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## On the Tensile Strength of Carbon Nanotubes

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The scattered strengths of carbon fibres with diameters on the scale of micrometres can be well described by using classical Weibull statistics. As materials such as carbon nanotubes approach the nanoscale, however, some principal assumptions like continuum in linear elastic fracture mechanics are unsuitable. Similar problems appear in the statistical analysis of fracture strengths by Weibull statistics because there are only a few rather than numerous defects in quasi-perfect carbon nanotubes [1]. Also, these defects are completely different from those such as microcracks in continuum mechanics. Therefore, effectiveness of Weibull statistics is often in question and there is still no consensus on its validity [1,2]. Simple and uncritical extrapolation may result in overestimation or underestimation on the strength of carbon nanotubes and even a misunderstanding on intrinsic fracture mechanisms.

Carbon nanotubes are a new form of carbon that consist of graphite sheets, rolled into hollow tubes and capped with C<sub>60</sub> fullerene hemispheres on both ends. There are two main types: single-walled carbon nanotubes (SWCNTs) with diameters of 0.5–2 nm and multiple-walled carbon nanotubes (MWCNTs) with diameters of 2–50 nm. In contrast to carbon fibres, these nanotubes have unique and promising properties such as low density, superior electrical and thermal conductivity, and extremely strong stiffness and strength (but relatively ductile with fracture strains of 5–20%). Combined with their high aspect (length-to-diameter) ratios, carbon nanotubes are considered as an ideal reinforcer in composites. It is not surprising that the strength of carbon nanotubes has been of great interest; however, their values have remained elusive both experimentally and theoretically. Early measurements on the tensile strength of carbon nanotubes were done indirectly through their nanocomposites, which leads to an unexpected low tensile strength of ~1.2 GPa. With the new technique of a “nanostressing stage” located within a scanning electron microscope, direct strength measurements on carbon nanotubes were possible in recent years, and a few strength data sets for SWCNTs and MWCNTs are available [3,4].

For simplicity, the influence of sizes (such as diameters or lengths) of carbon nanotubes on their strengths was usually ignored in statistical analysis, and a two-parameter Weibull distribution was used to fit strength data sets. The results showed that the Weibull distribution seems to accurately fit these data sets with a small Weibull modulus (~ 3) [2]. As is well known, the scaling law (or size effect) on strength is a direct consequence of Weibull statistics. That is, the strength of a brittle specimen decreases with increasing its size,  $\sigma(V) \sim V^{-1/m}$ , where  $m$  is the Weibull modulus and  $V$  is an effective volume (or length). This provides us a simple, intuitive method to check out the feasibility of Weibull statistics at the nanoscale. For example, in the case of SWCNTs under tensile load, the dependence of strengths ( $\sigma_e$  or  $\sigma_p$ ) on their diameters is shown in Fig. 1. It is obvious that there is no size effect as expected. In other words, classical Weibull distributions cannot well describe the real nature of strengths of SWCNTs. The same behaviour has also been found for strengths of MWCNTs.

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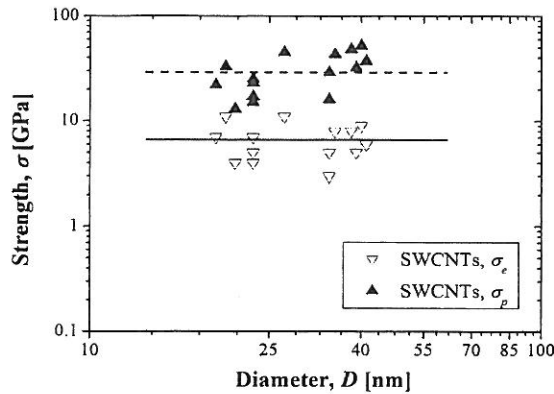


Fig. 1 Strengths of SWCNTs versus their diameters, where the strength  $\sigma_e$  was calculated by considering all SWCNTs in a rope to be carrying the applied load, and  $\sigma_p$  was calculated by considering only nanotubes in the perimeter of the rope to be carrying the load [3]. Their mean strengths are 6.7 GPa (solid line) and 30 GPa (dashed line), respectively

In the view of strength statistics, Weibull distribution should be considered as an empirical one on an equal footing with other possible candidates such as normal and lognormal distributions. To determine which one fits strength data better, a minimum (or Akaike) information criterion (AIC) is used, which is defined as  $AIC = (-2)\log(\text{maximum likelihood}) + 2(\text{number of independently adjusted parameters within a model})$  [5]. The numerical results show that, relative to Weibull and normal distributions, lognormal distribution seems to be a better choice for strength data sets of SWCNTs and MWCNTs.

The lognormal distribution implies that there be a characteristic strength (or length scale) in carbon nanotubes such as their mean strength due to the collective (multiplicative) interaction of numerous independent (or quantized) factors (not limited to flaws in the traditional sense) rather than a critical one that triggers the final failure as supposed in classical Weibull statistics. This provides a possible answer on the question as to the nature of nano-defects and their degree of severity (or sensitivity) [6], which is consistent with the conclusion from nanoscale Weibull statistics, a modification of classical Weibull statistics that was proposed by Pugno and Ruoff [1] based on their early work on quantized fracture mechanics. Furthermore, the present analysis indicates that there is a special kind of flaw tolerance at nanoscale and the scattered strengths of carbon nanotubes are independent of their sizes.

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