School of Economics and Finance

TESTING FOR SUBADDITIVITY IN THE AUSTRALIAN TELECOMMUNICATIONS INDUSTRY 1954-1990

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Declaration

This thesis contains no material, which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

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31 December 2005

Abstract

This study applies a test for subadditivity (natural monopoly) to Australian telecommunications industry data for the period 1954 to 1990. If an industry exhibits subadditive cost, a monopoly can provide total industry output at a lower cost than multiple firms. The test for subadditivity is dependent on econometric estimation of a theoretically valid cost function. The cost function employed in this study is a multiple output variation of the symmetric generalised McFadden cost function. The main advantage of this specification is the ability to impose concavity on the cost function with respect to the input prices without imposing *a priori* restrictions on the input substitution elasticities.

While there have been numerous previous subadditivity studies, this study is novel in two respects. First, this study contains the results of a direct test involving the provision of data carriage services provided by Australia's monopoly carrier from 1970 to 1990. Thus, the test for subadditivity is applied to a relatively new service at a time when demand is in its infancy. Second, the approach to modelling makes explicit allowance for radical technological changes and lags in adjustment.

The results indicate cost complementarity between data-aggregate output and large economies of scale. However, these effects are not strong enough to guarantee subadditivity. Analysis suggests that the most likely cause of subadditivity is the extent of network duplication between competitors. Evidence of subadditivity is found for firms that duplicate more than 30% of the network's fixed cost. This implies that at the national level, competition policy is the right choice. This suggests that regulated competition is likely to be no more costly than monopoly.

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This study could not have been possible without the assistance of many people and I am grateful for their diligence and support. Many of those that have provided help share a passion for research, which is never more apparent than during endless hours of tedious and painstaking work.

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Finally, this study is the result of years of after hours work searching through library archives, careful documentation, auditing, and model development. Much of it was done through late nights and hundreds, if not thousands, of hours. All of this effort impinged on time that could have been spent with my young family. Throughout all of that time, my wife Fiona has been a source of unswerving support in so many ways. From single-handedly taking care of our children Tara and Ryan, constantly finding ways of giving me quiet time to study, shouldering the burden of family responsibilities and through it all being a source of passionate encouragement. Your sacrifice is an eternal source of inspiration. Thank you also to Fiona's parents, the truly marvellous Frank and Rita Horgan.

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CHAPTER 1—INTRODUCTION

The primary motivation for this thesis is to test for subadditivity in the Australian telecommunications industry as represented by data provided in the Postmaster General’s Department (1901 to 1975) and Telecom (1975 to 1990) annual reports. Subadditivity is the mathematical term for natural monopoly. If an industry is subadditive, it is less costly to produce the entire industry output via a single firm than more than one firm. The alternatives to subadditivity are additivity and superadditivity. In the additivity case, production via one or more firms yields no gain or loss in efficiency while superadditivity indicates that supply via more than one firm yields an efficiency gain over monopoly. This thesis applies the test for subadditivity using data reported by Australia’s monopoly telecommunications service provider against the alternative of duopoly.

Testing for subadditivity requires estimation of a cost function that satisfies all of the restrictions implied by received theory. This has proven to be a challenging prospect. Standard cost function specifications proved inadequate, an outcome that is mirrored in the subadditivity literature. Satisfactory results could only be achieved with a dynamic multiple output version of the symmetric generalised McFadden cost function. The dynamic cost function is required to allow for short-run adjustment costs, which turn out to be substantial.

Estimation is further complicated by radical changes in technology that occurred throughout the 20th century. Beginning largely as a telegraph network in the 19th century, the Australian telecommunications network evolved to provide telephone services, telex, data and mobile telephony. Coinciding with this service innovation were radical changes in switching and transmission technology. Development in switching progressed from manual call switching through to fully automated computer controlled digital networks. Up to five distinct switching technologies were in use simultaneously across the network. Transmission technology developed from single open wire to twisted copper cable with increasing use of coaxial and later fibre optic cable in major traffic routes. The information about these changes in technology proved useful in developing sensible econometric results.

The data presented in this thesis relates to the cost of inputs such as capital, materials and labour, the volume and type of service provided and information about the types of technology employed. These data are available on public record from 1920 to 1990. Unfortunately, the transition from state-owned monopoly to the contemporary competitive regime has coincided with a marked deterioration in the quality of publicly available data necessary to test for subadditivity. Consequently, it has not yet been possible to sensibly examine the period since 1990. This thesis thus presents the results of painstaking data collation along with a reasonably comprehensive account of the history of the evolution of the Australian telecommunications network from 1920 to 1990. This information has not previously been widely accessible and thus, publishing these data provides the opportunity for further research.

The majority of previous studies have confined analysis to local and toll telephone calls. This thesis tests for subadditivity in a two-output model that specifies subscription to data services as one output and aggregates the remaining services, such as telegrams, telex, local and toll calls, cellular telephone calls, and telephone subscription, in the other output. Australian data services began with the Common User Data Network in 1970, which used the existing telecommunications network to link computers via point-to-point connections. Since then, data services have become progressively more sophisticated and accessible, leading to rapid expansion in the use of networked computing. Finding subadditivity would suggest that the monopoly service provision of data services led to a more efficient outcome than a competitive regime. Further, it is informative to examine whether subadditivity in the data-aggregate output configuration is dependent on the scale of output. If so, it might be appropriate to withhold the introduction of competition until a crucial threshold in output is reached. Another implication that is particularly important in the Australian context is the size of net losses in providing telecommunications service in areas of low population density under a competitive regime vis-à-vis monopoly.

The relevance of analysis based on data that are at least 15 years past requires discussion. Applying received theory reveals fundamental attributes of the Australian telecommunications industry, such as the relative importance of the share of fixed and variable costs, cost complementarities between outputs, and technological change in determining the degree of subadditivity. If these factors are largely time invariant, a cost function based on old data might provide a reasonable basis on which to base realistic expectations of alternative policies. More importantly, however, this thesis demonstrates the information that can be gleaned from the application of cost function theory to industry data. By demonstrating the method, it is hoped that Australian policy makers will mandate public disclosure of the data necessary to analyse the cost structure of the contemporary telecommunications industry.

The remainder of this thesis is organised as follows. Chapter 2 presents a brief overview of technological progress in the Australian telecommunications industry. There are relatively few sources available that provide a comprehensive overview of the technology. However, as demonstrated in the results, a reasonable knowledge of technological evolution is necessary to develop sensible results. Section 1 begins the chapter with a short description of telegraph technology. The main features discussed are telegraph's progressive automation and decentralisation. Section 2 and Section 3 discuss the two fundamental dimensions of telephone technology: switching and transmission. Switching refers to the call routing technology contained within telephone exchanges, with technological advances increasing switching speed and flexibility. As discussed in Section 2, there were a total of six generations of exchange technology with up to five technologies in operation simultaneously. Transmission refers to the types of cables used with the development of twisted copper cable, coaxial cable and ultimately fibre optic cable raising capacity exponentially. Section 4 then provides a short section describing the development of the Australian data network followed by Section 5, which concludes the chapter.

Chapter 3 provides a review of the received literature, beginning with a review in Section 1 of past applied studies. Two key insights drawn from the review are that: the chosen functional form of the cost function is crucial to producing a cost function that is consistent with received theory; and that the standard atemporal cost function invariably induces serial correlation. These two factors receive particular attention in developing the estimating strategy adopted in this thesis. Section 3 provides a short summary of cost function theory and concludes with an outline of flexible functional forms. Section 4 discusses the symmetric generalised McFadden cost function, while Section 5 considers the impact of technological change and established methods of controlling for it. Section 6 then discusses the reality of adjustment cost. Dynamic specification is introduced to address the serial correlation problem believed to be induced by adjustment lags as input composition changes. Section 7 follows with a discussion of serial correlation, adjustment cost and strategies developed in the time-series literature to deal with these issues.

With the general issues identified in Chapter 3, Chapter 4 outlines the estimation strategy in detail. Section 1 describes the modified generalised McFadden cost function based on Kumbhakar’s (1994) specification. Section 2 focuses on controls for technological progress and Section 3 is focused on the Box-Cox transformation, which is applied to the output arguments. Section 4 then specifies a dynamic version of Kumbhakar’s (1994) modified generalised McFadden (MGM) cost function and Section 5 provides a specific illustration to show how the Box-Cox transformation, technological change and dynamic specifications combine. Section 6 provides a short description of the MGM in revenue share form and Section 7 outlines the generalised error-correction model in revenue share form as an alternative strategy to the dynamic structure presented in Section 4 and Section 5. Section 8 then presents ancillary analysis derived from the equilibrium specification.

Chapter 5 provides a detailed description of the source data, discusses the evolution of the Australian telecommunications network, and explains the methods used to construct the required variables. Section 1 provides a summary of the source material and explains the methods employed to construct each of the required variables. Section 2 provides the resulting summary statistics and a detailed description of the evolution of Australia's telecommunications network since 1901. The detailed account of technological changes provides a useful guide for estimation and identified changes in technology suggest additional variables that may provide useful controls in the estimating model. Section 3 describes data transformation processes.

Econometric results are presented in Chapter 6. Section 1 discusses the preferred econometric model and results. Section 2 provides analysis of the ancillary variables such as marginal costs, cost elasticities and fixed cost estimates. Section 3 presents results for the subadditivity test and briefly discusses the implications. While the preferred results are presented in the main body of the chapter, alternative specifications are reported in the appendices. In addition, the chapter reports the difficulties in achieving satisfactory results with final estimation limited to the years 1950 to 1990. While the cost function specifications presented in this thesis are quite flexible, it seems that the production of satisfactory results over a longer period require models that permit parameter variation to accommodate radical changes in technology and major disturbances such as the World War I, the Great Depression, and World War II.

In short, the results suggest that the Australian telecommunications industry is generally not subadditive, though there is some variation depending on cost function specification. However, the results also show that subadditivity can be induced if competitors are forced to duplicate more than 30% of the incumbent's network. When subadditivity is found, it is due to high fixed cost. Thus, subadditivity depends on the extent to which competitors are forced to duplicate the incumbent's network. Chapter 7 concludes the thesis.

Overall, this thesis demonstrates that the applied tests for subadditivity are sensitive to controls for the technology employed. One implication of this is that subadditivity tests may need to be reapplied as subsequent generations of technology are deployed.

CHAPTER 2 — TECHNOLOGICAL CHANGE IN AUSTRALIAN TELECOMMUNICATIONS

The Australian telecommunications network has evolved radically throughout its history. This network began as a telegraph network in the 1800s with telephone services beginning in the 1880s. Further service innovation continued with the subsequent introduction of telex, dedicated data links connecting computers in each of Australia's capital cities, mobile telephony, facsimile transmission and, ultimately, the contemporary development of the Internet.

This extensive service innovation coincided with fundamental innovation associated with the ability to consistently increase message throughput throughout the 20th century. Indeed, as this chapter shows, there is no single point in time where the Australian telecommunications network can be said to be in equilibrium. This presents a serious challenge to estimating the cost of operating the network in a way that permits application of the test for subadditivity. Doing so requires the development of appropriate controls for technological progress.

Consequently, this chapter documents a brief outline of the Australian telecommunications technological evolution. Section 1 begins the chapter with a short description of telegraph technology. The main features discussed are telegraph's progressive automation and decentralisation. Section 2 and Section 3 discuss the two fundamental dimensions of telephone technology: switching and transmission. Switching refers to the call routing technology contained within telephone exchanges, with advances increasing switching speed and flexibility. As discussed in Section 2, there were a total of six generations of exchange technology with up to five technologies in operation simultaneously. Transmission refers to the types of cables used with the development of twisted copper cable, coaxial cable and ultimately fibre optic cable raising capacity exponentially. Section 4 then provides a short section describing the development of the Australian data network followed by Section 5, which concludes the chapter.

1. TECHNOLOGICAL CHANGE IN THE TELEGRAPH NETWORK

In 1901, the relatively mature telegraph network was accessible to a large proportion of the Eastern colonial population with, for example, 110 telegraph stations in Victoria. According to Moyal (1984), the first facsimile service began with the deployment of a Siemans-Karolous picturegram in the late 1920s "…with the first photographs transmitted between Sydney and Melbourne in 1929 and the first radio-picturegram was transmitted from London to Melbourne in 1934…" (Caslon Analytics, 2004: 4). Picturegram transmissions between PMG sites peaked at 6,280 in 1958 as media organisations, news services and large corporations operated privately owned picturegram equipment.

PMG annual reports indicate that Australia's telex network, which enabled telegram transmission from customer premises, began operation in 1954. The next most significant development, transition to the automated TRESS network occurred over the period 1960 to 1964. Cannibalisation of telex and picturegram transmissions began in 1970s with the proliferation of facsimile machines.

2. TELEPHONE NETWORK SWITCHING EVOLUTION

According to Power (1978), the first manually operated exchanges opened in Sydney and Melbourne in 1880. By 1911, the total number of manually switched telephone services reached 100,000. In the same year, the first Strowger public automatic exchange was installed in Geelong, Victoria using step-by-step technology. In subsequent years Strowger technology was progressively installed until 1937, when the first 2000-type step-by-step equipment replaced Strowger as the local exchange standard. Newstead (1995) reports that the step-by-step system utilised ‘direct’ switching with the calling path established step-by-step through a series of bi-motional selectors in control of a single digit. Consequently, step-by-step exchanges maintain network numbering and call routing in a fixed relationship, limiting their actual telephone number capacity 50-60% below the theoretical telephone number limit. With this technology, spare telephone numbers in one exchange area cannot be used to serve other areas that have reached their numbering capacity. The number of manual services in operation peaked in 1957, servicing approximately 400,000 subscribers in a network of 1.3 million.

Faced with rapid demand growth for telephone services, the Australian Post Master General’s Department (PMG) adopted the L.M. Ericsson ARF/ARK crossbar switching system as the local exchange standard in 1959, with the first installation in 1962.[1] New orders for step-by-step equipment subsequently ceased. Newstead describes the crossbar

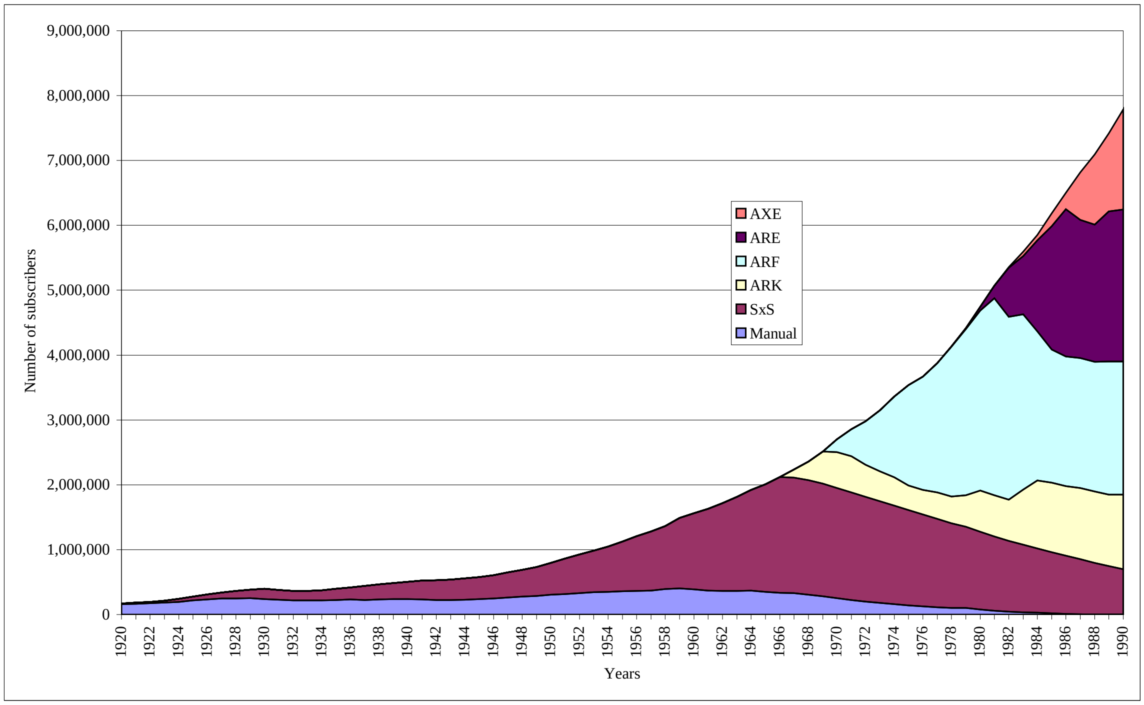
[1] Newstead (1995) reports 14% p.a. traffic growth for trunk calls.

equipment technology as semi-electronic, comprising electronic switch controls and analogue interconnection of voice circuits via relay, crossbar and glass-sealed reed physical contact technology. Crossbar technology decoupled dialled digits and switching through the use of a register control and tandem switch matrices. As this efficiency gain is scale-related, the number of step-by-step equipped ends continued to grow, peaking in 1964 at 1.8 million services. Crossbar equipment upgrades to ARE 11 began in 1976. The successive crossbar redesigning ultimately led to the substitution of crossbar common control equipment for ‘front-end’ computer control. With the emergence of computer stored program control (SPC) switching systems, all of the exchange functions could be controlled by the central processing unit.

Computer control offered a wider range of facilities, enhanced routing flexibility through software control and increased call-handling capacity. In addition, computer control reduced equipment space requirements and addressed some of the inherent inefficiencies in addressing and throughput constraints associated with the crossbar switching system.

The decision to adopt the completely electronic AXE system as the new local switching standard occurred in 1977, with the first installation planned for 1980 (Power, 1978). The main benefits of AXE are reduction in required floor space compared to ARE/ARF equipment (up to 50%), increased routing analysis and remote control switching.

Figure 2.1. Telephone subscribers by exchange technology



Note. This chart is based on a similar illustration provided in Power (1978). Post 1978 data is constructed to correspond to data points provided in source material.

Figure 2.1 illustrates the composition of the telephone network with respect to switching equipment. According to Power (1978), equipment changes occurred at 25-30 year intervals, although individual equipment installations typically remained in service for 40 years. For example, in 1978 there were 4.1 million services in the Australian telecommunications network, comprising 100,000 manual and 4 million automatic exchange services. Most (68%) of the automatic services were connected to crossbar equipment and approximately 1.2 million services were connected to step-by-step equipment. The network contained 1,425 manual exchanges, which served mostly small rural towns, and 4,350 automatic exchanges.

The number of exchanges peaks at 7,326 in 1959, just prior to the introduction of crossbar technology. With services in operation continuing to grow rapidly, this implies an increase in average exchange capacity. Indeed, the period 1960 to 1990 is distinguished by a trend reversal in total exchange numbers to just over 5,000 by the end of the period. Exchange reduction occurred at the expense of tandem exchanges utilised as overflow exchanges during peak loads. Tandem exchanges were strategically located between local exchanges so that differences in peak times could be exploited to extract greater efficiency in terms of exchange utilisation. However, as these tandem exchanges are geographically defined, their reduction implies a scale expansion of the local exchanges they served.

3. TRUNK NETWORK AND TRANSMISSION TECHNOLOGY EVOLUTION[2]

Innovation in transmission technology is reflected mainly in trunk telephone services. Australia's principal transmission route is the Melbourne to Sydney trunk line, reflecting the concentration of Australia's population in the southeast portion of the continent. According to Moyal (1984), the Melbourne to Sydney trunk line was completed in 1907 with extension to Adelaide completed in 1914, to Brisbane in 1923, Perth in 1930 and Hobart in 1935. Smith (1981) confirms that, from its inception, the single trunk circuit between Sydney and Melbourne represented the backbone of the national trunk telephone network.

Rapid traffic growth prompted construction of a second trunk circuit, which was completed in 1921. Both circuits were provided on physical pairs of wires on poles. A further three circuits were added in 1925, provided via a carrier system with repeaters at Goulburn, Wagga and Wangaratta. Additional four 3-circuit carrier systems were added

[2] All of the information relating to evolution of the trunk network is reproduced from Smith (1981).

by 1937 and the first 12-circuit carrier system in 1939, resulting in a total of 30 circuits. The use of higher frequency (150kHz) meant that six additional repeater stations were required. After the establishment of Canberra in 1927, a 3-circuit Melbourne-Canberra carrier system was installed in 1938, followed by a Sydney-Canberra system in 1939.

No further significant development occurred until after World War II. By 1947 there were 48 Sydney-Melbourne, 4 Melbourne-Brisbane, 12 Sydney-Canberra, 22 Melbourne-Canberra and 4 Sydney-Adelaide circuits. By this stage, the single Sydney-Melbourne open-wire pole route along the railway line was nearing full capacity, leading to the construction of a second Sydney-Melbourne pole route commencing in 1948. Erected between Blayney and Seymour via Cowra, Narrandera, Deniliquin, Echuca and Bendigo, the new trunk route connected with the existing Sydney-Blayney-Orange and Melbourne-Seymour carrier cables. The second route added a 12-circuit system in 1950, another 12-circuit system in 1951 and a third 12-circuit system in 1952. An additional two 12-circuit systems were added in 1953. The 1951 recession coincided with a temporary levelling in telephone traffic levels and then a period of rapid growth during the Korean War. Following widespread floods, the PMG adopted a policy of route and plant diversity to improve network reliability. Consequently, a 24-circuit radio system was installed in Goulburn from Sydney in 1952.

By the late 1950s, the open-wire routes reached their maximum capacity of 200 circuits. PMG management decided on a long-term solution to upgrade the trunk route with a six-tube coaxial system on a route that passed through Canberra. As the project was anticipated to take three years to complete, the PMG installed an interim Sydney-Canberra microwave radio system to operate at 4GHz. The radio terminals were installed at Redfern Exchange and Red Hill with coaxial cable linking the City South, Central Sydney (East Block) and the Canberra trunk exchanges, respectively. Using the four existing repeater stations, the microwave radio system began in July 1960. Completion of the Sydney-Melbourne coaxial cable project added two 6MHz valve-operated line systems on two pairs of tubes, one for telephony and one for both-way television relays. A third pair of tubes was retained as spare for future development.

By this stage, the Sydney-Melbourne route length was 965km with 103 unmanned repeaters spaced at 9km intervals. An additional 12 ‘main repeaters’ were sited at intermediate towns along with a ‘back-to-back’ terminal in Canberra. Maximum capacity was set at 960 telephone circuits with the capability to expand to 1,260 circuits. Of the 960 circuits, 500 were set as interstate trunk circuits with the remainder assigned for termination in intermediate towns. The new Sydney-Canberra telephone system was operational in July 1961 with Sydney-Melbourne operational by April 1962. The creation of network television led to television system activation in December 1963 when Channel 9 leased television relay facilities for a maximum 70 hours per week.

The trunk system expanded again when the construction of the Wollongong ‘spur’ microwave radio system began in January 1962. Later in the same year, a one-way television bearer was provided between Sydney and Canberra (with a ‘spur’ to Knights Hill from Maddens Plains for the Wollongong area transmitter). The Canberra national transmitter building and tower were established on Black Mountain. Coaxial cable linked Red Hill (Canberra) to Central, the ABC Studios and the Black Mountain transmitter.

Following a fire at the Civic exchange (Canberra) in 1961, the PMG adopted a policy of security and diversity in telecommunications for Canberra. The outcome was the construction of a new building to accommodate the ARM trunk exchange for Canberra and the trunk carrier equipment associated with the radio bearer. The expansion in trunk circuits for telephone users located in Canberra permitted Subscriber Trunk Dialling (STD) to be activated in Canberra in 1962, with Sydney subscribers given STD access to Canberra by 1965. STD from Sydney to Melbourne was operational from November 1964 and from Melbourne to Sydney by October 1965.

A new tower and building extension at Red Hill (Canberra) permitted regional television relay extension to the Wagga national television station by April 1965. The relay was subsequently extended to Griffith via radio to Wagga and then coaxial cable by July 1966. A radio relay system was established within the Victorian section of the route to provide television relays from Melbourne for the national television stations serving the Shepparton and Albury areas. In January 1965, the Sydney-Canberra-Melbourne coaxial cable TV relay was leased for a further 2 years for 60 hours per week in each direction to the Channel 9 network.

Deployment of the coaxial cable network permitted a marked rise in telephone traffic and by June 1966, 430 intercapital circuits, including 354 Sydney-Melbourne circuits were operational. A Canberra-Cooma-Brown Mt. radio relay system was established in 1966 to provide telephony circuits to Cooma and a television relay to the national transmitter for the Cooma-Bega area followed by installation of a second Sydney-Canberra one-way TV bearer in 1966. The radio system required construction of two new repeater stations. Activated in November 1967, the system enabled STD access between Melbourne and Brisbane. Total intercapital circuits (excluding Canberra) numbered 700, comprising 500 for Sydney-Melbourne, 90 for Melbourne-Brisbane and 80 for Sydney-Adelaide. The new interstate television bearer on the radio system enabled the television relay facility to be transferred from the coaxial system, thus allowing the second cable system to be used for telephone service, adding 300 circuits to the total of 1,260.

By 1971, all of the 4GHz Sydney-Canberra radio bearers were in use. Two-tube coaxial cables equipped with 12MHz solid-state line systems were installed, providing 2,700 new telephone circuits with 1,800 circuits dedicated to interstate traffic. A 12MHz line system was installed in 1973 on the third pair of tubes followed by a second 12MHz line equipment installation in 1975 on this pair of tubes. The higher frequency line required double the existing repeaters. Rapid traffic expansion prompted plans for an additional Sydney-Canberra-Melbourne 6.1GHz radio system with capacity of 1,800 telephone circuits, requiring four new repeaters.

In 1978, the Melbourne-Canberra section with the last remaining 6MHz valve-operated line system was decommissioned to reduce power costs and maintenance. The valve-operated circuits in the Sydney-Canberra route were retained due to a shortage of available telephone circuits. The 6.1GHz radio system installation was completed at the Black Mountain tower in 1978, permitting demolition of the Red Hill radio terminal building and tower in 1979. Additional broadcast capacity was added in late 1978 by adding a 6.1GHz bearer to support television relay for Channel 10. Finally, a 1,800 circuit bearer replaced the existing 1,200 circuit 6.1GHz bearer.

By 1981, a third Sydney-Canberra telephony bearer (6.1GHz radio relay system) was being planned for completion by 1983. According to forecasts, additional radio bearers were expected to be required for 1984, 1985 and 1986, anticipated to exhaust the bearer capacity of the 6.1GHz radio system. The plan also included a 140Mbps digital radio bearer system to be operational by 1985.

The last significant transmission innovation relevant to the sample period covered by this study is fibre optic cable. Fibre optic cable permits substantially greater traffic carrying capacity than coaxial cable and requires a reduced number of repeaters. The Australian Telecommunications Commission *Service and Business Outlook for 1983/84* (page 8) states that network field trials began from 1981 to provide Telecom with experience in installation and operation of fibre optic cable. By the end of financial year 1984, transmission of ‘live’ traffic began in a 34 Megabit per second junction system in Brisbane. Fibre optic networks were progressively installed in high traffic areas of Australia such as metropolitan inter-exchange routes and short-haul inter-urban links. Subsequent Service and Business Outlooks document increases in fibre optic capacity.

4. THE EMERGENCE OF THE AUSTRALIAN DATA NETWORK

Moyal (1984) reports that data and voice transmission via a common network began in Australia when computer operated data services commenced with the PMG’s Datel service in 1969, transmitting data over telephone lines. By 1973, Datel modems numbered 2,500 (Caslon Analytics, 2004: 6). In order to satisfy growing demand for access to lines for data transmission, the PMG established the Common User Data Network (CUDN), a packet switching service that permitted simultaneous access by multiple users (Caslon Analytics, 2004: 6). The CUDN offered a higher quality service (in terms of transmission speed and accessibility) at premium prices. There is relatively little discussion of later data services such as the Digital Data Service and Austpac. Both of these services employed digital technology. Austpac, which employed packet switching technology, helped to fund the infrastructure necessary for Internet access in Australia.

5. CHAPTER CONCLUSION

As documented in this chapter, the Australian telecommunications network evolved radically throughout the 20th century. The purpose for documenting technological progress in this study is to identify changes that may be relevant to cost function estimation. Standard theoretical treatment presents the cost function in an equilibrium setting. This chapter clearly indicates that the telecommunications network is not in equilibrium with respect to technology. Consequently, subsequent development in this thesis builds in controls for network evolution.

CHAPTER 3—LITERATURE REVIEW

Testing for subadditivity requires estimation of a cost function that is consistent with received economic theory. This has proven to be a challenging prospect in the literature with few past studies reporting entirely satisfactory results. Consequently, there has been a proliferation of alternative models developed that seek to address identified modelling issues such as appropriateness of functional form, the impact of technological change and adjustment cost.

This chapter begins Section 1 with a brief explanation of what is meant by subadditivity and an exploration of how subadditivity works. Section 2 reviews applied subadditivity studies with a particular focus on the telecommunications industry. Section 3 then discusses the test for subadditivity in detail and motivates the requirement to estimate a valid cost function. Section 4 provides a short summary of cost function theory and concludes with an outline of flexible functional forms. Section 5 discusses the symmetric generalised McFadden cost function, while Section 6 considers the impact of technological change and established methods of controlling for it. Section 7 then discusses the reality of adjustment cost. Section 8 follows with a discussion of serial correlation, adjustment cost and strategies developed in the time series literature to deal with these issues. Section 9 concludes the chapter.

Chapter 3

1. SUBADDITIVITY—CONCEPT AND OVERVIEW

According to György Pólya and Gábor Szegö (1976), a function is subadditive if

$$f(x+y) \leq f(x) + f(y) \tag{3.1.1}$$

Baumol (1977) applied the concept of the subadditive function to formally analyse natural monopoly. Baumol's proposition is that a multiple output firm is a natural monopoly if it is able to satisfy the entire market demand at a production cost that is less than all combinations of smaller, more specialised demand (Jamieson, 1997). Hence, in analysing natural monopoly, the function represented in (3.1.1) is total cost.

Natural monopoly can arise in many ways. Baumol analyses economies of scale and economies of scope. Economies of scale occur when an equi-proportionate 1% increase in the volume of inputs results in a more than 1% increase in the volume of output. That is, output increases faster than cost (Binger and Hoffman, 1988: 259). Panzar and Willig (1981) define economies of scope when the cost of jointly producing two or more outputs is less than separate production. Importantly, Baumol points out that, "*…scale economies are neither necessary nor sufficient for monopoly to be the least costly form of productive organization…*" (Baumol, 1997: 809). Rather, the sufficient conditions for multiproduct subadditivity must include complementarity in the production of different outputs.

The sufficient condition for cost subadditivity requires the satisfaction of two conditions:

1. strictly declining ray average cost; and

2. the cost function is transray convex.

The term 'ray average cost' refers to "…the special case in which output quantities all happen to vary proportionately *but input quantities follow the least-cost expansion path*…" (Baumol, 1977: 811). This concept is an analytical device necessary for dealing with cost subadditivity and is the multiproduct analogue of average cost in the single output case. The cost function is transray convex if it is no more expensive to produce goods in combination rather than separately (Baumol, 1977: 811). Hence, there is some kind of cost complementarity between outputs.

A likely example of declining ray average cost is when fixed cost is large relative to variable cost. Assume for a moment that variable cost is zero so that there is only fixed cost. Now allow a fixed output bundle (i.e. a weighted average of the individual outputs) to increase. Dividing fixed cost by the increasing output bundle results in declining ray average cost. Now allow non-zero variable cost. Ray average cost will decline until marginal cost increases by at least the rate of decline in fixed cost. The likelihood of this occuring increases as the ratio of fixed cost to output approaches zero.

Transray convexity is closely related to economies of scope (Baumol, 1977: 811). One example is a shared fixed cost as in the joint production of skins and meat; the single act of separating skin from meat results to create two saleable products. Economies of scope may also arise through the use of spare capacity, such as room in a building, to produce two or more outputs. Another common example is the sale of an output once treated as waste.

On page 819, Baumol (1977) points out that while strictly declining average cost and transray convexity are sufficient conditions for subadditivity, they are not necessary. Transray concavity can be overcome by a rapidly declining ray average cost. This could occur if fixed costs are similar regardless of output combination and are sufficiently large relative to variable costs. An example in the telecommunications industry would be the construction of separate tunnels and conduits for cables carrying voice and data traffic in the same area. In addition, Baumol states that even if some outputs have separate fixed costs, the cost function will still be subadditive if variable cost exhibits both strictly declining average cost and transray convexity.

2. THE RECEIVED LITERATURE

For most of Australia's history (1901 to 1992), and like many other countries, telecommunications services have been provided by a mandated public monopoly. From 1992 onwards, the Australian telecommunications industry has been progressively deregulated, allowing non-government (private sector) service providers to offer telecommunications services in competition with the publicly owned incumbent. The change in Australian telecommunications policy coincides with similar policy changes in many other countries.

A question that naturally arises from this policy reversal is what impact does competition have on the costs of providing telecommunications services? Does competition lead to unnecessary and costly duplication of the incumbent's telecommunications network and does this duplication then create a net cost increase for the industry? The answer to these questions is provided by the test for subadditivity and a considerable body of empirical research is concerned with the question of subadditivity in telecommunications.

Industry cost is subadditive if service provision via a single firm is less costly than two or more firms. In this case, competition will increase industry cost. The prospect of industry cost increases is particularly important for areas of low population density — there is a threshold population density below which the provision of telecommunications services can only be provided at a net loss. If industry cost is subadditive, facilities-based competition will require a larger subsidy to service a net loss service area than monopoly. On the other hand, if industry cost is superadditive, competition will lead to reduced cost and a lower subsidy for net loss service areas.

Among the most influential studies are Evans and Heckman (1983, 1984), who develop and apply a local test for subadditivity for multiple output industries to the US Bell system.[3] The test requires estimation of an industry two-output cost function, which specifies cost as a function of local and toll call output, input prices and technological change. Evans and Heckman's conclusion that the US Bell system is not subadditive for local and long-distance calls aroused considerable controversy.[4]

A subsequent study by Charnes et al. (1988) contradicts Evans and Heckman's conclusion. Röller (1990) and Diewert and Wales (1991) also challenge Evans and Heckman's (1983, 1984) conclusions. Röller estimated a CES-Quadratic rather than the translog employed by Evans and Heckman (1983, 1984) and modified the method to

[3] The local test confines analysis to the observed output range.
[4] The Evans and Heckman (1983, 1984) studies followed the US antitrust case against AT&T, in which AT&T agreed to divest a substantial portion of its network. Their results supported the case for breaking up AT&T.

incorporate the concept of a 'proper' cost function. Röller's method reversed Evans and Heckman's (1983, 1984) conclusions. Diewert and Wales (1991) further show that the cost function estimated by Evans and Heckman (1983, 1984) violates the theoretical requirement that cost is nonnegative in output and, therefore, question the validity of Evans and Heckman's (1983, 1984) test results.

Addressing the limitations of earlier studies, Shin and Ying (1992) develop a new dataset consisting of a pooled cross-section, time-series sample of 58 US local exchange companies (LECs) for the years 1976 to 1983. Using a translog cost function, they conclude that LECs cost functions are not subadditive over the output range that satisfies Röller's criteria for a proper cost function.

Sung and Gort (2000) seek to clarify the issue by exploring the relationship between subadditivity and economies of scale and scope. They find subadditivity in 58.7% of simulated cases based on data from eight LECs, which are used to estimate a translog cost function.[5] Sung and Gort (2000) report modest economies of scale as well as cost complementarity between local and toll calls. Auxiliary analysis shows that economies of scope are largely uncorrelated with subadditivity, but a strong positive relationship between subadditivity and economies of scale exists as well as a statistically significant negative relationship between subadditivity and firm size. Overall, the evidence presented by Sung and Gort (2000) suggests that a threshold point for subadditivity exists, influencing competitor market entry and exit.

[5] Sung and Gort (2000) state that their cost function generates negative marginal costs. Restricting analysis to the output region that generates positive marginal cost reduces the proportion of subadditive cases from 58.7% to 37.4%, while average subadditivity test measure changes from 0.4% to –3.3%.

Enter Wilson and Zhou (2001), who test for subadditivity based on data representing 71 LECs for the years 1988 to 1995. Comparing their study to Shin and Ying (1992), Wilson and Zhou (2001) find evidence of superadditivity when they estimate cost without controls for firm-specific heterogeneity. Adding controls for firm-specific heterogeneity reverses the result. Given the sensitivity of the subadditivity to changes in cost-function specification, they conclude that Shin and Ying's (1992) claim that LECs are unnatural monopolies is questionable.

Fuss and Waverman (2002) continue the debate by challenging the credibility of Röller's (1990) study, arguing that Röller's cost function biases the test toward finding subadditivity. Their criticism focuses on the assumption, imposed by Röller's (1990) model, that a firm specialising in long-distance calls incurs the same fixed cost as a firm that produces both long-distance and local calls. In effect, this assumption implies that the long-distance firm cannot avoid the fixed cost of operating the so-called last mile telephone network from the local exchange to customer premises. Fuss and Waverman (2002) thus conclude that Röller's (1990) study fails to provide sufficient evidence of subadditivity.

The conflicting results in US studies make the literature analysing telecommunications markets outside the US all the more relevant. Gentzoglanis (1993) analyses a relatively small publicly-owned Canadian telecommunications carrier called Alberta Government Telephone (AGT) for the period 1974 to 1985. Based on the translog model with two outputs (again, local and toll calls) and three inputs, Gentzoglanis (1993) finds evidence of subadditivity, concluding that AGT monopoly provision of local and toll calls is more efficient than would have been the case had AGT been exposed to competition. He notes, however, the cost savings appear to decrease with technological change.

Serafica (1998) analyses cost data based on the Philippine Long Distance Telephone Company (PLDT) from 1951 to 1993. The reported subadditivity test results indicate substantial efficiency would be realised by introducing competition. These results, however, possibly reflect PLDT's chronic technical inefficiency, which is discussed in Serafica's (1998) study.

Two Australian studies which analyse the cost structure of Australia's monopoly period, conducted by Bloch, Madden and Savage (2001) and Bloch et al. (2001) further highlight the sensitivity of subadditivity results to functional form of the estimated cost function. Using a composite cost function, Bloch, Madden and Savage (2001) find that the cost function for Australian telecommunications is subadditive for the years 1959 and 1991. On the other hand, Bloch et al. (2001) present test results that indicate the Australian telecommunications industry is not subadditive, based on a translog cost function that permits technology parameters to vary periodically. In reconciling these studies, it should be noted that Bloch et al. (2001) present output elasticity estimates that clearly imply negative marginal cost in toll calls throughout the sample period. Thus, as Röller (1990) previously highlighted, it seems the translog cost function exhibits degenerate behaviour and probably should not be relied upon for subadditivity tests.

In the UK, Correa (2003), finds evidence of subadditivity based on data obtained from 29 local infrastructure providers (similar to LECs) for the years 1990 to 1997. With subadditivity estimates on average ranging between 0.69% to 1.08%, she concludes that the higher cost increase associated with competition is likely to be outweighed by the benefits of an increased rate of innovation stimulated by competition. However, Correa (2003) also reports substantial difficulty in obtaining positive marginal costs. Most respecification efforts seem to fail to produce completely satisfactory results. Correa (2003) thus advises caution when evaluating the subadditivity evidence presented.

Acknowledging the widespread difficulty in obtaining sensible marginal cost estimates, Cooper et al. (2003) develop a new functional form. They then demonstrate their chosen function, which is intrinsically non-linear in parameters, by applying the model to the same US Bell system data employed by Evans and Heckman (1983, 1984). The model, which is a variant of the symmetric generalised McFadden cost function suggested by Diewert and Wales (1987), has the advantage that concavity in input prices can be imposed without destroying its ability to flexibly approximate the substitution possibilities between inputs. The reported results indicate that the estimated model satisfies all of the requirements for a proper cost function. However, Cooper et al. (2003) report that the Durbin-Watson statistics reject the null hypothesis of no serial correlation. This is a serious problem as serial correlation spuriously increases the statistical significance of the estimated parameters, thus limiting the ability to make inferences about the true cost structure. Attempts to eliminate serial correlation are reported to cause the cost function to generate negative marginal costs. Hence, Cooper et al. (2003) present ancillary analysis based on the serially correlated model.[6] They conclude that the US Bell system is not subadditive.

[6] It should be noted that the presence of serial correlation does not necessarily invalidate the model. If the estimated cost function accurately approximates the true cost function and all of the coefficient point

The above analysis of the literature reveals that the difficulty in estimating a proper cost function is a serious impediment to determining whether telecommunications is subadditive. Given the degenerate behaviour of the translog cost function, it is somewhat surprising that relatively few alternatives to the translog have been estimated. The results provided by Röller (1990), Bloch, Madden and Savage (2001) and Cooper et al. (2003) provide evidence those alternative functional forms perform better than the translog. In particular, the model presented by Cooper et al. (2003) is attractive as it provides a theoretically acceptable means of restricting the cost function to be globally concave in input prices. The model also permits identification of fixed cost, thus opening the way to a deeper analysis of the underlying causes of subadditivity. The use of the composite cost function by Bloch, Madden and Savage (2001) shows that the Box-Cox transformation might also be helpful.

3. TESTING FOR SUBADDITIVITY

Baumol (1977) defines a network as subadditive if, for $\sum_j q_i = \mathbf{q} \quad j = \{1,2,3,\ldots\}$, the cost of aggregate production is strictly less than the cost of producing the same output separately, i.e.

$$C(\mathbf{q},\mathbf{t},\mathbf{w}) < \sum_j C(q_j, \dot{\mathbf{q}}, \mathbf{t}, \mathbf{w}), \;\; j \notin \dot{\mathbf{q}}\,. \qquad (3.3.1)$$

estimates are correct, then the ancillary analysis provided is accurate. However, the presence of serial correlation means that the researcher cannot know with confidence that all of the parameters are statistically significant. If the estimated cost function includes spurious parameters, the ancillary analysis might be biased.

where $\mathbf{q}$ represents aggregate output, $\mathbf{t}$ denotes technological change variables and $\mathbf{w}$ is a vector of input prices.[7]

Following Evans and Heckman (1984), (3.3.1) can be adapted to compare monopoly joint production of the two outputs against the hypothetical n firm alternative, i.e.

$$C(q_1, q_2, \mathbf{t}, \mathbf{w}) \geq, \leq \sum_n C(\phi_n q_1, \omega_n q_2, \mathbf{t}, \mathbf{w}), \; n = \{1,2,3,\ldots\}, \phi \geq 0, \; \omega \geq 0 \tag{3.3.2}$$

where $\sum_n \phi_n = 1$, $\sum_n \omega_n = 1$, ensuring that the combined output of the n hypothetical firms equals the monopolist output. Since the test is primarily concerned with the efficiency impact of breaking up an incumbent, it is sufficient to restrict $n = 2$.

Evans and Heckman (1984) modify (3.3.2) to ensure that outputs remain within the monopolist's range actually observed. By confining analysis to the observed range, Evans and Heckman's (1984) adaptation is, in effect, a local test for subadditivity. The local test compares the actual monopoly cost to the costs that may realistically be expected to apply in the two-firm case.

In the Evans and Heckman (1984) model, the output level for firm A is determined by

$$q_{1At} = \phi(q_{1t} - 2q_{1M}) + q_{1M}, \tag{3.3.3}$$

$$q_{2At} = \omega(q_{2t} - 2q_{2M}) + q_{2M}, \tag{3.3.4}$$

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[7] Note that $\dot{\mathbf{q}}$ is $\mathbf{q}$ with element q_j removed.

where q_{1M}, q_{2M} denote the minimum observed output levels at time t for outputs q_{1t} and q_{2t}, respectively. The market share parameters ϕ, ω are set according to $0 \le \phi \le 1$ and $0 \le \omega \le 1$. Firm B output levels are then determined according to

$$q_{1Bt} = q_{1t} - q_{1At}, \text{ and} \tag{3.3.5}$$

$$q_{2Bt} = q_{2t} - q_{2At}. \tag{3.3.6}$$

Now define the following costs,

$$C_A(\phi, \omega, \mathbf{q}, \mathbf{t}, \mathbf{w}) = C(q_{1At}, q_{2At}, \mathbf{t}, \mathbf{w}), \text{ and} \tag{3.3.7a}$$

$$C_B(\phi, \omega, \mathbf{q}, \mathbf{t}, \mathbf{w}) = C(q_{1Bt}, q_{2Bt}, \mathbf{t}, \mathbf{w}). \tag{3.3.7b}$$

The degree of subadditivity is measured by

$$SUB_t = \frac{C_{Mon,t}(\mathbf{q}, \mathbf{t}, \mathbf{w}) - (C_{At}(\phi q_{1At}, \omega q_{2At}, \mathbf{t}, \mathbf{w}) + C_{Bt}(q_{1Bt}, q_{2Bt}, \mathbf{t}, \mathbf{w}))}{C_{Mon,t}(\mathbf{q}, \mathbf{t}, \mathbf{w})}. \tag{3.3.8}$$

where $C_{Mon,t}$ is the cost of monopoly output. $SUB < 0$ implies subadditivity for the specific output shares considered between the two firms. $SUB = 0$ defines the point of additivity and implies no difference in cost between monopoly and duopoly. $SUB > 0$ means costs are superadditive and there is a cost advantage to breaking up the monopoly.

Should evidence of subadditivity be found, it is helpful to determine its underlying cause. Baumol et al. (1988) show that subadditivity in the multiple output case may be due, individually or through some combination, of economies of scale and economies of scope. [8] Panzar (1989) shows that there are scale economies if

$$Scale = \frac{C(\mathbf{q},\mathbf{t},\mathbf{w})}{q_1 MC_1(\mathbf{q},\mathbf{t},\mathbf{w}) + q_2 MC_2(\mathbf{q},\mathbf{t},\mathbf{w})} > 1, \tag{3.3.9}$$

while economies of scope occur when the cost of jointly producing q_1 and q_2 is less than the cost of producing the outputs separately. That is,

$$C(\mathbf{q},\mathbf{t},\mathbf{w}) < C(q_1, 0, \mathbf{t},\mathbf{w}) + C(0, q_2, \mathbf{t},\mathbf{w}). \tag{3.3.10}$$

Equation (3.3.9) shows that economies of scale occur if output-weighted marginal cost is less than total cost. By partitioning total cost into fixed and variable components,

$$Scale = \frac{FC(\mathbf{t},\mathbf{w})}{q_1 MC_1(\mathbf{q},\mathbf{t},\mathbf{w}) + q_2 MC_2(\mathbf{q},\mathbf{t},\mathbf{w})} + \frac{VC(\mathbf{q},\mathbf{t},\mathbf{w})}{q_1 MC_1(\mathbf{q},\mathbf{t},\mathbf{w}) + q_2 MC_2(\mathbf{q},\mathbf{t},\mathbf{w})} \tag{3.3.11}$$

where $FC(\mathbf{t},\mathbf{w})$ is fixed cost and $VC(\mathbf{q},\mathbf{t},\mathbf{w})$ is variable cost, it can be seen (in 3.3.11) that economies of scale can arise if fixed cost comprises a sufficiently large portion of cost. However, if marginal cost is an increasing function of output, scale economies due to shared overheads will dissipate as output expands. Setting fixed cost close to zero, it is clear that scale economies occur if variable cost is greater than the sum of output-weighted marginal cost.

One way in which a value of scale greater than one might occur is if there are cost complementarities between outputs. To see this, consider the following two-output

[8] See Panzar (1989) for a more formal analysis of the relationship between subadditivity and economies of scale and scope.

example in which technology and input prices are fixed. A general second-order linear-in-parameters quadratic approximation of variable cost is defined as

$$VC = \alpha_1 q_1 + \alpha_2 q_2 + \tfrac{1}{2}\beta_1 q_1^2 + \tfrac{1}{2}\beta_2 q_2^2 + \gamma_{12} q_1 q_2 \qquad (3.3.12)$$

Cost complementarity exists when

$$\frac{\partial VC}{\partial q_1 \partial q_2} = \gamma_{12} < 0\,. \qquad (3.3.13)$$

Note that in this example,

$$MC_1 = \alpha_1 + \beta_1 q_1 + \gamma_{12} q_2 \qquad (3.3.14)$$

and

$$MC_2 = \alpha_2 + \beta_2 q_2 + \gamma_{12} q_1 \qquad (3.3.15)$$

Consequently, $\frac{\partial MC_1}{\partial q_2} = \gamma_{12}$ and $\frac{\partial MC_2}{\partial q_1} = \gamma_{12}$. With a change in, say q_1, scale economies requires

$$\frac{\partial MC_1}{\partial q_1} q_1 + MC_1 \frac{\partial q_1}{\partial q_1} + \frac{\partial MC_2}{\partial q_1} q_2 + MC_2 \frac{\partial q_2}{\partial q_1} < \frac{\partial VC}{\partial q_1} \qquad (3.3.16)$$

where $\frac{\partial VC}{\partial q_1} = \alpha_1 + \beta_1 q_1 + \gamma_{12} q_2$, $\frac{\partial MC_1}{\partial q_1} = \beta_1$ and $\frac{\partial q_2}{\partial q_1} = 0$. Holding q_2 fixed implies $\left(\frac{\partial MC_2}{\partial q_1}\right) q_2 = (\gamma_{12}) q_2 = \gamma_{12} q_2$. Hence, the inequality in (3.3.16) becomes

$$\beta_1 q_1 + \alpha_1 + \beta_1 q_1 + \gamma_{12} q_2 + \gamma_{12} q_2 < \alpha_1 + \beta_1 q_1 + \gamma_{12} q_2 . \tag{3.3.17}$$

Simplification yields

$$\beta_1 q_1 + \gamma_{12} q_2 < 0 . \tag{3.3.18}$$

Note that if $\gamma_{12} q_2 < 0$ and $\beta_1 q_1 > 0$, the inequality in (3.3.16) holds if $\left|\gamma_{12} q_2\right| > \beta_1 q_1$. In this case, cost complementarity may be sufficient to sustain economies of scale.

Baumol (1977) has shown that economies of scale are neither a necessary nor sufficient condition for subadditivity. In other words subadditivity might result if economies of scope due, for example to shared fixed overhead, are large enough. To assess the impact of this, modify (3.3.10) by partitioning total cost into fixed and variable components such that

$$\begin{aligned} C_M^{FC}(\mathbf{t},\mathbf{w}) + C_M^{VC}(q_1, q_2, \mathbf{t}, \mathbf{w}) < C_A^{FC}(\mathbf{t},\mathbf{w}) + C_A^{VC}(q_1, 0, \mathbf{t}, \mathbf{w}) \\ + C_B^{FC}(\mathbf{t},\mathbf{w}) + C_B^{VC}(0, q_2, \mathbf{t}, \mathbf{w}) \end{aligned} \tag{3.3.19}$$

where $C_M^{FC}(\mathbf{t},\mathbf{w})$ represents monopoly fixed cost, $C_A^{FC}(\mathbf{t},\mathbf{w})$ is firm A's fixed cost and $C_B^{FC}(\mathbf{t},\mathbf{w})$ is firm B's fixed cost. Note that in (3.3.19) firm A and firm B respectively specialise in the production of q_1 and q_2. Hence, the respective fixed costs reflect only those fixed costs necessary to produce each of firm's specialised output.

Subadditivity confined to shared fixed cost suggests that

$$C_M^{FC}(\mathbf{t},\mathbf{w}) < C_A^{FC}(\mathbf{t},\mathbf{w}) + C_B^{FC}(\mathbf{t},\mathbf{w}) . \tag{3.3.20}$$

This might occur when the specialist firms need to duplicate management. However, if the impact of fixed cost on subadditivity dissipates with scale, (3.3.20) suggests that there is a threshold point where fixed costs are additive. Define the point of cost additivity as the minimum viable scale for competition, i.e.

$$C_M\left(q_1^*, q_2^*, \mathbf{t}, \mathbf{w}\right) = C_A\left(\phi q_1^*, \omega q_2^*, \mathbf{t}, \mathbf{w}\right) + C_B\left((1-\phi)q_1^*, (1-\omega)q_2^*, \mathbf{t}, \mathbf{w}\right). \tag{3.3.21}$$

For scale $q_1 < q_1^*$, $q_2 < q_2^*$, monopoly service provision is more efficient than two or more firms. By contrast, when $q_1 > q_1^*$, $q_2 > q_2^*$ efficiency is not sacrificed by introducing competition.

Another possibility, which is closely related to economies of scale, is the concept of economies of size. While the two concepts overlap, distinctions can be made between them. Define increasing economies of scale as the situation in which a 1% equi-proportionate increase in inputs results in more than a 1% increase in output. Alternatively, a 1% equi-proportionate increases in inputs results in a less than 1% increase in total cost. Now define economies of size as occuring when an increase in output leads to a decrease in average total cost. The more general definition of economies of size allows for situations in which inputs cannot increase in fixed proportions. This may arise when the amount of one of the inputs, such as land or buildings, cannot easily or quickly adjust. Another example is an input that can only be increased or decreased in large increments (i.e. it is ‘lumpy’) relative to the other inputs. The amount of the ‘lumpy’ input may adjust in occasional steps rather than in continuous and infinitesimally small increments. Indeed, Morrison Paul (1999) refers to economies of size as biased or restricted scale economies (Morrison Paul, 1999: 6).

There are other examples of economies of size that warrant consideration. These include the ability to reduce input prices as the firm grows. One way this may occur is if the firm receives volume discounts through bulk purchases. Alternatively, the firm may be able to exercise monopsony power. This occurs where the firm becomes so large that it can successfully demand a reduction in input prices irrespective of volume purchased.

4. COST FUNCTION THEORY

A crucial step in testing for subadditivity is estimating the unknown cost function, defined as a function of input prices, output quantities, and a set of exogenous cost shifters. Following Silberberg and Suen (2001), the cost function is derived from the constrained optimisation problem,

$$C = \sum_{i=1}^{n} w_i x_i \tag{3.4.1}$$

subject to

$$\begin{gathered} f_1(x_1, x_2, x_3, \ldots, x_n, t_1) = q_1 \\ f_2(x_1, x_2, x_3, \ldots, x_n, t_2) = q_2 \\ \vdots \\ f_q(x_1, x_2, x_3, \ldots, x_n, t_q) = q_q \end{gathered} \tag{3.4.2}$$

where C is total cost, w_i is the price of input i, x_i is the quantity of input i, and $f_k(x_1, x_2, x_3, \ldots, x_n)$ is a general production function describing the transformation of inputs x_1 to x_n into output quantity q_k, $k = \{1,2,3,\ldots,q\}$. Thus, the firm uses inputs x_1 to x_n and technology t_1 to t_q to produce outputs q_1 to q_q in a way that minimises cost.

In the present context, the quantity of each output, input prices and technology are determined exogenously. Thus, the firm minimises cost by choosing the quantity of inputs given output quantity, input prices and technology. Under these circumstances, a general cost function can be derived by forming the Lagrangian function,

$$L = \sum_{i=1}^{n} w_i x_i + \sum_{k=1}^{q} \lambda_k \left(f_k (x_1, x_2, x_3, \ldots, x_n, t_k) - q_k \right) \tag{3.4.3}$$

where the λ_k are known as the Lagrange multipliers. Each Lagrange multiplier indicates the impact of varying the production constraint $f_k(x_1, x_2, x_3, \ldots, x_n, t_k) - q_k$.

A system of general factor demand equations can be derived, according to Shephard's lemma,

$$\frac{\partial C^*}{\partial w_i} = \frac{\partial L}{\partial w_i} = x_i = x_i^* \left(q_1, q_2, q_3, \ldots, q_q, t_1, t_2, t_3, \ldots, t_q, w_1, w_2, w_3, \ldots, w_n,\right) \tag{3.4.4}$$

where C^* denotes the minimum cost incurred in producing output q_1 to q_q given inputs prices w_1 to w_n and x_i^* denotes the optimal quantity of input i chosen achieve C^*. The corresponding cost function is then determined as,

$$C^* = \sum_{i=1}^{n} w_i x_i^* \left(q_1, q_2, q_3, \ldots, q_q, t_1, t_2, t_3, \ldots, t_q, w_1, w_2, w_3, \ldots, w_n\right) \tag{3.4.5a}$$

or simply

$$C^* = \left(q_1, q_2, q_3, \ldots, q_q, t_1, t_2, t_3, \ldots, t_q, w_1, w_2, w_3, \ldots, w_n\right) \tag{3.4.5b}$$

The envelope theorem implies that

$$\frac{\partial L}{\partial q_k} = \frac{\partial C^*}{\partial q_k} = \lambda_k \tag{3.4.6}$$

Hence, the λ_k is the marginal cost of producing output q_k.

Assuming that the cost function defined in (3.4.5b) is twice differentiable,

$$\frac{\partial^2 C^*}{\partial w_i\, \partial w_j} = \frac{\partial x_i^*}{\partial w_j} = \frac{\partial x_j^*}{\partial w_i} = \frac{\partial^2 C^*}{\partial w_j \partial w_i}. \tag{3.4.7}$$

The relationship in (3.4.7) is known as the symmetry restriction. In order to ensure that the cost function in (3.4.5b) is concave with respect to input prices, the twice continuously differentiable cost function must be negative semi-definite (Chiang, 1984: 347). That is,

$$H = \begin{bmatrix} \frac{\partial^2 C^*}{\partial w_1 \partial w_1} & \cdots & \frac{\partial^2 C^*}{\partial w_1 \partial w_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 C^*}{\partial w_n \partial w_1} & \cdots & \frac{\partial^2 C^*}{\partial w_n \partial w_n} \end{bmatrix} \tag{3.4.8}$$

where the determinants of the Hessian matrix alternate in sign. That is,

$$|H_1| = \frac{\partial^2 C^*}{\partial w_1 \partial w_1} \le 0\,,\ |H_2| = \begin{bmatrix} \frac{\partial^2 C^*}{\partial w_1 \partial w_1} & \frac{\partial^2 C^*}{\partial w_1 \partial w_2} \\ \frac{\partial^2 C^*}{\partial w_2 \partial w_1} & \frac{\partial^2 C^*}{\partial w_2 \partial w_2} \end{bmatrix} \ge 0\,,$$

$$\left|H_3\right| = \begin{bmatrix} \frac{\partial^2 C^*}{\partial w_1 \partial w_1} & \frac{\partial^2 C^*}{\partial w_1 \partial w_2} & \frac{\partial^2 C^*}{\partial w_1 \partial w_3} \\ \frac{\partial^2 C^*}{\partial w_2 \partial w_1} & \frac{\partial^2 C^*}{\partial w_2 \partial w_2} & \frac{\partial^2 C^*}{\partial w_2 \partial w_3} \\ \frac{\partial^2 C^*}{\partial w_3 \partial w_1} & \frac{\partial^2 C^*}{\partial w_3 \partial w_2} & \frac{\partial^2 C^*}{\partial w_3 \partial w_3} \end{bmatrix} \leq 0, \ldots$$

In total, the necessary conditions for a theoretically consistent cost function are (see Varian, 1992: 72):

1. nondecreasing in input prices;
2. homogenous of degree 1 in input prices, implying that the factor demand functions are homogenous of degree 0 in input prices;
3. concave in input prices; and
4. continuous in input prices.

Property 1 means that, holding input quantities constant, a price increase for at least one of the inputs will result in an increase in total cost. Property 2 implies that an equi-proportional price increase in inputs will cause total cost to increase in the same proportion. For example, doubling the price of inputs will cause total cost to double. Concavity (property 3) is a direct outcome of optimising behaviour and implies that an increase in one of the input prices compared to the remaining input prices (i.e. a change in relative prices) will cause cost to increase at a decreasing rate as the firm substitutes away from the relatively more expensive input. Property 4 is mathematically required to ensure constrained optimisation. Properties 1 to 4 provide helpful guidance in construction and estimating the unknown cost function (3.4.5).

The foregoing description can be expressed in a more concise way. Given an $n \times 1$ vector of input prices $\mathbf{w}$, a $q \times 1$ vector of output quantities $\mathbf{q}$, and a $t \times 1$ vector of proxies for technology $\mathbf{t}$ (allowing $t \neq q$), the cost function $C^* = C(\mathbf{q}, \mathbf{t}, \mathbf{w})$ is dual to the production function $\mathbf{q} = f(\mathbf{x}, \mathbf{t})$ where $\mathbf{x}$ is an $n \times 1$ vector of input quantities. The cost function is the outcome of cost-minimising (optimisation) behaviour, which implies theoretical restrictions with respect to input prices. That is, C^* is globally concave, linear homogenous, and nondecreasing in input prices. In addition, the cost function yields positive marginal costs and satisfies Young's theorem (referred to as the symmetry restriction).

Identifying the unknown cost function C^* necessarily implies approximation. Assuming it is twice differentiable with respect to its arguments, Diewert and Wales (1987) show that C^* at the point $(\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*)$ can be approximated to the second order by a function that has a sufficient number of free parameters. That is, a function is deemed to be a flexible approximation of C^* if the following equations can be satisfied (Diewert and Wales, 1987: 45):

$$C\ (\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) = C^*(\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) \tag{3.4.9a}$$

$$\nabla C\ (\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) = \nabla C^*(\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) \text{ and} \tag{3.4.9b}$$

$$\nabla^2 C\ (\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) = \nabla^2 C^*(\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*) \tag{3.4.9c}$$

where ∇C denotes the vector of first-order partial derivatives of C with respect to all of its arguments and $\nabla^2 C$ is the matrix (dimension $(n+q+t) \times (n+q+t)$) of second-order partial derivatives of C with respect to all of its arguments. Ensuring this number of arguments means that the first and second derivatives of the approximate cost function C and the unknown cost function C^* will coincide at the point $\left(\mathbf{q}^*, \mathbf{t}^*, \mathbf{w}^*\right)$ (Diewert and Wales, 1987: 46).

Collectively define the combined set of properties 1 to 4 and the flexible functional form criteria defined in (3.3.9) as the minimum set of criteria (MSC). Diewert and Wales (1987) survey cost functions that satisfy the MSC: the translog, generalised Leontief, the generalised McFadden (also known as the normalised quadratic) and the generalised Barnett. A cost function not surveyed by Diewert and Wales (1987) is known as the quadratic cost function. The quadratic cost function is widely applied in the cost function literature and also satisfies the MSC. In principle, any of these functions may be used to approximate C^*.

Ultimately, however, choice of functional form may be determined by additional criteria to the MSC. In this study, the translog does not permit zero output values and, therefore, cannot be applied. An additional criterion, namely the ability to impose concavity on the cost function without destroying its flexibility properties rules out the quadratic cost function and the generalised Leontief. The symmetric generalised McFadden can accommodate zero output values and can be constrained to satisfy the concavity restriction *a priori* without destroying its flexibility property. However, the symmetric generalised McFadden cost function, as defined by Diewert and Wales (1987), is a single output model and therefore requires modification to the multiple output version, such as the one proposed by Cooper et al. (2003) or Kumbhakar (1994).

A general cost function can be approximated by high-order polynomial functions, such as the Taylor series expansion. For example, many cost functions are assumed to be special cases of the following equation:

$$
\begin{aligned}
\zeta[C(\mathbf{q},\mathbf{t},\mathbf{w})] = \alpha_0 &+ \sum_{g=1}^{G}\alpha_g f(q_g) + \sum_{k=1}^{K}\beta_k f(t_k) + \sum_{i=1}^{n}\beta_i f(w_i) \\
&+ \sum_{g=1}^{G}\sum_{h=1}^{G}\alpha_{gh} f(q_g, q_h) + \sum_{k=1}^{K}\sum_{l=1}^{L}\beta_{kl} f(t_k, t_l) + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} f(w_i, w_j) \\
&+ \sum_{g=1}^{G}\sum_{i=1}^{n}\gamma_{gi} f(q_g, w_i) + \sum_{g=1}^{G}\sum_{k=1}^{K}\gamma_{gt} f(q_g, t_k) + \sum_{i=1}^{n}\sum_{k=1}^{K}\gamma_{ik} f(w_i, t_k) \\
&+ \sum_{g=1}^{G}\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ig} f(q_g, w_i, w_j) + \sum_{g=1}^{G}\sum_{k=1}^{K}\sum_{i=1}^{n}\gamma_{git} f(q_g, t_k, w_i) \\
&+ \sum_{g=1}^{G}\sum_{k=1}^{K}\sum_{i=1}^{n}\gamma_{git} f(q_g, f(t_k), w_i) \\
&+ \sum_{g=1}^{G}\sum_{h=1}^{H}\sum_{i=1}^{n}\gamma_{ig} f(q_g, q_h, w_i)
\end{aligned}
\qquad (3.4.10)
$$

where g,h denote discrete outputs, and i,j denote discrete inputs. The term ζ denotes functional transformations than could be applied to total cost, for example the logarithmic transformaton. The terms α, β and γ represent the unknown parameters of the cost function. Equation (3.4.10) is sufficient to permit parametric tests of separability, nonjointness, homotheticity, homogeneity, symmetry and functional form.

In summary, approximation to the unknown cost function is a matter of specifying a functional form for total cost that satisfies all of the theoretical restrictions of the MSC and has any other properties deemed appropriate for the particular application. Factor demand equations are derived by Shephard's lemma,

$$\frac{\partial C_t}{\partial w_{it}} = x_{it} = f(\mathbf{w}, \mathbf{q}, \mathbf{t}). \tag{3.4.11}$$

The factor demand equations implied by (3.4.11) are then jointly estimated to identify the unknown parameters consistent with the chosen functional form.

5. SYMMETRIC GENERALISED MCFADDEN

The symmetric generalised McFadden (GM) as defined by Diewert and Wales (1987) is a somewhat different formulation to (3.5.11), which is defined by

$$\begin{aligned} C(q,t,\mathbf{w}) = \sum_{i=1}^{n} b_i w_i \; &+ \left(\sum_{i=1}^{n} b_{ii} w_i \; + \frac{1}{2}\frac{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} + \sum_{i=1}^{n} b_{it} w_i \, t + b_{tt} \left(\sum_{i=1}^{n} \gamma_i w_i \right) t^2 \right) q \\ &+ b_t \left(\sum_{i=1}^{n} \alpha_i w_i \right) t + b_{yy} \left(\sum_{i=1}^{n} \beta_i w_i \right)^2 \quad . \end{aligned} \tag{3.5.1}$$

The advantage of (3.5.1) is that it can be rendered globally concave with respect to input prices w_i without destroying its ability to flexibly approximate the unknown cost function. This is a significant advantage over other commonly applied cost functions such as the translog, generalised Leontief, and the quadratic cost function. However, as it stands, the term $\frac{1}{2}\frac{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}$, contains too many parameters and is thus under-identified. To ensure the parameters are exactly identified, Diewert and Wales (1987) select $\mathbf{w}^* >> \mathbf{0}_n$ so that

$$\mathbf{\Sigma}\mathbf{w}^* = \mathbf{0}_n \, . \tag{3.5.2}$$

and $\boldsymbol{\theta}'\mathbf{w}^* > 0$. Note that to achieve the restriction $\boldsymbol{\Sigma}\mathbf{w}^* = \mathbf{0}_n$, Diewert and Wales (1987) set $\mathbf{w}^* = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_n$ and chose arbitrary values of α_i, β_i and γ_i such that $\boldsymbol{\alpha}'\mathbf{w}^* \neq 0$, $\boldsymbol{\beta}'\mathbf{w}^* \neq 0$ and $\boldsymbol{\gamma}'\mathbf{w}^* \neq 0$.

If the restrictions in (3.5.2) are satisfied, then the symmetric GM cost function is a flexible cost function at the point $\mathbf{w}^*$.

A key advantage with the symmetric GM is that cost function concavity can be imposed without destroying its flexibility properties. Concavity is imposed by specifying

$$\boldsymbol{\Sigma} = -\mathbf{A}\mathbf{A}' \tag{3.5.3}$$

where $\mathbf{A}'$ is an upper triangular matrix. Diewert and Wales (1987) recommend setting $\theta_i = \bar{x}_i$.

6. TECHNOLOGICAL CHANGE

As indicated in previous references to technological change, the unobserved production function underlying the unknown cost function is not static. Consequently, equation (3.2) explicitly allows for output to vary with changes in technological progress. This section provides a formal definition of technology and describes how the impact of technological progress is measured within the cost function framework.

In formal terms, the general production function described in equation (3.4.2) is a transformation function relates "…inputs to outputs of production processes when all inputs are efficiently used…" where 'efficiently' means "…that no more than is necessary of any input is used in the production of a given level of output…" (Heathfield and Wibe, 1987: 118). Hence, technological progress implies that the production function is changing. In turn, this implies that the efficient amount of any input used to produce a given output at two distinct points in time is likely to be different.

Heathfield and Wibe (1987) define two categories of technological progress: process and production innovation. Process innovation changes the efficient level of inputs to produce a given level of output. Product innovation yields changes in output that require some adjustment on the part of the consumer.[9] Such product innovation may be observed by the appearance of new attributes of a product or the appearance of an entirely new product.

Process innovation is further categorised as embodied and disembodied technological progress. Disembodied technological progress describes production processes that change the way inputs are combined to produce outputs without any underlying change to the quality of the inputs used. In general, disembodied technological progress can be taken to represent a change in the producing firm's knowledge. Embodied technological progress describes changes in the attributes embodied in production inputs. For example, recognising embodied technological progress in capital implies that a new vintage of capital is more efficient than an existing or older vintage of capital. As Heathfield and Wibe (1987: 119) point out, efficiency of production will depend on both the current state of knowledge and the rate of investment in new machines.

[9] In this thesis, the term 'product' includes services.

Of course, the concept of embodied technological progress may be extended to include labour as changes in capital vintage may require changes in skill. Thus, human capital theory can be included by recognising the level of skilled and unskilled labour and by categorising the level of skill embodied in total labour input.

The reality of technological progress invokes the concept of heterogenous inputs in production. This can be a particularly problematic issue given that expenditure data are often not sufficiently detailed to capture differences in vintages of inputs. Ideally, input quantities and prices should reflect differences in vintages. This implies that a given stock of inputs is the weighted sum of vintages employed in production. That is,

$$x_{it} = \kappa_{1it} x_{1it} + \kappa_{2it} x_{2it} + \kappa_{3it} x_{3it} + \ldots + \kappa_{mit} x_{mit} \tag{3.6.1}$$

where κ_{mit} represents the share of input i of vintage m (where $m = \{1,2,3,\ldots,M\}$) at time t in total aggregate quantity x_{it}. Similarly, aggregate input price should be a weighted sum of the replacement cost of each vintage employed in production,

$$w_{it} = \vartheta_{1it} w_{1it} + \vartheta_{2it} w_{2it} + \vartheta_{3it} w_{3it} + \ldots + \vartheta_{mit} w_{mit} \tag{3.6.2}$$

where ϑ_{mit} is the share of input price w_{mit} of input i of vintage m at time t in total aggregate price w_{it}. In practice, it is sufficient to have either the quantity or price recorded if expenditure by vintage is recorded, since

$$\frac{w_{mit} x_{mit}}{x_{mit}} = w_{mit} \text{ and } \frac{w_{mit} x_{mit}}{w_{mit}} = x_{mit}. \tag{3.6.3}$$

Given expenditure by vintage, the weights can be expenditure shares, i.e.

$$\kappa_{mit} = \vartheta_{mit} = \frac{w_{mit} x_{mit}}{w_{it} x_{it}} \tag{3.6.4}$$

so that the construction of aggregate price and quantity indexes create a constant-quality (or homogenous) production input.

Unfortunately, this level of data is often not available meaning that aggregation of heterogenous inputs will lead to bias in the identified aggregate price or quantity. That is, the identified price or quantity may be greater than or less than the corresponding constant-quality index.

A range of remedies has been suggested in the cost function literature. Heathfield and Wibe (1987) suggest that inputs can be indexed according to the date of employment in production. This suggestion is based on the idea that new vintages are always more efficient than older vintages. An efficiency index can then be specified that controls for changes in the efficiency of different vintages of input. Such an index develops a base or reference vintage and, in effect, converts each vintage of input into the reference base quantity or price. For example,

$$\tilde{x}_{it} = x_{it}^{a} e^{p_i t} \tag{3.6.5}$$

where e is the natural exponent, p_i represents the percent gain in efficiency per time period and x_{it}^{a} denotes the actual quantity of input i employed at time t. Thus, (3.6.5) converts the actual quantity of the input into a standardised quantity of input i. This conversion recognises that observationally equivalent units of input i introduced in two different time periods (e.g. x_{it}^{a} and x_{it-1}^{a}) will, in effect, be different effective quantities. If $p_i > 0$, then $\tilde{x}_{it} = x_{it}^{a} e^{p_i t} > \tilde{x}_{it-1} = x_{it-1}^{a} e^{p_i t-1}$.

Note that conversion of observed quantities of x_{it}^{a} into efficiency (i.e. constant-quality) units x_{it} also implies conversion of actual (or observed) input price w_{it}^{a}. Since expenditure has not changed that the constant-quality price $\tilde{w}_{it} < w_{it}^{a}$. Thus, the effective cost of input i is less than its observed cost, which is consistent with the maintained hypothesis that a change in the share between capital inputs of different qualities is induced by a change in relative prices. Hence

$$\tilde{w}_{it} = w_{it}^{a} e^{-p_i t} \tag{3.6.6}$$

An alternative treatment suggested by Cooper et al. (2003) is

$$\tilde{w}_{it} = w_{it} (1 + p_i)^{t_0 - t}, \tag{3.6.7}$$

where w_{it} denotes the constant-quality price of input i employed in production at time t referenced to base period t_0. These approaches assume that efficiency gain is some smooth function of a constant parameter and time, i.e.

$$\tilde{x}_{it} = f(p_i, t) \text{ or } \tilde{w}_{it} = f(p_i, t). \tag{3.6.8}$$

The equations in (3.6.6) or (3.6.7) can be entered directly into a cost function as arguments to control for biases induced by technical change. In addition, note that there may be variables that are better correlated with efficiency change than a simple time index. In this case, it is preferable to use such variables in place of time.

Entering proxies for technological change in a cost function model suggests that efficiency improvements may have implications for the demand for inputs (Heathfield and Wibe, 1987: 120). For example, increasing efficiency in one input may increase or reduce the demand for other inputs. This suggests that technological change may have specific biases. Indeed, the literature on technological progress as summarised by Heathfield and Wibe (1987) has developed the concepts of Hicks, Harrod and Solow input saving, input-neutral, and input-using technological progress.

Hicks neutrality refers to technological progress that leaves factor ratios unchanged if factor price remains constant. In other words, technological progress is "…input-saving, -using or –neutral depending on whether the marginal rate of substitution increases, decreases, or stays constant with a change in time while holding the capital-labour ratio constant…" (Morrison Paul, 1999: 51). Harrod neutrality refers to technological progress that does not change the proportion of capital used to produce a unit of output if the price of capital does not change. Solow neutrality suggests that technological progress will not change the labour to output ratio if the price of labour remains unchanged. Note that Hicks technological progress encompasses both Harrod and Solow technological progress.

As shown by Morrison Paul (1999: 51), Hicks technological progress bias is measured in the cost function framework by the term

$$B_{it} = \frac{\partial S_i}{\partial t} = S_i \left(\frac{\partial \ln x_{it}}{\partial t} - \frac{\partial \ln C_t}{\partial \ln w_{it}} \right) \qquad (3.6.9)$$

where $S_i = \dfrac{w_{it}x_{it}}{C_t}$, C_t is observed total cost of production at time t. If $B_{it} < 0$, technological progress is input i-saving and decreases total cost at the expense of employing less of input i.

As highlighted by Morrison Paul (1999: 52) the empirical complexity of these bias terms depends on the functional form chosen. In the popular translog cost model, the function reduces to the term

$$\frac{\partial S_i}{\partial t} = p_i t. \tag{3.6.10}$$

Thus, controlling for bias induced by embodied technological progress can be as simple as entering an argument for a time trend in the translog factor demand equations. Alternatively, (3.6.9) can be calculated once a cost function augmented with time arguments has been estimated. For example, the symmetric GM could be augmented with,

$$C(q,t,\mathbf{w}) = \left(\sum_{i=1}^{I} \gamma_i w_i + \frac{1}{2}\frac{\mathbf{w}'\mathbf{S}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} \right) q + \sum_{i=1}^{I} w_i e^{p_i t} \tag{3.6.11a}$$

or, following Nakamura (2001)

$$C(q,t,\mathbf{w}) = \left(\sum_{i=1}^{I} \gamma_i w_i q^{\beta_i} e^{\delta_i t} + \frac{1}{2}\frac{\mathbf{w}'\mathbf{S}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} \right) q e^{At} \tag{3.6.11b}$$

7. DYNAMIC COST FUNCTIONS

Thus far, the theory of cost functions has been presented in an equilibrium setting. Shephard's lemma implies that the level of individual inputs can be instantly adjusted to the levels necessary to meet the prescribed production target at minimum long-run cost. In practice it may be more reasonable to assume that inputs are quasi-fixed, implying the possibility that the observed short-run quantity of inputs used in production may be different to the long-run cost-minimising quantity.

The 'wedge' between short- and long-run cost minimising combinations of inputs is widely considered to be the result of adjustment cost. That is, adjusting inputs in response to exogenous shocks is neither instantaneous or costless. A large and growing literature, surveyed by Hamermesh and Pfann (1996), has expended a considerable amount of intellectual effort in analysing and measuring the magnitude of adjustment. The motivation for a substantial amount of this effort is the need "…to predict the effects of proposed policies or the likely impact of external shocks…" (Hamermesh and Pfann, 1996: 1264).

Hamermesh and Pfann (1996) define two types of adjustment cost referred to as net costs and gross costs. Net costs reflect the cost of changes in the quantity of a specific input across consecutive time periods, such as the number of employees in the firm, while holding other inputs constant. Changes in the level of capital services, induced by changes in the level of utilisation or capital stock, also generate net adjustment cost. Gross costs reflect the difference in the amount of an input's inflows and outflows within the current time period. Gross capital costs are incurred when the purchase and installation of new capital shifts other inputs away from current production. The net change in an input is identically equal to the gross change (Hamermesh and Pfann, 1996: 1266). Hamermesh and Pfann (1996) point out that circumstances such as the lack of secondary markets for capital goods "…means that uncertainty about future shocks makes firms hesitant to purchase new capital, thus creating substantial costs of adjustment attached to changing the stock (Dixit and Pindyck 1994)…" (Hamermesh and Pfann, 1996: 1267). Adjustment cost invariably induces lags in adjusting inputs in response to exogenous shocks and, in turn, necessarily implies specification of dynamic factor demand equations.

Though adjustment costs have typically been modelled as symmetric (so that positive and negative shocks have the same impact on cost) and convex, Hamermesh and Pfann (1996) point out that there is no necessary reason why the marginal cost of increasing an input would be the same as an equivalent decrease. Indeed, Hamermesh and Pfann (1996) survey a growing variety of adjustment cost models that allow asymmetric adjustment cost, such as piecewise linear costs and lumpy adjustment. More complex forms of adjustment cost models surveyed by Hamermesh and Pfann (1996) permit a combination of asymmetry, convexity and lumpiness at the cost of no general solution for the time path of the demand for inputs. Hamermesh and Pfann (1996) conclude their survey by noting that the standard assumption of convex and symmetric adjustment costs are not supported by microeconomic data. Indeed, other functional forms better describe adjustment of individual production inputs. Thus, the approach adopted in this thesis is to permit adjustment to vary across inputs and allow for possibly lumpy adjustment responses to exogenous shocks.

Surveying the literature more closely, it is apparent that there are three classes of dynamic cost functions. One class is the family of short-run or variable cost function (e.g. Schankerman and Nadiri, 1986), which assumes that one or more inputs are costly to adjust (quasi-fixed) and others are free to adjust (variable). Cost is minimised by choosing the optimal level of variable inputs, given the levels of the quasi-fixed inputs.

Another class of dynamic cost functions (e.g. Bernstein, 1989) extends the variable cost function to explicitly capture adjustment costs incurred by changing the level of quasi-fixed inputs. This approach specifies an intertemporal constraint in the form of an Euler equation by setting the cost of a quasi-fixed factor in a given time period to the expected discounted cost reduction that results from having a different level of the factor equal in subsequent periods. The Euler equation is then jointly estimated with the cost function and the variable input demand equations.

A shortcoming of both of the above approaches is that the researcher is required to know which inputs are quasi-fixed and which are variable. Further, Euler equation estimation requires imposition of simplifying assumptions about the appropriate discount rate, expectations about future output and technological change. These assumptions necessarily require the researcher to make judgements that may have a material impact on subsequent inferences.

The third class of dynamic cost function (e.g. Allen and Urga, 1999) allows all inputs to exhibit fixities that incur adjustment cost while preserving the cost-minimisation (optimisation) framework. The researcher is not required to determine *a priori* which variables are quasi-fixed and which are variable. This approach recognises lags in adjustment as an empirical reality, but does not attempt to explicitly explain the time-path to equilibrium. The development of a dynamic modified generalised McFadden (MGM) cost function in this thesis pursues this third approach to dealing with partial adjustment within an optimisation framework.

8. ECONOMETRIC ESTIMATION, SERIAL CORRELATION AND ADJUSTMENT COST

According to Anderson and Blundell (1982), serial correlation is a problem commonly encountered in cost function estimation. This is a major concern as it suggests model misspecification and violates the assumptions underlying classical regression analysis. Indeed, the difficulty encountered in estimating a proper cost function within the subadditivity literature may be a result of model misspecification. Unfortunately, there are a range of reasons why econometric models exhibit serially correlated residuals, such as inappropriate functional form, omitted variables, and mispecified dynamics. Perhaps the most serious situation occurs when there are multiple causes of serial correlation. In this case, cost function estimation becomes a non-linear process of elimination.

Thus, it is prudent to anticipate the likely causes of serial correlation. The issue of functional form is a serious issue that has been extensively analysed in the subadditivity literature and in the more general cost function literature. However, given that the family of so-called flexible functional forms are capable of approximating virtually any true but unknown underlying function to the second-order, the risk of serial correlation due to inappropriate functional form appears to be lower than other likely causes. Indeed, it seems that the greatest risk is violation of cost function concavity and/or the generation of negative marginal cost. Thus, given the possibility of multiple causes of serial correlation, it is prudent to choose a variant of the symmetric generalised McFadden cost function as this provides a valid way of ensuring concavity of the cost function with respect to input prices. The possibility of omitted variables appears unlikely as the relevant variables are provided by the theory of cost functions.

As discussed in Section 4, embodied technical change can lead to deterministic trends in the data, which may be linear or non-linear. Given the rapid rate of technological change documented in Chapter 2, it is prudent to include trend variables in the estimating model. The discussion of adjustment cost in Section 6 motivates the autoregressive specifications. This can be seen by reference to Hendry and Mizon (1978), who show that an autoregressive error structure can be interpreted as a convenient way of specifying dynamic relationships. This imposes a distinction between short-run disequilibrium and long-run equilibrium, which in the context of cost function estimation, seems likely to be due to the existence of adjustment costs.

Given that there is no theoretical guidance with respect to the magnitude of adjustment costs, it is prudent to consider other strategies to deal with adjustment cost. An alternative approach is to estimate the factor demand equations in error-correction form. The general error-correction model is then specified as

$$\Delta \mathbf{x}_t = \boldsymbol{\pi} \mathbf{x}_{t-1} + \sum_{p=1}^{P} \boldsymbol{\alpha}_p \Delta \mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t \tag{3.8.1}$$

where $\mathbf{x}_t = \begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \end{bmatrix}$ is a vector of endogenous variables, $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \end{bmatrix}$ is the vector of regression residuals and $\boldsymbol{\pi}$, $\boldsymbol{\alpha}_p$ are matrices of parameters to be estimated. The elements of the matrix product $\boldsymbol{\pi}\mathbf{x}_{t-1}$ are defined as

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1n} \\ \pi_{21} & \pi_{22} & & \\ \vdots & & \ddots & \\ \pi_{nn} & & & \pi_{nn} \end{bmatrix} \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \\ \vdots \\ x_{3t-1} \end{bmatrix} = \begin{bmatrix} \pi_{11}x_{1t-1} & \pi_{12}x_{2t-1} & \dots & \pi_{1n}x_{nt-1} \\ \pi_{21}x_{1t-1} & \pi_{22}x_{2t-1} & \dots & \pi_{2n}x_{nt-1} \\ \vdots & & \ddots & \\ \pi_{n1}x_{1t-1} & & & \pi_{nn}x_{nt-1} \end{bmatrix} \tag{3.8.2}$$

with each row in the right hand side of (3.8.2) capturing both the long-run relationship between variables and speed of adjustment. The speed of adjustment parameter is common to all of the x_{it-1} variables in each row. Hence, identification of the speed of adjustment parameter is a matter of normalising each row with respect to one of the x_{it-1} variables. For example, taking the first row and normalising with respect to x_{1t-1} yields

$$\pi_{11}\left(x_{1t-1} + \frac{\pi_{12}}{\pi_{11}} x_{2t-1} + \dots + \frac{\pi_{1n}}{\pi_{11}} x_{nt-1} \right). \tag{3.8.3}$$

Note that the term in brackets represents the equilibrium relationship

$$x_{1t-1} = -\frac{\pi_{12}}{\pi_{11}} x_{2t-1} - \dots - \frac{\pi_{1n}}{\pi_{11}} x_{nt-1}. \tag{3.8.4}$$

Referring back to (3.8.3), it is likely that some of the parameters are opposite in sign so that the linear combination represents a difference equation. For example, suppose $\pi_{11} < 0$ and remaining parameters are greater than zero, i.e. $\frac{\pi_{12}}{\pi_{11}} > 0, \ldots, \frac{\pi_{1n}}{\pi_{11}} > 0$. Then

$$\pi_{11}\left(\frac{\pi_{12}}{\pi_{11}} x_{2t-1} + \ldots + \frac{\pi_{1n}}{\pi_{11}} x_{nt-1} - x_{1t-1}\right) \tag{3.8.5}$$

Thus, in this example, when there is a positive deviation from long-run equilibrium, x_{1t-1} decreases and the remaining variables increase until equilibrium is restored.

The error-correction representation provides a strategy that allows estimation of factor demand estimation in first differences instead of levels. This strategy may offer an alternative means of controlling for serial correlation induced by adjustment costs, particularly if the data share common stochastic trends. All that remains is adaptation of the error-correction model to the cost function framework.

The system of equations (3.8.1) treats all of the variables as endogenous. Within the cost function framework, output quantities and input prices are taken to be exogenous. This allows Anderson and Blundell (1982) to adapt (3.8.1) by restricting the number of equations to the endogenous input quantities. However, in doing so, Anderson and Blundell base their long-run relationship on singular demand systems in which the endogenous variables are expenditure shares. In their example, Anderson and Blundell specify the equivalent of

$$\Delta S_{it} = \gamma_i\left(\sum_{i=1}^{n} \beta_i w_{it-1} - S_{t-1}\right) + \sum_{p=1}^{P}\sum_{i=1}^{n} \alpha_p \Delta S_{it-p} + \sum_{p=1}^{P}\sum_{i=1}^{n} \beta_p \Delta w_{it-p} + \varepsilon_{it} \tag{3.8.6}$$

where S_{it} is the expenditure share of variable i and w_{it} is the input price. Since expenditure shares necessarily add to one, estimation requires that one of the cost share equations is dropped from the estimating equations. This creates a problem in identifying the short-run dynamics, as discussed by Anderson and Blundell. Allen and Urga (1999) resolve the identification problem in the translog cost function case by estimating the dynamic cost function jointly with the cost share equations.

Falk and Koebel (2001) note Allen and Urga's (1999) development and adapt Anderson and Blundell's (1982) approach to the non-singular symmetric generalised McFadden factor demand system, which they refer to as the Generalised Error Correction Model (GECM). Falk and Koebel's approach avoids the identification problem of expenditure share systems and offers the additional benefit of being able to impose concavity on the Hessian matrix of second-order derivatives of the cost function with respect to input prices.

The GECM can be written as

$$\Delta \mathbf{x}_t = \mathbf{B}_1\left(\mathbf{x}_t^* - \mathbf{x}_{t-1}^*\right) + \mathbf{B}_2\left(\mathbf{x}_{t-1}^* - \mathbf{x}_{t-1}\right) \tag{3.8.7}$$

where $\mathbf{x}^*$ is a vector of optimal (cost-minimising) input quantities and $\mathbf{B}_1, \mathbf{B}_2$ are matrices of adjustment parameters. For a three-input system (3.8.7) becomes,

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} {}^1b_{11} & {}^1b_{12} & {}^1b_{13} \\ {}^1b_{21} & {}^1b_{22} & {}^1b_{23} \\ {}^1b_{31} & {}^1b_{32} & {}^1b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}^*\right) \\ \left(x_{2t}^* - x_{2t-1}^*\right) \\ \left(x_{3t}^* - x_{3t-1}^*\right) \end{bmatrix} + \begin{bmatrix} {}^2b_{11} & {}^2b_{12} & {}^2b_{13} \\ {}^2b_{21} & {}^2b_{22} & {}^2b_{23} \\ {}^2b_{31} & {}^2b_{32} & {}^2b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}\right) \\ \left(x_{2t}^* - x_{2t-1}\right) \\ \left(x_{3t}^* - x_{3t-1}\right) \end{bmatrix} \tag{3.8.8}$$

In scalar form (3.7.8) is

$$\Delta x_{1t} = {}^1b_{11}\left(x_{1t}^* - x_{1t-1}^*\right) + {}^1b_{12}\left(x_{2t}^* - x_{2t-1}^*\right) + {}^1b_{13}\left(x_{3t}^* - x_{3t-1}^*\right) + {}^2b_{11}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{12}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{13}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.9a}$$

$$\Delta x_{2t} = {}^1b_{21}\left(x_{1t}^* - x_{1t-1}^*\right) + {}^1b_{22}\left(x_{2t}^* - x_{2t-1}^*\right) + {}^1b_{23}\left(x_{3t}^* - x_{3t-1}^*\right) + {}^2b_{21}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{22}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{23}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.9b}$$

$$\Delta x_{3t} = {}^1b_{31}\left(x_{1t}^* - x_{1t-1}^*\right) + {}^1b_{32}\left(x_{2t}^* - x_{2t-1}^*\right) + {}^1b_{33}\left(x_{3t}^* - x_{3t-1}^*\right) + {}^2b_{31}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{32}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{33}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.9c}$$

The GECM nests several models. When $\mathbf{B}_1$ is diagonal, the GECM reduces to the partial ECM,

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} {}^1b_{11} & 0 & 0 \\ 0 & {}^1b_{22} & 0 \\ 0 & 0 & {}^1b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}^*\right) \\ \left(x_{2t}^* - x_{2t-1}^*\right) \\ \left(x_{3t}^* - x_{3t-1}^*\right) \end{bmatrix} + \begin{bmatrix} {}^2b_{11} & {}^2b_{12} & {}^2b_{13} \\ {}^2b_{21} & {}^2b_{22} & {}^2b_{23} \\ {}^2b_{31} & {}^2b_{32} & {}^2b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}\right) \\ \left(x_{2t}^* - x_{2t-1}\right) \\ \left(x_{3t}^* - x_{3t-1}\right) \end{bmatrix} \tag{3.8.10}$$

Which in scalar form is

$$\Delta x_{1t} = {}^1b_{11}\left(x_{1t}^* - x_{1t-1}^*\right) + {}^2b_{11}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{12}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{13}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.11a}$$

$$\Delta x_{2t} = {}^1b_{22}\left(x_{2t}^* - x_{2t-1}^*\right) + {}^2b_{21}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{22}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{23}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.11b}$$

$$\Delta x_{3t} = {}^1b_{33}\left(x_{3t}^* - x_{3t-1}^*\right) + {}^2b_{31}\left(x_{1t-1}^* - x_{1t-1}\right) + {}^2b_{32}\left(x_{2t-1}^* - x_{2t-1}\right) + {}^2b_{33}\left(x_{3t-1}^* - x_{3t-1}\right) \tag{3.8.11c}$$

When both $\mathbf{B}_1$ and $\mathbf{B}_2$ are diagonal (3.7.8) reduces to the simple error correction model

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} {}^1b_{11} & 0 & 0 \\ 0 & {}^1b_{22} & 0 \\ 0 & 0 & {}^1b_{33} \end{bmatrix} \begin{bmatrix} \left(x^*_{1t} - x^*_{1t-1}\right) \\ \left(x^*_{2t} - x^*_{2t-1}\right) \\ \left(x^*_{3t} - x^*_{3t-1}\right) \end{bmatrix} + \begin{bmatrix} {}^2b_{11} & 0 & 0 \\ 0 & {}^2b_{22} & 0 \\ 0 & 0 & {}^2b_{33} \end{bmatrix} \begin{bmatrix} \left(x^*_{1t} - x_{1t-1}\right) \\ \left(x^*_{2t} - x_{2t-1}\right) \\ \left(x^*_{3t} - x_{3t-1}\right) \end{bmatrix} \tag{3.8.12}$$

with corresponding scalar representation

$$\Delta x_{1t} = {}^1b_{11}\left(x^*_{1t} - x^*_{1t-1}\right) + {}^2b_{11}\left(x^*_{1t-1} - x_{1t-1}\right) \tag{3.8.13a}$$

$$\Delta x_{2t} = {}^1b_{22}\left(x^*_{2t} - x^*_{2t-1}\right) + {}^2b_{22}\left(x^*_{2t-1} - x_{2t-1}\right) \tag{3.8.13b}$$

$$\Delta x_{3t} = {}^1b_{33}\left(x^*_{3t} - x^*_{3t-1}\right) + {}^2b_{33}\left(x^*_{3t-1} - x_{3t-1}\right) \tag{3.8.13c}$$

When $\mathbf{B}_1 = \mathbf{I}$ and $\mathbf{B}_2 = \mathbf{I} - \mathbf{R}$ the GECM reduces to the simple autoregressive model

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \left(x^*_{1t} - x^*_{1t-1}\right) \\ \left(x^*_{2t} - x^*_{2t-1}\right) \\ \left(x^*_{3t} - x^*_{3t-1}\right) \end{bmatrix} + \begin{bmatrix} \left(1 - r_{11}\right) & -r_{12} & -r_{12} \\ -r_{21} & \left(1 - r_{22}\right) & -r_{23} \\ -r_{31} & -r_{32} & \left(1 - r_{33}\right) \end{bmatrix} \begin{bmatrix} \left(x^*_{1t} - x_{1t-1}\right) \\ \left(x^*_{2t} - x_{2t-1}\right) \\ \left(x^*_{3t} - x_{3t-1}\right) \end{bmatrix} \tag{3.8.14}$$

In scalar form, (3.7.14) is

$$\begin{aligned} \Delta x_{1t} &= \left(x^*_{1t} - x^*_{1t-1}\right) \\ &\quad + \left(1 - r_{11}\right)\left(x^*_{1t-1} - x_{1t-1}\right) - r_{12}\left(x^*_{2t-1} - x_{2t-1}\right) - r_{13}\left(x^*_{3t-1} - x_{3t-1}\right) \end{aligned} \tag{3.8.15a}$$

$$\begin{aligned} \Delta x_{2t} &= \left(x^*_{2t} - x^*_{2t-1}\right) \\ &\quad - r_{21}\left(x^*_{1t-1} - x_{1t-1}\right) + \left(1 - r_{22}\right)\left(x^*_{2t-1} - x_{2t-1}\right) - r_{23}\left(x^*_{3t-1} - x_{3t-1}\right) \end{aligned} \tag{3.8.15b}$$

$$\begin{aligned} \Delta x_{3t} &= \left(x^*_{3t} - x^*_{3t-1}\right) \\ &\quad - r_{31}\left(x^*_{1t-1} - x_{1t-1}\right) - r_{32}\left(x^*_{2t-1} - x_{2t-1}\right) + \left(1 - r_{33}\right)\left(x^*_{3t-1} - x_{3t-1}\right) \end{aligned} \tag{3.8.15c}$$

Rearranging (3.8.15a) to (3.8.15c),

$$\Delta x_{1t} = x_{1t}^{*} - x_{1t-1}^{*} + (1 - r_{11})x_{1t-1}^{*} - (1 - r_{11})x_{1t-1} - r_{12}\left(x_{2t-1}^{*} - x_{2t-1}\right) - r_{13}\left(x_{3t-1}^{*} - x_{3t-1}\right) \tag{3.8.16a}$$

$$\Delta x_{2t} = x_{2t}^{*} - x_{2t-1}^{*} + (1 - r_{22})x_{2t-1}^{*} - (1 - r_{22})x_{2t-1} - r_{21}\left(x_{1t-1}^{*} - x_{1t-1}\right) - r_{23}\left(x_{3t-1}^{*} - x_{3t-1}\right) \tag{3.8.16b}$$

$$\Delta x_{3t} = x_{3t}^{*} - x_{3t-1}^{*} + (1 - r_{33})x_{3t-1}^{*} - (1 - r_{33})x_{3t-1} - r_{31}\left(x_{1t-1}^{*} - x_{1t-1}\right) - r_{32}\left(x_{2t-1}^{*} - x_{2t-1}\right) \tag{3.8.16c}$$

That is

$$\Delta x_{1t} = x_{1t}^{*} - r_{11}x_{1t-1}^{*} - (1 - r_{11})x_{1t-1} + r_{12}\left(x_{2t-1} - x_{2t-1}^{*}\right) + r_{13}\left(x_{3t-1} - x_{3t-1}^{*}\right) \tag{3.8.17a}$$

$$\Delta x_{2t} = x_{2t}^{*} - r_{22}x_{2t-1}^{*} - (1 - r_{22})x_{2t-1} + r_{21}\left(x_{1t-1} - x_{1t-1}^{*}\right) + r_{23}\left(x_{3t-1} - x_{3t-1}^{*}\right) \tag{3.8.17b}$$

$$\Delta x_{3t} = x_{3t}^{*} - r_{33}x_{3t-1}^{*} - (1 - r_{33})x_{3t-1} + r_{31}\left(x_{1t-1} - x_{1t-1}^{*}\right) + r_{32}\left(x_{2t-1} - x_{2t-1}^{*}\right) \tag{3.8.17c}$$

When $\mathbf{B}_2 = \mathbf{B}_1$ (3.8.8) becomes the partial adjustment model

$$\Delta \mathbf{x} = \mathbf{B}_1\left(\mathbf{x}^{*} - \mathbf{x}_{-1}\right) \tag{3.8.18}$$

That is,

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} {}^1b_{11} & {}^1b_{12} & {}^1b_{13} \\ {}^1b_{21} & {}^1b_{22} & {}^1b_{23} \\ {}^1b_{31} & {}^1b_{32} & {}^1b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}\right) \\ \left(x_{2t}^* - x_{2t-1}\right) \\ \left(x_{3t}^* - x_{3t-1}\right) \end{bmatrix} \tag{3.8.19}$$

In scalar form (3.8.19) is

$$\Delta x_{1t} = {}^1b_{11}\left(x_{1t}^* - x_{1t-1}\right) + {}^1b_{12}\left(x_{2t}^* - x_{2t-1}\right) + {}^1b_{13}\left(x_{3t}^* - x_{3t-1}\right) \tag{3.8.20a}$$

$$\Delta x_{2t} = {}^1b_{21}\left(x_{1t}^* - x_{1t-1}\right) + {}^1b_{22}\left(x_{2t}^* - x_{2t-1}\right) + {}^1b_{23}\left(x_{3t}^* - x_{3t-1}\right) \tag{3.8.20b}$$

$$\Delta x_{3t} = {}^1b_{31}\left(x_{1t}^* - x_{1t-1}\right) + {}^1b_{32}\left(x_{2t}^* - x_{2t-1}\right) + {}^1b_{33}\left(x_{3t}^* - x_{3t-1}\right) \tag{3.8.20c}$$

When $\mathbf{B}_1$ is diagonal, the partial adjustment (3.8.20a) to (3.8.20c) reduces to

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \Delta x_{3t} \end{bmatrix} = \begin{bmatrix} {}^1b_{11} & 0 & 0 \\ 0 & {}^1b_{22} & 0 \\ 0 & 0 & {}^1b_{33} \end{bmatrix} \begin{bmatrix} \left(x_{1t}^* - x_{1t-1}\right) \\ \left(x_{2t}^* - x_{2t-1}\right) \\ \left(x_{3t}^* - x_{3t-1}\right) \end{bmatrix} \tag{3.8.21}$$

In scalar form (3.8.23) is

$$\Delta x_{1t} = {}^1b_{11}\left(x_{1t}^* - x_{1t-1}\right) \tag{3.8.21a}$$

$$\Delta x_{2t} = {}^1b_{22}\left(x_{2t}^* - x_{2t-1}\right) \tag{3.8.21b}$$

$$\Delta x_{3t} = {}^1b_{33}\left(x_{3t}^* - x_{3t-1}\right) \tag{3.8.21c}$$

Thus, the GECM form represents a convenient way of incorporating short-run dynamics in a way that readily allows identification of the long-run cost function.

9. CHAPTER CONCLUSION

This chapter has provided an outline of the literature relating to tests for subadditivity in the telecommunications industry. The literature surveyed in Section 1 indicates that applied subadditivity tests require estimation of a theoretically valid cost function. However, analysis of the literature also reveals that estimating valid cost functions is difficult. Section 2 provides a brief outline of the underlying cost function theory and, since the true cost function is not observed, also discusses the theory of flexible approximation. Remaining sections provide an outline of ancillary issues focused on the difficulties reported in the literature, their underlying causes and established strategies designed to resolve the difficulties. Given the extensive literature on cost functions, this chapter has not sought to provide an exhaustive account of cost function estimation. Instead, discussion has focused on modelling guidelines, which direct the development of the remainder of this thesis.

CHAPTER 4 — MODEL DEVELOPMENT

This chapter builds on Chapter 3 by developing a dynamic version of the multiple output symmetric generalised McFadden cost function. As demonstrated in this chapter, the dynamic model reflects: (i) lags in adjustment, particularly as the telecommunications network expands and (ii) technological progress. Controlling for these simultaneously presents a challenging problem, as lags in adjustment and technological progress impact on the ability to accurately measure the individual effects of changes in input price relativities and changes in output. In addition, applying the Box-Cox transformation function to the output arguments augments the model. The cost function is then converted to a revenue share model as shown by Cooper et al. (2003). It is believed that the combination of the Box-Cox transformation and revenue share version help preserve the maintained hypotheses of stationarity and homoskedasticity.

Section 1 describes the modified generalised McFadden (MGM) cost function based on Kumbhakar’s (1994) specification. Section 2 focuses on controls for technological progress and Section 3 is focused on the Box-Cox transformation, which is applied to the output arguments. Section 4 then specifies a dynamic version of Kumbhakar’s (1994) model and Section 5 provides a specific illustration to show how the Box-Cox transformation, technological change and dynamic specifications combine. Section 6 provides a short description of the MGM in revenue share form and Section 7 outlines the generalised error-correction model in revenue share form as an alternative strategy to the dynamic structure presented in Section 4 and Section 5. Section 8 then presents ancillary analysis derived from the equilibrium specification and Section 9 concludes.

1. THE MODIFIED GENERALISED MCFADDEN COST FUNCTION

Defining $\mathbf{z}$ as an $m \times 1$ vector that includes $\mathbf{q}$ and $\mathbf{t}$ ($m = q + t$), Kumbhakar's (1994) MGM is,

$$C = \boldsymbol{\alpha}'\mathbf{w} + \tfrac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}(\boldsymbol{\beta}'\mathbf{q})^2 + \mathbf{w}'\boldsymbol{\Lambda}\mathbf{z} + \tfrac{1}{2}(\boldsymbol{\theta}'\mathbf{w})\mathbf{z}'\boldsymbol{\Gamma}\mathbf{z} \tag{4.1.1}$$

where the vectors $\boldsymbol{\alpha}$ (with dimension $n \times 1$), $\boldsymbol{\beta}$ ($q \times 1$), and matrices $\boldsymbol{\Sigma}$ (symmetric and negative semi-definite $n \times n$), $\boldsymbol{\Lambda}$ ($n \times m$), $\boldsymbol{\theta}$ ($n \times 1$) and $\boldsymbol{\Gamma}$ ($m \times m$) contain the unknown parameters. All parameters are freely estimated, except for the parameters contained in $\boldsymbol{\theta}$, which are fixed prior to estimation.[10]

Note that Kumbhakar's (1994) model (4.1.1) can be generalised by building in the features suggested by Cooper et al. (2003). That is, allowing substitution possibilities to vary with individual outputs as in,

$$C = \boldsymbol{\alpha}'\mathbf{w} + \tfrac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}_1\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}H(q_1) + \tfrac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}_2\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}H(q_2) + \mathbf{w}'\boldsymbol{\Lambda}\mathbf{z} + \tfrac{1}{2}(\boldsymbol{\theta}'\mathbf{w})\mathbf{z}'\boldsymbol{\Gamma}\mathbf{z}\,. \tag{4.1.2}$$

Reconciling (4.1.1) and (4.1.2), it is clear that (4.1.1) is nested within (4.1.2) if, given two outputs, $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2$ and $H^1(q_1) = H^2(q_2) = \beta_1^2 q_1^2 + 2\beta_1\beta_2 q_1 q_2 + \beta_2^2 q_2^2$, $i \neq j$. The expansion of the substitution matrix $\boldsymbol{\Sigma}$ permits the substitution elasticities to vary with

[10] The parameter vector $\boldsymbol{\theta}$ can be arbitrarily set since the elasticities generated by cost function are invariant to changes in scale. Diewert and Wales (1987) suggest choosing $\theta_i = \bar{x}_i$ where $\bar{x}_i$ is sample average quantity of input i.

changes in individual outputs. This feature may prove particularly useful where the introduction of a new output signifies changes in the underlying technology.

However, note that the effects of technological changes may be independent of output. For example, in the production function case, Diewert and Lawrence (2002, 2005) suggest allowing the expansion of the parameter matrix $\boldsymbol{\Sigma}$ to

$$\boldsymbol{\Sigma} = \left(1 - \frac{t}{T}\right)\boldsymbol{\Sigma}_1 + \frac{t}{T}\boldsymbol{\Sigma}_2 \tag{4.1.3}$$

where $t = \{1, 2, \ldots, T\}$. Indeed (4.1.2) can be generalised further by allowing $\frac{1}{2}\dfrac{\mathbf{w}'\boldsymbol{\Sigma}_m\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}$ to interact with any of the elements of $\mathbf{z}$. In this case (4.1.2) becomes

$$C = \boldsymbol{\alpha}'\mathbf{w} + \frac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}_0\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} + \sum\nolimits_m \frac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}_m\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} z_m + \mathbf{w}'\boldsymbol{\Lambda}\mathbf{z} + \frac{1}{2}(\boldsymbol{\theta}'\mathbf{w})\mathbf{z}'\boldsymbol{\Gamma}\mathbf{z} \tag{4.1.4}$$

This specification adds considerably more parameters, but maximises the flexibility in functional form to allow the substitution matrix to vary with time, outputs and other proxies for changes in technology.

Another attractive feature of these variants of the symmetric generalised McFadden is that concavity in input prices can be imposed without destroying the flexibility of the cost function (Diewert and Wales, 1987: 52-53). This is achieved by setting

$$\boldsymbol{\Sigma} = -\mathbf{A}\mathbf{A}' \tag{4.1.5}$$

where, in the three input case, $\mathbf{A}$ is a lower triangular matrix,

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{4.1.6}$$

Hence,

$$-\mathbf{AA}' = -\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix} \tag{4.1.7}$$

Multiplying (4.1.7) yields

$$-\mathbf{AA}' = -\begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{22}a_{32} \\ a_{11}a_{31} & a_{21}a_{31} + a_{22}a_{32} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix} \tag{4.1.8}$$

For identification purposes, Diewert and Wales (1987) impose the restriction

$$\mathbf{\Sigma w}^* = \mathbf{0} \tag{4.1.9}$$

where $\mathbf{w}^*$ is a positive vector of input prices at an arbitrary point. The restriction in (4.1.9) can be guaranteed by ensuring that the σ_{ij} elements of $\mathbf{\Sigma}$ where

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \tag{4.1.10}$$

are arranged such that

$$\sum_j \sigma_{ij} = 0\,,\ \sigma_{ij} = \sigma_{ji}\,. \tag{4.1.11}$$

That is,

$$\mathbf{\Sigma w}^* = \begin{bmatrix} \sigma_{11} & \sigma_{12} & -(\sigma_{11}+\sigma_{12}) \\ \sigma_{12} & \sigma_{22} & -(\sigma_{12}+\sigma_{22}) \\ \sigma_{13} & \sigma_{23} & -(\sigma_{13}+\sigma_{23}) \end{bmatrix} \begin{bmatrix} w_1^* \\ w_2^* \\ w_3^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.1.12}$$

where $w_1^* = w_2^* = w_3^* = 1$. Multiplying (4.1.12) yields

$$\mathbf{\Sigma w}^* = \begin{bmatrix} \sigma_{11}w_1^* + \sigma_{12}w_2^* - (\sigma_{11}+\sigma_{12})w_3^* \\ \sigma_{12}w_1^* + \sigma_{22}w_2^* - (\sigma_{12}+\sigma_{22})w_3^* \\ -(\sigma_{11}+\sigma_{12})w_1^* - (\sigma_{12}+\sigma_{22})w_2^* + (\sigma_{11}+2\sigma_{12}+\sigma_{22})w_3^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.1.13}$$

Imposing concavity and the identification restriction simultaneously requires that

$$\begin{aligned} &\begin{bmatrix} \sigma_{11} & \sigma_{12} & -(\sigma_{11}+\sigma_{12}) \\ \sigma_{12} & \sigma_{22} & -(\sigma_{12}+\sigma_{22}) \\ -(\sigma_{11}+\sigma_{12}) & -(\sigma_{12}+\sigma_{22}) & \sigma_{11}+2\sigma_{12}+\sigma_{22} \end{bmatrix} = \\ &\quad -\begin{bmatrix} a_{11}^2 & a_{11}a_{21} & -(a_{11}^2 + a_{11}a_{21}) \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & -(a_{11}a_{21} + a_{21}^2 + a_{22}^2) \\ -(a_{11}^2 + a_{11}a_{21}) & -(a_{11}a_{21} + a_{21}^2 + a_{22}^2) & a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2 \end{bmatrix} \end{aligned} \tag{4.1.14}$$

2. OTHER CONTROLS FOR TECHNOLOGICAL CHANGE

Allowing substitution possibilities with changes in output composition, output volume, time or some other proxy may be necessary, but insufficient to yield sensible cost function estimates. Indeed, given technological change, it may be that the demand for individual factors of production changes as technology changes. Such autonomous changes occur since new vintages of capital employed in production embody attributes that older vintages of capital do not possess. Consequently changes in the capability of underlying inputs are reflected in autonomous shifts in demand for individual inputs.

Kumbhakar's (1994) model (4.1.1) captures autonomous shifts in demand induced by both embodied and disembodied technical change in the terms: $\mathbf{w}'\boldsymbol{\Lambda}\mathbf{z} + \frac{1}{2}(\boldsymbol{\theta}'\mathbf{w})\mathbf{z}'\boldsymbol{\Gamma}\mathbf{z}$. By Shephard's lemma, this implies that (using (4.1.1) as an example) the demand for input i is,

$$x_i = \alpha_i + \frac{\partial}{\partial w_i}\left(\frac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}}(\boldsymbol{\beta}'\mathbf{q})^2\right) + \boldsymbol{\Lambda}\mathbf{z} + \frac{1}{2}(\boldsymbol{\theta}')\mathbf{z}'\boldsymbol{\Gamma}\mathbf{z}. \qquad (4.2.1)$$

In assessing the impact of technological change, one solution is to specify functions that transform actual (or observed) input quantities or prices into constant-quality variables. However, Power (1978) provides the number of subscribers by exchange technology, which can be included in $\mathbf{z}$ as autonomous demand shifters.

3. BOX-COX TRANSFORMATION OF THE OUTPUT ARGUMENTS

The MGM embodies the assumption that the constraint implied by the first-order and second-order output arguments can be adequately approximated by a quadratic function that is linear in parameters. Indeed, Box and Cox (1964) suggest that it is less restrictive to assume that the true cost-output relationship is linear in parameters after a suitable transformation.[11] This avoids inducing heteroskedasticity and/or serial correlation in the regression errors by imposing an inappropriate functional form. In this study, Box and Cox's suggested transformation translates to

[11] Changes in underlying output composition may introduce changes in the aggregate cost-output relationship.

$$q_t = \begin{cases} \dfrac{q_t^{\lambda} - 1}{\lambda} & (\lambda \neq 0) \\ \log q_t & (\lambda = 0) \end{cases} \tag{4.3.1}$$

where λ is a parameter whose value may be fixed prior to estimation or included as one of the parameters to be estimated. In either case, the conditions for a particular estimate for λ to be deemed appropriate for the system of equations implied by (4.1.1) are that (1) the relevant statistical tests fail to reject the null hypotheses of homoskedasticity and no serial correlation and (2) the implied marginal costs are non-negative.[12]

4. THE DYNAMIC MGM COST FUNCTION

Development of the dynamic MGM cost function begins by recognising that observed total cost (C_t) consists of the long-run equilibrium for the cost-minimising combination of inputs (C_t^*) and additional total adjustment cost (C_t^A). Hence, total cost is defined as

$$C_t = C_t^* + C_t^A. \tag{4.4.1}$$

By definition, observed cost is equal to the sum of input expenditures,

$$C_t = w_{1t}x_{1t} + w_{2t}x_{2t} + w_{3t}x_{3t} \tag{4.4.2}$$

and by Shephard's lemma, long-run equilibrium cost is defined as

$$C_t^* = C_t^*(\mathbf{w}, \mathbf{q}, t). \tag{4.4.3}$$

---

[12] The Box-Cox transformation could, in principle, be applied to all variables.

All that remains is to define C_t^A. Following Hendry and Mizon (1978), consider the general demand equation

$$x_{it} = x_{it}^* + u_{it} \tag{4.4.4}$$

where $u_{it} = \rho_{ii} u_{it-1} + v_{it}$, where v_{it} has zero mean, a constant variance σ^2 and is serially independent. This first-order autoregressive error structure is commonly encountered in empirical demand studies (e.g. Berndt and Christensen, 1974; Deaton and Muellbauer, 1980). The error structure implies

$$u_{it} = \frac{v_{it}}{1 - \rho_{ii} L} \tag{4.4.5}$$

where L denotes the lag operator. Substituting (4.4.5) for u_{it} in (4.4.4),

$$x_{it} = x_{it}^* + \frac{v_{it}}{1 - \rho_{ii} L} \tag{4.4.6}$$

Multiply (4.4.6) by $1 - \rho_{ii} L$,

$$(1 - \rho_{ii} L) x_{it} = (1 - \rho_{ii} L) x_{it}^* + v_{it} \tag{4.4.7}$$

Simplifying and rearranging (4.4.7) yields

$$x_{it} = x_{it}^* + \rho_{ii} \left(x_{it-1} - x_{it-1}^* \right) + v_{it} \tag{4.4.8}$$

Hence, it is clear that the autoregressive error structure implies a partial adjustment model in which $x_{it-1} - x_{it-1}^* > 0$ suggests that the previous period's total demand for input

i is in excess of the previous period's long-run (equilibrium) demand for input i. The excess demand likely reflects an additional component required to adjust inputs. Hence $\rho_{ii}\left(x_{it-1} - x_{it-1}^{*}\right) + v_{it}$ indicates the additional component of input i required in the current period.

The implications for the long-run demand for input i depends on whether $\left|\rho_{ii}\right| < 1$, $\left|\rho_{ii}\right| = 1$ or $\left|\rho_{ii}\right| > 1$. In any case $u_{it} = \dfrac{v_{it}}{1 - \rho_{ii} L}$ implies the infinite series

$$u_{it} = v_{it} + \rho_{ii} v_{it-1} + \rho_{ii}^{2} v_{it-2} + \rho_{ii}^{3} v_{it-3} + \ldots \qquad (4.4.9)$$

Given $\left|\rho_{ii}\right| < 1$, the impact of a shock in period t will geometrically decline over time. If there are no new exogenous shocks, so that $\sum_{p=1}^{\infty} v_{it+p} = 0$, then demand for input i in excess of the equilibrium demand will follow a geometrically declining time path. Over time, $x_{it-1} - x_{it-1}^{*}$ approaches zero and the observed demand for input i will be equal to the long-run or cost-minimising demand. Thus, the $\sum_{p=1}^{\infty} \rho_{ii}^{p}\left(x_{it-p} - x_{it-p}^{*}\right)$ captures the total quantity of input i employed to achieve the long-run quantity.

If $\left|\rho_{ii}\right| = 1$ then $u_{it} = \dfrac{v_{it}}{1 - \rho_{ii} L}$ implies

$$u_{it} = v_{it} + v_{it-1} + v_{it-2} + v_{it-3} + \ldots . \qquad (4.4.10)$$

Accordingly, equation (4.4.10) states that past shocks never die out. In turn, this implies $\sum_{p=1}^{\infty}\left(x_{it-p} - x^{*}_{it-p}\right)$ and if there are no new shocks in v_{it}, the demand for x_{it} will remain at its current level. Finally, if $\left|\rho_{ii}\right| > 1$ then $u_{it} = \dfrac{v_{it}}{1-\rho_{ii}L}$ implies

$$u_{it} = \rho_{ii}u_{it-1} + v_{it} \tag{4.4.11}$$

With $u_{it-1} = \rho_{ii}u_{it-2} + v_{it-1}$ this implies

$$u_{it} = \rho_{ii}^{2}u_{it-2} + v_{it} + \rho_{ii}v_{it-1} \tag{4.4.12}$$

Continuous iteration yields

$$u_{it} = \sum_{p=0}^{\infty}\rho_{ii}^{p}v_{it-p} \tag{4.4.13}$$

With $\left|\rho_{ii}\right| > 1$, this implies a divergent series in which the impact of past shocks increases through time.

Now suppose that an exogenous increase in demand for another input j induces adjustment in input i. This implies a system of two demand equations

$$u_{it} = \rho_{ii}u_{it-1} + \rho_{ij}u_{jt-1} + v_{it} \tag{4.4.14a}$$

$$u_{jt} = \rho_{jj}u_{jt-1} + v_{jt} \tag{4.4.14b}$$

Rearranging (4.4.14) yields

$$u_{it} = \frac{\rho_{ij} u_{jt-1}}{1-\rho_{ii} L} + \frac{v_{it}}{1-\rho_{ii} L} \tag{4.4.15a}$$

$$u_{jt} = \frac{v_{jt}}{1-\rho_{jj} L} \tag{4.4.15b}$$

That is

$$u_{it} = \frac{\rho_{ij}}{1-\rho_{ii} L} \frac{v_{jt}}{1-\rho_{jj} L} + \frac{v_{it}}{1-\rho_{ii} L} \tag{4.4.16a}$$

$$u_{jt} = \frac{v_{jt}}{1-\rho_{jj} L} \tag{4.4.16b}$$

Hence the structural equations are

$$x_{it} = x_{it}^{*} + \frac{\rho_{ij}}{1-\rho_{ii} L} \frac{v_{jt}}{1-\rho_{jj} L} + \frac{v_{it}}{1-\rho_{ii} L} \tag{4.4.17a}$$

$$x_{jt} = x_{jt}^{*} + \frac{v_{jt}}{1-\rho_{jj} L} \tag{4.4.17b}$$

Multiply (4.4.17a) by $(1-\rho_{ii} L)$

$$(1-\rho_{ii} L)x_{it} = (1-\rho_{ii} L)x_{it}^{*} + \frac{\rho_{ij} v_{jt}}{1-\rho_{jj} L} + v_{it} \tag{4.4.18}$$

That is,

$$x_{it} = x_{it}^{*} + \rho_{ii}\left(x_{it-1} - x_{it-1}^{*}\right) + \frac{\rho_{ij} v_{jt}}{1-\rho_{jj} L} + v_{it} \tag{4.4.19}$$

Now multiply (4.4.19) by $(1-\rho_{jj}L)$

$$(1-\rho_{jj}L)x_{it} = (1-\rho_{jj}L)x_{it}^{*} + (1-\rho_{jj}L)\rho_{ii}\left(x_{it-1} - x_{it-1}^{*}\right) + \rho_{ij}v_{jt} + (1-\rho_{jj}L)v_{it} \qquad (4.4.20)$$

That is

$$\begin{aligned}\left(x_{it} - \rho_{jj}Lx_{it}\right) = \left(x_{it}^{*} - \rho_{jj}Lx_{it}^{*}\right) + \left(\rho_{ii}\left(x_{it-1} - x_{it-1}^{*}\right) - \rho_{jj}L\rho_{ii}\left(x_{it-1} - x_{it-1}^{*}\right)\right) \\ + \rho_{ij}v_{jt} + \left(v_{it} - \rho_{jj}Lv_{it}\right)\end{aligned} \qquad (4.4.21)$$

Simplify (4.4.21) to yield,

$$x_{it} = x_{it}^{*} + \left(\rho_{ii} + \rho_{jj}\right)\left(x_{it-1} - x_{it-1}^{*}\right) - \rho_{ii}\rho_{jj}\left(x_{it-2} - x_{it-2}^{*}\right) + v_{it} - \rho_{jj}v_{it-1} + \rho_{ij}v_{jt} \qquad (4.4.22)$$

Equation (4.4.22) shows that in the short run, the demand for input i in excess of the equilibrium demand is a combination of disequilibrium (excess) demand for input i over the previous two periods plus exogenous demand for inputs i and j. Since, in equilibrium $x_{it-1} = x_{it-1}^{*}$, $x_{it-2} = x_{it-2}^{*}$ and $E(v_{it}) = E(v_{it-1}) = E(v_{jt}) = 0$, equilibrium demand is again $x_{it} = x_{it}^{*}$. Note that the system can be further generalised to accommodate any number of cross-equation adjustment interactions.

For example, define the three input demand system as

$$x_{1t} = x_{1t}^{*} + u_{1t} \qquad (4.4.23a)$$

$$x_{2t} = x_{2t}^{*} + u_{2t} \qquad (4.4.23b)$$

$$x_{3t} = x_{3t}^{*} + u_{3t} \qquad (4.4.23c)$$

where

$$u_{1t} = \rho_{11}u_{1t-1} + \rho_{12}u_{2t-1} + \rho_{13}u_{3t-1} + v_{1t} \tag{4.4.24a}$$

$$u_{2t} = \rho_{21}u_{1t-1} + \rho_{22}u_{2t-1} + \rho_{23}u_{3t-1} + v_{2t} \tag{4.4.24b}$$

$$u_{3t} = \rho_{31}u_{1t-1} + \rho_{32}u_{2t-1} + \rho_{33}u_{3t-1} + v_{3t} \tag{4.4.24c}$$

The autoregressive term can be rearranged to,

$$x_{1t} = x_{1t}^{*} + \rho_{11}\left(x_{1t-1} - x_{1t-1}^{*}\right) + \rho_{12}\left(x_{2t-1} - x_{2t-1}^{*}\right) + \rho_{13}\left(x_{3t-1} - x_{3t-1}^{*}\right) + v_{1t}\,, \tag{4.4.25a}$$

$$x_{2t} = x_{2t}^{*} + \rho_{21}\left(x_{1t-1} - x_{1t-1}^{*}\right) + \rho_{22}\left(x_{2t-1} - x_{2t-1}^{*}\right) + \rho_{23}\left(x_{3t-1} - x_{3t-1}^{*}\right) + v_{2t}\,, \tag{4.4.25b}$$

$$x_{3t} = x_{3t}^{*} + \rho_{31}\left(x_{1t-1} - x_{1t-1}^{*}\right) + \rho_{32}\left(x_{2t-1} - x_{2t-1}^{*}\right) + \rho_{33}\left(x_{3t-1} - x_{3t-1}^{*}\right) + v_{3t}\,. \tag{4.4.25c}$$

The convenient aspect of the foregoing dynamic specification is that it permits a parsimonious estimation of the equilibrium demand functions directly with autoregressive error terms. This approach allows the data to reveal the adjustment mechanism, rather than imposing restrictions prior to estimation.

It is clear that the demand for input i is comprised of a structural equilibrium component x_{it}^{*} and a composite adjustment component u_{it}. The corresponding expenditure on input i is thus,

$$w_{it}x_{it} = w_{it}x_{it}^{*} + w_{it}u_{it} \tag{4.4.26}$$

Since the adjustment component u_{it} is comprised of exogenous (pre-determined) variables, the expenditure component $w_{it}u_{it}$ represents the portion of 'fixed' expenditure associated with adjusting input i. In turn, total observed cost is

$$C_t = \sum_3 w_{it}x_{it}^* + \sum_3 w_{it}u_{it} \tag{4.4.27}$$

The component $\sum_3 w_{it}u_{it}$ is the portion of fixed cost associated with total input adjustment.

5. THE DYNAMIC MGM FACTOR DEMAND EQUATIONS

The dynamic MGM cost function is derived by simply augmenting (4.1.1) with autoregressive errors. For example,

$$\begin{aligned}
C(\mathbf{q},t,\mathbf{w}) = a_1 w_{1t} + a_2 w_{2t} + a_3 w_{3t} + g(\mathbf{q},\mathbf{w}) \\
&+c_{11} f(q_{1t})w_{1t} + c_{12} f(q_{1t})w_{2t} + c_{13} f(q_{1t})w_{3t} \\
&+c_{21} f(q_{2t})w_{1t} + c_{22} f(q_{2t})w_{2t} + c_{23} f(q_{2t})w_{3t} \\
&+c_{1t} w_{1t} tech + c_{2t} w_{2t} tech + c_{3t} w_{3t} tech \\
&+\frac{1}{2}(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)\left(d_{11} f(q_{1t})^2 + 2d_{12} f(q_{1t}) f(q_{2t}) + d_{22} f(q_{2t})^2\right) \\
&+\frac{1}{2}(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)(e_1 f(q_{1t}) + e_2 f(q_{2t}))tech \\
&+\frac{1}{2}(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3) f_1 tech^2 \\
&+w_{1t}u_{1t} + w_{2t}u_{2t} + w_{3t}u_{3t}
\end{aligned} \tag{4.5.1}$$

where

$$g(\mathbf{q},\mathbf{w}) = \frac{-\frac{1}{2}\begin{pmatrix} a_{11}^2 w_{1t}^2 + 2a_{11}a_{21}w_{1t}w_{2t} - 2\left(a_{11}^2 + a_{11}a_{21}\right)w_{1t}w_{3t} \\ + \left(a_{21}^2 + a_{22}^2\right)w_{2t}^2 - 2\left(a_{11}a_{21} + a_{21}^2 + a_{22}^2\right)w_{2t}w_{3t} \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_{3t}^2 \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)},$$
$$\times\left(b_{11} f\left(q_{1t}\right) + b_{22} f\left(q_{2t}\right)\right)^2$$

$$f\left(q_{jt}\right) = \begin{cases} \dfrac{q_{jt}{}^{\lambda_j} - 1}{\lambda_j} & \left(\lambda_j \neq 0\right) \\ \log q_{jt} & \left(\lambda_j = 0\right) \end{cases}, \quad j = \{1,2\} \text{ and}$$

$$u_{1t} = \rho_{11}u_{1t-1} + \rho_{12}u_{2t-1} + \rho_{13}u_{3t-1} + v_{1t}$$

$$u_{2t} = \rho_{21}u_{1t-1} + \rho_{22}u_{2t-1} + \rho_{23}u_{3t-1} + v_{2t}$$

$$u_{3t} = \rho_{31}u_{1t-1} + \rho_{32}u_{2t-1} + \rho_{33}u_{3t-1} + v_{3t}$$

The corresponding factor-demand equation for input 1 is

$$\begin{aligned} \frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial w_{1t}} = x_{1t} &= a_1 + \frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_{1t}} + c_{11} f\left(q_{1t}\right) + c_{21} f\left(q_{2t}\right) + c_{1t} tech \\ &+ \frac{\theta_1}{2}\left(d_{11} f\left(q_{1t}\right)^2 + 2d_{12} f\left(q_{1t}\right) f\left(q_{2t}\right) + d_{22} f\left(q_{2t}\right)^2\right) \\ &+ \frac{\theta_1}{2}\left(e_1 f\left(q_{1t}\right) + e_2 f\left(q_{2t}\right)\right) tech + \frac{\theta_1}{2} f_1 tech^2 \\ &+ \rho_{11}u_{1t-1} + \rho_{12}u_{2t-1} + \rho_{13}u_{3t-1} + v_{1t} \end{aligned} \qquad (4.5.2)$$

where

$$\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_1}=\left(\begin{array}{l}\dfrac{-\left(a_{11}^2 w_{1t}+a_{11}a_{21}w_{2t}-\left(a_{11}^2+a_{11}a_{21}\right)w_{3t}\right)}{\left(w_1\theta_1+w_2\theta_2+w_3\theta_3\right)}\\ +\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11}^2 w_{1t}^2+a_{11}a_{21}w_{1t}w_{2t}-\left(a_{11}^2+a_{11}a_{21}\right)w_{1t}w_{3t}\\ +\frac{1}{2}\left(a_{21}^2+a_{22}^2\right)w_{2t}^2-\left(a_{11}a_{21}+a_{21}^2+a_{22}^2\right)w_{2t}w_{3t}\\ +\frac{1}{2}(a_{11}^2+2a_{11}a_{21}+a_{21}^2+a_{22}^2)w_{3t}^2\end{array}\right)\theta_1}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right)\times\left(b_{11}f\left(q_{1t}\right)+b_{22}f\left(q_{2t}\right)\right)^2$$

The factor-demand equation for input 2 is

$$\begin{aligned}\frac{\partial C(\mathbf{q},t,\mathbf{w})}{\partial w_{2t}}=x_{2t}=a_2+\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_{2t}}&+c_{12}f\left(q_{1t}\right)+c_{22}f\left(q_{2t}\right)+c_{2t}tech\\ &+\frac{\theta_2}{2}\left(d_{11}f\left(q_{1t}\right)^2+2d_{12}f\left(q_{1t}\right)f\left(q_{2t}\right)+d_{22}f\left(q_{2t}\right)^2\right)\\ &+\frac{\theta_2}{2}\left(e_1 f\left(q_{1t}\right)+e_2 f\left(q_{2t}\right)\right)tech+\frac{\theta_2}{2}f_1 tech^2\\ &+\rho_{21}u_{1t-1}+\rho_{22}u_{2t-1}+\rho_{23}u_{3t-1}+v_{2t}\end{aligned}\tag{4.5.3}$$

where

$$\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_{2t}}=\left(\begin{array}{l}\dfrac{-\left(a_{11}a_{21}w_{1t}+\left(a_{21}^2+a_{22}^2\right)w_{2t}-\left(a_{11}a_{21}+a_{21}^2+a_{22}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}\\ +\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11}^2 w_{1t}^2+a_{11}a_{21}w_{1t}w_{2t}-\left(a_{11}^2+a_{11}a_{21}\right)w_{1t}w_{3t}\\ +\frac{1}{2}\left(a_{21}^2+a_{22}^2\right)w_{2t}^2-\left(a_{11}a_{21}+a_{21}^2+a_{22}^2\right)w_{2t}w_{3t}\\ +\frac{1}{2}(a_{11}^2+2a_{11}a_{21}+a_{21}^2+a_{22}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right)\times\left(b_{11}f\left(q_{1t}\right)+b_{22}f\left(q_{2t}\right)\right)^2$$

The factor-demand equation for input 3 is

$$
\begin{aligned}
\frac{\partial C(\mathbf{q},t,\mathbf{w})}{\partial w_{3t}} = x_{3t} &= a_3 + \frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_{3t}} + c_{13} f(q_{1t}) + c_{23} f(q_{2t}) + c_{3t} tech \\
&+ \frac{\theta_3}{2}\left(d_{11} f(q_{1t})^2 + 2d_{12} f(q_{1t}) f(q_{2t}) + d_{22} f(q_{2t})^2\right) \\
&+ \frac{\theta_3}{2}\left(e_1 f(q_{1t}) + e_2 f(q_{2t})\right) tech + \frac{\theta_3}{2} f_1 tech^2 \\
&+ \rho_{31} u_{1t-1} + \rho_{32} u_{2t-1} + \rho_{33} u_{3t-1} + v_{3t}
\end{aligned} \tag{4.5.4}
$$

where

$$
\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_3} = \left(
\begin{array}{l}
-\dfrac{\begin{pmatrix} -\left(a_{11}^2 + a_{11}a_{21}\right) w_{1t} - \left(a_{11}a_{21} + a_{21}^2 + a_{22}^2\right) w_{2t} \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2) w_{3t} \end{pmatrix}}{\left(w_{1t}\bar{x}_1 + w_{2t}\bar{x}_2 + w_{3t}\bar{x}_3\right)} \\
+\dfrac{\begin{pmatrix} \frac{1}{2} a_{11}^2 w_{1t}^2 + a_{11}a_{21} w_{1t} w_{2t} - \left(a_{11}^2 + a_{11}a_{21}\right) w_{1t} w_{3t} \\ + \frac{1}{2}\left(a_{21}^2 + a_{22}^2\right) w_{2t}^2 - \left(a_{11}a_{21} + a_{21}^2 + a_{22}^2\right) w_{2t} w_{3t} \\ + \frac{1}{2}(a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2) w_{3t}^2 \end{pmatrix} \theta_3}{\left(w_{1t}\bar{x}_1 + w_{2t}\bar{x}_2 + w_{3t}\bar{x}_3\right)^2}
\end{array}
\right) \times \left(b_{11} f(q_{1t}) + b_{22} f(q_{2t})\right)^2
$$

Note that, since the cost function in (4.5.1) contains the same information as the factor-demand equations, it is sufficient to estimate (4.5.2), (4.5.3) and (4.5.4) without (4.5.1).

6. REVENUE-SHARE FORM

Cooper et al. (2003) advise that, to reduce the effects of time trends in estimation the factor-demand equations, it may be prudent to estimate the system of equations in revenue-share form. In addition, estimation in share form may assist with maintaining the null hypothesis of homoskedasticity. Defining factor inputs as revenue shares simply requires that

$$S_{it} = \frac{w_{it} x_{it}}{R_t} \tag{4.6.1}$$

where R_t denotes total revenue. This implies that (4.2.1) is converted to

$$S_{it} = \frac{w_{it}}{R_t}\left(\alpha_i + \frac{\partial}{\partial w_{it}}\left(\tfrac{1}{2}\frac{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}{\boldsymbol{\theta}'\mathbf{w}} H(\mathbf{q}_t)\right) + \boldsymbol{\Lambda}'\mathbf{z}_t + \tfrac{1}{2}\theta_i \mathbf{z}_t'\boldsymbol{\Gamma}\mathbf{z}_t\right) \tag{4.6.2}$$

where $H(\mathbf{q}_t)$ is a general function of output quantities.

As pointed out by Cooper et al. (2003), a fourth implied revenue-share equation can be identified, but not estimated, by the adding up identity. The residual revenue-share equation is

$$S_{\Pr ofit} = 1 - \sum_i S_{it} \tag{4.6.3}$$

Note that, in contrast to the above revenue share form, the more widely used translog cost function uses cost shares as dependent variables. Using cost shares necessitates the double logarithmic transformation. While feasible, the degenerate behaviour of the double logarithm as cost shares approach zero makes this strategy unattractive. An alternative is to simply replace total revenue with total cost in (4.6.1) and (4.6.2). However, this amounts to a mispecification since total cost (which is endogenous) would appear on the exogenous (right hand side) of (4.6.2).

7. ESTIMATION IN ERROR-CORRECTION FORM

As discussed in Chapter 3, the uncertainty with respect to the appropriate dynamic specification suggests that it is prudent to also consider estimation in error-correction form. Assuming the model is estimated in revenue shares, the generalised error-correction model will be in the form of

$$\Delta\mathbf{S}_t = \mathbf{B}_1\left(\mathbf{S}_t^* - \mathbf{S}_{t-1}^*\right) + \mathbf{B}_2\left(\mathbf{S}_{t-1}^* - \mathbf{S}_{t-1}\right) \tag{4.7.1}$$

where the elements of $\mathbf{S}_t = \begin{bmatrix} S_{1t} \\ S_{2t} \\ S_{3t} \end{bmatrix}$ are the actual revenue shares and $\mathbf{S}_t^* = \begin{bmatrix} S_{1t}^* \\ S_{2t}^* \\ S_{3t}^* \end{bmatrix}$ with the S_{it}^* elements corresponding to the right hand side of (4.6.2).

8. ANCILLARY ANALYSIS OF THE EQUILIBRIUM MGM COST FUNCTION

A cost function that satisfies the MSC will yield non-negative marginal costs and should also imply positive fixed costs that are less than total cost. Marginal costs are defined as the first-derivative of total cost with respect to outputs q_1 and q_2, i.e.,

$$\begin{aligned} MC_1 &= \frac{\partial C(\mathbf{q}, t, \mathbf{w})}{\partial q_1} \\ &= \frac{\partial g(\mathbf{q}, \mathbf{w})}{\partial q_1} + c_{11} f_1(q_{1t}) f(q_{1t}) w_{1t} \\ &\quad + c_{12} f_1(q_{1t}) f(q_{1t}) w_{2t} + c_{13} f_1(q_{1t}) f(q_{1t}) w_{3t} \\ &\quad + (w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)\begin{pmatrix} d_{11} f_1(q_{1t}) f(q_{1t}) \\ + 2d_{12} f_1(q_{1t}) f(q_{1t}) f(q_{2t}) \end{pmatrix} \\ &\quad + \frac{1}{2}(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)(e_1 f_1(q_{1t}) f(q_{1t}))tech \end{aligned} \tag{4.8.1}$$

where

$$\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial q_1} = -2b_{11} f_1(q_{1t})(b_{11} f(q_{1t}) + b_{22} f(q_{2t}))$$
$$\times \frac{\begin{pmatrix} a_{11}^2 w_1^2 + 2a_{11}a_{21}w_1w_2 - 2(a_{11}^2 + a_{11}a_{21})w_1w_3 \\ + (a_{21}^2 + a_{22}^2)w_2^2 - 2(a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_2w_3 \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_3^2 \end{pmatrix}}{(w_1\theta_1 + w_2\theta_2 + w_3\theta_3)}$$

and

$$f_1(q_{jt}) = \frac{\partial f(q_{1t})}{\partial q_{1t}}.$$

$$\begin{aligned} MC_2 &= \frac{\partial C(\mathbf{q},t,\mathbf{w})}{\partial q_2} \\ &= \frac{\partial g(\mathbf{q},\mathbf{w})}{\partial q_2} + c_{21} f_2(q_{2t}) f(q_{2t}) w_{1t} + c_{22} f_2(q_{2t}) f(q_{2t}) w_{2t} + c_{23} f_2(q_{2t}) f(q_{2t}) w_{3t} \\ &\quad + (w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)(d_{12} f(q_{1t}) f_2(q_{2t}) f(q_{2t}) + d_{22} f_2(q_{2t}) f(q_{2t})) \\ &\quad + \frac{1}{2}(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)(e_2 f_2(q_{2t}) f(q_{2t}))tech \end{aligned} \tag{4.8.2}$$

where

$$\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial q_2} = -2b_{22} f_2(q_{jt})(b_{11} f(q_{1t}) + b_{22} f(q_{2t}))$$
$$\times \frac{\begin{pmatrix} a_{11}^2 w_1^2 + 2a_{11}a_{21}w_1w_2 - 2(a_{11}^2 + a_{11}a_{21})w_1w_3 \\ + (a_{21}^2 + a_{22}^2)w_2^2 - 2(a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_2w_3 \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_3^2 \end{pmatrix}}{(w_1\theta_1 + w_2\theta_2 + w_3\theta_3)}$$

and

$$f_2(q_{jt}) = \frac{\partial f(q_{2t})}{\partial q_{2t}}.$$

Finally, the fixed costs are

$$\begin{aligned} C_t^{FC} &= a_1 w_{1t} + a_2 w_{2t} + a_3 w_{3t} + c_{1t} w_{1t} tech + c_{2t} w_{2t} tech + c_{3t} w_{3t} tech \\ &\quad + \frac{1}{2}\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right) f_1 tech^2 + w_{1t}u_{1t} + w_{2t}u_{2t} + w_{3t}u_{3t} \end{aligned} \tag{4.8.3}$$

Fixed cost is therefore a function of input prices, technology and the given input stocks. The technology arguments, as noted by Cooper et al., (2003) provide the effective quality-adjusted factor prices. Further, note that the lagged stocks of inputs 1, 2, and 3 form a component of fixed cost and thus, indicate that fixed cost is proportional to the given stocks of the inputs.

9. CHAPTER CONCLUSION

This chapter outlines the modelling strategy adopted in this thesis based on the received literature as presented in Chapter 3. Section 1 presents a variant of the MGM cost function and discusses measures designed to control for technological progress via input substitution possibilities. Section 2 then provides additional measures for controlling embodied technological progress and Section 3 provides a brief discussion of the Box-Cox transformation applied to the output arguments. Section 4 discusses the issue of dynamic specification at length, which is motivated by the likelihood of substantial adjustment cost. Section 5 provides an illustrative example. Sections 6 and 7 discuss ancillary specification strategies and Section 8 presents remaining ancillary measures required to check the theoretical validity of the equilibrium specification implied by the dynamic MGM specification.

With the modelling strategy now outlined, Chapter 5 presents a detailed description of the required data.

CHAPTER 5—DATA AND VARIABLE CONSTRUCTION

This chapter is concerned with the data used to estimate the factor demand equations outlined in Chapter 4. The detail provided in this chapter provides the context for the econometric analysis presented in Chapter 4 and goes some way in explaining the form of the preferred econometric model. The remainder of this chapter is as follows. Section 1 provides a summary of the source material and explains the methods employed to construct each of the required variables. Section 2 provides the resulting summary statistics and Section 3 describes variable transformation processes. Section 4 concludes.

1. VARIABLE CONSTRUCTION

Data required for econometric cost function estimation are total revenue, total cost, input price and quantity for capital, labour and materials and output. Primary data sources are Postmaster General (PMG) annual reports 1920 to 1975, Telecom annual reports 1976 to 1990, *Telecom Service and Business Outlook* 1981 to 1987, and the PMG *Financial and Statistical Bulletin* 1956 to 1973. Secondary sources include the Australian Bureau of Statistics, Public Service Board and Public Service Commissioner's annual reports 1922 to 1976, the International Telecommunication Union, along with a variety of published books and papers.

The annually reported financial statements, i.e. the profit and loss and balance sheet statements, provide the basis for developing consistently measured expenditure series for the three main inputs: capital, labour and materials. The primary advantage of using the financial statements to separately identify expenditure by input is that the financial statements are consistently and separately reported for the PMG's postal and telecommunications divisions. A secondary advantage is that these financial statements are reported on an accruals basis throughout the entire history of the PMG and Telecom. This means that expenditure is recorded in the time period in which it is incurred.[13] However, a key challenge in using the financial statements is that the profit and loss statements do not report expenditure by input. Thus, the profit and loss statements need to be deconstructed and reassembled into labour and materials.

The Postmaster General's Department provides both telecommunications and postal services for the years 1901 to 1975. Throughout this period, the telecommunications and postal services divisions reported separate financial statements, forming the basis for analysing the telecommunications division separately from the postal division. In 1975, the Postal and Telecommunications divisions were separated to create Australia Post and Telecom. Consequently, post-1975 source statistics are sourced from Telecom annual reports.

1.1 Capital expenditure

Capital expenditure for telecommunications is reported on the balance sheet (called the Detailed Statement of Fixed Assets in PMG annual reports). These statements consistently and conveniently report gross annual investment in buildings, communications plant, motor vehicles and other plant and equipment. In principle, a net

[13] By contrast, expenditure by input type (e.g. capital, materials, labour) reported in the statistical annexes and appendices are reported on a cash basis. Cash accounting records the time period in which the inputs are paid for, rather than used. From an economic analysis perspective accrual accounting is conceptually superior to cash accounting. Aside from this difference, expenditure by input type is often reported on a consolidated basis, which includes both the postal and telecommunications divisions. This obviously makes separating telecommunications inputs from post inputs impossible.

annual investment series could be created for each type of capital input by deducting depreciation and adjustments for changed service lives. However, analysis of accounting policy changes over the sample period indicates a consistent and substantial understatement of the depreciation expense. This is offset by periodic and substantial adjustments to changed service lives. Consequently, jumps occur in the estimated capital stock series that reflect inconsistencies in the accounting treatment of assets rather than changes in the quantity of the underlying asset. Therefore, the capital stock calculations in this thesis apply a constant asset-specific depreciation rate to each asset category after deflating annual investment.

1.2 Labour expenditure, staff numbers and average wages

Labour expenditure is the sum of salaries and wages, annual pension liability and long-service leave provision. PMG salaries, wages expenditure and staff statistics are reported in aggregate across postal and telecommunications division in both the PMG Statistical Appendix contained in annual reports and in the *Financial and Statistical Bulletin.* Pension and long-service leave expenditure are reported according to division in the divisional (Postal, Telephone and Telegraph) profit and loss and balance sheet statements. Notes to the financial accounts report the amount of pension and long-service leave expensed on the profit and loss statement and the amount charged to the fixed assets account. The amount charged to the profit and loss statement is associated with staff required to operate the telecommunications network as a going concern, while the amounts charged to fixed assets is associated with staff employed in the construction of the telecommunications network. The notes to the financial statements also make clear that pension and long-service leave provisions are calculated as a proportion of salaries and wages expenditure. This provides the basis for separately identifying operational and construction staff.

The annual average wage for the years 1922 to 1975 is calculated from aggregate statistics reported in Commonwealth Public Service Board annual reports. Salaries reported elsewhere provide a single annual expenditure for the postal and telecommunications divisions, but do not include construction staff salaries expenditure before the year 1969.[14] Total salaries and wages expenditure published in Public Service Board annual reports include basic salary and wages, extra duty and overtime payments for both operational and construction staff. The average wage is therefore calculated by dividing total salaries expenditure by total aggregate staff employed as reported by the Public Service Board annual reports.

Separating the postal and telecommunications division salaries and wages expenditure proceeds as follows. The separate pension provisions for operational staff for the postal and telecommunications divisions are summed to create the total annual pension provision for PMG operational staff. Total annual pension provision is then calculated by adding the pension provision for operational staff to the provision for construction staff. The total annual pension provision is divided by the total salaries and wages expenditure to establish the ratio of pension provision to total salaries and wages. Total salaries and wages for telecommunications operational staff is identified by dividing the pension provision for telecommunications operational staff by the total pension to total PMG salaries and wages expenditure ratio. Similarly, total salaries and wages for PMG

[14] The equivalence of salaries and wages expenditure for the years 1969 to 1975 reported in the Public Service Board annual report and the PMG *Financial and Statistical Bulletin* is confirmed through comparing the amounts contained in the two reports.

construction staff is identified by dividing the pension provision for construction staff by the total pension to total PMG salaries and wages expenditure ratio. Dividing telecommunications salaries and wages expenditure by the PMG average wage identifies the number of telecommunications operational staff.[15] Post 1975, the average wage is calculated by dividing total salaries and wages by total staff as reported in the Telecom annual statistical appendix.

After calculating the average wage, total telecommunications labour expenditure is calculated by adding pension and long-service leave provisions for telecommunications operational staff to operational salaries and wages. Labour expenditure for construction staff is omitted to avoid double counting as the construction labour is contained in the annual fixed assets investment expenditure.

1.3 Materials expenditure

Total materials expenditure is comprised of the amount expended on the profit and loss statement and the component charged to the asset account. PMG annual reports identify annual cash expenditure for materials from 1969, leaving the period 1920 to 1968 unidentified.[16] As with the labour component, total reported materials do not distinguish between the expensed and capitalised components. The alternative is to identify materials by deducting labour expenditure for operational staff from total expenses (as

[15] This approach assumes that the average wages for the postal and the telecommunications divisions are the same. The processes by which public service wages are determined is based on education, training, experience and responsibility. Centralised wage determinations by the Public Service Board prevented wages competition between government departments and divisions. This means that, in principle, recruitment competition between divisions and departments did not materialise in the form of bidding up wages offered to staff. Consequently, the assumption that the average wage is the same across postal and telecommunications divisions is deemed reasonable.

[16] Reported materials expenditure includes both the postal and telecommunications divisions for the years 1969 to 1975, inclusive.

reported on the profit and loss statement produces) less depreciation and interest expense. This residual materials series is an aggregate component, which includes expenditure on fuels, energy and materials such as office stationery, advertising and purchases of external services.

1.4 Capital price deflators and stocks

Statistical annexes and appendices contained in PMG and Telecom annual reports provide capital stock records for buildings, communications plant and motor vehicles. Buildings are reported in total and by type, e.g. number of post offices, exchange buildings, etc. Mileage (kilometres after 1969) of conduit and cable and number of exchanges provide capital stocks for communications plant. Total number of motor vehicles is separately reported for the engineering and postal divisions for the years 1955 to 1976. Telecom reports continue these capital stock records after 1975. There is no consistently measured capital stock for other plant and equipment.

Capital price deflators for buildings, motor vehicles, and other plant and equipment are obtained from the Australian Bureau of Statistics (ABS). The buildings price deflator is the ABS non-dwelling construction price series (ABS code: NUQA.PD_NDC_PC_GJ), which spans the years 1920 to 1990. Electronic equipment price (ABS codes: NUQA.PD_ELE_PC_GJ), motor vehicles (ABS code: NUQA.PD_RVH_PC_GJ) and other plant (ABS code: NUQA.PD_OPL_PC_GJ) price series span the years 1949 to 1990. All series obtained from the ABS correspond to Commonwealth Public Trading Enterprises, of which the PMG comprises a substantial portion.

Electronic equipment, other plant and road vehicles are spliced to capital price series constructed from PMG data. Electronic equipment price before 1949 is an aggregate series based on cable and total telephone and telegraph service plant expenditure, reported in the Detailed Statement of Fixed Assets, PMG annual reports. These series are divided by the annual additions to total cable length (kilometres) and annual additions to the number of exchanges. Using the Törnqvist method, the resulting aggregate price series and electronic equipment price are spliced to produce a combined log-change series that consists of the PMG capital price series for the years 1920 to 1948, simple average of both series for 1949 to 1962 and electronic equipment for 1963 to 1990. The spliced log-change series is converted to index form by calculating the cumulative sum after setting the base year, 1920, equal to one.

A motor vehicles price series for the years 1920 to 1965 is obtained based on source series from the US Bureau of Economic Analysis (BEA code: i2ncm011et20 and BEA code: i3ncm011et20). These data are investment expenditure series in motor vehicles by the US Telecommunications and Telegraph industry at historical and fixed cost (1996$), respectively. A new car price series is created by dividing the historical cost by the fixed price series. The new car price series is then spliced with the ABS motor vehicles series.[17] Although mass production of cars began in Australia after World War II, it is evident that Australia had a substantial pre-war motor vehicle assembly industry. These data provide value and quantity of imported motor bodies and chassis, which are used to create a composite Australian motor vehicles import price index. The Australian motor

[17] The reasonableness of this approach is tested against empirical and historical information. Pre-World War II manufacturing data are collated from Commonwealth Bureau of Statistics yearbooks for the years 1927 to 1938.

vehicle import price index is then compared with the US car price index. The series compare favourably, with pair-wise correlation of 0.23 in log-change form for the years 1927 to 1939. Secondly, anecdotal evidence sourced from archival material and PMG annual reports indicates that pre-World War II vehicles were US made (e.g. Model 'T' Ford postal wages and Chevrolet trucks). In the interests of consistency, it is the US car price series for the telecommunications industry that is spliced with the Australian series, as this is likely to better reflect the composition of automobiles appropriate for the telecommunications industry.

The pre-1949 price deflator for other plant and equipment series is developed from the value and capacity (in horsepower) of plant and machinery statistics by industry published in Commonwealth Bureau of the Census yearbooks. Selected industries include furniture making, metal works and machinery, and the heat, light and power industry. Price series for machines and equipment for each industry is obtained by dividing the value of plant and machines by the total effective horsepower. These prices are aggregated using the Törnqvist method and spliced with the other plant and equipment price series obtained from the ABS. Aggregating motor vehicles and other plant and equipment then creates an aggregate price series called machinery and equipment. Finally, since materials expenditure is the residual after deducting depreciation, interest and labour expenses, there is no single price deflator available. Instead, the materials price series is obtained by dividing total annual materials expense by communications plant stock.[18]

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[18] Since, it is reasoned, that required materials should be approximately proportional to the size of the capital stock. Note that Wilson and Zhou (1997) follow a similar strategy.

All capital stocks (buildings, communications plant, and machinery and equipment) are calculated by the perpetual inventory method. Buildings stock is obtained by using the non-dwelling construction price index to deflate annual building expenditure and cumulating using a constant depreciation rate of 2.04% (the depreciation rate is the inverse of service life estimates published in Chapter 16 of the ABS *Australian National Accounts: Concepts, Sources and Methods*). The initial stock is the reported number of telephone exchange buildings. Communications plant stock uses an annual depreciation rate of 6.62% and the initial capital stock is estimated by dividing the total value of the network by the aggregate price where total value is the price of cable (as at 1920) multiplied by total cable kilometres plus the 1920 price of exchange equipment multiplied by the total number of exchanges. The depreciation rate for machinery and equipment is 5.78% and the initial stock is deflated expenditure for 1920 divided by the depreciation rate.

1.5 Rental Price of Capital

Given price deflators by capital type, the rental price (user cost) is the product of the respective capital price deflators and the sum of the capital specific depreciation rate and the bond yield for 10-year Australian government bonds. Bond yields for the years 1920 to 1973 are obtained from Butlin (1977), with later years obtained from the Reserve Bank of Australia.

1.6 Total cost

Total cost is the sum of labour and materials expenses, and capital services. The resulting total cost is higher than the series reported in PMG and Telecom annual reports, which is mainly due to applying current replacement prices to the entire capital stock, whereas the reported annual total cost values capital services as the sum of depreciation and interest expense (based on historical book value). Interest expense in the annual reports corresponds to capital financed from borrowings, with an implied zero required rate of return applied to capital stock financed internally. Finally, before 1960 the PMG applied a zero depreciation rate to large portions of the capital stock.

1.7 Outputs

The quantity of output is comprised of both subscription and use, ranging across voice, text and data services, and are as reported in source material. Voice consists of the number of fixed-line and mobile telephone subscribers, as well as the volume of local, trunk and mobile phone (cell phone) calls.[19] Text is the number of telegrams together with the number of telex subscribers and telex calls. Data is the total number of subscribers for Datel, Digital Data Service and Austpac services. Revenue consists of fixed-line voice revenue (for calls and subscription), mobile phone revenue (for calls, accounts (subscription fee) and connections), telegraph revenue; telex revenue (for calls and subscription); revenue for other telecommunications (which corresponds to Digital Data Service and Austpac services) and residual revenue not elsewhere reported. Datel revenue is included in residual revenue not elsewhere reported.

1.8 Technical change

Technical change is reflected predominately in the type of services provided throughout the sample period. However, within wireline telephone additional technical change variables are provided. For wireline telephone, technical change is conceptually divided

[19] Use of the term mobile phone is used to describe wireless telephone services such as cellular telephone and its predecessor.

into transmission and switching. Technical change is captured within transmission by the total kilometres of both coaxial and optic fibre cable. Switching is defined by exchange technology, namely manual, step-by-step, crossbar (ARF, ARK and ARE exchanges), and digital (AXE) exchange technology.

2. SUMMARY STATISTICS

2.1 Inputs—capital expenditure

Capital expenditure is comprised of buildings, communications plant, and machinery and equipment. As shown in Figure 5.1, communication plant is the largest expenditure item for throughout the sample period followed by buildings. Machinery and equipment, which is comprised of other plant, equipment and vehicles expenditure represents a relatively small portion of total expenditure. Figure 5.2, which provides a detailed breakdown of other plant and equipment, is largely comprised of engineers' movable plant and workshop plant and equipment. Miscellaneous plant (Misc. Plant in Figure 5.2) comprises electrical generation plant, mechanical aids, research and development equipment, ADP machines and other unspecified expenditure items. The share of miscellaneous plant began to increase from the mid-1960s. A conclusive explanation for the increase in expenditure share is not possible because a breakdown of miscellaneous plant is not provided for every year. However, the PMG Financial and Statistical Bulletin 1967-1976 shows the quantity of: accounting machines; adding and listing machines; cash accounting machines, copying machines; data processing; dictating/recording machines; typewriters; and other miscellaneous machines. Most of these items increase over the 1967-1976 period. The Financial and Statistical Bulletin also shows the number of computers. It is likely that the increased share of miscellaneous plant is due to increases in these items.

Figure 5.1. Components of capital expenditure

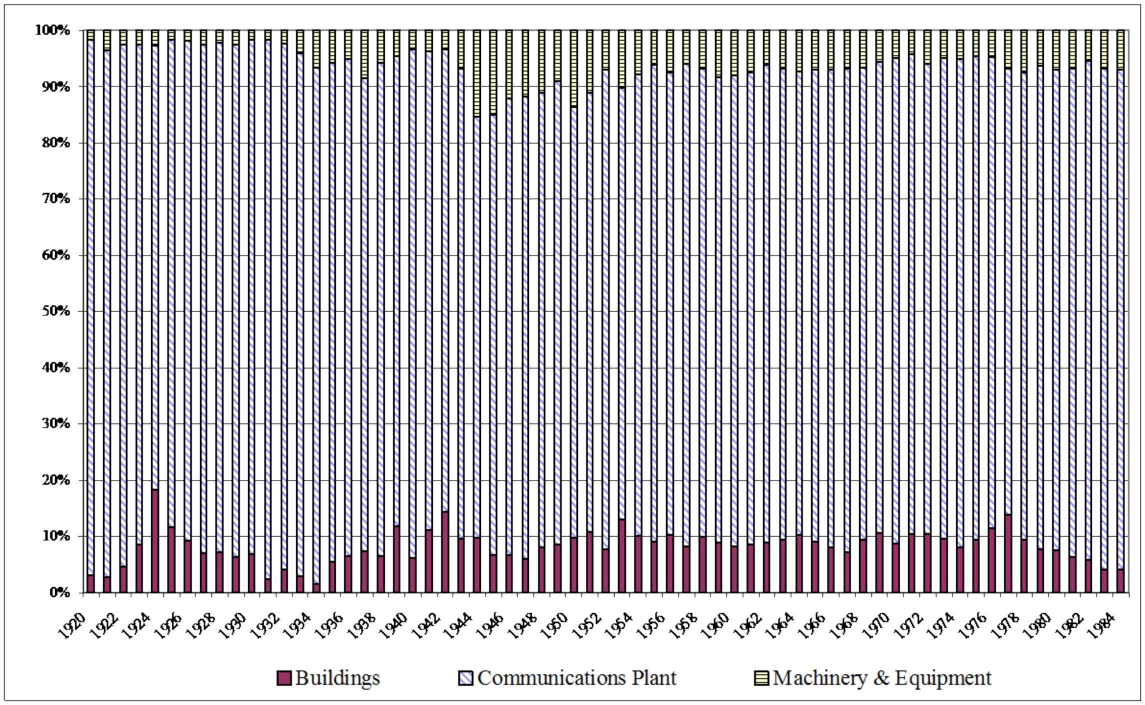
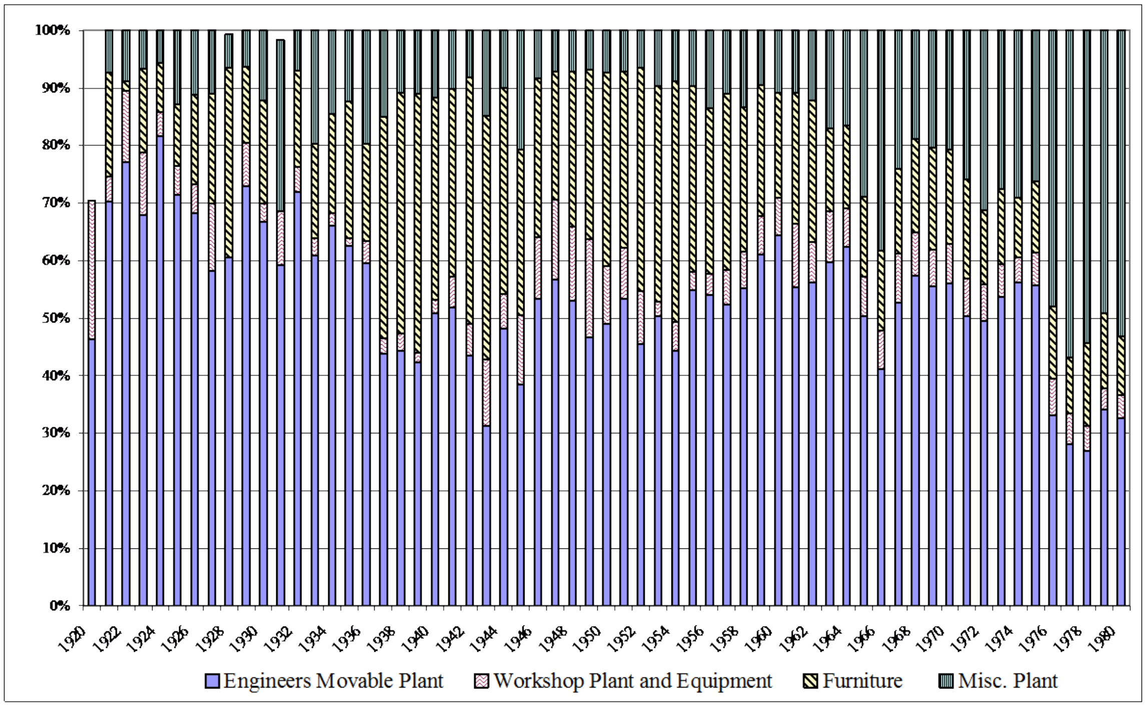


Figure 5.2. Component shares of other plant and equipment



2.2 Inputs—Operating cost and input composition

Table 5.1 presents summary statistics for total cost, labour expenditure and materials and Figure 5.3 shows cost shares for aggregate inputs. Total cost (telecommunications) is the sum of buildings, communication plant, labour, machinery and equipment, and materials expenditure. Casual inspection reveals that input cost shares exhibit considerable variation.

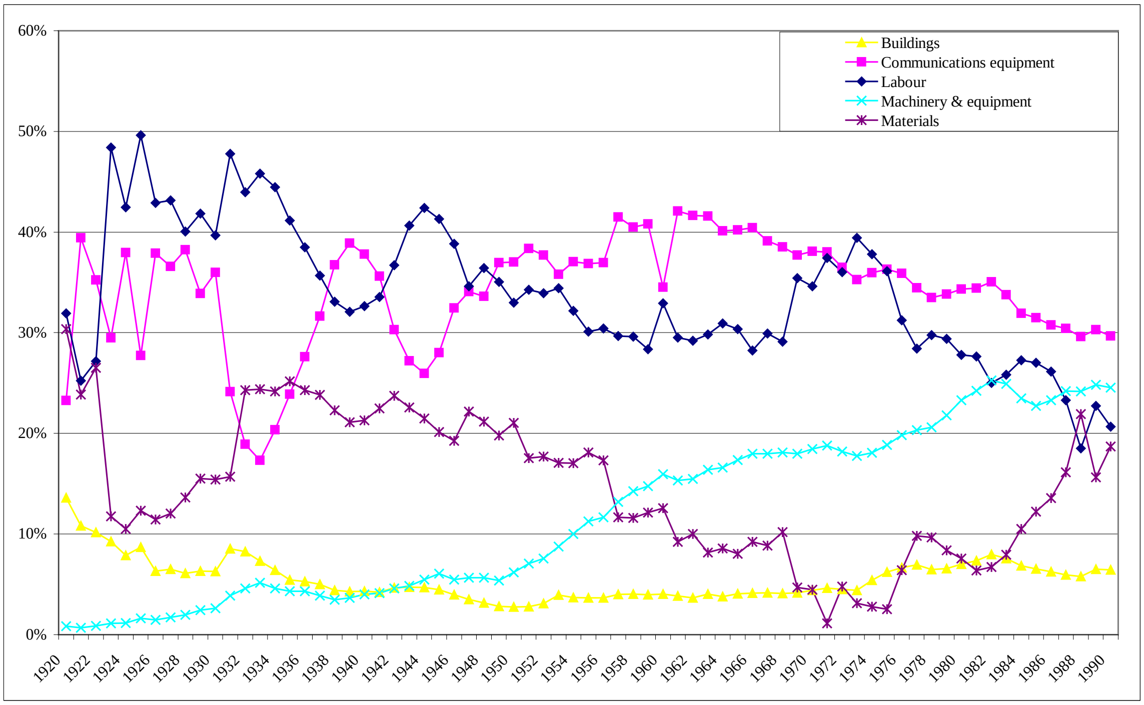
Buildings exhibit a downward trend in cost share until 1962 and then exhibit an upward trend thereafter. Machinery and equipment expenditure shares exhibit a strong upward trend. Communications plant and labour expenditure exhibit horizontal oscillation before 1947, thereafter exhibiting relatively stable decreases. Communications plant registers its minimum share in 1932, while the minimum for labour occurs in 1988.

Machinery and equipment increases monotonically and more rapidly than buildings. Aggregating the capital shares reveals relatively stronger trends with the capital share oscillating until 1944 and then exhibiting a positive trend, albeit slowing, for the remainder of the sample period. Materials expenditure represents 30% of total cost in 1920 with a trough evident between 1922 and 1932 where expenditure drops as low as 10% of total expenditure. Subsequent years exhibit a downward trend, with the minimum share occurring in 1971. After 1971, materials expenditure share increases sharply. Labour and materials appear negatively correlated for the 1922-1933 and 1968 to 1990 sub-periods and positively correlated for the intermediate period.

Table 5.1. Total cost, expenditure shares, prices and stocks

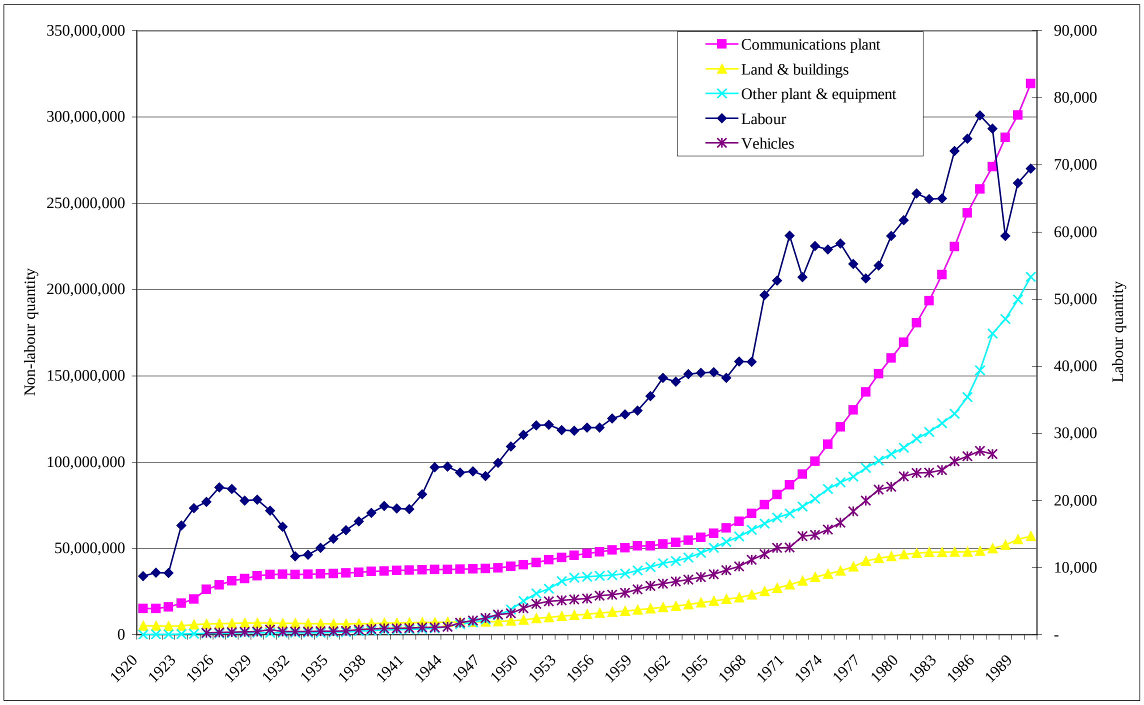
	Average	Std. Deviation	Minimum	Maximum	Coef. of Var.
Total cost	1,509,500,000	2,966,600,000	7,790,100	12,542,000,000	1.956
Cost Shares					
Buildings	0.054	0.020	0.030	0.134	0.373
Communications Plant	0.353	0.056	0.182	0.395	0.193
Labour	0.332	0.067	0.183	0.489	0.200
Machinery and Equipment	0.116	0.081	0.007	0.250	0.702
Materials	0.144	0.069	0.011	0.300	0.476
Capital rental prices					
Buildings	2.143	3.621	0.129	13.993	1.689
Communications Plant	2.868	3.496	0.066	11.913	1.219
Machinery and Equipment	5.999	7.594	1.186	29.257	1.266
Materials	0.948	1.622	0.066	7.700	1.711
Labour (wage)	6,206	9,720	281	38,136	1.566
Stocks					
Buildings	19,975,000	16,605,000	5,003,600	57,140,000	0.831
Communications Plant	82,689,000	77,405,000	15,087,000	319,320,000	0.937
Labour	36,359	19,457	8,735	77,388	0.535
Machinery and Equipment	22,692,000	29,146,000	20,441	104,210,000	1.284

Figure 5.3. Cost shares of aggregate inputs



Inspection of time series plots in Figure 5.4 reveals that the capital stocks are characteristically smooth, while labour exhibits a degree of oscillation around an exponential trend. A 'bubble' is evident between the years 1923 and 1933 and above-trend growth is evident between 1968 and 1977. The latter half of the 1980s are characterised by a decline in staff numbers.

Figure 5.4. Quantity measures of the main inputs



2.3 Output volume and service mix

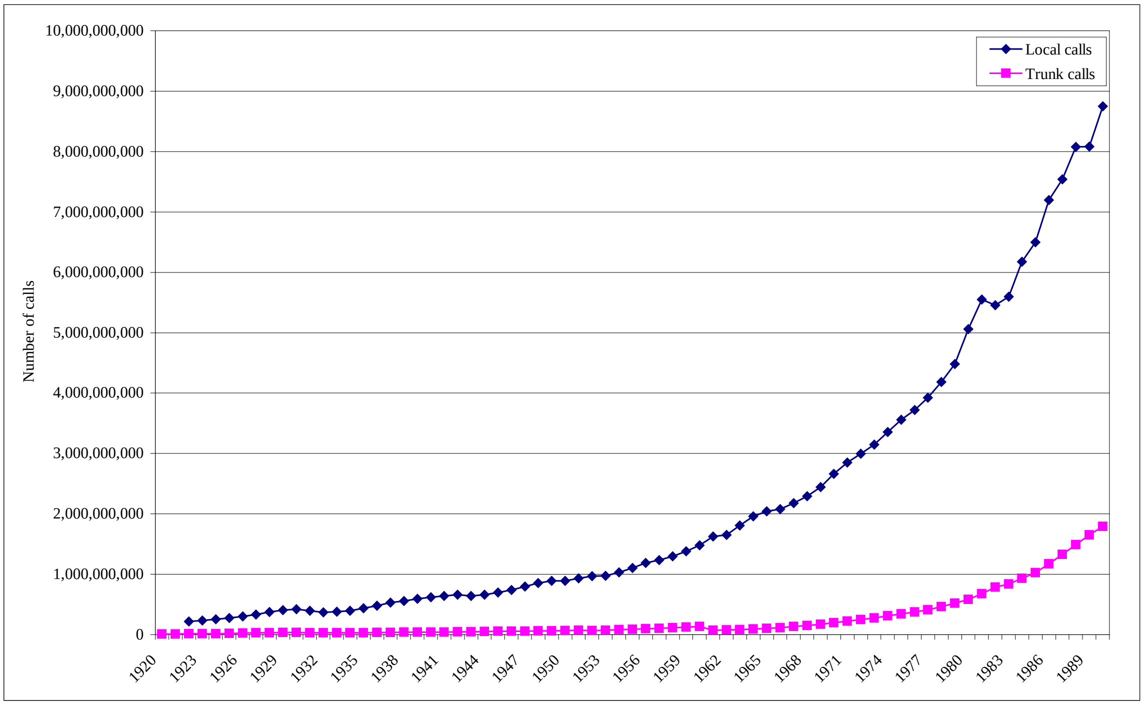
Table 5.2. Output volume and service mix 1920-1990

Services	Average	Std. deviation	Minimum	Maximum	Coef. of Var.
Text					
Telegrams	20,573,000	11,004,000	0	40,980,000	0.535
Telex					
Calls	9,768,500	15,892,000	0	51,301,000	1.627
Subscribers	7,986	13,677	0	46,423	1.713
Telephone					
Wireline					
Local calls	2,170,300,000	2,279,800,000	220,640,000	8,750,200,000	1.050
Trunk calls	264,600,000	412,090,000	12,420,000	1,794,200,000	1.557
Subscribers	2,066,700	2,119,400	172,110	7,786,900	1.026
Wireless					
Calls	3,676,100	20,579,000	0	156,000,000	5.598
Subscribers	4,444	24,715	0	184,940	5.562
Data (subscribers)					
Datel	13,632	30,592	0	1,039,600	2.244
Digital Data Service	3,869	14,794	0	84,189	3.824
Austpac	439	1,777	0	9,664	4.045

Source. PMG and Australian Telecommunications Commission annual reports.

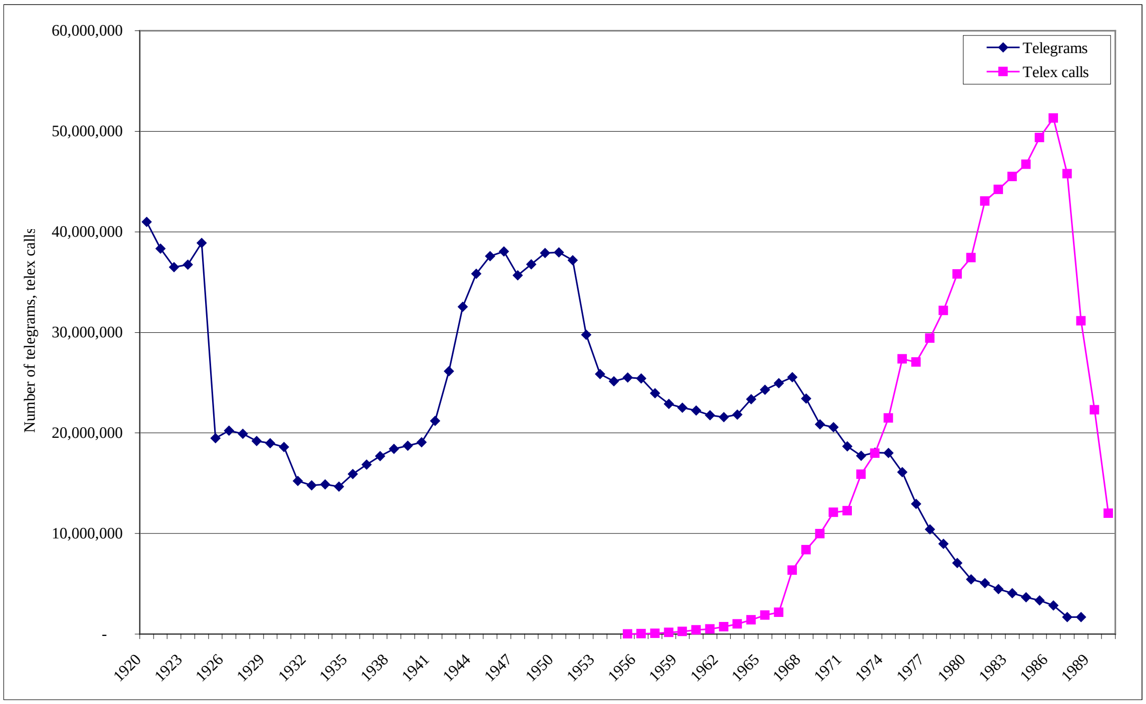
Table 5.2 presents summary statistics for the three main outputs, i.e. text-based, telephone and data for the period 1920 to 1990. Text-based services consist of telegrams and telex. Telephone is divided into wireline and wireless, while data is divided across Datel, Digital Data Services and Austpac. In scale terms, wireline telephone is clearly the largest in both call volume and subscribers. Within wireline telephone, the magnitude of local calls is an average call volume eight times larger than trunk calls. Both of these outputs exhibit exponential growth through time, with the difference between local and trunk call volumes halving by 1990. Telephone subscribers also exhibit strong growth, albeit at a substantially slower rate than call volume.

Figure 5.5. Number of local and trunk calls



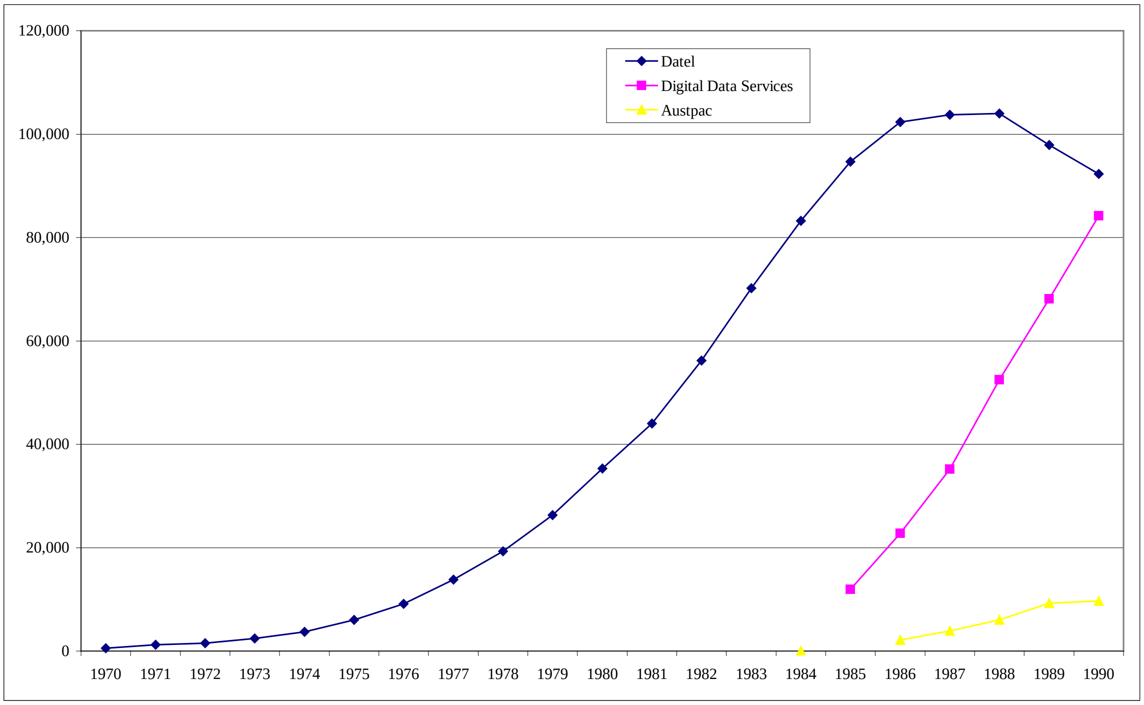
Compared to telephone, text services represent relatively minor shares of total output. Annual telegram traffic peaks in 1946 and exhibits a declining trend thereafter. The decline is explained somewhat by telex calls, which exhibit rapid growth from their introduction in 1957 until 1986, with the remaining observations declining rapidly. The number of telex subscribers follows a similar trend, peaking in 1986 and declining rapidly to 18,006 subscribers by 1990. Coefficient of variation statistics indicate that wireless telephone, first introduced in Melbourne and Sydney in 1981, is the most volatile series.

Figure 5.6. Number of telegrams and telex calls



Data services, which appear to have been provided predominantly on a subscription basis, represent a minor though important service category. Datel service begins in 1970 with 565 subscribers with subsequent subscriber adoption follows a ‘s’-shaped diffusion pattern through time. This diffusion path is an empirical regularity identified in the marketing literature as the typical time path following introduction of a new, successful service.

Figure 5.7. Data subscribers by service type



The two other data services, Digital Data Service (DDS) and Austpac mark the deployment of digital technology in the Australian telecommunications network. The Digital Data Service was introduced in 1982, providing point-to-point service improvements over the modem-linked Datel service, *viz.*, faster service and improved reliability.[20] Austpac is packet switched, providing subscribers with the ability to utilise flexible networking as a precursor to the Internet.

2.4 Technical change

Technical change variables, summarised in Table 5.3, are divided into transmission and switching components. The technologies summarised here embody changes in network flexibility (in terms of the type of services provided) and throughput (speed of processing). Thus, the benefits of technology manifest in direct cost reduction per unit of

[20] Data circuits instead of analogue modems connected subscribers.

output and indirect cost reduction by sharing fixed costs over an increasing range of services.

The technical change summarised in the table reflects what Newstead (1995) describes as a process of continual improvement conducted through an overlapping sequence of major innovations and equipment upgrades, each taking at least a decade to complete. These improvements include innovations in both network switching and transmission with each era of technological change resulting in an increased level of automation.

Table 5.3. Technical change variables

Network segment	Average	Std. deviation	Minimum	Maximum	Coef. of Var.	Unit of measurement
Transmission						
Coaxial cable	13,556	19,268	0	50,403	1.421	Kilometres
Fibre optic cable	6,883	31,478	0	182,000	4.574	Kilometres
Exchange switching						
Manual	220,560	111,290	0	402,660	0.505	Subscribers
Step-by-step	763,620	576,310	13,845	1,785,400	0.755	Subscribers
Crossbar						
ARF	555,120	960,500	0	3,035,700	1.730	Subscribers
ARK	223,450	362,610	0	1,150,000	1.623	Subscribers
ARE	231,210	635,520	0	2,345,900	2.749	Subscribers
Digital (AXE)	72,697	274,190	0	1,540,900	3.772	Subscribers

3. VARIABLE TRANSFORMATION

This section documents a number of necessary variable transformations and adjustments. In essence, this section provides the link between the raw form data described in the preceding sections of this chapter and the final estimation form used to develop the econometric model presented in Chapter 6.

The raw form data are contained in a single flat file (called PMG.dif) containing 47 variables and 72 rows. Variable names and a short description are presented in Table 5.4.[21]

Table 5.4. Estimation variables

Variable	Description	Unit of measurement
YEAR	Range is 1920 to 1990	Time
TELECOMR	Total telecommunications revenue	Dollar value
LOCCALLS	Total number of wireline local calls	Quantity
STDCALLS	Total number of trunk telephone calls	Quantity
CALLREV	Total wireline call revenue	Dollar value
CELLCALL	Total number of mobile telephone calls	Quantity
CELCALRV	Total mobile call revenue	Dollar value
CELLSUB	Total number of mobile telephone subscribers	Quantity
CELACREV	Mobile telephone access revenue	Dollar value
CELCNFEE	Mobile telephone connection fee revenue	Dollar value
SUBSCRIB	Number of wireline telephone subscribers	Quantity
RENTREV	Wireline telephone rental revenue	Dollar value
TOTTELGR	Total number of telegrams	Quantity
TELGRREV	Telegram revenue	Dollar value
TELEXCAL	Number of telex calls	Quantity
TLXCALRV	Telex call revenue	Dollar value
TELEXSRV	Number of telex services in operation	Quantity
TLXRENT	Telex subscription revenue	Dollar value
DATELSUB	Number of Datel subscribers	Quantity
DDS	Number of Digital Data Service subscribers	Quantity
AUSTPAC	Number of AUSTPAC subscribers	Quantity
OTHTELEC	Other telecommunications revenue	Dollar value
MANUAL	Number of wireline telephone subscribers connected to a manual exchange	Quantity
SXS	Number of wireline telephone subscribers connected to a step-by-step exchange	Quantity
ARK	Number of wireline telephone subscribers connected to an ARK exchange	Quantity

[21] See Chapter 5 Appendix 2 for a more expansive description and detailed account of how each variable is sourced.

Table 5.4. Estimation variables (continued)

Variable	Description	Unit of measurement
ARE	Number of wireline telephone subscribers connected to an ARE exchange	Quantity
AXE	Number of wireline telephone subscribers connected to an AXE exchange	Quantity
DDSC	Number of Digital Data Service centres	Quantity
AUSTPACX	Number of AUSTPAC exchanges	Quantity
XCHGQ	Total number of telephone exchanges	Quantity
COAX_KM	Total number of kilometres of coaxial cable	Quantity
OPTIC_KM	Total number of kilometres of fibre optic cable	Kilometres
COST	Total expenditure as reported on the Profit and Loss statement for the telecommunications division	Dollar value
WAGES_E	Total wages expenditure	Dollar value
WAGES_P	Average wage	Dollar value
COMPLTIN	Annual expenditure on communications plant	Dollar value
COMM_P	Spliced ABS electronic equipment price (ABS code: NUQA.PD_ELE_PC_GJ)	Indexed dollars
COMM_S	Total stock of communications plant	Indexed quantity
BUILD	Annual expenditure on buildings	Dollar value
BUILD_S	Total stock of buildings	Indexed quantity
NDC_P	Buildings price deflator is the ABS non-dwelling construction price series (ABS code: NUQA.PD_NDC_PC_GJ)	Indexed dollars
MCHE_P	Spliced and aggregated series — motor vehicles price deflator (ABS code: NUQA.PD_RVH_PC_GJ) and other plant (ABS code: NUQA.PD_OPL_PC_GJ) price series span the years 1949 to 1990	Indexed price
MCHE_E	Other plant and equipment expenditure	Dollar value
MCHE_S	Other plant and equipment stock	Indexed quantity
BOND_YLD	Annual 10 year bond yield for Commonwealth of Australia (Government) bonds	Decimal
DEPTELE	Depreciation expense	Dollar value
INTTELE	Interest expense	Dollar value

The above 47 source variables are read into SHAZAM. Missing observations for AUSTPAC (observation 66) and TLXRENT (observations 42 to 50, inclusive) are interpolated. An aggregate wireline telephone call variable (FONECL_Q) is created by summing LOCCALLS and STDCALLS. Average wireline telephone call price and subscription price are created by dividing the respective revenue variable by the quantity variable. Average prices for telegrams, telex calls and telex rent are similarly calculated. Telecom stopped reporting the annual telegram revenue after 1984, so the last calculated average price for telegrams is carried forward for the years 1985 to 1987, inclusive. Total number of telegrams is zero for 1988 to 1990, inclusive. Similarly, the missing average telex call price for 1955 to 1958, inclusive are set equal the average price for 1959, while average telex call price for 1985 to 1990, inclusive are set equal to the average price for 1984. The same applies to the average telex rental price. Total mobile telephone rental price is calculated by summing CELACREV and CELCNFEE. Average prices for mobile telephone rental and call prices are then calculated. All output price and quantity variables (except DATELSUB, DDS and AUSTPAC) are then used to calculate Fisher ideal aggregate price and quantity indexes using SHAZAM's index algorithm. The second aggregate output, DATA, is the sum of DATELSUB, DDS and AUSTPAC.

The stock of labour is calculated by dividing wages expenditure by the average wage. Materials expenditure is calculated by deducting the sum of wages expenditure, depreciation expense and interest expense from COST. Rental prices for buildings, communications plant and other plant and equipment are calculated by multiplying the respective price deflator by the sum of depreciation rate and the annual bond yield. The asset-specific depreciation rate is the inverse of expected operational life as reported by the ABS. That is, average building life is 49 years, communication plant is 15.1 years and other plant and equipment is 17.3 years. Total annual capital expense is then calculated summing the product of the rental price and the capital stock for each of the three categories of capital. Total cost is then calculated by summing total annual capital expense, materials expenditure and wages expenditure. Törnqvist rental price and quantity for aggregate capital are calculated using SHAZAM's index algorithm.

Finally, casual inspection of aggregate price and quantity variables for each of the three inputs revealed a substantial difference in scale between price and quantity. Since, the econometrics literature suggests that heteroskedasticity can be induced by scale, price and quantity are both rescaled so that price and quantity are approximately equal for a specific year. By construction, the rescaling procedure ensures that the product of price and quantity is equal to the original expenditure series.

4. CHAPTER CONCLUSION

The main purpose of this chapter is to provide a detailed outline of how the variables used in model estimation are sourced and constructed. The long time series, spanning the years 1920 to 1990, ensures that capital stocks are estimated as accurately as possible. Accurate measures of capital stock are crucial as they are integral to accurate measures of aggregate expenditure and price series. Finally, the documented rapid and continuous change in technology also suggests that model parameters may be subject to variation. This in turn, influences the sample period used in model estimation.

CHAPTER 6—ESTIMATION RESULTS

This chapter is organised as follows. Section 1 discusses the preferred econometric model and results. Section 2 provides analysis of the ancillary variables, such as marginal costs, cost elasticities and fixed-cost estimates. Section 3 presents results for the subadditivity test and briefly discusses the implications. Section 4 then provides concluding remarks.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The model developed in Chapter 4 places an explicit interpretation on the autoregressive error terms, viz., that statistically significant autoregressive parameters provided evidence of adjustment lags. The modified generalised McFadden (MGM) demand system, estimated in revenue share form, is

$$
\frac{w_{1t}x_{1t}}{R_t} = \frac{w_{1t}}{R_t}\left\{
\begin{array}{l}
a_1 + \left(
\begin{array}{l}
\dfrac{-\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_1\theta_1 + w_2\theta_2 + w_3\theta_3\right)} \\[2ex]
+\dfrac{\left(
\begin{array}{l}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{array}\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2}
\end{array}\right) \\[2ex]
\times\left(1+\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\right)^2 \\[2ex]
+ f_Z\, exchange_{Digital,t} \\[2ex]
+\dfrac{\theta_1}{2}\left(d_{11}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2 + 2d_{12}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) + d_{22}\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)
\end{array}\right\}
$$

$$
+\rho_{11}u_{1t-1} + \rho_{12}u_{2t-1} + \rho_{13}u_{3t-1} \tag{6.1.1}
$$

$$
\frac{w_{2t}x_{2t}}{R_t} = \frac{w_{2t}}{R_t}\left\{\begin{array}{l}
a_2 + \\
+\left(\begin{array}{l}
\dfrac{-\left(a_{11,2}a_{21,2}w_{1t} + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}\right)}{\left(w_1\theta_1 + w_2\theta_2 + w_3\theta_3\right)} \\
+\dfrac{\left(\begin{array}{l}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,A}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{array}\right)\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2}
\end{array}\right) \\
\times\left(1 + \left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2 \\
+ f_{Z1}exchange_{ARK,t} + f_{Z2}exchange_{old,t} \\
+\dfrac{\theta_2}{2}\left(d_{11}\left(\dfrac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\dfrac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2\right)
\end{array}\right\}
\tag{6.1.2}
$$

$$
+\rho_{21}u_{1t-1} + \rho_{22}u_{2t-1} + \rho_{23}u_{3t-1}
$$

$$
\frac{w_{3t}x_{3t}}{R_t} = \frac{w_{3t}}{R_t}\left\{\begin{array}{l}
a_3 + \\
+\left(\begin{array}{l}
\dfrac{-\left(\begin{array}{l}
-\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\
+(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_3
\end{array}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+\dfrac{\left(\begin{array}{l}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{array}\right)\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2}
\end{array}\right)\times\left(1 + \left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2 \\
+ f_Z exchange_{Digital,t} + c_{t3}time_t \\
+\dfrac{\theta_3}{2}\left(d_{11}\left(\dfrac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\dfrac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\dfrac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2\right)
\end{array}\right\}
$$

$$
+\rho_{31}u_{1t-1} + \rho_{32}u_{2t-1} + \rho_{33}u_{3t-1}
\tag{6.1.3}
$$

where x_{1t}, x_{2t} and x_{3t} correspond to the capital stock, materials volume, and labour stock, respectively. Similarly, input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively, while R_t is total revenue. Multiplying each equation by its respective input price and dividing by total revenue converts the system (6.1.1) to (6.1.3) into revenue share equations (see Cooper et al., 2003). Doing so reduces the scale differences in magnitude between variables and probably makes the maintained assumption of homoskedasticity more plausible. Output q_{1t} is a composite Fisher ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. The variable $exchange_{Digital,t}$ is the number of digital telephone exchanges (both ARE and AXE type), while $exchange_{old,t}$ is the combined number of manual and step-by-step exchanges and $exchange_{Ark,t}$ is the number of ARK type cross-bar exchanges. The variable $time_t = t$ where $t = \{0,1,2,\ldots,36\}$ for the years 1954 to 1990, inclusive. The quadratic specification of a single $tech$ variable is dropped, as it is apparent during estimation that quadratic arguments of $time_t$ and other proxy variables appear to coincide with less precise standard error estimates and detection of serial correlation.

Equations (6.1.1) to (6.1.3) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates, 2001), which allows the equations to be coded in the same way as presented in this thesis, thereby allowing concavity of the cost function with respect to input prices to be imposed by construction. The cost-function equation corresponding to the system of factor-demand equations (6.1.1) to (6.1.3) is not estimated, since it does not contain any additional information. Note that the Box-Cox transformation is applied to the outputs and that autoregressive error terms are added to each equation.

Box-Cox coefficients, λ_1 and λ_2, values are selected prior to estimation. Since convergence is not guaranteed, manual selection facilitates estimation and increases the chance that the estimation routine functions properly. By contrast, attempts to include λ_1 and λ_2 in the estimated parameters typically lead to convergence failure. The parameters θ_1, θ_2, and θ_3 are also set prior to estimation, so that the sum of the parameters equals one. Setting the Box-Cox parameters a priori is quite arbitrary and limits the attractiveness of the model. In an attempt to overcome the problem, the parameter originally freely estimated are set at the reported estimated values and the Box-Cox parameters then estimated freely. This process results in $\lambda_1 = 0.004$ and $\lambda_2 = 0.26$. The original model is re-estimated setting *a priori* at the values $\lambda_1 = 0.004$ and $\lambda_2 = 0.26$ a priori. The resulting estimated coefficients are completely different in magnitude from the original model as well as exhibiting some serial correlation and system heteroscedasticity.

Given the value $\lambda_1 = 0.004$ is close to zero, the logarithmic transformation is applied to aggregate output instead of the Box-Cox transformation. The model is then re-estimated. The resulting estimated parameters are again completely different to the original model. Thus, this process has failed to confirm the original model and suggests that the results are sensitive to the values that are set arbitrarily and those that are freely estimated. However, it should also be noted that setting $\lambda_1 = 1$ and $\lambda_2 = 1$, as does the standard quadratic model or applying the logarithmic transformation to the output variables is just as arbitrary as the process adopted here. In the final assessment, two models that result from the Box-Cox parameters being set *a priori* at the values shown in Table 6.1a and Table 6.1b are preferred since they yield results that are plausible and coincide with satisfactory auxillary test statistics.

Table 6.1a and Table 6.1b provide coefficient estimates and associated standard errors, the function value and Box-Cox parameters for the model estimated on sample data corresponding to the years 1954 to 1990, inclusive. The models presented reflect the variation caused by changing the value of λ_1 from 0.25 to 0.6. Varying the parametre is considered necessary since the ancillary calculations, such as marginal cost and returns to scale turn out to be sensitive to the value of λ_1. Table 6.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table 6.3 provides results of the remaining diagnostic statistics.

Estimating a model that does not violate fundamental conditions of nonnegative marginal, nonnegative variable and fixed costs, and the null hypothesis for a selected set of auxiliary statistics proves to be difficult. The models presented in this chapter are considered to be the best approximation of the true underlying cost function, rather than a unique representation. Other models that could also be deemed plausible approximations are presented in Appendix 2 to 6.

The estimated models vary in a number of important aspects. Appendix 2 allows additional flexibility in substitution possibilities by allowing $\mathbf{\Sigma} = \mathbf{\Sigma}_0 + \mathbf{\Sigma}_2 \times \left(b_2 \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2} \right)$.

Appendix 3 specifies $\mathbf{\Sigma} \times \left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1} + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2} \right)$, includes first-order output arguments and imposes the restriction that autoregressive parameters $\rho_{ij} = 0$, $i \neq j$. Appendix 4 retains $\mathbf{\Sigma} \times \left(1 + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2} \right)^2$ and imposes the restriction that autoregressive parameters $\rho_{ij} = 0$, $i \neq j$. Another important difference is that the cost function is estimated, while the materials share equation is dropped. While it is possible to estimate the materials equation, doing so provides no advantage in estimation efficiency since it contains no additional information. Appendices 5 and 6 present estimated error-correction models. These models provide reasonable estimates of the equilibrium cost function, although implied returns to scale measures appear to be highly sensitive to the specification. Experimentation with the error-correction model yields plausible estimates at the cost of violating the null hypotheses of homoskedasticity and no serial correlation. However, the presented error-correction models do confirm slow adjustment.

All models satisfy the criteria for proper cost functions and the null hypotheses of serially independent and homoskedastic errors are maintained. Sample periods chosen are the largest that maintain these null hypotheses and provide plausible model results.

The model presented in Table 6.1a converged within 234 iterations, with coefficient starting values left at Shazam’s default setting. The Shazam vector autoregressive errors option is utilised, allowing for different rhos (ρ_{ij}) in each equation. The estimation results indicate that the values for $\rho_{11}, \rho_{12}, \rho_{22}, \rho_{31}, \rho_{33}$ are statistically significant, while ρ_{23} is close to statistical significance. Table 6.1a shows that 10 of the 13 estimated structural parameters are statistically significant at conventional levels. The statistical significance of the autoregressive parameters is consistent with a substantial degree of inflexibility in adjusting all inputs. The degree of inflexibility in capital is not surprising given the large and long-planning horizon and ongoing capital programs. Public sector awards and conditions seem a plausible explanation for the adjustment lag in labour.

Table 6.1a Model 1—estimated parameters 1954-90

	COEFFICIENT	ST. ERROR	T-RATIO
a_1	379.92	2.07	**183.44**
a_2	1,021.40	5.71	**178.93**
a_3	286.46	1.73	**165.94**
$a_{11,2}$	173.45	56.06	**3.09**
$a_{21,2}$	-103.25	11.46	**-9.01**
$a_{22,2}$	-0.00	58.58	-0.00
f_Z	758.69	426.42	1.78
f_{Z1}	258.56	72.08	**3.59**
f_{Z2}	10.80	6.72	1.61
c_{t3}	3,575.80	17.36	**205.95**
d_{11}	6,234.30	906.08	**6.88**
d_{12}	-11,313.00	2,626.20	**-4.31**
d_{22}	456,440.00	2,163.20	**211.00**
ρ_{11}	0.72	0.12	**6.14**
ρ_{12}	-0.01	0.00	**-1.97**
ρ_{13}	0.03	0.07	0.43
ρ_{21}	0.37	1.44	0.26
ρ_{22}	0.82	0.08	**9.87**
ρ_{23}	0.60	0.66	0.91
ρ_{31}	0.46	0.20	**2.33**
ρ_{32}	0.02	0.01	**1.69**
ρ_{33}	0.93	0.11	**8.50**
λ_1	0.25		
λ_2	0.25		
θ_1	0.55		
θ_2	0.30		
θ_3	0.15		
Function value	288.91		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table 6.1b Model 2—estimated parameters 1954-90

	COEFFICIENT	ST. ERROR	T-RATIO
a_1	469.97	9.80	**47.96**
a_2	2,166.3	45.07	**48.06**
a_3	215.84	4.58	**47.15**
$a_{11,2}$	198.04	66.48	**2.98**
$a_{21,2}$	-98.24	17.38	**-5.65**
$a_{22,2}$	0.00	61.92	0.00
f_Z	606.63	366.00	1.66
f_{Z1}	252.23	83.70	3.01
f_{Z2}	46.29	2.83	16.38
c_{t3}	5,414.50	112.49	**48.13**
d_{11}	0.06	0.01	**4.62**
d_{12}	-55.72	11.75	**-4.74**
d_{22}	527,560.00	10,954.00	**48.16**
ρ_{11}	0.66	0.12	**5.32**
ρ_{12}	-0.02	0.00	**-2.06**
ρ_{13}	0.01	0.06	0.22
ρ_{21}	0.21	0.57	0.37
ρ_{22}	0.80	0.04	**18.76**
ρ_{23}	0.07	0.27	0.27
ρ_{31}	0.61	0.24	**2.58**
ρ_{32}	0.03	0.02	**1.62**
ρ_{33}	0.94	0.12	**7.75**
λ_1	0.60		
λ_2	0.25		
θ_1	0.55		
θ_2	0.30		
θ_3	0.15		
Function value	274.05		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table 6.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	0.04	0.25	1.61	1.61	1.79
	(0.85)	(0.88)	(0.66)	(0.81)	(0.89)
Equation 2	2.73	3.42	3.79	4.45	4.85
	(0.10)	(0.18)	(0.29)	(0.35)	(0.44)
Equation 3	0.20	0.21	2.43	3.08	3.14
	(0.65)	(0.90)	(0.49)	(0.54)	(0.68)

Note. P-values in parentheses.

Table 6.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	2.43
	(0.30)
Equation 2 $\chi^2(2)$	1.80
	(0.41)
Equation 3 $\chi^2(2)$	1.10
	(0.58)
System Test Statistics	
Serial Correlation $\chi^2(9)$	9.41
	(0.40)
Serial Correlation $\chi^2(27)$	9.07
	(0.99)
Serial Correlation F(27,37)	0.13
	(1.00)
Heteroskedasticity $\chi^2(36)$	46.79
	(0.11)
Heteroskedasticity F(36,71.5)	1.37
	(0.13)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table 6.2 and Table 6.3 show that the tests fail to reject the null hypotheses of no serial correlation and homoskedasticity at conventional levels of significance. Shazam calculates the Ljung-Box-Pierce test for serial correlation presented in Table 6.2 from the first five autocorrelations (see Whistler, White, Wong and Bates, 2001: 122). The null hypothesis is that the first five autocorrelations are all zero. Under the null hypothesis, the test statistic has an asymptotic chi-squared distribution.

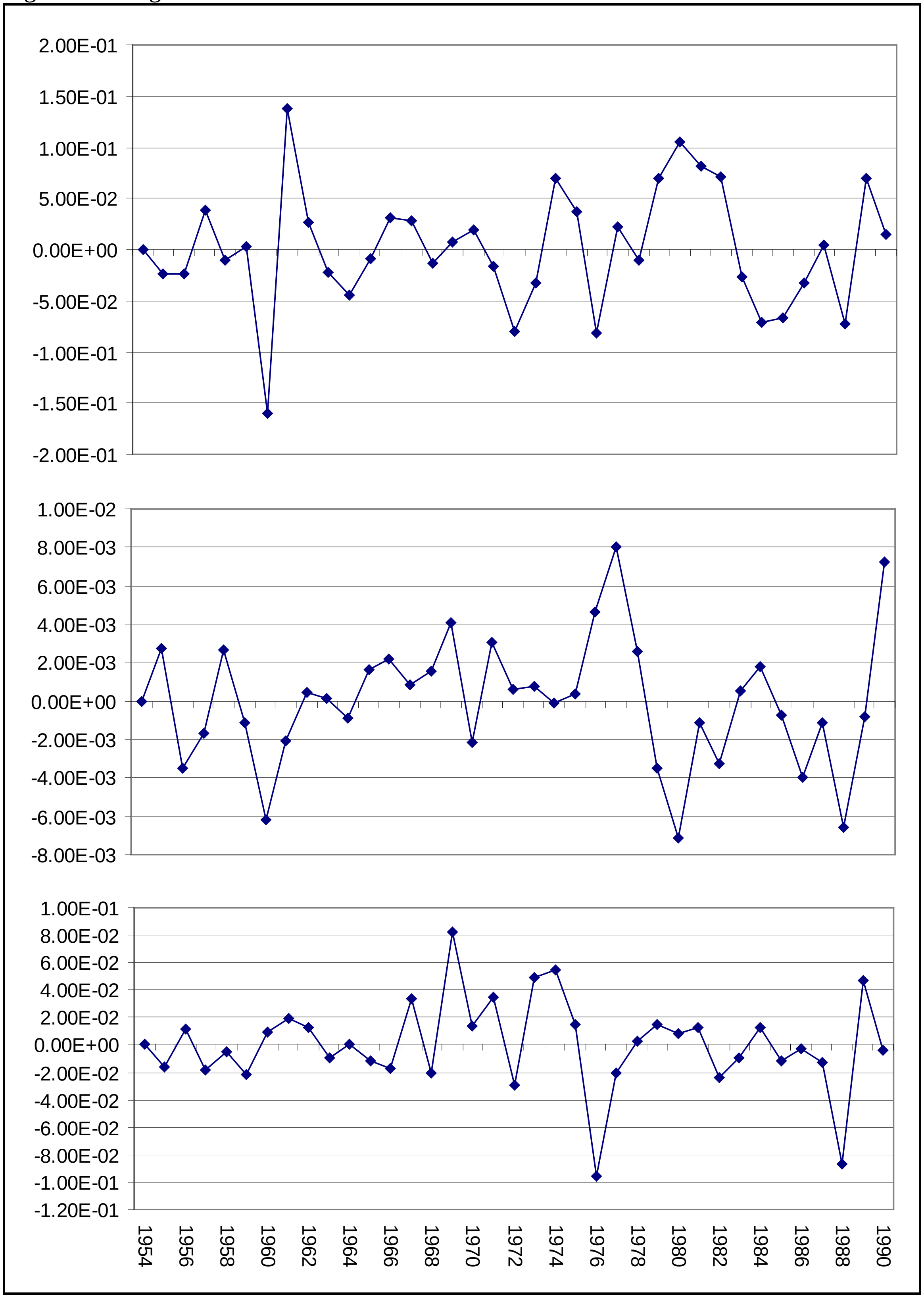
Regression residuals are provided in Figure 6.1. The single equation heteroskedasticity test (reported in the top portion of Table 6.3) is a modified version of the test proposed by White (1980) in which the levels and squares of the fitted values are regressed on the residuals of each estimated factor-demand equation.[22] That is, let $\hat{x}_{if}$ be the fitted variable corresponding to the quantity of input i and $\hat{\varepsilon}_{it}$ denotes the regression residuals calculated following estimation of the factor demand equation corresponding to input i. Then the modified version of White's test is

$$\hat{\varepsilon}_{it}^2 = \mu + \varphi \cdot \hat{x}_{if} + \nu \cdot \hat{x}_{if}^2 + \varepsilon'_{it} \qquad (6.1.4)$$

where $\mu,\ \varphi,\ \nu$ are regression coefficients and ε'_{it} represents regression errors. The test statistic is calculated according to $h_{mw} = nR^2$ where n is the number of observations and h_{mw} is chi-squared distributed with two degrees of freedom.

[22] The modified version of White's test reduces the dimensionality of the test compared to White's original test.

Figure 6.1. Regression residuals



The first system-serial-correlation test is the likelihood ratio test explained in Enders (1995: 313), which is applicable to any type of cross-equation restriction. In Step 1, the fitted variables (x_{if}) and lagged residuals ($\hat{\varepsilon}_{it-1}$) are calculated from estimation of equations (1.1) to (1.3). These variables become the regressands in an auxiliary SURE system in which the dependent variables from equation system (6.1.1) to (6.1.3) are the regressors. That is,

$$x_{1t} = \alpha_{10} + \alpha_{11}x_{1f} + \beta_{11}\hat{\varepsilon}_{1t-1} + \beta_{12}\hat{\varepsilon}_{2t-1} + \beta_{13}\hat{\varepsilon}_{3t-1} + \varepsilon''_{1t} \tag{6.1.5a}$$

$$x_{2t} = \alpha_{20} + \alpha_{21}x_{2f} + \beta_{21}\hat{\varepsilon}_{1t-1} + \beta_{22}\hat{\varepsilon}_{2t-1} + \beta_{23}\hat{\varepsilon}_{3t-1} + \varepsilon''_{2t} \tag{6.1.5b}$$

$$x_{3t} = \alpha_{30} + \alpha_{31}x_{3f} + \beta_{31}\hat{\varepsilon}_{1t-1} + \beta_{32}\hat{\varepsilon}_{2t-1} + \beta_{33}\hat{\varepsilon}_{3t-1} + \varepsilon''_{3t} \tag{6.1.5c}$$

where the α_{ij} and β_{ij} are coefficients to be estimated and ε''_{it} are the regression errors, $i = \{1,2,3\}$. In Step 2 the $\hat{\varepsilon}_{it-1}$ are removed and the auxiliary system

$$x_{1t} = \alpha_{10} + \alpha_{11}x_{1f} + \varepsilon''_{1t} \tag{6.1.6a}$$

$$x_{2t} = \alpha_{20} + \alpha_{21}x_{2f} + \varepsilon''_{2t} \tag{6.1.6b}$$

$$x_{3t} = \alpha_{30} + \alpha_{31}x_{3f} + \varepsilon''_{3t} \tag{6.1.6c}$$

are estimated.

The natural logarithm of the determinant of the unrestricted covariance matrix of the residuals from equation system (6.1.5a) to (6.1.5c) is then calculated and labelled $\ln|\Sigma_u|$. The natural logarithm of the determinant of the restricted covariance matrix of the residuals from equation system (6.1.6a) to (6.1.6c) is calculated and labelled $\ln|\Sigma_r|$. If serial correlation is present in the original system (6.1.1) to (6.1.3), then the $\hat{\varepsilon}_{it-1}$ should provide some explanatory power in the system (6.1.5a) to (6.1.5c). If there is no serial correlation, the β_i will be jointly zero. In this case, a likelihood ratio test of $\beta_{11} = \beta_{12} = \ldots = \beta_{33} = 0$ is

$$LR = (T - c)(\ln|\Sigma_r| - \ln|\Sigma_u|) \qquad (6.1.7)$$

where $T = 37$ denote the number of usable observations and $c = 5$ is the total number of parameters estimated in each equation of the unrestricted system, and LR is chi-squared distributed with three degrees of freedom. Table 6.3 shows that there is no evidence of system serial correlation. The degrees of freedom is equal to the total number of restrictions in the equation system. In this case, there are nine degrees of freedom.

The remaining system serial correlation tests are from Doornik (1996). The tests are modified slightly because of difficulty in re-estimating the original system with lags of the residuals. Instead, the system is re-estimated by regressing the dependent variables on the fitted regression. That is, the original system is estimated as in (6.1.6) above. The auxiliary system is as presented in (6.1.5). In both cases, the residuals are saved and used to calculate,

$$R_r^2 = 1 - |\hat{\mathbf{V}}'\hat{\mathbf{V}}||\hat{\mathbf{V}}_0'\hat{\mathbf{V}}_0|^{-1}$$

$$R_m^2 = 1 - \frac{1}{n} tr\left\{(\hat{\mathbf{V}}'\hat{\mathbf{V}})(\hat{\mathbf{V}}_0'\hat{\mathbf{V}}_0)^{-1}\right\}$$

Next, Doornik's LM statistic is calculated according to $LM = TgR_m^2$ is $\chi^2(sn^2)$ distributed, where s is the number of added terms in each equation and n is the number of equations in the original system. An LMF statistic is also calculated according to $LMF = \frac{1-(1-R_r^2)^{\frac{1}{r}}}{(1-R_r^2)^{\frac{1}{r}}}\frac{Nr-q}{np}$ which is approximately $F(np, Ns-q)$ distributed, where

$$r = \left(\frac{g^2h^2-4}{g^2+h^2-5}\right)^{½},\ q = \tfrac{1}{2}np-1,\ N = T-k-p-\frac{1}{2}(n-p+1).$$

k is the number of regressors in the original system.

T is the number of time observations.

The system heteroskedasticity test statistics presented in Table 6.3 are as suggested by Doornik, which in turn are based on Kelejian's (1982) extension of White's (1980) test to the simultaneous equations case.

Doornik's description of the test begins by rewriting equations (6.1.1) to (6.1.3) in matrix form as

$$\mathbf{Y}' = \mathbf{\Pi W}' + \mathbf{V}' \tag{6.1.8}$$

where $\mathbf{Y}'$ is $n \times T$, $\mathbf{W}'$ is $k \times T$, $\mathbf{\Pi}$ is an $n \times k$ matrix of estimated coefficients and $\mathbf{V}'$ is an $n \times T$ matrix of error terms. Specifically,

$$\mathbf{V}' = \begin{bmatrix} v_{11} & \cdots & v_{1t} \\ v_{21} & \cdots & v_{2t} \\ \vdots & & \vdots \end{bmatrix}_{n\times T} \tag{6.1.9}$$

The test entails applying generalised least squares to the auxiliary equation system,

$$\mathbf{V}' = \boldsymbol{\beta}\mathbf{P}' + \mathbf{E}' \tag{6.1.10}$$

where matrix $\mathbf{P}'$ contain various functions (e.g. squares and cross-products) of the original exogenous regressors contained in $\mathbf{W}'$. That is, estimate

$$\boldsymbol{\Psi}' = \boldsymbol{\beta}\breve{\mathbf{P}}' + \mathbf{E}' \tag{6.1.11}$$

where

$$\boldsymbol{\Psi}' = \begin{pmatrix} \hat{v}_{11}^2 - \bar{v}_1 & \dots & \hat{v}_{T1}^2 - \bar{v}_1 \\ \hat{v}_{12}^2 - \bar{v}_2 & \dots & \hat{v}_{T2}^2 - \bar{v}_2 \\ \hat{v}_{13}^2 - \bar{v}_3 & \dots & \hat{v}_{T3}^2 - \bar{v}_3 \\ \hat{v}_{11}\hat{v}_{12} - \bar{v}_{12} & \dots & \hat{v}_{T1}\hat{v}_{T2} - \bar{v}_{12} \\ \hat{v}_{11}\hat{v}_{13} - \bar{v}_{13} & \dots & \hat{v}_{T1}\hat{v}_{T3} - \bar{v}_{13} \\ \hat{v}_{12}\hat{v}_{13} - \bar{v}_{23} & \dots & \hat{v}_{T2}\hat{v}_{T3} - \bar{v}_{T3} \end{pmatrix} \tag{6.1.12}$$

where $\boldsymbol{\Psi}'$ is dimension $\frac{1}{2}n(n+1)\times T$, each of the $\bar{v}_{ij} = \dfrac{\left(\hat{v}_{1i}\hat{v}_{1j} + \hat{v}_{2i}\hat{v}_{2j} + \hat{v}_{3i}\hat{v}_{3j} + \dots + \hat{v}_{Ti}\hat{v}_{Tj}\right)}{T}$ and $i, j = \{1,2,3\}$. $\breve{\mathbf{P}}'$ is defined by taking deviations from the means of the variable contained in $\mathbf{P}'$. Since there is a large number of variables contained in the factor-demand system (6.1.1), (6.1.2) and (6.1.3), dimensionality of the test is reduced by defining $\mathbf{P}'$ as the levels and squares of the fitted equations (6.1.1), (6.1.2) and (6.1.3). That is,

$$\breve{\mathbf{P}}' = \begin{pmatrix} \hat{y}_{11} - \bar{\hat{y}}_1 & \dots & \hat{y}_{T1} - \bar{\hat{y}}_1 \\ \hat{y}_{12} - \bar{\hat{y}}_2 & \dots & \hat{y}_{T2} - \bar{\hat{y}}_2 \\ \hat{y}_{13} - \bar{\hat{y}}_3 & \dots & \hat{y}_{T3} - \bar{\hat{y}}_3 \\ \hat{y}_{11}^2 - \bar{\hat{y}}_1^2 & \dots & \hat{y}_{T1}^2 - \bar{\hat{y}}_1^2 \\ \hat{y}_{12}^2 - \bar{\hat{y}}_2^2 & \dots & \hat{y}_{T2}^2 - \bar{\hat{y}}_2^2 \\ \hat{y}_{13}^2 - \bar{\hat{y}}_3^2 & \dots & \hat{y}_{T3}^2 - \bar{\hat{y}}_3^2 \end{pmatrix}. \tag{6.1.13}$$

The test proceeds as follows:

1. Calculate $\hat{\mathbf{V}}[\boldsymbol{\psi}_t] = \frac{1}{T}\boldsymbol{\Psi}'\boldsymbol{\Psi}$ for the null hypothesis and label $\hat{\mathbf{V}}_0{}'\hat{\mathbf{V}}_0$ (dimension $g \times g$) where $g = \frac{1}{2}n(n+1)$.

2. Estimate (6.1.8) or equivalently $\boldsymbol{\Psi} = \breve{\mathbf{P}}\boldsymbol{\beta} + \mathbf{E}$

3. Label $\hat{\mathbf{V}} = \mathbf{E}$ (dimension $T \times g$) and calculate $\hat{\mathbf{V}}{}'\hat{\mathbf{V}}$ (dimension $g \times g$).

4. Calculate $R_r^2 = 1 - \left|\hat{\mathbf{V}}'\hat{\mathbf{V}}\right|\left|\hat{\mathbf{V}}_0'\hat{\mathbf{V}}_0\right|^{-1}$

5. Calculate $R_m^2 = 1 - \frac{1}{n}tr\left\{\left(\hat{\mathbf{V}}'\hat{\mathbf{V}}\right)\left(\hat{\mathbf{V}}_0'\hat{\mathbf{V}}_0\right)^{-1}\right\}$

6. Calculate $LM = TgR_m^2$, which is $\chi^2(gh)$ distributed where h is the number of regressors in auxiliary system equation i.

7. Calculate $LMF = \frac{1-\left(1-R_r^2\right)^{\frac{1}{r}}}{\left(1-R_r^2\right)^{\frac{1}{r}}}\frac{Nr-q}{gh}$, which is approximately $F(gh, Ns-q)$ distributed where s is the number of lags in the system, $q = \frac{1}{2}gh - 1$, $k_1 = 1$, $r = \left(\frac{g^2h^2-4}{g^2+h^2-5}\right)^{½}$, and $N = T - k_1 - h - \frac{1}{2}(g-h+1)$

For the heteroskedasticity test statistics reported in Table 6.2, the relevant parameters are

$g = \frac{3}{2}(3+1) = 6$, $h = 6$, $q = \frac{1}{2}\times 6\times 6 - 1 = 17$, $r = \left(\frac{6^2 6^2 - 4}{6^2+6^2-5}\right)^{½} = 4.39$,

$N = 37 - 1 - 6 - \frac{1}{2}(6-6+1) = 30 - 0.5 = 29.50$, $Ns - q = 29.5\times 3 - 17 = 71.5$,

$R_r^2 = 0.80$, $R_m^2 = 0.168$, $\chi^2(36)$ and $F(36,71.5)$.

Table 6.5. $\mathbf{\Sigma}$ matrix

-30,084.81	17,908.78	12,176.06	$\lvert H_1 \rvert$	-30,084.81
17,908.75	-10,660.64	-7,248.11	$\lvert H_2 \rvert$	0.21E-06
12,176.06	-7,248.11	-4,927.95	$\lvert H_3 \rvert$	0.00

In addition to the absence of serial correlation and heteroskedasticity, the estimated system (6.1.1) to (6.1.3) must reflect the properties of a proper cost function. The remaining necessary conditions for a proper cost function are: (1) negative semi-definiteness of the Hessian matrix of second-order derivatives with respect to the input prices; and (2) non-negative marginal costs.

The Hessian matrix is defined as

$$H = \begin{bmatrix} \dfrac{\partial^2 C_t}{\partial w_{1t}\partial w_{1t}} & \dfrac{\partial^2 C_t}{\partial w_{1t}\partial w_{2t}} & \dfrac{\partial^2 C_t}{\partial w_{1t}\partial w_{3t}} \\ \dfrac{\partial^2 C_t}{\partial w_{2t}\partial w_{1t}} & \dfrac{\partial^2 C_t}{\partial w_{2t}\partial w_{2t}} & \dfrac{\partial^2 C_t}{\partial w_{2t}\partial w_{3t}} \\ \dfrac{\partial^2 C_t}{\partial w_{3t}\partial w_{1t}} & \dfrac{\partial^2 C_t}{\partial w_{3t}\partial w_{2t}} & \dfrac{\partial^2 C_t}{\partial w_{3t}\partial w_{3t}} \end{bmatrix} \tag{6.1.14}$$

Before proceeding further, define the following:

$$g(\mathbf{q},\mathbf{w}) = \frac{f(\mathbf{w})}{h(\mathbf{w})} i(q_{2t})$$

$$f(\mathbf{w}) = -\left(\begin{array}{l} \frac{1}{2} a_{11,2}^2 w_{1t}^2 + a_{11,2} a_{21,2} w_{1t} w_{2t} - \left(a_{11,2}^2 + a_{11,2} a_{21,2}\right) w_{1t} w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right) w_{2t}^2 - \left(a_{11,2} a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right) w_{2t} w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2} a_{21,2} + a_{21,2}^2 + a_{22,2}^2) w_{3t}^2 \end{array} \right)$$

$$i(q_{2t}) = \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$h(\mathbf{w}) = \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)$$

By the quotient rule,

$$\frac{\partial g(\mathbf{q},\mathbf{w})}{\partial w_{it}} = \left(\frac{f_i(\mathbf{w})}{h(\mathbf{w})} - \frac{f(\mathbf{w}) h_i(\mathbf{w})}{h^2(\mathbf{w})} \right) i(q_{2t}) \tag{6.1.15}$$

The second-order derivative of the cost function with respect to input price w_i is

$$\frac{\partial^2 C_t}{\partial w_{it}\partial w_{jt}} = \frac{\partial^2 g(\mathbf{w})}{\partial w_{it}\partial w_{jt}} i(q_{2t}),\ i, j = \{1,2,3\} \tag{6.1.16}$$

$$\frac{\partial^2 g(\mathbf{q},\mathbf{w})}{\partial w_{it}\partial w_{jt}} = \left(\frac{\partial}{\partial w_{jt}} \left(\frac{f_i(\mathbf{w})}{h(\mathbf{w})} \right) - \frac{\partial}{\partial w_{jt}} \left(\frac{f(\mathbf{w}) h_i(\mathbf{w})}{h^2(\mathbf{w})} \right) \right) i(q_{2t}) \tag{6.1.17}$$

where

$$\frac{\partial}{\partial w_{jt}} \left(\frac{f_i(\mathbf{w})}{h(\mathbf{w})} \right) = \frac{f_{ij}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_i(\mathbf{w}) h_j(\mathbf{w})}{h^2(\mathbf{w})} \tag{6.1.18}$$

$$\frac{\partial}{\partial w_{jt}} \left(\frac{f(\mathbf{w}) h_i(\mathbf{w})}{h^2(\mathbf{w})} \right) = \frac{\partial}{\partial w_{jt}} \frac{\left(f(\mathbf{w}) h_i(\mathbf{w}) \right)}{h^2(\mathbf{w})} - \frac{2 f(\mathbf{w}) h_i(\mathbf{w}) h_j(\mathbf{w})}{h^3(\mathbf{w})} \tag{6.1.19}$$

By the product rule,

$$\frac{\partial}{\partial w_{jt}} f(\mathbf{w}) h_i(\mathbf{w}) = f_j(\mathbf{w}) h_i(\mathbf{w}) + f(\mathbf{w}) h_{ij}(\mathbf{w})$$

However, since

$$h_{ij}(\mathbf{w}) = 0,$$

$$\frac{\partial}{\partial w_{jt}} \frac{\left(f(\mathbf{w}) h_i(\mathbf{w}) \right)}{h^2(\mathbf{w})} = \frac{\left(f_j(\mathbf{w}) h_i(\mathbf{w}) \right)}{h^2(\mathbf{w})}.$$

Therefore

$$\frac{\partial}{\partial w_{jt}}\left(\frac{f(\mathbf{w})h_i(\mathbf{w})}{h^2(\mathbf{w})}\right)=\frac{\left(f_j(\mathbf{w})h_i(\mathbf{w})\right)}{h^2(\mathbf{w})}-\frac{2f(\mathbf{w})h_i(\mathbf{w})h_j(\mathbf{w})}{h^3(\mathbf{w})}$$

Therefore (6.1.17) is

$$\frac{\partial^2 g(\mathbf{q},\mathbf{w})}{\partial w_{it}\partial w_{jt}}=\left(\frac{f_{ij}(\mathbf{w})}{h\ \ (\mathbf{w})}-\frac{f_i(\mathbf{w})h_j(\mathbf{w})}{h^2(\mathbf{w})}-\frac{f_j(\mathbf{w})h_i(\mathbf{w})}{h^2(\mathbf{w})}+\frac{2f(\mathbf{w})h_i(\mathbf{w})h_j(\mathbf{w})}{h^3(\mathbf{w})}\right)i(q_{2t}) \tag{6.1.20}$$

In the case where $i = j$, (6.1.20) becomes

$$\frac{\partial^2 g(\mathbf{q},\mathbf{w})}{\partial w_{it}^2}=\left(\frac{f_{ii}(\mathbf{w})}{h\ \ (\mathbf{w})}-2\frac{f_i(\mathbf{w})h_i(\mathbf{w})}{h^2(\mathbf{w})}+2\frac{f(\mathbf{w})h_i^2(\mathbf{w})}{h^3(\mathbf{w})}\right)i(q_{2t}) \tag{6.1.21}$$

Specifically, define the $f_i(\mathbf{w})$

$$f_1(\mathbf{w})=-\left(a_{11,2}^2 w_{1t}+a_{11,2}a_{21,2}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{3t}\right)$$

$$f_2(\mathbf{w})=-\left(a_{11,2}a_{21,2}w_{1t}+\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}\right)$$

$$f_3(\mathbf{w})=-\begin{pmatrix}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}\\+(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}\end{pmatrix}$$

Thus define $f_{ij}(\mathbf{w})$

$$f_{11}(\mathbf{w})=-a_{11,2}^2,\ \ f_{12}(\mathbf{w})=-a_{11,2}a_{21,2},\ \ f_{13}(\mathbf{w})=a_{11,2}^2+a_{11,2}a_{21,2}$$

$$f_{21}(\mathbf{w})=-a_{11,2}a_{21,2},\ \ f_{22}(\mathbf{w})=-\left(a_{21,2}^2+a_{22,2}^2\right),\ \ f_{23}(\mathbf{w})=a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2$$

$$f_{31}(\mathbf{w}) = a_{11,2}^2 + a_{11,2}a_{21,2}, \ f_{32}(\mathbf{w}) = a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2,$$

$$f_{33}(\mathbf{w}) = -(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)$$

Define the $h_i(\mathbf{w})$

$$h_1(\mathbf{w}) = \theta_1, \ h_2(\mathbf{w}) = \theta_2, \ h_3(\mathbf{w}) = \theta_3$$

Hence from (6.1.21)

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{1t}\partial w_{1t}} = \left(\frac{f_{11}(\mathbf{w})}{h\ (\mathbf{w})} - 2\frac{f_1(\mathbf{w})h_1(\mathbf{w})}{h^2(\mathbf{w})} + 2\frac{f(\mathbf{w})h_1^2(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$\frac{\partial^2 C_t}{\partial w_{1t}\partial w_{1t}} = \left(\begin{array}{l} \dfrac{-a_{11,2}^2}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)} \\ + 2\dfrac{\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)\theta_1}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)^2} \\ - \dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array} \right)\theta_1^2}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{1t}\partial w_{2t}} = \left(\frac{f_{12}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_1(\mathbf{w})h_2(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_2(\mathbf{w})h_1(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_1(\mathbf{w})h_2(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence,

$$\frac{\partial^2 C_t}{\partial w_{1t}\partial w_{2t}} = \left(\begin{array}{l} \dfrac{-a_{11,2}a_{21,2}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ + \dfrac{\left(a_{11,2}a_{21,2}w_{1t} + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ + \dfrac{\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ - \dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array} \right)\theta_1\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{1t}\partial w_{3t}} = \left(\frac{f_{13}(\mathbf{w})}{h\ \ (\mathbf{w})} - \frac{f_1(\mathbf{w})h_3(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_3(\mathbf{w})h_1(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_1(\mathbf{w})h_3(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$\frac{\partial^2 C_t}{\partial w_{1t}\partial w_{3t}} = \left(\begin{array}{l} \dfrac{a_{11,2}^2 + a_{11,2}a_{21,2}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ + \dfrac{\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ + \dfrac{\left(\begin{array}{l} -\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{array}\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ - \dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array}\right)\theta_1\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{2t}\partial w_{1t}} = \left(\frac{f_{21}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_2(\mathbf{w})h_1(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_1(\mathbf{w})h_2(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_2(\mathbf{w})h_1(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$
\frac{\partial^2 C_t}{\partial w_{2t}\partial w_{1t}} = \begin{pmatrix}
\dfrac{-a_{11,2}a_{21,2}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+\dfrac{\left(a_{11,2}a_{21,2}w_{1t} + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\
+\dfrac{\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\
-\dfrac{2\begin{pmatrix}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{pmatrix}\theta_1\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3}
\end{pmatrix}\left(1+\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\right)^2
$$

$$
\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{2t}\partial w_{2t}} = \left(\frac{f_{22}(\mathbf{w})}{h\ (\mathbf{w})} - 2\frac{f_2(\mathbf{w})h_2(\mathbf{w})}{h^2(\mathbf{w})} + 2\frac{f(\mathbf{w})h_2^2(\mathbf{w})}{h^3(\mathbf{w})}\right)i(q_{2t})
$$

Hence

$$
\frac{\partial^2 C_t}{\partial w_{2t}\partial w_{2t}} = \begin{pmatrix}
\dfrac{-\left(a_{21,2}^2 + a_{22,2}^2\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+2\dfrac{\left(a_{11,2}a_{21,2}w_{1t} + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}\right)\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\
-\dfrac{2\begin{pmatrix}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{pmatrix}\theta_2^2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3}
\end{pmatrix}\left(1+\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\right)^2
$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{2t}\partial w_{3t}} = \left(\frac{f_{23}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_2(\mathbf{w})h_3(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_3(\mathbf{w})h_2(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_2(\mathbf{w})h_3(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$\frac{\partial^2 C_t}{\partial w_{2t}\partial w_{3t}} = \left(\begin{array}{l} \dfrac{a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)} \\ + \dfrac{\left(\begin{array}{l} -(a_{11,2}^2 + a_{11,2}a_{21,2})w_{1t} - (a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{2t} \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{array} \right)\theta_2}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)^2} \\ + \dfrac{\left(a_{11,2}a_{21,2}w_{1t} + (a_{21,2}^2 + a_{22,2}^2)w_{2t} - (a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}\right)\theta_3}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)^2} \\ - \dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - (a_{11,2}^2 + a_{11,2}a_{21,2})w_{1t}w_{3t} \\ + \frac{1}{2}(a_{21,2}^2 + a_{22,2}^2)w_{2t}^2 - (a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array} \right)\theta_2\theta_3}{(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{3t}\partial w_{1t}} = \left(\frac{f_{31}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_3(\mathbf{w})h_1(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_1(\mathbf{w})h_3(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_3(\mathbf{w})h_1(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$
\frac{\partial^2 C_t}{\partial w_{3t}\partial w_{1t}} = \left(
\begin{array}{l}
\dfrac{a_{11,2}^2 + a_{11,2}a_{21,2}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+\dfrac{\left(\begin{array}{l} -\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\ +\,(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{array}\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\
+\dfrac{\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\
-\dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array}\right)\theta_3\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3}
\end{array}
\right)\left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2
$$

$$
\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{3t}\partial w_{2t}} = \left(\frac{f_{32}(\mathbf{w})}{h\ (\mathbf{w})} - \frac{f_3(\mathbf{w})h_2(\mathbf{w})}{h^2(\mathbf{w})} - \frac{f_2(\mathbf{w})h_3(\mathbf{w})}{h^2(\mathbf{w})} + \frac{2f(\mathbf{w})h_3(\mathbf{w})h_2(\mathbf{w})}{h^3(\mathbf{w})}\right)i(q_{2t})
$$

Hence

$$\frac{\partial^2 C_t}{\partial w_{3t}\partial w_{2t}} = \left(\begin{array}{l} \dfrac{a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ + \dfrac{\left(\begin{array}{l} -\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{array} \right)\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ + \dfrac{\left(a_{11,2}a_{21,2}w_{1t} + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}\right)\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ - \dfrac{2\left(\begin{array}{l} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{array} \right)\theta_3\theta_2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

$$\frac{\partial^2 g_2(\mathbf{q},\mathbf{w})}{\partial w_{3t}\partial w_{3t}} = \left(\frac{f_{33}(\mathbf{w})}{h\ (\mathbf{w})} - 2\frac{f_3(\mathbf{w})h_3(\mathbf{w})}{h^2(\mathbf{w})} + 2\frac{f(\mathbf{w})h_3^2(\mathbf{w})}{h^3(\mathbf{w})} \right) i(q_{2t})$$

Hence

$$\frac{\partial^2 C_t}{\partial w_{3t}\partial w_{3t}} = \left(\begin{array}{l} \dfrac{-(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ +2\dfrac{\begin{pmatrix} -\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\ +(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{pmatrix}\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \\ -\dfrac{2\begin{pmatrix} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{pmatrix}\theta_3^2}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^3} \end{array} \right) \left(1 + \left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2$$

The cost function satisfies condition (1) by construction. However, confirmation that concavity is imposed in Table 6.5. Note that since input prices, output quantity and θ_i are strictly positive, it is the matrix of estimated coefficients $\mathbf{\Sigma}$ that determines concavity of the Hessian matrix. Hence, it is sufficient to report the determinants of $\mathbf{\Sigma}$ rather than the Hessian matrix. The calculated determinants as reported in Table 6.5 clearly indicate that the cost function is concave in input prices.[23]

Table 6.6 presents the calculated marginal costs for the years 1954 to 1990, which are calculated according to,

[23] A necessary and sufficient condition for negative semi-definiteness of the 3×3 Hessian matrix is that the determinants alternate in sign, i.e. $|H_1| \le 0$, $|H_2| \ge 0$ and $|H_3| \le 0$.

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}} = \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{11}q_{1t}^{\lambda_1-1}\left(q_{1t}^{\lambda_1}-1\right)+d_{12}q_{1t}^{\lambda_1-1}\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \tag{6.1.22}$$

$$\begin{aligned}\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = & -\frac{\begin{pmatrix} a_{11}^2w_1^2 + 2a_{11}a_{21}w_1w_2 - 2\left(a_{11}^2 + a_{11}a_{21}\right)w_1w_3 \\ + \left(a_{21}^2 + a_{22}^2\right)w_2^2 - 2\left(a_{11}a_{21} + a_{21}^2 + a_{22}^2\right)w_2w_3 \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_3^2 \end{pmatrix}}{\left(w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3\right)} \\ & \times 2q_{2t}^{\lambda_2-1}\left(1+\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \\ & +\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{12}q_{2t}^{\lambda_2-1}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_{22}q_{2t}^{\lambda_2-1}\left(q_{2t}^{\lambda_2}-1\right)\right)\end{aligned} \tag{6.1.23}$$

Table 6.6 confirms that marginal costs are positive for the years 1954 to 1990. Thus, the estimated cost function satisfies the necessary conditions for a proper cost function.

Table 6.6. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{x}_{1t}/x_{1t}$	$\hat{x}_{2t}/x_{2t}$	$\hat{x}_{3t}/x_{3t}$
1954	1.61	-	1.00	1.00	1.00
1955	1.72	-	1.02	0.99	1.03
1956	1.74	-	1.02	1.01	0.98
1957	1.63	-	0.97	1.01	1.03
1958	1.64	-	1.01	0.99	1.01
1959	1.69	-	1.00	1.01	1.04
1960	1.60	-	1.18	1.03	0.98
1961	1.73	-	0.87	1.01	0.96
1962	1.77	-	0.98	1.00	0.97
1963	1.59	-	1.02	1.00	1.02
1964	1.52	-	1.05	1.01	1.00
1965	1.54	-	1.01	0.99	1.03
1966	1.64	-	0.97	0.98	1.04
1967	1.61	-	0.97	0.99	0.92
1968	1.72	-	1.02	0.99	1.05
1969	1.41	-	0.99	0.93	0.82
1970	1.30	14,200.87	0.98	1.04	0.97
1971	1.20	7,345.61	1.02	0.79	0.93
1972	1.35	12,140.46	1.11	0.99	1.07
1973	1.23	9,210.35	1.05	0.98	0.90
1974	1.50	10,251.70	0.92	1.00	0.90
1975	1.81	10,603.76	0.96	0.99	0.97
1976	2.28	11,945.43	1.09	0.95	1.23
1977	2.80	12,616.99	0.97	0.94	1.05
1978	2.69	10,870.49	1.01	0.98	0.99
1979	2.84	10,126.97	0.92	1.03	0.96
1980	3.08	10,036.51	0.90	1.06	0.98
1981	3.19	9,859.82	0.92	1.01	0.97
1982	3.76	10,894.65	0.94	1.03	1.05
1983	3.82	10,490.94	1.02	1.00	1.02
1984	3.89	10,442.79	1.07	0.99	0.97
1985	3.90	9,897.03	1.07	1.00	1.03
1986	4.08	10,104.95	1.04	1.02	1.01
1987	4.56	11,107.20	1.00	1.00	1.04
1988	4.08	11,743.01	1.09	1.02	1.34
1989	3.83	10,830.61	0.92	1.00	0.86
1990	4.05	11,523.99	0.98	0.97	1.01

Note. $\hat{x}_{it}/x_{it}$ denotes the ratio of estimated to actual factor demand. Data services introduced in 1970. Hence, marginal cost for data services is zero prior to 1970.

2. ANALYSIS OF THE EMPIRICAL RESULTS

Having demonstrated that the reported cost function is proper, this section considers the implications of some of the parameters presented in Table 6.1. The final specification is the outcome of an iterative approach to identify a proper cost function. In that context, the *exchange* variables serve primarily as controls for time-based heterogeneity caused by changes in technology and subscriber numbers. Estimation without these variables leads to detection of serial correlation in the residuals corresponding to (6.1.1) and a negatively signed intercept in (6.1.2).

As explained by Wilson and Zhou (1997) access lines (or in this case subscribers) can be treated as either an input or an output. Given the *exchange* variables separate subscribers by technology, the variables also have a technical change interpretation. The first *exchange* variable, appearing in (6.1.1) and (6.1.3) is the number of subscribers connected to digital telephone exchanges in the network. The coefficient f_Z indicates that the demand for aggregate capital and labour increase with the number of digital exchange subscribers compared with subscribers connected with non-digital exchange technology.[24] Thus, digital technology is relatively capital-intensive while the additional labour reflects increased effort associated with installing a new technology.

Technical change parameters f_{Z1} and f_{Z2} indicate that the demand for materials varies in direct proportion to the number of cross-bar and old (manual and step-by-step) exchanges. Positive sign on the coefficients again suggests that decreasing the number of subscribers connected to cross-bar and old exchange technologies decreases materials

[24] Although the coefficient is not statistically different from zero at conventional levels.

cost. The relatively high materials demand is consistent with the older technologies requiring a higher level of maintenance relative to digital technology. However, since the number of non-digital technology exchanges is decreasing throughout the sample period, it is clear that the reduction in utilisation of non-digital exchange technology reduces the demand for materials.

Coefficient c_{t3} suggests that, controlling for substitution between inputs, the demand for labour increases with time. Since there is no explicit labour-quality differentiation through time, the positive c_{t3} coefficient may reflect a need for more highly trained labour, which in this model, is translated to a higher quantity of homogenous labour.

With respect to the output arguments, it is clear that there is a non-linear input-output relationship. Since the form of the outputs is crucial in this thesis, experimentation is conducted to determine if other plausible specifications yield contradictory results. Setting $\lambda_1, \lambda_2 = 1$ yields negative marginal cost for data while setting the parameters arbitrarily close to zero generates negative marginal cost for aggregate output. Expanding the output arguments to include first-order terms generates negative marginal cost for aggregate output. Simultaneously adjusting the λ_1, λ_2 and including the first-order output arguments also fails to yield a proper cost function. Retaining the first-order output arguments and making other adjustments results in the model presented in Appendix 3. Comparing the two models, it is clear that despite important specification differences, both suggest increasing long-run returns to scale and cost complementarity.

With respect to the model presented in this chapter, cost complementarity, defined as $\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}\partial q_{2t}} = d_{12} q_{1t}^{\lambda_1 - 1} q_{1t}^{\lambda_2 - 1} < 0$, is evident since $d_{12} < 0$. Indeed, an increase in output q_{1t} decreases the marginal cost of producing q_{2t} and vice versa. Since data is measured as the number of customers subscribing to data services, d_{12} indicates that the cost of aggregate output is reduced as the number of data subscribers increases. However, given that the d_{22} coefficient is 40 times larger than d_{12}, cost increase associated with adding data subscribers dominates.

Table 6.7 presents the proportion of total short-run fixed cost to total actual cost (C_t^F / C_t), variable cost to equilibrium cost (VC_t / C_t^*), equilibrium cost to actual cost (C_t^* / C_t), short-run returns elasticity of scale (RS_t) and equilibrium elasticity of scale (RS_t^*) for the models presented in Table 6.1a and Table 6.1b. The elasticities of scale are calculated according to

$$RS_t = \frac{1}{\varepsilon_{CQ_{1t}} + \varepsilon_{CQ_{2t}}} \text{ where } \varepsilon_{CQ_{it}} = \frac{\partial C_t}{\partial Q_{it}} \frac{Q_{it}}{C_t},\ i = \{1,2\}$$

while

$$RS_t^* = \frac{1}{\varepsilon_{CQ_{t1}}^* + \varepsilon_{CQ_{t2}}^*} \text{ where } \varepsilon_{CQ_{it}}^* = \frac{\partial C_t^*}{\partial Q_{it}} \frac{Q_{it}}{C_t^*},\ i = \{1,2\}.$$

Thus RS_t shows the short-run percentage change in combined output given a uniform one percent increase in the volume of inputs. Similarly, RS_t^* shows the equilibrium percent change in output given a uniform one percent increase in equilibrium inputs. The returns to scale numbers should be interpreted as single point observations of the ratio of average to marginal cost by year. That is, these numbers correspond to average and variable cost functions that shift according to changes in input prices, quantities and technological changes. As shown, fixed cost accounts for 55% to 86% of total cost, peaking in 1971 and trending down thereafter. The difference in short-run and equilibrium cost is reflected in a substantial difference in short-run and equilibrium returns to scale. This difference may be the result of economies of size, where one of the inputs, such as land, is adjusted at a much slower rate than the other inputs. The equilibrium returns to scale reflects the full adjustment of all inputs, thus resulting in a proportionally lower fixed cost.

Although substantially smaller in magnitude, equilibrium returns to scale is substantially higher than measures typically reported in UK and the US studies. For example, Bernstein (1989) reports returns to scale range between 0.92 for 1955 to 2.91 for 1978. Cooper et al. (2003) report returns to scale that range from 1.28 in 1947 to 1.17 in 1977. Correa (2003) reports average returns to scale of 1.03. Diewert and Wales (1991) report a returns to scale range of 0.62 to 1.41. Shin and Ying (1992) report average returns to scale of 1.04. This is not surprising given both the UK and the US telecommunications networks produce substantially larger volumes of output than the Australian telecommunications network. The relatively small market size prevents the Australian telecommunications network from exhausting available economies of scale.

However, given the high returns to scale measures presented in Table 6.7, it is worthwhile to investigate the underlying cause. Subsequent analysis reveals that the most significant factor in determining the magnitude of the returns to scale elasticity is the value of λ_1. In order to demonstrate the impact, the average and marginal cost curves corresponding to the year 1990 are presented in Figure 6.2 and Figure 6.3. The only difference between these two charts is the value of λ_1. Figure 6.2 shows the average and marginal cost curves with $\lambda_1 = 0.25$. Figure 6.3 shows the average and marginal cost curves with $\lambda_1 = 0.60$. The charts show that the slope of both the average and marginal cost curves depends on the value of λ_1. Indeed, setting $\lambda_1 = 0.50$ results in horizontal average and marginal cost curves.

Hence, the value of λ_1 impacts on the returns to scale measure, since returns to scale is equivalent to the ratio of average over marginal cost. The average returns to scale for Figure 6.2 is 2.09, whereas the average returns to scale for Figure 6.3 is 0.83. Setting $\lambda_1 = 0.50$ results in an average returns to scale of 0.99. However, despite this sensitivity, the results in Table 6.7 clearly show increasing returns to scale within the range reported by Bernstein (1989).

In summary, the results reveal a cost structure that exhibits both increasing returns to scale and cost complementarity. Inspection of the alternative specifications presented in the appendices indicates that increasing returns to scale and cost complementarity is robust to specification changes. Technical change has apparently caused a shift away from materials in favour of an increase in the demand for capital and labour.

Table 6.7. Fixed and variable cost contribution to total cost

	Model 1			Model 2		
YEAR	C_t^F/C_t	RS_t	RS_t^*	C_t^F/C_t	RS_t	RS_t^*
1954	0.70	6.74	2.17	0.95	16.84	2.66
1955	0.70	6.63	2.19	0.95	16.11	2.78
1956	0.70	6.70	2.19	0.95	15.49	2.71
1957	0.74	7.57	2.15	0.95	17.23	2.29
1958	0.74	7.72	2.16	0.95	17.14	2.33
1959	0.74	7.57	2.17	0.95	16.04	2.40
1960	0.73	7.54	2.19	0.94	15.04	2.44
1961	0.75	8.00	2.15	0.95	15.34	2.06
1962	0.75	7.97	2.17	0.94	15.05	2.18
1963	0.76	8.32	2.15	0.94	14.87	1.95
1964	0.76	8.28	2.16	0.94	14.05	1.97
1965	0.76	8.47	2.15	0.94	13.74	1.88
1966	0.76	8.32	2.17	0.94	13.07	1.99
1967	0.77	8.75	2.20	0.94	13.16	1.93
1968	0.78	8.89	2.26	0.94	13.22	2.07
1969	0.83	11.45	2.18	0.95	16.58	1.53
1970	0.82	11.40	2.16	0.93	17.29	1.81
1971	0.86	13.77	2.04	0.94	20.85	1.41
1972	0.82	11.01	2.13	0.91	15.23	1.84
1973	0.83	11.80	2.08	0.91	15.66	1.71
1974	0.82	11.37	2.06	0.90	14.39	1.69
1975	0.82	10.96	2.04	0.88	13.54	1.75
1976	0.78	9.01	2.08	0.84	10.83	1.96
1977	0.74	7.79	2.10	0.81	9.16	2.10
1978	0.73	7.52	2.09	0.79	8.62	2.09
1979	0.73	7.48	2.09	0.78	8.43	2.11
1980	0.72	7.16	2.12	0.76	7.91	2.13
1981	0.72	7.15	2.19	0.75	7.77	2.16
1982	0.70	6.69	2.52	0.73	7.15	2.45
1983	0.68	6.28	2.58	0.71	6.63	2.50
1984	0.66	5.94	2.76	0.69	6.18	2.64
1985	0.64	5.50	2.91	0.66	5.65	2.76
1986	0.62	5.21	2.96	0.63	5.28	2.78
1987	0.59	4.89	2.97	0.61	4.92	2.78
1988	0.50	4.03	2.86	0.53	3.58	2.37
1989	0.56	4.56	3.03	0.58	4.04	2.47
1990	0.55	4.43	3.03	0.57	3.84	2.43

Figure 6.2 Average, marginal cost for 1990. $\lambda_1 = 0.25$

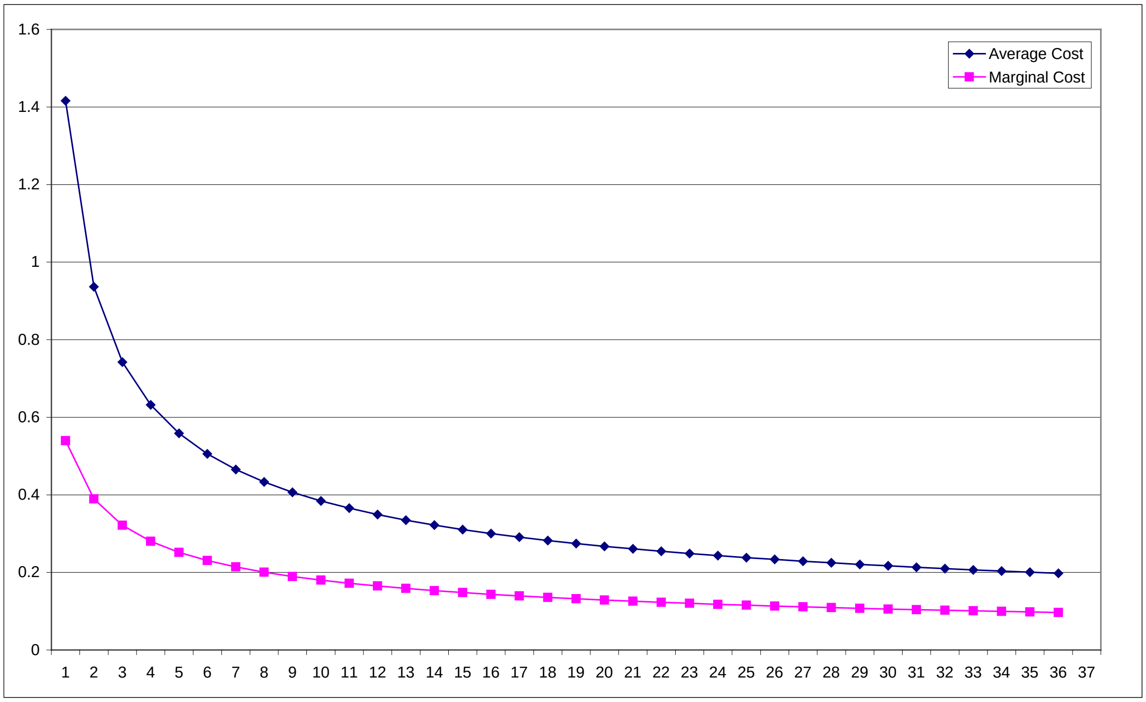
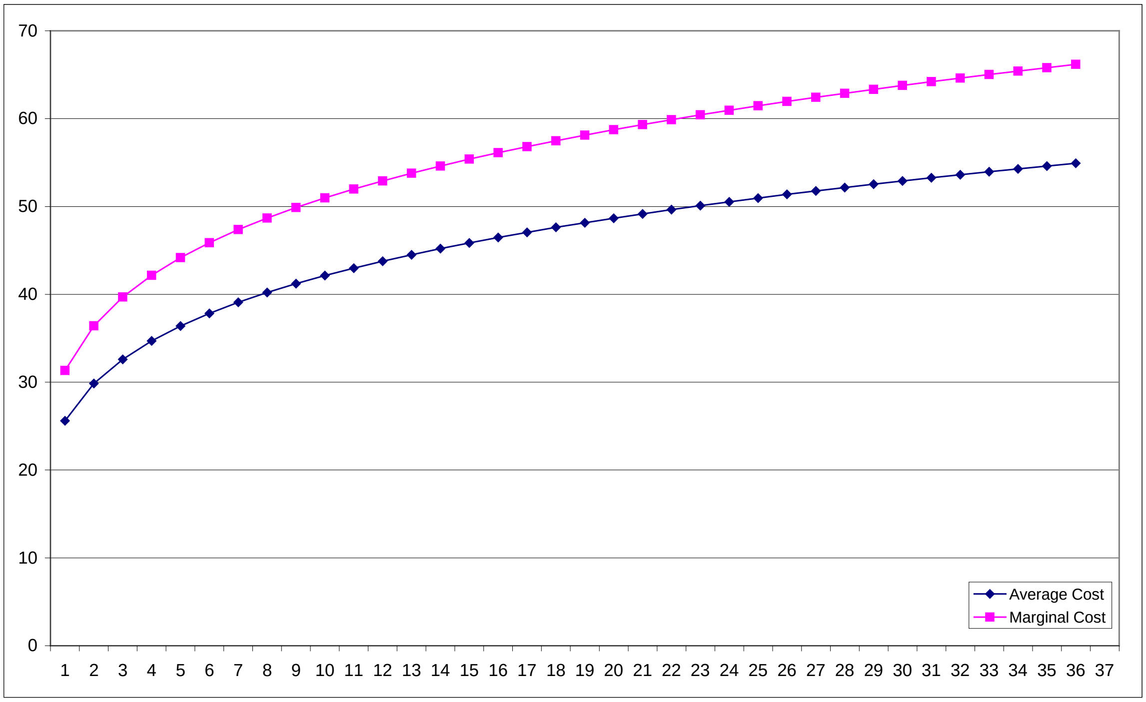


Figure 6.3 Average, marginal cost for 1990. $\lambda_1 = 0.60$



3. SUBADDITIVITY TEST RESULTS

A summary of the subadditivity calculations is presented in Table 6.8 to Table 6.10, inclusive. Table 6.8 assumes no change in fixed cost between the monopoly and hypothetical duopoly cases. In contemporary policy terms, this can be considered an 'open access regime' in which the incumbent telecommunications carrier provides non-discriminatory access to its competitor. Note that technology and input prices are fixed across columns and vary across rows. The parameter ϕ corresponds to market share of aggregate output for firm A and ω corresponds to Firm A's data output. For example $(\phi,\omega)=50{,}10$ indicates that Firm A has 50% market share of aggregate output and has 10% market share in data. Underlying calculations specify no change in fixed cost for monopoly and duopoly. Inspection across rows suggests that duopoly could have provided an efficiency gain from 1960 onwards regardless of Firm A and Firm B market shares of output. Inspection across columns reveals that SUB is highest for $(\phi,\omega)=50{,}50$ indicating that the efficiency gain through duopoly is maximised when total output volumes are split evenly across competitors.

Table 6.8. Subadditivity calculations (%) — no difference in fixed cost

Year	(ϕ,ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	24.71	24.71	24.71	**29.16**	**29.16**	22.21	22.21
1970	19.51	21.44	20.26	21.92	**23.21**	19.99	17.42
1975	19.51	23.13	19.52	21.21	**24.10**	20.59	16.93
1980	26.70	33.76	25.36	28.43	**34.55**	28.36	22.28
1985	29.96	38.15	27.33	31.03	**38.52**	31.34	25.14
1990	37.88	46.88	33.49	38.16	**46.92**	38.40	32.80

Note. Maximum SUB in each row is printed in bold type. SUB is calculated according to equation (3.2.8). SUB>0 indicates duopoly industry cost is less than the industry cost incurred via a monopoly provider.

Table 6.9 provides more detail for the case in which Firm A is the dominant carrier for aggregate output, but has varying market share in data. Reading from left to right, Firm B market share in data declines across columns. For example, the left column (i.e. when $(\phi,\omega)=100{,}10$) corresponds to the case in which Firm B is the dominant firm for data while in the furthest right hand column (i.e. $(\phi,\omega)=100{,}100$), Firm B produces zero data output. The results show that efficiency gain to duopoly peaks between 30% to 50% of market share in data.

Table 6.9. Subadditivity calculations for $(100,\omega)$ (%)

Year	(ϕ,ω)									
	(100,10)	(100,20)	(100,30)	(100,40)	(100,50)	(100,60)	(100,70)	(100,80)	(100,90)	(100,100)
1960	22.21	22.21	22.21	22.21	22.21	22.21	22.21	22.21	22.21	22.21
1970	19.99	20.34	20.54	**20.58**	20.45	20.17	19.72	19.12	18.35	17.42
1975	20.59	21.63	22.31	**22.62**	22.58	22.17	21.40	20.27	18.78	16.93
1980	28.36	30.75	32.37	33.22	**33.31**	32.64	31.20	28.99	26.02	22.28
1985	31.34	34.39	36.51	37.70	**37.95**	37.26	35.63	33.07	29.57	25.14
1990	38.40	42.16	44.82	46.39	**46.86**	46.24	44.52	41.71	37.80	32.80

Note. Maximum SUB in each row is printed in bold type.

Finally, Table 6.10 presents subadditivity results with the assumption that the duopoly case incurs a 30% increase in fixed cost compared to the monopoly case. This scenario could be considered to be a more realistic outcome as the market entrant partially duplicates the incumbent's telecommunications network. This result indicates that, even with partial duplication, competition in the Australian telecommunications network could still yield an efficiency gain from 1975 onwards.

Table 6.10. Subadditivity calculations (%) — 30% increase in fixed cost

Year	(ϕ,ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	6.37	6.37	6.37	**10.82**	**10.82**	3.87	3.87
1970	-1.24	0.70	-0.49	1.18	**2.46**	-0.76	-3.32
1975	-0.89	2.73	-0.88	0.81	**3.70**	0.20	-3.47
1980	10.52	17.58	9.18	12.25	**18.37**	12.18	6.10
1985	15.36	23.56	12.74	16.44	**23.93**	16.74	10.54
1990	26.64	35.64	22.25	26.91	**35.67**	27.15	21.55

Note. Maximum SUB in each row is printed in bold type.

Given that other results are presented in the appendices, it is prudent to compare the variation in the subadditivity results with those reported in this chapter. Appendix 2 broadly confirms the results, that is that duopoly offers efficiency gain provided fixed cost increases by no more than 30% after the introduction of competition. The remaining results indicate varying degrees of subadditivity. In some cases, there is an efficiency loss associated with competition. However, the trends evident in the alternative results suggest that natural monopoly is diminishing with time to the point where additivity is a plausible conclusion beyond 1990.

The subadditivity test results are broadly comparable to Evans and Heckman (1984) and Diewert and Wales (1991) in conclusion, *viz*. that the Australian telecommunications network, like that of the US Bell System is not subadditive. However, the results presented in this thesis qualifies the conclusion by showing that subadditivity can be induced of competitors are forced to duplicate a substantial portion of the fixed elements of the network. As shown in Table 6.10, global subadditivity would require duplication well in excess of 30%. Indeed, experimentation based on the model presented in this chapter suggests that a fixed cost would have to double to induce global subadditivity.

4. CHAPTER CONCLUSION

This chapter provides an estimated proper cost function for the Australian telecommunications industry. The presented model is novel in two respects: it specifies the number of data subscribers as an output that is separate from aggregate output; and it applies a dynamic version of the modified generalised McFadden augmented by Box-Cox transformed output variables. Econometric tests show no evidence of heteroskedasticity or serial correlation. Overall, experimental results indicate that natural monopoly in the Australian telecommunications network largely depends on the extent to which competitors must duplicate the incumbent’s fixed cost. A regulatory regime can be considered effective provided an increase in fixed cost associated with competitive supply does not exceed 30% of the fixed cost incurred under mandated monopoly. Note, however, that an effective open access regime in which fixed costs are not duplicated at all offers the largest possible efficiency gain.

CHAPTER 7—CONCLUSION

This thesis applies the Evans and Heckman (1984) test for subadditivity to Australian telecommunications industry data corresponding to the years 1954 to 1990, inclusive. As explained in Chapter 1, the test for subaddivity is a formal test for natural monopoly. The study is concerned with determining whether allowing competition in the Australian telecommunications industry results in a net economic cost or benefit. The study also permits an evaluation of alternative strategies within competition policy. For example, the study measures how much duplication in Australia's network can be tolerated before competition becomes economically more costly than monopoly. This in turn, informs policy makers on the matter of how strictly the incumbent telecommunications provider should be regulated to ensure preservation of the economic benefits possible through competitive mechanisms.

A subsidiary aim of the study is to assess the impact of fixed and variable cost on industry cost and to what extent these influence the cost outcomes of competition vis-à-vis monopoly. In doing so, it is important to isolate various possible influences such as technological change, economies of size, scale and scope on the cost structure of the telecommunications industry.

Given the multitude of possible influences on industry cost, it is necessary to develop a thorough understanding of the telecommunications industry. To facilitate such understanding, Chapter 2 provides a detailed account of Australia's telecommunications industry. The historical account provided in Chapter 2 shows that the telecommunications network has been in continual evolution. Beginning with the telegraph network, section 1 highlights the transition from manual operations to automation. Section 2 describes the developments in switching technology in the telephone network, while section 3 provides an account of the changes in transmission technology. Switching and transmission developments are the two fundamental drivers of telephone efficiency and are likely to have contributed to economies of scale over the years. Section 4 provides a brief account of the data network.

Following from the historical outline, Chapter 3 sets out the analytical framework. This begins with Baumol's seminal paper on the proper test for natural monopoly in a multiproduct industry. Section 1 explains what is meant by subadditivity and sets out the sufficient conditions. The section provides some examples of the two concepts necessary to analyse the nature of subadditivity: ray average cost; and transray convexity. The section also points out the empirical nature of subadditivity, in particular that satisfaction of the sufficient conditions for subadditivity is not necessary.

Section 2 draws together the insights of previous empirical tests for subadditivity in the telecommunications industry conducted around the world. The literature review shows that estimation of a cost function that is consistent with received economic theory has proven to be challenging. The main points drawn from this analysis is that: the subadditivity test is highly sensitive to the econometric specification, particularly functional form; and estimated cost functions have been found to violate the conditions of a proper cost function, particularly the generation of negative marginal cost.

The remainder of the chapter sets out the strategy adopted in the thesis. First, section 3 explains the application of the test for subadditivity as proposed by Evans and Heckman. The section also traces the possible sources of subadditivity analytically. Having defined the test, section 4 sets out the cost function theory, including the theoretical properties of a valid cost function, and explains how it can be estimated. Section 5 outlines the preferred function, Diewert and Wales's symmetric generalised McFadden. Section 6 considers various augmentations of the model necessary to adequately control for technological change. Section 7 introduces notions of adjustment cost, which leads to a detailed discussion of modelling strategies designed to adequately control for adjustment cost.

The estimation strategy is outlined in greater detail in Chapter 4. This begins in section 1 with a discussion of how the symmetric generalised McFadden cost function can be modified to allow a multiple output specification. The section also shows how concavity of the cost function with respect to input prices can be imposed. Sections 2 and 3 outline the other model augmentations needed to control for technological change and for the functional form applied specification associated with the output arguments. Section 4 provides a detailed description of the controls for adjustment cost. Sections 5, 6 and 7 derive the factor demand equations and show how the augmentations combine to create an estimable set of equations. Section 8 subsequently shows how the marginal and fixed cost components of the cost function can be calculated once the factor demand equations have been estimated.

A fully specified cost function model and strategy thus identifies the variables required in order to estimate the model. Chapter 5 discusses the source data and explains how these are used to construct the variables. The inputs (capital, labour and materials) and outputs are described in section 1 along with the decomposition of expenditure categories into price and quantity. Section 2 then presents summary statistics to examine how the inputs and outputs have varied over the sample period. Section 3 follows with a complete list of the constructed variables.

Chapter 6 presents the estimation results. The chapter shows that the estimated cost function satisfies all of the theoretical requirements for a cost function and auxiliary test statistics fail to reject the null hypotheses of no serial correlation and homoskedasticity. Subadditivity test results indicate that single firm provision of aggregate and data services in the Australian telecommunications industry is less efficient than two hypothetical firms if fixed cost remains unchanged. However, even a 30% increase in fixed cost fails to induce subadditivity. In reality, the most likely cause of an increase in fixed cost is network duplication.

Another important finding is that measures of subadditivity are sensitive to controls for technological change. Hence, an important caveat with respect to applying inferences to contemporary telecommunications technology is that the estimated cost function does not capture the impact of packet switching, which would have substantially enhanced the incumbent's operating efficiency from 1991 onwards.

The share of fixed cost in total cost, reported in Table 6.7, Chapter 6 also has policy implications. The sample period, 1954 to 1990, coincides with a substantial expansion of the telecommunications network and increasing network density. The rate of increase in network size, however, slows as time progresses and it is likely that expenditure on structures would have continued to trend down.

Utilisation of new vintages of capital employed since 1990 may also have helped reduce the fixed cost associated with operating the network. Notwithstanding these considerations, fixed cost is still likely to account for a substantial share of total cost.

Since fixed cost can be largely avoided by ceasing operations, the failure to provide subsidy is likely to lead to the complete removal of services in parts of Australia.

Overall, the thesis shows that had the Australian Government decided to allow regulated competition during the sample period (1954-1990) the cost of telecommunications services would not have been higher than service provision via the government-imposed monopoly. However, it is important to point out that it is likely that had the Australian Government allowed excessive network duplication, competition would have been more costly than the monopoly that was actually in place. Thus, it appears that some form of economic regulation in order to prevent excessive duplication would have been necessary. That is not to say, however, that the current regulatory regime would have been effective at preventing excessive duplication or have been better than alternative regulatory regimes. In particular, it is not clear that the Australian Government's current price control regulation regime would have been effective at all. Indeed, it may have been more effective to simply mandate competitor access to the incumbent's sunken infrastructure. This would have allowed sharing of exchange buildings, tunnels, conduits, towers etc.

There is substantial scope for future work. The current study says nothing about the impact of customer density on subadditivity. Despite substantial effort, it simply was not possible to capture customer density data. Some other ideas for future research based on the database presented in this thesis include an examination of the effects of internal service cannibalisation, the efficiency effects of joint provision of telegraph and telephone services, and advanced productivity analysis. To date, there appear to be no publicly available studies that have examined the efficiency of jointly producing telegraph, telex and telephone services. Anecdotal evidence suggests that

Australian telegraph and telephone services operated via separate networks. Perhaps these networks could have been constructed and operated more efficiently by separate firms. The data included in this thesis also provide the opportunity for development of econometric models that permit parameter variation as a way of estimating the efficiency impact of network heterogeneity.

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APPENDIX—DATA

Year	TelecomR	Loccalls	STDCalls	CallRev	CellCall	CelCalRv	CellSub	CelAcRev	CelCnFee	Subscrib
1920	6946654	249749892.6	12420205	2539151	0	0	0	0	0	172106
1921	7105174	245235542.2	12910956	2829030	0	0	0	0	0	183363
1922	8387508	220636629	13968169	3170419	0	0	0	0	0	195886
1923	8897752	233062627	15914582	3493791	0	0	0	0	0	214435
1924	9653364	257232340	18135444	3869448	0	0	0	0	0	242601
1925	10441838	273704896	21675450	4224122	0	0	0	0	0	278116
1926	11433456	300346103	26085231	4808164	0	0	0	0	0	309206
1927	12397648	335092110.5	29022130	5444392	0	0	0	0	0	338001
1928	13401888	372957519	32002070	6126424	0	0	0	0	0	363076
1929	14202626	407771550	34741671	6696360	0	0	0	0	0	384358
1930	14995670	420639538	35388715	7255882	0	0	0	0	0	395812
1931	13901270	397729885	30164077	6784116	0	0	0	0	0	376243
1932	13160080	368582696	28908085	6343376	0	0	0	0	0	363572
1933	13366224	378567877	29210983	6396946	0	0	0	0	0	363776
1934	13942586	396684056	30997010	6807784	0	0	0	0	0	372621
1935	14936250	436426094	32704626	7402946	0	0	0	0	0	396025
1936	16165304	478300566	35941033	8065156	0	0	0	0	0	417661
1937	17372150	530735347	37740865	8874466	0	0	0	0	0	440471
1938	18621294	558642667	40235501	9519784	0	0	0	0	0	465498
1939	19598750	596194307	41152181	10097742	0	0	0	0	0	487535
1940	20814332	621383964	42503383	10762856	0	0	0	0	0	506612
1941	21774294	642548833	43266460	11003190	0	0	0	0	0	525547
1942	24884618	664348028	45317116	12377354	0	0	0	0	0	531075
1943	28785784	639651471	47975812	12920896	0	0	0	0	0	540261
1944	31070256	663819693	51608901	14311810	0	0	0	0	0	557071
1945	32182928	698350609	54828671	15277906	0	0	0	0	0	577777
1946	33494670	741835516	56512978	16022160	0	0	0	0	0	607475
1947	34299846	798208781	58493664	17524428	0	0	0	0	0	647876
1948	36537146	855624347	62284405	18929008	0	0	0	0	0	688348
1949	38367852	891150906	64975715	19870706	0	0	0	0	0	734427
1950	49735986	888529700	68201635	26712094	0	0	0	0	0	794594
1951	59622598	931536467	72501591	31782448	0	0	0	0	0	864088
1952	78264890	967937315	69417771	41523608	0	0	0	0	0	927273
1953	81713708	972543269	74970977	44264656	0	0	0	0	0	985267
1954	87822602	1032179831	81772798	48746744	0	0	0	0	0	1049626
1955	94296584	1103364189	89895709	53085586	0	0	0	0	0	1127386
1956	103094144	1186338621	99111662	58807552	0	0	0	0	0	1206625
1957	115581666	1233644266	105999770	68306600	0	0	0	0	0	1278871
1958	125506590	1295307275	112554043	73245866	0	0	0	0	0	1361924
1959	137680866	1380000000	123500000	79820700	0	0	0	0	0	1491317
1960	164651272	1478000000	134000000	97843398	0	0	0	0	0	1561968
1961	179446284	1625000000	75000000	105452764	0	0	0	0	0	1631084
1962	185790498	1650000000	76500000	106955362	0	0	0	0	0	1718569
1963	203093574	1809000000	84500000	117569938	0	0	0	0	0	1812181
1964	222716610	1958000000	95700000	129735846	0	0	0	0	0	1918880
1965	257854690	2043000000	106500000	142722482	0	0	0	0	0	2010124
1966	284527708	2081000000	116600000	154303778	0	0	0	0	0	2120378
1967	311499880	2179000000	134200000	171099532	0	0	0	0	0	2234703
1968	364477843	2295000000	151400000	211812258	0	0	0	0	0	2358837
1969	412271682	2442000000	172200000	245570549	0	0	0	0	0	2511231
1970	463378054	2661956165	198443835	280756865	0	0	0	0	0	2703668
1971	560014123	2848000000	225280273	312111198	0	0	0	0	0	2857010
1972	645128824	2996000000	248668000	387500000	0	0	0	0	0	2977767
1973	710564619	3146000000	276268000	435404000	0	0	0	0	0	3147070
1974	853435000	3357000000	312500000	517375000	0	0	0	0	0	3361026
1975	1068624000	3560000000	345000000	637803895	0	0	0	0	0	3539020
1976	1424189000	3720000000	373000000	839580151	0	0	0	0	0	3667270
1977	1674991000	3924000000	412000000	967330577	0	0	0	0	0	3874229
1978	1856499000	4186000000	462000000	1078829751	0	0	0	0	0	4133407
1979	2044403653	4483000000	523005000	1211056626	0	0	0	0	0	4417220
1980	2280809521	5060819000	582325000	1355373305	0	0	0	0	0	4742662
1981	2609440222	5552136000	677866000	1533171743	0	0	0	0	0	5069342
1982	3084377000	5456219000	786182000	1761610000	0	0	0	0	0	5356572
1983	3635848000	5596916000	837171000	2011546000	0	0	0	0	0	5591667
1984	4220731000	6174595000	933621000	2328977000	0	0	0	0	0	5850594
1985	4764900000	6500000000	1026126000	2640434000	0	0	0	0	0	6186835
1986	5471700000	7195244000	1172076000	3003764000	0	0	0	0	0	6501468
1987	6047500000	7538893000	1328149000	3282900000	3000000	2400000	4423	200000	300000	6816301
1988	7199500000	8074682000	1488487000	3904200000	28000000	21800000	31622	1600000	1500000	7091549
1989	7976800000	8081400000	1652800000	4278200000	74000000	57700000	94529	4300000	3200000	7419982
1990	8878900000	8750200000	1794200000	4704200000	156000000	121700000	184943	7400000	4100000	7786889

APPENDIX—DATA

Year	RentRev	TotTelgr	TelgrRev	TelexCal	Tlxcalrv	TelexSrv	Tlxrent	DatelSub	DDS	Austpac	OthTelec
1920	611735	40980116	1990670	0	0	0	0	0	0	0	0
1921	801906	38333896	2295320	0	0	0	0	0	0	0	0
1922	921447	36501894	2361308	0	0	0	0	0	0	0	0
1923	924311	36747399	2368710	0	0	0	0	0	0	0	0
1924	1018632	38891906	2346320	0	0	0	0	0	0	0	0
1925	2749944	19472366	2582360	0	0	0	0	0	0	0	0
1926	3087930	20221789	2482974	0	0	0	0	0	0	0	0
1927	3403942	19903305	2436276	0	0	0	0	0	0	0	0
1928	3696080	19191554	2431014	0	0	0	0	0	0	0	0
1929	3924018	18964550	2393506	0	0	0	0	0	0	0	0
1930	4243212	18609226	2243140	0	0	0	0	0	0	0	0
1931	4110070	15212084	1819058	0	0	0	0	0	0	0	0
1932	3952710	14776315	1767116	0	0	0	0	0	0	0	0
1933	4003700	14875115	1780054	0	0	0	0	0	0	0	0
1934	4003754	14658885	1869128	0	0	0	0	0	0	0	0
1935	4126348	15928474	2062824	0	0	0	0	0	0	0	0
1936	4443286	16841849	2181924	0	0	0	0	0	0	0	0
1937	4676404	17707882	2265102	0	0	0	0	0	0	0	0
1938	5104922	18415519	2324690	0	0	0	0	0	0	0	0
1939	5448554	18713520	2326548	0	0	0	0	0	0	0	0
1940	5728284	19066008	2317934	0	0	0	0	0	0	0	0
1941	6139942	21184405	2429282	0	0	0	0	0	0	0	0
1942	6714524	26137036	3048066	0	0	0	0	0	0	0	0
1943	7910442	32534267	3912588	0	0	0	0	0	0	0	0
1944	8186358	35819311	4273198	0	0	0	0	0	0	0	0
1945	8254292	37569516	4479696	0	0	0	0	0	0	0	0
1946	8291134	38039788	4849584	0	0	0	0	0	0	0	0
1947	8377236	35660525	4404302	0	0	0	0	0	0	0	0
1948	8881344	36762080	4423340	0	0	0	0	0	0	0	0
1949	9400928	37893150	4742844	0	0	0	0	0	0	0	0
1950	11644291	37961156	6137750	0	0	0	0	0	0	0	0
1951	14288672	37184538	7258582	0	0	0	0	0	0	0	0
1952	21199540	29766244	7961014	0	0	0	0	0	0	0	0
1953	22874994	25852234	6984950	0	0	0	0	0	0	0	0
1954	24270076	25127053	7228964	0	0	0	0	0	0	0	0
1955	26272734	25511130	8549798	5178	0	95	0	0	0	0	0
1956	28389194	25415093	8516260	26534	0	171	0	0	0	0	0
1957	31977078	23964656	9499704	74323	0	257	0	0	0	0	0
1958	34288794	22884341	9798722	157290	0	359	0	0	0	0	0
1959	37784344	22508037	9775934	235000	0	502	0	0	0	0	0
1960	46561104	22240890	9829920	410744	691058	684	342000	0	0	0	0
1961	52340878	21766266	9536300	512333	738036.472	976		0	0	0	0
1962	56007668	21589343	9663834	732692	903710.164	1215		0	0	0	0
1963	59912508	21830562	9830694	1000552	1056646.22	1439		0	0	0	0
1964	64422296	23353951	10641186	1412338	1277058.55	1815		0	0	0	0
1965	82175125	24309138	11423484	1867701	1445978.18	2179		0	0	0	0
1966	93855778	24953359	11639031	2161353	1432723.27	2444		0	0	0	0
1967	100822801	25558825	11867762	6336464	3596381.91	3154		0	0	0	0
1968	108293388	23406733	14172254	8377816	4071288.46	4054		0	0	0	0
1969	116973726	20869489	14711297	9977018	4151299.2	5067		0	0	0	0
1970	126669177	20559966	15120253	12092737	4308143	6430	4931659	565	0	0	0
1971	153657905	18651404	17368885	12246157	5169817	7988	5798966	1200	0	0	0
1972	184974587	17734881	17284392	15868800	6632734	9235	6906661	1527	0	0	0
1973	192244264	18037209	18184679	17992780	7697572	10774	8137832	2458	0	0	0
1974	235816114	18001640	19499222	21489560	8512878	12857	9606573	3728	0	0	0
1975	301465905	16093032	25775255	27356000	11758528	14766	11481223	6019	0	0	0
1976	416830652	12930292	29423378	27036000	17417169	17047	14089828	9139	0	0	0
1977	454111430	10397494	31510930	29435000	20021322	19601	16218711	13792	0	0	0
1978	495420119	8966042	31303359	32177000	26081061	22724	18655331	19286	0	0	0
1979	533294622	7075878	35186617	35797000	24890447	25901	21495195	26291	0	0	0
1980	602654760	5430895	32497184	37415000	28936051	29731	25644357	35299	0	0	0
1981	674101787	5061011	33005500	43057000	29344968	33975	29830232	43995	0	0	0
1982	833201000	4458616	30165000	44209000	32488000	37802	40020000	56196	0	0	0
1983	1001212000	4050740	28998000	45492000	34556000	40810	56748000	70191	0	0	0
1984	1165912000	3668907	27424000	46725000	35012000	42186	59443000	83249	0	24	0
1985	1290545000	3357874	0	49371000	0	44851	0	94672	11959		174347000
1986	1407679000	2856248	0	51301000	0	46423	0	102367	22753	2104	234275000
1987	1542400000	1678391	0	45770000	0	43029	0	103726	35210	3924	262600000
1988	1819500000	0	0	31147000	0	33479	0	103962	52489	6015	331400000
1989	2003500000	0	0	22300000	0	24987	0	97890	68117	9235	1389900000
1990	2034900000	0	0	12000000	0	18006	0	92310	84189	9664	1794300000

APPENDIX—DATA

Year	Manual	SxS	ARK	ARF	ARE	AXE	DDSC	Austpacx	XchgQ	Coax_km
1920	158260.5064	13845.49361	0	0	0	0	0	0	2409	0
1921	166542.4912	16820.50881	0	0	0	0	0	0	2432	0
1922	175257.8834	20628.11662	0	0	0	0	0	0	2585	0
1923	184429.3638	30005.63625	0	0	0	0	0	0	3010	0
1924	194080.8	48520.2	0	0	0	0	0	0	3428	0
1925	219711.64	58404.36	0	0	0	0	0	0	4572	0
1926	232522.912	76683.088	0	0	0	0	0	0	5095	0
1927	245895.7275	92105.2725	0	0	0	0	0	0	5482	0
1928	247508.9092	115567.0908	0	0	0	0	0	0	5698	0
1929	251419.3094	132938.6906	0	0	0	0	0	0	5911	0
1930	239204.3138	156607.6862	0	0	0	0	0	0	6094	0
1931	228811.313	147431.687	0	0	0	0	0	0	6086	0
1932	219233.916	144338.084	0	0	0	0	0	0	6069	0
1933	217901.824	145874.176	0	0	0	0	0	0	6071	0
1934	217277.1584	155343.8416	0	0	0	0	0	0	6087	0
1935	224239.8754	171785.1246	0	0	0	0	0	0	6117	0
1936	230548.872	187112.128	0	0	0	0	0	0	6166	0
1937	222437.855	218033.145	0	0	0	0	0	0	6236	0
1938	235150.8197	230347.1803	0	0	0	0	0	0	6291	0
1939	238802.2722	248732.7278	0	0	0	0	0	0	6333	0
1940	236081.192	270530.808	0	0	0	0	0	0	6362	0
1941	230610.0236	294936.9764	0	0	0	0	0	0	6364	0
1942	223051.5	308023.5	0	0	0	0	0	0	6383	0
1943	221507.01	318753.99	0	0	0	0	0	0	6371	0
1944	230336.3101	326734.6899	0	0	0	0	0	0	6350	0
1945	239517.5474	338259.4526	0	0	0	0	0	0	6380	0
1946	249064.75	358410.25	0	0	0	0	0	0	6391	0
1947	262389.78	385486.22	0	0	0	0	0	0	6419	0
1948	274449.7079	413898.2921	0	0	0	0	0	0	6476	0
1949	288189.1548	446237.8452	0	0	0	0	0	0	6584	0
1950	304011.6644	490582.3356	0	0	0	0	0	0	6720	0
1951	317552.34	546535.66	0	0	0	0	0	0	6851	0
1952	328347.3693	598925.6307	0	0	0	0	0	0	6990	0
1953	342872.916	642394.084	0	0	0	0	0	0	7060	0
1954	349189.5638	700436.4362	0	0	0	0	0	0	7154	0
1955	357381.362	770004.638	0	0	0	0	0	0	7233	0
1956	361987.5	844637.5	0	0	0	0	0	0	7275	0
1957	370872.59	907998.41	0	0	0	0	0	0	7284	0
1958	394957.96	966966.04	0	0	0	0	0	0	7326	0
1959	402655.59	1088661.41	0	0	0	0	0	0	7326	0
1960	390492	1171476	0	0	0	0	0	0	7246	0
1961	368624.984	1262459.016	0	0	0	0	0	0	7161	0
1962	366055.197	1352513.803	0	0	0	0	0	0	7056	4981
1963	366060.562	1446120.438	0	0	0	0	0	0	6982	7440
1964	368424.96	1550455.04	0	0	0	0	0	0	6888	8903
1965	349761.576	1660362.424	0	0	0	0	0	0	6759	10083
1966	335019.724	1785358.276	0	0	0	0	0	0	6645	11882
1967	328501.341	1783557.25	122644.409	0	0	0	0	0	6519	13911
1968	306648.81	1762694.574	289493.6155	0	0	0	0	0	6358	15860
1969	281257.872	1734673.526	495299.6024	0	0	0	0	0	6230	20812
1970	250728.7675	1700513.626	550152.8789	202272.7273	0	0	0	0	6128	22722
1971	223513.4413	1661172.159	556415.3084	415909.0909	0	0	0	0	6060	27650
1972	199252.1996	1617542.772	490517.4831	670454.5455	0	0	0	0	5979	29057
1973	177624.3917	1570454.079	460355.1656	938636.3636	0	0	0	0	5916	29817
1974	158344.1718	1520668.269	434286.2861	1247727.273	0	0	0	0	5858	30977
1975	141156.7213	1468879.708	378983.5707	1550000	0	0	0	0	5772	31774
1976	125834.8806	1415713.542	377994.3049	1747727.273	0	0	0	0	5756	34910
1977	112176.1475	1361724.303	407146.7315	1993181.818	0	0	0	0	5686	36873
1978	100000	1307394.514	410103.3955	2311363.636	4545.454545	0	0	0	5638	39043
1979	100000	1253133.291	486857.2482	2561363.636	15865.82491	0	0	0	5576	41598
1980	75478.42314	1199274.949	636363.6364	2773997.086	57547.90584	0	0	0	5513	46904
1981	57859.52332	1146077.606	636363.6364	3035740.576	193300.6585	0	0	0	5460	47424
1982	43000	1093721.787	636363.6364	2815369.123	757117.4545	11000	0	0	5411	48037
1983	34000	1042309.028	850000	2700000	900357.9723	65000	8	0	5353	49134.464
1984	27000	991860.4813	1050000	2300000	1401733.519	80000	70	0	5294	50257
1985	20000	942315.5192	1072727.273	2050000	1901792.208	200000	260	12	5227	50403
1986	12000	893530.3377	1072727.273	2000000	2273210.39	250000	350		5036	50403
1987	6900	845276.5617	1100000	2000000	2134124.438	730000		31	5156	50403
1988	0	797239.8489	1100000	2000000	2114309.151	1080000	500	48	5156	50403
1989	0	749018.4942	1100000	2050000	2316418.051	1204545.455	500	48	5156	50403
1990	0	700122.0338	1150000	2050000	2345857.875	1540909.091	500	48	5156	50403

Year	Optic_km	Cost	Wages_E	Wages_P	ComPltIn	Comm_P	Comm_S	Build	Build_S
1920	0	6133178	2456712.302	281.2386537	1836872	1	15087343.66	58518	5108424.102
1921	0	7284844	3022208.65	326.3426076	2957820	2.349119999	15205230.42	84614	5034061.253
1922	0	7821914	3163833.148	344.2364043	3992244	2.000188505	16062447.22	198014	5003627.866
1923	0	8695760	5556308.437	341.3790424	5300178	1.506860824	18283627.71	509092	5101099.092
1924	0	9929994	6261433.858	331.834172	8582382	2.260728645	20618227.34	1990228	5777132.435
1925	0	11413424	7021555.883	355.0956615	9402232	1.247724201	26289955.29	1272286	6157967.988
1926	0	12644086	7906449.238	360.4715288	10054104	2.157850313	28900428.46	1034030	6437646.956
1927	0	13633628	8323505.973	383.1474078	8889956	1.996742866	31144711.3	678404	6562731.478
1928	0	14670914	8483947.364	425.0288073	8036626	2.178062342	32528471.62	628748	6666496.079
1929	0	15153316	8404741.929	417.727806	6962418	1.776997941	34033788.8	486568	6714404.911
1930	0	15754112	8407559.929	454.5500576	6267410	1.890196452	34876994.94	470352	6755207.334
1931	0	15548138	8226798.319	511.3194679	2506344	0.951381016	34933769.21	63450	6641382.942
1932	0	14284996	5845172.928	501.5117229	1702016	0.674876324	34878784.67	73602	6534747.992
1933	0	13969950	5757674.712	484.1753309	1699726	0.626588264	35007415.54	53328	6423145.682
1934	0	13486060	5948823.882	459.635994	2132932	0.807181919	35115126.51	37918	6307547.596
1935	0	14101548	6077541.105	424.5730949	3155706	1.072838568	35338366.22	193640	6257692.562
1936	0	14266468	6342832.607	407.5635507	4452028	1.309469673	35674226.53	331032	6264778.405
1937	0	14977652	6579512.284	389.3875479	5069630	1.611744357	36142677.99	445486	6318306.883
1938	0	15907886	7109151.165	391.3139823	6916010	2.229087566	36580411.34	512692	6383192.716
1939	0	16710528	7712034.427	401.7417232	7781396	2.557270944	36923258.23	1098194	6668053.754
1940	0	17320002	7980263.308	423.9107056	7467258	2.535267771	37199471.04	511578	6718716.168
1941	0	18214876	8319169.927	445.1261071	7473738	2.596641741	37410706.11	970868	6902738.648
1942	0	19796668	9358998.132	447.0598735	6495706	2.219081029	37641583.18	1142690	7117601.896
1943	0	22131252	11431703.76	458.3094494	6151928	2.197927846	37717726.83	698230	7166859.901
1944	0	23099610	12494356.31	499.0808328	5998858	2.195493295	37709488.29	778988	7232041.371
1945	0	23723092	13092410.76	542.4550035	7524092	2.533531713	37945214.85	645218	7255216.303
1946	0	25388262	14375506.32	590.919334	9856100	3.412201735	38089966.14	813622	7317223.62
1947	0	28935294	15257444.46	645.7704023	12839618	4.255453677	38318116.72	924564	7395502.54
1948	0	34689456	19554818.49	764.1730303	16386912	5.082708915	38740027.57	1619366	7617194.131
1949	0	41219996	23951658.46	854.4994898	24897724	6.998862263	39497334.05	2586234	8009205.737
1950	0	49738394	27972668.41	939.3597754	33082136	8.493389376	40519803.8	4220986	8631616.181
1951	0	61310280	37811400.4	1212.965079	46684076	11.05057924	41782307.28	6468780	9477853.883
1952	0	75849858	46925404.15	1500.188399	58888012	12.31184923	43482717.09	5321064	9990139.878
1953	0	78756014	50336102.49	1651.902218	48974978	11.11911047	44717155.2	8314838	10786621.43
1954	0	83817994	52590960.14	1729.236198	56555298	12.67991441	45921839.6	6985280	11397353.11
1955	0	90087580	53949637.69	1748.970091	59635224	13.33449751	47058002.1	6378244	11907357.73
1956	0	99140864	59864885.96	1939.257518	63098110	14.38004902	48040176.42	7906396	12554858.9
1957	0	106295238	63365537.68	1967.774265	73958346	16.26259629	49106612.8	7031994	13050377.77
1958	0	113579064	66997359.09	2040.365074	79149936	16.54308522	50323495.44	9444812	13765059.08
1959	0	123892844	69158746.03	2070.135406	85190260	17.67549916	51492697.76	9110812	14411966.09
1960	0	165251202	82837402.26	2331.131865	96147326	15.60387858	51388524.97	9505922	15066669.37
1961	0	175910328	89972413.59	2351.486368	102856982	21.57651013	52438105.3	10584244	15787078.35
1962	0	188960216	92880133.01	2462.050391	110902456	22.76248863	53516324.49	11648166	16586215.2
1963	0	205669062	96292118.06	2479.965358	118080098	22.84729033	54799638.89	13249970	17422236.2
1964	0	224266546	102367786.6	2622.192497	128587774	22.880677	56419792.24	15849166	18513788.44
1965	0	251024108	110850811.9	2834.563133	152081448	23.19166536	58808280.15	16481348	19589025.52
1966	0	274310484	113218704.1	2958.769424	174230974	23.52590109	61830821.39	16377758	20624472.77
1967	0	309422041	131623674.9	3234.349162	200971335	23.80884715	65619860.4	16714880	21565624.02
1968	0	353965382	141166762.8	3472.431564	227940657	23.99521178	70146386.46	25545074	23206820.58
1969	0	395534147	188925806.3	3734.632197	256499411	24.47135731	75290430.24	32676570	25260519.73
1970	0	441527656	214007647.9	4055.23275	290642230	24.89733476	81207037.81	29416617	26936747.18
1971	0	506168217	271398429.5	4564.020083	300280528	25.61384822	86778411.46	36565988	29002674.39
1972	0	574076497	283665863.9	5325.480456	340321821	26.37267455	93083748.43	42256262	31236498.62
1973	0	648449945	341561650.4	5896.638493	392073104	26.8897175	100537138.3	43942638	33370366.94
1974	0	794243000	452694782.2	7886.272986	488298613	27.76782371	110302501	45107021	35111810.35
1975	0	973498000	574113230.3	9849.151561	576668803	30.90831372	120422756.5	63053385	37064705.68
1976	0	1271795499	597461791.3	10809.84788	625638048	32.99720604	130155929.9	85363372	39422661
1977	0	1510589428	662348982.9	12476.49981	712530811	34.79464891	140662124.5	124602620	42614712.55
1978	0	1671580412	745773198	13557.42803	778740861	36.75972095	151132493.6	88649881	44391826.95
1979	0	1853885089	859909827.8	14469.47686	793310904	38.5536676	160342133.4	71294738	45475069.54
1980	0	2069264190	986240000	15968.65336	856627009	40.46724045	169494542.5	76065216	46472734.86
1981	0	2376953614	1166274757	17738.94114	1012133596	42.11023482	180718199.5	73081813	47172120.85
1982	0	2809260000	1350216964	20807.33093	1154000000	43.67091416	193430236.2	75000000	47710355.59
1983	0	3373181000	1533069580	23578.76452	1381000000	46.10013603	208598560.1	62080000	47829460.53
1984	4606	3911949000	1789471134	24825.42126	1522000000	47.14511931	224935480.3	70000000	48011255.32
1985	4847	4379667000	1950484962	26381.4727	1757000000	47.65724861	244471552.2	64000000	48026479.83
1986	11998.37998	5007102000	2148374405	27761.23443	1654000000	51.66495514	258182174.8	105000000	48505538.68
1987	29701.07742	5603100000	2203577171	29214.55421	1817000000	56.41106732	271168204.9	185000000	49899548.84
1988	73522.759	6419900000	1868414286	31448.29092	2165000000	57.71388124	288246172.2	260000000	52055919.32
1989	182000	6961300000	2565888000	38135.64159	1994117784	58.21472508	301151151.5	376199930.8	55306616.46
1990	182000	7530700000	2567480405	36957.95765	2427714548	59.49569752	319318538.3	274000000	57140181.45

APPENDIX—DATA

Year	NDC_P	MchE_P	MchE_E	MchE_S	Bond_ yld	DepTele	IntTele
1920	2.5917789	27.24048245	32186	20440.82006	5.87	325510	1014616
1921	2.776906	26.74944135	110744	23766.11158	7.22	312050	1093014
1922	2.6843425	27.58124743	106578	29960.89341	6.78	369962	1197988
1923	2.4992154	26.23644468	151732	40296.74616	6.29	438810	1350074
1924	2.4992154	24.49330324	291168	59125.49242	6.01	527176	1590276
1925	2.4992154	24.03425219	178816	81583.0914	5.96	722188	1928208
1926	2.4992154	22.65029115	202690	108148.9627	5.2	369302	2260210
1927	2.5917789	21.22890669	241374	139853.8044	5.34	437302	2548470
1928	2.5917789	21.4714363	191354	174442.5125	5.43	519644	2780144
1929	2.5917789	21.06963595	193614	211387.461	5.26	543920	3087968
1930	2.5917789	19.913231	118766	247667.0806	5.57	583270	3496224
1931	2.5917789	19.27919693	45214	280720.0651	6.51	645098	3973432
1932	2.4992154	18.75399661	43694	310874.4793	4.68	546890	4662518
1933	2.4066519	19.68944729	74322	339620.2613	3.92	552844	4592242
1934	2.4066519	17.66328996	152724	370898.8869	3.61	574488	3730396
1935	2.4066519	17.26529548	204018	406913.8276	3.31	567684	3737582
1936	2.4066519	16.59463178	252384	449576.5937	3.74	589008	3330106
1937	2.4066519	14.26327285	518038	513134.8487	4.02	639646	3363740
1938	2.5917789	13.33849804	454578	593158.3868	3.68	663034	3345424
1939	2.5917789	13.34304795	426552	681922.4052	3.9	723582	3200920
1940	2.6843425	13.22393933	280916	771715.3365	3.8	758554	3376700
1941	2.9620331	13.36597957	330486	864551.5576	3.09	811912	3512600
1942	3.1471601	13.57621171	271050	956992.8845	3.25	872250	3512340
1943	3.5174142	14.25970143	500122	1060901.049	3.23	920350	3427600
1944	3.6099778	14.73844891	1221076	1213648.199	3.24	948304	3329614
1945	3.7025413	15.04969576	1448878	1413953.169	3.24	985744	3264894
1946	3.7951048	13.55086883	1475156	1658492.773	3.25	1016396	2866872
1947	3.9802319	14.12464884	1836400	1961058.904	3.21	1101728	2806344
1948	4.2579225	14.53604365	2254420	2339286.293	3.17	1194072	2578076
1949	4.6281766	14.72755274	2742816	2806748.532	3.13	1297926	2448178
1950	5.2620096	16.95216796	5865444	3471544.069	3.12	1508946	2425868
1951	6.1984509	20.2456813	6594478	4294110.073	3.17	1400220	2755082
1952	7.386992	21.20930858	4862498	5181729.268	3.75	1680600	2737728
1953	8.142932	20.02386038	6585480	6198757.828	4.53	850290	2588434
1954	8.2365855	22.11873043	5408434	7264074.344	4.4	1026644	2344182
1955	8.4148666	23.50510477	4228106	8320278.935	4.52	1190112	2539418
1956	8.6983462	23.67129687	5727272	9411362.114	4.53	3440924	1749992
1957	9.1647062	24.6672959	5091794	10503146.97	5.09	16334292	1695330
1958	9.432208	25.71352404	6579094	11628944.39	5	18726434	1634012
1959	9.6203672	26.18960375	8560594	12834367.41	4.93	23591414	1567974
1960	9.8154619	26.80778661	9224950	14119368.1	4.83	21246342	29538472
1961	10.0881862	27.13774839	9310272	15469941.11	5.34	23764300	34049410
1962	10.1771424	27.4144719	7918648	16833593.25	4.88	26383546	37825488
1963	11.0523621	27.62320692	9627194	18248998.46	4.72	43224756	39822568
1964	10.7299304	27.64172569	11520100	19756741.58	4.29	47874880	45688104
1965	11.1122063	28.11003791	12712954	21368006.54	4.76	59887132	50987060
1966	11.1797514	28.55880734	14504867	23107467.3	5.15	66521329	57507366
1967	12.022917	29.30888357	15764393	24977376.43	5.02	74077275	64807647
1968	12.0241844	29.87762251	17723656	26998296.03	5.1	91017546	72306669
1969	12.6665887	30.78796017	17035695	29116644.43	4.91	100478831	81059085
1970	13.1489688	31.91629917	16776919	31293680.06	5.64	109776569	90134772
1971	13.6956248	33.40631679	15091144	33460177.08	6.41	123474935	103090917
1972	14.6503433	34.87634563	23944504	35785975.25	5.71	135243895	117606959
1973	15.5340185	36.31803942	22743696	38200237.91	5.3	148716125	130979128
1974	18.2424481	38.35859977	28312446	40768763.99	8.05	165073000	143248000
1975	23.1408832	45.17118626	30664164	43421266.81	9.5	185656000	173696000
1976	26.852902	52.12392322	35995738	46148249.43	9.99	312358000	239588000
1977	30.5436343	59.49166055	60790493	49180380.24	10.41	340817000	278629000
1978	32.8142509	66.09463679	69909294	52500153.46	9.1	366514000	317288000
1979	35.1161837	72.31747578	58559211	55878439.39	10	410411000	338090000
1980	38.7011323	79.39694722	70433923	59355332.76	11.76	447383000	367127000
1981	43.4561428	86.03248822	79200184	62933223.01	13.1	533680000	408125000
1982	48.9621776	92.48027025	70300000	66468647.26	16.4	577216000	518187000
1983	55.6698827	102.3520247	92040000	70058751.23	14.85	711853000	656172000
1984	59.240661	107.0316237	97534000	73694402.27	13.75	570829000	861937000
1985	63.0313581	109.5574851	149000000	77694500.63	13.5	647825000	898915000
1986	70.5086838	124.2695071	208000000	82239810.23	12.95	816001000	927464000
1987	76.0328254	140.8256405	275000000	87462929.84	12.8	847400000	1027800000
1988	80.2363649	148.0827385	193205825.3	92784864.46	11.95	1184500000	1156200000
1989	85.452445	147.7328436	224615894.1	98338324.68	13.5	1479000000	1153100000
1990	90.6234691	152.5356339	257988393.1	104206796.3	13.4	1479400000	1161600000

CHAPTER 5 APPENDIX 2—DATA SOURCES

The following provides an outline of data sources and describes procedures used to derive the estimating variables for the sample period 1920 to 1990.

TELECOMR

General Description: Total telecommunications revenue

Source:

1920-1963 Sum of FoneRev and TgrphRev

1964-1965, 1971-1972 PMG Annual Report, Page 26. Telecommunications Service-Statement of Profit and Loss

1966 PMG Annual Report, 1965-66. Page 24. Telecommunications Services-Statement of Profit and Loss

1967-1968 PMG Annual Report, 1966-67. Page 28. Telecommunications Service-Statement of Profit and Loss

1969, 1973 PMG Annual Report, Page 30. Telecommunications Service, Profit and Loss Statement

1970 PMG Annual Report, 1969-70. Page 38. Telecommunications Service, Profit and Loss Statement Year Ended 30 June 1970

1974 PMG Annual Report, 1973-74. Page 29. Telecommunications Service-Profit and Loss Statement Year Ended 30 June 1974.

1975 PMG Annual Report, 1974-75. Page 45. Telecommunications Service-Profit and Loss Statement Year Ended 30 June 1975

1976 Australian Telecommunications Commission, 1975-76. Page 59. Australian Telecommunications Commission, Profit and Loss Statement Year Ended 30 June 1976

1977 Australian Telecommunications Commission, 1976-77. Page 25. Australian Telecommunications Commission, Profit and Loss Statement Year Ended 30 June 1977

1978 Australian Telecommunications Commission, 1977-78. Page 31. Australian Telecommunications Commission, Profit and Loss Statement Year Ended 30 June 1978

FONEREV

General Description: Total Earnings. From 1915 to 1956 this is the sum of: RentRev, CallsRev, Public, STDRev and OtherRev. From 1957 to 1964 this is the sum of: RentRev, CallsRev and OtherRev.

Source:

1915-1925 Bulletin of Transport and Communications 1924-25. Page 42. No. 58. Postmaster-General's Department Revenue

1926-1932 Bulletin of Transport and Communications 1922-33. Page 44. No. 64. Postmaster-General's Department Earnings. Telephone Earnings

1933-1943 Bulletin of Transport and Communications 1942-43. Page 44. No. 53. Postmaster-General's Department Earnings. Telephone Earnings, Total Australia

1944-1948 Bulletin of Transport and Communications 1947-48. Page 51. No. 53. Postmaster-General's Department Earnings. Telephone Earnings, Total Australia.

1949 PMG Annual Report 1948-49. Page 37. Appendix A Table No. 4. Profit and Loss Account-Telephone Branch (Including Exchanges, Trunk Lines and Non-Exchange Lines) For Year Ended 30th June 1949

1950-1952 PMG Annual Report 1949-50. Page xx. Appendix A Table No. 3. General Profit and Loss Account For Year Ended 30th June 1950. Row: Revenue as per Branch Accounts-Telephone.

1953-1955 PMG Annual Report Appendix 4 All Branches-Profit and Loss Account Row: Earnings as per Branch Accounts-Telephone

1956-1960 PMG Annual Report. Appendix 3 All Branches-Profit and Loss Account, Row: Earnings: as per Branch Accounts. Telephone.

1961-1962 PMG Annual Report, 1961-62. Page 35. Telephone Service-Statement of Profit and Loss, 1961-62. Column: 1961/62. Row: Total Earnings

1963 PMG Annual Report, 1962-63. Page 24. Consolidated Statement of Profit and Loss, 1962/63. Column: Telephone. Row: Earnings.

1964 PMG Annual Report, 1963-64. Page xx. Consolidated Statement of Profit and Loss, 1963-64. Column: Telephone. Row: Earnings.

TGRPHREV

General Description: Total Earnings. From 1913 to 1923, this is the sum of: TelgrRev, CBWRev and MiscTgrR. From 1924, 1926 to 1940, this is the sum of: TelgrRev, CBWRev, BeamWRev, MiscTgrR, MetTlgrR and ShipTlgR. From 1950-1954, this is the sum of: TelgrRev, CBWRev, MiscTgrR and MetTlgrR. From 1955 to 1964 this is the sum of: TelgrRev, MiscTgrR and Leasetlg.

Source:

1915-1925 Bulletin of Transport and Communications 1924-25. Page 42. No. 58. Postmaster-General's Department Revenue, 1914-15 to 1924-25.

1926-1932 Bulletin of Transport and Communications 1922-33. Page 44. No. 64. Postmaster-General's Department Earnings

1933-1943 Bulletin of Transport and Communications 1942-43. Page 60. No. 53. Postmaster-General's Department Earnings

1944-1948 Bulletin of Transport and Communications 1947-48. Page 51. No. 53. Postmaster-General's Department Earnings.

1949 PMG Annual Report 1948-49. Page 38. Appendix A Table No. 5.

1950-1952PMG Annual Report 1949-50. Page xx. Appendix A Table No. 3

1953-1955 PMG Annual Report 1952-53. Page 35. Appendix 4

1956-1960 PMG Annual Report 1955-56. Page 43. Appendix 3

1961-1962 PMG Annual Report, 1961-62. Page 36. Telegraph Service-Statement of Profit and Loss, 1961-62. Column: 1961/62. Row: Total Earnings. Doubled to convert from pounds to dollars.

1963 PMG Annual Report, 1962-63. Page 24. Consolidated Statement of Profit and Loss, 1962/63. Column: Telegraph. Row: Earnings

1964 PMG Annual Report, 1963-64. Page xx. Consolidated Statement of Profit and Loss, 1963-64. Column: Telegraph. Row: Earnings. Doubled to convert from pounds to dollars.

LOCCALLS

General Description: Total annual number of Local calls.

Source:

1922, 1925 PMG Annual Report, Appendix T, Telephone Exchanges-Daily Calling Rates and Effective Paid Local Calls, Total for Commonwealth.

1923-1924 PMG Annual Report. Appendix U. Telephone Exchanges-Daily Calling Rate and Effective Paid Local Calls for the Year Ended 30th June 1924. Effective paid local calls-Total. Commonwealth.

1926-1928 PMG Annual Report. Appendix V. Telephone Exchanges-Daily Calling Rate and Effective Paid Local Calls. Effective paid local calls-Total. Total for Commonwealth.

1929 PMG Annual Report Appendix U. Telephone Exchanges-Daily Calling Rate and Effective Paid Local Calls. Effective paid local calls-Total. Total Calls for Commonwealth.

1930-1941 PMG Annual Report Appendix V. Telephone Exchanges-Daily Calling Rate and Effective Paid Local Calls. Effective paid local calls-Total. Total Calls for Commonwealth.

1942-1947 PMG Annual Report Appendix N. Telephone Exchanges-Daily Calling Rates and Effective Paid Local Calls-. Effective Paid Local Calls, Total. Total Calls for Commonwealth.

1948 PMG Annual Report Appendix J. Telephone Exchanges-Daily Calling Rates and Effective Paid Local Calls-1947-48. Effective Paid Local Calls, Total. Total Calls for Commonwealth.

1949-1952 PMG Annual Report Appendix K. Telephone Exchanges-Daily Calling Rates and Effective Paid Local Calls. Effective Paid Local Calls, Total. Total Calls for Commonwealth.

1953-1955 PMG Annual Report Appendix 16. Telephone Exchanges-Daily Calling Rates and Effective Paid Local Calls. Effective Paid Local Calls, Total. Total Calls for Commonwealth.

1957-1974 PMG Annual Report. Facts at a Glance. Commonwealth. Telephone Local Calls.

1976 Australian Telecommunications Commission Annual Report Table 18.

1977-1981 Australian Telecommunications Commission Annual Report Table 9

1982-1988 Australian Telecommunications Commission Annual Report Table 1

1989-1990 Australian Telecommunications Corporation Annual Report 1990-91, page 61.

STDCALLS

General Description: Number of long distance calls (trunk excluding international calls).

Source:

1914-1915 PMG Annual Report Appendix O. Telephone Trunk Line Traffic and Revenue.

1922 PMG Annual Report (1921-22), Page 63, Appendix R., Trunk Line Traffic and Revenue, Total column (Total calls for year).

1975, p71 table 30

1976 Australian Telecommunications Commission Annual Report 1976, page 84, Table 18.

1977-1981 Australian Telecommunications Commission Annual Report 1980-81, page 88 Table 9, Trunk calls.

1982-1988 Australian Telecommunications Commission Annual Report Table 1, Trunk Calls.

1989-1990 Australian Telecommunications Corporation Annual Report.

CALLREV

General Description: Total telephone call revenue

Source:

1920-1924 Sum of Local Call Revenue and Trunk Revenue

1925-1927 PMG Annual Report Appendix A. Table 6. Profit & Loss Account-Telephone Branch, Revenue

1927-1952 PMG Annual Report Appendix A. Table 4. Profit & Loss Account-Telephone Branch, Revenue.

1953-1955 PMG Annual Report Appendix 6. Profit & Loss Account-Telephone Branch, Revenue.

1956-1962 PMG Annual Report Appendix 5. Telephone Branch-Profit & Loss Account, Earnings.

1963-1966 PMG Annual Report Page 24. Telephone Branch-Profit & Loss, Earnings

1967-1968 PMG Annual Report 1967-68. Page 28. Telecommunications Services-Statement of Profit & Loss, Earnings

1969-1970 PMG Annual Report 1969-70. Page 38. Telecommunications Services- Profit & Loss Statement, Earnings.

1971-1972 PMG Annual Report 1971-72. Page 26. Telecommunications Service- Profit & Loss Statement, Earnings

1973-1974 PMG Annual Report 1973-74. Page 29. Telecommunications Service- Profit & Loss Statement, Earnings.

1975-1978 PMG Annual Report Page 81. Telecommunications Service- Profit & Loss Statement, Earnings.

1979-1981 PMG Annual Report Table 5. Earnings

1982 PMG Annual Report 1981-82. Page 20. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1983-1988 PMG Annual Report Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings

1989-1990 Australian Telecommunications Commission Annual Report 1989-90. Page 60 2. Operating Revenue.

1991 Australian Telecommunications Commission Annual Report 1990-91. Page 68. 2. Operating Revenue.

LOCAL CALL REVENUE

General Description: Total revenue from calls originating and terminating in the local exchange area.

Source:

1920-1924 Product of the number of local calls and average local call price

TRUNK REVENUE

General Description: Total revenue from call originating in Australia and terminating in Australia outside the local exchange area.

Source:

1920 PMG Annual Report 1919-20. Page 37. Table 4. Profit & Loss Account-Telephone Branch, Revenue. Column: Total. Row: Trunk Line Fees. Figure doubled to convert to dollars.

1921 PMG Annual Report 1920-21. Page 33. Table 4. Profit & Loss Account-Telephone Branch, Revenue. Column: Total. Row: Trunk Line Fees. Figure doubled to convert to dollars.

1922 PMG Annual Report 1921-22. Page 33. Table 4. Profit & Loss Account-Telephone Branch, Revenue. Column: Total. Row: Trunk Line Fees. Figure doubled to convert to dollars.

1923 PMG Annual Report 1921-22. Page 33. Table 4. Profit & Loss Account-Telephone Branch, Revenue. Column: Total. Row: Trunk Line Fees. Figure doubled to convert to dollars.

1924 PMG Annual Report 1923-24. Page 27. Appendix A Table 4. Profit and Loss Account-Telephone Branch, Revenue. Row: Trunk Line Fees. Figure doubled to convert to dollars.

CELLCALL

General Description: Total number of wireless (mobile) telephone calls.

Source:

1987-1994. Bureau of Transportation & Communication Economics (1994), Statistical Summary of the Communications, Entertainment & Information Industries, Attachment 1, Table 4.4 pg 128.

CELCALRV

General Description: Total revenue derived from wireless (mobile) telephone calls.

Source:

1987-1990 Telecom Annual Reports.

CELLSUB

General Description: Total number of wireless (mobile) telephone subscribers

Source:

1987-1993 Bureau of Transportation & Communication Economics (1994), Attachment 1 to Paper, Table 4.4 pg 128

CELACREV

General Description: Cellular Mobile Telephone Access Revenue.

Source:

1987-1993 Bureau of Transportation & Communication Economics (1994), Attachment 1 to Paper, Table 4.4 pg 128.

CELCNFEE

General Description: Cellular Mobile Telephone Connection Fees Revenue.

Source:

1987-1993 Bureau of Transportation & Communication Economics (1994), Attachment 1 to Paper, Table 4.4 pg 128

SUBSCRIB

General Description: Annual Number of telephone subscribers 1920 to 1990.

Source:

1920. PMG Annual Reports, Appendix Q. Telephone Statistics for the Commonwealth. Total. 2. Number of lines connected.

1923-1926. PMG Annual Reports Appendix R. Telephone Statistics for the Commonwealth. Total. 2. Number of lines connected.

1927-1929. PMG Annual Report, 1926-27, Appendix S. Telephone Statistics for the Commonwealth. Total. 2. Number of lines connected.

1930-1938. PMG Annual Reports, Appendix T. Total. Number of lines connected.

1939-1941. PMG Annual Reports, Appendix R. Total. Number of lines connected.

1942-1945. PMG Annual Reports, Appendix L. Total. Number of lines connected.

1946. PMG Annual Report 1955-56, page 10. Telephone Services. Subscribers' Services.

1947. PMG Annual Report, Appendix L. Total. Number of lines connected.

1948-1949. PMG Annual Reports, Appendix I. Column: Total. Row: Number of lines connected.

1950. Australian Telecommunications Commission Annual Report 1974-75, page 66, Table 21 Telephone Services in Operation. Total.

1951-1955. PMG Annual Reports, Appendix I Telephone Statistics for the Commonwealth. Total. Number of lines connected.

1956. PMG Annual Reports, Telephone Services. Subscribers' Services.

1957-1958. Australian Telecommunications Commission Annual Reports. Facts at a Glance, Telephone Exchange Lines.

1959. Australian Telecommunications Commission Annual Report 1958-59, page 13, Subscribers' Services. Total. Total.

1960. Australian Telecommunications Commission Annual Report 1974-75, page 66, Table 21 Telephone Services in Operation. Total. Total.

1961. Australian Telecommunications Commission Annual Report 1960-61, page 14, Telephones.

1962. PMG Annual Report 1961-62, page 13, Telecommunication Services. Telephones.

1963. PMG Annual Report 1962-63, page 5, Facts at a Glance. Telephone Services.

1964. PMG Annual Report 1963-64, page 4. The Year in Brief, Telephone Services.

1965-1966. PMG Annual Reports, Telecommunications Services.

1967-1969. PMG Annual Reports, Telecommunications Services. Subscribers' Services.

1970-1975. Australian Telecommunications Commission Annual Report 1974-75, page 66, Table 21 Telephone Services in Operation. Total.

1976-1980. Australian Telecommunications Commission Annual Reports Table 22 Telephone Services in Operation-Continuous and Non-continuous. Total.

1981-1984. Australian Telecommunications Commission Annual Reports Table 14 Telephone Services in Operation-Continuous and Non-continuous. Total.

1985-1988. Australian Telecommunications Commission Annual Reports Table 7 Telephone Services in Operation. Column: Australia, Total.

1990. International Telecommunication Union. Telecommunications Database 2001.

RENTREV

General Description: Earnings derived from telephone line rentals.

Source:

1913-1924 PMG Annual Report Table 4. Profit & Loss Account-Telephone Branch, Revenue. Column: Total. Row: Rentals & Calls

1925-1927 PMG Annual Report Appendix A. Table 6. Profit & Loss Account-Telephone Branch

1928-1952 PMG Annual Report. Appendix A. Table 4. Profit & Loss Account-Telephone Branch, Revenue.

1953-1955 PMG Annual Report. Appendix 6. Profit & Loss Account-Telephone Branch, Revenue.

1956-1962 PMG Annual Report Appendix 5. Telephone Branch-Profit & Loss Account, Earnings

1963, 1971-1972 PMG Annual Report. Page 26. Telephone Branch-Profit & Loss, Earnings.

1964-1966 PMG Annual Report Page 24. Telephone Branch-Profit & Loss, Earnings.

1967-1968 PMG Annual Report 1967-68. Page 28. Telecommunications Services-Statement of Profit & Loss, Earnings.

1969-1970 PMG Annual Report 1969-70. Page 38. Telecommunications Services- Profit & Loss Statement, Earnings.

1973-1974 PMG Annual Report 1973-74. Page 29. Telecommunications Service- Profit & Loss Statement, Earnings.

1975-1978 PMG Annual Report 1977-78. Page 81. Telecommunications Service- Profit & Loss Statement, Earnings.

1979-1981 PMG Annual Report 1980-81. Page 87. Table 5. Earnings.

1982 PMG Annual Report 1981-82. Page 20. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1983-1988 PMG Annual Report Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1989-1990 Australian Telecommunications Commission Annual Report 1989-90. Page 60. 2. Operating Revenue.

TOTTELGR

General Description: Total number of International plus Domestic telegrams

Source:

1920-1939 PMG Annual Report 1915-16. Appendix L. Statement Giving Particulars Concerning Telegraph Business.

1940-1941 PMG Page 58 1939-40 Annual Report.

1952 PMG 1951-42 Annual Report, Appendix E.

1953-1954 PMG Annual Report 1953-54, page 64, Telegraph Statistics.

1955 PMG Annual Report 1954-55, page 62, Telegraph Statistics.

1956 PMG Annual Report 1955-56, Telegraph Services, Telegraph Traffic.

1957 PMG Annual Report 1956-57, page 20, Telegraph Services, Telegraph Traffic.

1958 PMG Annual Report 1957-58, page 16, Telegraph Services, Telegraph Traffic.

1959, 1961 PMG Annual Report page 19, Telegraph Services, Telegraph Traffic.

1960 PMG Annual Report 1959-60, page 23, Telegraph Services, Telegraph Traffic.

1962 PMG Annual Report 1962-63, page 17, Telegraph Services.

1963-1964, 1967 PMG Annual Report page 11, Telegraph Services, first paragraph.

1965, 1968 PMG Annual Report page 10, Telegraph Services, first paragraph.

1966 PMG Annual Report 1965-66, page 8, Telegraph Services, first paragraph.

1969-1975 Telecommunications Commission Annual Report Table 40.

1976-1978 Australian Telecommunications Commission Annual Report Table 11.

1979-1981 Australian Telecommunications Commission Annual Report Statistical Supplement, Table 10.

1982-1986 Australian Telecommunications Commission Annual Report Statistical Supplement, Table 2.

1987-1988 ITU 1998.

TELGRREV

General Description: Total telegram revenue.

Source:

1913, 1915-1923 PMG Annual Report Table 5. Profit & Loss Account of Telegraph Branch, Revenue.

1914 PMG Annual Report 1913-14. Table 6. Profit & Loss Account of Telegraph Branch, Revenue.

1924-1952 PMG Annual Report Appendix A. Table 5. Profit & Loss Account of Telegraph Branch, Revenue.

1953-1954 PMG Annual Report 1952-53. Page 8. Appendix 7. Profit & Loss Account of Telegraph Branch, Revenue.

1955-1957 PMG Financial and Statistical Bulletin 1968-69. Page xx. Table 11 Earnings.

1958, 1960 PMG Annual Report Appendix 6. Profit & Loss Account of Telegraph Branch, Revenue.

1959, 1961-1962 PMG Annual Report Appendix 5. Profit & Loss Account of Telegraph Branch, Revenue.

1963-1964 PMG Annual Report 1963-64. Page 25. Telegraph Branch-Profit, Earnings.

1965-1966 PMG Annual Report 1965-66. Page 24. Telecommunications Services-Statement of Profit & Loss, Earnings.

1967-1968 PMG Annual Report 1967-68. Page 28. Telecommunications Services-Statement of Profit & Loss, Earnings.

1969-1970 PMG Annual Report 1969-70. Page 38. Telecommunications Services-Profit & Loss Statement, Earnings

1971-1972 PMG Annual Report 1971-72. Page 26. Telecommunications Service-Profit & Loss Statement, Earnings.

1973-1974 PMG Annual Report 1973-74. Page 29. Telecommunications Service-Profit & Loss Statement, Earnings.

1975 PMG Annual Report 1974-1975. p57, Table 5, Earnings

1976-1978 PMG Annual Report 1977-78. Page 81. Telecommunications Service- Profit & Loss Statement, Earnings.

1979-1981 PMG Annual Report 1980-81. Page 87. Table 5. Earnings

1982 PMG Annual Report 1981-82. Page 20. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1983-1984 PMG Annual Report 1983-84. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

TELEXCAL

General Description: Number of Telex Calls.

Source:

1955 Australian Telecommunications Commission Annual Report 1977-78, page 85 Table 12.

1956 PMG Annual Report 1956-57, page 21 second paragraph

1957 PMG Annual Report 1957-58, page 16, Teleprinter Exchange Service, second paragraph.

1959, 1960 PMG Annual Report page 20, Teleprinter Exchange Service, second paragraph.

1960 Australian Telecommunications Commission Annual Report 1977-78, page 85 Table 12.

1962 PMG Annual Report 1961-62, page 18, first paragraph.

1963 PMG Annual Report 19621-63, page 12, Expanding Telex Network, first paragraph.

1964 PMG Annual Report 1963-64, page 11, Teleprinter Exchange Service, first paragraph.

1965-1969 PMG Annual Report 1974-75, Page 76, Table 41.

1970-1978 Australian Telecommunications Commission Annual Report 1977-78, page 85 Table 12.

1979-1981 Australian Telecommunications Commission Annual Report 1980-81, Statistical Supplement, Table 11.

1982-1986 Australian Telecommunications Commission Annual Report 1982-83, Statistical Supplement, Table 3.

1987-1988 Australian Telecommunications Commission 1987-88, Statistical Supplement, Table 2.

1989-1990 Bureau of Transportation & Communication Economics (1994), Statistical Summary of the Communications, Entertainment & Information Industries, Attachment 1 to Paper, Table 4.9 pg 135.

TLXCLRV

General Description: Telex Calls Revenue

Source:

(1960), (1970-1981) PMG Annual Report Table 5. Telecommunications Service-Profit & Loss Statement, Earnings.

1982 PMG Annual Report 1981-82. Page 20. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1983-1984 PMG Annual Report 1983-84. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings

TELEXSRV

General Description: Number of Telex subscribers. Service began in 1955.

Source:

1955-1975. Australian Telecommunications Commission Annual Report 1974-75, page 76, Table 41 Telex Network. Column: Australia.

1976-1980. Australian Telecommunications Commission Annual Reports Table 27 Telex Services in Operation. Column: Australia.

1981-1985. Australian Telecommunications Commission Annual Reports, Table 18 Telex Services in Operation. Column: Australia.

1986-1990. Bureau of Transportation & Communication Economics (1994), Statistical Summary of the Communications, Entertainment & Information Industries, Attachment 1 to Paper, Table 4.9 pg 135.

TELEXRNT

General Description: Telex rental revenue.

Source:

1960, 1970-1981 PMG Annual Report Page 81. Table 5. Telecommunications Service- Profit & Loss Statement, Earnings.

1982 PMG Annual Report 1981-82. Page 20. Australian Telecommunications Commission Profit and Loss Statement, Earnings.

1983-1984 PMG Annual Report 1983-84. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings

DATELSUB

General Description: Total number of DATEL subscribers

Source:

1970 PMG Annual Report 1970-71, page 10. Data Services . First paragraph, fourth line.

1971 PMG Annual Report 1970-71, page 10. Data Services . First paragraph, sixth line.

1972 PMG Annual Report 1971-72, page 11. Datel Services. Second paragraph, second line.

1973 PMG Annual Report 1973-74, page 72. Table 25 Datel Services. Row: Total.

1974 PMG Annual Report 1973-74, page 72. Table 25 Datel Services. Row: Total.

1975 PMG Annual Report 1974-75, page 74. Table 36 Datel Services. Row: Total.

1976 Australian Telecommunications Commission Annual Report 1977-78, page 96. Table 34 Datel Services. Row: Total.

1977 Australian Telecommunications Commission Annual Report 1977-78, page 96. Table 34 Datel Services. Row: Total.

1978 Australian Telecommunications Commission Annual Report 1977-78, page 96. Table 34 Datel Services. Row: Total.

1979 Australian Telecommunications Commission Annual Report 1980-81, page 97. Table 31 Datel Services (a) Data Modems in Operation. Row: Total.

1980 Australian Telecommunications Commission Annual Report 1980-81, page 97. Table 31 Datel Services (a) Data Modems in Operation. Row: Total.

1981 Australian Telecommunications Commission Annual Report 1980-81, page 97. Table 31 Datel Services (a) Data Modems in Operation. Row: Total.

1982 Australian Telecommunications Commission Annual Report 1982-83, page 74. Table 22 Datel Services (a) Data Modems in Operation. Column: Australia.

1983 Australian Telecommunications Commission Annual Report 1983-84, page 73. Table 22 Datel Services (a) Data Modems in Operation. Column: Australia.

1984 Australian Telecommunications Commission Annual Report 1983-84, page 73. Table 22 Datel Services (a) Data Modems in Operation. Column: Australia.

1985 Telecom Annual Report 1989-90. Page 53 Ten Year Statistical Summary. Row: Data Services-Data Modems in Operation. Column: 1984-85.

1986 Telecom Annual Report 1989-90. Page 53 Ten Year Statistical Summary. Row: Data Services-Data Modems in Operation. Column: 1985-86.

1987 Telecom Annual Report 1989-90. Page 53 Ten Year Statistical Summary. Row: Data Services-Data Modems in Operation. Column: 1986-87.

1988-1990 Telecom Annual Report 1989-90. Page 53 Ten Year Statistical Summary. Row: Data Services-Data Modems in Operation. Column: 1987-88.

DDS

General Description: total number of Digital Data Subscribers

Source:

1985 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1984-85.

1986 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1985-86.

1987 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1986-87.

1988 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1987-88.

1989 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1988-89.

1990 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - Digital Data Service - Network Terminating Units. Column: 1989-90.

AUSTPAC

General Description: total number of AUSTPAC subscribers

Source:

1983 Telecom Service and Business Outlook 1983/84, page 12. Data Communications, fourth paragraph, 3rd line.

1984 Not available

1985-1990 Australian Telecommunications Commission Annual Report 1989-90. Ten Year Statistical Summary. Row: - AUSTPAC Service - Number of Outstations. Column: 1985-86

OTHTELEC

General Description: telecommunications revenue not elsewhere defined

Source:

1985 PMG Annual Report 1985-86. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings. Row: Other Network Services.

1986 PMG Annual Report 1985-86. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings. Row: Other Network Services.

1987 PMG Annual Report 1987-88. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings. Row: Other Network Services.

1988 PMG Annual Report 1987-88. Page 22. Australian Telecommunications Commission Profit and Loss Statement, Earnings. Row: Other Network Services.

1989 Australian Telecommunications Commission Annual Report 1989-90. Page 60. 2. Operating Revenue. Row: Other Telecommunications Revenue.

1990 Australian Telecommunications Commission Annual Report 1989-90. Page 60. 2. Operating Revenue. Row: Other Telecommunications Revenue.

MANUAL

General Description: total number of subscribers connected to manual exchanges.

Source:

Entire Sample: Power 1978

SXS

General Description: total number of subscribers connected to step-by-step exchanges.

Source:

Entire Sample: Power 1978

ARK

General Description: total number of subscribers connected to manual exchanges.

Source:

Entire Sample: Power 1978

ARE

General Description: total number of subscribers connected to ARE exchanges.

Source:

Entire Sample: Power 1978

AXE

General Description: total number of subscribers connected to AXE exchanges.

Source:

Entire Sample: Power 1978

DDSC

General Description: total number of subscribers connected to digital data Services.

Source:

1983. Telecom Service and Business Outlook for 1983/84, page 12. Data Communications, DDS. Beginning of service. Refers to service being available to all capital cities including Canberra.

1984. Telecom Service and Business Outlook for 1984/85, page 16. Data Communications, DDS.

1985. Telecom Service and Business Outlook for 1985/86, page 19. Data Communications, Digital Data Service.

1986. Telecom Service and Business Outlook for 1986/87, page 22. Dedicated Network Services, Digital Data Service.

1988. Telecom Service and Business Outlook for 1986/87, page 22. Dedicated Network Services, Digital Data Service.

AUSTPACX

General Description: total number of subscribers connected to Austpac.

Source:

1985. Australian Telecommunications Commission, Service and Business Outlook for 1985/86, page 19. Network Services, Data Communications, -Austpac.

1987. Australian Telecommunications Commission, Service and Business Outlook for 1987, page 16. Initiatives for large business customers, additional packet switching exchanges in 1988.

1988. Australian Telecommunications Commission, Service and Business Outlook for 1985/86, page 19. Network Services, Data Communications, -Austpac.

XCHGQ

General Description: total number of exchanges reported annually.

Source:

1920-1931. PMG Annual Reports, Appendix Q.

1932-1938. PMG Annual Reports, Appendix T.

1938-1940. PMG Annual Reports, Appendix R.

1942-1943. PMG Annual Reports, Appendix L.

1944-1955. PMG Annual Reports, various appendices.

1956. PMG Financial Statistics, Telephone Lines and Instruments, Number of exchanges.

1957-1963. Facts at a Glance, Telephone Exchanges.

1964. PMG Annual Report. The Year in Brief, Telephone Exchanges.

1965-1969. PMG Annual Report 1972-73. Table 29. Telephone Exchanges, Telephone Offices, Subscribers' Services and Public Telephones.

1970-1975.PMG Annual Report 1975, page 70, Table 27.

1976-1987. Telecom Annual Reports, Statistical Supplement.

COAX_KM

General Description: Total coaxial cable in network. Miles multiplied by 1.60934 to convert to kilometres.

Source:

1960-1961. PMG Annual Reports, Coaxial Cable Projects.

1962-1973. PMG Annual Report 1972-73, Table 41 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Cable

1974-1976. PMG Annual Reports, Table 29 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Cable.

1977-1981. Australian Telecommunications Commission Annual Report 1981-82, page 100, Table 26 Cable, Conduit, Aerial Wire, Pole and Radio Routes.

1982-1985. Australian Telecommunications Commission Annual Reports, Table 17 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Coaxial Cable in Tube Kilometres.

OPTIC_KM

General Description: total kilometres of optic fibre cable

Source:

1984 Australian Telecommunications Commission Annual Report 1983-84, page 72, Table 17 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Optical Cable in Fibre km.

1985 Australian Telecommunications Commission Annual Report 1984-85, page xx, Table 17 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Optical Cable in Fibre km.

1986 Not available

1987 Not available

1988 Not available

1989 Australian Telecommunications Commission Annual Report 1988-89, page 3. A year of major achievement. Second last bullet point.

1990 Not available

COST

General Description: total cost of operating the telecommunications division

Source:

1920-1961 Sum of IntFone, Costfone, IntData and CostData.

1962-1963 Sum of Costfone and CostData.

1964-1990 CostTele

INTFONE

General Description: interest of borrowings assigned to the telephone division.

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June, for the year. Interest and Exchange Charges as per Branch Accounts-Telephone. Figure doubled to convert to dollars.

1953-1955 PMG Annual Report Appendix 4 - All Branches Profit and Loss Account for the year. Interest and Exchange Charges as per Branch Accounts-Telephone. Figure doubled to convert to dollars.

1956-1961 PMG Annual Report Appendix 5 - Telephone Branch - Profit and Loss Statement for the year. Row: Interest. Figure doubled to convert to dollars.

COSTFONE

General Description: total cost for telecommunications. Largely taken from the profit and loss statement.

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June of year. Expenditure as per Branch Accounts-Telephone.

1953-1961 PMG Annual Report - All Branches - Profit and Loss Account for the year. Working Expenses as per Branch Accounts-Telephone. Figure doubled to convert to dollars.

1962-1963 PMG Annual Report Telephone Service - Statement of Profit and Loss for the year. Total Expenses. Figure doubled to convert to dollars.

1964-1975 PMG Annual Report Telecommunications Service - Statement of Profit and Loss for the year. Total Expenses. Figure doubled to convert to dollars.

1976-1978 Australian Telecommunications Commission Annual Report for the year. Table 6 Expenses. Row: Total Expenses. Column: year.

1979-1981 Australian Telecommunications Commission Annual Report for the year. Table 6 Expenses. Column: Total.

1982-1984 Australian Telecommunications Commission Annual Report for the year. Australian Telecommunications Commission Profit and Loss Statement Year Ended 30 June for the year. Column: the year. Row: Total Expenses.

INTDATA

General Description: interest paid on borrowings assigned to the telegraph division.

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June, for the year. Interest as per Branch Accounts-Telegraph. Column: Total. Figure doubled to convert to dollars.

1953 PMG Annual Report Appendix 4 - All Branches - Profit and Loss Account 1952/53. Interest and Exchange as per Branch Accounts-Telegraph. Figure doubled to convert to dollars.

1954-1955 PMG Annual Report Appendix 4 - All Branches - Profit and Loss Account for the year. Interest and Exchange as per Branch Accounts-Telegraph. Figure doubled to convert to dollars.

1956-1961 PMG Annual Report. Appendix 6 - Telegraph Branch - Statement of Profit and Loss - for the year. Row: Interest. Figure doubled to convert to dollars.

COSTDATA

General Description: total cost of the telegraph division as reported on profit and loss statements.

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June, for the year. Expenditure as per Branch Accounts-Telegraph. Figure doubled to convert to dollars.

1953-1961 PMG Annual Report Appendix 6 Telegraph Branch - Statement of Profit and Loss – for the year. Total Working Expenses (excluding Interest). Figure doubled to convert to dollars.

1962-1963 PMG Annual Report Telegraph Branch - Statement of Profit and Loss – for the year. Total Expenses. Figure doubled to convert to dollars.

COSTTELE

General Description: total cost for the telecommunications division (telephone and telegraph division combined).

Source:

1964 PMG Annual Report 1964-65, page 26. Telecommunications Service - Statement of Profit and Loss - 1963/64. Total Expenses. Figure doubled to convert to dollars.

1965 PMG Annual Report 1964-65, page 26. Telecommunications Service - Statement of Profit and Loss - 1964/65. Total Expenses. Figure doubled to convert to dollars.

1966 PMG Annual Report 1965-66, page 24. Telecommunications Service - Statement of Profit and Loss - 1965-66. Total Expenses. First year of dollar currency.

1967 PMG Annual Report 1967-68, page 28. Telecommunications Service - Statement of Profit and Loss - 1967-68. Row: Total Expenses. Column: 1966/67.

1968 PMG Annual Report 1967-68, page 26. Telecommunications Service - Statement of Profit and Loss - 1967-68. Row: Total Expenses. Column: 1967/68.

1969 PMG Annual Report 1969-70, page 38. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1970. Row: Total Expenses. Column: 1969.

1970 PMG Annual Report 1969-70, page 38. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1970. Row: Total Expenses. Column: 1970.

1971 PMG Annual Report 1970-71, page 26. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1971. Row: Total Expenses. Column: 1971.

1972 PMG Annual Report 1971-72, page 26. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1972. Row: Total Expenses. Column: 1972.

1973 PMG Annual Report 1972-73, page xx. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1973. Row: Total Expenses. Column: 1973.

1974 PMG Annual Report 1974-75, page 45. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1975. Row: Total Expenses. Column: 1974.

1975 PMG Annual Report 1974-75, page 45. Telecommunications Service - Profit and Loss Statement Year Ended 30 June 1975. Row: Total Expenses. Column: 1975.

1976 Australian Telecommunications Commission Annual Report 1976-78, Profit and Loss Statement Year Ended 30 June 1976, page 59. Table 6 Expenses. Row: Total Expenses. Column: 1976.

1977 Australian Telecommunications Commission Annual Report 1977-78, page 81. Table 6 Expenses. Row: Total Expenses. Column: 1977.

1978 Australian Telecommunications Commission Annual Report 1977-78, page 81. Table 6 Expenses. Row: Total Expenses. Column: 1978.

1979 Australian Telecommunications Commission Annual Report 1980-81, page 87. Table 6 Expenses. Column: Total.

1980 Australian Telecommunications Commission Annual Report 1980-81, page 87. Table 6 Expenses. Column: Total.

1981 Australian Telecommunications Commission Annual Report 1980-81, page 87. Table 6 Expenses. Column: Total.

1982 Australian Telecommunications Commission Annual Report 1982-83, page 18. Australian Telecommunications Commission Profit and Loss

Statement Year Ended 30 June 1983. Column: 1982. Row: Total Expenses.

1983 Australian Telecommunications Commission Annual Report 1983-84, page 22. Australian Telecommunications Commission Profit and Loss Statement Year Ended 30 June 1984. Column: 1983. Row: Total Expenses.

1984 Australian Telecommunications Commission Annual Report 1983-84, page 22. Australian Telecommunications Commission Profit and Loss Statement Year Ended 30 June 1984. Column: 1983. Row: Total Expenses.

1985 Australian Telecommunications Commission Annual Report 1985-86, page xx. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1986. Row: Expenses, Total Expenses.

1986 Australian Telecommunications Commission Annual Report 1984-85, page xx. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1985. Row: Expenses, Total Expenses.

1987 Australian Telecommunications Commission Annual Report 1986-87, page xx. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1987. Row: Expenses, Total Expenses.

1988 Australian Telecommunications Commission Annual Report 1987-88, page xx. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1988. Row: Expenses, Total Expenses

1989 Australian Telecommunications Corporation and Subsidiaries Annual Report 1989, page 68. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1989. Row: Expenses, Total Expenses.

1990 Australian Telecommunications Corporation and Subsidiaries Annual Report 1990, page 60. Australian Telecommunications Commission Profit and Loss Statement For The Year Ended 30 June 1990. Row: Operating Expenses, Total Expenses.

WAGES_E

General Description: Total wages expenditure.

Source:

1920-1975. Calculated by deducting the Post division wage expenditure from total wage expenditure.

1976-1990. Total wage expenditure.

Total wage expenditure

General description: Total expenditure on wages.

Source:

1920-1942. Parliamentary Papers, Finance, Miscellaneous statistics, Schedule-Salaries and Allowances. These figures include salaries for temporary workers.

1943-1949. PMG Annual Report. Financial Turnover, found under salaries and payments in the nature of salary.

1950-1968. Parliamentary Papers, Finance, Miscellaneous statistics, Schedule-Salaries and Allowances. These figures include salaries for temporary workers.

1969-1975. PMG Annual Report, Table 8, Cash Expenditure.

1976-1988. Australian Telecommunications Commission Annual Report. Reported on page 5, 6, 7 or 8.

1989. Telecom Annual Report 1988-89, page 14 (Pie chart).

1990. Telecom Annual Report 1989-90, page 20 (Pie chart)

Post division wage expenditure

General description: Total expenditure on wages by Post division.

Source:

1920-1975. PMG Annual Report.

WAGES_P

General Description: Total wage expenditure divided by Employees.

Employees (Telecom.)

General Description: Total number of full-time staff employed in all three divisions of the Postmaster General's Department.

Source:

1920-1975. The calculated number of full-time staff employed in the Telegraph and Telephone divisions of the PMG 1920-1975. This calculation is the ratio of Labour Share to Operating Cost divided by the Average Wage.

1976-1975. Equal to the variable Employees as previously reported.

COMPLTIN

General Description: the name ComPltIn is derived from the descriptive name Communications Plant Investment and is the total annual communications capital expenditure. The series is also reported directly in financial accounts. Since this series is the sum of sub-aggregates, considerable effort was expended in ensuring that the aggregate series is consistently measured and that there were no inexplicable 'jumps' by changes in measurement.

Source:

1920-1970. This is the sum of: DataPL2 and FoneP1.

1971-1981. Sum of: DataPL1, DataPL2 and FoneP1.

1982-1990. As reported in financial statements. For example, Telecom Service and Business Outlook for 1883/84. Page 14. Increase in Fixed Assets/Stores Holdings. Column: 1982/83. Row: Communications Plant/Stores Holdings.

DATAPL1

General Description Capital Programme. Telegram & Data

Source

1971-1981 Australian Telecommunications Commission Table 3 Investment in Fixed Assets, Column: Data Transmission Plant

1982 Australian Telecommunications Commission Annual Report 1981-82. Page 45. Datel Equipment.

1983 Australian Telecommunications Commission Annual Report 1982-83, page 8, Capital Programme. Telegram & Data

1984 Australian Telecommunications Commission Annual Report 1983-84, Expansion of the Network, page 47.

1986 Telecom Annual Report 1985-86, page 50.

DATAPL2

General Description: Data Transmission Equipment +Packet Switching

Source

1913 PMG Annual Report: p123, 1912-13

1916-1922 PMG Annual Report. Table No. 8 Detail Statement of Fixed Assets. Expenditure column. Row: Total Telegraph Equipment.

1923-1931, 1950, 1952 PMG Annual Report 1922-23. Page 361. Appendix A Table No. 8 Detail Statement of Fixed Assets. Expenditure column. Row: Total Telegraph Equipment

1932-1949 PMG Annual Report 1943-44. Page 35. Appendix A Table 9. Detailed Statement of Fixed Assets.

1953 PMG Annual Report 1952-53. Appendix 2. Detailed Statement of Fixed Assets.

1955-1959, 1961-1969 Financial and Statistical Supplement, Year Ended 30 June 1973. Page 7 Investment in Fixed Assets

1960, 1970 Australian Telecommunications Commission 1974-75, Table 3 Investment in Fixed Assets, page 56.

1971-1977 Australian Telecommunications Commission 1977-78, Table 3 Investment in Fixed Assets, page 80.

1979-1981 Australian Telecommunications Commission Table 3 Investment in Fixed Assets.

1982 Australian Telecommunications Commission Annual Report 1981-82. Page 45.

1983 Australian Telecommunications Commission Annual Report 1982-83, page 8, Capital Programme.

1984 Australian Telecommunications Commission Annual Report 1983-84, Expansion of the Network, page 47. Datel Equipment + Digital Data Equipment + Teleprinters.

1986 Telecom Annual Report 1985-86, page 50, Materials Purchasing Expenditure, Data Transmission Equipment +Packet Switching.

FONEP1

General Description: Investment in Fixed Assets. Capital Programme. Subscribers' equipment, Property, Plant and Equipment.

Source

1916-1922 PMG Annual Report 1915-16. Page 50. Table No. 8 Detail Statement of Fixed Assets

1923-1931, 1949-1952 PMG Annual Report. Appendix A Table No. 8 Detail Statement of Fixed Assets

1932-1948 PMG Annual Report. Appendix A Table No. 9 Detailed Statement of Fixed Assets

1953-1954 PMG Annual Report. Appendix 2. Detailed Statement of Fixed Assets

1955-1959, 1961, 1963-1969 Financial and Statistical Supplement, page 7

1960 PMG Annual Report 1974-75. Page 56. Table 3 Investment in Fixed Assets.

1962 PMG Financial and Statistical Bulletin 1961-62, Table 7. Investment in Fixed Assets.

1970-1975 PMG Annual Report. Table 3 Investment in Fixed Assets.

1976-1981 Australian Telecommunications Commission 1975-76, Table 3 Investment in Fixed Assets

1982 Telecom Annual Report 1981-82. Page 38

1983 Australian Telecommunications Commission Annual Report 1982-83, page 8, Capital Programme. Subscribers' equipment.

1984 Australian Telecommunications Commission Annual Report 1983-84, Expansion of the Network, page 47.

1986 Telecom Annual Report 1985-86, page 50, Materials Purchasing Levels

2000 Australian Telecommunications Commission Annual Report 2000-2001. Page 214. 12. Property, Plant and Equipment.

2001-2002 Australian Telecommunications Commission Annual Report 2000-2001. Page 215. 12. Property, Plant and Equipment.

CABLE_KM

General Description: Calculated variable. Measures total quantity of cables and conduits in kilometres.

Source:

1920-1927. PMG Annual Report, 1923-24. Appendix V. Cables and Conduits. Total. 2. Telephones-Mileage of conductors in underground cables (pairs, I.e., loop mileage). Figure converted to kilometres by multiplying by 1.60934.

1929. TpConQ, TpJuncQ, TPUndLQ, ConduitQ, ConRoutQ, TunnelQ, TrunkQ, JointQ, TgraphQ, SubXchgQ, SubXcWQ, NonXchQ, JuncQ, TgMorseQ, PoleXchQ, PolenXcQ.

1930-1931. TpConQ, TpJuncQ, TPUndLQ, ConduitQ, ConRoutQ, TunnelQ, TrunkQ, JointQ, TgraphQ, SubXchgQ, SubXcWQ, NonXchQ, JuncQ, TgMorseQ, PoleXchQ, PolenXcQ, ConSubQ.

1932-1935. TPUndLQ, ConduitQ, ConRoutQ, TunnelQ, TrunkQ, JointQ, TgraphQ, SubXchgQ, SubXcWQ, NonXchQ, JuncQ, TgMorseQ, PoleXchQ, PolenXcQ, ConSubQ, TpJunQ.

1936-1939. ConduitQ, ConRoutQ, TunnelQ, TrunkQ, JointQ, TgraphQ, SubXchgQ, SubXcWQ, NonXchQ, JuncQ, TgMorseQ, PoleXchQ, PolenXcQ, ConSubQ, TpJunQ, TPCondQ.

1940. ConduitQ, ConRoutQ, TunnelQ, TrunkQ, JointQ, TgraphQ, SubXchgQ, SubXcWQ, NonXchQ, JuncQ, TgMorseQ, PoleXchQ, PolenXcQ, ConSubQ, TpJunQ, TPCondQ, AerLineQ, JcableQ.

1941-1957 Source: PMG Financial and Statistical Bulletin 1956-57, page 55, Table 48 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Single Wire Mileage of Cables. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

1958-1959. PMG Financial and Statistical Bulletin 1970-71, Table 40 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Single Wire Mileage of Cables. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

1960 PMG Annual Report ending 30 June 1960, page 58.

1961 PMG Annual Report ending 30 June 1961, page 46.

1962 PMG Annual Report ending 30 June 1962, page 32.

1963-1970. PMG Annual Report 1972-73, Table 41 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Cable.

1971-1975 Australian Telecommunications Commission Annual Report 1974-75, page 73, Table 34 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Cable.

1976-1979 Australian Telecommunications Commission Annual Report 1978-79, page 100, Table 26 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Cable.

1980-1983 Australian Telecommunications Commission Annual Report Table 16, 17, Cable, Conduit, Aerial Wire, Pole and Radio Routes. Cable.

1984 Australian Telecommunications Commission Annual Report 1983-84, Expansion of the Network, page 47.

1985. Australian Telecommunications Commission Annual Report 1984-85, Table 17 Cable, Conduit, Aerial Wire, Pole and Radio Routes. Column: Length of Cable Conductors.

TpConQ

General Description: Telephones-Mileage of conductors in aerial cables (pairs, i.e., loop mileage). Figure converted to kilometres by multiplying by 1.60934.

Source:

1920-1925. PMG Annual Reports, Appendix U. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in aerial cables (pairs, i.e. loop mileage).

1926-1928. PMG Annual Reports, Appendix V. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in aerial cables (pairs, i.e. loop mileage).

1929-1932. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in aerial cables (pairs, i.e. loop mileage).

TpJuncQ

General Description: Telephones-mileage of conductors in cables for junction circuits (pairs, i.e. loop mileage). Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1925. PMG Annual Report 1918-19. Page 73. Appendix U. Cables and Conduits. Column: Total. Row: 3. Telephones-mileage of conductors in cables for junction circuits (pairs, i.e. loop mileage).

1926-1928. PMG Annual Report 1927-28. Page 65. Appendix V. Cables and Conduits. Column: Total. Row: 3. Telephones-mileage of conductors in cables for junction circuits (pairs, i.e. loop mileage).

1929-1932. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: 3. Telephones-mileage of conductors in cables for junction circuits (pairs, i.e. loop mileage).

TPUndLQ

General Description: Telephones-mileage of conductors in underground cables (pairs, i.e. loop mileage). Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1921. PMG Annual Reports, Appendix U. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in underground cables (pairs, i.e. loop mileage).

1922-1928. PMG Annual Reports, Appendix V. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in underground cables (pairs, i.e. loop mileage).

1929-1935. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: Telephones-mileage of conductors in underground cables (pairs, i.e. loop mileage).

ConduitQ

General Description: Conduits - length of, in duct miles. Figure converted to kilometres by multiplying by 1.60934.

Source:

1920-1925. PMG Annual Reports, Appendix U. Cables and Conduits. Column: Total. Row: Conduits - length of, in duct miles.

1926-1929. PMG Annual Reports, Appendix V. Cables and Conduits. Column: Total. Row: Conduits - length of, in duct miles.

1930-1940. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: Conduits - length of, in duct miles.

ConRoutQ

General Description: Conduits-route mileage. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1925. PMG Annual Reports, Appendix U. Cables and Conduits. Column: Total. Row: Conduits-route mileage.

1926-1940. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: Conduits-route mileage.

TunnelQ

General Description: Mileage of tunnels. Figure converted to kilometres by multiplying by 1.60934.

Source:

1920-1921. PMG Annual Reports, Appendix U. Cables and Conduits. Column: Total. Row: Tunnels, mileage of.

1922-1928. PMG Annual Reports, Appendix V. Cables and Conduits. Column: Total. Row: Tunnels, mileage of.

1929-1940. PMG Annual Reports, Appendix W. Cables and Conduits. Column: Total. Row: Tunnels, mileage of.

TrunkQ

General Description: Telephone trunk lines. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1921. PMG Annual Reports, Appendix V. Open Conductors. Column: Total. Row: Telephone trunk lines only.

1922. PMG Annual Report 1923-24. Page 66. Appendix W. Open Conductors. Column: Total. Row: Telephone trunk lines only.

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix V. Open Conductors. Column: Total. Row: Telephone trunk lines only.

1926-1939. PMG Annual Reports, Appendix X. Open Conductors. Column: Total. Row: Telephone trunk lines only.

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Column: Total. Row: 1. Telephone trunk lines only.

JointQ

General Description: Telegraph and telephone purposes. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1925. PMG Annual Reports, Appendix V. Open Conductors. Total. Telegraph and telephone purposes.

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix W. Open Conductors. Total. Telegraph and telephone purposes.

1929-1939. PMG Annual Reports, Appendix X. Open Conductors. Total. Telegraph and telephone purposes.

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 2. Telegraph and telephone purposes.

TgraphQ

General Description: Open conductors for telegraph only. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1921. PMG Annual Reports, Appendix V. Open Conductors. Total. Telegraph purposes only.

1922-1928. PMG Annual Reports, Appendix W. Open Conductors. Total. Telegraph purposes only.

1929-1939. PMG Annual Report 1930-31. Page 72. Appendix X. Open Conductors. Total. Telegraph purposes only.

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 3. Telegraph purposes only.

SubXchgQ

General Description: Subscribers' Exchange lines (metallic circuit). Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1921. PMG Annual Reports, Appendix V. Open Conductors. Total. Subscribers' Exchange lines (metallic circuit).

1922. PMG Annual Report 1923-24. Page 66. Appendix W. Open Conductors. Total. Subscribers' Exchange lines (metallic circuit).

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix V. Open Conductors. Total. Subscribers' Exchange lines (metallic circuit).

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix W. Open Conductors. Total. Subscribers' Exchange lines (metallic circuit).

1929-1932. PMG Annual Report 1930-31. Page 72. Appendix X. Open Conductors. Total. Subscribers' Exchange lines (metallic circuit).

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix X. Open Conductors. Total. 4. Subscribers' Exchange lines.

1936-1939. PMG Annual Report 1938-39. Page 77. Appendix X. Open Conductors. Total. 4. Subscribers' Exchange lines.

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 4. Subscribers' Exchange lines.

SubXcWQ

General Description: Open Conductors for subscribers' exchange lines (single wire circuit). Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix V. Open Conductors. Total. Subscribers' Exchange lines (single wire circuit).

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix V. Open Conductors. Total. Subscribers' Exchange lines (single wire circuit).

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix W. Open Conductors. Total. Subscribers' Exchange lines (single wire circuit).

1929. PMG Annual Report 1930-31. Page 72. Appendix X. Open Conductors. Total. Subscribers' Exchange lines (single wire circuit).

1930-1932. PMG Annual Report 1931-32. Page 68. Appendix X. Open Conductors. Total. 5. Subscribers' Exchange lines (single wire circuit).

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix X. Open Conductors. Total. 5. Subscribers' Exchange lines (single wire circuit).

1936-1939. PMG Annual Report 1938-39. Page 77. Appendix X. Open Conductors. Total. 5. Subscribers' Exchange lines (single wire circuit).

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 5. Subscribers' Exchange lines (single wire circuit).

NonXchQ

General Description: Open Conductors, Non-exchange circuits. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix V. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1923. PMG Annual Report 1923-24. Page 66. Appendix W. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1924-1925. PMG Annual Report 1924-25. Page 69. Appendix W. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix W. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1929. PMG Annual Report 1930-31. Page 72. Appendix X. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1930-1932. PMG Annual Report 1931-32. Page 68. Appendix X. Open Conductors. Total. 6. Non-exchange circuits (fire alarms, private wires, &c.).

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix X. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1936-1939. PMG Annual Report 1938-39. Page 77. Appendix X. Open Conductors. Total. Non-exchange circuits (fire alarms, private wires, &c.).

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 6. Non-exchange circuits (fire alarms, private wires, &c.).

JuncQ

General Description: Open Conductors, Junction Circuits. Figure converted to kilometres by multiplying by 1.60934.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix V. Open Conductors. Total. Junction Circuits.

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix V. Open Conductors. Total. Junction Circuits.

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix W. Open Conductors. Total. Junction Circuits.

1929-1931. PMG Annual Report 1930-31. Page 72. Appendix X. Open Conductors. Total. Junction Circuits.

1932. PMG Annual Report 1931-32. Page 68. Appendix X. Open Conductors. Total. 7. Junction Circuits.

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix X. Open Conductors. Total. Junction Circuits.

1936-1939. PMG Annual Report 1938-39. Page 77. Appendix X. Open Conductors. Total. Junction Circuits.

1940. PMG Annual Report 1939-40. Page 67. Appendix V. Open Conductors. Total. 7. Junction Circuits.

TgMorseQ

General Description: Telegraph Cable, Submarine Cable, and Pole Routes, Telegraph-mileage of conductors in Morse cable. Figure converted to kilometres by multiplying by 1.60934.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix X. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1929-1931. PMG Annual Report 1930-31. Page 72. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1932. PMG Annual Report 1931-32. Page 68. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1936-1939. PMG Annual Report 1938-39. Page 80. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

1940. PMG Annual Report 1939-40. Page 67. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Telegraph-mileage of conductors in Morse cable.

PoleXchQ

General Description: Telegraph Cable, Submarine Cable, and Pole Routes. Mileage of pole routes - (a) Within Exchange networks. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix X. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1929-1931. PMG Annual Report 1930-31. Page 72. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1932. PMG Annual Report 1931-32. Page 68. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. 3. Mileage of pole routes - (a) Within Exchange networks.

1936-1939. PMG Annual Report 1938-39. Page 79. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. 3. Mileage of pole routes - (a) Within Exchange networks.

1940. PMG Annual Report 1939-40. Page 67. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. 3. Mileage of pole routes - (a) Within Exchange networks.

PolenXcQ

General Description: Telegraph Cable, Submarine Cable, and Pole Routes. Mileage of pole routes - (b) Outside Exchange networks. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1920-1922. PMG Annual Report 1921-22. Page 68. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1923-1925. PMG Annual Report 1924-25. Page 69. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (a) Within Exchange networks.

1926-1928. PMG Annual Report 1927-28. Page 67. Appendix X. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

1929-1931. PMG Annual Report 1930-31. Page 72. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

1932. PMG Annual Report 1931-32. Page 68. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

1936-1939. PMG Annual Report 1938-39. Page 79. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

1940. PMG Annual Report 1939-40. Page 67. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of pole routes - (b) Outside Exchange networks.

ConSubQ

General Description: Telegraph Cable, Submarine Cable, and Pole Routes. Mileage of conductors in submarine cable (statute miles). Figure converted to kilometres by multiplying by 1.60934.

Source:

1930-1932. PMG Annual Report 1931-32. Page 68. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of conductors in submarine cable (statute miles).

1933-1935. PMG Annual Report 1934-35. Page 77. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of conductors in submarine cable (statute miles).

1936-1939. PMG Annual Report 1938-39. Page 79. Appendix Y. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of conductors in submarine cable (statute miles).

1940. PMG Annual Report 1939-40. Page 67. Appendix W. Telegraph Cable, Submarine Cable, and Pole Routes. Total. Mileage of conductors in submarine cable (statute miles).

TpJunQ

General Description: Cables and Conduits. Column: Total. Row: 3. Telephones-mileage of working conductors in cables for junction circuits (pairs, i.e., loop mileage). Figure converted to kilometres by multiplying by 1.60934.

Source:

1932-1935. PMG Annual Report 1934-35. Page 76. Appendix W. Cables and Conduits. Total. 3. Telephones-mileage of working conductors in cables for junction circuits (pairs, i.e., loop mileage).

1936-1939. PMG Annual Report 1938-39. Page 76. Appendix W. Cables and Conduits. Total. 2. Telephones-mileage of working conductors in cables for junction circuits (pairs, i.e., loop mileage).

1940. PMG Annual Report 1939-40. Page 66. Appendix U. Cables and Conduits. Total. 2. Telephones-mileage of working conductors in cables for junction circuits (pairs, i.e., loop mileage).

TPCondQ

General Description: Cables and Conduits. Telephones-mileage of conductors in aerial and underground cables. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1936-1938. PMG Annual Reports, Appendix W. Cables and Conduits. Total. Telephones-mileage of conductors in aerial and underground cables.

1939-1940. PMG Annual Reports, Appendix U. Cables and Conduits. Total. Telephones-mileage of conductors in aerial and underground cables.

AerLineQ

General Description: Aerial Wires and Pole Routes. Column: Total. Row: 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.). Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1940. PMG Annual Report 1940-41. Page 66. Appendix P. Aerial Wires and Pole Routes. Total. 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.).

1941. PMG Annual Report 1940-41. Page 66. Appendix V. Aerial Wires and Pole Routes. Total. 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.).

1942-1945. PMG Annual Reports, Appendix P. Aerial Wires and Pole Routes. Total. 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.).

1946-1952. PMG Annual Reports, Appendix M. Aerial Wires and Pole Routes. Total. 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.).

1953-1955. PMG Annual Reports, Appendix 23. Aerial Wires and Pole Routes. Total. 2. Exchange and non-exchange service lines (including subscribers' lines, public telephone lines, junction lines, fire alarms, private lines, &c.).

1956-1969. PMG Financial Bulletins, Aerial Wires and Pole Routes. Commonwealth. 1. Exchange and non-exchange lines.

JCableQ

General Description: Aerial Wires and Pole Routes. 1. Telephone, trunk, and/or telegraph purposes. Figure converted to kilometres by multiplying by 1.60934. Figure halved to convert single wire to pair kilometres.

Source:

1940-1941. PMG Annual Report 1940-41. Page 66. Appendix V. Aerial Wires and Pole Routes. Total. 1. Telephone, trunk, and/or telegraph purposes.

1942-1945. PMG Annual Reports, Appendix P. Aerial Wires and Pole Routes. Total. 1. Telephone, trunk, and/or telegraph purposes.

1946-1952. PMG Annual Reports, Appendix M. Aerial Wires and Pole Routes. Total. 1. Telephone, trunk, and/or telegraph purposes.

1953-1955. PMG Annual Reports, Aerial Wires and Pole Routes. Total. 1. Telephone, trunk, and/or telegraph purposes.

1956-1969. PMG Financial Statistical Bulletins, Aerial Wires and Pole Routes as at 30th June 1956. Column: Total. Row: 1. Single wire mileage of aerial wires - (b) Trunk and/or telegraph.

COMM_S

General description: communications equipment capital stock. Calculated by first ComPltIn series by price deflator, Comm_P. The resulting annual quantity is then used to calculate Comm_S according to the Perpetual Inventory Model. That is,

$$K_{i,t} = K_{i,t-1} + \sum_{\tau=1}^{T} k_{i,\tau}\left(1-\delta\right)^{t-\tau-1}$$

where

$K_{i,t}$ = real gross capital stock (aggregate) for capital class *i* at time *t*.

δ = depreciation rate

k = each period's addition to capital

COMM_P

General description: Price deflator for communications capital. Prior to 1949 the series is aggregate price based on: cable expenditure divided by Cable_KM; FoneP2 plus DataPl2 divided by annual change in the number of exchanges. The resulting two price series are aggregated according to the Törnqvist formula. COMM_P is then spliced with ABS electronic price series in log change form. Overlapping series are combined as a weighted average.

Cable expenditure

General Description: Expenditure on telegraph lines, telephone trunk lines and underground cable.

Source:

1920-1922 PMG Annual Report Table 8. Detail Statement of Fixed Assets.

1923-1931 PMG Annual Report Appendix A Table No. 8 Detail Statement of Fixed Assets.

1932-1948 PMG Annual Report Appendix A Table No. 9 Detailed Statement of Fixed Assets.

1953-1955, 1957-1959 PMG Annual Report, Appendix 2, found under the heading Total Joint Plant.

1956 PMG Financial Statistical Bulletin 1956. Page 12. Table 6 Detailed Summary of Fixed Assets.

1957. Annual Report year ending June 1957, Appendix 2, pg 52, found under Total Joint Plant but now moved into Telephone and Telegraph Plant.

1958. Annual Report ending 30 June 1958, Appendix 2, pg 46.

1959. Annual Report ending 30 June 1959, Appendix 2, pg 52.

1960. PMG Annual Report ending 30 June 1960, pg 58

1961 PMG Annual Report ending 30 June 1961, pg 46

1962 PMG Annual Report ending 30 June 1962, pg 32

BUILD

General Description: expenditure on land and buildings.

Source:

1920-1954. Sum of Buildings and Land

1950 PMG Annual Report 1974-75. Page 56. Table 3. Investment in Fixed Assets. Column: Land & Buildings.

1955-1959, 1961-1969 Financial and Statistical Supplement , Year Ended 30 June 1973. Page 7 Investment in Fixed Assets. Column: Land and Buildings

1960, 1970-1978 PMG Annual Report. Table 3. Investment in Fixed Assets. Column: Land & Buildings.

1979-1981 Australian Telecommunications Commission 1978-79, Table 1 Summary of Transactions Affecting Fixed Assets, page 90.

1982 Australian Telecommunications Commission 1981-82, Balance Sheet, page 21. Land and Buildings total row

1983 Australian Telecommunications Commission Annual Report 1982-83, page 8, Capital Programme. Land & Buildings

Buildings

General description: annual expenditure on buildings

Source:

1916-1922 PMG Annual Report 1915-16. Page 51. Table 8 Detail Statement of Fixed Assets. Expenditure column. Row: Buildings

1923-1931,1949-1952 PMG Annual Report Appendix A Table No. 8 Detail Statement of Fixed Assets. Expenditure column. Row: Buildings.

1932-1946, 1948 PMG Annual Report Appendix A Table No. 9 Detailed Statement of Fixed Assets. Expenditure column. Row: Buildings.

1947 PMG Financial and Statistical Bulletin 1963-64. Page 18. Table 11. Cash Expenditure on Capital Works. Column: Buildings

1953-1961 PMG Annual Report Appendix 2 Detailed Statement of Fixed Assets.

1962, 1968, 1973 PMG Annual Report, Page 32. Detailed Summary of Fixed Assets. Expenditure column, Buildings row.

1963 PMG Annual Report 1962-63, Page 18. Capital Development. Buildings

1964, 1966 PMG Annual Report, Page 21. Balance Sheet. Buildings

1965 PMG Annual Report 1964-65, Page 23. Balance Sheet. Buildings

1967 PMG Annual Report 1966-67, Page 25. Balance Sheet. Buildings.

1969 PMG Financial Statistical Bulletin 1968-69, Page xx. Table 5 summary of Transactions Affecting Fixed Assets

1970 PMG Annual Report 1969-70, Page 40. Fixed Assets. Buildings.

1971-1972 PMG Annual Report 1970-71, Page 28. Fixed Assets. Buildings.

1974 PMG Annual Report 1973-74, Page 31. Fixed Assets. Buildings

1975 PMG Annual Report 1974-75, Page 47. Fixed Assets. Buildings

1976-1980 Australian Telecommunications Commission Table 1 Summary of Transactions Affecting Fixed Assets.

1984 Australian Telecommunications Commission Annual Report 1983-84, page 29, Note 10. Fixed Assets.

1985, 1987 Telecom Annual Report Note 7. Fixed Assets. Buildings.

1986 Telecom Annual Report 1984-85, page 32, Note 11. Fixed Assets. Buildings.

1988-1989 Telecom Annual Report Notes 13. Fixed Assets.

1990 Telecom Annual Report 1989-90. Page 63. Note 18. Non-current assets.

Land

General description: expenditure acquiring land.

Source:

1916-1922 PMG Annual Report Table No. 8 Detail Statement of Fixed Assets

1923-1931, 1950-1952 PMG Annual Report Appendix A Table No. 8 Detail Statement of Fixed Assets.

1932-1949 PMG Annual Report Appendix A Table No. 9 Detailed Statement of Fixed Assets

1953-1961 PMG Annual Report Appendix 2 Detailed Statement of Fixed Assets.

BUILD_S

General description: Land and buildings capital stock. Buildings expenditure is divided by NDC_P to derive volume (quantity) of buildings added annually. Buildings capital stock then created by applying the Perpetual Inventory Method. Land is identified by deducting Buildings expenditure from Land and Buildings expenditure. Land quantity series is then added using the weighted average formula to buildings capital.

NDC_P

General description: ABS Non-dwelling construction. NUQA.PD_NDC_PC_GJ. Non-dwelling construction, Commonwealth Public Trading Enterprises.

Source:

Australian Bureau of Statistics.

MCHE_P

General description: capital price deflator for machinery and equipment. This is the weighted sum of Other Plant and Equipment price and Road Vehicles price for the years 1920 to 1948. The price series from 1949 is sourced from the Australian Bureau of Statistics.

Source:

1920-1948. Other Plant and Equipment deflator plus Road Vehicles deflator

1949-1990. Australian Bureau of Statistics series NUQA.PD_OPL_PC_GJ. Other Plant, Commonwealth Public Trading Enterprises.

Other Plant and Equipment Price Deflator

General description: price deflator derived by regressing the series NUQA.PD_OPL_PC_GJ on a time trend and the series NUQA.PD_NDC_PC_GJ. The series is then backcast to 1920.

Road Vehicles Price Deflator

General description: price deflator for road vehicles.

Source:

1920-1948. US National Bureau of Economic Research series National Bureau of Economic Research www.nber.org. Investment at historical cost index divided by Fixed $ investment index. Both series relate to the telecommunications industry.

1949-1990. Australian Bureau of Statistics series NUQA.PD_RVH_PC_GJ. Road motor vehicles, Commonwealth Public Trading Enterprises.

MCHE_E

General description: expenditure on machinery and equipment.

Source:

Road vehicle expenditure plus Other Plant and Equipment Expenditure

Other Plant and Equipment Expenditure

General description: expenditure on variable capital such as furniture and engineers' movable plant.

Source:

1920-1949. Sum of Furniture and Other.

1950, 1955-1959, 1961-1969. Financial and Statistical Supplement, Year Ended 30 June 1973. Page 7 Investment in Fixed Assets. Column: Other Plant and Equipment

1960 PMG Annual Report 1975-76. Page 74. Table 3 Investment in Fixed Assets. Other Plant & Equipment. Includes Engineers' Movable Plant

1970-1975 PMG Annual Report 1977-78. Page 80. Table 3 Investment in Fixed Assets. Other Plant & Equipment. Includes Engineers' Movable Plant

1976 PMG Annual Report 1975-76. Page 74. Table 3 Investment in Fixed Assets. Other Plant & Equipment. Includes Engineers' Movable Plan

1977-1981 PMG Annual Report 1980-81. Page 86. Table 3 Investment in Fixed Assets. Other Plant & Equipment. Includes Engineers' Movable Plant

1986 Australian Telecommunications Commission 1985-86. Page 32. Page 11. Fixed Assets. Other Plant & Equipment. Net investment for the year

Furniture expenditure

General Description: expenditure on furniture and office equipment.

Source:

1920-1922 PMG Annual Report Table No. 8 Detail Statement of Fixed Assets.

1923-1931, 1950 PMG Annual Report. Appendix A Table No. 8 Detail Statement of Fixed Assets.

1932-1949 PMG Annual Report. Appendix A Table No. 9 Detailed Statement of Fixed Assets.

Other

General Description: other plant and equipment plus engineers movable plant minus furniture and office equipment.

Source:

1916-1922 PMG Annual Report. Table 8 Detail Statement of Fixed Assets.

1923-1931 PMG Annual Report Appendix A Table No. 8 Detail Statement of Fixed Assets

1932-1935 PMG Annual Report 1931-32. Page 32. Appendix A Table No. 9 Detailed Statement of Fixed Assets

1936-1949 PMG Annual Reports.

Road vehicles

General Description: Expenditure on motor vehicles.

Source:

1920-1922 PMG, Annual Report Table 8, Detail Statement of Fixed Assets.

1923-1926, PMG, Annual Report, Appendix A, Table 8.

1927, PMG Annual Report, Appendix A, Table 9.

1928-1930, 1948 PMG, Annual Report, Appendix A, Table 8.

1931-1947, PMG, Annual Report, Appendix A, Table 9.

1948 PMG, Annual Report, Appendix A, Table 8.

1949-1952 PMG, Annual Reports, Appendix A, Table 8.

1953-1954 PMG, Annual Report, Appendix 2.

1960 PMG Annual Report 1974-75. Page 56. Table 3 Investment in Fixed Assets

1955-1959, 1961, 1963-1969 PMG, Financial and Statistical Supplement, Year Ended 30 June 1973. Page 7 Investment in Fixed Assets

1962 PMG Annual Report 1961-62, Page 32. Detailed Summary of Fixed Assets

1970-1975 PMG Annual Report. Table 3 Investment in Fixed Assets

1976-1981 Australian Telecommunications Commission, Table 1 Summary of Transactions Affecting Fixed Assets

1982 Australian Telecommunications Commission Annual Report 1981-82, page 45

1983 Australian Telecommunications Commission Annual Report 1982-83, page 8, Capital Programme.

1984 Australian Telecommunications Commission Annual Report 1983-84, page 29, 10. Fixed Assets.

1985 Telecom Annual Report 1984-85, page 29, Note 7. Fixed Assets.

1986 Telecom Annual Report 1984-85, page 32, Note 11. Fixed Assets

1987-1988 Telecom Service and Business Outlook for September 1987. Page 23. Application. Column: 1987/88. Row: Motor Vehicles.

1988-1990 Extrapolated.

MCHE_S

General description: machinery and equipment capital stock. Calculated by deflating MCHE_E and applying the Perpetual Inventory Method.

BOND_YLD

General description: bond yield on 10 year Commonwealth Government bonds.

Source:

1920-1973 Butlin (1977).

1974-1990 Reserve Bank of Australia.

DEPTELE

General description: depreciation expense charge as recorded on the Profit and Loss statement.

Source:

1920-1964 Sum of DepFone and DepData.

1965-1975. PMG Annual Reports. Telecommunications Service - Statement of Profit and Loss. Row: Depreciation.

1976-1990. Australian Telecommunications Commission Annual Reports, Profit and Loss Statement Year Ended 30 June. Row: Depreciation.

DepFone

General Description: telephone division depreciation expense.

Source:

1920-1964 PMG Annual Report Profit and Loss Account - Telephone Branch (Including Exchanges, Trunk Lines, and Non-Exchange Lines) For Year Ended 30th June Row: Depreciation. Column: Total. Figure doubled to convert to dollars.

DepData

General description: telegraph division depreciation expense

Source:

1920-1964 PMG Annual Report - Profit and Loss Account - Telegraph Branch For Year Ended 30th June Row: Depreciation. Column: Total. Figure doubled to convert to dollars.

INTTELE

General description: interest expense as charged on the profit and loss statement.

Source:

1920-1963 Sum of IntFone and IntData.

1964-1975 PMG Annual Reports. Telecommunications Service - Statement of Profit and Loss. Row: Interest.

1976-1990 Australian Telecommunications Commission Annual Reports Profit and Loss Statement Year Ended 30 June. Row: Interest.

IntFone

General Description: interest expense as charged on the telegraph division profit and loss statement

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June, for the year. Interest and Exchange Charges as per Branch Accounts-Telephone. Figure doubled to convert to dollars.

1953-1955 PMG Annual Report Appendix 4 - All Branches Profit and Loss Account for the year. Interest and Exchange Charges as per Branch Accounts-Telephone. Figure doubled to convert to dollars.

1956-1963 PMG Annual Report Appendix 5 - Telephone Branch - Profit and Loss Statement for the year. Row: Interest. Figure doubled to convert to dollars.

IntData

General Description: interest expense as charged on the telegraph division profit and loss statement.

Source:

1914-1952 PMG Annual Report Table No. 3 - General Profit and Loss Account for Year Ended 30th June, for the year. Interest as per Branch Accounts-Telegraph. Column: Total. Figure doubled to convert to dollars.

1953 PMG Annual Report Appendix 4 - All Branches - Profit and Loss Account 1952/53. Interest and Exchange as per Branch Accounts-Telegraph. Figure doubled to convert to dollars.

1954-1955 PMG Annual Report Appendix 4 - All Branches - Profit and Loss Account for the year. Interest and Exchange as per Branch Accounts-Telegraph. Figure doubled to convert to dollars.

1956-1963 PMG Annual Report. Appendix 6 - Telegraph Branch - Statement of Profit and Loss - for the year. Row: Interest. Figure doubled to convert to dollars.

CHAPTER 6—APPENDIX 1A

```
                              Grant Coble-Neal 2005
     Estimation of the two-output, three-input Modified Generalised McFadden cost function
                          Code file name: Final Model.Sha
                            Data file name: PMG.Dif



                                  SUMMARY STATISTICS

 NAME        N   MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM  COEF.OF.VARIATION CONSTANT-
DIGITS
 BUILD_S      71  0.19975E+08 0.16605E+08 0.27573E+15  0.50036E+07  0.57140E+08  0.83130       0.0000
 COMM_S       71  0.82689E+08 0.77405E+08 0.59915E+16  0.15087E+08  0.31932E+09  0.93609       0.0000
 MCHE_S       71  0.22692E+08 0.29146E+08 0.84951E+15   20441.      0.10421E+09   1.2844       0.0000
 LABOUR       71   36359.      19457.     0.37856E+09   8735.3       77388.      0.53512       0.0000
 NAME        N   MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM  COEF.OF.VARIATION CONSTANT-
DIGITS
 TOTTELGR     71  0.20573E+08 0.11004E+08 0.12110E+15   0.0000      0.40980E+08  0.53490       0.0000
 LOCCALLS     71  0.21703E+10 0.22798E+10 0.51974E+19  0.22064E+09  0.87502E+10   1.0505       0.0000
 STDCALLS     71  0.26460E+09 0.41209E+09 0.16982E+18  0.12420E+08  0.17942E+10   1.5574       0.0000
 TELEXCAL     71  0.97685E+07 0.15892E+08 0.25257E+15   0.0000      0.51301E+08   1.6269       0.0000
 TELEXSRV     71   7986.1      13677.     0.18707E+09   0.0000       46423.       1.7127       0.0000
 DATELSUB     71   13632.      30592.     0.93590E+09   0.0000      0.10396E+06   2.2442       0.0000
 DDS          71   3869.3      14794.     0.21886E+09   0.0000       84189.       3.8235       0.0000
 AUSTPAC      71   439.31      1777.0     0.31576E+07   0.0000       9664.0       4.0449       0.0000
 CELLCALL     71  0.36761E+07 0.20579E+08 0.42351E+15   0.0000      0.15600E+09   5.5982       0.0000
 CELLSUB      71   4443.9      24715.     0.61081E+09   0.0000      0.18494E+06   5.5615       0.0000
 SUBSCRIB     71  0.20667E+07 0.21194E+07 0.44917E+13  0.17211E+06  0.77869E+07   1.0255       0.0000
 NAME        N   MEAN        ST. DEV      VARIANCE     MINIMUM      MAXIMUM  COEF.OF.VARIATION CONSTANT-
DIGITS
 COAX_KM      71   13556.      19268.     0.37127E+09   0.0000       50403.       1.4214       0.0000
 OPTIC_KM     71   6882.7      31478.     0.99087E+09   0.0000      0.18200E+06   4.5735       0.0000
 MANUAL       71  0.22056E+06 0.11129E+06 0.12386E+11   0.0000      0.40266E+06  0.50458       0.0000
 SXS          71  0.76362E+06 0.57631E+06 0.33213E+12   13845.      0.17854E+07  0.75470       0.0000
 ARK          71  0.22345E+06 0.36261E+06 0.13149E+12   0.0000      0.11500E+07   1.6228       0.0000
 ARF          71  0.55512E+06 0.96050E+06 0.92256E+12   0.0000      0.30357E+07   1.7302       0.0000
 ARE          71  0.23121E+06 0.63552E+06 0.40389E+12   0.0000      0.23459E+07   2.7486       0.0000
 AXE          71   72697.     0.27419E+06 0.75181E+11   0.0000      0.15409E+07   3.7717       0.0000



 ...NOTE..SAMPLE RANGE SET TO:    35,    71
   32 VARIABLES IN  3 EQUATIONS WITH  13 COEFFICIENTS
 WITH   9 AUTOREGRESSIVE COEFFICIENTS
 ..ALGORITHM USES NUMERIC DERIVATIVES
         37 OBSERVATIONS

 REQUIRED MEMORY IS PAR=     147 CURRENT PAR=    4000

 COEFFICIENT STARTING VALUES
 A1         1.0000      D11        1.0000      D21        1.0000
 D22        1.0000      D11_       1.0000      D12_       1.0000
 D22_       1.0000      FZ         1.0000      A2         1.0000
 CD2        1.0000      FZ1        1.0000      A3         1.0000
 CT3        1.0000      RHO        0.0000      RHO        0.0000
 RHO        0.0000      RHO        0.0000      RHO        0.0000
 RHO        0.0000      RHO        0.0000      RHO        0.0000
 RHO        0.0000
     10000 MAXIMUM ITERATIONS, CONVERGENCE =   0.100000E-04

 INITIAL STATISTICS :
 TIME =     0.0200 SEC.   ITER. NO.     0   FUNCT. EVALUATIONS    23
 FUNCTION VALUE=    40.52080
 COEFFICIENTS
    1.000000       1.000000       1.000000       1.000000       1.000000
    1.000000       1.000000       1.000000       1.000000       1.000000
    1.000000       1.000000       1.000000       0.000000       0.000000
    0.000000       0.000000       0.000000       0.000000       0.000000
    0.000000       0.000000
 GRADIENT
  -0.7929657E-07  0.1040348E-04 -0.4855056E-03 -0.4741759E-03  0.3823049E-02
   0.4986811E-03  0.4945235E-04  0.4898143E-02  0.3811351E-06  0.4050753
   0.2399487E-02  0.1210765E-06 -0.7335643E-06   33.50083       9.153997
    4.314865      -1.580426       34.61329       4.346308       1.791657
    4.517059       32.38006

 FINAL STATISTICS :

 TIME =    4.406 SEC.   ITER. NO.   234   FUNCT. EVALUATIONS  7833
 FUNCTION VALUE=    288.9053
 COEFFICIENTS
```

```
    379.9215       173.4497      -103.2504     -0.2634027E-05   6234.326
   -11312.55       456442.1       758.6928       1021.394       10.80305
    258.5649       286.4551       3575.832       0.7172431     -0.1406701E-01
   0.2798700E-01  0.3741529      0.8156270      0.6041397      0.4606952
   0.2022011E-01  0.9269561
 GRADIENT
   0.4206413E-07 -0.7389644E-08  0.3524292E-07  0.5684342E-09  0.3979039E-08
   0.5684342E-09 -0.1705303E-08  0.1705303E-08  0.1853095E-06  0.1813305E-06
   0.1421085E-07  0.2955858E-07  0.1347189E-06  0.6028245E-04  0.9138773E-03
  -0.5223342E-04 -0.6423306E-06 -0.1722356E-05 -0.1477929E-06  0.3136051E-04
   0.5148877E-03 -0.3041691E-04

 SIGMA MATRIX
  0.33178E-02
 -0.10088E-04  0.10858E-04
  0.68846E-03  0.38728E-05  0.10688E-02
 GTRANSPOSE*INVERSE(H)*G  STATISTIC  -  =  0.37823E-11

       COEFFICIENT    ST. ERROR   T-RATIO

 A1        379.92      2.0711      183.44
 D11       173.45      56.058      3.0941
 D21      -103.25      11.461     -9.0085
 D22     -0.26340E-05  58.585    -0.44961E-07
 D11_      6234.3      906.08      6.8806
 D12_     -11313.      2626.2     -4.3076
 D22_     0.45644E+06  2163.2      211.00
 FZ        758.69      426.42      1.7792
 A2        1021.4      5.7084      178.93
 CD2       10.803      6.7206      1.6074
 FZ1       258.56      72.078      3.5873
 A3        286.46      1.7263      165.94
 CT3       3575.8      17.363      205.95
 RHO      0.71724     0.11683      6.1394
 RHO     -0.14067E-01 0.71250E-02 -1.9743
 RHO      0.27987E-01 0.65383E-01 0.42805
 RHO      0.37415      1.4445     0.25902
 RHO      0.81563     0.82634E-01  9.8703
 RHO      0.60414     0.66500     0.90848
 RHO      0.46070     0.19727      2.3354
 RHO      0.20220E-01 0.11952E-01  1.6917
 RHO      0.92696     0.10901      8.5036
```

Single Equation Serial Correlation Test Results

```
     ARIMA MODEL
 NUMBER OF OBSERVATIONS =  37
 ...NOTE..SAMPLE RANGE SET TO:    35,    71

 REQUIRED MEMORY IS PAR=      90 CURRENT PAR=    4000
      IDENTIFICATION SECTION - VARIABLE=U1
 NUMBER OF AUTOCORRELATIONS =    5
 NUMBER OF PARTIAL AUTOCORRELATIONS =   5

              0     0 0
 SERIES  (1-B) (1-B  )  U1

   NET NUMBER OF OBSERVATIONS =   37
 MEAN=   0.11433E-02   VARIANCE=   0.34086E-02   STANDARD DEV.=   0.58383E-01

     LAGS                                   AUTOCORRELATIONS                                   EST. STD
ERROR
                                                                                                  FOR ROW
     1  -5   0.031 -0.071 -0.179  0.001 -0.063                                                     0.164

  MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
     LAG    Q    DF  P-VALUE       LAG    Q    DF  P-VALUE
      1    0.04   1  .846           3    1.61   3  .658
      2    0.25   2  .884           4    1.61   4  .808
                                    5    1.79   5  .878


     LAGS                               PARTIAL AUTOCORRELATIONS                               EST. STD
ERROR
                                                                                                  FOR ROW
     1  -5   0.031 -0.072 -0.176  0.006 -0.091                                                     0.164


ARIMA MODEL
 NUMBER OF OBSERVATIONS =  37
 ...NOTE..SAMPLE RANGE SET TO:    35,    71
```

```
 REQUIRED MEMORY IS PAR=      90 CURRENT PAR=    4000
      IDENTIFICATION SECTION - VARIABLE=U2
 NUMBER OF AUTOCORRELATIONS =   5
 NUMBER OF PARTIAL AUTOCORRELATIONS =   5

             0     0 0
 SERIES  (1-B) (1-B  )  U2

   NET NUMBER OF OBSERVATIONS =   37
 MEAN=  -0.40936E-05   VARIANCE=   0.11160E-04   STANDARD DEV.=   0.33406E-02

     LAGS                                AUTOCORRELATIONS                                     EST. STD
ERROR
                                                                                                 FOR ROW
     1  -5   0.261 -0.129 -0.092 -0.124 -0.093                                                   0.164

  MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
     LAG    Q    DF  P-VALUE       LAG    Q    DF  P-VALUE
      1    2.73   1  .098           3    3.79   3  .286
      2    3.42   2  .181           4    4.45   4  .348
                                    5    4.85   5  .435


     LAGS                            PARTIAL AUTOCORRELATIONS                                 EST. STD
ERROR
                                                                                                 FOR ROW
     1  -5   0.261 -0.212  0.004 -0.138 -0.039                                                   0.164

     ARIMA MODEL
 NUMBER OF OBSERVATIONS =  37
 ...NOTE..SAMPLE RANGE SET TO:    35,    71

 REQUIRED MEMORY IS PAR=      90 CURRENT PAR=    4000
      IDENTIFICATION SECTION - VARIABLE=U3
 NUMBER OF AUTOCORRELATIONS =   5
 NUMBER OF PARTIAL AUTOCORRELATIONS =   5

             0     0 0
 SERIES  (1-B) (1-B  )  U3

   NET NUMBER OF OBSERVATIONS =   37
 MEAN=   0.45145E-03   VARIANCE=   0.10983E-02   STANDARD DEV.=   0.33140E-01

     LAGS                                AUTOCORRELATIONS                                     EST. STD
ERROR
                                                                                                 FOR ROW
     1  -5  -0.071 -0.007 -0.229  0.122  0.037                                                   0.164

  MODIFIED BOX-PIERCE (LJUNG-BOX-PIERCE) STATISTICS  (CHI-SQUARE)
     LAG    Q    DF  P-VALUE       LAG    Q    DF  P-VALUE
      1    0.20   1  .651           3    2.43   3  .489
      2    0.21   2  .902           4    3.08   4  .544
                                    5    3.14   5  .678


     LAGS                            PARTIAL AUTOCORRELATIONS                                 EST. STD
ERROR
                                                                                                 FOR ROW
     1  -5  -0.071 -0.012 -0.231  0.094  0.046                                                   0.164
```

Single Equation Heteroskedasticity Test Results

```
CHI-SQUARE PARAMETERS- DF=   2.0000
MEAN=   2.0000     VARIANCE=   4.0000     MODE=   0.0000

               DATA        PDF        CDF        1-CDF
  M_WHIT1
 ROW     1     2.4341     0.14805     0.70390     0.29610
CHI-SQUARE PARAMETERS- DF=   2.0000
MEAN=   2.0000     VARIANCE=   4.0000     MODE=   0.0000

               DATA        PDF        CDF        1-CDF
  M_WHIT2
 ROW     1     1.8020     0.20308     0.59385     0.40615
CHI-SQUARE PARAMETERS- DF=   2.0000
MEAN=   2.0000     VARIANCE=   4.0000     MODE=   0.0000

               DATA        PDF        CDF        1-CDF
  M_WHIT3
 ROW     1     1.1020     0.28818     0.42364     0.57636
```

Equation System Serial Correlation Test Results, see Doornik 1996

```
RR2
  0.2452394
RM2
  0.8398325E-01
T_
   36.00000
TR_
   2.748050
D_V0V0
  0.1659042E-05
D_VV
  0.1252180E-05
CHI-SQUARE PARAMETERS- DF=   27.000
MEAN=   27.000     VARIANCE=   54.000     MODE=   25.000

               DATA        PDF        CDF        1-CDF
  LM
 ROW     1     9.0702     0.50536E-03 0.48826E-03 0.99951
DF1_
   27.00000
DF2_
   37.00000
F DISTRIBUTION- DF1=   27.000     DF2=   37.000
MEAN=   1.0571     VARIANCE=  0.15553     MODE=  0.87844

               DATA        PDF        CDF        1-CDF
  LMF
 ROW     1    0.13367     0.27602E-04 0.33932E-06  1.0000
N_
   16.50000
T_
   36.00000
Q
   12.50000
N
   3.000000
P
   9.000000
```

Alternative Equation System Serial Correlation Test Results, see Enders 1995

```
CHI-SQUARE PARAMETERS- DF=   9.0000
MEAN=   9.0000     VARIANCE=   18.000     MODE=   7.0000

               DATA        PDF        CDF        1-CDF
  LR_
 ROW     1     9.4072     0.87920E-01 0.59943     0.40057
T_
   36.00000
```

System Heteroskedasticity Test Results, See Doornik 1996

```
G
   6.000000
DETERMINANT SHOULD BE MULTIPLIED BY 10**  -30
DETERMINANT SHOULD BE MULTIPLIED BY 10**  -30
Q
   17.00000
NA
   28.50000
T_
   36.00000
HB
   6.000000
G
   6.000000
K1
   1.000000
DF_1
   36.00000
DF_2
   68.50000
RM2
  0.1881841
RR2
  0.7595443
CHI-SQUARE PARAMETERS- DF=   36.000
MEAN=   36.000     VARIANCE=   72.000     MODE=   34.000

               DATA        PDF        CDF        1-CDF
  LM_H
 ROW     1     40.648     0.36096E-01 0.72697     0.27303
F DISTRIBUTION- DF1=   36.000     DF2=   68.000
MEAN=   1.0303     VARIANCE=  0.93989E-01 MODE=  0.91746

               DATA        PDF        CDF        1-CDF
  LMF_H
 ROW     1     1.1519      1.0488     0.69704     0.30296
```

Hessian Matrix of Second-order Input Price Parameters

```
HA2
    3 BY     3 MATRIX
  -30084.81       17908.75       12176.06
   17908.75      -10660.64      -7248.110
   12176.06      -7248.110      -4927.949
```

Calculated Determinants

```
D21_DET
  -30084.81
D22_DET
  0.2188958E-06
D23_DET
  0.4479403E-18

Rounded determinants
D21_DET
  -30084.81
D22_DET
  0.2188958E-06
D23_DET
   0.000000
```

If the cost function is concave in input prices, variable Concave2 will equal 1 and zero otherwise

```
CONCAVE2
   1.000000
```

Year	Fixed Cost	Variable Cost	Marginal Cost: Aggregate	Marginal Cost: Data
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
0.	0.	0.	0.00	0.00
1954.	117456237.	49576702.	1.61	0.00
1955.	127795830.	55199083.	1.72	0.00
1956.	141079455.	60059380.	1.74	0.00
1957.	160785058.	57708851.	1.63	0.00
1958.	171388362.	59965342.	1.64	0.00
1959.	183620562.	65924472.	1.69	0.00
1960.	188525611.	68096077.	1.60	0.00
1961.	233928326.	77956228.	1.73	0.00
1962.	244029513.	81719427.	1.77	0.00
1963.	251118083.	79464830.	1.59	0.00
1964.	257270495.	81876744.	1.52	0.00
1965.	285321505.	88185227.	1.54	0.00
1966.	311612784.	98627130.	1.64	0.00
1967.	346979251.	102825090.	1.61	0.00
1968.	384184911.	111458723.	1.72	0.00
1969.	449450604.	95100440.	1.41	0.00
1970.	520275369.	110537946.	1.30	14200.87
1971.	632214810.	106893968.	1.20	7345.61
1972.	657046871.	145804592.	1.35	12140.46
1973.	733404909.	149441986.	1.23	9210.35
1974.	1002521967.	213716458.	1.50	10251.70
1975.	1318795626.	294134766.	1.81	10603.76
1976.	1509336437.	430688515.	2.28	11945.43
1977.	1754724008.	605801699.	2.80	12616.99
1978.	1864340648.	675547449.	2.69	10870.49
1979.	2170197089.	792716937.	2.84	10126.97
1980.	2587721074.	1002711857.	3.08	10036.51
1981.	3074609651.	1194052732.	3.19	9859.82
1982.	3821894184.	1630950612.	3.76	10894.65
1983.	4089281062.	1909838834.	3.82	10490.94
1984.	4396023354.	2233192699.	3.89	10442.79
1985.	4638980605.	2649982342.	3.90	9897.03
1986.	5112622007.	3188705207.	4.08	10104.95
1987.	5641004323.	3909542985.	4.56	11107.20
1988.	5127074182.	5062299544.	4.08	11743.01
1989.	6397612524.	4994835254.	3.83	10830.61
1990.	6883569131.	5658351196.	4.05	11523.99

Year	Ratio: Fixed to Total Cost	Ratio: Fitted Cost to Actual Cost
1954.000	0.7031921	1.000000
1955.000	0.6983573	1.000000
1956.000	0.7014034	1.000000
1957.000	0.7358789	1.000000
1958.000	0.7408066	1.000000
1959.000	0.7358213	1.000000
1960.000	0.7346441	1.000000
1961.000	0.7500478	1.000000
1962.000	0.7491337	1.000000
1963.000	0.7596221	1.000000
1964.000	0.7585805	1.000000
1965.000	0.7638992	1.000000
1966.000	0.7595867	1.000000
1967.000	0.7714004	1.000000
1968.000	0.7751233	1.000000
1969.000	0.8253599	1.000000
1970.000	0.8247692	1.000000
1971.000	0.8553745	1.000000
1972.000	0.8183916	1.000000
1973.000	0.8307272	1.000000
1974.000	0.8242808	1.000000
1975.000	0.8176395	1.000000
1976.000	0.7779985	1.000000
1977.000	0.7433615	1.000000
1978.000	0.7340247	1.000000
1979.000	0.7324536	1.000000
1980.000	0.7207268	1.000000
1981.000	0.7202747	1.000000
1982.000	0.7008991	1.000000
1983.000	0.6816468	1.000000
1984.000	0.6631287	1.000000
1985.000	0.6364390	1.000000
1986.000	0.6158801	1.000000
1987.000	0.5906472	1.000000
1988.000	0.5031785	1.000000
1989.000	0.5615661	1.000000
1990.000	0.5488449	1.000000

```
UNIT 88 IS NOW ASSIGNED TO: C:\Shazam\Ancillary.dif
UNIT 88 IS NOW ASSIGNED TO: C:\Shazam\Fitted.dif
TYPE COMMAND
```

CHAPTER 6—APPENDIX 1B SHAZAM CODE

This appendix presents the computer code necessary for the model presented in Chapter 6.

```
=Format(//38X,'Grant Coble-Neal 2005'/ &
        7X, 'Estimation of the two-output, three-input Modified Generalised McFadden cost
function'/ &
        31X,'Code file name: Final Model.Sha'/37X,'Data file name: PMG.Dif'//)
=Print / Format

=Set NOECHO
Set Missvalue=-99999
SIZE 2500
Set NODOECHO
Set NOWARN

* This file estimates a two-output, three-input Modified Generalised McFadden cost
function

?Read(C:\Shazam\PMG.dif) / Dif

* Create time trend
Genr T=Time(-35)
Set Wide

* Interpolate missing observations
Genr Austpac:66=(Austpac:67/Austpac:65)**0.5*Austpac:65
Sample 42 50
Genr Tlxrent=(Tlxrent:51/Tlxrent:41)**(1/(51-41))*Lag(Tlxrent)

Sample 1 71
Genr TR=TelecomR
* Create telephone call output variable
Genr foneCl_Q=LocCalls+STDCalls

* Create average prices
Sample 1 71

Genr foneCl_P=CallRev/foneCl_Q
Genr fonesb_P=RentRev/Subscrib
Genr Tlgrph_P=0
Sample 1 68
Genr Tlgrph_P=TelgrRev/Tottelgr
Sample 65 68
Genr Tlgrph_P=Tlgrph_P:64

Sample 1 71

Genr TlxCl_P=0
Sample 41 65

Genr TlxCl_P=Tlxcalrv/TelexCal
Sample 36 40

Genr TlxCl_P=Tlxcl_P:41
Sample 66 71

Genr TlxCl_P=Tlxcl_P:65
Sample 1 71

Genr Tlxrnt_P=0
Sample 41 65

Genr Tlxrnt_P=Tlxrent/TelexSrv
Sample 36 40

Genr Tlxrnt_P=Tlxrnt_P:41
Sample 66 71
```

```
Genr Tlxrnt_P=Tlxrnt_P:65
Sample 1 71

Genr CelCal_P=0
Genr CelSubRv=CelAcRev+CelCnFee
Genr CelSub_P=0
Sample 68 71

Genr CelCal_P=CelCalRv/CellCall
Genr CelSub_P=CelSubRv/CellSub

Sample 1 71

* Aggregate outputs
?Index Tlgrph_P Tottelgr foneCl_P foneCl_Q Tlxcl_P TelexCal CelCal_P CellCall foneSb_P
Subscrib CelSub_P CellSub Tlxrnt_P TelexSrv  / Fisher=Tel_P1 QFisher=Tel_Q1
?Index foneSb_P Subscrib CelSub_P CellSub Tlxrnt_P TelexSrv / Fisher=Tel_P2
QFisher=Tel_Q2
?Index Tlgrph_P Tottelgr Tlxcl_P TelexCal CelCal_P CellCall Tlxrnt_P TelexSrv CelSub_P
CellSub foneSb_P Subscrib  / Fisher=Tel_P3 QFisher=Tel_Q3

Genr Data=DatelSub+DDS+Austpac

* Create labour quantity
Genr Labour=Wages_E/Wages_P

* Construct Materials expenditure, price
Genr Mat_E=Cost-Wages_E-Deptele-Inttele
Genr Mat_P=Mat_E/Comm_S

* Construct rental prices, i.e. multiply price deflator by the sum of the depreciation
rate and the bond yield
Genr Build_r=NDC_P*(1/49+Bond_yld/100)
Genr Comm_r=Comm_P*(1/15.1+Bond_yld/100)
Genr Mche_r=Mche_P*(1/17.3+Bond_yld/100)

* Produce summary statistics of capital stock and number of staff
Format(//40X,'SUMMARY STATISTICS'/)
Print / Format

Stat Build_S Comm_S Mche_S Labour / wide

* Construct total cost
Genr TC=Build_r*Build_S+Comm_r*Comm_S+Mche_r*Mche_S+Mat_E+Wages_E

* Calculate expenditure shares
Genr Build_ES=Build_r*Build_S/TC
Genr Comm_ES=Comm_r*Comm_S/TC
Genr Mche_ES=Mche_r*Mche_S/TC
Genr Mat_ES=Mat_E/TC
Genr Wages_ES=Wages_E/TC

* Produce summary statistics of expenditure shares
?Stat TC Build_ES Comm_ES Mche_ES Mat_ES Wages_ES /wide

* Produce summary stats of output quantities
Stat Tottelgr Loccalls stdcalls telexcal telexsrv datelsub DDS Austpac cellcall cellsub
subscrib / Wide

* Produce summary stats of technical change variables
Stat Coax_KM Optic_KM Manual SxS ARK ARF ARE AXE / wide

Format(//)
Print / Format

*****************************************************************************************
***************
** Aggregate inputs

?Index Build_R Build_S Comm_R Comm_S Mche_R Mche_S / Divisia=Cap_P QDivisia=Cap_Q
```

```
*****************************************************************************************
***************


* Produce summary statistics of input prices
?Stat Cap_P Mat_p Wages_P / wide Means=Mean_r


*** Create cost function variables
* Outputs

Sample 1 71
?Stat Tel_Q1 Data / Means=Q_means
Genr Q1=Tel_Q1
Genr Q2=Data



=************************************ Output functions

** Output 1
Gen1 L1=0.25
Genr f_Q1=(Q1**L1-1)/L1+1/L1
Genr f1_Q1=L1*Q1**(L1-1)/L1

** Output 2

Sample 51 71

Gen1 L2=0.25
Genr f_Q2=(Q2**L2-1)/L2+1/L2
Genr f2_Q2=L2*Q2**(L2-1)/L2

Sample 1 71

******************************** Input prices
Genr f_w1=(Cap_P)/cap_P:58
Genr f_w2=(Mat_P)/Mat_P:58
Genr f_w3=(Wages_P)/Wages_P:58

?Stat f_W1 f_W2 f_W3 / Means=w_mean

Genr X1=Cap_Q*Cap_p/f_w1
Genr X2=Mat_E/f_w2
Genr X3=Wages_E/f_w3


* Check expenditures: Difference between sum of expenditures and total cost should be
zero
Genr Check=X1*f_w1+x2*f_w2+x3*f_w3-TC
Genr Check1=X1*f_w1-Cap_P*Cap_Q
Genr Check2=X2*f_w2-Mat_E
Genr Check3=X3*f_W3-Wages_E
*Print Year Check1 Check2 Check3

* Technical change

Sample 1 71
Genr D=0

Sample 1 71
Genr D=manual+sxs

Sample 1 71
Genr z=ARE+AXE
Genr Z1=ARK
Genr TECH=t

* Means
Sample 1 71
```

```
?Stat x1 x2 x3 / Means=Mean_x
*Genr x1bar=Mean_x:1
*Genr x2bar=Mean_x:2
*Genr x3bar=Mean_x:3

Genr x1bar=0.55
Genr x2bar=0.3
Genr x3bar=1-x1bar-x2bar


* Coefficient switches
*Genr a2=0
*Genr ct3=0
Genr CD1=0
*Genr CD2=0
Genr g1=0
Genr g2=0
Genr g3=0
*Genr fZ=0
Genr g12=0
Genr g13=0
Genr b1=1
Genr b2=1
Genr a11=0
Genr a21=0
Genr a22=0
Genr R1=1
Genr R2=2

Genr W1=Lag(f_W1)
Genr W2=Lag(f_W2)
Genr W3=Lag(f_W3)

Genr base1=1
Genr base=(Q1+Q2)
Genr Lambda=1
Genr Lambda1=1
Genr Lambda2=1
Genr Lambda3=1

Genr X1a=((X1))*f_W1/TR
Genr X2a=((X2))*f_W2/TR
Genr X3a=((X3))*f_W3/TR

* Create instruments
Genr X1lag=Lag(X1,1)/TR
Genr X2Lag=Lag(X2,1)/TR
Genr X3LAG=Lag(X3,1)/TR


* Estimate the two-output three-input Modified Generalised McFadden cost function
Gen1 Beg1=35
Gen1 End1=71
Gen1 k=13

Sample Beg1 End1
NL 3  /NCOEF=k ITER=10000 PITER=10000 RESID=V0 PREDICT=YFit ACROSS
Eq x1a=g1*X1lag+g12*x2lag+g13*x3lag+f_W1/TR*(a1 &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                          &
                   +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
```

```
      *(1+b2*f_Q2)**R2                                          &
      +cd1*D                                                         &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))         &
      +0.5*x1bar*(fz*z))

Eq x2a=g2*X2lag+f_W2/TR*(a2 &
                    +(-(a11*a21*f_W1+(a21**2+a22**2)*f_W2-
(a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                       &
                    +(-(d11*d21*f_W1+(d21**2+d22**2)*f_W2-
(d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +cd2*D                                                      &
      +0.5*x2bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))      &
      +0.5*x2bar*(fz1*z1))

Eq x3a=g3*X3Lag+f_W3/TR*(a3 &
                    +(-(-(a11**2+a11*a21)*f_W1-(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                            &
                    +(-(-(d11**2+d11*d21)*f_W1-(d11*d21+d21**2+d22**2)*f_W2 &

+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +ct3*TECH                                                      &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))         &
      +0.5*x3bar*(fz*z))

End

Dim U1 End1 U2 End1 U3 End1
Copy V0 U1 / FROW=1;End1 FCOL=1;1 TROW=1;End1
Copy V0 U2 / FROW=1;End1 FCOL=2;2 TROW=1;End1
Copy V0 U3 / FROW=1;End1 FCOL=3;3 TROW=1;End1

Dim YFit1 End1 YFit2 End1 YFit3 End1
COPY YFit YFit1 / FROW=1;End1 FCOL=1;1 TROW=1;End1
COPY YFit YFit2 / FROW=1;End1 FCOL=2;2 TROW=1;End1
COPY YFit YFit3 / FROW=1;End1 FCOL=3;3 TROW=1;End1

Genr Y1sq=YFit1**2
Genr Y2sq=YFit2**2
Genr Y3sq=YFit3**2

Genr U1sq=U1**2
Genr U2sq=U2**2
Genr U3sq=U3**2

Format(//,'__________________________________________________________________________________________
________________________________' &
//35X,'Single Equation Serial Correlation Test Results' &
/'________________________________________________________________________________________________
_______________________'//)
Print / Format
```

```
Gen1 BEg2=BEg1+1
ARIMA U1 / NLAG=5 ALL
ARIMA U2 / NLAG=5 ALL
ARIMA U3 / NLAG=5 ALL

Sample beg2 End1
SET NOOUTPUT
?OLS U1sq Yfit1 Y1sq
Gen1 M_Whit1=$n*$R2

?OLS U2sq Yfit2 Y2sq
Gen1 M_Whit2=$n*$R2

?OLS U3sq Yfit3 Y3sq
Gen1 M_Whit3=$n*$R2

Format(//,'______________________________________________________________________________
________________________________' &
//35X,'Single Equation Heteroskedasticity Test Results' &
/'_______________________________________________________________________________________
_______________________'//)
Print / Format

Set OUTPUT

DISTRIB M_Whit1 / TYPE=CHI DF=2
DISTRIB M_Whit2 / TYPE=CHI DF=2
DISTRIB M_Whit3 / TYPE=CHI DF=2

SET NOOUTPUT

Format(//,'______________________________________________________________________________
________________________________' &
//20X,'Equation System Serial Correlation Test Results, see Doornik 1996' &
/'_______________________________________________________________________________________
_______________________'//)
Print / Format


* See Doornik (1996) working paper
* Create lags of the residuals
*Gen1 Beg2=Beg1+5
Sample Beg1 End1

Genr U1_1=Lag(U1,1)
Genr U1_2=Lag(U1,2)
Genr U1_3=Lag(U1,3)
Genr U1_4=Lag(U1,4)
Genr U1_5=Lag(U1,5)

Genr U2_1=Lag(U2,1)
Genr U2_2=Lag(U2,2)
Genr U2_3=Lag(U2,3)
Genr U2_4=Lag(U2,4)
Genr U2_5=Lag(U2,5)

Genr U3_1=Lag(U3,1)
Genr U3_2=Lag(U3,2)
Genr U3_3=Lag(U3,3)
Genr U3_4=Lag(U3,4)
Genr U3_5=Lag(U3,5)

If(U1_1 .EQ. -99999)U1_1=0
If(U1_2 .EQ. -99999)U1_2=0
If(U1_3 .EQ. -99999)U1_3=0
If(U1_4 .EQ. -99999)U1_4=0
If(U1_5 .EQ. -99999)U1_5=0

If(U2_1 .EQ. -99999)U2_1=0
If(U2_2 .EQ. -99999)U2_2=0
If(U2_3 .EQ. -99999)U2_3=0
```

```
If(U2_4 .EQ. -99999)U2_4=0
If(U2_5 .EQ. -99999)U2_5=0

If(U3_1 .EQ. -99999)U3_1=0
If(U3_2 .EQ. -99999)U3_2=0
If(U3_3 .EQ. -99999)U3_3=0
If(U3_4 .EQ. -99999)U3_4=0
If(U3_5 .EQ. -99999)U3_5=0



* Re-estimate the system using lags of the residuals
System 3 / Resid=V0 NOCONSTANT
OLS x1a YFit1
OLS x2a Yfit2
OLS x3a Yfit3
End

System 3 / Resid=V NOCONSTANT
OLS x1a YFit1 U1_1 U2_1 U3_1
OLS x2a Yfit2 U1_1 U2_1 U3_1
OLS x3a Yfit3 U1_1 U2_1 U3_1
End

COPY V0:1 V01
COPY V0:2 V02
COPY V0:3 V03

COPY V:1 V1
COPY V:2 V2
COPY V:3 V3

Sample Beg1 End1
Dim V0_ 36 3 V_ 36 3

DELETE V0 V

COPY V01 V02 V03 V0_ /FROW=36;71 TROW=1;36
COPY V1 V2 V3 V_ /FROW=36;71 TROW=1;36

MATRIX V0V0=V0_'V0_
MATRIX VV=V_'V_

MATRIX D_V0V0=Det(V0V0)
MATRIX D_VV=Det(VV)
Gen1 Rr2=1-D_VV*(D_V0V0**(-1))

MATRIX I_V0V0=Inv(V0V0)
MATRIX VV_IV0=VV*I_V0V0
MATRIX Tr_=Trace(VV_IV0)

Gen1 n=3
Gen1 s=3
Gen1 k1=3

MATRIX Rm2=1-Tr_/n

Print Rr2 Rm2
Gen1 T_=End1-Beg1
Gen1 LM=T_*n*Rm2
Gen1 Df_=s*n**2
Print T_ Tr_ D_V0V0 D_VV

SET OUTPUT
DISTRIB LM / TYPE=CHI DF=Df_
SET NOOUTPUT

Gen1 p=n*s
Gen1 N_=T_-k-p-0.5*(n-p+1)
Gen1 q=0.5*n*p-1
Gen1 r=((n**2*p**2-4)/(n**2+p**2-5))**0.5
```

```
Gen1 LMF=(1-(1-Rr2)**(1/r))/((1-Rr2)**(1/r))*(N_*r-q)/(n*p)

Gen1 DF1_=n*p
Gen1 DF2_=N_*s-q
Print Df1_ Df2_

SET OUTPUT
DISTRIB LMF / TYPE=F DF1=DF1_ DF2=DF2_
Print N_ T_ q n p

SET NOOUTPUT

*****************************************************************************************
********************************
***************************  ALTERNATIVE SERIAL CORRELATION TEST
*****************************************************
*****************************************************************************************
********************************

Format(//,'__________________________________________________________________________________
________________________________' &
//20X,'Alternative Equation System Serial Correlation Test Results, see Enders 1995' &
/'_______________________________________________________________________________________
_______________________'//)
Print / Format


* Re-estimate the system using lags of the residuals
System 3 /
OLS x1a YFit1 U1_1 U2_1 U3_1
OLS x2a Yfit2 U1_1 U2_1 U3_1
OLS x3a Yfit3 U1_1 U2_1 U3_1

End

Gen1 Sig_U=$SIG2

* Take the lags of the residuals away and estimate again

System 3 /

OLS x1a YFit1
OLS x2a Yfit2
OLS x3a Yfit3

End

Gen1 Sig_R=$SIG2

* Define the set of usable observations
Gen1 T_=End1-Beg1

* Define the number of parameters in each of the equations in the unrestricted system
(includes a constant)
Gen1 c=5

* Calculate the statistic
Gen1 LR_=(T_-c)*(Sig_R-Sig_U)

* Define the degrees of freedom (equal to the number of restrictions in the system)
Gen1 DF_=9

Set Output
DISTRIB LR_ / TYPE=CHI DF=DF_
Print T_

SET NOOUTPUT

*****************************************************************************************
***************************
```

```
************************************  END OF ALTERNATIVE SERIAL CORRELATION TEST
*********************************
******************************************************************************************
***************************


* Heteroskedasticity test (Kelejian 1982)

Format(//,'_________________________________________________________________________________
________________________________' &
//25X,'System Heteroskedasticity Test Results, See Doornik 1996' &
/'_______________________________________________________________________________________
_______________________'//)
Print / Format

* Calculate the within equation variances
Sample Beg1 End1
Genr V01sq=V01**2
Genr V02sq=V02**2
Genr V03sq=V03**2

* Calculate the cross-equation covariances for each time period
Genr V01_V02=V01*V02
Genr V01_V03=V01*V03

Genr V02_V03=V02*V03

* Calculate the means
Stat V01sq V02sq V03sq V01_V02 V01_V03 V02_V03 / Means=V_mean

* Calculate the deviations from the mean
Genr V01Dev=V01sq-V_mean:1
Genr V02Dev=V02sq-V_mean:2
Genr V03Dev=V03sq-V_mean:3
Genr V012dev=V01_V02-V_mean:4
Genr V013dev=V01_V03-V_mean:5
Genr V023dev=V02_V03-V_mean:6

* Create Psi
COPY V01Dev V02Dev V03Dev V012Dev V013Dev V023Dev Psi
MATRIX V_psi1=(T_**(-1))*Psi'Psi


* Calculate squares of the regressors
Genr f_W11=f_W1**2
Genr f_W22=f_W2**2
Genr f_W33=f_W3**2
Genr f_Q1sq=f_Q1**2
Genr f_Q2sq=f_Q2**2
Genr ft2=z**2

* Calculate the cross-products
Genr f_W12=f_W1*f_W2
Genr f_W13=f_W1*f_W3
Genr f_W23=f_W2*f_W3

Genr f_W1Q1=f_W1*f_Q1
Genr f_W1Q2=f_W1*f_Q2
Genr f_W2Q1=f_W2*f_Q1
Genr f_W2Q2=f_W2*f_Q2
Genr f_W3Q1=f_W3*f_Q1
Genr f_W3Q2=f_W3*f_Q2

Genr f_W1t=f_W1*z
Genr f_W2t=f_W2*z
Genr f_W3t=f_W3*z

Genr f_Q1t=f_Q1*z
Genr f_Q2t=f_Q2*z

* Calculate the means
```

```
Stat f_W1 f_W2 f_W3 f_Q1 f_Q2 z f_W11 f_W22 f_W33 f_Q1sq f_Q2sq ft2 f_W12 f_W13 f_W23
f_W1Q1 f_W1Q2 &
f_W2Q1 f_W2Q2 f_W3Q1 f_W3Q2 f_W1t f_W2t f_W3t f_Q1t f_Q2t / Means=P_mean

* Calculate the deviations from the means
Genr f_W1d=f_W1-P_mean:1
Genr f_W2d=f_W2-P_mean:2
Genr f_W3d=f_W3-P_mean:3
Genr f_Q1d=f_Q1-P_mean:4
Genr f_Q2d=f_Q2-P_mean:5
Genr zd=z-P_mean:6

Genr f_W11d=f_W11-P_mean:7
Genr f_W22d=f_W22-P_mean:8
Genr f_W33d=f_W33-P_mean:9
Genr f_Q1sqd=f_Q1sq-P_mean:10
Genr f_Q2sqd=f_Q2sq-P_mean:11
Genr ft2d=TECH-P_mean:12

Genr f_W12d=f_W12-P_mean:13
Genr f_W13d=f_W13-P_mean:14
Genr f_W23d=f_W23-P_mean:15

Genr f_W1Q1d=f_W1Q1-P_mean:16
Genr f_W1Q2d=f_W1Q2-P_mean:17
Genr f_W2Q1d=f_W2Q1-P_mean:18
Genr f_W2Q2d=f_W2Q2-P_mean:19
Genr f_W3Q1d=f_W3Q1-P_mean:20
Genr f_W3Q2d=f_W3Q2-P_mean:21

Genr f_W1td=f_W1t-P_mean:22
Genr f_W2td=f_W2t-P_mean:23
Genr f_W3td=f_W3t-P_mean:24

Genr f_Q1td=f_Q1t-P_mean:25
Genr f_Q2td=f_Q2t-P_mean:26

Stat Yfit1 Yfit2 Yfit3 Y1sq Y2sq Y3sq / Means=Y_mean

Genr Yfit1d=Yfit1-Y_mean:1
Genr Yfit2d=Yfit2-Y_mean:2
Genr Yfit3d=Yfit3-Y_mean:3
Genr Y1sqd=Y1sq-Y_mean:4
Genr Y2sqd=Y2sq-Y_mean:5
Genr Y3sqd=Y3sq-Y_mean:6


* Estimate the unrestricted system

Sample Beg1 End1
System 6 / RESID=E_Psi
OLS V01Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
OLS V02Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
OLS V03Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
OLS V012Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
OLS V013Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
OLS V023Dev Yfit1d Yfit2d Yfit3d Y1sqd Y2sqd Y3sqd
END


* Create separate vectors for the residuals, remove missing value markers and recompile
COPY E_Psi:1 E_Psi1
COPY E_Psi:2 E_Psi2
COPY E_Psi:3 E_Psi3
COPY E_Psi:4 E_Psi4
COPY E_Psi:5 E_Psi5
COPY E_Psi:6 E_Psi6

IF(E_Psi1 .EQ. -99999)E_Psi1=0
IF(E_Psi2 .EQ. -99999)E_Psi2=0
IF(E_Psi3 .EQ. -99999)E_Psi3=0
```

```
IF(E_Psi4 .EQ. -99999)E_Psi4=0
IF(E_Psi5 .EQ. -99999)E_Psi5=0
IF(E_Psi6 .EQ. -99999)E_Psi6=0

*Copy back into the matrix
COPY E_Psi1 E_Psi2 E_Psi3 E_Psi4 E_Psi5 E_Psi6 E_PsiM

Matrix V0V0=V_psi1
MATRIX VV=(1/T_)*E_PsiM'*E_PsiM

Gen1 g=0.5*3*(3+1)
Print g
Gen1 hb=6

SET OUTPUT

MATRIX Rr2=1-Det(VV)*(Det(V0V0)**(-1))
MATRIX Rm2=1-(1/g)*Trace(VV*INV(V0V0))
Gen1 df_=g*hb
Gen1 LM_h=T_*g*Rm2

Gen1 k1=1
Gen1 q=0.5*g*hb-1
Print q
Gen1 s=3

Gen1 Na=T_-k1-hb-0.5*(g-hb+1)
Print Na T_ hb g k1

Gen1 DF_1=g*hb
Gen1 DF_2=Na*s-q
Print Df_1 Df_2
Gen1 r=((g**2*hb**2-4)/(g**2+hb**2-5))**0.5
Gen1 LMF_h=(1-(1-Rr2)**(1/r))/((1-Rr2)**(1/r))*(Na*r-q)/(g*hb)

* Print results for the heteroskedasticity test


Print Rm2 Rr2

DISTRIB LM_h / TYPE=CHI DF=df_
DISTRIB LMF_h / TYPE=F DF1=DF_1 DF2=DF_2


Sample Beg1 End1
Set NOOUTPUT
NL 3  /NCOEF=k ITER=10000 PREDICT=YFit GENRVAR ACROSS

Eq x1a=g1*X1lag+g12*x2lag+g13*x3lag+f_W1/TR*(a1 &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                           &
                   +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                           &
      +cd1*D                                                          &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))         &
      +0.5*x1bar*(fz*z))

Eq x2a=g2*X2lag+f_W2/TR*(a2 &
                    +(-(a11*a21*f_W1+(a21**2+a22**2)*f_W2-
(a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
```

```
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                    &
                    +(-(d11*d21*f_W1+(d21**2+d22**2)*f_W2-
(d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                            &
      +cd2*D                                                        &
      +0.5*x2bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))      &
      +0.5*x2bar*(fz1*z1))

Eq x3a=g3*X3Lag+f_W3/TR*(a3 &
                    +(-(-(a11**2+a11*a21)*f_W1-(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                        &
                    +(-(-(d11**2+d11*d21)*f_W1-(d11*d21+d21**2+d22**2)*f_W2 &

+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                            &
      +ct3*TECH                                                       &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))        &
      +0.5*x3bar*(fz*z))

End

Set OUTPUT

* Check the Hessian matrix of second order input price terms
Dim HA2 3 3

* Second Hessian matrix
Matrix HA2(1,1)=-d11**2
Matrix HA2(1,2)=-d11*d21
Matrix HA2(1,3)=(d11**2+d11*d21)

Matrix HA2(2,1)=-d11*d21
Matrix HA2(2,2)=-(d21**2+d22**2)
Matrix HA2(2,3)=(d11*d21+d21**2+d22**2)

Matrix HA2(3,1)=(d11**2+d11*d21)
Matrix HA2(3,2)=(d11*d21+d21**2+d22**2)
Matrix HA2(3,3)=-(d11**2+2*d11*d21+d21**2+d22**2)


FORMAT(//,'_______________________________________________________________________________
________________________________'/// &
30X,'Hessian Matrix of Second-order Input Price Parameters'/ &
'_______________________________________________________________________________________
______________________'//)

Print / Format

Print HA2

FORMAT(//)
Print / Format

* Sigma matrix
```

```
Matrix HA12=HA2(1,1)
Matrix HA22=HA2(1;2,1;2)
Matrix HA32=HA2(1;3,1;3)

Matrix D21_det=Det(HA12)
Matrix D22_det=Det(HA22)
Matrix D23_det=Det(HA32)

FORMAT(//,'______________________________________________________________________________
________________________________'// &
42X,"Calculated Determinants"/ &
'_________________________________________________________________________________________
______________________'//)
Print / Format

Format(//,' ')
Print / Format

Print D21_det D22_det D23_det

If1 (ABS(D21_Det).LT.0.00000000001)D21_Det=0
If1 (ABS(D22_Det).LT.0.00000000001)D22_Det=0
If1 (ABS(D23_Det).LT.0.00000000001)D23_Det=0

FORMAT(/1X,'Rounded determinants')
Print / Format

Print D21_det D22_det D23_det
Format(//,' ')
Print / Format
If1 ((D21_Det.LE.0).AND.(D22_Det.GE.0).AND.(D23_Det.LE.0))Concave2=1

FORMAT(//'_______________________________________________________________________________
_______________________________'// &
15X,'If the cost function is concave in input prices, variable Concave2 will equal 1 and
zero otherwise'/ &
'_________________________________________________________________________________________
______________________'//)

Print / Format
Print Concave2

Sample BEG1 END1

Genr g_wa=-((0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2)

Genr g_wd=-((0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2)



Genr h_w=(f_w1*x1bar+f_w2*x2bar+f_w3*x3bar)



* Calculate factor demand equations
Genr x1_=g1*X1lag+g12*x2lag+g13*x3lag+f_W1/TR*(a1 &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                            &
                   +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
```

```
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +cd1*D                                                         &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))         &
      +0.5*x1bar*(fz*z))

Genr x2_=g2*X2lag+f_W2/TR*(a2 &
                    +(-(a11*a21*f_W1+(a21**2+a22**2)*f_W2-
(a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                       &
                    +(-(d11*d21*f_W1+(d21**2+d22**2)*f_W2-
(d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +cd2*D                                                      &
      +0.5*x2bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))      &
      +0.5*x2bar*(fz1*z1))

Genr x3_=g3*X3Lag+f_W3/TR*(a3 &
                    +(-(-(a11**2+a11*a21)*f_W1-(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                            &
                    +(-(-(d11**2+d11*d21)*f_W1-(d11*d21+d21**2+d22**2)*f_W2 &

+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +ct3*TECH                                                      &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))         &
      +0.5*x3bar*(fz*z))



* Calculate marginal costs
Genr MC_Q1=g_Wa/h_W*R1*b1*f1_Q1*(b1*f_Q1)**(R1-1)+h_W*f1_Q1*(d11_*f_Q1+d12_*f_Q2)

Genr MC_Q2=g_Wd/h_w*R2*b2*f2_Q2*(1+b2*f_Q2)**(R2-1)+h_W*f2_Q2*(d12_*f_Q1+d22_*f_Q2)


* Calculate fixed cost due to input 1
Genr C_F1=(x1-( &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                            &
                   +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
```

```
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2)) ))


* Calculate fixed cost due to input 2
Genr C_F2=g2*X2lag+f_W2*(cd2*D                                                        &
      +0.5*x2bar*(fz1*z1)+0.82*U2_1)



* Calculate fixed cost due to input 3
Genr C_F3=(x3-( &
                    +(-(-(a11**2+a11*a21)*f_W1-(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                          &
                    +(-(-(d11**2+d11*d21)*f_W1-(d11*d21+d21**2+d22**2)*f_W2 &

+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2)) )   )



* Calculate variable cost
Genr C_VC=(g_Wa)/h_W*(b1*f_Q1)**2+(g_Wd)/h_W*(1+b2*f_Q2)**2 &
          +0.5*h_W*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))

* Calculate fixed cost
Genr C_Fixed=TC-C_VC

Genr
T_C=C_VC+f_W1*(a1+cd1*D+0.5*x1bar*(fz*z))+f_W2*(a2+cd2*D+0.5*x2bar*(fz1*z1))+f_W3*(a3+ct3
*TECH+0.5*x3bar*(fz*z))

FORMAT(//'________________________________________________________________________________
_______________________________'// &
45X,'Ancillary Analysis'/ &
'_________________________________________________________________________________________
______________________'//)

Print / Format


Dim M 71 5
COPY Year C_Fixed C_VC MC_Q1 MC_Q2 M

Format(//1X,'Year',6X,'Fixed Cost',7X,'Variable',6X,'Marginal Cost:',4X,'Marginal
Cost:'/30X,'Cost',10X,'Aggregate',11X,'Data')

Print / Format
Format(1F6.0,2F15.0,1F15.2,10F17.2,20F15.2//)
Print M / FORMAT NONAMES


Genr TC_=C_Fixed+C_VC

* Calculate elasticities of scale
Genr ES_Q1=MC_Q1*Q1/TC
Genr ES_Q2=MC_Q2*Q2/TC
Genr ES_Q1a=MC_Q1*Q1/T_C
Genr ES_Q2a=MC_Q2*Q2/T_C
```

```
Genr ES=ES_Q1+ES_Q2
Genr ESa=ES_Q1a+ES_Q2a

* Calculate returns to scale
Genr RS=1/ES
Genr RSa=1/ESa

Genr Port1=C_Fixed/TC
Genr Port2=TC_/TC


Format(/6X,'Year',5X,'Ratio: Fixed to',1X,'Ratio: Fitted Cost'/18X,'Total Cost',4X,'to
Actual Cost')
Print / Format
Print Year Port1 Port2 / NONAMES

Write(C:\Shazam\Ancillary.dif)Year t MC_Q1 MC_Q2 C_Fixed C_VC TC_ TC C_F1 C_F2 C_F3 RS
RSa ES_Q1 ES_Q2 f_Q1 f_Q2 Q1 Q2 &
f_W1 f_W2 f_W3 x1_ Yfit1 x1a x2_ Yfit2 x2a x3_ Yfit3 x3a x1bar x2bar x3bar U1 U2 U3/ Dif
names


Write(C:\Shazam\Fitted.dif)Year MC_Q1 MC_Q2 TC RS RSa ES_Q1 ES_Q2 f_Q1 f_Q2 f_W1 f_W2
f_W3 &
x1_ x1a x2_ x2a x3_ x3a X1a X2a X3a x1bar x2bar x3bar Tech Z U1 U2 U3 / Dif names

STOP
```

CHAPTER 6—APPENDIX 1C SUBADDITIVITY CODE

This appendix presents the computer code necessary for the subaddivity calculations presented in Chapter 6.

```
=Format(//38X,'Grant Coble-Neal 2005'/ &
        7X, 'Estimation of the two-output, three-input Modified
Generalised McFadden cost function'/ &
        31X,'Code file name: Final Model - sub.Sha'/37X,'Data file
name: PMG.Dif'//)
=Print / Format

=Set NOECHO
Set Missvalue=-99999
SIZE 2500
Set NODOECHO
Set NOWARN

* This file estimates a two-output, three-input Modified Generalised
McFadden cost function

?Read(C:\Shazam\PMG.dif) / Dif

* Create time trend
Genr T=Time(-35)
Set Wide

* Interpolate missing observations
Genr Austpac:66=(Austpac:67/Austpac:65)**0.5*Austpac:65
Sample 42 50
Genr Tlxrent=(Tlxrent:51/Tlxrent:41)**(1/(51-41))*Lag(Tlxrent)

Sample 1 71
Genr TR=TelecomR
* Create telephone call output variable
Genr foneCl_Q=LocCalls+STDCalls

* Create average prices
Sample 1 71

Genr foneCl_P=CallRev/foneCl_Q
Genr fonesb_P=RentRev/Subscrib
Genr Tlgrph_P=0
Sample 1 68
Genr Tlgrph_P=TelgrRev/Tottelgr
Sample 65 68
Genr Tlgrph_P=Tlgrph_P:64

Sample 1 71

Genr TlxCl_P=0
Sample 41 65

Genr TlxCl_P=Tlxcalrv/TelexCal
Sample 36 40
```

```
Genr TlxCl_P=Tlxcl_P:41
Sample 66 71

Genr TlxCl_P=Tlxcl_P:65
Sample 1 71

Genr Tlxrnt_P=0
Sample 41 65

Genr Tlxrnt_P=Tlxrent/TelexSrv
Sample 36 40

Genr Tlxrnt_P=Tlxrnt_P:41
Sample 66 71

Genr Tlxrnt_P=Tlxrnt_P:65
Sample 1 71

Genr CelCal_P=0
Genr CelSubRv=CelAcRev+CelCnFee
Genr CelSub_P=0
Sample 68 71

Genr CelCal_P=CelCalRv/CellCall
Genr CelSub_P=CelSubRv/CellSub

Sample 1 71

* Aggregate outputs
?Index Tlgrph_P Tottelgr foneCl_P foneCl_Q Tlxcl_P TelexCal CelCal_P
CellCall foneSb_P Subscrib CelSub_P CellSub Tlxrnt_P TelexSrv  /
Fisher=Tel_P1 QFisher=Tel_Q1
?Index foneSb_P Subscrib CelSub_P CellSub Tlxrnt_P TelexSrv /
Fisher=Tel_P2 QFisher=Tel_Q2
?Index Tlgrph_P Tottelgr Tlxcl_P TelexCal CelCal_P CellCall Tlxrnt_P
TelexSrv CelSub_P CellSub foneSb_P Subscrib  / Fisher=Tel_P3
QFisher=Tel_Q3

Genr Data=DatelSub+DDS+Austpac

* Create labour quantity
Genr Labour=Wages_E/Wages_P

* Construct Materials expenditure, price
Genr Mat_E=Cost-Wages_E-Deptele-Inttele
Genr Mat_P=Mat_E/Comm_S

* Construct rental prices, i.e. multiply price deflator by the sum of
the depreciation rate and the bond yield
Genr Build_r=NDC_P*(1/49+Bond_yld/100)
Genr Comm_r=Comm_P*(1/15.1+Bond_yld/100)
Genr Mche_r=Mche_P*(1/17.3+Bond_yld/100)

* Produce summary statistics of capital stock and number of staff
Format(//40X,'SUMMARY STATISTICS'/)
Print / Format
```

```
Stat Build_S Comm_S Mche_S Labour / wide

* Construct total cost
Genr TC=Build_r*Build_S+Comm_r*Comm_S+Mche_r*Mche_S+Mat_E+Wages_E

* Calculate expenditure shares
Genr Build_ES=Build_r*Build_S/TC
Genr Comm_ES=Comm_r*Comm_S/TC
Genr Mche_ES=Mche_r*Mche_S/TC
Genr Mat_ES=Mat_E/TC
Genr Wages_ES=Wages_E/TC

* Produce summary statistics of expenditure shares
?Stat TC Build_ES Comm_ES Mche_ES Mat_ES Wages_ES /wide

* Produce summary stats of output quantities
Stat Tottelgr Loccalls stdcalls telexcal telexsrv datelsub DDS Austpac
cellcall cellsub subscrib / Wide

* Produce summary stats of technical change variables
Stat Coax_KM Optic_KM Manual SxS ARK ARF ARE AXE / wide

Format(//)
Print / Format

***********************************************************************
*********************************
** Aggregate inputs

?Index Build_R Build_S Comm_R Comm_S Mche_R Mche_S / Divisia=Cap_P
QDivisia=Cap_Q

***********************************************************************
*********************************


* Produce summary statistics of input prices
?Stat Cap_P Mat_p Wages_P / wide Means=Mean_r


*** Create cost function variables
* Outputs

Sample 1 71
?Stat Tel_Q1 Data / Means=Q_means
Genr Q1=Tel_Q1
Genr Q2=Data



=************************************ Output functions

** Output 1
Gen1 L1=0.25
Genr f_Q1=(Q1**L1-1)/L1+1/L1
Genr f1_Q1=L1*Q1**(L1-1)/L1
```

```
** Output 2

Sample 51 71

Gen1 L2=0.25
Genr f_Q2=(Q2**L2-1)/L2+1/L2
Genr f2_Q2=L2*Q2**(L2-1)/L2

Sample 1 71

******************************** Input prices
Genr f_w1=(Cap_P)/cap_P:58
Genr f_w2=(Mat_P)/Mat_P:58
Genr f_w3=(Wages_P)/Wages_P:58

?Stat f_W1 f_W2 f_W3 / Means=w_mean

Genr X1=Cap_Q*Cap_p/f_w1
Genr X2=Mat_E/f_w2
Genr X3=Wages_E/f_w3


* Check expenditures: Difference between sum of expenditures and total
cost should be zero
Genr Check=X1*f_w1+x2*f_w2+x3*f_w3-TC
Genr Check1=X1*f_w1-Cap_P*Cap_Q
Genr Check2=X2*f_w2-Mat_E
Genr Check3=X3*f_W3-Wages_E
*Print Year Check1 Check2 Check3

* Technical change

Sample 1 71
Genr D=0

Sample 1 71
Genr D=manual+sxs

Sample 1 71
Genr z=ARE+AXE
Genr Z1=ARK
Genr TECH=t

* Means
Sample 1 71
?Stat x1 x2 x3 / Means=Mean_x
*Genr x1bar=Mean_x:1
*Genr x2bar=Mean_x:2
*Genr x3bar=Mean_x:3

Genr x1bar=0.55
Genr x2bar=0.3
Genr x3bar=1-x1bar-x2bar


* Coefficient switches
*Genr a2=0
```

```
*Genr ct3=0
Genr CD1=0
*Genr CD2=0
Genr g1=0
Genr g2=0
Genr g3=0
*Genr fZ=0
Genr g12=0
Genr g13=0
Genr b1=1
Genr b2=1
Genr a11=0
Genr a21=0
Genr a22=0
Genr R1=1
Genr R2=2

Genr W1=Lag(f_W1)
Genr W2=Lag(f_W2)
Genr W3=Lag(f_W3)

Genr base1=1
Genr base=(Q1+Q2)
Genr Lambda=1
Genr Lambda1=1
Genr Lambda2=1
Genr Lambda3=1

Genr X1a=((X1))*f_W1/TR
Genr X2a=((X2))*f_W2/TR
Genr X3a=((X3))*f_W3/TR

* Create instruments
Genr X1lag=Lag(X1,1)/TR
Genr X2Lag=Lag(X2,1)/TR
Genr X3LAG=Lag(X3,1)/TR


* Estimate the two-output three-input Modified Generalised McFadden
cost function
Gen1 Beg1=35
Gen1 End1=71
Gen1 k=13

Sample Beg1 End1
NL 3  /NCOEF=k ITER=10000 PITER=10000 RESID=V0 PREDICT=YFit ACROSS
GENRVAR
Eq x1a=g1*X1lag+g12*x2lag+g13*x3lag+f_W1/TR*(a1 &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
```

```
      *(b1*f_Q1)**R1                                          &
                  +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                   +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                    +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                   /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +cd1*D                                                         &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))
&
      +0.5*x1bar*(fz*z))

Eq x2a=g2*X2lag+f_W2/TR*(a2 &
                   +(-(a11*a21*f_W1+(a21**2+a22**2)*f_W2-
(a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                    +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x2bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                       &
                   +(-(d11*d21*f_W1+(d21**2+d22**2)*f_W2-
(d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                    +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x2bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                          &
      +cd2*D                                                      &
      +0.5*x2bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))     &
      +0.5*x2bar*(fz1*z1))

Eq x3a=g3*X3Lag+f_W3/TR*(a3 &
                   +(-(-(a11**2+a11*a21)*f_W1-
(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3b
ar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                    +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                          &
                   +(-(-(d11**2+d11*d21)*f_W1-
(d11*d21+d21**2+d22**2)*f_W2 &
```

```
+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3b
ar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                            &
      +ct3*TECH                                                        &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))
&
      +0.5*x3bar*(fz*z))

End

Gen1 End1=71

Dim YFit1 End1 YFit2 End1 YFit3 End1
COPY YFit YFit1 / FROW=1;End1 FCOL=1;1 TROW=1;End1
COPY YFit YFit2 / FROW=1;End1 FCOL=2;2 TROW=1;End1
COPY YFit YFit3 / FROW=1;End1 FCOL=3;3 TROW=1;End1

* Calculate adjustment cost component
Genr x1_=g1*X1lag+g12*x2lag+g13*x3lag+f_W1/TR*(a1 &
                   +(-(a11**2*f_W1+a11*a21*f_W2-
(a11**2+a11*a21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                            &
                   +(-(d11**2*f_W1+d11*d21*f_W2-
(d11**2+d11*d21)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                    +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x1bar &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                            &
      +cd1*D                                                           &
      +0.5*x1bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))
&
      +0.5*x1bar*(fz*z))


Genr x2_=g2*X2lag+f_W2/TR*(a2 &
                    +(-(a11*a21*f_W1+(a21**2+a22**2)*f_W2-
(a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
```

```
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                        &
                    +(-(d11*d21*f_W1+(d21**2+d22**2)*f_W2-
(d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x2bar &
                      /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                           &
      &
      +0.5*x2bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))      &
      +0.5*x2bar*(fz1*z1)+cd2*D)

Genr x3_=g3*X3Lag+f_W3/TR*(a3 &
                    +(-(-(a11**2+a11*a21)*f_W1-
(a11*a21+a21**2+a22**2)*f_W2 &

+(a11**2+2*a11*a21+a21**2+a22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3b
ar) &
                     +(0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &

+0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(b1*f_Q1)**R1                                           &
                    +(-(-(d11**2+d11*d21)*f_W1-
(d11*d21+d21**2+d22**2)*f_W2 &

+(d11**2+2*d11*d21+d21**2+d22**2)*f_W3)/(f_W1*x1bar+f_W2*x2bar+f_W3*x3b
ar) &
                     +(0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &

+0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2)*x3bar &
                     /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar)**2) &
      *(1+b2*f_Q2)**R2                                           &
                                                            &
      +0.5*x3bar*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))
&
      +0.5*x3bar*(fz*z)+ct3*TECH)


Genr Adj1=(Yfit1-x1_)*TR
Genr Adj2=(Yfit2-x2_)*TR
```

```
Genr Adj3=(Yfit3-X3_)*TR


Set Output

* Subadditivity tests

Sample 1 71

=Genr Q1_min=f_Q1:35
=Genr Q2_min=f_Q2:35

* Restrict analysis to the region where hypothetical costs are positive
Sample 35 71

Do !=1,10
  Do #=1,10
    =Genr Phi=!*10/100
    =Genr Omega=#*10/100


=* Calculate Firm A output
=Genr f_Q1a=phi*(f_Q1-2*Q1_min)+Q1_min
=Genr f_Q2a=omega*(f_Q2-2*Q2_min)+Q2_min

=* Calculate Firm B output
=Genr f_Q1b=f_Q1-f_Q1a
=Genr f_Q2b=f_Q2-f_Q2a

Genr g_wa=-((0.5*a11**2*f_W1**2+a11*a21*f_W1*f_W2-
(a11**2+a11*a21)*f_W1*f_W3 &
                     +0.5*(a21**2+a22**2)*f_W2**2-
(a11*a21+a21**2+a22**2)*f_W2*f_W3 &
                     +0.5*(a11**2+2*a11*a21+a21**2+a22**2)*f_W3**2) &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar))

Genr g_wd=-((0.5*d11**2*f_W1**2+d11*d21*f_W1*f_W2-
(d11**2+d11*d21)*f_W1*f_W3 &
                     +0.5*(d21**2+d22**2)*f_W2**2-
(d11*d21+d21**2+d22**2)*f_W2*f_W3 &
                     +0.5*(d11**2+2*d11*d21+d21**2+d22**2)*f_W3**2) &
                    /(f_W1*x1bar+f_W2*x2bar+f_W3*x3bar))

=Genr h_w=(f_w1*x1bar+f_w2*x2bar+f_w3*x3bar)

=Genr C=a1*f_W1+a2*f_W2+a3*f_W3 &

+g_wa/h_W*(b1*f_Q1)**R1+g_Wd/h_W*(1+b2*f_Q2)**R2+0.5*h_W*(f_Q1*(d11_*f_
Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2)) &
      +0.5*h_W*(f_Q1*(d11_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))
&
               +0.5*x1bar*(fz*z)*f_W1+cd1*D*f_w1
&
               +0.5*x2bar*f_W2*(fz1*z1)+f_W2*(cd2*D)
&
               +f_W3*(0.5*x3bar*(fz*z)+ct3*TECH)+Adj1+Adj2+Adj3
```

```
=Genr C_Fixed=a1*f_W1+a2*f_W2+a3*f_W3
&
               +0.5*x1bar*(fz*z)*f_W1+cd1*D*f_w1
&
               +0.5*x2bar*f_W2*(fz1*z1)+f_W2*(cd2*D)
&
               +f_W3*(0.5*x3bar*(fz*z)+ct3*TECH)+Adj1+Adj2+Adj3

=Genr
C_VC=g_wa/h_w**(b1*f_Q1)**R1+g_Wd/h_W*(1+b2*f_Q2)**R2+0.5*h_W*(f_Q1*(d1
1_*f_Q1+2*d12_*f_Q2)+f_Q2*(d22_*f_Q2))

=Genr
C_VCa=g_wa/h_w**(b1*f_Q1a)**R1+g_Wd/h_W*(1+b2*f_Q2a)**R2+0.5*h_W*(f_Q1a
*(d11_*f_Q1a+2*d12_*f_Q2a)+f_Q2a*(d22_*f_Q2a))

=Genr
C_VCb=g_wa/h_w**(b1*f_Q1b)**R1+g_Wd/h_W*(1+b2*f_Q2b)**R2+0.5*h_W*(f_Q1b
*(d11_*f_Q1b+2*d12_*f_Q2b)+f_Q2b*(d22_*f_Q2b))


=Genr Sub!#=(C-(C_Fixed*1.3+C_VCa+C_VCb))/C

  Endo
Endo


Write(C:\Shazam\Sub.dif)Year Sub11 Sub12 Sub13 Sub14 Sub15 Sub16 Sub17
Sub18 Sub19 Sub110   &
                             Sub21 Sub22 Sub23 Sub24 Sub25 Sub26 Sub27
Sub28 Sub29 Sub210   &
                             Sub31 Sub32 Sub33 Sub34 Sub35 Sub36 Sub37
Sub38 Sub39 Sub310   &
                             Sub41 Sub42 Sub43 Sub44 Sub45 Sub46 Sub47
Sub48 Sub49 Sub410   &
                             Sub51 Sub52 Sub53 Sub54 Sub55 Sub56 Sub57
Sub58 Sub59 Sub510   &
                             Sub61 Sub62 Sub63 Sub64 Sub65 Sub66 Sub67
Sub68 Sub69 Sub610   &
                             Sub71 Sub72 Sub73 Sub74 Sub75 Sub76 Sub77
Sub78 Sub79 Sub710   &
                             Sub81 Sub82 Sub83 Sub84 Sub85 Sub86 Sub87
Sub88 Sub89 Sub810   &
                             Sub91 Sub92 Sub93 Sub94 Sub95 Sub96 Sub97
Sub98 Sub99 Sub910   &
                             Sub101 Sub102 Sub103 Sub104 Sub105 Sub106
Sub107 Sub108 Sub109 Sub1010 / Names


STOP
```

CHAPTER 6—APPENDIX 2

This appendix presents an alternative model to the one presented in Chapter 6. In contrast to the Chapter 6 model, this model allows additional flexibility in substitution possibilities by allowing $\mathbf{\Sigma} = \mathbf{\Sigma}_0 + \mathbf{\Sigma}_2 \times \left(b_2 \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2} \right)$. In addition, the model contained in this appendix uses different controls for technological change. Section 1 discusses the econometric model and results. Section 2 provides analysis of the ancillary variables such as marginal costs, cost elasticities and fixed-cost estimates. Section 3 presents results for the subadditivity test and briefly discusses the implications. Section 4 then provides concluding remarks.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The MGM demand system, estimated in revenue-share form, is

$$
\begin{aligned}
\frac{w_{1t}x_{1t}}{R_t} = \frac{w_{1t}}{R_t}\Bigg\{ & a_1 + \left(\frac{-\left(a_{11,0}^2 w_{1t} + a_{11,0}a_{21,0}w_{2t} - \left(a_{11,0}^2 + a_{11,0}a_{21,0}\right)w_{3t}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \right. \\
& \left. + \frac{\left(\begin{aligned} & \frac{1}{2}a_{11,0}^2 w_{1t}^2 + a_{11,0}a_{21,0}w_{1t}w_{2t} - \left(a_{11,0}^2 + a_{11,0}a_{21,0}\right)w_{1t}w_{3t} \\ & + \frac{1}{2}\left(a_{21,0}^2 + a_{22,0}^2\right)w_{2t}^2 - \left(a_{11,0}a_{21,0} + a_{21,0}^2 + a_{22,0}^2\right)w_{2t}w_{3t} \\ & + \frac{1}{2}(a_{11,0}^2 + 2a_{11,0}a_{21,0} + a_{21,0}^2 + a_{22,0}^2)w_{3t}^2 \end{aligned} \right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \right) \\
& + \left(\frac{-\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \right. \\
& \left. + \frac{\left(\begin{aligned} & \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ & + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ & + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{aligned} \right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \right) \times \left(b_2\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2 \\
& + g_Z D_t \\
& + \frac{\theta_1}{2}\left(d_{11}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \right) \Bigg\} \\
& + \rho_{11}u_{1t-1} + \rho_{12}u_{2t-1} + \rho_{13}u_{1t-1}
\end{aligned}
\qquad \text{(A6.1-1.1)}
$$

$$\frac{w_{2t}x_{2t}}{R_t}=\frac{w_{2t}}{R_t}\left\{\begin{array}{l} a_2+\left(\begin{array}{l}\dfrac{-\left(a_{11,0}a_{21,0}w_{1t}+\left(a_{21,0}^2+a_{22,0}^2\right)w_{2t}-\left(a_{11,0}a_{21,0}+a_{21,0}^2+a_{22,0}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\ +\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,0}^2w_{1t}^2+a_{11,0}a_{21,0}w_{1t}w_{2t}-\left(a_{11,0}^2+a_{11,0}a_{21,0}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,0}^2+a_{22,0}^2\right)w_{2t}^2-\left(a_{11,0}a_{21,0}+a_{21,0}^2+a_{22,0}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,0}^2+2a_{11,0}a_{21,0}+a_{21,0}^2+a_{22,0}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right) \\ +\left(\begin{array}{l}\dfrac{-\left(a_{11,2}a_{21,2}w_{1t}+\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\ +\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,2}^2+2a_{11,A}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right)\times\left(b_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\right)^2 \\ +g_ZD_t+f_{Z1}exchange_{ARK,t} \\ +\dfrac{\theta_2}{2}\left(d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2+2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)\end{array}\right\}$$

$$+\rho_{21}u_{1t-1}+\rho_{22}u_{2t-1}+\rho_{23}u_{3t-1} \tag{A6.1-1.2}$$

$$\frac{w_{3t}x_{3t}}{R_t} = \frac{w_{3t}}{R_t}\left\{ \begin{array}{l} a_3 + \left(\begin{array}{l} \dfrac{-\begin{pmatrix} -\left(a_{11,0}^2 + a_{11,0}a_{21,0}\right)w_{1t} - \left(a_{11,0}a_{21,0} + a_{21,0}^2 + a_{22,0}^2\right)w_{2t} \\ +(a_{11,0}^2 + 2a_{11,0}a_{21,0} + a_{21,0}^2 + a_{22,0}^2)w_{3t} \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ + \dfrac{\begin{pmatrix} \frac{1}{2}a_{11,0}^2 w_{1t}^2 + a_{11,0}a_{21,0}w_{1t}w_{2t} - \left(a_{11,0}^2 + a_{11,0}a_{21,0}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,0}^2 + a_{22,0}^2\right)w_{2t}^2 - \left(a_{11,0}a_{21,0} + a_{21,0}^2 + a_{22,0}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,0}^2 + 2a_{11,0}a_{21,2} + a_{21,0}^2 + a_{22,0}^2)w_{3t}^2 \end{pmatrix}\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \end{array} \right) \\ + \left(\begin{array}{l} \dfrac{-\begin{pmatrix} -\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\ +(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t} \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ + \dfrac{\begin{pmatrix} \frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{pmatrix}\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \end{array} \right) \times \left(b_2\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\right)^2 \\ + g_Z D_t + c_{t3} time_t \\ + \dfrac{\theta_3}{2}\left(d_{11}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \right) \end{array} \right\}$$

$$+\rho_{31}u_{1t-1} + \rho_{32}u_{2t-1} + \rho_{33}u_{3t-1}$$

(A6.1-1.3)

where x_{1t}, x_{2t} and x_{3t} correspond to the capital stock, materials volume, and labour stock, respectively. Similarly, input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively while R_t is total revenue. Multiplying each equation by its respective input price and dividing by total revenue converts the system (A6.1-1.1) to (A6.1-1.3) into revenue share equations (see Cooper et al., 2003). Doing so reduces the scale differences in magnitude between variables and probably makes the maintained assumption of homoskedasticity more plausible. Output q_{1t} is a composite Fisher Ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. A dummy variable is also included, $D_t = 1$ for the years 1969 to 1979 inclusive and zero otherwise. The variable $exchange_{ARK,t}$ is the number of telephone exchanges using ARK type cross-bar technology. The variable $time_t = t$ where $t = \{0,1,2,\ldots,46\}$ for the years 1945 to 1990, inclusive.

Equations (A6.2-1.1) to (A6.2-1.3) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates: 2001), which allows the equations to be coded in the same way as presented in this thesis. This allows concavity of the cost function with respect to input prices to be imposed by construction. The cost function equation corresponding to the system of factor demand equations (A6.2-1.1) to (A6.2-1.3) is not estimated since it does not contain any additional information. Note that the Box-Cox transformation is applied to the outputs; and single period lags of the dependent variables are added to each equation.

Table A6-2.1 provides coefficient estimates and associated standard errors, the log-likelihood statistic and Box-Cox parameters for the model estimated on sample data corresponding to the years 1954 to 1990, inclusive. Table A6-2.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table A6-2.3 provides results of the remaining diagnostic statistics.

The presented models converged within 186 iterations with coefficient starting values left at Shazam's default setting. The Shazam vector autoregressive errors option is utilised, allowing for different rhos (ρ_{ij}) in each equation. The estimation results indicate that the values for $\rho_{11}, \rho_{22}, \rho_{31}, \rho_{33}$ are statistically significant while ρ_{12} is close to statistical significance. Table A6-2.1 shows that 16 of the 17 estimated equilibrium model parameters are statistically significant at conventional levels. The statistical significance of the autoregressive parameters indicates a substantial degree of inflexibility in adjusting all inputs. The significance of the parameter g_Z suggests a higher fixed cost throughout the 1970s compared with the decades before and since. Parameter f_{Z1} suggests higher fixed materials expenditure associated with crossbar exchange technology. Parameter c_{t3} indicates increasing demand for labour through time, which in turn tended to increase total labour cost.

Table A6-2.1 Estimated parameters 1954-90

	COEFFICIENT	ST. ERROR	T-RATIO
a_1	2.33	1.01	**2.31**
a_2	13.16	1.38	**9.55**
a_3	2.13	1.00	**2.13**
$a_{11,0}$	6.40	1.27	**5.03**
$a_{21,0}$	103.05	10.42	**9.89**
$a_{22,0}$	138.28	13.92	**9.94**
$a_{11,2}$	-0.13	0.02	**-5.24**
$a_{21,2}$	0.05	0.00	**11.02**
$a_{22,2}$	-0.00	0.02	-0.00
b_2	273.20	7.96	**34.33**
c_{D2}	11.15	4.05	**2.76**
f_{Z1}	409.18	65.00	**6.30**
g_Z	14.79	1.45	**10.18**
c_{t3}	13.42	1.67	**8.03**
d_{11}	24,712.00	1,710.60	**14.45**
d_{12}	10,466.00	925.25	**11.31**
d_{22}	-1,796.40	418.90	**-4.29**
ρ_{11}	0.54	0.14	**3.72**
ρ_{12}	-0.04	0.01	**-3.24**
ρ_{13}	-0.15	0.06	**-2.38**
ρ_{21}	-5.75	2.06	**-2.79**
ρ_{22}	0.08	0.15	0.54
ρ_{23}	-3.74	0.87	**-4.29**
ρ_{31}	0.60	0.19	**3.09**
ρ_{32}	0.04	0.02	**2.54**
ρ_{33}	1.11	0.08	**13.19**
λ_1	0.20		
λ_2	0.50		
θ_1	0.55		
θ_2	0.30		
θ_3	0.15		
Function value	283.33		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table A6-2.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	1.11	1.31	1.67	2.13	2.57
	(0.29)	(0.52)	(0.64)	(0.71)	(0.77)
Equation 2	0.19	0.31	2.30	2.44	4.04
	(0.66)	(0.88)	(0.51)	(0.00)	(0.54)
Equation 3	1.13	1.13	4.69	5.06	5.48
	(0.29)	(0.57)	(0.20)	(0.28)	(0.36)

Note. P-values in parentheses.

Table A6-2.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	1.40
	(0.50)
Equation 2 $\chi^2(2)$	1.62
	(0.44)
Equation 3 $\chi^2(2)$	0.03
	(0.98)
System Test Statistics	
Serial Correlation $\chi^2(3)$	1.82
	(0.61)
Heteroskedasticity $\chi^2(36)$	30.82
	(0.71)
Heteroskedasticity F(36,71)	0.74
	(0.77)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table A6-2.2 and Table A6-2.3 show that the tests fail to reject the null hypothesis of no serial correlation and homoskedasticity at conventional levels of significance.

Regression residuals are provided in Figure A6-2.1 in which the left hand axis corresponds to Capital and Materials and the right hand axis correspond to the residuals for the Labour equation. The single equation heteroskedasticity test (reported in the top portion of Table A6-2.3) is a modified version of the test proposed by White (1980) in which the levels and squares of the fitted values are regressed on the residuals of each estimated factor demand equation.[1] That is, let $\hat{x}_{if}$ be the fitted variable corresponding to the quantity of input i and $\hat{\varepsilon}_{it}$ denotes the regression residuals calculated following estimation of the factor demand equation corresponding to input i. Then the modified version of White's test is

$$\hat{\varepsilon}_{it}^2 = \mu + \varphi \cdot \hat{x}_{if} + \nu \cdot \hat{x}_{if}^2 + \varepsilon_{it}' \qquad \text{(A6.1-1.4)}$$

where $\mu,\ \varphi,\ \nu$ are regression coefficients and ε_{it}' represent regression errors. The test statistic is calculated according to $h_{mw} = nR^2$ where n is the number of observations and h_{mw} is chi-squared distributed with two degrees of freedom.

[1] The modified version of White's test reduces the dimensionality of the test compared to White's original test.

Figure A6-2.1. Regression residuals

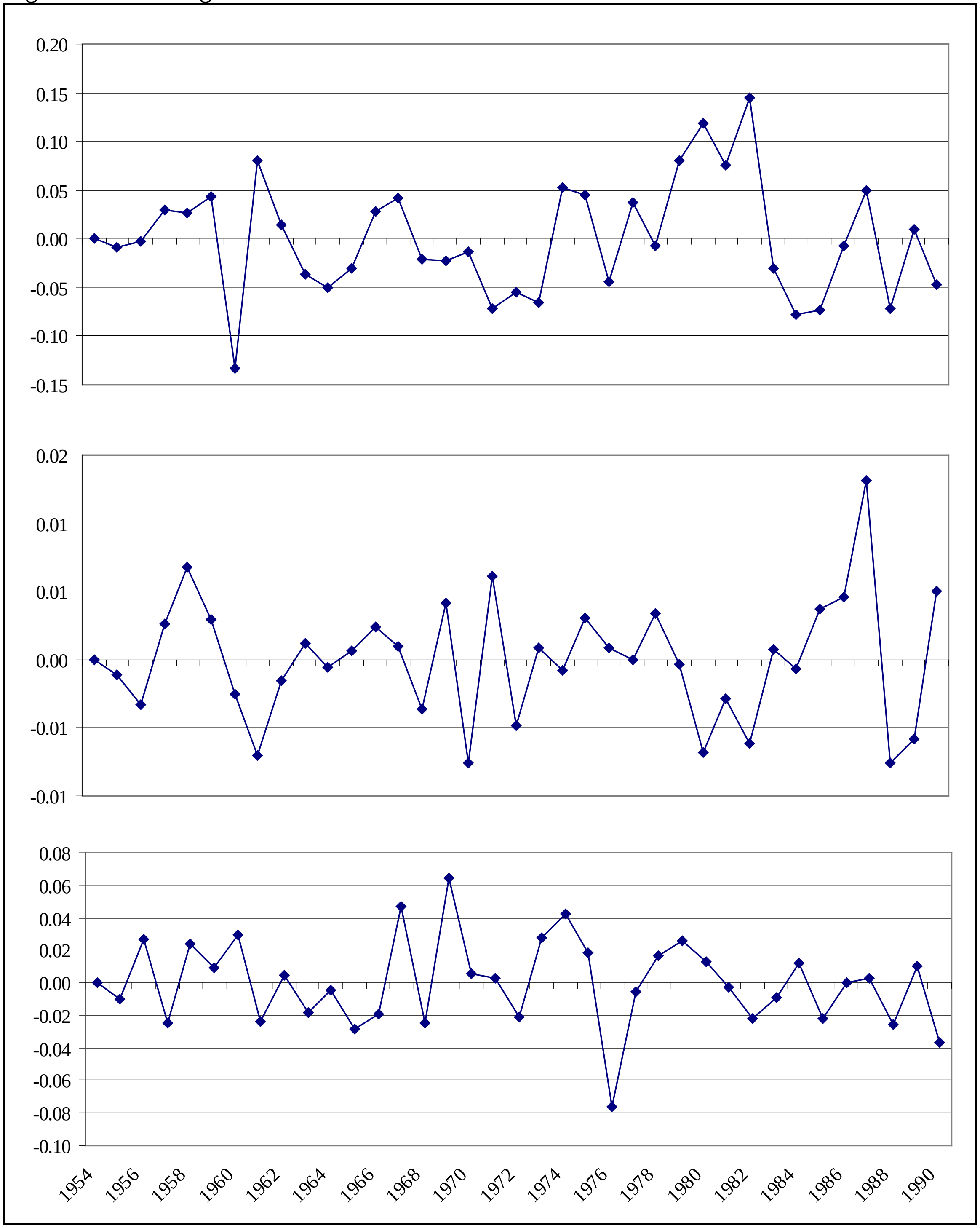

Table A6-2.5a. First Hessian matrix

-40.91	-659.15	700.06	$\lvert H_1 \rvert$	-40.92
-659.15	-29,742.54	30,401.69	$\lvert H_2 \rvert$	782,343.30
700.06	30,401.69	-31,101.75	$\lvert H_3 \rvert$	-0.28E-05

Table A6-2.5b. Second Hessian matrix

-0.017	0.007	0.010	$\lvert H_1 \rvert$	-0.02
0.007	-0.003	-0.004	$\lvert H_2 \rvert$	0.00
0.010	-0.004	-0.006	$\lvert H_3 \rvert$	0.00

In addition to the absence of serial correlation and heteroskedasticity, the estimated system (A6.1-1.1) to (A6.1-1.3) must reflect the properties of a proper cost function. The necessary conditions for a proper cost function are: (1) negative semi-definiteness of the Hessian matrix of second-order derivatives with respect to the input prices; and (2) non-negative marginal costs. The cost function satisfies condition (1) by construction. The estimated elements of the Hessian matrix along with the calculated determinants as reported in Table A6-2.5a and Table A6-2.5b. Clearly, the cost function is concave in input prices.

Table A6-2.6 presents the calculated marginal costs for the years 1953 to 1990, which are calculated according to,

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}} = \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{11}q_{1t}^{\lambda_1-1}\left(q_{1t}^{\lambda_1}-1\right)+d_{12}q_{1t}^{\lambda_1-1}\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \tag{A6.1-1.5}$$

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = -\frac{\begin{pmatrix} a_{11}^2 w_1^2 + 2a_{11}a_{21}w_1w_2 - 2\left(a_{11}^2 + a_{11}a_{21}\right)w_1w_3 \\ + \left(a_{21}^2 + a_{22}^2\right)w_2^2 - 2\left(a_{11}a_{21} + a_{21}^2 + a_{22}^2\right)w_2w_3 \\ + (a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 + a_{22}^2)w_3^2 \end{pmatrix}}{\left(w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3\right)} \\ \times 2b_1 q_{2t}^{\lambda_2 - 1}\left(b_1\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1} + b_2\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) \\ + \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{12}q_{2t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right) + d_{22}q_{2t}^{\lambda_2 - 1}\left(q_{2t}^{\lambda_2} - 1\right)\right) \qquad \text{(A6.1-1.6)}$$

Table A6-2.6 confirms that marginal costs are positive for the years 1953 to 1990. Thus, the estimated cost function satisfies the necessary regulatory conditions for a proper cost function.

Table A6-2.6. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{x}_{1t}/x_{1t}$	$\hat{x}_{2t}/x_{2t}$	$\hat{x}_{3t}/x_{3t}$
1954	1.46	-	1.00	1.00	1.00
1955	1.56	-	1.01	1.00	1.02
1956	1.57	-	1.00	1.01	0.95
1957	1.47	-	0.97	0.99	1.04
1958	1.47	-	0.98	0.97	0.96
1959	1.50	-	0.96	0.99	0.98
1960	1.42	-	1.16	1.01	0.94
1961	1.52	-	0.93	1.04	1.05
1962	1.56	-	0.99	1.01	0.99
1963	1.39	-	1.04	0.99	1.04
1964	1.32	-	1.05	1.00	1.01
1965	1.33	-	1.03	0.99	1.07
1966	1.41	-	0.97	0.98	1.05
1967	1.37	-	0.95	0.99	0.89
1968	1.46	-	1.03	1.03	1.06
1969	1.20	-	1.03	0.93	0.86
1970	1.38	22,613.31	1.02	1.13	0.99
1971	1.38	13,145.65	1.09	0.59	0.99
1972	1.58	15,576.00	1.07	1.08	1.05
1973	1.52	11,403.26	1.09	0.98	0.94
1974	1.95	11,893.54	0.94	1.02	0.92
1975	2.58	11,180.67	0.95	0.92	0.97
1976	3.55	11,551.30	1.05	0.99	1.18
1977	4.83	11,185.42	0.96	1.00	1.01
1978	5.05	9,004.99	1.01	0.97	0.96
1979	5.93	7,603.59	0.91	1.00	0.94
1980	7.02	6,932.40	0.88	1.06	0.97
1981	7.72	6,400.79	0.93	1.03	1.01
1982	9.81	6,472.86	0.88	1.05	1.05
1983	10.71	5,700.73	1.03	0.99	1.02
1984	11.28	5,372.78	1.08	1.00	0.97
1985	12.27	4,379.33	1.08	0.98	1.05
1986	13.41	4,022.52	1.01	0.98	1.00
1987	15.43	4,056.12	0.95	0.95	0.99
1988	11.65	5,683.16	1.08	1.02	1.10
1989	11.18	4,945.66	0.99	1.03	0.97
1990	11.86	5,171.63	1.05	0.98	1.13

Note. $\hat{x}_{it}/x_{it}$ denotes the ratio of estimated to actual factor demand. Data services introduced in 1970.

2. ANALYSIS OF THE EMPIRICAL RESULTS

Having demonstrated that the reported cost function is proper, this section considers the implications of some of the parameters presented in Table A6-2.1. As indicated in equations (A6.2-1.1) to (A6.2-1.3) estimation, the coefficients corresponding to technical change variables are different across factor demand equations, implying that the impact of technical change on factor demand differed across inputs. The first technical change variable, appearing in (1.2) is the number of telephone ARK exchanges in the network, in which the demand for materials varies in direct proportion to a variation in the exchange numbers. Since the number of exchanges is declining throughout the sample period, the parameter indicates that the decline stimulates an increase in all three inputs. The dummy variable corresponds to the years 1969 to 1979, suggests an autonomous increase in fixed cost across all three inputs. The upward shift in fixed cost probably captures the combined effects of relatively high inflation during the 1970s and the break-up of the PMG. Cost complementarity, defined as $\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}\partial q_{2t}} = d_{12} q_{1t}^{\lambda_1 - 1} q_{1t}^{\lambda_2 - 1} < 0$, is not evident since $d_{12} > 0$. Indeed, an increase in output q_{1t} increases the marginal cost of producing q_{2t} and vice versa.

Table A6-2.7 presents the proportion of total short-run fixed cost to total actual cost (C_t^F / C_t), variable cost to equilibrium cost (VC_t / C_t^*), equilibrium cost to actual cost (C_t^* / C_t), short-run returns elasticity of scale (RS_t) and equilibrium elasticity of scale (RS_t^*). The elasticities of scale are calculated according to

$$RS_t = \frac{1}{\varepsilon_{CQ_{1t}} + \varepsilon_{CQ_{2t}}} \text{ where } \varepsilon_{CQ_{it}} = \frac{\partial C_t}{\partial Q_{it}} \frac{Q_{it}}{C_t},\ i = \{1,2\}$$

while

$$RS_t^* = \frac{1}{\varepsilon^*_{CQ_{t1}} + \varepsilon^*_{CQ_{t1}}} \text{ where } \varepsilon^*_{CQ_{it}} = \frac{\partial C_t^*}{\partial Q_{it}} \frac{Q_{it}}{C_t^*},\ i = \{1,2\}.$$

Thus RS_t shows the short-run percentage change in combined output given a uniform one percent increase in the volume inputs. Similarly, RS_t^* shows the equilibrium percent change in output given a uniform one percent increase in equilibrium inputs. As shown, short-run fixed cost accounts for between 53% to 81% of total cost, peaking in 1971 and trending down thereafter. In equilibrium, however, variable cost accounts for accounts for more than 80% of equilibrium cost. The ratio of equilibrium to actual cost, which ranges between 23% and 55%, reveals the magnitude of adjustment cost. Note that the share of total adjustment cost trends down as time progresses (coinciding with slowing growth in network size). The difference in short-run and equilibrium cost is reflected in a substantial difference in short-run and equilibrium returns to scale. Although substantially smaller in magnitude, equilibrium returns to scale is substantially higher than measures typically reported in UK and the US studies. This is not surprising given both the UK and the US telecommunications networks produce substantially larger volumes of output than the Australian telecommunications network.

Table A6-2.7. Fixed and variable cost contribution to total cost

YEAR	C_t^F / C_t	VC_t / C_t^*	C_t^* / C_t	RS_t	RS_t^*
1954	0.67	0.93	0.35	7.39	2.60
1955	0.67	0.92	0.36	7.30	2.62
1956	0.68	0.92	0.35	7.42	2.63
1957	0.71	0.93	0.31	8.41	2.58
1958	0.72	0.93	0.30	8.59	2.59
1959	0.72	0.92	0.31	8.48	2.61
1960	0.72	0.92	0.31	8.51	2.63
1961	0.73	0.93	0.29	9.09	2.59
1962	0.73	0.93	0.29	9.07	2.61
1963	0.75	0.93	0.27	9.54	2.59
1964	0.75	0.93	0.27	9.56	2.60
1965	0.75	0.93	0.26	9.84	2.60
1966	0.75	0.92	0.27	9.70	2.63
1967	0.76	0.86	0.28	10.26	2.83
1968	0.77	0.76	0.30	10.45	3.17
1969	0.82	0.79	0.23	13.50	3.05
1970	0.79	0.83	0.26	9.98	2.57
1971	0.81	0.95	0.20	10.99	2.20
1972	0.78	0.86	0.26	9.20	2.37
1973	0.79	0.91	0.23	9.55	2.19
1974	0.79	0.93	0.23	9.09	2.10
1975	0.78	0.95	0.23	8.68	2.01
1976	0.74	0.90	0.29	7.17	2.07
1977	0.71	0.87	0.34	6.26	2.10
1978	0.70	0.89	0.34	6.13	2.06
1979	0.71	0.89	0.33	6.28	2.05
1980	0.71	0.88	0.33	6.19	2.07
1981	0.71	0.90	0.32	6.31	2.03
1982	0.70	0.91	0.33	6.10	2.03
1983	0.69	0.88	0.36	5.92	2.12
1984	0.67	0.84	0.39	5.69	2.25
1985	0.66	0.83	0.41	5.57	2.31
1986	0.64	0.83	0.43	5.44	2.35
1987	0.62	0.82	0.46	5.23	2.42
1988	0.48	0.83	0.63	3.77	2.36
1989	0.55	0.86	0.53	4.35	2.29
1990	0.53	0.84	0.55	4.23	2.34

3. SUBADDITIVITY TEST RESULTS

A summary of the subadditivity calculations are presented in Table A6-2.8 to Table A6-2.10. Table A6-2.8 assumes no change in fixed cost between the monopoly and hypothetical duopoly cases. In contemporary policy terms, this can be considered an ‘open access regime’ in which the incumbent telecommunications carrier provides non-discriminatory access to its competitor. Note that technology and input prices are fixed across columns and vary across rows. The parameter ϕ corresponds to market share for aggregate output for firm A and ω corresponds to Data output. For example $(\phi,\omega)=50{,}10$ indicates that Firm A has 50% market share of aggregate output and has 10% market share in Data. Underlying calculations specify no change in aggregate fixed cost for monopoly and duopoly. Inspection across rows suggests that an efficiency gain could have been derived from 1960. Note that this hypothetical gain in competitive (duopoly) supply increases over time. Inspection across columns reveals that for the years 1960 and 1970, SUB is highest for $(\phi,\omega)=50{,}10$ and $(\phi,\omega)=50{,}50$. After 1970, SUB reaches a maximum at $(\phi,\omega)=100{,}100$, where one supplier is confined to minimum output in both aggregate and data services. SUB tends to be lowest at an approximate 50%-50% split in output.

Table A6-2.8. Subadditivity calculations (%) — no difference in fixed cost

Year	(ϕ,ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	6.83	6.83	6.83	**13.27**	**13.27**	3.20	3.20
1970	9.02	8.07	7.02	**10.60**	10.55	5.59	8.15
1975	12.27	9.95	7.93	11.79	11.48	6.90	**12.71**
1980	20.58	14.61	11.35	17.48	15.99	9.73	**23.17**
1985	23.55	15.50	14.50	19.46	16.24	12.27	**27.66**
1990	29.94	21.52	25.49	26.89	21.73	22.47	**34.56**

Note. Maximum SUB in each row is printed in bold type.

Table A6-2.9 provides more detail for the case in which Firm A is the dominant carrier for aggregate output, but has varying market share in Data. Reading from left to right, Firm B market share in Data declines across columns. For example, the left column (i.e. when $(\phi,\omega) = 100{,}10$) corresponds to the case in which Firm B is the dominant firm for Data while in the furthest right hand column (i.e. $(\phi,\omega) = 100{,}100$), Firm B produces zero Data output. The results clearly show a progressive increase in potential efficiency gain to duopoly as Firm A's dominance increases.

Table A6-2.9. Subadditivity calculations for $(100,\omega)$ (%)

Year	(ϕ,ω)									
	(100,10)	(100,20)	(100,30)	(100,40)	(100,50)	(100,60)	(100,70)	(100,80)	(100,90)	(100,100)
1970	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20
1975	5.59	5.86	6.12	6.39	6.67	6.96	7.25	7.54	7.85	8.15
1980	6.90	7.39	7.92	8.49	9.10	9.74	10.43	11.15	11.91	12.71
1985	9.73	10.48	11.41	12.53	13.84	15.33	17.01	18.88	20.93	23.17
1990	12.27	12.37	12.88	13.78	15.09	16.80	18.91	21.42	24.34	27.66

Finally, Table A6-2.10 presents subadditivity results with the assumption that the duopoly case incurs a 30% increase in fixed cost compared to the monopoly case. This scenario could be considered to be a more realistic outcome as market entrant partially duplicates the incumbent's telecommunications network. The results indicate this difference in fixed cost produces subadditivity up to 1985. This result indicates that, even with partial duplication, competition in the Australian telecommunications network could still yield an efficiency gain from 1990 onwards.

Table A6-2.10. Subadditivity calculations (%) — 30% increase in fixed cost

Year	(ϕ, ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	-15.21	-15.21	-15.21	-8.77	-8.77	-18.84	-18.84
1970	-14.65	-15.60	-16.65	-13.07	-13.12	-18.07	-15.51
1975	-10.84	-13.16	-15.18	-11.33	-11.63	-16.22	-10.40
1980	0.17	-5.80	-9.06	-2.93	-4.42	-10.68	2.76
1985	3.29	-4.75	-5.76	-0.80	-4.01	-7.98	7.40
1990	12.98	4.56	8.53	9.93	4.77	5.51	17.60

Chapter 6—Appendix 3

This appendix presents an alternative model to the one presented in Chapter 6. Unlike the other models presented in this thesis, only two factor demand equations were estimated and both outputs are included in the sigma-output interaction terms. Two other important differences are the first-order output arguments and the restriction that autoregressive parameters $\rho_{ij} = 0$, $i \neq j$. The model also includes both outputs in the sigma-output interaction terms. Section 1 discusses the econometric model and results. Section 2 provides analysis of the ancillary variables such as marginal costs, cost elasticities and fixed cost estimates. Section 3 presents results for the subadditivity test and briefly discusses the implications.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The MGM demand system, estimated in revenue-share form, is

$$\frac{w_{1t}x_{1t}}{R_t}=\frac{w_{1t}}{R_t}\left\{\begin{array}{l} a_1 \\ \left(\begin{array}{l}\dfrac{-\left(a_{11,2}^2 w_{1t}+a_{11,2}a_{21,2}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\ +\dfrac{+\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2 w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{array}\right)\theta_1}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right)\times\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}+\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \\ +f_{Z1}exchange_{ARE,t}+f_{Z2}exchange_{Crossbar,t}+f_{Z3}coaxial_t+f_{Zt}t \\ +\dfrac{\theta_3}{2}\left(\begin{array}{l} d_1\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2 \\ +2d_{12}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\end{array}\right)\end{array}\right\}$$

$$+\rho_1 u_{1t-1}$$

(A6.3-1.1)

$$\frac{w_{3t}x_{3t}}{R_t}=\frac{w_{3t}}{R_t}\left\{\begin{array}{l} a_3+\left(\begin{array}{l}\dfrac{-\left(\begin{array}{l}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t} \\ +(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}\end{array}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\ +\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2 w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ +\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t} \\ +\frac{1}{2}(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{array}\right)\theta_3}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\end{array}\right)\times\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}+\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \\ +f_{Z1}exchange_{ARE,t}+f_{Z2}exchange_{Crossbar,t}+f_{Z3}coaxial_t+f_{Zt}t \\ +\dfrac{\theta_3}{2}\left(\begin{array}{l} d_1\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2 \\ +2d_{12}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\end{array}\right)\end{array}\right\}$$

$$+\rho_3 u_{3t-1}$$

(A6.3-1.2)

where x_{1t}, x_{2t} and x_{3t} correspond to the capital stock, materials volume, and labour stock, respectively. Similarly, input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively while R_t is total revenue. Each equation is estimated in revenue share form. Output q_{1t} is a composite Fisher Ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. The variable $exchange_{ARE,t}$ is the number of telephone exchanges using ARE digital technology while $exchange_{crossbar,t}$ captures the number of exchanges employing cross-bar (ARF+ARK) technology. The variable $time_t = t$ where $t = \{0,1,2,\ldots,46\}$ for the years 1954 to 1990, inclusive.

Equations (A6.3-1.1) and (A6.3-1.2) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates: 2001), which allows the equations to be coded in the same way as presented in this thesis, thereby allowing concavity of the cost function with respect to input prices to be imposed by construction. The cost function equation corresponding to the system of factor demand equations is not estimated. Note that the Box-Cox transformation is applied to the outputs; and single period lags of the dependent variables are added to each equation.

Table A6-3.1 provides coefficient estimates and associated standard errors, the function value statistic and Box-Cox parameters for the model estimated on sample data corresponding to the years 1954 to 1990, inclusive. Table A6-3.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table A6-3.3 provides results of the remaining diagnostic statistics.

The presented models converged within 169 iterations with coefficient starting values left at Shazam's default setting. The Shazam autoregressive errors option is utilised, allowing for different rhos (ρ_i) in each equation. Table A6-3.1 shows that only one coefficient is statistically insignificant at conventional levels. The statistical significance of the autoregressive parameters indicate that a substantial degree of inflexibility in adjusting all inputs.

Table A6-3.1 Estimated parameters 1954-90

	COEFFICIENT	ST. ERROR	T-RATIO
a_1	61,002.00	13.90	**4,387.10**
a_3	5,467.80	1.59	**3,432.80**
$a_{11,2}$	471.76	61.25	**7.70**
$a_{21,2}$	-83.93	38.75	**-2.17**
$a_{22,2}$	0.00	105.89	0.00
f_{Z1}	113.98	54.83	**2.08**
f_{Z2}	221.40	32.63	**6.78**
f_{Z3}	3,701.60	1,605.10	**2.31**
f_{Zt}	20,357.00	4.73	**4,299.60**
d_1	6,464,900.00	1,469.80	**4,398.60**
d_2	3,943,000.00	896.11	**4,400.10**
d_{11}	-1,139.40	440.55	**-2.59**
d_{12}	-1,726.40	1,062.20	**-1.63**
d_{22}	5,711.70	2,035.80	**2.81**
ρ_1	0.85	0.05	**17.92**
ρ_2	0.96	0.03	**30.39**
λ_1	0.3		
λ_2	0.5		
θ_1	0.55		
θ_2	0.30		
θ_3	0.15		
Function value	157.67		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table A6-3.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	0.12	0.49	0.52	4.72	5.26
	(0.73)	(0.78)	(0.92)	(0.32)	(0.38)
Equation 2	0.02	1.50	1.78	1.79	1.94
	(0.89)	(0.47)	(0.62)	(0.77)	(0.86)

Note. P-values in parentheses.

Table A6-3.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	0.26 (0.88)
Equation 2 $\chi^2(2)$	0.33 (0.85)
System Test Statistics	
Serial Correlation $\chi^2(3)$	1.83 (0.61)
Heteroskedasticity $\chi^2(36)$	7.87 (0.80)
Heteroskedasticity F(36,21)	0.57 (0.84)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table A6-3.2 and Table A6-3.3 show that the tests fail to reject the null hypothesis of no serial correlation and homoskedasticity at conventional levels of significance.

Table A6-3.5 and Table A6-3.6 provide evidence that the estimated cost function is conforms to the theoretical requirements of concavity in input prices and positive marginal cost.

Table A6-3.5. $\mathbf{\Sigma}$ matrix

-222,558.30	39,594.82	182,963.50	$\lvert H_1 \rvert$	-222,558.30
39,594.82	-7,044.22	-32,550.60	$\lvert H_2 \rvert$	0.05
182,963.50	-32,550.60	-150,412.90	$\lvert H_3 \rvert$	0.00

The marginal costs for the years 1960 to 1990, are calculated according to,

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}} = -\frac{\begin{pmatrix} a_{11,2}^2 w_1^2 + 2a_{11,2}a_{21,2}w_1w_2 - 2\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_1w_3 \\ + \left(a_{21,2}^2 + a_{22,2}^2\right)w_2^2 - 2\left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_2w_3 \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_3^2 \end{pmatrix}}{\left(w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3\right)} \times \lambda_1 q_{1t}^{\lambda_1-1}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)$$
$$+\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{11}q_{1t}^{\lambda_1-1}\left(q_{1t}^{\lambda_1}-1\right) + d_{12}\lambda_1 q_{1t}^{\lambda_1-1}\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)$$

(A6.3-1.3)

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = -\frac{\begin{pmatrix} a_{11,2}^2 w_1^2 + 2a_{11,2}a_{21,2}w_1w_2 - 2\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_1w_3 \\ + \left(a_{21,2}^2 + a_{22,2}^2\right)w_2^2 - 2\left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_2w_3 \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_3^2 \end{pmatrix}}{\left(w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3\right)}$$
$$\times\lambda_2 q_{2t}^{\lambda_2-1}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)$$
$$+\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_2 + d_{12}q_{2t}^{\lambda_2-1}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right) + d_{22}q_{2t}^{\lambda_2-1}\left(q_{2t}^{\lambda_2}-1\right)\right)$$

(A6.3-1.4)

Table A6-3.6. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{x}_{1t}/x_{1t}$	$\hat{x}_{3t}/x_{3t}$
1960	11.07	-	1.00	1.00
1961	11.49	-	1.01	0.99
1962	11.80	-	1.02	1.01
1963	10.27	-	1.00	1.03
1964	9.70	-	1.00	0.99
1965	9.62	-	0.98	1.03
1966	10.25	-	0.98	1.04
1967	9.88	-	1.00	0.97
1968	10.61	-	0.99	1.08
1969	8.24	-	0.98	0.91
1970	8.24	37,527.08	1.02	1.05
1971	7.39	24,470.40	0.99	1.00
1972	8.61	27,156.00	1.02	1.11
1973	7.68	20,623.50	1.00	0.97
1974	9.24	22,131.40	0.99	1.06
1975	11.31	22,736.16	0.98	1.05
1976	14.98	26,320.53	0.99	1.13
1977	19.23	29,488.36	1.00	1.12
1978	18.50	26,351.01	1.01	1.06
1979	19.84	25,418.20	1.00	1.04
1980	21.30	26,213.97	1.02	1.12
1981	21.51	26,489.67	1.01	1.05
1982	25.11	30,700.84	0.99	1.08
1983	25.45	31,242.43	1.01	1.05
1984	25.47	32,891.04	1.00	0.90
1985	25.17	33,423.39	1.01	1.00
1986	25.57	35,932.24	1.01	1.00
1987	28.20	41,250.86	0.99	1.04
1988	19.86	45,481.85	1.02	1.26
1989	17.83	41,614.93	0.99	1.02
1990	18.53	45,693.81	0.97	0.94

Note. $\hat{x}_{it}/x_{it}$ denotes the ratio of estimated to actual factor demand.
Data services introduced in 1970.

2. ANALYSIS OF THE EMPIRICAL RESULTS

This section considers the implications of the estimated model in more detail. As indicated in equations (A6.3-1.1) and (A6.3-1.2), the coefficients corresponding to technical change variables are different across factor demand equations, implying that the impact of technical change on factor demand differed across inputs. Cost complementarity, defined as $\frac{\partial C_t(\mathbf{q}, t, \mathbf{w})}{\partial q_{1t} \partial q_{2t}} = d_{12} \lambda_1 \lambda_2 q_{1t}^{\lambda_1 - 1} q_{1t}^{\lambda_2 - 1} < 0$, is evident since $d_{12} < 0$, although the corresponding t-statistic suggests that it is not significantly different to zero at conventional levels of significance.

Table A6-3.7 presents the proportion of total short-run fixed cost to total actual cost (C_t^F / C_t), variable cost to equilibrium cost (VC_t / C_t^*), equilibrium cost to actual cost (C_t^* / C_t), and elasticity of scale (RS_t). The elasticity of scale is calculated according to

$$RS_t = \frac{1}{\varepsilon_{CQ_{1t}} + \varepsilon_{CQ_{2t}}} \text{ where } \varepsilon_{CQ_{it}} = \frac{\partial C_t}{\partial Q_{it}} \frac{Q_{it}}{C_t},\ i = \{1,2\}$$

As shown, short-run fixed cost varies from 0% in 1960 to 43% in 1990. Variable cost accounts for accounts for more than 80% of equilibrium cost. The ratio of fitted cost to actual cost is substantially greater than one for most of the sample, suggesting that the model is not as plausible as the others presented in this thesis. Despite this, returns to scale implied by the model are within a plausible range.

Table A6-3.7. Fixed and variable cost contribution to total cost

YEAR	C_t^F / C_t^*	VC_t / C_t^*	C_t^* / C_t	RS_t
1960	0.00	1.00	1.52	1.66
1961	0.00	1.00	1.38	1.66
1962	0.03	0.97	1.43	1.71
1963	0.05	0.95	1.34	1.74
1964	0.06	0.94	1.35	1.75
1965	0.06	0.94	1.30	1.76
1966	0.07	0.93	1.33	1.78
1967	0.12	0.88	1.31	1.87
1968	0.17	0.83	1.39	2.00
1969	0.23	0.77	1.10	2.16
1970	0.28	0.72	1.14	2.24
1971	0.32	0.68	0.97	2.33
1972	0.36	0.64	1.20	2.47
1973	0.39	0.61	1.10	2.56
1974	0.42	0.58	1.11	2.65
1975	0.44	0.56	1.10	2.69
1976	0.46	0.54	1.32	2.73
1977	0.48	0.52	1.52	2.77
1978	0.50	0.50	1.54	2.81
1979	0.51	0.49	1.49	2.80
1980	0.52	0.48	1.51	2.79
1981	0.52	0.48	1.46	2.74
1982	0.50	0.50	1.47	2.59
1983	0.50	0.50	1.54	2.54
1984	0.50	0.50	1.62	2.47
1985	0.48	0.52	1.69	2.32
1986	0.47	0.53	1.75	2.23
1987	0.45	0.55	1.84	2.15
1988	0.43	0.57	2.14	2.10
1989	0.43	0.57	1.84	2.08
1990	0.43	0.57	1.90	2.05

3. SUBADDITIVITY TEST RESULTS

A summary of the subadditivity calculations are presented in Table A6-3.8 and Table A6-3.9. Both cases assume no change in fixed cost between the monopoly and hypothetical duopoly cases. Note that technology and input prices are fixed across columns and vary across rows. The parameter ϕ corresponds to market share for aggregate output for firm A and ω corresponds to Data output. For example $(\phi,\omega)=50{,}10$ indicates that Firm A has 50% market share of aggregate output and has 10% market share in Data. Underlying calculations specify no change in aggregate fixed cost for monopoly and duopoly. Inspection of Table A6-3.8 suggests mild subadditivity, reaching a minimum in 1985.

Table A6-3.8. Subadditivity calculations (%) — no difference in fixed cost

Year	(ϕ,ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	-1.66	-1.66	-1.66	-2.89	**-0.97**	**-0.97**	-1.66
1970	-2.78	-2.54	-2.37	-3.06	**-2.07**	-2.65	-2.78
1975	-3.59	-2.80	-2.85	-3.44	**-2.50**	-3.89	-3.59
1980	-4.85	**-1.98**	-3.94	-4.17	-2.70	-6.33	-4.85
1985	-5.34	**0.47**	-6.00	-4.51	-3.20	-8.56	-5.34
1990	-6.40	**0.65**	-10.75	-6.54	-6.70	-10.37	-6.40

Note. Maximum SUB in each row is printed in bold type.

Table A6-3.9 provides more detail for the case in which Firm A is the dominant carrier for aggregate output, but has varying market share in Data. Reading from left to right, Firm B market share in Data declines across columns. For example, the left column (i.e. when $(\phi,\omega)=100{,}10$) corresponds to the case in which Firm B is the dominant firm for Data while in the furthest right hand column (i.e. $(\phi,\omega)=100{,}100$), Firm B produces zero Data output. The results show slight subadditivity across all simulated output ranges.

Table A6-3.9. Subadditivity calculations for $(100, \omega)$ (%)

Year	(ϕ, ω)									
	(100,10)	(100,20)	(100,30)	(100,40)	(100,50)	(100,60)	(100,70)	(100,80)	(100,90)	(100,100)
1970	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97
1975	**-2.07**	-2.12	-2.16	-2.22	-2.28	-2.34	-2.41	-2.48	-2.57	-2.65
1980	-2.50	**-2.47**	-2.48	-2.54	-2.65	-2.81	-3.01	-3.26	-3.55	-3.89
1985	-2.70	-2.12	-1.79	**-1.70**	-1.86	-2.26	-2.91	-3.80	-4.94	-6.33
1990	-3.20	-1.35	-0.11	0.51	**0.53**	-0.07	-1.27	-3.09	-5.52	-8.56

Note. Maximum SUB in each row is printed in bold type.

Chapter 6—Appendix 4

This appendix presents an alternative model to the one presented in Chapter 4. In this model, the cost function (in revenue share form) was estimated with two of the revenue share equations and imposes the restriction that autoregressive parameters $\rho_{ij} = 0$, $i \neq j$. Another important difference is that the cost function is estimated, while the materials share equation is dropped. Section 1 discusses the econometric model and results. Section 2 provides analysis of the ancillary variables such as marginal costs, cost elasticities and fixed cost estimates. Section 3 presents results for the subadditivity test and briefly discusses the implications.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The MGM demand system, estimated in revenue-share form, is

$$
\begin{aligned}
\frac{TC_t}{R_t} = \frac{1}{R_t}\Bigg\{ & a_1 w_{1t} + a_2 w_{2t} + a_3 w_{3t} \\
& - \frac{\left(\begin{aligned} & \frac{1}{2} a_{11,2}^2 w_{1t}^2 + a_{11,2} a_{21,2} w_{1t} w_{2t} - \left(a_{11,2}^2 + a_{11,2} a_{21,2}\right) w_{1t} w_{3t} \\ & + \frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right) w_{2t}^2 - \left(a_{11,2} a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right) w_{2t} w_{3t} \\ & + \frac{1}{2}\left(a_{11,2}^2 + 2a_{11,2} a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right) w_{3t}^2 \end{aligned}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \times \left(1 + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \\
& + f_{Z1}\left(w_{1t} + w_{3t}\right) coaxial_t + f_{Z2} w_{2t} coaxial_t \\
& + \frac{1}{2}\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(+ d_{11}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2\right)\Bigg\} \\
& + \rho_0 u_{0t-1}
\end{aligned}
$$

(A6.4-1.1)

$$
\frac{w_{1t}x_{1t}}{R_t} = \frac{w_{1t}}{R_t}\left\{
\begin{aligned}
& a_1 \\
& + \left(\frac{-\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)}
+ \frac{\begin{pmatrix}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{pmatrix}\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \right) \times \left(1 + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \\
& + f_{Z1} w_{1t} coaxial_t \\
& + \frac{\theta_1}{2}\left(d_{11}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \right)
\end{aligned}
\right\}
$$

$$
+\rho_1 u_{1t-1} \qquad \text{(A6.4-1.2)}
$$

$$
\frac{w_{3t}x_{3t}}{R_t} = \frac{w_{3t}}{R_t}\left\{
\begin{aligned}
& a_3 + \left(\frac{-\begin{pmatrix}
-\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t} - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t} \\
+ (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}
\end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)}
+ \frac{\begin{pmatrix}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{pmatrix}\theta_3}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2} \right) \times \left(1 + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \\
& + f_{Z1} coaxial_t \\
& + \frac{\theta_3}{2}\left(d_{11}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)^2 \right)
\end{aligned}
\right\}
$$

$$
+\rho_3 u_{3t-1}
$$

(A6.4-1.3)

where x_{1t}, x_{2t} and x_{3t} correspond to the capital stock, materials volume, and labour stock, respectively. Input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively while R_t is total revenue. Each equation is estimated in revenue share form. Output q_{1t} is a composite Fisher Ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. The variable $coaxial_t$ is the number of kilometres of coaxial cable installed in the network.

Equations (A6.4-1.1) and (A6.4-1.3) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates: 2001), thereby allowing concavity of the cost function with respect to input prices to be imposed by construction. The cost function equation corresponding to the system of factor demand equations are estimated while the equation for materials is not estimated. The Box-Cox transformation is applied to the outputs; and single period lags of the dependent variables are added to each equation.

Table A6-4.1 provides coefficient estimates and associated standard errors, the function value statistic and Box-Cox parameters for the model estimated on sample data corresponding to the years 1954 to 1990, inclusive. Table A6-4.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table A6-4.3 provides results of the remaining diagnostic statistics. The presented models converged within 131 iterations with coefficient starting values left at Shazam’s default setting. The Shazam autoregressive errors option is utilised, allowing for different rhos (ρ_i) in each equation.

Table A6-4.1 shows that only one coefficient is statistically insignificant at conventional levels.

Table A6-4.1 Estimated parameters 1960-90

	COEFFICIENT	ST. ERROR	T-RATIO
a_1	89.91	1.01	**89.24**
a_2	607.90	1.31	**462.87**
a_3	41.550	1.00	**41.47**
$a_{11,2}$	-208.70	53.02	**-3.94**
$a_{21,2}$	67.01	5.11	**13.10**
$a_{22,2}$	-0.00	57.42	-0.00
f_{Z1}	7,923.50	316.59	**25.03**
f_{Z2}	7.06	0.86	**8.25**
d_{11}	12.32	3.75	**3.28**
d_{12}	-200.25	154.23	**-1.30**
d_{22}	236,120.00	359.18	**657.38**
ρ_0	0.94	0.02	**52.68**
ρ_1	0.95	0.02	**52.88**
ρ_3	0.92	0.02	**50.79**
λ_1	0.40		
λ_2	0.25		
θ_1	0.50		
θ_2	0.20		
θ_3	0.30		
Function value	232.66		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table A6-4.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	0.70	1.99	3.24	3.26	3.26
	(0.40)	(0.37)	(0.36)	(0.52)	(0.66)
Equation 2	0.03	0.76	2.44	2.51	2.68
	(0.86)	(0.68)	(0.49)	(0.64)	(0.75)
Equation 3	0.68	0.97	2.60	2.73	2.74
	(0.41)	(0.62)	(0.46)	(0.61)	(0.74)

Note. P-values in parentheses.

Table A6-4.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	0.11 (0.95)
Equation 2 $\chi^2(2)$	0.34 (0.84)
Equation 3 $\chi^2(2)$	0.45 (0.80)
System Test Statistics	
Serial Correlation $\chi^2(9)$	7.96 (0.54)
Heteroskedasticity $\chi^2(36)$	43.31 (0.19)
Heteroskedasticity F(36,6)	1.34 (0.38)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table A6-4.2 and Table A6-4.3 show that the tests fail to reject the null hypothesis of no serial correlation and homoskedasticity at conventional levels of significance.

Table A6-4.4 provides evidence that the estimated cost function is conforms to the theoretical requirements of concavity in input prices and positive marginal cost.

Table A6-4.4. $\mathbf{\Sigma}$ matrix

-43,577.61	13,986.19	29,571.42	$\lvert H_1\rvert$	-43.557.61
13,986.16	-4,490.92	-9,495.28	$\lvert H_2\rvert$	0.11E-02
29,571.42	-9,495.28	-20,076.14	$\lvert H_3\rvert$	0.00

The marginal costs for the years 1960 to 1990, are calculated according to,

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})=}{\partial q_{1t}} \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{11}q_{1t}^{\lambda_1-1}\left(q_{1t}^{\lambda_1}-1\right)+d_{12}q_{1t}^{\lambda_1-1}\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \qquad \text{(A6.4-1.4)}$$

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = -\frac{\begin{pmatrix} a_{11,2}^2 w_1^2 + 2a_{11,2}a_{21,2}w_1 w_2 - 2\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_1 w_3 \\ + \left(a_{21,2}^2 + a_{22,2}^2\right)w_2^2 - 2\left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_2 w_3 \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_3^2 \end{pmatrix}}{\left(w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3\right)} \\ \times q_{2t}^{\lambda_2 - 1}\left(1 + \frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) \\ + \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{12}q_{2t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right) + d_{22}q_{2t}^{\lambda_2 - 1}\left(q_{2t}^{\lambda_2} - 1\right)\right) \tag{A6.4-1.5}$$

Table A6-4.6. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{C}_t / C_t$	$\hat{x}_{1t} / x_{1t}$	$\hat{x}_{3t} / x_{3t}$
1960	0.76	-	1.00	1.00	1.00
1961	0.77	-	1.19	1.30	1.08
1962	0.81	-	0.94	0.94	0.94
1963	0.72	-	0.96	0.95	0.97
1964	0.72	-	0.96	0.95	1.00
1965	0.73	-	0.99	1.00	0.98
1966	0.81	-	1.00	1.02	0.95
1967	0.81	-	1.02	1.00	1.06
1968	0.89	-	0.98	0.97	0.95
1969	0.66	-	1.02	0.95	1.13
1970	0.63	19,889.05	1.02	1.04	1.01
1971	0.52	12,000.88	0.99	0.98	1.01
1972	0.69	15,365.50	0.95	0.96	0.92
1973	0.62	11,568.80	1.05	1.02	1.09
1974	0.76	12,272.33	1.09	1.14	1.03
1975	0.91	12,205.11	1.06	1.09	1.01
1976	1.27	14,243.74	0.92	0.94	0.87
1977	1.66	15,318.84	0.99	1.01	0.95
1978	1.62	13,120.38	0.99	0.99	1.00
1979	1.64	12,080.11	1.04	1.06	1.01
1980	1.76	11,842.76	1.01	1.05	0.94
1981	1.82	11,436.72	1.05	1.07	1.02
1982	2.18	12,605.82	1.04	1.08	0.96
1983	2.34	12,330.63	0.97	0.96	0.99
1984	2.61	12,516.42	1.00	0.95	1.11
1985	2.77	11,907.63	1.00	0.98	1.02
1986	3.05	12,208.87	1.01	1.01	1.04
1987	3.59	13,487.16	1.02	1.04	0.98
1988	4.22	14,273.14	0.91	0.94	0.79
1989	3.78	13,119.90	1.07	1.09	1.08
1990	4.23	14,009.48	1.02	1.02	1.04

Note. $\hat{C}_t / C_t$ denotes estimated total cost to actual total cost. $\hat{x}_{it} / x_{it}$ denotes the ratio of estimated to actual factor demand. Data services introduced in 1970.

2. ANALYSIS OF THE EMPIRICAL RESULTS

This section considers the implications of the estimated model in more detail. In this model, there is only one technical change variable, $coaxial_t$, in which the coefficient for materials is allowed to be different from the coefficient corresponding to capital and labour. Cost complementarity, defined as $\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}\partial q_{2t}} = d_{12}\lambda_1\lambda_2 q_{1t}^{\lambda_1-1}q_{1t}^{\lambda_2-1} < 0$, is evident since $d_{12} < 0$, although the corresponding t-statistic suggests that it is not significantly different to zero at conventional levels of significance.

Table A6-4.7 presents the proportion of total short-run fixed cost to total actual cost (C_t^F / C_t), variable cost to equilibrium cost (VC_t / C_t), equilibrium cost to actual cost (C_t^* / C_t^*), and elasticity of scale (RS_t). The elasticities of scale are calculated according to

$$RS_t = \frac{1}{\varepsilon_{CQ_{1t}} + \varepsilon_{CQ_{2t}}} \text{ where } \varepsilon_{CQ_{it}} = \frac{\partial C_t}{\partial Q_{it}}\frac{Q_{it}}{C_t},$$

$$RS_t^* = \frac{1}{\varepsilon_{CQ_{1t}}^* + \varepsilon_{CQ_{2t}}^*} \text{ where } \varepsilon_{CQ_{it}} = \frac{\partial C_t^*}{\partial Q_{it}}\frac{Q_{it}}{C_t^*}, \text{ and } i = \{1,2\}$$

As shown, short-run fixed cost varies from 92% in 1960 to 53% in 1990. Variable cost accounts for accounts for more than 80% of equilibrium cost. The ratio of equilibrium cost to actual cost is substantially less than one for most of the sample, suggesting that adjustment cost accounts for a substantial portion of fixed cost. The share of variable cost increases as the adjustment cost component declines over time. Returns to scale are consequently estimated to be very high.

Table A6-4.7. Fixed and variable cost contribution to total cost

YEAR	C_t^F / C_t	VC_t / C_t	C_t^* / C_t	RS_t	RS_t^*
1960	0.92	0.08	0.17	15.82	2.64
1961	0.93	0.07	0.14	17.92	2.43
1962	0.93	0.07	0.26	17.37	4.43
1963	0.93	0.07	0.29	18.38	5.35
1964	0.93	0.07	0.32	17.55	5.70
1965	0.93	0.07	0.34	17.81	6.03
1966	0.93	0.07	0.38	16.85	6.37
1967	0.93	0.07	0.39	17.40	6.85
1968	0.93	0.07	0.42	17.10	7.15
1969	0.95	0.05	0.42	24.43	10.29
1970	0.93	0.07	0.45	18.44	8.28
1971	0.93	0.07	0.48	21.78	10.36
1972	0.91	0.09	0.53	15.61	8.25
1973	0.91	0.09	0.51	16.12	8.22
1974	0.90	0.10	0.52	15.04	7.75
1975	0.89	0.11	0.51	13.96	7.17
1976	0.84	0.16	0.58	10.25	5.91
1977	0.80	0.20	0.62	8.20	5.11
1978	0.78	0.22	0.64	7.69	4.96
1979	0.77	0.23	0.65	7.54	4.91
1980	0.75	0.25	0.69	7.15	4.94
1981	0.75	0.25	0.67	7.16	4.81
1982	0.73	0.27	0.68	6.60	4.47
1983	0.70	0.30	0.70	6.00	4.22
1984	0.67	0.33	0.71	5.49	3.89
1985	0.64	0.36	0.72	4.99	3.61
1986	0.61	0.39	0.73	4.65	3.39
1987	0.58	0.42	0.74	4.31	3.18
1988	0.49	0.51	0.82	3.44	2.82
1989	0.55	0.45	0.76	3.94	3.01
1990	0.53	0.47	0.75	3.78	2.84

3. SUBADDITIVITY TEST RESULTS

A summary of the subadditivity calculations is presented in Table A6-4.8 and Table A6-4.9. Both cases assume no change in fixed cost between the monopoly and hypothetical duopoly cases. Note that technology and input prices are fixed across columns and vary across rows. The parameter ϕ corresponds to market share for aggregate output for firm A and ω corresponds to Data output. For example $(\phi,\omega)=50{,}10$ indicates that Firm A has 50% market share of aggregate output and has 10% market share in Data. Underlying calculations specify no change in aggregate fixed cost for monopoly and duopoly. Table A6-4.8 shows initially strong subadditivity, which declines through time.

Table A6-4.8. Subadditivity calculations (%) — no difference in fixed cost

Year	(ϕ,ω)						
	(10,10)	(10,50)	(10,100)	(50,10)	(50,50)	(100,10)	(100,100)
1960	-10.88	-10.88	-10.88	-10.82	-10.82	-10.91	-10.91
1970	-3.37	-3.34	-3.37	-3.36	-3.34	-3.36	-3.38
1975	-2.23	-2.19	-2.25	-2.23	-2.19	-2.23	-2.26
1980	-0.88	-0.80	-0.92	-0.88	-0.80	-0.88	-0.93
1990	-0.14	-0.03	-0.21	-0.14	-0.02	-0.14	-0.20

Note. Maximum SUB in each row is printed in bold type.

Table A6-4.9 provides more detail for the case in which Firm A is the dominant carrier for aggregate output, but has varying market share in Data. Reading from left to right, Firm B market share in Data declines across columns. For example, the left column (i.e. when $(\phi,\omega)=100{,}10$) corresponds to the case in which Firm B is the dominant firm for Data while in the furthest right hand column (i.e. $(\phi,\omega)=100{,}100$), Firm B produces zero Data output. The results show the same initially strong subadditivity across all simulated output ranges, which then decline over time.

Table A6-4.9. Subadditivity calculations for $(100, \omega)$ (%)

Year	(ϕ, ω)									
	(100,10)	(100,20)	(100,30)	(100,40)	(100,50)	(100,60)	(100,70)	(100,80)	(100,90)	(100,100)
1960	-10.91	-10.91	-10.91	-10.91	-10.91	-10.91	-10.91	-10.91	-10.91	-10.91
1970	-3.36	-3.36	-3.35	-3.35	-3.35	-3.35	-3.35	-3.36	-3.37	-3.38
1975	-2.23	-2.21	-2.20	-2.19	-2.19	-2.19	-2.20	-2.22	-2.24	-2.26
1980	-0.88	-0.85	-0.82	-0.81	-0.80	-0.81	-0.82	-0.85	-0.88	-0.93
1990	-0.14	-0.09	-0.06	-0.03	-0.03	-0.03	-0.05	-0.09	-0.14	-0.20

CHAPTER 6—APPENDIX 5

This appendix presents an error-correction model as an alternative to the one presented in Chapter 6. In this model, a multivariate three input error-correction cost function (in revenue share form) was estimated.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The three input error-correction model, estimated in revenue-share form, is

$$\Delta S_{1t} = \gamma_{11}\left(S_{1t}^{*} - S_{1t-1}^{*}\right) + \delta_{11}\left(S_{1t-1}^{*} - S_{1t-1}\right) \tag{A6.5-1.1}$$

$$\Delta S_{2t} = \gamma_{22}\left(S_{2t}^{*} - S_{2t-1}^{*}\right) + \delta_{22}\left(S_{2t-1}^{*} - S_{2t-1}\right) \tag{A6.5-1.2}$$

$$\Delta S_{3t} = \gamma_{33}\left(S_{3t}^{*} - S_{3t-1}^{*}\right) + \delta_{33}\left(S_{3t-1}^{*} - S_{3t-1}\right) \tag{A6.5-1.3}$$

where

$$S_{1t} = \frac{w_{1t}x_{1t}}{TR_t},\ S_{2t} = \frac{w_{2t}x_{2t}}{TR_t},\ S_{3t} = \frac{w_{3t}x_{3t}}{TR_t}$$

$$
S_{1t}^{*}=\frac{w_{1t}}{R_t}\left\{
\begin{aligned}
& a_1 \\
& +\left(
\begin{aligned}
& \frac{-\left(a_{11,1}^{2}w_{1t}+a_{11,1}a_{21,1}w_{2t}-\left(a_{11,1}^{2}+a_{11,1}a_{21,1}\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\
& +\frac{\left(
\begin{aligned}
& \frac{1}{2}a_{11,1}^{2}w_{1t}^{2}+a_{11,1}a_{21,1}w_{1t}w_{2t}-\left(a_{11,1}^{2}+a_{11,1}a_{21,1}\right)w_{1t}w_{3t} \\
& +\frac{1}{2}\left(a_{21,1}^{2}+a_{22,1}^{2}\right)w_{2t}^{2}-\left(a_{11,1}a_{21,1}+a_{21,1}^{2}+a_{22,1}^{2}\right)w_{2t}w_{3t} \\
& +\frac{1}{2}(a_{11,1}^{2}+2a_{11,1}a_{21,1}+a_{21,1}^{2}+a_{22,1}^{2})w_{3t}^{2}
\end{aligned}
\right)\theta_1}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^{2}}
\end{aligned}
\right)\times\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_2}\right) \\
& +\left(
\begin{aligned}
& \frac{-\left(a_{11,2}^{2}w_{1t}+a_{11,2}a_{21,2}w_{2t}-\left(a_{11,2}^{2}+a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)} \\
& +\frac{\left(
\begin{aligned}
& \frac{1}{2}a_{11,2}^{2}w_{1t}^{2}+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^{2}+a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
& +\frac{1}{2}\left(a_{21,2}^{2}+a_{22,2}^{2}\right)w_{2t}^{2}-\left(a_{11,2}a_{21,2}+a_{21,2}^{2}+a_{22,2}^{2}\right)w_{2t}w_{3t} \\
& +\frac{1}{2}(a_{11,2}^{2}+2a_{11,2}a_{21,2}+a_{21,2}^{2}+a_{22,2}^{2})w_{3t}^{2}
\end{aligned}
\right)\theta_1}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^{2}}
\end{aligned}
\right)\times\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \\
& +\frac{\theta_1}{2}\left(d_1\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^{2}+2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^{2}\right)
\end{aligned}
\right\}
$$

$$
S_{2t}^{*}=\frac{w_{2t}}{R_t}\left\{\begin{array}{l}
a_2+\\
+\left(\begin{array}{l}
\dfrac{-\left(a_{11,1}a_{21,1}w_{1t}+\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}\\
+\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}^2-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\end{array}\right)\times\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\\
+\left(\begin{array}{l}
\dfrac{-\left(a_{11,2}a_{21,2}w_{1t}+\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}\\
+\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}(a_{11,2}^2+2a_{11,A}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\end{array}\right)\times\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\\
\\
+f_{Z1}exchange_{old,t}\\
+\dfrac{\theta_2}{2}\left(d_1\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2+2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right.
\end{array}\right.
\qquad (A6.5\text{-}1.4)
$$

$$S_{3t}^{*}=\frac{w_{3t}}{R_t}\left\{\begin{array}{l}a_3+\left(-\frac{\begin{pmatrix}-\left(a_{11,1}^2+a_{11,1}a_{21,1}\right)w_{1t}-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}\\+\left(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{3t}\end{pmatrix}}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}+\frac{\begin{pmatrix}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}^2-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}\left(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{3t}^2\end{pmatrix}\theta_3}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\right)\times\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\\+\left(-\frac{\begin{pmatrix}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}\\+\left(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}\end{pmatrix}}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}+\frac{\begin{pmatrix}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}\left(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}^2\end{pmatrix}\theta_3}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}\right)\times\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\\+\frac{\theta_3}{2}\left(d_1\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2+2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)\end{array}\right\}$$

(A6.5-1.6)

where x_1, x_2 and x_3 correspond to the capital stock, materials volume, and labour stock, respectively. Input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively while R_t is total revenue. Each equation is estimated in revenue share form. Output q_{1t} is a composite Fisher Ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. The variable $exchange_{old,t}$ is the combined number of manual and step-by-step exchanges.

Equations (A6.5-1.1) to (A6.5-1.3) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates: 2001), thereby allowing concavity of the cost function with respect to input prices to be imposed by construction. The cost function equation corresponding to the system of factor demand equations is estimated while the equation for materials is not estimated. The Box-Cox transformation is applied to the outputs; and single period lags of the dependent variables are added to each equation.

Table A6-5.1 provides coefficient estimates and associated standard errors, the log-likelihood statistic and Box-Cox parameters for the model estimated on sample data corresponding to the years 1950 to 1990, inclusive. The parameters θ_1, θ_2 and θ_3 are set equal to the cost shares of the respective inputs. Structural adjustment parameters δ_{11}, δ_{22} and δ_{33} are of plausible magnitude and suggest a substantial degree of inflexibility in adjusting inputs. Table A6-5.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table A6-5.3 provides results of the remaining diagnostic statistics. The presented models converged within 647 iterations with coefficient starting values left at Shazam's default setting.

Table A6-5.1 Estimated parameters 1950-90

	COEFFICIENT	ST. ERROR	T-RATIO
γ_{11}	0.92	0.04	**23.20**
γ_{22}	1.05	0.04	**26.93**
γ_{33}	0.90	0.05	**19.08**
δ_{11}	0.49	0.08	**6.04**
δ_{22}	0.59	0.10	**5.60**
δ_{33}	0.31	0.10	**3.21**
a_1	-6,252,100.00	2,179,300.00	**-2.87**
a_2	8,050,500.00	1,090,500.00	**7.38**
a_3	8,956,300.00	4,220,000.00	**2.12**
$a_{11,1}$	-46.62	13.59	**-3.43**
$a_{21,1}$	-3.23	7.14	-0.45
$a_{22,1}$	-0.00	11.30	-0.00
$a_{11,2}$	-192.90	20.33	**-9.49**
$a_{21,2}$	-170.95	16.64	**-10.28**
$a_{22,2}$	0.00	75.21	0.00
f_{Z1}	-1.40	0.70	**-2.00**
d_1	22,365.00	13,297.00	1.68
d_2	2,130,600.00	199,130.00	**10.70**
d_{11}	51.98	7.30	**7.12**
d_{12}	-291.10	40.69	**-7.15**
d_{22}	727.84	231.03	**3.15**
λ_1	0.40		
λ_2	0.50		
θ_1	0.60		
θ_2	0.11		
θ_3	0.29		
Log-likelihood	366.08		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table A6-5.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	1.32	1.33	1.56	1.56	2.88
	(0.25)	(0.51)	(0.67)	(0.82)	(0.72)
Equation 2	0.34	0.37	0.44	0.45	1.21
	(0.56)	(0.83)	(0.93)	(0.98)	(0.94)
Equation 3	2.40	3.19	5.29	5.59	5.72
	(0.12)	(0.20)	(0.15)	(0.23)	(0.33)

Note. P-values in parentheses.

Table A6-5.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	1.32
	(0.52)
Equation 2 $\chi^2(2)$	0.83
	(0.66)
Equation 3 $\chi^2(2)$	0.96
	(0.62)
System Test Statistics	
Serial Correlation $\chi^2(9)$	4.98
	(0.84)
Heteroskedasticity $\chi^2(36)$	39.65
	(0.31)
Heteroskedasticity F(36,80)	1.07
	(0.39)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table A6-5.2 and Table A6-5.3 show that the tests fail to reject the null hypothesis of no serial correlation and homoskedasticity at conventional levels of significance.

Table A6-5.4 provides evidence that the estimated cost function is conforms to the theoretical requirements of concavity in input prices and positive marginal cost.

Table A6-5.4a. $\mathbf{\Sigma}_1$ matrix

-2,173.01	-150.76	2,323.77	$\lvert H_1 \rvert$	-2,173.01
-150.76	-10.46	161.22	$\lvert H_2 \rvert$	0.35E-09
2,323.77	161.22	-2,484.99	$\lvert H_3 \rvert$	-0.10E-21

Table A6-5.4b. $\mathbf{\Sigma}_2$ matrix

-37,209.37	-32,976.02	70,185.39	$\lvert H_1 \rvert$	-37,209.37
-32,976.02	-29,224.31	62,200.33	$\lvert H_2 \rvert$	0.65E-05
70,185.39	62,200.33	-132,385.70	$\lvert H_3 \rvert$	0.00

The marginal costs for the years 1950 to 1990 are calculated according to,

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}} = -\frac{\begin{pmatrix} a_{11,1}^2 w_{1t}^2 + 2a_{11,1}a_{21,1}w_{1t}w_{2t} - 2\left(a_{11,1}^2 + a_{11,1}a_{21,1}\right)w_{1t}w_{3t} \\ + \left(a_{21,1}^2 + a_{22,1}^2\right)w_{2t}^2 - 2\left(a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2\right)w_{2t}w_{3t} \\ + (a_{11,1}^2 + 2a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2)w_{3t}^2 \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_{3t}\right)}$$
$$\times \lambda_2 q_{1t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right)$$
$$+\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_1 + d_{11}q_{1t}^{\lambda_1 - 1}\left(q_{1t}^{\lambda_1} - 1\right) + d_{12}\lambda_1 q_{1t}^{\lambda_1 - 1}\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)$$

(A6.5-1.7)

$$\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = -\frac{\begin{pmatrix} a_{11,2}^2 w_{1t}^2 + 2a_{11,2}a_{21,2}w_{1t}w_{2t} - 2\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - 2\left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + (a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2 \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)}$$
$$\times \lambda_2 q_{2t}^{\lambda_2 - 1}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)$$
$$+\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_2 + d_{12}q_{2t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right) + d_{22}q_{2t}^{\lambda_2 - 1}\left(q_{2t}^{\lambda_2} - 1\right)\right)$$

(A6.5-1.8)

Table A6-5.5. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{C}_t/C_t$	$\Delta\hat{S}_{1t}/\Delta S_{1t}$	$\Delta\hat{S}_{2t}/\Delta S_{2t}$	$\Delta\hat{S}_{3t}/\Delta S_{3t}$
1950	11,187.13	-	1.31	-0.28	3.38	1.75
1951	14,128.69	-	1.29	1.52	0.71	0.95
1952	16,947.01	-	1.24	1.29	1.08	1.61
1953	17,430.18	-	1.23	-0.31	1.41	1.74
1954	18,942.33	-	1.23	0.97	0.64	-0.02
1955	19,949.83	-	1.21	0.55	0.71	0.82
1956	21,402.67	-	1.24	1.60	1.14	5.08
1957	23,050.28	-	1.22	0.63	1.01	1.41
1958	23,569.13	-	1.21	2.44	1.99	0.91
1959	24,514.37	-	1.22	3.98	0.00	0.55
1960	24,256.85	-	1.27	0.92	0.81	-12.37
1961	28,564.01	-	1.25	1.07	0.97	3.03
1962	29,090.94	-	1.24	-8.70	0.82	-11.87
1963	28,743.02	-	1.28	0.76	0.95	0.64
1964	28,535.34	-	1.31	0.80	0.52	-0.70
1965	30,394.17	-	1.32	0.74	1.01	0.19
1966	32,081.81	-	1.32	-0.04	0.74	-0.10
1967	33,195.01	-	1.30	1.51	1.83	1.79
1968	34,698.35	-	1.26	1.93	-0.03	0.79
1969	34,772.99	-	1.17	2.56	1.01	0.01
1970	37,859.49	3,500,541.45	1.24	1.78	0.98	2.96
1971	41,349.60	3,860,141.67	1.21	1.55	0.95	-1.75
1972	44,299.75	4,143,353.00	1.27	1.13	0.93	0.03
1973	45,768.61	4,297,169.25	1.27	0.83	0.94	0.03
1974	60,382.23	5,683,979.34	1.29	1.11	0.76	1.43
1975	75,834.83	7,158,757.53	1.27	0.80	1.35	-2.08
1976	86,405.05	8,172,341.62	1.26	1.54	0.89	0.91
1977	99,574.66	9,433,224.46	1.26	-0.74	0.84	0.03
1978	102,715.65	9,740,506.27	1.28	1.08	0.69	-2.18
1979	112,372.81	10,666,374.50	1.26	0.79	0.98	-0.33
1980	128,783.30	12,231,785.46	1.27	1.03	0.57	1.95
1981	144,738.24	13,752,194.37	1.28	1.07	0.89	0.19
1982	175,909.28	16,720,686.63	1.30	1.06	0.92	-3.84
1983	187,387.25	17,817,610.31	1.35	0.71	1.48	1.89
1984	191,981.46	18,257,226.37	1.33	0.90	1.11	-17.05
1985	199,679.61	18,995,244.56	1.36	0.05	1.38	1.73
1986	213,681.31	20,330,249.09	1.35	-0.93	1.25	0.98
1987	232,460.35	22,119,089.99	1.33	0.95	0.85	-0.69
1988	244,659.81	23,265,618.97	1.36	1.15	0.80	0.30
1989	274,460.27	26,101,906.78	1.38	2.28	0.73	-0.37
1990	277,210.01	26,363,474.69	1.29	3.23	0.63	0.29

Note. $\hat{C}_t/C_t$ denotes estimated total cost to actual total cost. $\hat{S}_{it}/S_{it}$ denotes the ratio of estimated change in revenue share to actual revenue share. Data services introduced in 1970.

Table A6-5.5 indicates that the error-correction model leads to substantially higher estimates of marginal and total cost. The ratio of the change in estimated revenue share to actual revenue share show that the model fit is substantially less accurate than the levels model presented in Chapter 6. Overall, the model enabled estimation of a cost function that is consistent with economic theory, but less plausible than the preferred model.

CHAPTER 6—APPENDIX 6

This appendix presents an error-correction model as an alternative to the one presented in Chapter 6. In this appendix, a multivariate three input error-correction cost function (in revenue share form) is presented. This model differs from the model presented in Chapter 6—Appendix 5 in the controls used for technological change.

1. PARAMETER ESTIMATES, AUXILIARY STATISTICS AND PROPERNESS

The three input error-correction model, estimated in revenue-share form, is

$$\Delta S_{1t} = \gamma_{11}\left(S_{1t}^{*} - S_{1t-1}^{*}\right) + \delta_{11}\left(S_{1t-1}^{*} - S_{1t-1}\right) \tag{A6.6-1.1}$$

$$\Delta S_{2t} = \gamma_{22}\left(S_{2t}^{*} - S_{2t-1}^{*}\right) + \delta_{22}\left(S_{2t-1}^{*} - S_{2t-1}\right) \tag{A6.6-1.2}$$

$$\Delta S_{3t} = \gamma_{33}\left(S_{3t}^{*} - S_{3t-1}^{*}\right) + \delta_{33}\left(S_{3t-1}^{*} - S_{3t-1}\right) \tag{A6.6-1.3}$$

where

$$S_{1t} = \frac{w_{1t}x_{1t}}{TR_t},\; S_{2t} = \frac{w_{2t}x_{2t}}{TR_t},\; S_{3t} = \frac{w_{3t}x_{3t}}{TR_t}$$

$$
S_{1t}^{*} = \frac{w_{1t}}{R_t}\left\{
\begin{array}{l}
a_1 \\
+\left(
\begin{array}{l}
\dfrac{-\left(a_{11,1}^2 w_{1t} + a_{11,1}a_{21,1}w_{2t} - \left(a_{11,1}^2 + a_{11,1}a_{21,1}\right)w_{3t}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+\dfrac{\left(
\begin{array}{l}
\frac{1}{2}a_{11,1}^2 w_{1t}^2 + a_{11,1}a_{21,1}w_{1t}w_{2t} - \left(a_{11,1}^2 + a_{11,1}a_{21,1}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,1}^2 + a_{22,1}^2\right)w_{2t}^2 - \left(a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,1}^2 + 2a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2)w_{3t}^2
\end{array}
\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2}
\end{array}
\right)\times\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_2}\right) \\
+\left(
\begin{array}{l}
\dfrac{-\left(a_{11,2}^2 w_{1t} + a_{11,2}a_{21,2}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{3t}\right)}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\
+\dfrac{\left(
\begin{array}{l}
\frac{1}{2}a_{11,2}^2 w_{1t}^2 + a_{11,2}a_{21,2}w_{1t}w_{2t} - \left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\
+\frac{1}{2}\left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - \left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\
+\frac{1}{2}(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2)w_{3t}^2
\end{array}
\right)\theta_1}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)^2}
\end{array}
\right)\times\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) \\
+ f_t t + f_{11}exchange_{old,t} + f_{12}exchange_{crossbar,t} \\
+\dfrac{\theta_1}{2}\left(d_1\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right) + d_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) + d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2 + 2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right) + d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)
\end{array}
\right\}
$$

$$S_{2t}^{*}=\frac{w_{2t}}{R_t}\left\{\begin{array}{l}
a_2+\\
\left(\begin{array}{l}
\dfrac{-\left(a_{11,1}a_{21,1}w_{1t}+\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}\\
+\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}^2-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\end{array}\right)\times\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\\
+\left(\begin{array}{l}
\dfrac{-\left(a_{11,2}a_{21,2}w_{1t}+\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{3t}\right)}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}\\
+\dfrac{\left(\begin{array}{l}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\+\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t}\\+\frac{1}{2}(a_{11,2}^2+2a_{11,A}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{array}\right)\theta_2}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\end{array}\right)\times\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\\
\\
+f_{21}exchange_{old,t}+f_{22}coaxial_t\\
+\dfrac{\theta_2}{2}\left(d_1\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2+2d_{12}\left(\dfrac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\dfrac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)
\end{array}\right\}$$

$$
S_{3t}^{*}=\frac{w_{3t}}{R_t}\left\{
\begin{aligned}
& a_3+\left(
-\frac{\begin{pmatrix}-\left(a_{11,1}^2+a_{11,1}a_{21,1}\right)w_{1t}-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}\\ +(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2)w_{3t}\end{pmatrix}}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}
+\frac{\begin{pmatrix}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\ +\frac{1}{2}\left(a_{21,1}^2+a_{22,1}^2\right)w_{2t}^2-\left(a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2\right)w_{2t}w_{3t}\\ +\frac{1}{2}(a_{11,1}^2+2a_{11,1}a_{21,1}+a_{21,1}^2+a_{22,1}^2)w_{3t}^2\end{pmatrix}\theta_3}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\right)\times\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\\
& +\left(
-\frac{\begin{pmatrix}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}\\ +(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}\end{pmatrix}}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)}
+\frac{\begin{pmatrix}\frac{1}{2}a_{11,2}^2w_{1t}^2+a_{11,2}a_{21,2}w_{1t}w_{2t}-\left(a_{11,2}^2+a_{11,2}a_{21,2}\right)w_{1t}w_{3t}\\ +\frac{1}{2}\left(a_{21,2}^2+a_{22,2}^2\right)w_{2t}^2-\left(a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2\right)w_{2t}w_{3t}\\ +\frac{1}{2}(a_{11,2}^2+2a_{11,2}a_{21,2}+a_{21,2}^2+a_{22,2}^2)w_{3t}^2\end{pmatrix}\theta_3}{\left(w_{1t}\theta_1+w_{2t}\theta_2+w_{3t}\theta_3\right)^2}
\right)\times\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)\\
& +\frac{\theta_3}{2}\left(d_1\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)+d_2\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{11}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)^2+2d_{12}\left(\frac{q_{1t}^{\lambda_1}-1}{\lambda_1}\right)\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)+d_{22}\left(\frac{q_{2t}^{\lambda_2}-1}{\lambda_2}\right)^2\right)
\end{aligned}
\right\}
\tag{A6.6-1.6}
$$

where x_{1t}, x_{2t} and x_{3t} correspond to the capital stock, materials volume, and labour stock, respectively. Input prices w_{1t}, w_{2t}, and w_{3t} correspond to capital, materials and labour, respectively while R_t is total revenue. Each equation is estimated in revenue share form. Output q_{1t} is a composite Fisher Ideal quantity index consisting of calls (for local, toll, cellular telephone service and telex), subscribers (fixed-line telephone, cellular telephone and telex), and telegrams. Output q_{2t} represent the total number of data subscribers for the years 1970 to 1990, inclusive, and is zero otherwise. The variable $exchange_{old,t}$ is the combined number of manual and step-by-step exchanges. Variable $crossbar_t$ is the total number of kilometres of operational coaxial cable in the network.

Equations (A6.6-1.1) to (A6.6-1.3) are estimated using the maximum likelihood estimation routine available in SHAZAM (Whistler, White, Wong and Bates: 2001), thereby allowing concavity of the cost function with respect to input prices to be imposed by construction. The cost function equation corresponding to the system of factor demand equations are estimated while the equation for materials is not estimated. The Box-Cox transformation is applied to the outputs; and single period lags of the dependent variables are added to each equation.

Table A6-6.1 provides coefficient estimates and associated standard errors, the log-likelihood statistic and Box-Cox parameters for the model estimated on sample data corresponding to the years 1950 to 1990, inclusive. The parameters θ_1, θ_2 and θ_3 are set equal to the cost shares of the respective inputs. Structural adjustment parameters δ_{11}, δ_{22} and δ_{33} are of plausible magnitude and suggest a substantial degree of inflexibility in adjusting inputs. Table A6-6.2 reports the Ljung-Box-Pierce test statistic for serial correlation by each equation. Table A6-6.3 provides results of the remaining diagnostic statistics. The presented models converged within 802 iterations with coefficient starting values left at Shazam's default setting.

Table A6-6.1 Estimated parameters 1950-90

	COEFFICIENT	ST. ERROR	T-RATIO
γ_{11}	0.98	0.03	**35.02**
γ_{22}	1.03	0.00	**123.53**
γ_{33}	0.97	0.02	**59.06**
δ_{11}	0.52	0.15	**3.37**
δ_{22}	0.11	0.04	**2.78**
δ_{33}	0.12	0.08	1.58
a_1	51,874,000.00	5,954,300.00	**8.71**
a_2	27,408,000.00	3,095,500.00	**8.85**
a_3	25,471,000.00	2,783,600.00	**9.15**
$a_{11,1}$	39.31	11.18	**3.52**
$a_{21,1}$	3.87	14.21	0.27
$a_{22,1}$	-0.00	55.42	-0.00
$a_{11,2}$	-141.35	34.12	**-4.14**
$a_{21,2}$	-93.33	38.31	**-2.44**
$a_{22,2}$	0.00	65.12	0.00
f_t	9,974,800.00	903,500.00	**11.04**
f_{11}	-39.58	11.36	**-3.48**
f_{12}	15.99	2.99	**5.36**
f_{21}	-6.39	2.22	**-2.88**
f_{22}	746.48	117.60	**6.35**
d_{11}	0.06	3.42	0.02
d_{12}	4.17	18.53	0.23
d_{22}	565.37	138.68	**4.08**
λ_1	0.40		
λ_2	0.50		
θ_1	0.60		
θ_2	0.11		
θ_3	0.29		
Log-likelihood	368.81		

Note. Bolded t-ratio indicates coefficient is statistically significant at conventional levels.

Table A6-6.2. Ljung-Box-Pierce test for serial correlation

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Equation 1	0.51	0.56	0.56	5.62	7.75
	(0.47)	(0.76)	(0.91)	(0.23)	(0.17)
Equation 2	0.31	0.58	0.70	0.83	0.87
	(0.58)	(0.75)	(0.87)	(0.93)	(0.97)
Equation 3	0.10	2.16	3.02	3.06	3.07
	(0.75)	(0.34)	(0.39)	(0.55)	(0.69)

Note. P-values in parentheses.

Table A6-6.3. Diagnostic statistics

Heteroskedasticity (modified White's test)	Statistic
Equation 1 $\chi^2(2)$	4.09
	(0.13)
Equation 2 $\chi^2(2)$	1.62
	(0.45)
Equation 3 $\chi^2(2)$	0.68
	(0.71)
System Test Statistics	
Serial Correlation $\chi^2(9)$	6.35
	(0.70)
Heteroskedasticity $\chi^2(36)$	42.55
	(0.21)
Heteroskedasticity F(36,80)	1.26
	(0.19)

Note. P-values in parentheses.

Single-equation and system diagnostic statistics presented in Table A6-6.2 and Table A6-6.3 show that the tests fail to reject the null hypothesis of no serial correlation and homoskedasticity at conventional levels of significance.

Table A6-6.4 provides evidence that the estimated cost function is conforms to the theoretical requirements of concavity in input prices and positive marginal cost.

Table A6-6.4a. $\mathbf{\Sigma}_1$ matrix

-1,545.30	-152.10	1,697.40	$\lvert H_1 \rvert$	-1,545.30
-152.10	-14.97	167.07	$\lvert H_2 \rvert$	0.00
1,697.40	167.07	-1,864.47	$\lvert H_3 \rvert$	0.00

Table A6-6.4b. $\mathbf{\Sigma}_2$ matrix

-19,978.52	-13,191.17	33,169.69	$\lvert H_1 \rvert$	-19,978.52
-13,191.17	-8,709.71	21,900.88	$\lvert H_2 \rvert$	0.00
33,169.69	21,900.88	-55,070.57	$\lvert H_3 \rvert$	0.00

The marginal costs for the years 1950 to 1990 are calculated according to,

$$\begin{aligned}\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{1t}} = & -\frac{\begin{pmatrix} a_{11,1}^2 w_{1t}^2 + 2a_{11,1}a_{21,1}w_{1t}w_{2t} - 2\left(a_{11,1}^2 + a_{11,1}a_{21,1}\right)w_{1t}w_{3t} \\ + \left(a_{21,1}^2 + a_{22,1}^2\right)w_{2t}^2 - 2\left(a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2\right)w_{2t}w_{3t} \\ + \left(a_{11,1}^2 + 2a_{11,1}a_{21,1} + a_{21,1}^2 + a_{22,1}^2\right)w_{3t}^2 \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_{3t}\right)} \\ & \times q_{1t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right) \\ & + \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_1 + d_{11}q_{1t}^{\lambda_1 - 1}\left(q_{1t}^{\lambda_1} - 1\right) + d_{12}q_{1t}^{\lambda_1 - 1}\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right)\end{aligned} \tag{A6.6-1.7}$$

$$\begin{aligned}\frac{\partial C_t(\mathbf{q},t,\mathbf{w})}{\partial q_{2t}} = & -\frac{\begin{pmatrix} a_{11,2}^2 w_{1t}^2 + 2a_{11,2}a_{21,2}w_{1t}w_{2t} - 2\left(a_{11,2}^2 + a_{11,2}a_{21,2}\right)w_{1t}w_{3t} \\ + \left(a_{21,2}^2 + a_{22,2}^2\right)w_{2t}^2 - 2\left(a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{2t}w_{3t} \\ + \left(a_{11,2}^2 + 2a_{11,2}a_{21,2} + a_{21,2}^2 + a_{22,2}^2\right)w_{3t}^2 \end{pmatrix}}{\left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)} \\ & \times q_{2t}^{\lambda_2 - 1}\left(\frac{q_{2t}^{\lambda_2} - 1}{\lambda_2}\right) \\ & + \left(w_{1t}\theta_1 + w_{2t}\theta_2 + w_{3t}\theta_3\right)\left(d_{12}q_{2t}^{\lambda_2 - 1}\left(\frac{q_{1t}^{\lambda_1} - 1}{\lambda_1}\right) + d_{22}q_{2t}^{\lambda_2 - 1}\left(q_{2t}^{\lambda_2} - 1\right)\right)\end{aligned} \tag{A6.6-1.8}$$

Table A6-6.5. Marginal cost and equation fit

Year	Aggregate Output	Data Services	$\hat{C}_t / C_t$	$\Delta\hat{S}_{1t} / \Delta S_{1t}$	$\Delta\hat{S}_{2t} / \Delta S_{2t}$	$\Delta\hat{S}_{3t} / \Delta S_{3t}$
1950	0.01	-	1.06	1.00	2.58	1.19
1951	0.01	-	1.04	1.02	1.15	0.72
1952	0.01	-	1.04	1.09	1.34	0.83
1953	0.01	-	1.06	0.91	0.96	2.36
1954	0.01	-	1.07	0.96	0.93	-0.20
1955	0.01	-	1.07	0.86	0.86	0.61
1956	0.01	-	1.08	1.23	1.30	3.17
1957	0.01	-	1.08	0.95	0.98	1.16
1958	0.01	-	1.07	1.56	1.79	0.73
1959	0.01	-	1.07	3.20	-0.80	0.86
1960	0.01	-	1.08	0.91	0.91	-35.42
1961	0.01	-	1.07	1.08	0.97	9.09
1962	0.01	-	1.10	9.79	2.16	-13.15
1963	0.01	-	1.11	0.81	0.73	1.29
1964	0.01	-	1.12	0.93	0.31	0.72
1965	0.01	-	1.12	0.92	0.95	0.80
1966	0.01	-	1.12	-0.15	0.83	0.42
1967	0.01	-	1.11	1.28	0.41	0.54
1968	0.02	-	1.11	1.09	1.05	0.92
1969	0.01	-	1.06	1.10	0.80	-0.20
1970	0.03	5,167.80	1.06	1.24	-0.45	-5.90
1971	0.05	5,189.11	1.03	0.56	0.90	-1.68
1972	0.05	5,461.94	1.07	0.97	1.01	0.18
1973	0.06	5,429.69	1.05	0.67	0.96	-0.21
1974	0.09	7,026.16	1.05	1.02	0.17	0.89
1975	0.14	8,596.19	1.05	0.91	1.33	-1.04
1976	0.19	9,611.18	1.06	1.28	0.90	0.77
1977	0.26	10,889.65	1.07	0.19	0.88	0.13
1978	0.30	11,111.52	1.08	0.97	0.40	-2.12
1979	0.39	12,033.14	1.07	1.02	0.65	-0.01
1980	0.50	13,700.59	1.09	1.14	-1.64	1.26
1981	0.59	15,340.79	1.09	1.23	0.92	0.01
1982	0.78	18,566.79	1.09	0.87	0.80	-2.74
1983	0.89	19,709.36	1.12	0.87	0.99	0.95
1984	0.93	20,162.93	1.09	1.10	0.84	-13.54
1985	1.04	20,904.06	1.10	0.94	0.81	0.22
1986	1.14	22,337.59	1.11	0.82	0.76	0.29
1987	1.27	24,278.06	1.11	0.66	0.70	-0.61
1988	0.98	25,739.63	1.17	1.03	0.73	0.16
1989	1.12	28,844.82	1.20	1.44	0.71	0.61
1990	1.12	29,135.51	1.14	2.33	0.51	0.57

Note. $\hat{C}_t / C_t$ denotes estimated total cost to actual total cost. $\hat{S}_{it} / S_{it}$ denotes the ratio of estimated change in revenue share to actual revenue share. Data services introduced in 1970.

Table A6-6.5 indicates that the error-correction model provides plausible estimates of marginal and total cost. The ratio of the fitted to actual change in factor demand (revenue share form) show that the model fit is substantially less accurate than the levels model presented in Chapter 6. Overall, the model enabled estimation of a cost function that is consistent with economic theory, but less plausible than the preferred model.