Science and Mathematics Education Centre

A Case Study of the Use of Technology in Secondary Mathematics with Reference to the Dimensions of Learning Framework

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This thesis is presented for the degree of

Doctor of Mathematics Education

of

Curtin University

March 2012

Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signed: Ian Thomson

Date: 11/2/12

Abstract

In this thesis a case study was conducted which concerned the use of technology in secondary mathematics. Within a particular setting, the research evaluates the contribution that the use of technology makes to the process of learning mathematics and to students' perceptions about learning mathematics.

In the study, Year 8 students used technology to work on problems involving algebra and geometry. The students had individual access to touch screen calculators. A tablet computer attached to a digital projector was available for use in the classroom. The research was carried out by a teacher working as a participant-researcher. The students involved in the research were drawn from a group who had been selected to take part in mathematics enrichment lessons. They were extracted from their usual mathematics classes for one lesson per week over a four month period for this purpose.

The study took place in a school which had committed itself to two main initiatives. One of these initiatives was to introduce technology into teaching and learning. The other initiative was to upgrade the teaching practices at the school by adopting the Dimensions of Learning framework as a pedagogical model. The tasks that the students were given to work on in the study were selected to align with different aspects of the Dimensions of Learning framework. The analysis used the Dimensions of Learning framework as a reference tool for both the design of the tasks and the outcomes of implementing them with the students.

It was found that the use of technology could be productively aligned with procedural ("know how") knowledge and declarative ("know what") knowledge. The contribution that the use of technology made to complex reasoning processes associated with extending knowledge and using knowledge meaningfully was also evaluated. The students' feelings about the use of technology were sometimes positive and sometimes negative but they had stronger feelings about other aspects of their experience in learning mathematics aside from the use of technology.

Acknowledgements

I wish to thank most sincerely my supervisor Associate Professor Bill Atweh, of the Science and Mathematics Education Centre of Curtin University, for his advice and encouragement throughout this project. Additionally, I would like to acknowledge and thank Dr Christine Howitt, Dr John Malone and Dr Tony Rickards for their guidance and support in the coursework that preceded this project. I would also like to thank my wife Cathy and sons Mark and Greig for their encouragement and unwavering support.

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CHAPTER 1 INTRODUCTION

The Rationale

As a secondary school mathematics teacher, I am keen to find ways to improve the learning and teaching process in my classroom. The use of technology in mathematics education is a recognised way of enhancing the learning experience. The presence of technology motivates students and provides a means of producing alternative representations of mathematical concepts. The availability of technology in mathematics education also supports enquiry based learning. In my professional capacity as a mathematics teacher, therefore, I have a desire to take advantage of technology enhanced teaching and learning methods. Exploring the use of technology in the mathematics classroom is significant to both my professional activities and my educational research.

The Focus of the Research

This thesis explores the use of technology in secondary mathematics. It focuses on the contribution that technology makes to students' learning in mathematics. This includes the ways that students acquire mathematical knowledge as well as the perceptions that they hold about learning mathematics. The use of technology is explained by associating it with a well-structured pedagogical model.

Problems have been encountered throughout the world in integrating technology into education. These problems indicate that considerable thought should be invested in finding ways to use technology appropriately in education. This thesis sheds light on this issue by conducting a case study of the use of technology in a specific classroom context embedded within the setting of a particular school. This is achieved by first of all providing an account of the circumstances and events that led to the increased use of technology at the school, and then focusing in to a micro level to examine an example of how technology was used in the classroom.

The Case Study

The research in this project is carried out by examining the use of technology in a mathematics classroom setting. Year 8 mathematics students were introduced to new technology in the form of a powerful calculator. The calculator, known as a ClassPad (CASIO Computer Co., 2011), has a touch screen and is operated by the use of a stylus. The eighteen students involved in the case study were extracted from their normal mathematics classes for one lesson per week. During these lessons the students worked on mathematics enrichment materials and used the calculators for activities in algebra, geometry and problem solving. The data for the research came in the form of lesson transcriptions of audio recordings, teacher reflections, semi-structured interviews and screen capture recordings of students working on a tablet computer. The analysis is carried out using qualitative techniques.

A series of mathematical activities (see appendices) which involved the use of technology was prepared for the study and given to students to work on in class. The outcomes of this form the central focus of the research, with the researcher adopting the dual role of researcher and teacher/practitioner. The context within which the research took place will be described in greater detail in Chapter 2.

Ethical considerations

Ethical aspects of the research were considered carefully in accordance with the guidelines and requirements of the Curtin University Human Research Ethics Committee. Approval for the research to take place was granted from this committee. A copy of the memorandum confirming this (Approval Number SMEC-05-09) can be found in the appendices. The Head of the Senior School, students and parents were fully informed of the purpose of the research. All of these people were provided with an information sheet that clearly stated the purpose of the research and signed consent forms were gathered from all of them. A copy of the information sheet and the consent form template are included in the appendices. In addition to being provided with an information sheet about the research, the purpose of the research was also explained to the students verbally. The students had the right to withdraw from the research at any time without any negative consequences. The school was named in the thesis. Signed consent for this was obtained from the Headmaster. The anonymity of the students, however, was protected through the use of pseudonyms.

The research was planned in such a way that the students who were subjects of the research would not have their progress in the Year 8 Mathematics course disrupted. This was achieved in two ways. First, only students who were able to stay ahead of the Year 8 Mathematics course were invited to take part in the research. Second, the students participating in the research were still doing and learning mathematics, although the focus of the lessons that they attended was more directed towards problem-solving and the use of technology. No aspect of the research was used to determine school grades. The process of selecting students is described more fully in Chapter 4 along with more details about the mathematics tasks that were prepared for them to work on.

The Aims

The overall aim of the research is to investigate ways to improve the learning of mathematics when technology is used in the classroom. More specifically this study, using the Dimensions of Learning framework as a reference tool, aims to investigate:

- the contribution that the use of technology makes to students' perceptions about learning mathematics
- the contribution that the use of technology makes to the learning process

The Pedagogical Model

The pedagogical model that is used in the thesis is the Dimensions of Learning model (Marzano, 2008). The features of this model are described in the outline below. A detailed description of the Dimensions of Learning model and its relevance to mathematics education is given in the Literature Review.

An outline of the Dimensions of Learning framework

Dimensions of Learning is a practical learning-based model of instruction (Marzano, 1992). It originated from research at the Association for Supervision and Curriculum Development in Alexandria in the United States. It is a development of a theoretical model known as Dimensions of Thinking (Marzano et al., 1988). The Dimensions of Learning framework is widely used in Queensland, Australia in both public and private schools across all subject areas. It is also used as the main model for learning at Central Queensland University. The Dimensions of Learning model provides a framework for learning which teachers can use to inform their teaching methods, plan courses and develop assessment tasks. The five dimensions which make up the learning framework are: (1) attitudes and perceptions; (2) acquiring and integrating knowledge; (3) extending and refining knowledge; (4) using knowledge meaningfully; and (5) habits of mind. The Dimensions of Learning framework was developed to improve teaching and learning in all subject areas.

The five dimensions of the Dimensions of Learning framework can be illustrated in a diagram as shown overleaf:

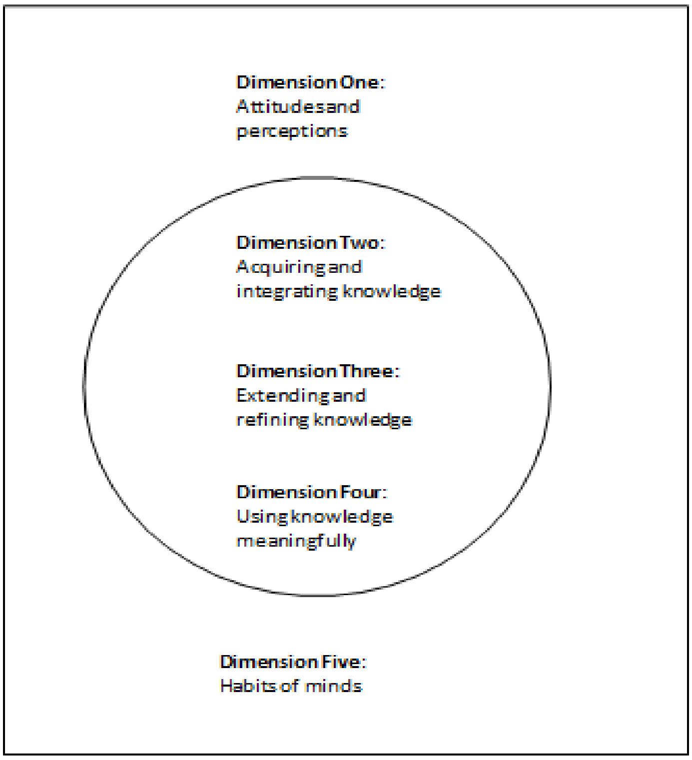


Figure 1: The Dimensions of Learning Framework (Marzano, 1992)

Dimensions Two, Three and Four are placed in the circle in the centre of the diagram. These three dimensions of the framework are primarily related to knowledge. They are concerned with acquiring knowledge, extending and refining knowledge and using knowledge meaningfully. Knowledge is central to the dimensions of learning framework and it is therefore fitting that it is placed in the centre of the diagram. The Dimensions of Learning framework, however, encompasses more than just dimensions that are directly related to knowledge. The affective aspects of learning are also included. Dimension One is concerned with the attitudes and perceptions that students have about their learning. Dimension Five is concerned with the ways that students tackle their learning. Dimensions One and Five are placed in the backdrop of the diagram. This is symbolic of the fact that they form a foundation on which effective learning of new knowledge can take place.

Using the Dimensions of Learning framework

The Dimensions of Learning model can be applied in a variety of ways. It can be incorporated into the curriculum planning process. Specific strategies which address various dimensions can be written into the curriculum. As well as using the Dimensions of Learning framework at this overarching level in curriculum planning, it can also be used to assist the aims of a particular lesson. This may be, for example, to extend the students' knowledge of a particular sub-topic.

In this thesis, the Dimensions of Learning model underpins the thinking that is applied in the planning of lesson activities for the research. It is used to analyse and interpret the data collected in the research project. The Dimensions of Learning model is also used to evaluate the effectiveness of the activities on student learning.

Outline of the Rest of the Thesis

The rest of the thesis begins in Chapter 2 with a description of the context. This provides background information about the school setting and the development of educational priorities in pedagogy and the use of technology. The study of what takes place in the classroom context is a snapshot taken on a journey in which the school plans for and develops the integration of technology into teaching and learning. The classroom study is inter-related with the saga of the development of the school's educational priorities on a wider scale. The story begins at the point when the school is developing a long term strategic plan.

Chapter 3 contains the Literature Review which is divided into two sections. The first section explores issues associated with the use of technology in mathematics.

These include descriptions of problems encountered when introducing technology, the various types of technology available and appropriate ways to use technology in mathematics. The second section examines the connections between the Dimensions of Learning framework and mathematics teaching and learning. This section justifies the use of the Dimensions of Learning framework in the implementation and analysis of the case study.

The Methodology and Research Methods are outlined in Chapter 4. This chapter explains how the research was conducted and how the data were collected. Chapter 5 is the Analysis chapter. In this chapter the data are analysed using the Dimensions of Learning framework. This is done in relation to the design and implementation of the tasks given to the students and in alignment with the aims of the research. Finally, in Chapter 6, the Conclusion draws together the emergent themes and articulates the findings of the research. The limitations of the research are acknowledged in this chapter. Implications are discussed and recommendations are made for further research.

CHAPTER 2 CONTEXT

This thesis draws together matters that concern the use of technology with those that relate to pedagogy. The data used comes from a specific classroom setting in which tasks are designed and implemented in line with the Dimensions of Learning framework. Through this process the contribution that technology makes in this setting is able to be explained. Before entering into the scenarios where the action takes place and the data are gathered, however, it will be helpful to outline the context from which the desire to use technology in association with a pedagogical model is derived. And so the story begins with a description of the context within which this project is based.

Ormiston College is a coeducational private school located between Brisbane and the Gold Coast in Queensland. The school has approximately 1300 students from reception to year 12. In late 2006 the Executive team at Ormiston had developed a five year strategic plan. Two main initiatives were incorporated in this plan. The first initiative was to develop the use of educational technology. The second initiative was to develop the use of up to date teaching and learning methods. To aid the development of the second initiative, it was decided that the school should introduce a teaching and learning model that would provide practical strategies for effective instruction, learning, curriculum planning and assessment. The model that was chosen for this was called “Dimensions of Learning” (Marzano, 1992). The dual initiatives related to technology and pedagogy were considered by the Executive to be of equal importance. Substantial funding was allocated to both.

Developing the Use of Technology

In keeping with the school’s initiative to develop technology, each department considered appropriate ways to contribute to this overall plan. The mathematics faculty at Ormiston devised a plan in late 2006 to integrate technology into the mathematics curriculum and everyday classroom practices. The mathematics team in the senior school (years 8 to 12) at this point comprised of six teachers who taught mathematics exclusively and four teachers who taught mathematics along with other responsibilities namely Year 12 Coordinator, Dean of Studies, Head of Technology and a Year 8 Class Teacher.

The Mathematics Department at that time made limited use of technology in its subject. Graphics calculators were available for use in some Year 11 and 12 courses. Year 9 students received one timetabled spreadsheet lesson per week. Students of the Mathematics A course in Years 11 and 12 had access to a computer room for one lesson a week. There were five rooms that were used predominantly for mathematics teaching. Two of these rooms had computers and digital projectors which were rarely used. A classroom management system had been installed on the school network but had been virtually unused.

There was, however, a clearly expressed desire from the Executive to upgrade the use of technology in the school. The intention to make better use of technology was documented in the school's strategic plan for 2007. In keeping with this policy, the mathematics department had to carefully consider how to use technology more effectively.

Following discussions between the Head of Technology and the Executive, it became clear that the school was willing to make a significant investment in technology for teaching and learning. In my capacity as Head of Mathematics, I needed to assess the best configuration of technology for the Mathematics Department and provide advice to the Head of Technology who then reported to the Executive. Factors that needed to be taken into account in this situation included finances, the technology skill levels of teachers, and implications for teaching with technology.

Upgrading the use of technology in the mathematics department

Drawing on the experience of others and ideas in mathematics education literature

From mathematics education literature it was recognized that there were many ways that the internet and other forms of technology could be integrated into secondary mathematics education and that there were growing indications that mathematics teachers were generally supportive of the use of computer technology in the classroom (Forgassi, 2006). There were several key factors however, that appeared to determine whether computer use would be encouraged or inhibited. For a successful outcome, mathematics teachers would require sufficient access to hardware, adequate levels of technical support, quality software and appropriate professional development (Forgassi, 2006).

In this stage of planning to upgrade the use of technology in the Mathematics Department, there was an opportunity to make use of the experience of others in integrating technology into educational settings. The approach taken by the Brewster Academy in New Hampshire involved substantial investment in hardware with every student being issued with a laptop computer (Bain, 1996). This was well beyond the financial commitment that Ormiston College would have been able to make at that time. The changes that were made at Brewster, however, were based upon two main principles, universal access and connectivity, and curricular embedding. Even with a less expensive solution, these principles were borne in mind.

It is important to consider the attitudes of teachers towards the use of technology and their skills in this area. Depending on the skills and attitudes of the staff involved, the technology may take on a variety of roles (Goos, Galbraith, Renshaw, & Geiger, 2006). The technology may be a *master*. In this role, the teachers are subservient to the technology. They may feel pressure to implement the use of the technology using whatever basic skills have been acquired through some initial training. The technology may be a *servant*. In this role, the technology is embraced and used in an informed knowledgeable way. As a servant, however, the technology is used in ways that perpetuate existing ways of producing knowledge.

The technology may be a *partner*. In this role, the technology can be used to do more than transmit prescribed knowledge and procedures using a firmly teacher directed approach: the technology may be used, for example, as a means to explore conjectures. Finally, the teacher may use the technology as an *extension of self*. In this role, the technology becomes an integral part of the teacher's teaching style and complements high levels of pedagogical and mathematical skill.

Teachers face challenges in coming to terms with these various roles of technology and their impact on the way that knowledge is produced in the classroom. Not least among these challenges is becoming familiar with the basics of operating the technology. As Goos et al. point out:

> Introducing new mathematical and communication technologies into classrooms can change the ways that knowledge is produced. Implicit in these changes are a number of challenges for teachers, the most obvious of which involves becoming familiar with the technology itself. Goos et al. (2006, p.318)

An essential component of professional development would be focused on familiarizing the staff with the technology. Apart from the challenge of using the new technology itself, however, it was understood by me and the Head of Technology that the teachers might experience challenges related to teaching and learning methods. In a general sense, technology was being embedded into a human system. This can alter the behaviour of the system and new properties may emerge from the system. In the context of the mathematics classroom, these emergent properties arise through changes in the communication structure. The patterns of social interaction that exist in the classroom may be altered. This may lead to knowledge being produced in more student-centred ways. It may also allow less vocal students the opportunity to contribute.

Identifying options

In 2006, the mathematics department at Ormiston had a variety of options to choose from in seeking to upgrade the use of technology which were affordable at that time. The deliberations above led to a more informed evaluation of these options. The options were outlined as follows:

Option 1:

- One computer and one digital projector would be installed in all five mathematics classrooms

- Teachers would have access to the internet and curriculum resources packaged together and available through an online portal.
- A tablet would be attached to each computer to allow teachers to write and draw freehand.
- Students would have access to Casio graphics calculators and occasionally have individual access to a "ClassPad 300" calculator that not only operates as a graphics calculator but also incorporates a computer algebra system and an interactive geometry system.
- Teachers would be able to use a Casio graphics calculator emulator that would display a working graphics calculator through the digital projector. Teachers would also have a geometry calculator emulator.

Option 2:

- As for option 1 above but in addition teachers would have eight computers in the classroom for student use.

Option 3:

- As for option 1 above but in addition an extra computer lab would be available to the mathematics department.

Evaluating the options

Each option was considered carefully and informal consultations took place with all the teachers individually. In considering these options, there was no doubt that Option 2 was very attractive. This option carried the opportunity for students to interact with computer technology in the classroom. With suitable classroom management methods, students would be able to share access by way of a rotation system. This option would have been adaptable and lent itself to a more student-centred approach to learning. This option however, required the highest level of change in practice from the teachers. The teachers would have had to learn to use the new technology for themselves and they would also have to cope with significant changes in teaching methods.

The danger with Option 2 was that busy teachers would have found themselves hard pressed to take on multiple dimensions of professional development at the same time, and the computers would have been left idle. This would have engendered a negative perception of technology usage in the school community. If technology were to be conspicuously under-utilised then this would have conveyed a message that technology was not valued. It would have been a waste of school resources. Any attempts to coerce the teachers into using the technology more fully would have led to a “technology as a master” situation. This option would have been too much too soon.

Option 1 would have involved the least amount of change for the teachers. Writing freehand on the tablet is straightforward. Accessing resources by way of a few clicks on the school website would also have been managed easily. The teachers were already used to working with graphics calculators and the use of the emulator would have made the task of instructing the students easier. For a relatively small amount of professional development, technology could have been introduced into the classroom that would engage the interest of the students. The highly visible presence of technology in the mathematics classrooms would also enhance the image of the mathematics department as one that was progressive with respect to the use of technology.

The danger with Option 1 was that it could have been used to perpetuate intense teacher directed methods. With technology being used as a servant of outmoded teaching methods, Option 1 could have become a serious “weapon of maths instruction”. Care would have to be exercised in the way that this option was implemented to ensure that the students were meaningfully engaged. The students would have access to electronic technology that interfaces with the technology available to the teacher. This would allow scope for exploratory methods and collaborative inquiry to take place. If implemented appropriately, this option could have made technology a partner to teaching and learning.

Option 3 provided similar benefits and required similar care in implementation as Option 1. The only difference would have been that there would be increased access to computer laboratory time. This would mainly be given to financial and application mathematics classes to work on spreadsheets. This would not have involved any radical changes to teaching methods.

Taking into account the capacity of teachers to change methods and learn new technology, Option 3 appeared to be the most attractive option. It provided the opportunity to move ahead with new technology and increase the amount of student interaction with technology without overburdening the teachers. This option was also perceived to be extensible. It had the potential to be a first step that could later lead to the implementation of Option 2 and possibly, in the fullness of time, a system along the lines of the School Design Model at Brewster Academy.

Ensuring access, connectivity and relevance to the curriculum

In the meantime, the two main principles of the Brewster model (i.e. universal access and connectivity along with curricular embedding) were examined more closely in relation to technology development at Ormiston. These principles were first, universal access with connectivity and second, curricular embedding. Universal access would be addressed since the technology would be installed at a consistent level throughout the department and from the students perspective there would be equity. Connectivity could be achieved by making use of the classroom management system (CMS). Teachers and students would be able to access resources from this system in or out of the classroom. My previous experience with a CMS would be helpful here.

The CMS installed at Ormiston is called MOODLE (Modular Object-Oriented Dynamic Learning Environment). The experience of Oak Ridge High School with this software suggests that it enhances connectivity in the educational community and even includes parental access (Perkins & Pfaffman, 2006).

The second principle of the Brewster method is curricular embedding (Bain, 1996). Ormiston would do well to address this aspect also. If new technology was to be introduced then it must be relevant to the curriculum rather than an extra that was merely added on. The mathematics curriculum at Ormiston was being reviewed prior to the start of the 2007. Integrating technology into the curriculum could take place at the same time and ensure that the new technology was used in meaningful ways in relation to the curriculum.

These deliberations led to an outline of the policy of the Mathematics Department with respect to the integration of technology into teaching and learning. In the strategic plan for the Mathematics Department for 2007 it was stated that the Department planned to 1. make use of MOODLE to provide a comprehensive online portal to teaching and learning resources which can be accessed in and out of the classroom; 2. use digital projectors in mathematics classroom to ensure consistent and equitable student access to instruction; and 3. significantly increase student access to graphics calculators.

Outcomes

Three years later

Three years later, the development of technology in the mathematics faculty at Ormiston College had proven to be a resounding success. The use of tablet technology in the classroom by the teachers had become the norm. In addition, almost all of the mathematics staff members had taken advantage of a subsidised lease scheme to purchase their own tablet laptops so that they could prepare lessons from home. Microsoft OneNote had become the main software used by teachers in the classroom. The CMS was being used extensively by students and staff. Teachers were regularly posting resources onto the CMS including handwritten classroom notes. Student use of the CMS had risen dramatically. In 2009, ClassPad calculators were distributed to Year 10 students and these calculators became the main calculator used by senior students. The focus of the department was shifting towards finding ways to enhance pedagogy in a technology-rich environment.

Five years later

Five years later, the development of technology in the mathematics faculty at Ormiston had become even more developed. Fifteen classes out of twenty in years 10, 11 and 12 had been issued with ClassPad calculators and students were using them regularly. The remaining five classes were studying mathematics related to everyday finances, measurement and statistics and the students in these classes were using graphics calculators regularly. The CMS was being used extensively by staff and students. There were sites on the CMS for each level which provided a wide variety of resources to staff and students. These resources included programs, revision materials, videos and weblinks. Some assessment had begun to take place using the CMS whereby students accessed tasks from the CMS and then uploaded their responses.

Meanwhile, by a curious twist of fate, thoughts about emulating the Brewster Model had been transformed from dream to possibility. In the midst of the global financial crisis of 2010, an election promise to implement the "Educational Revolution" was activated resulting in an injection of funds from the federal government of Australia into technology in schools. This meant that a one-to-one laptop policy could become a reality. Students in years 8, 9 and 10 had all been issued with laptops. Online ebooks had been introduced and students in year 8 were no longer carrying textbooks to class. It was clear that significant progress had been made over the five year period in introducing technology into the Mathematics Department at Ormiston. In this respect, one of the two main initiatives declared in 2006 had been addressed.

Adopting a Model for Teaching and Learning

The need for a model

The second initiative declared by the executive of Ormiston in 2006 was to develop modern teaching and learning practices at the school by making use of the Dimensions of Learning framework. This represented an attempt to steer a new pedagogical direction for the school. The underlying reasons for this had emerged from a desire to improve the Year 12 results. In order to do this some essential aspects of the Queensland educational system had to be considered.

The method of assessment of Year 12 students in Queensland is idiosyncratic in comparison to the other state systems in Australia. In Queensland, there are no external examinations. The students' achievements are determined through a school based externally moderated assessment system. The assessment tasks are developed and graded by the individual schools. Final grades in the various subjects are then agreed upon through a moderation process. This process involves consultations with panels of teachers from other schools. The overall performance (OP) of a student is then determined through a combination of the results achieved in their subjects in combination with the results that they achieve in a standardised state-wide test known as the Queensland Core Skills (QCS) Test. The QCS test is based only on content knowledge from Years 8 to 10 but it requires students to apply a wide range of skills including for example, comparing and contrasting, translating from one representation to another, and estimation.

At Ormiston, a mis-match was appearing in the results between those derived from subject-based assessment and those achieved in the QCS test. Even when students performed highly in the subject based assessment, if they did not also perform highly in the QCS test then their overall performance score was not as high as it might have been. It seemed that the students were well prepared for subject based assessment but they were having difficulty with the higher-order thinking skills necessary for success in the QCS test. It was in response to this problem area that the adoption of a learning model such as Dimensions of Learning (formerly known as Dimensions of Thinking) was investigated.

The Dimensions of Learning model suits the existing philosophy of the school

The decision to introduce the Dimensions of Learning model was also in line with other indicators of the educational philosophy of the school. For many years it had been the practice of the school to carry out assessments of the intellect of the students at yearly intervals. These tests were aimed at identifying the learning potential of the students and are referred to as the Structure of the Intellect (SOI) tests (Guilford & Hoeptner, 1971).

By making use of the SOI tests, Ormiston had to some extent been tapping into a school of thought which promoted the idea that intelligence may grow if nurtured appropriately. The tradition of this perspective dates back several decades. At that time, views began to emerge in which intelligence was considered to be learnable. This view is exemplified by Whimbey (1975) who claims that "mental capacity depends in large part on experiential and environmental factors". He could then see that if intelligence was learnable then it would be helpful to find ways of training people to increase their levels of intelligence. From this perspective, he states that "regarding intelligence as a skill centers attention on the potential efficacy of structured training" (Whimbey, 1975).

As well as the idea that intelligence can be nurtured, the notion that there are different types of intelligence was expressed. The work that was done on the structures of the intellect was more about differences in types of intelligence rather than differences in amounts of intelligence (Guilford & Hoeptner, 1971). The view that intelligence is multi-varied was subsequently developed by others to include qualities beyond those that are cognitive. The Multiple Forms of Intelligence that relate to problem solving and creativity are noteworthy here (Gardner, 2006). The boundaries of what constitutes intelligence were widened to include qualities beyond the cognitive such as emotions. As Styles (1999) explains.

> As others have stated, intelligence is not simply cognitive in nature, it is affective and conative as well: it involves emotional, motivational, and volitional aspects. In fact it is merely for convenience-to try to handle complexity- that we separate these aspects: In practice they are inseparable. (Styles, 1999)

There was a movement, therefore, towards intelligence being regarded as a multi-faceted quality of human beings that could be nurtured. This spawned new training methods. For example, the Habits of Mind (Costa & Kallick, 2008) are a set of learnable behaviours that are reputed to foster the growth of intelligence. They are designed to be integrated into all school subjects and it is intended that they will be taught, modelled and assessed.

Habits of Mind are a vital component of the Dimensions of Learning model which encompasses the acquisition of knowledge against a backdrop of attitudes and perceptions and habits of mind (R. Marzano, 2007). The Dimensions of Learning model follows in the wake of a movement towards learnable intelligence and it is this model that Ormiston sought to exploit to bring about improved educational outcomes for its students.

Integrating Dimensions of Learning into teaching and learning

Having made a commitment at the end of 2006 to adopt Dimensions of Learning as the model for teaching and learning in the Senior School, the process of introducing the staff to Dimensions of Learning began in earnest early in 2007. All teaching staff members in the Senior School were issued with a Dimensions of Learning manual (Marzano et al., 1997). Consultants from Central Queensland University and guest speakers were brought in make presentations and run workshops.

During 2007, all Senior School teaching staff members were rotated through a two day training course in Dimensions of Learning which was run by a representative from Central Queensland University. This training was referred to as “Initial DoL

Training”. The purpose of this training was to familiarise staff with the Dimensions of Learning framework. Some of the practical techniques associated with Dimensions of Learning such as the use of graphics organisers and comparison matrices were introduced in workshops.

Staff members were not expected to write curriculum units based on Dimensions of Learning during this initial training phase. There was a second level of training, however, which was carried out the following year. This second level of training was referred to as “Advanced DoL Training” and was provided to all staff members who had completed the initial training. The “Advanced DoL Training” also involved two days of professional development, but, unlike the “Initial DoLTraining”, the “Advanced DoLTraining” was not carried out over consecutive days.

In the first day of “Advanced DoL Training” the staff members were encouraged to bring an idea for a curriculum unit, or a series of lessons, for which a plan was to be developed based on Dimensions of Learning. These ideas were discussed in collaboration with colleagues and with advice from an educational consultant from Central Queensland University. Examples of units and templates for planning lessons were provided. The second day of “Advanced DoL Training” took place several weeks after the first day. This gave staff members time to work on developing a unit of work or series of lessons which they then had to present to the other members of the group.

The process of putting staff through these two levels of training continued in subsequent years. In 2009, the Deputy to the Head of the College was given the development of Dimensions of Learning as one of her major responsibilities and this included both Senior School and Junior School. The training of Senior School staff members continued and was expanded to include Junior School teachers, teacher aides and music tutors. The use of Dimensions of Learning became an integral part of the staff appraisal process.

In 2010, a group of staff members was drawn together by the Deputy to the Head of the College from a cross-section of the College to form a group referred to as the “DoL Key Team”. The role of this team was to promote the development of Dimensions of Learning by giving advice, encouragement and mentoring support to other staff members.

In 2011, the College purchased software to assist staff in the production of curriculum units. The software is called Unit Planner (EduTect, 2011) and is designed to provide templates for producing curriculum units which draw together not only aspects of Dimensions of Learning but also the Australian Curriculum. The Ormiston College Year 8 Mathematics Course for the Australian Curriculum was produced with the aid of this software.

Ormiston College continues to be committed to implementing Dimensions of Learning. There have been mixed levels of enthusiasm exhibited from staff members since it was first introduced. Some staff members have remained resistant to change. Others have independently taken initiatives and put great efforts into developing curriculum units based on Dimensions of Learning. Overall, the progress in integrating Dimensions of Learning into teaching and learning has been gradual and evolutionary.

CHAPTER 3 LITERATURE REVIEW

This thesis makes connections between the use of technology and a pedagogical model. In this respect it responds to literature which warns us about the problems of developing the use of technology without due regard to pedagogy. In the first section of the literature review, the longstanding difficulties that educators have faced in integrating technology into the classroom are examined. The idea that technology must go hand in hand with an appropriate pedagogy emerges as a recurring theme. The theme of technology linked to pedagogy is also examined in relation to ways of using new or specific types of technologies as they become available in the classroom. Connections between technology and pedagogy are also explored in relation to the behaviour of the teacher and the perceptions of the students.

If it is so vital, however, that technology be linked to pedagogy, then it will be useful to have a pedagogical framework to refer to. There may be many ways that technology can successfully be connected with an appropriate pedagogy but rather than have a collection of loose ideas about this it would be helpful to have a model of learning that provides some structure to the process. The Dimensions of Learning framework is explored as a means to providing such a structure. The Dimensions of Learning framework provides a generic model, however, intended to service all subjects and not just mathematics. The appropriateness of the Dimensions of Learning framework to the subject of mathematics is explored and justified in the second section of the literature review. This is achieved by connecting the Dimensions of Learning framework as described in the literature with practical examples drawn from my classroom practice. This connection between literature and practice serves to justify the use of the Dimensions of Learning framework in the practice-research project that follows in Chapter 4.

This chapter is therefore divided into two sections. In the first section the use of technology in the mathematics classroom is explored. Issues regarding the integration of technology are considered including those associated with specific types of technology. In the second section, connections are made between the Dimensions of Learning framework and the teaching and learning of mathematics.

The rationale for making these connections is given and then supported through the use of examples.

The Use of Technology in the Mathematics Classroom

Issues with integrating technology

Reflecting on the progress in the integration of technology into education in the last two decades of the twentieth century, Watson (2001) observed that technology had become ubiquitous in the business world but had not emerged fully in education. Certainly, for several decades, there has a been a long standing tension between on the one hand the enthusiasm of pioneers trying to encourage the use of technology, and on the other hand the everyday practices of teachers in the classroom (Laborde & Sträßer, 2010). In searching for reasons for the tardiness in the uptake of technology, Watson recognised that teachers can be threatened by change and that this may have impeded progress. He also felt that teachers were unimpressed by change that was about technology rather than about learning. He argued that technology can change knowledge and how we gain access to it but he emphasised that policies should be directed towards the teaching and learning that is required to gain this new knowledge (Watson, 2001) .

Watson further argued that in the United Kingdom the claims of success in the development of technology were restricted to political announcements. In reality, he felt these statements were merely political hype based on statistics showing increases in the number of computers in schools and increases in reports of these computers being used. He bemoaned the lack of depth in these claims and cited a body of research that indicated that progress with integrating technology into education was in fact disappointing. Watson did not agree, however, with the notion that the lack of real progress in integrating technology could be attributed to teachers' reluctance to accept change. Watson defended teachers in this regard by pointing to a lack of clear policy objectives and questioned the value of technology being promoted as a skills based subject in its own right when it might better be viewed as a means of teaching other subjects.

Watson concluded that interventions in an education system should be based on educational philosophy rather than technocentric considerations. Furthermore, technology should no longer be viewed as the agent for change. Instead it should be thought of as a tool which can assist with the process of change. From this perspective, change would be initiated and driven by educational ideas and not by technological advancements.

This principle is no less true in the mathematics classroom. A variety of successful approaches to mathematics teaching may be supported by the use of technology as opposed to being driven by the use of technology. These approaches may include an *exploratory approach* in which students establish and experiment with conjectures. The amount of time available, however, may dilute the exploratory approach to one of guided discovery (Brown, 2005). Technology may also be used to provide *multiple strategies for* solving mathematics problems. This enhances the view of mathematics as an activity that has purpose and offers students options for solving problems in the future. Technology may also be used to *promote discourse* that leads to deeper understanding. The capacity of the technology to represent movement is also helpful. For example, dynamic graphical representations of functions help students develop central concepts (Brown, 2005).

From this perspective, the integration of technology into the mathematics classroom brings many benefits to the students. As a supportive tool serving sound educational principles of enquiry, alternative methods and deepening of understanding through discourse it ought not to be ignored. Hence, in the current era, a competent mathematics teacher should be able to integrate technology into instruction (Erbas, Cakiroglu, Aydin, & Beser, 2006).

Types of recent technology used in mathematics

Touch screen technology

Before elaborating further on the ramifications of introducing technology into the mathematics classroom, it is appropriate at this juncture to provide some descriptions of the types of technology that may be involved. A very important development in this regard has been the emergence of touch screen technology. This form of technology is evident in a variety of forms but its most significant feature is that it has made it possible for the user to interact directly with the screen. Since no they do not require the use of a keyboard, the user experiences a close link between control and feedback (Lee, 2010). In the classroom, this type of technology can be present in the form of a large interactive white board (IWB).

Figure 2: An interactive whiteboard (2012, SMART Technologies Inc. All rights reserved).

It can also come in the form of a computer with a touch sensitive tablet operated by a stylus and connected to a digital projector. Both of these forms of touch screen technology allow the teacher to write by hand on the computer screen. Hand written notes can be stored on a computer and retrieved. The technology can also be used to bring a variety of computer resources into the classroom via the internet.

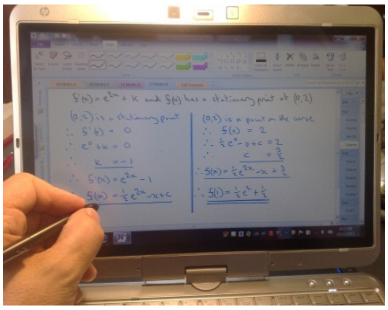


Figure 3: A tablet computer

Touch screen technology can also come in the form of a calculator with a touch sensitive screen. An example of this is the ClassPad calculator (CASIO Computer Co., 2011). A ClassPad calculator is essentially a hand held computer with a touch sensitive screen that is operated with a stylus. The ClassPad calculator has many features, including a graphics calculator, a spreadsheet and a geometry application, which can be accessed through its computer-like operating system.

Figure 4: The ClassPad calculator

In this project the teacher will use a computerised version of the calculator (an emulator) which is operated from the tablet computer and displayed by the digital projector. This allows the teacher to demonstrate the use of the calculator to the students.

Much of the research into teachers' use of touch screen technology in the classroom refers to interactive white boards but the principles involved apply equally to the use of computer tablets. It has been observed that research into the use of interactive whiteboards was initially focused on the technology itself rather than the associated pedagogy. Effective change in educational terms, however, is not solely about the technology (Glover, Miller, Averis, & Door, 2007). Teachers that have shown leadership in the use of this technology in the classroom have ventured further than using the technology to merely engage the students. These "mission teachers" have moved beyond the "wow" factor and capitalized on the interactive potential of the technology to enhance the learning process. This leads to a creative use of IWB technology in ways that surpass ordinary boards or projectors

IWB usage can be classified into three categories. The use of an IWB as a visual support to bolster a teacher directed approach is referred to as *supported didactic*. In this situation the IWB is used as an illustrative tool as opposed to a means of developing concepts. A second category of IWB use is referred to as *interactive*. In this category, teachers have generally acquired more technological skill and are willing to experiment with the IWB and its software. They are also more willing to share resources with other members of staff.

A third category of IWB usage is referred to as *enhanced interactive*. At this level, the use of technology has been fully incorporated into the teaching and learning process. The development of concepts is enhanced and the capacities of the technology are exploited to aid cognition. There is a good deal of sharing of resources and ideas amongst staff and this often involves the use of the school network. In the interactivity that takes place in the classroom, teachers are able to improvise more readily.

In terms of interactivity, tablet computers have some advantages over interactive whiteboards. Using a tablet computer connected to a projector, the teacher can face the students when writing on the screen. This makes communication between the teacher and the students easier. The teacher can maintain eye contact with the students without the need to constantly turn round to write on a board. The teacher can then more easily deal with questions as they arise (Galligan, Loch, McDonald, & Taylor, 2010).

Computer algebra systems

The use of a computer algebra system (CAS) in mathematics may appear to be a relatively recent advancement, but it has actually been in existence since 1955. Indeed the concept of a computer performing algebraic manipulation can be dated back even further since it is recorded that Countess Lovelace suggested to Charles Babbage that his "Analytical Engine" could be adapted to produce algebraic notation (Leadbetter & Thomas, 1989).

In a secondary school mathematics classroom, should a CAS be available, it would most likely come in the form of an application within a handheld calculator such as the ClassPad or the TI-89 (Texas_Instruments, 2011). In such a device, the CAS would be one of a set of stand -alone applications including a scientific calculator, a programming facility, a graphics calculator and a spreadsheet. (Cuoco, 2002). This is different from the environment of software such as *Mathematica*, where the user experiences a relatively seamless interface and the applications are not so obviously separated. When using calculators such as the ClassPad or the TI-89, if the user has already gained experience on a graphics calculator the CAS application will appear to the user as an additional affordance. The most important difference to the user is that the calculator will be able to manipulate algebraic expressions and functions. It is this difference that brings both opportunities and risks (Cuoco, 2002).

The practical benefits of having a CAS housed in a handheld calculator are the same as those that emerged with the arrival of graphics calculators in the 1990s. The inconvenience of going to a computer laboratory for an entire lesson even when that was not desirable could be set aside. Instead, the technology could be used for as long as was appropriate, and students could work individually or in group settings. In addition, considerable control over the learning process became possible for the students because the technology was in their possession and readily accessible (Trouche & Drijvers, 2010).

Another benefit that was accrued through the introduction of the graphing calculator was that it took the burden of some of the more mechanical procedures off the students and allowed them to concentrate on more complex work in forming mathematical connections or solving problems. In addition, the ability of the graphing calculator to display multiple representations helped students construct mathematical connections and thereby enhances their ability to solve problems (Bostic, 2010).

Mathematical modelling was another area that gained from the introduction of graphics calculators. Mathematical modelling can be thought of as a process which begins by describing an aspect of the real world by way of mathematical representations. Results are derived from the model, interpreted and then the model is revised if necessary. Graphics calculators aid the process of mathematical modelling through the multiple representations they afford and their ability to facilitate recursion and regression (Geiger, 2010).

In research conducted by Schmidt (2010) on upper secondary school students, the benefits of using a CAS were considered from the students' perspective. It was found that students in advanced courses enjoyed the possibilities afforded by a CAS to check results and were more likely to use the technology in other subjects. In general, students who achieved highly in mathematics assessment were more likely to perceive the use of a CAS as being beneficial to their education in mathematics.

The benefits of using a CAS have also been examined in relation to students' achievement in mathematics assessment. Tests over a five year period on upper secondary school students in the state of Thuringia in Germany revealed that the use of hand-held CAS devices had a postivie effect on student performance in mathematics (Schmidt, Kohler and Moldenhauer, 2009). The benefits were found to be more pronounced with students in advanced courses than for basic level courses. This difference was consistent with the students' perceptions of benefits of using a CAS researched by Schmidt (2010).

A CAS may play an important role in relation to procedural knowledge in algebra. Procedural knowledge in algebra traditionally involves a large amount of symbolic manipulation by handwith oencil and paper. If a CAS is used to perform some of this procedural work then it may provide three benefits. First, the use of a CAS to perform symbolic manipulation may save time which can be more productively spent on developing concepts. Second, a CAS can display the results of symbolic manipulation and allow students to observe patterns more readily. Third, the use of a CAS eliminates many of the errors that beset symbolic manipulation carried out using pencil and paper (Abdullah, 2007).

The risks that come into play when a new tool emerges often emanate from the fact that the tool may have been developed for one field, such as business, and then applied to another, such as mathematics education Cuoco (2002) sees this danger as being pertinent even when the tool emerges first in the field of mathematics and then is introduced into the field of mathematics education. When a CAS is introduced into an educational setting, there is a danger that problems will arise similar to those that emerged when scientific calculators were first introduced into the mathematics classroom. When scientific calculators first became available in the classroom, students were often asked to just perform the same calculations that they would have been asked to perform without technology.

Without planning aforethought teachers could fall into a similar trap with a CAS by asking their students to simply perform the same algebraic manipulations that they would previously have carried out using pencil and paper. A more productive approach with a CAS might be to address the criticism that formal algebra work in high school mathematics is overly concerned about the details of algebraic manipulation at the expense of higher order mathematical thinking. The use of a CAS may be applied here in the same way that scientific calculators were used to address the over emphasis on detailed paper and pencil skills in arithmetic (Cuoco, 2002).

In a study of the use of a CAS with Year 9 students undertaken by Artigue and Lagrange (1997), it was found that the uses of a CAS with junior secondary students can be supportive of the learning of mathematics. This is provided, however, that the CAS is used in a way that is relevant to mathematical learning at Year 9 level. Although the use of CAS can enliven mathematical activity, a note of caution must be expressed. If a CAS is not used in suitable conditions it can actually have a detrimental effect. One of the risks involved is the possible emergence of the *expertise reversal effect* (Kalyuga, Ayres, Chandler, & Sweller, 2010). The expertise reversal effect arises when experienced learners are presented with instructional guidance that is more appropriate for novice learners. For the novice learners the instructional guidance is beneficial and improves their performance. For the experienced learners, however, the guidance is redundant and results in an inefficient use of working memory which reduces their level of performance. An example of the occurrence of this effect can be seen when guidance is available in diagrammatic form and in text form. When these two forms of guidance are separated this is disadvantageous to the novice learner who has to devote working memory to making the connections between the two forms. Combining the diagrammatic representation with the text representation is helpful to the novice learner but unhelpful to the experienced learner who has to use up working memory in order to filter out redundant information. The negative consequences to the experienced learner of this split in attention can be ameliorated if the instructional guidance is given in visual and auditory form instead of diagrams and written text.

Dynamic geometry software

The introduction of dynamic geometry software (DGS) has also brought new opportunities. When such software is available to students they are able to interact with a figure in a technological environment as opposed to a drawing on pencil and paper. Using dynamic geometry software the student can, for example, click and drag on the vertices of a polygon in order to alter its shape and explore some essential properties. This would be a tedious and time-consuming activity if it were reliant on drawing and redrawing using pencil and paper. A relationship then develops between the technological tool and the user. The tool aids the quick and easy manipulation of the figure and the user capitalises on this facility in order to investigate more freely (Trouche & Drijvers, 2010).

In a study exploring students' understanding of geometric transformations when using dynamic geometry software it was found that the distinction between drawing and figure was significant. Students who thought in terms of a drawing tended to see transformations as actions on the one shape visible whereas those conceived of a figure tended to be able to predict more easily the outcome of the transformations (Hollebrands, 2003). Manipulating a figure in a dynamic geometry software environment can also be helpful because it can alleviate students of the need to make another representation of the figure. As an alternative to conventional proof-based explorations in geometry, for example, students can directly manipulate figures without having to make an algebraic or formal representation of the figure (Nathan, Penelope, Eva, & Barry, 2010).

The use of dynamic geometry software can help students develop spatial understanding and even make connections between two-dimensional and three-dimensional representations (Mammana & Pennisi, 2010). In an example of this, Mammana and Pennisi (2010) provided students with a series of activities in which they explored the connections between quadrilaterals and tetrahedrals. Through the use of DGS many properties and theorems common to both quadrilaterals and tetrahedrals emerged.

When reflecting on these capabilities offered by DGS, it is helpful to consider different perspectives on what is meant by the term geometry. A distinction can be made between, on the one hand, deductive geometry and, on the other hand, geometry which relies on observations. In deductive geometry there exists a system involving objects and relations. In a geometry of observations, the status of the objects is restricted to the visual evidence. There is value to be found for the students in both of these geometries; observations may be useful when seeking proof and the process of solving may not be purely deductive (Laborde, 2000).

Another perspective on what is meant by geometry is that it is a set of truths that exist independently and in isolation from people. From this point of view, a DGS would take the role of a tool like any other, including pencil and paper. If, however, by geometry we mean the activity of doing geometric work, we may incorporate the users in our perception of geometry. From this standpoint, DGS has a more significant role than pencil and paper because DGS offers more scope to see and explore relations in a flexible structure (Straesser, 2001).

The teacher may adopt several different roles in a DGS environment. At times the teacher may be a counsellor who points out students' misconceptions. At other times the teacher may play the role of technical assistant. Yet another significant role played by the teacher may to be work in a collaborative role alongside the students to explore concepts that emerge from constructions (Lew & Yoon, 2010).

The use of CAS in secondary mathematics challenges educators to find new methods of teaching and learning mathematics. It is anticipated that the response from teachers to the use of CAS in Australian schools will be varied. The variations in the uptake of the technology may range from those with a conservative approach to new strategies through to those who embrace the change in a rapid and radical manner (Garner, 2004).

Connections between technology and educational theory

New forms of knowledge

Research by Hoyles and Noss (2009), into the technological mediation of mathematics and its learning, undoubtedly targets the nexus of technology, mathematical content and associated pedagogy. They point out, for example, that the presence of computers introduces a new brand of knowledge related to the hardware and the interface through the software. In fact, "the knowledge instantiated in a computer system is no longer *the same* knowledge" (p. 132). Notwithstanding the importance of finding ways to adapt technological tools to facilitate and improve learning, there is also a need to re-assess the "mathematics-to-be-learned" in an environment replete with technology (Hoyles & Noss, 2009).

The presence of technology may also introduce some reassessments of matters related to pedagogy. For example, let us suppose that the main purpose of mathematical learning is to acquire the ability to express ideas abstractly using mathematics (Hoyles & Noss, 2009). This being the case, the use of algebra as a vehicle for learning about abstraction may have become so taken for granted that the notions of learning about algebra and learning about abstraction have been taken to mean the same thing. Digital technology, however, may bring novel representations and tools into play that can allow learning about abstraction to take place without the use of algebra. In other words, mathematical ways of thinking may be fostered using alternative means. This challenges existing ideas of what constitutes pedagogy and what constitutes content. From a traditional perspective, it may be assumed that the content is algebra and the pedagogy is the method of teaching algebra. The emergence of alternative technological representations may cause such assumptions to be turned around, however. The content may now be viewed- as perhaps it should have been along-as learning about abstraction. The pedagogy may then become the use of algebra or, the use of alternative representations afforded through the use of technology. This is a bold concept, and not one which will be pursued in this thesis, but it does serve to highlight the powerful significance that technology may play in mathematics education.

New epistemologies

Instead of just recycling old activities through new technology, teachers can take advantage of new epistemologies. In this context, the term "*new epistemologies"* means new ways of acquiring knowledge emerging from the introduction of the new technology. These new epistemologies may arise through conscious decisions to make use of the technological representations that the new technology affords. An example of this which came from the introduction of graphics calculators was the action of graphing the "left-hand side" of an equation and graphing the "right-hand side of the equation". This led to a useful visual image of the equation rather one represented by algebra (Trouche & Drijvers, 2010). It is often the case that artifacts have an influence on the construction of knowledge in mathematics (Trouche, 2003). In this example of solving equations graphically there is an interdependent relationship between the mathematical knowledge that is sought and the representational forms available to express it (Hoyles & Noss, 2009).

Other epistemologies that arose with the introduction of graphing calculators were not deliberate. Sometimes they resulted from problems that students encountered when using the technology, for example seeing a blank screen when they expected a graph to appear. The students would then have to come to terms with the fact that inappropriate scale selections might bring into focus an empty portion of an essentially infinite plane (Trouche & Drijvers, 2010).

So too, with the introduction of a CAS, new epistemologies may be taken advantage of regardless of whether they arise deliberately or accidentally. Cuoco (2002) argues strongly for the use of a CAS in a mathematics education setting to be directed towards conceptual development rather than mere symbol crunching. As a case in point he offers an example of using a CAS to illustrate the distinction between function and form in polynomials. He explains that two functions can be considered to be equal if they have the same function even if they differ in form. For example, $f(x) = x^2 - 1$ is the same function as $f(x) = (x - 1)(x + 1)$ since they would both generate the same table and graph assuming that we are referencing the real number system. The form $f(x) = (x - 1)(x + 1)$ is useful, however, because it makes more obvious the inputs required for an output of zero. He also provides a subtle example of the distinction between form and function using the expressions $x^5 - 2x + 1$ and $1 - x$ which appear decidedly different in form but are equal in function if we reference the set of integers modulo 5 (Cuoco, 2002).

The use of a CAS to explore concepts such as form and function, serve to point out that developing symbol sense is important when using a CAS. Zehavi (2004) explains that it is important to develop symbol sense when using a CAS because the CAS can work differently from what we expect. For example, if a CAS such as Derive[T] were to simplify the expression $4(3x - 2) + 3(x - 4)$ it would produce the result $15x - 20$ but if the same CAS were to simplify $4(3x - 2) + 6(x + 2)$ it might produce the result $2(9x + 2)$ because it automatically factorised the original expression (Zehavi, 2004).

Due to the presence of the new epistemologies, the student benefits from engaging in a meaningful semiotic interaction with the technology. Frequently when using technology it may be the case that the outsourcing of processing power is unproblematic. This may be so when the goal of using the technology is simply to find an algorithmic or numerical result. But, whilst it may be good to benefit from the pedagogic gain of outsourcing tedious calculation, the student may still profit from understanding some of the underlying techniques employed by the technology

(Hoyles & Noss, 2009). An example which illustrates this distinction is provided by Cuoco (2004). In solving the equation $x^2 - x - 1 = 0$, a numerical approximation may well be all that is required if all that the user is interested in is finding the values that make this equation true. But if the form of the roots in an algebraic sense is of interest, then it may be better to operate the CAS in such a way as to produce the roots in the form exemplified below:

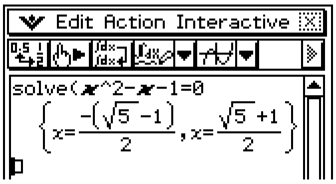


Figure 5: A CAS is used to find the roots of an equation in algebraic form

It would then be helpful to have the symbol sense to appreciate that this result is equivalent to $x = \frac{1 \pm \sqrt{5}}{2}$. The student would then have the result in a form which could be used, for example, to establish a formula for the *n*th term of the Fibonacci sequence. This example also serves to support the claim by Berger (2010) that working with a CAS can be viewed as a semiotic activity because it involves constructing, transforming and interpreting signs (Berger, 2010).

Behaviour of teacher and students when using technology

Interaction between teacher and students

Tanner and Jones (2007) have examined the interactions between teacher and students and evaluate them in terms of the amount of control that the teacher shares with the students and the quality of the dialogue that takes place. They delineate the nature of the interaction into five levels. The *lecture* is considered to have the highest degree of teacher control but affords no interaction. *Funneling* is the second level of interaction and is characterized by the way that the teacher uses questioning to heavily influence the direction of the dialogue. Tanner and Jones describe the third level of interaction as *probing*. At this level the teacher continues to direct the flow of the interaction but asks more probing questions to evoke deeper thinking from the students. The students exercise more control at this level of interactivity but the teacher is still in charge of the overall direction.

The next level of interactivity is characterized by the use of *focusing* questions. A significant level of skill is required from the teacher in highlighting strategies, providing insights and assisting students to build on their ideas. Finally, at the level referred to as *collective reflection*, the students are immersed in the interactivity. Teachers establish social conditions that allow the students to engage in activities such as self-evaluation, compiling their own revision notes and providing summaries to their peers (Tanner & Jones, 2007).

Creativity

Ultimately, the goal of using technology in the classroom should be to establish a truly creative teaching and learning environment. Features of the technology itself contribute a great deal to this aim. Facilities such as speed of delivery, range of materials, and interactivity make a significant contribution to the teaching and learning process. Notwithstanding these undeniable benefits of the technology itself, the skill of the teacher to orchestrate the classroom interaction and develop student inputs is of paramount importance. According to Wood and Ashfield (2008), "it is the skill and the professional knowledge of the teacher which is critical to the enhancement of the whole-class teaching and learning processes." (p. 84).

Creativity involves teachers and students. On the one hand the teachers may be creative when using technology to produce resources and on the other hand the students may exhibit creativity when learning through technology. The technology gives the teacher the opportunity to be creative in producing materials for the class using a wide range of media including video, graphics and links to websites (Wood & Ashfield, 2008). This may produce engaging presentations but does not necessarily mean that the students are participating in the creative process. Ideally, the teacher should be teaching creatively in a way that elicits a creative response from the students. In fact, the focus of truly creative teaching may well be less about the teachers' presentation and more about the students' learning. This being the case, support for students' learning will be the appropriate emphasis (Oncu, Delialioglu, & Brown, 2008).

Students actively involved

Used appropriately technology can support a high energy social climate in which productive class-room interaction can take place. Wood and Ashfield (2008) examine the use of technology, such as an interactive whiteboard, for the purposes of whole-class direct teaching. They express concern that the process of using technology in this way may result in skills and procedures taking precedence over higher order thinking activities: they warn against a purely transmission-based approach to teaching. They alert us to the dangers of developing passive and dependent learners.

Touch screen technology, as well as coming in the form of an IWB or a tablet computer, may also come in the form of hand-held devices such as the ClassPad. Devices such as these can be operated by the students individually, and this may help to involve the students more actively in the learning process. A technology setup in the classroom that involves hand-held devices operated by the students combined with a tablet computer operated by the teacher may produce a truly creative learning environment in which the students are actively involved.

Students' attitudes and perceptions about the use of technology

Students' attitudes can be described as the set of dispositions that they have towards the various components of their learning experience. These components include everything from ways of understanding, objects such as textbooks or possibly technology, and the people that they work with (Reed, Drijvers, & Kirschner, 2009). Attitudes have an effect on the perceptions that students hold about their ability to complete tasks successfully. These perceptions in turn give rise to behaviours which can either result in increased effort or dis-engagement. Reed et al describe interesting and sometimes unexpected effects of students' attitudes when technology is brought into the learning situation. In a study of the effects of attitudes and behaviours on the learning of mathematics using computer tools they found, as would be expected, that students with a more positive attitude towards mathematics achieved more highly in tests. With able students who had positive attitudes to the use of computer tools, however, they found what they refer to as the *interest reversal effect*. A lowering of test scores was recorded for these students. It was conjectured that the positive attitudes exhibited by these students towards the use of mathematical computer tools resulted in them diverting energy into technical matters at the expense of mathematical learning.

Reed et al found that self-reported behaviours were not related to the test scores that the students in their study achieved. They suggested that one reason for this was that the use of computer tools encourages more exploration to take place as students engage in exploration and investigation aided by the computer tools. In this more interactive environment the dialogue between students would differ from that which would take place in the more traditional mathematics classroom environments. It is possible that in the learning environment altered by the introduction of technology the students did not behave in the same way that they would in the regular classroom with its established codes of conduct.

Appropriate use of technology

Since students' attitudes to learning in the presence of technology are so significant, it is vitally important that teachers use technology in appropriate ways. Garafalo, Drier, Harper, Timmerman and Shockey (2009) give some assistance in this regard by providing five guidelines for the appropriate use of technology in mathematics classrooms. The first of these is to *use technology in context*. Rather than teaching skills first and then applying these skills afterwards to mathematical topics, it is preferable to introduce the technology along with meaningful content-based activities. This approach brings purposefulness to the use of technology. It is also more likely to result in teachers seeing the potential benefits of the technology for future activities (Garofalo, 2000).

The second guideline given by Garafalo et al is to *address worthwhile mathematics with appropriate pedagogy*. Here, it is deemed important that technology use should not compromise or distract from the mathematical concepts and procedures that are being addressed. Nor should technology be allowed to create false impressions in students' minds about what constitutes a proof in mathematics. Distinctions should be drawn between empirical results obtained from the use of technology and those derived from formal mathematics.

The third guideline offered by Garofalo et al is to *take advantage of technology*. Technology provides the capability to tirelessly perform many calculations and hence in areas such as recursion and regression has an advantage over by hand methods. If technology is used to carry out procedures that could be achieved equally well without technology then learning may in fact be retarded. Connections may be made here between this misuse of technology and the interest reversal effect described by Reed et al.

The fourth guideline is to *connect mathematics topics*. This guideline has a two-fold meaning in that it advocates both making connections between mathematics topics such as algebra and geometry, and also between mathematics and other disciplines such as the social sciences. These connections encourage students to construct a more holistic view of mathematics and to perceive the potential in associating mathematics with other real world pursuits.

The fifth guideline is to *incorporate multiple representations*. Digital technology can be used here to help students overcome difficulties in connecting information that appears, for example, in tabular, graphical and algebraic representations. It can be helpful to merge this use of digital technology with the use of hands-on technology using manipulatives and pencil and paper.

Summary

In this review the slow progress of the integration of technology into the classroom has been considered. This slow progress has been apparent despite technological advancements and political initiatives (Watson, 2001). The main lesson learned from this is that educational matters should take precedence over considerations about technology. This principle applies to the way that success in the integration of technology is evaluated. Progress in the integration of technology should be measured not by the amount of technology introduced but by the effectiveness of its use in educational terms.

The types of technological hardware currently available for use in the mathematics classroom were described. Special reference was made of touch screen technology in its various forms. These included various forms of touch screen technology such as interactive whiteboards, tablet computers and calculators. Approaches for using touch screen technology were reviewed in relation to pedagogy. These approaches comprised of supported didactic, interactive and enhanced interactive.

The potential for the presence of technology to raise new perspectives on the relationship between mathematical knowledge and pedagogy was touched upon (Hoyles & Noss, 2009). The emergence of new epistemologies through the introduction of new technology was also explored. These new epistemologies were considered with respect to the use of computer algebra systems (CAS) and dynamic geometry systems (DGS). Potential benefits of the use of these technologies were explored and some pitfalls were described. The effect of introducing these new systems echo the epistemological changes brought about by the introduction of graphics calculators and scientific calculators in previous years.

The types of learning that take place in a technology-rich environment were discussed. The learning theories of constructivism, social constructivism and social culturalism were found to be relevant (Gadanidis & Geiger, 2010). The potential for collaboration afforded by the use of technology meant that ideas from both social culturalism and social constructivism were prominent. However it was noted that both of these theories may need to be updated to incorporate new aspects of technology usage.

Finally, and importantly, the needs and attitudes of the students in a technology- rich setting were considered. A focus on the students' use of the technology and on the students' learning with technology were found to be important in fostering a truly creative learning environment. Some pitfalls were described in which technology if not introduced with care can actually have detrimental effects on students' attitudes and behaviours. These effects were identified as the interest reversal effect (Reed et al., 2009) and the expertise reversal effect (Kalyuga et al., 2010).

All of the literature pointed to the fact that when technology is introduced into the mathematics classroom is should be used in ways that are appropriate to the needs of the students. With this in mind, practical advice was provided which advocated the use of technology in context, addressing worthwhile mathematics with appropriate pedagogy, taking advantage of what the technology can offer, connecting mathematical topics and incorporating multiple representations (Garofalo, 2000).

Connections between the Dimensions of Learning Framework and the Teaching and Learning of mathematics

Introduction

In this thesis I am able to explain the part that technology plays in my students' learning of mathematics using the language of a pedagogical model. The Dimensions of Learning framework provides such a model, and it is one that I can use to develop tasks for my students and analyse the outcomes of these tasks when they are put into practice. By doing this, I gain an understanding of the role that technology plays in the learning that takes place.

The Dimensions of Learning framework is not the only pedagogical model in existence. Other excellent models include, for example, *Productive Pedagogies* (Chinnapan, 2006). It is not the purpose of this thesis to compare and contrast pedagogical models, however. Dimensions of Learning is a logical choice for the purposes of this project because, as explained in Chapter 2, it is the model that imbues the teaching and learning context within which the case study is embedded. In addition, this section will show that Dimensions of Learning is an appropriate choice because it can be connected very effectively with the teaching and learning of mathematics.

The Dimensions of Learning framework helps to enhance pedagogy through the use of a variety of strategies. These strategies include, for example, the use of graphic organisers to aid the acquisition and storage of knowledge. Course planning is enhanced by focusing on more than just the knowledge content of the topics. In planning courses, therefore, consideration is also given to the students' attitudes and perceptions about learning the subject and their learning habits as well as the types of knowledge to be acquired and how they can be extended and used meaningfully. The five dimensions which make up the learning framework are: (1) attitudes and perceptions; (2) acquiring and integrating knowledge; (3) extending and refining knowledge; (4) using knowledge meaningfully; and (5) habits of mind. The Dimensions of Learning framework was developed to improve teaching and learning in all subject areas.

McEwan (2008) explains that a key feature of the Dimensions of Learning framework is that it offers a way to nurture thinking skills. Educators aspire more than ever to help students to acquire higher order thinking skills that aid them to construct meaning rather than just recall information. Strategies that aim to develop higher order thinking through the use of "how" and "why" questions have long been hallmarks of inquiry-based learning. These general approaches can be taken a step further by providing explicit instruction in thinking skills. This involves setting out specific steps that explain to students how to perform the skills (McEwan, 2008).

The importance of thinking can be explored further by considering knowledge in more detail. Brandt (1988) explains that declarative knowledge can be described as information. This does not mean that declarative knowledge is restricted to facts. There are levels of generality within declarative knowledge which extend beyond the factual through to generalisations and concepts. Facts are still important, however, since generalisations and concepts are reliant on the facts that support them. Procedural knowledge also has hierarchical levels associated with it. These levels range from algorithmic skills through to strategies. Strategies are of a more general level and they draw on the lower level algorithmic skills where necessary (Brandt, 1988).

Hence, just as there is an interdependence of facts and concepts in declarative knowledge, there is interdependence of skills and strategies in procedural knowledge. This has implications for teaching in that teachers need to address both ends of the spectrum of generality in meaningful ways. Practice drills should be linked to tasks with a purpose. Cognitive skills should be applied to gaining meaningful information.

These connections between levels of generality within declarative knowledge and procedural knowledge sit well with the notion that knowledge and thinking are inextricably linked. An obvious way to capitalise on this link is to teach thinking skills in tandem with the content matter of subjects (Marzano, Pickering and McTighe, 1993). In the exploration that follows, the Dimensions of Learning framework and the thinking that underpins it will be considered specifically in relation to the teaching and learning of mathematics.

Dimension One: Attitudes and Perceptions

In Dimension One, teachers aim to make learning more effective by considering the students' attitudes and perceptions about the learning environment and the work that they are given to do. It is expected that learning will improve if students perceive tasks to have value to them. It also helps if the students understand clearly what is required of them in the tasks that are set. In addition, students need to feel equipped to carry out the tasks. If students are to feel equipped then they need to be confident that they have the necessary equipment, or what might be termed the external resources. The students also need to feel that they have the inner resources of ability and effort that will lead them to successfully completing the tasks. Hence, teachers can improve learning in the classroom by focusing on task value, task clarity and resources both external and inner.

Task Value

Task value can be enhanced if tasks are aligned in some sense with students' goals. For example, a class of Year 8 students are asked to carry out a statistics investigation entitled "Our Class". The students are to gather data by surveying their class mates. It would not be unlikely that the students would like to find out more about their classmates. Students who have recently arrived to the school in Year 8 may well like to know more about students who have attended the school since Year 1 and vice versa. The value of the task can in this way be aligned productively with the students' goals of deepening the bonds of social relationship within the class. Moreover, if the students can negotiate with their teacher over the questions that they will ask in their surveys then they will feel a greater sense of ownership of the task.

Consider another example of a task given to Year 10 students. All the students in the class have struggled with mathematics in the past, especially the more abstract aspects of the subject and subsequently they lack confidence in the subject. The students are asked to apply their skills in the topic of Measurement by designing the house of their dreams. The value of the task to the students can be enhanced in the way that it is introduced to them. Suppose the students are asked to look into the future and imagine that they are now twenty four years old. What kind of career will they be pursuing? What kind of interests will they have? Who else will be living in the house? By presenting the task in this way, the value of the task as perceived by the student can be improved. The students are likely to feel a connection with their personal dreams, ambitions and goals.

Marzano (1992) explains that another way to enhance task value is to tap into students' curiosity. Students have a natural tendency to want to know how things work and why things happen. Curiosity such as this can be made use of in the mathematics classroom. For example, at the beginning of a course on the topic of statistics I displayed the data shown below on the screen in the classroom to my Year 11 class.

Name	PClass	Age	Gender	Survived
Allen, Miss Elisabeth Walton	1st	29	female	Yes
Allison, Miss Helen Loraine	1st	2	female	No
Allison, Mr Hudson Joshua Creighton	1st	30	male	No
Allison, Mrs Hudson JC (Bessie Waldo Daniels)	1st	25	female	No
Allison, Master Hudson Trevor	1st	0.92	male	Yes
Anderson, Mr Harry	1st	47	male	Yes
Andrews, Miss Kornelia Theodosia	1st	63	female	Yes
Andrews, Mr Thomas, jr	1st	39	male	No
Appleton, Mrs Edward Dale (Charlotte Lamson)	1st	58	female	Yes

Figure 6: Data is displayed on the screen

The students are then asked questions about the data. Who are these people? They have such old-fashioned names. What could be so special about them? Perhaps the headings, *Name, PClass, Age, Gender* and *Survived* could provide a clue...

The students become curious and they guess eventually that they are looking at the passenger list of the Titanic. The list will serve as a useful dataset that will exemplify different classifications of data used in statistics. For example, Passenger Class would be associated with ordinal data; Gender would be associated with categorical data and Age with numerical data. Curiosity can further be fuelled by posing questions such as Who do you think would have had a better chance of survival, a first class female passenger or a third class male passenger? With motivation kindled through curiosity, students will hypothesise and can be encouraged to investigate the real data to test their hypotheses.

Marzano (1992) also recommends that teachers provide the students with interesting "tidbits" alongside the content of the topic. There are numerous examples of this type of intriguing incidental information amongst the many stories that are linked to the passenger list of the Titanic. One such example would be the moving story of the fate of Mrs Lucien Smith and her husband Mr Roger Smith. As newly-weds they honeymooned on the Titanic travelling first class. Sadly, Roger did not survive the ill-fated journey. Lucien did survive, however, and gave birth to a child later in the year. Stories such as these bring meaning to the data and maintain enthusiasm for learning.

Task Clarity

As well as perceiving tasks to have value, students also need to be clear about what is expected of them in a given task. Marzano (1992) explains that one of the best ways to communicate expectations about a task is to provide students with a model of a completed task. Returning to the example of the passenger list of the Titanic, let us assume that students are asked to select the name of the person on the list whose name is closest to their own name and then investigate that person's chances of survival. If the students are asked to write a report on their findings then, especially if it is the first time they have been required to write a report for mathematics, they will benefit from clear guidelines and a model of a completed report.

Providing the students with a suitable model of a completed report in this case can be achieved by using a different set of data. Here, for example, a suitable dataset to use would be a list of breakfast cereals located on the first, second and third shelves of a supermarket along with details of their nutritional content. An extract from the dataset is shown in Figure 7. The question to be investigated could be "Are the less healthy cereals located on the middle shelf close to the eye level of young children?" The example report can be viewed in appendix 9.

name	calories	protein	fat	sodium	fibre	carbo	sugars	potass	vitamins	shelf	rating
100%_Bran	70	4	1	130	10	5	6	280	25	3	68
100%_Natural_Bran	120	3	5	15	2	8	8	135	0	3	34
All-Bran	70	4	1	260	9	7	5	320	25	3	59
All-Bran_with_Extra_Fiber	50	4	0	140	14	8	0	330	25	3	94
Almond_Delight	110	2	2	200	1	14	8	-1	25	3	34
Apple_Cinnamon_Cheerios	110	2	2	180	1.5	10.5	10	70	25	1	30
Apple_Jacks	110	2	0	125	1	11	14	30	25	2	33
Basic_4	130	3	2	210	2	18	8	100	25	3	37
Bran_Chex	90	2	1	200	4	15	6	125	25	1	49
Bran_Flakes	90	3	0	210	5	13	5	190	25	3	53
Cap'n'Crunch	120	1	2	220	0	12	12	35	25	2	18
Cheerios	110	6	2	290	2	17	1	105	25	1	51
Cinnamon_Toast_Crunch	120	1	3	210	0	13	9	45	25	2	20
Clusters	110	3	2	140	2	13	7	105	25	3	40
Cocoa_Puffs	110	1	1	180	0	12	13	55	25	2	23

Figure 7: Extract from the cereals dataset (StatLib, 1996)

Resources

Marzano (1992) also explains that students must feel that they have the necessary resources to carry out a given task. Resources such as time, materials and equipment are referred to as external resources. For students of mathematics these external resources are increasingly becoming related to technology. In the Titanic example, students would need a spreadsheet with the details of over 1300 passengers. They would also need to have access to a computer.

Inner resources are less obvious and not entirely related to students' perception about their ability to perform a task. Effort is a vital criterion. In the example of the Dream House Project, the students are given an assessment rubric at the start of the project. Work ethic is one of the criteria in the rubric. This means that the inner resource of work ethic is valued to such an extent that it contributes to the student's overall grade. This brings methods of assessment into alignment with ways of learning. An extract from the rubric is shown in Figure 8, where it can be seen that work ethic is considered on an ongoing basis throughout the project. The asterisk indicates that it is one of the criteria that will be rewarded with a certificate at the end of project "House Warming Party" where all the students share their projects.

Criteria	A	B	C	D
Work ethic *	Always demonstrates a high level of work ethic is always on task and engaged, is always prepared for doing work.	Usually demonstrates a high level of work ethic is normally on-task and engaged, is always prepared for doing work.	Work ethic is satisfactory, is usually on-task and engaged, shows a consistent effort to be prepared for doing work.	Shows some attempt to work effectively and stay on task. Attempts to improve or maintain engagement and be prepared for work.

Figure 8: Extract from the assessment rubric for the Dream House Project

The preceding examples in this section serve to illustrate, therefore, that in Dimension One of the Dimensions of Learning framework the aspects of *task value*, *task clarity* and *resources* can be addressed in the teaching and learning of mathematics.

Dimension Two: Acquiring and Integrating Knowledge

Whereas Dimension One focuses on students' attitudes and perceptions to learning, Dimensions Two, Three and Four are concerned with the acquisition of knowledge. In the second dimension, knowledge is divided into two categories, *procedural knowledge* and *declarative knowledge*. Procedural knowledge relates to the skills and processes that are relevant to a particular content area whereas declarative knowledge is concerned with concepts and facts. There are three phases associated with acquiring procedural knowledge. The first phase is called *model construction*. In this phase, the learner develops an idea of the steps involved. The second phase is called *shaping*. This phase involves the elimination of errors and the identification of efficient techniques. The third phase is called *internalizing*. In this phase the learner engages in practice until the procedure can be performed with relative ease. Declarative knowledge also has three phases. The first phase of acquiring declarative knowledge is called *constructing meaning*. In this phase the learner uses existing knowledge to build a new mental framework. The term *constructing meaning* may sound a like a process that would be associated with an advanced stage of learning, but, in this context, it is more to do with the early stages of assimilating knowledge. The second phase is *organizing*. In this phase, the learner summarises and generalizes the new information. This could be interpreted as a process of accommodating new knowledge. The third phase of acquiring declarative knowledge involves *storing* information in such a way that it can be easily retrieved. It is concerned with the organisation of new knowledge. These phases are summarised in Figure 9 below.

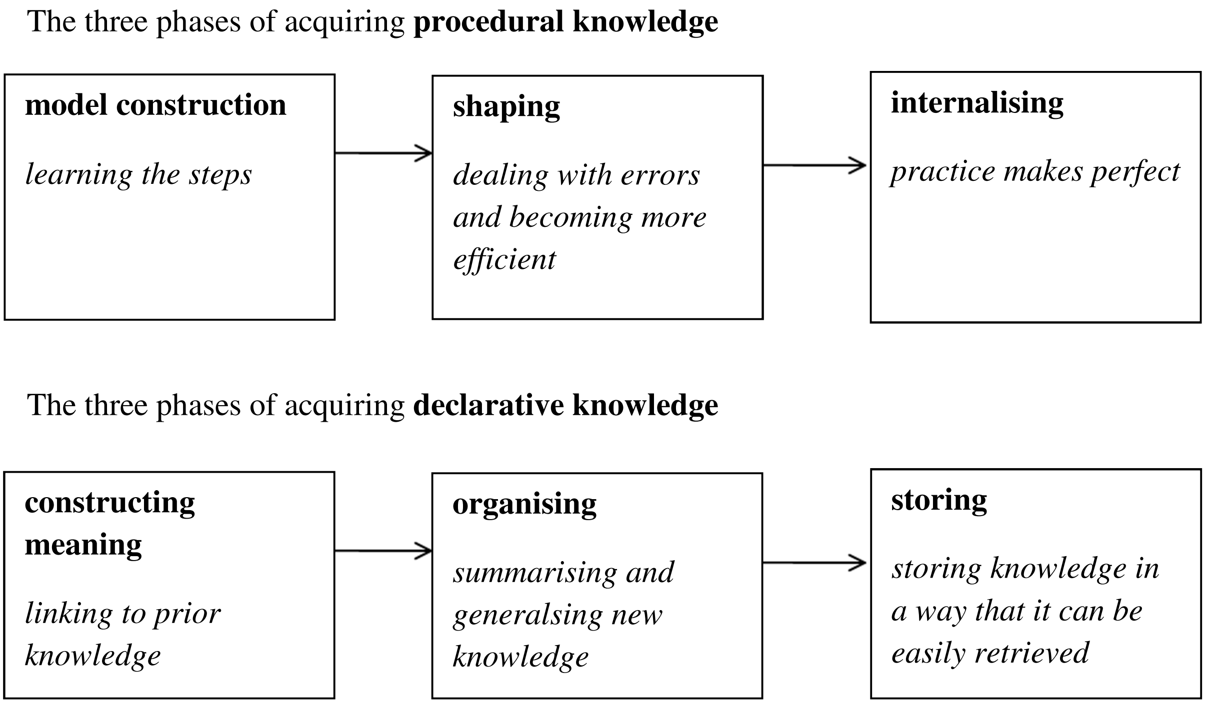


Figure 9: The phases of acquiring knowledge

A hierarchy of declarative knowledge according to levels of generality

It is possible to organise declarative knowledge into a hierarchy which moves from the specific to the general. From this perspective, descriptions, including vocabulary terms and facts, would be placed low on the hierarchy. In geometry, for example, vocabulary terms would include words such as quadrilateral, diameter and line segment. An example of a fact would be that a particular triangle with sides of length 5cm, 8 cm and 11 cm has a perimeter of 24 cm. Generalisations are placed further up the hierarchy. Whereas facts relate to specific objects or processes, generalisations relate to whole classes or categories of such objects or processes. A principle is a kind of generalisation that provides a relationship that can be used in a variety of instances. A principle is a powerful type of generalisation when considered in the context of mathematics. Examples include, the interior angles of a triangle add up to 180 degrees, the area of a rectangle can be found by multiplying the length by the width. The process of generalisation is noted as being a very powerful activity in mathematics (Skemp, 1971). It is a process that can not only contribute to growth in mathematical knowledge but can also bring unity to the subject by tying together familiar results with unfamiliar results (Sawyer, 1955).

Beyond generalisations and principles, however, concepts lie at the pinnacle of the hierarchy of generalised thinking. In a mathematical context, an example of a concept is associativity. In basic arithmetic, for example, calculating (15 + 11) +6 yields the same result as 15 + (11 + 6). This property is known as associativity. The operation of adding numbers can be described in general as being associative. The concept of associativity can be placed at a higher level of generality, however, when we consider how it can be transferred to other situations. In matrix arithmetic, for example, matrices of the same order can be added and the property of associativity will hold as exemplified below:

$$\left(\begin{bmatrix} 3 & 2 & 0 \\ 1 & 9 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 11 & 2 \\ -1 & 3 & -6 \end{bmatrix}\right) + \begin{bmatrix} 2 & -3 & 14 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 10 & 16 \\ 0 & 16 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & 9 & 4 \end{bmatrix} + \left(\begin{bmatrix} 5 & 11 & 2 \\ -1 & 3 & -6 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 14 \\ 0 & 4 & 2 \end{bmatrix}\right) = \begin{bmatrix} 10 & 10 & 16 \\ 0 & 16 & 0 \end{bmatrix}$$

Figure 10: Matrix arithmetic

On the other hand, it can be helpful to consider when a property that holds in one situation will not necessarily hold in another. In basic arithmetic, for example, $4 \times 5 = 20$ and $5 \times 4 = 20$. This illustrates the fact that in basic arithmetic the operation of multiplication is commutative. The order can be swapped and the result will remain unchanged. In matrix arithmetic this is not generally true as is illustrated below.

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 6 & -1 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} 36 & -1 \\ 38 & -1 \end{bmatrix}$$

whereas

$$\begin{bmatrix} 6 & -1 \\ 8 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 16 & 24 \end{bmatrix}$$

This shows, therefore, that the concept of commutativity can be considered in relation to different areas of mathematics. In some situations the property of commutativity will hold and in others it may not. The point is that students are learning how to abstract the concept of commutavitity without it being something that is contextually bound.

Procedural knowledge comprises skills and processes

Procedural knowledge encompasses both skills and processes. Examples of skills that a student studying algebra might be expected to acquire would include adding like terms, expanding brackets, and factorising a quadratic expression. The distinction between a skill and a process can be illustrated by considering the process of solving a quadratic equation through the use of factorisation and application of the null factor law. In this case there are steps to be followed and some decisions to be made. Skills are used within the process.

Macroprocesses are situated at a still higher level of generality. Examples of macroprocesses include, giving a speech or driving a car (Marzano et al., 1997). In mathematics, the use of optimisation techniques to find the best solution to a problem could be an example of a macroprocess. In such a macroprocess differential calculus could be involved along with other associated algebraic processes. The context of the problem would also have to be taken into account when assessing the results obtained. As is the case in other fields, a macroprocess in mathematics assumes a level of generality in which skills and processes become subservient to an overriding goal.

Helping students acquire declarative knowledge

In order to help students acquire declarative and procedural knowledge, it is useful to know the phases that the learner goes through in gaining these types of knowledge.

The acquisition of declarative knowledge has three phases comprising *constructing meaning*, *organising* and *storing*.

In the first phase of acquiring declarative knowledge, constructing meaning requires an active engagement with the new knowledge on the part of the learner in order to connect their prior knowledge with the new information. A useful instructional technique to help students construct meaning is to deliberately include a few minutes pause for reflection. In the process of approaching a problem in mathematics, or in learning a new concept, such a pause can allow students to respond from their intuition. For example, in the Ned Kelly statue problem described earlier, students can be encouraged to reflect and give an estimate based on intuition as to where the optimal angle will occur. This is an example of encouraging students to be actively involved in accommodating new knowledge rather than just being passive recipients.

It can also be helpful to students if, in the process of constructing meaning of new declarative knowledge, the content is presented in different ways. Multiple representations of problems and concepts in mathematics can be achieved for example through the use of diagrams, written descriptions, symbolic representations, concrete models and even physical movement of the students themselves. Consider an example where students are exploring the mathematics behind the spread of a disease. As an initial step in gaining an understanding of how the number of infected people might increase, the students could be asked to rise from their chairs and participate in a simulation. By generating random numbers from their calculators and physically moving in the classroom, the students can be engaged in an activity which helps them to understand the changes in the growth rate of the spread of the disease. This representation of the process combined with graphical and algebraic representations gives the students multiple opportunities to construct meaning of concepts such as growth rate, optimal value and limiting value.

Sometimes the teacher purposefully asks the students to draw on their existing knowledge. A familiar example of this in the mathematics classroom would be to build on the students understanding of length in guiding them to the discovery of pi. As the students compare the circumference and the diameter of a wide variety of circular objects they discover pi. They also gain an understanding of pi being associated with a generalised principle, that is, that the circumference of any circle divided by its diameter is a universal constant. Moreover, through their physical involvement in measuring the objects and observing the results, they are more likely to experience a sense of wonder and awe at the elegant simplicity of a mathematical principle which has been made manifest in the natural world.

Helping students organise declarative knowledge

Once students have acquired declarative knowledge, graphic organisers are very powerful tools for assisting students to organise this new found knowledge. When, in mathematics, generalisation or principles are involved, a graphic organiser can be used to effectively display relevant patterns. The example shown below in Figure 11 illustrates the use of a graphic organiser which was presented to the students in class to display the features of the slope-intercept form of the equation of a straight line. The main generalisation is shown in the top box and examples which illustrate varying aspects of the generalisation are shown underneath leading off from the main generalisation.

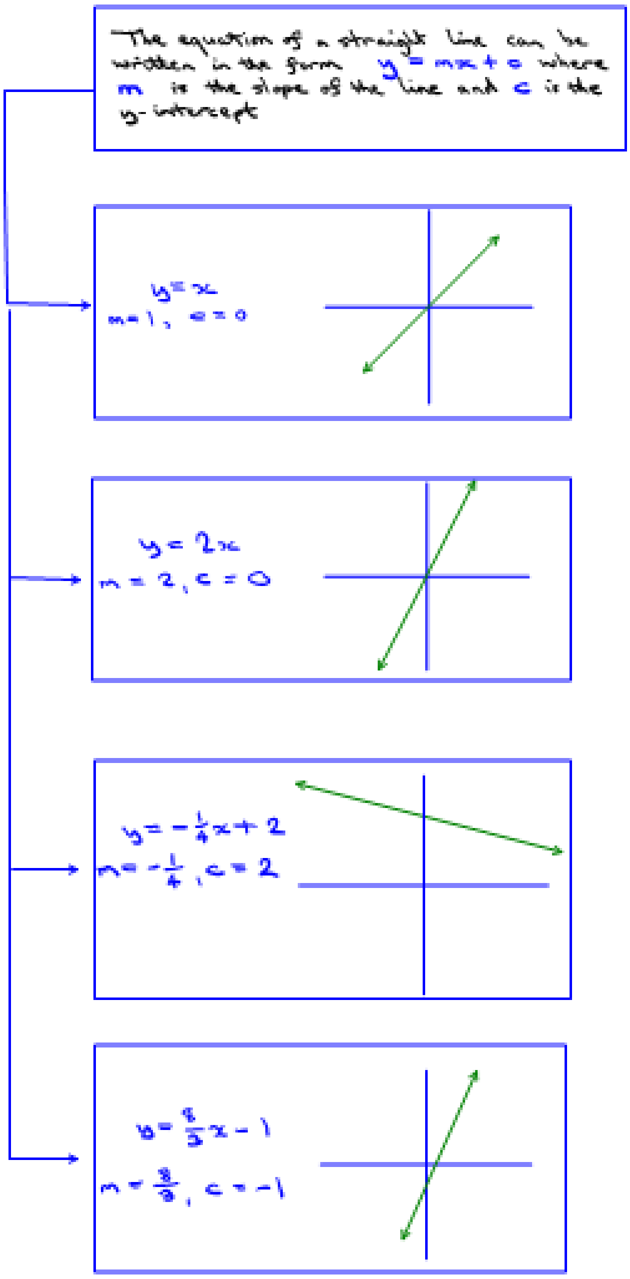


Figure 11: A graphic organiser showing a generalisation along with examples

In the example shown below in Figure 12, a graphic organiser is being used to display a concept pattern. In this case the overarching concept is “interest” in the financial meaning of the word. The main concept has characteristics which branch off from it. Here the graphic organiser helps the students not only to find an appropriate formula to use in a certain situation but also to appreciate for example that a future value annuity is associated with compound interest and involves regular payments.

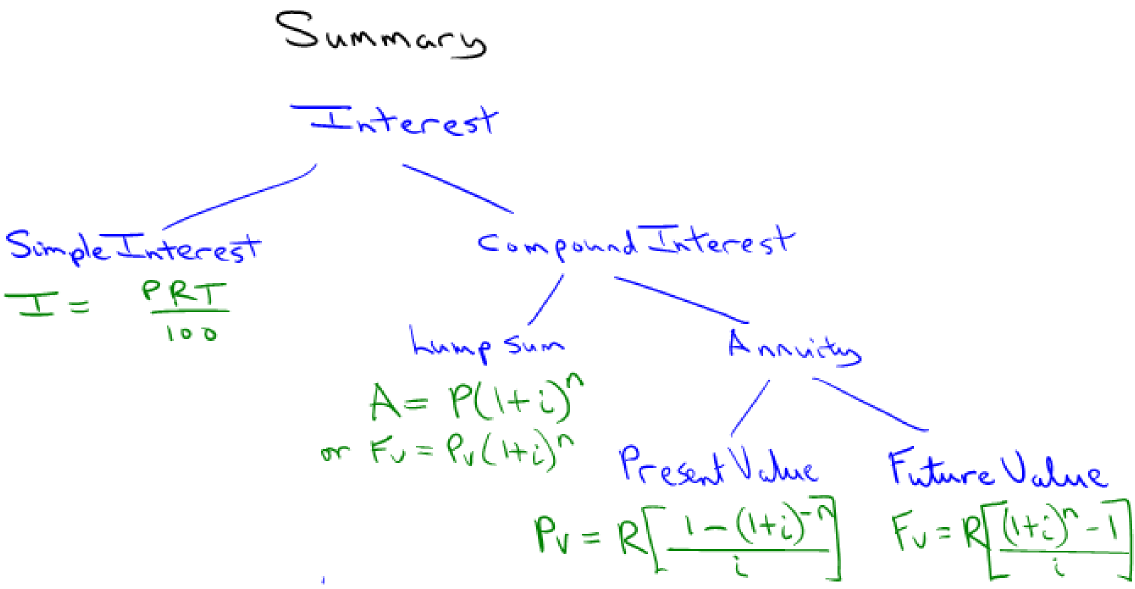


Figure 12: A graphic organiser for financial formulae

Helping students store declarative knowledge

The extent to which students should be compelled to memorise large tracts of mathematical facts is a subject of debate. Many mathematics teachers would argue that understanding should be emphasised more than memorisation. Certainly problems do manifest themselves when students rely on a process of memorisation without an underlying conceptual schema. Often, in the early stages of learning, a mechanical approach may appear to be successful but when, inevitably, the conceptual demands increase this approach breaks down leaving the student distressed and unable to cope (Skemp, 1971). Nevertheless, there are many pieces of information in mathematics that students need to be able to recall automatically. Times tables are notable examples.

It is helpful for students to have certain facts associated with trigonometry at their fingertips. Symbols linked to mnemonics are used effectively in this area. For example, the strange sounding word SOHCAHTOA is commonly used to help students remember the ratios involved in right-angled trigonometry. Symbols linked to a mental image are also helpful. For example, the diagram below helps students recall the sign of the trigonometric ratios for angles between 0 and 360 degrees.

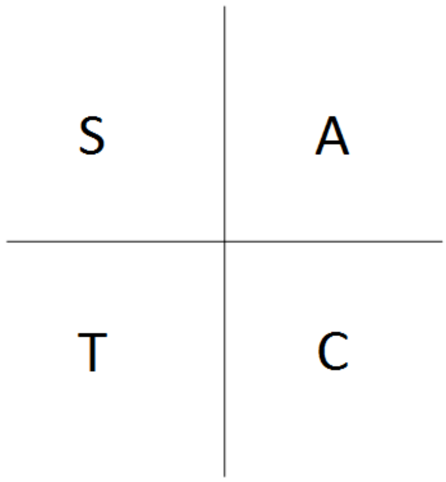


Figure 10: A mnemonic for the sign of trigonometric ratios

The diagram reminds students in which quadrant the ratios are positive. An accompanying mnemonic is also useful, for example, "**A**ll **S**tations **T**o **C**leveland ". On a cautionary note, however, it should be pointed out that whilst these memory tools are helpful and efficient, the students still require a sound understanding of the theory of the unit circle in order to make meaningful progress in the study of trigonometry and periodic functions.

Methods of acquiring procedural knowledge

The three phases associated with acquiring procedural knowledge are *model construction*, *shaping* and *internalising*. The first phase, model construction, entails finding a rough idea of the steps that are required. A helpful instructional technique during this phase is for the teacher to use a "think-aloud" approach when demonstrating the new skill or process (Marzano et al., 1997). It is helpful for students who are learning a new skill to have instructions presented in two forms, in this case visual and auditory. The combination of visual and auditory is also better for students who already have some experience. For these students the combination of visual and auditory is less of a distraction on their working memory than would be a combination of visual and written text. In mathematics teaching the "think-aloud" approach could be adopted in order to demonstrate a procedure involving the use of a graphics calculator. This allows more of the thinking associated with a particular context to be included rather than just a mechanical description of the keystrokes involved. The "think-aloud" approach can be incorporated into screen capture recordings which students can access over the internet at a later date. The accompanying voice-over provides instructions not just on how to use the calculator but also includes interpretive comments comparing the sets of data involved.

Another instructional technique to help students in the model construction phase of acquiring procedural knowledge is to give the students a set of written steps or a flow chart (Marzano et al., 1997). In the example below, a set of steps is displayed for solving a worded problem involving the use of differential calculus. It is important to note that an example such as this refers only to procedural knowledge in mathematics. There is no intention to promote a recipe-based approach to the doing of mathematics.

1. *Express the quantity to be maximised or minimised in terms of one variable*
2. *Find the derivative of the expression*
3. *Equate the derivative to zero to find the maximum or minimum*
4. *Use the first derivative test to test for a maximum or a minimum*
5. *Answer the question*

Figure 14: Suitable steps for solving a worded problem using differential calculus

A flow chart can also be helpful. A simple flow chart is shown below to help students follow the steps in factorising a quadratic expression.

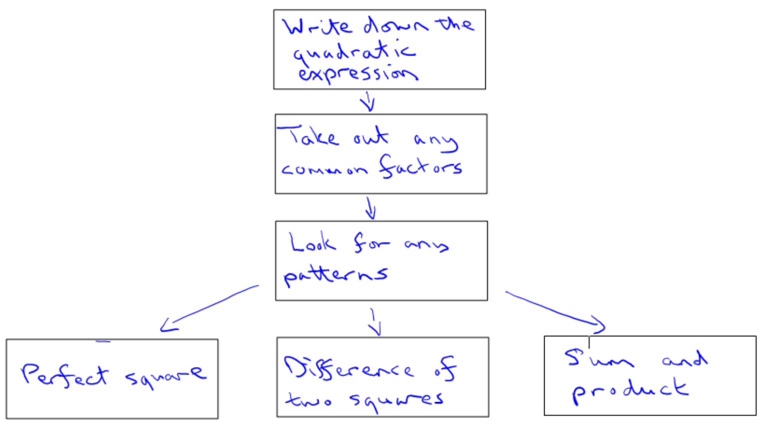


Figure 15: A simple flow chart for factorising a quadratic expression

The shaping phase of acquiring procedural knowledge is highly significant because it is here that errors can be eliminated before they become entrenched and hence more difficult to correct. During this phase the teacher can assist the students by identifying common errors and demonstrating how they can arise. Errors in this context are related to mistakes or slips in carrying out a procedure as opposed to errors which require analysis with critical thinking and which will be discussed in the next section on Dimension Three. It is also very important to provide feedback to students as they are working to make them aware of errors and correct them in progress. , (Marzano et al., 1997). In the process of solving equations, for example, there are times when positive and negative signs can become confused. When thinking aloud about the error illustrated in the second line of working shown below, the teacher would comment on the fact that the negative sign outside the brackets applies to everything inside the brackets and not just the first term.

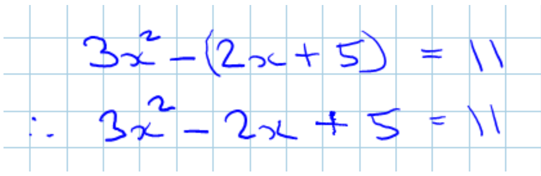


Figure 16: A typical error in solving equations

If students are to become sufficiently skilled in executing their procedural knowledge then they need to be given a variety of situations and contexts to deal with so that they can make any necessary adjustments to their methods. In the shaping phase of acquiring procedural knowledge it is important to check that the students have a firm grasp of the associated declarative knowledge. The teacher can check for this by periodically asking the students to explain their methods (Marzano et al., 1997). The example shown below illustrates the need for checking steps. In the example a correct answer is obtained using incorrect methods (Maxwell, 1959).

$$(5-3x)(7-2x) = (11-6x)(3-x)$$

$$\therefore\ 5-3x+7-2x = 11-6x+3-x$$

$$\therefore\ 12-5x = 14-7x$$

$$\therefore\ 2x = 2$$

$$\therefore\ x = 1$$

Figure 17: A typical error in solving equations

The third and final phase of acquiring procedural knowledge is *internalisation*. In this phase students are expected to achieve fluency and in some cases the skills need to be at a level referred to as automaticity (Marzano et al., 1997). In mathematics, number facts are most likely to fall into this category. Practice schedules are helpful in this area and a wide variety of software packages is available to provide students with practice material and give them feedback on their progress.

Dimension Three: Extending and Refining Knowledge

In the second dimension, knowledge both declarative and procedural is acquired and consolidated. In the third dimension, however, a greater depth of understanding is sought. Students are expected to do more than recall definitions and perform procedures. In order to extend and refine knowledge, eight complex reasoning processes are outlined as defined by Marzano et al (Marzano et al., 1997) and listed below. These complex reasoning processes are deliberately taught with a view to them becoming learned ways of thinking.

Comparing: Identifying and articulating similarities and differences among items

Classifying: Grouping things into definable categories on the basis of their attributes

Abstracting: Identifying and articulating the underlying theme or general pattern of information

Inductive reasoning: Inferring unknown generalisations or principles from information or observations

Deductive reasoning: Using generalisations and principles to infer unstated conclusions about specific information or situations

Constructing support: Building systems of support for assertions – providing justification for claims that are made

Analysing errors: Identifying and articulating errors in thinking

Analysing perspectives: Identifying multiple perspectives on an issue and examining the reasons or logic behind each

In order to gain the full worth of these complex reasoning processes, it is recommended that students are taught them explicitly. The processes are described to the students through the use of models including steps and flow charts where appropriate. Ultimately, the aim is for students to develop capabilities in applying complex reasoning processes to a wide variety of situations. This development is not left to chance, however. It is driven by overt and intentional teaching of the steps involved in the processes combined with a consistent use of terminology. This would mean, for example, that in teaching the complex reasoning process of analysing errors, types of errors would be classified and steps would be given to carry out the analysis. Students are taught, therefore, what the complex reasoning processes are and how to use them. From this perspective, the teaching approach is not merely a matter of providing examples and expecting learning to follow by osmosis but rather a more deliberate approach is preferred in which five aspects of each complex reasoning process are included in the teaching. First, the students are assisted in understanding the purpose of the process. Second, the students are given a model for the process and opportunities to practise its use. Third, students study the process and consider important steps. Fourth, graphic organisers are used to help the students understand and use the process. Fifth, a shift in emphasis is made from teacher-structured tasks to student-structured tasks thereby encouraging the students to become more independent in the use of the complex reasoning process (Marzano et al., 1997).

During the course of studying mathematics, students can be taught all of these complex reasoning processes. Their understanding of the processes and their understanding in mathematics can be enhanced simultaneously through the use of a variety of well-chosen examples and contexts. All of these complex reasoning processes can be related to the teaching and learning of mathematics. Illustrations are given below for two of them as examples.

Comparing

Essentially, comparing involves stating how things are the same and how they are different. The process of comparing can first be tackled with students at a basic level which allows for connections with their prior knowledge. Before embarking on a process of comparing in an overtly mathematical context, some examples can be considered from everyday experience. For example, two sports such as basketball and netball or hockey and soccer could be compared. Examples such as these provide students with the opportunity to learn how to compare things without being encumbered with new mathematical concepts.

After this, examples with a mathematical context can be introduced. These could include, for example, a comparison of a selection of polygons. The characteristics on which the comparison is based in this case could be, for example, the number of sides on the polygon, whether the polygon is convex or concave, and whether the polygon is regular or irregular.

Having exposed the students to the meaning and purpose of comparing, this complex reasoning process can be integrated into a topic in mathematics that the students are studying. For example, if a class of Year 11 students are studying the topic of Statistics then it will be helpful for them to be able to compare two sets of data using a variety of statistical measures. This activity of itself illustrates the difference between Dimension Two and Dimension Three. In Dimension Two the students would have learned how to calculate statistical measures such as mean, median, standard deviation, quartiles, range and inter-quartile range. In Dimension Three, however, the students are required to think at a higher level of complexity. Rather than just performing an algorithmic process, they are drawing comparisons and making inferences.

The story shown below could be read in class and the students could be issued with associated data to investigate. This activity could be used as an appropriate means of studying the process of comparing. The students will be able to see that comparing is a relevant process to adopt in exploring the two sets of data. There are key characteristics from a statistical perspective that can be used for the purpose of the comparison and there will be an opportunity to reflect on insights that are gained through the process.

A school story:

Ms Vera Ance is a Year 11 Geography teacher. Her colleague, Mr. Norman, teaches the other Year 11 Geography class at the school. It was with some trepidation that Ms Ance knocked on the office door of the Principal, Mr. Hastie. She had come to ask for extra materials and teacher assistance for her Geography class. Mr. Hastie was a man of few words and somewhat prone to jumping to conclusions.

Ms Ance explained how her nerves were "like piano wires" as she struggled to cope with her Geography class. Some of the students seemed to be bored whilst others needed everything explained over and over again.

Mr Hastie characteristically brought the meeting to an abrupt end.

"I can't see what the problem is. The results from the last common test show that your class average is about the same as Mr. Norman's and the range of marks is identical. You'll just have to pull yourself together Ms. Ance!"

Ms. Ance is aware of your skills in statistical analysis and she has come to you for assistance. She has a copy of the results for the last test from her class and from Mr. Norman's class.

An appropriate type of graphic organiser to help the students compare the two sets of data in this case is a comparison matrix. The essential components of a comparison matrix are that it has (1) a column for characteristics on which the comparisons are to be based; (2) columns headed with a description of the items to be compared; and (3) a column in which similarities and differences are to be recorded by the students.

Comparison Matrix of Class Results

Characteristics	Ms Ance's Class Results (%)	Mr Norman's Class Results (%)	Comments on similarities/differences
Mean	67.8	66.3	
Median	83	66	
Standard deviation	32.4	15.0	
Range	87	87	
Interquartile Range	69	20	

Reflection

Write a paragraph explaining whether or not Ms Ance has a valid complaint. Use the information from the comparison matrix above in your answer.

Figure 18: A comparison matrix

The comparison matrix is a useful tool in this activity. It shows that the means and ranges are very similar but there are significant differences in the standard deviations and inter-quartile ranges. The students can then be asked to use the results of this comparison to offer some insight into the problem described in the story. Having gained some confidence in the complex reasoning process of comparing through the use of teacher-structured tasks, the students will be better equipped to conduct comparison tasks of their own.

Analysing errors

In today's society people are receiving increasing amounts of information designed to persuade them to consume goods or support causes. Often there can be errors in the thinking behind the information. It is important that people can identify these errors and explain why the thinking is at fault. This process can be referred to as analysing errors. In the use of the word error there is a distinction here between errors in information and the use of the word error to mean mistakes or slip-ups in carrying out a procedure as discussed in the previous section on Dimension Two.

In the first instance, students can be made aware of the importance of analysing errors through examples from advertisements and articles found in the media. Humorous examples can also be included such as the one described below. In this example the trend in crime figures in Gotham City is exaggerated since the vertical scale on the graph does not begin at zero. It appears that Gotham city desperately need the services of Batman and Robin, but when the vertical scale is adjusted to begin at zero the situation seems much less dramatic. A few powerpoint slides with suitable sound effects can illustrate this point in an amusing way to the students.

Figure 19: Powerpoint slides illustrating misinformation by distortion

The students can be assisted further in the process of analysing errors if they are given a model for the process along with other opportunities to practise the process. In summary, the steps in the process are (1) Ask yourself if the information is important or designed to convince you of something; (2) Ask yourself if something seems wrong with the information; (3) Identify any errors; and (4) Seek more information (Marzano et al., 1997).

Students can be provided with further assistance in the complex reasoning process of analysing errors through the use of a graphic organiser. This can be provided in the form a flow chart as shown below in Figure 21. The flow chart gives a visual description of the thinking involved and also identifies four common types of errors that might be at play. There may be logical flaws in the thinking involved. For example, order of events may be used to justify causality. References may be inadequate as is often the case in the use of the internet. A claim may be asserted and supported by an appeal to force. Finally, the errors may emanate from misinformation, either by distorting the facts or by misapplying a principle (Marzano et al., 1997). In the Gotham City Crime Figures example, the error was misinformation through distortion of the facts and in the story of Sally Clarke the error was in the form of misinformation by misapplying a principle.

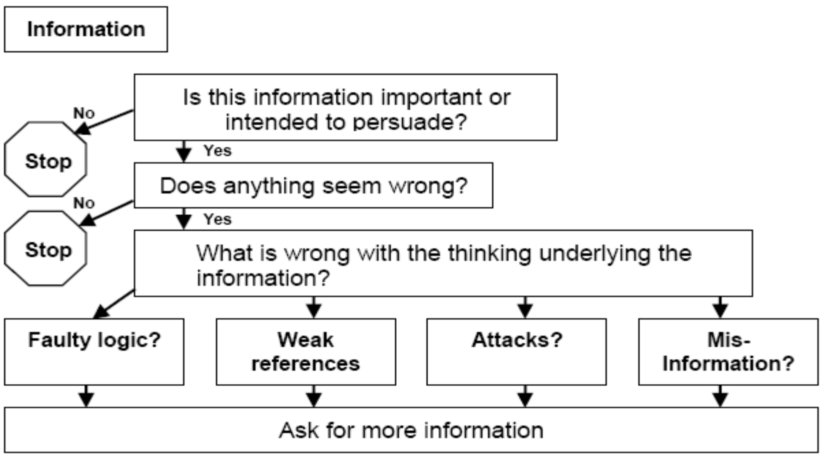


Figure 20: A process for analysing errors (Marzano et al., 1997)

Having been provided with a model of the process for analysing errors and opportunities to follow this process through examples, the students can be given less structured tasks. They could be shown a video, for example, in which a man appears to slide down a large waterslide and then miraculously land safely in a small pool of water(*Longest waterslide in the world*, 2011) (*Longest waterslide in the world*, 2011) The students may be right in saying this is a fraud but they would be expected to provide an analysis!

Figure 21: Frames from a video of a man appearing to go down a slide and land in a pool

Dimension Four: Using Knowledge Meaningfully

In Dimension Four, using knowledge meaningfully is regarded to be the main purpose of acquiring knowledge. As such it has tremendous power to engage learners especially when the knowledge is being used in a way that is of interest to them. As is the case with Dimension Three, complex reasoning processes are developed and used to elevate the students' level of thinking. For Dimension Four, the six complex reasoning processes involved are

Decision making: Generating and applying criteria to select from among seemingly equal alternatives

Problem solving: Overcoming constraints or limiting conditions that are in the way of pursuing goals

Invention: Developing unique products or processes that fulfil perceived needs

Experimental inquiry: Generating and testing explanations of observed phenomena

Investigation: Identifying and resolving issues about which there are confusions or contradictions

Systems analysis: Analysing the parts of a system and the manner in which they interact

As with Dimension Three, five aspects of each of the complex reasoning processes are considered. Students are first of all given assistance to understand the process. They are then provided with a model and practice. The important steps are examined in more detail. Graphic organisers are used to help the students understand and use the process. Finally, students are encouraged to use the process in a more student-structured way. The complex reasoning process of *Investigation* as an example is examined in more detail below.

Investigation

Investigation can be regarded as a process of finding ways to dispel confusion about ideas or events. There are three types of investigation, namely (1) definitional investigation; (2) historical investigation; and (3) projective investigation. A definitional investigation involves creating an exact definition of a concept. An historical investigation involves providing an explanatory account of an event or situation from the past. In a projective investigation, an event from the future or the past is explored by asking the question "What would happen if...?" or "What would have happened if...?" (Marzano et al., 1997).

A projective investigation is one which lends itself well to mathematical investigation. Students can appreciate the purpose and need for projective investigations in relation to, for example, the environmental future of the planet. It is helpful to give the students a model for the process of an investigation. In the case of a projective investigation, the steps in this model would be to (1) identify the event to be investigated; (2) clarify what is known already about the event; (3) describe the points of confusion or controversy; and (4) explain with justification how the matter can be resolved (Marzano et al., 1997).

The outbreak of a disease is an example of a real world phenomenon which students can investigate with a view to answering the question "What would happen if...?" Mid 2009, when news of the swine flu pandemic was being widely reported throughout the world, my class of year 12 students gathered data to investigate this situation. The investigation was carried out in such a way that it began as a teacher-structured activity but ultimately became a student-structured task. A simulation was carried out in the classroom to help students gain an understanding of the situation (Thomson, 2010b).

The sample results shown below were obtained by one of the students after he had averaged five runs of the simulation.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Total number infected	1	2	3.4	5.2	8	10.5	11.75	13	13.5	13.75	14	14

Figure 22: A student's table of results from the simulation

He then used an Excel spreadsheet to produce the scatterplot shown below.

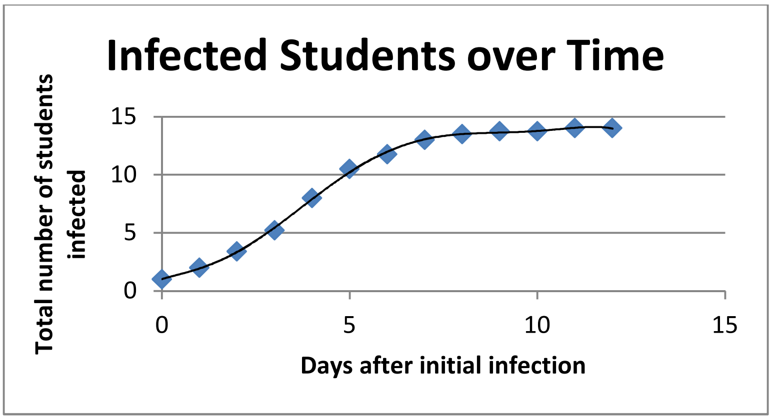


Figure 23: A student produces a scatterplot of results from the simulation

This scatterplot resembles the graph of the logistic model for growth. The logistic model has an equation of the form $y = \frac{C}{1 + ae^{-bx}}$ where y in this case represents the number of infections and x represents units of time. A typical graph of the logistic model for growth is shown below with the "point of inflection" (the point where the growth rate peaks) highlighted.

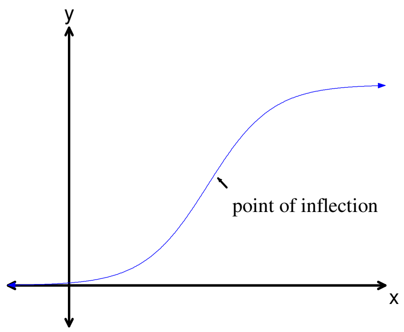


Figure 24: A graph of the logistic model

Using actual data from the World Health Organisation, the students constructed a graph of the growth of the number of swine flu cases as shown below. The question now asked of them was “What would happen if the growth in the number of swine flu cases followed a logistic model for growth?”

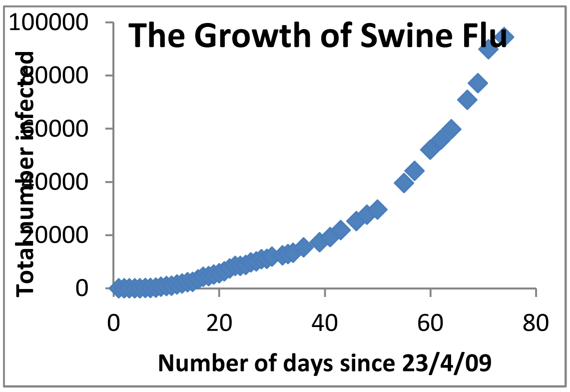


Figure 25: Graph showing the growth in cases of swine flu

The class went on to carry out an investigation into the spread of swine flu using the logistic model for growth. The students each submitted a three part report. In part (a) they presented their results from the simulation. In part (b) they used a computer algebra system to explore the application of the logistic model for growth and verified their results by manually applying their skills in calculus and algebra. Part (c) entailed researching an aspect of the growth of swine flu. Each student negotiated an individual focus for their research with me. For example, one student compared the growth of swine flu with the growth of other flu epidemics. Another student compared the mortality rates from swine flu between rich and poor countries. Other examples of areas of research included, growth in relation to population density, swine flu in large cities, and hospital admissions. The task was thus transformed from its teacher-structured beginnings to being much more student-structured. This is in keeping with the recommendations of Marzano et al (1997) in finding ways to assist students in the use of complex reasoning processes.

Marzano (1992) classifies the tasks that are used to promote the use of knowledge meaningfully into the following three categories: (1) application-oriented tasks; (2) long-term tasks; and (3) student-directed tasks. Application-oriented tasks are tasks that expect the students to accomplish a goal or apply their knowledge to answer a specific question. The One Son Policy investigation described above (McGivney-Burelle, 2004) is an example of this type of task. Long-term tasks take several periods of class time at least. “Living daylights” is an example of a long-term task which involves comparing minutes of daylight in Brisbane and Melbourne. It is one of many excellent long-term tasks produced by Peter Galbraith (2009). With student-directed tasks, students have control over the construction and product of the task although some negotiation with the teacher is recommended. Part (c) of the Modelling the Growth of Swine Flu investigation (Thomson, 2010b) is an example of this type of task.

Dimension Five: Habits of Mind

Dimensions Two, Three and Four are all related to the acquisition of knowledge. Dimension One: Attitudes and Perceptions and Dimension Five: Habits of Mind form a supporting framework on which learning can take place effectively. Indeed, without due regard to Dimensions One and Five, little progress is made in Dimensions Two, Three and Four. (Marzano, 1992). Habits of mind are a collection of characteristics and behaviours that intelligent people exhibit when they tackle problems which have no immediate solution (Costa & Kallick, 2008). In the Dimensions of Learning framework, the habits of mind are grouped into three categories as listed below (Marzano et al., 1997).

Critical thinking

- Be accurate and seek accuracy
- Be clear and seek clarity
- Maintain an open mind
- Restrain impulsivity
- Take a position when the situation warrants it
- Respond appropriately to others' feelings and level of knowledge

Creative thinking

- Persevere
- Push the limits of your knowledge and abilities
- Generate, trust, and maintain your own standards of evaluation
- Generate new ways of viewing a situation that are outside the boundaries of standard conventions

Self-regulated thinking

- Monitor your own thinking
- Plan appropriately
- Identify and use necessary resources
- Respond appropriately to feedback
- Evaluate the effectiveness of your actions

This list of habits of mind is not meant to be exhaustive, however, and it is recommended that people who use the Dimensions of learning model should adapt and expand the list according to their needs. An alternative list of the habits of mind is shown below (Costa & Kallick, 2000).

The Habits of Mind

- Persisting
- Managing impulsivity
- Listening with understanding and empathy
- Thinking flexibly
- Thinking about thinking (metacognition)
- Striving for accuracy
- Questioning and posing problems
- Applying past knowledge to new situations
- Thinking and communicating with clarity and precision
- Gathering data through all senses
- Creating, imagining, innovating

- Responding with wonderment and awe
- Taking responsible risks
- Finding humour
- Thinking interdependently
- Remaining open to continuous learning

Progress in the study of habits of mind followed in the wake of developments in ideas about intelligence. Research since the 1970s has brought about a paradigmatic shift in the way that that intelligence is viewed. Intelligence has begun to be viewed less as an innate static attribute and more as a human quality that can be learned. It has been suggested that learned intelligence can be enhanced through suitable experiences and direct vocabulary instruction (Marzano, 2003). Following from this perspective, the habits of mind are a collection of behaviours that can be learned and which constitute intelligence (Costa & Kallick, 2008).

From a cursory look through the habits of mind listed above, some immediate connections can be made with mathematics teaching and learning. In relation to problem solving in mathematics, for example, the quality of *persistence* has merit. Problem solving after all is non-routine and requires some perseverance and determination. *Managing impulsivity* is also relevant to problem solving in mathematics since it is often beneficial to cogitate deliberately over a problem rather than reach a conclusion with too much haste. The habit of mind of *striving for accuracy* can also clearly be connected with mathematics teaching and learning. Mathematics as a subject utilises concise notation in which minute differences can be significant and, when applying mathematical principles, mathematicians aim for precision.

Academic problems as a tool for fostering habits of mind

It is laudable to perceive the value of the habits of mind to learners and to recognise that these habits are relevant to mathematics teaching and learning. If it is accepted that the habits of mind are an expression of intelligence which can be taught, however, then mathematics teachers need to become more than just favourable observers. Some deliberate strategies need to be considered which will foster the habits of mind in students. One example of these strategies is the use of *academic problems*. Academic problems are problems that are structured well and are often associated with mathematics and science. They often take the form of puzzling posers which activate the brain (Marzano, 1992). The puzzles and diversions of

Martin Gardner (1959, 1990) or the intriguing curiosities produced by Ian Stewart (2008) are popular examples of these types of problems.

Some examples of academic problems suitable for use in a mathematics classroom are given in Figure 26 below.

1. John picked a bag of oranges to give to his friends. To the first of his friends, he gave half of the oranges he had and another one beside. To his second friend he gave half of the remaining oranges and another one beside. By this time, John had one orange left. How many did he start with?
2. A river 50m wide is spanned by a straight bridge which overlaps both banks of the river. One quarter of the bridge overlaps one bank and one half of the bridge overlaps the other bank. What is the total length of the bridge?
3. Two footballs and three tennis balls cost $60 while three footballs and two tennis balls cost $80. What would be the cost of one football and one tennis ball (together not singly)?

Figure 26: ("School Mathematics Competition," 2004)

Problems such as these relate well to creative thinking and critical thinking. Marzano (1992) notes three significant benefits in the use of these types of problems. First, they are inherently engaging. Students are drawn into them and desire a solution. This is despite the fact that the problems may not have a connection with any long-term goal. The key factor in terms of engaging the students is perhaps that the problems kindle students' curiosity in the same way that puzzles and crosswords attract the interest of so many human beings (R. Marzano, 2007). They are cognitively incomplete and this is motivating because it taps into the natural human desire to find missing information (Bormouth, 1968). Second, it is relatively easy to place these problems into the curriculum. They do not necessarily rely on new content and they can be slotted into the curriculum acting like a sponge to soak up spare minutes (Hunter, 1973). Third, these problems challenge the mind and relate to multiple habits of mind, especially those associated with critical and creative thinking (de Bono, 1990).

Implementation of the Dimensions of Learning Framework

In the preceding sections, Dimensions One, Two, Three, Four and Five were described in a sequential manner. When planning units of work using the Dimensions of Learning model it is not necessarily the case, however, that the planning process will take place in a strict sequential manner. Moreover, although all of the dimensions should be considered in the planning process, sometimes one dimension may have a greater emphasis than another. With these points in mind, three main models are offered for planning the implementation of a unit of work based on the Dimensions of Learning framework as discussed by Marzano et al (1997)..

In Model 1 there is a focus on knowledge. The process begins in Dimension Two by identifying the declarative and procedural knowledge central to the topic of work. The model then moves into Dimension Three where activities are created that are designed to extend and refine the knowledge acquired in Dimension Two. Finally, the task moves into Dimension Four. A task is designed which will allow the students to use knowledge meaningfully. Care is taken to ensure that the knowledge that is being applied is the same declarative and procedural knowledge that was introduced in the first step of the model.

In the teaching of mathematics there are various examples that can be found that illustrate the approach taken in Model 1. Essentially the students are taught the content first and then they are taught how to apply it. For example, Pythagoras' theorem is introduced and then students are taught how to use the theorem to find unknown sides of right angled triangles. Another example would be where students are taught the quadratic formula and then instructed on how to use the formula to solve quadratic equations. For many mathematical topics Model 1 offers a clear and logical approach to new topics.

The approach taken in Model 1 may not always be suitable, however. Let us suppose that students are to be introduced to the concept of logarithms. Assuming that the Model 1 approach is taken to teaching this topic then this would imply that the students would first be taught the declarative and procedural knowledge associated with the theory of logarithms. In other words, a logarithm would be defined without any reference to any real-life context with which it could be associated and students would be guided to work through exercises in using the laws of logarithms. Only after this exposition and practice had taken place would the students be asked to consider the connection between logarithms and a real-life context such as the magnitude of an earthquake. The Model 1 approach may be seen to fall short here. An opportunity may have been missed to raise curiosity and instil motivation for learning. A more imaginative approach could be to first of all seek some connection between mathematics and the real world. A question could be posed perhaps about how we can compare the magnitudes of earthquakes. The teacher might ask the class, "The recent earthquake in Japan was much more destructive that the one that took place in New Zealand. How can we use mathematics to compare the destructive power of earthquakes?" This approach is not consistent with Model 1 since the application of the mathematics is being considered prior to the introduction of the mathematical content and techniques. This justifies the use of a second model of implementation in which issues are explored prior to the exposition of the theory.

In Model 2 there is a focus on issues. This model begins in Dimension Four. An issue is identified that relates to the declarative and procedural knowledge associated with a particular topic. The advantage that this model has over Model 1 is that students can perceive a greater sense of purpose for using the knowledge involved in the unit of work. An example of this approach can be found in the section *Dimension One: Attitudes and Perceptions*, which includes a description of how an investigation into the passenger list of the Titanic is used to begin a unit of work on Statistics. Issues are raised about the passengers' chances of survival in relation to, for example, social class or gender. Questions such as, "*Who had a better chance of survival, a first class female passenger or a third class male passenger?"* can be resolved through analysis using statistical techniques. The investigation justifies the need to gain knowledge about Statistics. In a sense, the age old question "*When are we ever going to use this?*" is transformed into "*How can we do that?*"

In Model 3 there is a focus on student exploration. The declarative and procedural knowledge is identified and ways to extend and refine this knowledge are delineated. Instead of the teacher providing a task that focuses the students on using knowledge meaningfully, however, the students themselves are encouraged to conduct an exploration in which they use the knowledge from Dimensions Two and Three plus any other relevant knowledge. In this model more responsibility is given to the students and they are encouraged to carry out a project that uses knowledge meaningfully to them. The teacher assists the students in choosing a project which will use knowledge that emanates naturally from the subject matter of the topic. A student of a course which includes the topic of permutations and combinations, for example, may choose to investigate the mathematics of gambling machines. Through the investigation the students will draw on and extend knowledge from the coursework.

The example of the investigation into the spread of a disease given in the section *Dimension Four: Using Knowledge Meaningfully* is perhaps a hybrid of Models 2 and 3. The investigation begins in Dimension Four with activities initiated by the teacher. These activities lead into work in Dimensions Two and Three in acquiring declarative and procedural knowledge associated with the topic of Exponential Functions. This approach is in line with the steps involved in Model 2. In the third part of the investigation, however, the students negotiate an aspect of the spread of swine flu that they would like to explore in depth. This part of the investigation is more aligned with Model 3.

CHAPTER 4 METHODOLOGY AND RESEARCH METHODS

In this chapter, the methodology which underpins the research is described as well the techniques that were used to undertake the research. The paradigms that support the methodology are outlined and the appropriateness of the case study approach is justified.

The students involved in the research were drawn from a group of students who had been selected to take part in mathematics enrichment lessons. The selection process is explained and this provides a clearer picture of the types of students who are involved.

The classroom environment in which the research is conducted is described and reference is made to the technology that was available. The manner in which the teaching was conducted is also outlined. The activities that were specially prepared for the students are an additional source of data. As such, their connection with the Dimensions of Learning framework is mapped out in preparation for the analysis which is provided in Chapter Five.

The techniques that were used to collect data are explained in practical terms. The approaches that were developed to analyse the data are also described. The relevance of grounded research to the data analysis is referred to and the way that the data were coded from their raw beginnings is described.

The choice of methodology and the techniques used in the research are intended to support the overall aim of the research which is to improve the learning of mathematics when technology is used in the classroom. The strong links that are made with the Dimensions of Learning framework assist in this regard to shed light on the contribution that the use of technology makes to students' perceptions about learning mathematics and the contribution that the use of technology makes to the learning process.

Methodology

The methodology employed in this thesis is one that aligns with qualitative research. It takes the form of a case study carried out by a participant-researcher. The suitability of this methodology is justified by (1) considering the underlying paradigms to which qualitative research and the needs of the thesis relate; (2) recognising the need to deal with complexity and (3) appreciating the interpretive power of a case study conducted by a participant-researcher.

Underlying paradigms

The paradigmatic trail may be traced back to a perception of reality. This perception of reality is not based upon the notion that there is one and only one reality which embodies the way things are and how they work. If this were to be accepted then we would expect this same unique form of reality to be revealed by different inquirers. Instead, however, the ontological assumption on which the research methodology of this thesis rests is of a more relativist stance. From this viewpoint it is expected that different inquirers would produce different interpretations, although it might be hoped that through discourse these interpretations would subsequently merge into a more enlightened form (Lincoln & Guba, 1994).

Dealing with complexity

The ontological and epistemological paradigms described above connect well with qualitative research because qualitative research seeks to cope with the complexity of fluid realities and multiple perspectives. A qualitative methodology is also appropriate because of the inherent complexity of the matter that the thesis deals with. Through the design and implementation of mathematical tasks using a pedagogical model, this thesis sheds light on the contribution of technology in secondary mathematics. This is a complex situation. If a quantitative methodology were to be employed involving experimental research then this would be problematic with respect to complexity. In experimental research the number of variables examined is deliberately reduced and this denudes the situation under investigation of potentially informative layers of complexity (Shulman, 1986) and as Corbin & Strauss (2008) point out:

> The world is very complex. There are no simple explanations for things. Rather, events are the result of multiple factors coming together and interacting in complex and often

> unanticipated ways. Therefore any method that attempts to understand experience and explain situations will have to be complex. (p. 8)

A qualitative methodology addresses the need for complexity by preserving the richness and subtleties of the data, and as such is the one that suits the needs of this thesis.

A case study conducted by a participant-researcher

A key aspect of the research is to understand the contribution of technology when a situation is set up in which Dimensions of Learning designed tasks are put into action. This situational aspect is served by an interpretive approach (Cohen, Manion, & Morrison, 2000). A case study is a suitable method for carrying out such an approach. It is a useful way of gathering evidence about a specific phenomenon and often takes place in a natural setting (Anderson, 1998). When a case study is carried out by a participant-researcher, the participant –researcher engages in the activity, collects data, reflects and analyses data.

A case study differs from other types of research that may involve a participant-researcher. In evaluation research, for example, a participant-researcher may enter into a natural context with the intention of comparing the findings of the research with what was planned. The role of the participant-researcher in a case study, however, is to gain an in-depth understanding of a complex situation from a variety of perspectives. Through the case study approach explanations are found for how and why things occur (Anderson, 1998). A case study approach is appropriate to this thesis because the aim is to explain the contribution of technology to the students' perceptions about learning mathematics and to the process of learning mathematics.

A qualitative research approach permits the researcher to enter the situation as a participant-observer. This is despite the fact that the researcher will bear an individual worldview emanating from an environment dependent framework of beliefs, or *welttanschauung* to use the German term (Beishton & Peters, 1983). Using this approach, the researcher does not need to isolate variables in a quest for impartiality, and so valuable layers of complexity are allowed to remain intact. This facilitates a richer understanding of the situation.

This methodology is based on a relativist view on ontology and a subjectivist view on epistemology. This means that appropriate ways of evaluating the research need to be called upon. Evaluation methods that are associated with reliability and validity are more suited to quantitative research (Golafshani, 2003). Alternatives to these ways are used in the evaluation of this qualitative research project. Instead of *generalisation* as an evaluation criterion for the research, the alternative of *transferability* is used. In essence, transferability involves the researcher in providing sufficient description to enable the receiver of the research to make a judgement. Careful descriptions of the context of the research make these judgements easier to make. The context provided in Chapter 2 addresses this criterion along with the descriptions provided in the analysis and discussion in Chapter 5. The *credibility* criterion is offered as an alternative to *triangulation*. It would not be relevant to try to relate the findings to an objective reality when a multi-perspective approach is being adopted. Instead the credibility of a match between the data and the interpretations thereof is made using techniques including prolonged engagement and persistent observation (Guba & Lincoln, 1989).

Methods

Subjects

The student population

The student population at Ormiston College is divided into a Junior School and a Senior School. Both of these schools are coeducational. Students begin their studies in Senior School at Year 8 at which time they are approximately 13 years old. The Year 8 student population comprises of two groups of students. First, there are those students who have already been studying in the Junior School of the College in Year 7 and who automatically progress on to Year 8 in the Senior School. Second, there are those students who have been studying at other primary schools in the region and who move to Ormiston College for their senior years of schooling.

The students in Year 8 at Ormiston College are grouped into five classes of approximately thirty students. These classes are of mixed ability and each class is assigned to a “Form” teacher who provides pastoral care for the students. Unlike the situation in the Junior School, where the students will have experienced one single teacher for most subjects, the students in Senior School go to specialist teachers for various subjects. The students in each class stay together as a group, however. This means, for example, that the class list for English is exactly the same as the class list for Mathematics, and this applies to all the subjects that the students study.

The structuring of the Year 8 classes whereby each Form class stays together when moving from one subject teacher to another, is an integral part of the ethos of Year 8 at Ormiston College. Forming relationships with one another is seen to be a very important step in the journey through Senior School. All the Year 8 students attend a camp, the theme of which is “Campus Connections”. The activities in the camp are geared toward helping the students forge relationships and developing a sense of connectedness with the Senior School campus.

The fact that the students in Year 8 have been drawn together from different backgrounds and that they are not grouped according to ability means that there are some implications regarding their academic development which need to be considered. Some students have more experience in mathematics than others and/or are able to move at a faster pace through the Year 8 course. Separating these students from the rest of the cohort and putting them into a “top class” would mean splitting up the form classes and this would be antithetical to the pastoral aims of the College. Instead, a strategy has been in place for several years which attempts to cater for the needs of those Year 8 students who require a challenge.

The method of providing suitable experiences in mathematics for these able students is to run mathematics enrichment classes for them. Students who are identified as being in need of further challenges are extracted from their usual classes for one lesson per week. During these lessons the students go to a separate room and are taught by a teacher who focuses on mathematics enrichment. Mathematics enrichment in this context means that the students gain experience in mathematics at a greater depth without necessarily moving on to new content. Mathematics enrichment is distinct from mathematics acceleration in this respect. Mathematics acceleration would involve the students in moving on to higher year levels of content, whereas mathematics enrichment has an emphasis on problem solving at a higher level of difficulty than they would normally experience in their usual mathematics class.

Selecting students for mathematics enrichment lessons

Two main criteria are addressed when selecting students to be offered a place in the mathematics enrichment classes. First, the student should be an able and enthusiastic student of mathematics. Second, the student should be judged to be one that is able to miss one lesson out of six per week from their usual mathematics class and be able to catch up easily. In assessing the suitability of the students in relation to these criteria, a number of sources of information are drawn upon. These include results in standardised tests, results from class tests in mathematics, and the opinions of the students' mathematics teachers.

All the Year 8 students are tested at the start of the year using a standardised test which is administered by a consultant educational psychologist. The test is based on the work that was pioneered in the 1970s on the structures of the intellect (Guilford & Hoeptner, 1971). The test is designed to measure the learning potential of the students with respect to various types of intellect. Included in these types of intellect are those that would normally be associated with mathematics such as spatial sense and number sense. Students who score highly in these categories are often selected to take part in the mathematics enrichment lessons.

As well as scoring highly in relevant parts of the structures of the intellect test, students who are selected for maths enrichment would also usually be scoring in the top ten per cent of the cohort in common mathematics class tests. The questions in these tests are divided into two categories. Some questions focus on knowledge and procedures whilst others are concerned with applying mathematical skills to solve unrehearsed problems. Performance in the problem solving questions would be deemed to be more significant in relation to selection for maths enrichment.

The most important source of information in assessing the suitability of students for involvement in mathematics enrichment is their mathematics classroom teacher. This teacher has a first -hand relationship with the student. In some cases, the mathematics classroom teacher will be aware, for example, that a student who is performing highly in assessment may be working very hard to do so and may not benefit from missing out on one lesson of the normal mathematics lessons per week. On the other hand, the classroom teacher may know of a student who is not "top of the class" but who in class discussions is impressive in their ability to cope with higher order concepts. Sometimes the classroom teacher can intuit that students such as these may need an extra challenge to ignite their interest and improve their overall performance.

At the end of the fourth week of Term 1 in Year 8, consultations take place between the head of the mathematics department and the individual mathematics classroom teachers in order to decide on a list of students who would be suited to, and who would benefit from being offered the opportunity to take part in mathematics enrichment lessons. By this time the students have settled into their classes and formed relationships with their teacher and their peers. Sufficient quantitative and qualitative data is also available by this time based on which sensible informed decisions can be made.

Having selected students for the mathematics enrichment lessons, the students and their parents are informed of the opportunity and with everyone's agreement the lessons commence. The students are divided into two groups. The reason for this is purely due to timetabling constraints. The Year 8 students all have six lessons of mathematics per week but the classes are not all timetabled at the same time. Separate rooms are then allocated for the mathematics enrichment lessons and the students involved go to these rooms instead of their normal mathematics classroom.

Students who participated in the study

The subjects in the research project were students who attended the mathematics enrichment lessons and who had therefore gone through the selection process outlined above. In order to preserve the anonymity of the subjects, the following pseudonyms were used:

Female
Vera, Theresa, Deborah, Sue, Liz, Angela, Jaimie, Beth, Linda

Male
Joe, Roger, Will, Simon, Steve, George, Fraser, Robert, David

In the thesis, students from Year 10, Year 11 and Year 12 are also mentioned. These students were not subjects of the research, however, and I did not collect data directly from them. Where relevant, my experiences with these students were written down and described.

Teaching

The enrichment lessons took place in a technology-rich learning environment in a classroom which was separate from the normal mathematics classrooms. I had a tablet laptop which I took to the classroom and connected to a digital projector. The tablet laptop had Microsoft One Note software installed which allowed the user to write on the screen of the computer using a stylus. Screen capture software was also installed which allowed some recordings to be made of students working on the computer.

A class set of ClassPad calculators was also available. There were sufficient calculators in the set for the students to be able to use one each during the lessons. The ClassPad calculators were operated by the students using touch screen technology with a stylus. The tablet computer was installed with an emulator (computerized version of the calculator) which meant that the calculator could be displayed via the digital projector in the classroom. Using this equipment, I could demonstrate the operation of the calculator to the students.

Tasks

In the project, a series of tasks were used that were intended to relate to various parts of the Dimensions of Learning framework. These included a series of geometry tasks, a set of linear equations for solving and a collection of short competition type problems. All of the tasks are included in the appendices at the end of the thesis. Table 1 below shows the relationships between the tasks and the Dimensions of Learning framework.

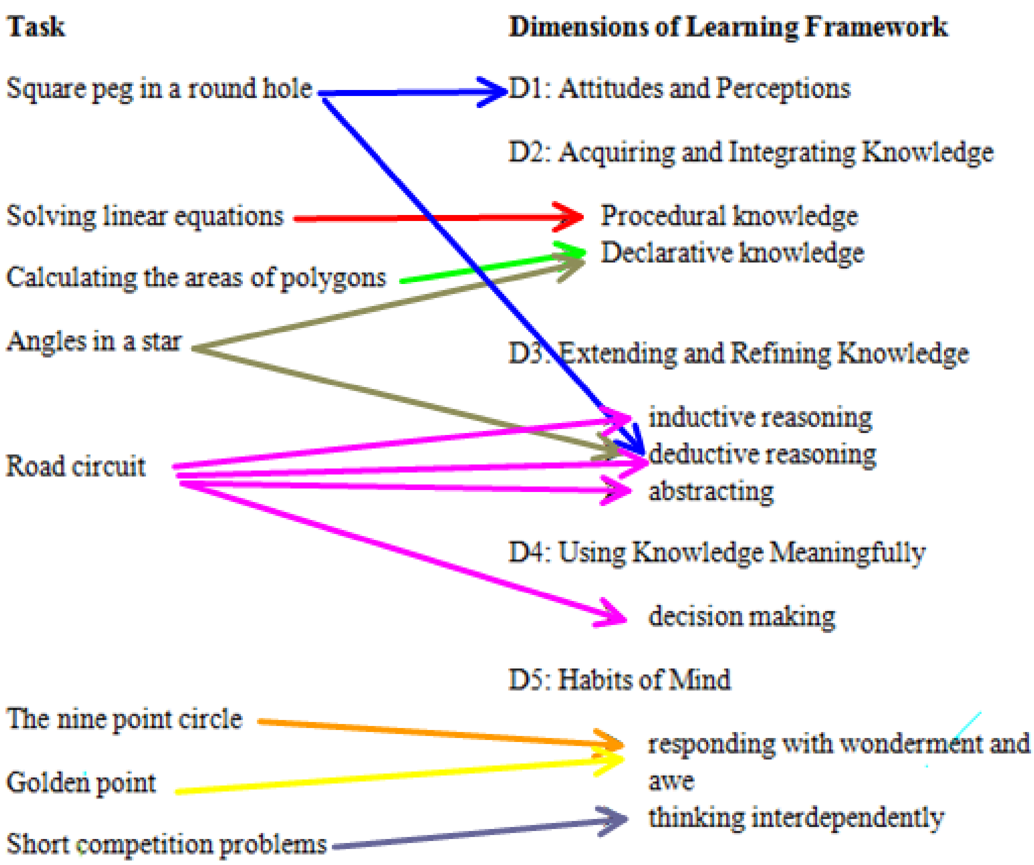


Table 1: Mapping between tasks and DoL framework

Procedures

Teaching methods

A variety of teaching methods were used in the project. These included (1) teaching all the students at the same time from the front of the class; (2) walking amongst the students as they worked and offering occasional help; and (3) working as a co-learner with the students. When adopting a teacher-directed approach, I would place the tablet computer on a lectern and teach from the front of the class facing the students. From this position I could orchestrate class discussions on problems, writing on the tablet screen as required. From this standpoint I could also use the calculator emulator on the computer to provide the students with demonstrations and instructions on the use of the calculator.

At other times I would allow the students to work through problems at their own pace conferring with one another if they so desired. When the students were working in this way, I would walk amongst them providing occasional assistance which could either be to do with mathematics or with the operation of the technology.

When I adopted the role of a co-learner, this would usually be because I had encouraged individual students to demonstrate to the others how they had gone about solving a problem. This promoted an understanding in the group that problems can be solved in a variety of ways. Occasionally, I would play the part of a co-learner in exploring a problem along with the class which I had not rehearsed personally.

Data collection

The data that was collected was qualitative in nature and was gathered in several forms. These comprised of my teacher reflection notes, audio recordings of semi-structured interviews with individual students or pairs of students, audio recordings of lessons, audio recordings of focus group interviews with groups of students, and screen capture recordings of students working on problems using technology on computer with an accompanying audio recording of their voices.

My teacher reflection notes were written up after lessons. They expanded on notes taking during lessons and also included ideas that emerged after contemplating what had taken pace in the lessons. A set of twelve teacher reflections were made in this way. The semi-structured interviews with individual students or pairs of students took place when the students were working on problems in the classroom. A small digital recorder was used for this purpose. A set of ten semi-structured interviews were recorded. It was useful that these interviews took place in the natural setting of the classroom because this allowed for more spontaneity. It also meant that the questioning could be meaningful to the students since it was directly related to work that they were doing at that instant.

The audio recordings of lessons captured the sound of my voice as well as the questioning and answering that took place during the lesson. These recordings, of which eleven were made, were particularly useful for lessons that involved a considerable amount of teacher direction from the front of the classroom. The focus group interviews, four in all, were recorded either outside of lesson time or aside from the class. One of these interviews involved a group of six students and the other interviews were carried out with small groups of two and three students. These interviews were valuable for gathering data about the students' general perceptions about their learning in mathematics and the use of technology.

The screen capture recordings were made in the classrooms. Seventeen of these recordings were made. The students used the touch screen capability of the tablet computer to work on problems. The screen capture program which had been installed on the tablet computer recorded what the students were doing on the computer screen and simultaneously recorded what they were saying about their work. For the most part, the students used the ClassPad emulator (computer version of the ClassPad calculator) when making these recordings.

An NVivo database was used to store all the data in its various forms. Transcriptions of the audio recordings were typed out and saved in the database. The screen capture recordings were also stored in NVivo.

Data Analysis

Grounded research

The data analysis was conducted initially using a grounded theory approach which entailed beginning with the raw data itself. (Corbin, 2008). The first step in analysing the data involved reading it through in order to become familiar with it. At this preliminary phase of analysis the analytical tool of *questioning* was being employed at a basic level. Beginning in this way I endeavoured to avoid any regimented way of thinking which might cause me to overlook information that could be embedded in the data in its raw form. I probed the data with loosely formed *sensitising questions* (Corbin, 2008) such as "How are the students responding to these tasks?" and "In what way am I behaving as a teacher in this context?". Ultimately, I hoped that I would be able to associate the data with higher level categories corresponding to the five dimensions of the Dimensions of Learning framework. Initially, however, I wanted to remain open to the nuances of the data in the raw.

Coding the data

My first steps in coding the data involved using conceptual labels. These codes were not In-Vivo codes since they were not exact words used by the subjects but rather they were low level conceptual labels of my devising which gave some meaning to the data. In order to make connections between concepts, I used the analytical tool of *making comparisons* by considering the similarities between various episodes described in the data. Differences between episodes also became apparent through this approach. The analytical tool of *looking for the negative case* was also helpful in seeking to explain why some students exhibited negative attitudes and perceptions towards technology in contrast with the more positive attitudes and perceptions of other students.

During the process of reviewing the data a variety of open codes were developed that represented chunks of the data. These open codes included codes such as "cooperation" and "support from technology". When progressing to the process of axial coding, it became apparent that the five dimensions of the Dimensions of Learning framework provided very suitable axial codes. For example, the open codes of "awesome", "boredom", "engagement" and "need for challenge" related very well to Dimension One Attitudes and Perceptions. It became clear that the analysis of the research could be more efficient if the Dimensions of Learning framework was utilised to categorise the data.

Making use of the Dimensions of Learning framework in this way is consistent with the view of Corbin (2008) who acknowledges that the use of a predefined theoretical framework may be useful after the researcher has studied a topic and finds that it is closely aligned to a pre-existent framework. The departure from a purely inductivist approach to data interpretation may relieve the researcher from having to reinvent the wheel. This combination of grounded theory and existing theory can be productive and efficient (Goldkuhl & Cronholm, 2003).

CHAPTER 5 ANALYSIS AND DISCUSSION

In this chapter the analysis of the data and relevant discussions are presented together. This allows the connections between the analysis and the discussions to be conveniently connected within each of the dimensions of the Dimensions of Learning framework. A variety of tasks are analysed and discussed in relation to the Dimensions of Learning framework. This analysis and discussion addresses both the design of the tasks and their implementation. Using the Dimensions of Learning framework as a reflective tool, I was able to paint a picture of what took place in the learning environment when the tasks were implemented. Having done this, it was easier for me to see where technology fitted in to the overall picture. An image then emerged of the contributions that technology can play in the process of learning mathematics in my classroom. Examples of these contributions are described following the structure of the Dimensions of Learning framework, looking at each of the five dimensions in turn.

Beginning with Dimension One, the perceptions that the students expressed about the use of technology were sometimes negative and sometimes positive. The students' perceptions were considered in relation to their general experience in learning mathematics, however, and not just specifically about the use of technology for mathematics. This meant that the perceptions about the use of technology could be considered in terms of their relative importance when compared with perceptions about other aspects of learning.

As far as Dimension Two was concerned, the distinction between declarative and procedural knowledge was an important factor in the way that technology was used. Ways of matching the use of technology to support the development of these types of knowledge emerge from the analysis and discussion. Dimension Three, which concerns the extension and refinement of knowledge, is explored in part by encouraging the Year 8 students to reach higher levels of understanding but also by giving the problems to older students and then examining the methods that they used to solve the problems.

The analysis and discussion concerning Dimension Four: Using Knowledge Meaningfully focused on the process of decision making and appropriate uses of technology. Two particular habits of mind are analysed and discussed in relation to Dimension Five. The habits of mind involved are *responding with wonderment and awe* and *thinking interdependently*.

Dimension One: Attitudes and Perceptions

Introduction

As was explained in Chapter 1 of the thesis, Dimensions Two, Three and Four of the Dimensions of Learning framework are all concerned with the core business of gaining and using knowledge. It is understood from the framework, however, that learning does not take place in a vacuum. There are other influences which come to bear on learning which are not directly related to the knowledge itself. One source of these other influences come from the attitudes and perceptions that the students hold about the learning process. Another source is the ways of working, or Habits of Mind that the students employ when they are engaged in learning. This model in which learning knowledge is seen to be influenced by these factors is depicted in Figure 1 shown below which was previously given in Chapter 1. In the diagram the knowledge dimensions are symbolically placed in a circle set against a backing which incorporates Attitudes and Perceptions and Habits of Mind.

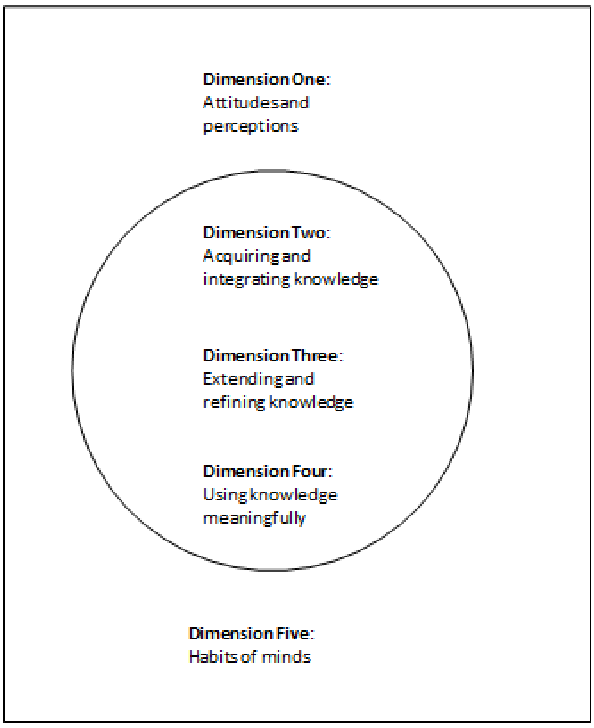


Figure 1: The Dimensions of Learning Framework (Marzano, 1992)

In Dimension One of the Dimensions of Learning framework it is recognised that there is a need to address the affective aspects of learning. The ways that students perceive the work they are asked to do and the attitudes that they subsequently develop have an impact on their learning. Students glean perceptions about learning mathematics from the learning environment that they work within. This learning environment has many facets including the tasks that are set, the way that the students are arranged and interact, the way that the teacher behaves and the way that technology is used.

Students' perceptions about their mathematics lessons

All Year 8 students at the school were timetabled to spend six lessons per week studying mathematics. The students in the case study, however, were taken out of their various normal mathematics classes for one lesson per week and brought together to work on mathematics enrichment. A recurring theme that arose from the data was that students had been experiencing boredom in their normal mathematics classes. The students felt that the work they were given in normal classes was easy and this made them bored.

> Joe: Yeah, seeing as well we really need to be extended I guess, 'cause like the usual work is a little bit easy.
> David: And boring
> Theresa: Yes
> [Focus group interview 18th November 2009]

They found that when the teacher set work for the class, they would finish ahead of their class-mates. They then found themselves with time on their hands and tedium set in.

> Robert: You finish the work …and we need the chance to work on something
> Teacher: So you would finish early and…
> Theresa: And you just sit there
> [Focus group interview 18th November 2009]

At times this could lead to some interesting behaviour. Robert confided in me that he had spent one of his normal mathematics lessons carrying out an investigation of his own devising into a way of remembering pi to a high number of decimal places.

> Robert: Sir, do you want me to show something pretty awesome that I spent like two hours working out… well just a maths lesson. You go 21.99114857divided by 7… it equals pi to the nearest 10 digits

Teacher: Wow
Steve: He worked that out during like some sort of maths test
Teacher: Hold that thought there {I finished helping another student}
Teacher: Now tell me again
Robert: If you go 21.99114857 divided by 7 it equals pi to the nearest 10 digits
Teacher: Oh…how did you figure that out?
Robert: Aw well I did it for like ages. I just guesstimated it 'cause I knew that 22 was close and I just took it down and then went up and down and all this kind of stuff. But then I realized if you just go pi times 7 and you get the same answer. Yeah that's how I worked out that…where's the other piece…{fumbles through jacket}… yeah, I wrote this down yesterday… it's in there somewhere…it's been through the wash…Well I worked out that…oh there it is…{finds a crumpled scrap of paper}…197.9203372 divided by 63 equals pi…but the thing is I think that number will be much more easy to remember than pi…so I just remember…let's see if I know the first one off by heart...err it's 21.9911 divided by 7 equals similar number to pi but it's not because you only have to add the last 4 digits of this to make it exactly pi to more digits
Teacher: So you've found a number that's easier to remember…
Robert: Yeah
Teacher: …that you can use to work out pi to a reasonable level of accuracy?
Robert: Yeah, 'cause 9911 is easier than 3.1415…I don't know the rest
Teacher: Interesting!
[Lesson transcription 24th November 2009]

Situations like the one above suggest that these students in their state of boredom were seeking a challenge. This sentiment was echoed by others.

Joe: We need to *challenge* our minds
[Focus group interview 18th November 2009]

Will: Sometimes class work is a bit easy so extension is really good for some challenges.
[Focus group interview 25th November 2009]

The students expressed their appreciation of the enrichment lessons. They viewed these lessons as opportunities to be challenged mathematically.

Theresa: Because in normal maths you just don't have to…like you already know so you just do it but here you actually get to think and stuff
[Focus group interview 18th November 2009]

Robert: It's better because you just get bored in class
George: Class is quite repetitive and it's good to get a new thing every lesson…
[Focus group interview 24th November 2009]

David: And this is the only time I like maths…usually, I don't usually like maths…
[Focus group interview 18th November 2009]

It seemed that the students were not only seeking challenges but they were also desirous of the company of other students with like minds.

> Will: Oh, it's really good because we get to work with people like of the same ability
> [Focus group interview 25th November 2009]

> Theresa: I think that maths extension is just generally better than maths because like everyone here is like pretty much the same and we all get along and stuff…and like we're not just all bored in class. We are actually having fun doing what we are doing.
>
> Joe: So that's basically the thing to it. We are working with people who can work with us and we are working at solutions to problems which will challenge us
> [Focus group interview 18th November 2009]

The principal cause of boredom that the students experienced in their normal mathematics lessons was that the teaching was not differentiated to suit their needs. In a large class of thirty Year 8 students it is no easy task to deal with all the students' individual needs. Essentially, the students in the class were offered the same thing at the same time. This led to a situation where students who were able and motivated would complete the set work early and then find themselves at a loose end. Understandably, they developed a perception of mathematics lessons as being boring.

It emerged from the data that the effect of this boredom manifested itself in three main ways. First, the students were seeking a challenge. They did not want to just sit there in class waiting for new work or put their pens down and listen to explanations that they did not need. Second, the students were pining for interaction with peers who matched their ability in and enthusiasm for mathematics. Third, the students wanted the freedom to work at their own pace. They felt frustrated when having to work at the same pace as everyone else in the class.

> David: …when someone gets stuck the whole class has to stop…which I don't like
> [Focus group interview 24th November 2009]

The students appreciated the enrichment lessons because they were able to move ahead at their own pace.

> Will: … we don't have to wait for the rest of the class to catch up. We can just get on to some harder work.
> [Focus group interview 25th November 2009]

David: You can just move along at your own pace…do you know what I mean?
[Focus group interview 25th November 2009]

Theresa: Like you do it at your own pace.
[Focus group interview 18th November 2009]

A careful examination of the data reveals the interesting fact that although the students' perceptions about their normal mathematics lessons were adversely affected by boredom, their attitudes to mathematics as a subject were still positive. Their negative perceptions about the learning environment had not translated into poor classroom behaviour. The love of the subject was still strong. This was exemplified by the actions of Robert who, having finished a test early, set about inventing a method for memorising pi. He did not just doodle. As another example, consider the words of David who said, "…I don't usually like maths". Arguably, these words could be translated to mean "… I don't usually like the mathematics learning environment that I find myself in" for this was the same student who leapt out of his chair in excitement pointing to the screen when he saw an interesting geometric construction appear. David was passionate about mathematics.

The students said frequently that they felt the need for a challenge. This was expressed directly in the responses they made in interviews. It was also expressed indirectly in situations such as the one in which Robert was inventing a method of memorising pi. Not through words but through his actions, Robert made it clear that he needed a challenge. If we examine his investigation closely we see that his method of memorising pi was to find two numbers that would be more easily memorised than pi but which would multiply together to give a close approximation to pi. This type of thinking about numbers has links to applications in encryption. Robert's creative thinking is impressive. Teachers should foster this type of thinking in their students. Students such as these need a challenge to alleviate their boredom and satisfy their thirst for learning. But also, we teachers need them to be challenged so that their talent can be developed and so that society can gain from this valuable human resource.

From the interview data it emerged that the students had positive attitudes and perceptions about the enrichment lessons. They appreciated the challenges afforded to them through the problem solving tasks. They enjoyed being able to interact with others who shared their enthusiasm and passion for mathematics and they were grateful for the freedom to work at their own pace. The part that technology had to play in this situation is of particular interest in this study and will now be considered in more detail.

Students' perceptions about using technology

Positive perceptions

The students expressed many positive attitudes and perceptions about the use of technology for doing work in geometry. They were impressed by the accuracy which they could obtain when they were using the technology to make geometric constructions. The ease with which they could make these constructions using the technology was also appealing. They were responding positively, therefore, to the fact that the calculator could alleviate them from the painstaking work that would have been required using pencil and paper. The effort they had to expend to acquire the procedural knowledge required to operate the calculator was well worth the benefits that they accrued in terms of speed and accuracy. This was especially true when they could use the calculator to animate a construction.

The students commented on the advantages of making geometric constructions on the calculator as opposed to using by hand (pencil and paper) methods.

> Teacher: How do you feel about doing geometry on a calculator rather than on paper?
> Steve: It's different
> George: It's easier
> Fraser: It's better.
> Steve: It works it all out.
> George: It's all set up for you and you can just choose the dimensions
> Fraser: It's easier 'cause you don't have to measure everything
> [Focus group interview 24th November 2009]

> Roger: Well you can accurately spatialise any geometry and it's quite useful
> [Focus group interview 18th November 2009]

> Teacher: So what sorts of things can you do on the ClassPad on the geometry part of it that you found useful.
> Roger: Questions that require you to deduct angles
> Teacher: Mm, mm right
> David: And shapes…it's more exact
> Joe: So basically…
> Theresa: It's just cooler 'cause like you get to see it work

> Joe: It's lot easier too because you can animate it to cue to different points. I know we had a question involving a clock last time we used the geometry to find out what angle it would be and it's a lot easier than ruling out all the different angles
> Theresa: Yes it is
> [Focus group interview 18th November 2009]

Joe offered the explanation that the brain works quicker than the hand. It seems that drawing constructions by hand would have slowed him down whereas the calculator could keep up with the speed of his brain.

> Joe: Also do you know the expression "the brain works quicker than the hand"?
> Teacher: Sounds good
> Joe: Basically I found that it's true because like you can do all the equations and all that but then you have to write it all down. Like with tools like the geometry for instance, basically writing it all down is quick, easy, it's…it's just there, and then you are still doing the same problem. You are still putting the same amount of effort into it but the thing is you can do it quicker because your brain's working rather than your hand.
> [Focus group interview 18th November 2009]

Since the use of the calculator speeded up the procedural work involved in tackling the geometry problems, this allowed more time for the students to focus on acquiring declarative knowledge. Joe articulated this in his own words when he talked about the brain being quicker than the hand. He perceived his efforts in thinking about the problem to be paramount. In his view, the procedural work in constructing figures and taking measurements was subservient to higher order thinking. Moreover, he felt that because he could interact speedily with the technology he was functioning mentally rather than physically. He was using his brain rather than his hand. He felt that this actually speeded up his thinking. In the language of the Dimensions of Learning framework, technology was not only speeding up procedural work, which is helpful but not of itself surprising, but it was also speeding up the acquisition of declarative knowledge.

Several students expressed their liking for the way that technology can assist them in the steps of their working and the location of errors.

> David: The computer knows. It also helps you it actually shows you the working out. Because in class sometimes our teacher just skips some parts assuming we already know them but yeah sometimes we don't
> [Focus group interview 18th November 2009]

> David: And the calculator's also good because it like corrects you in I can't remember what it is but…
> Joe: Algy

David: Algy, yeah. It has a bit where it corrects you
Teacher: Yes
Theresa: And it's good because it tells you like if your method was right even if your answer was wrong
David: It shows you like where you went wrong
[Focus group interview 18th November 2009]

The positive comments from David and Theresa about the ClassPad indicated that they perceived it to be a source of support. This support came in the form of assistance with steps in their working and in finding their errors. There was also a suggestion that technology had an elevated status which at times was even above that of their teacher. A telling phrase from David in this regard was "The computer knows." This suggests that technology was not only perceived to be helpful but it could also be regarded as an ally worthy of their respect.

Students also made positive comments about the enjoyment they experienced in using the calculators and the benefits in terms of image with their peers.

David: And I like using the computerlibs
[Focus group interview 18th November 2009]

Joe: Besides, it makes us feel good
[Focus group interview 18th November 2009]

Vera: Two really pointless other reasons but good none the less reasons why the ClassPad is good. First of all if you don't want to be just stuck in class and second forth once you get out of extension, all your friends that aren't in extension ask "how was it?" and you can just say...yeah we played around and it was such a good calculator
David: And they're like that's gay, you're all nerds and stuff like that but yeah…
[Focus group interview 18th November 2009]

In these comments made by Joe, Vera and David they expressed their feelings of enjoyment in using the ClassPad. This was helpful in terms of their affective learning and was evident in their engagement in their learning. Only those students who attended the enrichment lessons were able to access the calculators. These students felt that this made them "feel good". It afforded them some status. Interestingly, in the data that was collected there was no suggestion that being selected for enrichment lessons was a source of status amongst peers. It had emerged from the data that alleviation from boredom was more significant to the students than status. In fact, there was a suggestion that attending enrichment lessons could even have been regarded as undesirable in terms of teenage image, or, to use the students' vernacular, it was "gay" and for "nerds". But, as Vera pointed out, the fact that they were allowed access to an attractive calculator gave them a rebuttal to any playground jibes about image.

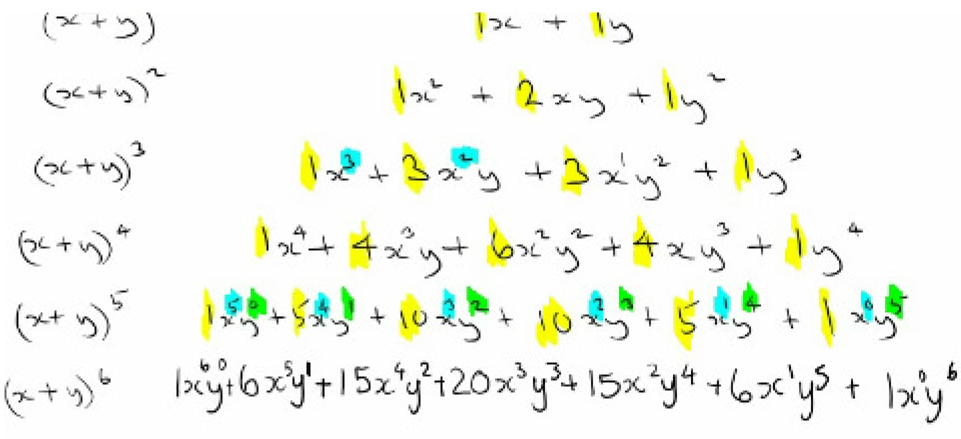
Negative perceptions

Not all students were enthusiastic about the use of technology. A few students expressed reservations about working with the calculator. One of the students, Linda, who had missed several lessons which had been devoted to instruction about and use of the calculator due to her sporting commitments was doubtful about its value.

> Teacher: And what about using the calculators?
> Linda: I don't know. I don't actually…
> Teacher: You don't? What would you rather do…say with the geometry? How would you rather do it?
> Linda: I don't really know to be honest. I don't know. I don't think I've figured how to use the calculators properly yet so it doesn't really work yet
> [Focus group interview 24th November 2009]

Linda's disinclination to use the calculator continued to be evident long after the enrichment lessons had concluded for the year. Even when she entered higher year levels in which she and all her class-mates were issued with a ClassPad calculator, Linda did not gain confidence in the use of this form of technology. She carried an older model scientific calculator with her in her pencil case at all times and used it as her main calculator. She only used the ClassPad when she could not avoid doing so, for example, when the course required the use of the graphing calculator application in the ClassPad.

Linda's negative reaction to the use of the ClassPad calculator did not seem to be linked to any kind of negative response to the use of technology in general. For example, she had shown competence and enthusiasm in working on the tablet computer. With this form of technology she had shown perseverance and aptitude. This was evident when she worked on an extension activity in which she displayed her mastery of the binomial expansion. Despite initial setbacks, she persisted and, using tablet technology, she displayed her ability to write out a binomial expansion to the power of six on a computer screen whilst simultaneously providing a voice over explaining her method.



[Screen capture 25th August 2009]

Linda's negative reaction to the use of the ClassPad calculator could not be attributed to a lack of enthusiasm for mathematics as a subject. She was a high achieving "A grade" student in assessment and continued to be so in subsequent years. She participated in extra activities such as "Maths Teams Challenges" and was awarded Distinctions in national mathematics competitions. Nor did it seem that her negativity towards the ClassPad was associated in any way with her relationships with her peers or the teacher. Her close friend Angela was a devotee of technology. In an interview session in which Linda and Angela took part, Angela was very positive about the use of technology and the ClassPad calculator in particular. Angela remarked,

> Angela: I just like it because you get to interact with the technology and I like technology a lot. If we continue it and we get a calculator to take home that'd be great. [Focus group interview 24th November 2009]

Linda's negativity towards the ClassPad did not seem to stem from any dislike of her teacher. Evidence of this came in the form of a birthday card which she made by hand and presented to me. Inside the card she had painstakingly written out the binomial expansion up to the power 24.

Strangely, however, despite the fact that Linda had a love and enthusiasm for mathematics, and despite the positive attitudes of her friends and teacher towards the ClassPad calculator, she continued to be averse to using it. The only reason I could speculate on to explain her negativity was that her confidence had been affected by missing out on the initial lessons with the calculator. When the calculator was first introduced to the group, Linda was unable to attend due to her commitments to training for diving. When she returned to the maths enrichment lessons her classmates had already gained considerable experience in the use of the calculator. Despite being a resilient student in other ways, she never seemed to recover from this setback.

Some other students who were competent in the use of the calculator were not always enthusiastic about the use of this technology. Deborah and Liz expressed reservations about the use of the calculator. This arose when they were using the algebra error locater software, Algy (Harradine, 2011), in their work on solving linear equations. Deborah and Liz had already acquired considerable skill in solving linear equations using pencil and paper. When questioned about the use of the calculator to help with solving linear equations, the tone of their voices was polite but unenthusiastic.

> Teacher: Now have you used the calculator to solve some equations
> Liz: Yeah but I've mostly done it on paper, so yeah
> Deborah: Me too
> Teacher: What did you prefer?
> Liz: Well, it's good to check it on the calculator to see if it's right but normally first I do it on the paper – so yeah.
> Teacher: What about you? {addressing Deborah}
> Deborah: Me too
> Teacher: Alright, so how much algebra have you done in class already?
> Liz and Deborah together: A lot
> Teacher: Lots of algebra?
> Liz and Deborah together: Yeah
> [Focus group interview 11th August 2009]

The source of the negative reaction from Liz and Deborah appeared to be that they could see little purpose in using the calculator to help them solve linear equations when they were already proficient in doing this with pencil and paper.

Yet another student, Will, expressed negativity about the use of the ClassPad for work that he felt he could do quite easily without technology.

> Teacher: Sometimes we've used technology mainly the ClassPad calculators. What do you think about using that sort of technology?
> Will: I don't know, they're a bit annoying sometimes because sometimes we use them for problems that you could just sort of work out on paper, but when you use them to work out like bigger sums and stuff they're good because you know the calculator's got it right
> Teacher: Right, so there are some things that you can do by yourself anyway?
> Will: Yep
> [Focus group interview 25th November 2009]

As far as Will was concerned, technology was a useful support when complex computations were involved. The reliability of technology was helpful in these situations because it would not be as error prone as calculations carried out using pencil and paper. Will was critical, however, of the use of technology for tasks where he felt that the use of technology was unnecessary. In fact in these situations he experienced the use of technology as a source of irritation. It was “annoying”.

This interpretation of Will’s perspective was supported by observations of his disinclination to use the calculator to help solve simple linear equations. This contrasted with his unbridled enthusiasm when pursuing the golden number, using a spreadsheet, at the conclusion of which he cried,

> Will: And would you look at that! We’ve got the Golden Ratio!
> [Lesson transcription 25th August 2009]

Will preferred to be able to select when to use technology and when not to. This was evident when the students were working on a set of problems including the one shown in Figure 27 below.

4. In the figure the dots form a square 4 x 4 unit grid,
ie the adjacent horizontal and vertical dots are 1 unit apart.
What is the area of the triangle ABC?

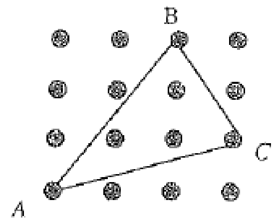


Figure 27: Short problem ("School Mathematics Competition," 2004)

I noticed that some of the students had used technology to them solve this problem. They had drawn the figure on the geometry screen of the calculator and used the calculator to display areas. When I asked Will if he was interested in seeing how technology had helped with this problem or how it could be used to check the answer, he was not interested and replied in a definite tone,

> Will: No, I’m alright thanks. I’ve already figured that one out.
> [Lesson transcription 1st September 2009]

Allowing for the interests and abilities of individual students

It became apparent how vitally important it was to take into account the interests and abilities of the individual students when incorporating technology into lessons. In situations where I had imposed a structure on the lesson which involved all the students doing the same tasks and using technology in a prescribed way, negative responses from some of the students had emerged. In lesson time devoted to the use of the algebra error locater software, Algy, for example, there were mixed reactions. Whilst Theresa was positive about this use of technology and found it helpful, Deborah and Liz had been put off by what they perceived to be an unnecessary use of technology. Roger had also expressed a desire to go beyond the prescribed use of the technology when he asked about the computer algebra capabilities of the calculator. At other times, however, when the students were using the calculators to do work involving geometry, Deborah and Liz displayed positive attitudes and were engaged in using the technology. Meanwhile Will was irritated by any trivial use of technology but used it avidly to explore mathematical ideas.

On reflection, it seemed that there was a danger of negative consequences when all the students were made to use the technology in the same way at the same time because sometimes individual students did not need the support of the technology for certain tasks. Two possible manifestations of these negative outcomes could have been first the *interest reversal effect* (Reed et al., 2009) and second the *expertise reversal effect* (Kalyuga et al., 2010). The interest reversal effect, which involves students becoming undesirably preoccupied with the workings of the technological tool and hence diverted from the main direction of the learning, was not evident in the data. This is not to say that it could not have become an issue to consider. Robert, for example, had given a detailed account of his exploration into a method of estimating pi which he confessed that he had become absorbed in after finishing a class test early. If Robert was in possession of an advanced technological tool in such a situation, we may speculate that he could have become diverted by its workings.

Some more definite connections could be drawn, however, between the data and the expertise reversal effect. The expertise reversal effect is a detrimental effect caused by an inappropriate use of technology in instruction. It arises when able learners are provided with more scaffolding than is necessary. This causes these learners to be distracted, leading to an inefficient use of their working memory and a retardation of their progress. Will’s annoyance over the use of technology for problems that he felt could have been tackled more easily without technology could possibly have been associated with the expertise reversal effect. Deborah and Liz's polite yet indifferent tone when questioned on the worth of the use of technology to support them in solving linear equations could also have been linked to this effect. These students were quite possibly experiencing a painful distraction in their learning, which according to the expertise reversal effect, the brain of the able learner cannot ignore despite the redundant nature of the extra information (Kalyuga et al., 2010).

From all of these observations and reflections, a picture emerges of the students' attitudes and perceptions towards their learning environment. The perception that the students expressed most strongly was that their normal mathematics lessons were boring. The positive perceptions about the enrichment lessons were to a large extent an expression of relief from this boredom. The students appreciated the challenge, the intellectual interaction, and the freedom to move at their own pace, all of which they perceived to be lacking in their normal mathematics classes.

The contribution that technology made to the students' attitudes and perceptions can be put into perspective within this context. The students' feeling and thoughts about the use of technology were not the main focus of the students' perceptions overall. Certainly, there were positive perceptions about the capabilities of the technology and the support that it provided for solving problems. There were even some positive perceptions about the esteem gained from being allowed to use an advanced calculator, although these were really more to do with providing protection from negative comments from other students outside of the group. There were also some negative perceptions about the use of technology which were related to individual needs and abilities. The overriding perception, however, was that the normal mathematics lessons were boring and that the enrichment lessons provided some relief from this with the use of technology forming a part of that relief package.

Designing tasks to improve students' perceptions about mathematics lessons

In order to improve students' perceptions about mathematics lessons some careful consideration can be given to way that tasks are designed. In the second section of the Literature Review it was explained that learning improves when students perceive tasks to have value to them. There are many ways in which tasks can be designed in order to make connections with this sense of *task value*. Tasks can be constructed, for example, that are intended to appeal to students because they are linked with the students' long term goals.

Another way of enhancing this sense of *task value*, however, is to appeal to the students' sense of curiosity (Marzano, 1992). The Square Peg in a Round Hole task (see appendix 1) was designed with this in mind. In this task the initial question posed to the students is "What fits better, a square peg in a round hole or a round peg in a square hole?" The purpose of beginning the task with this question is to arouse the students' curiosity. From the outset, therefore, the task is designed in a way that relates to Dimension One: Attitudes and Perceptions.

When using this task, the link to the affective aspect of learning can be capitalised on further. This occurs when the teacher questions the students asking them why it is that people sometimes say "I feel like a square peg in a round hole?" It is pointed out to the students that people say this when they feel as if they do not fit in. Perhaps they have just arrived at a new school and do not have any friends. The final question put to the students is "Why do people say *I feel like a square peg in a round hole*. Why do they not say *I feel like a round peg in a square hole?*" By this time the students have been drawn into the problem through a mixture of emotions and they are engaged.

When implementing this task, I presented it to a group of twelve Year 8 students. I realised that it would benefit the students to engage with this problem by constructing shapes and performing calculations. Bearing this in mind, I issued the students with ClassPad calculators. By following instructions contained in the booklet of activities I had prepared for them, the students were able to use the geometry application of the calculator to construct the shapes. They could also use the calculators to calculate areas and proportions. Figure 28 below, shows the display that the students produced when comparing the areas of a circle within a square. By selecting the circle the students could see that from the calculator display that the area in this case was 59.57911 units squared. Similarly, by selecting all the sides of the square, they could see that the area of the square was 75.85848 units squared.

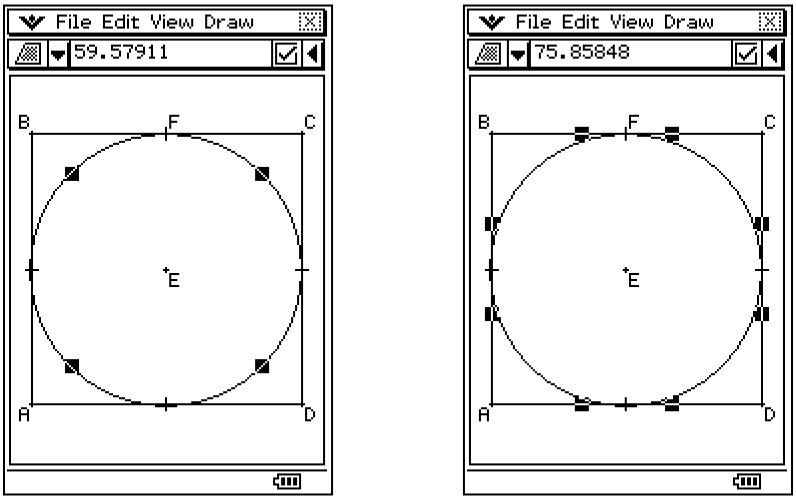


Figure 28: The area of a circle is compared with the area of the square that surrounds it

In this case, the percentage of the square covered by the circle was found to be

$$\frac{59.57951}{75.85848} \times 100\% \cong 78.54\%$$

In the example shown below in Figure 29, the student has displayed a square of area 64 units squared encompassed by a circle of area 100.531 units squared.

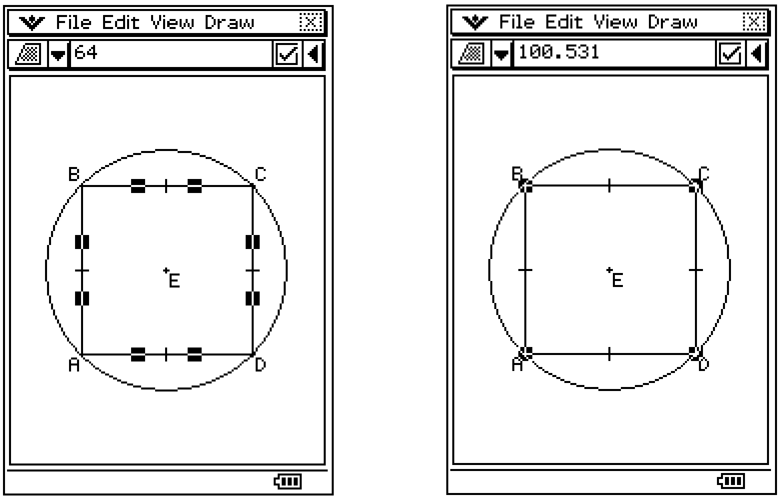


Figure 29: The area of a square is compared with the area of the circle that encompasses it

In this case, the percentage of the circle covered by the square was found to be

$$\frac{64}{100.531} \times 100\% \cong 63.66\%$$

These results suggested that a round peg fits better in a square hole than a square peg in a round hole.

This task is associated with Dimension One: Attitudes and Perceptions because of the initial intention to encourage engagement by arousing the students' curiosity. The engagement is meaningful, however. It is not based on technological wizardry. There is no shallow "WOW" factor involved. A simple question has been used as kinder to ignite the students' thinking. It is after this has taken place that the students apply the use of technology to help them find an answer to the question. The technology is introduced at an appropriate time. Also, the technology has a sensible purpose. It is meant to assist with the enquiry process into the problem. Students in Year 8 generally do not have levels of skill that are sophisticated enough to answer a question such as this using a formal algebraic method. They can, however, conduct an enquiry by drawing diagrams, taking measurements and performing calculations. The technology helps with these types of processes by making them fast, accurate, efficient and clear. This is an appropriate use of technology embedded within a sound pedagogical approach.

Dimension Two: Acquiring and Integrating Knowledge

Introduction

In this thesis, by designing and implementing tasks with aspects of the Dimensions of Learning framework in mind, the contribution that technology makes to these aspects is brought to light. In this section of the analysis, the aspect that is considered is Dimension Two. Dimension Two of the Dimensions of Learning framework focuses on acquiring and integrating knowledge. Two types of knowledge are involved and these are referred to as declarative knowledge and procedural knowledge. Declarative knowledge is the type of knowledge that is associated with knowing facts or understanding concepts, whereas procedural knowledge is about knowing how to perform tasks using skills and processes. The distinction that is made in the Dimensions of Learning framework between declarative knowledge and procedural knowledge proves to be useful when identifying the role that technology plays in the learning process (Thomson, 2010a).

Technology supports the acquisition of procedural knowledge

Technology was used to support the students in their work with solving linear equations (see appendix 7). To this end, error location software called Algy (Harradine, 2011) was installed in the ClassPad calculators. Using Algy, the students were able to enter their working steps for solving equations into the calculator. The software was then used to feedback to the students whether or not the steps were correct and pin-point any errors they may have made. This means that the technology was being used to support the students as they were in the process of acquiring procedural knowledge. The procedural knowledge in this case involved mastering the algorithmic process of solving linear equations. As explained in the second section of the Literature Review, procedural knowledge has three phases associated with it. The second of these phases is known as the *shaping phase*. In the shaping phase of acquiring procedural knowledge students practise their skills and try to eliminate errors in the procedures. It is here that the use of technology was designed to play a supporting role.

An important aspect of this shaping phase is the elimination of errors and Algy helped in two main ways. First, Algy could accurately pin-point errors. This made learning more efficient because the students were able to focus on where they had gone wrong and make a correction. Second, Algy provided immediate feedback either in locating an error or by affirming that the work was correct. In these ways, therefore, Algy provided the students with an improved way of correcting their work. They were able to access specific and immediate feedback instead of relying on just a final answer from the back of a textbook or having to wait for the teacher.

The procedural knowledge that the students were working on involved solving linear equations. As is often the case, the students had difficulty in finding their errors. If their final answer did not agree with the answer in the textbook then they would have to wade through their steps to find the error or wait for the teacher to help them. I introduced the students to the Algy application which is designed to help students carry out the steps correctly when they are solving equations (simple linear equations in this case). Algy allows the students to enter into the calculator the standard algorithmic steps that are required to solve the equation.

Using Algy, the students were able to check their working in two ways. One method produces a response from Algy which indicates whether or not their final answer is correct. This method makes no allowance for working that follows from the previous line. In the example shown in Figure 30, one line of working has been entered and checked using this method. Algy indicates that there is an error by displaying the symbol ×.

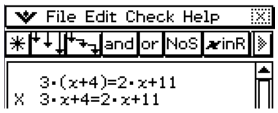


Figure 30: Algy indicates an error

In the example shown below, the student has entered four lines of working before checking the answer. Although several lines of working follow from the previous lines, each line of working has been marked with the symbol ×.

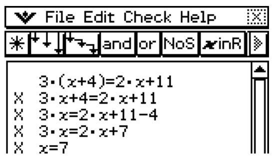


Figure 31: Algy marks all lines as incorrect

In the example shown in Figure 32, the algebra has been done correctly throughout. The calculator indicates that a particular line of algebra is correct by displaying the symbol "->". In the recording, the student in this case was heard to conclude by saying "And I got that one all right!"

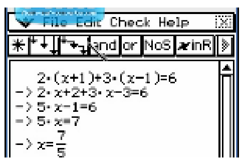


Figure 32: All correct

The students could choose another method of checking their working, however. Algy can display the symbol × where there is an error, but display the symbol "- >" when a line follows from a previous line even if the final answer is incorrect. In the example below, there is one line of working that is incorrect. This is indicated by the symbol “×”. Taking this error into account, however, the subsequent lines are marked as correctly following from the previous line.

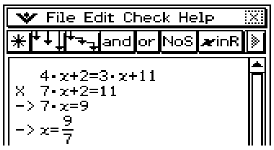


Figure 33: Algy takes errors into account

Essentially, therefore, the calculator could tell the students where they had gone wrong. In this example, the student understood that only one mistake had been made in the working. This mistake had been identified by the calculator. The students can relate this type of feedback to the feedback they would receive normally from their teachers. In a test, for example, they might expect their teacher to give partial marks rather than full marks or no marks at all. The students can obtain the feedback from the calculator in an instant, however, and in this sense the feedback from the calculator is more immediate than the teacher’s feedback. It is also interesting to note that the feedback the students are receiving is oriented to the task. The purpose of the feedback is to aid the process of learning rather than to provide normative comparisons. This type of feedback is deemed to be beneficial because it is intended to develop an interest in learning for its own sake (Jagacinski, 1992).

The students found Algy to be helpful and responded positively to using it as exemplified below.

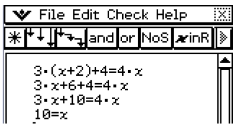
> Teacher: So what’s the difference then when you’re using Algy?
> Theresa: It helps you more
> David: It tells you where you’ve gone wrong. Like it still says if you’ve done this instead of that then it’s still right what you’ve done [Lesson transcription, 11th August, 2009]

A note of caution needs to be expressed at this point, however. It is possible for students to perform procedures successfully but in a shallow way that lacks conceptual understanding (Marzano, 1992). I received a salutary reminder of this when interviewing some of the students.

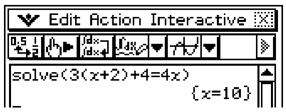
> Teacher: How do you usually know if you’ve got an algebra equation correct?
> Theresa: Emm, well you only have *x* and a number left
> David: And it’s usually a whole number

Theresa: Yeah it's usually a whole number – or a fraction [Lesson transcription, 11th August, 2009]

Clearly I needed to reflect on the depth of understanding that the students had acquired despite their demonstrated ability to perform procedures correctly. A difference in conceptual understanding amongst the students came to light when another student, Roger, who had successfully been using Algy had asked me to show him how to use the computer algebra system (CAS) that is built into the ClassPad. Whereas Algy is an application that has been specially designed to support students as they carry out algebraic procedures, a CAS can provide the student with an answer without displaying any intermediate steps. This distinction is illustrated below using the equation $3(x+2) + 4 = 4x$ as an example.



When using Algy, the user inputs the solution to the question which can then be checked by Algy.



When using the CAS, the calculator displays the answer

Figure 34: The difference between using Algy and using a CAS

I showed Roger how to use the CAS and after he had used it to solve an equation I asked him questions about what he had done.

Teacher: How do you know that you have got the right answer?
Roger: I've done the equation and it's right?
Teacher: You can check it by hand?
Roger: Yeah
Teacher: How do you know that is the right answer to that equation...$x=2$?
Roger: I did it using Algy
Teacher: Alright, how else could you do it if you look at the original equation? How could you tell that that is the correct answer?
Roger: I could replace x with 2
Teacher: And does that work?
Roger: Give me a sec. That side is 6 and that would be 6.
Teacher: So it works out?
Roger: Yeah [Lesson transcription, 11th August 2009]

Roger had displayed a better understanding than some of the other students of what it means to solve an equation. He understood the concept of balance in an equation and used this to check his answer by substitution. He was able to display this understanding despite the fact that he did not perform the steps to solve the equation (the CAS did this for him). On the other hand, some students who performed all the procedural steps successfully still only had a shallow understanding of what it means to solve an equation. All of this served to remind me that a successful display of procedural knowledge does not guarantee a high level of conceptual understanding.

Technology supports the acquisition of declarative knowledge

In other situations, the purpose of the tasks was to encourage the development of declarative knowledge in the students. In these cases it was appropriate to allow the technology to perform any procedural work that was associated with the tasks and hence allow the students to focus on the acquisition of the declarative knowledge. In the Square Peg in a Round Hole task which was described in the previous section on Dimension One, the declarative knowledge involved was the fact that a square takes up a smaller proportion of the circle that encompasses it than does a circle of the square that it encompasses it. In the pursuit of this declarative knowledge, the students were asked to draw and find the areas of squares and circles. The technology alleviated the students of the burden of this procedural work which, although helpful to the investigation, was not considered to be the main focus of the students' learning.

Another task that was designed to use technology to take care of the procedural work for the students and allow them to concentrate on acquiring declarative knowledge was the Calculating the Areas of Polygons task (see appendix 3). The declarative knowledge that was aimed for through this task was that the area of a polygon drawn on a lattice grid can be calculated from a formula involving the number of grid points inside the polygon and the number of grid points on the boundary of the polygon. The main aim of the task was not to practise calculating the areas of polygons using traditional methods. Rather the aim of this task was to identify a pattern connecting the grid points on the boundary of the polygon and the grid points within the polygon to the area of the polygon. As was described in the second section of the Literature Review, a graphic organiser is a useful tool to aid students in the search for a generalised pattern and such a tool was used in the classroom in this task. The generalised pattern in this situation was the formula connecting the number of interior and boundary grid points with the area of the polygon. The role of technology here was to automate the procedure of finding the area of the polygon in a fast and reliable way thereby allowing the students to remain focused on the graphic organiser and the search for a generalised pattern.

When implementing this task in the classroom, I began by introducing a group of twelve Year 8 students to the geometry application of the ClassPad. Initially, I allowed the students to experiment with the geometry tools. During this time they learned how to draw points, lines, and circles, and how to make composite shapes of their own devising. They also picked up how to clear the screen and delete parts of their drawings.

Using the emulator displayed through the digital projector, I demonstrated how to display grid points on the screen and draw a randomly shaped polygon. The area of the polygon could be calculated by the calculator and displayed at the top of the screen as illustrated in Figure 35.

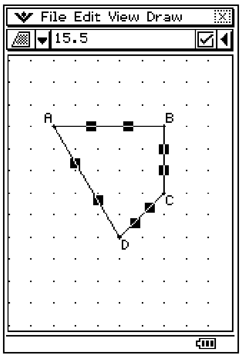


Figure 35: Calculating the area of a polygon

The students then drew polygons in the way that I had shown them and they all recorded the number of grid points in the interior of the polygon, the number of grid points on the boundary of the polygon, and the area of the polygon. A table was made on the tablet computer with students coming forward to contribute their results. The table was displayed through the digital projector. The start of the table is shown below. Could there be a pattern that connected the number of interior points, the number of boundary points and the area?

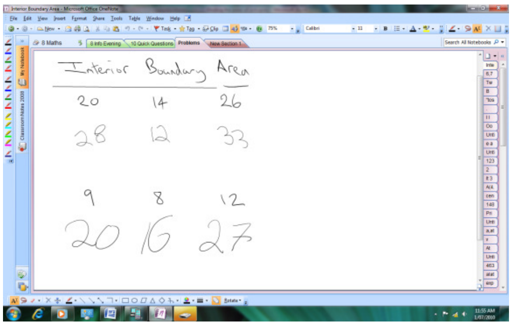


Figure 36: Students contribute results

In the class sharing discussion that took place, students made suggestions as to what the rule for the pattern might be. The first two suggestions that were made were discounted because they only fitted some of the results and not all. Eventually a suggestion was made that the rule could be "subtract one from the number of boundary points, divide by two, add the number of interior points and then take away a half". The class tested this rule against the results and found it to be satisfactory. A simpler version was also offered which was "the number of interior points add half the number of boundary points then take away one". I congratulated the class on coming up with a convincing rule and encouraged them to think about how they might prove that this rule would always work.

By the close of the session, the students had made significant progress in the organizing phase of acquiring declarative knowledge. The declarative knowledge in this case was the rule to calculate the area of polygon drawn on a grid. The results contributed by the students onto the tablet computer screen formed the graphic organiser from which a generalised pattern was developed.

The advantage of using technology in this task was that the students were able to use the technology to calculate the areas of the polygons. In this way, the students were relieved from the drudgery of calculating areas and could instead focus their attention on finding a generalized pattern. The technology aided in the organizing phase of acquiring declarative knowledge by alleviating the use of procedural knowledge that was not directly relevant to the main objective.

Another example of a task that was designed to help students acquire declarative knowledge by allowing technology to perform some of the procedural work for them was the Angles in a Star task (see appendix 3). In this task the affordances of touch screen technology were used to a greater degree than in the Calculating the Areas of Polygons task. Not only were figures able to be constructed, but through “point and drag” motions of the stylus the students could manipulate the figures and see immediate changes on the screen. In the Angles in a Star task the declarative knowledge being sought after was that the angles at the vertices of a five-pointed star will always add up to 180 degrees.

When implementing this task I introduced a group of ten Year 8 students to the problem involving the angles in a five-pointed star. The problem posed was to find the sum of the angles at the vertices of a five-pointed star. I drew an example of a five-pointed star on the screen as shown in Figure 38 below, and highlighted the angles that were to be added. I explained that the star did not need to be regular in shape. In the case of an irregular star, the angles would differ, but the question asked was what would the *total* of these angles be? The answer to this problem is that the angles in a five-pointed star add up to 180 degrees. This can be proven using deductive geometry with the aid of the exterior angle of a triangle theorem. Many students, however, would not be ready to make the leap to this level of formal proof. I expected that students would benefit from drawing stars and taking measurements in the hope of finding a generalised pattern.

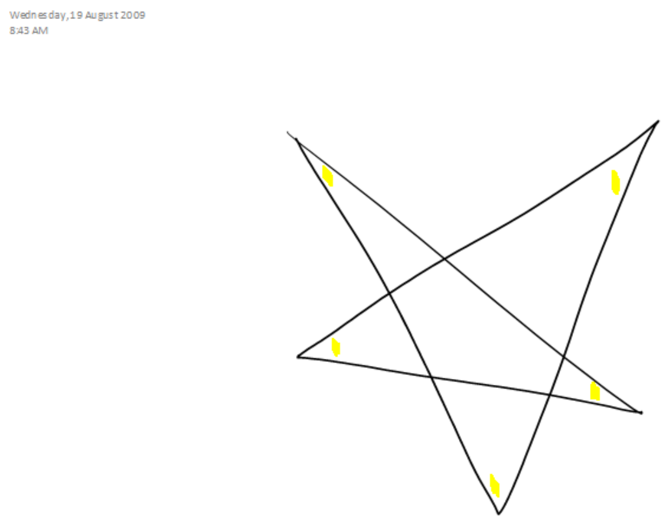


Figure 37: A five-pointed star is drawn on the screen

The problem was included in the booklet of activities, I had written for the students. I directed the students to the relevant pages in the booklet and set them to work on the problem. The students were able to follow the instructions in the booklet without any great difficulty. They helped one another occasionally. They each wrote down the measurements for the angles in the star that they had drawn and added up the measurements to find the total. Having allowed the students to work at their own pace using the booklet, I did not need to remain at the front of the room giving step by step instructions. This left me free to circulate around the students giving help and encouragement when the need arose. Angela asked for help and she benefited from some advice to take things one step at a time. She had successfully drawn a five-pointed star but appeared to be trying to move too quickly through the procedures and was becoming confused. The extract shown below from the transcription of the interchange between myself and Angela shows how I tried to persuade her to take a steadier approach.

> Teacher: Mm – so you've drawn the star
> Angela: Drawn the star
> Teacher: What does it say next?
> Angela: It says click anywhere in free space
> Teacher: Yep, good
> Angela: Then you "use the same process to measure the angles at another vertex". Find the total of the angles and the vertex- and I tried – "writing out proof using deductive geometry"
> Teacher: Oh well why don't we take it a step at a time? Why don't we just measure an angle first?
> Angela: Yep, measure and angle- so that's….
> Teacher: OK you could write that down somewhere… and then trying measuring all the angles
> Angela: And then add them up?
> Teacher: And then add them up
> Angela: OK thank you
> Teacher: OK
> Angela: All the angles
> Teacher: Well one at a time
> Angela: Yeah {little laugh} not all angles at one time … oh that's a good idea,{noticing another student's calculator} putting the angles on the screen… {continues to work on the problem for a little while} OK I think I've got it now.
>
> [Lesson transcription, 19th August 2009}

Apart from providing assistance with some practical aspects of the use of the calculator, the purpose of my intervention in this situation was to patiently guide the student to continue taking measurements in a methodical manner.

I waited until all of the students had completed the exercise of drawing the star, measuring the angles and finding the total. I then engaged the students in a whole class discussion about the results of the investigation. Most students concluded that the total was 180 degrees. Some students found that the calculated total was not exactly 180 degrees, for example, in one case the total was 179.99998 degrees. This was considered to be convincingly close to 180 degrees and the slight difference was attributed to an accumulation of rounding errors in individual measurements. Using the emulator and the projector screen I then engaged all of the students in an exploration of the construction. Using the interactive power of the calculator I demonstrated how vertices of the star could be moved. This was done by tapping with the stylus on a vertex in order to select it and then dragging the vertex to another location on the screen.

By exploiting the interactive power of the calculator in this way it was possible to draw multiple configurations of the star. In each configuration, it was found that, although the individual angles changed, the sum of the angles was always calculated to be 180 degrees. This was an example of the technology allowing the user to experiment with reconstructions with a speed and efficiency that allowed the main focus to remain on the outcomes of the changes as opposed to the mechanics of the changes. Clearly, by comparison, the drawing of multiple configurations of the star using pencil and paper and measuring the angles with a protractor would have taken an inordinate amount of time. The geometry mode of the calculator was providing an experimental advantage similar to that afforded by the "what if" power of a spreadsheet. Examples of various configurations of the star are shown below.

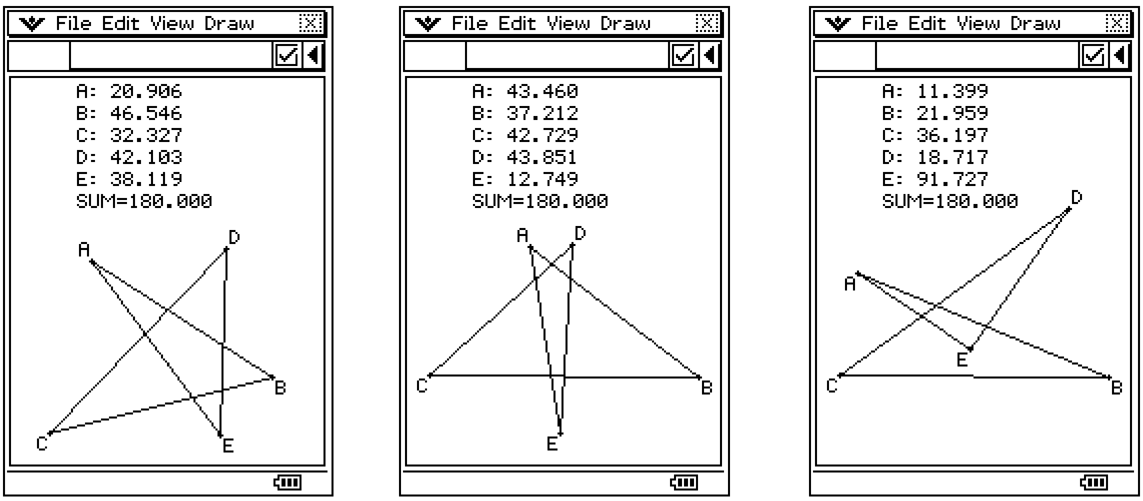


Figure 38: Different configurations of the star

This is consistent with the benefit which was noted in the Literature Review of using dynamic geometry software (DGS) to facilitate freer investigation (Trouche & Drijvers, 2010). The technology in these tasks was also being used to present the constructions as figures which could be manipulated as opposed to static drawings and, as is noted in the Literature Review this helps students to more easily predict outcomes of transformations (Hollebrands, 2003).

The period of familiarisation with the DGS is referred to as the development of an "instrumental genesis". It is recognised that this may be a long term process (Lagrange & Tran, 2010). The potential difficulty of it taking a long time for the students to adapt to the use of the DGS did not transpire in the implementation of the tasks, however. This assertion is supported from the data in the analysis because in all of the geometry tasks it was possible to begin with the mathematical problem associated with the task without any lengthy instruction time on the operational skills of using the technology.

The students found this to be a convincing demonstration, which, when combined with their own findings seemed to provide clear evidence that the angles in any five-pointed star add up to 180 degrees. I found it necessary to extend the problem and lead the students into a discussion about mathematical proof. I posed the question "It seems as if the total is 180 degrees, even when we change the star, but will the total *always* be 180 degrees? How can we *prove* that this is *always* true?" Questions such as these lead us on into Dimension Three of the Dimensions of Learning framework which is concerned with extending and refining knowledge.

Dimension Three: Extending and Refining Knowledge

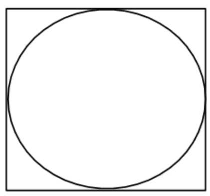
Introduction

In this section of the analysis, the outcomes of tasks designed and implemented with Dimension Three of the Dimensions of Learning framework are examined. This illuminated the part that technology can play in helping my students extend and refine their knowledge. Dimension Three is intended to build upon the work that has taken place in Dimension Two. The aim is to take students beyond a foundational level of understanding and skill. As explained in the second section of the Literature Review, there are eight complex reasoning processes which may be used in order to achieve this aim. Two of these complex reasoning processes involve reasoning. One of these processes is referred to as *inductive reasoning* and the other as *deductive* *reasoning*. In inductive reasoning, unknown generalisations or principles are inferred from information or observations. In deductive reasoning, generalisation or principles are used to infer unstated conclusions about specific information or situations. This section of the analysis will show that although technology did not play a pre-eminent role in encouraging the students to develop deductive reasoning, it contributed support for the students' contextual understanding of problems. By revisiting a problem using different methods to obtain a solution, it will also bring to light the contribution that technology makes to a variety of complex reasoning processes associated with Dimension Three.

Encouraging students to use deductive reasoning

On several occasions I endeavoured to extend the students' understanding by guiding them towards solutions based on deductive reasoning. One example of this arose when I revisited the Square Peg in a Round Hole task (see appendix 1) with the same group of students three months later. When the students were given the task for the first time the students had been able to conclude correctly that a round peg fits better into a square hole than a square peg in a round hole, and they had reached this conclusion by drawing shapes on the calculator and making measurements. The limitation of this conclusion was that it was derived from one specific square and one specific circle. I now wished to guide them to a higher level of thinking which would lead to a proof by deductive means that *any* circle will take up a greater proportion of the area of a square that encompasses it than *any* square will take up of the area of the circle that encompasses it. One possible version of such a proof is shown below:

In a plan view, let the round peg be represented by a circle and the square peg be represented by a square.



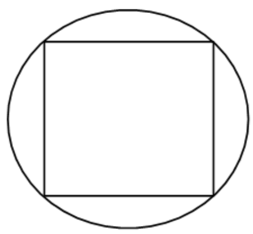
Now let a circle of radius r units be encompassed by a square

The area of the circle = $\pi \times r^2$ *units squared*

The area of the square = $2r \times 2r = 4r^2$ *units squared*

The proportion of the square covered by the circle = $\dfrac{\pi r^2}{4r^2} = \dfrac{\pi}{4}$

Now let a square of side length x units be encompassed by a circle



The area of the square = x *units squared*

The diameter of the circle = the length of the diagonal of the square

By Pythagoras' Theorem, the length of the diagonal = $\sqrt{x^2 + x^2} = \sqrt{2x}$

This means the radius of the circle = $\frac{\sqrt{2x}}{2}$ *units*

The area of the circle = $\pi \times r^2 = \pi \times \left(\frac{\sqrt{2x}}{2}\right)^2 = \frac{\pi x^2}{2}$ *units squared*

The proportion of the circle covered by the square = $\frac{x^2}{\frac{\pi x^2}{2}} = \frac{2}{\pi}$ *units squared*

Since $\pi > 3$ *then* $\frac{\pi}{4} > \frac{3}{4}$ *and* $\frac{2}{\pi} < \frac{2}{3}$

Therefore $\frac{\pi}{4} > \frac{2}{\pi}$

This means that the proportion of the square covered by the circle is greater than the proportion of the circle covered by the square. Hence a round peg fits better in a square hole than a square peg in a round hole.

When I revisited the problem with the students they were able to quickly revise the method of constructing the shapes and performing the calculations. Once again they reached the conclusion that the round peg fitted better into the square hole than did the square peg in the round hole. It was noted that even when individual students started with different sizes of squares or circles the proportions remained the same in the answer. About fifteen minutes was spent on this activity during which time the students worked referring to the booklet I had prepared for them and with little assistance from me. I then proposed to guide the students towards a more generalised solution to the problem.

I approached this by way of a class discussion. This was helpful in drawing together various pieces of mathematical knowledge that were required. Only some students knew about Pythagoras' Theorem for example, whilst others had to be reminded about the process of division of fractions. Eventually, through the discussion and the questioning of the students in the group it was evident that a shared understanding had been reached about the problem. The method used was more general than the previous method of construction and calculation. Figure 39 below shows what was displayed on the screen in the classroom, culminating from the discussion. It can be seen in the screenshot that the method was not a fully generalised one in that there were still some details that were specific. For example, the radius of the circle within the square was set at one unit in length rather than being labelled as a variable.

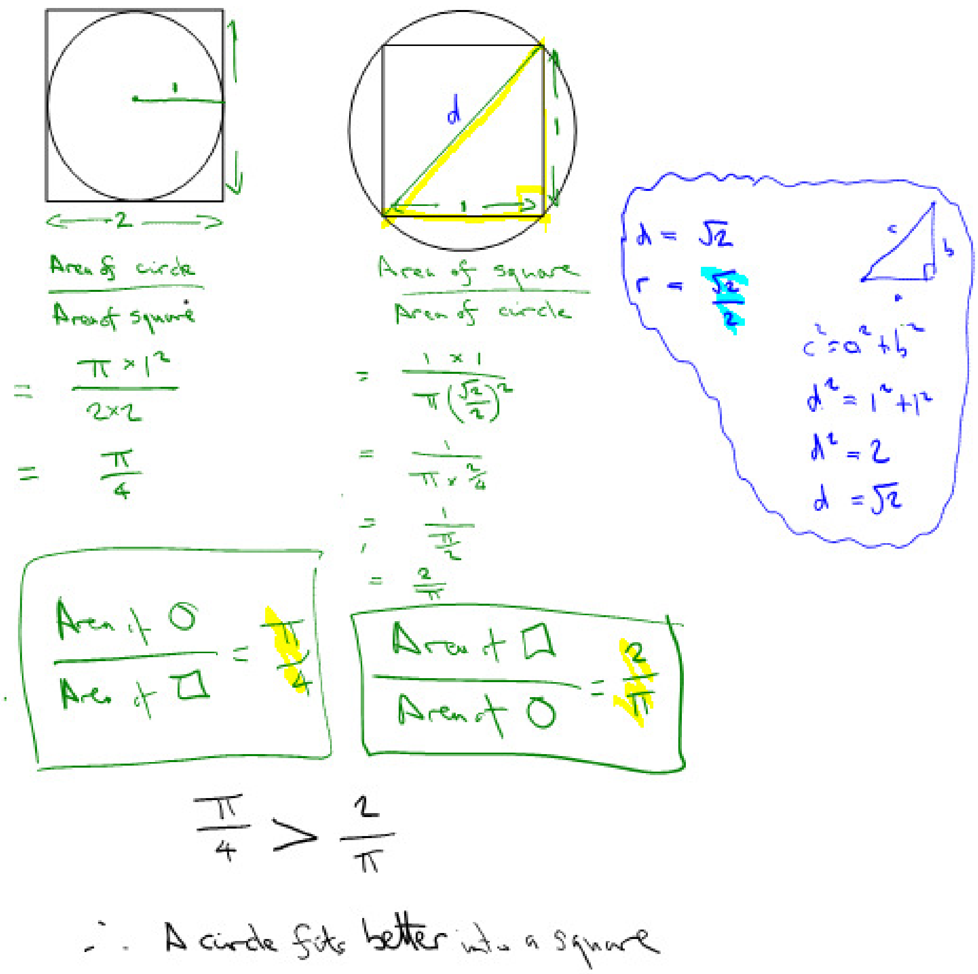


Figure 39: Screenshot of the teacher's computer showing the culmination of class discussion.

When I revisited the Square Peg in a Round Hole task, I had hoped to guide the students to a proof based on deductive reasoning. Despite the fact that the method used in the class discussion was not as formal as it ideally could have been, the students had been taken on a journey which took them closer to a formal deductive proof. Technology really only played its part in helping students to comprehend the context of the problem by drawing the squares and the circles. In Dimensions of Learning terms, the technology had provided support in the constructing meaning phase of acquiring declarative knowledge. Beyond this the technology took a less prominent role.

I found that leading students towards proofs using deductive reasoning was not always an easy task. This was most evident in the situation where students were working on the problem of finding the sum of the internal angles of a five-pointed star (see appendix 3). Using technology, the students had been able to quickly construct multiple versions of five-pointed stars and measure the angles. Each time the total appeared to be 180 degrees. This was the correct answer but it had been obtained by observations rather than having been produced from a rigorous proof. Now that the students, aided by technology, had gained a sound understanding of the problem, my goal was to lead them to a proof using deductive geometry.

Making this leap from observations to formal proof can involve some additional pre-requisite knowledge. I made the mistake of overlooking this fact when I assumed that the students knew the external angle of a triangle theorem. This theorem was a key component of the deductive proof and because the students were unaware of this theorem my explanations were very misunderstood. On reflection afterwards, I felt that the students had at least made progress in the constructing meaning phase of acquiring declarative knowledge although my faux pas had hindered their progress in the organising phase of acquiring declarative knowledge. My ultimate goal had been to lead students to a higher level of mathematical thinking, in this case a deductive proof. I had learned that to achieve this it is helpful to spend time with the support of technology in the constructing meaning phase of acquiring declarative knowledge. I had also learned, however, that I needed to be careful in my assumptions about the students' existing knowledge.

Applying different complex reasoning processes to the same problem

The Year 8 students were tackling problems in ways that were constrained by the knowledge, both procedural and declarative, that they possessed at that stage in their education in mathematics. In order to explore the possibilities for further learning in problem solving, I gave the Year 8 students a task to work on and then gave the same task to students at older year levels. It was interesting to see how the task could be tackled in different ways. Of more significance, however, was the variety of complex reasoning processes that could be applied to the same task.

The Road Circuit task

The task chosen for this exercise was the Road Circuit task (see appendix 4). This task begins with a situation involving four towns *A*, *B*, *C* and *D* which are already connected by roads as shown in Figure 40.

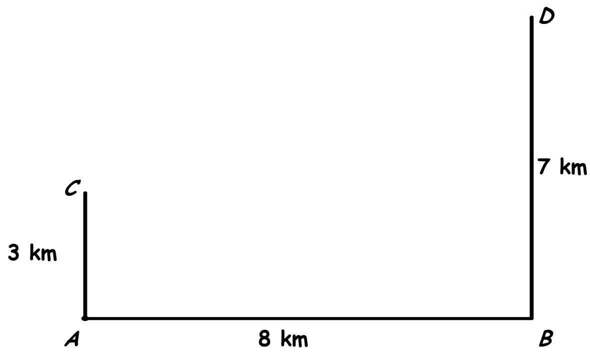


Figure 40: Diagram of roads connecting the four towns

To help with traffic flow a roundabout has to be placed somewhere along the road between *A* and *B*. Connecting roads are to be constructed from the roundabout to towns *C* and *D*. But the question is where should the roundabout be placed in order to minimise the cost of building the extra roads?

A solution using scale diagrams

Presenting this task to the Year 8 students was done through a teacher-led classroom discussion with an accompanying hand-out (see appendix 4). Using the geometry application in the ClassPad calculator, the students, with my guidance, constructed a scale diagram of the road circuit as shown in Figure 41. To begin with they placed the roundabout at some point near the centre of the road connecting towns *A* and *B*. The calculator was used to display the perimeter of the circuit for various positions of the roundabout along the road from *A* to *B*. The students were able to display two measurements on the calculator screen. The first measurement was the perimeter of the circuit and the second measurement was the distance from town *A* to the roundabout at *E*.

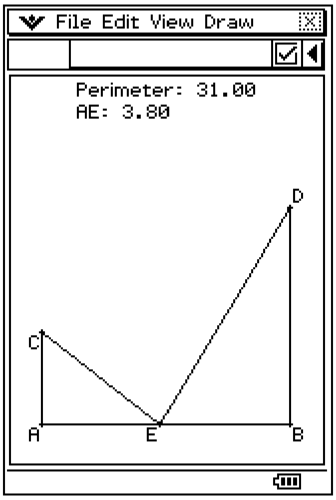


Figure 41: A scale diagram of the road circuit on the ClassPad

The students then ran an animation in which the roundabout was placed at regular intervals along the road from *A* to *B*. The measurements displayed on the screen were automatically updated. By watching the animation the students could see approximately where the roundabout should be placed in order to minimise the total length of the roads, i.e. the perimeter of the road circuit. The animation is illustrated in the screenshots in Figure 42.

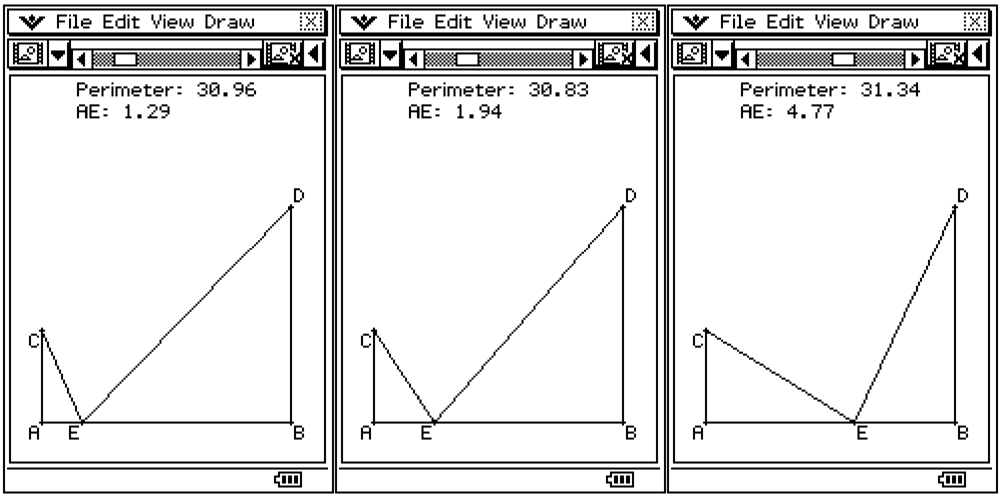
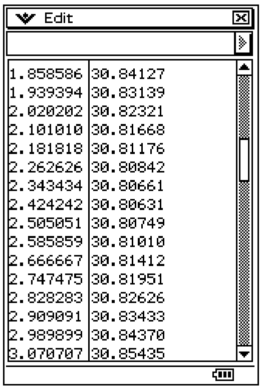


Figure 42: Screenshots of the animation

After watching the animation the students were able to produce a table of results from it. The table showed the distance from town *A* to the roundabout and the perimeter of the road circuit. In this way, the students now had a chance to connect more directly with the numbers involved. They could determine from the table (see

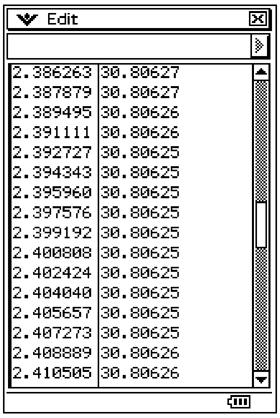
Figure 43) that the best answer would be obtained if the roundabout was placed at approximately 2.4 km from *A*.



1.858586	30.84127
1.939394	30.83139
2.020202	30.82321
2.101010	30.81668
2.181818	30.81176
2.262626	30.80842
2.343434	30.80661
2.424242	30.80631
2.505051	30.80749
2.585859	30.81010
2.666667	30.81412
2.747475	30.81951
2.828283	30.82626
2.909091	30.83433
2.989899	30.84370
3.070707	30.85435

Figure 43: Screenshot of the results

With some further guidance, they were then able to adjust the animation so that it collected measurements from around this region in order to gain a more accurate result. The technology in this sense satisfied the students' natural inclination to "zoom in" on the numbers. In this way the students obtained an answer for the minimum perimeter at 30.81 km when the roundabout was placed at 2.4 km along the road from *A* to *B* as shown below in Figure 44.



2.386263	30.80627
2.387879	30.80627
2.389495	30.80626
2.391111	30.80626
2.392727	30.80625
2.394343	30.80625
2.395960	30.80625
2.397576	30.80625
2.399192	30.80625
2.400808	30.80625
2.402424	30.80625
2.404040	30.80625
2.405657	30.80625
2.407273	30.80625
2.408889	30.80626
2.410505	30.80626

Figure 44: A more accurate answer is obtained

This was a satisfactory answer for my Year 8 students and it was appropriate to their current level of mathematical knowledge. I wondered how they could be extended in the future to embrace other methods of finding the answer. I decided to explore the road that lay ahead for them in their learning journey through school. To do this, I presented the same problem to older students in the school. These older students were in my Year 10, Year 11 and Year 12 classes. As explained in Chapter 4, they were not subjects of the research. It was useful, however, for me to recount my experiences as a teacher when presenting the same problem to these classes. I sought a crystal ball which I could look into and gain an image of how the Year 8 students' knowledge might be extended in the future.

A solution using similar triangles

When I presented the same problem to a class of Year 10 students they used the same method as the Year 8 students. In addition, however, they were able to use another completely different method for solving the problem. By interacting with the diagram on the screen the students were able to produce an adaptation of it as shown below. (The students were shown how to set the diagram so that the fixed lengths, the right angles and the slope of the line from *A* to *B* would remain unchanged). The problem now focused on minimising the distance from *C* through *E* to *D*. It was clear to them that the shortest distance would be found when *C*, *E* and *D* all lay on a straight line as shown in Figure 45. This was intuitively obvious to the students given their existing knowledge that the shortest distance between two points (*C* and *D* in this case) is a straight line.

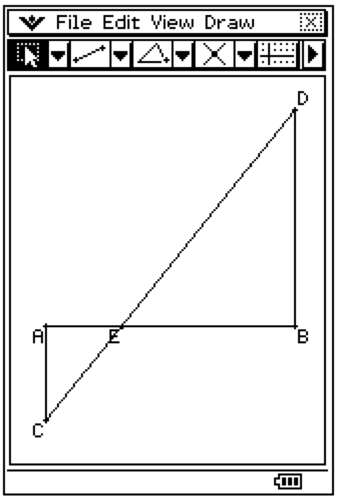


Figure 45: The shortest distance between two points is a straight line

The problem then became a useful example of the use of similar triangles which was demonstrated to the students. A tablet computer connected to a digital projector was used for this purpose. A screenshot of what was displayed on the board is displayed in Figure 46 below.

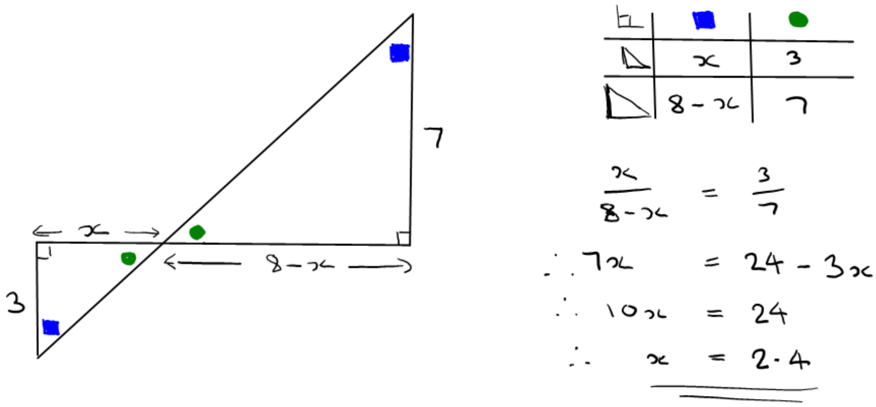


Figure 46: A solution using similar triangles is demonstrated to the students

Hence, an exact answer to the problem emerged by deductive means, which corroborated the previous answer obtained by repeated measurements.

It is worth reflecting on how the solution obtained by the Year 8 students using a table of values compares with the solution found by using similar triangles. The table of values method relies essentially on a trial and error approach and cannot with certainty produce an exact answer. In contrast with this, the similar triangles method is exact and unequivocal. The methods are conceptually different, however, and it is wiser to allow the Year 8 students the opportunity to adopt an approach involving numbers before attempting a more abstract method.

A solution using Pythagoras' Theorem

The problem was presented once more, this time to my Year 11 class. The method of repeated measurements with the aid of the animation was carried out as before. In addition, however, the students were guided towards another approach to the problem that entailed the use of Pythagoras' Theorem. In the course of a class discussion, a labelled diagram was developed by the students like that shown in Figure 48 below. The diagram supported a method of solving the problem which combined symbolic manipulation and graphing.

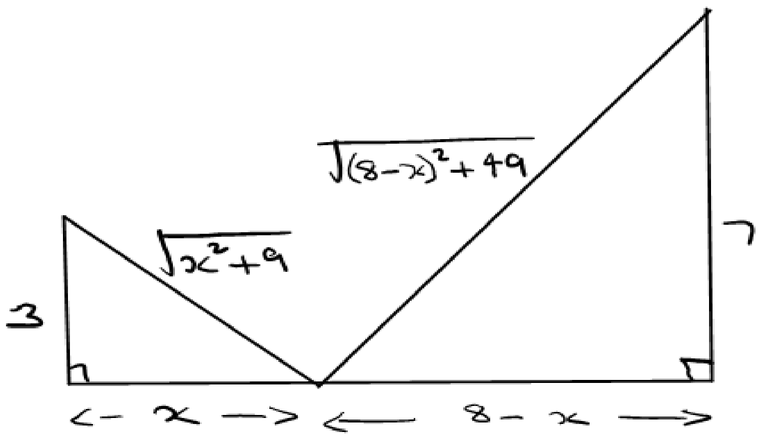


Figure 47: Using Pythagoras' Theorem

By entering a formula for the perimeter into the graphing calculator application of the ClassPad, the students were able to produce a graph from which the best position for the roundabout could be identified along with the minimum perimeter of the road circuit as shown in Figure 48.

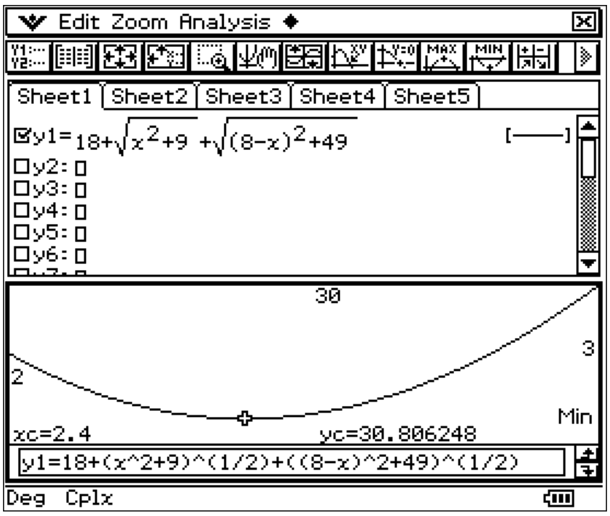


Figure 48: Year 11 students find a graphical solution

This method using Pythagoras' Theorem involves a good deal of algebraic manipulation which was not present in the method that the Year 8 students employed. The Year 8 students produced an answer from their tables that was "probably exact". It may appear that the Year 11 students were not producing an exact answer either since they were reading their answer from a graph. The important distinction that needs to be made, however, is that the Year 11 students were able to derive an exact function on which to base an answer. This method could therefore be considered to be more robust than the method used by the Year 8s. The Year 11 students used technology to convert the function to a table and then to a graph but this should not overshadow the underlying rigour of their method.

A solution using differential calculus

Finally, the problem was given to my Year 12 class. With characteristic zeal they strove to establish an algebraic relationship and use differential calculus to solve the problem. They first obtained a formula for the perimeter in terms of the distance from A to the roundabout as shown below:

$$P = \sqrt{x^2 + 9} + \sqrt{((8 - x)^2) + 49} + 18.$$

Using the computer algebra system (CAS) in the ClassPad, they found the derivative of the perimeter function and equated it to zero as shown in Figure 49.

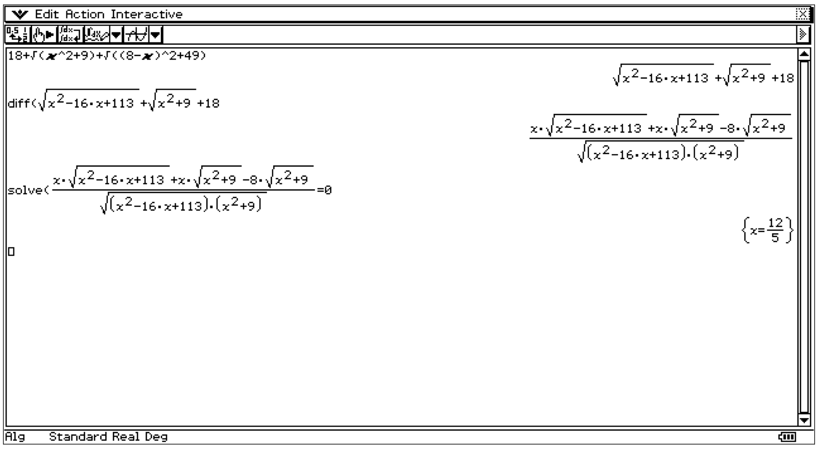


Figure 49: Year 12 students use differential calculus

This produced the exact answer of $\frac{12}{5}$ or 2.4 in agreement with the other answers.

The complex reasoning processes that were used

The various methods that were used can be related to Dimension Three: Extending and Refining Knowledge of the Dimensions of Learning framework. These connections can be made by considering the different types of complex reasoning processes that were involved. When the Year 8 students were constructing diagrams and tables they were working in a way that aligns with the complex reasoning process of *constructing support*. Their use of diagrams and tables, whilst not providing an exact proof, gave strong supporting evidence to help validate the answer they reached. The method using similar triangles that the Year 10 students used was one that aligned with the complex reasoning process of *deductive reasoning*. This connection is justified since existing principles, in this case the properties of similar triangles, were used to reach a conclusion about the problem. The method used by the Year 11 students can also be aligned with the complex reasoning process of deductive reasoning since Pythagoras' Theorem was used to formulate an equation for the perimeter from which a solution was reached. Once again an existing principle was employed in the process of obtaining a solution. The connection to deductive reasoning is still valid here despite the fact that the final answer was arrived at using a graphical method because the graphical method was based on an equation derived from existing principles.

The solution obtained by the Year 12 students could be aligned with the complex reasoning process of *abstracting*, for in this case the use of the calculus tapped into an underlying theme connected with finding an optimal solution to a system involving rates of change. The general pattern of information in the problem allowed them to classify the problem as an optimization problem as opposed to a problem specifically about perimeter and length. Having abstracted the information from the problem they then went on to solve the problem using the methods of differential calculus. They would have equally applied this process of abstraction to, for example, a financial problem in which they were required to find the optimal number of items a manufacturer should produce in order to maximize profits.

It would not have been possible for the Year 8 students to have undertaken all these different approaches to solving the road problem because they had not yet acquired the procedural and declarative knowledge that would have been required. The effect of taking the same problem on to higher year levels, however, illustrated the fact that there need not be a hard and fast connection between a problem and any particular complex reasoning process. When, in due course, the Year 8 students move on to higher year levels it will be possible for a teacher to point this out to them – same problem as before but different complex reasoning process.

The contribution of technology to the complex reasoning processes

The contribution that technology made to the various complex reasoning processes can now be highlighted. When the Year 8 students were *constructing support* for their solution, the technology contributed to this by facilitating the production of scale diagrams which were accurate and could be animated. Tables were quickly produced which could be updated instantly when parameters were changed. When the Year 10 students used *deductive reasoning*, the technology allowed the diagram of the road circuit to become a figure which could be manipulated on the screen. As was noted in the Literature Review, the technology user can capitalise on this affordance and is able to investigate more freely (Trouche & Drijvers, 2010). It was also noted in the Literature Review that the representation of the drawing as a figure which can be manipulated makes it easier for students to make predictions (Hollebrands, 2003). It was further noted in the Literature Review that by directly manipulating a figure students can bypass the need for an algebraic or formal representation (Nathan et al., 2010). This was evident when, with one stroke of the stylus, the figure was manipulated into a form from which a solution could be obtained based on the simple principle that the shortest distance between two points is a straight line.

Interestingly, when the Year 11 students developed a solution based on Pythagoras' Theorem, the contribution from technology came after the deductive reasoning had taken place using pencil and paper. The students produced a function in exact form which represented the perimeter of the road circuit, and then the graphing application of the ClassPad was used to find the minimum perimeter. The contribution of technology in this case was to perform the procedural work of graphing in order to complete a solution that was based on deductive reasoning.

When the Year 12 students used differential calculus they obtained a solution that came out of a process of symbolic manipulation. A CAS was used to implement this which allowed the students to focus on the problem rather than what would have been a lengthy by hand manipulation (Broline, 2007). Students appreciate the use of a CAS in this situation (Pierce, 2001). As with the method using similar triangles, the approach using differential calculus produced an exact answer. The students used the complex reasoning process of abstracting in order to reformulate the specific problem into a more general form of an optimization problem. The contribution made by technology was to automate the symbolic manipulation that was required to complete the solution.

Dimension Four: Using Knowledge Meaningfully

Introduction

Dimension Four of the Dimensions of Learning framework is concerned with using knowledge meaningfully. It takes a prominent place in the framework because it provides an indisputable reason for acquiring knowledge in the first place. This section of the analysis will show the contribution that technology made to complex reasoning processes associated with using knowledge meaningfully. As explained in the second section of the Literature Review, Dimension Four has six complex reasoning processes associated with it, namely, *decision making*, *problem solving*, *invention*, *experimental inquiry*, *investigation* and *systems analysis* (Marzano, 1992). These complex reasoning processes provide the defining properties of the term "using knowledge meaningfully". In other words knowledge is used meaningfully when decisions are made, when problems are solved, when inventions are produced, when experiments are conducted, when investigations are carried out, or when systems are analysed.

Examples were given in the Literature Review of the complex reasoning process of *investigation*. One particular type of investigation known as a *projective investigation* (Marzano et al., 1997) was referred to using examples related to the population of China (McGivney-Burelle, 2004) and the growth of swine flu (Thomson, 2010b). These investigations involve using knowledge meaningfully where there is a connection with the "real world". Connections with the real world are not strictly necessary when using knowledge meaningfully in mathematics, however. Investigations into Kurschak's Tile (Thomson, 2011) and the Shoemaker's Knife (Thomson, 2010c) for example, are based on inventions from the minds of mathematicians Kurschak and Archimedes respectively. This section of the analysis will show the contribution that technology made when a task, based on a fictitious situation was designed and implemented in line with the complex reasoning process of *decision making*.

A task designed for decision making

The Road Circuit task (see appendix 4) referred to in the previous section is an example of a task that was designed to relate to this complex reasoning process. In this task a problem about a road circuit is presented to the students. In the problem there are four towns *A*, *B*, *C* and *D* which are already connected by roads as shown in Figure 50.

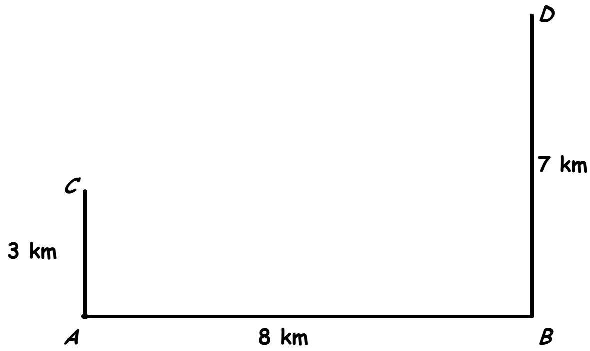


Figure 50: Diagram of roads connecting the four towns

To help with traffic flow a roundabout has to be placed somewhere along the road between *A* and *B*. Connecting roads are to be constructed from the roundabout to towns *C* and *D*. But the question is where should the roundabout be placed in order to minimise the cost of building the extra roads?

The problem as it stands has a practical component concerned with improving traffic flow. For example, travelling from town C to town D requires a long journey through towns A and B. Building a roundabout somewhere along the road between town A and town B will help with this situation. The mathematical problem of positioning the roundabout to minimise the amount of materials used has connections with environmental considerations and sustainability. Another valid connection here is that vehicles will use less fuel if the road length is minimised. The activity is designed therefore to have a sense of purpose about it and it may be classified as an application-oriented task as defined by Marzano (1992).

The road circuit task was suitable for inclusion in a short time frame of one lesson. When examining the full importance of Dimension Four: Using Knowledge Meaningfully, more long term investigations should be considered. The short time slots when data were collected, however, precluded the use of long term investigations with the Year 8 subjects in this research. This is acknowledged as a limitation but information and descriptions of more long term investigations are provided in the Literature Review.

The contribution of technology to decision making

In the Road Circuit task, the technology is used from the very beginning within the context of the problem. In this respect, the use of the technology is in line with the first guideline for the appropriate use of technology which was noted in the Literature Review as given by Garofalo et al (2000) which is to *use technology in context*. The use of the technology is also aligned with the third guideline which is to *take advantage of technology*. In keeping with this guideline, the capability of the technology to tirelessly recalculate and produce tables and graphs is utilised appropriately and in ways that rightfully take precedence over performing them with pencil and paper. There can also be no doubt that the use of the technology matches up with the fifth guideline which is to *incorporate multiple representations* for in this one task the technology provided representations in tabular, graphical, algebraic, geometric and dynamic forms.

These connections to the guidelines given by Garofalo et al (2000) indicate that the technology has been used appropriately in this task. These indicators of the appropriate use of technology are inextricably linked, however, with the contribution that the use of the technology makes to the complex reasoning process of *decision making*. By beginning with the problem in context the need for a decision to be made is emphasised. By taking advantage of the technology the decision making process is enhanced. By providing multiple representations a variety of perspectives on the problem is provided to which differing students can relate. The appropriate use of technology as defined by Garofalo et al (2000), therefore, sits very well with the complex reasoning process of decision making outlined by Marzano (2007) in Dimension Four of the Dimensions of Learning framework.

Dimension Five: Habits of Mind

Introduction

Dimension Five of the Dimensions of Learning framework is concerned with the students' ways of working or, as they are referred to in the Literature Review, Habits of Mind. As illustrated in Figure 51 below, Dimension Five, in common with Dimension One: Attitudes and Perceptions, is not linked to knowledge directly. Both of these dimensions form part of an essential backdrop, however, on which effective learning takes place.

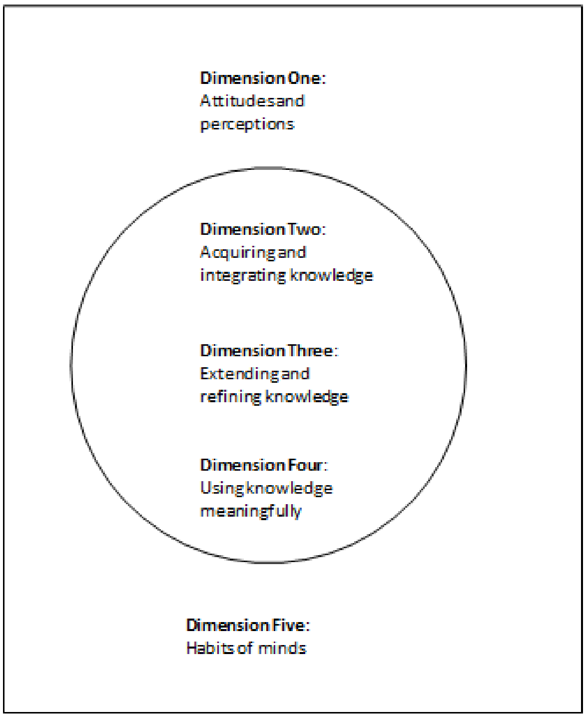


Figure 51: The Dimensions of Learning Framework (Marzano, 1992)

By designing tasks to connect with Dimension Five and describing the outcomes of implementing these tasks in relation to Dimension Five, the contribution that technology makes to this aspect of learning will be highlighted. This approach, as it did in the analysis of the previous four Dimensions, will illuminate the subtleties of the contribution that technology makes. Technology will not be the leading actor in the play as it unfolds. Instead, in the examples given, it will be the medium which facilitates the making of connections between mathematical ideas and enhances the interdependent thinking of students.

Responding with wonderment and awe

One of sixteen Habits of Mind referred to in the Literature Review is *responding with wonderment and awe* (Costa & Kallick, 2008). The language used to express this habit is extreme – I confess *wonderment and awe* does not fill every minute of every one of my lessons. But just as there are precious moments in life which sustain us and recharge our spiritual batteries, there are times when the love of learning is rekindled by a momentary experience. We may lose sight of what is wondrous and awesome, however, if we leave its emergence to chance. We need to tell ourselves, from time to time, to stop and look at the sunset, the tree, the flight of a bird. It is a good *habit*. As a teacher, I would like my students when they are learning mathematics to acquire this habit. With this in mind, I would like them to experience mathematical moments that are sparkling and interesting. In this respect I share the outlook of Lloyd who states that "My vision, shared by many others, is to nourish and support rather than stifle that wonder and joy" (Lloyd, 2009).

When seeking to nurture this habit in the students, I found in my planning that sometime it was helpful to preserve the element of surprise in a mathematical investigation. Even when the students had become very familiar with the technology, there were times when I felt it preferable to adopt a leading role in the classroom to achieve this. In these situations I used the calculator emulator displayed on the digital projector to lead students through an activity. An example of such an activity was the construction of the "Nine Point Circle" which can be found from the mid-points of a triangle, the feet of the altitudes of the triangle and the mid-points of lines drawn from the orthocentre to each vertex. This activity is an extension of the Euler Line task (see appendix 6). In this activity I behaved as a leader and the students travelled with me on a journey of investigation. By leading an activity such as this from the front of the class rather than providing a hand-out, I was able to maintain an element of surprise. The extract below illustrates how I revealed the outcome of the activity and the positive, enthusiastic response which it elicited from the students.

> Teacher: Oh good. Now what I want to do is hide some of the points. I'll just show you on the screen which ones we will hide. {now addressing whole group from screen} First of all we'll hide the A, B and C…the original triangle…OK?... we'll also hide the centroid, the orthocentre and circumcentre be careful not to hide that mid-point though between the circumcentre and the orthocentre. Hide those. And finally I'm going to hide the actual triangle. So I'm just going to carefully select the sides of the triangle…
> Will: Heh, it's a circle!

Sue: It’s a circle. That is *so* cool
David: Oh look a circle! {leaping to his feet and pointing to the screen}

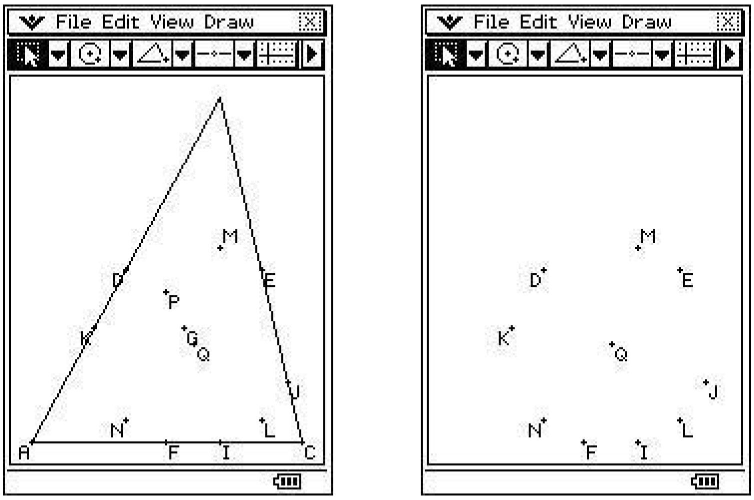


Figure 52: The Nine Point Circle

So now the place of technology in this design can be pin-pointed. The technology itself was not the source of marvel. It was the mathematics that evoked the responses of wonderment and awe from the students. It was a circle that was “so cool” and made the students say “Hey, look!” The technology merely helped reveal the circle in a more powerful immediate way than a printed text method would have done. The technology was the “mathemagician’s” assistant. Voila!

Undoubtedly the discipline of mathematics has a utilitarian aspect to it which has supported wondrous and awesome practical achievements such as the construction of the Pyramids or sending a human being to the moon. However, the connections that may be found within mathematics between its various branches are also a potential source of wonderment and awe. These connections, when revealed, exude a sense of wonderment and awe that is qualitatively different from that derived from practical sources. The Sydney Harbour Bridge is certainly a marvel and a tribute to engineering underpinned by applied mathematics. But an equation that connects branches of mathematics hitherto perceived to be disparate is also marvellous although in a way that somehow carries more mystique. Consider, for example, the equation $\mathbf{e}^{\mathbf{i}\pi} + 1 = 0$ which includes the five most significant mathematical numbers $\mathbf{e}$, $\mathbf{i}$, π, 1 and 0 all in one equation.

This notion of mathematical connections also appears in the Literature Review. One of the guidelines provided by Garofalo et al (2000) for the appropriate use of technology in mathematics is to *connect mathematics topics*. It is advocated that technology is used to illustrate the connections between topics such as algebra and geometry. Matching up the algebraic solutions to a quadratic equation with a graphical representation is one possible example of this. As well as these intra-connections, however, is also recommended that mathematics topics are connected not just to other mathematics topics but also to areas of the real world and disciplines outside of mathematics. The aim here is to promote within the students a more holistic view of mathematics and the surrounding world.

An example of a task that was directed towards wonderment and awe by connecting mathematics topics was the Golden Point task (see appendix 5). This task begins by introducing students to the concept of the golden ratio. The idea is that students are guided to construct a geometric shape from which emerges a “golden point”, a golden point being a point which divides a line in the golden ratio. In Figure 53 below this means that the distance from A to B divided by the distance from A to C equals the distance from A to C divided by the distance from C to B. This simple idea has connections with other areas of mathematics, for example the ratio of successive terms in the Fibonacci sequence conform to the golden ratio. It is also reputed to have connections with the ratio of human body parts and the dimensions of famous buildings and artworks.

Figure 53: A golden point

Once again when implementing a task with an objective related to the Dimensions of Learning framework, I would uncover an interesting perspective on the use of technology. Technology would prove to be a useful aid in illustrating a mathematical connection. The source of wonder, however, was the connection itself not the technology which aided its manifestation. Nor indeed, was the technological expertise of the teacher a source of marvel because, as it happens, I had to call on the help of the students to operate the technology. The appearance of the connection was to be greeted with great enthusiasm from the students.

To begin with in this task the students become familiar with the idea of a golden point. Technology is used as a medium through which the students achieve this by actually creating a golden point on the screen of the calculator. A reproduction of what the students constructed on the calculator is shown below with the golden point highlighted in bold.

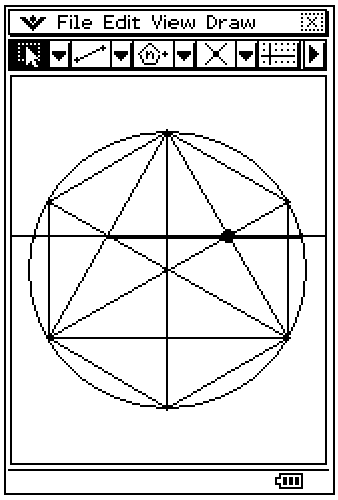


Figure 54: A golden point is created

I introduced the students to this task from the front of the classroom using the tablet laptop connected to the digital projector. I began by explaining what was meant by the Golden Ratio. I used a visual approach and made use of colour as shown in Figure 55 below. Pointing to the screen, I explained that the lengths of the green line and the red line are in the golden ratio if the length of the blue line divided by the length of the green line is the same as the length of the green line divided by the length of the red line. I repeated this slowly a couple of times making sure that the students understood. I told the students that when the green line and the red line were in the golden ratio, then the length of the green line divided by the length of the red line would be approximately 1.618

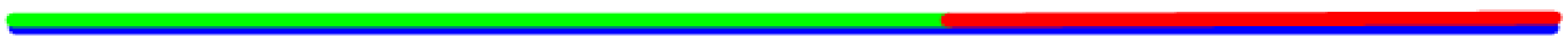


Figure 55: Extract from the teacher's screen when explaining golden ratio

I also explained that a golden point is a point that divides a line in the golden ratio.

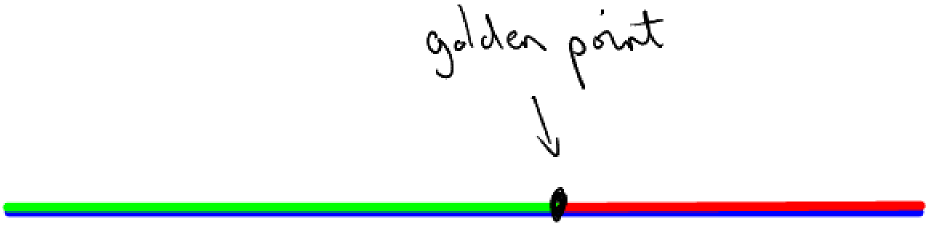


Figure 56: Extract from the teacher's screen when explaining golden point

I then distributed the geometry activities booklets and the ClassPad calculators and set the students to work on the Golden Point investigation (see appendix 5). I allowed the students to work independently on this following the instructions in the booklet. I moved around the class offering help where necessary. The students were able to complete the investigation and show, using measurements, that there appeared to be a golden point in the construction. Their measurements were accurate to five decimal places. They found a point which divided a line segment in the construction in the ratio 1.61803:1. I was aware of the fact that the students did not have the mathematical knowledge required to prove through deductive means that the ratio was exactly $\frac{1+\sqrt{5}}{2}:1$ which would have proved that the point they had found was indeed a golden point. I was pleased, however, that their decimally approximate measurements were consistent with the existence of a golden point.

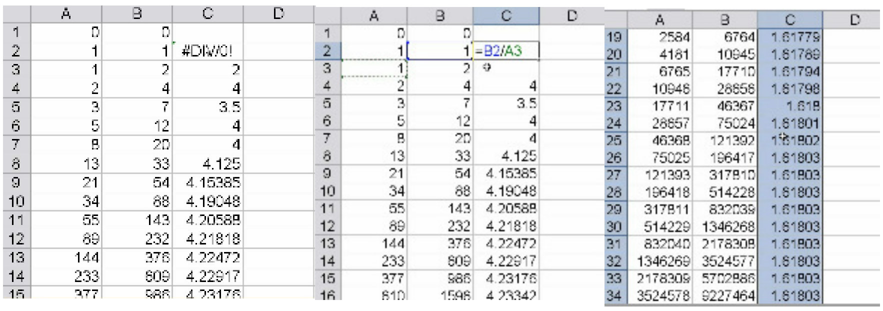
Returning to the front of the class, I decided I would take the opportunity to help the students see that the work that they had just undertaken had many connections in other areas of mathematics and real life. I displayed some examples from architecture and art where the golden ratio may exist. It also occurred to me to mention the connection with the Fibonacci sequence. Having become energetic and enthusiastic about all these amazing connections I acted on impulse and decided to lead the class on an exploration into this connection between the golden ratio and the

Fibonacci sequence. I began by reminding the students about the Fibonacci sequence.

> Teacher: If you took the Fibonacci – does anyone know what the Fibonacci sequence is? {I write the beginning of the Fibonacci sequence on the screen}
> David: 2 then3
> Will: then 5
> David: then 8
> Will: Oh yeah the last two equals the next one
> Together: yeah, yeah
> Teacher: That's the interesting thing. If you took the Fibonnacci sequence and added them up – right- for- you know -maybe did quite a few of them- a hundred of them- and added up the numbers -and divided it by the point you had reached in the sequence. Guess what you would get?
> David: 1.6083 [Lesson transcription 25th August 2009]

It then occurred to me that I could demonstrate this relationship through the use of a spreadsheet. I duly displayed a spreadsheet on the screen and invited the students to begin the exploration with me. Unfortunately, I made some mistakes when attempting to use the spreadsheet to illustrate this connection between the golden ratio and the Fibonacci sequence. Luckily when I asked for assistance Will came forward to help.

> Will: I am going to teach you the Fibonacci thingy. So, I have got the… the system so that the last two equal the one underneath. So one plus one equals two, one plus two equals three, and this is the progressive total. So up until here over on this side we've got two. Up until here…over here we've got thirty three. And this number over here is this divided by that. So there's a special golden ratio that should occur. So I'll just fix it up. Where's delete on this thing? Delete, delete, delete…OK, so we'll say that this here equals that divided by that. Now, we'll click on that formula and drag it down to that. And would you look at that! We've got the Golden Ratio! [Lesson transcription 25th August 2009]



	A	B	C	D
1	0	0		
2	1	1	#DIV/0!	
3	1	2	2	
4	2	4	4	
5	3	7	3.5	
6	5	12	4	
7	8	20	4	
8	13	33	4.125	
9	21	54	4.15385	
10	34	88	4.19048	
11	55	143	4.20588	
12	89	232	4.21818	
13	144	376	4.22472	
14	233	609	4.22917	
15	377	986	4.23176	

	A	B	C	D
1	0	0		
2	1	1	=B2/A3	
3	1	2		
4	2	4	4	
5	3	7	3.5	
6	5	12	4	
7	8	20	4	
8	13	33	4.125	
9	21	54	4.15385	
10	34	88	4.19048	
11	55	143	4.20588	
12	89	232	4.21818	
13	144	376	4.22472	
14	233	609	4.22917	
15	377	986	4.23176	
16	610	1596	4.23342	

	A	B	C	D
19	2584	6764	1.61779	
20	4181	10945	1.61789	
21	6765	17710	1.61794	
22	10946	28656	1.61798	
23	17711	46367	1.618	
24	28657	75024	1.61801	
25	46368	121392	1.61802	
26	75025	196417	1.61803	
27	121393	317810	1.61803	
28	196418	514228	1.61803	
29	317811	832039	1.61803	
30	514229	1346268	1.61803	
31	832040	2178308	1.61803	
32	1346269	3524577	1.61803	
33	2178309	5702886	1.61803	
34	3524578	9227464	1.61803	

[Video frames from screen capture, 25th August 2009]

To my delight, my plea for help with the use of technology from the students was answered; and not in a spirit of disdain but in an exclamation of joy, “We’ve got the Golden Ratio!” I rejoiced in the use of the first person plural here. “*We’ve* got the Golden Ratio!” This had been a team effort. The connection had been made amid a loud clamour of delight from the class.

My efforts in the design and implementation of the Golden Point Investigation can be viewed as an attempt to use technology appropriately to connect mathematics topics thereby aiding the students to construct a more integrated view of mathematics and other pursuits. In so doing I was trying to nurture the habit of mind of responding with wonderment and awe by revealing these various connections. The loud whoops of enthusiasm from the class audible on the screen capture recording suggest that this was achieved to some degree. The wonder was associated more with the connection than the technology. The technology provided effective support, however, in rapidly performing over thirty calculations and displaying a result to an accuracy of five decimal places.

Thinking interdependently

Another one of the sixteen Habits of Mind referred to in the Literature Review is *thinking interdependently* (Costa & Kallick, 2008). This habit is concerned with students working productively in collaborative settings. To examine this habit of mind, the students were set short problem-solving tasks. Some examples of these are given in Appendix 8. As explained in the Literature Review, these sorts of problems relate to Dimension Five of the Dimensions of Learning framework where they are classified as academic problems. Academic problems are well-structured short problems that can be fitted into available time slots and they are intended to activate students’ thinking (Marzano, 1992). I set these problems for my students in an environment where they were free to collaborate and could access technology whenever they wanted to. Resulting from this arrangement, some interesting contributions of technology came to light in relation to the interdependent thinking of the students. The use of technology assisted in identifying a misunderstanding about a problem which was preventing a group from reaching a satisfactory solution to the problem. Another contribution of technology that emerged was that it supported students in communicating their methods of solving problems. This was achieved through collaborative pairs of students making screen capture recordings of their work on solving problems using a tablet computer..

In these lessons that were devoted to short problem solving tasks, the students were content to work in collaboration with one another. They were appreciative of the fact that they could cooperate well together

> Vera: 'Cause we can like work together... [Focus group interview 18th November 2009]

There was also an understanding that they could learn from one another. In collaboration there was respect for the fact that problems could be solved in different ways.

> Linda: ...different people in the group have different ideas and ways to solve stuff. So like you can learn stuff from them as well and like the questions like we go over them and double check them and so can teach each other stuff as well [Focus group interview 24th November 2009]

> Theresa: Me and Sam had like two ways…like he could explain his way and we could learn from Sam and then like Sam could learn from my way [Focus group interview 18th November 2009]

It was interesting, however, to delve deeper into the social interactions that took place when technology was introduced into this collaborative working situation. In one of the problem solving sessions, Vera, Deborah and Liz were working together on a set of problems. They were having difficulty with the problem shown below.

4. In the figure the dots form a square 4 x 4 unit grid,
ie the adjacent horizontal and vertical dots are 1 unit apart.
What is the area of the triangle ABC?

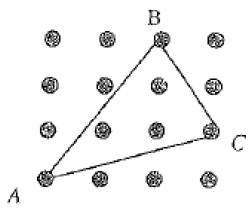


Figure 57: Short Problem ("School Mathematics Competition," 2004)

The answer they reached seemed unreasonable but they were not clear about what they were doing wrong. Vera spontaneously went across the room to the ClassPad bag. She took out a ClassPad and used the geometry application to draw the shape and find the area as shown below.

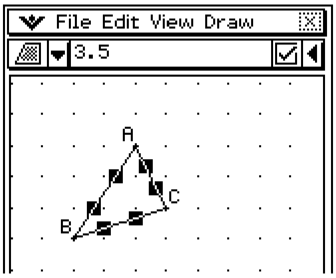


Figure 58: Shape drawn on ClassPad

She then returned to the others with the answer of 3.5 units squared obtained using the technology. Deborah and Liz were puzzled. How could the area of the triangle ABC be so small in comparison to the area of the square which they had reckoned to be 16 units squared. This was the trigger that helped them to realise that the diagram showed a 4 x 4 array of grid points giving a total of 16 grid points, but the area of the square was only 3 x 3 units squared. By considering the answer from the technology therefore, the students were able to spot the discrepancy and realised the error that they had been making. They were then able to complete the problem correctly. They were able to use their original approach of finding the area of the square and subtracting the area of the triangles surrounding the triangle ABC.

As Vera recalled:

> Vera: Oh yeah, we had to measure or figure out the area of an angle, not an angle, of a shape and because we were having trouble actually figuring it out because it was such a morphed shape. We decided…I went and drew it on the calculator and instead of having to go through all the calculations and all the ruling out what it couldn't have been and what it could have been. We got to the answer a lot quicker because actually if you click it actually it tells you the area.
>
> Teacher: And once you got that answer you seemed quite convinced by it but then what happened when you back to Deborah who had been doing it? What did she do, do you remember?
>
> Vera: Emm, well when I went back to them they were I think half way through the question and I told them the answer and they decided to check it by finishing the question [Focus group interview 18th November 2009]

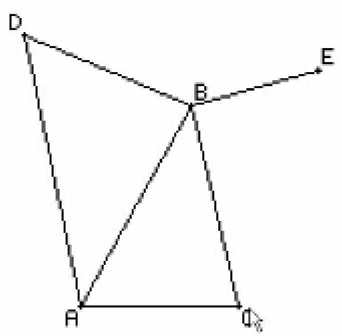
When Vera decided to draw on some assistance from technology she was able to make a contribution to the group to help solve the problem. This situation was an interesting example of what can take place when technology is brought into the mix in a collaborative process. When people collaborate, progress is made by way of combining contributions from individuals in the group. In this situation, the technology became an integral part of the collaborative working of the group. The group had realised that an error had been method in the process they were using to calculate an area but could not find the error. The contribution from one of the members of the group of an answer obtained using technology was helpful in overcoming this block. Interestingly, the contribution of the answer was significant not so much because it was the correct answer but because it supported the error analysis which led to the successful conclusion of the by hand method that the students were keen to complete. Technology in this context contributed to the students' interdependent thinking by helping them overcome an error which was preventing the group from completing their chosen method of finding a solution.

Another example of technology making a contribution to the students' interdependent thinking arose when students were working together using the geometry application of the ClassPad. This time the technology aided interdependent thinking by affording a means through which a shy student could demonstrate her understanding of a geometric construction. The classroom was equipped with a projector and a tablet computer which enabled individual students to demonstrate to the class how they had solved problems or made constructions on the geometry application of the calculator. Some girls, who were able to use the technology successfully to solve problems and create complex constructions, were shy about sharing with the whole class. When I asked a student who had successfully completed a geometric construction on the ClassPad to demonstrate this to the class, she was very reluctant. This was despite encouragement from her class-mates.

> Teacher: Ok, so now you've got to join those mid-points to the opposite side of the original…
> Sue: And you've got the…
> Teacher: And you've found the point?
> Sue: Yeh
> Teacher: Could you do that on the computer for me?
> Sue: Ah…{shy sounds from Sue, encouraging comments from Liz and Deborah}
> [Lesson transcription, 1st September 2009]

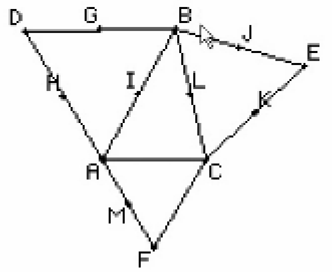
Some of these students were willing, however, to make use of the screen capture software on the computer to make a recording of the computer screen with an accompanying voice over. In the informal setting of the classroom, students were able to move to the computer, put on the headphones and make recordings. Students who I had encouraged to make a recording but who were somewhat daunted by the prospect would tend to take a class-mate with them.

A curious psychosocial phenomenon then emerged. As illustrated in the two examples below, pairs of girls made recordings. In each case, one girl operated the computer speaking softly at times, while the other girl, with all the confidence and panache of a games show host, provided a running commentary. In both examples the confident girl as well as playing to the gallery makes encouraging comments which are nurturing of the less confident girl. The first example is one in which Sue, with assistance and encouragement from Theresa, was demonstrating how to construct a Napoleon Point. (A Napoleon Point, allegedly discovered by Napoleon Bonaparte, is formed from the intersection of the lines drawn from the vertices of a triangle to the centroids of equilateral triangles constructed on the sides of the triangle.)



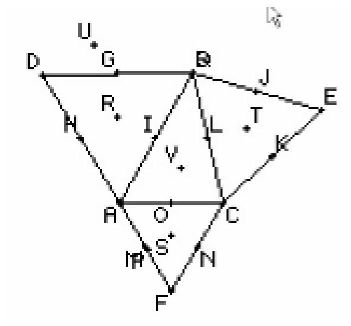
Theresa: We have to draw equilateral triangles around another triangle

[Screen capture video, 1st September 2009]



Theresa: We have to change the angles to 60 degrees... this is going really well...

[Screen capture video, 1st September 2009]



Theresa: ...we are done

[Screen capture video, 1st September 2009]

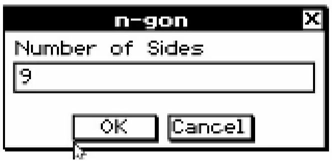
Humour is used by Theresa as part of the show but the use of humour is also intended to put Sue at her ease. Sue is deft in carrying out the procedures but when she gets a little muddled and has to retrace a few steps, Theresa makes interjections in a friendly tone of voice such as:

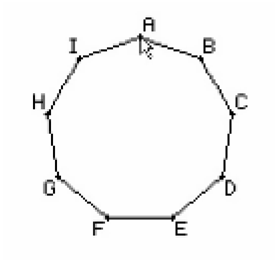
> Theresa: Oh no Sue! You have ruined the whole world Sue. The world is going to end Sue and it is all your fault.
>
> [Screen capture video, 1st September 2009]

And later in a comic serious tone:

> Theresa: Undergoing some technical difficulties once again
>
> [Screen capture video, 1st September 2009]

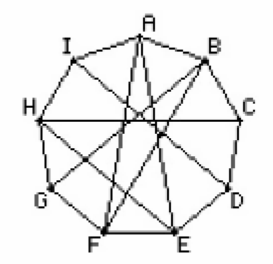
Beth and Jaimie also exhibited this type of behaviour. This pair of girls was working on a problem involving a regular nonagon. Whereas in the first example the students were working through a set of instructions, in this example the students were given less guidance. This meant that the girls in this situation were dealing with some higher order concepts. In order to achieve the goal of finding the required angle they would need to build for themselves some declarative knowledge about the properties of a nonagon. They had to find the measure of the angle IOD as illustrated below. Using the technology, the girls were able to draw a nonagon. As, Jaimie, the more outgoing girl, explained:





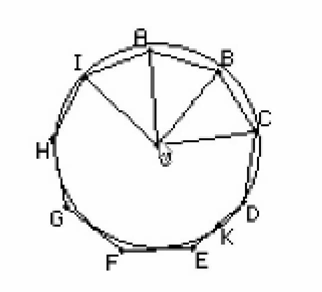
Jaimie: Then you press 9...and it will come up with the shape- the nonagon
[Screen capture video, 30th October 2009]

As in the previous example, the less outgoing of the two girls carried out the technical operations. An attempt was made to locate the centre of the nonagon by drawing diagonals in the mistaken belief that they would all intersect at the centre as would be the case for example with a rectangle. Both girls came to the realisation that this method would not work. Jaimie made the announcement:



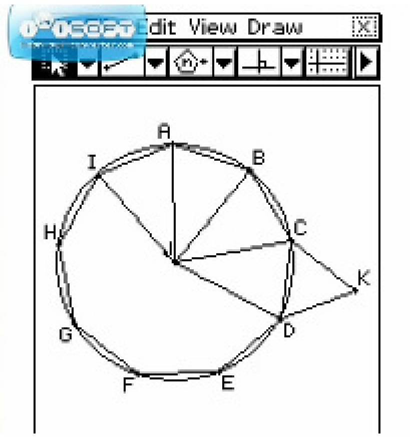
Jaimie: It does not seem to be going to the middle. Maybe because it has nine sides
[Screen capture video, 30th October 2009]

A second more fruitful exploration was then made in which a circle was drawn around the nonagon. The circle was only an approximate fit but it was adequate enough to help the girls explore drawing triangles inside the nonagon.



Jaimie: So we are going to make triangles and add them up
[Screen capture video, 30th October 2009]

Eventually, Beth who was operating the technology came to the correct answer and was able to explain her method. In the process of making a construction she was able to make comments which justified her answer. The comments were echoed more loudly by Jaimie.



Jaimie: … so four times 40 is 160 degrees

[Screen capture video, 30th October 2009]

Using technology, the girls had presented their working in solving the problem in a way that was less daunting than a presentation to the class from the front of the room.

Another surprising example of collaboration occurred in the classroom amongst three girls. The girls were working on the Road Circuit task (see appendix 4) and were constructing a diagram on the geometry application of the ClassPad. They were trying to construct the scale diagram shown below and animate the movement of point E along the line AB.

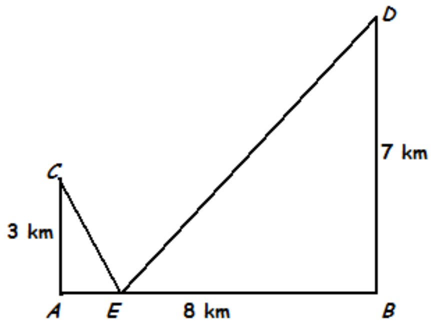


Figure 59: Animating the Road Circuit

In the process of doing this, the girls faced different difficulties. One had not constructed the lengths in the correct proportion, another had not set the right angles required and another had difficulty setting up the animation. When I came to see how the group was progressing, I was surprised to find that each girl in the group had ended up with another girl's calculator.

> Teacher: So… you're helping...
> Deborah: That's Liz's, that's mine and that one's Vera's
> {Deborah, Liz and Vera all chuckling}
> Liz: I'm trying to fix hers
> Teacher: Sorry, wait a minute… that one's…
> Together: That one's Liz's…that one's mine and that one's hers
> Teacher: Alright… a bit of a swap around! [Lesson transcription, 1st September 2009]

This complete swap around of the calculators in the group was not something that the girls had intended to happen. It had simply eventuated from them endeavouring to help one another as difficulties arose. The girls were amused themselves to realise that a complete swap around had taken place.

In this situation the three girls had swapped calculators when they were collaborating and unwittingly ended up each with another's calculator. One significant aspect of the technology in this situation which contributed to the students' interdependent actions was the fact that the technology was in a light and mobile form. The supportive collaborative behaviour that took place was facilitated by the fact that handheld devices are easy to orient and move. There was no need to lean over desks or stand up and swap seats as might be the case if larger computers were involved. In this situation the size and mobility of the devices which housed the technology played a part in enhancing the interdependent thinking amongst the students.

The examples of collaboration described above can be linked to the literature by viewing them as positive forms of social interdependence. They each fall into the category referred to as *means interdependence* as defined by Johnson and Johnson below:

> Means interdependence includes resource, role, and task interdependence. These methods are overlapping and not independent from each other. Resources can be divided among group members like a jigsaw puzzle. Roles such as reader, recorder, summarizer, and encourager of participation can be assigned to group members. The assigned task can be divided so that each group member is responsible for doing one aspect of the assignment. Johnson & Johnson (2009, p. 365)

When Theresa worked in collaboration with Sue she adopted the roles of summariser and encourager of participation. Interestingly, there was no need for Sue and Theresa to have another group member to take the role of recorder. Technology took this role. Beth and Jaimie worked together using technology in a similar way to Sue and Theresa.

When Deborah, Liz and Vera swapped their calculators with each other they were exhibiting the form of means interdependence referred to as *resource interdependence* (Johnson & Johnson, 2009). The resources in this case were handheld calculators which were able to be divided among the group just like pieces of a jigsaw puzzle.

Not all of the interactions that took place between students using technology could be interpreted as glowing examples of social constructivism in action, however. One example of students collaborating over the use of technology involved two boys who were making a geometric construction in which they were working on the Golden Point task (see appendix 5) and were in the process of constructing a golden point.

> Roger: OK press H and then I. Now press the doo-flicky
> Robert: Which doo-flicky?
> Roger: That guy. Now write that number down.
> Robert: Why?
> Roger: It's called the golden rule
> Robert: So what is the golden rule? What number?
> Roger: OK I'll just put it into my calculator
> Robert: What is the golden rule!
> Roger: OK I'll show you. Weren't you looking? OK give me the number 2.9…
> Robert: 72424
> Roger: 2.972424 right?
> Robert: Yes
> Roger: OK now we're going to click into an empty space. Click I and then J. Divided by…read it to me
> Robert: 1.83705
> Roger: There you go 1.618…you did it right Robert. So that's the golden rule, or the golden number rather
> Robert: I'm good! I probably had it about thirty times already. [Lesson transcription, 1st September 2009]

In this example, the interaction between the students was rapid, functional, and seemed to occur without any social graces. One student was helping another but the instructions were very direct, for example "*OK, give me the number*" and "*...read it to me*". There was little evidence of any nurturing of in depth understanding.

CHAPTER 6 CONCLUSION

This thesis showed the contribution that technology made in my mathematics classroom when tasks are designed and implemented with reference to an appropriate pedagogical model. This was achieved using tasks that were devised with the Dimensions of Learning framework in mind in an environment that was well-equipped with technology. When I put these tasks into action, I was able to describe the outcomes using qualitative data which were collected in a variety of forms. From the overall picture that emerged, I examined the way that technology was involved and thereby understood more clearly the part that it played.

The need for the use of technology to be considered in conjunction with pedagogy was identified in the Literature Review. Negative experiences of attempting to integrate technology into education in the UK were attributed to paying too much attention to the technology itself rather than placing a primary focus on educational aims (Watson, 2001). The desire to develop a productive combination of technology and pedagogy was implicit in the long term plans laid down by the Executive at my school. As described in Chapter Two, my school launched two initiatives simultaneously, one was to develop the use of technology and the other was to introduce the Dimensions of Learning framework as a model for teaching and learning.

The Dimensions of Learning framework provided a suitable pedagogical model to aid the research. In the Literature Review, connections were made between this model and the teaching and learning of mathematics and this justified its use. The five dimensions of the Dimensions of Learning framework comprise Dimension One: Attitude and Perceptions; Dimension Two: Acquiring and Integrating Knowledge; Dimension Three Extending and Refining Knowledge; Dimension Four: Using Knowledge Meaningfully and Dimension Five: Habits of Mind. The data were analysed in relation to each of these dimensions

Findings in Relation to the Aims

The contribution of technology to my students' perceptions about learning

The research uncovered a variety of perceptions that my students held about their experiences in learning mathematics and the use of technology. The data that were analysed in relation to students' perceptions came from a wide view of the situation. It was not narrowed down in the first instance to an examination of the students' perceptions about the use of technology. This was a deliberate strategy to preserve complexity and provide a richer picture. As a result, the findings pertaining to the students' perceptions about the use of technology were in due proportion to the overall situation.

Perceptions about the use of technology were not the strongest perceptions that the students expressed. Negative perceptions about their normal mathematics lessons were the most prominent. These negative perceptions centred round feelings of boredom and frustration with the pace of the lessons and the level of challenge. There were positive perceptions about the enrichment lessons in that the students found the work more challenging, they liked being able to work at their own pace and they enjoyed being with other students who were enthusiastic about mathematics.

There were positive and negative perceptions about the use of technology. The positive perceptions were in part associated with being in the enrichment class. The use of technology was perceived as part of that overall experience. Other positive perceptions about the use of technology were expressed in terms of its helpful support and interactive capabilities. Negative perceptions arose when technology, for some students, provided more scaffolding than was necessary. This was concerning because it could possibly have led to the expertise reversal effect. This is a detrimental effect which clogs the working memory of capable learners with information they do not need (Kalyuga et al., 2010).

As well as considering my students' perceptions about the overall situation, I also considered their perceptions about the tasks I had set for them. I found Dimension One of the Dimensions of Learning framework to be useful here in bringing together information from the research about this. In Dimension One there are three areas that relate to students' perceptions of tasks. These areas are (1) task value; (2) task clarity and (3) resources (Robert Marzano, 2007).

I found from the research that there were variations in my students' perceptions of task value when using technology. It is important to note that in the Dimensions of Learning framework, task value is considered from the viewpoint of the student not the teacher. Task value is not an objective measure of the inherent worthiness of a task, but instead it is the subjective assessment of the task as perceived by the student. This understanding of task value is consistent with the multi-perspective approach adopted in the methodology of the research. This supportive connection between the pedagogical model and the methodology can also be linked to outcomes from the analysis of the research. For a given task, individual students had different perceptions of task value. There were variations in students' perceptions about the use of error location software to aid the solving of linear equations. There were variations in perceptions about what type of technology was useful and when.

As far as task clarity is concerned, once again this is defined in the Dimensions of Learning framework from the viewpoint of the student. For the main part, I found that task clarity was not an area of difficulty for my students. They understood what they were asked to do with the technology. This was evident not so much through the words of the students, because it did not arise as an issue in any of the semi-structured interviews, but more through their demonstrated ability to perform tasks using technology. The lesson descriptions and transcriptions showed that, although at times I was moving around the room to provide occasional technical assistance, the students generally understood the instructions on how to use the technology.

It is fair to contend that this aspect of task clarity was positively affected by the efforts that went into selecting technology for use in the mathematics classrooms at the school as described in Chapter Two. The teacher's use of touch screen technology on the tablet computer displayed through the digital projector proved to be a suitable setup for giving the students instructions to operate the ClassPad calculators which also used touch screen technology.

There were some problems with task clarity, however, when I sought to extend and refine the students' knowledge. These problems arose when I made wrong assumptions about the depth of my students' prior knowledge of topics or when I embarked on the exploration of conjectures without preparing myself well enough on how I would use the technology. These aspects of task clarity required improvement on my part.

The meaning of resources in the Dimensions of Learning framework is also defined from the student's perspective. As explained in the Literature Review, its meaning is two-fold. First, the resources may be "external resources" which refer to the physical equipment that a student needs to tackle an area of learning. These external resources include technology and my students were fortunate to be in a technology-rich environment. Second, the resources may be "internal resources" which refer to the student's perception of their personal wherewithal to carry out the learning involved. Many of the tasks that I gave to my Year 8 students could have been tackled straight away using algebra and deductive reasoning. My Year 8 students would not have felt that they had the resources in terms of prior knowledge to do this. This was clear because it was challenging for them when tasks were extended to this level. They did, however, feel relaxed about tackling tasks such as the Square peg in the round hole task (see appendix 1), the Angles in a star task (see appendix 3) and the Road circuit task (see appendix 4) using diagrams and measurements. This approach was compatible with their internal resources. The contribution of technology was to enhance these internal resources by making the constructions and measurements quickly, accurately and in a way that could be manipulated easily on the touch screen of the calculator.

When designing tasks with Dimension One: Attitudes and Perceptions in mind a number of strategies can be employed. As outlined in the Literature Review, one of these strategies is to tap into the students' curiosity. When I took this approach with my students, I found that the technology itself did not make the students curious and hence keen to investigate. It was, in fact, the other way round, the curiosity stemming from the task encouraged my students to use the technology to seek a solution.

The contribution of technology to my students' learning

I gained an understanding of the contribution that technology made to my students' learning by considering the involvement of technology in relation to Dimensions Two, Three, Four and Five of the Dimensions of Learning framework. Most often, technology played a supporting role rather than a controlling one.

Considering Dimension Two: Acquiring and Integrating Knowledge first of all, I found that technology could make a contribution in the *shaping* phase of my students' acquisition of procedural knowledge by providing them with feedback in the form of an error analysis. When my students were acquiring declarative knowledge on the other hand, technology could provide support by performing procedural work for the students. This allowed my students to focus on the acquisition of the declarative knowledge, particularly in the *constructing meaning* and *organising* phases.

Dimension Three of the Dimensions of Learning framework is concerned with extending and refining knowledge. As outlined in the Literature Review, the process of extending and refining knowledge is achieved through the use of a variety of complex reasoning processes. The contribution that technology could make for my students with respect to some of these complex reasoning processes came to light through the research. The complex reasoning processes which were involved were *constructing support*, *deductive reasoning* and *abstracting*.

Investigating these complex reasoning processes began by implementing tasks that my students could approach with their existing mathematical knowledge. The same tasks were then revisited and more sophisticated levels of knowledge were applied to them. This revisiting was carried out through discussion and investigation in follow-up lessons with the same students. It was also carried out by taking a problem that my Year 8 students had been working on to older students in the school. In this way I made a quasi-longitudinal exploration of the road that lay ahead for my students in applying other complex reasoning process and more refined methods.

I found the process of extending and refining my students' knowledge to be challenging at times, especially when I underestimated their prior mathematical knowledge. My efforts were worthwhile, however, in that I was able to illuminate the part that technology played in addressing my students' use of complex reasoning processes. As far as *constructing support* was concerned, technology was helpful in situations that involved lending weight to conjectures by producing, graphs, tables, accurate scale diagrams and animations.

Technology played a lesser role with respect to *deductive reasoning*. The ability of the technology to allow the user to construct and manipulate a geometric figure was helpful to a degree but apart from this the contribution of technology to deductive reasoning was more restricted to aiding my students' understanding of the underlying context of a problem. This involved, for example, the production of geometric constructions on the calculator screen from which measurements could be taken and displayed. The contribution of technology to the complex reasoning process of *abstracting* was also limited as far as my Year 8 students were concerned. There were signs that technology might be helpful to them in developing this complex reasoning process later in their school careers, however. This came to light when older students in the school used the process of *abstracting* to solve a problem that the Year 8 students had tackled using the process of *constructing support*.

Another collection of complex reasoning processes, also detailed in the Literature Review, contain those that relate to Dimension Four: Using Knowledge Meaningfully. One of these processes, *decision making*, was examined in the research. I found that my students were assisted by the use of technology with respect to this complex reasoning process. This was evident when they were seeking optimal solutions such as finding the best place for a roundabout on a road circuit. The technology provided assistance by facilitating repeated measurements and calculations corresponding to changes in placement.

In Dimension Five: Habits of Mind, the focus is on encouraging students to develop productive ways of approaching their work. One of these habits is *responding with wonderment and awe*. I found encouraging the development of this habit in my students to be an ambitious but laudable aim. The use of technology was less significant to this pursuit than the mathematical aspects of the investigations. I observed that whatever degree of wonderment and awe my students could be said to have exhibited it was on account of interesting mathematical revelations and connections – it was not attributable to the use of technology per se. As occurred often in the research, the place of technology was to facilitate rather than direct.

With the habit of mind of *thinking interdependently*, however, technology did have a higher profile than at other times in the research. In tying together situations where my students used technology in collaborative settings, I noticed that the use of technology was to some extent an agent of change. It could provide information that made some of my students stop and rethink. And this is a contribution on a higher level than, say, performing calculations speedily to support problem solving. I also observed some of my students communicating their understanding in a collaborative way using technology in a way that would not have taken place if the technology had not been present.

Limitations

This research focuses on one subgroup of students from one particular school using specific types of technology and as such it cannot claim to convey a message that is of universal significance. The time constraint of one lesson per week with the students gave rise to some limitations. In particular, this meant that the analysis related to Dimension Four: Using Knowledge Meaningfully was restricted and could include application-oriented tasks (Marzano, 1992) but not long-term tasks such as those produced by Peter Galbraith (2009). The research was conducted with subjects who were extracted from their usual classroom settings in order to provide them with more challenging work. This meant that the findings were limited to those drawn from small group settings with quick and able students. Many of the difficulties of integrating technology into a classroom with large numbers of students of mixed ability were therefore removed from the field of study. Despite these limitations, it is hoped that the context and descriptions given will aid the discretion of the reader and provide some degree of transferability.

Implications and Recommendations for Further Research

In working through the project, the importance of the connection between the use of technology in association with two different types of knowledge, that is procedural and declarative knowledge, came to light. This connection has implications for practitioners in the classroom, authors of mathematics texts, and curriculum writers. Paying heed to this connection could make the work of classroom teachers more effective in their efforts to promote the development of procedural knowledge and the development of declarative knowledge. As far as procedural knowledge is concerned, this could be achieved by focusing on the use of technology on the shaping phase of acquiring procedural knowledge, that is the phase where procedural knowledge is refined and errors eliminated, rather than on a mechanistic use of technology which merely duplicates by hand methods.

In the same vein, if the instructions that authors of mathematical texts provide for carrying out mathematical procedures using technology are presented in parallel with by hand methods on the same mathematical procedures, then this could lead to redundancies in the pedagogical approach adopted by the user of the text. The author may not be intending to recommend this inefficient approach, and of course it is the responsibility of the user of the text to adopt an appropriate pedagogy. Nevertheless, it could be helpful if the presentation of the text suggested a use of technology that supported the acquisition of procedural knowledge in the shaping phase rather than just as an alternative to the by hand method.

As far as declarative knowledge is concerned, teachers could make effective use of technology by directing its use towards the organising phase of acquiring declarative knowledge, by which is meant the phase where students are involved in summarising and generalising knowledge. A use of technology in this way, which supports the acquisition of declarative knowledge rather than duplicating the processes of procedural knowledge, is one that curriculum writers could target. Meaningful uses of technology could then be embedded into curriculum documents.

The exploration that took place in the research into ways of guiding students from specific solutions to problems to generalised solutions has some potential links to curriculum design. Students at early stages in their high school mathematics training may not have acquired sufficient skills to allow them to solve problems in different ways. In the analysis section on Dimension Three: Extending and Refining Knowledge, the same problem was taken to older students and in so doing several different solutions were produced. But if a curriculum had a longitudinal design with respect to problems and methods then the same students could revisit problems over time. This could be a powerful way for the students to gain an appreciation of different methods and their relative mathematical worth without having to spend time on learning about new contexts.

During the course of the research, there were differing aspects of my behaviour as a mathematics teacher when technology was introduced into the classroom. This area was not focused on in any great depth in the analysis but it is one that is worthy of further research. If my tendency to adopt different roles as a teacher in the presence of technology is one that others might experience, then this may have implications for teachers' professional development. Some may need assistance in increasing the variety of roles that they adopt. There are also possible implications here for the type of hardware that is used. When behaving as a coach from the side, or as a co-learner, for example, it may be that hardware that is mobile would be more suitable than hardware that is fixed in location. There may also be implications for the design of the physical learning environment of the classroom to accommodate varied modes of teaching.

Gender-related issues would also merit further research. This could be useful in highlighting any differences in students' perceptions about the use of technology that were linked to gender. For example, gender differences could be studied with respect to the *inner resources* that students perceive they can bring to bear on tasks involving the use of technology. The habit of mind of *thinking interdependently* with the aid of technology could also be considered with gender in mind. It may be fruitful to consider possible gender differences in the ways that students collaborate in the presence of technology. This could bring a new layer of meaning to the term *thinking interdependently* and stretch the boundaries of learning theories such as social culturalism and social interdependence.

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APPENDICES

Appendix 1: Approval for Research

memorandum

Curtin
University of Technology

To	Ian Thomson, SMEC
From	Pauline Howat, Coordinator for Human Research Ethics, Science and Maths Education Centre
Subject	Protocol Approval **SMEC-05-09**
Date	12 March 2009
Copy	Bill Atweh, SMEC Divisional Graduate Studies Officer, Division of Science and Engineering

Office of Research and Development

Human Research Ethics Committee

TELEPHONE 9266 2784
FACSIMILE 9266 3793
EMAIL hrec@curtin.edu.au

Thank you for your "Form C Application for Approval of Research with Minimal Risk (Ethical Requirements)" for the project titled "*A CASE STUDY OF THE IMPACT OF NEW TECHNOLOGY ON THE LEARNING OF MATHEMATICS IN A SECONDARY SCHOOL CLASSROOM*". On behalf of the Human Research Ethics Committee I am authorised to inform you that the project is approved.

Approval of this project is for a period of twelve months **5th March 2009** to **4th March 2010**.

If at any time during the twelve months changes/amendments occur, or if a serious or unexpected adverse event occurs, please advise me immediately. The approval number for your project is **SMEC-05-09**. *Please quote this number in any future correspondence.*

PAULINE HOWAT
Coordinator for Human Research Ethics
Science and Maths Education Centre

Please Note: The following standard statement must be included in the information sheet to participants: *This study has been approved by the Curtin University Human Research Ethics Committee (Approval Number SMEC-05-09). If needed, verification of approval can be obtained either by writing to the Curtin University Human Research Ethics Committee, c/- Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth, 6845 or by telephoning 9266 2784.*

Appendix 2: Information sheets for students, parents and Head of School

Curtin University of Technology
Science and Mathematics Education Centre

Participant Information Sheet for Students

I am currently completing a piece of research for my Educational Doctorate degree in Mathematics Education at Curtin University of Technology.

Purpose of Research
I am investigating the benefits of using technology in the secondary mathematics classroom.

Your Role
I am interested in finding out how technology in the classroom can improve students' learning of mathematics.
I would like to find out how using technology can help students understand new concepts in mathematics and how they feel about using technology in the mathematics classroom. I will ask you questions about how you went about solving problems and seek your opinions on what it is like in the classroom when technology is present.
The interview process will take approximately 20 minutes.

Consent to Participate
Your involvement in the research is entirely voluntary. You have the right to withdraw at any stage without it affecting your rights or responsibilities. When you have signed this consent form I will assume that you have agreed to participate and allow me to use the students' data in this research.

Confidentiality
The information provided will be kept separate from students' personal details, and only myself and my supervisor will have access to this. The interview transcript will not have students' names or any other identifying information on it and in adherence to university policy, the interview tapes and transcribed information will be kept in a locked cabinet for at least five years, before a decision is made as to whether it should be destroyed.

Further Information
The research has been reviewed and given approval by Curtin University of Technology Human Research Ethics Committee (Approval Number SMEC 05 09). If you would like further information about the study, please feel free to contact me on 3870 4456 or by email i.thomson@ormiston.qld.edu.au. Alternatively, you can contact my supervisor Associate Professor Bill Atweh on (08) 9266 7073 or by email b.atweh@curtin.edu.au.

Thank you very much for your involvement in this research.
Your participation is greatly appreciated.

Curtin University of Technology
Science and Mathematics Education Centre

Participant Information Sheet for Parents

I am currently completing a piece of research for my Educational Doctorate degree in Mathematics Education at Curtin University of Technology.

Purpose of Research
I am investigating the benefits of using technology in the secondary mathematics classroom.

Your Role
I am interested in finding out how technology in the classroom can improve students' learning of mathematics.
I would like to find out how using technology can help students understand new concepts in mathematics and how they feel about using technology in the mathematics classroom. I will ask them questions about how they went about solving problems and seek their opinions on what it is like in the classroom when technology is present.
The interview process will take approximately 20 minutes. Students will be asked to complete questionnaires several times throughout the year.
Your role is to grant permission for your child to participate and allow me to use your child's data in this research.

Consent to Participate
Your involvement in the research is entirely voluntary. You have the right to withdraw at any stage without it affecting your rights or responsibilities. When you have signed this consent form I will assume that you have agreed to participate and allow me to use the students' data in this research.

Confidentiality
The information provided will be kept separate from students' personal details, and only myself and my supervisor will have access to this. The interview transcript will not have students' names or any other identifying information on it and in adherence to university policy, the interview tapes and transcribed information will be kept in a locked cabinet for at least five years, before a decision is made as to whether it should be destroyed.

Further Information
The research has been reviewed and given approval by Curtin University of Technology Human Research Ethics Committee (Approval Number SMEC 05 09). If you would like further information about the study, please feel free to contact me on 3870 4456 or by email i.thomson@ormiston.qld.edu.au. Alternatively, you can contact my supervisor Associate Professor Bill Atweh on (08) 9266 7073 or by email b.atweh@curtin.edu.au.

Thank you very much for your involvement in this research.
Your participation is greatly appreciated.

Curtin University of Technology
Science and Mathematics Education Centre

Participant Information Sheet for Head of Senior School

I am currently completing a piece of research for my Educational Doctorate degree in Mathematics Education at Curtin University of Technology.

Purpose of Research
I am investigating the benefits of using technology in the secondary mathematics classroom.

Your Role
I am interested in finding out how technology in the classroom can improve students' learning of mathematics.
I would like to find out how using technology can help students understand new concepts in mathematics and how they feel about using technology in the mathematics classroom. I will ask them questions about how they went about solving problems and seek their opinions on what it is like in the classroom when technology is present.
The interview process will take approximately 20 minutes. Students will be asked to complete questionnaires several times throughout the year.
Your role is to grant permission for this research to take place at Ormiston College.

Consent to Participate
Your involvement in the research is entirely voluntary. You have the right to withdraw at any stage without it affecting your rights or responsibilities. When you have signed this consent form I will assume that you have agreed to participate and allow me to use the students' data in this research.

Confidentiality
The information provided will be kept separate from students' personal details, and only myself and my supervisor will have access to this. The interview transcript will not have students' names or any other identifying information on it and in adherence to university policy, the interview tapes and transcribed information will be kept in a locked cabinet for at least five years, before a decision is made as to whether it should be destroyed.

Further Information
The research has been reviewed and given approval by Curtin University of Technology Human Research Ethics Committee (Approval Number SMEC 05 09). If you would like further information about the study, please feel free to contact me on 3870 4456 or by email i.thomson@ormiston.qld.edu.au. Alternatively, you can contact my supervisor Associate Professor Bill Atweh on (08) 9266 7073 or by email b.atweh@curtin.edu.au.

Thank you very much for your involvement in this research.
Your participation is greatly appreciated.

Appendix 3: Consent form for students, parents and Head of School

CONSENT FORM

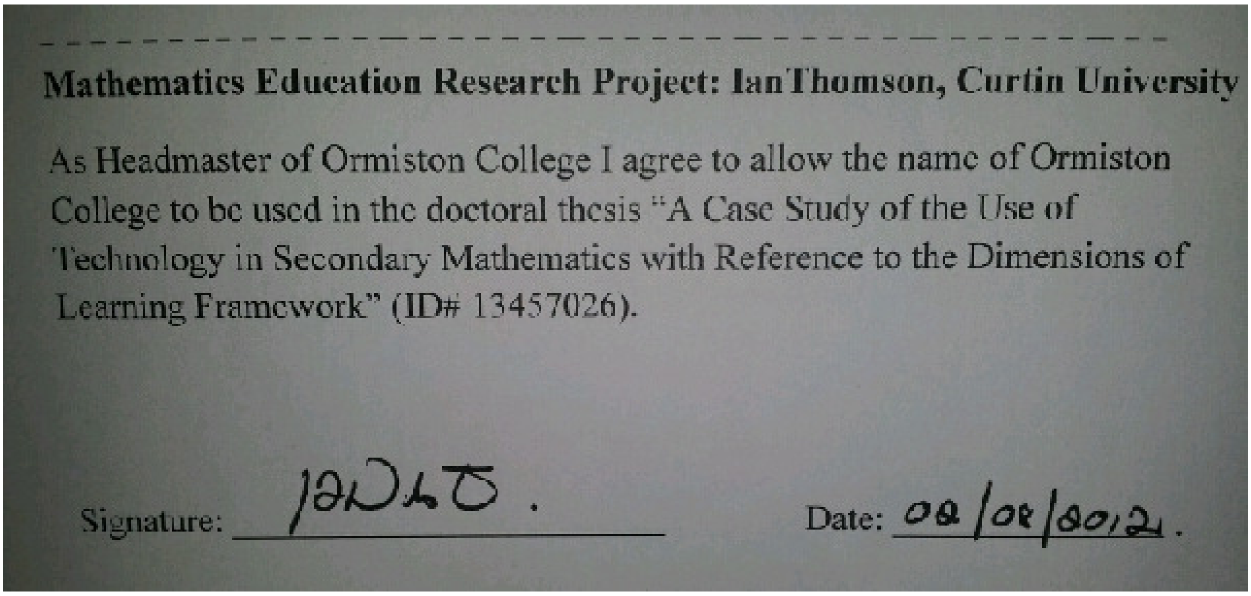
- I understand the purpose and procedures of this study.
- I have been provided with the participant information sheet.
- I understand that the procedure itself may not benefit me.
- I understand that my involvement is voluntary and I can withdraw at any time without problem.
- I understand that no personal identifying information like my name and address will be used in any published materials.
- I understand that all information will be securely stored for at least 5 years before a decision is made as to whether it should be destroyed.
- I have been given the opportunity to ask questions about this research.
- I agree to participate in the study outlined to me.

Name: ____

Signature: ______________________________

Date: ________________________

Appendix 4: Consent to use the school name in the thesis



Mathematics Education Research Project: Ian Thomson, Curtin University

As Headmaster of Ormiston College I agree to allow the name of Ormiston College to be used in the doctoral thesis "A Case Study of the Use of Technology in Secondary Mathematics with Reference to the Dimensions of Learning Framework" (ID# 13457026).

Signature: ______________________ Date: 02/08/2012.

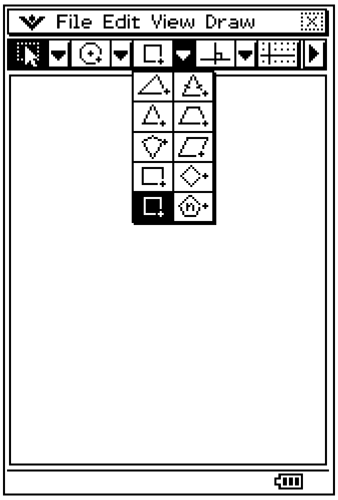
Appendix 5: The Square Peg in a Round Hole Task

A Square Peg in a Round Hole

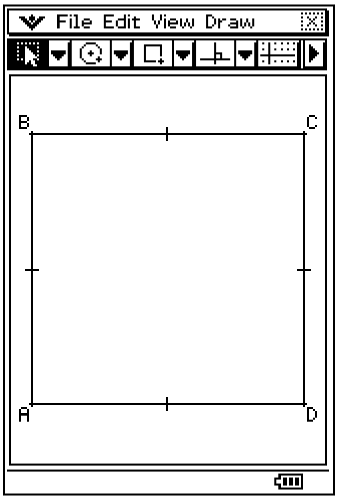
Which fits better, a square peg in a round hole or a round peg in a square hole?

To investigate this question, we can begin by drawing a square.

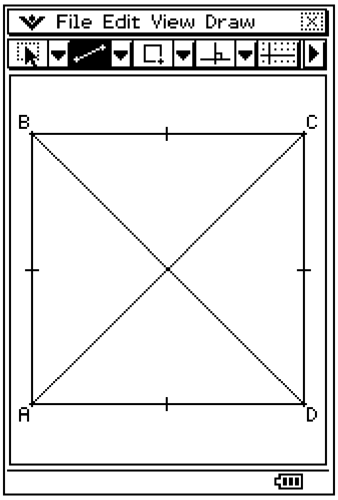
Select a square from the Polygons menu.



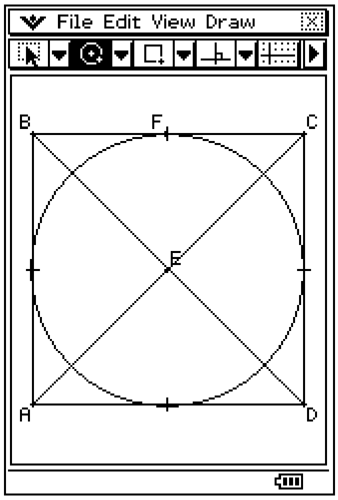
Tap anywhere on the screen to draw the square.



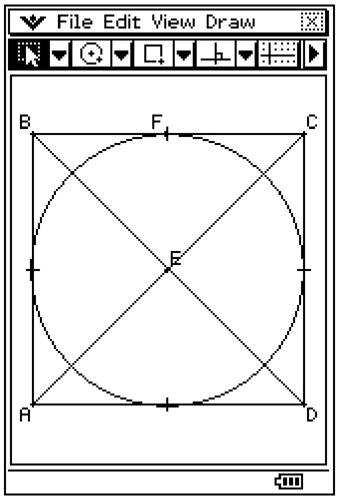
Draw in the diagonals of the square using line segments. This will find the centre of the square.



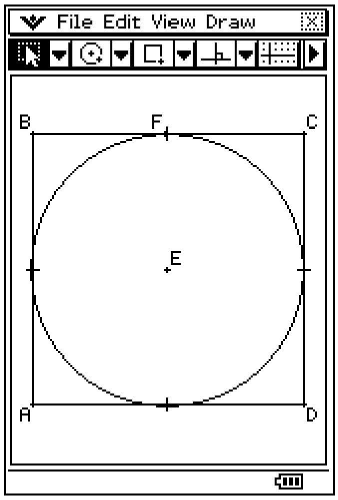
Now draw a circle inside the square using the intersection of the diagonals as the centre and one of the midpoints of a side of the square as a point on the circumference of the circle.

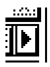


Tap the deselect tool at the top left of your screen.

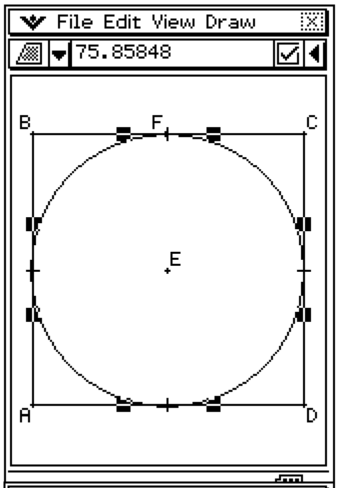


The diagonals are no longer required so select them and press clear button on your calculator keypad.



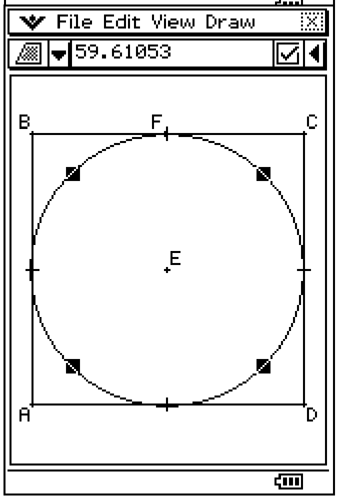
Tap on  to go *around the corner.*

Select the sides of the square. The area of the square will appear at the top of the screen as 75.85848 units squared.



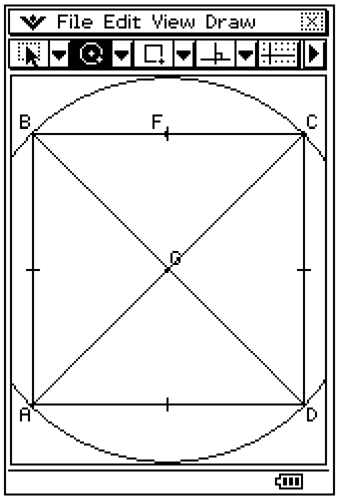
Tap anywhere in *free space* to deselect the square then select the circle. The area of the circle will be shown as 59.61053 units squared.

How good a fit is this?

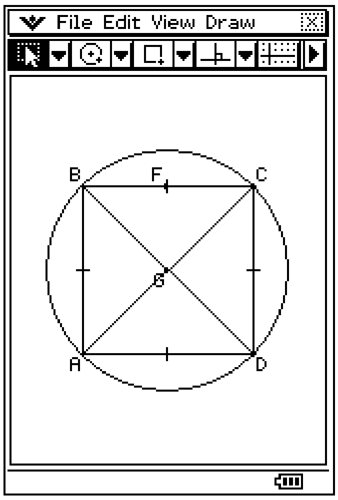


Now let's see how well a square peg fits into a round hole. Press Clear to delete the circle.

Draw in the diagonals of the square again and draw a circle around the square using the intersection of the diagonals and a vertex of the square.



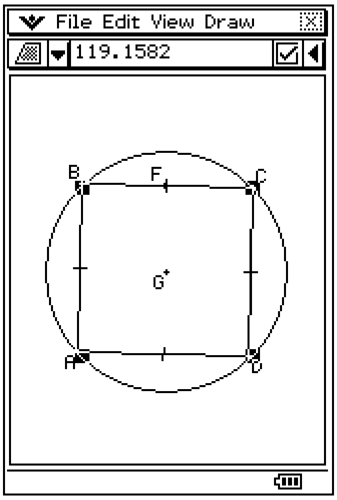
Select **Zoom to Fit** from the **View** menu.



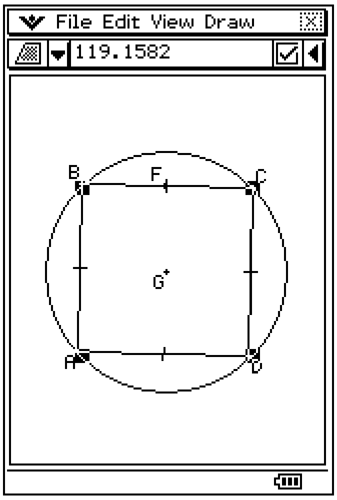
Delete the diagonals and select the circle.

The area of the circle is shown to be 119.1582.

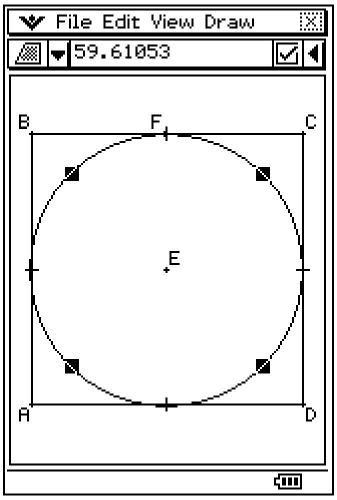
The area of the square was shown previously to be 78.85848 units squared. How well does the square fit into the circle?



What do you think now? Which fits better, a square peg in a round hole or a round peg in a square hole?



Square peg in a round hole



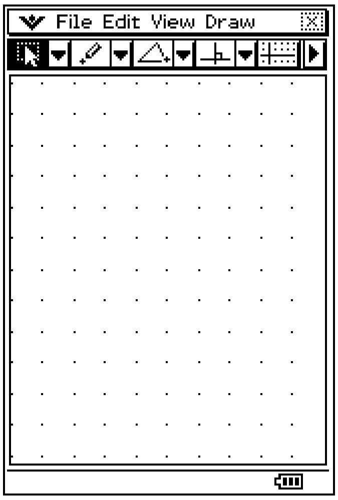
Round peg in a square hole

Appendix 6: Calculating the Areas of Polygons Task

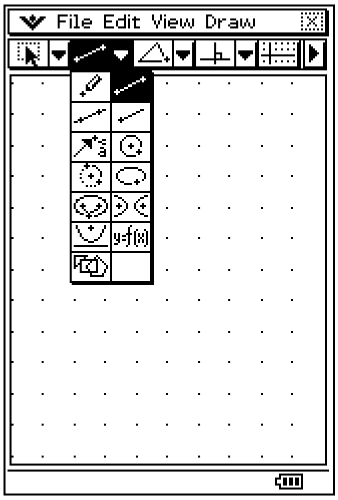
Calculating the Areas of Polygons

Start a new file by selecting **New** from the **`File`** menu.

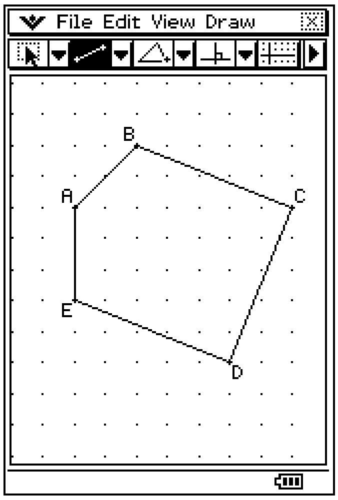
Select **`Integer Grid`** from the **`View`** menu.



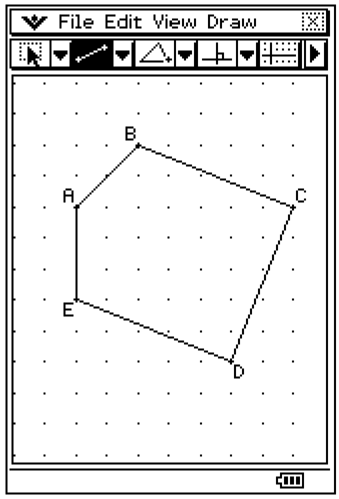
Select line segments from the drawing tools menu



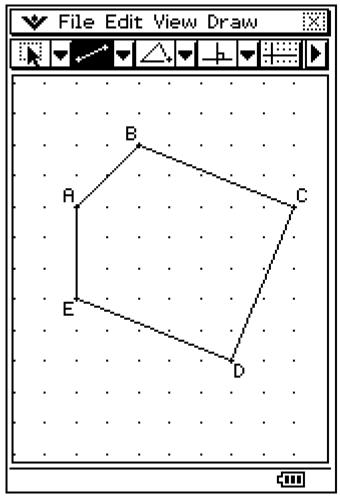
Tapping on grid points, draw a polygon of your choice. An example is shown on the right.



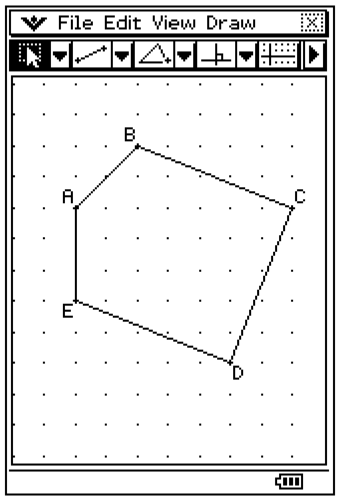
Count the number of grid points that can be seen inside the polygon. These are called interior points. In this example there are 29 interior points.

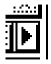


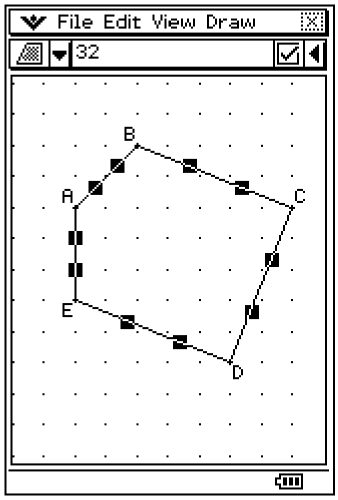
Now count the number of boundary points. Boundary points are points that are on the edges of the polygons and which are also on a grid point. In this example there are 8 boundary points.



Tap on the deselect arrow at the top left of your screen



Tap on  to go *around the corner*.



Select all the sides of the polygon. The area will be displayed at the top of the screen. In this example, the area of the polygon is 32 units squared.

Now see if you can find a connection between the number of interior points, the number of boundary points and the area of the polygon. Construct several different polygons and record the results in the table below. You might like get some friends to help you with this.

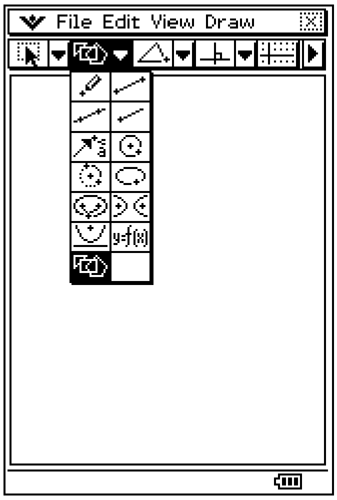
Number of interior points (I)	Number of boundary points (B)	Area of polygon

See if you can find a formula for the area of the polygon in terms of I and B.

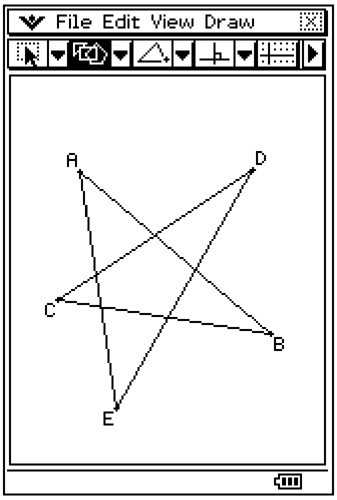
Appendix 7: Angles in a Star Task

Angles in a Star

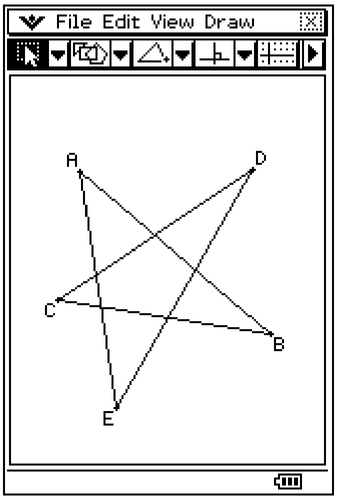
Select the polygon tool from the drop down menu as shown on the right.



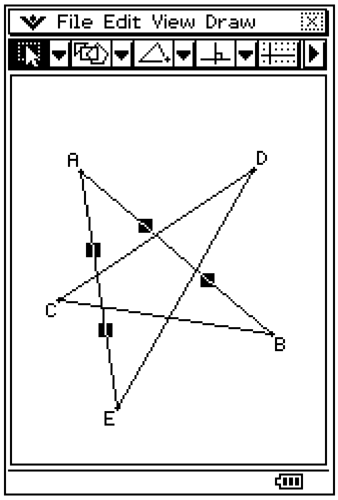
Tap five points on the screen to make a star shape. It does not have to be a regular shape.

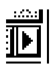


Tap on the deselect arrow at the top left of the screen

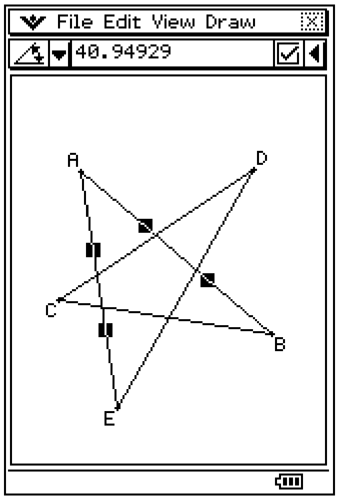


Select the lines AB and AE by tapping near the middle of each one



Tap on  to go *around the corner*.

The measure of the angle at **A** will appear at the top of the screen.



Click anywhere in *free space*.

You can use the same process to measure the angles at the other vertices.

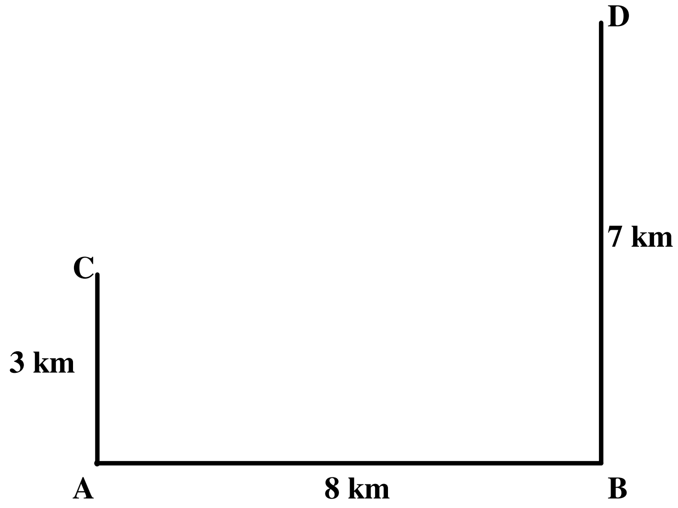
Find the total of all the angles at the vertices.

Now try writing out a proof using deductive geometry (hint: the exterior angle of a triangle theorem might be helpful).

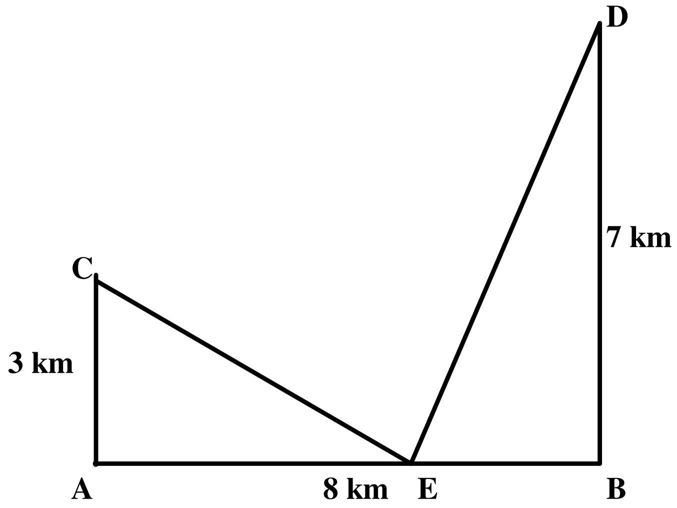
Appendix 8: Road Circuit Task

Road Circuit

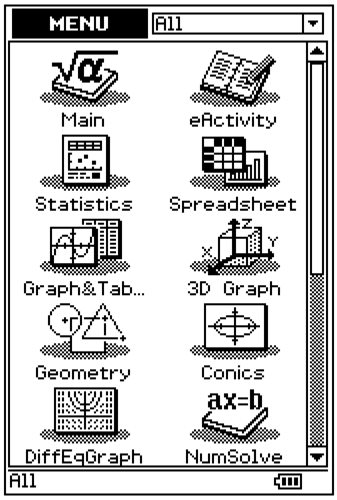
This investigation is about a road circuit. At present, four towns A, B, C and D are connected by road. AB is 8 km long. AC is at right angles to AB and is 3 km long. BD is also at right angles to AB and is 7 km long.



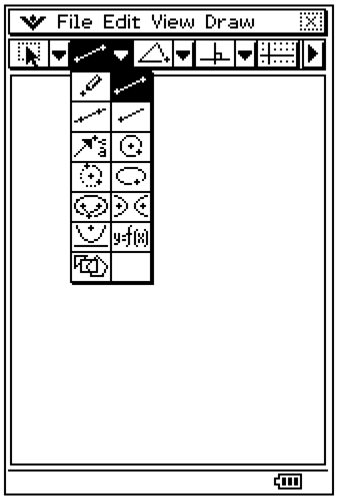
A roundabout with roads leading off to C and D is to be built somewhere along the road from A to B. The question is where should the roundabout be built in order to minimise the road building costs?



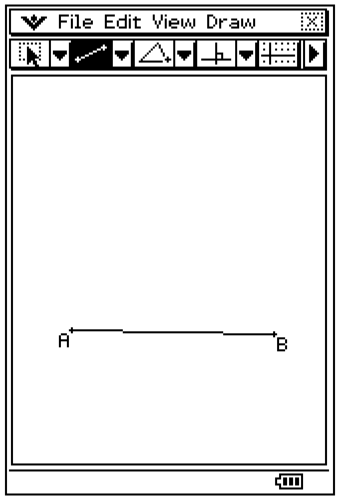
Time for some road construction!



Click on the Geometry icon.

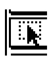


Select the Line Segment tool from the drop-down menu as shown.

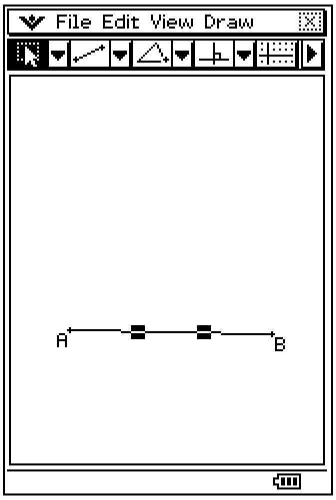


Click twice on the screen to create the line segment AB.

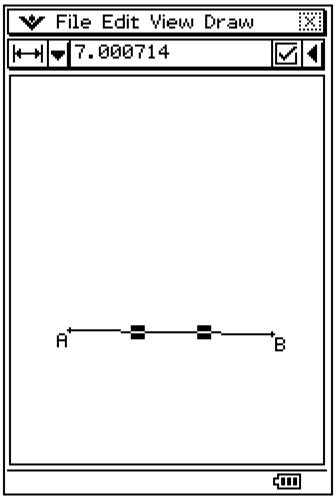
(Don't worry if it does not look perfectly straight)

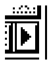


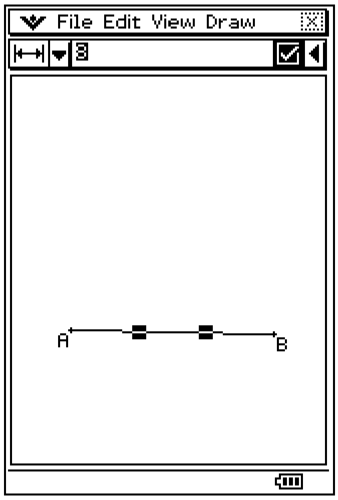
Click on near the top left of the screen. This will allow you to stop drawing line segments temporarily.



You can now select the line segment AB by clicking on it.



Now click on  at the top right of the screen and go *around the corner*. The length of the line segment will be displayed at the top of the screen. In this case it measures 7.000714. Your line segment will have a different length to the one shown in this screenshot.

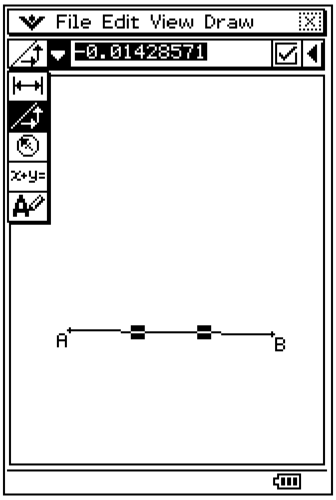


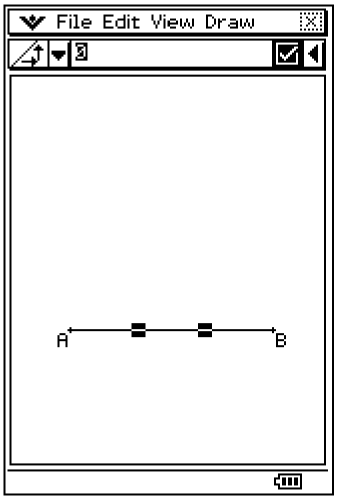
In order to make the line segment exactly 8 units long, you need to:

- Highlight the measurement by dragging the stylus across it
- Punch in 8 on the keyboard
- Click on the tick

The line segment is now 8 units long.

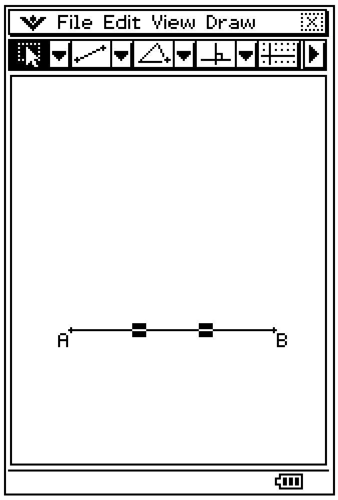
Now select the second option from the drop down menu. This will display the slope that the line segment is lying on.

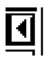




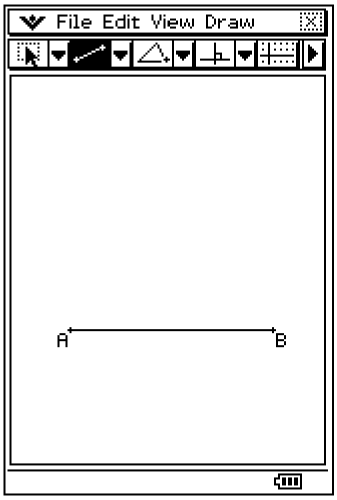
Punch in 0 on the keyboard and click on the tick

The line segment is now 8 units long and perfectly flat.

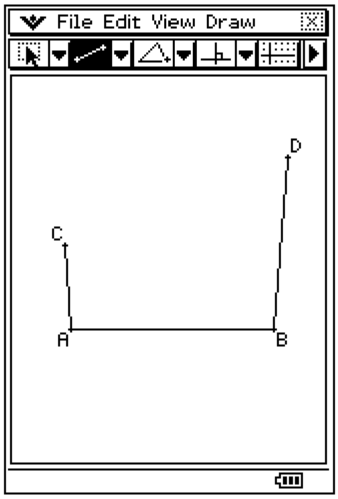


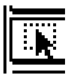
Click on  at the top right of the screen to return to the original menu

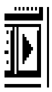
Now click on the line segment tool

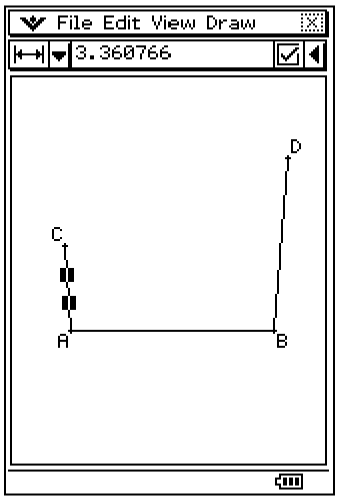


Create two more line segments AC and BD



Click on  near the top left of the screen in order to stop drawing line segments.

Then click on  to go *around the corner.*

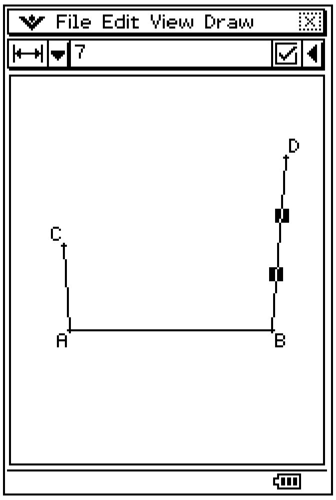


Click on the line segment AC and select the measuring tool from the drop down menu

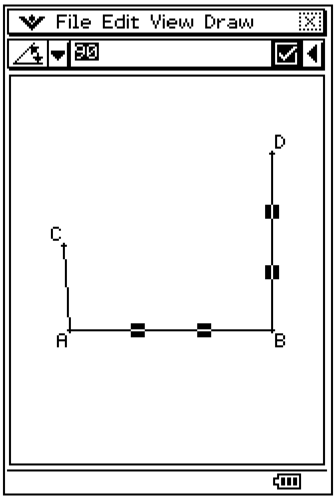
Make the line segment AC exactly 3 units long.

Deselect AC by clicking anywhere on the screen,

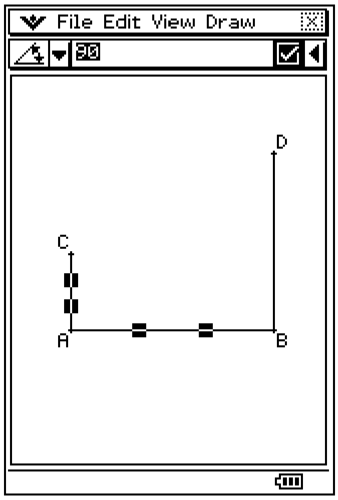
Select the line segment BD and make it exactly 7 units long.

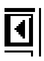


You need to make sure that BD is at right angles to AB. To do this, click on AB so that both BD and AB are selected. The angle between AB and BD will now be displayed at the top of the screen. Change this angle to 90 degrees.

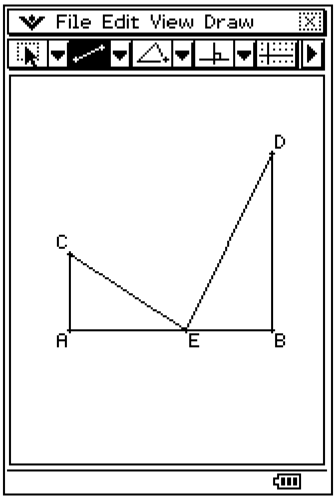


Deselect AB and BD by clicking anywhere in *free space*. Now select AB and BC and make sure that the angle between them is 90 degrees.

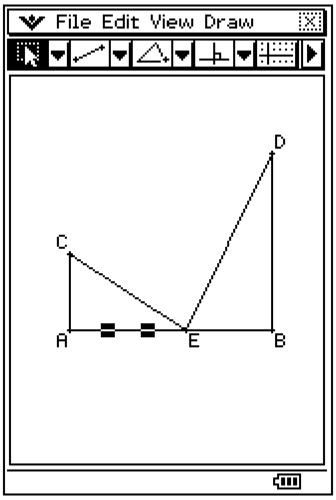


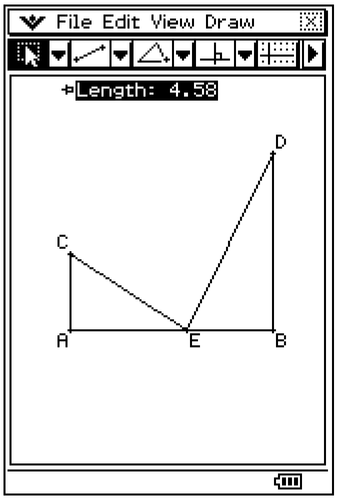
Click on  at the top right of the screen to return to the original menu

Once you have done this, click anywhere on the screen to deselect, then choose the line segment tool and draw line segments from C to E and from D to E. It does not matter exactly where you put E.



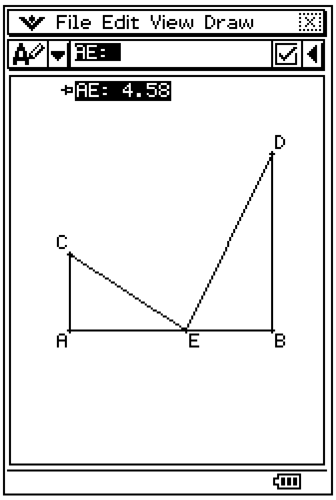
Now draw a line segment from A to E. This will not change the appearance of the screen but it will allow you to measure the length of AE.





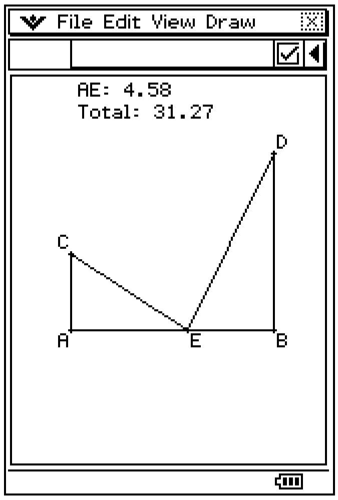
To display the length of AE on the screen, first of all select the line segment AE by clicking near the centre of it. Be careful not to select AB.

Now select **Measurement** then **Length** from the **Draw** drop down menu.



If you click on  you can go *around the corner* and change “Length” to AE

You can also display the total length of the road circuit on the screen by selecting all the roads and selecting **Measurement** then **Circumference** from the **Draw** drop down menu. You could rename "Circumference" as "Total"



You can now make the ClassPad repeatedly redraw the road circuit showing E at various positions along the road from A to B.

To achieve this:

1. Select the point E and the line segment AB.

2. Add an animation by selecting **Edit, Animate** then **Add animation.**

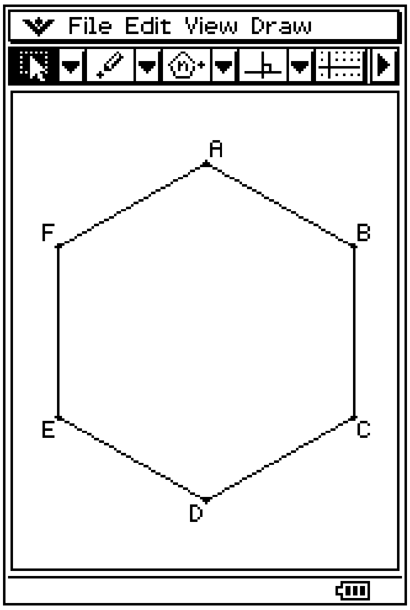
3. Start the animation by selecting **Edit, Animate** then **Go to and fro.**

4. To stop the animation, select **Edit, Animate** then **Stop.**

Appendix 9: Golden Point Task

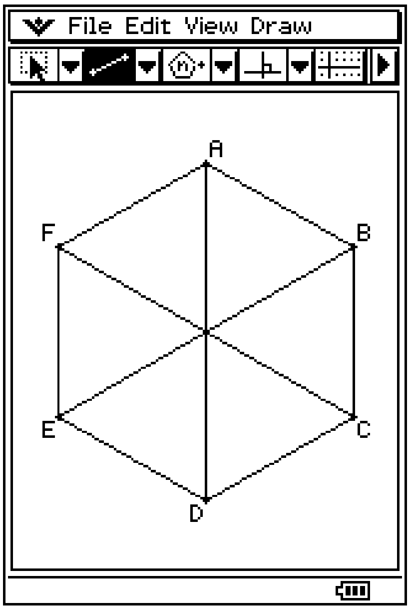
(Wells, 1991)

Golden Point Investigation

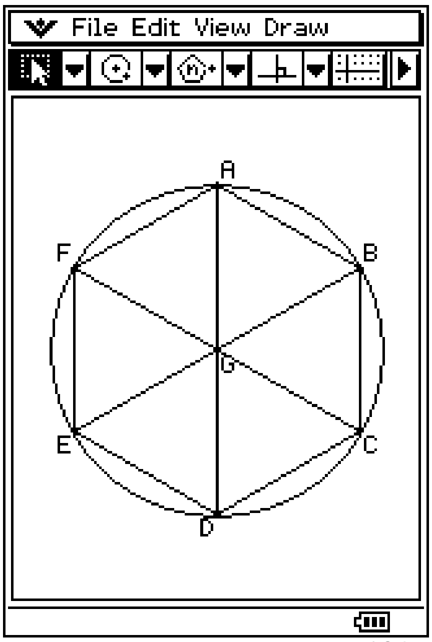


Select New from the File menu and select OK to clear all.

Draw a regular hexagon and label the vertices A, B, C, D, E and F



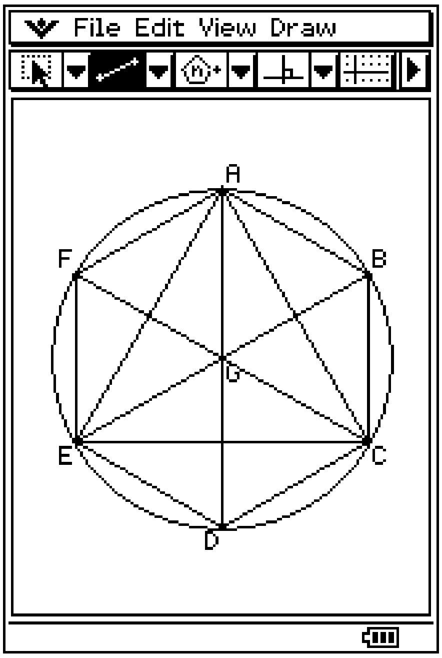
Add the diagonals of the hexagon using line segments



Draw a circle around the hexagon

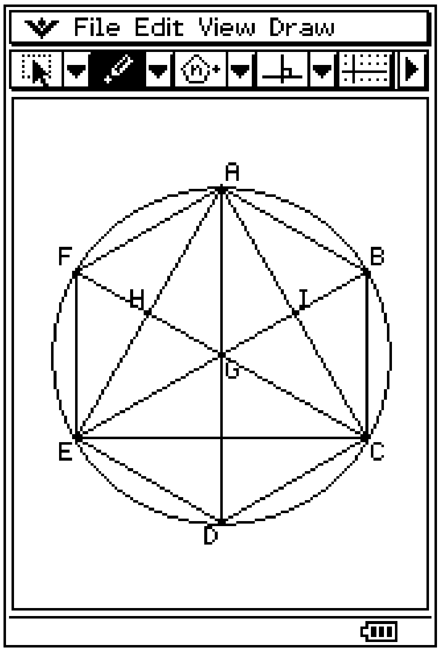
The point G is the centre of the circle and has centre (0,0)

Set the coordinates of the point A to be (0,1) and then zoom in. You now have a unit circle surrounding a hexagon

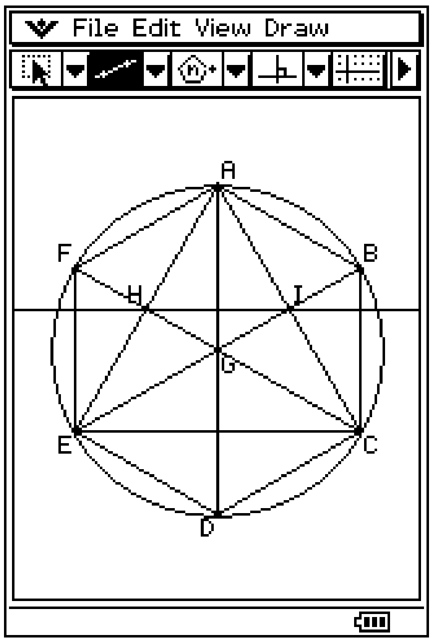


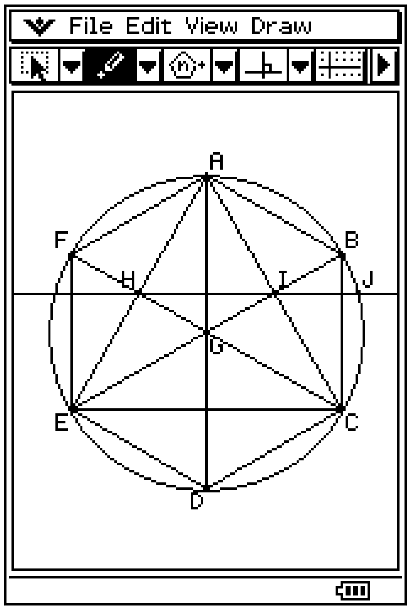
Construct an equilateral triangle with vertices A, C and E

Plot the intersection of AE and FC and the intersection of AC and BE



Draw an infinite line through the points H and I





Plot the point J where the line through the points H and I intersects the circle.

Tap on  at the top right of the screen to go *around the corner*. Tap on the points H and I and note the distance between them as displayed at the top of the screen. Tap in free space then tap on the points I and J and note the distance between them.

Calculate the ratio of the lengths of HI to IJ

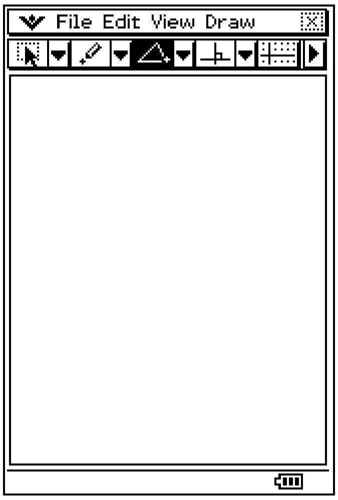
A golden point is a point that divides a line in the golden ratio. Explain why the point I appears to be a golden point.

Prove that the point I is a golden point.

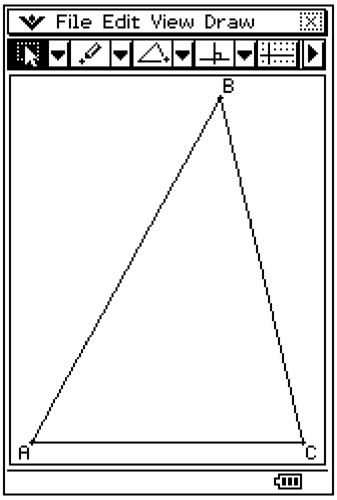
Appendix 10: Euler Line Task

The Euler Line

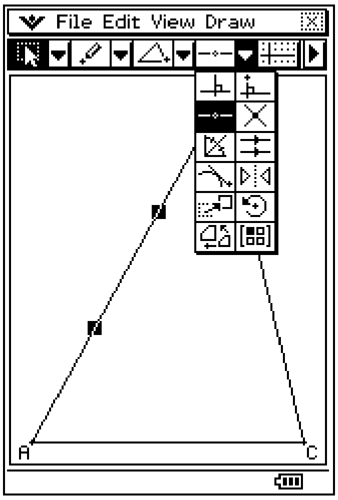
Finding the centroid of a triangle:



Select a triangle from the polygons menu



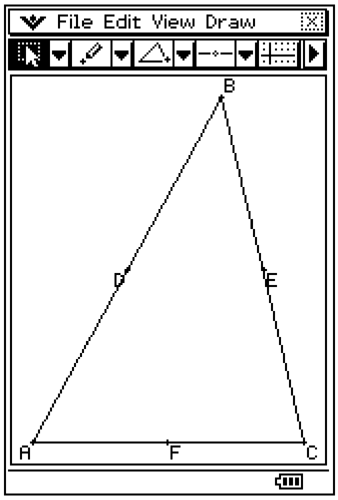
Tap anywhere on the screen to make the triangle appear



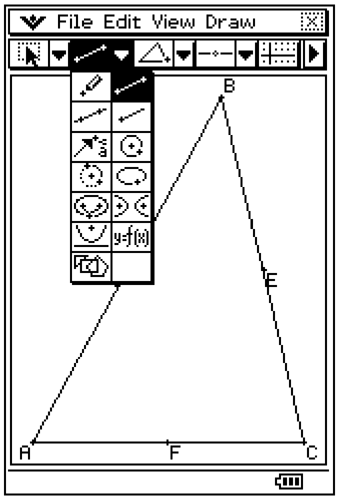
Select one side of the triangle and then select the mid-point tool from the tool s menu

Once you have created a mid-point, click anywhere in *free space* on the screen

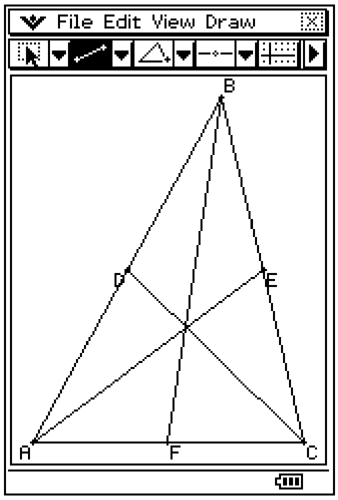
Create mid-points on the other two sides of the triangle using the same process



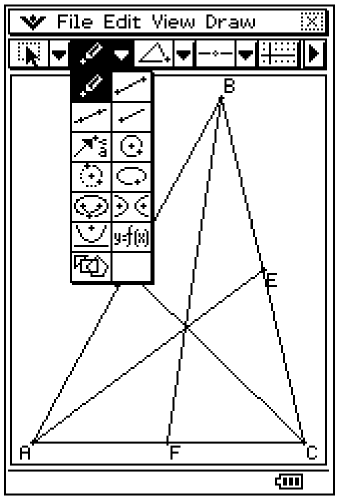
Select the line segment tool from the first drop down menu



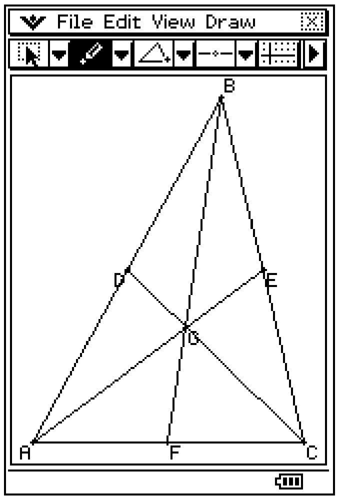
Use the line segment tool to join the mid-points to the opposite vertices as shown on the right



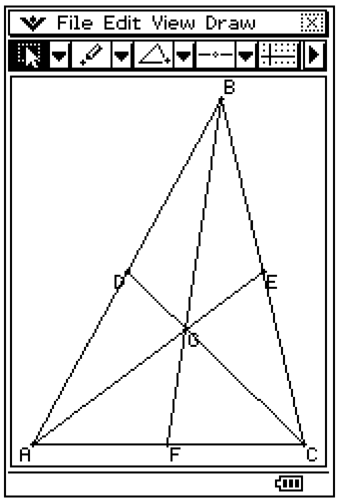
Select the point tool from the first drop down menu



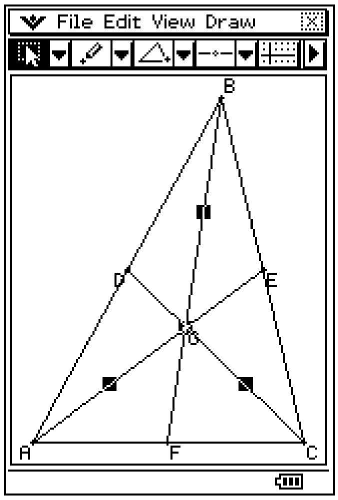
Use the point tool to create a point where the three medians meet. This point is called the centroid.



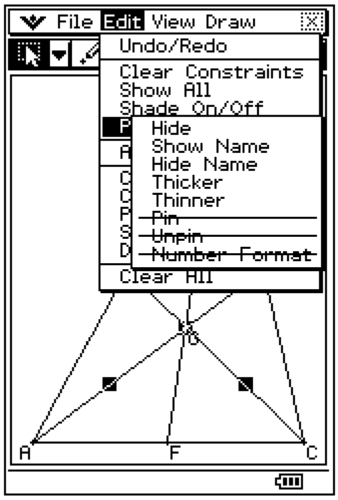
Tap on the deselect tool



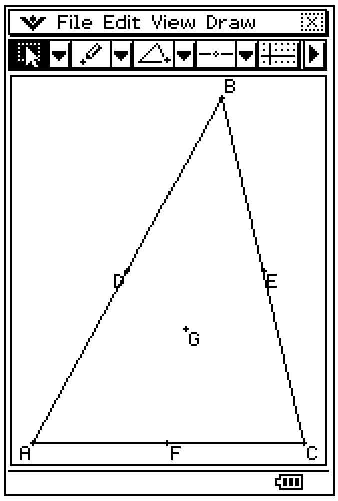
Select all three medians by clicking near the centre of each one



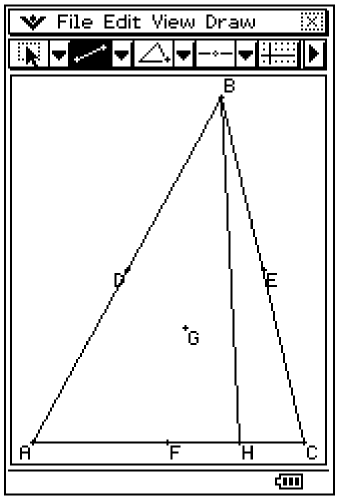
Hide the median lines by tapping on **Edit, Properties** then **Hide**



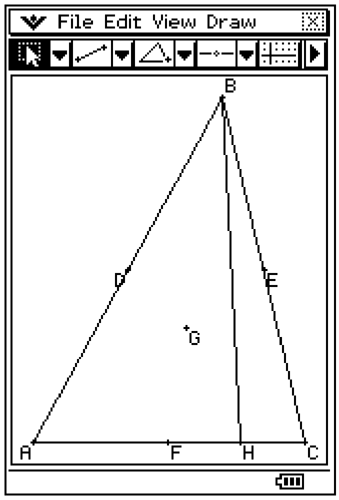
The centroid will remain visible



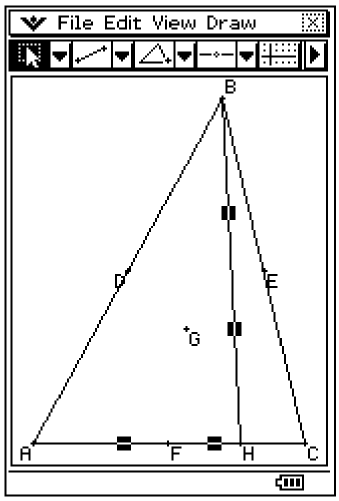
Finding the orthocentre:



Draw a line from the point B to the base AC.

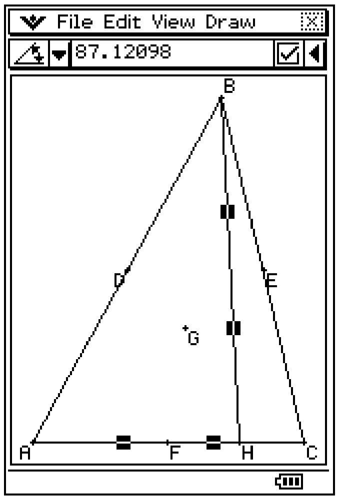


Tap on the deselect tool

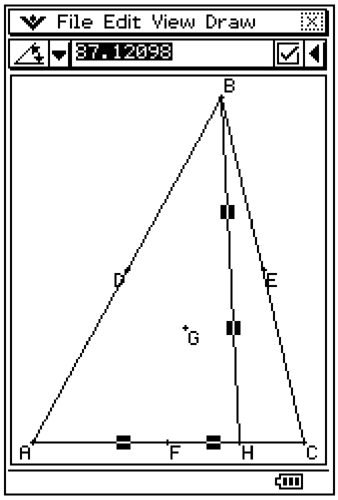


If BH is to be an altitude, we need to make it form a right angle with the base AC. In order to do this, select both lines BH and AC

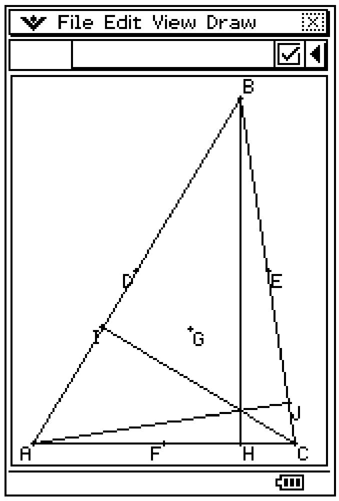
Go *around the corner.*



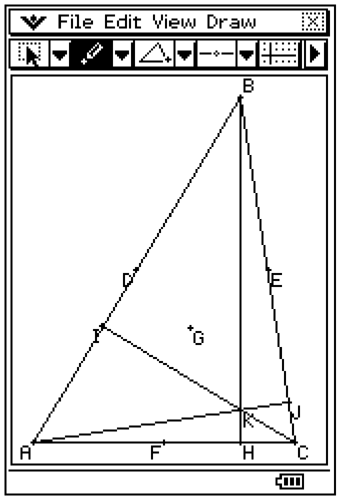
Highlight the measurement of the angle between BH and AC by dragging the stylus across it.



Key in 90 and tap on the tick. This will fix the angle at 90 degrees. BH will now be an altitude of the triangle.

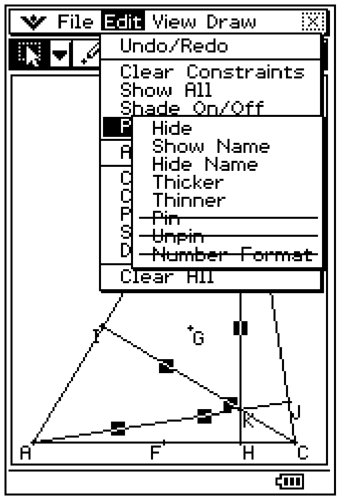


Follow the same procedure to construct the other two altitudes.

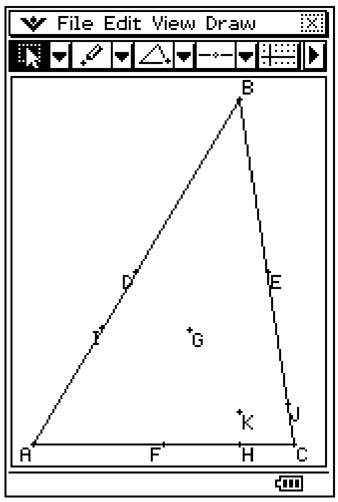


Use the point tool to create a point where the three altitudes meet. This point is called the orthocentre.

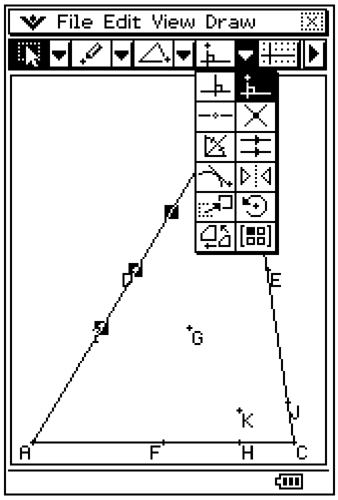
Tap the deselect tool. Select the three altitudes and hide them by selecting `Edit, Properties` and `Hide`.



The orthocentre will remain visible.

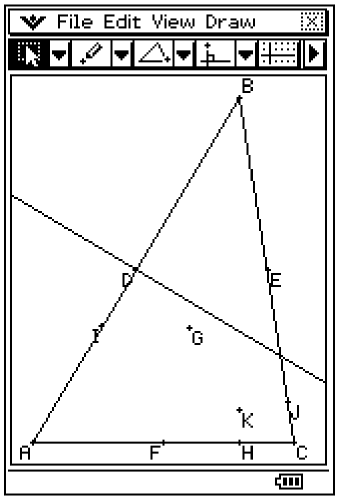


Finding the circumcentre:

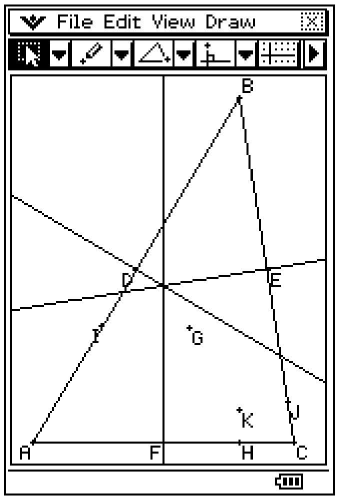


A perpendicular bisector cuts through the mid-point of a line at right angles.

Select a side of the triangle and select its mid-point. Now use the perpendicular bisector tool to construct the perpendicular bisector.

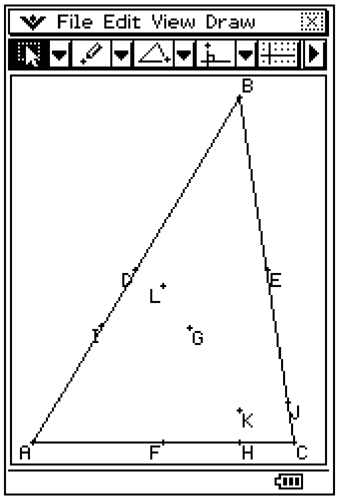


Once you have constructed the perpendicular bisector, tap anywhere in *free space* to deselect.



Follow the same procedure to construct the other two perpendicular bisectors. Don't forget to tap in *free space* to deselect.

As before, create a point where the lines meet then hide the lines. The circumcentre is now visible inside the triangle.



The circumcentre is equidistant from all three vertices. How could you illustrate this?

The centroid, the orthocentre and the circumcentre all lie on a straight line. How could you illustrate this?

Measure the distance from the centroid to the orthocentre and measure the distance from the centroid to the circumcentre. What do you notice?

What happens when you move one of the vertices?

What would happen if the triangle was equilateral?

Appendix 11: Linear Equations Tasks

1 Solve the following for x:

a) $3x + 4x = 14$
b) $4(3x + 4) - 6x = 46$
c) $3(x - 4) - x = 12$
d) $2(y - 5) + 3(y + 3) = 24$
e) $2(x + 1) + 3(x - 1) = 6$
f) $2(1 - 2x) + 8x = -7$

2 Solve the following for x:

a) $5x - 4 = x + 4$
b) $3x + 1 = 7x - 6$
c) $2x - 4 = 7 - x$
d) $3(x - 2) = x + 4$
e) $4(x - 3) = 3 - x$
f) $3(2x - 1) = 4x - 5$
g) $2 - x = 3(x + 3) + 1$
h) $2x + 1 = 2(1 - 3x) + 7$

3 Solve the following equations:

a) $12(x - 1) = 3(2x + 1) + 9$
b) $3(2p - 1) = 5(1 - p) + 3$
c) $2(5x + 1) + 2 = 3(x - 1)$
d) $6(x - 4) = 2(x + 5) = 3(x - 1) + 4$
e) $2(a - 1) + 3(a + 2) = a + 4$
f) $2x + 6(4 - x) = 2x + 3(x - 1)$
g) $4b + 2 - b = 7 + b - 3$
h) $5(3 - 4x) = 2(5 - 3x) - x$

Appendix 12: Academic Problems

("School Mathematics Competition," 2006)

1. In this addition sum a, b, c and d are different prime digits. What is the value of a, b, c and d?

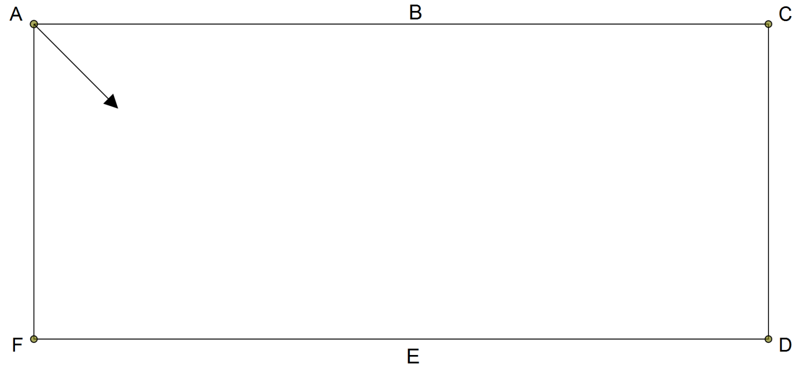
```
  a b
+ a c
+ a d
-----
  c b
```

2. The roosters on the ABC chicken farm want to buy an alarm clock. If each contributes 15c they are 70c short. If each contributes 20c they have 70c too much. How many roosters are on the farm?

3. Before a bicycle rally, Emma calculates that if she races at a steady 15 kph, she will pass the check point one hour too soon, but if she slows down to 10 kph she will arrive an hour late. How far is it to the first check point?

4. What is the largest number you can form if the first digit of the number is 8, i.e. the number is 8 …, and any two consecutive digits in the number gives a two digit number which is either divisible by 19 or 27?

5. There are six pockets A, B, C, D, E, and F on a billiard table 250 cm long and 100 cm wide. A ball is struck from A in a 45 degree direction, hits an edge and rebounds at 45 degrees. It continues to do this until it finally goes into a pocket. Which pocket will it eventually fall into?



6. It is your birthday and you want to have a small party. You invite 4 friends to your party. However they each invite 3 friends, each of whom invite 2 friends, who in turn each invite 1 friend. Assume that no one is invited by more than one person, and that everybody who is invited comes to your party, how many people will there be to wish you a happy birthday?

7. What is the smallest angle between the hands of a clock at 6:44 am?

Appendix 13: A Sample Report

Introduction

In this investigation I examined data concerning various types of breakfast cereals. I explored the assertion that the least healthy cereals were to be found on the middle shelf in the supermarket. I conducted the investigation by examining the nutritional value of the cereals in relation to their location on the supermarket shelves.

Analysis

The data contained information about the nutritional content of the cereals including calories, protein, fat, sugars, carbohydrates, potassium and vitamins. The location of the cereals was also noted as shelf one, two or three. In addition, a rating out of 100 was obtained for each type of cereal from a panel of health experts. I observed that the numbers of types of cereals were distributed amongst the three shelves as shown in Table 1 and Chart 1.

Types of Cereal	
Shelf	Total
1	20
2	21
3	36
Grand Total	77

Table 1: Numbers of cereal types per shelf

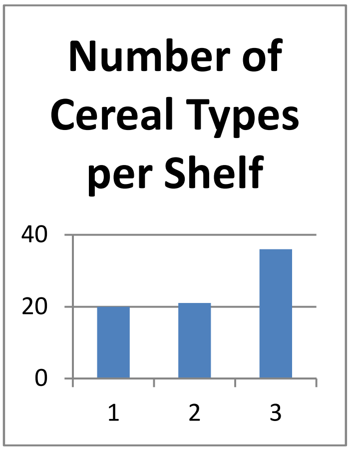


Chart 1: Numbers of cereal types per shelf

Clearly there are more varied types of cereals on the third shelf than on the other two shelves. Of more significance to the healthiness of the cereals, however, is the average number of units of sugar per shelf. This is shown in Table 2 and Chart 2 below. It is evident that shelf two contains the cereals that have the highest average sugar content.

Average of sugars	
Shelf	Total
1	4.8
2	9.6
3	6.5
Grand Total	6.9

Table 2: Average units of sugar per shelf

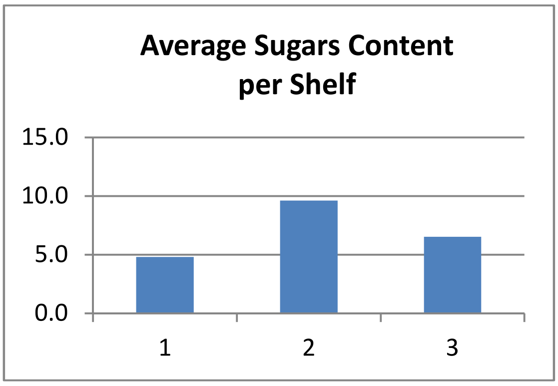


Chart 2: Average units of sugar per shelf

By way of contrast, the fibre content on the second shelf was observed to be the lowest as can be seen in the table 3 and chart 3 below:

Average of fibre	
Shelf	Total
1	1.7
2	0.9
3	3.1
Grand Total	2.2

Table 3: Average units of fibre per shelf

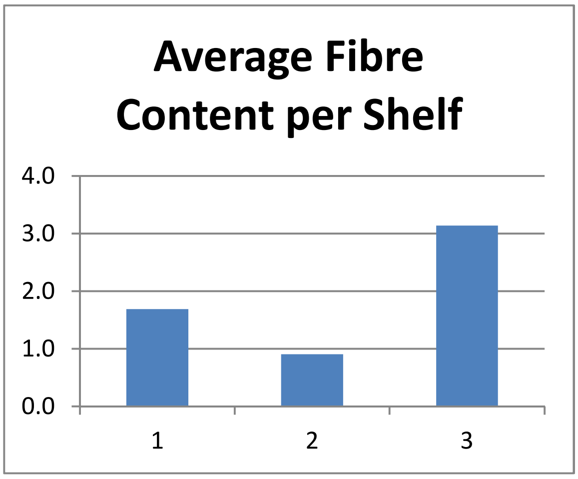


Chart 3: Average units of fibre per shelf

All of the cereals were given a rating out of 100 by a panel of health experts. A comparison of these ratings is given in the parallel box plots shown in Chart 4.

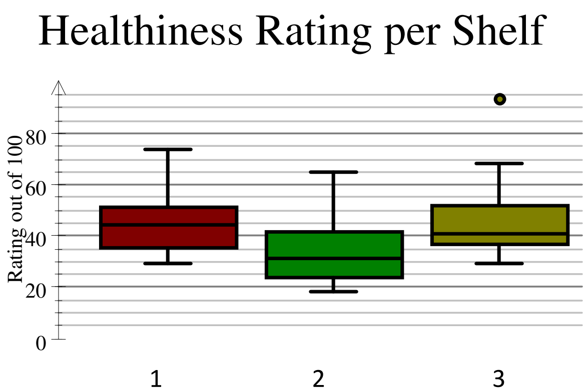


Chart 4: Healthiness ratings per shelf

Shelf two does not fare well in the ratings given by the health experts. It can be seen that three quarters of the ratings for cereals on shelf two are below the median ratings of shelves one and three.

Conclusion

The sugar content of the cereals was found to be highest on the second shelf. Since high sugar content can be taken to be undesirable, this is an indication that the cereals on shelf two are less healthy than the others. Fibre on the other hand is considered to be beneficial to a healthy diet. The comparatively low fibre content on the second shelf is another indication that the second shelf contains the less healthy cereals. These findings are consistent with the ratings from the health experts. The analysis of the ratings show the experts rated the cereals on the second shelf to be considerably lower than those on the other shelves. The analysis has produced evidence that supports the assertion that the cereals on the second shelf are less healthy than those on the other two shelves.

Assumptions and Limitations

It has been assumed that the data recorded was typical of the way that the cereals would usually be distributed on the shelves of the supermarket. The analysis only examined sugar content and fibre content as measures of healthiness. Other factors could have been examined such as fat content and carbohydrate content.

Concluding Comments

The comparison of the healthiness of the cereals, although not entirely comprehensive, agreed with the findings of the health experts. The findings of the investigation support the claim that the cereals on the second shelf are less healthy than the others.

Bibliography

References are included here

Appendix

Raw data are included here