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Time-Varying Skewness in Stock Returns: An Information-Based Explanation.

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There is evidence of regularities in the skewness of asset returns reported in the literature. The literature, however, offers no adequate explanations for these phenomena. Based on a simulation approach, we provide evidence that at least some aspects of skewness can be explained in terms of extant information-based theories in finance. Using a well-accepted model for generating asset returns, we demonstrate that when the effects of the uncertain information hypothesis and Kahneman and Tversky's prospect theory are incorporated in the return-generating process, the resulting return distributions can show negative skewness and variations of skewness with changing economic climates similar to what has been observed in empirical distributions.

Introduction

The presence of both positive and negative skewness in individual and portfolio stock return distributions is well documented in the literature (Beedles, 1979; Schwert, 1990; Aggarwal and Aggarwal, 1993; Alles and Kling, 1994; and Bekaert, Erb, Harvey, and Viskanta, 1998). Several papers have reported evidence of the presence of regularities in the variation of skewness, both cross-sectional and intertemporal. For example, Badrinath and Chatterjee (1991) report skewness properties across several stock groups, including industry groups, high and low risk stocks, and small and large stocks. Aggarwal and Aggarwal (1993) show that skewness properties differ across the organized stock markets, and Alles and Kling (1994) document a significant presence of negative skewness in return distributions and changes of the degree of skewness with the stages of the business and stock market cycles.

While evidence of skewness has been around for more than two decades, our understanding of the cause(s) of skewness, particularly negative skewness, is limited. The few contributions to this literature, such as Damodaran (1985), McNichols (1988), and Aggarwal and Rao (1990), provide explanations for only some aspects of the negative skewness phenomenon. Damodaran (1985) shows that a negative skewness can be introduced into stock return distributions when firms have a greater

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propensity to release good news versus bad news, and McNichols (1988) finds less positive skewness during earnings announcement periods versus non-announcement periods. Damodaran's skewness bias is predicted to be evident mostly over short time intervals. McNichol's negative skewness effect is a comparative bias (announcement periods versus non-announcement periods) and, as such, her results do not constitute an explanation for the existence of overall negative skewness in asset returns. Aggarwal and Rao (1990) demonstrate the existence of an inverse relation between the degree of institutional investors in a stock and its positive skewness. In overall terms, these papers do not provide an adequate explanation for the existence of negative skewness in the scale reported in the literature. In this paper, we demonstrate that some existing information-based theories can explain the presence of negative skewness in asset returns and its variation with changes in economic conditions. We demonstrate that the process of information revelation to investors under uncertainty, referred to as the "uncertain information hypothesis" (UIH) by Brown, Harlow, and Tinic (1988) can form the basis for an explanation of negative skewness in asset returns. We combine this hypothesis with hypotheses derived from Kahneman and Tversky's prospect theory (1979) to develop an explanation for the systematic variation of skewness observed as the economic climate changes.

Our examination of skewness in return distributions is based on a Monte Carlo simulation analysis. Our use of simulated data rather than market data enables us to introduce price shocks and uncertainty changes on the simulated stock process in a controlled manner. In particular, it enables us to incorporate the impact of information and behavioral hypotheses and observe their effects on skewness. We employ a widely used model to represent the formation of stock prices over time, the geometric random walk model (GRW), as a vehicle for our simulations. Then we introduce the effects of the uncertain information hypothesis and the behavioral hypotheses to the simulated returns process and examine the resulting changes to skewness properties of the return distributions. Based on our simulation results, we argue that the phenomenon of partially revealing market information contained in the uncertain information hypothesis is a possible cause of negative skewness in returns and that the uncertain information hypothesis coupled with our behavioral hypotheses can provide an explanation for the time-varying skewness phenomenon. Conversely, our results provide support to the uncertain information hypothesis as a plausible theory of the behavior of stock prices.

Simulation of Returns with the Geometric Random Walk Model and the Skewness Properties of Returns

In this section we briefly describe the features of the benchmark geometric random walk model that we later employ to demonstrate the effects of the information based hypotheses on skewness. The geometric random walk model is represented in levels as

$$P_t = P_{t-1}.e^k.a_t \tag{1a}$$

or in the natural logarithm of levels as

$$Ln(P_t) = Ln(P_{t-1}) + k + Ln(a_t) \tag{1b}$$

where:

P_t = The stock price at time t;
k = The expected continuously compounded return;
a_t = A log normally distributed random noise term; and
$\ln(a_t)$ = A normally distributed random noise term with mean zero and standard deviation σ.

The geometric random walk model is widely used to represent the path of stock prices because its properties are consistent with the empirical behavior of stock prices. [See Chapter 10 in Hull (2000).] It has several salient features. The stochastic noise term a_t is independently and identically distributed over each time period and, as such, the model is consistent with weak-form efficiency of financial markets. Because a_t is log normally distributed, it cannot take negative values; hence, the stock price is always positive. The continuously compounded rate of return, $R(c)_t = k + Ln(a_t)$ and the simple rate of return, $R(s)_t = e^k.a_t - 1$ are both independent of the level of the stock price. Notice that $R(c)_t$ is normally distributed with mean k and variance σ^2, while $R(s)_t$ is log normally distributed with mean, variance, and skewness (respectively)

$$E(R_t^s) = e^{(k+1/2\sigma^2)},$$

$$Var(R_t^s) = e^{(2k+\sigma^2)}(e^{\sigma^2} - 1),$$

$$Skew(R_t^s) = (e^{\sigma^2} - 1)^{1/2}(e^{\sigma^2} + 2)$$

While $R(c)_t$ obviously has no skewness, the simple return $R(s)_t$ is positively skewed with skewness increasing in the variance term σ^2. Thus, if the geometric random walk model is a fair representation of the process of stock price formation, then distributions based on simple returns should be positively skewed. Conversely, the negative skewness patterns of stocks reported in the literature could not be generated by an information mechanism that is represented by the simple version of the geometric random walk model.

The Uncertain Information Hypothesis and its Implications on the Skewness of Returns.

Brown, Harlow, and Tinic (1988) present a modification to the Efficient Market Hypothesis (EMH), which they refer to as the uncertain information hypothesis, to represent a world of partially revealed information. They suggest that it is unrealistic to assume that all the ramifications of major news events affecting stock prices are immediately and fully understood by investors and that they could revise prices gradually as the true impact of such events is revealed. Rather, the uncertainty and the incompleteness often associated with these events would more likely make investors form conditional probability distributions of the likely price outcomes and stock prices would be revised to the expected value of this distribution. According to the uncertain information hypothesis, however, investors would temporarily discount the stock price below this expected value of the price distribution to take account of the uncertainty associated with the news event. As the uncertainty associated with the news event is gradually resolved, the discount on the stock price also disappears, and the price reverts to the expected value. The temporary discounting of the stock price may follow good news as well as bad news as long as there is incompleteness or uncertainty associated with either type of news. Brown, Harlow, and Tinic contend that price discounting is a rational response of risk-averse investors and is fully consistent with an informationally efficient market.

The uncertain information hypothesis and the world of partially revealing prices can be incorporated and simulated in the geometric random walk model by the introduction of an appropriately characterized random variable representing stochastic news shocks that impact the price path. The geometric random walk model modified to accommodate the news shocks takes the following form:

$$P_t = P_{t-1}.e^k.a_t.S_t \tag{2a}$$

and the log version of the model is

$$Ln(P_t) = Ln(P_{t-1}) + k + Ln(a_t) + Ln(S_t) \tag{2b}$$

where S_t is the news shock at time t.

To perform our simulation, we use the log form of the geometric random walk model, equation (2b). A value of 50 is chosen for the initial stock price P_{t-1}, and a value .06 percent is chosen for k, the expected daily return that corresponds approximately to a 16 percent annual return. Based on a 250 day year, this is a reasonable rate of return for an average stock. Because $Ln(a_t)$ is normally distributed, values for $Ln(a_t)$ are drawn by sampling from a normal distribution. The parameters of the normal distribution are chosen to have a zero mean and a daily standard deviation of 1.25 percent that corresponds approximately to a 20 percent annual standard deviation, which is a reasonable level of volatility in an average stock.

The magnitude of the news shock S_t in equation (2b) is based upon the value of an indicator variable I which is drawn from a Poisson distribution.[1] If the value of I drawn is 1, then $Ln(S_t)$ is zero and there is no shock and therefore no impact on price. But when S_t takes a zero value, $Ln(S_t)$ is non-zero and there is a positive or negative shock on the price. The probability of either getting or not getting a shock

[1] The Poisson distribution is an appropriate process for simulating random shocks. An example where the Poisson distribution has been used in the past is in Cox and Ross (1975)

can be controlled by the parameter chosen for the Poisson distribution. In our simulations, if in a particular time period a value drawn for I is zero, then we also control the probability of the resulting shock S_t being a good news or bad news. This is done by allowing S_t to take either a negative or a positive value in accordance with the value taken by a random variable drawn from a uniform distribution.[2] The magnitude of the shock, whether positive or negative, is also allowed to take random values within a pre-specified range, based on sampling from a uniformly distributed range of values. Simulations were run at two ranges of values for the shocks. The first level is drawn from a uniformly distributed range of values between 0 and .5, and the second level is drawn from a range between .5 and 1.[3]

Two hundred and fifty daily prices are generated for each price path to correspond approximately to a one year period. From each year's price path the simple daily returns and the continuously compounded rates of returns, as well as the skewness, can be computed. In the simulation results presented in the following tables, each such cycle is simulated 100 times, and the mean and the standard deviation of the skewness values from the 100 cycles are reported. Simulations are repeated with several alternative values for the parameters of the expected daily rate of return and the standard deviation of the noise term in the geometric random walk model.

The phenomenon of partially revealing prices is then built into the geometric random walk model as follows. Suppose on day t a news shock occurs whose *full* impact on the price is S_B (if it is bad news) or S_G (if it is good news). Owing to the added uncertainty associated with the event, however, investors discount the price reaction of day t by an amount A_B or A_G (both negative) respectively, corresponding to a bad news shock and a good news shock, respectively. (In the simulations, the price discount is represented as a proportion of the shock S_B or S_G). The net price change on day t is thus S_B - A_B or S_G - A_G, respectively. When the uncertainty is subsequently resolved (for instance on day t+1), the price receives a positive adjustment of A_B or A_G on day t+1, in each case.

In Table 1, we present the results of introducing uncertain news shocks to the stock price path and the resulting effect on the skewness of the return distribution. Panel A presents average skewness values based on simple daily returns, while panel B presents average values based on continuously compounded returns. The left side results are based on an a_t standard deviation of .0125, while the right side results are based on a level of .05. The values in the first row of panels A and B represent the

[2] The purpose of determining the outcome of the shock as positive or negative on the basis of the outcome of a uniformly distributed variable is to be able to influence the probability of obtaining positive or negative shocks. We vary this probability in the analysis later in the paper.

[3] The variance of the shock at each level is .0208 which compares with the two alternative variances chosen for the random noise term of .0125 and .05. The mean shock at the two levels are .25 and .75 which are by design considerably larger than the zero mean noise level.

skewness values computed from the basic geometric random walk model, without news shocks. In panel A, the positive skewness of the distribution is higher on the right side (.1502 versus .0375), but the skewness of the continuously compounded returns in panel B does not change with the volatility level. The second and third rows in panels A and B show the skewness effects at two levels of news shocks applied in the simulations. Row 2 is based on news shocks drawn from the range 0 and .5, and the third row is based on the range .5 and 1. Across each row of the table moving from left to right, the discount on price due to uncertainty is gradually increased, from 0 percent to 10 percent, 30 percent, and 50 percent of the news shock magnitude. Across the rows, as the proportion of price discount increases, the skewness becomes increasingly negative. This is seen at both levels of noise volatility. These results confirm that the process of partially revealing prices tends to make return distributions negatively skewed and that the negative skewness is accentuated when the uncertainty associated with the shock is higher. Comparing skewness in rows 2 and 3 in both panels, we observe that the level of shock does not affect skewness nearly as much as the proportion of price discount to the shock.

Table 1—Simulation Results of the Effect of Partially Revealing Information on Skewness[a]

Panel A: Skewness Based on Simple Daily Returns

	Noise Volatility (a_t) = .0125				Noise Volatility (a_t) = .05			
	Proportion of Price Discount on News Shocks[b]							
	0%	10%	30%	50%	0%	10%	30%	50%
News Shock Levels (S_t)[c]								
Zero Shock	.0375				.1502			
Shock <.5	1.989	1.089	-1.128	-1.738	1.471	.657	-.472	-1.043
	(.117)	(.107)	(.084)	(.047)	(.078)	(.088)	(.069)	(.047)
.5 < Shock < 1	3.236	2.415	.516	-.258	3.136	2.337	.54	-.288
	(.029)	(.053)	(.053)	(.024)	(.033)	(.042)	(.043)	(.025)

Panel B: Skewness Based on Continuously Compounded Daily Returns

	Noise Volatility (a_t) = .0125				Noise Volatility (a_t) = .05			
	Proportion of Price Discount on News Shocks[b]							
	0%	10%	0%	10%	0%	10%	0%	10%
News Shock Levels (S_t)								
Zero Shock	0				0			
Shock <.5	-.116	-1.16	-2.80	-3.35	.009	-.886	-2.0	-2.488
	(.128)	(.109)	(.075)	(.054)	(.081)	(.073)	(.068)	(.051)
.5 < Shock < 1	.006	-.936	-2.262	-2.794	-.024	-.973	-2.242	-2.68
	(.071)	(.058)	(.037)	(.013)	(.056)	(.061)	(.033)	(.015)

[a]Results presented are the mean skewness values of 100 simulated skewness observations. Each observation is based on a path of 250 returns generated from the Geometric Random Walk model. The standard errors of the mean skewness values are given within brackets below the respective means

[b]This is the proportion of the news shock price change discounted by investors due to uncertainty

[c]This is the magnitude of the information shocks introduced to the geometric random walk model simulations. The uncertainty of news shock impacting on day t is assumed to be fully resolved on the following day (day t+1)

In the model of uncertainty resolution considered in Table 1, we assumed that the uncertainty of news shock impacting on day t is fully resolved on the day after the news shock (day t+1). We next examine the effect on skewness when the speed of uncertainty resolution is more gradual, which perhaps better reflects many real world situations. We let the price discount associated with the news shock impacting on day t to be resolved in two stages, partially on day t+1 and the rest on day t+2. In Table 2, we compare the effects on skewness of this two stage resolution process with the single stage uncertainty resolution process of Table 1. We select two levels of price discounts considered in Table 1, proportions 30 percent and 50 percent of shock. The 30 percent discount of the single stage resolution is compared with that of a two stage, 20 percent plus 10 percent, resolution process. The comparative results are given in the left side of Table 2. The 50 percent single stage resolution is compared with a two stage, 30 percent plus 20 percent, resolution process, and the comparative results are given in the right side. We see that the skewness of the two stage process is always more negative than the skewness of the single stage process. This is true whether simple returns or continuously compounded returns are used, as shown by a comparison between panel A and panel B. When the resolution of the price discount is more gradual, the negative skewness of the process is greater.

Table 2—Simulation Results Comparison of Slower and Faster Uncertainty Resolution Processes on Skewness[a]

Panel A: Skewness Based on Simple Daily Returns	30% Price Discount Process		50% Price Discount Process	
	One Stage Resolution Process	Two Stage Resolution Process	One Stage Resolution Process	Two Stage Resolution Process
News Shock Levels (S_t)				
Shock < .5	-1.128	-1.327	-1.738	-2.757
	(.084)	(.120)	(.047)	(.068)
.5 < Shock <1	.516	0.50	-.258	-1.239
	(.053)	.064	(.024)	(.031)
Panel B: Skewness Based on Continuously Compounded Daily Returns				
Shock <.5	-2.80	-3.01	-3.35	-4.24
	(.075)	(.089)	(.054)	(.052)
.5<Shock<1	-2.262	-2.67	-2.794	-3.543
	(.037)	(.027)	(.013)	(.017)

[a] Results presented are the mean skewness values of 100 simulated skewness observations. Each observation is based on a path of 250 returns generated from the geometric random walk model. The values given within brackets below the mean skewness values are the standard errors of the mean skewness

The Effect of Some Behavioral Factors Associated with Uncertain Information on Skewness

In this section we provide two alternative explanations for the variation of skewness patterns over changing market cycles documented in Alles and Kling (1994). The regularity they observed was that skewness tended to be less negative (or more positive) during periods of market downturns and economic recessions and more negative during expansions. Our first explanation for this is based on Kahneman and Tversky's prospect theory (1979). According to this theory, individuals regard gains or losses relative to a subjective reference point. Furthermore, the reference point can change under different contexts or different environments. This concept can be applied to the process of investors responding to uncertain market information. In the uncertain information hypothesis, investors have the ability to categorize uncertain information as either good news or bad news. Presumably, such a categorization is made relative to a reference point. Because it is common to observe a shift toward pessimism in the outlook of investors during periods of economic downturns, in terms of prospect theory, we can regard this shift in outlook as a shift in the reference point of investors relative to which investors categorize market news as good or bad. As a result, a news event which would have been regarded with indifference or as bad news in periods of prosperity could well be regarded as good news in times of hardship. For example, job growth in times of prosperity may be viewed as bad news (inflation), but would be viewed positively in times of an economic contraction. Given a shift in the reference point during recessions, investors may then experience relative good news more frequently than bad news in comparison to their experiences at times of prosperity. One can intuitively believe that this would be true when the economy is emerging from a recession.

To examine how such a shift in the reference point of investors might affect skewness in a world of partially revealing prices, we consider a series of geometric random walk model simulations in which the subjective reference point moves in a pessimistic direction. In these simulations, randomly impacting news shocks will be increasingly regarded as good news shocks. These results are shown in panel A of Table 3. Reading the skewness values across the rows, it is clear that the return distribution which was negatively skewed when compiled from an information process that received good and bad news shocks in equal measure becomes increasingly positively skewed as the frequency of good news impacts is increased relative to the frequency of bad news impacts. To compare these results with empirical results, Alles and Kling (1994) reported that over the 1962 to 1989 period, skewness of the NYSE index in bull market periods was -2.25 and in bear market periods .307 and the NASDAQ index -2.13 and -1.28 in the corresponding periods.

We offer a second hypothesis to explain the tendency of skewness to be less negative in periods of economic downturns, which is again based on the behavioral responses of investors to uncertain information. We hypothesize that in a climate of economic hardship or market decline, investors will have a tendency to accept negative news at face value and as a matter of course. On the contrary, positive news that goes against the grain of the prevailing economic climate will be accepted with a higher degree of skepticism. As a result, the degree of uncertainty attached to good news will, in general, be higher in comparison to the degree of uncertainty attached to bad news in periods of recessions. We incorporate this behavior in the price process simulations by introducing positive news shocks with a higher level of uncertainty, (reflected by a high price discount) which is subsequently resolved in two stages. Negative news, on the other hand, is introduced with no price discount. The results of this simulation are shown in Panel B of Table 3. The skewness of returns resulting from the process during bad times (with the higher uncertainty on good news) is compared with a process in which there is equal uncertainty toward positive and negative shocks. The comparisons are made at two levels of price discounts to shocks, 30 percent and 50 percent. The first two columns compare processes with a 30 percent price discount while the last two columns compare processes with a 50 percent discount. The comparisons show that the price process during bad times with less uncertainty associated with bad news is always less negatively skewed relative to the process with equal uncertainty shown toward good and bad news.

Table 3—Simulation Results of Investor Attitudes Toward Uncertainty on Skewness[a]

Panel A - Effect of Reference Point Shifts on Skewness

	Frequency of Positive Shocks to Negative Shocks			
	50:50	60:40	70:30	80:20
Skewness of Simple Returns	-1.327	-.08	.102	1.45
	(.120)	(.105)	(.127)	(.126)
Skewness of Cont. Comp. Returns	-3.01	-2.74	-2.01	-1.28
	(.089)	(.115)	(.134)	(.224)

Panel B: Effect of Different Uncertainties Associated with Good and Bad News on Skewness

	30% Price Discount Process		50% Price Discount Process	
	Equal Uncertainty on Good and Bad News	Less Uncertainty on Bad News	Equal Uncertainty on Good and Bad News	Less Uncertainty on Bad News
Skewness of Simple Returns	-1.33	-0.42	-2.76	-1.79
	(.120)	(.10)	(.068)	(.052)
Skewness of Cont. Comp. Returns	-3.01	-2.03	-4.24	-3.21
	(.089)	(.087)	(.052)	(.088)

[a] Results presented are the mean skewness values of 100 simulated skewness observations. Each observation is based on a path of 250 returns generated from the geometric random walk model. The simulations were carried out with a standard deviation of .0125 for the stochastic error term and random shocks drawn between the parameters 0 and 0.5. The values given within brackets below the mean skewness values are the standard errors of the mean skewness

Conclusions

The contributions of this paper are twofold. First, we demonstrate how return distributions can become negatively skewed as a consequence of the process of information assimilation as described in the uncertain information hypothesis. Therefore, the uncertain information hypothesis provides an explanation for the negative skewness observed in empirical return distributions and, conversely, the simulation results constitute support for the uncertain information hypothesis as a plausible theory of the formation of stock prices over time. Our results imply that negative skewness maybe a natural by-product, along the same lines as expected return and variance changes, of the influence of uncertain information on market prices.

Second, this paper investigates whether alternative behavioral theory-based explanations can provide answers for the systematic variation of skewness as economic environments change, documented in Alles and Kling (1994). We demonstrate that as economic conditions change, the changes that investors make in categorizing news as good or bad relative to a subjective reference point, and the changes they make to the levels of uncertainty attached to good and bad news can give rise to variation in skewness.

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