A low-frequency laboratory apparatus for measuring elastic and anelastic properties of rocks
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Summary:
In this paper we present a new version of low-frequency laboratory apparatus designed for measurements of complex Young’s moduli and extensional attenuation of rock samples at seismic (1÷400 Hz) and teleseismic (≤1Hz) wave frequencies. The device can operate at confining or uniaxial pressures from 0 to 70 MPa. The preliminary data obtained for sandstone quarried in Donnybrook, Western Australia, are presented.

Introduction:
There are a few low-frequency devices described in literature, which are reportedly able to measure simultaneously elastic and anelastic properties of rock samples at low (seismic and teleseismic waves) frequencies. These devices utilize a strain-stress relationship and differ by the type of the forced oscillations applied to a specimen under investigation: torsional (Jackson and Paterson, 1987; Paffenholz and Burhardt, 1989) and axial (Spencer, 1981; Paffenholz and Burhardt, 1989; Batzle et al, 1999). However, only the device proposed by Batzle et al (1999) is able to operate at confining pressure, which is set up by placing the entire device in a gas pressure vessel.

Using the gas pressure vessel puts serious constrains on the sizes of rock samples and the mechanical assembly of the apparatus. The latter should be massive enough to avoid spurious mechanical resonances (Batzle et al (1999) in the mechanical parts of the device.

Another essential disadvantage of the apparatus described by Batzle et al (1999) arises from the fact of using the electromechanical shaker. Because the electromagnetic shaker is unable to work under high load, uniaxial pressure cannot essentially exceed the pressure created by the gas in the pressure vessel and applied to the whole mechanical assembly of the apparatus. The last factor makes the experiments with separated uniaxial and confining pressures unfeasible.

In this paper we present the preliminary data obtained with a new low-frequency laboratory apparatus, operating at seismic and teseismic frequencies, which also utilizes the strain-stress relationship and measures the complex Young’s moduli of rock samples at confining or uniaxial pressures from 0 to 70 MPa. The apparatus is able to operate with the rock samples which have length up to 10cm and diameter up to 4cm.

Figure 1: The mechanical assembly of the low-frequency laboratory apparatus.
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Design of the low-frequency apparatus

The mechanical assembly of the apparatus is presented in Figure 1. The assembly comprises two massive steel platforms and a set of units between them, which includes a hydraulic actuator, a Hoek triaxial core holder, a piezoelectric stack actuator PSt 1000/35/60 (APC International Ltd) with the limit of maximum load of 70,000 N and with the frequency of its mechanical resonance >20 kHz, aluminum calibration standard, and two aluminum plugs having passages for a fluid injection.

The main purpose of using the platforms is to reduce the spurious mechanical resonances in the mechanical assembly.

A rock sample to be tested is placed inside a sleeve, which is mounted within the central passage of the triaxial core holder. The fluid passages in the aluminum plugs attached to the sample enable the flow of fluids through the sample and provide the means for pore pressure control. The central passage of the core holder and the hydraulic actuator are connected via fluid lines with two independent hydraulic pumps providing radial and axial static forces applied to the specimen. Therefore, the axial and radial forces can be changed independently, which is important for studying the physical characteristics of the sample under various conditions.

The schematic diagram of the electrical part of the apparatus is shown in Figure 2. The multilayer piezoelectric adaptor transforms the periodic voltage, applied by an oscillator, into axial mechanical stress, which causes displacements in the aluminum standard and the rock sample. The displacements modulate the conductivity of the strain gauges coupled with the aluminum standard and rock. A set of electrical bridges transforms the modulated conductivity into electrical signals, which, after digitizing by an analogue-digital converter, are received by an acquisition computer, where the signals are averaged and processed. The processing in the acquisition computer is synchronized with the oscillator by a triggering signal.

Method and Operation

Now we describe the process of measuring in more detail. The signals corresponding to the axial and radial components of the strain in the rock sample are detected by the strain gauges, one of which is aligned with axial direction and the second is orthogonal to the first one. These signals are used to calculate the Young’s modulus and Poisson’s ratio.

If we assume that periodical stress is applied along z-axis, then from Hooke’s law (e.g. Mavko et al, 2009)

\[ \varepsilon_y = \frac{1}{E} (1 + v) \sigma_y - v \varepsilon_x \sigma_x, \]  \hspace{1cm} (1)

where \( \varepsilon_y \) are the elements of the specimen strain tensor; \( \sigma_y \) are the elements of the stress tensor; \( \sigma_x = \sum_{i=1}^{3} \sigma_{ix} \) : we can find the Poisson’s ratio \( v \)

\[ v = \frac{c - \varepsilon_x}{1 + c - 2c \varepsilon_x}, \]  \hspace{1cm} (2)

The Young’s modulus of the specimen \( E \) can be found from Eq. (1):

\[ E = \sigma_y \frac{(1 + v)(1 - 2v)}{2\varepsilon_x + (1 - v)\varepsilon_x}, \]  \hspace{1cm} (3)

The stress \( \sigma_x \) can be expressed through the parameters of the aluminum standard as \( E_s \varepsilon_x^{sl} \), where \( E_s \) is the known Young’s modulus and \( \varepsilon_x^{sl} \) is a measured amplitude of axial strain. So, the Young’s modulus of the specimen is

\[ E = E_s \frac{\varepsilon_x^{sl}(1 + v)(1 - 2v)}{2\varepsilon_x^{al} + (1 - v)\varepsilon_x}, \]  \hspace{1cm} (6)

It can be shown that the coefficient \( c \), which is required for the determination of the Poisson’s ratio \( v \) in Eq. (2), is equal to
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\[ k = \frac{rL}{(R^2-r^2)L} \frac{\varepsilon_{xx}}{\varepsilon_{xx}}, \tag{7} \]

where \( k \) is the bulk modulus of the oil; \( L \) and \( r \) are the length and radius of the specimen; \( R \) is the internal radius of the core holder.

Using the Young’s modulus determined by Eq. (2), we can find compressional and shear velocities:

\[ V_p = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}}, \quad V_s = \sqrt{\frac{E}{2(1+\nu)\rho}}, \tag{8} \]

where \( \rho \) is the density of the sample.

The extensional attenuation \( Q_E \) in the sample can be estimated as follows. The periodic signals, which are detected by axial strain gages coupled with the aluminium standard and examination specimen, are averaged and subjected to Fourier transform. The resulting complex Fourier transform amplitudes \( A_\text{al} \) and \( A_\text{s} \), which are calculated at the frequency of the periodic voltage generated by the oscillator, are used to estimate the attenuation \( Q_E \):

\[ Q_E \approx \frac{\text{Im}(A)}{\text{Re}(A)}, \tag{9} \]

where \( A = \frac{A_\text{s}}{A_\text{al}} \), \( |A_\text{al}| \) and \( |A_\text{s}| \) are the absolute values of the amplitudes \( A_\text{al} \) and \( A_\text{s} \), which correspond to the signals obtained from the axial strain gages coupled with the aluminum standard and rock sample respectively.

The example of the signals obtained from this sample at the frequency of the periodical stress oscillations 10 Hz and at confining pressure 15 MPa, is given in Figure 3.

The preliminary results obtained for a sandstone sample at seismic frequencies are presented in Figures 4 and 5. experiments were carried out at room temperature (~22ºC) and at confining pressures from 3 to 40 MPa. The physical parameters of the water saturated sample are as follows: the density is 2245 kg/m\(^3\), porosity and permeability are 14.8% and 7.8 mD correspondingly. The diameter of the sample is 38mm, the length is 70 mm.

There is a comparison of \( V_p \) and \( V_s \) velocities in Figure 6 obtained for the water saturated sample at low and ultrasonic frequencies. The measurements were conducted at a confining pressure of 30 MPa.

Figure 3: Signals obtained from the water saturated sandstone at the frequency of the periodical stress oscillations 10 Hz and at confining pressure 15 MPa. The top signal was obtained from the gauge attached to the aluminum standard, the middle and bottom signals were obtained correspondingly from the axial and radial gauges attached to the rock sample. The number of averagings is 100.

Figure 4: The extensional attenuation \( Q_E \) measured for the water saturated sandstone at 10 and 50 Hz. Pore pressure is 0.1 MPa.
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Conclusions

We presented a new version of the low frequency laboratory apparatus operating at seismic frequencies and at either confining or uniaxial pressures from 0 to 70 MPa. The apparatus utilizes a stress-strain technique and measures the complex Young’s modulus, extensional attenuation and Poisson’s ratio of the rock sample at the range of strain amplitudes between $10^{-6}$ and $10^{-8}$.

The main peculiarities of the proposed design of this low-frequency device are as follows:

- A combination of the multilayer piezoelectric adaptor, having a high limit of maximum load, and the Hoek triaxial core holder provides the means to separate uniaxial and lateral pressures applied to the sample, which makes the experiments with uniaxial and confining pressures feasible.

- Two massive platforms as a part of the mechanical assembly completely reduce spurious mechanical resonances in the apparatus. The top limit of the frequencies of operation is limited exclusively by the frequency of the mechanical resonance in the piezoelectric actuator.

- The sizes of the rock samples, which can be measured in this apparatus, are determined by the sizes of the triaxial core holder only.

Figure 5: P- and S-wave velocities obtained for the water saturated sandstone. At seismic frequencies

Figure 6: P- and S-wave velocities obtained for the water saturated sandstone at seismic frequencies (1 – 150 Hz) in comparison with ultrasonic measurements at 1 MHz. Effective pressure is 30 MPa.
REFERENCES


