

EVIDENCE OF A NORTH-SOUTH TREND BETWEEN AUSGeoid98 AND THE AUSTRALIAN HEIGHT DATUM IN SOUTHWEST AUSTRALIA

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ABSTRACT

The AUSGeoid98 gravimetric geoid model has been compared with 48 GPS-levelling points at a ~50 km spacing across part of the southwest of Western Australia. This is arguably the best subset of GPS-derived ellipsoidal heights in Australia with an internally estimated precision of $<\pm 9$ mm. The spirit-levelled heights were tied to the Australian Height Datum (AHD) using class C techniques [12mm-root-km allowable misclose]. The comparisons show that AUSGeoid98 gives a GPS height transformation to the AHD with a precision of $\sim\pm 13$ cm, which is less than reported earlier ($\sim\pm 36$ cm) for a nationwide dataset. A clear north-south trend of ~ 0.81 mm/km [ppm] is also evident in the differences; of which approximately one-third is attributable to a north-south error in the AHD induced by dominant north-south sea surface topography effects at the nearby fixed tide gauges. After removal of this north-south trend, the standard deviation of the differences reduces to ~ 5 cm.

INTRODUCTION

It is common-practice to empirically validate gravimetric geoid models on land through comparisons with GPS-levelling data. Of course, this is inevitably subject to errors in the GPS and spirit levelled heights, but these are currently the only data with which to verify geoid models on land. Moreover, if the geoid models are to be subsequently used to transform GPS heights to the local vertical datum, then such an approach offers the most useful information to this majority of users. However, as geoid modelling and GPS positioning techniques continue to improve, deficiencies in local vertical datums are now becoming apparent [3].

This short note describes a comparison between the AUSGeoid98 regional gravimetric geoid model of Australia [6] and a set of 48 GPS minus Australian Height Datum (AHD [13]) ‘geoid-type’ heights in the southwest seismic zone (SWSZ [2]) of Western Australia. The main finding of interest here is a very clear north-south trend between AUSGeoid98 and the AHD, which is permitted because of the quality of the GPS data used. However, it remains difficult to discriminate this trend between AUSGeoid98 and the AHD.

Earlier comparisons between AUSGeoid98 and the AHD over the whole of Australia [5] indicated a general north-south trend of ~ 0.26 mm/km (i.e., ppm). However, this relied upon a nationwide GPS-levelling dataset that is now of questionable veracity [11]. For the present study, the internally estimated precision of the GPS-derived heights

is $\leq \pm 9$ mm, which allows for a more reliable identification of the north-south trend between AUSGeoid98 and the AHD, and thus corroborates the earlier studies.

DATA

AUSGeoid98 [6] uses data from EGM96 [12], Australian land and ship-track gravity observations, a 27'' regional digital elevation model, and satellite-altimeter-derived gravity anomalies in marine regions. The ship-track and altimeter gravity data were merged using least squares collocation. The residual (to EGM96) geoid heights were computed using a deterministically modified Stokes kernel in an adapted remove-compute-restore scheme. Topographical correction, downward continuation and indirect effect terms were computed approximately under the 'Moritzian' scheme, acknowledging that more sophisticated algorithms are now available.

The GPS data used in the present study were collected during May 2002 as part of a campaign to determine epoch-one coordinates for geodetic estimation of any surface deformation associated with the SWSZ belt of intra-plate [17] earthquake activity [8]. This collaborative venture involved funding and scientists from Geoscience Australia, the Western Australian Department of Land Administration (DOLA), the New Zealand Institute of Geological and Nuclear Sciences Ltd, Curtin University of Technology, and the University of Western Australia. While these GPS data were collected for geodynamic studies, they also offer a useful dataset for gravimetric geoid validation, assuming of course that the vertical geodynamic motion is small [18].

DOLA installed 48 new monuments comprising 5/8'' Whitworth threads on levelled base-plates set in firm bedrock using epoxy resin and concrete. Approximately 15 of these sites were occupied simultaneously with (Trimble, Ashtech and Leica) dual-frequency GPS instruments for at least five days, coupled with a 'backbone' of five receivers tracking GPS data for the entire campaign. These data were multi-baseline processed using Bernese v.4.2 [9] with respect to permanent IGS (International GPS Service) GPS trackers at Yarragadee, Perth and Alice Springs, using IGS 'final product' orbits, and according to IGS standards [1, 8].

	1 σ East (mm)	1 σ North (mm)	1 σ Up (mm)
Maximum	3.4 (1.4)	8.3 (1.5)	21.8 (8.8)
Minimum	0.6 (0.6)	0.7 (0.7)	3.5 (3.5)
Mean	1.09 (0.99)	1.36 (1.16)	6.80 (6.18)
STD	0.50 (0.16)	1.10 (0.17)	3.20 (1.06)

Table 1. Descriptive statistical summary of the internally estimated precision (1 σ in mm) of the GPS-derived coordinates (ITRF2000, epoch 2002.37) of the 48-point network across the SWSZ [parenthetic values exclude the two 'outliers' at SZ23 and SZ36]

The *internally estimated* precisions of the resulting ITRF2000 coordinates are shown in Figure 1 and summarised in Table 1 (stations SZ11 and SZ47 were not installed). Note the low precisions for stations SZ23 and SZ36 [the x axis in Figure 1 appears to be mis-registered because SZ11 is absent]. These two stations were coordinated using Leica CRS1000 instruments from the Australian GPS consortium [7], which

caused several serious problems during the campaign due to incorrect data recording during periods of low battery power. Therefore, an auxiliary aim of this study will be to determine whether, as a corollary, AHD and AUSGeoid98 can provide a validation of the GPS vertical position solutions; cf. [14].

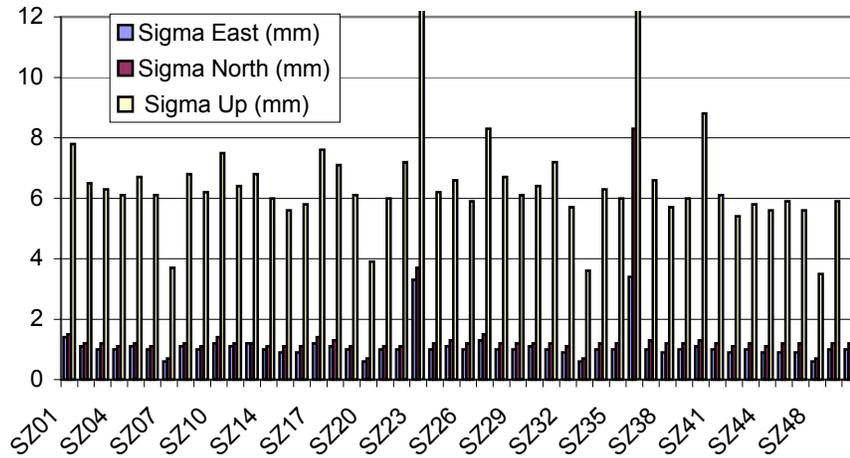


Figure 1. Internally estimated precision (1σ) of GPS-derived coordinates (ITRF2000 epoch 2002.37) of the 48-point network across the SWSZ (mm)

Previous GPS-levelling validations of AUSGeoid98 [5, 6] used ~1,000 nationwide GPS-derived ellipsoidal heights that are now of questionable veracity [11]. Specifically, they have been compiled over a number of years during which GPS data collection and processing algorithms have matured considerably. Indeed, some earlier GPS surveys may have been tied to ‘ellipsoidal heights’ derived from spirit-levelled benchmarks plus earlier geoid models (i.e., $h = H + N$, where h is the ellipsoidal height, H is the spirit-levelled height with respect to the AHD, and N is the geoid height), though this assertion cannot be verified as yet. If true, however, this is most unsatisfactory because subsequent attempts at geoid validation would only compare the new geoid model with an older one, thus not giving any validation.

DOLA geodetic surveyors spirit-levelled the 48 SWSZ stations with respect to the AHD according to the Australian class C standard, which allows for a misclose of 12 mm per square-root-km [10]. The distance traversed from existing AHD benchmarks to the new SWSZ monuments ranged from a few tens of metres to ~13 km. The spirit levelling was conducted soon after the GPS campaign, so it is reasonably safe to assume that no vertical ground deformation (albeit small anyway [18]) has occurred between the GPS and spirit-levelling observations.

The AHD is widely acknowledged to contain distortions of ~1.5m [3, 12], with a dominant north-south trend, which is attributed to the effects of sea surface topography on the 30 tide gauges fixed to mean sea level in the 1971 adjustment; e.g., [4]. Therefore, the major error in the levelled heights probably stems from the definition of the AHD and subsequent spirit levelling data. The difficulty of estimating errors from allowable miscloses is well known, so a “hand-waving argument” (i.e., without scientific or numerical rigour) is used to estimate the AHD height errors as ~20 mm over the SWSZ, excluding the systematic errors in the AHD.

METHODS, RESULTS AND DISCUSSION

Gravimetric geoid heights were computed at the ITRF2000 coordinates of the 48 stations (Appendix A) using on-line facilities that interpolate from precomputed geoid grids. AUSGeoid98 was bi-cubically interpolated via <http://www.auslig.gov.au/geodesy/ausgeoid/nvalcomp.htm>. EGM96 was spline interpolated from <http://164.214.2.59/GandG/egm96/intpt.htm>. These geoid heights were then subtracted from the GPS-levelling-derived ‘geoid-type’ heights, and summarised in Table 2.

	Maximum	Minimum	Mean	STD
GPS-AHD minus AUSGeoid98	0.196 (0.196)	-0.277 (-0.277)	-0.010 (-0.009)	±0.129 (±0.132)
GPS-AHD minus EGM96	1.175 (1.175)	-0.211 (-0.211)	0.512 (0.512)	±0.283 (±0.283)

Table 2. Descriptive statistical summary (in m) of the differences between the 48 SWSZ GPS-AHD heights and AUSGeoid98 and EGM96 (parenthetic values exclude the two ‘outliers’ at SZ23 and SZ36)

The larger mean value of the differences between the GPS-levelling and EGM96 geoid heights in Table 2 is due to the different zero-degree geoid terms. The zero-degree term in EGM96 is computed from the difference in mass and potential between it and the WGS84 normal ellipsoid, and its value has been determined as -53 cm [12]. The zero-degree term in AUSGeoid98 was computed from the mean difference from a nation-wide set of GPS-levelling data, and its value has been determined as -94 cm [6]. As such, the standard deviation (and trends, described later) should be interpreted as the more reliable statistics.

From Table 2, there is no evidence that the two ‘outliers’ (i.e., in terms of the larger internal error estimates; Figure 1) in the processed GPS data significantly degrade the comparisons. Therefore, it can be assumed that the GPS data processing has probably fixed the correct ambiguities for these stations, though the noise at each is higher. Accordingly, it could be argued that the use of GPS and levelling data provides a useful external check on GPS height solutions; cf. [14].

Figures 2, 3 and 4 show the (GPS-AHD minus AUSGeoid98) differences plotted as functions of ITRF2000 latitude, ITRF2000 longitude and AHD height, respectively. Figures 5, 6 and 7 show the same for EGM96. There is a clear ($R^2 = 0.82$; i.e., goodness of fit) north-south trend of ~ 0.81 mm/km (ppm) between the AHD and AUSGeoid98 (Figure 2), whereas there is no appreciable east-west trend (Figure 3) or elevation-dependent trend (Figure 4). The trends for EGM96 are far less clear (Figures 5, 6 and 7), simply because of the poorer overall agreement with the GPS levelling data (Table 1), but the north-south trend of ~ 0.95 mm/km for EGM96 is the ‘better’ defined ($R^2 = 0.23$). The small elevation-dependent trend in Figures 4 and 7 is due to the AHD heights increasing southwards in the study area.

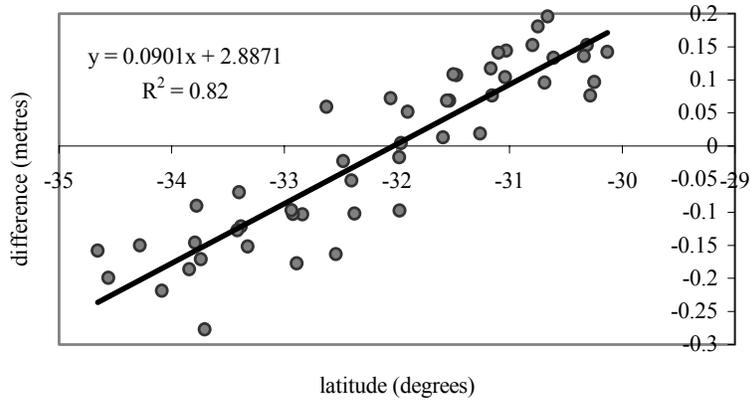


Figure 2. Differences (in m) between the 48 SWSZ GPS-AHD heights and AUSGeoid98 as a function of ITRF2000 latitude

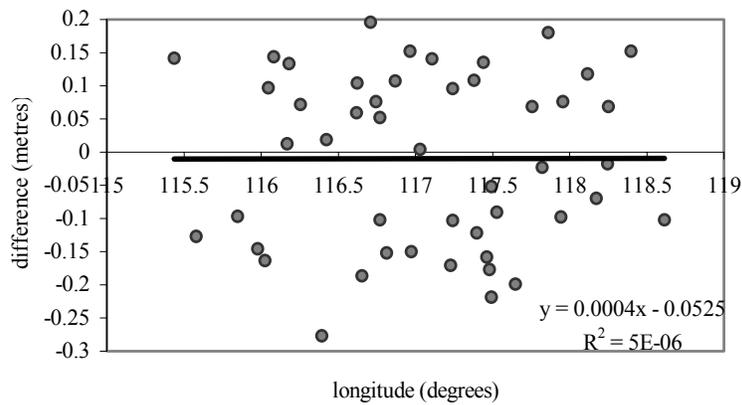


Figure 3. Differences (in m) between the 48 SWSZ GPS-AHD heights and AUSGeoid98 as a function of ITRF2000 longitude

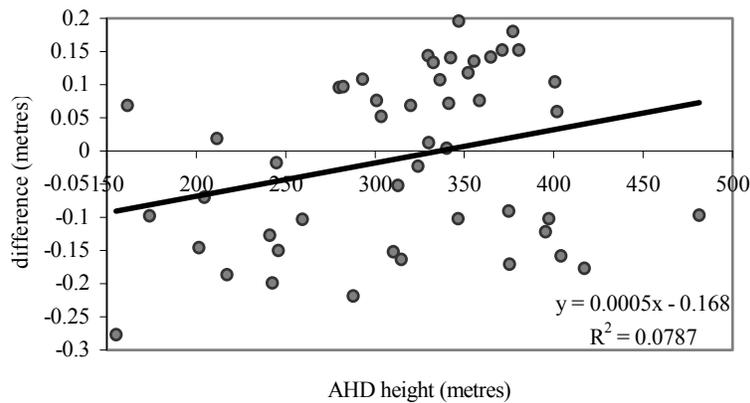


Figure 4. Differences (in m) between the 48 SWSZ GPS-AHD heights and AUSGeoid98 as a function of AHD height

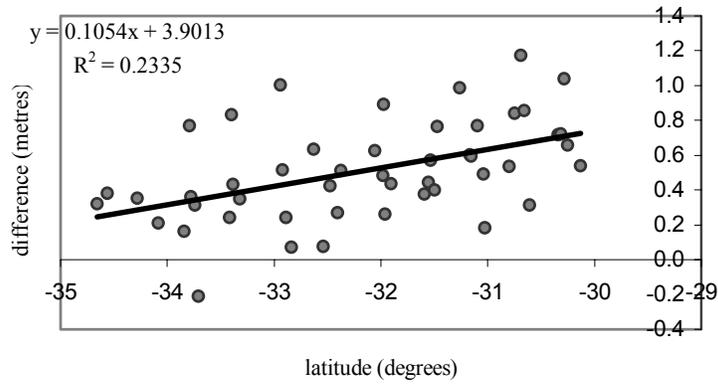


Figure 5. Differences (in m) between the 48 SWSZ GPS-AHD heights and EGM96 as a function of ITRF2000 latitude

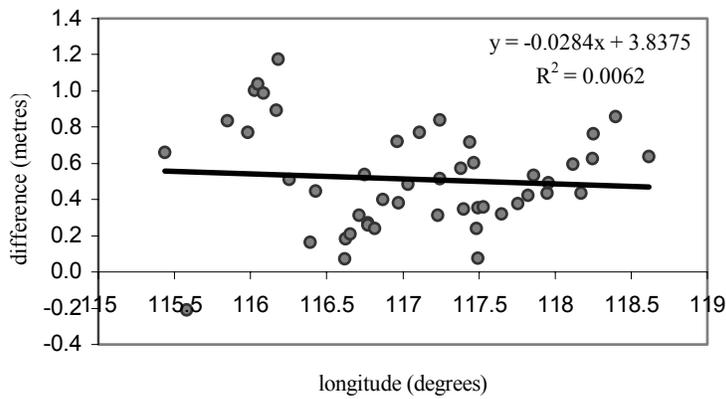


Figure 6. Differences (in m) between the 48 SWSZ GPS-AHD heights and EGM96 as a function of ITRF2000 longitude

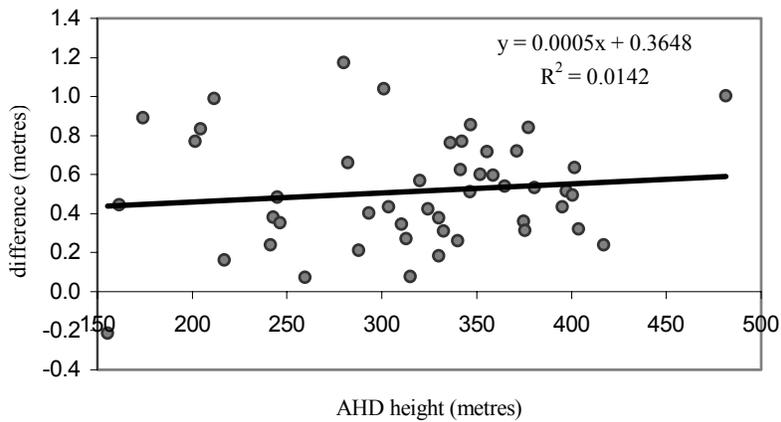


Figure 7. Differences (in m) between the 48 SWSZ GPS-AHD heights and EGM96 as a function of AHD height

Comparing Figures 2, 3 and 4 with Figures 5, 6 and 7 and the results in Table 1 show that AUSGeoid98 improves upon EGM96 (as is to be expected because of the inclusion of additional data to reduce the omission error) by reducing the standard deviation and the magnitude of the trends. However, the addition of regional gravity data cannot completely correct long-wavelength errors (the commission error) in the global geopotential model [16]. As such, the remaining ~ 0.81 mm/km north-south trend may be due to propagation of a long-wavelength error in EGM96 into AUSGeoid98.

This clear north-south trend concurs with previous studies by this author [4, 5, 6], but is more conclusive because of the good quality GPS data used in this study. Importantly, this only shows that the differences (assumed to be errors) are in one or all of the AHD, AUSGeoid98 and EGM96. However, these *cannot* be separated because of the correlations, lack of redundancy and likely errors in the AHD, AUSGeoid98 and EGM96. For instance, a ~ 0.81 - 0.95 mm/km tilt in one or all of the AHD, AUSGeoid98 and EGM96 is entirely plausible.

Figure 8 shows contours of the differences between the AHD and AUSGeoid98, computed using a tensioned spline interpolation [15]. The north-south trend, which is clear in Figure 2, is not so clear in Figure 8, thus showing the usefulness of a simple linear regression.

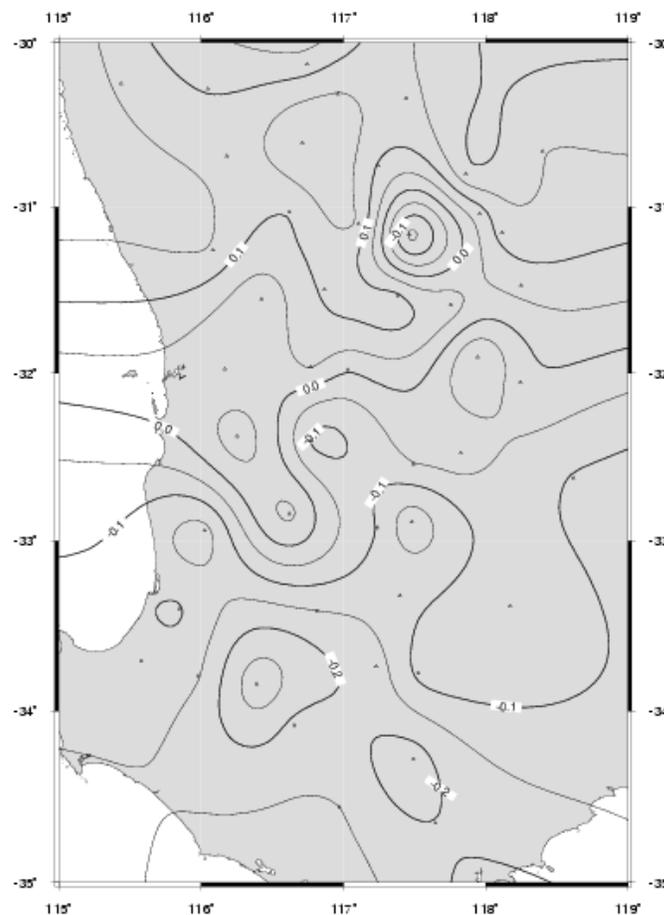


Figure 8. Contours (in m) of differences between the AHD and AUSGeoid98 (no trend removed)

Thus far, the systematic errors in the AHD have been omitted from the analysis and discussion. As stated, the ~ 1.5 m distortions in the AHD are dominated by a north-south trend, which is attributed to unmodelled sea surface topography (SST) effects on the tide gauges fixed in the realisation of the AHD. [4] shows that published SST models indicate a dominance in the north-south direction. Therefore, it is reasonable to expect that this will introduce a north-south distortion in the AHD.

In order to substantiate this, contours of the differences between the free-and fixed-network adjustments of the AHD ([13], Annex D) were ‘consulted’, and were used to estimate a north-south tilt of ~ 0.55 mm/km in the AHD across the SWSZ. Removing this estimate from the trend computed from Figure 2 indicates a north-south tilt of ~ 0.26 mm/km in AUSGeoid98, and a tilt of ~ 0.40 mm/km in EGM96. However, the many conditions outlined earlier, notably the inseparability problem, mean that these values must be treated with great caution.

Finally, the ~ 0.81 mm/km north-south trend between the AHD and AUSGeoid98 (Figure 2) was removed from the differences, and the descriptive statistics recomputed. Table 2 shows that the use of GPS in conjunction with the north-south tilted AUSGeoid98 model will yield AHD heights to a precision of ~ 5 cm in the SWSZ. This value is commensurate with results in many other parts of the world.

	Maximum	Minimum	Mean	STD
GPS-AHD minus tilted AUSGeoid98	117	-127	0	± 55

Table 2. Descriptive statistical summary (in mm) of the differences between the 48 SWSZ GPS-AHD heights and the tilted AUSGeoid98

SUMMARY AND CONCLUSIONS

This short note has presented a simple validation of the AUSGeoid98 regional gravimetric geoid model of Australia using a high-quality GPS dataset across part of southwestern Western Australia. It confirms earlier indications that there is a clear north-south trend between the AHD and AUSGeoid98, but the source of this trend cannot yet be reliably separated. Simple inspection of the free- versus fixed-network adjustments of the AHD in the study area suggests that fixing local tide gauges to mean sea level can account for approximately one-third of the observed trend. Before the trend was removed, the standard deviation of agreement was ~ 13 cm, which reduced to ~ 5 cm after trend removal. These values are more optimistic than previous studies (e.g., ~ 36 cm; [6]), which are attributed, in part, to the use of significantly improved GPS data for this study.

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Appendix A: SWSZ GPS-levelling data

Latitude (degrees)	Longitude (degrees)	Ellipsoidal height (m)	AHD height (m)
-30.2491617	115.438471	252.0994	282.230
-30.282882	116.047472	273.5354	300.856
-30.1316919	116.746192	340.3401	364.691
-30.3133847	116.963217	346.8963	371.104
-30.3398711	117.441334	331.7438	355.566
-30.6908414	116.180941	252.8395	279.965
-30.6106898	116.710647	307.2231	332.561
-30.7498066	117.243825	352.5656	377.236
-30.7978249	117.860203	356.1396	380.395
-30.6620563	118.399419	323.1680	346.911
-31.2604024	116.083051	182.5188	211.530
-31.0302283	116.621416	303.6589	329.846
-31.0994332	117.10764	316.6748	342.234
-31.0401382	117.957368	375.7898	400.476
-31.1542331	118.115965	333.6465	358.480
-31.5553041	116.425922	134.0393	161.614
-31.4978989	116.868133	266.6029	293.241
-31.5352052	117.382447	293.6077	319.917
-31.5900978	117.755766	303.8279	329.960
-31.4737531	118.249463	311.0133	336.229
-31.9749827	116.166668	144.7364	173.835
-31.9647709	116.771438	313.0803	340.019
-31.9800179	117.032417	218.3307	244.935
-31.9039321	117.9455	277.5698	303.685

Latitude (degrees)	Longitude (degrees)	Ellipsoidal height (m)	AHD height (m)
-32.0555522	118.245806	315.6509	341.395
-32.37691	116.25396	318.0106	346.348
-32.403708	116.768889	286.1540	312.962
-32.5429589	117.49173	288.2168	314.890
-32.4774548	117.824038	298.0209	324.217
-32.937729	116.024241	452.1572	481.392
-32.8375496	116.620429	232.2296	259.337
-32.9209674	117.240981	370.7064	397.160
-32.8879441	117.481825	390.5158	417.044
-32.6262863	118.616647	375.3281	401.732
-33.4009708	115.847875	173.795	204.411
-33.4162649	116.814997	213.9165	241.175
-33.3247531	117.398451	283.4584	310.311
-33.384376	118.172151	368.2373	395.294
-33.7073268	115.580636	122.5700	155.281
-33.8441164	116.391983	188.6789	217.066
-33.7398525	117.230842	347.9799	375.285
-33.7790254	117.527751	347.8387	374.858
-33.7941048	115.979122	171.6461	201.446
-34.0854923	116.655716	259.3983	287.767
-34.2813289	117.493036	217.6259	246.182
-34.5600101	116.972252	212.3189	242.746
-34.6551835	117.647355	373.3640	403.864
-31.1681761	117.464577	326.5092	351.908