Science and Mathematics Education Centre

Engaging Students in

Extended Learning Conversations

to Improve their Mathematical Understanding

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This thesis is presented for the Degree of
Doctor of Philosophy
of
Curtin University

March 2012

DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

Signature: 

Date: March 2012

Abstract

The study was undertaken to investigate classroom strategies focused on language that would facilitate cognitive processing and improve mathematical understanding along with examining the link between the strategies and elaborated, extended learning conversations. This involved developing strategies in the mathematics classroom for students to engage in extended learning conversations (elaborated discourse) to develop and demonstrate their understanding of mathematical concepts. The theoretical base for developing the strategies was grounded in the theories of Halliday, Bernstein and Vygotsky and influenced by those working in language development. The foundation for the study was an examination of teacher language usage in the classroom based on the belief that with language being effectively used one could actually make a difference to students' understanding of mathematics.

The study was conducted using a *Participatory Action Research* design with teacher as researcher working to improve the learning/teaching classroom practice and was carried out with two groups of students in a rural district high school. The first group consisted of year 8, 9 and 10 students and the second group consisted of year 6 and 7 students both in multi– aged classes. Data analysis was undertaken using a *Grounded Theory* approach. The study commenced early in Term 1 of 2010, with a follow up in the latter stages of Term 2 and was completed with collection of student responses late in Term 4.

The learning/teaching strategies that were identified and developed were the Shared Experience, Purposeful Discussion, Blended Instruction and Student Peer Teaching strategies. The identification of these strategies presented a strong case for a *Mathematical Linguistic Pedagogy* combining elements of mathematical content knowledge and linguistic pedagogy. The strategies presented an approach to teaching and learning that combined elements of constructivist philosophy alongside elements of traditional teaching practice that focus on elaborated use of language. The use of the strategies enabled diagnosis of misconceptions as well as enhancing learning, providing for deep understanding and empowering students to share reflections of their own thought development and processes.

ACKNOWLEDGEMENTS

Nothing of any great worth is accomplished in isolation or alone, and so it is with the utmost sincerity, humility and modesty that I wish to thank the many people who have seen me through this journey. I have been honoured to have many different people in my professional and personal life who have helped to guide, support and motivate me along the way.

I would like to thank Associate Professor Bill Atweh and Professor Emeritus John Malone who have guided, supported and offered encouragement to me during the good times and the more difficult. Professor John Malone commenced with me as my supervisor when I first started on this thesis. Professor Bill Atweh took over as my supervisor in the last few years. Both John and Bill have provided assistance and encouragement and guided me with this thesis. I thank Bill for pushing me to my limits in order that I could ‘be the best that I could be’ and John for teaching me how to make my writing even better.

I want to thank my family for supporting me through what seems like a long and arduous journey. I would like to take this opportunity to thank my husband Mark who has gently encouraged me and constantly reminded me of the goals I wanted to achieve and for his belief in me as an educator.

I would also like to thank my children – Jaessen, Holly, Graeme, Caillan and Maire for it is you who have given me the inspiration to want to continue working with young people and to improve the learning for all young people.

Thank you, Holly, for continually inspiring me with your journey, one that is also almost complete.

I must make special mention of all the young people I have worked with over my many years of teaching. You, the students in my classes, have also inspired me and driven me to search for ways to make classroom learning more effective for you. Thank you, to the students, who participated in this study; your learning has enlightened and delighted me.

Thank you too, to my teaching colleagues, who have provided support and encouragement, advice and feedback, offered challenges and taken up those challenges offered by me. Thank you to those colleagues who worked with me on this research study; thank you especially to my Cooperative Colleague. Your support was invaluable and I could not have completed this thesis without your assistance.

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Blended Instruction Strategies:

A group of instructional strategies that combine elements of discovery, inquiry-based learning with elements of explicit instruction that rely on *Shared Experience* and/or *Purposeful Discussion* to set them up, to lead into them and to prepare students to gain benefit from the direct instruction. The *Blended Instruction Strategies* included: Targeted Instruction; Responsive Teaching, and Guided Discovery.

Classroom Discourse:

Using Gee's (2008) and Christie's (2002) definitions, classroom discourse values knowledge, values the learner, and seeks to make available to learners as explicitly as possible, significant and useful information and ideas in socioculturally meaningful ways of speaking, listening, writing, reading, feeling, valuing, and believing.

Closed Learning Community:

A learning community, like a class, that has focus, shared purpose and shared language can be an effective environment for learning. Language can be used to create that learning community. Bernstein (1971, 1973) referred to the language in groups like these as Restricted Code. The language used in a closed learning community like a class does have a language of its own.

Cognitive Framework/Platform:

An anchor point, from which new learning, skills, knowledge and understanding can be developed. Language can be used to create a Cognitive Platform, like the Fractions Stand-up by something as simple as saying 'remember when we stood up to find fractions'. Using the right word, with the right tone and gesture can act as the seed to germinate new knowledge. Using appropriate language a teacher can set up base understanding and then that understanding can be built upon, hence the term cognitive platform as an anchor for embedding new knowledge. When combined with a 'Physical Activity' that cognitive platform becomes powerful.

Converging Purposeful Discussion:

Purposeful Discussion that is focused, purposeful and used to bring students' thinking and background knowledge to a common point for new learning that is structured and factual and where there is usually only one form of methodology or reasoning.

Elaborated Discourse:

Utterances that are connected by two or more discourse connectors to form an elaborated utterance, a component of elaborated discourse.

Established Student Peer Teaching

Once the skills have been achieved and practiced and students are confident with the process then effective peer teaching or tutoring provides the students with a means by which to display and share their learning and understanding of mathematical concepts. Minimal intervention is required and usually only to highlight and amplify for the students who are leading the learning.

Establishing Student Peer Teaching

Establishing is when the strategy is used for the first time with a class group of students: when commencing with students an informal approach coupled with a shorter preparation time and smaller audience yields more effective results. More teacher input is required when the strategy is first introduced and the accompanying skills are being developed.

Focused Task:

Any brief learning task that provides the focus for a following *Purposeful Discussion* and may involve students working individually or in small groups. Essentially each of the tasks provides focus and the means by which to retrieve and activate background knowledge during the

Purposeful Discussion that would follow. The Focused Tasks used in this study were Six Calculations, Mental Questions/Calculations, Feedback and Roundtable Reflection.

Guided Discovery:

A strategy that enables students to be stepped through a series of activities that lead them into 'discovering' for themselves relationships, formulas and so on.

Learning Conversation:

Like a conversation, a verbal exchange between two or more people, free flowing, with the exchange of ideas and thoughts dictating the pace and perhaps its direction, but as a more directed activity with a purpose, so that although it is informal, the conversation can be facilitated around a central purpose. The term 'conversation' implies that there is meaning and understanding on the part of both the sender and the receiver in the sharing of information and includes a purposeful, though informal exchange including both verbal and non-verbal exchanges. The purpose of the learning conversation, of the directed exchanges, is for students to develop their cognitive processes and then to mediate those cognitive processes with the language developed in learning conversations. As a conversation is about communication, the non-verbal components would include gestures, body language and tone and encompassing the needs of learners, which implies the concepts of scaffolding and modelling be included.

Mathematical Linguistic Pedagogy:

In a Venn diagram *Mathematical Linguistic Pedagogy* would be the intersection of linguistics, mathematical content knowledge and pedagogy. It is the knowledge, skills and expertise that a teacher possesses to use language to communicate; to build closed learning communities; to use language as a teaching tool; to enable elaborated discourse; and to effectively maximise the learning of students in their class(es).

Non-converging Purposeful Discussion:

Purposeful Discussion where there is no 'one' apparent correct method, process or answer, where students are encouraged to reflect on their own processes and the discussion opens up.

Physical Activity:

This strategy involved physical movement, with students having to make decisions, communicate with other students, reflect and work together. This provided a shared Cognitive Platform or Organiser that students could refer to like 'remember when we stood up to find fractions.

Purposeful Discussion:

Based on Steinbring's (1998) term, Purposeful Discussion hones the intent of a learning conversation, communicates and offers a definition of 'purposeful discussion' as follows –

teacher's question → students respond → teacher questions conviction or otherwise of the response as an alternative to the traditional 'interrogation' style and means of communication – from ordinary to technical mathematical language, language of gestures and icons, to symbols, from literal to metaphorical use of words, manipulating material objects to speaking about the possible outcomes of an imagined action on these objects as being significant contributors to 'purposeful discussion'.

Responsive Teaching:

Similar in many ways to Targeted Instruction; however, unlike Targeted Instruction it is often unplanned and arises out of the necessity to meet student needs. When a teacher recognises this opportunity it can be used to effectively facilitate learning.

Shared Experience:

Often in the form of a warm up activity such as a set of mental calculations, a brainstorm or similar learning activity which then became the reference point, or trigger, for *Purposeful Discussion*. The *Shared Experience Strategy* could occur on its own; however, *Purposeful*

Discussion might not occur, or may occur but might not be effective, without first having a reference point, or trigger referred to as a Cognitive Platform, which was provided by the *Shared Experience*.

Student Peer Teaching:

This strategy can be embedded within collaborative group-work, ; however, it can stand alone. *Student Peer Teaching* follows the principles and philosophy of Constructivism; students extending their knowledge from a familiar base and sharing this with their peers and younger students. Students work in collaborative groups to teach themselves or revise a concept that they could then teach to other students.

Targeted Instruction:

Is based on Huitt's (1996) format of Direct Instruction with active explication of the skill or subject matter being taught, linking and making concepts relevant, employing appropriate analogies and metaphorical stories and basing the new learning in a context familiar to students, with students' understanding being checked and assessed throughout the instruction. Opportunities for student participation is usually encouraged at relevant points and followed up with individual or small group practice. Targeted Instruction follows on from *Purposeful Discussion*: students' attention has been gained, and in a sense the preceding discussion has worked as an advanced organiser and retrieved relevant knowledge.

Chapter 1

Context and Background

This research study is one that has been the focus of my teaching practice for a number of years and one that has driven me to search for ways to enhance the mathematics learning of students and to improve their educational outcomes. The evolution of this study was a result of my continuing search to unlock the power of language in the learning of mathematics.

The purpose of this chapter is to introduce the problem and my concerns. Section 1 examines language use in the mathematics classroom, the links with language, thought development and conceptual understanding, and develops a definition of a learning conversation. Section 2 elaborates the focus of the study, provides the research questions, significance and the research methodology. Section 3 examines the context, including the school, characteristics of the researcher and the contemporary environment within which practitioners and researchers work. Section 4 introduces the factors that influenced the research direction, including modelling, scaffolding and Constructivism, along with the outlining of the linked areas that influenced the study. Finally, section 5 provides a very brief overview of the chapters that follow.

1.1 Background to the Problem

The relationship between language and thought as proposed by Vygotsky (1987) and the language development theories of Halliday (1985, 1993) and Bernstein (1971, 1973, 1975) suggest that teachers can make better use of the language they use and manage in their classrooms to enhance the learning of their students. Even with the plethora of research studies supporting the enhancement and greater use of classroom discourse for the improvement of mathematical learning, there remains an underdeveloped set of classroom strategies available for teachers to use, or to use as models for their own teaching. Whilst the results of such research studies continue to remain mainly theoretical, they will not attract the attention of practitioners to the use of classroom discourse and its role in learning and understanding mathematics.

1.1.1 Language in the Mathematics Classroom

Language use in the mathematics classroom has been a concern for me for over ten years now; it is a concern I have felt both as a teacher and as an administrator – a manager of teacher performance. I am a teacher who enjoys engaging students in conversation about mathematics, I like to clarify, expand and learn from the give and take of a productive, focused conversation. After reading Rowland (2000) I was impressed with the concept of 'approximation' or 'hedges' that students would use in the mathematics classroom, for some students a high-risk environment. Hedges or approximations provided students with a means to minimise those risks using vague language. Through engaging in 'contingent', or responsive, questioning a teacher could manoeuvre their way through those vague utterances and establish shared understandings. This was something I felt I was attempting to do and which I believed benefited students.

About six years ago, at a previous school to the current research setting, I questioned my group of year 11 students about their experiences with learning mathematics. Students entered the school at the beginning of year 11 and came from a range of different schools across both government and private systems. Most students had rather negative experiences to tell of how they never had 'proper maths' teachers and how they were 'bad at maths', implying they were still 'bad at maths' and I should not expect them to perform well. I was not looking for the horror stories but more the experiences remembered by students who had a positive story to tell about how their teacher had helped them learn mathematics. I was hoping to hear that their teachers engaged them in conversations; I didn't so much hear that; however, what I did hear was that those who felt their experiences were positive remembered teachers telling them stories that helped them understand their mathematics. They remembered the stories and they could relate them to me.

I realised that this was an important phenomenon to stumble on; however, I still did not know how I could observe other teachers' performances and examine them for examples of storytelling and conversations. I wanted to know that if this was common in mathematics classes, whether it made a difference and what that difference might look like. I spent considerable time unsuccessfully working out how I might achieve this until I discovered Participatory Action Research (Denzin & Lincoln, 2008; Kemmis, 2001) while searching for research design models. It was at this point that I began to believe my best approach would be through examining my own journey and how my teaching practices might change, given that I was the best person to examine my own performance. This is the point at which my journey began.

1.1.2 Classroom Conversations

My observation and reflection on my classes has led me to the belief that the conversations engaged in by teacher and students are crucial to the learning of mathematics. Learning is a social experience for most students. Learning does not happen in isolation; rather it comes about as a process that involves discussion, clarification, amplification and understanding. This all happens through the conversations that are developed. It is only through conversation, clarification and amplification that both the learner and the teacher can come to know that each has a similar understanding. This is an intellectual activity and one that is reliant on developing language skills.

I have also been concerned, for a corresponding number of years, with the significance of mathematical language and communication using that language to develop and enhance understanding of mathematical concepts. Students frequently misuse mathematical language and consequently have developed misconceptions through that misuse. For example, I have recently seen students coming into year 8 using the words '*x two*' to represent 'x^2' and subsequently, when they encountered expressions in the form of 3^2 they multiplied 3 by 2. Correction of the expression to '3 to the power of 2' has seen them more likely to interpret problems involving indices correctly. This raises an interesting dilemma: is the problem one where these students have not developed the language, or rather the concepts or both? Further, should my teaching focus first on how to read these terms, or how to understand them or both?

As a teacher I believe that students should be taught how to read the text, engage in use of the language at the same time that understanding of the concept is developed. I believe there is a strong correlation between language and concept development. This relates again to Vygotsky (1987) and his theories on the development of thinking and language and whether higher order thinking is dependent on the acquisition of language skills. I was confident that I could gain an insight into this perplexing issue with my students as I pursued my Action Research with extended learning conversations.

1.1.3 Language and Thinking

According to Hoffman (2009), being engaged in mathematical activity is an intellectual activity. Whether using the term cognitive functioning, intellectual activity or higher order thinking skills, the basic premise is the same. All three are predicated on the understanding that mathematical language is used to communicate thinking. Language is required for communication of intellectual activity and cognitive reasoning. Vygotsky (1987) demonstrated the link between language and thinking. He promoted the view that the most significant moment in intellectual development occurs when language and practical activity converge. Prior to this point, these two were viewed as completely independent lines of development. Language connects the constructed meaning with the interpersonal world. Intellectual activity (thinking) occurs through the use of language. Language is the tool for thought. Hence, if mathematical activity is seen as an intellectual activity, acquisition of mathematical language is required to communicate and engage in reflection and metacognition.

Much of mathematics teaching and learning has been based on practical tasks and developmental work that has seen students ‘doing’ mathematics. In the doing of mathematics; however, we sometimes have lost sight of the need for students to have time for intellectual activity, for cognitive reasoning and reflection (metacognition). Practical mathematics may be happening at the expense of investing in developing students’ mathematical understanding and language. Mathematics should be practical; however, it often focuses on the technical aspects without allowing sufficient time for students to reflect. Many students engage in mathematics without deeper understanding (Schoenfeld, 1994).

1.1.4 Language and Concept Development

Another example of use of language and the ‘suggestion’ of a lack of concept development was at a recent athletics carnival where one of my students was measuring the distance of the discus throws. She reported back to the recorder that the distance measured was ‘16 metres seven’. The supervising teacher repeated that as ‘16.7 metres’; however, the recorder turned to me and said that it was ‘16 metres and seven centimetres, 16.07, as the student had previously called out 16 metres 70 for another throw’. The recorder, a parent whose husband owned a local furniture production business then went on to tell me that the first thing her husband had to sort out with new young employees was how to measure and record the measurement. This example raises again the question: does mathematical language and its communication obstruct the development of mathematical concepts, or is its misuse a sign of a lack of understanding?

Quite by accident I had the opportunity to scrutinise a measuring tape used by ‘home builders’. I am informed that the measuring standard in the building industry is the millimetre, which I understand. When I read the tape, I understood how it is structured; however, it would be quite confusing to most people unfamiliar with that system. This re-emphasises my original premise: it is through conversation with clarification and amplification that a shared understanding is reached. This example also provided me with an opportunity to have students read an assortment of measuring tapes and to have that extended learning conversation about the building standard and how we might interpret measuring tapes that are presented in different formats. The process conformed to the actions of the owner of the furniture production business who clarified the context and format with his new employees through essentially what is a learning conversation.

I recognised another example during the teaching of rotation, reflection and translation of objects. In a conversation with students about an object that had been rotated I asked the question ‘what is it (the shape) rotated about?’ After some blank looks and some strange guesses I realised that students were focused on the word ‘about’ which we usually associated with approximation. For me this meant that I needed to watch my use of the word ‘about’ and that I needed to clearly discuss its use in rotation.

Oral language and communication will continue to have these inherent difficulties when the purpose and meaning of the sender does not equate with the understanding and meaning of the receiver. Clarification between sender and receiver is one outcome of having an extended learning conversation. In this study I brought my experience working with students as peer leaders in an effort to develop relationship and communication skills, and to overcome miscommunication by seeking clarification through shared conversations.

1.1.5 The Learning Conversation

So, like an orchestra, a learning conversation must include the component parts that will bring strength and diversity to the conversation, the purpose of which is to facilitate learning and deeper understanding. There must also be a vehicle for the purpose of mediating cognitive processes, which also has to be part of the reason for having a learning conversation. The director of this learning conversation operetta is the teacher in the first instance – who must know when to bring in each of the contributing sections; when to encourage students, and to empower those who may not be able to find their voice. The teacher as facilitator or director must also be able to introduce risk-taking into the process as, without risk, a teacher jeopardises moving students further ahead in their learning journey or process.

At this point my definition of a learning conversation includes a purposeful, though informal exchange including both verbal and non-verbal exchanges. The purpose of the learning conversation, of the directed exchanges, is for students to develop their cognitive processes and then to mediate those cognitive processes with the language developed in learning conversations. The learning conversation is also a time to practice those new found language skills in order to become a cyclical process. As a conversation is about communication, the non-verbal components would include gestures, body language and tone.

I am proposing that when a teacher goes into her/his classroom that she/he needs to be aware of the size and strength of the orchestra existing there. A conversation should not be seen as just a filler or transition from one concept to another. It is possibly the most vital aspect of the learning process when accompanied and set in context with the other apparatus available for the learning experiences of students of all ages.

Hence, a conversation – that is, a learning conversation – is the vehicle. Language is the component parts of the vehicle, specifically the engine, and the teacher is the driver. In the metaphor of an orchestra, the conversation is the overall production and the language is the musical notes which when put together form the powerful total of a performance. All the different sections contribute their own parts, sometimes large parts, sometimes seemingly insignificant but powerful parts that are heard as a single rendition which resonates with all who hear the message, for a musical piece does impart a message. Just as a metaphor has been used here, metaphors are very often used in the classroom. The metaphor, or story used by teachers in their classrooms reflects the orchestra coming together to resonate with all who hear it, all who have contributed, no matter the size of their part. The metaphor becomes a part of the learning conversation that teachers have with their students, enabling them to progress on their learning journeys.

What is important is to be part of the conversation, whether as a significant overt contributor, or as a quieter, reflective partner in the process. All who are part of the process are subject to the language, are subject to the mediation of their cognitive processes and are hence provided with opportunities for developing deeper understanding. There is a danger ; however, that, even though students are part of the conversation, they are not participants but mere observers, not influenced by the conversations and therefore not benefiting from the mediating processes of the language in use. The difficulty for all teachers is to ensure that there are no passengers, no mere observers among students using language through learning conversations to mediate cognitive processes in order to bring about true and deeper understanding.

Learning Conversation - Definition

I needed a definition for learning conversation. Why a definition? Most (Merriam-Webster, 2012; Oxford Dictionaries, 2012) would agree that a definition of conversation would include the phrase ‘verbal exchange between two or more people, with the focus on the verbal exchange itself. A conversation is, by its very nature, free flowing, with the exchange of ideas and thoughts dictating the pace and perhaps its direction. Here, if I refine the definition I am choosing of learning conversations, I would propose that a learning conversation be explained as being like a conversation, but as a more directed activity with a purpose, so that although it is informal, the conversation can be facilitated around a central purpose. With the introduction of the importance of gestures into language learning I believe it is also appropriate at this point to consider adapting the use of gestures into a definition of a learning conversation.

Any definition of learning conversation should encompass the needs of learners, which implies that the concepts of scaffolding and modelling, in a sense, must be included in that definition. Without bringing those concepts on board, many students would not be able to enter the learning conversation.

Elaborated Discourse

Elaborated Discourse is one measure of how well students are engaging in discourse, or an elaborated learning conversation; it is also a means of analysis of that discourse. Having students working towards achieving the goal of elaboration means focusing on the language used in the classroom, bringing it to the forefront of practice, developing students' skills in conveying their thoughts through oral language, inserting formal language into the classroom learning conversation at appropriate points, and linking the formal with the informal language being used by them. Without having the goal of elaborated discourse, discussions might lose the focus of language development and might remain at a very basic level of responses from students.

A Hybrid Approach – Possible Solution to the Problem

I have understood and used a constructivist approach to my teaching for approximately 25 years along with elements of a more traditional approach. Events and life have shaped the methodologies and strategies employed in my classrooms. This has led to a somewhat hybrid epistemology where strategies incorporating elements of a Constructivist approach are melded with some explicit, 'chalk and talk' teaching. I believed that the use of this hybrid approach with an emphasis on classroom strategies focused through language might produce elaborated discourse during learning conversations hence revealing students' cognition and metacognition and could lead to enhanced mathematical understanding.

1.2 Focus of the Study

The purpose of this research study was to place an emphasis on language and elaborated discourse and its use in improving understanding of mathematical concepts. The focus of the study is on:

- Investigating explicit teaching, elaboration and development of mathematical language;
- Developing extended learning conversations in mathematics classrooms; and enhancing cognitive reasoning and intellectual

activity through the use of classroom strategies focused through language.

The study proposes that the process of learning mathematics is far more effective, if it involves an extended learning conversation among teacher and learners.

1.2.1 Research Questions

The research questions are:

1. What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)?
2. What is the role of language in the application of those strategies to engage students in extended learning conversations?
3. What benefits are created and challenges encountered when those strategies are employed in the mathematics classroom?

1.2.2 Significance of the Study

The study is significant in that if a model set of strategies can be developed for teachers to adapt for use in their own classrooms, then the vision that the National Council of Teachers of Mathematics (1989, 1991) had for learning mathematics might be achieved more realistically. It could actually be implemented. Past research in the area has been highly theoretical in focus, so a research study that produces usable and adaptable strategies might lead to a greater uptake of classroom discourse. The focus of the study is pragmatic and relevant to practitioners.

The study is also significant in that cognition might also be enhanced through a broader application of classroom strategies focused through language and not just communication skills. This would suggest that a greater emphasis placed on *Mathematical Linguistic Pedagogy* could potentially enhance students' mathematical understanding and functioning.

1.2.3 Research Methodology

A Participatory Action Research design was chosen for the research methodology as I was involved with my class as a participant. This necessitated challenges for meeting the requirements of authenticity and trustworthiness (Denzin & Lincoln, 2008) to provide credibility for this research study. The use of a Grounded Theory approach (Glaser & Strauss, 2009; Strauss & Corbin, 1990) evolved during the course of the study and, through the use of this data analysis approach, much valuable information was yielded. The research design and methods for data analysis are discussed in detail in Chapter 3.

1.3 The Broader Context

Organisations like the National Council of Teachers of Mathematics (NCTM: 1991) have for many years been encouraging the use of discourse in mathematics classrooms to improve understanding, and they have focused on professional standards and developing teachers' skills in implementing and managing real classroom discourse. The NCTM has had classroom discourse at the forefront of their platform for at least the last 20 years. In that time education has become more politicised all around the world with education systems implementing standardised testing, with broad comparisons of literacy and numeracy levels, and with more and more accountability being sought for student achievement, or lack of achievement. Change is a constant factor, as is public scrutiny, which contribute to a challenging environment in which to operate.

Education in Australia, particularly in Western Australia, continues to operate in a politicised environment of nationalised, standardised testing amid calls for improvements in literacy and numeracy. A national curriculum in its early stages of implementation adds to the agitation. The state government in response has an emphasis on school leadership, developing instructional leadership and improving the quality of teaching. In this environment there is an inherent danger that the system might look to superficial quick-fix solutions as funding and a host of other benefits become attached to those high stakes one-off testing performances. School leadership is under increasing pressure to demonstrate improvement in educational outcomes as measured by standardised testing often with reduced resources. That is the current state of play and, as anyone who has been in teaching for more than a decade knows, change is always just a year or two away.

When I commenced teaching over 30 years ago, in the late 1970s, curriculum was centrally dictated and there was standardised normative based testing to determine the levels and grades a teacher could allocate in each year group. That system was replaced, in the early 1980s, by one that had some flexibility still; however, with a centrally determined curriculum and without normative based standardised testing. Then, in 1987, Unit Curriculum was introduced, another structured centrally based curriculum. Following this, in the early 1990s came what might be considered as a free-for-all curriculum with little direction, followed by an outcomes based system, in the late 1990s, centrally facilitated with students progressing through levels. This was replaced again by a centrally dictated curriculum, in 2009, first through state-based mechanisms and then through the implementation of the national curriculum with some uptake in 2011/2012, with nationalised standardised testing allowing comparisons of performance across schools and states. Interestingly, national testing commenced several years before implementation of the national curriculum; a curriculum that has to be broad enough to encompass all state requirements.

1.3.1 The Changing Education Platform

When I attended the United Kingdom's annual Association of Teachers of Mathematics conference in Loughborough in 2004 I was told by several attendees, when asked where I was from, that Australia was following suit with what the United Kingdom had done but was no longer doing. At that point the UK had commenced with standardised testing across all Key Stages. Five years later when I attended the same annual conference I attended a session where it was outlined that standardised testing was not going to occur for all Key Stages (Pope, 2009). According to the UK Department for Education (2010) 26% (that is, over 4,000 schools) did not administer the tests in 2010, and Science was not included in the National

Curriculum tests in 2010. Instead, a five per cent sample of schools took Science sampling tests to estimate national attainment in the subject. Like Australia, the UK is undergoing significant reform; however, it appears to be shifting in a different direction.

The point I am making here is that education platforms (that is, the environment in which students learn and teachers work) is constantly undergoing change, reform or whatever term is used to describe intervention by outside forces to fix the problems with students' perceived lack of achievement and performance in the education system. Whilst it would be convenient to be able to ignore this flux process it is impossible to not consider the impact that affects those of us undertaking research in learning or teaching.

At first I believed that the research I was undertaking was not dependent on which direction curriculum change was heading. It was not going to make a big difference whether we were curriculum directed or not. Although having said that, there is a context where there would have been an impact on my research study, and that is where the curriculum is heavily defined and where there is little room to manoeuvre. Fortunately, in Western Australia, that was not the case during my research study. This afforded me freedom to change my teaching approaches and experiment with novel pedagogies.

1.3.2 Evidence Based Approach

There is strong pressure on principals to accept and use an evidence based push to demonstrate improvement. Teachers are under pressure to focus on preparation for national testing and considerable teaching/learning time is required to be devoted to this preparation. For some, including me, this causes considerable conflict because, as an administrator, I understand the need to demonstrate improvement of ‘results’. As a teacher-researcher I also understand how my students learn and the benefit of providing learning experiences which allow time for collaborative group-work, for reflection and for metacognition.

Students not only need to learn the concepts and procedures of mathematics but must also learn to use such ideas to solve non-routine problems and to learn to mathematise in a variety of situations (Romberg, 2001). Learning to mathematise occurs as a consequence of building on prior knowledge and in conversations with other students and teachers as well as engagement in purposeful activities. The ability to mathematise is seen as a function of being mathematically literate. In our current environment the function of being mathematically literate is measured by performance on nationally based testing, National Assessment Program – Literacy and Numeracy (NAPLAN). As mentioned previously, this creates a dilemma as an administrator, teacher and researcher.

There is overt pressure on school administrators from parents, politicians and school systems for their schools to demonstrate improvement, and to show value adding with students. The current performance measure is the NAPLAN testing. Performances of similar or 'like' schools are compared and this information is made public, hence there is pressure on teachers to adhere to a strict regime of test preparation. There-in, lays the dilemma. As a school administrator I understand the importance of my students performing well on national tests and as a teacher-researcher I also understand the value of implementing strategies that would improve my students' mathematical understanding hence improving their performance on those tests. I managed this dilemma through focusing on the latter – developing and implementing classroom strategies focused through language for use in the mathematics classroom.

1.3.3 The School Context

District high schools in Western Australia are special educational institutions where opportunities arise in solving the issues of geographical isolation, small numbers of students in years 8, 9 and 10, lack of specialist teachers, and in many cases problems associated with generational poverty. Structuring of classes is an area where an opportunity exists for multi-age grouping where effective teaching and learning can take place provided that administrators and teachers manage the classroom environment effectively.

At the time of commencing this research study I was in a situation where the district high school I was in had 32 students in years 8, 9 and 10, with 16 of those students in year 10, nine in year 9 and seven in year 8. The students had been grouped according to ability and to their academic preference. Those students who had a strong 'hands on' learning style preference were grouped together and provided with some extra practical learning opportunities. The other students were grouped together in a more academic stream. The mathematics class that was the focus of this proposed study had three year 8, six year 9 and seven year 10 students at the commencement of the research study. Not only were they multi-aged, they were also of mixed ability, even though the class had an academic focus.

As an effective teacher I needed to find ways to teach and to have students learn according to their ability, knowledge and more importantly stage of development. Some concepts I could cover across all age groups, others I had to teach to small groups. Within the group of 16 students and across three year groups I had three small groups; the three year 8 students made up the first group, the six year 9 students and two of the year 10 students made up the second group, and lastly five year ten students made up the third group. I often used collaborative group-work to assist students in the learning process; the groups were sometimes self-selected and sometimes arranged by me. Most students worked cooperatively, with some arrangements more successful than others.

Teaching in the Middle School

Like me, teachers in the middle schooling years have searched for ways in which to make mathematics more practical and ways to make mathematics learning more meaningful (Vygotsky, 1987; Sherrin, Louis & Mendez, 2000; Grouws, 1992). Vygotsky (1987) highlighted the convergence of language use (speech) and practical activity, with the practical activity providing meaning on an intrapersonal level and language (speech) making the connection on an interpersonal level. Without this understanding in the classroom, students are often turned off mathematics, reluctant to engage in problem solving activities and view themselves as 'not good' at mathematics.

Switching them back on to learning mathematics is difficult, as is finding an approach to counter negative attitudes and lack of mathematical understanding.

Changes in mathematics learning must continue to occur in the middle schooling years; however, change must also happen in the early childhood years where fundamental attitudes and understandings are developing. In Western Australia, over the last two decades, considerable resourcing of programs like *First Steps – Reading, Writing, Oral Language, Mathematics (1993)*and *Getting it Right* – Mathematics (McDonald, 2009; National Literacy Trust, 2009; Tasmania Education Department, 2006) has gone into the early years to improve literacy; however, few gains have been made in sustained, improved mathematical understanding. NAPLAN results for Western Australian schools and the Department of Education's (DOE) response to those results supports a public perception that improvements in both literacy and numeracy are required in Western Australia.

Collaborative Group-work Strategies to Improve Learning

The use of these strategies is currently being employed in many government system schools in Western Australia. The focus for the employment of these strategies is the improvement of literacy and to some extent numeracy as national testing (NAPLAN, 2009) has demonstrated that WA lags behind the other Australian states in overall statistics.

One reason I examined the use of group-work strategies is because I wished to employ those same strategies in working with students on improving their deeper understanding of mathematics. I am proposing that the employment of strategies that empower students, that enable them to share their understandings and learning with each other, can be a powerful tool in improving understanding. This is in contrast to the use of group-work strategies designed just to expand content coverage – a reason some teachers and researchers (Gillies, 2007; Kohn, 1991a, 1991b; Slavin, 1991a, 1991b) have used to justify the use of these strategies.

A second reason why I am proposing the use of group-work strategies is that it leads naturally towards the outcome of students learning from each other (Kutnick, Ota & Berdondini, 2008). My goal is to have students learning from each other on a regular basis. This can be achieved by setting up strategies for group-work, embedding them and employing them for students to work together to teach another group of students. I am suggesting that this process then will see students working together and helping each other as an effective way of learning.

In smaller schools where students are grouped in classes with a range of ages and abilities, they can only benefit from employing these kinds of strategies – something our primary school colleagues have been encouraging their colleagues to use for some years. So where does one start with group-work strategies with a group of students who have not experienced this way of learning and doing mathematics?

To believe that students may not have experienced these kinds of strategies would be a fallacy, as students would have been exposed to group-work in the kindergarten. Back in that class, students would have shared and worked together as that is the way with younger children, and so all students should be familiar with the means of working together to achieve shared goals though they are probably not aware of the jargon, or the names of strategies being employed. However, they would gain the experience of working together and find out the benefits from such strategies.

Over the years many strategies have been introduced to improve learning for the less able students with an unexpected outcome being that higher achieving, more able students have also benefited. Group-work strategies are no different. Employing these strategies works for all students as those higher ability, higher achieving students are able to articulate their understanding which in turn mediates their thinking (as suggested by Vygotsky), and refines and hones their understanding, hence providing them with opportunities for achieving deeper and longer lasting understanding and improved cognitive functioning. Few would argue with employing group-work strategies if their benefit is available to all students – it would seem illogical. I am advocating the employment of group-work strategies to assist with mediated cognitive processes; hence deeper understanding and greater 'making meaning' for all students.

1.3.4 The Researcher

I have a strong belief in a Constructivist approach to learning, teaching and philosophy (Wertsch, 1985; Ernest 1990; von Glasersfeld, 1995; Prawat & Floden, 1994) and have attempted to use my beliefs and understandings to improve the mathematical learning and understanding of my students. Previously I worked in another district high school for eight years, followed by eight years working with senior students before commencing at my current school. Together with my strong Constructivist beliefs I have also had an interest in language development and the link with learning (Vygotsky, 1987; Brown, 2001; Chapman, 1993; Ferrari, 2004).

I first took an interest in the conversations that occur in the mathematics classroom over ten years ago when I was presented with the problem of students seeking my assistance with their teacher whom they could not engage other than to go over the algorithms required to solve problems. My interest was not in the social conversations that occur in the classroom, but more in the learning conversations. The reluctance of a teacher to engage in those extended learning conversations was the impetus for my journey into engaging in research surrounding talk in mathematics classrooms and the link with language and learning. This coupled with an interest in making mathematics in the classroom more realistic and practical demanding engagement from students has been a long term goal of mine. At that time I was influenced by those undertaking research in the area of conversations in the classroom like Corwin and Storeygard (1995), Corwin, Storeygard and Price (1995), Roehler and Cantlon (1997), Lampert and Blunk (1998), Rittenhouse (1998), and Sfard and Kieran (1999).

My first recollection was working on a linguistic based assignment as part of my undergraduate degree over thirty years ago: Noam Chomsky's writings (1972) intrigued me in terms of child related language development. Through the raising of five children and seeing first-hand the value of a language rich environment on their intellectual development raised concerns for me regarding the power and impact of language on learning, in particular on mathematics learning.

1.4 Factors Influencing the Research Direction

Many factors influence the direction of a research study: personal, professional and those that arise out of the context in which the researcher operates and the role that they undertake within that context or environment. Sometimes the influencing factors may appear diametrically opposed and it is within the process of bringing those divergent views together that new understandings become evident. The impact of social learning (Stephan, Cobb, Gravemeijer, 2003) coupled with the use of modelling and scaffolding in the contemporary classroom provides some intriguing observations for a researcher interested in how language is used to enhance mathematical understanding and functioning. One might surmise that social learning provides the basis or platform for modelling and scaffolding and for later higher level learning and understanding and, if for nothing else, provides value for this effect alone.

For the practitioner, the teacher in the classroom, the important fact is not which theory of language acquisition, or learning is responsible for the development of understanding in her or his students, but what processes, strategies or approaches make the difference. It matters not to a practitioner whether they cross the boundaries of Constructivism or explicit teaching. Making a difference to their students is what drives the teacher in the classroom.

As a 'Stepping Out' Trainer (Education Department of Western Australia, 2004) I am only too aware of the position and value of modelling in the teaching and learning process. Imitation learning as a component of Social

Learning (Bandura, 1969), provides us with a theoretical understanding for modelling which can be used to explain learning in a social situation, classroom or any learning experience where there is more than one; that is, where one is learning from at least one other human. Learning is a social experience (Vygotsky, 1987).

The terms modelling and scaffolding are used to explain the process of ‘doing’ with students and walking students through processes, skills and understandings in a way that supports, and then empowers them to do it for themselves. In the classroom, teachers have seen the value of modelling from employing it and seeing the improvement in students' understanding. In the first instance, as students are learning something new, the modelling aspect provides a safe environment for students to ‘have a go’. In the second instance the scaffolded approach provides an opportunity for students to develop confidence in preparation for deeper understanding.

1.4.1 Cognitive Mediation

Following on from, or alongside, modelling and scaffolding, deeper understanding is heightened with the learning conversation that will occur in the classroom. Vygotsky (1987) proposed that thinking, where deeper understanding occurs, is first platformed with language and then mediated through the use of language. If there is not a good fit between the quality of the modelling, scaffolding and the learning conversation, then there is the possibility that the mediation of thought will not be optimal. This of course is all embedded in the understanding that students' processing of thought and the uptake of new learning is all predicated on their prior knowledge, hence bringing into play Vygotsky's Zone of Proximal Development (1987).

The classroom teacher, therefore, should be aware of the place of modelling, scaffolding, learning conversations and position this within reach of students' prior experiences, knowledge and understanding. To position new learning in the context of students' experiences provides a prime opportunity for enhanced understanding. Making students aware of how they learn, based on their previous experiences also enhances how they take on board new learning, new knowledge. The bottom line still is that the learning is facilitated through the use of learning conversations in the mathematics classroom. The next stage for me will be to explore further the employing of the learning conversation and the link to cooperative group work.

1.4.2 Contributing Factors

I used the following areas to develop a series of learning and teaching strategies where students learn the language, learn the concept and the skills to then have mathematical conversations with their peers:

- The link between language and thinking (Vygotsky, 1987). I see a need to further investigate the link between the development of language and thinking. If the least that exists is a correlation between the two, then that is a significant contributor to my research study.
- Social Learning, modelling, embedding and language development (Bandura, 1969; Bernstein, 1971, 1973, 1975; Vygotsky, 1978). Research in the area of Social Learning, Sociocultural Theory provides a research basis for the use of modelling in classrooms.
- Metacognition and the BACEIS model (Hartman, 2001) and the use of scaffolding as a bridging tool. The BACEIS acronym stands for Behaviour, Affect, Cognition, Environment, Interacting and Systems. Metacognition, especially linked with reflection is a significant contributor in extended learning conversations. Reflective thinking is the essence of Metacognition (Hartman, 2001). To have a conversation with little or no reflection limits the value of having the conversation.
- Cooperative group learning (Gillies, 2007; Joliffe, 2007). A good context to have these extended learning conversations is through the use of cooperative group learning strategies.
- Use of language for communication (Queensland DET, 2004; Steinbring & Bussi, 1998). Substantive conversations have been a focus of learning and promoted as an effective tool for learning and for developing communication skills.

- Use of language for learning (Lampert & Blunk, 1998; Wertsch, 1985; Rowland, 2000). Some attention must be given to the use of language in the learning process; reading the written text, engaging in extended learning conversations along with developing conceptual understanding.
- Pedagogical understanding that learning is enhanced when a person teaches another. A different level of understanding is required to be able to confidently teach another person. As a beginning teacher I gained a greater understanding of many basic concepts when I first started to teach them to students.

1.5 Overview of the Chapters of This Study

In this first chapter the research study was introduced along with its background and contextual basis. Constructivist approaches to teaching supported with elements of explicit teaching morphing into a hybrid methodology.

In Chapter 2 the literature review revolves around the history of classroom discourse, with a particular focus on the NCTM's *Professional Standards for Teaching* (1991). The chapter examines past research, in order to put more recent research into perspective and then to highlight opportunities for further research. Literature around *Mathematical Linguistic Pedagogy* and discourse analysis, the teaching of ESL and the links to mathematics teaching is examined and the influence of Halliday, Bernstein and Vygotsky is examined in the light of learning the language of mathematics.

Chapter 3 details the methodology employed in the study, particularly the Participatory Action Research design and the use of a Grounded Theory approach for analysing the data. The chapter also considers the participants and the context in which they functioned.

Chapter 4 examines the data related to the set of strategies that I introduced to enhance learning, including the language focus of each of the strategies and the role that each of them played in the learning experience. The chapter also includes detailed descriptions of the strategies and provides examples of how they were used. Chapter 5 examines the explicit teaching strategies referred to as *Blended Instruction*, a set of instructional strategies that combine elements of discovery, inquiry-based learning with elements of explicit instruction. The chapter also includes detailed descriptions of the *Blended Instruction* strategies and examples of how they were used. Here, in both Chapter 4 and 5 the research questions are first answered.

Chapter 6 provides an overview of the research study, reflects on the research questions, outlines a set of recommendations, examines the way forward and looks at what opportunities there are for further research in this area.

Chapter 2

Literature Review

Language usage, classroom discourse and school success is not a new area for examination; there is at least fifty years' worth of published research into classroom discourse, conversation and language, language and sociocultural background and classroom talk in an effort to improve mathematics learning; however, questions still remain. What impact does classroom language usage, classroom discourse or in basic terms classroom talk have on students' understanding of mathematics and can specific classroom strategies focused through language assist in improving students' mathematical understanding and performance? With those questions in mind this chapter examines literature specifically focused on classroom discourse, predominantly that which has focused on spoken language usage in the mathematics classroom, placing past research into an historical perspective in order to then situate more recent research in a much broader landscape.

Much of that research has been theoretical, not easily engaged with by practitioners and lacking a real focus on language usage for teaching purposes. It is in this area of language use for the purpose of enhancing thought and language, as first proposed by Vygotsky (1978), that this study was designed to provide a new direction for research and also to provide practical pedagogical links with the more theoretical approaches. The study is thus highlighting gaps in the current literature base and has attempted to fill those gaps, emphasising its significance.

The chapter continues with an examination of research into second language learning, using the parallels with learning and using mathematical language, encompassing an examination of the work of Halliday and Bernstein, and other work subsequently generated from studies carried out in the 1970s. Among the latter is the research of Vygotsky concerning the impact of social learning, and this is also examined. As Brown (2001, p2) said 'it is no coincidence that the study of language has become so prominent in our examination of the social world'. The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991) provided the impetus for many research studies related to classroom discourse, which in turn has expanded the breadth and depth of investigations into oral language in the classroom setting through student/teacher and student/student conversations, classroom talk, classroom discourse, elaborated discourse and a re-examination of language codes.

Section 1 below provides an exploration of past research, with a brief, broad snapshot of research focused on oral discourse. Here a definition of discourse is presented to clarify what is being examined as the focus of classroom discourse research. Section 2 focuses on the research generated by the NCTM's *Professional Standards for Teaching Mathematics* (1991). These standards continue to be a focus and continue to generate research studies. Section 3 places the focus on teachers and the difficulty they face when implementing discourse based learning. Section 4 places the focus on doing mathematics as an intellectual activity. Section 5 examines learning conversations and the role of non-verbal aspects of communication, modelling, introduces '*Purposeful Discussion*' as a honed component of a learning conversation and examines the role of language on cognitive mediation. Section 6 examines the link between language and the learning of mathematics. Section 7 introduces *Mathematical Linguistic Pedagogy*, provides a brief discussion of this as an opportunity for further research and leads into the focus on language in the following section. This section also deals with the role of pedagogical content knowledge, especially important in the development of *Mathematical Linguistic Pedagogy*. Section 8 discusses the role of language being at the core of all learning and the role of schools as change agents, examines the parallels with second language learning and discusses ways that research in this area can be transferred into the area of mathematics learning. Section 9 examines the influence Halliday has had on learning theories and language development. Section 10 discusses the social aspects of learning, with an examination of the contribution of Bernstein. Section 11 discusses elaborated discourse and discourse analysis and the chapter concludes with Section 12, providing a brief summary of the research framework.

2.1 Past Research

As a practising teacher I was well aware that there was considerable research being conducted with an emphasis on classroom discourse in mathematics classrooms. My first 'review' was with Corwin and Storeygard (1995) and with Corwin, Storeygard and Price (1995) which I came across in 2000 when I began to take a serious interest in examining research surrounding classroom talk. More recently I have examined articles and papers from research conducted far earlier on language usage, language background and school experience (Bernstein, 1971; Chomsky, 1972; Halliday & Hassan, 1976; Wells, 1981) citing research as early as the mid-1960s – fifty years ago, examining reasons for differences in school success based on language backgrounds and social class.

2.1.1 Snapshot of Past Research Focused on Discourse

There has been considerable research into mathematical communication; however, the aspect that I am looking for in terms of the language concerns improving mathematical understanding. My thinking has been influenced by Brown (2001) who focused on the way in which language and interpretation underpin the teaching and learning of mathematics; Marton, Runesson and Tsui (2004) who examined the role of language in learning; Ferrari (2004) whose focus was on language in mathematics learning; Chapman (1993) on understanding language practices in the mathematics classroom; Zevenbergen (2000) who suggested that Constructivist epistemologies have placed aspects of language central to the learning process; and by Roehler and Cantlon (1997) who nominated learning conversations as the prime vehicle for learning and who highlighted the role of modelling oral discourse strategies by teachers.

Walshaw and Anthony (2008) assessed the kinds of human infrastructure that promotes mathematical discourse in the classroom and that allows students to achieve desirable outcomes. Mathematical discourse involving explanation, argumentation, and defence of mathematical ideas becomes a defining feature of a quality classroom experience. Sherin, Louis, and Mendez (2000) discussed a project in which they began to work together to develop a middle school classroom where students talk about mathematics. Their goals were to have students respond to other students' comments rather than just state their own ideas and that students use one another's ideas as the basis for thinking and learning about mathematics.

There has also been considerable research on the social and cultural aspects and influences on discourse and learning. Chapman (1993) focused on social semiotics which views meaning as an active process, generated through social interaction. Edelsky, Smith and Wolfe (2002) considered classroom contexts, including beliefs and values that both permit and are constituted by discourse; Applebaum (1995) used youth culture as a context for engaging students in classroom discourse. Wertsch (1985) explained Vygotsky's position on the cultural development of behaviour and its impact on learning.

Past research (Cobb, 1998; Lampert & Blunk, 1998) has also focused on observing discussions between students and student teacher with the teacher acting as a facilitator. Lampert (1998, p 8) used the term 'revoicing' to describe a process employed by teachers to shift student understanding. I am proposing more than just facilitating discussion. I am proposing initiating that discussion with the teacher, myself, acting as the director of the discussion, following a plan and with the purpose in mind of 'handing off' the direction of the discussion to the students in the class once they have mastered the language and the skills to continue the discussion that has been modelled for them. This can be achieved by using the principles of Vygotsky's (1978) Zone of Proximal Development, providing support and scaffolding the discussions so that they become more in-depth and sophisticated with students joining the conversation when they feel confident and are ready.

2.1.2 Definition of Discourse

Over the last two decades there has been much published research surrounding the topic of mathematics' classroom discourse. Much of the research has focused on the operational aspects of the classroom discourse and the importance of having students write and speak about their mathematics (Elliott & Garnett, 2008). Many authors have also focused on the communication aspect of the oral and written components of discourse. The authors stated that the term 'classroom discourse' has a very comprehensive coverage and in its broadest sense represents all the spoken, written and other means of communication that happens in the classroom.

Elliott and Garnett (2008) quoted Gee's definition of 'Discourse' as socioculturally meaningful ways of speaking, listening, writing, reading, feeling, valuing, and believing, and so on; as contrasted with 'discourse' with a lowercase d which just stands for language in use. Christie (2002) supported classroom discourse that values knowledge, values the learner, and seeks to make available to learners as explicitly as possible, significant and useful information and ideas.

D'Ambrosio and Prevost (1995) stated that classroom discourse is often understood as a process of engaging the members of the classroom community – students and teachers – in talking with one another. The term 'classroom discourse' is used to mean the process of engaging the classroom community in real dialogue, where meaning is negotiated and assumptions are questioned. An underlying assumption throughout the discussion is that classroom discourse can help shape the views of the nature of mathematics held by students.

2.2 The NCTM Standards as a Starting Point

The National Council of Teachers of Mathematics (NCTM) *Professional Standards for Teaching Mathematics* (1991) outlined standards for teaching mathematics, evaluating the teaching of mathematics, professional development for mathematics teachers and support for those teachers. The document put forth six *Standards for Teaching Mathematics*, and of the six standards three are focused on discourse: T*eachers' Role in Discourse; Students' Role in Discourse;* and *Tools for Enhancing Discourse*. The remaining three standards focus on the tasks employed, the classroom environment and analysis of teaching and learning. It is the three standards focused on discourse that have prompted considerable research into mathematics classroom discourse. Much of the research focused on the link between language and thinking, and also the view that through effective discourse students can internalise and understand new concepts.

The teacher's role was clearly outlined in the Standards, namely to orchestrate discourse, and the role, elaborated, focuses on ways teachers can manage classroom discourse that contributes to students' understanding of mathematics which requires an environment where everyone's thinking is respected and in which reasoning and arguing about mathematical meanings is the norm, to provoke students' reasoning about mathematics through the tasks provided and the questions students ask or by asking them to explain. Discourse centred on mathematical reasoning is establishing by doing this consistently, cultivating a tone of interest when asking a student to explain or elaborate tasks that focus on thinking and reasoning which serves to provide the teacher with ongoing assessment information. Teachers must encourage and expect students to do the talking, modelling, and explaining; teachers must filter and direct students' explorations in order that student activity and talk does not become too diffuse and unfocused.

Writing and talking about their thinking clarifies students' ideas and gives the teacher valuable information from which to make instructional decisions. Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking (NCTM, 2000).

The Standards clearly articulated the beliefs, values and demands on teachers in creating classrooms that are discourse focused and what this looks like; however, the vision still remains highly theoretical and there remains a need to articulate strategies that teachers can adapt into their own practice to make the vision a reality. The Standards *(NCTM, 1989, 2000)* continued the focus on communication and the link with thinking in the mathematics classroom. For Middle School students there are 13 standards with *Standard 2: Mathematics as Communication* clearly articulating the expectation that language, reflection and thinking are deeply embedded in learning. The 14 standards for years 9 – 12 and again, *Standard 2: Mathematics as Communication,* supported the expectation of students being engaged in a curriculum focused around language usage, reflection and thinking to develop greater mathematical understanding.

While the NCTM's focus on discourse was clearly articulated, the focus in the *Australian Curriculum: Mathematics* is not overt; however, it is included. The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem-solving skills (ACARA, 2010). It is equally important that students be able to describe how they reached an answer or the difficulties they encountered while trying to solve a problem, and that continually encourages students to clarify, paraphrase, or elaborate is one means by which teachers can acknowledge the merit of students' ideas and the importance of their own language in explaining their thinking. Students build understanding when they describe their thinking mathematically and when they interpret mathematical information and are reasoning mathematically, when they explain their thinking, when they deduce and justify strategies used and conclusions reached (ACARA, 2010). In the United Kingdom there is a focus on communicating and reflecting. Students should be able to communicate findings effectively and engage in mathematical discussion of results (Department for Education, 2011). The preceding examples highlight the focus on discourse across many countries and its contribution to enhancing mathematical understanding and developing higher levels of cognition.

2.3 Sociocultural Theory and Classroom Discourse

Elliott and Garnett (2008) captured the essence, focus and direction of research in the last 20 – 30 years in their compilation of articles integrating communication strategies like reading, writing, listening and speaking into mathematics classrooms. Besides reading, writing, listening and speaking terms such as discourse, dialoguing, voice, literacy, and conversations were chosen, all suggesting that some form of communication was in operation. The authors presented a view of situated and sociocultural meaning making in mathematics classrooms; as mentioned earlier Gee (1992, 2008) provided a definition of 'Discourse' as socioculturally meaningful ways of speaking, listening, writing, reading, feeling, valuing and believing; as contrasted with 'discourse' with lowercase 'd' which Gee said just stands for language in use.

Ernest's (1998) 'conversation' metaphor was used by Elliott and Garnett to embody and support their philosophical underpinnings. They stated that having clarified the theoretical framework of situated socioculturalism, they looked for a metaphoric expression of the theory that also captured their philosophical beliefs. Hence they turned to the writings of Paul Ernest and were convinced by his argument for making conversation the epistemological unit for a Social Constructivist philosophy of mathematic.

Elliott and Garnett stated that classroom conversations are contextual, values influenced, and dynamic. Steele (2001) described a sociocultural approach to teaching as one where the teacher involves students in explaining their thought processes. She stated that communication is central to learning using this approach. It was this very fundamental tenet that I embraced as a part of the use of learning conversations. Further examination of Steele (2001, p404) supports the belief that

> *students create their own knowledge and develop mathematical meanings as they learn to explain and justify their thinking to others. The mathematical language comes from society, and thought (concept) comes from the individual.*

Steele (2001) would argue that we need to talk to understand the role that communication plays in helping children construct links between informal notions and the abstract language and symbolism of mathematics and would begin with Vygotsky's (1978) Zone of Proximal Development.

My hypothesis is that the explicit teaching of the relevant mathematical language will enhance student engagement in the learning of mathematics which leads to improved understanding. As an analogy, my writing of this thesis and my engagement in learned discourse about mathematics learning has required me to learn the language of the discipline and of the academic community. For me to have credibility I have to be able to engage in that language otherwise I am excluded from that community as I lack that credibility. For me, the language was developed through conversations with my supervisors in discussions surrounding my study. Through modelling and scaffolding I have come to a better understanding and command of the language of the discipline.

In exactly the same way, as a student pilot I experienced lack of understanding, lack of being able to engage in the language of the profession and an inability of intellectual processing due to the stressful and high anxiety situation. I believe I experienced learning anxiety similar to the mathematics anxiety experienced by my students when I introduce a new concept. Until I experienced this I did not understand how a lack of language impacted on the learning of relevant concepts.

Steele (1999) said the past decade had been a time for much discussion about the influence of social interaction on the development of mathematical understanding. This discussion continued in the following decade and will most likely continue. The roots of this discussion can be traced back to the ideas of Vygotsky (1978) mentioned above which will be further examined in

Chapter 3. Steele (1999, p 38) asked the question *'What do communication and interaction in the classroom have to do with Vygotsky's idea about learning mathematical language?'* and followed this with '*In what ways must new words be learned to enrich a child's understanding of mathematics?'* She asserted that Vygotsky believed that as children talk, they internalise the meanings of the words that they say.

It is through communicating ideas that language can be internalised. Visual images can be created by effective use of language in the mathematics classroom. As Steele (1999) purported, children learn new words by reflecting on them, picturing the meanings of the words in their minds as they interact. Through the expression of thoughts, children begin to reason for themselves. She cited Vygotsky's (1978) Zone of Proximal Development where children learning new words in the presence of a knowledgeable other person find themselves in a place for learning that is located somewhere between the child's current understanding and potential understanding. In this place a knowledgeable person can add meaning to what is familiar to the child when he or she enters the child's Zone. This conception of the Zone of Proximal Development suggests a teacher can assist a child by providing the child with new information to assimilate with present knowledge, thus adding to the child's knowledge base; taking the student from the familiar to the unfamiliar (Steele, 1999) and a learning conversation is perhaps the perfect vehicle where this can be achieved. This is another reason why I have chosen to enhance the use of learning conversations as a part of the learning strategies used in this study.

Rowland's 'hedging' or 'approximation' (2000), described a similar approach, interpreting discourse in mathematics classrooms from an informal , vague use of language and understanding of concepts to a more formal and exact use as participants reduce the level of risk-taking through conversation. This situates itself within the transformation from informal to formal language which takes place naturally in a learning conversation. Elliott and Garnett (2008) stated that teachers must jump into conversations, but only when they have command of the role of translators who can go between the discourse community of mathematicians and Gee's (2008) 'primary discourses' of students learning mathematics. Steele (1999) also asserted that children develop language through their experiences; developing, clarifying, and generalising meanings of words by learning the words as symbols of experienced concepts, using the words, and having people around them react to their word use. Mathematical language is built on generalising ideas through communication.

Steele (1999) also stated that children are often confused by mathematical language when unfamiliar language is given to them without their being involved in the experiences described by the language.

> *Children often have difficulty learning new words not because of the word sounds but because their own understandings of the concepts behind the words have not fully developed. Word meaning is a combination of thought, language, experience, and communication. As students progress in school, their ability to reason abstractly matures, as does their ability to communicate mathematically.*
>
> (Steele, 1999, p38)

She cited Straker (1993) to support the assertion that children need active, physical experiences with tools or objects as they are exposed to the concepts for which the language will be used. In the early years of learning mathematics, experiences and language must be closely connected; exploration in concrete settings is a powerful way to develop meanings of new words.

When students use language to describe their thinking, their teachers gain valuable information about what they understand (Steele, 1999). Steele cited Corwin, Storeygard and Price (1996) to support the belief that teachers should make use of students' oral language and classroom activities to provide a meaningful context for learning. Steele (1999) continued with stating that we should not move students too quickly towards new mathematical language without giving them the opportunity to explore, investigate, describe and explain ideas and that the reorganisation of concepts goes on during these opportunities. She also said that giving students mathematical language in meaningful contexts helps them more clearly communicate their thinking to others. When students use correct mathematical terms, the teacher can attend to what is being said and, therefore, to what is being understood or learned (Steele cited Duckworth, (1987) to support this statement).

Steele (1999) said the fundamental concern for teachers is how and when to introduce new words to students. Many teachers have students keep a glossary of words and definitions which they are required to learn as a part of learning and using the vocabulary of mathematics. As Steele said teachers should not introduce new words by requiring that students memorise definitions to pass vocabulary tests. Steele cited Vygotsky (1994) to support the statement that individuals come to learn the meanings of technical terms by transforming them and being transformed by them in the process of internalisation. For Vygotsky (1987), only in the ZPD does an individual internalise the meanings of society. Social interaction is a big part of the use of language in the mathematics classroom. Elliott and Garnett (2008) stated that Social Constructivist Paul Ernest (1998) gave us a ‘conversation’ metaphor to encapsulate what we believe about knowledge acquisition (epistemology), what we value (axiology), and what we perceive as reality (ontology). They said they are convinced by researchers who say that ‘all learning is through conversation’ citing Ernest (1998), Gergen (1985) and Steele (2001). Elliott and Garnett believed we must take social responsibility for all utterances and the actions and thoughts produced by those utterances. They believed that what is real and has meaning is what is co-created through language with others and cited Wittgenstein’s *Philosophical Investigations* (1953) to support this belief.

Ball (1991) said the *Professional Teaching Standards* called unprecedented attention to the ‘discourse’ of mathematics classrooms, as embodied in three standards: *Teacher’s Role in Discourse*, *Students’ Role in Discourse*, and *Tools for Enhancing Discourse*. She said that ‘discourse’ an unfamiliar term to many, is used to highlight ways in which knowledge is constructed and exchanged in classrooms and used the following questions to make the reader reflect on the process of classroom discourse:

> *Who talks? About what? In what ways? What do people write down and why? What questions are important? Whose ideas and ways of knowing are accepted and whose are not? What makes an answer right or an idea true? What kinds of evidence are encouraged or accepted?* (Ball, 1991, p44)

Ball (1991) said that the discourse of the classroom is formed by students and the teacher and the tools with which they work. Teachers play a crucial role in shaping the discourse of their classrooms through the signals they send about the knowledge and ways of thinking and knowing that are valued. Ball offered a practical example to highlight the ways a teacher might respond to a seemingly incorrect student response and that in differing ways of responding, different messages are sent about the usefulness and validity of the student's response and how this might impact on the student's view of mathematics. She then went on to infer that the interactions between teacher and student influence students' ways of knowing.

> *without explicit attention to the patterns of discourse in the classroom, long established norms of school are likely to dominate - competitiveness, an emphasis on right answers, the assumption that teachers have the answers, rejection of non-standard ways of working or thinking, patterns reflective of gender and class biases. For example, in many mathematics classrooms, answers have traditionally been right because the teacher says so or because the teacher and the student together decipher what 'they' (the textbook authors) want.* (Ball, 1991, p 44)

Ball continued and said that even with careful attention to patterns of classroom discourse traditional norms will underlie the interactions of students and teachers. She asked the reader to consider the way in which correct answers are treated in a mathematics class; given an example of students solving the problem 'what is two thirds of nine?' and a student gives the answer as six. She said the teacher reflex is to hear it as a correct answer and either to move on, praise the student and/or agree and repeat the answer for the benefit of the rest of the class. She said that even if teachers are disposed to ask students to explain their answers, that in the way that teachers pose probing questions they provide cues for students as to the ‘correctness’ of their responses; inadvertently teachers use different types of probing questions dependent upon the correctness of a student response. Ball also discussed the point that when teachers accept a student’s answer as correct, implying a student understands the problem or question, an opportunity is missed to gain an insight into students’ thinking. The corollary is that teachers do the same when hearing an incorrect response.

So what Ball was saying is that by a better examination of the discourse in the classroom teachers can tell whether a student understands the concepts that are being taught, and they can also be aware of the values that are transmitted either overtly or covertly when responding to correct or incorrect answers. She went on to say that the classroom environment or culture that the students and teachers construct affects the discourse in some important ways; the environment shapes how safe students feel, whether and how they respect one another and themselves, and the extent to which serious engagement in mathematical thinking is the norm. She questioned whether students’ voices and thinking are valued by the teacher and other students and asked what norms are established for the exchange of ideas, how disagreements are expressed and handled, how much risk is involved in being wrong and to what extent can every student participate and learn in the class. These are questions that any teacher who wants to involve their students in classroom discourse should be asking themselves.

The opportunity here for teachers is to create an environment in their classrooms where discussion of solutions, approaches, processes and/or ways of reasoning are used by students. Using a learning conversation in this way counters the fall-back position of responding to students in a very limited fashion as outlined earlier in the chapter – an alternative revealed in this study.

2.4 Focus on Teachers

Ball (1991) stated that working to become as skilful as possible in our classrooms requires us to learn to see more broadly and deeper. She said we must examine the language of our work with students, reflect on the direction and tone of class discussions and consider the time we allow students to explore and investigate; all these endeavours being critical in achieving the discourse that is fostered. Facilitating worthwhile learning seems very much a matter of orchestration; of eliciting and interweaving multiple voices, threads, themes, and timing. Forced examining of alternative perspectives on the intellectual and social classroom environment can enhance the virtuosity of teachers' work. She says this is no easy task. One of the benchmarks that Corwin, Storeygard and Price (1996) employed to change teacher behaviour was to have teachers reflect on their own mathematical thinking as a way to find their mathematical identities. They also said that looking closely at student's work and listening attentively to their ideas can change a teacher's sense of mathematics and how they teach it.

Ball said that perhaps the most valuable part of her experience working on the *Professional Teaching Standards* was what contributed to growth in her own teaching. She gained new ways of looking at what she was doing as a teacher, which made her think about how she cast the tone of discourse in her classroom. Having a safe classroom environment was the attitude she was encouraging. By articulating thinking and concerns to ourselves and to others can increase our professional skills. She said that by raising new questions and issues that shaped the ways in which we see and think about our classrooms we can enhance our orchestration of interaction in our classes in ways that can contribute to the kinds of learning outlined in the curriculum and evaluation standards for school mathematics from the NCTM in 1989.

Hoffman (2009) described mathematical activity as an intellectual activity and if you think of it as an intellectual activity then there is the necessity for a language to be able to engage in that intellectual activity because without it the intellectual activity is limited. Kazemi (1998) said that as mathematics teachers, we want students to understand mathematics, not just to recite facts and execute computational procedures. We also know that allowing students to explore and have fun with mathematics may not necessarily stimulate deep thinking and promote greater conceptual understanding. Tasks that are connected to students' lives still may not challenge students to build more sophisticated understanding of mathematics. The actions of the teacher play a crucial role. Kazemi highlighted a study that demonstrates what it means to 'press' students to think conceptually about mathematics (Kazemi & Stipek, 1997), that is, to require reasoning that justifies procedures rather than statements of the procedures themselves in a study that assessed the extent to which 23 elementary teachers supported learning and understanding during whole class and small group discussions. Like researchers in other studies Kazemi observed that when teachers help students build on their thinking, student achievement in problem-solving and conceptual understanding increased. Whether it is called 'press' or 'purposeful discussion' the focus is the same – involving students in a focused discussion, in a learning conversation, reflecting on the mathematics they are using.

2.4.1 Discourse in the Classroom Causes Difficulty for Teachers

Van Zoest and Enyart (1998) said that discourse is one area of the NCTM's *Professional Standards* (1991) that causes many teachers particular difficulty; mathematics teachers have a long history as lecturers. The authors cited Richards' (1991) initiation-reply-evaluation sequences between a teacher and students which are not uncommon, acknowledging Weiss (1994) to support their assertion that genuine mathematical conversations are rare in most classrooms. This is a concern echoed throughout the readings and studies I examined; however, I believe that it can be addressed through the strategies identified and developed in this study. Discourse can be a problem area for teachers when they do not realise how important it is and have not seen or experienced dynamic classroom discourse. Once a teacher has seen students defending their mathematical ideas, questioning other students' ideas and helping clarify the mathematics to one another, the importance of discourse becomes clear. Teachers are pragmatic and will take on board strategies that they see making a difference as opposed to theoretical rhetoric which does not resonate with them as it is too far removed from their world. As one of those pragmatic practitioners I am continually searching for ways to make a difference as demonstrated in this study.

Van Zoest and Enyart (1998) suggested that knowing that this kind of student interaction is possible can strongly motivate teachers to improve the discourse in their classrooms. Once a teacher is convinced of the need for change, the biggest obstacle is his or her own awareness of how to make meaningful discourse a reality in the classroom; however, explicit strategies are not developed. They also suggested that teachers can look at their own practice and move towards the goal of dynamic and productive mathematical discourse. Van Zoest and Enyart went on to say that improving discourse in the classroom is not something that can be done without considerable effort. We cannot expect all our students to come to class with the communication skills necessary for participating in class discussions or working with other students. We can; however, expect middle school students to feel very strongly about things; channelling energies into engagement in mathematical tasks and mathematical discussions offers opportunities for students to develop mathematical power. Teachers need to become efficient mathematical conversationalists, and strategies that lead teachers into developing this skill are required and this study can provide those strategies.

Schifter (1996) was cited by Van Zoest and Enyart highlighting changes that teachers made in the mathematics classrooms, hence demonstrating that it is possible to change the discursive practices in mathematics classrooms. Stein (2001) said a pre-requisite for effective classroom discourse is a good task that is rich enough to elicit student thinking and discussion; however, even with good tasks some teachers found the classroom discussions fell into a rut. Chapin and Eastman (1996) talked about constructing learning environments; developing a community of learners is more complex than simply encouraging discourse or asking students to complete mathematical tasks. Teachers' habits of mind and their beliefs and attitudes are fundamentally linked to their ability to implement recommendations like those outlined in the professional standards of teaching mathematics (NCTM 1991).

Manouchehri and Enderson (1999), like other authors, started off by saying that the NCTM's *Professional Standards* (1991) has directed attention to discourse in the mathematics classroom. They recommended that mathematics instruction promote students' discourse by orchestrating situations in which each individual's thinking is challenged and by asking students to clarify and justify ideas. They cited Ball (1991) to demonstrate that discourse as described by the standards document, highlights the way in which knowledge is constructed and exchanged in the classroom. Teaching mathematics from the perspective of developing mathematical discourse requires building a new vision for mathematics classroom and poses a major challenge for mathematics teachers at all levels. This challenge was recognised by D'Ambrosio (1995) who identified the need to build environments in which students could construct a personal relationship with mathematics as one of the most important requirements for promoting and sustaining the type of discourse that was envisioned by the reform movement. She said that in such environments, students engage in authentic mathematical enquiries; act like mathematicians, ask for ideas and concepts; and negotiate the meanings, connections and ideas with others in class. Manouchehri and Enderson stated that the most visible aspects of the described classroom is the nature of interactions of the students during whole group discussions, the language used by students in describing their discoveries, and the constructive way in which they build ideas on the basis of one another's arguments and explanations, and the social nature of their mathematical activity.

Manouchehri and Enderson further explained that the role the teacher plays is important; although the teacher seems to be overshadowed by student talk and student interactions, a careful examination of the episodes highlights their crucial role in the flow of discourse in the classroom. The most prevalent features of the teacher's teaching appear to be the questioning techniques employed, ongoing attempts at fostering students' reliance on their peers, and the calculated interventions in creating classroom discourse. The authors stated there are several major challenges associated with building environments in which users are actively engaged in self-directed learning. The paramount tasks are encouraging student participation and involving all students. A vital factor in developing classroom discourse is creating a social environment in which students listen to one another, respect one another and themselves, accept opposing views, and participate in a genuine give-and-take of ideas and perspectives. Moreover, it demands establishing a classroom culture in which collaboration and active learning are emphasised, valued, and celebrated by both the teacher and students. This approach requires altering students' perception of the role of the teacher, the teacher's expectations, and their own role as learners within the classroom. These perceptions, for the most part, constitute the social norms and cultural learning environment. The authors went on to say that certainly undertaking discourse about a mathematical issue entails students taking the initiative to explore relationships, forming arguments, challenging one another's arguments, making gestures, testing the validity of these conjectures, and elaborating and justifying their positions to peers. Discourse within a socio-mathematical setting requires an ongoing sharing of ideas on the part of the participants and their willingness to listen to one another and to negotiate mathematical ideas and meanings.

As stated earlier the sociocultural argument for creating an environment in which learning conversations, focused discussions is a very strong and compelling argument. Manouchehri and Enderson would argue that creating such a culture within the mathematics classroom also involves altering students' perception about the nature of mathematics and the purpose of mathematics learning. Students need to be convinced that mathematics is a subject that involves more than applying algorithms and just finding the right answer. They should be doing mathematics as more than finding answers to dichotomous right or wrong questions. Manouchehri and Enderson used the preceding perceptions of the mathematical culture of the classroom to add support to their argument. They also said that if teachers promote mathematics as a sense-making activity then the classroom culture should reflect this idea as an alternative for the quick fix, correct or incorrect answer. Encouraging conversation provides the opportunity for students to share their understandings and can be the beginning of viewing mathematics through a new lens where discussion is important, is at the forefront of classroom operation and demonstrated quite overtly that it is valued as a part of the learning process and not just offered to provide a ‘feel good’ environment where teachers can answer in the affirmative to the question ‘Do I use discussions in my classroom for the purpose of improving students’ understanding of mathematics?’. It is a purpose of this study to demonstrate how classroom strategies focused through language can empower learning conversations used in the classroom to enhance mathematical understanding.

Manouchehri and Enderson said that they have illustrated an example of classrooms in which productive mathematical discourse occurred. Their example highlighted the notion that creating a classroom environment conducive to promoting discourse requires consideration of attention to both the social and mathematical elements within the learning environment; these elements and the culture of the classroom have an impact on the nature and quality of discourse that takes place among students. They determine the extent to which engagement in mathematical enquiries is achieved.

They also said that although the discourse of the classrooms was for students, the crucial role of the teacher in the process should be emphasised because the teacher sets the climate of the class, creates an environment safe enough for students to explore and negotiate and helps students build and share knowledge. It is the teacher who designed situations in which productive discourse is created and sustained. It is also the teacher who helps constitute the norms of a classroom environment in which enquiries are celebrated and mathematics learning is perceived as a social activity that involves conjecturing, sharing, supporting, and the negotiation of meaning.

D'Ambrosio and Prevost (1995) discussed three ways in which classroom discourse can promote and nurture students' understanding of mathematics as a humanistic activity – that is, as an activity engaged in by people in the community. First, classroom discourse can serve to involve students in defining the curriculum. Second, classroom discourse can help students build a personal relationship with mathematics by engaging in authentic mathematical inquiry. Students, as they move through the middle years of schooling, become increasingly less motivated to participate in the mathematical activities in school and are more alienated from what is considered mathematics and school. This can happen in large part because the environment has not been created in which students build a 'personal relationship' with mathematics. Although many cases where such alienation could be cited, I have not taken the time to analyse the situation here; however, instead work with the basic assumption that for too many students, school mathematics is an uninteresting irrelevant subject. As mathematics teachers, most of us have established a love of mathematics and find it difficult to understand why so many of our students are struggling and resisting engagement with it. We make the following comments about our students – 'if they would only put more time into it, if only they would do their homework, if only they would not miss classes'. D'Ambrosio and Prevost (1995) said that such comments reflect beliefs that what we are doing in class is appropriate and student struggles are due to their own disinterest and lack of motivation; however, if we imagined taking the perspective of the students who have led us to form these beliefs we could come to understand their perspective only through creating classroom environments in which students' voices are heard and in which the students interests are explored and in which the students are called on to give direction to the classroom activities.

Developing strategies that encourage student participation in valued learning conversations is one way that might address the issues identified by D'Ambrosio and Prevost. There is more than one reason to have learning conversations in the mathematics classroom. The socio-cultural aspects of teenagers developing their own identities can be accommodated by using their stories as part of the learning conversations along with the need for the teacher and the system to show progress in learning and understanding mathematical concepts.

D'Ambrosio and Prevost stated that a view of mathematics as a disciplined form of engaging in intriguing questions is worthy of exploration but does not characterise the view of mathematics held by many students. Understanding the evolution of mathematical ideas, raising questions, and challenging what is accepted as standard mathematical knowledge are issues deserving attention in the school curriculum. Students need to understand mathematics as an ever changing and growing field. They need to understand the changes that occur within the field as new questions are asked, as assumptions are challenged, and as new conventions are accepted. The learning conversation provides an entry into this world for students who are often alienated and find mathematics as very abstract.

D'Ambrosio and Prevost talked about discourse occurring as learners' negotiate meanings and understandings; classroom discourse is at the heart of supporting students as they build a relationship with mathematics and construct an understanding of mathematics as a humanistic activity. The nature of the discourse is to promote these two levels of understanding. Mathematics needs to be an inquiry-based approach with the curriculum promoted by student inquiry. The direction of students' inquiry is enhanced by the teacher's contributions to the enquiry process. It is a shared and negotiated process. The authors said that the difficulties inherent in the enquiry-based environment are numerous. The demands on the teacher to be a lifelong learner, to serve as a resource, to share authority for knowledge, to set the curriculum agenda aside when necessary, and to question and learn with students necessitates a major shift in focus on what constitutes a teacher's role. That role suggests that the relationship between the teacher and students be one of collaboration in dialogue, with both teacher and students working towards their own growth in understanding where the classroom environment can promote successful students. It is said that all students explore mathematical ideas to the degree that reflects their interest, excitement and to the degree that explorations become relevant and important. All contributions should be valued and respected. In D'Ambrosio's and Prevost's (1995) observations students' personal and collective history shaped the curriculum as interests are reflected in the investment and engagement in learning which further provides an argument for engaging and supporting students in learning conversations.

2.5 Learning Conversations

I want to distil the research and definitions of classroom discourse, particularly mathematics classroom discourse, down into a meaning for learning conversations. The term 'conversation' implies that there is meaning and understanding on the part of both the sender and the receiver in the sharing of information. Language, the words used, is not the only part of a conversation. A conversation can be accompanied by gestures and/or other forms of body language, changes in tone, pauses, placing emphasis on some words more than others which all work together to enhance the possibilities for understanding in that conversation. The term 'learning conversation' implies that learning is the purpose of the conversation.

2.5.1 Language and Gestures

Iacaboni (2008) made a case for language and gestures as being one system and he explained that gestures, being spontaneous arm and hand movements, are unique to an individual. He said that when we cannot find the words to express ourselves, hand gestures can help in the retrieval of the missing words and also, at other times, gestures provide information that the words themselves do not provide. He shared a mathematics example which illustrates the above point. He said that children often use a dual format to explain the concept they are learning; problem-solving procedures stated with words, a different procedure with gestures. In fact, these speech gesture mismatches indicate a transitional expected phase in the learning process. He gives the example of 5 + 4 + 3 = _ + 3. There may be an incorrect verbal response: added the five, the four, the three and three and got 15, which may not reveal any awareness of the concept of an equation. However, if the student's hand moves under the left side of the equation, then stops, then moves again under the right side of the equation the movement reveals that the student's mind is starting to grasp the concept that an equation has two sides that are separate.

The important aspect for a teacher here is to notice the mismatch between gesture and the spoken communication and then to do something about it. There is a link with Vygotsky's Zone of Proximal Development (1987), and links also with Rowland's hedging of language (2000). Iacaboni stated that speech-gesture mismatches seem to indicate which mental activity favours the grasping of new concepts in young children, and that there is much research to confirm that this is the case. Typically, gestures are ahead of speech in these childhood mismatches, as in the equation illustration, the gestures tend to convey more of an understanding of the concepts. Gestures facilitate learning during counting tasks, children are helped by pointing gestures and Iacaboni went on to say that the mismatches show a better ability to generalise recently acquired knowledge and concepts, that is, the children who proceed from incorrect explanations matched in speech and gestures directly to correct explanations matched in speech and gestures.

Iacaboni also said children are very sensitive to the gestures of teachers. With mathematics problems, children are more likely to correctly repeat the procedure when the teacher's speech is matched with an appropriate gesture, compared with no gesture at all. Gestures accompanying speech have a dual role of helping the speakers to express their thoughts and helping the listeners/viewers understand what is being said. It follows that mismatching gestures by the teacher get in the way of learning. Indeed, children are less likely to correctly repeat a procedure when the teacher's speech is accompanied by a mismatching gesture, compared with no gesture at all; consider the example of the mathematics equation, and the teacher pointing to each number on both sides of the equation, with a series of manual gestures such as children typically used in solving a simple addition problem. This mistake only encourages students to make the mistake made by the student (as given in the previous paragraph) in that example, adding up all of the numbers on both sides of the equation. Instead, the teacher's gestures should visually depict two sides of the equation, with perhaps a bracketing gesture with the left hand to the left side in the same gesture with a right hand for the right side. It could make a difference when teaching students.

Iacaboni said that as adults, gestures are unique to each of us, nevertheless they divide into two categories; eye contact and eight iconic gestures that reflect the content of the speech that they accompany. Gestures are important in face-to-face interaction. I refer back to the missing element idea proposed earlier in this chapter. Interestingly there is support for the use of ‘gestures’ as a support for language and to assist with understanding from more traditional areas (Bates & Dick, 2002; Marrongelle, 2007).

Iacaboni said we automatically and interactively negotiate the meaning of certain words with very precise meaning within the context of the specific conversation rather than the meaning we get from the dictionary. He said that non-verbal forms of communication easily fall into patterns. The listener really looks at the speaker's eyes and connections are made. The speaker tends to start a new sentence without completing one in progress almost as if assured of understanding in the listener. Both the words and the actions in a conversation tend to be part of a coordinated joint activity and a common goal and extensive dialogue is natural and easy; however, this is not an area that is generally studied by traditional linguists. Iacaboni stated that every conversation is a coordinated activity with a common goal, and all recreate to a degree the evolution of a new language.

This reaffirms for me the place of the learning community: a teacher and a group of students create a unique learning community with a shared language that is developed through the conversations employed as part of that teaching process. I witnessed an example of this when another experienced teacher came in and took my class and used different phrases. This confused my students who didn't understand what was being asked of them and it took quite some time to establish what was being asked. To an outside observer this might have seemed that the students did not actually understand the mathematics, but it was the language vehicle that was being used that caused the confusion.

The example of the mathematics equation brings together for me the importance of language accompanied by gestures and with ‘hedging’ (Rowland, 2000), which also sits quite well with Vygotsky's (1987) Zone of Proximal Development. This strengthens the underpinning theoretical basis for the role of gestures in early childhood language development and for potential continued relevance with older students to use as a first step in developing mathematical conversational skills. These skills can be used effectively in *Purposeful Discussions* for sharing, explaining ideas and processes, reflecting on thoughts and being the vehicle for enhancing cognition.

Possibly in early research studies the focus has been the language that was being used in the conversations and in promoting classroom conversations; however, there does not appear to have been much of a focus on the gestures that accompany language use. There is evidence from multiple sources and disciplines to support the belief that language used appropriately and effectively, including the accompanying gestures and other forms of non-verbal communication, can make a positive difference for students learning mathematics. The next step is to create a visual image of the bringing together of the elements from Sociocultural Theory, from Social Constructivism, from Vygotsky’s beliefs on cognition and layering underneath or around this the understanding that comes from the research of the evolution of language and gestures. In the early stages language development is supported by gestures. These gestures can get in the way if they are not congruent with what the speaker is sending as a verbal message.

The facilitator of learning – that is, the teacher – needs to be aware of the importance of gestures and their role in the development of language, hence the shaping of cognition which is the essence of understanding in mathematics learning. As stated above these understandings can be incorporated into learning conversations, specifically *Purposeful Discussions*, classroom strategies focused through language that are structured, grounded in a theoretical background that are potentially the missing element from earlier research. The misuse of gestures can explain some misconceptions students have when learning mathematics, for example, the adding up all numbers across an equation when a consistent sweeping gesture has been made in relation to the equation which might be written on the board. Another example might be where students encounter improper fractions. Here they confuse the rule that when you are dividing by a fraction you turn it upside down and multiply. Because a strong gesture has been used in turning the fraction upside down it assumes a powerful message which is retained and misused by students in situations where it is highly inappropriate. This is a fundamental understanding that all teachers of mathematics should be exposed to, it is not a fact that should be kept secret.

Purposeful Discussion

Where some authors have used discussion or conversation others like Steinbring (1998) have used the term ‘purposeful discussion’ to really focus the intent of the learning conversation in the mathematics classroom. Steinbring stated that the way we communicate partly determines what we communicate and offered a definition of ‘purposeful discussion’ as follows –

> teacher’s question → students respond → teacher questions conviction or otherwise of the response as an alternative to the traditional ‘interrogation’ – teacher’s question → students answer → teacher’s evaluate.

It is this use of purposeful discussion that I used as a basis for the development of the *Purposeful Discussion* strategy that forms a major part of this study. Steinbring (1998) also commented on style and means of communication – from ordinary to technical mathematical language, language of gestures and icons, to symbols, from literal to metaphorical use of words, manipulating material objects to speaking about the possible outcomes of an imagined action on these objects as being significant contributors to ‘purposeful discussion’. I will adopt the use of *Purposeful Discussion* to hone the intent of a learning conversation.

Modelling

As a Stepping Out Trainer (1995) I was only too aware of the position and value of modelling in the teaching and learning process. Learning is a social experience. The terms modelling and scaffolding are used to explain the process of ‘doing’ with students and walking students through processes, skills, understanding in a way that supports, then empowers them to do it for themselves. In the classroom teachers have seen the value of modelling from employing it and seeing the improvement in students' understanding. In the first instance, as students are learning something new, the modelling aspect provides a safe environment for students to ‘have a go’. In the second instance the scaffolded approach provides an opportunity for students to develop confidence in preparation for deeper understanding. The same can be said of developing learning conversation skills with students – the teacher models and scaffolds the language skills and the conversation skills.

Cognitive Mediation

Following on from or alongside modelling and scaffolding deeper understanding is heightened with the learning conversation that will occur in the classroom. Vygotsky (1987) proposed that thinking, where deeper understanding occurs, is first platformed with language and then mediated through the use of language. If there is not a good fit between the quality of the modelling, scaffolding and the learning conversation then there is the possibility that the mediation of thought will not be optimum. This of course is all embedded in the understanding that students' processing of thought and the uptake of new learning is all predicated on their prior knowledge, hence bringing into play Vygotsky's Zone of Proximal Development (1987).

The classroom teacher therefore should be aware of the place of modelling, scaffolding, learning conversations and position this within reach of students' prior experiences, knowledge and understanding. To position new learning in the context of students' experiences provides a prime opportunity for new learning. Making students aware of how they learn, based on their previous experiences also enhances how they take on board new learning, new knowledge. The conclusion remains that the learning is facilitated through the use of learning conversations in the mathematics classroom.

2.6 Language and Learning Mathematics

It is now pertinent to consider learning mathematics without a great understanding of language. I can consider my situation in learning to fly as an analogy of learning mathematics. Trying to have an extended conversation when you don't have the language skills is probably almost impossible; I believe it limits you to just the basics and you cannot have that deep, elaborated conversation that is required for understanding and the learning of mathematics. To engage in conversation with my instructor I needed to first of all learn the language that is used in aeronautics. The aeronautical language is a specialised language just like mathematics which is a specialised language and if you do not have an understanding of the language then you cannot engage in that conversation. I am reminded of many mathematics classrooms where mathematics is taught through a series of ‘do this,’ followed by ‘do this,’ followed by ‘do that’ and then you get your answer. Deep meaning and understanding comes from having a conversation. Students who must ask questions to clarify their understanding must have the language skills that go with that.

I have been contemplating whether the language skills required for deeper mathematical understanding along with having a requirement for subject specific language skills contributes to a significant drop in understanding when students go from a primary school setting to the first year of high school. The language requirements in high school, where student learning is managed by a subject specialist, are probably at a very different level to those that may be required in primary school and if we have gone from an experiential type of learning to an algorithmic type of learning, or even the converse, then the gap in language skills required is probably too big to breach for many students.

There are probably many other factors that would contribute to a perceived drop in achievement from year 7 to year 8. Behavioural problems may come about through becoming a teenager, and a number of other interests might develop and so there are other factors that could contribute to that lack of interest or a lack of engagement in the mathematics lesson. However, I do believe that language, or those aspects of language skills in mathematics, contributes in that situation going from year 7 to year 8. This study focused on students in years 6 – 10 and so bridged the gap where students move from a primary setting into a secondary setting.

Over the years, as stated previously, there have been many studies into language in the mathematics classroom. Now what I am proposing is a way of developing language skills which are required to develop a higher level of understanding and to be able to engage in the thinking that is required for developing understanding of mathematics. According to Piaget (1964) students in the primary years go through looking at mathematics in a concrete operational procedure and then shift into a more abstract form. The abstract form requires language skills and students who don't have those language skills find their mathematics knowledge suffers.

2.7 Mathematical Linguistic Pedagogy

Brown (2001) was concerned with the way in which language and interpretation underpinned the teaching and learning of mathematics. In particular, he focused on issues of language, understanding, communication and social evolution, all of which he said have been tackled by mathematics education research under the banner of Constructivism and related areas, central themes in post-war western thinking on philosophy, and the social sciences, yet research in mathematics education seems to under-utilise the resource of work done in the broader context. He said that he sought to show how language is instrumental in developing mathematical understanding, and also how both chronological and spatial dimensions of classroom experience condition ideas being met.

The push in the mid-1990s was to find good mathematical investigations where students could record their ideas and then communicate them (Corwin, Storeygard & Price 1995). There is little opportunity for students to communicate and express themselves mathematically in some of the ways that would be supported in their literacy classes (Corwin, Storeygard & Price 1996). The problem with this is that only using discussion to communicate the ideas of an investigation limits the possibilities for discussion as having a prominent role in the learning process. They also stated that to communicate mathematics language that matches ideas and actions the language needs to be easily accessible. They made the recommendation that mathematics be taught as a form of communication. There is support for this from Vygotsky's (1987) link between language and thinking. Corwin, Storeygard and Price (1996) said that the medium of mathematical expression is human language; this may not take a unique form; however, it ranges from informal to formal and as it becomes more formal mathematical talk assumes specialised characteristics that are rarely present in children's talk. They said that talk in students' development is underrated and that talk is both individual and social. Teachers' use of mathematical terms is a powerful model for students who gradually internalise words and terms that 'are put out there where students can take hold of them' (Corwin, Storeygard & Price 1996). Students internalise mathematical conversations as models for both thinking and problem solving.

2.7.1 A Linguistic Approach

Cazden (2001) stated that spoken language is the medium by which much teaching takes place and in which students demonstrate to teachers much of what they have learned. She went on to say that in classrooms one person, the teacher, is responsible for controlling all the talk that occurs. The teacher controls not just negatively but also positively to enhance the purposes of education. She highlighted the 'linguistic' point that differences in how and when something is said can be a temporary adjustment matter for a student or it can be a matter of seriously impairing effective learning. Hence it is essential to consider the classroom communication system as a problematic medium that should not be ignored. She also identified three linguistic features of classroom life – ‘propositional, social and expressive functions’ (Cazden, 2001, p 3); all functions of language as a whole, not of separate utterances. She asked three questions:

> *How do patterns of language use affect what counts as “knowledge,” and what occurs as learning? How do these patterns affect the quality or inequality of students’ educational opportunities? What communication competence to these patterns presume and/or foster?* (Cazden, 2001, p 3)

She pointed out that other aspects of language in education result from non-deliberate, non-conscious choice at the point that it happens. Her focus was the ‘non-conscious’ aspects of language usage in the classroom, specifically, carefully attending to who is speaking and who is receiving thoughtful responses.

Cazden (2001, p60) provided us with insight into the use of language as a scaffolding tool posing the following question related to the goal of learning – a change within each student: ‘How do the words spoken in classrooms affect this learning?’ She also reminded us that scaffolds based on Vygotsky’s Zone of Proximal Development (1987) need to be continually changing as a student’s conception grows. There is a linguistic understanding required by teachers to recognise the time for shifts in language use, along a continuum from very informal to more formal and where each student is at, in terms of their conceptual development, to be able to effectively ‘shift’ a student. This was referred to earlier, matching the conceptual development of a student with the appropriate language.

2.7.2 Language as a Medium for Learning

I have now arrived at the point where language is being employed as a medium for learning which is discussed in the following section. The impact of linguistics on mathematics learning should not be underestimated. The use of linguistic pedagogy related to mathematics teaching appeared quite recently (Bailey, Chang, Heritage & Huang, 2010). These authors hypothesised that teachers’ use of imprecise vocabulary and structures that don’t support mathematical thinking may negatively impact student achievement in mathematics. The spotlight is on teacher's language use and their understanding of the importance of a linguistic pedagogy. The authors suggested that further research in the area would be needed to substantiate the hypothesis and specifically to investigate the direct effects of teacher linguistic pedagogy on student achievement, focusing on the capacity of students to understand and explain their understanding of important concepts and their application. I refer to *Mathematical Linguistic Pedagogy* as the linguistic pedagogy of teachers of mathematics.

I am not a linguist, nor do I have a linguistic background; however, I have an avid interest in the application of linguistics applied in the mathematics classroom. The study referred to above, that examined teacher linguistic pedagogy, suggested to me that there should be further research conducted in the area of *Mathematical Linguistic Pedagogy*. I believe that the term could apply to the overlap of mathematical pedagogical content knowledge (PCK) and linguistic application in the mathematics classroom. I can examine the use of language at a practical level; however, in addition I believe it is time for a study in *Mathematical Linguistic Pedagogy*. Later sections in this chapter provide an examination of the potential application of the work of Halliday, Bernstein and others to *Mathematical Linguistic Pedagogy*.

2.7.3 The Link with Pedagogical Content Knowledge

The kind of knowledge – knowing to anticipate specific student understandings and misunderstandings in specific learning contexts, and having strategies ready to employ when students demonstrate those understandings or misunderstandings, is an example of pedagogical content knowledge (PCK) (Schoenfeld, 2005). This links in with the importance of the learning conversation and having discussion points for clarification and amplification and for mediation of cognitive processing. Schoenfeld stated that there is a need to include a substantial number of ways of giving meaning to mathematical operations and concepts, and seeing and fostering connections among them.

Schoenfeld (2005), on reviewing the work of *the Developing Mathematical Ideas Professional Development Program for Elementary Teachers* outlined four critical skills, upon which the program developers focused. The first skill is attending to the mathematics in what one's students are saying and doing. He said that this may sound obvious, 'focusing on student thinking is actually a learned skill – and not necessarily one that teachers have when they emerge from their teacher preparation programs.' The second skill is assessing the mathematical validity of students' ideas. 'The issue is that even if the work looks non-standard, is the mathematics correct?' The third skill is listening for the sense in students' mathematical thinking – even when something is amiss. Once one is alerted to the mathematical possibilities in student thinking, one can often find a core of a correct mathematical approach in something that produces an incorrect answer. This gives something to build on. The fourth skill is identifying the conceptual issues the students are working on. Schifter (1996) provided an example of a student responding to a problem with a strange combination of arithmetic operations. Upon closer examination, the student's work is seen to represent an incorrect generalization of a strategy that was useful in a different context. This provided the basis for an interesting mathematical conversation with the student. Pedagogical content knowledge is essential to be able to have an effective conversation with a student and, as stated above, the four skills as identified by Schoenfeld (2005) are also essential to be able to make use of that pedagogical content knowledge in a way that takes into account Vygotsky's (1987) Zone of Proximal Development, identifying the problems, misconceptions and opportunities to investigate those misunderstandings and turn them into understandings.

2.8 Language at the Core of All Learning

This study is proposing the bringing together of disparate philosophies of learning, language, thinking and metacognition in order to build comprehensive strategies which facilitate learning and understanding of mathematics. As a part of this presentation, the focus is on advanced language learning and thinking. Byrnes (2006) advocated the use of Sociocultural Theory (SCT) and Systemic Functional Linguistics (SFL) as the components of an approach to advanced second language learning. An adaptation of her work and reasoning could fit in with the learning of mathematics treating it as a language to be learned. Even though it is not a second language it has the features of language learning that might be explored. After reviewing the writing of Byrnes I was inspired to search for the writings of Halliday. I agreed with the proposals put forward by Byrnes and was then inspired to reread Vygostky (1987); however, with more of a focus on thinking and how language mediates cognition rather than viewing language solely as a vehicle for communication. Language serves to assist learning through the sharing of ideas and processes as well as strengthening our own internal understanding through shaping our thinking.

School as the Place to Make Change

Generally children who are enculturated into a language have on average better-developed language skills when they enter formal schooling (Gee, 2008, p33); a view held by Bernstein (1971, 1975) although other early research disputed the view that language skills were socially based (Wells, 1985). Regardless of whether one believes in the advantage/disadvantage of social class, the argument put forward by Gee (2008) – that the answer rests in schools where students are socialised or enculturated into a certain social practice and that schools are the place to make change – is hard to dispute; however, he questioned whether schools can be changed. Logically, one would assume the same to be true of mathematical language and understanding. Programs like First Steps (1993) have demonstrated that when successfully implemented they can make a difference and develop the language skills of most children. Again, logically one would assume the same to be true of mathematical language and understanding. It should happen, but does not for a variety of reasons as highlighted earlier in this chapter.

Colleagues have commented to me that they had explained a mathematical concept to a student several times without the student 'understanding', to then have a fellow student explain the concept in a way that made sense to the student. What I am saying here is not about the formal mathematical language of the mathematics text, but rather the interpretive language that makes sense to students at a time that it needs to make sense to them. Rowland (2000) calls this *hedging* or *approximation*, based on Vygotsky's (1987) Zone of Proximal Development.

As mentioned previously, programs like *First Steps* (1993) and *Stepping Out* (1994) focused on improving students' literacy skills through whole language learning, through immersion in real experiences and then by employing a holistic, integrated approach to learning aimed at moving children through clearly identified stages of development. The *Curriculum Framework* (1998) supported a similar approach to learning. These programs were successful in initiating change in the ways that many classrooms in Western Australian schools are structured and the ways in which language learning is planned. The programs had some success in improving literacy skills for some students.

Programs in the United States like the *Talking Mathematics* project (Corwin, 1995), funded by the National Science Foundation, explored the ways in which teachers supported mathematical discourse in elementary classrooms. Corwin and Storeygard (1995) suggested that differences in instruction arose from a lack of experience in creating mathematics. My own experience, as a student like many of my generation and for several generations following, was to memorise procedures for solving relatively trivial problems. The tendency for teachers is to revert to the style in which they learned their mathematics (Corwin & Storeygard, 1995).

My early observations clearly demonstrated similar trends (McClellan, 2000). I found discernible differences in backgrounds and the ways in which teachers approached mathematical problems. Out of a group of nine, one teacher was comfortable with tackling problems with students and with constructing mathematics. This teacher's background as a student involved exposure to a discovery type approach in years 8 and 9 where mathematics was created and 'investigation' was encouraged. Other teachers who had been exposed to a more rigid, rote learning approach reverted to that approach when dealing with mathematical problems with their students.

As mentioned earlier, my recent experiences as a trainee recreational pilot highlighted for me the importance of knowing the language of what is being learned, to be able to enter into a meaningful conversation with the instructor which then facilitated the learning of the theory required for flight. I experienced anxiety just like my students experience in the classroom when new concepts are introduced and students do not have a grasp of the language required to enter, in a meaningful way, into the conversations surrounding those new concepts. My turn as a learner in a highly emotionally charged environment where my life was on the line if I made a mistake really honed the focus of this study for me. The importance of classroom strategies focused through language to provide students the opportunities to enter the learning conversation and join in the *Purposeful Discussion* in a meaningful way cannot be overstressed.

2.8.1 What Can Be Learned From the Research on Second Language Learning?

An examination of work on second language learning shows many links and/or implications for the learning of mathematical language and puts forward ways that could be used to enhance the understanding of mathematics. It is worth considering the parallels between learning mathematics and learning a second language.

An examination of Byrnes (2006) writing on advanced second language learning demonstrates strong links with learning mathematics, especially learning the language of mathematics. In her work there is some direction and lessons to be learned by those of us who are mathematics educators. Byrne referred to Vygotsky (1987) and his belief relating to thinking, and it is that precise belief that I want to make use of as well. Systemic Functional Linguistics (SFL) as developed by Halliday (1980) and Vygotsky's Sociocultural Theory (SCT) (1987) are put forward as proponents by Byrnes. As Halliday (1980) proposed, children go through stages: learning language, learning through language and then learning about language. Vygotsky's Sociocultural Theory, including the Zone of Proximal Development and scaffolding, also have application in learning mathematics. Vygotsky (1987) proposed that language mediates cognitive processes, that 'language is one of the most mediating tools of the mind'. So, speaking and writing shape and reshape cognition.

If the goal of the teaching/learning process is to create greater understanding then it follows that shaping and reshaping cognition ought to be a goal; a goal that can be achieved through the use of conversation which is, after all, language in the form of give and take, ebb and flow where the opportunity for shaping and reshaping is high. The strategies developed in this study provide the opportunity for students, during '*Purposeful Discussion*,' to share their thoughts and reflect on those thoughts in an environment that encourages, if not expects, all students to do that when they are ready as encapsulated in the following:

> *we are in complete agreement that helping learners to construct mathematics knowledge in the classroom will require teachers to be open to the voices, experiences, and sociocultural realities that meet in each classroom. A sociocultural focus that promotes discourse requires a conscious and enduring commitment to culturally responsive teaching. Mathematics with this focus is neither culture free nor culturally elite. Culturally responsive teaching celebrates the diverse cultures that create mathematics and take steps to highlight the works or mathematical texts of mathematicians from all cultural groups. Most important, it eliminates barriers to communication that arise because of ignored historical realities, shared cultural meanings, or dismissive linguistic practices.*
>
> *(Garnett, 2008, p85)*

Lampert and Blunk, (1998) cited the contribution of Russian activity theory to the way we formulate the relationship between thought and language stating that it underscores the connection between words and worldview. Activity theory has the child taking a central role in making language meaningful as it is acquired. The speaker of a new language is not a receiver of conventional definitions and ways of knowing, but rather an appropriator. Hicks (1996, pp. 8, 9) stated that 'as the child moves within the social world of the classroom, she appropriates (internalises) but also reconstructs the discourses that constitute the social world of her classroom. This creative process is what I would term "learning"'. Bauersfield, (1995) would add that just as the child appropriates from the culture of the classroom, the teacher puts things out there to be appropriated, functioning as a partner in the conversation but with a special mission and power to ensure that the classroom culture is rich in offerings, challenges, alternatives, and models, including language. Byrne (2006) made use of 'languaging' as a word to describe language produced not just as language but as in making meaning and making sense of the language which is being used to learn the second language. Hence I want to take that literally and transfer it into my situation where I was treating the learning and making sense of mathematics as being exactly like learning a language.

In reviewing the work of Halliday and Vygotsky, Byrnes (2006) proposed that when children learn language, they are not simply engaging in one kind of learning among many; rather, they are learning the foundation of learning itself. This is an extremely important and fundamental statement. According to Halliday (1993) human learning is a semiotic process; language development is learning how to interpret, hence learning language is a meaning making process. Interestingly Halliday (1993) also stated that most theories of learning, including those that take account of language learning, come from outside the study of language and either ignore language development or confine it to a one-dimensional aspect of the learning process.

Byrne (2006), examining the work of Halliday affirmed that language is not a domain of human knowledge. Language is the essential condition of knowing, the process by which experience becomes knowledge (Halliday, 1993). Halliday (1993) proposed an alternative; exploring pedagogies that are based around language and interpreting learning as something that is inherently a semiotic process. Halliday made a very good case for a pedagogy based upon language development and this comes through in his list of 21 features, aspects of child language development that are critical to a language-based theory of learning (see Appendix D).

Byrne (2006) captured the essence of Halliday's approach and added that it not only explores the relationship between the semiotic tool 'language' and the human capacity to learn; but also draws on much earlier insights gained by educational practitioners that educational failure is primarily linguistic failure. Byrne (2006) added that it is critical to emphasise that a language-based theory of knowing and learning investigates the nature of the language resources needed for enabling such ways of knowing, rather than focusing on the settings or content. It is grammar that reconstructs experience from common-sense ways of knowing to metaphorical ways of knowing and understanding.

2.9 Influence of Halliday

Halliday (1993) requires more examination, especially on his 21 features in the evolutionary process of child language development (see Appendix D) as this can only make for a stronger case for the use of conversation in the mathematics classroom when that language is being used to shape cognition, that is, thought and understanding and to build on that understanding. This supports precisely the Social Constructivist view of building on what is known and moving into the unknown; this also conforms with Vygotsky's Zone of Proximal Development (1987), which also fits with the work of Rowland (2000) on 'hedging' and approximation with language use in mathematics.

Earlier in the chapter numerous readings and studies were cited where discussion had been employed in the classroom for all the reasons that have been mentioned. However, as I have stated earlier, there has been at least 50 years of research into the area of conversation, discussion and talk in the classroom which to date has not produced the results expected. There has to be something missing and I believe that one of the elements missing is the understanding of language development in younger children which can lead to a better understanding of continued language development in older children and a better understanding of how that language might be used to enhance learning. Strategies that consider this missing element may result in more success than previous studies have shown. A re-examination of Halliday's (1993) process of language development can show the way.

Very pertinent to my argument is a statement by Halliday (1993) that most theories of learning come from outside the study of language and this includes those that take account of language learning. He stated that language is the essential condition of knowing, the process by which experience becomes knowledge. He further went on to propose that we interpret learning as a semiotic process.

> *When children learn language, they are not simply engaging in one kind of learning among many, rather they are learning the foundation of learning itself. The distinctive characteristic of human learning is that it is a process of making meaning – a semiotic process; and the prototypical form of human semiotic is language. Hence the ontogenesis of language is at the same time the ontogenesis of learning.*
>
> (Halliday, 1993, p 93)

Halliday made an interesting point in his elucidation of the 21 features related to child language development. He outlined the strategy in language learning of 'the trailer', a preview of what is coming, like an advance organiser of a child's learning, and explained that in learning children take a step forward and then back off for a while before consolidating this step and then building it into their overall learning process. The observer often refers to the first foray as a fluke, not understanding the place in the learning process of that first foray into new learning. Halliday is referring to younger children; however, the same could be said with older students learning the language and the practise of mathematics.

2.10 Social Aspects of Learning

Many studies of the classroom, as shown earlier in the chapter, look at the social aspects of language interactions. It is understood that the learning of mathematics is a social experience and that the social relationships based around language usage in the classroom is an important aspect in the learning of mathematics. I do not want to discount the importance of the social aspects of language in the mathematics classroom, moreover what I want to do is focus on the way that the language enhances, or in some cases excludes students from the learning of mathematics. Students are excluded because they don't have the 'right' set of language skills for 'talking' mathematics; however, the question remains 'How do you assist students to develop the mathematical language skills to be able to learn mathematics?' Another question would be whether students have grasped the language in its approximate informal code at this stage, and then how do you assist them as they progress through more 'formal language' learning and more formal concepts? I have been in classrooms with six year old students, seeing them go through the 'concrete' learning stage and have seen the students use a semiformal language, used in a very informal way. Now if teachers are to relate the mathematics to students' experiences, then my experience suggests that this be in a very intuitive, informal way. So perhaps the use of an 'informal language' and an informal language register when students first start to learn mathematical concepts is needed to be recognised in primary years. So what we're looking at is a very informal mathematical language register associated with learning concepts at a concrete level moving through to a formalised mathematical language register associated with learning abstract concepts. It was not the intent of this study to formulate such a register; however, the synthesis of past research studies, both in classroom discourse and linguistics suggests that it is required and could be the focus of a study built upon the current one.

2.10.1 Influence of Bernstein

Bernstein has influenced many aspects of learning especially learning based on linguistic principles. In this study it is necessary to reflect on some of Bernstein's (1971; 1973) prior work, particularly his ideas related to language codes; his elaborated language code and his restricted code may have a significant influence on the strategies developed as part of this study. His restricted language code referred to an intimate register that 'draws on a store of shared meanings and background knowledge, a restricted code carries a social message of inclusion, of implicitly acknowledging that the person addressed is "one of us"' (Atherton, 2011a). Restricted code applies to restricted communities whereas elaborated code spells everything out.

Bernstein suggested that 'if students cannot manage the elaborated language code' then they will not be successful in an educational setting (Atherton, 2011a). Gee (2008) used the term 'social languages' to refer to sub languages within language and the intent of these 'social languages' is similar to Bernstein's (1971, 1973) restricted code, where the language excludes and includes the group to which it is formed around.

It is appropriate at this point to suggest that strategies could be developed using a combination of Bernstein's (1971, 1973) restricted and elaborated language code; however, there may be a negative impact to developing a restricted code with a cohort of students in that they may learn a significant amount while they are there in that class; however, the following year if the teacher that they then have does not participate or is not a member of that restricted code then that can cause problems for students so that is something that needs to be considered. So when developing strategies that use a model of elaborated language code and restricted code the strategies need to be developed so that use of the restricted code is maximised to create a cohesive classroom group and the elaborated language used needs to be explicitly built on and linked to the former. Within the strategies developed the elaborated language comes into play when new concepts are developed; however, as part of that development of new concepts the restricted code is called into play when metaphors are used so it is a dual elaborated code/restricted code that is in play.

> *The restricted code works better than the elaborated code for situations in which there is a great deal of shared and taken-for-granted knowledge in the group of speakers. It is economical and rich, conveying a vast amount of meaning with a few words, each of which has a complex set of connotations and acts like an index, pointing the hearer to a lot more information which remains unsaid. … because it draws on a store of shared meanings and background knowledge, a restricted code carries a social message of inclusion, of implicitly acknowledging that the person addressed is 'one of us'. It takes one form within a family or a friendship group, and another with the use of occupational jargon within a work group. Its essential feature is that it works within, and is tuned to, a restricted community.* (Atherton, 2011b)

2.11 Elaborated Discourse and Discourse Analysis

As an extension of Bernstein's elaborated code I moved to examine elaborated discourse, what it looked like and how it might be used as a part of this study. Bernstein focused on the formal language used in educational settings and this he referred to as elaborated code. Elaborated discourse is outlined below.

Elaboration

In Section 5 I explained and defined *Purposeful Discussion*. As part of the study I wanted to be able to classify students' responses offered in *Purposeful Discussion* as elaborated and wanted to develop a system for classification. To class students' responses as elaborated I first adapted discourse connector examples from Bauer-Ramazani (2005), as displayed in Table E.1 (see Appendix E) and used this list to identify the discourse connectors in my students' responses. I adopted the practice of Bailey, Chang, Heritage & Huang (2010) to define elaborated discourse as 'discourse using more than two discourse connectors'. Table 2.1 is a further adaptation containing those examples that students were most likely to use, which provided me with a distilled list of discourse connectors that was relevant to the students who participated in the study.

Table 2.1 – Discourse Connector Examples

which is	but	an example	when
that's how	instead	even if	until
so	First	just like	whenever
Still	Second	Before / after	the next time
Since	Afterward	That	who
As	After that	Whose	until
now that	Later	When	the answer
so that	Then / Next	Where	I did
that	while	Just went	I done
such...that	If..., then	I went	Because
If it was	Like	different to	Because of

(Adapted from Bauer-Ramazani, 2005)

There are other recent examples of discourse analysis (Gee, 2011), thematic analysis of mathematics' classroom discourse (Herbel-Eisenmann & Otten, 2011); however, discourse analysis is not a major focus of this study. The intent of including elaborated discourse connectors and defining elaborated discourse is to provide a practical means to evaluate the contributions of students during *Purposeful Discussions*.

2.12 Summary

To restate, the research focus is based around creating strategies that enable students to engage in *Purposeful Discussion* leading to reflection and metacognition. The strategies are framed around the impact of language on learning and the influence of modelling and scaffolding. The framework encompasses the work of second language learning, its applications in learning mathematics language as a second language and in general the impact of language on learning and mediation of cognitive processes. Linguistic pedagogy synthesised with pedagogical content knowledge leads into an area that could be called *Mathematical Linguistic Pedagogy*, an approach to the teaching and learning of mathematics that could provide answers to why the implementation of classroom discourse strategies has not been as successful as initially expected.

The theoretical framework comes down to language mediating cognitive processing; Venn diagrams would probably be the best way to visualise this grammatically or pictorially. Language is the universal set and cognitive processing is a subset, along with metacognition, raising the question 'is metacognition language mediating cognition?' That is a question that needs examining with an answer that could change the way that teachers talk in their classrooms.

Chapter 3

Methodology

In Chapter 2, the examination of research from the last 50 or so years revealed a lack of structured strategies that could make a difference with the language employed in the mathematics classroom setting, hence influencing the learning, understanding and thought processes associated with that mathematics learning. Returning to the research questions provided a lens through which the choice and development of a research design and methodology might effectively be examined. The research questions follow:

1. What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)?
2. What is the role of language in the application of those strategies to engage students in extended learning conversations?
3. What benefits are created and challenges encountered when those strategies are employed in the mathematics classroom?

Using the research questions as a lens, a Participatory Action Research design was employed as a basis for this study. It took many years to get to the point of selecting this research design, or rather it choosing me. As stated previously, I have long held an interest in, and belief in the idea that there is a strong link between language development and mathematical understanding. Prior to commencing this study I attempted to look at the behaviours displayed by other teachers who successfully engaged in learning conversations with their students. I began with a small sample of students in year 11 to try to elicit what those behaviours might look like; this proved extremely difficult and really provided me with no new information. I attempted to look at the contexts where learning conversations might successfully occur; I examined personality types and the profiles of teachers that students responded to in those learning conversations. Again this provided me with no answers to questions I was seeking.

In the six months prior to the research study commencing I focused on my own behaviours in the mathematics classroom and what I could learn by examining how I interacted with my students. This led to a far more fruitful examination. I had control over my behaviour, over the context in which my students learn and over the reflective processes in which I engaged as teacher and researcher; hence, the natural progression to the Participatory Action Research design. This research study focused on my classroom, my practice and my processes of planning for improvement in students' understanding of mathematical concepts. It focused on the opportunities I might find to introduce and manage extended learning conversations.

As a participant in the study I was able to interpret and analyse my behaviours and those of my students with the assistance of a Cooperative Colleague who shared the same classes. The findings were extrapolated in order to share what I learned. My research is unique in that I am bringing together the following areas in a model in which I found opportunities for students to engage in extended learning conversations with both me and with their peers: Vygotsky's (1987) link with language and thinking; metacognition; cooperative group learning; use of language for learning and communication, and a pedagogical understanding that learning is enhanced when a person teaches another.

Section 1 below discusses the development of the practice, the factors that influenced the choice of classroom strategies that could be focused through language and early planning prior to data collection. Section 2 examines the research methodology focusing around a discussion of Participatory Action Research. Section 3 focuses on the research design, including researcher background, student background and instruments and procedures. Section 4 focuses on a discussion of the data whilst section 5 focuses on data analysis including a discussion of the choice of a Grounded Theory Approach to analyse data. Section 6 provides a brief overview of the difficulties of recording the data, section 7 deals with the ethical issues and finishes with a summary of the chapter in section 8.

3.1 Development of the Practice

In Chapter 1 (Section 1.4.2) seven areas were outlined that shaped and guided the decisions I made in choosing, developing and using the strategies where students learned the language, learned the concept and engaged in mathematical conversations in the classroom. Early planning, prior to collecting data for the study involved bringing together those six areas as described in Chapter 1 and using them to research and develop a series of comprehensive classroom strategies focused through language. I began with Reflection and Metacognition as opportunity allowed me to introduce this aspect to students before the others. Six months prior to the data collection commencing I started mental arithmetic exercises with my students three times a week. I started the lesson with a set of ten mental questions. Mental provided me with a vehicle to have students discuss their strategies for solutions to the questions that I posed. I started with simple numerical questions and expanded into more difficult problems and into measurement, space and algebra questions. The purpose of those questions was to provide revision of concepts taught, to introduce new concepts by building on what was known as well as the discussion of strategies and techniques, and even to the use of language and spelling of mathematical terms.

I worked on the Reflection and Metacognition element of the strategies which I considered to be the first stage of development of a set of classroom strategies focused through language. That continued throughout the following stages as did specific teaching of mathematical language, as the opportunity arose. The discussion of strategies was the beginning of the learning conversations in an informal manner.

I worked on providing students with the opportunity to demonstrate their understanding of a concept that they had learned. Through negotiation with me students chose a concept (or several concepts) about which they would demonstrate their understanding. Some chose to do this through a

PowerPoint presentation while others produced a word document which I then asked them to explain to me. Throughout this process students were helping each other clarify understanding when they noticed a misconception and were beginning to have conversations with each other about their chosen concepts and making comparisons with their own. The conversation at the end of the process was providing students with the opportunities to engage in extended conversations with me, which in turn was providing them with practice in which to engage with their peers.

I then focused on group-work and working collaboratively; in this stage I wanted students to develop and to understand the purpose of working collaboratively so that when they took on the task of sharing their understanding with younger students they would have the skills to support and assist those students. Year 10 students left the class at the end of the school year, so for the research study period the year 8 and 9 students took on the leadership roles.

Finally students were asked to identify a concept that they could teach to their younger peers. They had a choice of medium through which to teach this concept in a mini presentation. Students worked in collaborative groups where they assisted and helped each other refine their presentations. Presentations were carried out and discussed. These events were the precursor to data collection in the research study.

3.2 Research Methodology

The research design employed in this study was qualitative in nature and focused on an Action Research model; Denzin and Lincoln (2011) stated that Action Research is about being pragmatic. The authors declared that it is grounded in local knowledge and co-created, that ‘research becomes praxis – practical, reflective, pragmatic action – directed to solving problems in the world’ (Denzin & Lincoln 2011, p 21). The authors revealed that Action Research is focused on problems that arise from the ground up, that are not delivered from above and that validity and credibility is measured by the willingness of local stakeholders to be involved and to act on the basis of the results. Action Research, as defined by Levin and Greenwood (2011, p 29) citing Greenwood and Levin, (2007, p1) stated Action Research is ‘a set of self-consciously collaborative and democratic strategies for generating knowledge and designing action in which trained experts in social and other forms of research and local stakeholders work together’. The authors stated that the research focus is chosen collaboratively by the stakeholders and the action researchers and the focus is on doing ‘with’ rather than doing ‘for’. Action Research is, by its very nature, cyclical, involving planning, action, evaluation and reflection.

Participatory Action Research (PAR), a subset of Action Research involving the researcher as a participant, describes the research methodology adopted for this study. Brydon-Miller, Kral, McGuire, Noffke and Sabhlok (2011) provided a definition of Participatory Action Research as one being distinct from Action Research due to its focus on ‘collaboration, political engagement, and an explicit commitment to social justice.’ This hardly seems to differentiate between the two; however, the authors stated that participation is a major characteristic of this type of research and not just in the sense of collaboration, but in the sense that all who are involved in the context, that is, the stakeholders, need to be involved in the ‘whole of the project undertaken’ (Brydon-Miller, Kral, McGuire, Noffke & Sabhlok, 2011, p 388).

I was part of the normal classroom environment and as the teacher of the class of students involved in the study I was a participant, which can be viewed as both an advantage and a limitation. As Denzin and Lincoln (2008) stated qualitative researchers study things in their natural settings using a variety of empirical materials and multi-methods. The issues of credibility, authenticity and trustworthiness needed to be addressed and the research was required to stand up to the scrutiny of both my peers and examiners. The research also needed to clearly demonstrate that the findings are both authentic and trustworthy. It was important that how I addressed the issues was clearly articulated. Denzin and Lincoln (2011) reminded the reader of their images and representations of qualitative research triangulation methodology as that of a montage, bricolage or as bricoloeur, or quilt-maker.

The authors suggested another more encompassing image is one of a 'crystal' using multiple lenses to engender a sense of attempting to 'secure an in-depth understanding of the phenomenon in question' (Denzin & Lincoln, 2011, pp. 5, 6). The combination of multiple methodologies, practices, perspectives, observers, and empirical data or materials can be understood as a 'strategy that adds rigour, breadth, complexity, richness, and depth to any inquiry' (Denzin & Lincoln, 2011, pp. 5, 6).

Lincoln, Lyneham and Guba (2011, p122) stated that the 'hallmarks of authentic, trustworthy, rigorous or 'valid' Constructivist or phenomenological inquiry – were fairness, ontological authenticity, educative authenticity, catalytic authenticity and tactical authenticity'. The authors proposed that fairness may be addressed by the 'quality of balance', including all stakeholders views, perspectives, values and so on. Omission was seen as a form of bias, although not directed at objectivity. The authors described ontological and educative authenticity as criteria for a raised level of awareness and stated that catalytic and tactical authenticities refer 'to the ability of a given inquiry to prompt, initial, action on the part of research participants and, then, the involvement of the researcher/evaluator in training participants in specific forms of social and political action if they desire such training' (2011, p122). Kincheloe, McLaren and Steinberg (2011, p171) further added that catalytic authenticity is 'research that possesses catalytic validity displays the reality-altering impact of the inquiry process and directs this impact so that those under study will gain self-understanding and self-direction'.

Altheide and Johnson (2011) stated that when discussing validity in Action Research or Participatory Action Research the focus is on making explicit what is taken for granted, and that researchers share an ethical obligation to make public their claims, to show why they should be trusted. The authors added that 'What is common to each of these approaches, and by implication all forms of inquiry, is a process of acquiring information, organising it as data, and then analysing and interpreting those data with the help of refractive (conceptual, theoretical, perhaps political) lenses' (Altheide & Johnson, 2011, p. 584).

Field notes, research diaries, interviews, conversations, photographs, recordings and notes to oneself are all representations of the world and participant qualitative research involves an interpretive, naturalistic approach to the world (Denzin & Lincoln, 2008). I used a range of representations including some of those mentioned above as well as working with a Cooperative Colleague. Denzin and Lincoln (2008) described qualitative research as a situated activity that locates the observer in the world and consists of a set of interpretive material practices that not only make the world visible, but transform it. As the teacher-researcher I was situated within the research context and used recordings and conversations with participants in an effort to make that world visible and transform it.

My role provided me with a unique view of the studied classroom environment as well as of the processes in that environment. As Kemmis (2001) stated, Action Research is a practical and self-educational process for the practitioner. Davis (2007) said that Action Research is a dynamic, circular and evolving research process. As she described, it is not neatly defined and often does not fit the architecture of the standard thesis format. Kemmis, (2001) described a spiral of continuous and overlapping cycles of planning, action, observation, reflection and critical analysis as representing the key characteristics of Action Research.

The research was carried out in a rural district high school in wheat-belt Western Australia. These schools have students from pre-primary to year 10. The participants were the students in years 6 – 10, in the school. The limitations in interpreting the results of this study related to the nature of district high schools in terms of their rural setting, student numbers and my role as teacher and administrator as a participant researcher. The steps taken to minimise the impact of those limitations had to be clearly articulated in the context of the research site, the participants as the sample, having a Cooperative Colleague working with the same students and having the results available for scrutiny. Using a research methodology that was grounded in theory as well as practice helped provide credibility.

3.3 Research Design

Developing a theoretical framework from the research questions and using those areas that shaped and guided the research practice, as mentioned in the introduction to this chapter, impacted on the chosen methodology. I examined past research (of which there is not a lot) on language and its use in the mathematics classroom to enhance learning through deeper understanding which comes about through mediation of cognitive processes as proposed by Vygotsky (1987). I examined work on cooperative group strategies to bring all of the above together in a way that works for student learning and enhanced understanding.

I wanted to use the parallels with advanced language learning to bring another aspect of support to my belief that mathematics learning and doing can, to a certain extent, be considered as a second language, and much of the pedagogy employed for second language learning can be applied to the learning and understanding of mathematics. For my research approach I employed a Participatory Action Research model, where the focus was on improving my processes and my application of pedagogy. It has taken me about seven years to arrive at that final, probably most important, decision to approach the research design from this perspective to allow or enable me to enter into the research design without having to filter the data through someone else's experiences.

In effect, because of the design, I experienced the teacher-researcher's data first hand. This can be a positive and a negative, as in the first instance I was fully aware of the risk to avoid collecting and immediately recording the data because it was happening to me – I may have lost some of the data. However, to avoid this I had the video camera turned on during the lessons; to both collect and record, and collaborate my data through hearing the students' voices. I collected around 20 hours of lessons with students in years 6 – 10.

3.3.1 The Researcher Background

Every situation is a means of telling a story related to what I am doing, whether it is teaching or researching or just reading. This journey of self-exploration is of utmost significance in terms of my research as it is the lens through which I make all judgements about my observations, it is what influences any decisions that I make and the processes I employ to go about actioning those decisions. Unlocking my thoughts is a vehicle I can use to my advantage; where it will take me I probably already know. It will, or it is taking me where I want to go and that is to continue to write and to write with enthusiasm and passion in the way that I am passionate and enthusiastic about my subject area of mathematics.

I am not singularly passionate; I have many passions and I want to be able to pursue them all; however, mathematics education is a driving force in my life. Reflective, yes I am. It is at my core of existence. It is who I am. I have a need to reflect on everything that I do and I have had a lot of practice with it, which means I believe that I can examine all my experiences and actions dispassionately and objectively and quite often find that I need to alter the way that I do things. That returns full circle again to my way of being and having a story for what I am working on. Being reflective is part of my doctoral research design. I focused on my actions as part of the research process. It is my actions that I have control over and through a reflection process I analysed my actions and could then either improve them or support them. I could observe others but I have no control over their actions, over how they would interpret my observations and communication of those observations.

At the least, I am able to interpret my actions, explain my actions; however, there is a need to outline the context they were made in. Hence I need to explain myself; my values and beliefs about teaching and learning, my pedagogy and the way that I go about my craft of teaching. I had to consider what aspects of myself I needed to divulge, eliciting those aspects that provide background for my contextual descriptions and explanations; that is extremely important to the validity of my research.

My preference as a researcher was to approach my research from the viewpoint of practitioner, rather than purely from the perspective of a theorist, as I was as an administrator and teacher, a stakeholder in the process. As a school administrator I had a responsibility to students and to parents to ensure that the curriculum was delivered in the most appropriate manner and that, in my classroom and the classrooms under my management, opportunities were provided for the best possible learning to occur and to demonstrate improvement in national testing results. My preference; however, must be overridden by my duality of roles; I am a teacher and a researcher. As such, through my research project I am seeking to understand and theorise as well as find practical solutions to the problems associated with the use of mathematical language and demonstrated understanding of mathematical concepts. It is only through having sufficiently clear contextual descriptions and explanations about me that I can satisfy the conditions for authenticity and trustworthiness.

Cooperative Colleague

I had a colleague who shared the teaching load with the younger group of students. We had to regularly meet and plan for the class and during that time we discussed the data collected and my interpretations of that data. We met informally three or four times a week, to discuss planning, issues with students, what had happened in my lessons and how my colleague could follow up in his lessons. We also met, formally, once a week to discuss students' progress, planning for a seamless program, researching ways we might consolidate what students were learning. The main issue for us was that we use consistent language and approaches, hence the frequent, informal meetings. It was important that I share the data with a colleague who was familiar with the students in the group.

3.3.2 Students' Background

As outlined earlier I worked with two class groups; the first class consisted of year 8, year 9 and year 10 students and the second class group consisted of year 6 and year 7 students. Table 3.1 displays the makeup of Class group A and Table 3.2 displays the makeup of Class group B.

At the commencement of the study there were three year 8 students, four year 9 students and four year 10 students. All three year 8 students were girls; one was from Zimbabwe and has a non-English speaking background. English is her second language. At the commencement of the study she had only been in Australia for 3 to 4 months. Hence, there was a language problem. There were three year 9 boys and one girl, with another girl and boy joining the class for the later recordings and one other boy leaving. There were three year 10 girls and one boy with another girl and boy joining the group for the later recordings. So this class group consisted of seven girls and three boys. In smaller district high schools it is quite common to run a class of students of both mixed ability and year groups. Transiency rates were quite high, as demonstrated in Table 3.1 and Table 3.2.

Table 3.1 – Makeup of Class Group A

	Commencement		**Change during the study**		**End of study**	
	Girls	Boys	Girls	Boys	Girls	Boys
Year 8	3	0	-	-	3	0
Year 9	1	3	+1	-1, +2	2	4
Year 10	3	1	+1	+1	4	2
Totals	7	4	+2	+2	9	6

The second class group, detailed in Table 3.2, started with 22 students of mixed ability in year 6 and year 7. In the year 6 group there were 12 students; six boys and six girls. In the year 7 group there were 10 students to start with; six boys and four girls. One of the year 7 boys left the school at the end of first term and does not appear in the later recordings.

Table 3.2 – Makeup of Class Group B

	Commencement		**Change during the study**		**End of study**	
	Girls	Boys	Girls	Boys	Girls	Boys
Year 6	6	6	-1	-1	5	5
Year 7	4	6		-1,	4	5
Totals	10	12	-1	-2	9	10

For most of the study there were 21 students in the class, with 11 boys and 10 girls. The class was very mixed ability, with two students on Individual

Education Plans; one female year 6 student and one male year 7 student. Another student, a boy, had Education Assistant time.

The Year 6/7 Class

I took on the task of teaching mathematics to a combined year 6/7 class; I had not taught that year group previously and, while it seemed like an opportune time to take this on while I was researching language usage in mathematics, the task was more daunting than I was expecting. The class consisted of 22 students aged between 11 and 13 years. There were 12 male and 10 female students in the group. There was also a very wide range of abilities within the group. When I took on the class there were two students with Individual Education Plans and one student who received Education Assistant support time.

The students had a mathematics text to work from so if all else failed I had a fall-back plan, which was to use the mathematics book. At the beginning of the year I started finding out what the students could and could not do. From there I focused on the Measurement unit, particularly practical measurement and accuracy, conversion of basic units and some perimeter and area of simple shapes. I also focused on the four operations with whole numbers, basic fractions and decimals.

At the time of the recorded observations I had been teaching the class for five weeks. The class had developed routines and working relationships. One of the tasks I had students working on was related to the Winter Olympics in Canada which had given us the opportunity to look at time zones as part of the Measurement strand. Students constructed a time-zone sheet with times around the world and most had some understanding that there were differences in times around the world depending on geographic location.

We had also worked through length of days as students had little understanding that a day consisted of 24 hours. It took some time to overcome that misconception: a day was the daylight hours for many students. We also worked through the earth travelling around the sun and the length of a year, a month, lunar month, weeks, hours, minutes and seconds.

While we were working on this I was looking at the maps that I had printed as part of the time-zone exercise and was reminded of the problems associated with having a flat map of a spherical shape. I put to students that there was quite some error in the translation of the maps. This seemed quite appropriate to focus on as I had made accuracy an important skill to achieve.

To demonstrate this I took an orange and cut the peel around a hemisphere. I was able to slip the peel in two parts off the orange. I then showed the students what happened when you make cuts in the orange to flatten it and how this happens when you look at the earth and make a flat map. To explore this we started working with triangles and that is where the recorded lesson starts.

There is a considerable difference in teaching mathematics to year 6 and 7 students and to teaching year 8, 9 or 10. Before I started teaching the class I was expecting that there would be differences and I knew that I would have to do some exploratory work before I could fully appreciate the differences between the two groups. This is what I undertook to do in the first four weeks. I also knew that I would probably need a lot more practical applications if I was to involve students more in their learning. I was also expecting that language application would not be the same; probably not as sophisticated. I was expecting that I would need to develop their mathematical conversational skills.

I had some expectations about these students being inquisitive and more open to learning than many of my secondary students. Over the years I had made comments that there seemed to be a falling off in ability or interest between year 7 and year 8 in Western Australia where students moved from one school setting to another. This is not part of my study but it was something that I was aware of and expecting to see. Whether it is something that I will make a difference with is an issue that is outside the range of this current study. Suffice to say that it was probably part of what we were trying to achieve as a middle school model at our school. In 2010 we changed the configuration of class groupings with the Year 6/7 class group being taught by secondary specialist teachers rather than as previously taught by one primary teacher. The students stay in the same classroom for most of their classes with teachers changing.

Year 8/9/10 Class

I continued working with the group I had in the previous year. With a significant (contextually) number of year 10 students leaving at the end of year 10 in the previous year we lost a large proportion of the class. The year 10 group of students in the previous year made up just under half of the students in the group. This is the third year that the year 10 students have been taught by me, the second year the year 9 students have been taught by me and the first year the year 8 students have been taught by me.

I have a very strong teacher - student relationship with this group. My daughter was one of the year 10 students. Having worked over the years with three of my children in my classes I have developed a policy that while we are at school I am the teacher, the deputy and not a parent. The older students know how I work and I know how they work and learn. The differences in the relationship in the class is not just attributable to the fact that older students have a different relationship with their teacher, but also to the fact that I have worked with them for several years.

3.3.3 Instruments and Procedures

I catalogued the observations into two groups; the first group of observations, seven recorded lessons, is of the year 6/7 class. The second group, consisting of nine recorded lessons, is of the year 8/9/10 class with one brief recording of an individual student. Within each of those groups there is a range of recorded lessons; these include practical lessons, discussion based lessons and lessons where students are presenting their own lessons. For each of the recorded observations that I will refer to in Chapters 5 and 6 I will provide an outline of the lessons, the setting and the context. I will explain what the lesson is and what I am trying to achieve in the lesson. I experimented with several different formats for recording the observations; I tried using tables and matrices with the intent of creating some quantitative data; however, I found this extremely difficult as it obstructed interpreting my observations. After creating problems for myself I arrived at the conclusion that the best format for recording data from the observations was simply to translate and record and then to come back and make notes on what I had recorded.

As stated above, a research journal, notes, digital video recordings, conversations and descriptive narrative were used as tools as part of the research methodology. Student participants were initially requested to keep a journal recording their participation in activities; however, this proved to be too big an intrusion into the learning time so it was abandoned in favour of digital video recordings which captured the essence of what students were learning and conversations that occurred spontaneously. The reflection aspect was captured as part of the lesson by asking students to write one, two or three sentences about what they had learned. My research journal became a series of notes and recordings. Each of the lessons in the research study was first transcribed in narrative form.

The data was collected over a six month period, beginning in February 2010 until June 2010. Data from two follow up lessons was collected in November with the year 8/9/10 class and in early December with the year 6/7 class. As this was an Action Research model the planning, action, observation, reflection and critical analysis were ongoing during that period. Reflective notes logged developments and changes in my behaviour with ongoing reflection and analysis with which the Cooperative Colleague assisted.

3.4 The Data

A catalogue of the data is displayed in Table 3.3 below.

Table 3.3 – Catalogue of Recorded Observations

	Date	Group	Length	Lesson	General Comments
1	08/03	Yr 6/7	48:23	Area of Triangles	Mainly practical lesson with some discussion
2	10/03	Yr 6/7	40:55	Drawing circles and segments	Mainly practical - Compass work – students' voices
3	10/03	Yr 8/9/10	48:42	SURDS	Mostly students working in groups, with some student explanations and write ups on the whiteboard and the last four minutes teacher led, students looking at questions and talking through answers
4	12/03	Yr 6/7	30.09	Rule of Order	Teacher led, student reflection
5	15/03	Yr6/7	18:57	Flat Maps	Discussion of mapping discovery of Australia
6	15/03	Yr 8/9/10	5:21	Mental	Discussion, reflection
7	15/03	Yr 8/9/10	25:12	Factorising	Common factors, factorising binomials and trinomials
8	17/03	Yr 8/9/10	66:22	Orange Problem	Setting the scene with the globe and flat maps, sharing the orange problem and the year 6/7 solution This one is important for both tone and body language as well as the total discussion
9	17/03	Yr 8/9/10	32.20	Practical - Orange Problem	Practical, students voices
10	19/03	Yr 6/7	46:20	Fractions	Teacher led, student reflection
11	21/03	Yr 6/7	39:22	Fractions 2	Equivalent Fractions, multiples first
12	23/02	Yr 8/9/10	21:24	More Factorising	Difference of Squares for year 10
13	31/03	Yr 8/9/10	43:34	Feedback on group-work	Here feedback is sought from the College class regarding working in groups
14	28/06	Yr 8/9/10	49:32	Student choice to teach a concept	Practical, student voices and discussions with me
15	28/06	Sara	1:27	Sara's choice of concept	Clarification, cognitive mediation
16	29/06	Yr 6/7	12:28	Choice	Choice of game or concept from Maths book to learn and share
17	29/06	Yr 8/9/10	47:14	Choice 2	Mental plus working on choice of concept
18	30/06	Yr 8/9/10	52:39	Student Presentations	Student led discussion
19	12/09	Yr 10	19:42	Student Feedback	Feedback presentation organised by four Yr 10 students
20	14/11	Yr 8/9/10	50:06	Socks On Socks Off	Targeted Teaching using visual imagery
21	6/12	Yr 6/7	12:04	Revisit Rule of Order	Assessment of Rule of Order of Operations Understanding
22	7/12	Yr 6/7	30:32	Student Feedback	General feedback from 10 students

Data was collected from 20 separate data sessions, comprising 18 occasions of lesson data, two occasions of feedback, one from each class group and two occasions of follow up lesson data recorded almost six months after the initial data collection. There is approximately nine and a half hours of recorded observations from 15 lessons. Of those lessons seven of the observations are of the year 6/7 class while the remaining eight are from the year 8/9/10 class. There are two additional recordings of the year 8/9/10 class; one is the beginning of another lesson with just the ‘mental’ segment recorded and another is of an individual student. There is 340 plus minutes of the year 8/9/10 from the eight separate lessons with a 19.5 minute feedback presentation from four year 10 students plus 50 minutes of follow up explicit learning lesson using visual imagery and metaphor. There is 234 minutes of the year 6/7 class from the seven separate lessons of one hour each with 30.5 minutes of semi-structured interviews conducted in one lesson time plus one 12 minute follow up Rule of Order of Operations assessment lesson.

Thirteen of the observations were made in March while five were recorded in June. There was approximately 12 weeks between the recorded observations. Two of those weeks were holidays, one week was while I was on leave, one week was taken out due to NAPLAN testing and the two weeks prior to testing were taken out for NAPLAN preparation. The other six weeks would have been related to completing work for reporting including testing.

My first round of data interrogations was to view the video files. I had converted them all into digital video files and was able to view them on my computer. For some of them I had a setup where I would watch on my stand-alone computer which has a surround sound system and have my laptop set up with my voice activated software so that I could view and make notes at the same time. This worked quite well; with some training I found this quite efficient. I also found that I could view the video files on my laptop while I had my voice activated software set up and could switch from one window to another. Again with a little practice this was quite effective.

I also tried viewing and hand recording notes. I am pedantic and like to have things ordered and systematic. For efficiency I needed to go through this process. I also needed to organise and catalogue all of the files, record their length, their lesson title and the date of recording. I have recorded this in Table 3.3 so that I could see exactly what I had. Sometimes I missed lessons where some interesting teaching happened. As with all teaching you sometimes do not know what is going to be brilliant and what is going to be hum drum.

When one records students the intrusion of the camera can be quite unsettling. I found the younger students handled this intrusion much better than the older students. Although having said that I have evidence of younger students pulling faces and making comments that I would hear later while I was watching the files. There is nothing nasty, just some very mild silly behaviour. Those little performances for the camera did not impact on the students working, so for most intents and purposes I have ignored the impact of the camera.

When I set the camera up in the classrooms I was really only looking for voice data. I soon found after viewing the videos that I needed more than just voices. Some of the lessons where I have just my voice and that of the students' voices I have probably captured most of what I was looking for; however, I have missed the gestures and writing on the whiteboard; I have missed the body language; however, I was able to pick up on the change of voice tone. Even though I missed the body language and gestures with some recordings I do have evidence of both in other lessons. Given the range of lessons, I captured sections where students and I had been engaged in discussion in a class group, in a one-on-one situation and in small groups. I also captured evidence where students had been presenting to the class, and there was some interesting data to emerge from those observations.

It was possible to discern some other differences between the two groups and even within groups. I recorded lessons where the students were completing practical work; one lesson where students were providing me with feedback and other lessons where I was directing and/or facilitating discussions. I was concerned that with 15 and a half hours of video recordings that there was not enough and I was quite tempted to go and record more; however, I have now come to the conclusion that I probably would just have recorded more of the same. More does not necessarily equate to better. So at that stage I was satisfied that I had sufficient data recorded to be able to draw some conclusions.

I found it very difficult while I was scrutinising the data to start at the beginning and analyse each one. I think this has more to do with the way I read than anything else. It took some time, viewing the data whilst obtaining a sense of the broader picture before I could engage with the data and commence a strategic analysis.

3.5 Data Analysis

The data was organised, in the first instance, by type of data, that is, video recordings, reflections, student interviews, and student reflections and also into date and time order. Digital video recordings were transcribed verbatim and then placed into tables for more efficient and easier analysis. I developed a protocol for transcribing digital video recordings of classroom conversations and of student presentations; however, I found it just as efficient to keep the transcribed records as the verbatim narrative, which was then transferred into a table which separated my utterances from those of the students.

3.5.1 *Emergent Data using Grounded Theory Approach*

Initially I preferred a narrative record of the observed data as this empowered me to record, as literally as I possibly could, all that I observed. Through the writing of the narrative, patterns of data emerged which could be classified into specific, individual strategies. Employing a Grounded Theory approach (Glaser & Strauss, 2009; Strauss & Corbin, 1990) those specific strategies were reclassified to conform to a design which I gave expression to and transformed into the major strategies I reported in the following chapters.

A Grounded Theory approach (Strauss & Corbin, 1990) was used to guide analysis of data. Once the data had been transcribed and organised, 'open coding' or level 1 coding (Strauss & Corbin, 1990) was used to break down and examine the data to compare, to conceptualise and to categorise the data. Following this process 'axial coding' or level 2 coding was used to put the data back together after the connections between the codes had been identified. 'Selective coding' or level 3 coding followed, where the process focused on the selection of core categories and then relating those to other categories, validating the relationships between the categories. Diagrams and flowcharts were drawn to show the identified relationships between categories establishing causal and effect categories as a means to assist in the process of analysis.

Following an early examination of the data using the Grounded Theory approach the data appeared to fall into three different categories; namely, using a whole class approach, small group work and peer teaching. Using these three categories revealed usable data in the whole class approach and peer teaching categories. The data yielded with the small group work setting or category was focused more on the benefits of using small groups in multi-year and age-group settings. At that point a re-coding of the data was required; that took many conversations with my supervisor and searching for the strategies that were represented in the data. It also required a step back and examination of the big picture aspects of the data and it was at that point that the data yielded to a recoding and strategies emerged.

Examining the big picture and using a two-dimensional table, as displayed in Appendix F yielded the set of workable categories. Those categories were more related to the language focus of the research study. I commenced the examination of the data through the lens of the classroom contexts as it provided me with the means to group and identify the strategies I was employing with my students; both the intended strategies and those that arose and/or developed opportunistically.

3.6 Recording Techniques and Difficulties

Any teacher who has recorded student performances in class presentations knows there is quite often something that goes wrong with the technology. I did experience problems with both the technology and with my ability to teach and to record my own lessons. The Cooperative Colleague was not always available to assist; there were times when he had his own class to teach. I had problems with where to position the camera so that it achieved the best result for me. At other times when I was recording students working and was looking for examples of their conversations it was unimportant to actually have the camera focused and so I walked around the room with the camera pointing at the floor or wall as I found students were less nervous when I wasn't pointing the camera at them. Some of the camera work then looks very amateurish. I have shots of students' feet and images of the walls and doors, etc. It probably came about purely by accident, although it was more likely trial and error that the best spot for the camera was the back left corner where I could get most of the students in the frame; most forgot the camera was on and it also included me.

Initially, I was not concerned about having the camera on me, as I only wanted to record my voice and the language and expressions I was using; however, it became apparent that one of the things I was really seeking to examine was my body language and the gestures that I was using. This didn't really resonate with me until I was through recording so it was fortunate that I did have some footage of my body language. I needed this as one of the aspects I was examining in terms of conversation was the role of gestures. After analysing the data it became apparent to me that there was also a role for tone in that definition of conversation that I had developed. I had recorded what I considered a good example of tone and how that changed a lesson.

Being a teaching administrator put additional pressures on being a researcher and I am not sure that I would recommend it to others. Often in the recorded observations I would be interrupted by office staff, the principal or other teachers. Any teaching administrator knows that this happens on a regular basis. Just like others in the situation I have developed coping skills for both myself and my students so that they can continue to work uninterrupted.

Other difficulties encountered were to do with setting up the recording equipment. Often I would leave one class and go to another and that would mean I would be taking the equipment with me. Other times the class I was going to record would be in the classroom with another teacher which meant that I might be able to set up the equipment and leave it in the storeroom but I would then need to start it at the same time as the students and I entered the room. There were times when my colleague could set up the equipment; however, he was not always available due to his own teaching demands.

After recording I would quite often pack up the equipment while I was still talking to students and then carry the camera to my office and leave it as I was required to be out on duty, at a meeting or wanted by staff. Hence there were a number of significant demands on the teaching administrator. Always there is a balance to be achieved and I had to make a decision about what to trade off and what to leave in so that the research was not impacted, watered down or altered in some unintended way.

The timing of the recording of observations was probably not optimal either. I started recording lessons in Term 1 in the last month of the term. That in itself was not a problem; however, there was preparation for NAPLAN (National Assessment Program – Literacy and Numeracy) beginning with students in both groups. There was also a new middle school structure being trialled in response to staffing issues and constraints.

Another difficulty I experienced was what I did with the tapes after I had recorded them. Again I was lacking a required skill; transferring videotapes to digital medium. The early recordings were transferred by my colleague; the benefit of this was that it allowed us to discuss the data during the transfer process. Initially I had viewed the tapes from the camera because all I was looking for, at the time, were the audio recordings, and the spoken language. Fortunately after the first few weeks I acquired the skills to transfer the videorecording to digital medium. Capturing the video to disk enabled me to see more of the recordings and it made them give up a lot more data than I had previously anticipated.

I found, on viewing the tapes that the lessons proceeded exactly as any normal lesson would and so at the beginning of each of the observations, there was no real introduction, other than what would be in a normal lesson. That made it difficult for someone viewing the tapes. This is where, as a participant in the lesson, I was able to add in information that set the scene for the recorded observation. In many of the lessons that were recorded the camera was focused on the students, and not on me as it was mainly students I was looking for and so some of the information that was written on the whiteboards did not appear on screen. Here again, as the participant in the lesson I was able to add in the information that is not seen but was referred to in the lesson.

3.7 Ethical Issues

As this research study formed part of the normal program for the students involved there was no risk in terms of ‘harm from an external source’. As the teacher and deputy principal of the students involved I had a responsibility and duty of care for maintaining security of both their personal information and of their school achievement. Originally, I had intended for students to keep a journal of their work in the study; however, this proved too cumbersome to manage and took far too much time. Students participated in writing a reflection after many of the lessons included in the study and at the beginning of a lesson were often asked to reflect on a prior lesson. It felt like an unnecessary imposition to tie students up in recording data that I could find through the digital recordings and through conversations in the classroom; this was a far more effective and efficient way of collecting data and formed a natural part of the lessons.

There was; however, a serious concern with the potential identification of students. To maintain security of students’ personal data I randomly selected gender appropriate names. The translated list of names was kept secure and was only available to the researcher.

3.7.1 Permission, Privacy and Confidentiality

Before the research project commenced a letter with pertinent information was provided to all potential participants notifying them that they were being asked to take part in a research project; that the project did form part of their normal program, that no individual would be identified when the project data was reported and that the focus of the research was on the value adding that came from participating. If permission was not granted, students still took part in the study as it was part of their normal program; however, their individual data was not reported in the research. In each of the classes there were only two students who did not return their permission forms and one of those left early in the study. Potential participants and their parents/caregivers were also notified that students had the opportunity to withdraw their results at any stage of the project. A written guarantee of privacy and confidentiality to individuals from whom data are reported was provided.

Written permission to report results from the research study was sought from the principal of the school, from parents/caregivers of my students and from the students themselves.

3.7.2 Data Security

All raw data is kept securely as part of the normal resulting that schools carry out and the data specifically related to the research questions is being kept securely at SMEC, Curtin University. Data will be kept at SMEC for a period of five years and at the school site for a minimum of seven years, after which it will be destroyed.

Students, parents/caregivers and the school principal were kept informed of the progress of the research study and students were involved in reviewing the progressive data and responding to that data. Data collection was completed by digital video recording of classroom activities and from interviews and conversations that occurred with the participants in the mathematics classroom so that there was as little disruption as possible to the overall teaching/learning program.

3.7.3 Acknowledgement

Letters acknowledging participation were sent to all students at the completion of the research study. Participants were acknowledged in the research report with care being given that individual students were not identified.

3.8 Summary

In summary, this chapter revisited the research questions and made a case for the choice of Participatory Action Research design. The researcher, cooperative colleague, class groupings and student background were highlighted and instruments and procedures were discussed. Data collection methods and types of data were examined and discussed along with the difficulties and limitations with the data collection. The reasons for choosing the Grounded Theory approach to data analysis were examined along with a discussion of how the data emerged. The ethical issues were examined and a discussion of the ethical issues were dealt with was provided. In the following two chapters the data will be fully examined.

Chapter 4

Results and Reflections Part 1

Through a filter of how language was used in the classroom to enhance student learning the data was classified using a Grounded Theory Approach (Glaser & Strauss, 2009; Strauss & Corbin, 1990). The data are inextricably linked across the categories; however, I separated them using a two-dimensional table, as displayed in Appendix F, matching data against categories and sub-categories then extricating each of the different aspects for the purposes of clarification. Four strategies emerged, namely *Shared Experience*, *Purposeful Discussion*, *Blended Instruction* and *Student Peer Teaching*. In this chapter I will define and examine the *Shared Experience Strategies* and *Purposeful Discussion*. In the following chapter *Blended Instruction Strategies* and *Student Peer Teaching* will be defined and examined. Before commencing the examination of the strategies a brief description of the lessons that yielded the data is provided in the following subsection.

The focus is on the strategies that I believe enhanced learning given the language focus of each of the strategies and the role that each of them played in the learning experience. The *Shared Experience Strategy* was often in the form of a warm up activity such as a set of mental calculations, or a brainstorm or similar learning activity which then became the reference point, or trigger, for the *Purposeful Discussion*. *Purposeful Discussion*, where elaborated discourse was promoted, was a major strategy employed. The *Shared Experience* Strategy could occur on its own; however, *Purposeful Discussion* might not occur without first having a reference point, or trigger that will be referred to as a Cognitive Platform, which was provided by the *Shared Experience*. This was then followed by a *Blended Instruction Strategy*: Targeted Instruction, Responsive Teaching or Guided Discovery which might then include the use of metaphorical stories, analogies or acronyms. The *Student Peer Teaching Strategy* displayed some of the knowledge, skills and understandings that students had achieved. In this one strategy it was possible to see the cumulative effect of the previous strategies in a classroom where mathematical language was embedded.

A discussion of Elaborated Discourse within each of the strategies is included as that was a goal I intended to achieve with students. Just taking a measurement of how well students could demonstrate their achievement in elaborating their utterances only provided part of the picture. Broadening it out to include how and what was happening in the classroom in working towards achieving Elaborated Discourse better described this activity.

4.1 Background to the Lessons

I have focused on the data in seven lessons; four where the context was *whole of class interaction*, one where the context was *small group-work* and two where the context was *student peer teaching*. Four of the lessons were conducted with the younger group of students, i.e. Class B, consisting of year 6/7 students, and three were with the older class group, Class A, year 8/9/10 students. A brief outline of the lessons that produced the data is provided in order to create a reference point for the reader.

Lesson 1 – Rule of Order of Operations – Class B – Year 6/7

My desired outcome for this lesson was to have students being able to correctly follow the rule of order of operations to complete some simple calculations. I had planned to commence the lesson with a 15 minute warm up focused on six mental calculations. This was to be followed by a discussion with students explaining their methodology. After the discussion I had planned to give them the rule of order acronym, BIMDAS (standing for Brackets, Indices, Multiplication, Addition, Subtraction), written ‘down the board’ and to give them a hierarchical metaphor to assist their understanding. My plan was to then have the students work on the task sheet that I had constructed (see Appendix C).

Lesson 2 – Fractions – Class B – Year 6/7

In this lesson I wanted to obtain a sense of what students knew and understood about fractions. I wanted them to be able to calculate common fractions, like $^{1}/_{10}$, ½, ⅓ and ¼ of whole numbers. I wanted students to reflect on the processes they might employ when calculating the basic fractions mentioned above. I believed that a good place to start was with a brainstorm type activity centred on the word 'fraction' and what the word meant to students. I then planned to conduct a physical activity with students finding common fractions of their student group, that is, having students stand up to represent a half of the group, or a quarter, or other common fractions that could be represented. The fractions represented would be dependent on the number of students in attendance in the class. The lesson would conclude with students individually calculating some common fractions of whole numbers followed by a discussion about their methodology.

Lesson 3 – Equivalent Fractions – Class B – Year 6/7

In this lesson I wanted to follow on from the previous lesson and introduce the concept of equivalent fractions. I used group feedback as the starting point as there had been a number of students absent in the previous lesson. After the feedback, the plan was to quickly revisit the activity from the previous lesson to provide a similar experience for those students who had been absent. The next step was to have students produce some equivalent fractions by using multiples. Students needed to practice and understand multiples and then use that information to produce equivalent fractions of common fractions like ½, ⅓, ¼ and so on.

Lesson 4 – Indices and Word Problems – Class A – Year 8/9/10

I included this lesson as the *Shared Experience* was of a very different nature to all the other lessons. I had been absent from my class due to administrative requirements and a relief teacher was working with my students. The lesson provoked considerable discussion and concern about what the students were and were not learning. Some older students had come to talk to me before the lesson concerned that the younger students did not understand what they had been working on, so I had to change the way I approached the lesson. Hence the plan then was to use the beginning of the lesson to let students air their concerns and to then focus them back onto the mathematics that they had been learning and to follow this with a discussion regarding that mathematics. What followed was revision of Indices, followed by revision of Word Problems.

Lesson 5 – The Orange Problem – Class A – Year 8/9/10

In this lesson students were preparing to commence work on a practical problem, revolving around the difficulties creating a flat map of the earth. I had shown the year 6/7 students how to cut the peel from an orange in one complete sphere by cutting around half the circumference and sliding a knife in under the skin and then together we had worked out how to make the sphere flat. The students in Class A were aware of this through siblings and were aware of what I had been doing with the other class. This lesson was included to discuss the data at the commencement of the lesson. My focus was not on the collaborative group-work as such; simply on the *Shared Experience*.

Lesson 6 – Algebra – Class A – Year 8/9/10

The Class A students were given the brief to find a concept that they could teach to a student in the year below them, i.e. the year 10 students chose concepts to teach the year 9 students; the year 9 students chose concepts they could teach the year 8 students and the year 8 students had a modified brief to choose concepts that they would have liked to have understood when they started year 8. The students were allocated three hours of class time (three one-hour lessons) to prepare their 'lessons'. There were three year 8 students, four year 9 students and five year 10 students in the group when the data was collected.

Most times students were able to choose their own concepts that they either wanted to strengthen or with which they felt confident. I believed that it was better to empower students to choose their own concepts rather than artificially force a concept on them for which they were not ready or prepared. Usually when I employed this strategy, at least three to four hours class time was spent in preparation. During this time students consulted with each other and with me. Students had access to text books, the school library, the computer network and the Internet to assist them. They also had the choice of working with others to assist them with either their preparation or with joint presentations. The culture of assisting each other was established with this class group: there was no restriction on who students chose to work with or where they sat in the classroom during the preparation for their presentations.

Lesson 7 – Games – Class B – Year 6/7

The task and my expectations were different with the younger students. I had asked the students in Class B to select a concept, or a game from their workbook, Primary Maths for WA Books 6 and 7 (Neale, Webber & Lilburn, 2006); something that they liked and could share with other students. I used their workbook as a support for their learning and at that point in time had not used many lessons. Students could choose from any lesson page or from one of the games. I was not concerned about what they chose – I was more interested in how they would convey their understanding of the concept or game in a conversation with me and their peers.

The students were able to do this on their own or in small groups. All students chose to work in small groups; girls working with girls and boys working with boys. The choice of group members and concept to share was their decision. The class had a 40 minute lesson to make their selection and become familiar with how they wanted to share the information with other students. This was the first time that I had asked this class to perform this way. I really wanted the experience to be a gentle introduction into a sharing of understanding with their peers. I needed it to be a relatively pleasant experience and one where risks for students were minimised.

The boys chose games; however, the girls chose from the lessons. The boys needed to read the rules and play the games to be confident enough to share their understanding. The girls needed to learn more about the concepts they had chosen; one group of girls had chosen to learn more about factors, and they appeared to be comfortable exploring their chosen concepts.

Lesson 8 – Area of Triangles – Class B – Year 6/7

In this lesson, with Class B, students were working on finding the area of 16 triangles inscribed in a rectangle (see Figure 5.4 in Chapter 5). Eight pairs of triangles represented sections of orange peel that when laid side by side could be inscribed in a rectangle whose length was the circumference of the circle and whose height was half the circumference. This lesson followed on from an earlier lesson and was being used to find the approximate percentage of area that is added when a sphere is flattened, like making a map of the earth. In this lesson one of the aims was to find the relationship between the area of a triangle and the surrounding rectangle.

4.2 Shared Experience Strategy

I commenced the examination of the strategies with the *Shared Experience*. I refer to the *Shared Experience Strategy* as one that all students were required to experience and to participate in with the expectation that a focused discussion would follow. This strategy usually occurred at the beginning of a lesson and was used as a precursor to the *Purposeful Discussion Strategy*. A warm up activity, a brainstorm, a set of mental calculations or similar would be used. When used as the precursor to *Purposeful Discussion* the criteria for employing this strategy was not just that it could be used as a discussion reference point; equally importantly, it needed to become the Cognitive Platform for the discussion that would follow and that possibly would be referred to in the following teacher instruction. Basically any activity or learning form that focused discussion could be used. The main learning formats I used were sets of ten mental calculations; other longer calculations; group feedback; physical movement of students; brainstorms and, with small classes, a roundtable discussion group. These can be organised into three categories; Focused Tasks, Physical Activity and Cognitive Organisers.

4.2.1 Focused Tasks

A focused task is any brief learning task that provides the focus for a following *Purposeful Discussion* and may involve students working individually or in small groups. The focused tasks I used were a set of six calculations; a set of ten mental calculations/questions; a group quiz with four questions and group feedback. Essentially each of the tasks provided focus and the means by which to retrieve and activate background knowledge during the *Purposeful Discussion* that would follow.

Example 1: Set of Six Calculations

In Lesson 1, Rule of Order of Operations the *Shared Experience Strategy* encompassed the completion of six calculations by students and the collection of their solutions which were written on the whiteboard next to the appropriate calculation. As I started to collect their answers I realised that there were many misconceptions with understanding ‘the order’ of Rule of Order of Operations; there were four different responses to the first calculation. I determined that there was a wide range of experience and that using the six calculations brought out the students’ prior knowledge and understanding, or lack of, of this concept. I used a basic exercise with six calculations; each calculation involving the four basic operations with four 3’s and might include brackets. The six different calculations are displayed in Figure 4.1.

When I collected student responses I did not comment on whether they were correct or incorrect using a Responsive Teaching strategy; in this instance passive teaching. I believed that it was important to create an environment where students were comfortable providing their solutions, correct or incorrect and that it was important to acknowledge that some calculations would be wrong and that it was important for us to work out what was happening so that we could correct the methodology.

Complete the following:
3 + 3 – 3 + 3 =
3 + 3 x 3 + 3 =
3 x 3 + 3 – 3 =
3 x 3 ÷ 3 + 3 =
(3 + 3) x 3 – 3 =
3 + 3 x (3 – 3) =

Figure 4.1 – Rule of Order of Operations Calculations

I did not validate the correct responses; I went through the exercise of collection of responses with no indication of whether the response was correct. At that point I wanted all the different responses so that I could ask students to explain their method of calculation, during the *Purposeful Discussion* and diagnose misconceptions before teaching them the correct rule of order of operations.

I wanted to find out if students were just working left to right, or as I suspected following the acronym literally B I M D A S. Just collecting answers and telling students that their answer was correct or incorrect did not serve my purpose. I believed that if I stopped collecting answers as soon as I found a correct response then I would not be able to identify the misconceptions with the rule; that is, I would not be able to identify what it was students were doing so that I could provide a situation that would challenge their incorrect methodology. In itself, having a range of answers for each of the calculations would possibly challenge students' conceptions and possibly cause some disequilibrium. Table 4.1 displays the answers I collected for all of the calculations.

In Table 4.1 the responses were recorded as they were given to me. I have included the correct answer in the table; however, students were not given the correct answer until all the student explanations had been provided in the *Purposeful Discussion*.

Table 4.1 – Student Responses to Calculations

Calculation	Student Responses	Correct Answer	% Students with correct answer
3 + 3 – 3 + 3	9, 6, 0, 3	6	58
3 + 3 x 3 + 3	24, 21, 18, 15, 14	15	37
3 x 3 + 3 – 3	9, 3	9	84
3 x 3 ÷ 3 + 3	6, 3, 12, 13, 5, 16	6	63
(3 + 3) x 3 – 3	15, 6, 12	15	84
3 + 3 x (3 – 3)	0, 6, 3, 9	3	16

After collecting all the responses I believed that students had most of the order correct; however, they were literally calculating multiplication before division because that is how the rule states it and they were doing the same thing with addition and subtraction. The only way I knew what they were doing was to have them explain their method of calculation. Interestingly the third calculation only had two different responses. The one incorrect response came from a student who I believe transposed the subtraction and addition signs. Students' reasoning would be followed up in the *Purposeful Discussion* that followed.

Example 2: Mental Questions/Calculations

Students were regularly asked 10 mental questions where they would record their answers and then a discussion would follow where students explained the mental reasoning they employed to deduce their solution. This approach was often used as a springboard for new material and in some cases as revision and checking for understanding.

In the lesson that introduced 'The Orange Problem' with the older group of students, that is, in group A, the lesson commenced with 10 mental questions written on the whiteboard (See Figure 4.2). The ensuing discussion that followed where solutions were put forward was what provided the platform for the introduction of a guided discovery task. The subtle difference with this *Shared Experience* was the length of time for completing the questions and the subsequent discussion; in this situation approximately ten minutes of the lesson was used for the *Shared Experience* and the following *Purposeful Discussion*. The purpose of asking students to answer a specific set of questions, followed by a brief discussion of their solutions, was to create a cognitive platform or framework that could be referred to in the subsequent introduction of a new task.

1. How do you spell circumference?
2. What is the formula for finding circumference?
3. How do we find the area of a circle?
4. How do we find the area of a triangle?
5. How do you spell hexagon?
6. What are the sizes of the angles in a hexagon? (regular hexagon)
7. What are the sizes of the angles in an equilateral triangle?
8. What's the rule for Pythagoras?
9. Is a triangle with sides of 3, 4 and 5 a right angled triangle?
10. What is the relationship between the radius and the diameter in a circle?

Figure 4.2 – Orange Problem Mental Questions

The mental questions which students worked on, on their own, formed the basis for new learning. In 'The Orange Problem' with the year 8/9/10 students the mental questions included some spelling of circumference and hexagon, calculations involving Pythagoras, stating the relationship between radius and diameter, stating the formula for finding the area of a triangle and area of a square (see Figure 4.2).

As stated above ten mental questions were used as a *Shared Experience* for many lessons encompassing those that were based around collaborative group-work. Many teachers use mental questioning as a warm up activity before commencing the work for that lesson; the term warm-up implies preparing for something; however, it is not necessarily used to provide a link or platform into new work. As a result of my interest in language and cognition and linking previous knowledge with new learning the questions were carefully selected to provide a basis for recall and for preparation for new work. The teacher's role was one of enabling, of facilitating, guiding and managing. Of more importance was how the *Shared Experience* was set up and how students were prepared for the following *Purposeful Discussion*.

Example 3: Feedback

The strategy referred to as Feedback is one where students report information back to their group, back to the class or back to the teacher. It may originate out of small group or individual work. It may follow group discussion or may occur without prior discussion. Lesson 3 was a follow-up lesson on Fractions with Class B, which followed on from the previous lesson where had been introduced to this group of students. As several students had been absent for the previous lesson it was considered appropriate to commence the lesson with a review of what students had learned. Students were in groups of 3 to 5 and were provided with approximately two minutes to share what they had learned in the previous lesson with the students in their groups who had been absent. They were required to nominate a spokesperson who would report in one or two sentences what they had learned.

In groups, the students shared the information about standing up to 'show' one-fifth, two-tenths, one-third, three-ninths; however, in one group a student was explaining that there was ten boys in the group and that if she (the teacher) had said 'half the boys stand up', then five would stand-up. The student continued with the explanation giving an example of finding a quarter of ten. The other student replied with two and a half, to which the first student showed what it might look like if one student 'half stood up'. This did not happen. Only fractions that would give whole number answers were asked to be represented.

As part of the class feedback the first group reported back that they had written fraction down on a piece of paper and what that meant and then they had to find fractions of numbers, easy ones first and then some hard ones. The second group reported back about the students standing up to represent fractions of a group. 'Say half the boys stand up.' The remainder of the groups provided similar feedback about the practical activity, using different fractions, and about finding fractions of numbers.

Example 4: Roundtable Reflection

In Lesson 4 with the students from Class A, a Roundtable Reflection of a lesson with a relief teacher was used as a reference point to bring students' discussions to a focus on what they had learned and how they could help each other learn. Students were quite agitated and frustrated and wanted their concerns heard. Rather than just listen to their concerns, which I did, I then used that as a springboard into *Purposeful Discussion* where the students explained to me what they had been learning. Had I dismissed the students' concerns and not addressed them then very little meaningful learning would have occurred. Sometimes it is wiser to digress, to address immediate needs of students and then make the most of the situation and turn it into the beginning of a learning experience.

The following excerpt from Lesson 4 highlights the instance of listening to students' concerns and then using that Roundtable Reflection as a strategy to focus students back on to their learning.

Teacher: *What have the year nines been helping you with?*

Cameron: *Index notation and stuff. It was mainly index notation ... yeah, but they were doing .... they were doing forms of ...*

Cameron and Dale:
They were doing like, they didn't understand and Caitlin came over and helped but we were doing all right by ourselves. She, I mean, Caitlin had to ...

Cameron: *Take time off her work, because the teachers were all focused on… so we were here trying to figure it out, by the way you give them really hard work.*

Dale: *Then we had to go see Belle, because she didn't understand the thing.*

Cameron: *And we were like sitting there for nearly half an hour.*

Teacher: *Is there a problem when you work together in a group?*

Cameron: *No, it's actually fun, you learn more.*

Teacher: *Do you learn more when you teach someone else?*

Dale: *You have an experience of a younger person.*

Cameron: *You have both your view and the other person's view.*

Caitlin: *And you are repeating it so many times, it gets stuck in your head.*

Teacher: *The other thing too, and this is what I found when I first started teaching maths. I had to really understand what I was doing before I could teach someone else, because you can't ... to have to explain to someone else. If you don't know what you're doing, it's difficult.*

Lily: *Even though we do have fun talking to each other. That's no exception for you….*

The concerns had been heard; students were working on Indices and in this instance the *Purposeful Discussion* flowed on immediately following the airing of student concerns.

4.2.2 Physical Activity

This strategy involved physical movement, with students having to make decisions, communicate with other students, reflect and work together. This provided a shared Cognitive Platform or Organiser that students could refer to like 'remember when we stood up to find fractions …'

Example 1: Fraction Stand-up

In Lesson 2 with Class B, I followed up the brainstorm with an intended secondary *Shared Experience*; however, it was probably the more important part of the total shared experience of that lesson. The brainstorm had provided an opportunity to recall what students knew about the language associated with the concept of Fractions; however, I wanted to test out another approach to find out what students really understood about equivalent fractions, or fractions of a total. This had loosely been introduced by students and I thought I could use it to probe further.

There were 19 students in the class that day; an odd number, a prime number and a number not conducive to fraction finding, so I looked at the numbers of boys and girls. There were 10 boys in class and nine girls, so, I had numbers I could work with.

In this part of the *Shared Experience* the boys were asked to have ½ of their group stand up, followed by 1/10, 1/5, 2/10, etc. This proved to be an interesting experiment and took a little time with the boys to get them to understand that the number of boys to stand up when asked for 1/5 was the same as asking for 2/10. The girls found it a little easier as they were working second, had observed the boys in action and were working with a total of nine students. They were asked to present 1/3, 2/3, 3/9, 6/9 etc. The benefit of using a physical activity that involved all students was that it required participation from all students and encouraged some discussion between students as to how many students should stand up, sit down, and so on. There were some discussions amongst students on how many students should be standing up; however, this was limited to one word utterances and pointing.

In that Fractions lesson there was some thinking and practice time at the beginning followed by an active session where students were standing up and working together to produce a correct response. The very obvious and concrete examples impacted on the way that students were working out answers and participating. This proved to be highly effective in involving all students and in the follow up *Purposeful Discussion* I believed more students participated in elaborated discourse.

4.2.3 Cognitive Organisers

This strategy involves information and/or responses being solicited from students using techniques such as Brainstorming; to get ideas flowing and to gather those ideas and/or information. This is more a cognitive activity, providing an avenue for reflection and sorting of ideas.

Example 1: Brainstorm

In Lesson 2 with Class B, on Fractions I commenced with a Brainstorm. I asked students to write the word ‘Fraction’ on their page and to write down what that word meant for them. They were allowed about a minute to write down meanings. Students then had to tell me what they had written down. Students provided a wide range of responses to indicate that they had a reasonable understanding of fractions and their practical application. Some of the responses provided were agricultural in nature as that was the context in which we lived and learned.

Some of the initial responses were theoretical in nature like:

Harrison: *Fraction is a part of a whole or a few parts of a whole.*

Other responses were more personal with students offering the following:

Lachlan: *Like it’s separate from that,.. that…, you all get something.. like you get a half and the other person gets a half.*

Lachlan: *Birthday cakes.*

Teacher: *Birthday cakes, right. Important because …... why?*

Lachlan: *‘Cause then if Riley has too small a piece and I have a big piece he might ‘chuck a sookie’.*

The previous response also included an Australian colloquialism – ‘chuck a sookie’, similar to ‘chuck a sad’, referring to the other student becoming angry if the pieces of cake were not cut into the same fractional sizes.

More than half of the responses contained an agricultural flavour. Given that the brainstorm was being employed to bring out students' prior knowledge it was expected and anticipated that this would happen. I knew that many of my students helped out on their farming properties and hence had a strong practical knowledge of the language applied to fractions in use. There was even a reference to mixed units of measurement which that particular student found quite natural to use. The three following responses typically displayed students' contextual understanding:

Jamie: *Harvesting.*
Like you want two-thirds of a paddock done but you can't do the other third because it's too wet and ….

Riley: *Seeding.*

[Prompted by me, with, 'Can you think of an example?']

Riley: *… Like you might need to measure how much barley you want on the bar, or whatever. You might want half a tonne of barley and half a tonne of canola or something.*

Jamie: *Spraying.*

Teacher: Spraying, yeah, *when you've got to mix chemicals together?*

Jamie: *Or like when you're spraying a large area, say you're spraying 200 hectares and you've got 100 and you've got five inches too wet or boggy and you cut through there or something.*

My role was to accept responses, offer support, prompt with expressions like 'Can you think of an example?' and even to offer an interpretation which in the above case turned out to not be the one the student wanted to share.

Other students stayed with responses that demonstrated that they made the link within the school curriculum. These responses were not elaborated or developed beyond a very simple one word, like the following:

Science, maths, cooking.

Although a student followed up on the previous response with

Kitchen, measuring things.

It is well understood that participants in a brainstorm exercise will bounce ideas off each other and one person's response will trigger another participant to respond. This *Shared Experience* was not about spending a lot of time further developing students' basic understandings of fractions. It was more about bringing out some of the understanding and knowledge students had about the language associated with fractions and how that language might be used in contexts that were familiar to them. It worked very well in doing just that.

4.2.4 Elaborated Discourse related to the Shared Experience Strategy

In the Rule of Order lesson there was no opportunity for elaborated responses as I had limited students' responses to answers only. I believed it was important for students to 'see' the range of answers to the six calculations. That was the reasoning behind me not providing any indication, at that stage, of whether answers were correct or incorrect.

Out of 18 responses provided for the brainstorm in the Fractions lesson (Lesson 2) only four were elaborated, with two of those being prompted by me. (22% of responses were elaborated). Eight students contributed the responses. In Table 4.2 I have provided a breakdown of students' responses from the Fractions Lesson which occurred on March 19, 2010. I was concerned, at that point, with the high number of non-participating students (59%), especially the high proportion of non-participating girls. Rather than pressuring them to respond I preferred to gently persist and believed that with time more students would participate willingly.

Table 4.2 – Elaboration – 'Fractions' from Lesson 2

Non-verbal participants	Students responding	Number of Responses	Number Elaborated	Percentage Elaborated
13	8	18	4	22%

There were few opportunities for students to provide elaborated responses in the *Shared Experiences*. I was not concerned with the lack of opportunities; however, I was concerned, as stated above, with the high number of non-participating students and knew I would need to incorporate strategies to empower more students to enter discussions.

4.2.5 Reflections on Shared Experiences

There may be little, or minimum discussion during the *Shared Experience Strategy*. To a certain extent this was determined by the chosen format and as stated above there may have been a blurring between the two strategies. Before I carried out a lesson I had to examine what it was that I was wanting to achieve, examining the language requirements, thinking about where my students were at in relation to those requirements and then devising an appropriate activity or other similar learning form to bring about what I was looking for, and more specifically setting the scene for the *Purposeful Discussion* that would follow.

I used five different approaches, or rather as I prefer to call them *Shared Experience* learning situations to act as stimulants for the following-on *Purposeful Discussion*. The type of learning situation was not the important factor; it was what I could make of that situation to stimulate a follow-on focused, purposeful discussion and to create Cognitive Platforms for new learning

Benefits of the Shared Experience Strategy

The first benefit, providing the focus for *Purposeful Discussion*, would have to be the main goal for the *Shared Experience* and having said that, it can provide the focus for a rich discussion. The focused activity with the six Rule of Order calculations provided a seamless transition into the following *Purposeful Discussion* where students were required to explain how they got their answers. The 'Stand up' Physical Activity provided a source for the following *Purposeful Discussion* surrounding the explanations for finding fractions of whole numbers. The 'Fractions' brainstorm, used as a Cognitive Organiser, also served the same purpose.

The second benefit of the *Shared Experience* strategy would be that it also provides students with an anchor point, which I call a Cognitive Platform, a reference point from which to develop new understandings, skills and knowledge. The ten mental questions set the scene and acted as a

Cognitive Platform upon which a Guided Discovery lesson followed as did the Fractions Feedback.

The third benefit of the *Shared Experience* is addressing the needs of students. There is always a great deal of uncertainty when one approaches a lesson prepared to be flexible in terms of meeting the current needs of a group of students. Responding to meet the needs of students does create tension for a teacher; however, the rewards from creating the appropriate *Shared Experience* outweigh any challenges or difficulties that might be encountered. This was evident in the Roundtable Reflection where students' concerns were aired. The concerns needed to be heard and validated otherwise I would have 'lost' the group. Having listened to their concerns the lesson was then able to proceed, not necessarily in the direction I had planned but reviewing what students had learned hence addressing their needs at the point of need.

The fourth benefit and major improvement that I see for students is that I am constantly receiving feedback in terms of where students are at in terms of their prior knowledge and understanding which enables me to better target the language I am using along with the way I introduce new concepts. This was evident with the answers offered to the Rule of Order calculations. Most evident was the literal following of the rule of order.

A fifth benefit would be in making the links for students. The ten mental questions I chose for the *Shared Experience* at the beginning of Lesson 5 – The Orange Problem were linked to finding areas of triangles, circles and understanding the link with radius, diameter and circumference. I wanted students to be able to see the development and calculation of the area of a circle segment, hence the question on hexagons.

Challenges of the Shared Experience Strategy

The first challenge was dealing with the need for flexibility. I cannot be curriculum driven and I needed to be flexible as to how I used the information that came out of a *Shared Experience* that I used as a reference point for

Purposeful Discussion and to use that reference point as a cognitive platform into new work. I could usually get close to judging where I thought the group was at, in terms of their knowledge and understanding; however, there were times when I interpreted their responses and actions incorrectly and had to be a little more flexible than I had planned. A simplistic example of this was when as student offered ‘seeding’ as one of his responses in the Fractions brainstorm and I responded with mixing chemicals. That was not what he wanted to share, and so I had to give him another opportunity to expand on his contribution.

In as much as there is a benefit in meeting the needs of students it also creates challenges. There was always a great deal of uncertainty when one approached a lesson prepared to be flexible in terms of meeting the current needs of a group of students. Responding to meet the needs of students creates tension for a teacher; however, creating the appropriate *Shared Experience* and observing student behaviour can empower both students and teacher.

Managing unwanted outcomes also presents a challenge. In a *Shared Experience* students cannot always be relied upon to participate in an altruistic manner. In Lesson 3 students were requested to engage in group discussions, preparing feedback to the class on what they had learned in the previous lesson, discussing in groups prior to my collecting the feedback.

However, in one group, this discussion was quite misleading as a student misrepresented what they had learned, probably for social reasons. The student discussed and physically represented a student half way between being seated and standing up to show the fractional representation of a half, which was not one of the examples I had asked students to represent. I had deliberately only chosen fractional representations of either nine or 10 that would end up as whole numbers.

Another challenge is managing the internal tension created. Responding to meet the needs of students created tension for me as the teacher; however, creating the appropriate *Shared Experience* and observing student behaviour empowered both students and teacher.

4.3 Purposeful Discussion Strategy

The second major strategy that was employed with my students was a strategy that I referred to as *Purposeful Discussion*. Engaging students in conversation with a purpose that was focused around a shared or common experience was the strategy developed and used in my classrooms. The strategy was based on the model suggested by The Mathematical Association (2001) where students were given a problem or set of problems to work on and were then asked to describe the methods they used to get their solution. This strategy has also been referred to as 'purposeful discussion' (Steinbring, 1998). The two definitions/explanations were used as the basis for my expanded definition of *Purposeful Discussion*. The dimension of a focused discussion following a *Shared Experience* was added so that *Purposeful Discussion* really did have purpose.

Purposeful Discussion may be used by the teacher to assist in determining a students' developmental stage, students' understandings of concepts and/or to identify students' misconceptions. It may be used by students to model for their peers and to learn from their peers, for self-correction through elaboration of their methodology and thinking processes and for practice of mathematical language. It may also be used to build rapport and to establish credibility between teacher and student. The strategy that referred to as *Purposeful Discussion* does not stand alone; it must have a precursor activity that creates a *Shared Experience*, that is, the reference point around which the discussion is focused. The focused discussion can then be followed by a teacher-led activity such as Direct Instruction with the use of metaphorical stories, analogies and/or acronyms; or any suitable classroom teaching tools.

I also made use of the difference Steinbring (1998, p1) purported in reference to 'purposeful discussion'; that is, it was different to the traditional teacher 'interrogation' where the teacher asks a student a question, the student responds with an answer and the teacher evaluates the student's response, before moving on to the next student and repeating the process. This interrogation → response → evaluation, sometimes referred to as 'guess what is in the teacher's head' is different to *Purposeful Discussion* where students explain or describe their methods leading to elaborated responses which was demonstrated by the data in the lessons where this strategy was employed.

Purposeful Discussion is defined as a language derived activity that involves a group of students engaged in a focused classroom discussion related to a pre-cursor *Shared Experience* that provided the focal point(s) for the discussion. *Purposeful Discussion* promotes elaborated discourse with students providing explanations of their methodologies and reflection on their metacognitive processes.

4.3.1 Converging Purposeful Discussion

I am referring to Converging *Purposeful Discussion* as discussion that is focused, purposeful and used to bring students' thinking and background knowledge to a common point for new learning that is structured and factual and where there is usually only one form of methodology or reasoning as outlined in the following examples.

Example 1: Rule of Order

One of the benefits of using *Purposeful Discussion*, that is, having students explain their methodology, is their realisation of an incorrect process or calculation. An example of this was a student's response in Lesson 1(Rule of Order): Jamie's response to Calculation 1: (3 + 3 – 3 + 3) was 9, which was an incorrect response. He realised as he was explaining his process that his answer was incorrect as demonstrated below:

Jamie: *I used the brackets ... the BIMDAS,*
I went addition ... first,
which is three plus three, then you minus it, subtraction,
which … [unclear what he says next] ... I did it wrong

This example demonstrated how students self-corrected when they were articulating their solution. One of my fundamental beliefs and philosophies in teaching is that if I can get students to see their error in process or methodology then I am more likely to have success changing the way they do things; they are more likely to accept support from me and take on board what I am showing them. I believe that if a student does not see an error in what they are doing, then they are not going to value what I am saying.

There is great benefit to a teacher in listening to what students are saying while they are engaged in *Purposeful Discussion*. While the discussion is very focused it is possible for a teacher to diagnose students' misconceptions and to plan for ways to counter these. I found it beneficial to listen to students and to really focus on what their discussion was telling me.

In Lesson 1 (Rule of Order) I found that most students followed the rule literally. I came to understand from what students were saying that the rule was important. I continued the focus on the rule with this class; however, with older students I would examine why the rule was the way it was. Tyler, a year 7 student, provided an example of literally following the rule:

Tyler: *well, I did BIMDAS, three plus three and the other three plus three, which is the six, … , six minus six, which is zero.*

This is an example of following the rules literally; brackets, indices, multiplication and then addition and subtraction, so addition is performed before subtraction.

Eleven of the 19 students did get the correct answer for the first calculation. I surmised that many of these arrived at the correct answer purely by working from left to right. I would need to examine their responses to the other calculations to be sure of whether they understood the order. Like Harrison who stated:

Harrison: *I just ... went three plus three, take three, then add three*

This was a literal transaction, from left to right.

There were responses that concerned or confused me (like Tim's) that I was not sure how to interpret; he started with adding the two lots of 3 + 3 but then changed midstream and went to the beginning to go 6 – 3. The initial response suggested a literal observance of the rules, followed by a partitioning which effectively dropped off +3. There is a hint that he corrected his earlier incorrect calculation to then miscalculate due to having confused himself. I would need to see how he processed another calculation before I could really be sure what was going on. I have included his response below:

> Tim: *I done addition, the three plus three and three plus three. I went forward, to the front [with a hand gesture points from right to left], and then 'cause three plus three equals six, take three equals three.*

Example 2: Mental Discussions

The lesson introducing 'The Orange' problem is typical of the discussion following mental questions used as a *Shared Experience*. Most of the language I used was directing or facilitating language, with some probing by means of asking a pertinent question. There were also examples of me repeating what students were saying as means of confirming and modelling. Where it was possible students were made aware of alternative strategies and solutions.

The following excerpt is taken from a passage where the class was discussing the formula for area of a circle:

Teacher:	*Why do I get you to do radius squared times π and not π times radius squared, as it is in the book?*
Caitlin:	*Because some people square the π as well as the radius.*
Teacher:	*Kids will do π times r and then hit the squared button on their calculator. So if you're going to hit the squared button it's better to do it where you're supposed to … r squared!*

One of my concerns with mathematical language was that equation for finding the area of a circle and I had made a very conscious decision years ago to teach that formula as $A_{\odot} = r^2 \times \pi$ (The area of a circle is equal to the radius squared multiplied (or times) by pi.) I also made use of language with the circumference formula and taught that as the diameter multiplied by pi, which then translated to twice the radius multiplied by pi. With both of the examples students had been guided through the activity of measuring circular solids and their diameters to gain an understanding of the ratio of pi. Students had also cut up a circular area and rearranged it to arrive at the formula for area. Hence the rearranged formulas had followed naturally. I also wanted to make the point that students understood that the way they used the formula was no different to the way they saw the formula in the book and that they also understood the reasons why they used the formula in a rearranged form.

This is one example of using *Mathematical Linguistic Pedagogy* to enhance student learning by understanding how language sometimes in its purest form creates misconceptions. Taking the example of the formula for area of a circle, the language or words, give it as pi multiplied by the radius squared with the emphasis, unintentionally on squared. Hence students who have not fully grasped the concept do exactly that; they perform the operation $(\pi \times r)^2$. Understanding and then translating the knowledge into a workable form that students are able to relate to, can remove the point at which the misconceptions are created. Carefully analysing language, seeking out the parts that can cause possible misconceptions and then finding alternative ways to use the language which still enables the correct teaching of the concept empowers students to understand that concept and then at a later stage pure language can be used. I will follow this up in the next chapter where I examine the data related to linguistic pedagogy.

Another example, discussing the formula for area of a triangle after a student had offered the correct solution and I had written it on the whiteboard illustrates the use of language as both a source for misconception and a means for clarifying understanding.

The formula was written:

$$A_{\Delta} = \frac{1}{2} \text{ base} \times \text{height}$$

Teacher: *What does it mean?*

Cameron: You're halving the base and timesing it by the height.

Teacher: *What's another way?*

Cameron: *You can halve the height or you can halve the base or you can halve the whole thing at the end.*

Dale: *Ah, hah! You times the whole thing and then just halve it.*

Cameron: *Divide it by 2.*

Teacher: *Divide it by 2.*

Repetition is something I have seen myself do many times. As in the example above after a student provided a viable solution I would repeat it as stated and often would repeat it again in formal language if the student's solution was informal.

The previous comments were employed as a point to further extend the concept and to re-link it with previous knowledge in that we had used a rectangle as the base object and that the triangle's area was half of the area of the rectangle surrounding the triangle. *Purposeful Discussion* enabled the means to use language to create the visual image of the triangle and rectangle:

Teacher: *The concept we want people to understand is that the area of a triangle is equivalent to half of the area of the rectangle that surrounds it. That's the concept, but the maths that goes with it is that you can halve the height or half the base but don't halve... what happens if you halve both?*

Lily: You have to times that by two again.

In the above instance the strategy was not intended as a lengthy discussion but more as a quick recall confirming knowledge and understanding of the skills that would be required later in the lesson. However, even within a 10 minute session, there were opportunities to use the language to further develop and enhance conceptual understanding and to make students aware of their cognitive functioning through a reflective process.

It is important to remember that the students in this group had a range of abilities and ranged in age from 13 to 15 and that the younger students were able to learn from the discussion that was carried on with more confident students. Of course there could have been a situation where students were not engaged, and were not listening and did not receive the benefit of being a part of a conversation even as a spectator.

Rather than just having the teacher provide answers for the questions, students were invited and encouraged to provide their solutions and to offer explanations. Where there was confusion or disparity amongst answers I stepped in with a few key words or asked a probing question that redirected students.

4.3.2 Non-verbal Communication Enhances Purposeful Discussion

The action of air drawing/writing is more than a gestural support for oral language. Gestures aligned with spoken language act as a powerful tool to both support and strengthen the spoken word. Air writing assists students to clarify and explain their actions. This was an unintended strategy that was evident within the older group of students, in Lesson 4 where its use enhanced their explanations.

I associated Air writing with *Purposeful Discussion* as its use appeared to be as a support, and to clarify and extend the spoken words. In my own case I use it to support reflections on my cognitive processes.

Defined, Air Writing is the action of writing in the air the mathematical equation, statement or calculation at the same time as the oral language it accompanies. After observing this in my students there was an awareness that it was something I did, as solutions were explained and as I shared reflections of my cognitive processes.

There was considerable evidence of students using hand gestures and/or air writing to support, enhance or even clarify what was being said. In Lesson 2, in explaining his process the student had the correct answer, reflected on his thinking and was beginning to use air writing/drawing as a supportive gesture.

Lachlan: *I did 15. I did three plus three [He has his right hand in front of his face. He has his hand going up and down as he is saying three plus three] which is six and then I did six, wait, I know how to ....*

There was the beginning of air writing in the lesson on rule of order with Class B. Tim used his hands to gesture returning to the beginning of the calculation when he confused himself.

Air writing really became apparent in Lesson 4, with Class A. At least three students used it continuously to assist and support their explanations of working with index problems. This is highlighted in the following extract from Lesson 4 where the discussion was referring to fractions raised to a power.

Caitlin: *Fractions, when you do fractions.*

[Holding up her left-hand, thumb and index finger, spread apart.]

[Her hands move in a circular motion.]

Teacher: *So like if I had three over four and it's all to the power …*

Here I was writing three over four and used my hands to put brackets around that and then put my right hand up to my upper right of the brackets to show where the power would be.

Teacher: *So, ... okay, ..., what would you do?*

[followed by more hand gestures]

Caitlin: *Power of four.*

She was air writing three quarters multiplied by three quarters, multiplied by three quarters multiplied by three quarters.

Caitlin: *You times the numerators and you times the denominators.*

Teacher: *So, if you had …*

My left hand is up in front of me again.

Teacher: *… all to the power of four.*

I wrote three over four with my right hand, while my left hand was the bracket. I used both hands to go three to the power of four; my right hand indicated the power. I moved my right hand left to right underneath that. I then wrote four to the power of three. I employed this strategy and quite possibly used it frequently.

4.3.3 Non-Converging Purposeful Discussion

In this form of *Purposeful Discussion* there was no one apparent correct method or process, students were encouraged to reflect on their own processes and the discussion opened up rather than becoming limited and stalling as outlined in the following example.

Example 1: Reflective Thinking

An observation that came out of Lesson 2 is that when students are engaged in conversation about their thinking eventually they start to use the language of thought and state *'I was thinking,'* and there is evidence of metacognition being explained in what they are saying.

The pauses in my utterances also were aimed at students' thinking and mediation of thought processes. That was another deliberate strategy aimed at modelling or at least scaffolding a way into thought mediation. There was evidence in the Fraction lesson that this was happening. I also saw evidence of students using the same processes, of issuing utterances, pausing, starting again, repeating themselves and then clarifying what they were saying. Maybe this is part of a process that we all go through. Irrespective, it was seen as an 'okay' process in the classroom. A student did not have to give a glib or confident answer to be heard. There were one or two students who started to explain their thought processes. They were heard to say '*I might have done it like*', or '*I thought ....*'

In the Fractions lesson (Lesson 2) students were asked to find a half, a quarter and a tenth of one hundred. They were given time to work on this. After the elapsed time they were asked to offer solutions. They were expected to explain their reasoning if they offered an answer. They were then asked to find a half, a quarter and a tenth of 162. Girls who had not previously engaged in the discussion provided very detailed accounts of their methodology and their thinking as demonstrated in the following examples:

Chloe, finding ½ of 162

> *I got 81, I halved 62, which is 31, then I halved a hundred which is 50 then I added 31 and 50 together.*

Amelia, finding ½ of 162

> *I got 81, and I divided two into 2 which is 1 then divided six into two which is 3, then divided 100 into 2 which is 50 and added them together.*

The notion of two into two, six into two and 100 into 2; the language is not strictly correct; however, the intent and the purpose is understood. The student is partitioning 162 into two groups. Later this might present some problems for the student and one that I would need to be aware of and correct indirectly rather than confront. With this student, if I confronted her with her language use as being incorrect she would not have believed me and I would have made no change. However if I modelled the correct use of the language or paraphrased, something like 'oh, you went six divided by 2, and so on' I might have more success in improving her explanation of her reasoning.

Harrison, also explaining how he found ½ of 162:

> *I got 81 an I did it, I took 6…, 80 from 6, … I divided 160, which is 80, then I divided the two, which is 1 and that's 81.*

Not only is this an elaborated response, it displays self-correction as the student is reflecting on his own process.

In this next excerpt one of the students introduced the slow train metaphor, explaining how people worked out the fractions in different ways. Accepting the metaphor empowered students to have different explanations validating the different ways of working out solutions.

Jamie: *um, I, ... I ... I put this into like a real-life situation. Say Ella's way ... she how, she divided it up [yep] and that's the long way so that's something like going Cunderdin to York back into Perth… but if you go like Logan's way that's straight to Perth.*

If you go some other people's way it's to Northam, then you go out to York and then go into Perth.

Student A: *So how did you do it Jamie?*

Mia: *He took the long way.*

Jamie: *Like Logan, I took the express.*

Harrison: *Did I take the express?*

Teacher: *... no you didn't, you were close to it.*

Student A: *You went to York*

Keith: *He went to Cunderdin, He was on the Prospector (train). I did 160 divided by two is 80, because 16 … divided by two is eight, … and two divided by … one, I mean divided by two is um one.*

As much as the above excerpt may appear to have represented a side step in the lesson, it was invaluable as a means of encouraging students to enter the conversation. It was also a ‘safe’ way to acknowledge that students performed their calculations and reasoned in different ways and that was ‘okay’.

4.3.4 Elaborated Discourse and Purposeful Discussion

Of the18 explanations provided in the early part of *Purposeful Discussion* of the Fractions lesson (Lesson 2), only five were elaborated. (So, only 28% were elaborated). Nine students contributed to the responses. In the second half of the *Purposeful Discussion* lesson, out of 18 explanations 14 were elaborated. (Here 78% were elaborated.) The questions were more complex than finding a half, a quarter or tenth of one hundred; however, students were able to give quite elaborated responses, inferring that having practiced elaborating less complex problems, students were then able to offer elaborated explanations. This time more girls responded and they were deliberately chosen first so that they were included and were heard.

I inferred from the data that students improved the rate of elaborated discourse after they had been given examples and after they had practiced as a class. After practice, the percentage of elaborated responses increased from 22% to 78%.

Table 4.3 provides information about the number of students who provided responses during *Purposeful Discussion*s in three of the lessons. Lesson 2 had two separate *Purposeful Discussion*s. There were 19 students in the class for Lesson 1 and 18 students in the class for Lesson 2. In the first *Purposeful Discussion* in Lesson 2 (2a in the Table), nine out of the 18 students responded. In the second *Purposeful Discussion* (2b in the Table), 13 out of the 18 responded. As mentioned elsewhere, the value of practice could have contributed to the increase in participation of students and also to the increase in the number of elaborated responses. In Lesson 4, four students out of seven provided the responses.

Table 4.3 – Elaboration in Purposeful Discussion

Lesson	Number of responders	Number of Responses	Number Elaborated	Percentage Elaborated
1	10	21	10	48%
2a	9	18	5	28%
2b	13	18	14	78%
4	4	40	10	25%

Table 4.4 provides information about the number and percentage of students in the class who participated in the *Purposeful Discussion*s. So, in Lesson 1, ten students out of a total of 19, provided 21 responses and of those responses just under half were elaborated. In Lesson 2a, nine students out of a total of 18, provided 14 responses, of which just over a quarter were elaborated. In Lesson 2b, 13 out of 18 students provided responses of which about three-quarters were elaborated. In Lesson 4, four out of seven students provided 40 responses, of which one-quarter were elaborated.

Table 4.4 – Number of Students Responding in Purposeful Discussion

Lesson	Number of responders	Total Students	Percentage Participating
1	10	19	53%
2a	9	18	50%
2b	13	18	72%
4	4	7	57%

Purposeful Discussion supports and encourages elaborated discourse. Whilst the discussion may appear free-flowing and the teacher's role is one of managing and encouraging the participants' conversations, there is structure and focus as a consequence of the precursor *Shared Experience*. There was rich data related to students explaining their processing; that is, in the way that they had gone about searching for and working out an answer in Lessons 1 and 2. The language I used was, at times, very probing and may even have appeared to frustrate the students as they knew what the answer was and answered 'just because I know'.

4.3.5 Reflections on Purposeful Discussion

From what I have read about previous and contemporary research related to classroom discourse is that in most classrooms the teacher 'guided' the discourse to a convergent point to get students to see a 'truth' or point that she/he was trying to make to the students. So this previous research has reported that classroom discourse has been aimed at being convergent.

In Lesson 2, on Fractions, the classroom discourse was in no way aimed at producing a convergent result. In that lesson the *Shared Experience* focused the *Purposeful Discussion*, which revolved around students explaining and reflecting on their methodology of finding common fractions of whole numbers, like 1/10 of 150, ¼ of 162 etc. There was no 'one right way' to find an answer. I have an abhorrence of algorithmically driven teaching and try to move away from it wherever and whenever I can.

What I was hoping students would do was share the way they found their answers; reflecting especially on the methodology that worked for them. I did not want them to perform the operation in a specific way. I really did not care how they did it; all I cared about was that they understood how they got their answers and were able to share that. Interestingly that was the lesson where more students became involved in the *Purposeful Discussion*.

That then lead me to reflect on what else was happening in those five lessons. In the first lesson the *Shared Experience* pointed students in the direction of discussing their methodology for finding the answers for a set of six calculations based on the Rule of Order of Operations. The *Purposeful Discussion* in that instance was most beneficial for self-correction and for me to diagnose the errors students were making and the possible reasons for those errors. In some ways that *Purposeful Discussion* was convergent as there is only one way to complete a calculation if you are following the Rule of Order correctly. That led to Targeted Instruction.

In Lesson 2, The Fractions lesson, the *Purposeful Discussion* was divergent; students introduced the slow train metaphor (See section 4.3.3, Reflective

Thinking). There are many ways to complete a calculation and get the correct answer. In Lesson 4, students were demonstrating their understanding of indices and operations with indices. This was neither a convergent or divergent discourse. Students were offering the examples that were discussed. It did not lead to Targeted Instruction but did lead to me talking about misunderstanding language in word problems. In Lesson 5, The Orange Problem introduction, students were asked to reflect on the way we used formulas and to make cognitive links between area calculations for various two dimensional shapes in preparation for new learning. In this instance the process was more convergent.

Convergent or not convergent discourse; both were evident in the *Purposeful Discussion*s. A teacher could have reason to use either, dependent upon what they were hoping to achieve. I believe it is important for teachers to reflect on how they might use a *Purposeful Discussion* and how a non-convergent discussion might lead to improved student confidence in their processes and reasoning and better understanding by students of those processes purely by the teacher allowing a divergence of discussion and through acceptance of student practice that might be different, even if not incorrect.

Of course it has to be accepted that discussions or conversation in the classroom is a valuable learning tool before any deeper analysis has meaning or value for anyone other than me. There appeared to be value in looking at the pauses in the discussions and where they were used as part of encouraging thought and for students entering and joining the discussion. This could be seen as evidence of scaffolding in thought mediation where the teacher and students are finishing off a teacher's sentence together and I have seen this happen on several occasions; especially in lessons with the older students. With the older students this appeared to be a well-established strategy.

Increasingly students demonstrated aspects of metacognition in their explanations and with practice students became more adept at elaborating their responses. As stated above, as students were able to explain their reasoning or process I gained a better understanding of what they were doing which did two things – application in diagnosing misconceptions and supporting good reasoning or procedures. Secondly it encouraged student to better understand their processes and reasoning and I believe it empowered students as well as lifting their confidence. There was an indirect benefit of peers modelling elaborated responses. All students could benefit.

In the Lesson on Rule of Order of Operations the nature of the explanation lent itself to more elaborated responses whereas the nature of the explanations for using fractions and for finding fractions of a hundred or other numbers required a different level of thought processing from students. Choosing the Rule of order of Operations lesson as the basis for building elaborated discussions was a fortuitous decision as mentioned above: the nature of the explanations enabled students to experiment with describing their methods as the structure of the response was provided in the sense that they started with a number followed by a mathematical operation, and this action and explanation was replicated for each step of the problem.

Benefits of the Purposeful Discussion Strategy

The first benefit of the *Purposeful discussion* strategy would have to be encouraging mediation of thought. From the examples in this previous section it is apparent that provided students are offered opportunities and encouraged then mediation of cognitive processes can occur as demonstrated in the 'I was thinking…', 'I thought…' or 'I was thinking … but …' responses. Students reflected on their processes when they were given the opportunity.

In the Fractions lesson (Lesson 2) approximately 2/3 of my language was directed at thought mediation with about 1/3 directed at clarification and/or expansion. The main aim here was to find how much time was spent with language that mediated students' cognition and what words or phrases invited students into the conversation. A teacher's actions, what is said and how and when it is said can empower, exclude or facilitate extended conversations and participation in *Purposeful Discussion*. A second benefit would be that of the facility for self-correction. One of the benefits of using *Purposeful Discussion*, that is having students explain their methodology, is their realisation of an incorrect process or calculation. An example of this was a student's response in Lesson 1(Rule of Order). Jamie's response to Calculation 1: (3 + 3 – 3 + 3) was 9, which was an incorrect response. He realised as he was explaining his process that his answer was incorrect as demonstrated below:

Jamie: *I used the brackets ... the BIMDAS, I went addition ... first, which is three plus three, then you minus it, subtraction which [unclear what he says next] ... I did it wrong.*

This example demonstrated how students self-corrected when they were articulating their solution. One of my fundamental beliefs and philosophies in teaching is that if I can get a student to see their error in process or methodology then I am more likely to have success changing the way they do things; they are more likely to accept support from me and take on board what I am showing them. I believe that if a student does not see an error in what they are doing then they are not going to value what I am saying.

A third benefit would be that of a teacher 'hearing' what students were saying. As stated earlier there is great benefit to a teacher in hearing what students are saying while they are engaged in *Purposeful Discussion*. Here, where the discussion was very focused, it was possible to diagnose students' misconceptions and to plan for ways to counter these. There are also occasions where students' comments or intentions may have been misinterpreted and through discussion it was possible to clarify that misunderstanding.

One feature of the extended 'whole of class' conversation strategy was to identify or diagnose problems with students' understanding of concepts. Hence the fourth benefit would have to be diagnosing students' misconceptions. Through my response to students' explanation and elaboration of processes it became apparent that I had identified possible misconceptions and then created opportunities to challenge those misconceptions. This was highlighted in the data from the lesson on Rule of Order of Operations after I had collected answers to the warm up questions. *Purposeful Discussion* enabled students to explain their methodology hence enabling me to pinpoint the errors that students made in their calculations. It became apparent that they followed some of the rules correctly but had different interpretations for other parts of the rules. Just getting their answers would not have identified how they were working and what they were thinking.

I had recognised from their responses that students were interpreting the rules in different ways. I elicited all responses in order to understand what they were doing. In order to do this the environment had to be safe for risk-taking. This was something that I had been working with the students on for five weeks; however, it still needed to be established each lesson.

As stated earlier in this chapter, there may be a sequential link between the three major strategies; however, in some of the lessons there did appear to be a cyclical approach. Whilst, at times, there is a very clear and distinct transition from one strategy to another there were times when this was a little more fluid.

Just as *Purposeful Discussion* follows the pre-cursor *Shared Experience* strategy I believe there is a need to follow up with an appropriate teaching strategy to maximise the learning opportunities that come out of the elaborated discussions on process and the explanations of reflections on metacognition.

Challenges of the *Purposeful Discussion* Strategy

The greatest challenges were those associated with engaging non-participating students. Most of the difficulties I encountered were frustration in the early lessons with non-participation of many students. There appeared to be a pattern of domination within the group that had carried over from the previous year. It was something I was very conscious of and deliberately worked on by encouraging other students to participate. It appeared that as students became more confident and the classroom environment supported students ‘having a go’ more students became involved.

The other observation that was clear was that it was a challenge to involve more girls in the discussions. The boys tended to dominate the conversations and through not calling on the girls specifically and embarrassing them or putting them on the spot, I chose to avoid directly engaging them until they were ready. Knowing how difficult it was to involve more students, I found ways in later lessons to only allow one person per group to speak rotating through every group before another person in the group could speak. I also found ways to ask a group for an answer; without putting students on the spot too much and with some choice as to who in the group answered.

For me as the teacher I sometimes found it a challenge to not jump in and ‘takeover’ the conversation rather than gently encourage and guide it to where it needed to go. Being responsive to meet the needs of a diverse classroom of students is also difficult. Listening to students explaining their methodology and attempting to interpret that instantly before moving on to listen to the next student is difficult, if not almost impossible; however, it is possible to get a sense of what the students are revealing.

As the teacher, my role was required at times to be a seemingly passive role and to be restricted to that of facilitator, managing, encouraging, clarifying and guiding students’ contributions. When the strategy had been established with a group of students my role changed to one of participant; however, this was dependent on the group and how well the strategy was established.

Time was a challenge in several ways. One of the difficulties I felt was that benefits might not be immediately evident. Hence a teacher could experience the feeling that change was not happening. Interrogation of the data revealed how little time was spent in each elaborated utterance. Sometimes, just a few seconds were required to have an elaborated utterance; however, it is the sum of all the utterances that make up an extended conversation. The parts of the conversation may have been very brief; however, the whole could extend over a few minutes, or more often longer.

Intense interrogation of the data was a difficult task and I don't believe that, for me, there was an easy way to do it other than to transcribe word for word, in order, with some recognition of the time-frame. Initially I was using the time reference purely as a reference point to be able to go back and to re-interrogate the data or to be able to check that what I was seeing was actually what was happening. After doing this for a few of the lessons I became aware of how short the time span was for the utterances that made up the conversation, as stated above, and that was when I really discovered the importance of the time-frame as a measure of how much time was spent on specific dialogue and how short the time was for each person's contribution in a meaningful conversation in the rapid-fire utterances that occurred in the classroom environment. There was a real need to be aware of allowing 'think' time and not rushing students.

The pace of lessons also presented a challenge. Even when the lesson involved mainly discussion and in the Fractions lesson I believed I really laboured on a point of understanding to the point of appearing to frustrate some of the students that only took up a couple of minutes. The real time as opposed to the discussion/conversation time and the importance of that time did not seem to correlate. In the classroom extended conversations can be managed without them taking up an entire lesson where the benefits are tangible.

4.4 Reflections Focused on the Research Questions

Each of the research questions is examined in turn with a discussion provided for each of the questions. The reflection focuses on the strategies in Research Question 1, the role of language in Research Question 2 and the benefits and challenges in Research Question 2.

4.4.1 Research Question 1

An appropriate place to begin a reflection would be with Research Question1: What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)? The strategies involved in answering this research question are the two discussed in this chapter – namely the *Shared Experience* and *Purposeful Discussion* strategies. It was only through the use of the Grounded Theory Approach that I could see for myself what the strategies were, how they were used and how I arrived at the points where the strategies were used.

The two strategies were predicated on the constructivist philosophy that we all construct our own knowledge based on our prior experiences. This would have been the starting point where I would have begun to look for ways to use classroom strategies focused through language, but how does one intervene to try and simulate a shared experience that all students might experience? My response to that question would have been through the use of language to embed a framework, or platform, that all students can take up. There may have been reasons why that might not happen as outlined previously in the challenges associated with the strategies.

Within each of the strategies there were a range of ‘sub-strategies’ namely; for the *Shared Experience* there were those I referred to as Focused Tasks, Physical Activity and Cognitive Organisers. The examples provided for Focused tasks were Six Calculations, Mental Questions/Calculations, Feedback and Roundtable Reflection. The example provided for Physical Activity was the Fraction Stand-up and for the Cognitive Organiser was a Brainstorm. The sub-strategies for the *Purposeful Discussion* strategy were divided in two – Converging *Purposeful Discussion* and Non-converging *Purposeful Discussion*. The examples offered for Converging *Purposeful Discussion* were Rule of Order and Mental Discussions. The example provided for Non-converging *Purposeful Discussion* was Reflective Thinking.

4.4.2 Research Question 2

The next step is to reflect upon Research Question 2: What is the role of language in the application of those strategies to engage students in extended learning conversations? As the data has demonstrated language can at times play a critical role in how students interpret and understand the information that has been shared with them. Language has the power to be persuasive but can also cause misunderstanding and confusion.

The requirement that an understanding of how language is used for communication is taken for granted. The requirement that how language is used creates an inclusive or excluded class group is less understood and needs to be taken seriously. Without this understanding the strategies that would be created would be less effective. Having an understanding of how language can be used as a scaffolding and modelling tool would also be an advantage. The response to research Question 2 is provided in three parts – language used to create a closed learning community, language used as a teaching tool and the role of elaboration.

Language Used to Create a Closed Learning Community

Language can be used to create a closed learning community. Bernstein (1971, 1973, 1975) referred to this as Restricted Language code where language relevant to a sociocultural subgroup had its own ‘language’. Through developing a shared language that occurs during extended conversations the group effectively acts like a closed group with a language of its own. This is represented in the table below. In creating this closed community it is about the use of language to create an environment that is conducive to risk-taking on the part of students, hence increasing their willingness to participate and to share. In focused discussions with students their contributions can be accepted, acknowledged and validated without recrimination.

Table 4.5 – Strengthening the Learning Experience
Using Classroom Strategies Focused Through Language

Shared Experience	• Focused Tasks • Physical Activity • Cognitive Organiser	Groundwork for Closed Learning Community (Restricted Code)
Purposeful Discussion	• Convergent Purposeful Discourse • Divergent Purposeful Discourse	• First create the Closed Learning Community • Second, can bridge the gap between Closed Learning Community and Formal (Elaborated) Codes • Elaborated Discourse

Language Used as a Teaching Tool

The way in which the language is being used either supports or hinders learning through the use of language to embed a framework, or platform, that all students can take up. Language for learning is focused purely on how language is employed in the classroom; this is the mechanics of language use. There needs to be a constant cycle of using the language as a teaching/learning tool, listening to students and reflecting on what they really are saying. A teacher would need to continually reflect on their own language usage as well as on how their students responded. This is a demanding aspect of the teaching/learning process; however, not one that could be overlooked. A teacher would also need to examine how every aspect of their 'teaching' using classroom strategies focused through language contributes to student learning.

The 'Orange Lesson' is an example of this; where the warm up activity – a set of ten mental questions – formed the basis for a discussion surrounding the concepts that students would be required to use in the Guided Discovery activity that followed. Referring to the formula for finding the area of a circle,

$A_\odot = \pi \times r^2$, used with students initially as $A_\odot = r^2 \times \pi$, and stated as 'the radius squared timesed by pi', then transitioned to $A_\odot = \pi \times r^2$, where students square the radius first and then finally where students use the formula as it is given. Similarly, with finding the circumference of a circle, students use $C_\odot = d \times \pi$, then the diameter being twice the radius is developed with students, using those words, hence the formula becomes '$C_\odot = 2r \times \pi$, twice the radius timesed by pi'.

Language is used to make the links, set up for students to engage with. The language used assists students to make the cognitive link and the jump through the formula 'shifts'. Here also the language is used to create visual imagery which also assists with creating the link cognitively. Along with creating visual imagery language can be used to create a springboard or platform for new material. It can be used to check for understanding through listening to students' responses, for recall of information, knowledge, understanding and/or skills levels. It can be used for sharing and consolidating, assessing the mood and readiness for learning by listening to the language used by students to express their thoughts and processing.

Language can be used as a Cognitive Platform, like the Fractions Stand-up by something as simple as saying 'remember when we stood up to find fractions'. As well as testing students' understanding language should be modelled by teachers linked to concrete and/or physical activities which occurred with that same Fractions lesson. The language modelled would have been similar to 'one-fifth is the same as two-tenths' during and/or after students had completed the activity. Language in the Brainstorm created a classroom context using contexts that were familiar to students, bringing in student experiences and making links with what they know. Here students modelled language use for their peers, self-corrected and were encouraged to use correct mathematical language through teacher modelling.

The *Purposeful Discussion*s were language derived activities that encouraged and actively promoted reflection and metacognition. Brown's (2001) 'hedging' or approximation may have been seen in the early stages when students used safe words like ‘I think’; however, this was evident later as examples of students reflecting on their thought processes. In the *Purposeful Discussion* activities language was also used for picking up errors in how students used language to explain, inferring a potential misconception in subsequent years when students would be working on more complicated problems. The student who explained her methodology for finding half of 181 stated she ‘divided two into two, which is one, then divided 100 into two which is 50 and then added’. She was partitioning; however, did not seem to be aware that she was doing that.

It was also very evident that non-verbal means were used to support the language being used. Air drawing was a good example of how gestures were used. The non-verbal supports appeared to be employed to enhance, clarify, or extend and for some appeared to make the links back to previous visual imagery that had been created and linked to that language.

Visual imagery was supported, or linked with language by using metaphor and analogy. The student introduction of the train metaphor to explain differences in processing an answer provided validation for different ways of doing things. When it was accepted by the group it sent the message that it was ‘okay’ to have different ways to do things.

The Role of Elaboration

The strategies incorporate the use of language as a teaching tool, language used for communication and language used as a means of creating a restricted community, where inclusion and acceptance are implicit. Atherton’s (2011) comments on Bernstein’s codes, captures the essence and value of creating a closed or restricted community as outlined in Chapter 2. So what purpose does elaborated discourse serve? The following list provides substantial reasons for employing elaborated discourse as a part of the classroom strategies focused through language:

- to reflect understanding/thinking,
- to help develop understanding,
- to enable and demonstrate self-correction,

- to reflect problems,
- to reflect learning as being a social experience, and
- to move students from participation in a closed, inclusive group to being participants in the broader mathematics community.

For students to be able to elaborate their responses they must learn the skills of elaboration to be able to engage in conversation and to convey what they have learned. Hence teachers need to model the skills and provide time and opportunities to engage in extended learning conversations that require elaborated responses.

4.4.3 Research Question 3

Finally, reflecting on Research Question 3: What benefits are created and challenges encountered when those strategies are introduced into the mathematics classroom to engage students in extended learning conversations? A fitting starting point would be to examine how the strategies were introduced, what they looked like and what could go wrong when implementing the strategies. Basically this question is examining how the strategies are adopted into the classroom and what benefits and challenges one might encounter. Examination of the benefits is a logical place to start. Benefits created through use of the *Shared Experience* and *Purposeful Discussion* strategies are first that the *Shared Experience* strategies provide the focus for the following *Purposeful Discussion*. Both strategies create anchor points or Cognitive Platforms for further learning. The strategies address the needs of students and the teacher receives almost instant feedback about students' understanding.

Purposeful Discussion , both converging and non-converging encourage mediation of thought with considerable evidence to show students using the language of 'I think', 'I thought', 'I went', and then often changing direction of an explanation after they realised they were incorrect. So self-correction is also a benefit. There is also benefit in hearing what students are saying, as it is here that teachers can diagnose errors and misunderstanding. It is then possible to find ways to undo a misconception or correct an error in a way that does not discourage a student from further participation.

Examination of the challenges cited for each of the strategies reveals some insights into the answer to this question. The following difficulties were associated with the two strategies – *Shared Experience* and *Purposeful Discussion*:

- the need for flexibility;
- meeting the needs of students;
- managing unwanted outcomes;
- managing the internal tension;
- non-participation of students;
- the teacher holding back;
- time and pace of progress; and
- pace of lessons.

The challenges were outlined previously in this chapter. They are not insurmountable; however, they should be addressed when preparing to implement the strategies. The difficulties were centred on the unpredictability of the direction and pace of a lesson when these types of strategies are employed which can cause major internal tension and discomfort for a teacher. A teacher may feel that the risk of implementing these strategies is too great when there is a requirement to adhere to the timeframe of a content based curriculum. In my case I was able to make the choices of what aspects of the curriculum were covered and how they were covered. In hindsight, the control and management of what happens in an individual teacher's classroom would also appear to be a major difficulty.

The following chapter examines *Student Peer Teaching* and *Blended Instruction* strategies which may ease some of the concerns a teacher might have with risks being outweighed by the benefits.

Chapter 5

Results and Reflections Part 2

This chapter follows on from the previous chapter where the strategies *Shared Experience* and *Purposeful Discussion* were introduced and examined. The *Blended Instruction Strategies* and *Student Peer Teaching Strategy* are introduced and examined in this chapter. The strategies were identified and extricated in a similar manner to the strategies examined in the previous chapter using a two-dimensional table matching data against categories and sub-categories. The examination and discussion of the strategies follows the same format as Chapter 4 with the *Blended Instruction Strategies* examined and discussed first, followed by the *Student Peer Teaching Strategy*. Within each of the strategies the sub-strategies are examined with examples provided. A discussion of elaboration is provided for each of the strategies as is a reflection on each of the strategies. The chapter concludes with a reflection on the research questions.

A brief outline of each of the lessons referred to in this chapter was provided in Chapter 4 to place the strategies into context. Table 5.1 displays the links between the examples and the strategies.

Table 5.1 – Strategy Examples

Example	Lesson	Strategy
Metaphoric Links	1	*Blended Instruction*
Creating Visual Links	3	
Problem Words	4	Responsive Teaching
Foreground or Background	8	
Establishing the Strategy	6	*Student Peer Teaching*
The Established Strategy	7	

The sub-strategy examples relate to the following lessons: Metaphoric Links refers to Lesson 1; Creating Visual Links refers to Lesson 3; Problem Words was part of Lesson 4 and Foreground or Background refers to Lesson 8. The *Student Peer Teaching* sub-strategy examples refer to Lessons 6 and 7.

5.1 Blended Instruction Strategies

In this section the focus is on the explicit teaching strategies, referred to as *Blended Instruction*, that I believe enhanced learning given the language focus of each of the strategies and the role that each of the them played in the learning experience. Specifically, *Blended Instruction* is a group of teaching strategies that rely on *Shared Experience* and/or *Purposeful Discussion* to set them up, to lead into them and to prepare students to gain benefit from the direct instruction. *Blended Instruction* is a set of instructional strategies that combine elements of discovery, inquiry-based learning with elements of explicit instruction. The *Blended Instruction Strategies* referred to are:

Targeted Instruction;
Responsive Teaching, and
Guided Discovery.

5.1.1 Targeted Instruction

Targeted Instruction was based on Huitt's (1996) format of Direct Instruction with active explication of the skill or subject matter being taught, linking and making concepts relevant, employing appropriate analogies and metaphorical stories and basing the new learning in a context familiar to students, with students' understanding being checked and assessed throughout the instruction. Opportunities for student participation was usually encouraged at relevant points and followed up with individual or small group practice.

Targeted Instruction comfortably follows on from *Purposeful Discussion*: students' attention has been gained, and in a sense the preceding discussion has worked as an advanced organiser and retrieved relevant knowledge. There can be a natural flow into employing Targeted Instruction with the use of *Purposeful Discussion* as a precursor as that focused discussion strengthens the targeted strategy. Compare this to a traditional teaching strategy where the teacher introduces a concept, provides the algorithm, possibly follows with some discussion and then has students work on related problems. The Targeted Instruction strategy targets specific skills that students have been cognitively prepared for in advance with *Purposeful Discussion*.

Purposeful Discussion acts in a similar manner to an advanced organiser; however, it is different in that it provides a cognitive platform, created through appropriate use of language, on which to then build new knowledge. Students need to make sense of new learning in ways that they are able to relate to, so the use of metaphorical stories is one way that empowers them to make this link. Targeted Instruction follows *Purposeful Discussion*, is targeted and specific and employed when there is a convergence of knowledge and /or skills that students need to learn. The following examples are taken from Lesson 1 (Rule of Order of Operations) and Lesson 3 (Equivalent Fractions). I have included these two examples as they demonstrate different aspects of the *Blended Instruction* strategy. The Rule of Order of Operations follows directly from a convergent *Purposeful Discussion* that has prepared students for a targeted and specific lesson that employs an analogy to develop the teaching points. The Equivalent Fractions lesson follows from a *Shared Experience* that has laid the foundation; however, this lesson uses a group activity that requires input from all students to develop the teaching points.

Example 1: Metaphoric Links

In Lesson 1: Rule of Order of Operations the Targeted Instruction Strategy followed a *Shared Experience* of six calculations and *Purposeful Discussion* focused on the methodology students used to complete the calculations. The focus in the Targeted Instruction was on the rules, where explicit instruction was given with the use of an analogy. I made a conscious decision to focus on the rules with this class group as it was the most effective way to get them to use the rules consistently. The students had not done any work on the distributive property of multiplication and I preferred to stay focused and specific. However, having said that I am disappointed that I missed an opportunity to lay some groundwork through the use of a few simple words, as will be explained later in this section.

Teacher: *Okay, everyone remember what I said about if everyone knows the rules. We can all operate basically on the same page. But if you don't know the rules you're going to feel a little bit left out at times.*

Language was employed as a tool to explain, as in the following example.

Teacher: *Okay, I am going to stop you all there. Some of you do some bits of it right, some of you don't. So let me just give you, what should happen in them. We will work out what the rules are, what the agreed rules are ... by mathematicians ... and this happened ages and ages ago. Okay! So, it's not our rules that we make up in the class. It's not your rules that we make up. It's not my rules. It's rules that have been agreed upon a long time ago so that when we all do maths we do it the same way.*

With the whole class I worked through each of the six calculations, preferring to use a mix of formal and informal language which was mostly procedural, diagrammed, pointed and gestured, and I provided extra examples for the calculations with the most errors. What I chose to do was expose the methods and talk through some of the reasons why students might have performed the calculations incorrectly.

As well as using language as a tool to explain, it was demonstrated to model its use in mathematics. In this next example I modelled, as well as used, visual cues for students. Figure 5.1 displays the visual cues that were used where students were provided with the steps in computing the solution. The calculation was written on the whiteboard and the number of the steps was written above the calculation with a coloured marker.

Teacher: *So, so this one here, three plus three is six, take three is three, plus three is six.*

① ② ③

3 + 3 – 3 + 3

Figure 5.1 – Steps to complete Calculation 1

Teacher: *So, there's my answer there. So, one, two, three steps there. Okay.*

[I pointed to six which was written to the right of the calculation.]

Teacher: *Now if you watch carefully, and listen I reckon you'll pick up some of the rules.*

I then used the same process to compute the answers to the remaining five calculations. Two of the calculations appeared to be much easier than the others with most of the students (16 out of 19) answering them correctly.

Teacher: Okay. Now ... let's have … a look at what was easier for question number three and question number five. Question number five had brackets in there and you pretty much all know that if you have some with brackets in them you need to do those first. And they just happen to be at the beginning. Okay. And then came the times, the multiply, and then came the subtract. So, you might have got that one right purely by accident.

① ② ③

(3 + 3) x 3 - 3

Figure 5.2 – Steps to complete Calculation 5

The focus was on the rule and that was provided to students, along with an analogy.

Teacher: Right, okay. I am going to give you the rule and I'm going to probably give it to you in a way that maybe you might not have seen before, and you may go 'hang on, that's not quite how I remember it.' But, if I give it to you this way it will help you just a little bit.

Students were asked if they knew what a hierarchy was, and they attempted to provide a response. Some of the students knew what a hierarchy was as they had learned about it in the previous year and a student offered a comment and put it into the context in which it had been learned. The following excerpt displays the analogy used and the focus on the rules.

Teacher: S*o hierarchy is what's at the top all the way down to what's at the bottom. So when I do rule of order, Kane, with my secondary students I tell them that the brackets, that's like Mrs X [the principal], then the indices that's like me. The multiplies and divides are like the teachers, and the adds and subtracts, the ones at the bottom are like the students. Okay. ... they tend to remember that one, because they always think they're at the bottom of the heap. So, brackets, first, because they are at the top of the order, indices next because they are the second most important. Then there is a little subtle change to what you know as the rules ... now, multiply and divide at the same level of operations, okay.*

So multiplies and divides ... multiply is not more important than divide. Multiplies and divides ... if you've got them in your sum they get done ... left-to-right. So if I have something that looks like [writes on the board 3 ÷ 3 x 3]. You would have, in the past, I reckon, three times three is nine divide by three is three. How many of you would have done that?

Students did indicate they would have done that, which on reflection, I realise I could have then said that it could have looked like three divided by nine [3 ÷ 9]. So that was an opportunity I missed.

Teacher: *Right. The way it should be done when you've just got multiplies and divides is just work them from left to right. So three divided by three is one, times three is three. So if you don't have the same rules, you will get different answers. So it is important that we all have the same rule.*

Jamie: *Is it taking it from a child image like dollars, like that sort of thing, like two dollars out of ten dollars, how much change you get up to more middle school or high school image?*

The above student was trying to make sense of the rule in a way that connected for him. He was suggesting that the rule was more sophisticated, less like primary and more like middle school.

Students were then left to create their own calculations using the digit 4 connected with three of the four mathematical operands or using brackets (like 4 + 4 – 4 x 4) in a similar way to what we had done with the 'Four 3s'.

When the Targeted Instruction was examined I wanted to understand how I might have done it better by using more informal language, like using 'lots of' instead of 'times by' or 'multiplied by' in order that students might start to get an understanding for the distributive property. My focus was setting up language so that base understanding happened that could be built on later. The better way in this instance would have been to pay more attention to the language I was using. Being aware of the impact of linguistics or language usage does not necessarily translate into effective action. In the calculation involving 3 + 3 x 3 + 3 I had the opportunity to make more of the language I was using and where I was placing the emphasis. Reading the calculation *'Three plus three times three plus three'* does not provide any hints for students; nor does it work as an anchor for embedding new knowledge. I believe that appropriate language use, that is, using the right word with the right tone and gesture, can act as the seed for new knowledge from which to germinate a concept later in a student's learning cycle. If I had used the words '**lots of**' instead of times or multiplication such as 'three plus **three**

lots of three plus three' and put tonal emphasis on the **three lots of thre***e* this would have set the seed for the concept of the distributive property later in students' learning. Even emphasising three plus three **times** three plus three could have had an impact on changing the misconception. Not only does this transport the student through the transition from informal to more formal language, it sets the seeds for the learning/understanding of the concept.

Mathematically there is only one way to complete this calculation; however, if the emphasis is put on 'three plus three take three plus three' then students may become quite confused. This was not noticed until I was going over my narrative transcripts and my voice activated software had recorded the calculation as '3 + 3 take 3 + 3.' When I reflected about what that meant, I understood that the emphasis was on 'take' and students would believe that meant '3 + 3' then take the other '3 + 3'. A point worth considering when teaching – Where does one place the emphasis in one's language? Are students confused with the emphasis on the incorrect part of the calculation or equation? This is, in essence, no different to having a gesture that does not match the oral speech. Students interpret this as something not intended, which then becomes very difficult to challenge.

The focus in the Targeted Instruction was on the rules, where explicit instruction was given with the use of an analogy. I went over each of the six calculations, preferring to use a mix of formal and informal language which was mostly procedural. I diagrammed, pointed and gestured, and also provided extra examples for the calculations with the most errors. What I chose to do was expose the methods and talk through some of the reasons that students might have performed the calculations incorrectly.

Example 2: Creating Visual Links

I had discarded this lesson originally, when I first examined all the data searching for examples of learning conversations. There were no examples of extended classroom conversations in this lesson and the lesson did not provide any valuable insights, or so I thought. It was not until I had gone through the process of identifying the strategies through the Grounded Theory approach that I came to see the value of this lesson in terms of the teacher contribution. At that point I had to go back and analyse the lesson through a different lens. The section of the lesson examined here is the part that follows after the *Shared Experience* which was a group, then whole of class, feedback from the previous lesson introducing Fractions. This section commences about five minutes into the video recorded data.

Teacher: *Okay so some of you mentioned that I counted the boys and then I counted the girls and then I did a fifth of the boys stand up then I did two-fifths of the boys stand up. Remember one of those caused a little bit of a dilemma and the boys couldn't quite get it ... but eventually they got it and I had a third of the girls stand up, two thirds of the girls stand up and that was when there was nine and then I changed it to 10 by adding me. So what I was getting you to do was actually to think about equivalent fractions. Now I don't know if you've heard a lot about equivalent fractions or even the word ... yes ... no ...* [I see some nods] *or forgotten maybe?*

As a direct result of my reading for the Literature Review and the journey that was taking me to reflect more on the way I used language to introduce new concepts. I was developing an understanding of equivalent fractions with students and then able to work on a better understanding of addition and subtraction of fractions. The logic behind introducing equivalent fractions through a prior physical activity was that the activity would build a cognitive platform from which to launch the concept of equivalent fractions. It was necessary to embed the concept of multiples at the same time to then be able to further develop the equivalent fraction concept.

The following excerpt builds on the *Shared Experience* through employing a targeted activity that has been used to engage all students to be involved in the development of the concept. The activity forms part of a very structured approach that started with the concept of equivalent fractions being introduced with the ‘Stand up’ *Shared Experience* where the language was linked to a visual image that all students shared. The lesson follows on with 'multiples' being used to make the next link for students.

Teacher: *Okay, ... Right ... Equivalent fractions are really, really important in terms of being able to add fractions together and subtract fractions but before we can even look at equivalent fractions, … that is fractions that are the same and you showed me on Friday that you knew, eventually, that one fifth of the boys and two-tenths of the boys the same number of people stood up. Remember that? umm ... 3/5 of the boys and 6/10 were the same. So I was just getting you to practice that and in the end you did it really quite well. So those are equivalent fractions. One fifth, two tenths, 3/5, 6/10 but as I said before we can go and do lots of work on equivalent fractions we are going to go back and look at multiples. Now multiples* [turning around and looking up at the tables above the whiteboard] *are pretty much just part of our tables, our times tables.*

Teacher: *Now I'm going to start with group 1 with this very first one and I'm going to do the first one and then I want you to follow on with the pattern and I'm just going to go in each group, group1, group 2, group 3, group 4, group 5, group 6, and we're going to go from the right, from my right, … the corner that is closest to me on the right and we're going to go clockwise in each group. Okay so ... to make sure that we've got that little bit sorted, so Tim that means you're starting your group. Who's starting group two?* [A hand goes up] *yep, this group, Chloe, umm group 5, Cameron, excellent and group 6, Keith, yep. Because you're in this position* [points to desk in front right hand corner] *so you are in a corresponding position in each of those groups … and then we are going to go round the group clock wise. …. I'm going to do the first one ... we are just going to go down the tables okay because they are multiples. I am just going to, ... say two, then Riley will say four and we're just going to go round the room very quickly, … okay except we're going to have a* *big problem once we get past ... if I start when we get past the 12th person … they are not up here anymore, right.* [pointing to the times tables chart above the whiteboard]

I started with two and the pattern was continued by students. The process was repeated with multiples of five and then after discussion the process was repeated with multiples of 12. With the multiples of 12 I wrote them on the board, so that students had a visual record of the last one called.

Students then had to find the next 5 multiples of 13 and write them in their files. It was completed with a group competition to see which group could finish theirs first. When students reported back how many in their group had correct responses they were asked to give it in a fraction form. The first group reported back ‘two out of five, or two-fifths, no it’s two out of four’. When questioned what two out of four was the student replied that it was a half. Each group reported back in terms of fractions and were heard using language like:

Harrison: *Three out of four, … three-quarters*

At that point it was time to move to equivalent fractions. I started by writing ½ on the board and asked the questions below:

$$\frac{1}{2} = \frac{\quad}{4} = \frac{\quad}{6} = \frac{\quad}{8} = \frac{\quad}{10} = \frac{\quad}{12}$$

Figure 5.3 – Equivalent Fractions

Teacher: *A half equals how many out of four?*

Students: *Two.*

Teacher: *And how many out of six?*

Students: *Three.*

Teacher: *How many out of eight?*

Teacher: *Now can you do the same with 1/3, 1/4, 1/5?*

Students then worked individually on the first few equivalent fractions for each of those written on the board, that is, one-third, one-quarter and one-fifth. The above activity was fairly brief as the concept had previously been created for students with the *Shared Experience* activity.

5.1.2 Responsive Teaching

Responsive teaching is similar in many ways to Targeted Instruction; however, unlike Targeted Instruction it is often unplanned and arises out of the necessity to meet student needs. When a teacher recognises this opportunity it can be used to effectively facilitate learning. The following example highlights the nature of Responsive Teaching. There are times when the teacher's role is very active as in the example, Problem Words, and there are times when the role is to ask a pertinent question which begins the facilitation of student discussion, which becomes a learning experience. The latter form may only take a few minutes and may be one-on-one teaching. I have included the whole of group example as I believe it details the strategy.

Example 1: Problem Words

This example was the last part of Lesson 4, described in the previous chapter. In the *Shared Experience* lesson, students fed back a problem that was encountered earlier in the day. The *Purposeful Discussion* that had followed focused on Indices; however, at the end of the discussion I thought it wise to return to the issue. This part of the lesson is included as it represents what could be Responsive Teaching. The issue that was raised earlier was that students did not understand some word problems and were not able to be helped. I had not expected this to be a concern for students. I had not taken into account the problems that might be encountered by a student for whom English was a second language. The following excerpts are taken from Lesson 4 and highlight the problem that some words can create.

Teacher: *When you guys did this … I don't know if you used this one when you were in year eight. But we did it … we still did the same things, but might have done it a different way ... the question goes take a number and multiply it by six okay*

Lily: *Yeah, Miss, Sara didn't get this question and I was helping out because the teachers weren't helping her and then, anyway, and then I didn't even get it like it was that one you just read out and then they go and look ahead for answers that they could have picked out and was … began, a number multiplied by six and then add another number..*

Bailey: S*omething, something plus h*

Lily: *Yeah and then put another letter on it, apparently that was the answer*

Teacher: *Before, I wouldn't even have thought about it, but recently, as I've started thinking about the way we structure questions and the language that we use in questions we've got take a number is the first three words. Now when you hear take what do you think about?*

Bailey: *Subtraction, subtract*

Teacher: *Yeah, so and I think that's what got in the way for Mrs Brown, and it got in the way for the year eights. So… to take a number that must mean minus something …*

Lily: *I didn't think of it like that. I did like take a number like that, [using her hands to show how you take a number to you] out of the book.*

Teacher: *Yeah, I'm not sure if it's a generational thing, or what it is but* *that was an expression that was used quite a lot, if you take this or if you take that it didn't mean minus*

I believed that the language had caused the concern for the student whose English was not strong; however, I had not anticipated that it might be of concern to other students. By responding to the learning needs of students a much greater concern had been identified and it became an opportunity to clarify some understandings. The other interesting aspect is the use of hand gestures to confirm students' understanding of 'take a number' and the way the gestures were used to clarify meaning for one of their peers. This is highlighted in the next few lines from the lesson.

Darcy: … *wait, so you just take a number [uses his hands to show taking].*

Caitlin: [Uses her hands] *Just take it.*

Teacher: [I use my hands to show the action of taking.*]* *Really, it should say, start with a number … and multiply by another number, and it changes everything.*

Darcy: *The whole perspective, … because you don't know the number, … oh,… because you don't really minus it.*

Darcy was a year 10 student and for him this had not previously been clarified. Interestingly, Miller and England (1989) also found that students confused the meaning of the word *take*. In their study students did not understand that it might be used in the sense of removing something and with that action being different to subtract. In both cases it highlights the importance of teachers understanding how language is used and interpreted. If the word 'start' had been used instead of 'take' there may not have been the same level of confusion. In this next segment I substituted the word 'start' to see if the problem then made sense for students who had not understood the question. I had a text with problems for students to translate.

Teacher: *No, so, okay ... okay, if I start with a number and multiply it by another number, like Lily and Bailey. What might I get? …*

One of the year 10 students helped to clarify the problem.

Caitlin: *So, they are just replacing the numbers with letters.*

Darcy: *No, because, … you don't know the umm … what they are?*

Teacher: *The answer's this one here.*

Dale: *But what number do you start with?*

At this point other concerns had been raised, so the learning issue that was raised appeared to be just one of translating the language; however, it became one of addressing a misconception.

Teacher: *Any number, … don't know, … so, … what they are trying to show is … when you're using variables … replace what you don't know. So, what operations are you doing?*

Dale: *Multiplication.*

Teacher: *So, here's another one um, let's see if we can do an easy one … start with a number.*

Lily: *Would the answer for that other one um ...*

Teacher: *For the one I am doing now, start with a number and multiply it by six…*

Lily: *Yeah that one.*

Teacher: *Start with a number call it … 'n', for a number.*

Lily: *Okay.*

Teacher: *Okay, start with the number and multiply by six.*

Lily: *Six n* [6n].

Teacher: *So you've got six n, then add another number to that answer.*

Caitlin: *So that would be six n plus …* [6n +].

Teacher: *Six n plus …*

Caitlin: *Six n plus … another number … but that's random … another letter …*

Teacher: *Yeah.*

Lily: A*nd that's the answer, that's what I said to Sara … and then Mrs Brown and Mrs Green said that it was like six, another letter plus another number. That's what they said.*

Teacher: *Yeah.*

Lily: *That's what I mean.*

Teacher: *It's that one here* [I showed them the answer in the book].

Lily: Y*eah it's d down the bottom.*

Teacher: *This one, sorry, which is what you just said to me. It was there.*

Lily: *Yeah.*

Teacher: *What was causing confusion to everyone was that one word. If you are not a maths person, and you get language that causes confusion then you have a problem.*

Responding to the learning needs is important, as is recognising in this example where the confusion was coming from. The other important aspect is to respect the learning that can come from peers, and allow time for students to discuss problems they have been working on.

5.1.3 Guided Discovery

The Guided Discovery strategy enables students to be stepped through a series of activities that lead them into 'discovering' for themselves relationships, formulas and so on. Here language, elaborated, can provide the links for students assisting them to make the discoveries. The following example, taken from Area of Triangles Lesson 8, highlights the connectivity of the *Shared Experience*, *Purposeful Discussion* and *Blended Instruction* strategies.

Example 1: Foreground or Background

Lesson 8: Area of Triangles commenced with a quiz as a *Shared Experience*. Students were asked two questions and after each question the group's answer sheet was collected. The first question was how many square centimetres were there in an 8cm x 24cm rectangle drawn on centimetre grid paper. Four out of six groups had this correct. The second question was how many square centimetres were there in the eight double triangles that were inside a rectangle. This lesson was one in the sequence of lessons that were preparing students for drawing a flat map representation of the Earth. Previously, I had cut an orange skin into 8 segments and shown how this would look laid out on a flat surface. Before attempting to draw the segments I had students working with triangles to approximate the area of the segments as displayed in Figure 5.4. Each of the triangles was 3cm across the base and 4cm in height.

The Guided Discovery task was to find the area of the triangles and to informally develop a rule for finding the area of a triangle along the way to working out approximately how much 'extra' was added into the area on a flat map. I had cut the orange skin so that it looked like the triangles displayed in Figure 5.4, except that the triangles were an approximate representation of the circular segments.

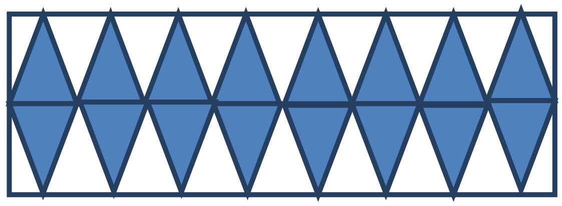

Figure 5.4 – Rectangle Surrounding Triangles

This example was included as it highlighted another issue with the way the language and terminology was used. Students often struggle to understand that area is a measure of a surface and I had started working with area with the year 6/7 students by counting square centimetres in rectangles that had been drawn on centimetre grid paper.

Taylor: *She said how many square centimetres. It's on the board. They're halves.*

Harrison: *Do we have to count the half ones as well?*

There was confusion because the triangles did not fit perfectly over the square centimetre grids. The class stopped for morning recess and then resumed. I asked the students to think about how they could count the square centimetres. After the break, students cut out their triangles with the purpose of discovering how to find the area of a triangle. Some students found the pieces fitted together when they cut out the triangles. One student cut the triangles out but left the rest of the rectangle intact. Effectively he had a template with the double triangles removed whereas other students had cut around the triangles and disposed of the paper that had surrounded the triangles. I stopped the class and held up the template.

Teacher: *Jonathon thought he was being clever, he has wasted time but ….. who can tell Jonathon what he has left behind.*

Harrison: *He left eight triangles.* [Meaning the eight double triangles.]

Keith: *He's taken half of the triangles out and you have half left, so just halve the rectangle. … Half of 192 is 96.*

Teacher: *Did it help to see it like that?*

I saw this as an opportunity to use the language to create a visual image that could reinforce what students understood. Students could focus on the foreground or the background and achieve the same level of understanding.

Students were then asked to write three sentences about the area of the triangles or the number of squares of a triangle.

Teacher: I could write … the area of the triangles was half the area of the rectangle or the area of the rectangle was double the area of the triangles. … Using numbers … don't write in words [this was written on the whiteboard as well].
A of Δ's = $96cm^2$, A of □ = $192\ cm^2$

In that lesson I had to undo a problem I had possibly compounded introducing area by counting square centimetres. I had to counter the understanding that students held that they were only counting whole square centimetres. Using an orange and its skin to represent the earth gave a concrete and visual object that we could talk about. In the following lesson I had students using a compass drawing intersecting circles with a radius the same as the height of the triangles. Part of the guided discovery was to get students using the language associated with area, with triangles and rectangles.

5.1.4 Elaborated Discourse and Blended Instruction Strategies

Teachers have enough to consider with pedagogical issues, teaching methodologies, teaching and learning philosophies and beliefs and classroom management issues without having to think about the words they use, about the tone of their utterances, about their body language and gestures and the messages sent in the communications and/or conversations that they have in their classroom. I was not aware of that level of detail and its impact on learning and I have been teaching for over 30 years. I knew it was important to have conversations; I knew it was important to use the language of my subject and to continue to bring the language to a more sophisticated level. In the *Blended Instruction* strategies it was possible to model elaborated discourse and to provide opportunities for students to learn the skills to also engage in elaborated discourse. There were fewer opportunities for students than in *Purposeful Discussion*; however, dependent upon the concepts being learned the previous transcripts show evidence of teacher and students engaging in elaborated discourse.

Perhaps there are times when it is easier to engage students in extended learning conversations where there is elaborated discourse. Perhaps there are times when it is more relevant for a teacher to model the skills of elaborated discourse and also to transition between formal and informal language use. It is possibly far more effective to use elaborated and extended conversations where it is appropriate and relevant rather than forcing those conversations to occur in every situation.

5.1.5 Reflection

Here I see value in the strategy of Responsive Teaching, one of facilitating and enabling students to engage in a way that is not dominated by the teacher. Rather than stating outright to students that their responses were incorrect, and so challenging them there in front of their peers, I preferred to ask another question like 'How would you show that?' or 'How would it work?' and let the student acknowledge that they had found the error in their methodology or thinking. This is; however, more relevant linked to the *Purposeful Discussion* strategy.

As outlined earlier *Blended Instruction* within the whole of classroom setting can seamlessly follow a period of *Purposeful Discussion*. This was exemplified in the lesson on Rule of Order of Operations. Targeted Instruction, a form of Direct Instruction was evident in at least one of the lessons, and combinations of Targeted Instruction and Guided Discovery were evident in other lessons, none more so than the introduction of the Orange problem where the initial part of the lesson following *Purposeful Discussion* was an introduction by me of what I had done with the younger group of students and how this group of students could use more sophisticated mathematics to check whether my guessed calculations were correct.

Whether it is Targeted Instruction, Guided Discovery or Responsive Teaching, the elements of the strategies are the same: a Shared Experience, followed by a *Purposeful Discussion*, followed by one of the *Blended Instruction* strategies. The teacher's role can be one of scaffolding and modelling of elaborated discourse with progressive matching of informal and formal mathematical language, developing a learning community using *Purposeful Discussion* to develop a relationship with students and using descriptive and connected language to support understanding.

It was interesting to note that towards the end of the Fractions Lesson, where I was explaining to students about why it was important to understand how they processed information and solved problems, my tone changed and became very quiet, and at this stage I have not found a word to describe the change in tone. I can recognise it, but I cannot say that I have ever been aware of doing it. Students, even after five weeks seemed to pick up on when there was a change in tone that that was a signal for something important that should be listened to. Reviewing the video data showed students turning their heads in my direction with those who may not have been 'paying attention' appearing to be listening attentively. Again something interesting, but was it something that contributed to enhanced learning?

As stated earlier there is data rich in language and it is the intense interrogation that provided an opportunity to evaluate this language data. I can register a difference in tone and I did see that half way through the Fractions lesson where I started to say to students that I wanted them to do something difficult. My tone became deeper and my voice projection became quieter. I am not naturally a loud person in the classroom and I found a long time ago that I did not need to get louder than students to be able to manage them I found that I could get very quiet and they would eventually stop and listen and 'pay attention' and this is what I did and students responded.

In drilling down to the word level of the discussion in my mathematics class I am obliged to consider more than just the words that were used. As mentioned previously I had become aware of the impact of tone in my oral communications. In interrogating the scripts and in viewing the video files again I am now very much aware of the pauses and of the tone that was being used. It appears to be quite critical - an analysis of the words being used is something that could be undertaken.

When considering the whole package of communication one might include an analysis of the types of words being used. I can only judge what is happening in my own classroom. In continuing to visit the ‘tone’ aspect I became aware of how I might use this even though I was not aware that tone was an integral part of the way I conducted a learning conversation in my classroom. To put this into context, I am very aware of not being a dominant force in my own classroom. I did not want to be seen as the teacher who knows everything and that my way was the only way to do things. I wanted to facilitate and let students develop their own learning without being forceful about it. I wanted students to take up learning for themselves rather than feel it was being imposed.

In reporting the data related to my role I chose to focus on its relevance within the *Mathematical Linguistic Pedagogy* context. It was here that the ‘language’ platforms were created and mathematical language was employed to help students to create visual links. The language used also empowered students to make choices, to create their own mathematics.

Benefits of the Blended Instruction Strategies

The first benefit of the *Blended Instruction* strategy would be in the focus that it provides for learning. The *Blended Instruction* strategy is a focused teaching strategy with the focus coming from both the *Shared Experience* and *Purposeful Discussion*. This was demonstrated in the Rule of Order of Operations lesson where the focus of the *Blended Instruction* was teaching the ‘rule’. The use of analogy or metaphor can further focus the Targeted Instruction and can be effective in making links for students with new learning or in undoing a misconception.

A second benefit of the *Blended Instruction* strategies is that the language, used thoughtfully, clarifies and makes explicit what is being learned/taught. Students are encouraged and provided with opportunities to use the language associated with the concept introduced with the strategy. The concepts that are being learned/taught are made explicit to students through the use of appropriate language.

A third benefit is that the *Blended Instruction* strategies can provide teaching at a point of need. The strategies can address a need that students present as a problem for them or their peers. As the previous strategies have prepared the students for one of the *Blended Instruction* strategies, there is reason to believe that the targeted teaching will be more effective and less 'hit and miss'.

A fourth benefit of the strategies is that they enable a teacher to 'act' on the feedback that has been received during the previous *Shared Experience* and/or *Purposeful Discussion* strategies. Hence one of the purposes of the Targeted Instruction is to home in on areas of need that have been identified and this links with addressing the needs of students.

Challenges of the Blended Instruction Strategies

One of the challenges with implementing the strategies is that it can take considerable practice to recognise the opportunity for Responsive Teaching and to have the confidence to follow that learning need. I could very easily have missed opportunities to follow through with areas of need that had been identified with the prior strategies.

A second challenge is that the strategies can look very algorithmic and teacher driven. The subtleties of the strategy may not be apparent to an outside observer. The *Blended Instruction Strategies* were not intended to take up a large part of any lesson. They do follow on from the relevant *Shared Experience*s and /or *Purposeful Discussion* strategies. The role of the *Blended Instruction Strategies* is to provide targeted and responsive instruction that builds on the cognitive platforms that were created with the prior strategies. The strategies provide structure or a framework and are not intended as a straightjacket.

A third challenge for a teacher is that the use of a *Blended Instruction Strategy* requires considerable planning and carefully thinking through the consequences and planning with how to deal with the unforeseen or unintended outcomes, problems, or uncovered misconceptions. Planning, in addition to what a teacher would usually go through is required. The planning has to be 'what if ….' type questions and reflection that a teacher might not otherwise have to do. The planning has to also incorporate being flexible in the direction a lesson might take and how to fit that within the big picture syllabus.

A fourth challenge involves having a good understanding of both mathematical content and pedagogy as both are required if the strategies are to be effective. Without the background, opportunities for using the strategy may not be recognised along with ways to use language to counter misconceptions.

A fifth challenge is in managing time. This would definitely be a challenge, an issue for teachers who are abiding by a content driven curriculum. A teacher has to work through the issue of time reflecting on what might be achieved with or without the use of the strategies.

5.2 Student Peer Teaching Strategy

This strategy was embedded within collaborative group-work; however, I believe that it can stand alone hence the reason for identifying it as a separate strategy. Like collaborative group-work, *Student Peer Teaching* follows the principles and philosophy of Constructivism; students extending their knowledge from a familiar base and sharing this with their peers and younger students. I found an opportunity to have students teach each other and learn from each other and had found this a rewarding learning experience. Previously I had used this strategy as a revision technique and to create opportunities for students to share their understanding, knowledge and skills with each other. Students worked in collaborative groups to teach themselves or revise a concept that they could then teach to other students.

Collaborative group-work was not as successful with younger students. Students working in collaborative groups in the lesson on ‘Drawing Circles and Segments’ were observed helping each other by pointing, using one or two word utterances like ‘here’, ‘there’, ‘like that’, and by ‘doing’, that is taking the student’s work and doing it for them. Students do not automatically have the skills to be able to share their knowledge, skills and understanding in front of an audience, that is, to peer teach. These are skills that require developing and can take considerable time. It is also important that they see value in the strategy in order to fully engage them. This was in evidence in the Roundtable Reflection example in the previous chapter. Just like the strategies, developing the skills with students to ‘teach’ their peers is also quite structured. The following examples or situations explain how the students go through stages in the development of the required skills and confidence. In the beginning, or when the strategy is being established a much greater proportion of teacher talk is required. When the skills are well-developed minimal teacher input is required.

5.2.1 Establishing the Student Peer Teaching

Establishing is when the strategy is used for the first time with a class group of students: when commencing with students an informal approach coupled with a shorter preparation time and smaller audience yields more effective results. More teacher input is required when the strategy is first introduced and the accompanying skills are being developed. This is demonstrated in the following example with Class B. In their first experience with this strategy students were reluctant to share their understanding. The boys participated; however, the girls delayed participation stating that they were not ready. My expectation was that the girls would be more willing to participate in future Peer Teaching/Sharing as they did observe the boys sharing their understanding in an informal, relaxed setting, sitting on the floor at the back of the classroom. The example that follows was based on a game that students had chosen, referred to as One Thousand. With the younger class students had been encouraged to work in groups of two or three as this would help minimise the risk-taking for students.

Example 1: Introducing the Student Peer Teaching Skills

The first group to share their understanding, Class B, consisted of three boys whose average age was 11½; Lachlan, Logan and Harrison; their selection was a game that they had chosen from their Mathematics workbook. Their audience consisted of three male students and the teacher. I had not seen the game, nor did I know the rules or how it was played or what the aim of the game was. We were seated at the back of the classroom with the audience seated on the floor and the three boys standing next to me. I was seated on a chair.

In response to me preparing the students by asking them what the aim of their game was the students responded as follows:

The first student explained the game literally by reading the rules. He read from the instructions.

Harrison: *The aim is to be the player with the higher score when one player reaches their opponents' home base.*

The second student attempted to explain the game in his own words. There were lots of pauses and indications of thinking, rethinking and refining as the student continued his explanation.

Logan: *What happens is there's, … you start on home base and then you go, … move on all the numbers and you add all the numbers up and then when you get to the other persons' home base you add all the numbers up and whoever's got the least, … least amount, umm, wins.*

The student provided an elaborated response and I could hear as well as see him pause, reflect and continue with his explanation; however, I still did not understand how the game was played or what its aim was. An elaborated response does not mean that it is one that conveys or shares meaning or understanding.

The third student wanted to provide an explanation of how the game was played; he offered a practical example, after being prompted by one of his peers, showing how a move might be made.

Lachlan: *So, say you're down on this bottom home base you might choose to go to 30 and then up to 40 and every time you … [interruption]*

Harrison: Demonstrate.

Lachlan: *… you might go to 30 and you keep on doing that and every time you change your … you move you write that number down on a piece of paper and then when you get to the other home base and that other person gets to their home base you count up how many, how much you have as your score as whoever has the less wins.*

In this group the first response was literal, the second built on the first and the third was quite an extensive response; however, it was unclear what mathematics was required or what were the strategies involved to win the game. Brown (2001) referred to 'hedging' or vagueness as ways students could 'keep their options open' to minimise risk-taking. I believe that the first student's literal response was a means of minimising the level of risk for him. I chose to intervene and to clarify even further and asked how moves were made just to ensure that our understandings of the game's rules were the same.

Teacher: *So, how can you move? You can move diagonally, did you say that?*

[I indicated a diagonal move with a hand gesture.]

Lachlan: *No, you can't move diagonally, you can only move up, down and left, right.*

I continued to probe:

Teacher: *So how do you know how many to move?*

Harrison: *You're only allowed to move one at a time.*

There was discussion amongst the three students as to how the moves worked. They attempted to clarify and convince each other that they knew how the game was played. Logan attempted to explain with another example and finally mentioned the 1000 points that players start with. This had not really been clarified and I believe the students thought the audience had that bit of information.

Lachlan: *You start with a thousand.*

Logan: *And then you minus.*

Lachlan: *And then after that you get the calculator and a thousand minus your score and …*

Logan: *What you want to do is 40, 70, … you wouldn't go 40, 70 …*

Harrison: *No, that's not right.*

The boys had another discussion amongst themselves, challenging each other.

Logan: *Yes it is.*

Harrison: *It says here that when one player reaches their opponent's home base the game ends. The player with the higher score wins.*

At this point I intervened again to clarify, to focus students and to send the message that what they were presenting was good.

Teacher: *The higher score …. Now … This is actually a good discussion to have because you start with a thousand points don't you? … Yep, and you said, and you weren't incorrect in* *what you'd actually said. You said you count up the points that you landed on and ….*

Logan: *And then you minus them and then you've got the highest score.*

Teacher: *Add them together and you have to subtract that from a thousand.*

Harrison: *Yeah.*

Teacher: *So the person who has the lowest score when that's subtracted from a thousand, that will give the highest score won't it?*

Harrison: *What if you get lower than a thousand?*

Lachlan: *Then you get lower than a thousand.*

The students, at this stage, did not appear to delve past the surface in explaining the mathematics and required further probing.

Teacher: *Will you always get lower than a thousand?*

Logan: *Yes!*

Teacher: *Because?*

Logan: There's numbers in the board.

Teacher: … *but why do you get lower than a thousand?*

Logan: *Because you're minusing numbers.*

From their explanations, and with me probing how points were calculated, agreement was reached that you could either add up all the points and subtract them from 1000 at the end or continuously subtract them starting from 1000. The player with the most points at the end won. This fitted with the explanation provided by Logan and Lachlan that the player with the lowest score at the end won, as they were adding their points up and then subtracting from 1000. The boys were using both methods and finally through discussion convinced each other that both methods produced the same result. They had not arrived at that conclusion during their preparation whilst playing the game, possibly because they were not discussing the strategy, just going through the motions, and it was not until I probed as to how the game was played did they have to confront their differing methodologies, which they then realised produced the same result.

Interestingly, I believe that had the students not explained their strategies to me they would not have understood that each used a different method – either continuous subtraction starting from 1000 or addition of all scores which were then subtracted from 1000. There was an understanding that both methods produced the same result. This was a ‘cognitive platform’ on which some higher level mathematics could be built. When teaching those students in the following year I could recall that experience and use it to relate to some basic algebraic skills.

The focus of this conversation for me was that without sharing the boys would not have made the connection with the two methods, and also that it took considerable clarifying before it really became clear what the game was and how it was played. Probing was important and an indication of the age and experience of this group of students. The level of probing was deeper than what would be required with students in Class A; however, I felt I needed to do it in an oblique way so as to not send the message to the students that ‘they were not competent in sharing their understanding’. At this level with students it was as much about becoming confident and comfortable with a higher level of personal risk-taking.

When younger students first begin to share their understanding the focus should be on developing confidence and having them value the experience.

The context in which students share their learning is far more informal, with small groups of students and with the teacher more involved to prompt and ask appropriate questions. Students needed to practice the skills in a minimal risk environment. With older students, practising skills and developing confidence is again the focus; however, at times it is important to have students explain their methodology in a one-on-one situation. This can minimise the risk somewhat for them as it provides an opportunity for them to develop the skills of explaining and sharing their learning. It is the skills related to explaining and sharing that are required, along with developing confidence and being willing to take a risk in front of peers.

5.2.2 The Established Student Peer Teaching Strategy

Once the skills have been achieved, practiced, confidence is established and students are confident with the process then effective peer teaching or tutoring provides the students with a means by which to display and share their learning and understanding of mathematical concepts. Minimal intervention is required and usually only to highlight and amplify for the students who are leading the learning. The three following examples were taken from the presentations by two year 10 students.

Example 1: BIMDAS

In Class A, Caitlin and Bailey, two year 10 girls worked together to teach year 9 students and their peers in year 10 how to expand difference of squares and perfect squares. This took approximately 40 minutes. Interestingly to do this they commenced with rule of order and practised first with some Rule of Order of Operation questions involving brackets and then Expanding Difference of Square Binomials followed by Expanding Perfect Squares. In the group were four year nine students and five year 10 students, including Caitlin and Bailey. I had played no role in the development of the girls' lesson other than to approve their choice.

During the girls' presentation there was considerable humour and some 'banter'. There was a relaxed atmosphere, students joked with each other whilst following the presentation. The boys were at the back of the group in front of the camera and there were some remarks made; however, I don't believe that the camera influenced their behaviour or comments. I sat at the back of the room with the camera. The two girls wrote their examples on the board and asked students to work through them. The discussion was free-flowing and lively. Students encouraged each other, sometimes talked at the same time and often finished each other's utterances. There was some elaborated discourse in the first lesson and it was apparent that language was in use to modify thought.

In the following extract there was an attempt by Caitlin to extract a response to demonstrate understanding, not just an answer. She continued with the expectation in the class that students explained their methodology and students were quite adept at processing this level of questioning mentally. The students performed two similar calculations without any problems.

Caitlin had written on the board:

$$(4 + 2)^2 - 8$$

Caitlin: *This is the first equation, just using BIMDAS, so what do you do first?*

Cameron: *Brackets, and yes, four plus two is six which equals 36*

Caitlin: *So, now what do you do?*

Dale: *It's squared, it's already squared and then you take eight,*

Cameron: *Eight from 36*

Dale: *Equals 28*

Interestingly the two year 9 boys worked together and quite often finished each other's sentences or spoke together at the same time.

Example 2: Difference of Squares

After working together on the questions on Rule of Order of Operations students were presented with the following, written on the board:

(a + 4) (a – 4)

The students were told that this was expanding by using the difference of squares.

Comments from the year 9 students were

So, so …so ‘a’ will equal four, ‘a’ will equal zero

Its 16a something

It equals minus16

There was some confusion amongst year 9 students. They had no experience with Difference of Squares and were attempting to apply their understanding of Rule of Order of Operations. Caitlin explained that you multiplied ‘this by this’, while she used her hand to refer to each of the brackets. Year 9 students had not yet seen this concept and so the two year 10 girls were actually teaching the year 9 students a new concept. The students were applying their understanding of Rule of Order and wanting to remove the brackets but were unsure of how to handle the ‘a’s. There were comments like:

Get rid of the brackets first

One of the other year 10 girls wanted to tell the students what to do.

Lily: *‘a’ times ‘a’*

[concurrently]

Dale: *You plus ‘a’, so it’s ‘a’ squared*

Dale: *That's what I said*

At that point there was the potential for a misunderstanding to develop as Dale had not 'said that' but possibly meant that; however, he believed that he had said the same. Rather than intervene I waited.

Dale: *And then… and then 'a' squared [a^2].*

Caitlin had written a^2 –

Cameron: *Take 16*

Dale: *How is it 16? Plus 4 and minus 4!*

Dale: *There will be 'a' squared*

Lily: *Minus a-four, 4a…plus 4a…*

Caitlin explained that it was a^2 minus 16: [$a^2 - 16$].

Caitlin: *Because this is a plus 4 [hand on the first bracket] and in the other bracket you have minus 4, the parts cancel each other out.*

The year 9 students still appeared confused.

Caitlin wrote up (2m – 1) (2m + 1) on the board

Bailey: *So with what we said before can you figure out this one?*

Dale: I*s it $4m^2$ or,… or is it $2m^2$?*

Cameron: *It's $4m^2$*

Dale: *But it's not a times.*

Caitlin: *It's two times two*

Caitlin explained using a hand gesture, touching each of the 2s.

Cameron: *Brackets, … brackets means timesing.*

[He stated to Dale with a very matter of fact tone.]

Caitlin: *And the ones will cancel each other out.*

Caitlin used a hand gesture again to refer to the middle terms and finished writing up $4m^2 – 1$.

I interrupted for the first time and asked,

Teacher: *Does it help if you just one time multiply the whole thing out, so they connect to see the two bits disappear?*

Caitlin led the boys through

$$4m^2 – 2m + 2m – 1$$

I asked them to do another one on the board.
Caitlin wrote up

$$(4a +2) (4a -2)$$

Dale: *Eight a squared.*

The students looked at each other.

Caitlin: *What is four times four?*

Dale: *Eight. Oh four times four is 16 so 16 a squared* [$16a^2$]. *I thought it was plus.*

There is evidence of modelling here in that Caitlin did not say 'no you are wrong'; she asked what is four times four, which led to a corrected response.

Caitlin connected 4a with -2 and led Dale through 4a times -2 is -8a and linked +2 times 4a (students said plus 8a) while following the FOIL algorithm [in each bracket, First terms, Outside terms, Inside terms and Last terms multiplied together], multiplying the first terms in the brackets, the outside terms, the inside terms and then the last terms. Caitlin linked plus 2 with minus 2 and students said that was four. Cameron corrected and said it was negative four.

Dale: *$16a^2$ negative 4*

Teacher: *$16a^2$ take 4*

I did intervene with Dale to correct his use of language; I used ‘take’; however, I could just as easily have used minus.

The next problem students worked on was (2 + d)(2 – d) written on the board. Caitlin asked students:

Caitlin: *What do you do first?*

Dale: *Two times two, and that is four, that equals four and then you do two times d which is 2d...*

Cameron: *Two times negative d which is negative 2d*

[He said that at the same time that Dale said 2d.]

Dale: *Negative 2d, take d its negative 2d and then umm and then you do umm, umm then you do the other 2d, …plus 2d*

He was helped by Cameron, who used his hands to show a plus sign. This was an example of Dale starting a solution, reflecting as he worked on it and self-correcting when necessary.

Caitlin asked them what they did next?

Dale: O*hh, ohh the two, the two and the two ...*

Cameron: *No, the d's. d squared.*

Dale: *The d's is d squared.*

Peter: *0h, I see it!*

[He said this as he used a hand gesture to show how the terms have been multiplied.]

Most of the discussion was with Cameron and Dale; however, there were two other year 9 students in the group; Peter and Belle. Both had been quietly participating along with the two boys; however, being quiet they were not always heard. As the observer I could see they were participating.

Cameron: *It's negative… I think.*

Dale: *So the negative overrules everything?*

Cameron: *It depends how it goes, if it is a positive times a negative it is a negative.*

Dale: *It's a negative, it is negative d squared.*

The two boys worked together frequently, correcting and updating with each other.

I interrupted:

Teacher: *What does it end up as?*

Dale: *Umm negative 4 d squared*

Cameron: *Four minus 2d plus 2d....*

[This was said at the same time.]

Cameron: *No, its umm*

Caitlin: *It's four*

There was confused looks.

Caitlin: *How about if we rearrange it?*

Cameron: *Negative d squared, [corrected himself] four minus d squared.*

Dale: O*h, … four minus d squared*
[and went oh… pointing at the board.]
I get it, because those two get eliminated, because you've got those two ... there is a positive four and a negative d squared.

That was an expression of reflecting and understanding which came through when students explained, corrected and shared their methodology.

At that point I stopped them and went through a very brief discussion of the squared terms and the minus sign often being referred to as a difference, hence the term Difference of Squares.

Example 3: Perfect Squares

The two girls took the students through multiplication of Perfect Squares in a similar manner to the previous examples. The students asked for the lines to be drawn to connect the terms when they were being multiplied using the FOIL algorithm.

Cameron: *It's repeated multiplication*

Cameron was referring to the multiplication of the two middle terms and I believe he would make the transition to the next stage where he would not need the two brackets written out and the two middle terms gathered. After that the students asked for another example, a request followed by laughter. It appeared funny to them that they had asked for more maths.

They worked on $(9 + x)^2$ which equalled $81 + 18x + x^2$. Caitlin had written $x^2 + 18x + 81$. I asked them what was wrong with having $81 + 18x + x^2$?

Caitlin: *We put the x-squared first.*

Dale: *Because it's x, x-squared, we always put the letters first.*

The above three excerpts, Rule of Order of Operations, Difference of Squares and Sum of Squares demonstrated that providing students with the opportunity to share their learning in a minimal risk environment does empower them to learn from each other, even identifying misconceptions. Two male year 9 students dominated the discussion; however, they attempted to encourage their peers and later in the session another year 9 male, Peter started to enter the discussion. The female year 9 student was participating; however, as she was very quiet her voice often went unheard.

5.2.3 Elaborated Discourse and Student Peer Teaching

I recorded 20 responses from the students in the example from Class B. Of those, nine were elaborated, that is, 45% were elaborated. I recorded 103 responses, with Class A, over the three cited examples and of those, 36 were elaborated, that is 35% were elaborated. With the older group of students there was minimal intervention or interruption by the teacher hence much of the interaction or student utterances were procedural in nature. Hence 36 elaborated responses would seem to imply that students were able to engage in elaborated discourse. Elaboration, or providing opportunities for students to develop the language skills and the confidence to provide elaborated responses, remains a worthy goal for me. It was the progression to elaborated responses that provided opportunities for learning/teaching mathematical concepts, and it also provided opportunities for clarification to reach a shared understanding.

As much as my interest has been in elaborated discourse, I do believe that in extended conversations there are opportunities that may highlight misconceptions and allow for the correction of those misconceptions. In the conversations for the first group, students were providing explanations of two different ways to record the scoring for the game. Both worked. However, clarification was necessary for me and probably other students. The value for students here was in making the personal link. I believed I had created a cognitive platform for students which could be used later by recalling the games, the experience or by how we had clarified our understanding.

In Class A, as stated above 35% of responses were elaborated; however, again, of more interest was the approximation to correct use of language and the ongoing conversation. In that instance, as in the games explanations of Class B, the entire conversation needs to be examined rather than just the individual utterances.

Discourse was not spontaneous so I needed to provide a structure for students to be able to participate in potential elaborated discourse in a peer teaching setting. Elaboration is not always required, at times simple short responses are required. I wanted students to have the ability, skill and opportunity to participate in elaborated discourse when appropriate.

5.2.4 Reflections of the Student Peer Teaching Strategy

As the teacher my role was required at times to be seemingly passive and to be constrained to that of facilitator, managing, encouraging, clarifying and guiding students' contributions. When the strategy had been established with a group of students my role changed to one of participant; however, this was dependent on the group and how well the strategy was established. Empowering students to display their skills and knowledge provided me, as the teacher, with a tool I could use to assess students both formally and informally. I chose informally as I did not want the students to feel the pressure of a formal assessment at that point in time.

The impact of teacher modelling on student language and behaviour was apparent in the expressions that were used by students, in the formatting of how students wrote up answers and in the way that students interacted with each other. Students reflected as they processed information and as they processed solutions to problems. The pauses, self-corrections and 'umms' were all evidence of self-reflection, as were the 'oh, I get it now.' This strategy is one that is particularly useful with multi age groups and/or classes where students can learn from each other in a fun and safe environment.

Benefits of the Student Peer Teaching Strategy

The first benefit of the *Student Peer Teaching Strategy* was undoubtedly students talking and sharing their mathematics. At a minimum level this provides students practice with mathematical language, modelling of language and skills. On a broader level the strategy provides students opportunities to engage in a social learning situation that can support meaningful learning and improved understanding. The second benefit, associated with the first would have to be students learning from each other. Students can often explain concepts in a manner that makes sense to their peers. They can interchange between formal and informal mathematical language and explain their understanding on a level different to how a teacher might explain.

A third benefit would be students taking a leadership role in learning mathematics. Students were able to choose the concept (or concepts) that they wanted to teach their younger peers, they planned their presentations and worked through the prior understandings needed to teach those chosen concepts. The students effectively took responsibility for their learning and deeper understanding of the concepts they had chosen. The fourth benefit would be the opportunity to develop deeper understanding. Having to explain mathematical concepts requires a reasonable level of understanding. This may mean that students have to spend time learning or ‘re-learning’ the concept in order to effectively explain it to others.

A fifth benefit would be with the opportunities for a teacher for real assessment. Given the opportunity to observe students, whether it is formative or summative assessment, while they are sharing their mathematical knowledge, skills and understanding can provide a teacher with valuable insights and information. Whilst listening to students and watching them explain a concept a teacher can judge their level of understanding and performance.

Challenges of the Student Peer Teaching Strategy

The first challenge would be in managing the strategy, as it needs careful management. As the examples demonstrated the skills for sharing learning need to be developed in an environment that supports risk-taking by students. In the early stages of sharing their learning students need considerable support from the teacher. Probing questions need to be employed to extract the information, or understanding that students want to convey. With the younger students I chose to do this in a manner that conveyed the message that I didn’t understand as I believed this was a safer approach, less threatening for students and would continue to engage them in the conversation. With the older group of students I only intervened to ask for more examples and on one occasion to clarify. As students develop the skills less intervention is required by the teacher. This is where careful management is required; knowing how much to intervene and the manner of intervention.

A second challenge is in overcoming the high level of risk for both students and teacher. The risks for students are how they might appear in front of their peers – image is important for most students and overcoming issues to do with confidence in speaking in public. The risks to be overcome for teachers are managing the student risks as well as overcoming the potential risk that students might not learn as anticipated. The latter risk is really one of perception and possibly not a real risk as students in this study demonstrated they were eager to engage in managing their own learning and were willing to undertake peer teaching. Perhaps the risk in this study was minimised by the preparation of students. Time spent in preparation can overcome both of the risks just described.

The third challenge would be with the time it takes to develop confident students. There was approximately 30 minutes preparation time with the younger class of students and approximately three hour lessons preparation time for the older group of students. This challenge links back to the two mentioned above; knowing how much time to allow before students should be ready to share their learning. Again this would be dependent upon the skills the students have and other purposes that a teacher might have such as the informal learning that would be occurring as students were progressing their own learning and discussing this with other students and the teacher. Given that the younger students had not yet developed the skills for sharing their learning there did not appear to be a good reason to allow more time for preparation.

A fourth challenge, linked to the one above, is with the preparation of students for peer teaching. Factors like how much time, how much freedom in the choice of concept, how much support to provide, whether students should work in small groups or individually all need to be considered. There would be no hard and fast rule. The decisions made would be totally dependent upon the makeup of the class group. A fifth challenge would be with engaging all students and this links to the previous challenge and the possible solution would be in how the preparation was managed and then how the peer teaching was managed. Knowing how a student would cope with peer teaching would be something a teacher could ascertain during the preparation and in observing student interaction.

5.3 Reflections Focused on the Research Questions

As with Chapter 4 each of the research questions is addressed in turn providing a discussion for each strategy.

5.3.1 Research Question 1

The reflection will begin with Research Question 1: What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)? The *Shared Experience* and Purposeful Discussion strategies were offered as part of that range in the previous chapter. In this reflection the *Blended Instruction* and *Student Peer Teaching Strategies* will be added to the range. Again there were a range of sub-strategies within each of the strategies: for the *Blended Instruction Strategies* there were those I referred to as Targeted Instruction, Responsive Teaching and Guided Discovery. The examples provided for Targeted Instruction were Metaphoric Links and Creating Visual Links. The example provided for Responsive Teaching was Problem Words and for Guided Discovery the example provided was Foreground or Background. The sub-strategies for the *Student Peer Teaching Strategy* were divided in two – Establishing the Peer Teaching Strategy and The Established Peer Teaching Strategy.

5.3.2 Research Question 2

There are three parts to the reflection of the second research question – language used to create a closed learning community, language used as a learning tool and the role of elaboration. A discussion of the three areas is provided below.

Language Used to Create a Closed Learning Community

A learning community that has focus, shared purpose and shared language can be an effective environment for learning. Bernstein (1971, 1973, 1975) referred to the language in these groups as Restricted Code.

Table 5.2 – Strengthening the Learning Experience

Using Classroom Strategies Focused Through Language – Part 2

Blended Instruction	• Targeted Instruction • Responsive Teaching • Guided Discovery	• Uses elaborated language within the closed learning community developed through the *Shared Experience* and *Purposeful Discussion*
Student Peer Teaching	• Establishing the Strategy • The Established Strategy	• Develops and builds on strengthening the closed learning community • Builds elaborated discourse • Demonstrates elaborated discourse

Suffice to say that the language used in a closed learning community like a class does have a language of its own. This language can be a powerful tool for learning. Table 5.2 builds on Table 4.5, details the strategies and their use in creating a closed learning community and the progression to elaborated and formalised language which links with the paradigm outlined below in Figure 5.5.

Language Used as a Teaching Tool

Blended Instruction strategies with a mix of formal and informal language, modelling language and the use of metaphor and analogy make links and use language to create visual links. The mathematics can be modelled with visual cues. In Example 1: Metaphoric Links, the analogy of school hierarchy was used to show how a hierarchy functioned. This was linked to the acronym BIMDAS written vertically down the whiteboard to enhance that understanding of hierarchy.

Using appropriate language a teacher can set up base understanding and then that understanding can be built upon, hence the term cognitive platform as an anchor for embedding new knowledge. Using the right word, with the right tone and gesture can act as the seed to germinate new knowledge. Like the Fractions Lesson (Lesson 3), where I introduced the mathematical concept of ‘corresponding’ which I knew was difficult for some students to grasp when they came to corresponding angles. This caused me to question where I should place the emphasis with the words that I use. In that same lesson, working with fractions, collecting correct responses in groups, students were asked to give their answer as a fraction. For example, three correct out of five was given as three-fifths; two out of four was given as a half. This was just another way in which students could practice with the use of language and with the concept of equivalent fractions.

A potential area of concern can be when the meanings of words change. I became aware of this in the Problem Words example with Responsive Teaching. Although most students understood the use of the word ‘take’ in a word problem, such as ‘take a number and then add six’, one student openly admitted that he understood the word to mean ‘subtract’. Other words can take on more emphasis to assist students with understanding and functioning like the example in the Set of Six Calculations where using ‘lots of’ instead of ‘times’ or ‘multiplied by’ could assist with developing understanding of the distributive property. There would be so many examples where a focus on language could place emphasis on specific words which would then assist with understanding.

The Role of Elaboration

The *Blended Instruction Strategies* provide opportunities for modelling and scaffolding of language by a teacher. There are also opportunities for the teacher to model elaborated discourse. There are far more opportunities for students to demonstrate their thinking, understanding and skill development through elaborated responses with the *Student Peer Teaching Strategy*. In the first example offered with younger students, 40% of responses were elaborated; however, there was considerable teacher probing to achieve those responses. With the older group of students 35% of responses, or utterances, were elaborated; however, many of the student utterances were related to procedure as the students were managing the lesson. The following list was offered in the previous chapter as being substantial reasons for employing elaborated discourse, and those reasons remain the same with the strategies examined and discussed in this chapter:

- to reflect understanding/thinking,
- to help develop understanding,
- to enable and demonstrate self-correction,
- to reflect problems,
- to reflect learning as being a social experience, and
- to move students from participation in a closed, inclusive group to being participants in the broader mathematics community.

To that list can be added:

- to encourage students to take a leadership role in the classroom, and
- to encourage students to take responsibility for their own learning and to share that learning.

The Language Paradigm

There is evidence to suggest that students progress through differing stages of language usage and that with support, practice and development of skills move to elaborated utterances without assistance. Figure 5.5 represents a paradigm or model of language usage as I believe happened in my class during the study. The top end of the figure represents the beginning of using the strategies with the positions of student and teacher outlined. The bottom of the figure represents the changed positions when the strategies have been implemented.

Student	Teacher
• *Language is basic, unsophisticated, linked to visual imagery* • *Descriptions are simple utterances*	• *Uses the language of the student with progressive introduction of more sophisticated language/terminology* • *Must keep in mind the linguistic endpoint and that understanding, knowledge and skill of the concept being learned is the 'point'.* • *Linguistic forays, descriptions in basic terms and correlated with the students 'stage' of understanding*
~	~
• *Language has become more sophisticated, utterances now more resemble sentences, and are mathematically correct* • *Here we see students have a greater command of the language, skills, knowledge and understanding both of the language and of the mathematical concept being learned*	• *Will back step as required, reclaiming the links of the past and re-establishing the concept in more sophisticated language*

Figure 5.5 – Classroom Strategies
Focused Through Language Paradigm

5.3.3 Research Question 3

Reflection on Research Question 3 leads us to the benefits and challenges with both the *Blended Instruction Strategy* and the *Student Peer Teaching Strategy*. The benefits created by using the strategies should start with the focus the strategies provide for learning. The *Blended Instruction Strategies* provide focus for targeted instruction, whilst the *Student Peer Teaching Strategies* provide focus for students in preparation and in the sharing of learning that happens during student presentations. Both strategies have the potential to clarify and make explicit what needs to be learned and can occur at the point of student need. A teacher can act on feedback that was provided in a preceding *Shared Experience* or *Purposeful Discussion* whilst the *Student Peer Teaching Strategy* offers genuine occasions for all forms of assessment.

The challenges created by the strategies would include management of the strategy along with planning and thinking through the various directions a lesson could potentially take. Management of the high level of risks for students is another challenge that must be managed carefully. Preparation of students and recognising when they are ready to participate in *Student Peer Teaching* would help minimise the risks and potentially overcome some of the problems associated with engaging all students. Management of time involves challenges; time allocated to preparation of students and time to deviate from a planned lesson to participate in Responsive Teaching; however, this can be balanced with the benefits provided by engaging in the strategy. Recognising opportunities, especially for Responsive Teaching is also a challenge.

Knowledge of mathematical content and pedagogy are essential as without this background opportunities for using the strategies, that are language focused, may not be recognised hence rendering the strategies ineffective. This knowledge also enables a teacher to easily translate between informal and formal language use. The strategies are closely linked and may appear very formulaic and this would seem to be a challenge in appearance. The strategies do provide structure and demonstrate how they can fit together to work effectively.

The following chapter provides an overview of the study and offers the conclusions and discussion. Recommendations are also offered.

Chapter 6

Conclusions and Discussion

Classroom discourse has been a focus of many classrooms and research studies for over 20 years (Applebaum, 1995; Brown, 2001; Chapman, 1993; Cobb, 1998; Corwin, Storeygard & Price, 1995; D'Ambrosio & Prevost, 1995; Edelsky, Smith & Wolfe, 2002; Elliott & Garnett, 2008; Ferrari, 2004; Lampert & Blunk, 1998; Marton, 2004; Rowland, 2001; Sherrin, Louis & Mendez, 2000; Steele, 2001; Walshaw & Anthony 2008; Wertsch, 1985;), a focus reflected in associations like the NCTM. There appeared to be much that could be gained by discourse experiences being introduced and supported in the classroom; however, this has not been borne out. The 'promise' of classroom conversations – classroom talk contributing to better understanding of mathematical concepts by students – has not translated into common practice. As highlighted in Chapter 2, introducing discourse into classrooms is difficult for teachers (D'Ambrosio, 1995; Manouchehri & Enderson, 1999; Van Zoest & Enyart, 1998). Teachers need to be convinced of the value of change: Habits of mind, beliefs and attitudes need to be changed. Consequently many teachers abandoned the search for improving the use of discourse as a part of learning/teaching which, in the flourish of the ensuing research and implementation, left many believing that the promise of learning benefits was nothing more than just a promise. This study has re-examined the 'promise' and has found that by taking a slightly different approach, the rewards that were promised can be achieved. The strategies used in this study were not new; however, language can be highlighted in them for different purposes.

In this final chapter the Research Questions are further discussed. Section 1 provides an overview of the thesis, followed by Section 2 where an in-depth reflection on the Research Questions is provided and findings are offered as a means to answer those questions. Section 3 outlines the distinctive contributions of this study, whilst Section 4 examines its limitations. Section

5 provides a number of recommendations, followed by concluding remarks in Section 6.

6.1 Overview of the Thesis

In preparing an overview of this study I found that the best approach was to review the learning journey I had taken. The roots, or origins, of this thesis go back approximately ten years, back when I first became concerned at how language was being used in mathematics classrooms. Examination of relevant literature and critical appraisal of other studies took me to the place where I started examining language usage and strategies focused through language that could be used in the classroom, and I began to believe that, with language being effectively used, I could actually make a difference to students' understanding of mathematics. Hence the following research questions were formulated:

1. What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)?
2. What is the role of language in the application of those strategies to engage students in extended learning conversations?
3. What benefits are created and challenges encountered when those strategies are employed in the mathematics classroom?

With the research questions firmly in mind, the study was planned with two groups of students in a rural district high school. The first group consisted of year 8, 9 and10 students and the second group consisted of year 6 and 7 students both in multi-aged classes. A colleague who also taught the same group of students acted as a Cooperative Colleague. The study commenced early in Term 1 of 2010, with a follow up in the latter stages of Term 2 and was completed with the collection of student responses late in Term 4. As this study involved a Participatory Action Research design (Brydon-Miller, Kral, McGuire, Noffke & Sabhlok, 2011; Denzin & Lincoln, 2008, 2011), a cycle of planning, undertaking of strategies, evaluation and reflection was carried out with my colleague, with refinements being made, based on the evaluation and reflection.

Intense data analysis was undertaken following the completion of the study. The first rounds of analysis yielded different contexts in which language usage occurred; however, the data yielded very little of the strategies in that format. At that point a different analysis approach was called for and Grounded Theory (Glaser & Strauss, 2009; Strauss & Corbin, 1990) was determined as the way to see past the surface. Without a change of approach to analysing the data I would have missed identifying and categorising the strategies. With a backwards form of analysis, matching the data in a two-dimensional table yielded the strategies *Shared Experience*, *Purposeful Discussion*, *Blended Instruction* and *Student Peer Teaching*.

I did experience difficulty identifying the strategies that were in use in my classroom. It took several months of examining the data, discussing my interpretations with my Cooperative Colleague and reflecting on what I was doing and what my students were doing before I could put names to the strategies developed and refined during the research study. Even though I had put together a lesson which included the strategies I still had not, at that point, given names to the strategies that were being consistently employed. As such I used them but did not say to students ‘Today we are going to do strategy … so that we can learn …’. Most of the time I would have said ‘Today we are going to talk about and use …’ The study provided definition for an intuitive approach that was based on using language as a teaching tool.

Teachers have a considerable arsenal of teaching strategies that they have developed over their teaching careers. They probably have not ever felt the need to take time to reflect on the bigger picture to identify and give names to each of the strategies that they used. The data analysis enabled me to identify and name the strategies that were being used.

Imitation learning, modelling and scaffolding seemed to fit nicely into that expanded ‘grab bag’ of philosophy, pedagogy and teaching/learning strategies that I possessed. It gave sense to the impact of gestures as an extension of language and its many uses by students, and by me, in the classroom while teaching and while communicating in a group or one-on-one.

Language use is very much part of the ‘bigger picture’ and how we deploy language in our classrooms is very much dependent upon our beliefs about teaching and learning and about the pedagogies to which we adhere. The strategies developed in this study encompass the beliefs and values of Bernstein’s (1971, 1973, 1975) language codes, elaborated and restricted, constructivist practices finding students’ prior knowledge and understanding as well as some traditional teaching experiences. Learning can be enhanced through the use of awareness of language codes and embedding them in the strategies used to facilitate learning.

Accepting and using the language arising from the *Purposeful Discussion* created a closed, or restricted, class group (learning community) whose foundation was created in the *Shared Experience Strategy*. There were opportunities to begin to bridge the gap into more formal language in the *Purposeful Discussions*; however, the ensuing *Blended Instruction Strategies* provided the means where the formal language was used in a dual sense, interchangeable and accepted.

6.2 Research Questions – Reflections and Findings

This section synthesises the reflections of the research questions offered in Chapters 4 and 5, and uses the same format as the previous reflections.

6.2.1 Research Question 1:

What range of classroom strategies can be used to engage students in extended learning conversations (elaborated discourse)?

As stated previously, the strategies, *Shared Experience*, *Purposeful Discussion* and *Student Peer Teaching*, were founded on the constructivist philosophy that we all construct our own knowledge, based on our prior experiences. The starting point was finding ways to use classroom strategies that could be focused through language which would embed a framework, or platform, that all students could take up. Table 6.1 displays the range of strategies, sub- strategies and examples used in this study.

Table 6.1 – The Classroom Strategies Focused Through Language

Strategies	Sub-strategies	Examples
Shared Experience	→ Focused Tasks	→ Six Calculations → Mental Questions/ Calculations → Feedback → Roundtable Reflection
	→ Physical Activity	→ Fraction Stand-up
	→ Cognitive Organiser	→ Brainstorm
Purposeful Discussion	→ Convergent Purposeful Discussion	→ Six Calculations → Indices
	→ Divergent Purposeful Discussion	→ Reflective Thinking
Blended Instruction	→ Targeted Instruction	→ Metaphoric Links → Creating Visual Links
	→ Responsive Teaching	→ Problem Words
	→ Guided Discovery	→ Foreground or Background
Student Peer Teaching	→ Establishing	→ Introducing
	→ Established	→ Rules → Differences → Perfect Squares

Just using classroom strategies does not infer success; there is a need to continually reflect on language usage as well as on how students responded.

This is a demanding aspect of the teaching/learning process; however, not one that can be overlooked. Every aspect of the 'teaching' process that contributes to student learning needed to be examined. This was in order to present a cohesive lesson plan where every aspect contributed to the same purpose. The Equivalent Fractions Lesson (Lesson 3) is an example of this; where the *Shared Experience* – a physical activity – formed the basis for the following *Purposeful Discussion* surrounding the concepts that students would be required to use in the Targeted Instruction that followed.

6.2.2 Research Question 2:

What is the role of language in the application of those strategies to engage students in extended learning conversations?

Language has the power to be persuasive but can also cause misunderstanding and confusion. As the data demonstrated, language can play a critical role in how students interpret and understand the information that has been shared with them. The strategies incorporate the use of language as a teaching tool, language used for communication and language used as a means of creating a restricted community where inclusion and acceptance are implicit. The requirement of how language is used to create an inclusive or excluded class group is less understood. Atherton's (2011) comments on Bernstein's codes, captures the essence and value of creating a closed or restricted community as outlined in Chapter 2. Insight into how language can be used as modelling and scaffolding tools is requisite. Without this understanding the strategies used would be less effective.

The response to research Question 2 is provided in three parts – language used to create a closed learning community, language used as a learning/teaching tool and the role of elaboration.

Language Used to Create a Closed Learning Community

A learning community with focus, shared purpose and shared language can be an effective environment for learning. Language can be used to create that learning community. Bernstein (1971, 1973) referred to this language as Restricted Language code where language relevant to a sociocultural subgroup had its own ‘language’ and the language used in a closed learning community like a class does have a language of its own. As highlighted in Table 6.2 the strategies and the way they are used create a closed learning community which can then be used to progress elaborated and formalised language.

Table 6.2 – Strengthening the Learning Experience

Using Classroom Strategies Focused Through Language

Shared Experience	• Focused Tasks • Physical Activity • Cognitive Organiser	• Lays groundwork for Closed Community (Restricted Code)
Purposeful Discussion	• Convergent Purposeful Discourse • Divergent Purposeful Discourse	• First creates the Closed Learning Community • Second, can bridge the gap between Closed Learning Community and Formal (Elaborated) Codes • Promotes student engagement in Elaborated Discourse
Blended Instruction	• Targeted Instruction • Responsive Teaching • Guided Discovery	• Teacher models, students develop elaborated language within the Closed Learning Community progressed through the *Shared Experience* and *Purposeful Discussion*
Student Peer Teaching	• Establishing the Strategy • The Established Strategy	• Develops and builds on strengthening the closed learning community • Builds and demonstrates elaborated discourse acquisition

Through developing this shared language during extended conversations the group effectively acts like a closed group with a language of its own that can be a powerful tool for learning.

In this Closed Learning Community language is used to create an environment that is conducive to risk-taking on the part of students, hence increasing their willingness to participate and to share. Their contributions can be accepted, acknowledged and validated without recrimination.

Language Used as a Learning/Teaching Tool

The way in which the language is being used either supports or hinders learning through the use of language to embed a framework, or platform, that all students can take up. Language for learning is focused purely on how language is employed in the classroom; this is the mechanics of language use. There needs to be a constant cycle of using the language as a teaching/learning tool, listening to students and reflecting on what they really are saying with a teacher continually reflecting on their own language usage as well as on how their students responded. This is a demanding aspect of the teaching/learning process and one that should not be overlooked. A teacher should also examine how every aspect of their ‘teaching’ using classroom strategies focused through language contributes to student learning.

The ‘Orange Lesson’ was an example of this; where the warm up activity – a set of ten mental questions – formed the basis for a discussion surrounding the concepts that students would be required to use in the Guided Discovery activity that followed. In that example the language used assisted students to make the cognitive link and the jump through the formula ‘shifts’, create visual imagery which also assisted with creating the link cognitively. Language can also be used to create a springboard or platform for new material. It can be used to check for understanding through listening to students’ responses, for recall of information, knowledge, understanding and/or skills levels. It can be used for sharing and consolidating, assessing the mood and readiness for learning by listening to the language used by students to express their thoughts and processing. Opportunities should be provided for students to practice using the language.

Language can be used as a Cognitive Platform, like in the Fractions Stand-up lesson, by something as simple as saying ‘remember when we stood up to find fractions’. Using the right word, with the right tone and gesture can act as the seed to germinate new knowledge. Using appropriate language a teacher can set up base understanding and then that understanding can be built upon, hence the term cognitive platform as an anchor for embedding new knowledge. Like the Fractions Lesson (Lesson 3) where I introduced the mathematical concept of ‘corresponding’ which I knew was difficult for some students to grasp when they came to corresponding angles. A potential area of concern could be when the meanings of words change. There would be so many examples where a focus on language could place emphasis on specific words which would then assist with understanding.

Language should be modelled by teachers linked to concrete and/or physical activities which occurred with that same Fractions lesson. The mathematics can be modelled with visual cues. Language in the Brainstorm lesson created a classroom context using contexts that were familiar to students, bringing in student experiences and making links with what they know. Here students modelled language use for their peers, self-corrected and were encouraged to use correct mathematical language through teacher modelling.

The *Purposeful Discussions* were language derived activities that encouraged and actively promoted reflection and metacognition. Brown’s (2001) ‘hedging’ or approximation may have been seen in the early stages when students used safe words like ‘I think’; however, metacognition was evident later as examples of students reflecting on their thought processes. In the *Purposeful Discussion* activities language was used for picking up errors in how students used language to explain, inferring a potential misconception in subsequent years when students would be working on more complicated problems.

It was also very evident that non-verbal means were used to support the language being used to enhance, clarify, or extend and for some appeared to make the links back to previous visual imagery that had been created and linked to that language. Air drawing was a good example of how gestures were used. The use of metaphor and analogy supported, or linked, language and visual imagery. *Blended Instruction Strategies* with a mix of formal and informal language, modelling language and the use of metaphor and analogy make links and use language to create visual links.

The Role of Elaboration

Examining patterns of discourse was of value in realising the role of elaboration. There appeared to be places in lessons where it was not as important to have elaboration and it is not realistic to want to have most of a lesson operating where the majority of the conversation was elaborated. The data revealed that as a lesson progressed, not only did more students participate, but the number of students participating in elaborated discourse, increased. One would expect that within *Purposeful Discussions* student responses would be more likely to be elaborated. This was borne out by the data.

Hearing what students are saying is a powerful learning/teaching tool. For students to be able to elaborate their responses they must learn the skills of elaboration to be able to engage in conversation and to convey what they have learned. Hence teachers need to model the skills and provide time and opportunities to engage in extended learning conversations that require elaborated responses. The *Blended Instruction Strategies* provide opportunities for modelling and scaffolding of language by a teacher. There are also opportunities for the teacher to model elaborated discourse.

It is the responsibility of a teacher to assess a student for a range of reasons; however, I believe it is essential that a teacher assesses students in terms of their understanding and knowledge and skills. Tests can assess what students can do in routine situations, and just because a student can follow an algorithm and end up with the correct result does not necessarily mean that a student has a very broad understanding of concepts that have been learnt. One way to test students' broad understanding is by engaging them in conversation about what they have learned and how they have learned it, and how they understand that and process information. So I believe it is the responsibility of a teacher to provide students with the skills to be able to engage in an elaborated conversation in order to convey their level of understanding.

There are far more opportunities for students to demonstrate their thinking, understanding and skill development through elaborated responses with the *Student Peer Teaching Strategy*. In the examples offered in Chapter 5, the percentage of student responses considered as elaborated ranged between 35 – 40%. The younger group of students achieved a higher percentage of elaborated responses; however, the format of their sharing was quite different to that of the older students. The older group of students managed their 'lesson' which had a mix of elaborated and very brief responses; however, many of the student utterances were related to procedure. There was evidence to suggest that students move through differing stages of language usage and that with support, practice and development of skills they move to elaborated utterances without assistance.

So what purpose does elaborated discourse serve? The following list provides substantial reasons for employing elaborated discourse as a part of the classroom strategies focused through language:

- to reflect understanding/thinking,
- to help develop understanding,
- to enable and demonstrate self-correction,
- to reflect problems with student understanding,
- to reflect learning as being a social experience,
- to move students from participation in a closed, inclusive group to being participants in the broader mathematics community.
- to encourage students to take a leadership role in the classroom,

- to encourage students to take responsibility for their own learning and to share that learning.

6.2.3 Research Question 3

What benefits are created and challenges encountered when those strategies are introduced into the mathematics classroom.

This question examined how the strategies translated into the classroom pedagogy. Examination of the benefits and challenges cited in Chapters Four and Five for each of the strategies revealed some insights into the answer to this question.

The Benefits

Benefits created through use of the *Shared Experience* and *Purposeful Discussion* strategies are first that the *Shared Experience* strategies provide the focus for subsequent *Purposeful Discussion*. Both strategies create anchor points or *Cognitive Platforms* for further learning. The strategies address the needs of students, and the teacher receives almost instant feedback about students' understanding.

Purposeful Discussion, both converging and non-converging encourage mediation of thought with considerable evidence to show students using the language of 'I think', 'I thought', 'I went', and then often changing direction of an explanation after they realised they were incorrect. So self-correction is a benefit. There is also benefit in hearing what students are saying, as it is here that teachers can diagnose errors and misunderstanding. It is then possible to find ways to 'undo' a misconception or correct an error in a way that does not discourage a student from further participation.

The benefits created by using the strategies – *Blended Instruction* and *Student Peer Teaching* commence with the focus the strategies provide for learning. The *Blended Instruction Strategies* provide focus for targeted instruction whilst the *Student Peer Teaching Strategies* provide focus for students in the preparation and sharing of learning that happens during student presentations. Both strategies have the potential to clarify and make explicit what needs to be learned and can occur at the point of student need. A teacher can act on feedback that was provided in a preceding *Shared Experience* or *Purposeful Discussion,* whilst the *Student Peer Teaching Strategy* offers genuine occasions for all forms of assessment. A summary of the benefits is offered in the following discussion.

Creation of Cognitive Platforms

The strategies provided focus, creating Cognitive Platforms from which new learning, skills, knowledge and understanding could be developed. Rich discussion came from the *Shared Experience*s; and from those experiences, students learned from each other and misconceptions were identified and corrected.

Providing Instant Feedback

The major improvement for students was the constant feedback in terms of their prior knowledge and understanding which enabled better targeting of the language used and the way new concepts were introduced.

Making the Links for students

The ten mental questions chosen for the *Shared Experience* at the beginning of Lesson 5 – The Orange Problem – were linked to finding areas of triangles, circles and understanding the link with radius, diameter and circumference. Students needed to be able to see the development and calculation of the area of a circle segment, hence the question on hexagons.

Encouraging Mediation of Thought

Provided students are offered opportunities and encouraged, then mediation of cognitive processes can occur as demonstrated in the 'I was thinking…', 'I thought…' or 'I was thinking … but …'. Students will reflect on their processes if they are given the opportunity and they will assimilate the language that is modelled and scaffolded in learning conversations. Much of the language used was directed at cognitive mediation. A teacher's actions and what is said, how and when it is said can empower, exclude or facilitate extended conversations and participation in *Purposeful Discussion*.

Self-Correcting

One of the benefits of using *Purposeful Discussion*, that is, having students explain their methodology, is their realisation of an incorrect process or calculation. Students self-corrected when they were articulating their solutions. As stated previously, one of my fundamental beliefs is that if I can get students to see their error in process or methodology then I am far more likely to have success changing the way they do things; they are more likely to accept support from me and take on board what I am showing them. I believe that if students do not see an error in what they are doing then they are not going to value what I am saying.

Hearing What Students Say Provided Valuable Insight

As stated earlier, there is great benefit to a teacher in hearing what students are saying while they are engaged in *Purposeful Discussion*. Here, where the discussion is focused, it is possible to diagnose students' misconceptions and to plan for ways to counter these. There are also occasions where students' comments or intentions may be misinterpreted, and through discussion it is possible to clarify misunderstanding.

Diagnosing Student Misconceptions

Identifying or diagnosing problems with student understanding of concepts was an outcome of *Purposeful Discussion*. Through responding to students' explanations and elaboration of processes it was possible to identify misconceptions and then create opportunities to challenge those misconceptions.

Addressing the Needs of Students and Teaching at the Point of Need

The *Blended Instruction Strategies* can address a need that students demonstrate as a problem for them or their peers. Students engaged in the *Student Peer Teaching Strategy* present an opportunity to develop deeper understanding. Creating the appropriate *Shared Experience* and observing student behaviour empowered both students and teacher.

Focusing Strategies

The *Shared Experience*, *Purposeful Discussion* and *Blended Instruction Strategies* come together to form a cohesive and focused set of teaching strategies.

Leadership, Empowering Students and Students Learning From Each Other

Students took a leadership role in mathematics. Students were empowered when they entered a *Purposeful Discussion.* Students talked about and shared their mathematics. Student responses were implicitly 'valued' when their responses were accepted. Students gained confidence when a structured approach was used to engage them in peer sharing.

The Challenges

Examination of the challenges cited for each of the strategies offers insights into the answer to this question. The following challenges were associated with the strategies:

- management of the strategy along with planning and thinking through the various directions a lesson could potentially take;
- management of the high level of risks for students;
- recognising opportunities;
- the need for flexibility; meeting the needs of students;
- managing unwanted outcomes;
- managing the internal tension;
- non-participation of students;
- the teacher holding back;
- time and pace of progress;
- pace of lessons; and,
- challenges centred on the unpredictability of the direction and pace of a lesson when these types of strategies are employed, causing internal tension and discomfort for a teacher.

Preparation of students, recognising when they are ready to participate in *Student Peer Teaching* would help minimise the risks and potentially overcome some of the problems associated with engaging all students. Management of time is a challenge; time allocated to preparation of students and time to deviate from a planned lesson to participate in Responsive Teaching. This can be balanced with the benefits provided by engaging in the strategy. Recognising opportunities for using the strategies that are language focused can be made easier by having knowledge of mathematical content and pedagogy which also enables a teacher to easily translate between informal and formal language use. Without this background the opportunities may not be recognised, hence rendering the strategies ineffective.

Further examination highlighted the dichotomous nature of the challenges. After initial identification the challenges were coded and regrouped. Clearly they fell into either one of the following categories: Management of Teaching and Learning, or Facilitating and Promoting Student Learning. The following discussion highlights the challenges experienced through the two categories.

Management of Teaching and Learning

Managing Tension Created From the Need For Flexibility

Flexibility was required in how a lesson proceeded following a *Shared Experience*. There were times when I thought I knew where the class group was at in terms of their prior understanding; however, I occasionally got that wrong and had to be more flexible than I had planned, which translated into spending more time on preparing students during either the *Shared Experience* or *Purposeful Discussion* strategies. The beginning of the ‘Orange Lesson’ is an example of this, where, during the *Purposeful Discussion* more time was spent on discussing the solutions to the ten mental questions. I had anticipated that this would be a quick review; however, judging from student responses, more time was needed to ensure that students had a better understanding. With the Rule of Order Lesson I realised far more time was required to engage students in *Purposeful Discussion* after I had collected all their responses.

Flexibility is an absolute requirement if one is going to be responsive to student needs. Lack of flexibility causes difficulty and internal tension for a teacher who must decide whether it is better to spend more time on ensuring that concepts are better understood or whether to press on with the curriculum.

Non-Participation of Students

Most of the difficulties encountered resulted from frustration with the non-participation of many students. Breaking patterns of dominance by specific students in a group can be difficult; however, not impossible to achieve. Deliberately encouraging other students to participate in any manner that works can support reluctant students to become more involved. Encouraging girls in a group that is dominated by boys is difficult and ways must be found to challenge male domination.

Time and Pace of Progress

Time was a concern in several ways; benefits might not be immediately evident. Hence, a teacher could experience frustration, feeling that change was not happening. In a rapid fire conversation with willing and eager students, often just a few seconds was required for individual students to offer elaborated utterances. As was often seen in the data elaborated discourse, with some students, could be achieved in a very short space of time. The converse of the above is that it takes considerable time to encourage students and to develop the skills required to participate in that extended learning conversation, which does appear to be a contradiction.

Frequently, the data showed that more students engaged in the learning conversation, with more elaborated utterances, as the lesson progressed inferring that time, modelling, scaffolding and practice is required.

Time is an issue in a content-driven curriculum and would definitely be an issue for teachers who are abiding by this type of curriculum. It takes time to develop confident students who will lead student peer teaching. The pace of lessons was difficult to manage, even when the lesson involved mainly discussion. The real time as opposed to the discussion/conversation time and the importance of that time do not always seem to correlate.

Planning and Preparation

Preparing students for *Student Peer Teaching* presented management difficulties related to resources – for example, movement of students to other areas like the computer lab – as students were working individually and at different levels. Planning was also required to address the 'what if …', 'where might it go …', types of questions so that opportunities were not missed due to a lack of preparation in an area that had not been considered.

Risk Levels

Overcoming the high level of risk for both students and teacher was a real issue and one that needed to be managed. A high level of confidence is required to step into a learning/teaching approach that may be outside that considered as normal.

Managing a Closed, Inclusive Class Group

The difficulty involved in managing a closed group was in handling the transition to another class or teacher, as evidenced when another colleague, not part of the study questioned the younger group of students about what they were doing and was confronted with confused looks. Being aware of the difficulty a teacher faces in transitioning students into formal language and preparing them for the duality of their informal and formal language understanding.

Facilitating and Promoting Student Learning

Dependent on Pedagogical Content Knowledge

A good understanding of both mathematical content and pedagogy are required if the strategies are to be effective, as without this background, opportunities for using the strategy may not be recognised along with ways to use language to counter misconceptions. Teaching can look like it is algorithmically driven to an outside observer. The subtleties of the strategy may not be apparent to that outside observer. There could be a high level of risk associated with other's perceptions hence having a broad pedagogical content knowledge might minimise this risk somewhat.

Planning and Preparation

The use of the *Blended Instruction Strategy* required considerable planning and thinking through the consequences of, and planning for how to deal with the unforeseen, unintended outcomes, problems, or uncovered misconceptions. Preparing students for *Student Peer Teaching* requires a different approach when students could all be working on a different concept. Preparation of students can become a one-on-one exercise with the teacher, or with peers.

Responsiveness

There is a great deal of uncertainty when one approaches a lesson prepared to be flexible in terms of meeting the current needs of a group of students. Responding to meet the needs of students creates tension for a teacher; however, creating the appropriate *Shared Experience* through Responsive Teaching, and observing student behaviour can empower both students and teacher. Being responsive to meet the needs of a diverse classroom of students is also difficult. Listening to students explaining their methodology and attempting to interpret that instantly before moving on to listen to the next student is difficult, if not almost impossible; however, it is possible to get a sense of what the student is revealing. It can take considerable practice to recognise the opportunity for Responsive Teaching and to have the confidence to follow that learning need.

The Teacher Holding Back – The Role of Teacher as Facilitator

As the teacher, my role was required at times to be a seemingly passive role and to be restricted to that of facilitator, managing, encouraging, clarifying and guiding students' contributions. When the strategy had been established with a group of students my role changed to one of participant; however, this was dependent on the group and how well the strategy was established.

For me as the teacher I sometimes found it difficult to not jump in and 'takeover' the conversation rather than gently encourage and guide it to where it needed to go (Mary Boole's construct of teacher lust [1931], Tyminski, 2010).

Acceptance of the Value of Creating a Closed, Inclusive Group

This would be very easily overlooked and not something a teacher might consider as being one of their learning/teaching strategies; however, its value cannot be overstated. Accepting student responses as they are offered is difficult as a teacher usually wants to provide a 'correct' response before hearing a student's deliberation, which is linked to the previous difficulty. A teacher may feel that the risk of using the strategies is too great when there is a requirement to adhere to the timeframe of a content based curriculum. In my case I was able to make the choices of what aspects of the curriculum were covered and how they were covered. In hindsight, the control and management of what happens in an individual teacher's classroom would appear to be a major difficulty.

6.2.4 Findings for the Research Questions

Finding 1: (RQ1)

Classroom strategies can be focused through language to engage students in extended learning conversations.

Finding 2: (RQ1)

The range of classroom strategies focused through language can include *Shared Experiences*, *Purposeful Discussion*, *Blended Instruction* and *Student Peer Teaching*. Within each of the strategies there exists a range of sub-strategies that include, but may not be restricted to, those offered in this study.

Finding 3: (RQ1)

A set of classroom strategies, focused through language, can work together to form a cohesive set of strategies targeted to improve student learning and understanding.

Finding 4: (RQ2)

Language can be used to create a closed learning community which can be effective in enhancing student understanding.

Finding 5: (RQ2)

Language can be used as an effective learning tool where gestures and tone support emphasis on precise parts of language, hence targeting a specific means to achieve cognition of concepts. Language can be used to create cognitive platforms for further learning. Language can be used to create visual links further broadening the mechanism for student understanding.

Finding 6: (RQ2)

Engaging students in extended conversations where elaborated discourse is an achievable goal provides students with opportunities to reflect, participate in metacognitive processes hence improving their understanding.

Finding 7: (RQ2)

Students will engage in elaborated discourse, where it is relevant and appropriate, when they share their learning whilst participating in Student Peer Teaching.

Finding 8: (RQ3)

The benefits created when the strategies were introduced into the mathematics classrooms include the following:

- Creation of cognitive platforms
- Provision of instant feedback
- Creation of the links for students
- Encouraging mediation of thought

- Self-correction by students when reflecting on their processing
- Hearing what students say provides valuable insight
- Diagnoses of student misconceptions
- Addressing the needs of students and teaching at the point of need
- Focusing of strategies
- Leadership, empowering students and students learning from each other.

Finding 9: (RQ3)

The challenges encountered when the strategies were introduced into the mathematics classroom fall into two categories:

Those associated with management of teaching and learning –

- Managing tension created from the need for flexibility
- Managing unwanted outcomes
- Managing non-participation of students
- Managing the time and pace of progress
- Managing the planning process
- Managing the risk levels
- Managing a closed, inclusive group.

Those associated with facilitating and promoting student learning –

- Dependence on pedagogical content knowledge
- Dependence on planning and preparation
- Dependence on student responsiveness
- The teacher holding back
- Accepting the value of creating a closed, inclusive group

6.3 Distinctive Contributions of this Study

The classroom strategies used in this study are quite possibly no different to the strategies in use in most mathematics classrooms. The distinctive difference is the focus that language offers the strategies. A major part of this study was examining classroom strategies focusing the role of language. As stated elsewhere, there have been many studies examining classroom discourse and the impact of classroom 'talk' on student understanding. These studies (Ball, 1991, 2007; Steele, 1999, 2001) have examined the need for, and the use of classroom discourse; however, this study actually provides real strategies that could be implemented in any mathematics classroom. Other studies (D'Ambrosio, 1995; Manouchehri & Enderson, 1999; Van Zoest & Enyart, 1998) highlight the difficulties for teachers in adopting discourse practices into their classrooms; hence a distinctive contribution would be a pragmatic solution which is what this study is offering. This study has focused on a range of classroom strategies, focused through language, that become highly effective in promoting elaborated discourse, engage students in reflective practices and position students to share and lead their learning.

The relevance of Mathematics Linguistic Pedagogy is a significant and distinctive contribution of this study. Another study (Bailey, Chang, Heritage & Huang, 2010) referred to teacher linguistic pedagogy in mathematics and whilst referring to this study I adopted the term '*Mathematical Linguistic Pedagogy*' inferring a distinctive linguistic pedagogy that applied in the learning and teaching of mathematics. If one accepts mathematics (or mathematical) linguistic pedagogy then it follows that embedding a mathematical concept can happen by an appropriate choice and use of mathematical language. I strongly believe that it is possible to create a 'cognitive platform' or 'cognitive framework' by choosing the appropriate language. I also believe that practical or physical activities like finding one-tenth or one-fifth of the student group of students also provided a cognitive platform. I refer here to scaffolding, using language and cognitive scaffolding. Teachers need to become efficient mathematical conversationalists, and strategies that lead teachers into developing this skill are required. This study has provided those strategies.

6.4 Limitations of this Study

The limitations in interpreting the results of this study related to the nature of district high schools in terms of their rural setting, student numbers and my role as teacher and administrator as a participant researcher. The study was undertaken in a rural district high school where student numbers are small and classes are required to be timetabled so that multi-aged groups operate in order to effectively maximise resources. It is much easier to organise students in these classes to work together so that older students can peer teach younger students. In a larger class in a larger school, differences in student ability and/or interest could allow for student peer teaching in a similar manner using the strategies developed in this study. Translating the principles of the study would seem to be possible

The sample, that is the students and the chosen classes, were not selected randomly. Those who participated in the study were the students in the two classes that I taught. The younger group of students had not previously been in any classes that I taught; however, the older class did include some students with whom I had worked for two years. This could be seen as a limitation; however, it could also be seen as a positive factor. There weren't any discernible differences between the two classes, other than those one would expect due to differences related to age. Transiency of students contributed to minor difficulties with the study, which in turn may add to the limitations of the study.

Another limitation would be the teacher's background, as previously outlined in the overview of the thesis. Having a background in teaching students about communication and misunderstanding encouraged an interest that I already had in terms of language and how it can be used effectively in the classroom. Others interpreting this study and translating it into their own classrooms would need to consider this limitation. My role as a teacher administrator could be seen as a limiting factor for others interpreting this study. I certainly was able to circumvent any issues that might arise out of having a flexible program. Being in that position I was less likely to be questioned by students, parents or colleagues as to the structure of my lessons, planning and programs. This would be a significant difficulty for others who might like to take on board the strategies if they were not in a position to determine what happens in their classrooms.

I have interpreted the results in a manner given my background, possibly biased, as I feel an ownership for the strategies used in my classroom and as such this could limit my objectivity. Following on from this I have named, defined and described the strategies, hence adding to the issue of personal bias being a limiting factor. I do not believe there were procedural problems; however, again my interpretation through potential ‘rose coloured glasses’ may have limited my objectivity. I have made every attempt to be aware of this potential bias and limiting factor and to be very aware that the study was not about me as a teacher, but about a set of strategies that could be used to improve learning for students.

6.5 Recommendations and Implications for Further Research

As much as I believe this study has answered questions about engaging students in extended learning conversations and using language as a teaching tool, it has also raised many other questions. The following discussion suggests recommendations and implications of this study.

Language Used as a Teaching Tool

This study opened up the area of using language as a teaching tool; however, there may be further opportunities for research in the area of using language as a teaching tool and not just for conversation. Language can be considered as a teaching vehicle and as a change agent as well as being considered as a way of mediating thought. As part of this suggestion, research could be carried out into how language enhances as well as impedes learning and the role of gestures in aiding or limiting the learning. Gestures and tone are part of the use of language, part of the teaching package.

Using Language to Create a Closed Learning Community

This was one aspect of the study and I believe there is merit in following up the benefit that can be created by using a shared language to create an effective learning community. In following up the benefit, the challenge of transition to a new teacher each year must be addressed. Other factors to be considered could be the age of students where this is most effective.

Teachers as Good Conversationalists

The focus of this study was on Learning Conversations and in Chapter 2 the suggestion was made that teachers would be required to be good conversationalists if they were to pursue classroom discourse strategies. Ways to encourage this skill could definitely be an area to follow up.

Conducting One's Own Research

There is great scope for teacher practitioners to undertake their own research. There are practical considerations that would make this difficult; however, the rewards can be quite significant in terms of improving one's own performance and in improving outcomes for all students.

Mathematical Linguistic Pedagogy

This study has highlighted a place for *Mathematical Linguistic Pedagogy* as an area that could be developed to make a difference with students' learning. Further investigation of the link between pedagogical content knowledge and teacher linguistic pedagogy could be an area for some ground-breaking and exciting research.

Develop a register, or continuum, of informal to formal mathematical language, along with strategies to shift students along that continuum.

This proved to be outside the scope of the current research study; however, one that could prove to be of immense value.

Combining Explicit Instruction with a Constructivist Approach

Does it have to be an either or? Why do the two approaches, Explicit Instruction and the opposing Constructivist approaches have to be dichotomous, separate? Is it time to start combining the two approaches? This study offered *Blended Instruction Strategies* as a means to effectively combine the two approaches. This would appear to be another fruitful area for research.

6.6 Concluding Remarks

Like any teacher who has many years' experience working with students, I have acquired a range of strategies that I employ in my teaching. About 25 years ago I was introduced to the constructivist epistemology. It is a philosophy of learning that I took on board and that became part of my educator persona. That, coupled with a three year period out of mathematics where I was working with students on communication and relationships with a view to developing leadership skills, added to the strategies that I employed.

I have always had an interest in language and its impact in the mathematics classroom. As an undergraduate student I undertook a research assignment into the influence of Noam Chomsky whose work with linguistics had a profound influence on me as a beginning teacher and probably something that has stayed with me since that time. The development of my beliefs related to pedagogy and linguistic understanding maybe theoretical; however, in my case it was no accident. A combination of all the teaching roles I have experienced brought me to a point where I not only had an interest, but had developed a repertoire of strategies brought from other learning areas and roles like the Stepping Out Literacy trainer role.

I consider myself first and foremost an educator in the field of mathematics with a passion for understanding and using language appropriately to advance the understanding of my students. When I commenced this study I had a strong belief that language in the form of extended conversation was a means of developing deeper understanding of mathematics concepts that my students were learning. We all adhere to a teaching and learning philosophy and to particular pedagogies, including those associated with our learning areas. From within that broad-based theoretical background we have instinctively developed, or acquired, strategies that meet the needs of our students.

Talk and conversation is the fluid with which cognition takes place and the fluid with which communication between humans takes place. Hence the question: Is language the network path that connects cognitive processing and the creation of deep understanding? Possibly, but it is probably too simplistic an answer. If it was the answer then some other scholarly person would surely have happened upon this long before I commenced my research.

Every situation is a means of telling a story related to what I am doing, whether it be teaching, researching or just reading. This journey of self-exploration is of utmost significance in terms of my research as it is the lens through which I made all judgements about my observations. It is what influenced any decisions that I made and the processes I employed to go about actioning those decisions.

Unlocking my thoughts is a vehicle I can use to my advantage. I am reflective. It is at my core of existence. It is who I am. I have a need to reflect on everything that I do and I have had considerable practice with it, which means that I can examine all my experiences and actions dispassionately and objectively and quite often find that I need to alter the way that I do things. Being reflective was part of my doctoral research design. I focused on my actions as part of the research process. It is my actions that I have control over and I can, through a reflection process analyse my actions and then either improve them or support them. I could observe others but I have no control over their actions, nor over how they would interpret my observations and in the communication of those observations.

Words, concepts and context are important in the conveyance of meaning, that is, from the sender to the constructor of meaning. In the classroom context the teacher can act as the filter; interpreting and reassigning meaning to the utterances of students. How this is achieved places value on the relevant utterances and can create levels of discrimination, or inclusion.

Can language be used in a specific way to enhance mathematical understanding with students in the middle years? *Mathematical Linguistic*

Pedagogy gives voice and provides the explanation for the success of some, and not other, students. This study gives legitimacy to *Mathematical Linguistic Pedagogy*. The strategies developed as part of this study are based in *Mathematical Linguistic Pedagogy*.

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Participant Information Sheet and Consent Form

Dear Parents/ Caregivers and Students,

Thank you for your time in considering your participation in a study on using extended learning conversations to improve mathematical understanding.

Allow me please to briefly explain the purpose of this study and what we are requesting of you. If you require more information please don't hesitate to contact either of us on the above emails.

Project Aims
The focus is on the use of mathematical language and how it can be used to improve understanding of mathematical concepts. The focus of the study is on:

- explicit teaching and development of mathematical language;
- developing extended conversations about mathematics in my mathematics classroom to help students learn better; and
- using that to help students to become better at thinking about the mathematics they are learning.

What we are doing in this research project?
Your participation in this research study involves participating in a series of strategies involving reflection and mental processing, having conversations in the mathematics classroom with other students and the teacher, collaborative group-work and presenting your work in a PowerPoint or something similar to demonstrate what you have learned. This is all part of the normal classroom program. Students will have to maintain a logbook/reflective journal and will have to give some feedback about the strategies. Students will be video and audio recorded so that data can be collected and analysed. You will be asked to give feedback to help me improve the strategies I am using so that we can share this information with other teachers. Most of this is what would happen in the regular classroom program. The strategies we use will be different and you will be required to spend 15 minutes per hour lesson completing a reflection and feedback sheet.

Use of data collected in this research study
We will be publishing our findings from this study in conference presentations and journal articles in mathematics education. We are happy to provide you with copies of such publications upon request.

Consent to Participate
Your permission to allow you to participate in this study is completely voluntary. Your refusal to participate will in no way affect your relationship with Curtin University.

Confidentiality
All data gathered in this research study will be dealt with in greatest confidentiality. Data gathered through semi structured interviews and through observation will be dealt with sensitively and all aspects of individual anonymity will remain a high priority.

Risks associated with participation
There are no associated risks for participation in this project.

Further information
This study has been approved by the Curtin University Human Research Ethics Committee (Approval Number SMEC-01-10). If needed, verification of approval can be obtained either by writing to the Curtin University Human Research Ethics Committee, c/- Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth, 6845 or by telephoning 9266 2784. If you would like further information about the study, please feel free to contact xxxxxxxxxxx on (08) xxxxxxxxxx (xxxxxxxxxx). This research study has met the policy requirements of the Department of Education.

Consent form
If you have had all questions about the project answered to your satisfaction, and are willing to participate, please complete the **Consent Form** on the following page.

This information letter is for you to keep.

______________________ ________________________

xxxxxxxxxx **xxxxxxxxxx** (Doctoral student)

1 March 2010

Science and Mathematics Education Centre

Parent / Caregiver and Student Consent Form

- I have read the participant information sheet and understand the purpose and procedures of the study as described therein.
- I understand that the procedure itself may not benefit me directly.
- I understand that my involvement is entirely voluntary and that I can decline without it affecting the relationship with the research team or Curtin University of Technology.
- I understand that no personal identifying information will be used.
- I have been given the opportunity to ask questions and, if I have asked questions, I am satisfied with the answers I received.
- I understand that this research may be published on a website or in a journal provided that the participants or the school are not identified in any way without prior permission.
- I understand that my school will be provided with a copy of the findings from this research upon its completion.

Parent / Caregiver Consent

I consent for my child ____ to participate.

Full Name (printed): ___________________________________

Signature: ___________________________________ Date: / /

Student Consent

I agree to participate in the project

Full Name (printed): ___________________________________

Signature: ___________________________________ Date: / /

Dear Principal,
Thankyou for your time in considering your school's participation in a study on using extended learning conversations to improve mathematical understanding.

Allow me please to briefly explain the purpose of this study and what we are requesting of your students. If you require more information please don't hesitate to contact either of us on the above emails.

Project Aims
The focus is on language and its use in improving understanding of mathematical concepts. The focus of the study is on:

- explicit teaching, and development of mathematical language;
- developing extended learning conversations in my mathematics classroom; and
- enhancing cognitive reasoning and intellectual activity.

The research proposes that the process of learning mathematics is far more effective, even enhanced if it involves an extended learning conversation including teacher and learners.

What we are doing in this research project?
Your students' participation in this research study involves participating in a planned, staged series of strategies involving reflection and metacognition, engaging in extended learning conversations, collaborative group-work and student presentations as part of their normal classroom program. Your students will be asked to maintain a logbook/reflective journal and will be asked to participate in semi-structured interviews to obtain feedback about the strategies. The project will involve approximately 30 students. The project demands an additional commitment from student participants in the form of recording in journals and providing feedback. All other activities would form part of students' regular programs. The actual demands are probably 15 – 20 minutes per hour lesson, over a term.

Use of data collected in this research study
We will be publishing our findings from this study in conference presentations and journal articles in mathematics education. We are happy to provide you with copies of such publications upon request.

Consent to Participate
Your permission to allow your students to participate in this study is completely voluntary. Your refusal to participate will in no way affect your relationship with Curtin University.

Confidentiality
All data gathered in this research study will be dealt with in greatest confidentiality. Data gathered through semi structured interviews and through observation will be dealt with sensitively and all aspects of individual anonymity will remain a high priority.

Risks associated with participation
There are no associated risks for participation in this research study.

Further information
This study has been approved by the Curtin University Human Research Ethics Committee (Approval Number SMEC-01-10). If needed, verification of approval can be obtained either by writing to the Curtin University Human Research Ethics Committee, c/- Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth, 6845 or by telephoning 9266 2784. If you would like further information about the study, please feel free to contact xxxxxxxxx on (08) xxxxxxxxxx (email xxxxxxxxxx). This research study has met the policy requirements of the Department of Education.

Consent form
If you have had all questions about the study answered to your satisfaction, and are willing for your school to participate, please complete the **Consent Form** on the following page.
This information letter is for you to keep.

______________________ ________________________

xxxxxxxxxx **xxxxxxxxxx** (Doctoral student)

1 March 2010

Science and Mathematics Education Centre

Principal Consent Form

- I have read the participant information sheet and understand the purpose and procedures of the study as described therein.
- I understand that the procedure itself may not benefit me directly.
- I understand that my involvement is entirely voluntary and that I can decline without it affecting the relationship with the research team or Curtin University of Technology.
- I understand that no personal identifying information will be used.
- I have been given the opportunity to ask questions and, if I have asked questions, I am satisfied with the answers I received.
- I understand that this research may be published on a website or in a journal provided that the participants or the school are not identified in any way without prior permission.
- I understand that my school will be provided with a copy of the findings from this research upon its completion.

Full Name (printed): ____________________________________

Signature: ____________________________________ Date: / /

Students were learning the Rule of Order of Operations as part of a program based on the syllabus documents of the Western Australian Department of Education (DOE). Figure C.1 is an extract from the DOE syllabus document for Middle Childhood Education.

the acronym BIMDAS represents the order:
o Brackets
o Indices
o Multiplication and
o Division (in the order they appear)
o Addition and
o Subtraction (in the order they appear)
(eg ***for year 6*** *use simple examples such as '26 – 5 x 4 = 26 – [5 x 4] = 26 – 20 = 6')*
(eg ***for year*** *7 use '45 – 36 + 8 x 5.8 = 45 – 36 + [8 x 5.8] =45 – 36 + 46.4 = 55.4)*

Figure C.1 Understanding Operations

(from the Department of Education's K – 10 Syllabus, 2008)

Figure C.2 is extracted from the Expected Standards: for C Grade Descriptors for year 7.

When problem solving, students calculate accurately with more than one operation. (from Expected Standards: C Grade Descriptors - Mathematics, 2010)

Figure C.2 – Link with expected C grade standard in Year 7

In the Rule of Order of Operations Lesson students, who sit in groups of up to five, had been given six warm up calculations at the commencement of the lesson as displayed in Figure 5.1. This was in preparation for having the students work on the activity as displayed in Figure C.3.

Rule of Order Activity Using brackets, +, -, x and ÷ make as many numbers as you can using four 4s.	
4 + 4 ÷ 4 + 4	**=**
4 x (4+4) + 4	**=**
	[the table continues for 14 lines]

Figure C.3 – Rule of Order Activity

Year 6	Year 7
Compare and find equivalences for simple fractions and key percentages using a range of models, including number lines	**Compare and find equivalences for simple fractions and key percentages using a range of models, including number lines**
compare fractions using models, drawings and number lines *(eg use fraction strips to show 1/3 = 2/6 = 3/9)*	read, write and order proper fractions *(eg 1/3)* improper fractions *(eg 7/5)* and mixed numerals* *(eg 3 ¾)*
partition an object or collection in a variety of ways to show equal parts *(eg three fifths of the class is girls, that is 0.6 or 60% of the class)*	quantity or collection of things *(eg find ¾ of a class of 28 students, 2/3 of a bag of marbles, 4/10 of a glass of water)*
recognise simple equivalences of fractions including mixed numerals and improper fractions through diagrams and models *(eg colour 1 6/8 grids to see it equals 1 ¾ or 7/4)*	recognise simple equivalences of fractions including mixed numerals and improper fractions through diagrams and models* *(eg colour 2 ¾ grids to see it equals 11/4)*
	the size of fractions are relative to the whole *(eg ' 1/3 of your pocket money might be more or less than 1/3 of mine')*
count in fractional amounts *(eg four-fifths, one whole, one and one-fifth or four-fifths, five-fifths, six-fifths)*	count in fractional amounts *(eg one and one-third, one and two-thirds, two, two and one-thirds)*

Figure C.4 – Fraction Learning Requirement Outcomes
(From the Department of Education and Training's K – 10 Syllabus, 2008)

Figure C.5 displays the Expected C Grade Outcomes for Year 6 and 7. I have included this to show the link between the syllabus, expected standards and my teaching program.

Year 6		**Year 7**
Use mental methods with * whole numbers * fractions * decimals	Add common fractions Find fractions of whole numbers ** 1/3 of 36	whole numbers, fractions and decimals

Figure C.5 – Year 6 and 7 'C Grade Expectations'

Summary of Features of Child Language Development

1. Symbolic Acts (“Acts of Meaning”):	Starting to construct signs.
2. Iconic (Natural) Symbols:	Constructing signs that resemble what they mean.
3. Systems of Symbolic Acts:	Organizing signs into paradigms (protolanguage).
4. The Lexicogrammatical Stratum:	Constructing a three-level semiotic system (language).
5. Non-Iconic (Conventional) Symbols:	Taking up signs that do not resemble their meanings.
6. “Trailer” Strategy:	Anticipating a developmental step that is to come.
7. “Magic Gateway” Strategy:	Finding a way into a new activity or to a new understanding.
8. Generalisation (Classifying, Taxonomising):	Naming classes (“common” terms) and classes of classes.
9. The “Metafunctional” Principle:	Experiential and interpersonal meanings (from single function utterances, either pragmatic [doing] or mathetic [learning], to multifunctional ones, both experiential and interpersonal).
10. Semogenic Strategies:	Expanding the meaning potential (refining distinctions, moving into new domains, deconstructing linked variables).
11. Construal of “Information”:	From rehearsing shared experience to imparting unshared experience.
12. The Interpersonal “Gateway”:	Developing new meanings first in interpersonal contexts.
13. Dialectic of System and Process:	Constructing language from text, constructing text from language.
14. Filtering and the “Challenge” Zone:	Rejecting what is out of range and working on what is accessible.
15. Probability – The Quantitative Foundation:	Construing relative frequencies.
16. Discourse-The Third Metafunction:	Construing a parallel world of semiosis.
17. Complementarities:	Construing experience from different angles of vision.
18. Abstraction and Literacy:	Understanding abstract meanings and moving .into the written mode.
19. Reconstruction and Regression:	Backing off to an earlier semiotic “moment” while re-construing both content and expression.
20. Grammatical Metaphor (Nominalising, Technologising):	From common-sense grammar to the grammar of objects and technical hierarchies.
21. Synoptic/Dynamic Complementarity:	Reconciling two semiotic models of human experience.

(Adapted from Linguistics and Education, Halliday 1993)

Table E.1 – Discourse Connector Examples
(The items bolded are those that students frequently used)

Therefore (**which is**) (**that's how**)	However (**but**)	in addition	in fact	
Thus (**so**)	on the other hand	furthermore	as a matter of fact	**the answer**
Consequently	**instead**	moreover	Indeed	**I did**
as a result	rather	besides	In conclusion	**(I done)**
Hence	**First**	additionally	To conclude	
Nevertheless	**Second**	similarly	In summary	
Nonetheless	**Afterward**	likewise	To summarise	**Just went**
Still	**After that**	otherwise	As we have seen	in other words
On the contrary	**Later**	For example	In short	that means (**that**)
Because	**Then / Next**	**an example**	**after**	**I went**
Since	On the whole	For instance	**before**	so
As	In general	To illustrate	**when**	well
due to the fact that	Generally	although	**while**	still
now that	Generally speaking	even though	since	also
so that	**while**	**even if**	as soon as	Yet
such...that	whereas (**where**)	despite the fact that (though)	**until**	
In contrast to (**If it was**)	**If..., (then**)	in spite of the fact that	by the time that	
different from	whether (or not)	just as	**whenever**	
Unlike	**when**	**just like**	**the next time**	
Similar to	In case that	In addition to	**who**	
Like	Provided that	**Before / after**	whom	
Unless	due to	**that**	**until**	
Because of	as a result of	**whose**	since	
	Despite in spite of	**when where**	during	

(Adapted from Bauer-Ramazani, 2005)

Table F.1 – Two-Dimensional Data Category Table

Lesson		Class	Shared Experience			Purposeful Discussion		Blen	
			FT	PA	CO	CD	NCD	Targeted	F
Rule of Order	1	6/7	Six Calculations			Rule of Order		Metaphoric Links	
Fractions	2	6/7		Fraction Stand-up	Brainstorm		Reflective Thinking		
Equivalent Fractions	3	6/7	Feedback					Creating Visual Links	
Indices and Word Problems	4	9/10	Round table Reflection			Indices			P W
The Orange Problem	5	8/9/10	Mental questions / Calculations			Mental Discussions			
Algebra	6	9/10							
Games	7	6/7							
Area of Triangles	8	6/7	Quiz						