Correction to Angus-Leppan, P.V. (1979) Refraction in levelling – its variation with ground slope and meteorological conditions, Australian Journal of Geodesy, Photogrammetry and Surveying, no. 31, pp. 27-42

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NOTE: Since the *Journal of Spatial Science* continues the *Australian Journal of Geodesy, Photogrammetry and Surveying*, it is logical to present this correction here.

BACKGROUND & MOTIVATION

Angus-Leppan (1979), hereafter AL79, derived equations for correcting for atmospheric refraction in geodetic spirit-levelling, based on estimates of the upward flux of sensible heat through the atmosphere H in units of Watts per square metre (W/m^2). This required the estimation of the cloud factor C and surface wetness factor W from AL79 Figure 1 and calculation of the Sun's zenith distance ζ using AL79's equation 13. However, as will be shown, the equation to compute heat flux H on an inclined surface H' (AL79, equation 17) is in error.

Although from an old paper, this equation is still used (e.g., Heer and Neimeier, 1985; Kearsley and Ahmad, 1993, Kearsley *et al.*, 1993; Johnston *et al.*, 2002; Robertson, 2003) and is convenient for its simplicity, or is cited (e.g., Angus-Leppan, 1980; Zeki Coşkun and Baykal, 2002). The refraction correction can be applied rigorously with measured temperature gradients. However, it can also be applied using C and W parameters estimated from meteorological conditions recorded by the field party. These parameters can also be estimated from weather stations in the vicinity of the levelling, highlighting its usefulness when attempting to apply refraction corrections to historical levelling data that has no meteorological data recorded at the time of survey.

THE INCORRECT EQUATION

The equation to compute H', the value for heat flux H scaled for the aspect of a slope and the direction of the Sun, is shown in AL79 equation 17 as

$$H' = H\cos\rho + \sin\rho\tan\rho\cos(\gamma-\alpha) \quad (1)$$

where ρ is the angle the between the surface normal and the local vertical, γ is the azimuth of the slope, and α is the Sun's azimuth. As will be proven below, this equation is incorrect.

THE CORRECTED DERIVATION

In AL79, the derivation starts with the energy balance at the ground surface, resulting in net radiation R (AL79, equation 8A)

$$R = S_D - S_U + L_D - L_U \quad (2)$$

where, respectively, S_D and L_D are shortwave and longwave radiation received, with S_U and L_U being the shortwave and longwave components reflected and re-radiated. This net radiation R is dispersed as (AL79, equation 8B)

$$R = G + F + H \quad (3)$$

where G is radiation conducted into the ground, F is dissipated as the latent heat of evaporation, and the remaining energy is heat flux H, which is moved upwards into the atmosphere by eddies. The parameter H can be estimated from (AL79, equation 9)

$$H = 450CW\cos\zeta \quad (4)$$

where ζ is the Sun's zenith distance. AL79 then investigated the effects on the value of H resulting from variations in the slope ρ and aspect $(\gamma-\alpha)$ of the topographic surface, finding that the ratio of the intensity of the incident shortwave radiation on an inclined plane S'_D to that on a horizontal plane S_D is equal to (AL79, equation 12)

$$\frac{\cos\zeta'}{\cos\zeta} \quad (5)$$

where ζ' is the Sun's angle of incidence with respect to the surface normal of the inclined plane S'_D. Note that the Sun's zenith distance ζ is equal to the angle of incidence when the vertical and surface normal are coincident (ie., on a level surface). From the heat balance equations 2 and 3 above, AL79 then obtained (equation 16)

$$H = S_D - S_U + L_D - L_U - G - F \quad (6)$$

AL79 showed that the components of the right hand side of equation (6) were all approximately proportional to S_D. He then concluded that the sum of the terms on the right-hand-side of equation 6 could, with a reasonable degree of accuracy, be considered to be approximately proportional to S_D, with H thus proportional to S_D. From this, and using H' as the value for H on an inclined surface, (with H' approximately proportional to S'_D) AL79 found

$$\frac{H'}{H} = \frac{S'_D}{S_D} = \frac{\cos\zeta'}{\cos\zeta} \tag{7}$$

Substituting AL79's equation 15, which is correct from spherical trigonometry

$$\cos\zeta' = \cos\rho\cos\zeta + \sin\rho\sin\zeta\cos(\gamma - \alpha) \tag{8}$$

equation (7) here then becomes

$$\frac{H'}{H} = \frac{\cos\rho\cos\zeta + \sin\rho\sin\zeta\cos(\gamma-\alpha)}{\cos\zeta} = \frac{\cos\rho\cos\zeta}{\cos\zeta} + \frac{\sin\rho\sin\zeta\cos(\gamma-\alpha)}{\cos\zeta} \tag{9}$$

Cancelling for $\cos\zeta$, using trigonometric identities, and multiplying both sides by *H*, gives the correct result for AL79's equation 17 as

$$H' = H(\cos\rho + \sin\rho\tan\zeta\cos(\gamma - \alpha)) \tag{10}$$

It can thus be seen that when our equation (10) is compared to AL79's equation 17 [also equation (1) here], $\tan\rho$ should actually be $\tan\zeta$ and the parentheses enclosing all terms after *H* ensure that *H* is not multiplied by only the first term $\cos\rho$.

As an example, where ρ is 1.3°, the sight length is 50 m, and *H* is 300 W/m^2, using equation (10) a refraction error of 0.67 mm over 5 km accumulates, which is equivalent to 0.57 mm per 100 m of height difference. Over a 1000 km distance, this accumulates to 0.134 metres, which is consistent with Table VI in AL79. Therefore, it appears that AL79's Table VI has been calculated using the correct formula (equation 10 here), and not the one presented as AL79's equation 17 (equation 1 here).

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