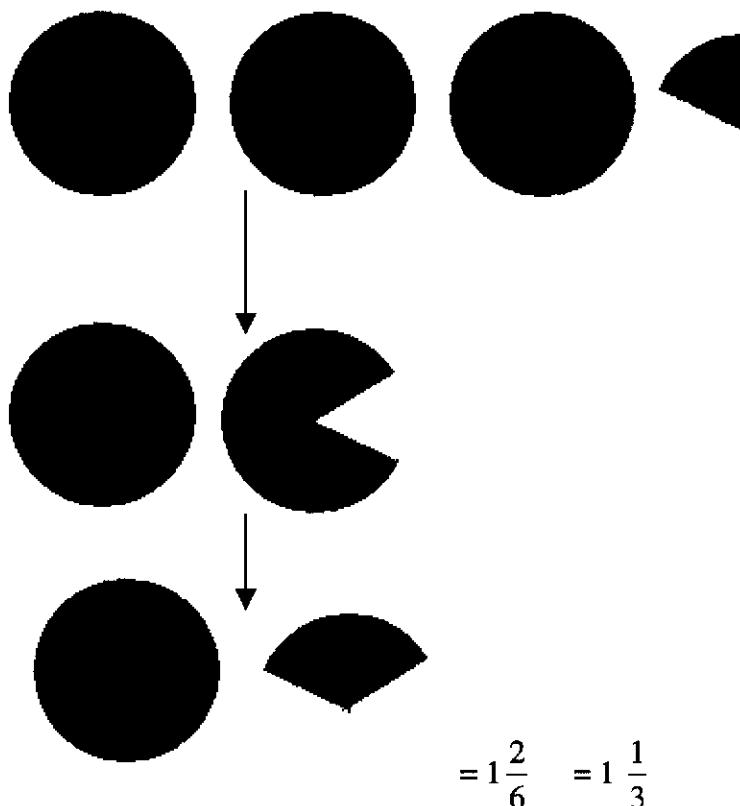
Pupil 2: (on black board): $4\frac{2}{9} - 1\frac{7}{9}$. The pupil took 1 whole from 4 wholes, thus changing 4 wholes into 3 wholes. The pupil then decomposed the one whole into 9 equal parts and added 2 thus he continued as $= 3\frac{11}{9} - 1\frac{7}{9} = 2\frac{4}{9}$

These pupils realised that since initially, they could not take away seven ninths from two ninths, they had to change one of the whole pieces into nine equal pieces, added the nine ninths to the two ninths making eleven ninths and so changed the four wholes into three wholes. Subsequently, during the development of mixed number procedures, her children became increasingly more independent in solving similar questions.

Some of Morris's other pupils solved the subtraction of fractions involving mixed numbers by using pictorial representations to verify the correctness of symbolic representations. For example:

Teacher: $3\frac{1}{6} - 1\frac{5}{6} =$

Pupil: Gives a pictorial solution as below:



The above example implies that with a developmentally appropriate approach, pupils can be helped to eliminate the conceptual difficulties that they encountered before.

Many pupils reported in literature (D'Ambrosio & Mewborn, 1994) tend to believe that it is impossible to subtract a fraction from a whole number. For an example, $1 - \frac{1}{3}$ may appear to be an easy task to many teachers, but a local study in Brunei by Leong *et al.*, (1997) shows that Primary 5 pupils still found it difficult. It proved to be a very difficult task for pupils without fractional understanding generally and with a poor understanding of the principle of connections of fractions. Only 13% of the sample in Leong *et al.*'s study could perform the above task, although most of the class work in their workbooks involved such manipulation of symbols. All sort of responses were given including $1\frac{0}{3}$ and $\frac{0}{3}$, showing conclusively that many pupils had difficulty with these concepts.

Other studies (Huinker, 1998; Johnson, 1999) have demonstrated pupils' problems in performing the subtraction of fractions. Initially, with the introduction of subtraction involving similar denominators, pupils seem to be capable of solving problems. As they moved onto the subtraction of fractions with different denominators however, difficulties started to appear. Ashlock (1994) showed that a number of pupils he studied solved as follows:

$$\frac{4}{6} - \frac{1}{3} = \frac{3}{3}$$

and $5\frac{2}{3} - 2\frac{5}{8} = 3\frac{3}{5}$.

Another pattern, which Ashlock found, consisted of the following error:

$$\frac{5}{6} - \frac{4}{5} = \frac{1}{30}$$

and $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$.

The current study set out to establish if the above two patterns are prevalent among Bruneian P5 and P6 pupils and if there was a difference between the results for Bruneian boys and girls.

2.9 Multiplication of Fractions

Some common errors in multiplication of fractions as reported in earlier studies such as those by Ashlock (1994) and Taber (1999) are shown in the following examples (p.48):

$$\text{a) } \frac{7}{8} \times \frac{3}{4} = \frac{7}{8} \times \frac{6}{8} = \frac{42}{8}$$

$$\text{b) } 4\frac{2}{3} \times \frac{1}{4} = 4\frac{2}{12}$$

$$\text{c) } 5\frac{1}{3} \times 6\frac{3}{4} = 30\frac{3}{12}$$

$$\text{d) } 2\frac{1}{2} \times 1\frac{3}{10} = 2\frac{5}{10} \times 1\frac{13}{10} = 2\frac{65}{10}$$

The above examples show different patterns of mathematical errors in multiplication of fractions. Various factors could have resulted in the pupils arriving at the above responses. The influence of the addition and subtraction processes with unlike denominators is demonstrated in examples a) and d) where the pupils tried to convert the second fraction so that both would have like denominators before they are multiplied. A problem of decomposition is evident in examples b), c) and d) though pupils have multiplied the common fractions correctly.

Understanding the multiplication of fractions involves understanding ideas about fractions and understanding ideas about multiplication. When working with fractions, pupils need to understand that equal-sized parts are needed and that the size of a part is based on the size of the unit (Cramer & Bezuk, 1991). Based on this recommendation, it implies that the pupils also need to be able to solve problems that involve equal sharing. Furthermore, pupils need to understand that each fraction has many equivalent representations: $\frac{9}{12}$ and $\frac{6}{8}$ are forms of $\frac{3}{4}$, for example.

Although multiplication can be viewed in several ways (such as repeated addition and arrays) as suggested by Cramer & Bezuk, (1991), many situations with fractions involve taking a part of a whole. For example, the following problem extracted from a typical mathematics workbook involves finding one-fourth of one-half of a whole cookie;

You have one-half of a giant chocolate chip cookie. You give your friend one fourth of the piece you have. How much of the whole cookie did you give your friend?

The problem can be represented mathematically by

“ $\frac{1}{4}$ of $\frac{1}{2}$ of 1 whole” or by “ $\frac{1}{4} \times \frac{1}{2}$ ”.

This “taking a part of a part of a whole” interpretation can be applied to the multiplication of fractions by whole numbers greater than 1; by 1 itself, and by fractions smaller than 1. For example, $\frac{3}{4} \times 6$ can be thought of as starting with six items or units; portioning the collection into four equal parts, or one-fourths; and selecting three of the one-fourths parts. The answer, $4\frac{1}{2}$, is expressed in terms of the original unit. Similarly, $\frac{3}{4} \times 1$ can be thought of as starting with one item or unit; portioning it into four equal parts, or one-fourths; and selecting three of the one-fourth parts. Here again, the answer $\frac{3}{4}$ is expressed in terms of the original unit. Finally, $\frac{3}{4} \times \frac{1}{2}$ can be thought of as starting with one-half of a unit, portioning the one-half into four equal parts, or one-fourths and selecting three of the one-fourth parts. The answer $\frac{3}{8}$ is expressed in terms of the original unit.

The fact that pupils have great difficulty in multiplication of fractions is well documented. As an example, Taber (1999) reported on his study, which documented pupils’ difficulties with multiplication problems containing rational numbers. He focused especially on studies demonstrating the work of younger pupils who were just beginning their study of operations with rational numbers. The focus was on pupils’ solutions of problems with fractional rather than decimal operators and it employed a system of problem categories that took into account and organised the

variety of problem actions that could be represented by the multiplication operation. In order to make a comparison with the above-mentioned studies and to investigate if primary level pupils in Brunei would have difficulty with the multiplication of fractions, a number of items of this type were incorporated in the instrument used in the current study.

Studies by Streefland (1991) and Greer (1994) showed that simply learning the rules for computing with rational numbers does not guarantee an understanding of the meaning of multiplication or division with fractions. As the researchers have pointed out, learning to multiply with fraction involves not only learning to multiply new kinds of numbers, but also learning to recognise new kinds of situations that can be expressed in terms of multiplication. Earlier, Kieren (1988) argued that understanding rational numbers and operation is not simply an extension of whole number knowledge but must be built up from rational number concepts such as partitioning, equivalency, and forming dividable units. As Hiebert and Behr (1988) pointed out, pupils first learn arithmetic operations in the environment of whole number quantities and operators. In order to construct knowledge about rational numbers, pupils must transform their knowledge of number and operations to include operations with rational numbers. An example of a familiar situation is taking one-fourth of 12 cookies which can be symbolized either by $12 \div 4$ or by $\frac{1}{4} \times 12$, where $\frac{1}{4}$ is a fractional operator.

2.10 Division of Fractions

Division of fractions is often considered the most mechanical and least understood topic in elementary school (Fender, 1987; Empson, 1995; Fong, 1995; Graebar, Anna & Tirosh, 1990). Pupils' success rates on various tasks related to such division are usually very low (Hart, 1981; Carpenter *et al.*, 1988). Although this statement might lead one to think about the usefulness of teaching children the meaning of and methods for dividing fractions, the standard algorithm for division of fractions continues to be included in the curricula of elementary schools in many countries (Tirosh, 2000).

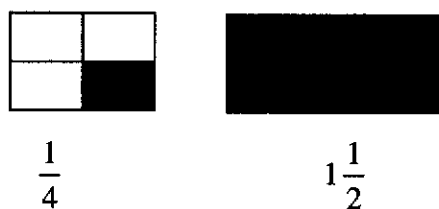
In the division of fractions, such as “What is $\frac{1}{2}$ divided by $\frac{1}{4}$?” traditionally, pupils were taught and trained to solve such problems by using the “invert and multiply” method, which most of them memorise, quickly forget, and almost never understand. Thus, pupils will write:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

Ideally, the above division of fractions should be taught as follows (p.51):

$$\begin{aligned}
 &= \frac{1}{2} \div \frac{1}{4} \text{ (i.e. How many } \frac{1}{4} \text{ s in a } \frac{1}{2} \text{ ?)} \\
 &= \frac{\frac{1}{2}}{\frac{1}{4}} \\
 &= \frac{\frac{1}{2}}{\frac{1}{4}} \times \frac{\frac{4}{1}}{\frac{4}{1}} \text{ Rationalising the denominators.} \\
 &= \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2 \text{ (i.e. there are 2 quarters in a half)}
 \end{aligned}$$

Battista (1999) commented that in contrast, pupils who have made sense of fractions and who understand the operation of division do not need a symbolic algorithm to compute an answer to this problem. Because they interpret the symbolic statement in terms of appropriate mental models of quantities, they are quickly able to reason that, because there are four fourths in a unit and because there are two fourths in a half, there are 6 fourths in $1\frac{1}{2}$. Younger pupils might need to draw a picture as shown on page 52 to support such reasoning.



Battista (1999) suggests pupils who truly make sense of this situation are not manipulating symbols, oblivious to what they represent. Instead, they are purposefully and meaningfully reasoning about quantities. They are not blindly following rules invented by others. Instead, they are making personal sense of the ideas. These pupils have developed powerful conceptual structures and patterns of reasoning that enable them to apply their mathematical knowledge and understanding to numerous real-world situations, giving them intellectual autonomy in their mathematical reasoning.

Battista (1999) further suggested that to develop powerful mathematical thinking in pupils, instruction must focus on guiding and supporting their personal construction of ideas. Such instruction encourages pupils to invent, test, and refine their own ideas rather than to blindly follow procedures given to them by others. As in the above example on the division of fractions, if pupils are to progress to a meaningful understanding of the symbolic manipulation of fractions, that understanding must come from pupils' reflections on their own work with physical quantities. Given appropriate experiences in mentally manipulating these quantities, pupils can, with proper guidance, derive strictly symbolic methods for dividing fractions. Pupils might invent the "invert and multiply" method, or they might devise a different symbolic procedure. (For instance, some pupils obtain a common denominator and then divide the numerators as shown in the earlier example). Since pupils derive these symbolic procedures through personally meaningful manipulation of quantities, their knowledge of the procedures becomes semantically rich in its connection to their reasoning about quantities. It is no longer inert and strictly syntactical. Research clearly shows that such "construction-focused" mathematics instruction produces more powerful mathematical thinkers (Cobb, Yackel & Wood, 1992).

The above studies have influenced me to observe if pupils in my sample will demonstrate similar strategies and techniques in carrying out the division of fractions. Therefore, the current study incorporated a number of items on division with both simple fractions and mixed numbers.

Literature also shows that some misconceptions may arise from pupils' own incorrect intuitions and informal experiences. Evidence is provided in the findings of Newstead and Murray's (1998) study with an example of pupils' inability to interpret the item " $2 \div \frac{1}{2}$ " as '*how many $\frac{1}{2}$'s are there in 2?*' This inability to interpret correctly provides an example of a limiting construction arising from pupils' own intuitions and real-life experiences. The division of fractions out of context conflicts with pupils' deep-seated ideas about division, as it produces an answer larger than the number to be divided and, unlike whole-number division, cannot be interpreted as a 'sharing' situation (Baroody & Hume, 1991). However, these authors suggested that an exposure to a wider variety of division situations at school would have provided an opportunity for this conflict to be resolved.

Hart (1981) reported that many pupils think that division is commutative and consequently argue that $1 \div \frac{1}{2} = \frac{1}{2}$ because $1 \div \frac{1}{2} = \frac{1}{2} \div 1 = \frac{1}{2}$. Hart further explained that the above error might have different sources. For example, pupils' belief that $\frac{1}{4} \div \frac{1}{2} = 2$ could originate from faulty ideas about the algorithm as mentioned by Tirosh (2000). An example like $\frac{1}{4} \div \frac{1}{2} = \frac{4}{1} \times \frac{1}{2} = 2$, could have originated from intuitive beliefs about this operation (e.g., in division, the dividend should always be greater than the divisor, therefore $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2} \div \frac{1}{4} = 2$), or from inadequate formal knowledge (e.g., division is commutative and therefore $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2} \div \frac{1}{4} = 2$). Other factors could lead to such a response as well. A teacher's acquaintance with various sources of error in children's responses would assist the

teacher in identifying both the specific source of a pupil's mistake and an appropriate choice of instruction.

Researchers such as Fischbein *et al.*, (1985) and Bell, Greer, Grimison and Mangan, (1989) have also written about the children's difficulty in choosing the correct operation and writing the correct arithmetic sentence for one-step multiplication and division problems with multipliers and divisors less than one or larger than the dividend. For example, $3 \div 6$ and $6 \div 3$ are treated as the same. Not only the above phenomenon was observed among young children in Bell *et al.* and Fischbein *et al.*'s studies but other researchers such as Greer and Mangan (1986) and Tirosh and Graeber (1990) have also observed this similar difficulty among college students.

In addition, many children studied by Brown (1981) and Ekenstam and Greger (1983), acquire certain incomplete or incorrect conceptions about multiplication and division that are consistent with Fischbein *et al.*, (1985). They believe that

- (a) multiplication always makes bigger,
- (b) division always makes smaller, and
- (c) dividing a smaller number by a larger number is not permissible.

Therefore, when problem numbers do not fit the model or understandings that pupils possess, incorrect choices of operations are frequently made (Bell *et al.*, 1989).

The items in the diagnostic pre- and post-tests used in the current study were used to investigate if primary level pupils in Brunei demonstrated similar problems as reported in the above studies. If P5 pupils demonstrated the problem, would they improve after they moved to P6 – that is, after they received further instruction on fractions?

2.11 Word Problems Involving Fractions

The diagnostic test included four items on one-step word problems involving fractions. Each question required pupils to either add, subtract, multiply or divide the fractions involved. Literature shows that this area is also of great difficulty to most pupils (Ekenstam & Greger, 1983; Bell *et al.*, 1989; Sainah, 1998; Taber, 1999) in solving mathematics word problems where nearly half of the difficulties encountered are linguistic in nature (Newman, 1977; Clements, 1991; Whitting,

1998). Accordingly, pupils who speak English as a second language could be expected to experience more difficulties in solving word problems because of their possible lack of knowledge of English (Whang, 1996). Sainah (1998) in her study, found that word problems involving addition and subtraction are difficult for Bruneian pupils to model and difficult for them to solve. These difficulties lead pupils to commit many transformation errors. She also indicated that pupils find problem-solving tasks more difficult than the symbol manipulation of computational tasks. However, her study did not involve any problems with fraction elements. In another study, Taber (1999) stated that although most pupils learn the rules for addition, subtraction, multiplication and division of decimal or fraction numbers, many have difficulty writing a mathematical expression that will solve a simple problem. For example, in a National Assessment of Educational Progress (NAEP) study (1990), 60% of the 13-year-olds could compute the product of two fractions, but only 17% could solve the problem such as:

George had $\frac{3}{4}$ of a pie. He ate $\frac{3}{5}$ of that. How much pie did he eat?

(Carpenter, Matthews, Lindquist & Silver, 1984, p.487).

Being able to use mathematical expressions to symbolise the relationships among quantities and the actions of operators in problem situations is an essential component of mathematical reasoning, particularly algebraic reasoning (Polya, 1962; Sowder, 1988; Perso, 1991). Studies show that children do not have too much difficulty in solving word problems involving addition and subtraction, but in the minds of many children, the concepts of fractions and multiplication mixed in a word problem is “like oil and vinegar” (Hardiman & Mestre, 1989). Many pupils think that if a fraction is involved, multiplication cannot be involved. In fact, children who identify multiplication as the appropriate operation to solve a problem when they are prevented from seeing the numbers involved frequently change their minds when they are shown the numbers and see a multiplier that is less than one. Greer (1987) referred this phenomenon as *nonconservation*. He suggested that the difficulty, however, does not seem to be that children believe that the computation cannot be done or that they are unable to perform the computation. Results from the National Assessment of Educational Progress study (Carpenter *et al.*, 1984) indicate that pupils’ computational abilities with fractions are far better than their ability to solve word problems involving fractions. The researchers believe that the source of the difficulty more likely lies in a child’s lack of understanding of the different ways

that multiplication can be embodied in a word problem (Bell *et al.*, 1989; Briars & Larkin, 1984).

To date, efforts directed toward understanding how children attempt to solve arithmetic problems have focused mainly on single-step addition and subtraction word problems. However, Carpenter *et al.*, (1998) argued that research on children's solutions of simple arithmetic word problems can provide insight into the development of more complex problem solving abilities. Several studies have shown that children's ability to solve these problems is influenced by the structure of the problem text (Briars & Larkin, 1984). Text structure has also been indicated as an important factor in multiplication and division problems (Bell *et al.*, 1989). Furthermore, studies such as those by Bell *et al.* (1989) and Greer (1987) suggested that the selection of an operation for solving multiplicative problems appears to be influenced also by the types of numbers (whole numbers vs. fractions or decimals) in the problem. Even when the problems are juxtaposed and attention is drawn to their similarity, children have been observed to choose different operations for identically structured problems that differ only in the types of numbers involved (Ekenstam & Greger, 1983).

A number of studies have documented pupils' difficulties with multiplication word problems that contain rational numbers (Bell *et al.*, 1989; Graeber *et al.*, 1990; Harel, Behr, Post, & Lesh, 1994). In Graeber *et al.*'s study, they focused on pupils' solutions of problems with fractional rather than decimal operators, and it employed a system of problem categories that took into account and organised the variety of problem actions that could be represented by the multiplication operation. Graeber *et al.*'s study made a comparison of the strategies used on a set of 30 one-step multiplication problems by 141 fourth grade pupils who had not studied multiplication of fractions with those of 194 sixth grade pupils who had studied multiplication of fractions during two schools years. They found that both groups of pupils considered multiplication a more appropriate operation for some problems than for others expressing the same mathematical relationships. These results suggest that a different instructional focus is needed in order to address directly pupils' beliefs about multiplication and division and help pupils extend their

understanding of multiplication of whole numbers to include multiplication by fractional operators less than one.

In light of the above difficulties in solving word problems involving fractions found among other pupils outside Brunei, the present study aimed to examine if Bruneian pupils would also encounter such difficulty. Word problem involving fractions comprised the sixth component tested in the diagnostic test used in this study.

A classical reference by Newman (1977) classified pupils' errors in solving word problems as errors of reading, comprehension, transformation, processing and encoding. The pupils' problem-solving process is like a hurdle race and to be successful in the race the pupils have to leap each hurdle. If pupils fail to perform a stage, they will not be able to perform the next stage (Ekenstam & Greger, 1983). For example, if they cannot comprehend a problem, they will not be able to perform the transformation stage. As a result, they will not be able to solve the problem (Saman & Suffolk, 2001). A study carried out by these researchers on solving word problems with Primary six Bruneian pupils revealed that the most common type of errors committed by the pupils in solving word problems involving multiplication were transformation errors, with almost half (49.1%) of the pupils interviewed at fault. Pupils made this type of error in every problem in the study. Comprehension errors (29.3%) were the second most common type of errors.

Newman (1977) suggested that errors could be a result of the question form, the pupils' carelessness or a lack of motivation. Lack of care can only be detected by listening to the pupils' second attempt. In this study, only 48 pupils were selected and given a second chance to attempt the problems that they did incorrectly initially. If the pupil was careless originally and the pupil's second attempt was correct, then the error would be classified as due to lack of care. Provided the form of the problem was clear and unambiguous and there was evidence that the pupil was trying to solve the problem, then the errors would result from failing to complete the five hurdles mentioned earlier.

A number of local studies have investigated pupils' performance on word problems (Sainah, 1998; Raimah, 2001; Saman & Suffolk, 2001). They found that Bruneian P6 pupils perform poorly on word problems, especially those involving

multiplication and division. The most common errors were process errors and transformational errors, as the pupils failed to choose the correct operations to solve the problems.

2.11.1 Use of Keywords Strategy in Solving Word Problems

Willoughby (1990) believes that the abundant books, pamphlets and courses on critical thinking and problem solving that have been propagated in the 1980s cannot be of help unless certain pedagogical misconceptions are clarified. This includes prescribed rules such as finding key words in a problem to decide the appropriate operations on the values given in the problem, or applying arithmetic algorithm to any word problem. Developing critical and analytical thinking through problem solving takes time and a lot of teacher's commitment and dedication. (Willoughby, 1990; Barb & Quin, 1997).

Past studies on word problems in Brunei (Veloo & Lopez-Real, 1994; Sainah, 1998; Saman & Suffolk, 2001) show that misinterpretations of the significance of key words such as "*altogether*", "*difference*", "*left*", "*each*", "*total*", "*how many*" and "*more*" is an important cause of the errors displayed by the pupils they studied. These researchers demonstrated that there is a strong indication in their studies that the transformation errors made by the pupils were due to the misuse of keywords. In their studies, to solve the problems, pupils often decided what operation to perform by the cue words. For an example, the words "*total*", "*left*", "*product*" and "*each*" often signalled the pupils to perform the *addition*, *subtraction*, *multiplication* and *division* operations respectively. However, this did not help them because many pupils were unable to carry out the correct process to solve the problems, as the use of indicators or keywords did not always work. These keywords were not only used in the current study but are commonly used in the pupils' everyday mathematics class exercises. For example, in the Brunei Darussalam Primary Mathematics textbooks, workbooks and PCE examination papers the words *altogether*, *total*, *difference*, *times*, *product*, *each* and *shared equally* are commonly used in problems involving the four arithmetic operations, *addition*, *subtraction*, *multiplication* and *division* (Ministry of Education, 1999). The cue words for addition such as "*total*", "*more*" and "*altogether*" is sometimes misleading.

Those above-mentioned studies carried out in Brunei, except that of Raimah (2001), examined pupils' performance on word problems that did not involve fraction elements and were carried out within smaller samples than used in the present study. Taking a larger sample and on a longitudinal basis, the current study set out to determine if similar problems were prevalent among P5 and P6 pupils in word problems involving fraction elements. The questions posed in this study; would the common errors remain process skill and transformational, or would they be of a comprehension or encoding nature? In addition, would the pupils resort to the use of keywords to solve the problems? This will be addressed more in the result chapter; Chapter 4 (p.109)

2.12 Misconceptions and Errors on Algebra

Besides fractions, difficulties in learning algebra have been also well-documented (Booth, 1988; Herscovics & Linchevski, 1994; Boulton-Lewis, Cooper, MacGregor, 1996; Pillay & Wilss, 1998). Perso (1991) made an extensive research on misconceptions on algebra. She identified some nineteen misconceptions in algebra held by students in her study group. She later grouped those misconceptions into four categories related to problems on:

- 1) basic understanding of letters and their place in mathematics, and more specifically, algebra;
- 2) manipulation of letters or 'variables';
- 3) the use of the rules of algebraic manipulation to solve equations; and
- 4) the use of the knowledge of algebraic structure and syntax to form equations.

Other literature on algebra indicates a number of concepts need to be understood before students can begin algebra study. For example to solve

$$\frac{2}{5}(x - a) = -\frac{3}{4}(2 - x),$$

the following conceptual understandings are necessary: the equality concept, directed numbers, fractions, order conventions, the variable concept (Johnson, 1985; Norton & Cooper, 1999). Norton and Cooper (1999) stressed that if any one of those understandings is lacking, the solving process is almost impossible. For example, misconceptions related to the equal sign might result in a pupil not carrying out the same operation to both sides of the equation. Pupils who do not understand directed

numbers are likely not to change the sign of 2 and $-x$ when they multiply through. Lack of understanding of fractions will probably result in students not seeking appropriate ways to convert the denominators to 1. Misconceptions in order convention (and the distributive law) will probably lead to problems related to the sequence in which the students try to solve the problem. Finally, if the students do not understand the variable concept it is likely that, even if they do solve the equation, their solution will probably lack meaning for them. That is, the pupils will see the exercise as an abstract activity involving the application of arbitrary rules.

The current study examined P5 and P6 pupils' performance on the learning of fractions. It was expected that their performance could be used to indicate if they had the prerequisites to learn algebra at the secondary level later. The results of the study could therefore be used to inform future teachers and teacher educators of appropriate teaching strategies.

2.13 Using Pupil Errors

The study of pupils' mathematical errors has significantly influenced the field of mathematics assessment and intervention, providing alternative perspectives as to what error patterns indicate (Ashlock, 1990, 1994; Borasi, 1985b, 1994). As mentioned earlier in this chapter, traditionally, pupils who made errors in their work were regarded as suffering from some type of learning disability (Kephart, 1960), and that they made errors because they lacked knowledge of "correct" algorithms. Such a deficit model of error production suggested that the pupil exhibiting the error had learned nothing as a result of the initial teaching effort. Baroody and Hume (1991) demonstrated that pupils' errors in using algorithms are usually not caused from failing to learn a particular idea but from learning or constructing incorrect mathematical ideas.

In the current research, pupils' mathematical errors identified in the first diagnostic test were analysed and shared with the teachers concerned. Even though the study did not engage any intervention strategy, those teachers whose pupils were involved in the study were alerted to pay special attention to the misconceptions and mathematical errors identified in the first diagnostic test in their subsequent instruction at the pupils' P6 level in the following year. Pupils' mathematical errors,

if detected at an early stage, would thus be corrected in order to prevent them from developing further (Borasi, 1985a, 1985b; Swedosh, 1996).

2.14 Pupils' Attitudes towards the Learning of Fractions

The study of attitudes has a long and complex history in social psychology. For the purposes of verbal measurement, most researchers seem to agree that an attitude is a state of readiness, a tendency to respond in a certain manner when confronted with certain stimuli (Oppenheim, 2001). Oppenheim further suggested that most of an individual's attitudes are usually dormant and are expressed in speech or behaviour only when the object of the attitude is perceived either visually or cognitively. He demonstrated that attitudes are reinforced by beliefs (the cognitive component) and often evoke strong feelings (the emotional component) that may lead to particular behavioural intents (the action tendency component). In the current study, pupils' attitudes towards fraction learning were measured by using a 26-item questionnaire that covered the three-mentioned components, with the aim to investigate if male and female pupils held different attitudes towards the learning of fractions.

While I was unable to locate a definition of attitudes that was specifically focused on fractions, there do exist a number of definitions of attitude relative to mathematics. Fennema and Sherman (1977) suggest that research supports the following "tentative conclusions":

- 1) A significant positive correlation exists between pupils' attitudes and their mathematics achievement – this relationship increases as pupils proceed through the grades.
- 2) Pupils' attitudes towards mathematics are quite stable, especially in Grades 7 – 12.
- 3) The middle grades are the most critical time period in the development of pupils' attitudes towards mathematics.
- 4) Extremely positive or negative attitudes tend to predict mathematics achievement better than attitudes that are more neutral.
- 5) Gender-related differences in attitudes towards mathematics exist, perhaps related to similar gender-related differences in confidence or anxiety measures relative to learning mathematics.

Past research has established that pupils' attitudes towards mathematics contributes an impact on mathematics achievement or was correlated with mathematics achievement (Kim & Hocevar, 1998; Lokan & Greenwood, 2000). Lokan and Greenwood (2000) studied the relationship between attitudes towards mathematics and mathematics achievement using TIMSS data, and found that a relationship exists between those two variables. Therefore, attitudes towards mathematics have been considered an important factor in influencing participation and success in mathematics.

Though fractions are one of the components of the larger mathematics field, an extensive literature search indicated a paucity of studies on pupils' attitudes towards fractions per se. One that does address this matter is the study by Raimah (2001) on Bruneian pupils' attitudes towards fractions. This study suggested that of the 260 Primary 6 pupils in her sample, 67% indicated that they liked learning about fractions, 28% disliked fractions while 5% were undecided. The pupils had offered various reasons for enjoying learning about fractions – for example, fraction tasks were easy to understand; they were important for future use; their inclusion in the upcoming PCE examination, their teachers were enjoyable and fractions were very interesting. Among the reasons given by her pupils for disliking fractions were: fractions were difficult to understand; they were confusing and boring and because the pupils considered themselves poor in mathematics generally.

Groff (1994) noted that it was common for middle grade mathematics textbooks to devote from 15 to 25% of their pages to fractions study. The Brunei Primary 6 (PCE) mathematics syllabus and related examination is not an exception. Furthermore, standardised tests of mathematics contain a multitude of items that measure fractions knowledge, thus emphasising to teachers that it is their responsibility to develop pupils' mastery of this mathematics topic. Groff commented that from his experience as a middle grade teacher, he had noticed that many of his pupils harboured a particular dislike for fractions study. Not only his pupils, but also he himself realised and sensed that learning to add, subtract, multiply and divide fractions was a "dead-end" activity. Students sensed too that fractions were irrelevant to the solution of mathematics problems in anyone's daily

life, consequently studying fractions thus seemed a foolish waste of time. Later on, when he worked as a teacher educator, he often encountered the same negative attitudes about fractions in the classes he supervised. Groff did acknowledge though that the above statements were only his premonition and were yet to be confirmed by empirical evidence.

Thus as Prior (2000) has demonstrated, even though mathematical abilities are perceived as less critical for educational and occupational success in the longer term compared to literacy skills, many children finish school with an inadequate competence in, and a negative attitude towards mathematics. Moreover, he commented that many adults feel inadequate and anxious when they are obliged to call upon their numeracy skills.

Pupils' attitudes towards mathematics have always been attributed to be a causal factor in determining their performance in mathematics (Shoenfeld, 1985; Borasi, 1990; Gadalla, 1999). As reported in some literature (e.g., Brandon *et al.*, 1987) boys surpass girls in mathematical performance and so do their attitudes towards mathematics. Relich (1996) reported that on affect, there is considerable evidence that males are more positive about personal aptitudes in mathematics when compared to females. However, he added that this could vary from study to study. Although biological reasons have been advanced for the gender difference (e.g., Benbow & Stanley, 1983), sociocultural reasons are more widely accepted, particularly sex role expectations and gender identity (e.g., Meece, Parsons, Kaczala, Goff & Futterman, 1982). In another study by Boli, Allen and Payne (1985), the authors suggested that the greatest differences between boys and girls could be found in their attitudes to and self-confidence in mathematics rather than in actual achievement. Marsh, Smith and Barnes (1985), in their Australian study with fifth-grade pupils, found that although the girls outperformed boys on a standardised mathematics test, they nevertheless had lower self-concepts than the boys did. Other studies such as those by Thomas and Costello (1988) also provide evidence of boys' perceived superior competence and girls undervaluing their achievements in mathematics.

In Brunei, a country with a different educational and cultural background from most studies reported in the literature, Yusof (1993) found that there was no significant

difference between secondary school girls and boys in terms of pupils' general attitudes towards mathematics. The present study, conducted a decade later, would be informative if it were to find out that there were any gender differences in terms of Bruneian P5 and P6 pupils' attitudes towards the learning of fractions. If there were gender differences reported in the current study, future research might be directed towards determining the possible reasons for such attitudinal differences among primary and secondary school pupils. The issue of gender differences is dealt with further in the next section.

2.15 Gender Differences

For more than two decades of research findings, performance in mathematics has frequently been associated with gender (Hyde *et al.*, 1990; Entwisle *et al.*, 1994; Manning, 1998; Freislich & Bowen-James, 2000). While girls and boys commence basic schooling on a generally equal footing across countries, by grade twelve girls are generally in a lower academic position than their male counterparts, particularly in the areas of math and science (Linn & Peterson, 1985; Rogers & Gilligan, 1988). The interest level, confidence, and subsequent achievement of girls in the area of science and mathematics drop by around the seventh grade (Linn & Hyde, 1989). Jones (1984) and Friedman (1989) also demonstrated that differences in mathematics performance between the genders have been found to increase from grade seven, and many studies have shown that by age 13, boys are significantly superior to girls in both their mathematical performance and their attitudes towards mathematics (Hanna *et.al.*1990). A study by Leeson (1995) demonstrated that the overall performance of Grade 6 boys in the Australian Primary Mathematics Competition for the years 1990 to 1992 was superior to that of girls. Leeson further suggests that significant gender differences were found on a number of mathematical topics: Boys performed better on items involving estimation (of numbers and measurement), rates, speed-time-distance, percentages and non-routine problems, while girls' performance was superior on reflection of shapes, figure identification and number patterns.

Literature suggests that several factors are said to be related to the said gender gaps in indicators such as curriculum design, teacher-student interaction, and the

variability of self-esteem levels (Rogers & Gilligan, 1988). The ratio of female mathematics teachers to male mathematics teachers has been said to affect the gender differences in pupils' mathematical performance and attitudes. Researchers such as Hanna *et al.*, (1990) believed that this might be an important factor in explaining sex differences in mathematical achievement, since it most likely affects the degree to which girls subscribe to the notion that mathematics is the preserve of men. Thus, according to this reasoning, countries with negligible sex differences in achievement would be expected to have higher proportions of female mathematics teachers than their highly sex-differentiated counterparts. Although this notion seems intuitively sound, the data from Hanna's study did not support it. They quoted two examples: in British Columbia, very small sex differences were observed even though only 3% of the mathematics teachers were female, while in Hungary very large sex differences emerged despite the fact that a majority of the mathematics teachers (60 %) were female. In Brunei Darussalam, the 1999 statistics shows that male teachers constitute only 30.2% of the total primary school teaching force of 3 858. The trend for the past ten years has shown a steady decline of male teachers of about 1.6% each year (Yong, 2001). With the number of female teachers dominating in Brunei primary schools, it was considered important to investigate if teacher-related differences in achievement and attitudes exist among the pupils. In the current study that involved four schools and fifteen teachers, only three of the classroom teachers involved males.

Many researchers believe that a female's perception of mathematics as a male domain may negatively affect her motivation to do well in the subject and hence affect her achievement (Leder, 1982; Hyde, Fennema & Lamon, 1990; Fennema, Carpenter, Jacobs, Franke & Levi, 1998). In fact, it is considered that girls often fear success in mathematics, believing that their social relationships with their male peers will suffer if they are perceived as superior in a sphere that they imagine to be forbidden to women (Moss, 1982). Finn, Dulberg and Reis (1979) have suggested that girls' performances improve most significantly in programs that rely on older girls to counsel, encourage, and tutor younger girls. In a study by Ethington and Wolfe (1986), it was suggested that a degree of the emergent male superiority in mathematical performance may be attributed to greater exposure to formal mathematics training, but evidence found in a study carried out by Barnes (1983)

suggests that males are not superior in all aspects of mathematics. While some differences in mathematical achievement, albeit small (Willis, 1989), do seem to exist, these vary from culture to culture, and though very small and on the side of male advantage, differences in performance decrease over the years (Benbow, 1992).

The above studies are cited in the current study because I am interested to investigate if such gender differences will exist in Brunei – a country with a different cultural background from those above-mentioned studies.

Another factor that may affect sex differences in mathematical achievement and attitudes is the degree of home and parental support the pupils received. Hanna *et al.* (1990) believed that since mathematics is often perceived as a male domain, it is probable that in certain environments parents give more support to mathematical learning for boys than for girls. In fact, Fennema and Sherman (1977) have reported that girls report less home support for their mathematical endeavours. Again, as Hanna *et al.* (1990) have suggested, it is conceivable that such differential support could affect the achievement of females. Perhaps in countries where large sex differences are observed, parents give more support to their male offspring than to their female offspring, while such differential treatment may be less evident in countries where no differences are observed between the sexes. However, the data from Hanna *et al.*'s study of mathematical achievement of Grade 12 girls in fifteen countries did not show any evidence of the differences.

More recently, Shalev, Auerbach and Gross-Tur (1995) studied a group of Israeli children who were two years behind in arithmetic in Grade 4 and they found no gender differences in a sub-group of the original sample they studied at the age of 11-12 years.

Though some researchers suggest that girls are said to be performing less well in mathematics as compared to boys, other researchers believe that this gender differences in mathematics is decreasing, except for some complex mathematics and in the selection of university majors or careers that involve mathematics (Fennema, 1994; Forgasz, Leder & Vale, 1999). There is now a changing perspective on whether mathematics is a male or female domain subject (Forgasz, Leder, Gardner,

1999). Linn (quoted in Franden, 2002), a leading researcher on gender differences in mathematics education, has studied this subject for over twenty years. In her earlier studies she found that there were a number of gender differences in mathematical processing and that boys did better than girls. She found in her later studies, however, that this was not the case but that the gap had closed substantially (Linn & Hyde, 1989). In fact, even though middle school students often think that boys are better than girls in mathematics and science, this is not true according to researchers. They are at least equal and in many cases, the girls surpass the boys. Linn and Hyde claim that the main gender difference is in the confidence level of the student (which may be influenced by gender).

As the above-cited studies were conducted outside Brunei, it was considered important to investigate if the pupils in the current study demonstrated gender differences in mathematical performance, particularly in the area of fractions and attitudes towards the learning of mathematics and fractions. If such differences did exist, in which particular areas of fractions, or on which attitude sub-scales did the differences occur?

2.16 Conclusion

In terms of the literature reviewed in this chapter, the current study sought to determine new information about Bruneian P5 and P6 pupils' mathematical error patterns on fractions. It was anticipated that this information would fill a number of gaps in the literature currently available. For example, past studies on fractions dealt with pupils who were taught with English (as the first language) as the medium of instruction while this particular study dealt with pupils with English (as the second or third language) as the medium of instruction only when they reach their fourth year of formal education. Therefore, since this study is the first to embark longitudinally upon in this area from the South-East Asian region and since it covered all the six components of the fractions unit stipulated in the Brunei upper primary mathematics syllabus, it will add more information to that currently available for future researchers.

More importantly, fractions have previously been studied either in one or two areas only. Two examples are *Difficulties in Learning Simple Addition Facts: A persistent Mathematical Errors in Fractions Work*

Problem (Hopkins, 2000) and An Investigation of Errors Made by Primary Six Pupils on Word Problems Involving Fractions (Raimah, 2001). The current study incorporates all six aspects of fractions, including word problems that introduce the fraction notion.

The instrument used for the fractions diagnostic tests was a specially designed instrument based on the Brunei Darussalam Upper Primary Mathematics syllabus, while the attitude test was adapted from an instrument that had been used locally, but which was revised extensively in order to conform to the current study's research questions. Thus, the instruments have not been used elsewhere and it is anticipated that future researchers will be able to make use of them so that a comparison of pupils' error patterns elsewhere can be made on the six identified areas on fractions.

Chapter 3 will describe the processes involved for gathering the data. It covers the research paradigm, research design, research questions, sample selection, the design of the diagnostics tests and the attitude questionnaires, measurement process, the coding of the responses from the diagnostic tests and attitude surveys, the pilot study, data analysis, quality controls and ethical considerations that were taken into account.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter describes the research methodology used in this study. Information will be presented in separate sections covering: the research paradigm, the research aims and questions, the design of the study, the methodological considerations, the instrumentation used, conduct of the pilot study, the sampling strategy, data analysis procedures, quality controls and ethical considerations. This chapter also discusses the rationale for the methodology adopted as well as the advantages and drawbacks of the data collection methods. To provide the readers a clear view of the research design adopted for this study, a research methodology matrix is also provided in this chapter. To place the design and methodology in context, it is first important to reiterate the research paradigm that guided this study.

3.2 Research Paradigm

Though this study is exploratory in nature, no specific research paradigm fits best to describe it wholly. Holistically, this study is highly quantitative because the data is mainly derived from paper-and-pencil diagnostic pre- and post-tests and pre- and post-attitude towards fractions questionnaires. However, this study also adapts some elements of qualitative research paradigm such as lesson observations and pupils' interviews. Qualitative paradigm is described by Silverman (2000, p.1) as “..... seems to promise that we will avoid or downplay statistical techniques and the mechanics of the kinds of quantitative methods used in, say, survey research or epidemiology”. However, since this study also employs some criteria of other research paradigms, the term mixed-method approach is considered more appropriate to describe the paradigm that guided this study. As Neto (2003) suggests that though regression-discontinuity is strong in internal validity and can parallel other non-equivalent designs in terms of validity threats, interpretation of results might be difficult. He therefore recommends that “adding qualitative flesh to the quantitative bones is a good strategy to overcome that difficulty” (p.4). Examples of

other research paradigm criteria adapted in this study are: identifying the problem and research purposes; deciding the focus of the study; selecting the research design and instrumentation; addressing validity and reliability; ethical issues; approaching data analysis and interpretation. More about the research design and methodology employed in this study is presented throughout this chapter.

3.3 Research Aims and Questions

Two purposes of the study were to determine if there are any error patterns in working with fractions made by P5 and P6 pupils in Brunei, and to identify the most prevalent errors held by these pupils. Researcher-designed diagnostic pre and post paper-and-pencil tests were used to assist in achieving these goals. The study also aimed to find out if P6 pupils continued to hold similar error patterns that were identified when they were at P5 and to investigate if female pupils exhibited/committed more errors in fraction as compared to their males counterparts or vice versa. Additionally, the study sought to examine pupils' attitudes towards the learning of fractions and to investigate if there were gender differences associated with these attitudes.

The study was longitudinal in nature, the same cohort of pupils being followed through and re-tested in two successive years. The following research questions provided a focus for the study:

1. What are the prevalent mathematical errors on fractions held by Bruneian P5 pupils?
2. Will the same errors made by P5 pupils still be prevalent after they move to P6?
3. In which component/s of the six identified areas of the fractions unit do P5 and P6 pupils exhibit the most errors?
4. Are there gender differences in the errors exhibited by Bruneian P5 and P6 pupils?
5. What are the attitudes of Bruneian P5 and P6 pupils towards the learning of fractions?
6. Are there any gender differences in P5 and P6 pupils' attitudes towards the learning of fractions?

3.4 Design of the Study

A summary of data sources and sample, data type, instrument developed, data collection strategy and data analysis for each research question is summarised in the research matrix shown in Table 3.1 (p.72).

Table 3.1 *Research Methodology Matrix*

Research Questions	Data Source and Sample	Data Type	Instrument Developed	Data collection strategy	Data Analysis
RQ#1: What are the prevalent mathematical errors on fractions held by Bruneian P5 and P6 pupils?	Schools 1,2,3,4 396 pupils 214 boys 182 girls Primary 5 (2002)	Quantitative and Qualitative	A 28 item (pre) diagnostic paper- and –pencil test	Written tests Pupils interviews	Frequencies, Statistical significance and graphical presentations
RQ#2: Will the same errors made by P5 pupils still be prevalent after they move to P6?	Schools 1,2,3,4 338 pupils 187 boys 151 girls Primary 6 (2003)	Quantitative and Qualitative	A 30 item (post) diagnostic paper- and –pencil test	Written tests Pupils interviews Lessons observations	Frequencies, Statistical significance and graphical presentations
RQ#3: In which component/s of the six identified areas of fractions unit do P5 and P6 pupils exhibit the most errors?	Schools 1,2,3,4 396 pupils Primary 5 (2002) Schools 1,2,3,4 338 pupils Primary 6 (2002)	Quantitative and Qualitative	A 28 item (pre) diagnostic paper- and –pencil test A 30 item (post) diagnostic paper- and –pencil test	Written tests Pupils interviews Lessons observations	Frequencies, Statistical significance and Graphical presentations

Research Questions	Data Source and Sample	Data Type	Instrument Developed	Data collection strategy	Data Analysis
RQ#4: Are there gender differences in errors exhibited by Bruncian P5 and P6 pupils?	Schools 1,2,3,4 396 pupils Primary 5 (2002) Schools 1,2,3,4 338 pupils Primary 6 (2003)	Quantitative and Qualitative	A 28 item (pre) diagnostic paper- and –pencil test A 30 item (post) diagnostic paper- and-pencil test	Written tests Pupils interviews Lessons observations	Frequencies, Statistical Significance and Graphical presentations
RQ#5: What are the attitudes of P5 and P6 pupils towards the learning of fractions?	Schools 1,2,3,4 396 pupils Primary 5 (2002) Schools 1,2,3,4 338 pupils Primary 6 (2003)	Quantitative and Qualitative	A 26 item pupil attitude pre- test Pupils' interviews	Written attitude questionnaire Pupils interviews	Frequencies, Statistical Significance

Research Questions	Data Source and Sample	Data Type	Instrument Developed	Data collection strategy	Data Analysis
RQ#6: Are there any gender differences in P5 and P6 pupils' attitudes towards the learning of fractions?	Schools 1,2,3,4 396 pupils Primary 5 (2002)	Quantitative and Qualitative	A 26 item pupil attitude post-test Pupils' interviews	Written attitude questionnaire Pupils interviews Lessons observations	Frequencies, Statistical Significance
	Schools 1,2,3,4 338 pupils Primary 6 (2003)				

3.4.1 Sample

Diagnostic Pre-test

The pupils participating in the diagnostic pre-test were 396 fifth graders (P5) from four schools in Brunei Darussalam. However, the number increased to 404 for the pupil attitude questionnaire that was administered a week later. Only those pupils who sat for both diagnostic pre-test and attitude pre-test were considered for inclusion in the samples for this diagnostic pre-test, the sample size totalling 396 (214 males and 182 females). The ages ranged from 10 to 12 years. Table 3.2 below shows the sample number by schools and gender for the pre-test.

Table 3.2

Number of Pupils by Schools and Gender for the Pre-test

School	Male	Female	Total
1	54	36	90
2	35	37	72
3	59	52	111
4	66	57	123
Total	214	182	396

Diagnostic Post-test

For the diagnostic post-test, the participating pupils were drawn from the same cohort of P5 pupils from the four participating schools who in the following academic year moved to the P6 level. However, the number decreased to 338, with a drop of fifty-eight pupils. This mortality rate was due to pupils changing schools at the beginning of the 2003 school term, while others were either absent for the administration of the diagnostic post-test or the attitude post-test. Only those pupils who sat for both the diagnostic and attitude post-test were considered for inclusion in the sample for this diagnostic post-test, which totalled 338 pupils (187 males and 151 females). The ages ranged from 10.5 to 12.5 years. Table 3.3 (p.76) displays the sample sizes by schools and gender for the post-test.

Table 3.3

Number of Pupils by Schools and Gender for the Post-test

School	Male	Female	Total
1	46	24	70
2	29	30	59
3	52	48	100
4	60	49	109
Total	187	151	338

All pupils participating in the diagnostic pre- and post-tests had been taught fraction topics in Primary 3 and 4. Their instruction included concept of fractions; simple fractions; equivalent fractions; comparison of fractions; addition and subtraction of fractions with common denominators; proper fractions, improper fractions and mixed numbers, and simple word problems on fractions. The topics for Primary 5, taught around April and May, included addition and subtraction of fractions with different denominators; simple multiplication and division of fractions; fractions of a quantity; expressing one quantity as a fraction of another; simple word problems on fractions; decimal fractions, and conversion between fractions and decimals. Since the study was longitudinal in nature, the same cohorts of pupils were studied a year later when they moved to P6. Here they were administered the diagnostic and attitude post-tests. Prior to be given the post-test, they received more instruction on fractions, included percentages, from the same teachers who taught them at their P5 level, following the normal P6 mathematics syllabus.

All four schools were government-funded, co-ed institutions in the Brunei Muara district, the largest, and the most densely populated among the four districts in Negara Brunei Darussalam. The schools were selected on the basis of two considerations. One consideration was pragmatic, namely, the distance I would have to travel to reach the schools. The second reason considered was the ease of entry due to my close associations with the four headmasters and headmistress and their schools. After receiving an affirmative response, the chosen schools were contacted in person and initial arrangements were made with the school heads who later

appointed one teacher as a co-ordinator, normally the Head of Mathematics Department of the school, to liaise with me throughout the study period. The distance of each of the four schools from my residence ranged from 20 - 30 km. School 1 is about 10 minutes driving from the city centre and has an enrolment of 600 pupils and 28 teachers. There were three classes of Primary 5 at the time with an average of 30 pupils in each class. School 2 is situated in the outskirts of the city centre and had an enrolment of about 550 pupils and 22 teachers. There were three classes of Primary 5 with an average of 24 pupils. School 3 is about 20 minutes drive from the city centre and is situated in the largest resettlement area in Brunei. The enrolment was about 1000 pupils and 38 teachers with five classes of Primary 5 with an average of 28 pupils each. School 4 is about 35 minutes drive from the city centre with an enrolment of about 750 pupils and 32 teachers. There were four classes of Primary 5 with an average of 30 pupils in each class. All the 15 classes from the four schools involved in the study were of mixed ability. The term “mixed ability” refers to a system in which there was no streaming carried out in terms of pupils’ cognitive ability. In each of the fifteen classes involved, there were a few above-average pupils, however the majority of the pupils were either of average or below average ability, as identified by their respective class teachers based on the pupils’ performance in their tests and term examinations. All the four schools conducted classes in the morning 7.15 am to 12.30 pm. In the afternoon, most children in Brunei go to their religious classes.

3.4.2 Instrumentation

Quantitative

Quantitative data for the study were collected through use of the following instruments:

- two diagnostic paper and pencil tests (pre- and post)
- the pupil attitude questionnaire (pre- and post)

The Diagnostic Pre- and Post-tests

The pre- test was a researcher-designed diagnostic paper and pencil test. The test consisted of 28 items covering all the six components of the fraction units covered in

P5 Brunei Darussalam mathematics syllabus. The six components were: understanding of the fraction concepts and the sequencing of fractions; manipulating fraction symbols (addition and subtraction); manipulating fraction symbols (multiplication and division); determining a fraction of a quantity; alternative forms of fractions and solving word problems with fraction elements (See Appendix A, p.199 for full text).

The post-test was similar to the pre-test, however two additional items covered only at the P6 level were added to the alternative form of fractions component, making a total of 30 items. (See Appendix B, p.205 for full text).

The relevance and validity of the items included in the diagnostic pre- and post-tests were dealt with during the pilot study stage and are further described in Section 3.6 (p. 96) of this chapter.

The Pupil Attitude Questionnaire

This instrument was developed to investigate the P5 and P6 pupils' attitude towards the learning of mathematics in general and fractions in particular. The instrument was adapted from Raimah (2001). Raimah's original instrument consisted of 12 items and was modified and validated in order to conform more closely to the current study's research aims and objectives. The revised instrument consisting of 26 statements used a 5-point Likert scale and aimed to measure pupils' attitudes in terms of the three components and four constructs : Cognitive (Confidence and Perception), Affective (Enjoyment) and Conative (Commitment). (See Appendix C, p.212 for full text).

Qualitative

Data collected qualitatively were gathered by the following means:

- pupil interviews (pre- and post)
- lesson observations

Pupil Interviews

Ashlock (1990) commented that written tests are helpful in diagnosis, but they are limited. He further added that interviews have long been recognised as an effective way to collect additional information about a child's mathematical concepts. Therefore, to support data obtained from the diagnostic pre- and post-tests as well as the pupil attitude pre- and post-questionnaires, semi-structured pupil interviews were planned and conducted with selected pupils to gather more data and to probe the pupils' understanding of the items and word problems on which they made errors in the pre- and post-tests. The interviews were conducted with forty-eight individual pupils in a room set aside for the purpose (either a library or a school resource room) and pre-arranged by the appointed co-ordinator of the schools chosen. All the interview sessions were audio taped with the pupils' knowledge and consent and supported by detailed field notes for transcription and analysis purposes.

Lesson Observations

To complement information obtained from the pupil interviews, and in a further effort to support data obtained earlier from the diagnostic tests and the pupil attitude questionnaires, twelve lesson observations were made on two P6 classes, whose teachers (one male and one female) volunteered to be observed teaching. Initially, I intended to observe at least four classes and to look for four volunteer teachers, that is one class/teacher from each participating school. However, due to time constraints and the difficulty in managing my available time and the teachers' timetables in the four classes of four different schools, I was restricted to observing two classes.

In the Brunei primary schools timetable, it is a common practice for mathematics lessons to be allocated during the first half of the morning. A sample of a class timetable is provided in Appendix G (p.250). The first class taken for the observation was from School 1 with the male teacher and the second class was from School 2 with the female teacher. Each observation lasted for between fifty minutes to an hour, depending on the punctuality of the teachers to come into their classes. All the lessons were on fractions and the observations were made at the end of February and early March 2003 when the fractions unit was scheduled to be taught according to the Brunei Darussalam P6 mathematics scheme of work. The scope of

the fractions unit for P6 syllabus was an extension of what the pupils had learnt at the P5 level. The detail of the Brunei Darussalam upper primary mathematics syllabus was described earlier in Chapter 1 and is provided in Appendix D, p.215.

Prior to the actual observation, the teachers were informed of my visits either via a phone call or a personal visit made on the day before. On the observation days and prior to their teaching sessions, the teachers' lesson plans for that particular lesson were first obtained from them with their consent. This was to assist me in examining their description of the activities the pupils would be undertaking and the fraction areas these activities covered in the lessons. I used field notes to record all the instructional activities carried out during the lesson observation periods. Evaluation of the pupils' participation and activities was guided by observing their familiarity, level of difficulty, enjoyment, involvement and understanding of the activities presented in the lessons. The observations were recorded in activity logs, including the time the lesson and presentation began and when the pupils began the task and activities. Though the focus of the observation was on the pupils' activities and participation which reflected their enjoyment, confidence, perception and commitment, what the teachers did during, after the presentation and before the lessons ended were also observed. This was done in order to justify if what the pupils perceived about their teachers in the questionnaires was reflected in the actual lessons – activities such as monitoring the pupils' work and helping the pupils with difficulties throughout the lessons.

3.4.3 Procedure

The multi-method approach involving both qualitative and quantitative methodologies as summarised in Figure 3.1 (p.81) was adopted in this study in line with current thought on the advantages of a mixed-method procedure (Guba & Lincoln, 1989). As mentioned earlier, the qualitative components that were used included interviews with individual pupils and lesson observations that were recorded in field notes. The quantitative components included the diagnostic pre- and post-tests on fractions, which the pupils had learnt at the P5 and P6 levels, and the pre- and post-questionnaires for pupils focusing on their attitudes towards the learning of fractions.

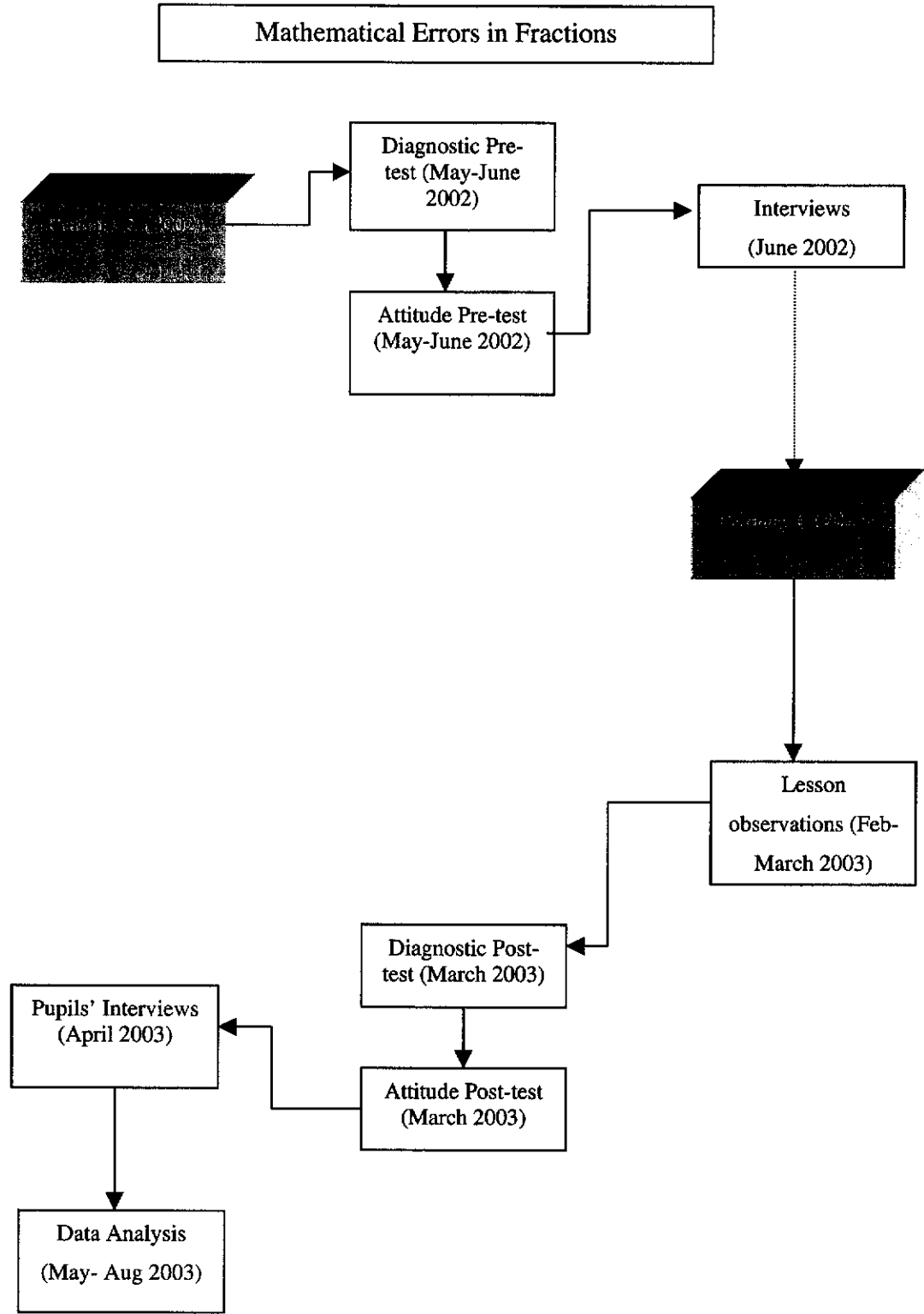


Figure 3.1 A summary of data collection procedures.

I administered the diagnostic pre- and post-tests as well as the pupils' attitude tests with the help of the classroom teachers or the mathematics teachers whose pupils were involved in the study as well as the appointed co-ordinator for each school. I also carried out the pupils' interviews and lesson observations. The attitude test was first administered to the cohort of P5 pupils in the week following that in which they sat for the first diagnostic test – in May and June 2002. The same test was re-administered to the same cohort of pupils about seven months later at the pupils' P6 level. The administration of the diagnostic post-test and post attitude test took place at the end of February and early March 2003.

The Administration of the Diagnostic Pre-test

In stage one, data was generated by administering to the 396 (214 males and 182 females) P5 pupils involved in the study the diagnostic test with 28 computational items covering a unit on fractions, based on the Brunei Darussalam Primary Mathematics Curriculum. The pre-test included four additional one-step word problems on addition, subtraction, multiplication and division. All pupils received prior instruction on fractions in their third, fourth and fifth grade mathematics classes. I conducted the first written test that was administered one class at a time over a period of three weeks to all 15 participating classes from the four chosen schools. I carried out this activity with the help of the class and mathematics teachers during the last week of May and the first two weeks of June 2002.

Before the pupils started answering the questions, the instructions were read to them. They were reminded to keep their answers to themselves, and that they should not discuss their responses with their fellow pupils nearby, nor copy the responses from their friends. The main advantage of administering the diagnostic pre- test by myself was that being present at the site had helped to resolve any problem that arose during the implementation of the test. I took charge of all the classes for the entire duration of the tests. Though I did inform the respective teachers that they could stay in their classes during the test if they wished to, all preferred to leave me alone after introducing me briefly to their pupils. For most pupils, it was not their first encounter with me as I have made some familiarisation visits to all the classes that took part in the study.

On average, the pupils took one hour to complete the test but those who could not do so were required to submit the test at the end of the period, as I did not want to encroach on the next subject period. In addition, during the planning and negotiation stages, it had been agreed that I would only take at most one hour to administer the test. Before collecting the test scripts, I ensured that all pupils had written down their names, class, age, sex and school clearly. Since the pupils were to be followed through when they moved to P6 in the next year, it was essential for me to retain those details in case any pupil changed class or school. In addition, each answer script was numbered from S001, which meant Sample number one up to S396, which meant Sample number 396 respectively. All the answer scripts were kept securely and separately in big envelopes according to the respective classes and schools. I handled all these exercises and the security of the answer scripts was guaranteed. On the cover of each envelope, the names of the pupils who were absent on the test administration days were noted clearly. This was double-checked with the class teacher as recorded in the class daily attendance.

Though the pupils were not informed beforehand about the diagnostic test, I expected them to attempt all the questions so that I would be able to obtain complete data. Unfortunately for some weaker pupils, they could not understand some questions and so chose to leave them unattempted. The nature of the questions in the diagnostic test was open-ended and computational, where the pupils needed to show and write their working in the space provided. Though they were given an hour to complete the 26 items, I believed that each pupil would spend different amounts of time on each question, therefore, in order to give them the chance to move at their own pace in dealing with each question, I decided to avoid reading each question to them. Also, it was not my intention to assist pupils during the test as it might have distracted other pupils' attention at that time and would have been unfair. The most I could do when I noticed pupils were labouring with questions that they could not solve was to advise them to proceed to the next question so they did not waste time on a particular question.

The Administration of the Diagnostic Post-test

The diagnostic post-test was administered at the end of February 2003 – after the pupils received further instruction on fractions during their first school term at P6. This was the same cohort of pupils who participated in the diagnostic pre-test when they were in P5 level. Included in the post-test were two additional items on alternative forms of fractions, (item 24 and 25) making a total of 30 items in the instrument (See Appendix B, p. 205). The two additional items were included in the post-test because these items were only covered in the P6 mathematics syllabus. The post-test was administered in order to investigate if the same cohort of pupils would display and repeat the same error patterns identified in the first diagnostic test at P5 level administered the year previously. Subsequently, the study examined the component of fractions in which the error patterns occurred more often. Similar to the diagnostic pre-test, the post-test was administered by myself with the cooperation of all 15 class and mathematics teachers whose pupils were involved in the study. As in the pre-test, the pupils were informed that the test would not be considered as a formal examination, and they were reminded to show all the necessary working in the space provided. Since the format and procedures were similar to that in the diagnostic pre-test, the questions were not read to the pupils.

The Administration of the Pupil Attitude Questionnaire

The attitude questionnaire was administered to pupils a week after they completed the diagnostic pre-test during the fourth week of June 2002. Special permission to administer the attitude test to the individual classes was obtained from the headmasters and the headmistress of the four selected schools. The classes' timetables were first obtained from the respective teachers in order to arrange the schedule of the test for all the fifteen classes. It was agreed that I would use the classes' mathematics periods to carry out the test. With the cooperation of the appointed co-ordinators, class and subject teachers, I administered the questionnaire myself by reading clearly each individual statement twice, and the pupils wrote their responses accordingly in the space provided on the questionnaire form. This procedure was not exercised in the diagnostic pre- and post-tests because the nature of the questions in those tests was computational and required pupils to show their

individual written working and solutions, whereas in the attitude pre- and post-tests, only a ticked response was required (Strongly Agree, Agree, Undecided, Disagree and Strongly Disagree). In reading the statements to the pupils, I allowed time for pupils to consider each item before they ticked their responses. While reading each item, I moved around the classroom to monitor the pupils' understanding and responses for each item. Prior to reading the 26 statements, all the instruction was read clearly to the pupils. Each class took around 20 minutes to complete the questionnaire. I ensured that all pupils finished answering the questionnaire in the time available, and thus the full return of the questionnaires was guaranteed.

During the administration of the questionnaires, I made certain that all the instructions and the explanation for the interpretation of each item within the questionnaires were provided with consistent language and intention to the pupils in all fifteen classes. By doing so, I could ensure optimum consistency of procedures during the data collection stage. The pupils were reminded that the questionnaires were not a test for them, but that the statements concerned matters requiring their opinions. Similar to the administration of the diagnostic pre-test, the pupils were reminded to keep their answers to themselves and that they should not discuss their responses with their fellow pupils nearby, nor copy the responses from their friends.

Before the pupils submitted their questionnaires, they were reminded to check if they had filled in all the necessary information such as their gender, age and class. With this approach, I could ensure that no information was missing. Compared to the diagnostic pre- and post-tests, the rate of responses for all the items was very satisfactory, the reason being that, with myself reading each statement for them, the pupils moved at about the same rate from one item to another, ticking the provided fixed responses. This is one obvious advantage of using the Likert scale format of the questionnaire. All questionnaires were then kept securely and separately in large envelopes according to the pupils' respective classes and schools. As for the diagnostic pre- and post-tests, all attitude questionnaires were also numbered according to the pupils' numbers in the diagnostic tests. This meant that each pupil in the study was identified by the same sample number regardless of the instrument they were completing in this study. The names of the pupils who were absent on the test administration day were noted on the envelope of each class in order to ensure

that only those pupils who sat for both the diagnostic pre-test and attitude pre-test were considered for inclusion in the samples for the first phase of the study.

The same attitude questionnaire was re-administered to the same cohort of pupils at the end of February 2003 – less than a year later at the P6 level. The attitude post-test was administered just after the pupils sat for the diagnostic post-test. The purpose of administering the attitude post-test was to examine if there would be changes in pupils' attitudes towards mathematics at the P6 level. I adopted the same administration procedure for the 15 participating classes at the four chosen schools. Though the pupils were not aware of being given the same attitude test for the second time, all concerned teachers were informed about the day and time when the test was to be administered. This was to avoid any clashes with any school events to be organised on that particular day.

The Pupils' Interviews I

To support data obtained from the diagnostic pre-, post and the pupil attitude questionnaires, interviews were conducted immediately after the diagnostic pre- and the post-tests. For the diagnostic pre-test, I carried out the interviews during the last week of June and the first two weeks of July 2002, whilst for the post-test, the interviews were conducted during the month of April 2003.

The one-to-one interview was conducted in each of the four schools' libraries or in a special room pre-arranged by the school co-ordinator. Special permission to allow the pupils to attend the interview sessions was obtained from the subject teachers in charge of the classes at the interview times. The 48 pupils were interviewed individually, and the duration of the interviews varied from 10 to 35 minutes depending on the number of items that the pupil had incorrect. Twelve pupils from each of the four schools were selected for the interviews. They represented the four top, four averages and four below average based on their total scores in the diagnostic pre-test.

I endeavoured to develop a positive rapport with the pupils by making the interview informal. To make the pupils at ease, I asked the individual pupils a few informal questions such as what they normally do on non-school days or what their favourite

television program was. In addition, I have also made some initial familiarisation visits to the four schools before the interview dates. Prior to each interview, I first showed each of the above average interviewees their test scores and complimented them on their achievement, though they were reminded again that their scores were not to be used as formal examination results. For the average and below average pupils, I showed and complimented them for any particular item that they have answered correctly. With this approach, I believe that I was able to obtain maximum cooperation and response from all pupils interviewed. For the high and average achievers, the interviews were conducted in English. However, for the below average pupils, the interviews had to be conducted in Malay as many of these pupils were unable to communicate effectively and confidently in English.

Each pupil was asked to solve again the selected items and problems that they had completed incorrectly in the written diagnostic pre-test. They did these one at a time in the space provided besides each item. Pupils were asked to describe what they had done to solve the items and problems. More able pupils were asked to provide explanations for their choice of answers, which in this case were their incorrect answers. Yackel and Cobb (1995) have noted that pupils who are encouraged to provide explanations to others for their mathematical actions create objects of reflection, both for themselves and for others. In other words, they make public their thinking, which can then be tested, accepted, clarified, or queried. Based on Yackel and Cobb's view, this study interview protocol contained the general introductory question "Can you please tell me how you came up with this answer?" Additional follow-up questions were asked as needed until I was satisfied that the pupil had given a complete explanation. In addition, I was free to use my own discretion about what to ask, depending on the pupils' responses. Often I focused my questions on particular workings that led them to the wrong answers. I took detailed written field notes and audio taped each interview. After the interviews, pupils' explanations were transcribed for discussion purposes. Some examples of pupils' interview transcripts are provided in Appendix E (p.217) and Appendix F (p.237).

Though the word problem component of the diagnostic test had been piloted earlier, I considered that a check on the difficulty of the terminology and the vocabulary used in the problems was necessary with the pupils, as this would provide

information and suggest whether or not the same word problems were to be used in future. Therefore, for the four additional questions on word problems, the interview dialogue was based on Newman's interview procedure described in the study of Marinas and Clements (1990). I asked each interviewee the following five questions:

1. "Read the questions to me. If you don't know a word, tell me." To identify *reading* errors.
2. "Tell me, what the question asked you to do." To identify *comprehension* errors.
3. "Now tell me what method you used to find the answer. Why?" To identify *transformation* errors.
4. "Now, go over each step of your working and tell me what you are thinking." To identify *process skill* errors.
5. "Tell me what the answer to the question is? Point to your answer." To identify *encoding* errors.

3.5 Data Collection

Data was collected using the two quantitative instruments, namely the diagnostic pre- and post-tests and the attitude pre- and post-questionnaires, at different stages of the study period. Over the duration of the study and with respect to the data collection, several quality controls (after Guba & Lincoln, 1989) were exercised. These included prolonged engagement with the sample; persistent observation; the use of a collaborative colleague; peer debriefing; attention to progressive subjectivity and ethical considerations. They were adopted in order to ascertain the quality of the study both qualitatively and quantitatively. A detailed account of each criterion is presented in Section 3.8 (p.103) of this chapter.

3.5.1 The Diagnostic Pre- and Post-tests

Error patterns were identified with a self-developed paper-and-pencil diagnostic pre-test. The same test was also used as a post-test administered to the same group of pupils a year later. Besides the pupils' interviews and lessons observations, the study

relied significantly on the paper-and-pencil tests to gather information – an approach that had both strengths and weaknesses.

McIntosh, Reys, Reys, Bana & Farrell (1997) reported that there is a growing concern, in both the United States (where perhaps the worst excesses of multiple choice testing have flourished) and elsewhere, against paper-and-pencil testing as a valid and reliable means of assessing pupils. At the same time, for this particular study while electronic means are being considered, a paper-and-pencil approach is the only way currently available for gathering a quantity of information quickly and cheaply.

Items generated for this diagnostic pre- and post-tests were those that commonly appeared in pupils' textbooks and workbooks. Initially, the pencil and paper test consisted of 41 open-ended items, but after pilot testing, only 28 items were used in the diagnostic pre-test and 30 items in the diagnostic post-test. The open-ended nature of the items allowed the pupils to show all the necessary workings before they arrived at a solution. This helped me to identify what strategies were being employed by the pupils in attempting to solve the items. Subsequently, it helped me to look for the error patterns exhibited by the pupils.

For this study, computational items were included because these are the types of items that were familiar to the pupils. Also, they were the type of exercises that pupils were exposed to in their daily class work and homework and they appeared in their monthly tests and term examinations. The format of the tests was also in line with their first public examination – the Primary Certificate Examination (PCE) – which they were to take at the end of their P6 level. The questions for this instrument were chosen after a careful analysis of the Brunei upper-primary mathematics curriculum and each question could be linked directly to the Brunei national upper-primary mathematics syllabus. With such familiar items, it was hoped that error patterns with fractions would be revealed and explored.

3.5.2 The Development of the Diagnostic Pre- and Post-test

The diagnostic test was developed and used to assess the pupils' prior knowledge and strengths in relation to fractions. To develop this test, a detailed content analysis

of the Brunei Darussalam upper-primary mathematics curriculum for P5 and P6 level specifically on the fractions unit was conducted. Other materials such as the pupils' textbooks, workbooks, teachers guide, school tests and similar tests used by other researchers both locally and overseas were also consulted (Australian Council for Educational Research, 1990; Doig, 1990; Booker, 1994; Bana *et al.*, 1997; Fatimah, 1998; See & Yusof, 2000). With items developed from resources familiar to P5 and P6 pupils and representing the types and sequences of fractions exercises they had been doing in their classes, it was hoped that the test would provide a good overview of the development of fractions through the pupils' schooling in the Brunei primary mathematics curriculum. The 28 and 30 items paper-and-pencil diagnostic pre- and post-tests used in the study focused on pupils' understanding, pupils' ability on computation and pupils' ability at solving mathematical word problems containing fraction elements. The classification of the 28 items for the pre-test and 30 items for the post-test according to the six components of the fractions unit is shown in Table 3.4 (p.91). The 28 items for the diagnostic pre-test and the 30 items for the post-test can be found in Appendix A (p.199) and Appendix B (p.205).

Table 3.4

Classification of Items to Fractions Components and Sample Items for Each Component of the Diagnostic Pre- and Post-tests

Fraction Components	Number Of Items	Item Number	Sample Item
Understanding of fractions concepts and sequencing fractions	8	1,2a,2b, 3,4,5,21,22	Complete the following equivalent fractions: $\frac{\quad}{10} = \frac{1}{2} = \frac{8}{\quad}$
Manipulating fraction symbols; addition and subtraction	6	6,7,8,9,10,11	Calculate $1\frac{2}{3} + \frac{4}{7}$
Manipulating fraction symbols; multiplication and division	5	12,13,14, 15,16	Calculate $\frac{6}{7} \times \frac{2}{3}$, giving your answer in its lowest terms.
Determining fraction of a quantity	4	17,18,19,20	What is $\frac{3}{4}$ of 60 kilograms?
Alternative forms of fractions	1 (pre-test) 3 (post-test)	23 (pre-test) 23,24,25 (post-test)	Express 0.85 as a fraction, giving your answer in its lowest terms.
Solving mathematical word problems with fraction elements	4	24a,24b,24c, 24d (pre-test) 26a,26b,26c, 26d (post-test)	A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?

3.5.3 *Semi-structured Interviews*

Studies have shown that interviewing pupils is one approach that helps researchers probe and delve more into pupils' thinking processes. McIntosh *et al.* (1997) stated that there is no doubt that on many occasions when researchers look at the answers given by individual pupils, or the overall responses of a cohort of pupils to an item, their first response is: "I wish we could probe that further through interviews". When McIntosh and his colleagues conducted interviews with American pupils, the responses were illuminating. For example, in their study, a pupil responded that there were 'lots' of fractions between $\frac{1}{5}$ and $\frac{2}{5}$; but when invited in an oral interview to name some, the pupil offered $\frac{1}{6}$ and $\frac{1}{7}$ as examples.

In the present study, to determine how and why the errors were made, semi-structured interviews were conducted with selected pupils and this data was shared with the teachers of those pupils. Even though no special intervention program was planned in the present study, the data obtained from the diagnostic pre-test was used to inform and alert the teachers concerned about the areas in which their pupils exhibited more errors. Special sessions were held with the concerned teachers for information sharing in their respective schools arranged by the appointed co-ordinators. Though the scope of the information shared was quite general, it was envisaged that the concerned teachers would pay special attention to the identified areas in which their pupils were weak during their next round of instruction on fractions at P6. The four chosen schools practise a policy wherein both class and mathematics teachers follow through their pupils from P4 to P5 and P6. This system enables the teachers to be more familiar with their individual pupils' strengths and weaknesses.

3.5.4 *The Interview Sample*

A total of 48 pupils, 25 males and 23 females and twelve from each of the four participating schools were interviewed (probed) on the items in which they were incorrect in the diagnostic pre-test. The 12 pupils represented the four top, four average and four below average performers from one selected class in each school.

The interviews were conducted with the aim of determining the pupils' source of difficulties and to obtain more information on how they could have arrived at the wrong or faulty answers that they wrote in their test paper. The same pupils were interviewed again after they sat for the post-test in 2003. However, due to the mortality rate, only 42 pupils were available for the interview in the diagnostic post-test. A number of examples of the interview transcripts of selected pupils are provided in Appendix E (p. 217) and Appendix F (p.237).

3.5.5 The Pupil Attitude Questionnaire

This instrument was developed to investigate the P5 and P6 pupils' attitude towards the learning of mathematics in general and fractions in particular. The same instrument was used as the pre- and post-test administered to the same cohort of pupils. The instrument was adapted from Raimah (2001). Raimah's original instrument consisting of 12 items was excessively modified and validated in order to be more appropriate with the current study's research aims and objectives. The validation processes of the instrument were dealt with during the pilot study and are described in Section 3.5 of this chapter. The revised instrument with 26 statements with 5-point Likert scale responses, aimed to measure pupils' attitudes in terms of three components and four scales namely; Cognitive (*Confidence* and *Perception*), Affective (*Enjoyment*) and Conative (*Commitment*). The allocation of items to scales and sample items for each scale are shown in Table 3.5 (p.94).

Table 3.5

Allocation of Items to Constructs and a Sample Item for Each Scale of the Pupils' Attitude Questionnaires.

Attitude Constructs	Number of items	Item Number	Sample Items
Confidence	7	9,10,11,12,13,14,15	I am good at fractions. I am confident when answering questions on fractions.
Perception	6	16,17,18,19,20,21	Fractions are useful for me to excel in mathematics. The easiest part of mathematics is fraction.
Enjoyment	8	1, 2, 3, 4, 5, 6, 7, 8	I enjoy answering fraction questions in class. I enjoy doing questions on addition of fractions than subtraction of fractions.
Commitment	5	22, 23, 24,25,26	I always make sure that I can do my work on fractions correctly. I always make sure that I finish my work on time.

Based on the pupils' responses to the five-point Likert scale of Strongly Agree (5), Agree (4) Undecided (3), Disagree (2) and Strongly Disagree (1), scores for all items in a given scale were added to form a scale score total. The higher the scale score, the more favourable attitude is perceived by a particular pupil on that particular dimension. The highest possible score was 130 and the lowest possible

score was 26. To avoid confusion among the pupils, all the 26 statements in the attitude questionnaire were positively worded and thus no item needed to be scored in reverse.

To determine the internal consistency of the items within each construct, a Cronbach alpha reliability was computed using the Statistical Package for Social Sciences (SPSS) program version 10. The alpha reliability coefficient is a measure of the mean correlation of items within a scale and indicates the extent to which the items within a scale measure the same dimensions. Table 3.6 below shows the different components, constructs, items number and Cronbach alpha value for the pupils' attitude questionnaire.

Table 3.6

Internal Consistency (Cronbach Alpha Reliability) of the Pupil Attitude Questionnaire

Components	Scale	Constructs	Items	Cronbach alpha
Affective	1	Enjoyment	1,2,3,4,5,6,7 and 8	0.69
Cognitive	2	Confidence	9,10,11,12,13,14 and 15	0.62
	3	Perception	16,17,18,19,20 and 21	0.60
Conative	4	Commitment	22,23,24,25 and 26	0.49

Coefficients of 0.7 or greater would be considered acceptable in that they indicate a sufficiently high extent to which the items within a scale measure the same dimensions within that scale (Green, 1994). Even though the Cronbach alphas for the four constructs were all <0.7 , the value was consistent with the three constructs (Enjoyment, Confidence and Perception) except for the Commitment construct. Even though some of the statements had been revised and reworded after the pilot study, the alpha reliability coefficients returned for the four constructs were still $<$

0.7. A possible reason for this could be that the sample size for the pilot study was very small. Only sixteen pupils took part in the pilot study. This will be described more in the section following.

3.6 Pilot Study

The two instruments, namely the diagnostic pre-test (which was also used as a diagnostic post-test) and the pupil attitude questionnaires were pilot tested at different times prior to the data collection of the main study. For familiarisation purposes, the pupils' interviews procedure was also trialled with nine pupils who took part in the pilot study. For the lesson observation practice, a one-hour mathematics lesson was also observed with one P5 class from a school that was not involved in the main study.

3.6.1 The Diagnostic Pre-and Post-tests

These instruments were validated and pilot tested to a class of twenty-five P5 pupils from a primary school in the Brunei-Muara district in June 2001. The tests were administered to 13 girls and 12 boys, with ages ranging from 10 to 12 years. At the time the pilot test was carried out, the pupils had just completed a unit on fractions. In addition, they had been introduced to simple fractions when they were in Primary 3 and learnt more about fractions when they were in Primary 4.

Special permission to conduct the pilot study was sought from the Headmistress of the school and the class teacher of the class chosen. Before the pilot test session, a mathematics education professor, four P5 mathematics teachers and two mathematics education colleagues from the Department of Science and Mathematics Education of the University of Brunei Darussalam determined the instruments' face validity. This was in order to confirm that the items selected were items that were familiar to the P5 and P6 pupils and also that the items were appropriate. The month of June, which was part of the school second term, was chosen for conducting the pilot test because it occurred two weeks after the P5 pupils had completed the unit on fractions.

I conducted the pilot study over a period of one hour during the pupils' mathematics period. I introduced myself and explained to the pupils that I was interested in learning how people solve computational and word problems, and that I wanted them to try to solve the items and problems in the booklets. The pupils were asked to show as much working as they were able on how they found the answer to each of the items and problems. They were to do this by writing all the necessary steps in the space provided in the booklets. Pupils were allowed as much time as they needed to complete each item and problem, but they were reminded that they should finish it within an hour. Five pupils completed the test in less than one hour, eight completed in about one hour, but twelve needed 20 minutes extra.

After an analysis of the pilot study, I decided to reduce the items to 28 for P5 pupils and 30 items for P6 (See Appendix A, p.199 and appendix B, p.205 for full text). The facility index for the items ranged from 0 to 0.9. Only items with facility index between 0.3 and 0.8 were taken for the actual test in the main study. These items would be used to identify the most prevalent error patterns. The 28 and 30 items paper-and-pencil test used in the main study contained both computation and more conceptually oriented items, which mainly were categorised to six main areas namely:

- Understanding fraction concepts and sequencing fractions (Items 1, 2a, 2b, 3, 4, 5, 21 and 22).
- Manipulating fraction symbols addition and subtraction (Items 6, 7, 8, 9, 10 and 11).
- Manipulating fraction symbols multiplication and division (Items 12, 13, 14, 15 and 16).
- Determining fractions of a quantity (Items 17, 18, 19 and 20).
- Alternative forms of fractions (Item 23 for the Pre-test and Items 23, 24 and 25 for the Post-test).
- Solving mathematical word problems with fraction elements (Items 24a, 24b, 24c and 24d for the Pre-test and Items 26a, 26b, 26c and 26d for the Post test).

3.6.2 The Pupil Attitude Questionnaire

The pupil attitude questionnaire was based on one developed by Raimah (2001). Two colleagues from University of Brunei Darussalam who were experts in mathematics education validated it in June 2002. Three P5 mathematics teachers and ten P5 pupils who were not involved in the main study determined the questionnaire's face validity. After the validation process, the revised version was piloted with a sample of sixteen P5 pupils from another school not involved in the main study. The 26 statements were all positively worded, a strategy adopted in order to avoid confusing the pupils so that they would be more focused on the direction of those statements. A response to an item was scored 5, 4, 3, 2 or 1, with a score of 5 corresponding to "strongly agree", 4 to "agree", 3 to "undecided", 2 to "disagree" and 1 to "strongly disagree". Since all the statements were positively worded, no reverse scoring was required. The 26-statement questionnaire can be found in Appendix C (p.212).

3.6.3 The Pupils' Interviews II

Based on the responses on the diagnostic test administered to the 25 pupils in the pilot study, nine pupils were selected to be interviewed for pilot purposes. They represented the top three, an average three and the lowest three of the 25 pupils who sat for the pilot test. The main purposes for piloting the interview were to ascertain the clarity of the instruction for each question in the diagnostic test and, most importantly for me to gain experience for the actual interview sessions in the main study.

3.6.4 The Lesson Observation

A one-hour lesson observation was carried out in an attempt to have a familiarisation session with a mathematics lesson in a class of P5 pupils in June 2002. Though the observed lesson was not on fractions, it was expected that the atmosphere of the lesson and the pupils' interaction would be similar to those lessons to be observed in the main study. As Burn wrote "Pilot observation and interviews need to be as close in context to the realities of the actual study as possible. The idea is not to get data but to learn" (Burn, 2000 p. 428).

3.7 Data Analysis

For all the diagnostic pre- and post-tests and the attitudes towards fractions pre- and post-tests, data were analysed using the Statistical Package for Social Sciences (SPSS) program. The following pupils' performances were computed, analysed and compared by using various tests in the statistical software package SPSS 10.0 program.

1. Primary 5 pupils' different types of mathematical errors from the diagnostic pre-test in terms of gender.
2. Primary 6 pupils' different types of mathematical errors from the diagnostic post-test in terms of gender.
3. Primary 5 and Primary 6 pupils' attitudes towards the learning of fractions.

Percentages and means of occurrences of each type of errors for all items in the diagnostic pre- and post-tests were calculated. The results are presented in Chapter 4. As for the attitude tests, the mean scores for each scale were also computed using the advanced computer statistical software package SPSS 10.0 program. Using the SPSS, a data file containing 285 variables was coded for each sample pupil. Variables 1 to 5 contained the sample pupils' profiles. Variables 6 to 68 and 101 to 253 contained the pupils' responses, type of identified errors and the total score for each component of fractions unit tested both for the diagnostic pre- and post-tests. Variables 69 to 100 and variables 254 to 285 contained the pupils' responses and the average scores for the four scales of the attitude pre- and post-tests. A system of SPSS coding was developed specifically for generating the statistics needed to answer the research questions in this study. The results and discussions of the findings are presented in Chapter 4 (p.109).

3.7.1 *Diagnostic Pre- and Post-tests*

For the diagnostic pre- and post paper-and-pencil tests, each correct response was given a score of one, and a zero was given for each incorrect response or unattempted item. The total number of items used in each test determined the total possible score. The highest possible score for the pre-test was 28, while for the post-test, the highest score was 30. All responses were entered into a spreadsheet using

the SPSS programme, described in 3.6. First, I analysed the pre- and post-tests achievement in order to look for pupils' overall performance in each test. The incorrect responses were further analysed and categorised. For this study, pupils' errors were identified from their written responses for each question in the test. Swedosh (1996) has demonstrated that the misconceptions, misunderstanding and mathematical errors employed by the pupils can lead to a patently absurd answer or conclusion, therefore the purpose of the written tests was to identify the most common error patterns among the entire sample. Five types of errors after Engelhardt (1977) were identified namely: 1) Grouping errors, 2) Basic fact errors, 3) Defective algorithm, 4) Incorrect operations, and 5) Careless errors. Below are the detailed description and examples for each type of identified errors.

Grouping errors: Grouping errors occurred in all categories of addition, subtraction and multiplication as well as in the one-step word problem operations. Some pupils did not regroup (borrow) but instead simply subtracted the smaller digit from the larger digit ignoring its position, or added and multiplied the fractions correctly but failed to regroup the final answer properly.

Examples: 1) $4\frac{3}{7} - 2\frac{5}{7} = 2\frac{2}{7}$

2) $\frac{5}{9} + \frac{1}{2} = \frac{10}{18} + \frac{9}{18} = \frac{19}{10}$

3) $\frac{3}{7} \times 1\frac{1}{2} = 1\frac{3}{14}$

4) $\frac{3}{8} \times 64 = \frac{192}{8}$

Basic fact errors: In all operations of fractions, many errors were found involving pupils' weaknesses in basic fact and conceptual understanding. Roberts (1968) described basic fact errors as "obvious computational errors" which he defined as a pupil's ability to apply the correct operation, but incorrect due to the response being based on the error in recalling basic number facts.

Examples: 1) $1\frac{2}{3} + \frac{4}{7} = 1\frac{6}{10}$

2) $\frac{7}{9} - \frac{2}{9} = \frac{5}{9} = \frac{1}{3}$

$$3) \quad \frac{6}{7} \times \frac{2}{3} = \frac{12}{21} = \frac{3}{7}$$

$$4) \quad 1 - \frac{2}{5} = \frac{5}{5} - \frac{2}{5} = \frac{2}{5}$$

These examples are the most common errors exhibited not only by the sample pupils in this study but also by other pupils elsewhere (Ashlock, 1994; McIntosh *et al.*, 1997; Shamsiah & Clements, 2002).

Defective algorithm: A number of pupils made mistakes involving the incorrect application of an algorithm. In this kind of error, pupils normally attempt to apply the correct operation but make mistakes other than number fact errors in carrying through the necessary steps. Therefore, though they started with the correct operation, they then seemingly lost track of what they were doing and resorted to a different operation.

Examples: 1) $\frac{1}{10} = \frac{1}{2} = \frac{8}{10}$

$$\frac{5}{10} = \frac{1}{2} = \frac{8}{14}$$

$$\begin{aligned} 2) \quad & 1\frac{2}{3} + \frac{4}{7} \\ &= \frac{5}{3} + \frac{4}{7} \\ &= \frac{12}{21} + \frac{7}{21} \\ &= \frac{19}{21} \end{aligned}$$

Incorrect operations: The pupils' errors in this category may have some sensible origins – perhaps a misinterpretation or some misunderstanding of standard procedure that the teacher has taught them. In trying to solve the questions, the pupils attempt to respond by performing an operation other than the one that is required to solve the problem.

Examples: 1) $\frac{2}{3} - \frac{1}{5} = \frac{2}{15}$

- 2) Express 4 kg as a fraction of 10 kg

$$= 4 \times 10 = 40 \text{ kg or } 4 + 10 = 14 \text{ kg or } 10 - 4 = 6 \text{ kg}$$

- 3) $\frac{6}{7} \times \frac{2}{3} = \frac{9}{21} \times \frac{9}{21}$ (multiplying 7 by 3 and 3 by 7 to get

common denominator 21) BUT (adding 6
by 3 and adding 2 by 7)

$$= \frac{18}{21} \text{ (Now, applying addition instead of multiplication)}$$

$$= \frac{6}{7} \text{ (changing the above into its lowest term)}$$

Careless errors: These types of errors occurred when pupils have successfully solved the questions but somehow recorded the answers carelessly.

Examples: 1) $\frac{2}{5} \div \frac{4}{5} = \frac{2}{5} \times \frac{5}{4}$ (but after cancelling 5 and 5 correctly, the pupils

recorded the answer as $\frac{2}{5}$ instead of $\frac{2}{4}$ or $\frac{1}{2}$)

2) $\frac{6}{7} \times \frac{2}{3}$ (after cancelling 6 and 3 correctly, the pupils recorded the

answer as $\frac{9}{7}$ instead of $\frac{4}{7}$).

3.7.2 The Pupils Attitude Questionnaire

With the attitude pre- and post-tests, pupils' responses were based on a five-point Likert scale of Strongly Agree (5), Agree (4) Undecided (3), Disagree (2) and Strongly Disagree (1). Scores for all items in a given scale were added to form a scale total score. The higher the scale score, the more favourably the attitude perceived by a particular pupil on that particular dimension. The highest possible total score for all dimensions was 130 and the lowest possible score was 26. All responses were entered into the spreadsheet using the SPSS program version 10, described in Section 3.7 (p.99) of this chapter.

3.8 Quality/Criteria Controls

Guba and Lincoln (1989) state that there should be clear criteria for assessing the quality of studies conducted qualitatively. The qualitative components of this particular study, namely the pupils' interviews, field notes and lesson observations gave me an opportunity to focus more on the insights and discoveries regarding the data rather than engaging in hypothesis testing which I regarded as inappropriate for this study.

As mentioned earlier, the criteria for legitimising qualitative research in terms of its credibility and authenticity, suggested by Guba and Lincoln (1989) included prolonged engagement, persistent observation, use of a collaborative colleague, peer debriefing and progressive subjectivity. Their role in this study is described below.

3.8.1 *Prolonged Engagement*

Lincoln and Guba (1985) asserted that substantial involvement of the researcher at the site of the inquiry helps to overcome the effects of misinformation, distortion, or presented "fronts", to establish the rapport and build the trust necessary to uncover constructions, and facilitate immersing oneself in and understanding the context's culture. Therefore, with myself as the researcher and key person present for almost three and a half months throughout the study period, any unnecessary misinformation and distortion related to the data collected throughout the fieldwork were minimised. The time period included the preliminary negotiations with the headmasters and headmistress, the class and mathematics teachers whose classes were involved in the study, the familiarisation visits to all the participating classes, the administration of the diagnostic pre- and post-tests, the pre- and post-attitude tests, the interview sessions, the lessons observation and the briefing sessions with the teachers concerned. The presence of myself at the study site provided an opportunity to make persistent and prolonged observation to add depth to the scope of the study. Adequate observation enabled me to "identify those characteristics and elements in the situation that are most relevant to the problem or issue being pursued and [to focus] on them in detail" (Lincoln & Guba, 1985, p.304).

3.8.2 Persistent Observation

I was involved administering, observing and interviewing in all the sample classes during the implementation of all the diagnostic pre-test, the administration of the pupil attitude questionnaire, the pupil interviews, the lessons on fraction observation and the administration of the diagnostic and attitude post-tests. Before the actual data collection, a number of preliminary visits, meetings and discussions were made with the four headmasters and headmistress as well as the 15 class teachers whose pupils were selected for the study. During those meetings, the class lists were obtained from the teachers as a check on the number of pupils involved in the study as well as other relevant profile details of the schools.

3.8.3 Collaborative Colleague

A female colleague from the Department of Educational Psychology was selected to be my collaborative colleague to share and test out the findings from the study. The appointment of a collaborative colleague assisted me considerably, especially in clarifying matters that seemed “blurred” to me initially. As an example, initially I was quite confused in identifying a few items that should be under the affective and cognitive sub-scales of the attitude questionnaire. However, after some discussion, I managed to differentiate the items and thus resolved the problem. My colleague was recruited to be an observer and confidant, and she assisted in ensuring that my interpretations of data collected from the pupils were not biased. This person was one whose opinion was well respected and neutral towards this study and her contribution assisted me in establishing the validity and reliability to the data collected. As Guba and Lincoln (1989) suggest, fairness requires the constant use of the member-check process, not only for the purpose of affirming that the constructions have been received 'as sent', but also for the purpose of commenting on the fairness process. The debriefing sessions commenced at the preliminary stage of the study and the discussion included the identification of the potential area of study. Thereafter, frequent meetings were held mostly in person (informally) and occasionally via phone calls, short message service (SMS) and email communications.

3.8.4 Peer Debriefing

The role of the recruited “collaborative colleague” was further extended with the peer debriefing sessions, which were held frequently during the study. Lincoln and Guba (1985, p.308) define peer debriefing as “a process of exposing oneself to a disinterested peer in a manner paralleling an analytic session and for the purpose of exploring aspects of the inquiry that might otherwise remain only implicit within the inquirer’s mind.” During the debriefing sessions, extensive discussions on the findings, conclusions and tentative analyses were held with my peers (some colleagues from the University of Brunei Darussalam and Science and Mathematics Centre), the purpose of which (as stated by Guba & Lincoln, 1989), was to “test out” the findings with persons who had no contractual interest in the situation. The regular debriefing sessions I had with my peers ensured that any bias that I may have held were neutralised.

3.8.5 Progressive Subjectivity

Since I had a mental conception of the sequence and development of the study, it was feasible for me to monitor those developments frequently, by keeping a check on the pre-arranged events and time line. This process is called progressive subjectivity (Guba & Lincoln, 1989). All events pertaining to my field study such as the administrations of the diagnostic pre- and post-tests as well as the pre- and post attitude tests to the fifteen classes were recorded in my special “diary”. The official Brunei school term calendar for the year 2002 and 2003 were always referred to in order to make sure that the schedules for the test administration would not coincide with any public holidays. In addition, the four participating schools’ own calendar were also sought from the headmasters and the headmistress, so that my tests and interview schedules would not fall on the days when they had their own school activities such as their reading or science week, monthly tests, parents day or cleaning campaign days.

As for the two teachers who volunteered for the lesson observations, I had made initial arrangement with them as on which days and time, I would come and observe their lessons. However, before each observation, they would be informed again a

day before either via a phone call or a personal visit to the school. This was to ensure that they would be “ready” for their lessons.

With the above-mentioned criteria in place to guide the study, I was confident that the credibility and authenticity of the study would be optimised.

3.9 Ethical Consideration

A number of ethical issues were considered throughout the study. The issues involved (I) the processes to be followed, (II) confidentiality matters and (III) anonymity. These are described below:

3.9.1 Process

The principle of informed consent is the most important ethical element in conducting research (Burn, 2000). It involves the right to participate and the right to refuse to take part (Cohen, Manion & Morrison, 2000). I was aware that in democratic countries such as Australia, participants must first give their consent before participating in any research activities. In this case, potential participants should sign an informed consent form that describes the purpose of the research and the right to withdraw (Burn, 2000). In Brunei, this does not actually happen. Since the implementation of all diagnostic and attitude tests, pupils’ interviews as well as the lesson observations were carried out within the school hours, a letter of consent from the parents is not required in Brunei. However, the pupils were encouraged to inform their parents that they had been sitting for the tests and that some were being selected to be interviewed. There was also no need for the Ministry of Education or the schools to inform parents or gain parental permission. By being given permission from the Brunei Education Authority, teachers were required to co-operate in the study as part of their teaching duties. The school children also were no exception. It was also continually emphasised that their scores in the tests were not taken for official examinations and that the interviews were not a form of test. Thus, even though the participants in this study did not have to sign a consent form, I had made the situation clear and was confident that the participants were fully informed and ready to take part in the study.

Special permission in a form of an official letter for me to have access to the chosen schools was sought from the head of the primary section of the Department of Schools, Ministry of Education, Brunei Darussalam. This letter was obtained in April 2002. Before the implementation of each test, the four schools concerned were visited informally as to find out if they were willing to participate in my study. The headmasters and headmistresses were shown the permission letter and they were informed of the purpose of the study and its duration. A special briefing with the fifteen teachers was also held as to inform them about the nature, purpose and duration of the study. On the test administration days, the pupils were informed of the aims of the study and the methods to be employed. The pupils and the teachers were also told that they were free to withdraw from the study at any time and that they were expected to participate without coercion (Burns, 2000). However, apart from those pupils who moved to other schools or those who happened to be absent on the day of the administration of the tests, no pupils withdrew during the study.

3.9.2 Confidentiality

An important ethical issue arising in the study involved the confidentiality of the pupils' achievement scores in all the tests given. Scores obtained by the individual pupils were only available to myself for the purpose of data analysis and discussion. Each of the forty-eight pupils chosen to be interviewed was shown his or her answer scripts and the scores obtained from the test. The overall performance and the areas in which their pupils were weak were shared with the fifteen teachers concerned, in order to inform them of the current performance of their respective pupils so that they might consider appropriate changes in their teaching the following year.

3.9.3 Anonymity

The names and locations of the four schools, the names of headmasters and the headmistress, the identity of the classes participating in the study, the names of the pupils selected to be interviewed and the names of the class teachers were kept anonymous. This was to ensure the confidentiality of all parties concerned. Pseudonyms were used throughout the study to protect the confidentiality of the schools and pupils chosen to be interviewed. The four schools were named as

School 1, School 2, School 3 and School 4. The fifteen classes were named as Class 1 to Class 15 respectively. The 396 sample pupils were numbered as S1 to S396 while the 48 pupils interviewed were named as Pupil 1, Pupil 2, and Pupil 3 to Pupil 48 respectively. The same numbering system was used for the diagnostic and attitude post-tests. With this system, only myself could identify the identity of the pupils who were either absent during the pre or post-tests.

3.10 Summary

This chapter has described the quantitative and qualitative methodology and the techniques employed for gathering data. The justification for using the methods and techniques has also been explained and their limitations considered.

Efforts to ensure the internal and external validity of the instruments used in the study were discussed. These included piloting the instruments and revising them based on the pilot data, the use of Cronbach's alpha with the items in the attitude questionnaire, the calculation of the Pearson Product Correlation Coefficient between scales of the attitude questionnaire, supporting the data by using a mixed-method approach, which also strengthened reliability (Burn, 2000), pupils' interviews, field notes and lessons observations. In addition, verification of data as well as carrying out a face validity exercise with a mathematics education professor, mathematics teachers and colleagues at the university were also carried out.

The following chapter, Chapter 4 presents the analysis of the diagnostic pre- and post-tests. The discussion will be supported by data obtained from the pupils' interviews, field notes and lessons observation.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

In this chapter, the data gathered using the methodology discussed in Chapter 3 are analysed and interpreted. The data sources consisted of a researcher-designed diagnostic test on fractions administered as both a pre- and a post-test and an adapted and modified attitude on the learning of fractions questionnaire. These data were supported by others obtained from pupils' interviews, field notes and lessons observations. The chapter first revisits the six research questions and then answers them one-by-one in order to guide the discussion.

4.2 Research Questions:

Regarding the sample of this study:

1. What are the prevalent mathematical errors on fractions held by Bruneian P5 pupils?
2. Will the same errors made by P5 pupils still be prevalent after they move into P6?
3. In which component/s of the six identified areas of the fractions unit do P5 and P6 pupils exhibit the most errors?
4. Are there gender differences in the errors exhibited by Bruneian P5 and P6 pupils?
5. What are the attitudes of Bruneian P5 and P6 pupils towards the learning of fractions?
6. Are there any gender differences in P5 and P6 pupils' attitudes towards the learning of fractions?

4.2.1 Research Question 1

What are the prevalent mathematical errors on fractions held Bruneian by P5 pupils?

This research question was answered by computing the means and standard deviations of occurrences for each type of errors using the Statistical Package for Social Studies (SPSS) version 10 program (Pallant, 2001). Two separate analyses were carried out to identify the type of errors exhibited by the pupils in the sample. The first analysis was designed to identify the type of errors exhibited by the P5 pupils in the first five components of the fractions unit tested, and the second analysis identified the type of errors exhibited by the pupils in the sixth component. Five types of mathematical errors were identified from the pupils' responses to the diagnostic pre-test. The most prevalent error patterns involved basic fact errors, followed by defective algorithm, grouping, incorrect operations and careless errors. As for the one-step word problems involving fractions, the most prevalent errors were those involving process errors, followed by transformation errors, comprehension errors, reading errors and encoding errors. The means and standard deviations of occurrences for each type of errors were calculated by using the SPSS program and are presented in Table 4.1 (p.111) and Table 4.2 (p.113) respectively. As explained in Chapter 3, Section 3.9.3, (p.107), pupils in the sample were numbered from S001 to S396. Therefore, in interpreting the information given in Table 4.1, "S294" refers to sample number 294, and "Item 7" refers to item number 7 in the diagnostic tests.

The results show that the mean for basic fact errors is significantly higher than all other types of errors means and this explains why the performance of the pupils on computational fractions items was very low. The pupils not only lacked meaningful knowledge, but they also lacked the mechanical knowledge about how to solve the problems. Evidence from the interviews shows that some pupils thought that their wrong answers were "correct" even after being probed. This scenario is evidenced by the interview transcripts with pupil No.25, in Appendix F (p.236). Only after

being alerted to and shown clearly where they went wrong did they accept the fact that their answers were incorrect. For example, in the current study a number of pupils found it difficult to explain why 0.85 and $\frac{17}{20}$ (Item 23) were different representations of the same number. This is justified by looking at the percentages of this particular item. In the pre-test only 7.6% of 396 pupils could respond to the item correctly though increased to 23.4% of 338 pupils in the post-test. Perhaps a reason for this difficulty was that pupils' knowledge of fractions was so basic and fundamental. In a study by Hunting, Oppenheimer, Pearn and Nugent (1998), Grade 6 pupils encountered similar difficulties. The pupils could not understand why $\frac{1}{2}$ and 0.5 were equivalent. In this study the P5 pupils, not only encountered this difficulty, but P6 pupils did so too.

Table 4.1

Means, Standard Deviations of Occurrences and Examples for Each Type of Errors Exhibited by Primary 5 among the Five Components of Fractions

Types of errors	Problem	Example of errors	Means	Standard Deviations
Grouping error	$1\frac{2}{3} + \frac{4}{7}$	$= 1\frac{14}{21} + \frac{12}{21} = (1\frac{26}{21})$ (S294, Item 7)	1.19	1.01
Basic fact error	Express 4 kg as a fraction of 10 kg.	$= \frac{4}{10} = \frac{1}{2}$ (S188, Item 19)	11.23	5.28
Defective algorithm	$\frac{1}{10} = \frac{1}{2} = \frac{8}{-}$	$\frac{5}{10} = \frac{1}{2} = (\frac{8}{4})$ (S187, Item 2b)	1.26	1.37
Incorrect operation	$\frac{13}{15} + \frac{8}{15}$	$= \cancel{\frac{5}{15}} = \frac{1}{3}$ (S291, Item 6)	0.64	0.99
Careless error	Calculate $\frac{3}{8} \times 64$	$= \frac{3}{\cancel{8}} \times \cancel{64} 8 = \frac{24}{1}$ (S059, Item 12)	0.56	0.91

The pupils' weakness in basic fact knowledge was a hindrance for them in proceeding further with the computational items. This is evidenced by the facts that most pupils interviewed in this study could perform the fractions operations because they were weak in basic mathematical operations. As stated by Swedosh (1996), in many areas of mathematics learning, success at a particular level depends heavily on previous mastery of basic concepts and also on being able to use certain skills with confidence. He further argued that the level of understanding of some of basic mathematical concepts which pupils are expected to acquire in secondary school determine, to a large extent, the preparedness of students to study tertiary mathematics. Accordingly, the level of understanding of the basic concepts in fractions at the elementary level does contribute to a sound foundation for the pupils when moving into the secondary school.

Of the four questions on one-step word problems (Items 24a, 24b, 24c and 24d for the pre-test and Items 26a, 26b, 26c and 26d for the post-test), five types of errors (after Newman 1977) were identified. The errors were of a reading, comprehension, transformation, process and encoding nature. The means, standard deviations and examples for each type of errors committed in the diagnostic pre-test are shown in Table 4.2 (p.113). The table shows that the P5 demonstrated that the pupils' process and transformation error means were the highest followed by the means for comprehension errors, reading errors and encoding errors respectively. This confirms the earlier findings by Raimah (2001), where in her study with P6 pupils in Brunei, the percentages for process skill and transformational errors were 29.00 and 27.6 respectively. Other local studies (Sainah, 1998; Saman & Suffolk, 2001) have also found that transformation errors were the most exhibited errors in solving word problems. However, the current study differs from the others because it dealt with word problems involving fractions and all the four operations of addition, subtraction, multiplication and division, whereas the others dealt with either word problems involving addition and subtraction only, or word problems involving multiplication and division only. In other words, this study was more extensive than what has been previously investigated. This present study suggests that Bruneian

pupils may have difficulty in transforming the given information in problems and thus are unable to process and solve them successfully.

Table 4.2

Means, Standard Deviations and Examples of Each Type of Errors Exhibited by Primary 5 Pupils in the Sixth Component (Word Problems) of Fractions

Types of errors	Problem	Example of errors	Means	SD
Reading error	A bag of sugar weighs $\frac{3}{4}$ kg and another bag weighs $1\frac{1}{2}$ kg. What is the total weight of the two bags of sugar?	(Indicated by pupils not responding to the written items or just wrote some meaningless responses. During the interviews, pupils were not able to read the questions fluently).	0.49	1.18
Comprehension error	A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?	(Indicated by pupils could read the problems but did not know what the questions asked them to do, thus they just rewrote the given figures). $2\frac{1}{2} - 1\frac{2}{3}$ (S024, Item 24b)	0.94	1.39
Transformation error	A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?	$4 \div 10 = 4 \overline{)10}$ $\underline{8}$ 2 $2\frac{2}{4} = 2\frac{1}{2}$ (S014, Item 24d)	1.09	1.03
Process error	Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.	$5 \times 1\frac{1}{2} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 5$ (S051, Item 24c)	1.1	1.01
Encoding error	A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?	$2\frac{1}{2} - 1\frac{2}{3} = \frac{5}{2} - \frac{5}{3}$ $= \frac{15}{6} - \frac{10}{6} = \frac{5}{6} = \frac{25}{6}$ (S059, Item 24b)	0.01	0.31

Therefore, in answering this research question, the P5 data shows that in the computational components, the most prevalent errors were those of basic facts, which shows a mean score of 11.23 as presented in Table 4.1, p.111. The results further show that the pupils did not exhibit many grouping errors, defective Mathematical Errors in Fractions Work

algorithms, incorrect operations and careless errors. As for the word problems, P5 pupils showed that they exhibited more process and transformational errors (mean scores of 1.1 and 1.09) than they did reading, comprehension and encoding errors.

4.2.2 Research Question 2

Will the same errors made by P5 pupils still prevalent after they move to P6?

This research question was answered by examining the means and standard deviations of each type of errors exhibited by the P5 pupils during the diagnostic pre-test, and again as they moved to P6 level during the diagnostic post-test. The examination was carried out by means of a paired-samples *t*-test. The purpose was to evaluate the impact of the further instruction on fractions given at the P6 level on the means of each type of errors exhibited by the pupils. Two separate analyses were carried out: The first analysis dealt with the five computational components of the diagnostic pre- and post-tests while the second analysis dealt with the sixth component (word problems) of the diagnostic pre- and post-tests. Initially, the Levene test for equality of variance was carried out. Since the significance value for all types of errors were larger than 0.05 and the sample size was more than 30, homogeneity of variance was not violated. The results are presented in Table 4.3 (p.115) and Table 4.4 (p.117) respectively.

The results show there was a statistically significant decrease in the means of basic fact errors from the diagnostic pre-test ($M=11.23$, $SD=5.28$) to the diagnostic post-test ($M=9.21$, $SD=5.59$), $t(337)=6.52$, $p<0.05$. The eta squared statistic (0.11) indicated a moderate effect size. The results also show that there was a statistically significant increase in incorrect operation means from the diagnostic pre-test ($M=0.64$, $SD=0.99$) to the diagnostic post-test ($M=1.66$, $SD=1.49$), $t(337)=9.90$, $p<0.05$. The eta squared statistic (0.24) indicated a large effect size. Besides the basic fact and the incorrect operation types of errors, the results also show that there was a statistically significant increase in means for the careless errors from the diagnostic pre-test ($M= 0.56$, $SD=0.91$) to the diagnostic post-test ($M=0.99$, $SD=1.09$), $t(337)=6.08$, $p<0.05$. The eta squared statistic (0.1) indicated a moderate effect size. Given that the eta squared value for the basic fact errors was 0.11 and

careless errors was 0.1, I can conclude that there was a moderate effect, with a quite substantial difference in the means obtained for the basic fact and careless types of errors in the diagnostic pre- and post-tests. As for the incorrect operation type of errors, the eta squared statistic (0.24), indicates that there was a large effect, with a substantial difference in the means obtained in the diagnostic pre- and post-tests. The results demonstrate that no significant effects between the diagnostic pre- and post-tests were found with respect to grouping errors and defective algorithm errors ($p > 0.05$).

Table 4.3

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value for Each Type of Errors for Components 1 -5

Types of errors	Time	Mean	Standard Deviations	t-value	df	p
Grouping	Pre-	1.19	1.01	0.88	337	0.38
	Post-	1.12	1.08			
Basic facts	Pre-	11.23	5.28	6.52	337	0.00*
	Post-	9.21	5.59			
Defective algorithm	Pre-	1.26	1.37	1.48	337	0.14
	Post-	1.42	1.59			
Incorrect operations	Pre-	0.64	0.99	9.90	337	0.00*
	Post-	1.66	1.49			
Careless	Pre-	0.56	0.91	6.08	337	0.00*
	Post-	0.99	1.09			

*significant at 0.05 level

In examining the main effects of the basic facts, incorrect operations and careless errors, the values of the mean scores indicate that at the P6 level, the pupils improved their performance on the basic fact errors, indicated by the significant decrease in means compared to their performance at the P5 level. However, the mean scores for the incorrect operations and careless errors increased from the pre-

test to the post-test (Incorrect operation: Pre-test $M=0.64$, Post-test $M=1.66$; Careless errors: Pre-test $M=0.56$, Post-test $M=0.99$). This indicated that not only the incorrect operations and careless type of errors were still prevalent at the P6 level; it seemed that the P6 pupils even committed more of these types of errors. Similarly, the grouping and defective algorithm types of errors remained prevalent at the P6 level.

Basic fact errors were the most dominating errors exhibited by the sample pupils when they were at the P5 level. However, as mentioned earlier, an improvement was seen with the decrease in means in the diagnostic post-test at the P6 level. A possible explanation for this improvement could be due to the pupils' maturity and because of the extra exposure and instruction on fractions at the P6 level. However, the improvement could not be attributed solely to these reasons. Other reasons such as the fact that some pupils may have remembered what they had done in a similar test before, especially the forty-eight pupils who were interviewed after the diagnostic pre-test, might have also contributed.

On the other hand, pupils' cognitive maturity and additional exposure to fraction units did not contribute to the improvement in the other four types of errors. This was indicated by the mean scores for the grouping and defective algorithm types of errors, which did not change significantly and thus suggested that the pupils still exhibited about the same amount of those errors. The mean scores for the incorrect operation and careless errors increased and the increase was statistically significant. This increase indicated that not only those two types of errors were still prevalent among the P6 pupils, but more of those errors were exhibited during the diagnostic post-test as compared to the pre-test. This finding suggests that though the pupils did not experience so many problems with basic fact of fractions, they still had problems with grouping and had problems in keeping track with the computation process. Many pupils in this study were observed to have used inappropriate algorithms in computing the items. They also had more problems in choosing the correct operations in the computations, and more careless errors were committed in encoding their final answers to the given questions though they had used correct algorithms, correct operations and knew the basic facts of those questions. This evidence supports the reports from sources such as the federally sponsored National

Assessment of Educational Progress (Mullis et al., 1991) which indicate that fractions are exceedingly difficult for pupils to master. As shown in this study P5 and P6 pupils learn disappointingly little about fractions year-by-year, often unable to recall in their P6 level most of the fractions information taught to them the previous year during their P5 level. This is justified by their unsatisfactory performance at the P6 level (shown by the low mean scores) even after they received more formal instructions on fractions after the pre-test which was given during their P5 level.

The next section deals with the sixth component (word problem) of the diagnostic pre- and post-tests. Table 4.4 below shows the Means, Standard Deviations, *t*-value, degree of freedom and significance value for each type of error in Component 6 of the diagnostic pre- and post-tests.

Table 4.4

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value for Each Type of Errors for Component 6

Types of errors	Time	Mean	Standard Deviations	<i>t</i> -value	<i>df</i>	<i>p</i>	Eta Squared
Reading	Pre-	0.49	1.18	2.79	337	0.01*	0.02
Comprehension	Post-	0.28	0.82	1.37	337	0.17	0.01
	Pre-	0.94	1.39				
Transformation	Post-	1.08	1.35	4.75	337	0.01*	0.06
	Pre-	1.09	1.03				
Process	Post-	0.77	0.89	2.38	337	0.02*	0.02
	Pre-	1.10	1.01				
Encoding	Post-	0.93	0.98	5.04	337	0.00*	0.07
	Pre-	0.09	0.31				
	Post-	0.01	0.08				

*significant at 0.05 level

The results show that between the diagnostic pre- and post-tests results, decreases were found in reading, transformation, process and encoding types of errors. The means of the reading errors decreased from the diagnostic pre-test ($M=0.49$, $SD=1.18$) to the post-test ($M=0.28$, $SD=0.82$), $t(337)=2.79$, $p<0.05$, while the eta squared statistic (0.02) indicated a small effect size (Burns, 2000). The means of the

transformation errors decreased from the diagnostic pre-test ($M=1.09$, $SD=1.03$) to the post-test ($M=0.77$, $SD=0.89$), $t(337)=4.75$, $p<0.05$ while the eta squared statistic (0.06) indicated a moderate effect size. Similarly, the means of the process errors decreased from the diagnostic pre-test ($M=1.10$, $SD=1.01$) to the post-test ($M=0.93$, $SD=0.98$), $t(338)=2.38$, $p<0.05$ while the eta squared statistic (0.02) indicated a small effect size. The means of the encoding errors decreased from the diagnostic pre-test ($M=0.09$, $SD=0.31$) to the post-test ($M=0.01$, $SD=0.08$), $t(337)=5.04$, $p<0.05$ while the eta squared statistic (0.07) indicated a moderate effect size (Burn, 2000). Given the eta squared value of the reading, transformation, process and encoding types of errors ranged from 0.02 to 0.07, I can conclude that there was a small to moderate effect in the means of those types of errors obtained from the diagnostic pre- to post-tests. This means that is the decrease and increase in means of the occurrences of the five types of errors in this word problem component were only very slight after the pupils received more formal instruction on fractions at P6 level.

In examining the above result for the word problem component, it shows that the pupils made less errors in reading, transformation and process types of errors (evidenced by the decrease in means). The greatest decrease of means was observed in the transformation errors, followed by reading and process errors. The decrease suggests that at their P6 level, the pupils experienced fewer problems in reading, transforming and processing the given information to solve the given problems. The pupils also made less encoding errors. However, some pupils at the P6 level still had difficulty in understanding the problems. This is evidenced by the increase of means for the comprehension errors (Pre-test $M=0.94$, Post-test $M=1.08$). This finding indicates that even though the pupils improved in their reading ability, many still were not able to comprehend the given information effectively. The reason for this could be that some pupils had language problems, an issue that is further discussed in Chapter 5. Though they could read the problems, a number of pupils could not understand the meaning or the message conveyed and simply tried to manipulate any given figures in the problems, directed only by the keywords available. This issue of pupils using key words as their problem solving strategy is further discussed in my response to Research Question 3 (p.125). Evidences provided in Appendices E and F (p.216 and p.236) obtained during the observations made after the diagnostic post-

test, and interviews conducted after the diagnostic pre- and post-tests confirmed this. Some of the pupils interviewed demonstrated that they could read the problems clearly but could not understand what the questions required them to do and thus resorted to any keywords available in the given problems to direct them in solving the problems.

Looking again at the overall results of the computational and word problem components, the results show that in components 1, 2, 3, 4 and 5, the pupils still demonstrated the five types of errors they had made in P5, namely: grouping errors, basic fact errors, defective algorithm, incorrect operation and careless errors. P6 pupils exhibited an improvement in basic fact errors (evidenced by the decrease in means). Grouping and defective algorithm types of errors were still persistent (evidenced by the similar means value), while incorrect operations and careless errors were not only persistent, but more of these types of errors were exhibited at the pupils' P6 level (evidenced by the increase in means). As for the word problem component, at their P6 level, the pupils showed improvement in reading, transformation, process and encoding errors (evidenced by the decrease in means shown in Table 4.4, (p.117) compared to their performance at their P5 level. The comprehension errors were still persistent.

The overall performance of the pupils in the diagnostic post-test improved significantly both in the computational parts (Components 1-5) as well as in the word problem section (Component 6). This information was obtained from pupils' total scores of the diagnostic pre- and post-tests. The means of the overall scores and standard deviations of the diagnostic pre- and post-test are presented in Table 4.5 (p.120), while Fig. 4.1 (p.123), Fig 4.2 (p.124) and Fig 4.3 (p.125) display the bar graphs for the mean scores for the computational, word problems and overall components of the diagnostic pre- and post-tests.

Table 4.5

Means, Mean scores Percentage Differences and Standard Deviations of the Diagnostic Pre- and Post-tests

Components	Mean	Mean score Percentage Differences	Standard Deviation
Components 1-5	Pre-test	6.09	4.32
	Post-test	9.75	5.97
Component 6	Pre-test	0.27	0.69
	Post-test	0.91	1.17
Overall	Pre-test	6.09	4.75
	Post-test	10.82	7.03

For the computational components (Components 1 – 5), the highest expected mean for the diagnostic pre-test was 24 and 26 for the post-test. Meanwhile for the word problem component (Component 6), the highest expected mean for both the pre- and post-test was 4. Though the pupils showed an improvement in their fractions performance, indicated by the percentage increase, the mean score was far below the expected full scores or at least 50 percent of the expected highest scores of the diagnostic pre- and post-tests. This can be observed in Table 4.5 above where the means for both the computational and word problems components were very low (Components 1- 5: Pre-test M=6.09, Post-test M=9.75); (Component 6: Pre-test M=0.27, Post-test M=0.91).

While the results shown are poor, there was a slight improvement over P5 level results. The pupils at both the P5 and P6 level found the tests to be difficult, especially when they were required to do the items on-the-spot, without prior information and help. Moreover, the test covered the whole unit of fractions stipulated in the Brunei upper primary mathematics syllabus. Though the pupils had just been taught the fractions unit prior to the tests, carrying out the daily class work exercises was not the same as sitting for a test administered by a person who was not

their teacher. Although they had been informed that the tests were informal and their scores would be treated informally, some pupils might have still been anxious about them. When the pupils were completing their class exercises on fractions, they were given help every now and then by their teacher, and the atmosphere was quite relaxed as they could consult their neighbours whenever they were not sure of what they were to do. Moreover, in their daily class exercises time was not limited, as those pupils who could not complete their work in class could always take it home to complete. However, when the pupils were doing their diagnostic pre- and post-test, no help was given and the time given was very limited. As a result, many pupils might have completed the test without taking too much care. A further investigation at the pupils' answer scripts provides some evidence for this. For examples, some pupils in this study were observed to have just written some meaningless responses to items such as

$$\text{Calculate } 3\frac{2}{7} - 1\frac{3}{4} = \frac{15}{28} \quad (\text{Item 10, Pupils 187})$$

Express 65% as a fraction in its simplest form = 56 (Item 24, Pupil 204)

A piece of ribbon of length 4 metres is cut into 10 equal lengths. What is the length of each piece? = 11 cut (Item 26d, Pupil 324)

These factors may have affected their performances in both the pre- and post-tests. In addition, the result also indicates that at the P5 level, the pupils still had difficulty in using their knowledge of fractions to solve problems that were regarded as being appropriate for that level of schooling despite having experienced the mathematics instruction and activities presented by their teachers as early as when they were at their P3 level.

However, at their P6 level, though some of them were still unable to understand fractions language effectively, as evidenced by their still unsatisfactory performance in the diagnostic post-test, many pupils could have acquired knowledge about sharing things as a division process and that would have provided a sound basis for the construction of future knowledge of rational numbers (Hunting & Sharpley, 1988). The number of pupils obtaining a pass mark of 50 % increased by about 21%. In the diagnostic pre-test, the highest score obtained was 93% and the lowest score was zero, while in the diagnostic post-test, the highest score obtained was 97% and

the lowest score was still zero. Only thirty-three pupils out of 396 (8.3%) in the diagnostic pre-test obtained 14 marks (or above 50%) while in the post-test, ninety-nine pupils out of 338 (29%) obtained 15 marks (or above 50%). Though the improvement was statistically significant, educationally it was not. Pupils' poor performance could be explained by the fact that no special intervention or coaching activities were performed with the pupils apart from them receiving the normal instruction on fractions unit from their respective mathematics teachers scheduled in their program for the year 2003. Moreover, as mentioned earlier, the pupils were not informed that they would be given a test, and so the diagnostic pre and post-tests were considered as on-the-spot tests for all the pupils. Even though the pupils had completed the same test at their P5 level, many of them might have forgotten about it, and this may explain why the pupils still tended to make similar errors.

Diagnostic Pre- and Post-test Performance by Gender

A further investigation was carried out to investigate the pupils' performance in terms of gender both for the diagnostic pre- and post-tests by looking at the means standard deviations and percentage differences. The result is presented in Table 4.6 (p.123).

Table 4.6 shows that the percentage differences of the mean scores increase in all components of the diagnostic pre- and post-tests for both male and female pupils, with females show a higher percentage increase in the overall component as compared to males. The differences in means of the pupils' performance in the diagnostic pre- and post-tests for Components 1-5, Component 6 and the overall components in terms of gender are further illustrated in the bar charts presented in Figure 4.1 (p.123), Figure 4.2 (p.124) and Figure 4.3 (p.125).

Table 4.6

Means, Mean scores Percentage Differences and Standard Deviations for the Diagnostic Pre- and Post-tests for Components 1-5, Component 6 and the overall components.

Components		Means		Mean scores	Standard
				Percentage	Deviations
				Differences	
1-5	Male	Pre-test	6.05	57.36 (+)	4.6
		Post-test	9.52		6.19
	Female	Pre-test	6.14	63.52 (+)	4.09
		Post-test	10.04		5.68
6	Male	Pre-test	0.31	180.65 (+)	0.77
		Post-test	0.87		1.17
	Female	Pre-test	0.21	180.65 (+)	0.63
		Post-test	0.95		1.17
Overall	Male	Pre-test	6.36	66.19 (+)	5.14
		Post-test	10.57		4.45
	Female	Pre-test	6.35	75.28 (+)	7.32
		Post-test	11.13		6.68

Figure 4.1 (p.124) shows that the means for both the female and male pupils in the five computational components of the diagnostic pre-test were relatively low and about the same. At the P6 level as shown in Fig. 4.1, both male and female pupils showed a significant improvement with female pupils showing more improvement than male pupils. However, the increase was statistically but not educationally significant. This is because the means for both the male and female pupils were still lower than the expected average mean of 24 (pre-test) or 26 (post-test), implying that even after further instruction on fractions, only very little improvement was observed. The pupils at the P6 level still found that the fractions unit was difficult and they still had problems with the basic facts of fractions.

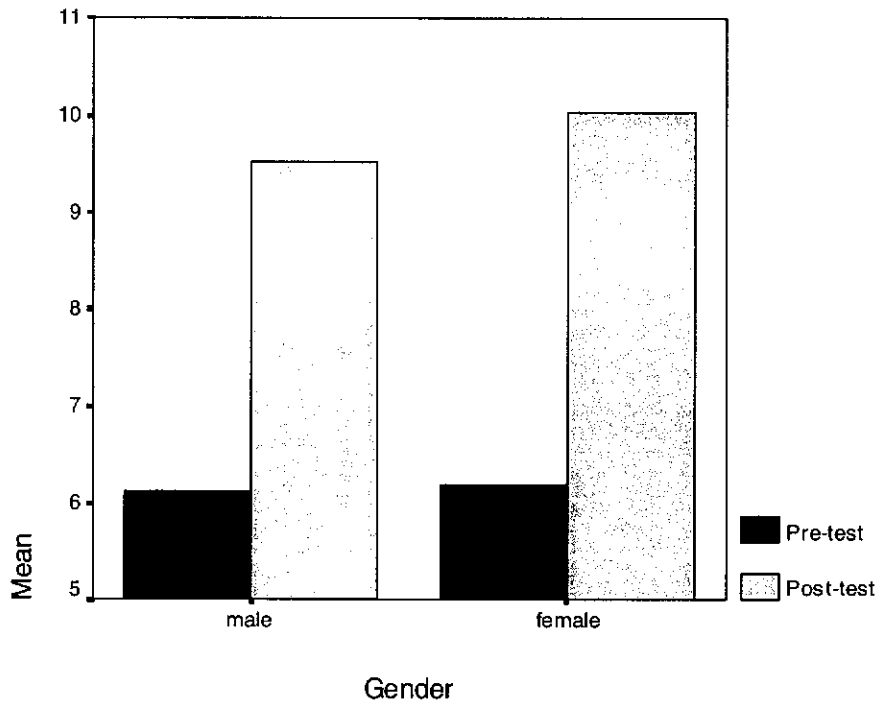


Figure 4.1 Means scores for diagnostic pre- and post-tests for Components 1-5 by gender.

In Figure 4.2 (p.125), it is clear that for the word problem component, in the diagnostic pre-test, the mean for male pupils was higher than that for the female pupils. However, in the diagnostic post-test, female pupils' score superseded those of the male pupils. Though both means were low, female pupils showed improvement in solving word problems while the male pupils improved to a lesser degree. Saman and Suffolk (2001) also found that Bruneian girls performed much better than boys in solving word problems involving multiplication and division.

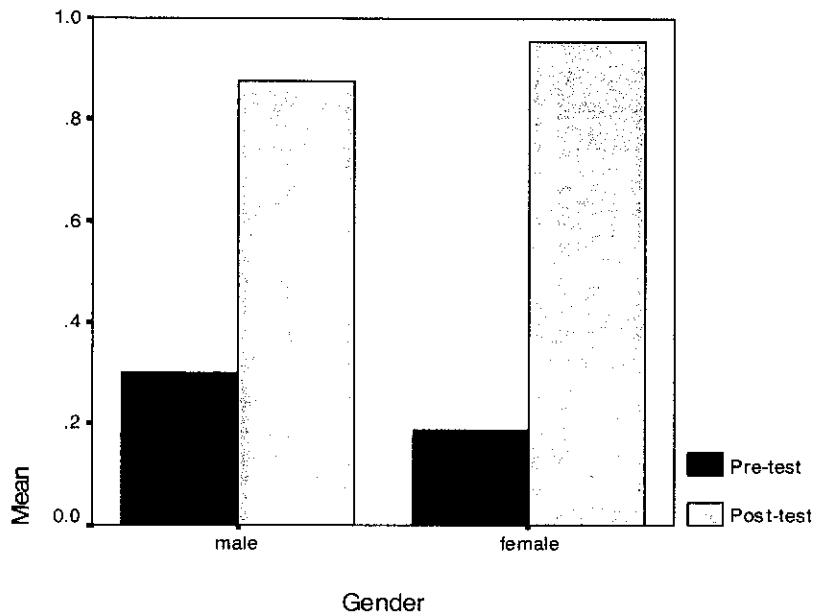


Figure 4.2 Means scores for diagnostic pre- and post-tests for Component 6.

Figure 4.3 below shows that for the pupils' overall performance, male and female pupils scored equally in the diagnostic pre-test, but female pupils showed a slightly higher mean as compared to their male counterparts in the diagnostic post-test. This indicates that at P6 level, female pupils made some improvement in the overall test compared to male pupils.

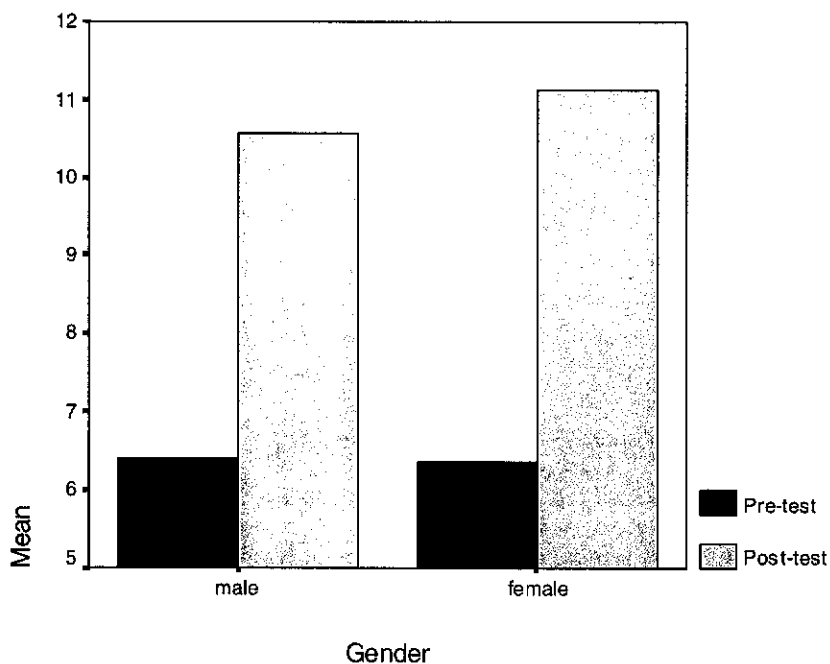


Figure 4.3 Overall means scores for the diagnostic pre- and post-tests by gender.

Therefore, in answer to the second research question: At their P6 level and after the pupils received more instruction on fractions, they still committed the same type of errors as when they were at P5 level. However, some improvements in terms of the errors exhibited were observed both in the computational and word problems components. At the P6 level, generally the pupils exhibited fewer basic fact errors, but grouping and defective errors were still prevalent and incorrect operations and careless errors increased. As for the word problem component, the pupils at the P6 level still made similar errors identified at their P5 level. Improvement was observed in that at the P6 level, the pupils exhibited less reading, transforming and processing errors, however comprehension and encoding errors were still prevalent, due possibly to reasons mentioned earlier.

4.2.3 Research Question 3

In which component/s of the six identified areas of the fractions unit do P5 and P6 pupils exhibit the most errors?

This research question was answered by examining the means of each type of error exhibited by the pupils at their P5 and P6 levels, using the paired-samples *t*-test. The analysis were made possible by examining pupils' responses to the 28-items diagnostic pre-test administered at the P5 level and their 30-items diagnostic post-test administered at the P6 level. The average means of each type of errors exhibited in each of the six components were computed with the purpose of examining which of the six tested components pupils made most errors. The means and average means of each type of errors for the individual six components of fractions unit held by the sample pupils at the P5 and P6 levels are presented in Table 4.7 (p.127).

Table 4.7

Means and Average Means of Each Type of Errors for the Six Components of Fractions Exhibited in the Diagnostic Pre-and Post-tests

Time	Error Types Compo nents	Grouping error	Basic Fact error	Defective algorithm	Incorrect operation	Careless error	Av. means
Pre-	1	0.00	4.08	0.39	0.00	0.00	0.89
Post-		0.00	3.16	0.36	0.17	0.00	0.74
Pre-	2	0.75	2.99	0.32	0.14	0.10	0.86
Post-		0.86	2.25	0.35	0.01	0.17	0.73
Pre-	3	0.42	1.89	0.35	0.29	0.21	0.63
Post-		0.22	1.18	0.42	0.34	0.13	0.46
Pre-	4	0.00	1.78	0.19	0.57	0.13	0.53
Post-		0.00	1.21	0.28	0.94	0.22	0.53
Pre-	5	0.00	0.48	0.00	0.00	0.19	0.13
Post-		0.00	1.42	0.00	0.01	0.44	0.37
		Reading	Comprehen sion error	Transfor mation error	Process error	Encoding errors	Av. means
Pre-	6	0.49	0.94	1.09	1.10	0.09	0.74
Post-		0.28	1.08	0.77	0.93	0.01	0.61

Note:

- Component 1 - Understanding of fractions concepts and sequencing fractions.
- Component 2 - Manipulating fraction symbols; addition and subtraction.
- Component 3 - Manipulating fraction symbols; multiplication and division.
- Component 4 - Determining fraction of a quantity.
- Component 5 - Alternative forms of fractions.
- Component 6 - Word problems involving fractions.

The result in Table 4.7 above shows that among the six identified components of fraction unit, on average the pupils exhibited higher mean scores in components 1, 2 and 6 in the diagnostic pre-test and in components 1 and 2 in the diagnostic post-test. This suggests that components 1 and 2 remained as the more difficult components for the pupils at the P6 level. The average means of errors for components 1, 2, 3 and 6 decreased in the diagnostic post-test as compared to the pre-test, while the average means of errors for component 4 remained the same and the average means of errors for component 5 increased. If one examines at the means for each type of errors for each component, it can be seen that at both the P5 and P6 levels, most pupils had problems with the basic facts of fractions even though they had been introduced to the concepts at P3 and received further instruction when they were at P4 and P5. Means for basic fact errors improved in the post-test (ranging from 1.18 to 3.16). They were consistently higher than the other

types of errors in all the five components of the fraction unit tested both in the diagnostic pre- and post-test (ranging from 0.48 to 4.08). The first component of the tests (Understanding fractions and sequencing fractions) was considered the easiest component, but this was where the mean of basic fact errors was the highest both in the diagnostic pre- and post-tests. This confirms the results of earlier local studies such as those by Jabaidah (2001) and Shamsiah and Clements (2002). Understanding fractions and the ability to order and compare fractions are the prerequisites for subsequent learning of other components of fractions. However if the pupils were weak in this component, it was no surprise that they also performed poorly in the subsequent components of the fractions unit tested. This is because pupils' success on a particular area in mathematics depends on their mastery of the basic concepts and facts acquired earlier (Swedosh, 1996).

A considerable number of pupils confused their fraction ideas with whole number concepts (Post, Cramer, Behr, Lesh and Harel, 1993). In the current study, this is evidenced by looking at the examples of pupils' responses to items such as the following:

$$4\frac{5}{9} + 2\frac{1}{2} = 6\frac{6}{11} \text{ (Item 8, Pupil 092)}$$

$$1\frac{2}{3} + \frac{4}{7} = 1\frac{6}{10} \text{ (Item 7, Pupil 160)}$$

$$3\frac{2}{7} - 1\frac{3}{4} = 2\frac{1}{3} \text{ (Item 10, Pupils 093)}$$

A possible reason for the above phenomenon is that the pupils' prior learning of whole numbers and the early introduction of fraction may have had a negative influence on most pupils' understanding of fractions and their operations., Bezuk, (1988) and Case (1992) have demonstrated that children's whole number schemes can interfere with their efforts to learn fractions and this is what probably occurred with the pupils in this study. This is because by the time fraction instruction commences at Primary 3 level, children have considerable knowledge of whole numbers and how they work.

A number of pupils in this study who were asked to explain their incorrect answers did not, or could not see anything amiss in their responses. This is evidenced by the interview transcripts with Pupils 25 and 22 provided in Appendix F, (p.236). These pupils need further attention from their teachers who should review with them the basic meanings of common fractions and their interrelationships with other equivalent representations such as decimals and percentages. Physical models such as area grids and number lines can be useful in this regard. Some experimental work (Case, 1992) that takes children's understanding of percent as a starting point for instruction in fractions and decimals holds promise. Evidences from the pupils' interviews (Appendices E, p.216 and F, p.236) show that most were unable to perform simple calculations, be it addition, subtraction, multiplication or division. For example, a number of them were unable to use multiplication tables, or they used inefficient counting or addition strategies when what was required was to multiply two figures. The following excerpt for item 8 from an interview with a pupil (Pupil 22, p.233) demonstrates the error:

Example: $4\frac{5}{9} + 2\frac{1}{2}$

This pupil could see that the lowest common multiple for nine and two was 18 and converted $4\frac{5}{9}$ into $4\frac{10}{18}$ and $2\frac{1}{2}$ into $2\frac{9}{18}$ correctly. Though this pupil could convert the two fractions into their equivalences successfully, the way he obtained 10 and 18 was not by multiplying 2 and 5 or 2 and 9 but by adding 5 and 5 in the numerator and adding 9 and 9 in the denominator. It was true that this pupil arrived at a correct answer this time, but with larger numbers such as 7×9 , he might have faced a difficulty that could have led him to arrive at an incorrect solution. It was not only that the pupils were unable to use the multiplication tables effectively, but also that some pupils could not multiply the given numbers correctly. For example, in many instances during the interviews, the pupils multiplied 6 by 6 to get 6 but not 36. However, after being probed further and told to write multiplication table 6 in the working space, they realized that 6×6 was 36. This phenomena shows that the pupils who responded in such a way lacked the mastery of the multiplication basic facts. Without the mastery of basic concepts and facts, pupils may not only have difficulty with fractions but with other areas of mathematics as well. Bryant (1995)

and Prior (2000) have shown that mastery of basic arithmetic underpins the learning of more complex mathematical capacities that depend on an accurate memory for basic facts.

The most common mistake found in both the diagnostic pre- and post-tests was with item 6) $\frac{13}{15} + \frac{8}{15}$. With this item, the most common answer many pupils wrote was $\frac{21}{30}$. Meanwhile, for the subtraction items, the most common mistake the pupils made was for the item $1 - \frac{2}{5}$. Many wrote either $\frac{1}{5}$ or $\frac{2}{5}$. Similar errors were detected in a study by McIntosh *et al.*, (1997) when they found that the most common answer given by his sample pupils across grades 3 – 10 for item $\frac{1}{2} + \frac{1}{4}$ was either $\frac{2}{6}$ or $\frac{1}{3}$. They also found that for item $1 - \frac{1}{3}$, the most common answers given was either $\frac{1}{4}$ or $\frac{3}{4}$. The examples show that the pupils in both this and the McIntosh *et al.*'s study have experienced conceptual problems. They did not treat fractions as distinct numbers but treated them as whole numbers. It shows that pupils in the elementary grades had more conceptual errors than procedural errors. A decrease of the means of basic fact errors in all the five computational components suggests that some pupils have improved their basic facts skills, but this problem persisted with a number of them. As McIntosh *et al.*, (1997) suggested, this type of error, after decreasing gradually in extent through Grades K to 3, appears to remain static at about 20% of errors in basic fact additions and subtractions in Grades 4 – 6.

In a further effort to investigate the types of errors exhibited in component 6, the percentages of occurrences of each type of errors on each question were computed. In examining the percentages presented in Table 4.8 (p. 130), the reader will notice that the percentages do not sum to 100. This is because the percentages of the correct and unattempted responses were excluded as they are irrelevant to the discussion.

The results show that in solving the four questions on one-step word problems in the diagnostic pre-test, the pupils demonstrated the highest percentage of process error for items 24a and 24b and transformation error for items 24c and 24d. The current study confirms the earlier findings by Saman and Suffolk (2001) and Raimah (2001). In the diagnostic post-test, the highest percentages were observed for process errors in items 26a, 26b and transformation errors in items 26c, and 26d. Table 4.7 shows the break down of the percentages of each type of errors committed for each question in component 6, both for the diagnostic pre- and post-tests.

Table 4.8

Types and Percentages of Errors for Each Question on Word Problems (Component 6) for the Diagnostic Pre- and Post-tests

	Question	Reading errors	Comprehension errors	Transformation errors	Process errors	Encoding errors
Pre-	24a	9.1	19.4	7.8	46.0	5.8
Post-	26a	3.5	19.2	3.0	38.4	0
Pre-	24b	12.1	19.7	11.1	49.5	0.8
Post-	26b	4.5	18.2	5.1	28.8	0
Pre-	24c	15.4	25.3	39.4	14.1	1.3
Post-	26c	8.1	23.2	25.5	13.4	0.5
Pre-	24d	16.2	29.3	49.2	1.0	1.5
Post-	26d	8.8	30.6	32.6	1.5	0.3

The percentages of the reading errors for all the four word problems both in the diagnostic pre- and post-tests were consistently low, showing that the pupils had not so much a problem in reading out the given examples, but that the percentage of the comprehension errors was increasing for all four word problems. Reading errors were characterised by the pupils failing to respond in writing to the given items, or writing some meaningless responses with no evidence of working at all. The interviews conducted with the selected pupils confirmed the conjecture. The low percentages of the reading errors at both their P5 and P6 levels showed that most pupils had little problem in reading the questions. However, the increase in the percentages of the comprehension errors for all the four questions indicates that though most pupils could read the questions, they were unable to comprehend what

the questions asked them to do. As explained in Chapter 3 (p. 82), due to the nature and format of the diagnostic tests, the questions were not read out to the pupils. When one examines the percentages of transformation errors for the word problems on addition and subtraction during the diagnostic pre- and post-tests (Items 24a, 24b, 26a and 26b), they were lower than the comprehension error. This is because even when the pupils were unable to understand the questions, many of them transformed the information given in the problems merely by manipulating the figures available.

Meanwhile, for the two word problems involving multiplication and division in the pre-and post-tests (Items 24c, 24d, 26c and 26d), the percentage of transformation errors was larger than that for process errors. Evidence from the interviews reveals that even when some pupils could understand what the questions asked them to do, they were unable to transform the information correctly. The findings of the study confirm the fact that the transformation errors made by the pupils were due to the use of keywords. The use of keywords (cue words) by pupils in Brunei was also found in other studies conducted by Veloo and Lopez-Real (1994), Sainah (1998), Saman and Suffolk (2001). In their workings, pupils in this study often decided what operation to perform by the cue words. For an example, the words “*total*”, “*left*”, “*product*” and “*each*” often signalled the pupils to perform the *addition*, *subtraction*, *multiplication* and *division* operations respectively. Naturally, the use of indicators or keywords did not always work. These keywords were not only apparent in this study but are commonly used in the pupils’ everyday mathematics class exercises. For example, in the Brunei Darussalam Primary Mathematics textbooks, workbooks and PCE examination papers the words *altogether*, *total*, *difference*, *times*, *product*, *each* and *shared equally* are commonly used in problems involving the four arithmetic operations, *addition*, *subtraction*, *multiplication* and *division* (Ministry of Education, 1999 & 2000). Hart (1981) found that 30% of the 13-year olds in her study sample used the “key word” strategy. The cue words for addition such as “*total*”, “*more*” and “*altogether*” is sometimes misleading. The fact that the percentage of encoding error for the word problems on addition and subtraction was low was probably not due to the fact that the pupils could not proceed to encode the solution but because they were already lost while processing the solution earlier, thus they committed process errors. For example, the word *total* is also used in problem 24c (pre-test) and 26d (post-test). *Ali bought 5 packets of*

chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought. This problem required *multiplication* to solve it most efficiently. However, many pupils used *addition* to solve the problem. It was possible to solve this problem by using *addition* if the pupils carried out *repeated addition*. In the interviews, the pupils who made errors and calculated the solution by using *addition* revealed that the word *total* meant *addition* to them. An example of an interview transcript shown below demonstrates that some pupils could not explain why sometimes *total* suggests *addition* and sometimes *multiplication*. Some pupils in the interviews said that they did not understand the problems because they did not understand the words and the English language used in the written problems. Therefore, the keywords strategy helped them to reach a solution, though it often led them to incorrect solutions.

Question 24c (pre-test) and 26c(post-test):

Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.

Interviewer: Your answer before was $6\frac{1}{2}$ kg. Can you tell me how you got the answer?

Pupil: I added 5 and $1\frac{1}{2}$ and so I got $6\frac{1}{2}$

Interviewer: But did you try to understand the question first before you did the addition?

Pupil: Yes..... but when I saw the word total, I thought that I needed to add the numbers.

Interviewer: Ok, now read the question to me. If you don't know a word, tell me.

Pupil: (read the question)

Interviewer: Tell me what the question asked you to do?

Pupil: To find the total weight of the 5 packets of chicken wing.

Interviewer: Now, tell me what method you will use to find the answer and why?

Pupil: (Quiet.....)

Interviewer: Ok, let me ask you some questions.

How many packets did he have altogether?

Pupil: 5 packets

Interviewer: What is the weight of each packet?

Pupil: $1\frac{1}{2}$ kg

Interviewer: (drew 5 packets of chicken wings and wrote $1\frac{1}{2}$ kg on each packet)

Ok...what operation do you think you need to use?

Pupil: Multiply?

Interviewer: Yes, Can you show me now?

Pupil: (The pupil wrote $5 \times 1\frac{1}{2} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$ kg)

Interviewer: Good!

(I explained to her that she also could do addition, provided that she did repeated addition which I showed her soon after).

A number of pupils interviewed did understand that they were required to calculate the total weight of the chicken wings. They said that they should do addition to solve the problem. I acknowledged that they could add and let them perform the operation. What most pupils did was to add the number of packets (5) and the given weight ($1\frac{1}{2}$ kg), thus $5 + 1\frac{1}{2}$ instead of carrying out the repeated addition as $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}$. This shows that the pupils had problems in transforming the information correctly from the written word version to the number version.

Evidence from the lesson observations confirmed that the use of those keywords by the pupils in this study was stressed by their teachers while teaching word problems and thus influenced the pupils to interpret the words as indicators of which operation to use when solving word problems. This strategy simplified considerably the difficult task of answering word problems but it was not always successful – as shown with item 24c (pre-test) or 26c (post-test).

Another example of the transformation problem was found with item 24d in the pre-test or 26d in the post-test: *A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?* Though the pupils knew that they needed to do a divide to solve the problem, they may have been directed by the keyword “each”, which meant to cut the ribbon into smaller pieces, they were dividing 10 by 4 instead 4 by 10. When probed why they did it that way, pupils interviewed said that it was because 10 were bigger than 4 and they believed that in division, the dividend should always be greater than the divisor. This suggests that

the pupils thought that it was impossible to divide a smaller number by a bigger number – an error caused by the pupils' inadequate knowledge about the basic facts of number. Another possible reason was that some pupils might have thought that, just like in addition and multiplication, division was commutative. Hart (1981) and Tirosh (2000) showed similar findings in their studies when they reported their pupils also thought that division was commutative. This possibly explains why the percentage of transformation errors for the word problem involving division was high (49.2% for the pre-test and 32.6% for the post-test).

Therefore, to answer this research question, both in the diagnostic pre- and post-tests, the pupils exhibited the most errors in component 1, 2 and 6 (Understanding of fractions concepts and sequencing fractions, addition and subtraction of fractions and word problems involving fractions). However, the overall result suggests that a decrease in the average means exists in four of the six components tested, which included components 1, 2 and 6. This suggests that when at the P6 level, and after receiving further instruction on fractions, the pupils demonstrated that they improved their overall cognitive skills in understanding fraction concepts and the sequencing of fractions; in performing addition, subtraction, multiplication and division of fractions and in solving word problems. However, the pupils still experienced problems in working with the alternative forms of fractions and in determining a fraction of a quantity.

4.2.4 Research Question 4

Are there gender differences in the errors exhibited by Bruneian P5 and P6 pupils?

This research question was answered by administering a 28-item diagnostic pre-test to 396 P5 pupils (214 males and 182 females) in May-June 2002. The same test, treated as a diagnostic post-test with two additional items was re-administered to the same cohort of pupils when they moved to P6 in March 2003. The two additional items on alternative forms of fractions were only covered after the pupils received further instructions at their P6 level. Due to school transfers at the beginning of the

2003 school term and pupils' absentees on the diagnostic post-test administration days, the sample size dropped from 396 to 338 (187 males and 151 females).

An independent-samples *t*-test was conducted to investigate if there was a significant difference between gender means scores in terms of error patterns exhibited by P5 and P6 pupils in Brunei. The result and analysis of the gender means scores will first deal with the data for P5 pupils and will be followed by data for P6 pupils. Initially, the F-test was carried out using the Levene's Test for Equality of Variance. Since the significant value for all types of errors were larger than 0.05 and the sample size was more than 30, homogeneity of variance was not violated. In reporting the result, the *t*-value for equal variances assumed is used. The results are presented in Table 4.9 (p.137).

Among the five types of errors depicted in Table 4.9, significant differences were found in error Type 1 (grouping errors) and error Type 4 (incorrect operation), $p < 0.05$. The mean for grouping errors of males ($M = 1.05$, $SD = 0.95$) is significantly different ($t = 2.46$, $df = 394$, two-tailed $p = 0.01$) from that of female pupils ($M = 1.29$, $SD = 1.04$). The mean scores for incorrect operations by males ($M = 0.49$, $SD = 0.84$) is also significantly different ($t = 2.93$, $df = 394$, two-tailed $p = 0.01$) from that of female pupils ($M = 0.78$, $SD = 1.09$). For both types of errors, females show higher mean scores than males, suggesting that females have committed more of these errors as compared to males in all components of the fractions unit tested. Though some researchers (Rogers & Gilligan, 1988; Leeson, 1995) indicated that boys performed better than girls in mathematics and science, the differences found in only two from the five components tested in this study suggests that the differences were not significant.

Table 4.9

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the five Types of Errors Exhibited by P5 Pupils by Gender for the Five Components

Type of errors	Gender	N	Mean	Std. Dev.	t-Value	df	p
Grouping error	Male	21	1.05	0.95	2.46	394	0.01*
	Female	41	1.29	1.03			
Basic fact error	Male	21	11.32	5.39	0.51	394	0.6
	Female	41	11.04	5.20			
Defective algorithm	Male	21	1.29	1.48	0.5	394	0.61
	Female	41	1.36	1.35			
Incorrect operation	Male	21	0.49	0.84	2.93	394	0.01*
	Female	41	0.78	1.09			
Careless error	Male	21	0.62	0.98	1.23	394	0.22
	Female	41	0.51	0.79			

*Significant at 0.05 level

A further independent-samples *t*-test was conducted for the two types of errors with the aim to investigate in which components any significance differences did occur. The results are presented in Table 4.10 (p.138).

Table 4.10 shows that even though a significant difference was found between genders for grouping errors among all the five components as a whole, there were no significant difference by gender for the grouping errors found in any of the five components when computed by individual categories ($p > 0.05$). Both P5 male and female pupils committed roughly equal numbers of errors in components 2, 3 and 4. No grouping errors were committed by either male or female pupils in Categories 1 or 5. The main reason for it was the nature of the questions in those two categories: the pupils did not need any grouping or decomposing knowledge to solve the items in those two categories.

Table 4.10

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the Grouping Errors for the Computational Components for P5 Pupils by Gender

Component	Gender	N	Mean	Std. Deviation	t- Value	df	P
1	Male	214	.00	.00			
	Female	182	.00	.00			
2	Male	214	0.66	0.83	1.63	394	0.1
	Female	182	0.79	0.78			
3	Male	214	0.38	0.50	1.85	394	0.07
	Female	182	0.48	0.56			
4	Male	214	0.01	0.09	1.02	394	0.31
	Female	182	0.02	0.02			
5	Male	214	.00	.00			
	Female	182	.00	.00			

*Significant at 0.05 level

A *t*-test was not computed for components 1 and 5 as there was no grouping errors committed in these two components, thus the standard deviations of both groups are zeros.

As for the incorrect operation type of errors, an independent-samples *t* test output shows that significant difference was found in category 3, ($p < 0.05$). Female pupils show a higher mean score than the males, indicating that more females committed incorrect operations for items in category 3. Table 4.11 (p.139) shows the mean, standard deviation, *t*-value, degree of freedom and significance value of the incorrect operations type of error for the five components of fractions unit tested.

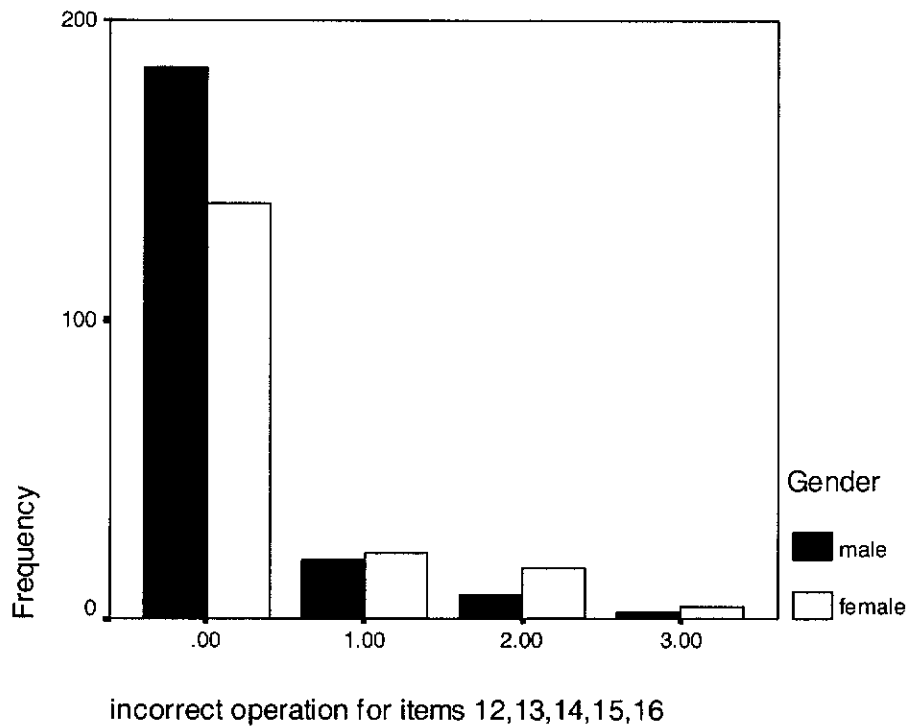
Table 4.11

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of Incorrect Operations Type of Error for the Five Computational Components

Component	Gender	N	Mean	Std. Deviation	t- value	df	P
1	Male	214	0.10	0.32	1.06	394	0.29
	Female	182	0.07	0.26			
2	Male	214	0.04	0.21	0.87	394	0.38
	Female	182	0.07	0.41			
3	Male	214	0.19	0.54	2.74	394	0.01*
	Female	182	0.37	0.75			
4	Male	214	0.50	0.83	1.76	394	0.08
	Female	182	0.66	0.91			
5	Male	214	0.00	0.00	1.88	394	0.06
	Female	182	0.01	0.13			

*Significant at 0.05 value

The difference in means for Items 12, 13, 14, 15 and 16 (Component 3) is illustrated in the bar graph in Figure 4.4 (p.140). The figure shows that males committed fewer errors for this Component 3 (multiplication and division) than females.



Keys: 0 – no error, 1 – 1 error, 2 – 2 errors, 3 – 3 errors

Figure 4.4 Frequencies of incorrect operations for Items 12, 13, 14, 15, 16 (Component 3) of P5 pupils by gender.

An independent-samples *t*-test was also conducted to investigate if P5 males and females differed significantly in the responses to the four word problems involving fractions. The results are presented in Table 4.12 (p.140). The result shows that the mean scores of overall errors for males for the four word problems ($M = 3.66$, $SD = 0.80$) was not significantly different ($t = 1.64$, $df = 394$, two-tailed $p = 0.10$) from that of female pupils ($M = 3.79$, $SD = 0.65$). However, when a further *t*-test was computed for each type of error, significant differences between males and females were found in comprehension and transformation errors. The mean score of comprehension errors for males for the four word problems ($M = 1.07$, $SD = 1.45$) is significantly different ($t = 2.07$, $df = 394$, two-tailed $p = 0.04$) from that of females ($M = 0.79$, $SD = 1.25$). Similarly, the mean score of transformation errors for males on the four word problems ($M = 0.90$, $SD = 0.97$) is significantly different ($t = 3.82$, $df = 394$, two-tailed $p = 0.00$) from that of females ($M = 1.29$, $SD = 1.09$). No significant differences were found in reading errors, process errors and encoding errors. Though males showed a higher mean score for comprehension errors (which

suggests that males committed more comprehension errors than females), the mean for the transformation errors is otherwise, in that females committed more transformation errors than the males. This finding is consistent with the study by Shalev, Auerbach and Gross-Tur (1995), when they reported that they found no gender differences in mathematics performance among the Grade four pupils they studied in Israel. The pupils in their study were at the average age of 11 – 12 years, which is in the same age group with the pupils in the current study. Though the findings of this study were consistent with the findings elsewhere, what is distinctive with the pupils in the current study is the education system they are in. Because of the bi-lingual system of education practised in Brunei since 1984, the pupils in Brunei only learnt mathematics in English, which is their second language when they were at P4. Prior to P4, the pupils were taught mathematics in Malay, which is their mother tongue. Therefore, the finding suggests that the medium of instruction used in mathematics teaching and learning does not create gender differences in the pupils' mathematical performance.

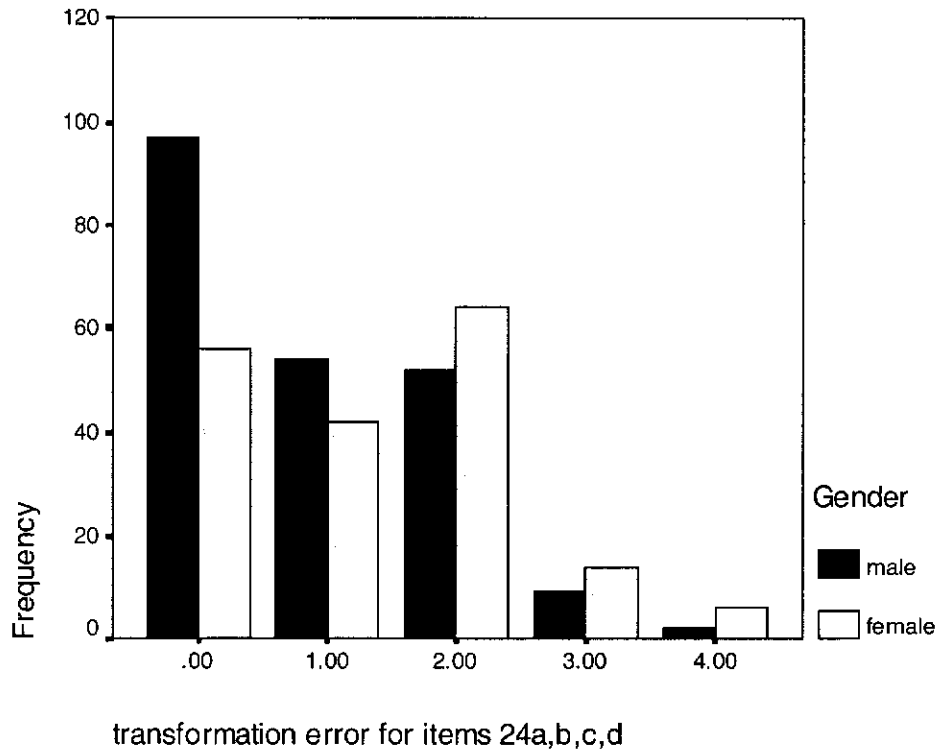
Table 4.12

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the Five Types of Errors Exhibited by P5 Pupils by Gender for the Sixth Component (Word Problems; Questions 24a, 24b, 24c and 24d)

Type of errors	Gender	N	Mean	Std. Dev.	t-Value	df	p
Reading error	Male	214	0.59	1.29	1.37	394	0.17
	Female	182	0.42	1.08			
Comprehension error	Male	214	1.07	1.45	2.07	394	0.04*
	Female	182	0.79	1.25			
Transformation error	Male	214	0.90	0.97	3.82	394	0.01*
	Female	182	1.29	1.09			
Process error	Male	214	1.03	1.03	1.37	394	0.17
	Female	182	1.17	0.95			
Encoding error	Male	214	0.07	0.26	1.31	394	0.19
	Female	182	0.11	0.35			

*significant at 0.05 level

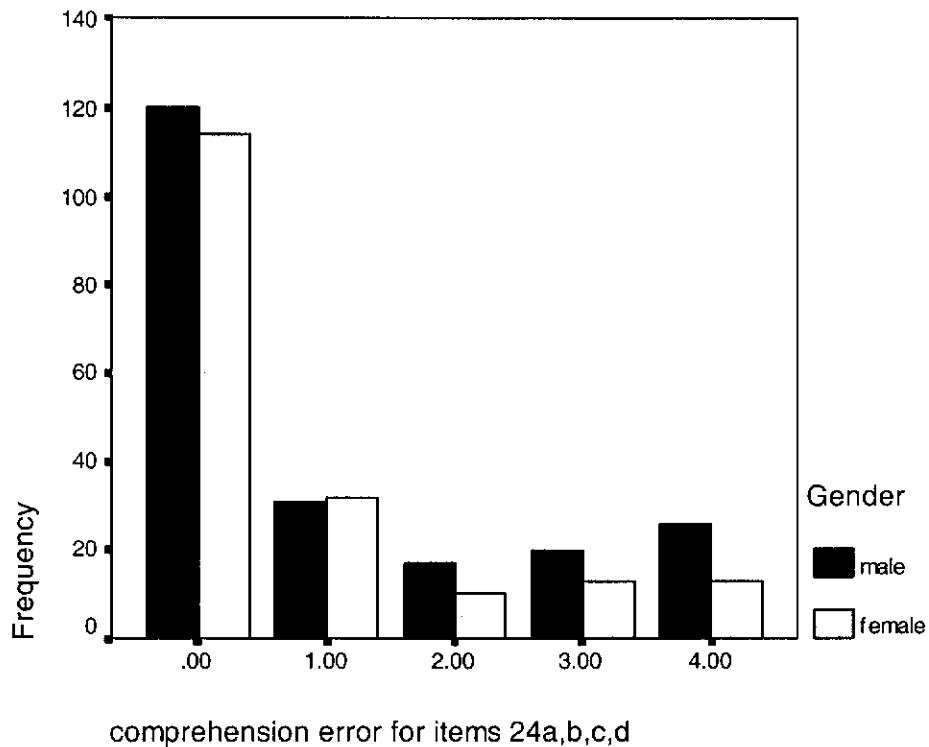
The frequencies for the transformation and comprehension errors among the four items on word problems between male and female pupils are shown in Figures 4.5 (p.141) and 4.6 (p.142) respectively.



Keys: 0 – no error, 1 – 1 error, 2 – 2 errors, 3 – 3 errors

Figure 4.5 Frequencies of the transformation errors for Items 24a, 24b, 24c and 24d (Component 6) of P5 pupils by gender.

Figure 4.5 above shows that fewer males committed transformation errors among the four items on word problems than females, with more females committing one, two, and three transformation errors but neither males nor females committed more than three transformation errors for this category.



Keys: 0 – no error, 1 – 1 error, 2 – 2 errors, 3 – 3 errors, 4 – 4 errors

Figure 4.6 Frequencies of the comprehension errors for items 24a, 24b, 24c and 24d (Component 6) of P5 pupils by gender.

Figure 4.6 above shows that males made fewer comprehension errors, but on the other hand, more males also committed two, three and four comprehension errors among the four items in the word problems category.

The next section deals with the result and analysis for the P6 pupils. The F-test was carried out using the Levene's Test for Equality of Variance. Since the significant value for all types of errors were larger than 0.05 and the sample size was more than 30, homogeneity of variance was not violated. In reporting the result, the t-value for equal variances assumed is used. The result is presented in Table 4.13 (p.144).

Table 4.13

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the Five Types of Errors Exhibited by P6 pupils by Gender for the Five Computational Components

Type of errors	Gender	N	Mean	Std. Deviation	t-Value	df	p
Grouping error	Male	187	1.08	1.01	0.67	336	0.51
	Female	151	1.16	1.15			
Basic fact error	Male	187	9.38	5.16	0.61	336	0.54
	Female	151	9.01	6.10			
Defective algorithm	Male	187	1.37	1.69	0.69	336	0.49
	Female	151	1.49	1.46			
Incorrect operation	Male	187	1.6	1.48	0.72	336	0.48
	Female	151	1.72	1.52			
Careless error	Male	187	0.94	1.06	0.93	336	0.35
	Female	151	1.05	1.15			

*significant at 0.05 level

Table 4.13 suggests that no significant differences were found between the mean scores for any of the five types of errors. Though female pupils showed slightly higher means in defective algorithms ($M = 1.49$, $SD = 1.46$), incorrect operation ($M = 1.72$, $SD = 1.52$) types of errors as compared to their male counterparts ($M = 1.37$, $SD = 1.69$ and $M = 1.6$, $SD = 1.48$), the differences may only have been due to chance and the t -test shows that they were not significant. That is, there was no significant difference in the number of errors made by male and female pupils in the five computational components tested. Therefore, generally both male and female pupils committed equally the same number of errors in the five computational components tested.

A further independent-samples t -test was conducted for each type of error with the individual five computational components tested. The aim was to investigate if there were any significant differences between male and female pupils' responses in those components. Among the five types of errors and five computational components, significant gender differences were found for the type 4 error (incorrect operations)

in Component 3 (Items 12,13,14,15 and 16). The results are presented in Table 4.14 below.

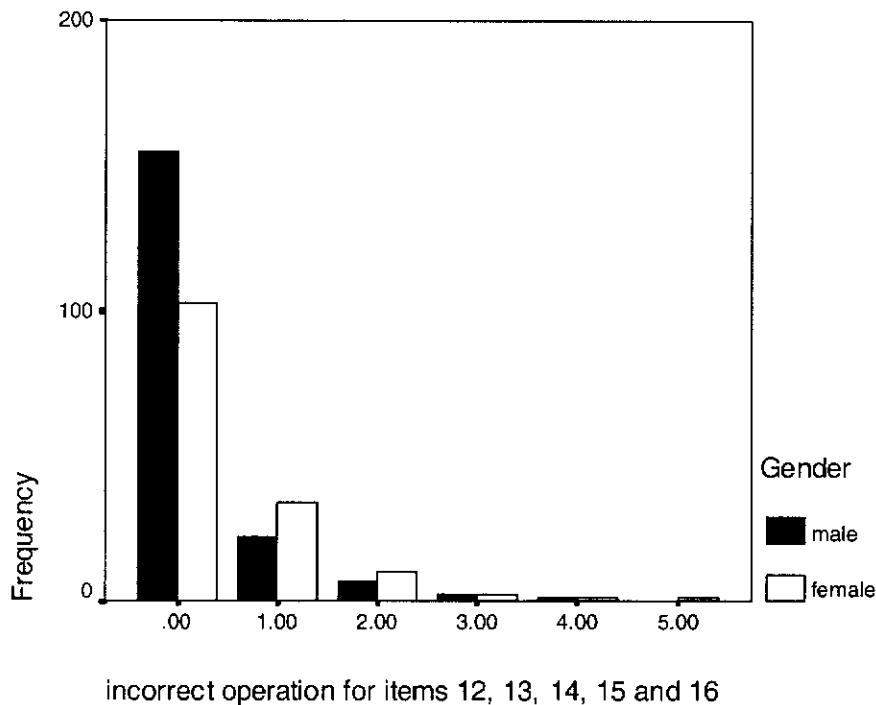
Table 4.14

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the Incorrect Operations Exhibited by P6 Pupils by Gender for Components 1 - 5

Component s	Gender	N	Mean	Std. Deviation	t- Value	df	p
1	Male	187	0.17	0.4	0.15	336	0.88
	Female	151	0.17	0.39			
2	Male	187	0.15	0.53	0.31	336	0.75
	Female	151	0.13	0.47			
3	Male	187	0.25	0.63	2.68	336	0.01*
	Female	151	0.46	0.82			
4	Male	187	0.97	0.94	0.77	336	0.44
	Female	151	0.89	0.94			
5	Male	187	0.69	0.28	0.11	336	0.91
	Female	151	0.66	0.27			

*significant at 0.05 value

Table 4.14 above shows that a significant difference was found in the incorrect operations type of errors for Component 3; multiplication and division of fractions (Items 12,13,14,15 and 16). The mean of for the incorrect operations for males ($M = 0.25$, $SD = 0.63$) is statistically different ($t = 2.68$, $df = 336$, $p = 0.010$) from that of females ($M = 0.46$, $SD = 0.82$). These results indicate that more female pupils committed incorrect operations in performing the multiplication and division of fractions than did the male pupils. The difference in means is further illustrated in Figure 4.7 (p.146).



Keys: 0 – no error, 1 – 1 error, 2 – 2 errors, 3 – 3 errors, 4 – 4 errors, 5 – 5 errors

Figure 4.7 Frequencies of incorrect operations for Component 3 (items 12, 13, 14, 15 and 16).

Figure 4.7 shows that for Component 3 (multiplication and division of fractions), males committed fewer incorrect operations errors compared to females. More females committed one, two and three incorrect operations errors, and about equal number of males and females committed three and four incorrect operations errors for questions 12, 13, 14, 15 and 16 in the diagnostic post-test. The results are similar to the data obtained from their P5 performance: the female pupils yielded the higher percentage of incorrect operations for items on the multiplication and division of fractions.

An independent-sample *t*-test was carried out to examine if there was any gender differences for the P6 pupils in the type of errors exhibited in performing the four items on word problems (Component 6). The results are presented in Table 4.15 (p.147).

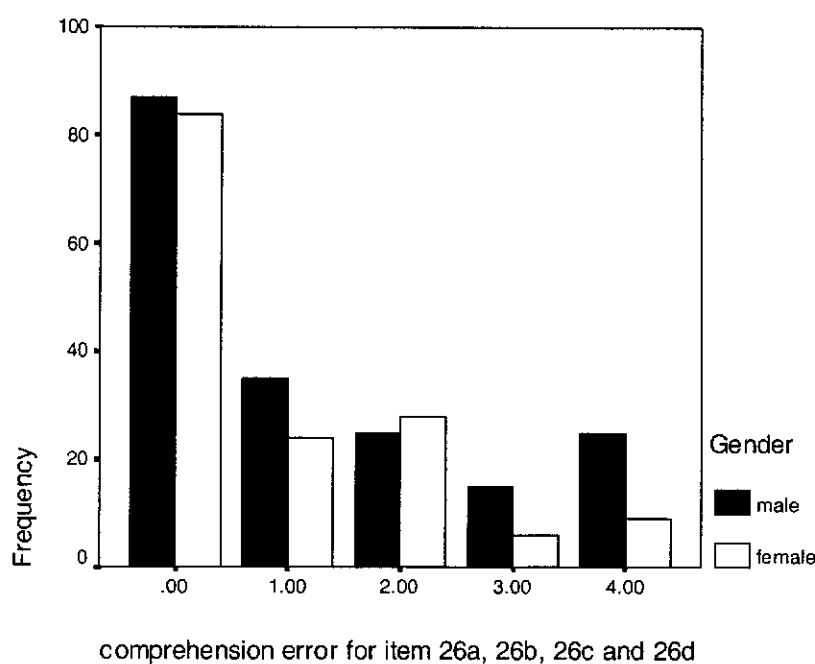
Table 4.15

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of the Types of Errors Exhibited by P6 Pupils by Gender for Component 6

Type of errors	Gender	N	Mean	Std. Deviation	t- Value	df	p
Reading	Male	187	0.32	0.86	0.79	336	0.43
	Female	151	0.25	0.77			
Comprehension	Male	187	1.23	1.44	2.34	336	0.02*
	Female	151	0.89	1.19			
Transformation	Male	187	0.7	0.87	1.59	336	0.11
	Female	151	0.85	0.89			
Process	Male	187	0.86	0.98	1.65	336	0.09
	Female	151	1.03	0.98			
Encoding	Male	187	0.00	0.00	1.58	336	0.11
	Female	151	0.01	0.11			

*significant at 0.05 value

Table 4.15 shows that a significant difference was found in the comprehension errors for the word problems. The mean for the comprehension errors for males ($M = 1.23$, $SD = 1.44$) is statistically different ($t = 2.34$, $df = 336$, $p = 0.02$) from that of females ($M = 0.89$, $SD = 1.19$). This result indicates that more male pupils committed comprehension errors in solving word problems involving fractions than did their female counterparts. Though there was a decrease in the mean of the reading errors, the t -test result indicates that it was not significant. The difference in frequencies is further illustrated in Figure 4.8 (p.148).



Keys: 0 – no error, 1 – 1 error, 2 – 2 errors, 3 – 3 errors

Figure 4.8 Frequencies of comprehension errors for Component 6 of P6 pupils by gender.

This finding contradicts that of Leeson (1995), who demonstrated that the overall performance of Grade 6 boys in the Australian Primary Mathematics Competition for the years 1990 to 1992 was greater than that of girls. However, this contradiction is expected because in Brunei, anecdotal evidence suggests (though not proven with empirical data yet) that girls are performing better, and are more interested in mathematics than are the boys. Leeson found that significant gender differences exist in relation to some mathematical topics: Boys performed better on items involving estimation (of numbers and measurement), rates, speed-time-distance, percentages and non-routine problems, while girls' performance was greater for the reflection of shapes, figure identification and number patterns. However in this current study, gender differences were only observed in comprehension and transformation errors, and were not observed in the other three types of errors in the word problem component.

Therefore, to answer research question four: For P5 pupils, significant gender differences were found only in the grouping and incorrect operations, not in the basic facts, defective algorithms and careless error categories among the five

computational components tested. The gender differences for the grouping errors were found in pupils' overall performance in the five computational components, where female pupils yielded higher mean scores than did the male pupils. No gender differences for grouping errors were found in the individual components. For the incorrect operations, gender differences were found both in the overall performance and in component 3; multiplication and division of fractions. Both overall and in component 3, female pupils yielded higher mean scores compared to male pupils. As for the word problems (Component 6), significant gender differences were found in the comprehension and transformation errors operations but not in the reading, process and encoding errors operations. Male pupils yielded higher mean scores for the comprehension errors compared to female pupils, but female pupils yielded higher mean scores for the transformation errors compared to male pupils. As they moved to their P6 level, no gender differences were observed in the pupils' overall performance in the five computational components. Following a further independent-sample *t*-test, gender differences were found in the incorrect operations type of errors in category 3; multiplication and division of fractions, where female pupils showed higher mean scores than the male pupils. No significant gender differences were found among the grouping, basic fact, defective algorithms and careless errors operations in any of the five computational components when analysed individually. For the sixth component, solving word problems involving fractions, gender differences were found in the comprehension errors where male pupils showed higher mean scores compared to females. No gender differences were found for the reading, process, transformation and encoding errors operations for Category 6 among the P6 pupils.

To summarise, it was observed that gender differences were observed when the pupils were at their P5 level, but became less obvious at the P6 level. The decrease is consistent with the findings of Fennema (1994) who suggested: gender differences in mathematics decrease except for some complex mathematics and in the selection of university majors or careers that involve mathematics.

4.2.5 Research Question 5

What are the attitudes of Bruneian P5 and P6 pupils towards the learning of fractions?

This research question was investigated by administering a 26-item attitude towards fractions questionnaire to the 396 sample pupils a week after they sat for the diagnostic pre-test at the end of June and early July 2002. The same attitude questionnaire was re-administered to the same cohort of sample pupils when they moved to P6 in the following year. The administration of the questionnaire was carried out at the end of March 2003, about a week after the pupils sat for their diagnostic post-test. Due to school transfers at the beginning of the 2003 school term and pupils' absences on the attitude post-test administration days, the number in the sample dropped to 338 (187 males and 151 females) from 396 (214 males and 182 females). Therefore, the actual sample for this study was only those pupils who sat for both the pre- and post-test, which were 338 (187 males and 151 females). The 26 statements used a 5-point Likert response scale and aimed to measure pupils' attitudes in terms of the three components and four constructs: Cognitive (Confidence and Perception), Affective (Enjoyment) and Conative (Commitment). (See Appendix C for full text, p.213).

Based on the pupils' responses to the five-point Likert scale of Strongly Agree (5), Agree (4) Undecided (3), Disagree (2) and Strongly Disagree (1), scores for all items in a given scale were added to determine the individual pupils' total score. The higher a particular pupil scored on that particular dimension, the more favourable was his or her attitude. The highest possible score was 130 and the lowest possible score was 26.

A statistical comparison of the combined mean scores of the pupils' score during their P5 and P6 levels on the 5-point Likert scale items was used to answer this research question. The highest expected mean was 5 and the lowest 1. The result is presented in Table 4.16 (p.151).

Table 4.16

Means and Standard Deviations of Pupils' Attitude Pre- and Post-tests

Items	Time	Mean	Std. Deviation
1 – 26	Attitude pre-test	3.97	0.38
1 – 26	Attitude post-test	3.94	0.42

Table 4.16 indicates that overall, the pupils showed a positive attitude towards the learning of fractions at both their P5 and P6 levels. When a repeated measure design using the General Linear Model (GLM) was carried out, the result indicated that there was no significant difference between the attitude pre- and post-test, ($p > 0.05$). At their P6 level, the pupils still held positive attitude towards the learning of mathematics and fractions. The t -test shows that the slight difference in mean scores between the pre- and post-tests (Pre-test $M = 3.96$, Post-test $M = 3.94$) was not statistically significant.

At the end of their P6 level, the pupils were to sit for their first public examination, the PCE which would determine their path through secondary education. Therefore, a very good result in mathematics was vital for all of them, for a failure in any one of the five examined subjects, namely English, Malay, Mathematics, Science and General Studies, would detain them for another year at the P6 level.

In a further effort to investigate if there were any significant effects over time in the four scales measured, a repeated measure design using GLM was carried out for the four individual scales. The means and standard deviations of the four individual scales of the attitude questionnaire were also used to investigate in which of the four constructs the pupils showed the most favourable attitude. The results are presented in Table 4.17 (p.152) and demonstrate that the pupils' mean scores on each of the four attitudinal constructs were about equal value, with a slightly higher mean scores for the perception sub-construct for both the pre- and post-tests. This indicates that both at their P5 and P6 levels, the pupils perceived the importance and usefulness of learning fractions if they were to excel in mathematics. The enjoyment, commitment and confidence sub-constructs also yielded equally high mean scores, which indicate that, on the whole, at both their P5 and P6 levels, the pupils enjoyed learning, had a

strong sense of commitment and were confident about learning mathematics and fractions.

Table 4.17

Means and Standard Deviations for the Four Constructs of Attitude Questionnaire for the Pre- and Post-tests

Constructs	Items	No of items	Time	Mean	Std. Dev.
1 – Enjoyment	1,2,3,4,5,6,7,and 8	8	Pre-	4.04	0.50
			Post-	4.01	0.56
2 – Confidence	9,10,11,12,13,14 and 15	7	Pre-	3.58	0.55
			Post-	3.51	0.61
3 – Perception	16,17,18,19,20 and 21	6	Pre-	4.28	0.47
			Post-	4.33	0.46
4 - Commitment	22,23,24,25 and 26	5	Pre-	4.02	0.49
			Post-	3.97	0.53

Therefore, to answer this research question, the overall results demonstrate that both at the P5 and P6 levels, the pupils showed an equal and positive attitude towards the learning of fractions. There was no change in pupils' attitude towards the learning of fractions at their P6 level as compared to their P5 level. This was indicated by their overall mean scores at their P5 level (3.97) and (3.94) at their P6 level. As for the four attitudinal sub-constructs, no significant differences in means were found among them. At both levels, the pupils perceived the importance and usefulness of learning mathematics and fractions probably because of their awareness of the importance of doing well in mathematics for their upcoming PCE examination, pupils admitted that they enjoyed learning fractions, held a high sense of commitment and maintained a high confidence level.

4.2.6 Research Question 6

Are there any gender differences in P5 and P6 pupils' attitudes towards the learning of fractions?

This research question was investigated by conducting an independent-sample *t*-test using the mean scores obtained from the 26-items attitude questionnaire administered when the pupils were at their P5 and P6 levels. The aim was to investigate if there were any differences in mean scores obtained by male and female P5 and P6 pupils in their attitude towards the learning of fractions. This section deals with the data obtained from both the P5 and P6 pupils. The results are presented in Tables 4.18 (p.153), 4.19 (p.154), 4.20 (p.155) and 4.21 (p.155) respectively.

Table 4.18

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of P5 Pupils' Attitude Towards Fractions by Gender

Gender	N	Mean	Std. Dev.	<i>t</i>	<i>df</i>	<i>P</i>
Male	214	3.95	0.39	0.091	394	0.927
Female	182	3.95	0.39			

*Significant at 0.05 level

The result reveals that there was no significant difference between the mean attitude score for males and that of the female pupils ($t = 0.091$, $df = 394$, $p > 0.05$). This result indicates that P5 male and female pupils held equally positive attitudes towards the learning of fractions. An earlier result discussed in Research Question 5 on page 138 indicated that generally, both male and female pupils held positive attitudes towards the learning of fraction.

An independent-samples *t*-test was further conducted to investigate if there were significant differences between male and female pupils' attitudes towards the learning of fractions in terms of the four attitude constructs (Enjoyment, confidence, perception and commitment) measured. Table 4.19 (p.153) shows the mean, standard deviation, degree of freedom, *t*-value and significance value of P5 pupils'

attitudes towards fraction in terms of the four attitude constructs measured by gender.

Table 4.19

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of P5 Pupils' Attitudes Towards Fractions in Terms of the Four Attitude Constructs by Gender.

Attitude Constructs	Gender	N	Mean	Std. Deviation	<i>t</i>	<i>df</i>	<i>p</i>
Enjoyment	Male	214	4.04	0.54	0.245	394	0.81
	Female	182	4.05	0.49			
Confidence	Male	214	3.62	0.5	1.841	394	0.07
	Female	182	3.51	0.61			
Perception	Male	214	4.25	0.47	0.196	394	0.85
	Female	182	4.26	0.48			
Commitment	Male	214	3.97	0.52	1.732	394	0.08
	Female	182	4.06	0.47			

*Significant at 0.05 value

The results show that the pupils' mean scores on each of these four attitudinal constructs were about equal, with the pre-test data for P5 pupils yielding the group means, standard deviation, *t*, *df* and significance value. There were no significant differences in the mean scores obtained by male and female pupils in any of the four attitudinal constructs measured. Though the mean score of boys in the confidence sub-construct (3.62) was slightly higher than the mean score of girls (3.51), the difference was not statistically significant ($t = 1.841$, $df = 394$, $p = 0.066$).

A similar independent-samples *t*-test was carried out to deal with data for the P6 level and the results are presented in Tables 4.20 and 4.21 (p.155).

Table 4.20

Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of P6 Pupils' Attitudes Towards Fractions by Gender

Gender	N	Mean	Std. Deviation	<i>t</i>	<i>df</i>	<i>p</i>
Male	214	3.95	0.37	0.24	336	0.809
Female	182	3.94	0.47			

*Significant value at 0.05

The result reveals that there was no significant difference between the mean attitude score for males and that of the female pupils ($t = 0.24$, $df = 336$, $p > 0.05$). This result indicates that, just as when the pupils were at their P5 level, P6 male and female pupils held equally positive attitudes towards the learning of fractions.

An independent-samples *t*-test was further conducted to investigate if there were significant differences between P6 male and female pupils' attitudes towards the learning of fractions in terms of the four attitude constructs (Enjoyment, confidence, perception and commitment) measured. Table 4.21 shows the mean, standard deviation, *df*, *t* and significance value of P6 pupils' attitudes towards learning of fractions in terms of the four attitude sub-constructs measured, by gender.

Table 4.21

Post-test Means, Standard Deviations, t-value, Degree of Freedom and Significant Value of P6 Pupils' Attitudes Towards Fractions in Terms of the Four Attitudes Sub-constructs by Gender

Attitude Constructs	Gender	N	Mean	Std. Deviation	<i>t</i>	<i>df</i>	<i>p</i>
Enjoyment	Male	186	4.03	0.58	0.76	336	0.44
	Female	152	3.98	0.55			
Confidence	Male	186	3.54	0.49	0.95	336	0.34
	Female	152	3.48	0.72			
Perception	Male	186	4.32	0.43	0.67	336	0.51
	Female	152	4.35	0.50			
Commitment	Male	186	3.94	0.48	1.15	336	0.25
	Female	152	4.01	0.59			

*Significant at 0.05 value

No change in mean scores was observed as the pupils moved to their P6 level. A similar result was found by Campbell (1995), when she demonstrated that no pre or post changes were found with her sample pupils with the mathematics general attitude scale, even after it was administered several years later.

The effect size for each construct was calculated and the following values were found: Enjoyment, $d = 0.02$; Confidence, $d = 0.19$; Perception, $d = 0.02$; Commitment, $d = 0.17$). All the effect sizes are d of .20 and are considered as small effects (Burns, 2000). It thus suggests no significant difference between boys and girls exist in all four constructs measured.

The above results obtained from the P5 and P6 pupils are consistent with the findings obtained by Linn and Hyde (1989) and Rogers and Gilligan (1988), namely that young boys' and girls' achievement, interest level, and confidence in mathematics and science are relatively equal. Some literature, such as that by Hanna *et al.*, (1990), suggests that the ratio of female teachers to male teachers has been said to have affected the gender differences in pupils' mathematical performance. However, unique to this current study and as mentioned earlier in Research Question 4, the larger proportion of mathematics female teachers to male teachers in Brunei primary schools did not seem to affect the pupils' attitudes towards fractions in general.

Therefore, in answer to research question six: The overall results show that there were no significant gender differences in terms of the sample pupils' attitudes towards the learning of fractions ($p > 0.05$). Both at their P5 and P6 levels, male and female pupils yielded higher means (P5 male mean = 3.95, female mean = 3.95, P6 male mean = 3.95, female mean = 3.94). It can be concluded that both the P5 and P6 male and female pupils showed equally positive attitudes towards the learning of fractions in general, and also towards the four sub-constructs measured.

4.3 Summary

The data that was used to answer the six research questions of this study were collected using two instruments. The first instrument, the diagnostic test, was used Mathematical Errors in Fractions Work

to gather the necessary information concerning the different types of errors exhibited by the sample pupils in learning about fractions. This instrument was also used to assess the pupils' cognitive achievement/performance on the fractions unit tested by examining the different types of errors they exhibited when working with fractions. The second instrument, the attitude towards fractions questionnaire was used to gather information about the pupils' perceived attitude towards the learning of fractions. Evidence obtained from the pupils' interviews and lesson observations supported the data obtained from these two instruments. Both the diagnostic test and the attitude questionnaire were administered twice to the same cohort of pupils over a year period. The first administration was treated as a pre-test and given to the pupils when they were at their P5 level. The second administration (treated as a post-test) was given after the pupils received further instruction on fractions at their P6 level. Comparisons of the different types of errors exhibited in the diagnostic tests and their perceived attitudes towards the learning of fractions during the pre- and post-tests enabled the preparation of a response to each of the six research questions.

All the resulting data was analysed by computer using the SPSS version 10 program, with subsequent results being represented both in tabular and graphical format. Analysis of the data collected using the first instrument showed positive results – that is to say, the pupils' achievement in the post-test improved. Though the average scores for both the computational and word problems components were considered low, the improvement was statistically, though not educationally, significant. In terms of the types of errors made both in the computational components, the pupils showed improvements in some types of the errors. This is evidenced by the decrease of mean scores for basic fact errors. It implies that at their P6 level, the pupils made less basic fact errors but grouping and defective errors were still prevalent, and incorrect operations and careless errors increased for the computational components. As for the word problem component, the pupils at their P6 level still held similar errors identified at their P5 level. It was observed that at their P6 level the pupils exhibited less reading, transforming and processing errors, but comprehension and encoding errors persisted.

The study reveals that there are several important points teachers have to consider in teaching the computation of fractions and word problems involving fractions first. Teachers should ensure that the pupils have mastered the basic concepts and basic facts of fractions. Pupils who do not master the basic concepts and facts may not only have difficulty with fractions but with other areas of mathematics as well (Byrant, 1995; Swedosh, 1996, 1999; Prior, 2000). It should be made clear to pupils that the computation of fractions is different from the computation of whole numbers, and teachers must be able to make the pupils aware that they are not to treat fractions as whole numbers in the computations of fractions. Though the most common type of errors among the P5 and P6 pupils in this study was basic fact errors, other types of errors should not be neglected – especially grouping, incorrect operations, defective algorithm and careless errors. Similarly, in teaching word problems, the teachers should be aware of the pupils' English language ability, reading ability, and vocabulary. Although transformation and process errors are the most common types of errors exhibited by the P5 and P6 pupils in this study, teachers should not neglect other types of mistakes especially reading, comprehension and encoding errors. The findings in this study indicate that each pupil made errors when dealing with fractions, and that the errors varied from pupil to pupil, even though they were of the same age group, gender, class and taught by the same teachers. The implication is that teachers must be alert and aware of this phenomenon and try to tackle the errors according to the individual pupil's needs. If the current common practice of the teachers in Brunei is simply to look for right or wrong answers, they should move away from this practice by examining more closely the types of errors their pupils make and to think of the possible causes of why the pupils arrive at incorrect solutions.

This study also indicates that some pupils tried to make their own strategies to solve word problems – for example, concentrating on keywords, which they associated with the four arithmetic operations. However, as shown in this current study and another studying Brunei by Ching and Yusof (1996), the use of keywords did not always help them to solve the problems correctly.

An analysis of the data collected using the second instrument indicated that the pupils held very favourable attitudes towards the learning of mathematics and

fractions both at their P5 and P6 levels. No gender differences were detected at either level. Both the P5 and P6 male and female pupils demonstrated equally positive attitudes towards the learning of mathematics and fractions in general, and towards the four sub-constructs measured.

The following chapter uses these results as a basis for drawing implications, conclusions and formulating a number of recommendations.

CHAPTER 5

CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

5.1 Introduction

This final chapter describes the outcomes of this study and uses them as a basis for the formulation of a number of conclusions and recommendations. The chapter begins with the summary of the thesis that overviews each chapter. It also includes the main findings of this study in relation to the six research questions that guided this study. A number of implications on the teaching and learning of mathematics, particularly on fractions in Brunei, were derived from the findings and these implications will be discussed in this chapter also. The chapter then outlines several recommendations for both the teaching and learning of fractions in primary schools and for further research in mathematics education. As with almost all studies, this investigation has its strength and limitations and these are also discussed. The chapter begins with a summary of the thesis.

5.2 Summary of the Thesis

Chapter 1 (Background, Context and Rationale) described the background of this longitudinal study of upper primary school pupils' mathematical errors in working with fractions. In order to provide the readers with a clear view of the research context, the current education policy and the bi-lingual education system practised in Brunei Darussalam since 1984 was presented first. It specifically highlighted the fact that the pupils in this study learnt mathematics in English (a second language to most pupils and even a third language to some others), from Primary 4 onwards though they had been learning the subject in Bahasa Melayu (the mother tongue) from pre-school to Primary 3 level. This transfer of language of instruction was thought to have adversely affected the pupils' performance in the two diagnostic tests administered to the pupils over the period of a year – a situation that will be described later in this chapter. My study aimed to identify the types of errors made by P5 pupils in working with fractions, and to further examine if such errors were still persistent as they moved to P6 about a year later after they were given more formal instruction on fractions. To justify the research aims, the rationale for the

study and the six research questions that guided the study were also provided as well as a brief summary of the methodology employed and the significance and scope of the study. Chapter 1 concluded with a summary of the whole thesis structure.

Chapter 2 (Literature Review) provided a review of the past literature and issues pertaining to mathematical errors, particularly on fractions relating to the research questions being investigated in this study. The issues included the learning difficulties and error patterns exhibited by pupils when working with fractions, which encompassed the basic introductory concepts of fractions and the four mathematical operations of fractions as well as the application of fractional concepts and operations in solving word problems. An important point, which arose from the literature, was that pupils' learning difficulties in working with fractions is a universal phenomenon.

The difficulties in fractions learning are not only evident among the second or third language speakers but also with the first speakers of English. Logically, if the first language speakers of English experience problems in learning fractions, how much greater is the problem for pupils whose language of instruction is not their mother tongue? In this study, this language problem was more apparent in the problem-solving component compared to the computational or mechanical components of the two diagnostic tests administered in this study.

Chapter 2 also reviewed pertinent literature concerning gender and attitudinal issues which were considered as factors in determining pupils' performance in mathematics. This was important because it provided a window for the readers to understand the issues related to pupils' difficulties in working with fractions.

Chapter 3 (Research Design and Methodology) described how the data sources for this study consisted of the same cohort of pupils followed through longitudinally over a year period. This chapter also outlined the mixed-approach research paradigm adapted for this study. The reasons for the choice of the methodology were also described. The diagnostic and attitude pre-tests were administered during the second half of the 2002 school year when the pupils were at the P5 level. The sample consisted of 396 pupils (214 males and 182 females) from four government co-ed primary schools in the Brunei-Muara district, the most populated district in Brunei

Darussalam. The same diagnostic and attitude tests treated as post-tests were re-administered during the first half of the 2003 school year – that is, at the pupils' P6 level. Due to school transfers and pupils' absentees on the diagnostic and attitude post-tests administration days, the number in sample dropped from 396 to 338 (187 males and 151 females). The research design provided comparative data if the errors identified earlier would still be prevalent at the pupils' P6 level and if they were, in which of the six tested components were the errors more prevalent. The reduced sample numbers were not a concern because in this 'cohort study', though the specific population was tracked over a specific period of time, selective sampling was used (Borg & Gall, 1979, p.291). This meant that some members of a cohort might not have been included each time.

Educational researchers have claimed that there are merits in supporting quantitative data with qualitative data (Lincoln & Guba, 1985; Cohen *et al.*, 2000) such as it increase the credibility, reliability and the validity of the data. To accomplish this, data obtained from both the diagnostic pre- and post-tests and attitude towards mathematics pre- and post-questionnaires were supported by data obtained from the pupils' interviews and lesson observations. Though the amount of qualitative information obtained in this study was limited, it was expected that it would provide a better understanding of the result by supporting the quantitative information with data obtained from the pupils' interviews and lessons observation (Cohen *et al.*, 2000). Therefore, the approach of complementing the quantitative data with the qualitative information afforded greater credibility to the findings of my study.

Chapter 3 also described the processes involved and the instruments developed and used for gathering the data. The details covered how the sample was selected, the design of the diagnostic test and the attitude questionnaire, measurement and coding of the different error patterns as well as the coding of the responses from the attitude questionnaire and statistical analysis. The reliability and validity of the diagnostic pre- and post-test, as well as the attitude questionnaire were investigated in terms of each scale's internal consistency reliability (using Cronbach's alpha coefficient) and discriminate validity (using the mean correlation of a scale with the other scales). Quality controls and ethical issues were also described in this chapter.

Chapter 4 (Results and Discussion) reported results from analyses of the quantitative data. These results were also interpreted in the light of qualitative data obtained from the pupils' interviews and lessons observations in order to answer several of the research questions. The analysis of the data gathered using the methodology discussed in Chapter 3 and the findings were discussed in Chapter 4 also. Data was analysed using the Statistical Package for Social Sciences (SPSS) version 10 and the percentages; means and standard deviations of occurrences for each type of errors for the overall components of the diagnostic tests were computed. Other statistical tests such as the independent-sample *t*-tests, paired-samples *t*-test and repeated measure design using General Linear Model (GLM) were also employed in analysing the data. The paired-samples *t*-test was employed with a purpose to evaluate the impact of the further instruction on fractions given at the P6 level on the mean scores of each type of errors exhibited by the pupils, while the GLM test was carried out in order to investigate if there were significant change in pupils' attitudes towards fractions from the pre- to the post-tests.

Though this study was developmental in nature, it was a type of cohort study (Cohen *et al.*, 2000), in which the pupils were followed through from their P5 to P6 level; therefore, Analysis of Variance (ANOVA) was not used. ANOVA is more appropriate to test between-groups such as different pupils in each group (Pallant, 2001). The relationships of the present study's findings with the findings of other research both locally and internationally along with the uniqueness of the current study were also highlighted in Chapter 4.

In the remainder of this chapter, a summary of the findings is presented, along with the implications for the teaching and learning of fractions particularly in Brunei schools; recommendations for future research; the strengths and limitations of the study, and a number of general conclusions arising from the study.

5.3 Summary of Findings

The outcome of this study suggests that generally, most P5 and P6 pupils in Brunei Darussalam experienced having difficulties in mastering the basic facts regarding fractions. This study further suggests that there was limited educational improvement in fractional understanding overall with the same range of errors being

repeated at the pupils' P6 level. Researchers such as Byrant, (1996); Swedosh, (1996); Hunting, Davis and Pearn (1997) and Andrew and Carroll, (1999) have shown that the mastery of basic facts of arithmetic underpins the learning of more complex mathematical capacities, which depend on accurate memory of these basic facts. For the computational components of the diagnostic tests, although a statistical improvement was observed in terms of the grouping and basic fact types of errors made in doing fractions as the pupils moved to P6, educationally the improvement was not very promising. This is because the mean scores of the pupils' correct responses for the computational component were still very low. Errors such as defective algorithms, incorrect operations and careless errors were not only persistent as they moved to P6 level but they tended to become more prevalent. As far as word problems involving fractions are concerned, although the pupils evidenced improvement in the reading, transformation, process and encoding types of errors, comprehension errors were still numerous. The findings in this study indicate pupils made errors, which were different among individual pupils, even though they were all of the same age group, gender, class and were taught by the same teachers. The implication is that teachers must be alert, become aware of this phenomenon, and tackle errors by adopting special intervention strategies and activities such as those described by Bell (1984) and Perso (1991).

The pupils' poor performances in the computational and word-problem components in the diagnostic pre- and post-tests provided information for teachers regarding pupils' learning across the time span from their P5 to P6 levels. In addition, the findings also have implications on the quality of the instructional activities given to the pupils in the study, the quality of teacher education programs in Brunei and the pupils' difficulties in learning mathematics in the English language. The language issue will be discussed further in the latter part of this chapter. It is also hoped that through my sharing in local seminar and conferences, the results obtained from this study will be transparent all teachers and concerned parties in Brunei Darussalam.

Generally, no significant gender differences were found in terms of the types of errors the pupils made at both the P5 and P6 levels. In terms of the pupils' attitudes towards mathematics, despite their poor performances in the diagnostic pre- and post-tests, the pupils held very positive attitudes towards the learning of fractions at

both the P5 and P6 levels. No significant gender differences were found in terms of pupils' attitudes towards the learning of fractions. The following section highlights the implications of the findings regarding the teaching and learning of mathematics and fractions.

5.4 Implications of Findings for Teaching and Learning

The findings of this study have implications for the quality of the instructional activities provided by teachers of mathematics, the impact of language transfer in the medium of instruction – that is from Bahasa Melayu to English, and the quality of the teacher training program in Brunei. Thus, as this study was the first formal longitudinal study on primary pupils' mathematical errors in working with fractions conducted in Brunei over a year period, its findings could provide useful information for teachers, teacher educators, curriculum planners and the Brunei Ministry of Education about the learning difficulties encountered by the pupils in this topic. As Dole *et al.*, (1997) demonstrated, the value of error pattern research could be seen to contribute to the development of specific intervention strategies to help pupils overcome learning difficulties and make progress towards mathematical achievement. They further suggest that error pattern research has pedagogic implications, and more importantly at another level, error pattern research has implications for teacher training programs. The teacher training program should emphasise the importance for student-teachers to be equipped with sound theoretical knowledge on how pupils learn mathematics and efforts should be taken to ensure that this knowledge is reflected in practice.

5.4.1 Quality of Instructional Activities

Ideally, fractions concepts should be introduced and taught using concrete materials and activities. However, teachers in Brunei have been observed to teach fractions using more procedural approaches rather than conceptual approaches. Data from my observations suggests they overuse direct instruction that focuses on content rather than for deep understanding. (Asmah, 2001) from her study with primary schools teachers in Brunei concludes that teaching methods in the Bruneian primary schools are mainly teacher-centred. In addition to that, Shamsiah and Clements (2002) suggested that in teaching fraction concepts, primary school teachers in Brunei

overemphasised area-model approaches to fractions, and they imply that this overemphasis was counterproductive so far as their pupils' learning was concerned. The fact that mathematics lessons in Brunei schools are teacher-centred is evidenced when the two teachers observed in this study noted to teach the pupils by introducing the rules rather than the processes. For example, in one of the lessons on fractions observed, I made a casual observation with the first teacher observed said:

“ Remember, remember..... before you add the two fractions, you must first find their L.C.M, I say it again..... L.C.M, L.C.M!” (Teacher 1, female, March 1, 2003)

“ O.k., listen, to divide two fractions, you need to change the division sign into multiplication, then invert the second fraction, look for any numbers that you can cancel and just do like what I have taught you in the doing the multiplication of fractions the other day”. (Teacher 1, female, March 7, 2003)

This rote learning style was also confirmed when one of the interviewed pupils said that he did not know why he needed to change the division sign into multiplication and to invert the second fraction, apart from the fact that his teacher said so. This pupil, just like other pupils in Brunei schools interviewed in Suffolk and Clements's (2003) study responded to fractions tasks by applying “rules without reasons”. In another lesson on addition of mixed number, the second teacher observed said:

“ O.k. Class... What did I tell you before? To find the next equivalent fraction of a given fraction, you just need to multiply or divide the numerator and the denominator with the same number (Teacher then showed two examples on the black board). (Teacher 2, male, February 22, 2003)

“I don't think you will have problems in doing this addition of mixed numbers because this is just the same as the addition of simple fractions which you have just learnt yesterday. Just follow the steps as yesterday, except now that you need to add the whole numbers first”. (Teacher 2, male, February 27, 2003)

These examples of teacher-centred approaches indicate the pupils are trained to learn concepts by rote, without the teacher demonstration regarding the processes

involved. This rule procedure approach is similar to pupils being taught to memorise multiplication tables without any understanding as to the processes involved in multiplication operations. The rule-driven approach used by the observed teachers in this study might have impacted from the pressure of the P6 exit examination (PCE). Whether they like it or not PCE exam must cast a shadow over the work teachers do in P5 and especially at P6. Getting pupils through the exam is seen as a priority. Therefore, teachers assume rule driven methods as being the most successful, or most in keeping with cultural expectation. However, the findings of this study provide evidence to the contrary.

In light of the above contrary, and in order to improve the quality of mathematics teaching and learning generally and on fractions unit particularly, teachers in Brunei must shift from the currently observed scenarios. The pupils were just unable to apply the rules without seeing the logical processes in the mathematical operations. Teachers should include more hands-on activities appropriately contextualised and make use of physical apparatus and equipment, such as those recommended by Womersley (2000) as described in Chapter 2.

During the lesson observations, it was noticed that the two teachers spent a considerable time on developing skills with algorithms. This practice seems to continue despite repeated assertions that premature emphasis of algorithmic learning will result an inability to internalise, operationalise, and apply this concept in an appropriate manner (Carpenter, Kepner, Corbitt, Lindquist & Reys, 1980). Teachers should be aware that to consolidate pupils' learning, their intuitive knowledge needs to be considered. Fishbein (1987) acknowledged that children's intuitive knowledge has a role in fraction learning. He asserted that bringing meaning to the language and symbols of fractions is a result of interweaving of children's intuitive and formal knowledge. Intuitive knowledge plays an important role in mathematical learning (Beth & Piaget, 1966). Beth and Piaget further suggest that intuitive knowledge consists of imagery, lived through-experiences, thought tools and constructive mechanisms, all of which provide a basis for formal knowledge. Kieren (1992) acknowledges the existence of intuitive mechanisms that interact with formal mathematical structures. Kieren adds that another possible reason relates to the level of abstraction at which much instruction is focused. Children are expected to

operate at the abstract symbolic level too often and too soon. Piaget (1973) suggests that children pass through qualitatively different stages of intellectual development in a predictable order but at varying rates. The stages are referred to as sensory-motor, preoperational, concrete operational, and formal operational. Children at the age where fractions are normally introduced and developed in classrooms are generally at the concrete operational level stage (Primary 3). Their ability to synthesise, make deductions, and follow *if/then* arguments very much depend on their personal experience and firsthand interactions with the environment.

As a result, those pupils who are at the concrete operational level stage could memorise the concepts that were introduced and taught at the time; however, after a while they tended to forget the earlier acquired concepts. Furthermore, they were not able to relate them to the newly introduced concepts. Confrey (1990), Kieren (1992) and Mack (1998) suggest that often teachers take no account of children's spontaneous attempts to make sense of rational numbers, thus discouraging children from attempting to understand these numbers on their own and encouraging them to adopt an approach based on the rote application of rules. The findings of this study indicate that generally the pupils' performances in the diagnostic tests were not very promising. The pupils' difficulties in working with fractions, if not dealt with professionally, provide a signal that these pupils will have further difficulties at their secondary level.

Researchers such as Ames (1992) demonstrate that the instructors' teaching style is one of the factors that contributed to the level of the pupils' performance. What contributed to such ineffective teaching approaches used by the teachers observed in this study? The reason could be related to the quality of the teacher education program available in Brunei.

5.4.2 Quality of Teacher Education

Error patterns research has implications for teacher training programs (Dole *et al.*, 1997, p.3). Effective teachers not only know in a relational way the mathematics content they are expected to teach but also what Shulman (1986) called "pedagogical content knowledge". All fifteen teachers participating in this current study, including both the observed teachers, were trained teachers at the University

of Brunei Darussalam. It is the objective of the teacher education program in Brunei to produce generalist and innovative teachers so that they can work effectively with primary school pupils in all subjects they are teaching (University Brunei Darussalam handbook, 2002, p.8). Throughout the three to four years teacher education program, opportunities are provided for student teachers to be trained in structured teaching and practices where they are encouraged to relate and apply their understanding and skills in a practical context.

Thipkong and Davis (1991) alerted teacher educators to the influence of teachers' own errors and misconceptions in mathematics upon their teaching of the subject matter, and thus on the pupils they are teaching also. In their research, they identified pre-service teachers' misconceptions in interpreting and applying decimals, noting that the misconception "multiplication makes bigger, division makes smaller" was extremely prevalent. Their findings suggest that teachers need to be aware not only of their own pedagogical knowledge but also their content knowledge. If teachers are aware of their own errors and misconceptions in particular mathematical topics, great care would need to be taken so that such errors and misconceptions are not transferred to learners (Maurer, 1987; Stefanich & Rokusek, 1992). There are at least two challenges for teacher educators in Brunei that arise from this study. Foremost is to develop strategies that will help teachers to identify pupils' mathematical errors with fractions. The second is to assist teachers to provide appropriate remediation within the multiple contexts of ordinary classrooms in Brunei. This study serves to inform mathematics educators of teacher training programs to at least re-examine the quality of the pre-service teachers' content and pedagogical knowledge in mathematics.

5.4.3 The Pupils' Language Difficulties

The findings of this study also have implications for pupils' language competencies. As mentioned in Chapter 1, from P4 onwards some subjects are taught in English and some are taught in Malay. Mathematics, which the pupils start learning at the Pre-school level, is taught in Malay. However, when the pupils move to Primary 4, mathematics is taught in English. This switching of the medium of instruction undoubtedly has an adverse effect on the pupils. Concepts, which are once learned

in Malay have to be relearned in English, and this may cause difficulties to many pupils. Although Brunei is practising the Bilingual system of education, many pupils in Brunei were observed to be non-fluent in either speaking or writing English (Liew *et al.*, 1993). Miller (1999) reported that teachers in his study have identified the acquisition of English as a second language as the major hurdle to mainstream integration of students of non-English speaking background. He added that mathematics has its own structures that are usually less flexible than common usage and the word choice is vitally important.

Though it was not the purpose of this study to evaluate the pupils' language competencies in learning mathematics, their language difficulty is obviously reflected in the pupils' understanding of mathematical word problems. The study indicates that a greater percentage of pupils tried to devise their own strategies to solve word problems – for example, concentrating on keywords, which they associated with the four arithmetic operations. Though there were only four questions on word problems in the diagnostic pre- and post-tests, Pupils' responses to Question 24c in the pre-test and Question 26c in the post-test (*Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought*) demonstrate that the pupils used the key word *total* and manipulated the digits by carrying out addition, thus $5 + 1\frac{1}{2} = 6\frac{1}{2}$ kg. Of the 396 pupils who sat for the diagnostic pre-test, 40% used this strategy, as did 42% of 338 pupils in the post-test. Though the use of keywords might have worked in other problems, it did not in this particular question. It seems that this phenomenon does not only happen with Bruneian pupils because Hart (1981) found out that 30% of the 13-year olds in her study sample used the same strategy. As mentioned earlier in Chapter 4, Ching and Yusof (1996) noted that problem solving is a complex activity for the pupils and it involves the co-ordination of knowledge, experience and attitude.

Key words were found to be an important cause of the errors displayed by the pupils in Brunei. Therefore, the implication is that practising teachers need to alert the pupils that the key word strategy will not always be helpful and applicable to certain problems (Lean, Clements & Campo, 1990). Teachers and pupils need to be made

aware that understanding the language problems is the foundation for solving the mathematical word problems successfully. As mentioned earlier, the pre-service teachers need to be alerted of this serious problem in language and fractions learning so that they will not teach this flawed process to their pupils. Teachers should be made realised that the use of keywords to assist pupils in solving word problems is a false strategy (Barb & Quin, 1997). This study will hopefully provide information that will assist teachers to understand the characteristics of language related difficulties among Bruneian-English-bilingual children in solving mathematical word problems, and pose questions such as “Do Bruneian pupils think in English or Bahasa Melayu when the word problems are presented in English?” Whang (1996) claimed that many Asian languages share many common features such as the vocabularies and structures, therefore the findings of this study will also provide an insight on other Asian bilingual children, such as Japanese, Korean and Chinese children. Beyond that, this study should contribute to the better understanding of the cognitive processes of pupils in Brunei who are brought up in the Bilingual system of education.

5.5 Strengths of the Study

The main strength of this study is that it is the first longitudinal study aimed at diagnosing primary school pupils’ mathematical errors when working with fractions in Brunei, following the same cohort of pupils over a year period. Though a few studies have already been carried out in the area of fractions, this study was more extensive in nature. Unlike earlier studies such as those by Bezuk, 1991; Brinker, 1998; Fatimah, 1998; Saman and Suffolk, 2001; Suffolk and Clements, 2003, this study covers all the six components of fraction unit stipulated in the Brunei upper primary mathematics syllabus. The study has yielded up-to-date information and thus adds to the available literature on the teaching and learning of fractions in Brunei schools. This information will inform education planners in Brunei regarding the current performance of primary pupils in studying fractions; some implications for the quality of the instructional activities given to the Bruneian pupils; pupils’ difficulties in learning fractions in English, which is a second or third language for the pupils, and implications for the quality of teacher education programs available in Brunei. The study employed a researcher-designed instrument that is in line with

the Brunei upper primary mathematics syllabus and hence reflects the pupils' everyday mathematical exercises. The instrument should be more appropriate for use than other instruments produced outside Brunei.

5.6 Limitations of the Study

As with other studies, this piece of research has its own limitations in terms of the sampling, instrumentation, data collection and analysis and the procedure used. These limitations are described below.

5.6.1 Sampling

This study is based on 15 classes (396 pupils in the pre-test and dropped to 338 pupils in the post-test) in four schools from one district in Brunei Darussalam, so its representativeness is limited and caution should be exercised in generalising the results. Nevertheless, the study provides strong evidence that Bruneian pupils perform quite badly on fractions. More specifically the research provides evidence that Bruneian pupils are weak in the basic facts of fractions and number sense in general. Very few pupils demonstrated high computation skill indicating that Bruneian pupils need to be encouraged to be more focused on the learning of basic mathematical facts, not only on fractions, but also general numbers sense. Sowder (1988) suggests teachers must examine more than the pupils' answers (in this case the wrong answers) and must demand from pupils more than just answers such as the pupils' verbal explanation on how they arrived at such answers.

5.6.2 Instrumentation

This study used a modified attitude questionnaire based on one by Raimah (2001). The Cronbach alphas for the four constructs were all <0.7 , however, the values were consistent among three constructs (Enjoyment, Confidence and Perception) but not for the Commitment construct. This could be due to the fact that there were only sixteen pupils involved for the pilot testing of the attitude questionnaire.

5.6.3 Data Collection

The scope of the qualitative component of this study was limited to interviews with only 48 pupils and twelve hours of lesson observations, so caution must again be exercised in generalising beyond this study. However, the same pupils interviewed represented fifth and sixth graders who had demonstrated low to high performance on both the diagnostic pre- and post-tests. The probes from the interviews suggested that sound and effective understanding of fractions were yet to be established by most of these Bruneian pupils. Evidence from the interviews revealed that all high-, middle- and low-performing pupils tended to rely heavily on computational techniques that had been taught in classes. The two classes observed were also considered to represent the typical mathematics lessons carried out in Bruneian schools.

Another limitation of this study concerns the validity of the pupils' responses to the pre- and post-attitude questionnaires. Though the pupils showed very favourable attitudes towards the learning of fractions, their performances in the diagnostic pre- and post-tests were not very satisfactory. As an outsider, the pupils in this study might have tried to impress me, thus compelling them to respond to the statements as positively as possible. This drawback was observed during the administration of the attitude test when some pupils simply ticked a response that might not have reflected their genuine feelings about the learning of fractions. As an example, in this study, when pupils could not actually decide their feeling about a certain statement, they simply either ticked "agree" or "undecided", seemingly so that they would not be left behind before I read the next statement. This problem might have been avoided if the pupils were allowed to take the questionnaire home where they could respond to it at their own time and pace. Fearing that the return rate would be low and the pupils might have just copied each other's answers, such a procedure was not exercised in this study on purpose.

The question about the originality of some pupils' responses to the attitude questionnaire was also doubtful. This suspicion is justified by evidence from the interviews when two female pupils, on being asked about their favourite subject, said that it was Bahasa Melayu (Malay) and Geography, but after a pause, they

changed their responses to mathematics indicating that their second responses might have not been genuine.

Another problem, which was beyond my control and that of the class teachers, was the classes' attendance rate. From the fifteen classes that participated in the study, only one class had a full attendance on the day the test was administered. The average number of absentees for each class was four, and to test the absent pupils on another day was impossible as I had fifteen classes to monitor. A further concern about testing absentees later was that other pupils in the class might have informed the absent pupils about the type of questions being asked in the test.

5.6.4 Procedures

The time span between the administration of the diagnostic pre- and post-tests could also be a factor that contributed to the pupils' poor performance as the pupils were required to learn other subjects during this period. As mentioned in Chapter 3, due to the nature and format of the diagnostic pre- and post-test, the questions were not read out to the pupils. This procedure might have affected the pupils' responses to the items also due to problems experienced in reading and understanding the questions asked in the tests. This can be seen by looking at the pupils' answer scripts for both the diagnostic pre- and post tests that some pupils just left some items unanswered. Moreover, though the class teachers of the participating pupils were informed about the administration of the second test (post-test), the pupils were not informed and therefore they did not prepare for the test, a situation that may have had the potential to affect the results of the diagnostic post-test.

In view of the above limitations, it is important to keep the findings of this study in context and one should not generalise them beyond the scope of the cohort examined.

5.7 Recommendations for Further Research

The focus of this study was on the mathematical errors and problems a sample of primary school pupils in Brunei had with learning fractions and on how the

outcomes of the study might inform the design of early mathematics instruction in the topic.

In closing, I want to emphasise that the notion of a developmental perspective (Sophian, 2000) is important. In general terms, the main recommendation I am making is that the design of mathematics instruction for the primary school level be informed by consideration of the long-range goals and challenges of mathematics education as well as by an analysis of what young children know and can learn. Thinking about what children will need to know in order to make sense of fractions served as a useful illustration of this idea because we know enough about the difficulties pupils have with fractions to make some tangible suggestions about how early elementary mathematics instruction could help. In other areas, too, we need to look closely at our educational system long-term instructional goals, at where difficulties arise as we undertake to meet those goals, and at how we might prepare primary school pupils for those challenges in the ways we teach them in the very first few years of formal schooling. Where we do not have sufficient knowledge to answer these questions, the prospective developmental perspective may serve as a useful tool in framing a research agenda as well as in formulating instructional goals for our pupils in Brunei.

Many of the basic mathematical concepts are learned in the primary grades. It will be a mistake, though, to assume that the initial learning will be retained without reinforcement. Concepts and skills from all the strands of mathematics must be continually reinforced and extended. (Perso, 1992; Swedosh, 1999). Many teachers believe pupils will not retain concepts and skills unless they are reinforced. Reinforcement for the pupils occurs through repetition and drill. The teachers credit the daily drill they use with playing a significant role in reinforcing the concepts they teach and for successfully guiding their pupils to, or above the grade level. In one sense, then, the teachers' emphasis on repetition can be construed as being in line with ideal teaching as they are reinforcing previously taught concepts and skills. In fact, their assumption that knowing the procedure implies understanding the concept would lead them to believe that by providing their pupils with repeated practice, they are reinforcing their understanding of mathematical concepts. On the other hand, their reinforcement consists of isolated practice of computational skills

without connecting them to a conceptual base or placing them within a meaningful context. Therefore, it is important for all the teachers and student teachers to be aware of the above situation.

This longitudinal study followed a cohort of P5 pupils through their P6 level. To understand better the pupils' conceptual development and the effectiveness of the instructional activities delivered to the pupils at every level of fraction teaching, a more extensive longitudinal study is recommended. A replication study is recommended and it may start when the pupils are at their P3 level, a level when fractions concepts are first introduced in the mother tongue, to be followed through P4, P5 and P6. With this approach, more data could be gathered as to determine the pupils' difficulties in learning fractions and more importantly, the pupils' difficulty in the language transfer from P3 and P4 could be better detected and explored.

In addition, the findings of this study provide an indication that more in-depth, extensive and longitudinal studies into the development of fraction concepts need to be carried out in a range of schools (urban and rural) in all the four districts in Brunei Darussalam. This first longitudinal study might be considered as the first of a series of needed studies and it is recommended that more extensive but similar research should be carried out in other schools so that the findings from this study could be made more transferable to other schools in the country.

5.8 General Conclusions

Fractions and decimals represent a significant extension of pupils' knowledge about numbers and children's rational number thinking is a complex phenomenon (Mullis, Dossey, Owen & Phillips, 1991). Complex, because rational numbers find expression in a variety of subconstructs and because the development of rational number knowledge has been shown to be linked to children's ideas about whole numbers. Children's whole number knowledge can inhibit rational number learning, but it can also be used to advance rational number understanding (Pitkethly & Hunting, 1996). When pupils possess a sound understanding of fraction and decimal concepts, they can use this knowledge to describe real-world phenomena and apply it to problems involving measurement, probability, and statistics. An understanding of fractions and decimals broadens pupils' awareness of the usefulness and power of

numbers and extends their knowledge of the number system. Therefore, it is vital in Primary 1-6 to develop concepts and relationships that will serve as a foundation for more advanced concepts and skills that the pupils need at their secondary level. This is because Zurina (2003) found that all the 47 Secondary 3 students from a school in Brunei who took a *Basic Fraction Test* in her study had a poor grasp of fraction concepts and skills though all of them had passed the Primary 6 (PCE) examination. In another more recent study by Suffolk and Clements (2003) with some 2000 Secondary 2 students in Brunei, they found that typically, the students adopted purely mechanistic (and often inappropriate) approaches to dealing with fraction tasks, with only the best-performing pupils thinking about fractions tasks in holistic ways. Therefore, if the learning difficulties pupils have in learning fractions can be identified and rectified or at least minimised at the primary school level, the same problems might not be repeated at the pupils' secondary level. The findings of this study suggests that the problem is becoming serious in Brunei and therefore means and measures should be taken to halt this deterioration and rectify the situation.

The primary school level instruction should aim to help pupils understand fractions and decimals, explore their relationships, and build initial concepts about order and equivalence. As evidence from the lesson observations suggests pupils construct these ideas slowly, it is crucial that teachers use physical materials, diagrams, and real-world situations in conjunction with ongoing efforts to relate their learning experiences to oral language and symbols. When fraction concepts are introduced at the P3 and P4 levels, instruction should emphasise basic ideas such as the basic concepts of fractions which can help to reduce the amount of time currently spent in the upper grades in correcting pupils' misconceptions and procedural difficulties.

This study highlights several mathematical errors in the P5 and P6 pupils' work in fractions. Of course, paper and pencil tests constitute only one of the many techniques of assessing pupils' mathematical performance in fractions. Nevertheless, this effort has contributed towards finding evidence that primary school pupils in Brunei have difficulties in learning fractions. Those difficulties in learning fractions and mathematical errors in other mathematical topics will need to be addressed by the teachers concerned. Efforts should be taken to overcome the errors by designing and developing specific intervention strategies and activities such as conflict or

diagnostic teaching (Bell, 1984; Perso, 1991) so that teachers can provide opportunities for the pupils to successfully overcome those fractions learning difficulties and thus progress towards mathematical achievement. However, I wish to state that this study does not claim absolutes in the results and that the findings should be used as general indicators of the performance of P5 and P6 pupils in Brunei.

In conclusion, this study has revealed that there are several important points teachers have to consider in teaching the computation of fractions and word problems involving fractions. In teaching fractions, teachers must ensure that pupils have mastered the basic concepts and basic facts of fractions. Pupils who do not do so will not only have difficulty with fractions but with other areas of mathematics as well (Swedosh, 1996; Prior, 2000). Teachers should make it clear that the computation of fractions is different from the computation of whole numbers. Furthermore, teachers must be able to convince pupils that they are not to treat fractions as whole numbers.

With the recommendations proposed for future studies, a more coordinated effort should be encouraged among the officers from the Department of Schools, Ministry of Education, the teacher educators, the heads of schools and teachers at the school level. They should be made aware of the current difficulties existing in the teaching and learning of fractions. Teachers' continuous professional commitment to improve fractions teaching should be emphasised. Not only that, but pre-service teachers' own mathematical content and pedagogical knowledge needs to be re-evaluated, with new emphasis placed on the teaching and learning of fractions.

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Diagnostic Pre-test



**Mathematical Errors on Fractions Among Primary
Five Pupils in Brunei Darussalam**

Dear Pupil,

I am a lecturer at Universiti Brunei Darussalam and I am currently enrolled in a Ph.D program at the Curtin University of Technology, Perth, Australia. My research in Brunei aims to find out which areas of fractions Primary 5 pupils find most difficult.

I would be grateful if you could help me by answering the enclosed test on fractions. There are 28 questions. There is no pass or fail mark for this test. I seek your kind cooperation to answer all the questions but please do them yourself.

Thank you very much for your time and cooperation.

Yours sincerely,

Hajah Jamilah Haji Mohd Yusof.
Lecturer, UBD/ Research Student

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Tel: 249001Ext.548 (Office) or 772795 (Home) E-mail jmyusof@shbie.ubd.edu.bn


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Before you start, please fill in the details below:

Name :..... School:.....
Age:..... Class..... Male/Female (please circle one).

Appendices

Please read the questions and write your answer in the space provided. Where appropriate, show your working in the space provided.

Questions	Space for working
<p>Question 1.</p>  <p>What fraction of the figure above is shaded?</p> <p>Answer:</p>	
<p>Question 2.</p> <p>Complete the following equivalent fractions:</p> <p>a) $\frac{4}{5} = \frac{\quad}{15}$</p> <p>b) $\frac{\quad}{10} = \frac{1}{2} = \frac{8}{\quad}$</p>	
<p>Question 3.</p> <p>Change $2\frac{3}{5}$ to an improper fraction.</p> <p>Answer:</p>	
<p>Question 4.</p> <p>Change $\frac{13}{4}$ to a mixed number.</p> <p>Answer:</p>	
<p>Question 5.</p> <p>Which fraction is larger, $\frac{3}{4}$ or $\frac{3}{5}$?</p> <p>Answer:</p>	

Questions	Answers
<p>Question 6.</p> <p>Calculate $\frac{13}{15} + \frac{8}{15}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 7.</p> <p>Calculate $1\frac{2}{3} + \frac{4}{7}$</p> <p>Answer:</p>	
<p>Question 8.</p> <p>Calculate $4\frac{5}{9} + 2\frac{1}{2}$</p> <p>Answer:</p>	
<p>Question 9.</p> <p>Calculate $\frac{7}{9} - \frac{2}{9}$</p> <p>Answer:</p>	
<p>Question 10.</p> <p>Calculate $3\frac{2}{7} - 1\frac{3}{4}$</p> <p>Answer:</p>	
<p>Question 11.</p> <p>Calculate $1 - \frac{2}{5}$</p> <p>Answer:</p>	
<p>Question 12.</p> <p>Calculate $\frac{3}{8} \times 64$</p> <p>Answer:</p>	

Questions	Answers for Working
<p>Question 13.</p> <p>Calculate $\frac{6}{7} \times \frac{2}{3}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 14.</p> <p>Calculate $\frac{3}{7} \times 1\frac{1}{2}$</p> <p>Answer:</p>	
<p>Question 15.</p> <p>Calculate $\frac{2}{5} \div \frac{4}{5}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 16.</p> <p>Calculate $\frac{5}{6} \div 6$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 17.</p> <p>Calculate: $\frac{2}{3}$ of 48</p> <p>Answer:</p>	
<p>Question 18.</p> <p>What is $\frac{3}{4}$ of 60 kilograms?</p> <p>Answer:</p>	

Questions	Original Text (Word/Number)
<p>Question 19.</p> <p>Express 4 kg as a fraction of 10 kg.</p> <p>Answer:</p>	
<p>Question 20.</p> <p>Express 45 minutes as a fraction of 1 hour. (Giving your answer in its lowest terms).</p> <p>Answer:</p>	
<p>Question 21.</p> <p>Arrange the following fractions from the smallest to the largest.</p> $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$ <p>Answer:</p>	
<p>Question 22.</p> <p>Arrange the following fractions from the largest to the smallest.</p> $1\frac{1}{10}, \frac{4}{5}, 1\frac{3}{4}$ <p>Answer:</p>	
<p>Question 23.</p> <p>Express 0.85 as a fraction, giving your answer in its lowest terms.</p> <p>Answer:</p>	

Question 24.

Solve the following problems:

- a) A bag of sugar weighs $\frac{3}{4}$ kg and another bag of sugar weighs $1\frac{1}{2}$ kg. What is the total weight of the bags?
- b) A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?
- c) Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.
- d) A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?

Diagnostic Post-test



Mathematical Errors on Fractions Among Primary Five Pupils in Brunei Darussalam

Dear Pupil,

I am a lecturer at Universiti Brunei Darussalam and I am currently enrolled in a Ph.D program at the Curtin University of Technology, Perth, Australia. My research in Brunei aims to find out which areas of fractions Primary 6 pupils find most difficult.

I would be grateful if you could help me by answering the enclosed test on fractions. There are 30 questions. There is no pass or fail mark for this test. I seek your kind cooperation to answer all the questions but please do them yourself.

Thank you very much for your time and cooperation.
Yours sincerely.

Hajah Jamilah Haji Mohd Yusof.
Lecturer, UBD/ Research Student

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Tel: 249001Ext.548 (Office) or 772795 (Home) E-mail jmyusof@Shbie.ubd.edu.bn

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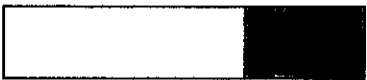
Before you start, please fill in the details below:

Name :

School: Date:

Class: Male/Female (please circle one) Age:

Please read the questions and write your answer in the space provided. Where appropriate, show your working in the space provided.

Questions	Space for working
<p>Question 1.</p>  <p>What fraction of the figure above is shaded?</p> <p>Answer:</p>	
<p>Question 2.</p> <p>Complete the following equivalent fractions:</p> <p>a) $\frac{4}{5} = \frac{\quad}{15}$</p> <p>b) $\frac{\quad}{10} = \frac{1}{2} = \frac{8}{\quad}$</p>	
<p>Question 3.</p> <p>Change $2\frac{3}{5}$ to an improper fraction.</p> <p>Answer:</p>	
<p>Question 4.</p> <p>Change $\frac{13}{4}$ to a mixed number.</p> <p>Answer:</p>	
<p>Question 5.</p> <p>Which fraction is larger, $\frac{3}{4}$ or $\frac{3}{5}$?</p> <p>Answer:</p>	

Questions	Space for working
<p>Question 6.</p> <p>Calculate $\frac{13}{15} + \frac{8}{15}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 7.</p> <p>Calculate $1\frac{2}{3} + \frac{4}{7}$</p> <p>Answer:</p>	
<p>Question 8.</p> <p>Calculate $4\frac{5}{9} + 2\frac{1}{2}$</p> <p>Answer:</p>	
<p>Question 9.</p> <p>Calculate $\frac{7}{9} - \frac{2}{9}$</p> <p>Answer:</p>	
<p>Question 10.</p> <p>Calculate $3\frac{2}{7} - 1\frac{3}{4}$</p> <p>Answer:</p>	
<p>Question 11.</p> <p>Calculate $1 - \frac{2}{5}$</p> <p>Answer:</p>	

Questions	Diagrams for Questions
<p>Question 12.</p> <p>Calculate $\frac{3}{8} \times 64$</p> <p>Answer:</p>	
<p>Question 13.</p> <p>Calculate $\frac{6}{7} \times \frac{2}{3}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 14.</p> <p>Calculate $\frac{3}{7} \times 1\frac{1}{2}$</p> <p>Answer:</p>	
<p>Question 15.</p> <p>Calculate $\frac{2}{5} \div \frac{4}{5}$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 16.</p> <p>Calculate $\frac{5}{6} \div 6$, giving your answer in its lowest terms.</p> <p>Answer:</p>	
<p>Question 17.</p> <p>Calculate: $\frac{2}{3}$ of 48</p> <p>Answer:</p>	

Questions	Answers (for working)
<p>Question 18.</p> <p>What is $\frac{3}{4}$ of 60 kilograms?</p> <p>Answer:</p>	
<p>Question 19.</p> <p>Express 4 kg as a fraction of 10 kg.</p> <p>Answer:</p>	
<p>Question 20.</p> <p>Express 45 minutes as a fraction of 1 hour. (Giving your answer in its lowest terms).</p> <p>Answer:</p>	
<p>Question 21.</p> <p>Arrange the following fractions from the smallest to the largest.</p> <p>$\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$</p> <p>Answer:</p>	
<p>Question 22.</p> <p>Arrange the following fractions from the largest to the smallest.</p> <p>$1\frac{1}{10}, \frac{4}{5}, 1\frac{3}{4}$</p> <p>Answer:</p>	
<p>Question 23.</p> <p>Express 0.85 as a fraction, giving your answer in its lowest terms.</p> <p>Answer:</p>	

Questions	Space for working
<p>Item 24.</p> <p>Express 65% as a fraction in its simplest form.</p> <p>Answer:</p>	
<p>Item 25.</p> <p>Change $\frac{3}{5}$ into a percentage.</p> <p>Answer:</p>	

Item 26. **Solve the following problems:**

a) A bag of sugar weighs $\frac{3}{4}$ kg and another bag of sugar weighs $1\frac{1}{2}$ kg. What is the total weight of the bags?

b) A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?

c) Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.

d) A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?

Appendix C

Pupil Attitude Questionnaire

Name: _____

Boy or Girl: _____

Class: _____

Age: _____

School: _____

Read the following statements. Then, for each statement, place a tick (/) in the column which describes your feelings about learning fractions.

	STATEMENTS	<i>Strongly Agree</i>	<i>Agree</i>	<i>Undecided</i>	<i>Disagree</i>	<i>Strongly Disagree</i>
1	I like mathematics.					
2	I enjoy answering fractions questions in class.					
3	I enjoy doing questions on equivalent fractions, comparing and ordering fractions.					
4	I enjoy doing questions on addition of fractions than subtraction of fractions.					
5	I enjoy doing questions on multiplication of fractions than division of fractions.					
6	I enjoy solving word problems on fractions.					
7	I have always enjoyed learning about fractions since Primary 3.					
8	I look forward to learning more about fractions next year in Primary 6.					
9	I am good at fractions.					
10	I am confident when answering questions on fractions.					

	STATEMENTS	<i>Strongly Agree</i>	<i>Agree</i>	<i>Undecided</i>	<i>Disagree</i>	<i>Strongly Disagree</i>
11	I usually get high marks for a mathematics test that consists of fractions questions.					
12	Fraction questions are easy.					
13	I am able to explain to my friends how to get correct answers to fractions questions.					
14	I am good at getting correct answers to fractions questions.					
15	I usually do well in fractions.					
16	The easiest part of mathematics is fraction.					
17	If I have to learn mathematics with lots of fractions in it, I will probably good at it.					
18	It is important for me to be good at fractions.					
19	Fractions are useful for me to excel in mathematics.					
20	My teacher is good in teaching mathematics.					
21	My teacher is good in teaching fractions.					
22	I always make sure that I can do my work on fractions correctly.					
23	I always ask my teacher if I don't understand anything on fractions.					

	STATEMENTS	<i>Strongly Agree</i>	<i>Agree</i>	<i>Undecided</i>	<i>Disagree</i>	<i>Strongly Disagree</i>
24	I always make sure that I finish my work on time.					
25	I always make sure that I do my homework each time my teacher gives me.					
26	I always get help from my parents if I have any problem with my mathematics work.					

Thank you.

Appendix D

Fractions Component Within Brunei Upper Primary Mathematics Syllabus

Primary 4 syllabus. Topic 3 within the syllabus for Primary 4 pupils is titled “Fractions”. The following details are provided:

- | | | |
|--------------|-----|--|
| 3. Fractions | 3.1 | Concept of fractions. |
| | 3.2 | Simple fractions with denominators not greater than 100. |
| | 3.3 | Equivalent fractions. |
| | 3.4 | Comparison of fractions. |
| | 3.5 | Addition and subtraction of fractions with common
denominators. |
| | 3.6 | Proper fractions, improper fractions and mixed numbers. |
| | 3.7 | Simple word problems on fractions. |

In addition, section 4 of Primary 4 syllabus on “decimals,” calls for Primary 4 pupils to be taught “decimal fractions in relation to common fractions” (p.7).

Primary 5 syllabus. Topic 3 within the Syllabus for Primary 5 pupils is titled “Fractions and Decimals.” The part of the entry concerned with fractions is as follows:

- | | | |
|------------------------------|-----|---|
| 3. Fractions
and Decimals | 3.1 | Addition and subtraction of fractions with different
denominators. |
| | 3.2 | Simple multiplication and division of fractions. |
| | 3.3 | Fractions of a quantity. |
| | 3.4 | Comparison of fractions. |
| | 3.5 | Expressing one quantity as a fraction of another. |
| | 3.6 | Simple word problems on fractions |
| | 3.7 | Decimal fractions up to 3 decimal places, including place
value. |
| | 3.8 | Conversion between fractions and decimals..... (p.13) |

Primary 6 syllabus. Topics 2 and Topic 3 within the Syllabus for primary 6 pupils are titled “Numbers and The Four Operations” and “Percentage” contain more components of fractions. The related sub-topics are:

- | | |
|---|--|
| 2. Numbers
and the Four
Operations. | 2.1 Introduction to integers |
| | 2.2 The number line; addition and subtraction of numbers

including integers and fractions on the number line. |
| | 2.4 The four operations on numbers, fractions and decimals

including the use of brackets..... (p.18). |
| 3. Percentage | 3.1 Concept of percentage (symbol: %) |
| | 3.2 Expressing fractions as percentages and vice versa. |
| | 3.3 Expressing decimals as percentages and vice versa.....(p.18) |

Appendix E

Pupils' Interview Transcripts (Pre-test)

Pupil No. 22

Sample 189 , Male (School 3)

<i>Interviewer</i>	Can you please read question 1 and tell me how you got the answer?
<i>Pupil</i>	What fraction of the figure above is shaded?
<i>Interviewer</i>	What was your answer?
<i>Pupil</i>	$\frac{1}{2}$
<i>Interviewer</i>	Why did you think that it was $\frac{1}{2}$?
<i>Pupil</i>	Because the rectangle is divided into two parts and one part is shaded.
<i>Interviewer</i>	Can you please look at the two parts again, are they of equal size?
<i>Pupil</i>	No, they are not.
<i>Interviewer</i>	So, do you think your answer is correct?
<i>Pupil</i>	No.
<i>Interviewer</i>	What do you think you can do?
<i>Pupil</i>	Er..... to divide this white part.....
<i>Interviewer</i>	Ok, please show me.
<i>Pupil</i>	The pupil took sometime to divide the white area into 2 equal parts.
<i>Interviewer</i>	So...now....how many equal parts are there?
<i>Pupil</i>	Three.
<i>Interviewer</i>	How many part is shaded?
<i>Pupil</i>	One part.
<i>Interviewer</i>	Therefore, how do write the answer?
<i>Pupil</i>	The pupil wrote $\frac{1}{3}$
<i>Interviewer</i>	Ok...so the shaded part is.....
<i>Pupil</i>	$\frac{1}{3}$
<i>Interviewer</i>	Is that the correct answer?
<i>Pupil</i>	Yes, I think so.
<i>Interviewer</i>	Ok let us have a look at the next question that you got wrong. Question 7. Can you please read the question?
<i>Pupil</i>	One two over three plus four over seven.
<i>Interviewer</i>	What was your answer?
<i>Pupil</i>	One twenty six over twenty one.
<i>Interviewer</i>	Why do you think that your answer was wrong?
<i>Pupil</i>	(smiling...) because I did not change twenty six over twenty one into mixed number.

- Interviewer* Good...ok can you do it now?
- Pupil* Pupil converted $\frac{26}{21}$ into $1\frac{5}{21}$ and added the whole number 1 to make $2\frac{5}{21}$
- Interviewer* So what is your answer?
- Pupil* Two five over twenty one.
- Interviewer* Do you think your answer is correct or can you change five over twenty one into its lowest term?
- Pupil* My answer is now correct and five over twenty one cannot be changed into its lowest term.
- Interviewer* Good, let us move to another question that you got wrong. Question 8. Can you please read the question.
- Pupil* Four five over nine plus two one over two.
- Interviewer* Good.... What was your answer?
- Pupil* Six one over three.
- Interviewer* Your answer was wrong. Can you please tell me how you got six one over three?
- Pupil* First, I got six six over eighteen. Then I divided it by two and got six three over nine and divided it again by three to get six one over three.
- Interviewer* But...how did you get six over eighteen first?
- Pupil* Er..... er.... Five plus one six and nine times two is eighteen. Er...yes, I was wrong.... I should not add five and one.
- Interviewer* Good...so what should you do now, can you please show me.
- Pupil* The pupil converted $\frac{5}{9}$ into $\frac{10}{18}$ and $\frac{1}{2}$ into $\frac{9}{18}$ added 4 and 2, then wrote $6\frac{19}{18}$.
- Interviewer* So what is your answer now?
- Pupil* $6\frac{19}{18}$
- Interviewer* Look at your answer again, do you think you can do something to it?
- Pupil* Pause..... er..... yes.... I should change $\frac{19}{18}$ into $1\frac{1}{18}$ and add 6.
- Interviewer* Can you please write your answer now?
- Pupil* The pupil wrote $7\frac{1}{18}$.
- Interviewer* That's correct. Can you now see where you went wrong?
- Pupil* Yes... I can.
- Interviewer* Next, can you re do question 10. Read the question for me first.
- Pupil* Three two over seven minus one three over four.
- Interviewer* Alright show me how you do it now.

- Pupil* The pupil first subtracted the whole number one from three and wrote 2. He then worked out the common multiple for seven and four which he finally got 28 and converted $\frac{2}{7}$ into $\frac{8}{28}$ and $\frac{3}{4}$ into $\frac{21}{28}$. Here...he stopped and looking for a cue from me.
- Interviewer* Right, what should you do next?
- Pupil* I cannot subtract 21 from 8.
- Interviewer* Therefore, what should you do now?
- Pupil* Borrow?
- Interviewer* Right, where can you borrow from?
- Pupil* 2
- Interviewer* How many will you borrow and what will happen to 2?
- Pupil* I will borrow one and 2 will become 1.
- Interviewer* Good... so what do you do with the one you borrowed?
- Pupil* Plus 8 to make 18
- Interviewer* Are you sure? Now what is the value of the one whole that you borrowed?
- Pupil* 10
- Interviewer* No it is not 10... (At this point I had to explain the value of 1)
- Pupil* The pupil then wrote $1\frac{28}{28} + \frac{8}{28} - \frac{21}{28}$
- $$= 1\frac{36}{28} - \frac{21}{28} = 1\frac{15}{28}$$
- Interviewer* Good, can we have a look at the next question that you went wrong. Question 14. Read the question, please.
- Pupil* Three over seven times one one over two
- Interviewer* Your answer was three over seven and it was incorrect. Can you tell me how you got your answer?
- Pupil* I changed $1\frac{1}{2}$ into $\frac{2}{2}$ and multiplied it with $\frac{3}{7}$, got $\frac{6}{14}$ and changed it into its lowest term, $\frac{3}{7}$.
- Interviewer* As you see, your answer was incorrect. Can you see where you went wrong?
- Pupil* Pause..... don't know.....
- Interviewer* Ok, can you look at $1\frac{1}{2}$, tell me how you got $\frac{2}{2}$?
- Pupil* 2 times 1 is 2, then 2 times 1 again equal 2 and 2 over 2.
- Interviewer* (At this point, again I had to explain how to convert a mixed number into improper fraction). Right tell me now what is $1\frac{1}{2}$ in its improper fraction form?

- Pupil* 2 times 1 is 2. 2 plus 1 is 3 so $\frac{3}{2}$
- Interviewer* Ok...can you please do the question again?
- Pupil* The pupil wrote $\frac{3}{7} \times \frac{3}{2} = \frac{9}{14}$
- Interviewer* Good, can we move to the next question now? Right... question 19. Read the question, please?
- Pupil* Express 4 kg. as a fraction of 10 kg.
- Interviewer* Show me how you did this question.
- Pupil* The pupil wrote $\frac{4}{10}$ and changed it to $\frac{2}{5}$.
- Interviewer* But your answer before was $\frac{2}{6}$. So what was wrong with your answer before?
- Pupil* Er...er..... I changed 10 into 6 but it should be 5.
- Interviewer* So what is the correct answer now?
- Pupil* $\frac{2}{5}$
- Interviewer* Good.... Next question is question 20. Read the question for me, please.
- Pupil* Express 45 minutes as a fraction of 1 hour. (giving your answer in its lowest terms)
- Interviewer* Right show me how you got this answer (pointing to the answer he wrote before $1\frac{1}{3}$).
- Pupil* The pupils correctly converted 1 hour into 60 minutes and wrote $\frac{45}{60}$, doing the cancellation of the numerator and denominator, wrote $\frac{9}{12}$ and next $\frac{3}{4}$. There, he stopped.
- Interviewer* Where did you get $1\frac{1}{3}$?
- Pupil* (Silent but smiling.....) I divided 4 by 3 but it should not.
- Interviewer* Can you see why you went wrong now? So what is the correct answer?
- Pupil* Yes, I can see now..... The correct answer is $\frac{3}{4}$ hr.
- Interviewer* Ok, let's move to the next question. Question 21, can you read the question?
- Pupil* Arrange the following fractions from the smallest to the largest.
 $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$

- Interviewer* Right... your answer was $\frac{3}{5}, \frac{4}{8}, \frac{1}{3}$. Can you tell me why your answer was wrong?
- Pupil* A long pause..... yes.... This is from largest to smallest. It is the other way round....
- Interviewer* Ok...what should it be then?
- Pupil* $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$
- Interviewer* Ok...now what about the next question. Read the question, please.
- Pupil* Arrange the following fractions from the largest to the smallest $1\frac{1}{10}, \frac{4}{5}, 1\frac{3}{4}$. I think I made the same mistake.....
- Interviewer* What do you mean by the same mistake?
- Pupil* It is the other way round. It should be $1\frac{3}{4}, 1\frac{3}{4}, \frac{4}{5}$.
- Interviewer* Good, I am glad that you could see your mistake. Let's look at the next question. Question 23. Please read the question.
- Pupil* Express 0.85 as a fraction, giving your answer in its lowest terms.
- Interviewer* What was your answer?
- Pupil* $\frac{1}{2}$
- Interviewer* Can you tell me how you got $\frac{1}{2}$?
- Pupil* I don't know..... I was only guessing.
- Interviewer* At this point again, I had to explain how to get the correct answer.
- Pupil* (He looked satisfied and came up with the correct answer, $\frac{17}{20}$).
- Interviewer* Right...may we now move to the next section on word problems? You got two wrong out of four. The first one is question 24b. Can you please read the question for me and tell me if there is any word that you don't know.
- Pupil* A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?
- Interviewer* Good.... Tell me now what the question asked you to do.
- Pupil* Er..... to minus?
- Interviewer* Yes... but what made you think that you have to minus?
- Pupil* Because it says "left" here (pointing to the word in the question)
- Interviewer* But how did you get $1\frac{1}{2}$ as your answer here?
- Pupil* I don't know..... (He seems to have no idea at all and there was no working shown)
- Interviewer* Alright.... Now you know that you have to do subtraction. Can you please show me and go over each step of your working and tell me what you are thinking.

- Pupil* The pupil wrote $2\frac{1}{2} - 1\frac{2}{3}$, worked out the common multiple for 2 and 3 as 6 and converted $2\frac{1}{2}$ into $2\frac{3}{6}$ and $1\frac{2}{3}$ into $1\frac{4}{6}$, thus wrote $2\frac{3}{6} - 1\frac{4}{6}$, then pause.....
- Interviewer* Right what do you have to do next?
- Pupil* I cannot take away 4 from 3 teacher.....
- Interviewer* So, what do you have to do?
- Pupil* Silent.....
- Interviewer* (At this point, I had to explain the concept of borrowing)
- Pupil* The pupil then cancelled 2 and changed it into 1, then wrote $\frac{6}{6} + \frac{3}{6} - \frac{4}{6}$. He further cancelled 1 and 1 indicating that he took away the whole number 1 from 1. Then he wrote $\frac{9}{6} - \frac{4}{6} = \frac{5}{6}$
- Interviewer* Right, therefore what is your answer now?
- Pupil* $\frac{5}{6}$ litres.
- Interviewer* Good... lets now move to the last question that you went wrong. Question 24d. Can you please read the question?
- Pupil* A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?
- Interviewer* What was your answer before?
- Pupil* 9 pieces..... (No working shown)
- Interviewer* You see that your answer was wrong.....Do you know what the question asked you to do?
- Pupil* I don't know teacher.....
- Interviewer* (I had to explain the process in the question, probing him at every stage what to do till he finally solved the problem)

Pupil No. 6

Sample 042, Female (School 1)

- Interviewer* Can you please look at the first question that you went wrong. Right..... question 3. Read the question and can you please tell me how you got the answer.
- Pupil* Change $2\frac{3}{5}$ to an improper question.
- Interviewer* What was your answer before?
- Pupil* $\frac{7}{3}$
- Interviewer* And ... your answer was wrong?
- Pupil* Yes..... it is wrong because I wrote $2\frac{1}{3}$ instead of $2\frac{3}{5}$. (This pupil actually had converted $2\frac{1}{3}$ into $\frac{7}{3}$ correctly but just carelessly rewrote $2\frac{3}{5}$ into $2\frac{1}{3}$ (careless error).
- Interviewer* Right... it is good that you see your mistake now. Can you please do this question, now?
- Pupil* The pupil wrote $2\frac{3}{5}$, then wrote a little multiplication sign between whole number 2 and denominator 5 and a little addition sign between the whole number 2 and numerator 3. Then she converted the fraction into $\frac{13}{5}$ successfully.
- Interviewer* Good... lets move to the next question now...question 19. Read the question, please
- Pupil* Express 4 kg as a fraction of 10 kg.
- Interviewer* How did you get 40 kg as your answer?
- Pupil* Em..... I multiplied 4 and 10 to get the answer.
- Interviewer* Did you understand the question?
- Pupil* Em.... No, teacher.....
- Interviewer* (Teacher explained the question and probed her to write the fractional form)
- Pupil* The pupil wrote $\frac{4}{10}$ and without hesitation, she did the cancellation and converted it to $\frac{2}{5}$
- Interviewer* Good.... Can you do the next question? (The next question, question 20 was in a similar form with qn.19) Read the question first, please.

- Pupil* Express 45 minutes as a fraction of 1 hour (Giving your answer in its lowest terms). (She wrote the answer as $\frac{2700}{60}$ minutes i.e. she multiplied 45 and 60 minutes)
- Interviewer* Show me how you redo this question, now.
- Pupil* (The pupil wrote $\frac{45}{60}$, cancelled 45 and 60 by writing a little 5 besides 45 and 60 and wrote $\frac{3}{4}$ as the answer.
- Interviewer* Good.... Next lets go the word problem. Question 24d) Read the question, please.
- Pupil* A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?
- Interviewer* How did you get $2\frac{1}{2}$ metres as your answer?
- Pupil* Er.... I divided 10 by 4
- Interviewer* But... what did the question want you to find out?
- Pupil* The length of each piece of ribbon.
- Interviewer* What is the length of the ribbon?
- Pupil* 4 metres
- Interviewer* The number of pieces?
- Pupil* 10
- Interviewer* So...what should you do now?
- Pupil* 10 divide by 4 because I could not divide 4 by 10
- Interviewer* Yes...you can.... (At this point I had to explain by drawing a long ribbon and o cut it into 10 small pieces) Now.... Do you think the length of the ribbon will be more or less than 4 metres?
- Pupil* Less
- Interviewer* Right.... Show me now how you can solve the problem.
- Pupil* (The pupil wrote $\frac{4}{10}$, cancelled the numerator and denominator, wrote a little 2 besides 4 and 10 and wrote $\frac{2}{5}$ metres.
- Interviewer* Good.... and thank you.

Pupil No. 30

Sample 292, Female (School 4)

- Interviewer* Can you please look at the first question that you went wrong. Right..... question 6. Read the question and can you please tell me how you got the answer?
- Pupil* Calculate $\frac{13}{15} + \frac{8}{15}$
- Interviewer* What was your answer before?
- Pupil* $\frac{1}{3}$ Oops...my answer was wrong... I thought it was minus.....
- Interviewer* Good... you can see that your answer was wrong. Can you tell me the correct answer, please?
- Pupil* Yes..... it should be $\frac{21}{15}$ equals to $1\frac{6}{15}$ or $1\frac{2}{3}$
- Interviewer* Right... may we now move to the next question that you went wrong, question 14. Can you please read the question and tell me why you went wrong.
- Pupil* Calculate $\frac{3}{7} \times 1\frac{1}{2}$ I was wrong because I converted the fractions into their equivalences. This is multiplication. So I just need to convert $1\frac{1}{2}$ into $\frac{3}{2}$ and multiply with $\frac{3}{7}$.
- Interviewer* Right show me now how you do it.
- Pupil* $\frac{3}{7} \times \frac{3}{2} = \frac{9}{14}$
- Interviewer* Good... now lets go to the word problems that you went wrong... question 24c. Can you read the question please?
- Pupil* Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.
- Interviewer* Your answer before was $6\frac{1}{2}$ kg. Can you tell me how you got the answer?
- Pupil* I added 5 and $1\frac{1}{2}$ and so I got $6\frac{1}{2}$
- Interviewer* But did you try to understand the question first before you did the addition?
- Pupil* Yes..... but when I saw the word total, I thought that I needed to add the numbers.
- Interviewer* Ok now can you read the question again?
- Pupil* (read the question)
- Interviewer* Tell me what the question asked you to do?
- Pupil* To find the total weight of the 5 packets of chicken wing.
- Interviewer* What is the weight of each packet?

- Pupil* $1\frac{1}{2}$ kg
- Interviewer* How many packets did he have altogether?
- Pupil* 5 packets
- Interviewer* (drew 5 packets of chicken wings and wrote $1\frac{1}{2}$ kg on each packet)
- Ok...what operation do you think you need to use?
- Pupil* Multiply?
- Interviewer* Yes, Can you show me now?
- Pupil* (The pupil wrote $5 \times 1\frac{1}{2} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$ kg)
- Interviewer* Good!
- (I explained to her that she also could do addition, provided that she did repeated addition which I showed her soon after).

Pupil No. 10

Sample 052, Female (School 1)

- Interviewer* Can you please look at the first question that you went wrong. Right..... question 1. Read the question and can you please tell me how you got the answer?
- Pupil* What fraction of the figure above is shaded?
- Interviewer* What was your answer before?
- Pupil* $\frac{1}{2}$ Oops...my answer was wrong... I know it is less than half.
- Interviewer* So... what do you think is the correct answer?
- Pupil* (The pupil drew a line in the middle of the white part and wrote $\frac{1}{3}$)
- Interviewer* Good... may we now move to the next question that you went wrong, question 10. Can you please read the question and tell me why you went wrong.
- Pupil* Calculate $3\frac{2}{7} - 1\frac{3}{4}$
- Interviewer* Rightcan you look at your working again and tell me where you went wrong.
- Pupil* (The pupil looked through his working and after a while he noticed what his mistake was) His mistakenly wrote $43 - 28$ as 5 instead of 15, resulting he encoded his final answer as $1\frac{5}{28}$ instead of $1\frac{15}{28}$.
- Interviewer* Right..... the next question that you went wrong was question 11. Please read the question.
- Pupil* $1 - \frac{2}{5}$
- Interviewer* Look at your working, why did you change 1 into $\frac{8}{1}$?
- Pupil* (Smiling..... I don't know teacher...just did it)
- Interviewer* Right you know that your answer was wrong, can you do the question again?
- Pupil* So what is 1 teacher?
- Interviewer* (At this point I had to explain and then let him solve the question. Without much problem he solved the question correctly)
- Pupil* The pupil wrote $1 - \frac{2}{5} = \frac{5}{5} - \frac{2}{5} = \frac{3}{5}$
- Interviewer* Ok... let's go to the next question now, question 17. Can you please read the question?
- Pupil* Calculate $\frac{2}{3}$ of 48
- Interviewer* Look at your working before, can you see why you went wrong?

Appendices

- Pupil* The pupil looked through his working and said.... Yes... I wrongly divided 48 by 3 as 11, it should be 16 and 16×2 is 32.
- Interviewer* Good... next question is no.19. Please look at your answer and tell me why you went wrong.
- Pupil* em..... yes... I didn't change $\frac{4}{10}$ into its lowest term. It should be $\frac{2}{5}$.
- Interviewer* Good the next questions now.... No.21. You wrote that $\frac{4}{8}$ is smaller than $\frac{1}{3}$ and $\frac{1}{3}$ is smaller than $\frac{3}{5}$. Why did you think it that way?
- Pupil* I don't know this teacher.....
- Interviewer* At this point again, I had to demonstrate by drawing three rectangular diagrams to show the fractions size. He paid attention and could see the order now as $\frac{1}{3}$, $\frac{4}{8}$, $\frac{3}{5}$. Right the next question, question 22 is similar to question 21. Can you show me how you will arrange these fractions from the largest to the smallest?
- Pupil* (The pupil drew rectangular figures to represent the given fractions and he successfully arranged the fractions from the largest to the smallest as $1\frac{3}{4}$, $1\frac{1}{10}$, $\frac{4}{5}$)
- Interviewer* Good and thank you.

Pupil No. 42

Sample 307, Male (School 3)

- Interviewer* Can you please look at the first question that you went wrong. Right..... question 1. Read the question and can you please tell me how you got the answer?
- Pupil* What fraction of the figure above is shaded?
- Interviewer* What was your answer before?
- Pupil* $\frac{1}{2}$
- Interviewer* How did you get $\frac{1}{2}$?
- Pupil* Because there are two parts and one part is shaded. So it is $\frac{1}{2}$.
- Interviewer* Ok...now please look at the two parts, are they of the same size?
- Pupil* Em..... no they are not.
- Interviewer* Therefore can you say that the shaded part is a half?
- Pupil* no.....
- Interviewer* Ok... look at the figure again. How many equal parts like the shaded part can you get?
- Pupil* (silent for a while.....) em... I think three teacher.
- Interviewer* Ok... goodcan you please show me?
- Pupil* (The pupil drew a line midway of the white part)
- Interviewer* Right..... can you say what fraction is the shaded part?
- Pupil* Yes..... it is one over three (The pupil wrote $\frac{1}{3}$)
- Interviewer* Good... let's go to the next question.... Question 6. Can you please see why your answer was wrong?
- Pupil* (The pupil took a while.....) I did not change $\frac{21}{15}$ into its lowest term.
- Interviewer* Ok can you do it now?
- Pupil* (Without much problem the pupil wrote $\frac{21}{15} = \frac{7}{5} = 1\frac{2}{5}$)
- Interviewer* Good..... next question is question 7. Can you see why you got it wrong?
- Pupil* (The pupil went through his working and after a while he noticed where he went wrong, he had wrongly subtracted 21 from 26 as 6 and he noticed that it should be 5) Yes.... It should be $2\frac{5}{21}$
- Interviewer* Good the next questions now..... Question 10. Please go through your working and see if you can see where you went wrong.
- Pupil* (The pupil went through his working and but he could not see where he went wrong)

- Interviewer* (At this point I had to point out his mistake and told him to redo the question)
- Pupil* He wrote $3\frac{2}{7} - 1\frac{3}{4} = 2\frac{8}{28} - \frac{21}{28}$ then said.... What should I do now, teacher?
- Interviewer* Right.... You see that you cannot subtract 21 from 8, what did you normally do if you have this situation?
- Pupil* Borrow?
- Interviewer* Right..... where can you borrow from?
- Pupil* From 2?
- Interviewer* Right... carry on now.....
- Pupil* He wrote $\cancel{2}1\frac{8}{28} - \frac{21}{28} = 1\frac{28}{28} + \frac{8}{28} - \frac{21}{28} = 1\frac{36}{36} - \frac{21}{28} = 1\frac{15}{28}$
- Interviewer* Ok... that's good....let's move to the next question, question 14. Can you tell me your mistake here?
- Pupil* (The pupil went through his working and noted his error of overgeneralization of converting the given fractions into their equivalences as in addition and subtraction of fraction with unlike denominators)
- Interviewer* Alright now can you do the question again?
- Pupil* The pupil wrote $\frac{3}{7} \times 1\frac{1}{2} = \frac{3}{7} \times \frac{3}{2} = \frac{9}{14}$
- Interviewer* Yes....that is now a correct answer. The next question is question 15. As you can see here, you have worked out the question correctly as $\frac{1}{2}$. However, you wrote the final answer as $\frac{2}{5}$. Where did you get $\frac{2}{5}$?
- Pupil* (Smiling.....) I don't know teacher.....
- Interviewer* It means you were being careless here. Ok.... Next question now is question 16. You have also shown the correct working but look now where you went wrong.
- Pupil* (He looked at his working and realized that 6×6 is 36 but he wrote 12)
- Interviewer* So, can you tell me the correct answer now?
- Pupil* Yes... it is $\frac{5}{36}$ not $\frac{5}{12}$
- Interviewer* Good.... the next one is question 20. Again you have shown the correct working but can you tell me why your answer $\frac{9}{12}$ was not accepted?
- Pupil* Silent..... emm..... because I didn't change it into its lowest term?
- Interviewer* Right!..... ok can you do it now?

- Pupil* He wrote $\frac{9}{12}$, crossed out 9 and 12 and wrote a little 3 besides the numerator and denominator and finally wrote $\frac{3}{4}$
- Interviewer* Ok...that's better The next question is question 21. Why did you say that $\frac{1}{3}$ is smaller than $\frac{3}{5}$ and smaller than $\frac{4}{8}$
- Pupil* Because I see that 1 is smaller than 3 and 3 is smaller than 4.
- Interviewer* (At this point, I had to draw 3 rectangular figures to represent the three given fractions and let him see the different sizes) Can you see now which fraction is the smallest?
- Pupil* Yes.... $\frac{1}{3}$
- Interviewer* Next should be?
- Pupil* $\frac{4}{8}$
- Interviewer* Good and what is the largest fraction?
- Pupil* $\frac{3}{5}$
- Interviewer* Ok.... You see that even though $1 < 3 < 4$ but in fractional form there are not always the cases. You have to look at the denominators and draw the figures to see the different sizes. Rightnow please write the three fractions in order from the smallest to the largest.
- Pupil* The pupil wrote $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$
- Interviewer* Good... the next question, question 22 is similar to this one. Can you read the question first?
- Pupil* Arrange the following fractions from the largest to the smallest
- $1\frac{3}{4}, 1\frac{1}{10}, \frac{4}{5}$
- Interviewer* Ok...can you do this now?
- Pupil* (The pupil drew five identical rectangular figures to represent the given fractions and he successfully arranged the fractions from the largest to the smallest as $1\frac{3}{4}, 1\frac{1}{10}, \frac{4}{5}$)
- Interviewer* Question 23 now.... You have written the decimal in correct fractional form but can you tell me why your answer was not accepted?
- Pupil* Yes... because I did not write it in its lowest term.
- Interviewer* Good.... please do it now.
- Pupil* He wrote $\frac{85}{100}$, crossed out 85 and 100 and wrote a little 5 besides the numbers..... worked out the division and finally wrote $\frac{17}{20}$

- Interviewer* Ok that is good and now I want you to look at the only word problem that you got wrong. Question 24c. Can you read the question please and if you don't know a word, tell me.
- Pupil* Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg.
Find the total weight of the chicken wings he bought.
- Interviewer* Your answer before was $3\frac{1}{2}$ but you did not show any working. Can you tell me how you got the answer?
- Pupil* Smiling..... I don't know....guessing.....
- Interviewer* Now after you read the question, can you tell me what the question asked you to do?
- Pupil* Yes.... To find the total weight of the 5 packets of chicken wing.
- Interviewer* Ok....tell me what method you should use to find the answer, Why?
- Pupil* Times?
- Interviewer* Ok.....can you try now and show all the workings.
- Pupil* The pupil wrote $5 \times 1\frac{1}{2} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$ kg.
- Interviewer* Tell me what is the answer?
- Pupil* $7\frac{1}{2}$ kg.
- Interviewer* Good and thanks.

Pupil No. 25

Sample 205, Female (School 3)

- Interviewer* Can you please look at the first question that you went wrong? Right..... Question 1. Read the question and tell me how you got the answer, please.....
- Pupil* What fraction of the figure above is shaded?
- Interviewer* What was your answer before?
- Pupil* $\frac{1}{2}$
- Interviewer* How did you get $\frac{1}{2}$?
- Pupil* Because there are two parts and one part is shaded. So it is $\frac{1}{2}$.
- Interviewer* Ok...now please look at the two parts, are they of the same size?
- Pupil* They are not.
- Interviewer* Therefore can you say that the shaded part is a half?
- Pupil* no.....
- Interviewer* Ok... look at the figure again. How many equal parts like the shaded part can you get?
- Pupil* (silent for a while.....) three teacher?
- Interviewer* Ok... goodcan you please show me?
- Pupil* (The pupil drew a line midway of the white part)
- Interviewer* Right..... can you say what fraction is the shaded part?
- Pupil* Yes..... it is one over three (The pupil wrote $\frac{1}{3}$)
- Interviewer* Good... let's go to the next question.... Question 2b. This is a two part question and you got the first part correct. Can you please see why your answer to the second part was wrong?
- Pupil* (The pupil took a while.....) Yes.... I was multiplying $\frac{1}{2}$ with 5 to get $\frac{5}{10}$ and so I think I should multiply $\frac{1}{2}$ with 8 to get 8 over something..... yes it should be 8 over 16 (The pupil then wrote $\frac{8}{16}$)
- Interviewer* Good the next questions now..... Question 10. Please go through your working and see if you can see where you went wrong.
- Pupil* (The pupil went through his working and but he could not see where he went wrong)
- Interviewer* (At this point I had to point out his mistake and told him to redo the question)
- Pupil* He wrote $3\frac{2}{7} - 1\frac{3}{4} = 2\frac{8}{28} - \frac{21}{28}$ then said.... What should I do now, teacher?

- Interviewer* Right.... You see that you cannot subtract 21 from 8, what did you normally do if you have this situation?
- Pupil* Borrow?
- Interviewer* Right..... where can you borrow from?
- Pupil* From 2?
- Interviewer* Right... carry on now.....
- Pupil* He wrote $\cancel{21} \frac{8}{28} - \frac{21}{28} = 1 \frac{28}{28} + \frac{8}{28} - \frac{21}{28} = 1 \frac{36}{28} - \frac{21}{28} = 1 \frac{15}{28}$
- Interviewer* Ok... that's good....let's move to the next question, question 14. Can you tell me your mistake here?
- Pupil* (The pupil went through his working and noted his error of overgeneralization of converting the given fractions into their equivalences as in addition and subtraction of fraction with unlike denominators)
- Interviewer* Alright, now can you do the question again?
- Pupil* The pupil wrote $\frac{3}{7} \times 1 \frac{1}{2} = \frac{3}{7} \times \frac{3}{2} = \frac{9}{14}$
- Interviewer* Yes....that is now a correct answer. The next question is question 15. As you can see here, you have worked out the question correctly as $\frac{1}{2}$. However, you wrote the final answer as $\frac{2}{5}$. Where did you get $\frac{2}{5}$?
- Pupil* (Smiling.....) I don't know teacher.....
- Interviewer* It means you were being careless here. Ok.... Next question now is question 16. You have also shown the correct working but look now where you went wrong.
- Pupil* (He looked at his working and realised that 6×6 is 36 but he wrote 12)
- Interviewer* So, can you tell me the correct answer now?
- Pupil* Yes... it is $\frac{5}{36}$ not $\frac{5}{12}$
- Interviewer* Good.... the next one is question 20. Again you have shown the correct working but can you tell me why your answer $\frac{9}{12}$ was not accepted?
- Pupil* Silent..... emm..... because I didn't change it into its lowest term?
- Interviewer* Right!..... ok can you do it now?
- Pupil* He wrote $\frac{9}{12}$, crossed out 9 and 12 and wrote a little 3 besides the numerator and denominator and finally wrote $\frac{3}{4}$
- Interviewer* Ok...that's better The next question is question 21. Why did you say that $\frac{1}{3}$ is smaller than $\frac{3}{5}$ and smaller than $\frac{4}{8}$
- Pupil* Because I see that 1 is smaller than 3 and 3 is smaller than 4.
- Interviewer* (At this point, I had to draw 3 rectangular figures to represent the three given fractions and let him see the different sizes) Can you see now which fraction is the smallest?

- Pupil* Yes.... $\frac{1}{3}$
- Interviewer* Next should be?
- Pupil* $\frac{4}{8}$
- Interviewer* Good and what is the largest fraction?
- Pupil* $\frac{3}{5}$
- Interviewer* Ok.... You see that even though $1 < 3 < 4$ but in fractional form there are not always the cases. You have to look at the denominators and draw the figures to see the different sizes. Rightnow please write the three fractions in order from the smallest to the largest.
- Pupil* The pupil wrote $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$
- Interviewer* Good... the next question, question 22 is similar to this one. Can you read the question first?
- Pupil* Arrange the following fractions from the largest to the smallest
 $1\frac{3}{4}, 1\frac{1}{10}, \frac{4}{5}$
- Interviewer* Ok...can you do this now?
- Pupil* (The pupil drew five identical rectangular figures to represent the given fractions and he successfully arranged the fractions from the largest to the smallest as $1\frac{3}{4}, 1\frac{1}{10}, \frac{4}{5}$)
- Interviewer* Question 23 now.... You have written the decimal in correct fractional form but can you tell me why your answer was not accepted?
- Pupil* Yes... because I did not write it in its lowest term.
- Interviewer* Good.... please do it now.
- Pupil* He wrote $\frac{85}{100}$, crossed out 85 and 100 and wrote a little 5 besides the numbers..... worked out the division and finally wrote $\frac{17}{20}$
- Interviewer* Ok that is good and now I want you to look at the only word problem that you got wrong. Question 24c. Can you read the question please and tell me if you don't know any word.
- Pupil* Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.
- Interviewer* You answer before was $6\frac{1}{2}$ kg. and it was incorrect. Now after you read the question, can you tell me what the question asked you to do?
- Pupil* Yes.... To find the total weight of the 5 packets of chicken wing.
- Interviewer* Now tell me what method you need to use to find the answer, why?
- Pupil* It should not be addition? Em..... I think it should be multiplication.

Appendices

- Interviewer* Ok.....can you go over each step of your working and tell me what you are thinking.
- Pupil* The pupil wrote $5 \times 1\frac{1}{2} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$ kg.
- Interviewer* Tell me what the answer to the question is, can you point to your answer?
- Pupil* $7\frac{1}{2}$ kg.
- Interviewer* That's good and thanks.

Appendix F

Pupils' Interview Transcripts (Post-test)

Pupil No. 30

Sample 292, Female (School 4)

- Interviewer* Can you please look at the first question that you went wrong? Right..... Question 6. Read the question and tell me how you got the answer, please.....
- Pupil* Calculate $\frac{13}{15} + \frac{8}{15}$, giving your answer in its lowest terms.
- Interviewer* What was your answer before?
- Pupil* $\frac{7}{5}$
- Interviewer* Can you tell me how you got $\frac{7}{5}$?
- Pupil* Em..... the two fractions are of the same denominators, so I just add 13 and 8 makes 21. So it is $\frac{21}{15}$ and change to $\frac{7}{5}$
- Interviewer* Ok... but can you tell me why $\frac{7}{5}$ was not accepted as a correct answer?
- Pupil* Silence.....
Aa.... Yes because I did not change it into mixed number.
- Interviewer* Yes... good. So can you do it now?
- Pupil* The pupils wrote $1\frac{2}{5}$
- Interviewer* Good, Can we look at the next question. Can you please read the question and tell me why your answer to was not correct?
- Pupil* Calculate $1\frac{2}{3} + \frac{4}{7}$. My answer was $1\frac{26}{21}$
- Interviewer* Ok... you have shown the correct working. Now tell me why $1\frac{26}{21}$ was not accepted?
- Pupil* Same as the last question. I did not change $\frac{26}{21}$ into mixed number.
- Interviewer* Right.... Can you write the correct answer now?
- Pupil* $1 + 1\frac{5}{21} = 2\frac{5}{21}$
- Interviewer* Good... let's go to the next question.... Question 8. Read the question, please.
- Pupil* Calculate $4\frac{5}{9} + 2\frac{1}{2}$

- Interviewer* Like the previous question, you have shown the correct working but somehow your answer $6\frac{1}{18}$ was incorrect, can you tell me why?
- Pupil* Pause..... Yes... actually I changed $\frac{19}{18}$ to $1\frac{1}{18}$ but I forgot to add 1 to 6. It should be $7\frac{1}{18}$ but not $6\frac{1}{18}$.
- Interviewer* Ok... that's very good. The last question that you went wrong was Question 24. Can you please read the question?
- Pupil* Express 65% as a fraction in its simplest form.
- Interviewer* What was your answer?
- Pupil* $\frac{65}{100}$
- Interviewer* Why do you think I did not accept that answer?
- Pupil* Because I did not change it into its simplest form.
- Interviewer* Good..... can you show me it simplest form now?
- Pupil* The pupil wrote $\frac{13}{20}$
- Interviewer* Good and thanks.

Pupil No. 25

Sample 205, Female (School 3)

- Interviewer* Can you please look at the first question that you went wrong? Right..... question 8. Read the question and tell me how you got the answer, please.....
- Pupil* Calculate $4\frac{5}{9} + 2\frac{1}{2}$
- Interviewer* What was your answer before?
- Pupil* $6\frac{6}{11}$
- Interviewer* Can you please tell me how you got that answer?
- Pupil* I added 4 and 2 to get 6, then 5 and 1 to get 6 and 9 and 2 to get 11.
- Interviewer* Alright.... But now you see that your answer was wrong. Can you tell me why?
- Pupil* Silence.....
- Interviewer* You cannot find your mistake?
- Pupil* No.
- Interviewer* Ok..... Let me now tell you why you went wrong. You see that you are adding fractions but not whole numbers. So you cannot treat addition of fractions as like addition of whole numbers. Ok... can you please write the question again?
- Pupil* The pupil wrote $4\frac{5}{9} + 2\frac{1}{2}$
- Interviewer* Ok... right you see that 4 and 2 are whole numbers. So you can add them up. What do you get?
- Pupil* 6
- Interviewer* Right now you need to add $\frac{5}{9} + \frac{1}{2}$ but you cannot add them right away. You see that these two fractions are of different denominators. So you need to change them into their equivalent fractions so that both fractions will have same denominators. To do that you need to think of a common multiple for 9 and 2. Can you find one?
- Pupil* Silence..... and tried to scribble something on the paper. 18?
- Interviewer* Yes.....that's right. How did you get that? 9
- Pupil* 18 is the smallest number that can be divided by both 9 and 2.
- Interviewer* Good... right can you change the two fractions into their equivalences?
- Pupil* The pupil wrote $\frac{5}{9} \times 2 = \frac{10}{18}$ and $\frac{1}{2} \times 2 = \frac{9}{18}$,
then she wrote $\frac{10}{18} + \frac{9}{18}$
- Interviewer* Ok... now you see that they have the same denominators. So you can add the numerators but keep the denominators as it is. What is the answer now?

Pupil $\frac{19}{18} = 1\frac{1}{18} + 6 = 7\frac{1}{18}$

Interviewer Ok. That's very good. Remember what I told you that you cannot add fractions like adding whole numbers. Can we move to the next questions now? Right Question 10. Can you read the question, please?

Pupil Calculate $3\frac{2}{7} - 1\frac{3}{4}$

Interviewer Your answer before was $4\frac{1}{11}$ because you were adding 3 and 1 to get 4, 7 and 4 to get 11 and then you subtracted 3 and 2 to get 1. You notice that it was incorrect. Right using what I showed you in the previous question, can you do the question again, please?

Pupil The pupil wrote $3\frac{2}{7} - 1\frac{3}{4}$. She subtracted 1 from 3 to get 2. She thought

for a while and then wrote $\frac{2}{7} \times 4 = \frac{8}{28}$, $\frac{3}{4} \times 7 = \frac{21}{28}$.

Then she wrote $2\frac{8}{28} - \frac{21}{28}$ Then she kept silent.....

Interviewer Right, what do you think you can do now?

Pupil Borrow from 2?

Interviewer Yes... that's right. Carry on....

Pupil The pupil wrote $2\frac{8}{28} - \frac{21}{28}$, crossed 2 and wrote

$$1\frac{28}{28} + \frac{8}{28} - \frac{21}{28}$$

$$1\frac{36}{28} - \frac{21}{28} = 1\frac{15}{28}$$

Interviewer Good.... do you think you can change it into its simplest form?

Pupil Think for a while and said "No"

Interviewer Ok... good... let's have a look at the next question.... Question 14. Can you please read the question?

Pupil $\frac{3}{7} \times 1\frac{1}{2}$

Interviewer Ok.. what was your answer before?

Pupil $\frac{6}{14}$

Interviewer Right, look at your working and can you please tell me why your answer was wrong?

Pupil The pupil looked at her working which she wrote as $\frac{3}{7} \times \frac{3}{2} = \frac{6}{14}$.

After a while she realized her mistake and said

3×3 is 9 not 6.

Interviewer Correct. So can you write the correct answer now?

- Pupil* She wrote $\frac{9}{14}$
- Interviewer* That's good. May we look at the next question that you went wrong? Ok... Question 15. This time it is on multiplication. Read the question, please.
- Pupil* $\frac{2}{5} \div \frac{4}{5}$
- Interviewer* Right, your working and answer before was $\frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$ and you can see that it is incorrect. Can you see where your mistake is?
- Pupil* Yes..... because I forgot to invert $\frac{4}{5}$ into $\frac{5}{4}$
- Interviewer* Ok... can you do the question again, please?
- Pupil* The pupil wrote $\frac{2}{5} \div \frac{4}{5} = \frac{2}{5} \times \frac{5}{4}$
(She then cancelled 2 and 4, wrote 1 next to 2 and wrote 2 next to 4 and cancelled 5 and 5 and wrote 1 next to both the 5s.
Finally she wrote $\frac{1}{2}$ as the answer.
- Interviewer* That's good. Ok... the next question is Question 17. Read the question, please.
- Pupil* $\frac{2}{3}$ of 48
- Interviewer* Right your working was correct but your answer 24 was wrong. Can you spot your mistake?
- Pupil* She looked at her answer and after a while she noticed it and said 48 divided by 3 is 16 but not 14.
- Interviewer* Ok.. good. So what should it be now.
- Pupil* The pupil wrote $\frac{2}{3} \times 48$, canceling 3 and 48. She wrote 1 besides 3 and 16 besides 48. She finally wrote 32.
- Interviewer* That' good. Ok... can we move to the next question, now? Right... Question 19. Can you please read the question?
- Pupil* Express 4 kg as a fraction of 10 kg.
- Interviewer* Your answer $\frac{4}{10}$ was partly correct but it is not accepted. Why do you think so?
- Pupil* Think for a while.....
Er.... I have to write in its simplest form?
- Interviewer* Yes.... You are right. Can you do it now?
- Pupil* The pupil wrote $\frac{4}{10}$, canceling 4 and 10. She wrote 2 besides 4 and 5 besides 10. She then wrote $\frac{2}{5}$.
- Interviewer* Good, next question now..... Question20. Please read the question for me.

- Pupil* Express 45 minutes as a fraction of 1 hour (Giving your answer in its lowest terms).
- Interviewer* Your answer was $\frac{1}{2}$ and you did not show any working. Can you tell me how you got the answer?
- Pupil* (Smiling..... but quiet.....) I don't know, I just guessed the answer.
- Interviewer* Alright..... you know that your answer was wrong. Let me tell you how to do this question. You see that the question asked you to express 45 minutes as a fraction of 1 hour. So first of all you must change 1 hour into minutes. Do you know how many minutes make one hour?
- Pupil* Yes.... 60 minutes.
- Interviewer* Good... right so now, both units are in minutes. Can you now write them in fraction form?
- Pupil* The pupil wrote $\frac{45}{60}$
- Interviewer* That's good..... what else can you do with it?
- Pupil* Change into its lowest terms?
- Interviewer* Yes.... good
- Pupil* The pupil cancelled 45 and 60, thought for a while and wrote 9 besides 45 and 12 besides 60. Then she wrote $\frac{9}{12}$.
- Interviewer* Do you think that is the answer or can you still change it into its simplest forms?
- Pupil* Silence..... But after a while she cancelled 9 and 12 and wrote 3 besides 9 and 4 besides 12. Then she wrote $\frac{3}{4}$ hrs.
- Interviewer* That is very good..... next question is Question 24. Can you please read the question?
- Pupil* Express 65% as a fraction in its simplest form.
- Interviewer* Ok... your answer was $\frac{65}{100}$ but I did not accept it as correct. Do you know why?
- Pupil* Because I did not write the answer in its simplest form.
- Interviewer* That's right. Can you do it now?
- Pupil* The pupil wrote $\frac{65}{100}$, cancelled 65 and 100. She wrote table 5 on the side of the paper until she got 100. Then she wrote 13 besides 65 and 20 besides 100. Finally she wrote $\frac{13}{20}$.
- Interviewer* Do you think you can still cancel the fraction?
- Pupil* No.
- Interviewer* Ok... right so that is the answer for this question. The next question is Question 26a. You see that you had it wrong. Can you read the question to me and tell me any words that you don't know.

- Pupil* A bag of sugar weighs $\frac{3}{4}$ kg and another bag of sugar weighs $1\frac{1}{2}$ kg. What is the total weight of the bags?
- Interviewer* Now after you read the question, can you tell me what the question asked you to do?
- Pupil* Yes.... To find the total weight of the 2 bags of sugar.
- Interviewer* Ok....what operation do you think you need to use?
- Pupil* Addition?
- Interviewer* Ok.....can you try now and show all the steps.
- Pupil* The pupil wrote $\frac{3}{4} + 1\frac{1}{2}$, changed $1\frac{1}{2}$ to $1\frac{2}{4}$
- $$= \frac{3}{4} + 1\frac{2}{4} = 1\frac{5}{4}$$
- Interviewer* Ok.. that is good, what else can you do with that?
- Pupil* The pupil wrote $1\frac{5}{4} = 2\frac{1}{4}$ kg.
- Interviewer* Can you point to the answer?
- Pupil* Pointed at $2\frac{1}{4}$ kg.
- Interviewer* Yes.. that is very good. So your answer is $2\frac{1}{4}$ kg but not $1\frac{1}{4}$ which you did before. Can you see where your mistake now is?
- Pupil* Yes... I forgot to add the 1 to $1\frac{1}{4}$ to make $2\frac{1}{4}$.
- Interviewer* Yes... good. Next question is 26 b). Can you please read the question?
- Pupil* A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?
- Interviewer* Do you understand what the question asked you to do?
- Pupil* (The pupil read the question again quietly....) Yes.... To find out how much orange juice is left after being drunk.
- Interviewer* Ok... that's right. So what operation do you need to do now?
- Pupil* Subtraction?
- Interviewer* Ok.... That's right. Can you please show me the operation now?
- Pupil* The pupil wrote $2\frac{1}{2} - \frac{2}{3}$.
- Interviewer* What do you need to do next?

- Pupil* The pupil wrote number 6 besides the fractions, then wrote

$$2\frac{3}{6} - \frac{4}{6},$$
crossed 2 and wrote 1 besides it, then she wrote

$$= 1\frac{6}{6} + \frac{3}{6} - \frac{4}{6}$$

$$= 1\frac{9}{6} - \frac{4}{6} = 1\frac{5}{6} \text{ litres.}$$
- Interviewer* Point to the answer, please.
- Pupil* The pupil pointed at $1\frac{5}{6}$ litres
- Interviewer* Ok... that is very good. May we go to the last question that you went wrong now... that is Question 26d). Can you please read the question?
- Pupil* A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?
- Interviewer* Ok... your answer before was $2\frac{2}{4}$ and you were dividing 10 by 4 to get the answer. Now can you tell me why you divided 10 by 4?
- Pupil* Because 10 is bigger than 4 and I don't think I can divide 4 by 10 to get the answer.
- Interviewer* Ok... read the question again to your self and tell me what are 4 and 10 for in the question.
- Pupil* (The pupil read the question to herself quietly.....)
4 metres is the length of the ribbon and 10 is the number of pieces.
- Interviewer* Ok... so you already knew that the operation you need to use is division. What do you need to divide?
- Pupil* (Quiet.....) the length of the ribbon by the number of pieces.
- Interviewer* Alright..... can you show it in fraction form now?
- Pupil* The pupil wrote $\frac{4}{10}$, crossed 4 and 10 and wrote 2 besides 4 and 5 besides 10. Finally she wrote $\frac{2}{5}$ metres.
- Interviewer* Can you please point to the answer?
- Pupil* The pupil pointed $\frac{2}{5}$ metres.
- Interviewer* Very good..... and I hope you will now not forget what you have done with the questions that you went wrong in this test. Thanks.

Pupil No. 22

Sample 189, Male (School 3)

- Interviewer* Can you please look at the first question that you went wrong? Right..... Question 7. Read the question and tell me how you got the answer, please.....
- Pupil* Calculate $1\frac{2}{3} + \frac{4}{7}$
- Interviewer* What was your answer before?
- Pupil* $1\frac{20}{21}$
- Interviewer* Can you please tell me how you got that answer?
- Pupil* I don't know.... May be I multiplied 3 and 7 and I got 21..... emm.... I don't know teacher.....
- Interviewer* Ok...Can you try it once more now? What do you need to do with the fractions?
- Pupil* Plus them?
- Interviewer* Yes.... But can you add them right away?
- Pupil* (Silence.....) No..... find the LCM of 3 and 7?
- Interviewer* Yes.... Do you know how to find the LCM of 3 and 7?
- Pupil* The pupil wrote 3,7.... Think quietly then wrote 3,6,9,12,15,18,21,24 then wrote 7,14,21,28. She then circled 21
- Interviewer* Ok... so why did you circle 21?
- Pupil* It is the LCM for 3 and 7.
- Interviewer* Ok... next what do you need to do?
- Pupil* Change 3 and 7 to 21.
- Interviewer* Ok... Can you please show me now?
- Pupil* The pupil wrote $\frac{2}{3} \times 7$ and $\frac{4}{7} \times 3$

$$= 1\frac{14}{21} + \frac{12}{21} = 1\frac{26}{21}$$
- Interviewer* Ok... that's right, what else can you do now?
- Pupil* Change $\frac{26}{21}$ into mixed number?
- Interviewer* Yes..... carry on, please.
- Pupil* The pupil wrote $1 + 1\frac{5}{21} = 2\frac{5}{21}$.
- Interviewer* Ok... that is correct now. Ok... let's have a look at the next question that you went wrong. Right.... Question 10. I think you had the same error here as before. Can you try the question once more?

Pupil

The pupil wrote $3\frac{2}{7} - 1\frac{3}{4}$

Slowly and carefully he wrote 2 then $\frac{2}{7} \times 4$ and $\frac{3}{4} \times 7$. He then wrote

$$= 2\frac{8}{28} - \frac{21}{28}$$

He then crossed 2, wrote 1 besides 2 and then wrote

$$= 1\frac{28}{28} + \frac{8}{28} - \frac{21}{28}$$

$$= 1\frac{36}{28} - \frac{21}{28}$$

$$= 1\frac{15}{28}$$

Interviewer

Ok... do you think that is the answer or can you change it into its lowest terms?

Pupil

Think for a while..... I think 15 and 28 cannot be divided by a same number. So the answer is $1\frac{15}{28}$

Interviewer

Yes, very good. Can we try the next question now.... Right.. Question 14. Your answer before was $1\frac{6}{7}$, but you did not show any working how you got the answer. Can you do the question again now?

Pupil

The pupil wrote the question $\frac{3}{7} \times 1\frac{1}{2}$. Then he asked me if she needed to change $1\frac{1}{2}$ into improper fraction.

Interviewer

Yes... you need to before you multiply them.

Pupil

Then she wrote $\frac{3}{7} \times \frac{3}{2}$. She kept quiet.....

Interviewer

What next you should do now?

Pupil

I cannot cancel any numbers now, Can I multiply them?

Interviewer

Right..... carry on.....

Pupil

She multiplied the fractions and wrote $\frac{9}{14}$ besides the fractions.

Interviewer

That's correct, very good. Next question is Question 16, that is $\frac{5}{6} \div 6$. Your answer was $\frac{5}{12}$. How did you get that?

Pupil

(Smiling..... Could be because he noticed her mistake)
I added 6 and 6 to get 12 and put 5 as the numerator.

Interviewer

Ok... so you see that your answer was wrong. Can you try it again now?

- Pupil* The pupil wrote $\frac{5}{6} \div 6$
- Interviewer* Ok... what does 6 mean here?
- Pupil* $\frac{6}{1}$
- Interviewer* Good.... carry on now
- Pupil* The pupil wrote $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$
- Interviewer* Yes... that's correct, the next question is Question 21.
- Pupil* Arrange the following fractions from the smallest to the largest. $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$
- Interviewer* Right... your answer was $\frac{1}{3}, \frac{3}{5}, \frac{4}{8}$. Can you tell me why you wrote this as the answer?
- Pupil* Because I looked at the denominators, 3 is the smallest and 8 the largest.
- Interviewer* But you see now that it is not the denominators which determine the size of the fractions. Right can you draw 3 rectangles of the same size now?
- Pupil* The pupil drew 3 rectangles.
- Interviewer* Can you divide the first rectangle into 3 equal parts and shade one part. Divide the second rectangle into 5 equal parts and shade 3 parts and divide the third rectangle into 8 equal parts and shade four parts.
- Pupil* (The pupil did the activity with my guidance)
- Interviewer* Right..... can you look carefully at the 3 fractions now, look at the shaded parts. Can you see now which of them is the smallest?
- Pupil* Yes.... $\frac{1}{3}$
- Interviewer* Next should be?
- Pupil* $\frac{4}{8}$
- Interviewer* Good... and the largest is
- Pupil* $\frac{3}{5}$
- Interviewer* Om... can you write them from the smallest now?
- Pupil* The pupil wrote $\frac{1}{3}, \frac{4}{8}, \frac{3}{5}$
- Interviewer* Good, now the next question is similar to this except that you need to arrange them in descending order. Can you read the question first?
- Pupil* Arrange the following fractions from the largest to the smallest.
- $1\frac{1}{10}, \frac{4}{5}, 1\frac{3}{4}$
- Interviewer* Right, do you want to draw 3 rectangles like before or can you now see which fraction is the largest?
- Pupil* I think $1\frac{3}{4}$ is the largest.

- Interviewer* That's good.... how did you know that?
- Pupil* Because both $1\frac{1}{10}$ and $1\frac{3}{4}$ but $\frac{1}{10}$ is so little compared to $\frac{3}{4}$. So $1\frac{3}{4}$ is the largest, next is $1\frac{1}{10}$ and $\frac{4}{5}$ is the smallest.
- Interviewer* That's correct. Can you write the 3 fractions from the largest now?
- Pupil* $1\frac{3}{4}, 1\frac{1}{10}, \frac{4}{5}$
- Interviewer* Very good. The next question that you went wrong was Question 24. Can you read the question, please?
- Pupil* Express 65% as a fraction in its simplest form.
- Interviewer* Your answer was $\frac{65}{100}$ and I did not accept it as correct. Do you know why?
- Pupil* Quiet for while..... because it is not in its simplest form?
- Interviewer* Yes... you are right. So can you improve your answer now?
- Pupil* The pupil crossed 65 and 100, thought for a while and then wrote 13 besides 65 and 20 besides 100. He then wrote $\frac{13}{20}$.
- Interviewer* Ok.. good. The next question is 26 a) Can you read the question, please.
- Pupil* A bag of sugar weighs $\frac{3}{4}$ kg and another bag of sugar weighs $1\frac{1}{2}$ kg. What is the total weight of the bags?
- Interviewer* Now after you read the question, can you tell me what the question asked you to do?
- Pupil* Yes.... To find the total weight of the 2 bags of sugar.
- Interviewer* Ok....what operation do you think you need to use?
- Pupil* I think it should be addition because I need to find the total weight of the 2 bags of sugar.
- Interviewer* Ok... good, can you show the operation now?
- Pupil* The pupil wrote $\frac{3}{4} + 1\frac{1}{2}$. Then he changed $\frac{1}{2}$ to $\frac{2}{4}$ and wrote $\frac{3}{4} + 1\frac{2}{4} = 1\frac{5}{4} = 2\frac{1}{4}$ kg.
- Interviewer* That's correct. The last question now is Question 26d). Can you please read the question?
- Pupil* A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?
- Interviewer* Ok... your answer before was $2\frac{2}{4}$ and you were dividing 10 by 4 to get the answer. Now can you tell me why you divided 10 by 4?
- Pupil* Because 10 is bigger than 4 and so I divided 10 by 4 to get the answer.
- Interviewer* Ok... read the question again to your self and tell me what are 4 and 10 for in the question.
- Pupil* (The pupil read the question to herself quietly.....)
4 metres is the length of the ribbon and 10 is the number of pieces.

Appendices

- Interviewer* Ok... so you already knew that the operation you need to use is division.
What do you need to divide?
- Pupil* Divide 4 by 10?
- Interviewer* Alright..... can you show all the steps of your working now?
- Pupil* The pupil wrote $\frac{4}{10}$, crossed 4 and 10 and wrote 2 besides 4 and 5 besides 10. Finally she wrote $\frac{2}{5}$ metres.
- Interviewer* Can you point to the answer?
- Pupil* The pupil pointed at $\frac{2}{5}$ metres.
- Interviewer* Yes..... that's correct now. Thanks.