

Department of Mathematics and Statistics

An Optimisation Approach
to
Materials Handling in Surface Mines

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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Christina N. Burt

Date

Abstract

In the surface mining industry the *equipment selection* problem involves choosing a fleet of trucks and loaders that have the capacity to move the materials specified in the mine plan. The optimisation problem is to select these fleets such that the overall cost of materials handling is minimised. The scale of operations is such that although a single machine may cost several million dollars to purchase, the cost of operation outweighs this expense over several years. This motivates the need for a purchase *and* salvage policy, so that the optimal equipment replacement cycle can be achieved.

Mining schedules often appear with multiple mining locations and dump-sites, where a dump-site can also represent a stockpile or a mill. Multiple periods must also be considered, which adds to the complexity of determining the optimal replacement policy for equipment. Further, some mines begin with a pre-existing set of equipment, and the subsequent fleet must be both compatible and satisfy the mill constraints. We also need to consider the possibility of a heterogeneous fleet.

The equipment selection problem is cursed with a cascade of inter-dependent variables and parameters. For example, the cost of operating a piece of equipment depends on its utilisation; the utilisation depends on the availability of the equipment; and the availability depends on the age of the equipment. We formally define the equipment selection problem in the Introduction (Chapter 1) and further discuss the complexities of the problem.

While numerous methods from *Operations Research* and *Artificial Intelligence* have been applied to this problem, optimal multiple period solutions remain elusive. Also, pre-existing equipment and heterogeneous fleets have largely been ignored. We present a comprehensive literature review in Chapter 2, outlining the methods applied and candidly discussing the successes and pitfalls of these approaches. We also organise the literature by linking related fields, such as *Shovel-Truck Productivity* and *Mining Method Selection*.

In Chapter 3 we extend the *match factor* ratio, an important productivity index for the mining industry. Previously this ratio was restricted to homogeneous fleets and single location/dump. We provide several alternative ratios that incorporate heterogeneous trucks, heterogeneous loaders and multiple locations. These extensions are then

applied to solutions in subsequent chapters to indicate the efficiency of the selected fleets in terms of the proportion of time they are working (rather than waiting).

In this thesis, we consider the equipment selection as an optimisation problem. We wish to purchase only whole units of trucks and loaders, which suggests integer variables are appropriate for this problem. Similarly, salvage occurs in whole units. As the productivity constraints (satisfying the mill requirements) are linear, we consider an *integer programming* approach.

In Chapter 4 we present a single location/dump multi-period integer program that provides a purchase and salvage policy for a surface mine. We demonstrate through a retrospective case study that the solutions are economically better than current methods. We also demonstrate the robustness of the model through a series of test cases. We extend this model to a *mixed integer linear program* (MILP) to optimise over multiple locations/dump-sites in Chapter 5, and test this model on two case studies. This model also produces an optimised allocation policy for the multiple mining locations and truck routes.

In Chapter 6 we consider the utilisation of the equipment in the objective function. This MILP model provides the purchase and salvage policy for a single-location multi-period surface mine. In this model we introduce constraints that capture the non-uniform piecewise linear ageing of the equipment. We test this model on a case study used in previous chapters.

All of the presented models allow for pre-existing equipment and heterogeneous fleets. Further, they all consider multiple period schedules, ensuring they are all innovative equipment selection tools.

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Dedication

I dedicate this thesis to the memories of our Opa, Kevin van Leest, and our handsome friend, Larry Burt - who were both excellent hobby gardeners in their own right.

*Over the ramparts you tossed
The scent of your skin and some foreign flowers
Tied to a brick
Sweet as a song
The years have been short but the days go slowly by
Two loose kites falling from the sky
Drawn to the ground and an end to flight*

The Shins, *Pink bullets*

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Chapter 1

Introduction

The ultimate goal of a mining operation is to provide a raw material for the community at the least expense. If the operation is successful in minimising the cost of material removal, then remaining profits can be used to effectively rehabilitate the mining site once all the material has been mined. The aspect of the mining operation which has the most influence on profit is the cost of materials handling. In this thesis, we focus on the problem of equipment selection for surface mines as an important driver for the overall cost of materials handling. In the mathematical branch of *Operations Research*, we interpret this problem in the context of an optimisation goal:

To optimise the materials handling such that the desired productivity is achieved and the overall cost is minimised.

In general, the equipment selection problem involves purchasing suitable equipment to perform a known task. It is essential that all owned equipment be compatible with both the working environment and the other operating equipment types. This equipment must also be able to satisfy production constraints even after compatibility, equipment reliability and maintenance is taken into account. By examining the equipment selection problem as an optimisation problem, we can begin to consider purchase and salvage policies over a succession of tasks or multiple periods. With this in mind, our objective for this research is:

Given a mining schedule that must be met and a set of suitable trucks and loaders, create an equipment selection tool that generates a purchase and salvage policy such that the overall cost of materials handling is minimised.

By considering the salvage of equipment in an optimisation problem, we are effectively optimising equipment replacement as well as the selection of the equipment. Throughout the remainder of this introduction we will introduce some necessary background for the equipment selection problem and outline our approach to solving it.

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1.1 Surface mining

A surface mine is used to extract mineral endowed rock (or **ore**) from the earth to a depth of 500m. We extract minerals such as iron, copper, coal and gold in this way. This method is employed, as an alternative to underground mining, when the ore is close to the surface ($< 500\text{m}$); the overlying soil (or **overburden**) is shallow; or the surrounding environment is too unstable to tunnel beneath. There are several methods of surface mining including open-pit, stripping, dredging and mountain-top removal. This thesis will focus on **open-pit** surface mining, which involves removing ore from a large hole in the ground (sometimes referred to as a **borrow-pit**). The largest open-pit mine in Australia is KCGM's "super pit", which encroaches on the regrettably positioned township of Kalgoorlie in Western Australia [Figure 1.1].



FIGURE 1.1: Kalgoorlie Consolidated Gold Mines super-pit (KCGM 2007a).

Surface mines are created by sequentially removing small vertical layers (or **benches**) of material at a time. These benches appear as graded contours in the "super pit" image [Figure 1.1]. Over time, these benches are removed and the borrow pit becomes wider and deeper. The height of the bench can vary from 4m to 60m and will dictate the type of equipment that can remove it.

The mined material can generally be categorised into ore and waste material, although they may be further categorised depending upon their quality or grade. These

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materials are sorted to a number of dump-sites, which can include mills and stockpiles. The ore will be refined at the mill, while the stockpiles are important for ensuring that the mill receives the correct mixing of ore grades to meet market demands. The related productivity requirements are optimised by the **mining schedule**. The schedule, alongside the **pit optimisation** (the optimisation of the shape of the pit), provides required productivities, bench sequences and the shape of the mine (including bench heights). For large scale open pit mining in particular, **trucks** and **loaders** are the preferred method of materials handling (Czaplicki 1992, Ta, Kresta, Forbes & Marquez 2005).

1.2 Loaders

Remark 1.2.1 *Throughout this thesis we consider a “loader” to be any type of high productivity excavating equipment, which may include a mining loader, shovel or excavator.*

Loaders are used to lift the ore or waste material onto the trucks or other equipment for removal from the mine. In an open-pit mine, loader types can include **electric rope** and **hydraulic excavators**, the **hydraulic backhoe excavator**, and **front-end loaders** (also **wheel loaders**) (Erçelebi & Kirmanlı 2000). This variety of machines differs significantly in terms of reliability, maintenance needs, compatibility with different truck types, volume capacity, and cost per unit of production.

The capacity for a loader is defined by the **bucket** size, and this in tandem with the **loader cycle time** (the time required to fill the bucket and drop its contents into a truck) defines the productivity of the loader. The productivity of the loader itself is therefore tied to the number of **passes** (or buckets of material) required to fill each truck. The number of passes will vary depending on the size of the truck. The truck capacity is rarely a round multiple of the loader bucket size. Depending on the policy of the mine, the loader may or may not pass an incomplete bucket to fill the last amount. A rule-of-thumb can be used to determine whether an additional pass is made. For example, in this thesis we adopt the rule that if the remaining truck capacity is greater than one third of the loader bucket capacity, then we make an additional pass. Otherwise we round the number of passes down.

The electric rope shovel [Figure 1.2] can vary from 25 tonnes to 110 tonnes in bucket size, and generally has a low cycle time of about 35 seconds. Cables control the arm and bucket of this loader. The hydraulic excavator is fashioned similarly but controls the bucket with the use of high pressure hydraulic fluid.

An alternative to the electric rope shovel is the hydraulic backhoe excavator [Figure 1.3]. The electric rope shovel, hydraulic loader and hydraulic backhoe excavator are

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FIGURE 1.2: An electric rope shovel (Mining-Technology 2007*b*).



FIGURE 1.3: A Liebherr R996 hydraulic backhoe excavator (Mining-Technology 2007*a*).

all capable of swinging the bucket from the mining area to a positioned truck without moving the base of the loader. The hydraulic backhoe can range from 19 tonnes to 86 tonnes in bucket capacity, and has the fastest cycle time of all the loader types. The backhoe loader (also known as **back-actor** or **rear-actor**) is so named because of the action of the bucket, which draws the bucket through the earth toward the loader.

The front-end loader is the most versatile of loaders, although limited in bucket capacity which can vary from 23 tonnes to 57 tonnes [Figure 1.4]. This loader type has the slowest cycle time as it must manoeuvre on its tyres to position the bucket over

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FIGURE 1.4: A CAT 992G front-end loader operating in a mine in Russia (Karelia Government 2007).

the truck.

The type of loader selected for use in a surface mine depends on the type of mineral to be extracted and other environmental conditions, such as the bench height. We must also take other considerations, in particular the compatibility of the loaders with selected truck fleets, into account in the equipment selection process. For example, some loaders cannot reach the top of the tray on the larger trucks. Conversely, some loader capacities exceed the capacity of the truck. If we are determined not to underpin the optimisation process, then we must model the problem such that we select the truck and loader types simultaneously.

1.3 Trucks

Mining trucks are used to haul the ore or waste material from the loader to a dumpsite. They are also known as **off-road** trucks or **haul** trucks [Figure 1.5]. More commonly, these vary from 36 tonnes to 215 tonnes. The size and cost of operating mining trucks is directly proportional to its tray capacity, while the speed the truck can travel is inversely proportional to its capacity. As with loaders, the variety of truck types differs according to their reliability, maintenance requirements, productivity and operating

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FIGURE 1.5: Large capacity mining trucks: one *travelling empty* down the ramp while the other *travels full* up the ramp (KCGM 2007b).

cost.

The performance of a truck is greatly affected by the mine environment. For example, the softness of the road soil creates an effect of **rolling resistance** which reduces the efficiency of the truck in propelling itself forward. The effects of rolling resistance can be controlled and reduced by wetting and compressing the roads regularly [Figure 1.6]. Rolling resistance varies a lot across the road and over time, and is notoriously difficult to estimate.

The forward motion of the truck is also affected by **rimpull**. Rimpull is the natural resistance of the ground to the torque of the tyre and is equal to the torque of the wheel axle multiplied by the wheel radius. Manufacturers supply **rimpull curves** for their trucks to enable a satisfactory calculation of truck cycle times. The rimpull curves map the increase in road resistance as the truck increases speed.

The effects of rolling resistance and rimpull are exacerbated by **haul grade**, which is the incline of the haul road. These parameters, alongside **haul distance**, are critical for the accurate calculation of the truck cycle time.

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FIGURE 1.6: A grading machine smoothing the road for the trucks to reduce the effect of rolling resistance (CAT 2007).

1.3.1 Truck cycle time

Definition 1.3.1 *The **truck cycle time** comprises of load time, haul time (full), dump time, return time (empty), queuing and spotting [Figure 1.7].*

A cycle may begin at a loader where the truck receives its load. The truck then **travels full** to the dump-site via a designated route along a haul road. The dump-site may be a stockpile, dump-site or mill. Once the load has been dumped, the truck turns around and **travels empty** back to the loader. The act of manoeuvring the truck under the loader to be served is called **spotting**. This can take several minutes. In a large mine the truck cycle time may be around 20-30 minutes in total, and can vary a lot over time as the stockpiles are moved and the mine deepens.

The truck cycle time is an important parameter because related parameters (that are not dependent on the final selected fleet) can be absorbed into it. Ultimately we wish to include intimate details of the mine, such as topography and rolling resistance, in the modelling process. These parameters can be estimated prior to modelling and incorporated into the truck cycle time. Further, the truck cycle time can be used to absorb parameters such as rimpull, haul grade and haul distance into one estimate. However, the level of queuing that occurs in a fleet is dependent on the number of trucks operating against each loader. This makes it difficult to accurately estimate

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truck cycle times before the fleet is determined.

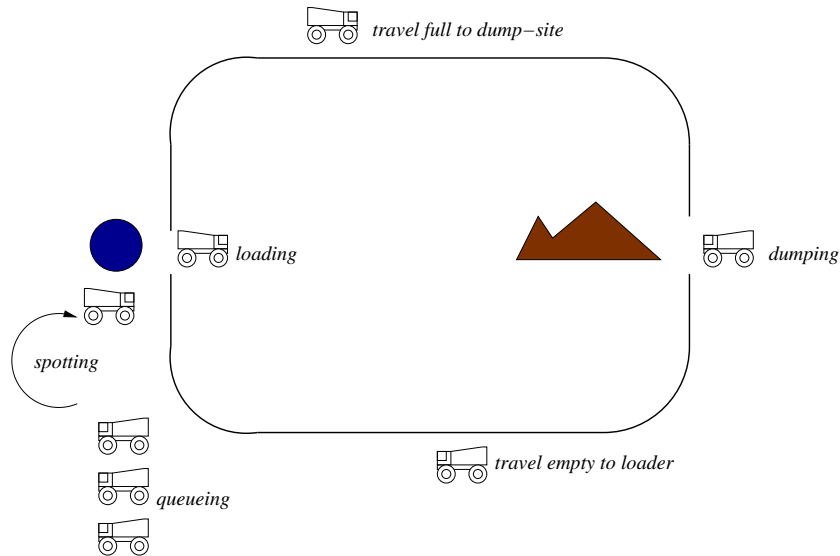


FIGURE 1.7: The truck cycle time is measured from the time the truck is filled at the loader, travels full to the dump-site, dumps the load, and travels empty to the loader to join a queue and positions itself for the next load (spotting). The truck cycle time includes queuing and waiting times at the dump-site and loader.

In industry, the common method of truck cycle time estimation is to estimate the speed of the trucks using manufacturers' performance guidelines (Smith, Wood & Gould 2000). These guidelines are simulation results that take into consideration engine power, engine transmission efficiency, truck weight, capacity, rimpull, and road gradients and conditions (Blackwell 1999). This is combined with topographical information to provide an estimate of the hauling route. The guidelines must also be used in combination with rolling resistance estimates to determine any lag in cycle time. Smith et al. (2000) provides a method for determining a rolling resistance estimate. Regression models can also be used to determine good truck cycle time estimates (Çelebi 1998).

In this thesis we make use of truck cycle time estimates provided by the industry partner.

1.4 Shovel-truck productivity

The ability to predict the productivity of a truck and loader fleet is an important problem for mining and construction, as the productivity of the fleet is intrinsic to equipment selection. A part of the equipment selection literature bases the selection entirely on productivity estimates of the fleets. This research usually comes under the banner of *shovel-truck productivity*, and focuses on "predicting the travel times on the haul and return portions of the truck cycle ... and the prediction of the interaction

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effect between the shovel and truck at the loading point” (Morgan & Peterson 1968). The shovel-truck productivity problem has been well established in construction and earthmoving literature (Kesimal 1998). This work aims to match the equipment (in both type and fleet size) such that the productivity of the overall fleet is maximised. However, much of the literature on shovel-truck productivity exists for construction case studies and little published research applies to surface mining. Nonetheless these methods must be addressed here as they represent the core ideas behind current industry practice in surface mining equipment selection (Smith et al. 2000, Erçelebi & Kırmanlı 2000).

Those methods deemed classical include **match factor** and **bunching theory**. The match factor is the ratio of truck arrival rate to loader service time, and provides an indication of the efficiency of the fleet. Bunching theory studies the natural variance in the truck cycle time due to bunching of faster trucks behind slower trucks. Shovel-truck productivity methods incorporate both match factor and bunching ideas into the solution. These methods use many assumptions, considerable expert knowledge/experience and rely on heuristic solution methods to achieve a solution. Modelling of the true bunching effect would be a helpful asset to the mining and construction industries, as the effect is not well studied and is currently unresolved. However, the derivation of such a model is beyond the scope of this thesis.

1.4.1 Match factor

The match factor itself provides a measure of productivity of the fleet. The ratio is so called because it can be used to match the **truck arrival rate** to **loader service rate**. This ratio removes itself from equipment capacities, and in this sense, potential productivity, by also including the loading times in the truck cycle times.

Douglas (1964) published a formula that determined a suitable number of trucks, M_b , to *balance* loader output. This formula is the ratio of loader productivity to truck productivity, but as it makes use of equipment capacity it is considering the *potential* productivity of the equipment. That is, if the loader is potentially twice as productive as the selected truck type, then we require two trucks to balance the productivity level. Let c_e denote the capacity of equipment type $e \in \mathbf{X} \cup \mathbf{X}'$, and t_e signify the cycle time of equipment type e , where \mathbf{X} is the set of all truck types and \mathbf{X}' is the set of all loader types. The productivity of equipment type e is represented by P_e and the number of trucks of type i in the fleet is x_i , where $i \in \mathbf{X}$, while we denote the loader types as i' where $i' \in \mathbf{X}'$. We denote the equipment efficiency by E_e (representing the proportion of time that the equipment is actually producing). We can write

$$P_{i'} = \frac{c_{i'} E_{i'}}{t_{i'}} \quad \forall i' \in \mathbf{X}', \quad (1.1)$$

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for a single loader operation. The productivity of the truck fleet is represented by:

$$P_i = \frac{c_i E_i x_i}{t_i} \quad \forall \quad i \in \mathbf{X}, \quad (1.2)$$

and the match balance is represented by:

$$M_b = \frac{P_{i'}}{P_i}. \quad (1.3)$$

Truck cycle time is defined for equation (1.2) as the sum of non-delayed transit times, and includes haul, dump and return times. Note that ratio (1.3) is restricted to one loader. This is a simple ratio that can be used to ensure that the truck and loader fleets do not restrict each other's capacity capabilities. Sometimes however, it is not necessary for the productivities of the truck and loader fleets to be perfectly matched. Morgan & Peterson (1968) published a simpler version of the ratio, naming it the match factor, $MF_{i,i'}$, for truck type i working with loader type i' is given as:

$$MF_{i,i'} = \frac{t_{i,i'} x_i}{\bar{t}_X y_{i'}}, \quad (1.4)$$

where x_i is the number of trucks of type i ; $y_{i'}$ is the number of loaders of type i' ; $t_{i,i'}$ is the time taken to load truck type i with loader type i' ; and \bar{t}_X is the average cycle time for the trucks excluding waiting times. This ratio uses the actual productivities of the equipment rather than potential productivities, and therefore achieves a different result to equation (1.3). In this thesis we consider only the Morgan & Peterson interpretation of match factor: we are interested in the actual productivities of the truck and loader fleets.

Remark 1.4.1 *The **match factor** is the ratio of actual truck arrival rate to loader service time.*

In this thesis we make use of the match factor as a productivity indicator, and contrary to the Morgan & Peterson interpretation, we assume that queue and wait times are included in the cycle times. With this idea of cycle time in mind, a match factor of 1.0 represents a balance point, where trucks are arriving at the loader at the same rate that they are being served. Typically, if the ratio exceeds 1.0 this indicates that trucks are arriving faster than they are being served. For example, a high match factor (such as 1.5) indicates over-trucking. In this case the loader works to 100% efficiency, while the trucks must queue to be loaded.

A ratio below 1.0 indicates that the loaders can serve faster than the trucks are arriving. In this case we expect the loaders to wait for trucks to arrive. For example, a low match factor (such as 0.5) correlates with a low overall efficiency of the fleet, namely

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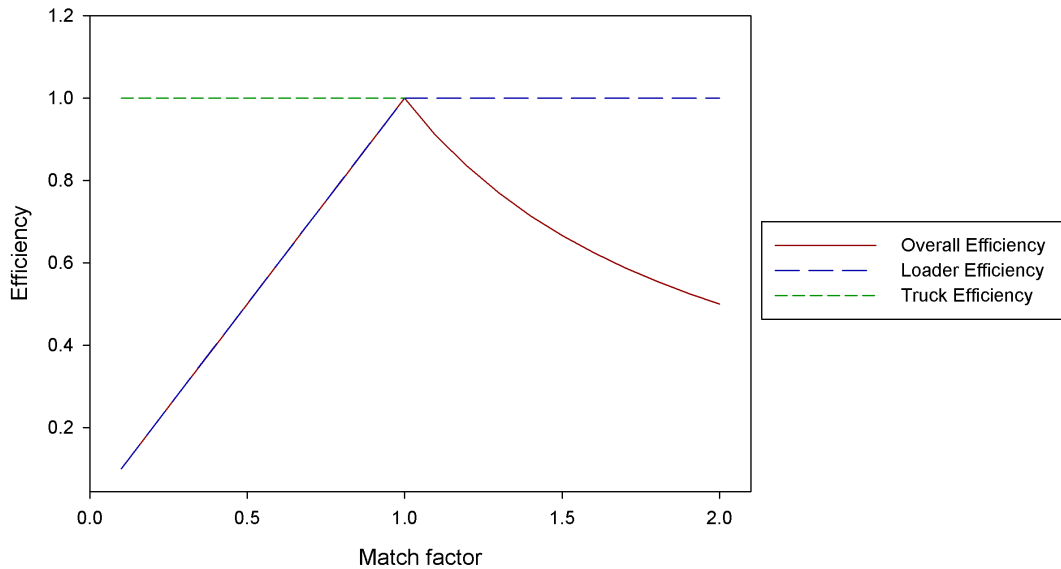


FIGURE 1.8: The match factor combines the relative efficiencies of the truck and loader fleets to create an optimistic efficiency for the overall fleet.

50%, while the truck efficiency is 100% [Figure 1.8]. This is a case of under-trucking; the loader's efficiency is reduced while it waits. Unfortunately, in practice a theoretical match factor of 1.0 may not correlate with an actual match factor of 1.0 due to truck bunching. In this sense, the calculated match factor value is optimistic.

The match factor ratio has been used to indicate the efficiency of the truck or loader fleet and in some instances has been used to determine a suitable number of trucks for the fleet (Smith, Osborne & Forde 1995, Cetin 2004, Kuo 2004). While the ratio can be used to give an indication of efficiency or productivity ratios, it fails to take truck bunching into account. Therefore caution must be taken in the interpretation of any calculated match factor values.

Match factor has been adopted in both the mining and construction industries (Morgan 1994*b*, Smith et al. 1995). The construction industry is interested in achieving a match factor close to 1.0, which would indicate that the productivity levels of the fleet are maximised. However, the mining industry may be more interested in lower levels of match factor (which correspond to smaller trucking fleets and increased waiting times for loaders) as this may correlate with a lower operating cost for the fleet. This can happen if equipment with greater productivity rates than required can perform the task with lower operating costs than equipment that perfectly matches the required production.

The match factor ratio relies on the assumption that the operating fleets are **homogeneous**. That is, only one type of equipment for both trucks and loaders is used

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in the overall fleet. When used to determine the size of the truck fleet, some literature simplify this formula further by assuming that only one loader is operating in the fleet, namely Morgan (1994*b*), Smith et al. (1995), and Nunnally (2000). Homogeneous fleets are desirable for the mine, as they simplify maintenance, training of artisans and the burden of carrying spare parts for different types of equipment. However, heterogeneous fleets may provide overall cost savings.

In practice, mixed fleets and multiple loaders are common due to **pre-existing** equipment or optimal fleet selection that minimises the cost of the project. A situation with pre-existing equipment can arise both at the start of a mining schedule, and when a new selection of equipment is desired part-way through the schedule. This highlights a need for a match factor ratio that can be applied to heterogeneous fleets.

1.5 Mining method selection

Two distinct but dependent problems are the *mining method selection* and *equipment selection* problems. The equipment selection problem arises immediately after we have obtained a solution to the mining method selection problem. In order to determine the most appropriate mining method, **diggability studies** are performed throughout the area to be mined. These studies look at the quality of the overburden, the **swell factor** of the soil and the soil compaction: effectively the ease at which the earth can be removed from the site. Once the diggability study is complete the mining managers select a suitable mining method, thereby influencing both the “mode of operation” as well as the types of available equipment from whence we make our selection (Chanda 1995). In a global sense, the mining method selection process will choose the type of surface mine that is to be developed, such as open-pit, stripping or dredging. Also, an expert considers climatic, geological, site, and geo-technical conditions to choose not only an appropriate mining method, but a sub-set of appropriate truck and loader types (Gregory 2003, Başçetin 2004). It is with this sub-set of equipment that we begin our search for an optimal equipment fleet solution.

1.6 Equipment selection

Equipment selection is a combinatorial problem (Hassan, Hogg & Smith 1985) that involves selecting an appropriate fleet of trucks and loaders to perform the task of materials handling. There are two industries for whom the truck and loader equipment selection problem is of critical importance: mining and construction. Both these industries have applied considerable effort to find a suitable solution strategy for their operations (Marzouk & Moselhi 2002*b*). In the selection of the fleet we choose the *number*, *type* and *size* of the equipment (Bozorgebrahimi, Hall & Blackwell 2003).

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Intuitively this problem is closely related to the *allocation* problem (which involves allocating equipment to defined tasks in the schedule) and the equipment *replacement* problem (where we optimise the replacement cycle for the chosen equipment).

There are several aspects of the equipment selection problem that have prevented tractable models in the past. The selection, or purchase, of equipment restricts the primary decision variables to integers. However, the allocation of this equipment to fleets and routes intuitively lends itself to non-integer decision variables. After the consideration of equipment types and equipment age, aspects such as multiple locations, multiple periods, and the desire for individual machine tracking further exacerbate the number of decision variables in the model, rendering it a *large scale problem*. Also, the set of available equipment can be large and characteristically different (Hassan et al. 1985). The greatest challenge of the equipment selection problem is to derive a tractable model which easily identifies with an optimal solution. For example, with the presence of many identical trucks and loaders which can be allocated to the various locations in a number of ways, we can create not only multiple optimal solutions but also large clusters of similar solutions near the optimal.

Other challenges present in the equipment selection problem include:

- **Heterogeneous fleets:** Within the pit itself, several loaders may operate at different locations with different objectives. In order to meet the needs of a particular location, different loader types may be operating in the same mine. Similarly, the trucking fleets that work between the loaders and the dump-sites may have mixed types. Typically, mixed fleets arise when new equipment is purchased and added to an older fleet of pre-existing equipment. However, it is possible that mixed fleets can provide a cheaper fleet if the productivity requirements do not evenly divide into the operating capacity of a particular truck or loader type.
- **Truck cycle time:** The number of trucks in the fleet can influence all equipment efficiency: under-trucking restricts the efficiency of the loader, but over-trucking restricts the efficiency of the trucks due to queuing and bunching. It is difficult to predict the cycle time of a fleet with different types of trucks or loaders (a heterogeneous fleet), or even with varying fleet size. Even the actual cycle time of trucks and loaders can vary significantly with the accompanying fleet without any changes to the fleet make-up (Edwards & Griffiths 2000). The interdependency of these aspects of a mining operation suggest that better solutions may be obtained by optimising the problem in its entirety rather than focusing on individual items or subsets of the problem (Atkinson 1992).
- **Uncertainty:** There is a desire to model the uncertainty and risk in the problem (Cebesoy, Gzen & Yahşi 1995). The primary parameters that are subject to

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variability are truck cycle time and equipment availability. Truck cycle time is deemed more important in terms of its variability than loader cycle time because it comparatively dwarfs the other. This has led to the modelling of the truck cycle time by probability distribution (Cebesoy et al. 1995). However, no probabilistic modelling of equipment availability presents itself in the literature.

- **Compatibility:** The trucks and loaders must be sufficiently compatible, which can involve restrictions such as the dumping height of a loader matching the truck tray height (Çelebi 1998). The existence of pre-existing equipment in the mine creates difficulties with equipment compatibility.
- **Equipment salvage:** Equipment salvage should also be considered as equipment exceeds its maximum age. This is particularly important if we are considering using pre-existing equipment that is close to its maximum age at the beginning of the schedule. Also, if salvage is permitted then we can optimise the replacement cycle of the equipment at the same time that we optimise the purchase policy.

In a bid to curb these complexities, models in the literature are typically restricted to assumptions such as homogeneous fleets, average truck cycle times for the whole period, and bunching or queuing is ignored. Costs are often considered to be constant, and pre-existing equipment has never been permitted. This research addresses the assumption of homogeneous fleets and the existence of pre-existing equipment. Further, we introduce a model that accounts for utilisation in the objective function and consider optimising over multi-period and multi-location schedules.

A fast and effective equipment selection tool is important due to the dynamic nature of the productivity requirements. A fast tool would allow a re-run of the equipment selection process whenever there is a significant change in production requirements or some other relevant parameter. This research will focus on deriving mathematical models and computational solutions for a surface mining application, although the models and derived formulas may be just as easily applied to a construction industry case study. The research may also be relevant for underground mine equipment selection - however, this application has not been considered in this body of work.

1.7 Equipment cost

The operating cost of mining equipment dominates the overall cost of materials handling over time. Typically these costs include maintenance, repairs, tyres, spare parts, fuel, lubrication, electricity consumption and driver wages into one estimate. The best way to account for the operating cost of mining equipment is, in itself, an important problem. Some equipment selection tools use life-cycle costing techniques to obtain

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an **equivalent unit cost** for the equipment (Bozorgebrahimi, Hall & Morin 2005). These costs estimate the average lifetime cost *per hour* or *per tonne*. Clearly this is not practical if we are considering salvaging equipment when it is no longer useful or has reached the end of its optimal replacement cycle. Industry improves the equivalent unit cost estimate by scaling the value depending on the age of the equipment. That is, if the equipment requires a full maintenance over-haul at the age of 25,000 hours then this cost bracket will reflect a greater expense through a scaled factor of the unit cost.

Equipment operating cost is highly dependent on the age of the equipment. That is, cost per tonne is determined by productivity; equipment productivity is dependent on equipment availability: equipment availability is dependent on equipment age. Operating cost can also be affected substantially by the simple addition of one loader to a single-loader fleet (Alkass, El-Moslmani & AlHussein 2003). Although the most obvious objective function for an equipment selection model is to minimise cost, as a function of utilisation and equipment age this adds great complexity to the problem and has the potential to introduce nonlinearities (Hassan et al. 1985).

Any mining operation is dynamic in nature and may be subject to considerable changes in the mine plan. In many cases, an equipment selection plan for a multi-period mine may be rendered inadequate as these changes come to light. The purpose of the tools derived in this thesis, however, aim to provide the best possible starting solution given the information available at the time. To add to this varying nature of the production parameter, the cost parameters may also change significantly and are themselves estimates (Hassan et al. 1985). Specifically, the capital expense data available at the time the equipment selection tool is run may differ from the time of purchase due to:

- the establishment of new contracts with the corresponding suppliers;
- improved historical data (accumulated through previously owned equipment) which may be combined with supplied data (from the equipment producers)(Smith et al. 2000, Edwards, Malekzadeh & Yisa 2001);
- a change in demand for second-hand equipment or scrap metal - thus affecting the salvage value of a piece of machinery;
- changes in the interest and depreciation rates used for the *net present value* calculations.

Using hire cost data is a simple alternative to using a mix of manufacturer supplied production costs and real data (Edwards et al. 2001), but this is not always possible or practical.

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With these examples as justification, we argue that it is not necessary to consider the cost objective function in its most natural and accurate form: nonlinear. As all the parameters of the objective function are themselves approximations, the objective function may be more wisely considered in piecewise linear format. Certainly in industry this is common practice where operating cost, for example, is considered to be a piecewise linear function of an age bracket, rather than a nonlinear function of unit age. By these arguments, the relative parameters of a linearised objective function can be considered to be sufficiently precise.

1.8 Optimisation

In an optimisation problem, we focus on a single **objective function**, $f(x)$, whose purpose is to measure the quality of the decision (Luenberger 2003). Mathematical programs look at the state of a system and its structure, and in considering a suitable objective determines how the system can move into the next state.

A general mathematical program can be formulated as follows:

$$\begin{aligned} &\text{Minimise} && f(x) \\ &\text{subject to} && h_i(x) = 0 \quad i = 1, 2, \dots, m \\ & && g_j(x) \leq 0 \quad j = 1, 2, \dots, r \\ & && x \in S \end{aligned}$$

For a **linear program** we consider the case where the objective function, $f(x)$, and all the constraints, $h_i(x)$ and $g_j(x)$ are linear.

Linear programming is a mathematical programming technique that aims to capture the behaviour of the problem within a linear objective function and linear constraints. This technique is credited with both explicit formulation of the problem and, through various solution methods, an efficient solution. The philosophy of linear programming is simply to derive a mathematical structure by observing the important components of the system and their essential interrelationships (Dantzig 1998). The “Transportation Problem” is a famous example of linear programming.

For **integer programming**, we have the additional restriction that all variables are integers. The appeal of integer programming as a modelling method is the compactness of model presentation, the existence of proof of optimality for many of the solution methods (such as branch and bound), and the ability to perform sensitivity analysis on the objective function and constraints post hoc. However, **mixed integer linear programs** (including integer programs with some binary 0-1 variables) are at times computationally difficult. Some aspects of the formulation have an enormous impact on the computation time, such as the integrality and formation of the constraint matrix (Taha 1975).

Mathematical modelling can bring more advantages in analysis than simply the

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concise and comprehensible structuring of the problem. The way in which a problem is modelled can help to identify “cause-and-effect relationships” (Hillier & Lieberman 1990). Further to this, the various relationships between the variables are considered simultaneously.

1.9 In this thesis

This research scans a broad topic that spills into several industries. From a mining industry perspective, we tackle the problem of clearly defining equipment selection and set about achieving optimal solutions from several compact models. As we seek a purchase and salvage policy of whole units of equipment and the productivity constraints are linear, we develop several mixed integer linear program mining models. The necessity for a logical condition in equipment compatibility reinforces the use of integer programming as a modelling method.

In this introduction we have defined the research problem, introduced some necessary background and described some complexities associated with the equipment selection problem. In addressing the problem we begin by reviewing the methods applied in this area and assess the extent to which these methods successfully address the problem we wish to solve [Chapter 2]. We also make particular note of any deficiencies in the literature that can be addressed in our research. Our main contribution from Chapter 2 is:

- a review and consolidation of the literature; refining the boundaries of the *mining method selection* and *equipment selection* problems. We pay particular attention to two seemingly disparate streams of research, *shovel-truck productivity* and *mining equipment selection*, and draw the relevant aspects together.

In this introduction we defined the match factor ratio for homogeneous fleets. This is an important index in the mining industry and is used to indicate the overall efficiency of the fleet. Recognising the match factor ratio as an important validation tool, we extend the ratio in several ways in preparation for use on our heterogeneous solutions in later chapters. In Chapter 3 we present:

- two ways to calculate the match factor for the case of heterogeneous truck fleets [Section 3.2];
- a ratio to calculate the match factor for the case of heterogeneous loader fleets [Section 3.3];
- two ways to calculate the match factor for the case of heterogeneous truck and loader fleets [Section 3.4];

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- extensions for all the above variations for the case of unique truck cycle times and multiple locations.

Our literature review indicates that modelling the equipment selection problem in a tractable way is a difficult task. We consider the existence of pre-owned equipment in all the developed models [Chapters 4, 5 and 6]. A consequence of this is that we must also consider and permit heterogeneous truck and loader fleets. All models presented here consider heterogeneous fleets and also allow equipment to be salvaged at the start of any period in the schedule. Heterogeneity of fleets and equipment salvage have not before been considered in optimisation models in this surface mining application. The inclusion of pre-existing equipment is also novel, which is surprising given the prevalence of pre-existing equipment at the start of a mining schedule and the frequent necessity to re-perform equipment selection mid-way through a schedule.

We begin this challenge by first addressing the case of single location equipment selection for a multiple period mine [Chapter 4]. We present a set of constraints that ensure satisfiability of production requirements after heterogeneous fleet compatibility is taken into account. We validate this model using a retrospective case study, where our solution obtains significantly better results than existing industry methods. We perform robustness testing on a series of test cases we synthesise from the case study. The important contribution from this chapter is:

- We develop a single-location, single-dump-site, multi-period MILP equipment selection model [Chapter 4] which provides the optimal purchase and replacement policy for trucks and loaders. This is a mixed-integer program that solves quickly using *Ilog Cplex* libraries. Interestingly, multi-period schedules have not been considered in the mining literature.

Motivated by the success of the single-location model, we extend this model to multiple-locations for a multiple period mine [Chapter 5]. It is necessary to introduce continuous variables to permit allocation of equipment to mining locations and routes, reducing the solvability of the model. We introduce additional constraints that strengthen the formulation and reduce computation time by shrinking the starting solution space. We test this new model on two industry case studies that describe two very different surface mining operations. In this chapter:

- We develop a multi-location, multi-dump-site, multi-period MILP equipment selection model [Chapter 5] which provides the optimal purchase and replacement policy for trucks and loaders. In addition, this model optimally allocates the trucks and loaders to routes and mining locations respectively. Previously, multiple locations/dump-sites have not been considered in the mining equipment selection literature.

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In a bid to create a more realistic cost objective function, we introduce a utilisation variable for the third model [Chapter 6]. This variable enables the model to account for the actual hours operated by each piece of equipment. We derive constraint sets to relate the non-uniform linear cumulative utilised hours to the equipment units in a linear manner. We test this model on a series of synthetic test cases. The main contribution from this chapter is:

- We develop a single location, single dump-site, multi-period MILP equipment selection model that attributes operating cost to equipment utilisation (Chapter 6). Utilisation based costing has also not been considered in any solvable models, although some theoretical models have been proposed.

In our concluding chapter [Chapter 7], we summarise our findings and look at opportunities for future research on the equipment selection problem.

Chapter 2

Literature review

The equipment selection problem is important to both the construction and mining industries. In spite of the vastly different needs of these two industries, similar methods are applied in both. The mining industry is interested in selecting a truck and loader fleet that can meet materials handling needs at minimum cost; the construction industry places importance on an additional objective: project end date. That is, the completion of one project early can have just as significant implications for the cost of the operation as a late completion. Additionally, the mining industry moves significantly larger quantities of material and over a longer period of time. From this standpoint, equipment retirement age, equipment salvage and heterogeneous fleets are more important considerations for a mining industry equipment selection model.

The equipment selection literature for these two industries is broad, and steps over into two closely related topics: **mining method selection** and **shovel-truck productivity**. The objective of mining method selection, sometimes known as preliminary selection (Cebesoy et al. 1995), is to select a sub-group of equipment that is suitable to operate in the given mining conditions (Oberndorfer 1992). The relationship to the equipment selection problem is that the mining method selected directly affects the equipment available to select. In mining method selection, a mining manager has several mining methods to choose from: each has its own risks and benefits. For example, the position of an ore deposit will determine whether surface or underground mining will be adopted. The mining method selected then implies a subset of suitable trucks and excavating equipment. Some research selects the mining method alongside the equipment, while others select the mining method before selecting the truck and loader fleets. Shovel-truck productivity is a research area propelled by the construction industry which aims to provide good productivity estimates. This will lead to improved fleet selection for a mine.

Due to the difference in problem definition and naming, much of this work appears to have been completed incognizant of the surface mining equipment selection research.

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That is to say, equipment selection research rarely directly references shovel-truck productivity work and vice versa. However, shovel-truck productivity methods are also used in the mining industry for the purpose of equipment selection in practice.

In this chapter we review the current literature on the equipment selection problem and provide:

- A collation of equipment selection research from both mining and construction industries;
- A summary of research categories and hitherto applied methods;
- A consideration of the successes and pitfalls of each applied method; and
- A discussion of how the key elements of the equipment selection problem may be captured.

In addition to direct equipment selection research, this review investigates the literature that applies to equipment selection from within the mining method selection and shovel-truck productivity areas. Figure 2.1 describes the distribution of equipment selection literature and also lists some techniques that have been applied to these problems. The solution methods for all of these problems have been varied in both complexity and success.

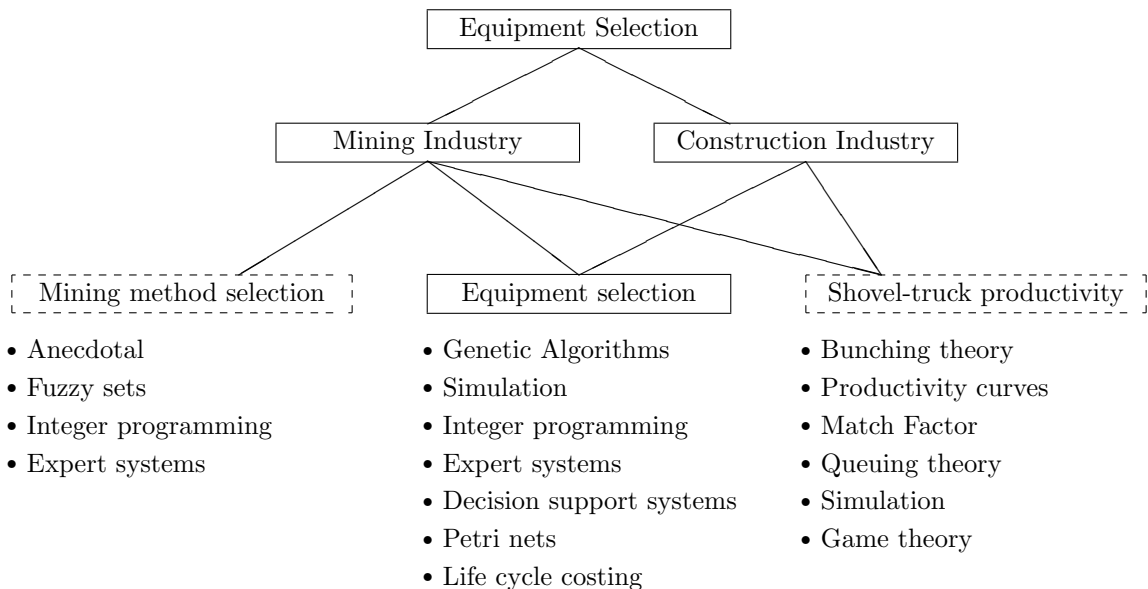


FIGURE 2.1: The distribution of literature and applied techniques for the *equipment selection* problem

Operations research techniques such as linear and integer programming have been applied in a bid to achieve an optimal solution [Section 2.3.3]. With many of these

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methods it is easy to demonstrate optimality, and so a successful program that yields even a small percentage improvement could represent a great windfall for the mining operation. These programming methods can compactly incorporate complexities of the equipment selection problem, which helps to describe a more realistic performance of a particular fleet than models that are overly restricted by assumptions. Artificial intelligence techniques such as expert systems, knowledge based methods and genetic algorithms have also been applied to equipment selection with some success, although optimality has not been demonstrated in these applications [Section 2.3.5].

For mining method selection, integer programming and artificial intelligence techniques have been important new developments, although anecdotal methods persist in the literature [Section 2.2]. For **equipment selection**, the methods applied are very broad [Section 2.3]. Linear programming, artificial intelligence techniques, simulation and life cycle costing techniques have dominated the literature. Some models have been developed using queuing theory, although these are largely underdeveloped. Shovel-truck productivity has seen some discussion of bunching theory¹, productivity curves², match factor³ and queuing theory. These methods are typically applied for the purpose of determining instantaneous productivity levels rather than equipment selection, and therefore progressive models do not exist.

The construction industry only considers equipment selection and shovel-truck productivity, but has been an important motivator for the latter research area. The haulage fleet can be significantly more expensive to run than the loading fleet, and consequently more attention has been offered to the derivation of sound haul fleet solutions rather than optimising the truck and loader fleet together (Bozorgebrahimi et al. 2003).

A closely related problem in surface mining is the dispatching problem, which involves finding the dynamic optimal allocation of equipment to tasks. There have been attempts to use dispatching methods to solve the equipment selection problem [Section 2.4]. The allocation of loaders to fleets is important in determining the productivity capabilities of the corresponding fleet. This is addressed in our research in the multi-location model in Chapter 5.

There are many more closely related problems such as mine production scheduling (Leschhorn & Rotschke 1989, Golosinski & Bush 2000, Caccetta & Hill 2003, Kumral & Dowd 2005), pit optimisation (Frimpong, Asa & Szymanski 2002), equipment costing (O'Hara & Suboleski 1992, Morgan 1994a, Leontidis & Patmanidou 2000), production

¹*Bunching theory* is the study of the jamming effect that can occur when equipment travel along the same route.

²*Productivity curves* are created via simulation or extensive data collection where estimated productivity levels can be compared to actual productivity performance to help understand efficiency loss.

³*Match factor* is the ratio of truck arrival rate to loader service time, and is used to estimate a suitable truck fleet size.

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sequencing (Western 1995, Halatchev 2002) and equipment replacement (Tomlinsong 2000, Nassar 2001). These problems are not within the scope of this study and will not be discussed here.

2.1 Shovel-truck productivity

The shovel-truck productivity research area focuses on estimating and optimising the productivity of a truck and loader fleet. This is based on the intuitive notion that improving productivity will translate into cost reductions (Schexnayder, Weber & Brooks 1999). Often these productivity optimisation methods extend in a simple way to become an equipment selection solution. The efficiency of the truck fleet is related to the number of trucks required to perform the materials handling task (Alarie & Gamache 2002). Match factor and bunching theory are deemed classical shovel-truck productivity methods, while queuing theory has received some attention. We discuss these three methods here as the dominant methods in this area.

The simplest method for determining fleet size based on productivity is as follows:

$$\text{Number of units required} = \frac{\text{Hourly production requirement}}{\text{Hourly production per unit}}.$$

Clearly the truck and loaders types must be pre-selected and the corresponding fleets must be homogeneous for this simple concept to be of use.

2.1.1 Match factor

The *match factor* ratio is an important productivity index in the mining industry. The match factor is simply the ratio of truck arrival rate to loader service time, and is used to determine a suitable truck fleet size. Smith et al. (1995) claimed that operations with low match factors are “inefficient”. Such comments must be interpreted carefully. That is to say, fleets with a low match factor can be very cheap and satisfy the productivity requirements of the operation. The use of the word efficient is used strictly in reference to the ability of both trucks and loaders to work to their maximum capacity. One must question why it is important for this to be so. When the match factor ratio is used to determine the suitability of a selected fleet, one must consider that the minimum cost fleet may not be the most productive or efficient fleet. In this way, a match factor of 1.0 should not be considered ideal for the mining industry, as this corresponds to a fleet of maximum productivity. That is, a loader operating at 50% capacity may be significantly cheaper to run than another loader that operates at 100% capacity under the same conditions.

Adopting the same concepts as the traditional match factor method, Gransberg (1996) described a heuristic method for determining the haulage fleet size.

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1. Determine the cycle times, T , for haul and return routes

$$T = \frac{d}{2} \left(\frac{1}{v_H} + \frac{1}{v_R} \right)$$

where v_H and v_R are haul and return velocities of the truck; d is the haul distance (metres), the divisor 2 averages the velocities.⁴

2. Obtain loading time, L , from load growth curves (Caterpillar 2003).
3. Estimate delay time, D , along route.
4. Calculate “instantaneous” cycle time, C :

$$C = L + T + D.$$

5. Determine “optimum” fleet size

$$N = \frac{C}{L}.$$

Note that no method for estimating delay times was provided by the authors. We can see that step 5 is using the truck arrival rate to truck loading rate ratio with a nominal match factor value of 1.0, and only one loader. When the match factor is 1.0, the truck arrival rate perfectly ‘matches’ the loader service rate and the overall fleet is said to be efficient (with respect to wasted capacity). It is very restrictive to force the match factor to be any value, and is unlikely to result in an optimum solution with respect to fleet cost.

2.1.2 Bunching

The productivity of the overall fleet is limited by the lowest productivity of either the truck or loader fleets. Recall the sketch of truck and loader fleet efficiency [page 11]. Before the intersection, the productivity of the fleet is limited by the capacity of the truck fleet, and the loader will have additional waiting periods. After the intersection, the productivity of the fleet is limited by the capacity of the loader, and the trucks will have additional waiting periods. The intersection itself is the theoretical “perfect match point” (Morgan & Peterson 1968). This match point is also influenced by the natural variation in haul cycles, which can lead to further queuing. This is known

⁴Gransberg (1996) cited the following equation with no justification for the divisor 88:

$$T = \frac{d}{88} \left(\frac{1}{v_H} + \frac{1}{v_R} \right).$$

It is possible that it was intended to represent a conversion from miles per hour to metres per second, although no mention of units was made in the paper.

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as bunching. This is usually due to some of the objects moving more efficiently than others. It can also be due to small, unpredictable delays.

When trucks are operating in a cycle, the truck cycle time will tend toward the slowest truck cycle time, unless overtaking is permitted. That is, faster trucks will bunch behind the slower trucks, causing a drop in the average cycle time. When trucks queue at the loader or dumpsite waiting for their next load, this has the effect of *resetting* the truck cycles and reduces the effect of bunching. Bunching in off-road trucks is not well studied, and typically, reducing factors are used to shrink the efficiency to account for bunching (Douglas 1964, Morgan 1994*b*, Smith et al. 2000).

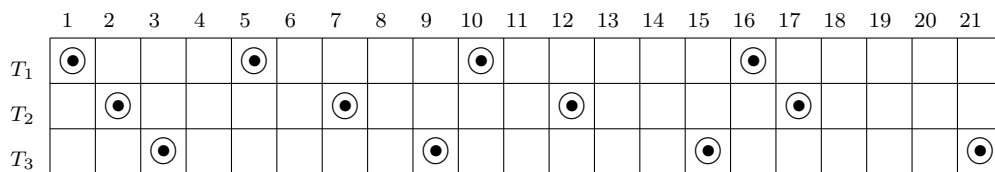


FIGURE 2.2: An illustrative delay effect of bunching with just three trucks.

Consider three trucks with cycle times 4, 5 and 6 minutes [Figure 2.2]. If we do not permit overtaking, then the fastest truck will be delayed by the slower trucks and its cycle time will converge to the slowest truck cycle time.

Bunching is known to reduce a fleet's ability to utilise its maximum capacity. Nagatani (2001) has studied the problem of modelling bunching in general traffic flow and bus routes. Bunching in the truck cycle may be modelled in the same manner.

Bunching certainly occurs in a system of a loader and its correlating fleet of trucks (Smith 1999). This relationship is not as complex as that of buses and passengers: if some trucks have bunched behind a slower truck, then the time headway between the slower truck and the next truck in line will be restored to some extent after the trucks queue at the loader or dump-site (Smith et al. 1995). From this standpoint, the cycle times of all of the trucks approaches the cycle time of the slowest truck, but the time headway is reset before the times converge. This means that the actual average cycle time for the trucks will be lower than the estimated average cycle time (Smith et al. 2000). This is a conservative measure and is not adopted in practice: industry generally uses the average estimated cycle time, often with a reducing factor to account for the efficiency lost to bunching.

The effects of bunching can be significant. In fact, Smith et al. (1995) found in a case study that actual travel times were 21% longer than the calculated times, although they attributed this difference to overestimation of machine efficiency and poor rolling resistance estimates. However, some of the difference would probably have been due to the effect of bunching. The interactive effect of equipment types as well as the size of the trucking fleet can lead to overestimations of fleet efficiency (Smith et al. 2000). It

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is important to consider the effects of queuing and bunching if the proposed schedule is to be met by the selected equipment.

Smith et al. (2000) suggested that the effects of bunching can be curbed by providing accurate equipment speeds before the selection of the equipment and fleet sizes. For homogeneous fleets, it is unlikely that factors such as rolling resistance would influence an individual truck alone, but rather the whole fleet would become slower. In turn, this may have no effect on the bunching of the trucks. In heterogeneous fleets, however, different truck types will be affected differently by poor estimates of rolling resistance, which in turn could have an exacerbating effect on the bunching of the trucks.

2.1.3 Queuing theory

Queuing theory is the study of the waiting times, lengths, and other properties of queues. Waiting time for trucks and loaders has been the focus of some shovel-truck productivity research. Although this has not resulted in good equipment selection solutions, it may provide suitable bounds for truck behaviour in an equipment selection model. Queuing theory was first notably applied to shovel-truck productivity by O'Shea (1964). In this work, he used queuing theory to predict the productivity of trucking fleets in an attempt to account for productivity lost when the trucks queue at a loader. Further on, Karshenas (1989) outlined several improvements that were incorporated into an equipment selection program. These models use the inter-arrival time of one truck instead of the inter-arrival time of the entire fleet. However, the model requires the times between any arrivals, which restricts it to homogeneous fleets.

Huang & Kumar (1994) have continued this work in an attempt to select the size of the trucking fleet using a more accurate productivity estimate. They developed a fleet size selection $M_1 \setminus M_2 \setminus N \setminus FIFO \setminus n_1 \setminus n_2$ model to minimise the cost of idle machinery. Here M_1 and M_2 assert that the customer arrival rate and service rate are exponentially distributed. There are N parallel servers and the service discipline is *First-In-First-Out*. The upper bound of customers allowed in the system is n_1 , while n_2 is the maximum number of potential customers. This is an interesting approach and relies on the assumption that idle machinery contributes to the cost. Their model recommended the selection of fleet sizes that match the maximum efficiency for both location and haulage equipment. Though it is questionable that such a method would improve the economic result, it is useful to consider the variability in some of the parameters of the equipment selection problem such as truck cycle times and queue length.

2.2 Mining method selection

The mining method selection problem is an approach to equipment selection that reasons that the environmental conditions will imply a particular mining method, and that the selection of loader and truck types will follow intuitively from there. This problem then focuses on choosing the correct excavation method for the given conditions. Generally, this research is based on anecdotal methods, and seeks a feasible solution rather than an optimal solution. Much of the literature on mining method selection does not discuss equipment selection modelling in enough detail to be discussed here, but is nonetheless an important area to research when considering pre-selection procedures.

Atkinson (1992) acknowledges the interdependency of ground preparation, excavation and loading, transport and mineral treatment: that “the optimum cost per ton may not be obtained by attempting to minimize each of the individual operational costs”. It is the complexity of combining these factors into one problem that has led many engineers to the primarily anecdotal and knowledge-based solution methods applied to mining method selection. In this way, the loader types and loader fleet size are selected based on diggability studies; the truck type is selected based on the loader; and, the truck fleet size is selected based on all the above information.

Başçetin, Oztaş & Kanlı (2004) applied a *multi attribute decision making* (MADM) model to the mining method selection problem. This methodology is justified for problems where significant expert knowledge is required that cannot be translated directly into a quantity (Bandopadhyay & Nelson 1994). These qualitative parameters are captured in an ordinal ranking. This transformation, from qualitative to ordinal, results in a loss of information.

Fuzzy set theory has also been used to interpret the uncertainties in the decision making process (Başçetin & Kesimal 1999, Wei, Fan & Xu 2003, Bitarafan & Ataei 2004, Başçetin et al. 2004, Başçetin 2004). Fuzzy logic allows black-and-white decisions to blend at the periphery to form a grey area with known probability, so that decisions can be made that satisfy certain goals or constraints. In this method, ranks, probabilities or priority weightings must be allocated to each decision. This can be a considerably complicated step if it is substantiated. However, this method does not consider the size of the fleets or pre-existing equipment.

Amirkhanian & Baker (1992) designed an expert system with 930 rules to perform the task of mining method selection. Although published under the guise of equipment selection, their research does not endeavour to select a particular truck or loader type but rather provides a sub-set of suitable equipment from which a suitable fleet may be chosen.

2.3 Equipment selection

The equipment selection problem aims to select an appropriate set of trucks and loaders subject to various objectives and constraints. The assumptions and types of constraints that are included in the models are varied. Subsequently, an interesting array of methods has been applied to this problem with varying success, such as heuristic, statistical, optimisation, simulation and artificial intelligence techniques.

2.3.1 Heuristic methods

The use of heuristic methods persists in industry, with spreadsheets employed to aid iteration rather than optimisation (Eldin & Mayfield 2005). Smith et al. (2000) recommended the construction industry darling match factor formula as a means of determining the appropriate fleet size. However, selecting the best equipment types must be performed by an expert before applying the formula. The match factor ratio, published by Morgan & Peterson (1968) as an efficiency measurement tool has been, until recently, restricted to homogeneous fleets or, at best, heterogeneous truck fleets (Burt & Caccetta 2007). The earthmoving industry still uses this ratio to determine an appropriate truck fleet size once the loader fleet and truck type has been established (Smith et al. 2000).

The flow chart described in Figure 2.3 illustrates the technique traditionally used by equipment selectors. However, it relies heavily on one or more experts in equipment selection. This increases the likelihood of an “attractive alternative” being missed in the analysis (Webster & Reed 1971). Also, the expert cannot possibly determine the quality of their solution in relation to the large number of alternatives over the solution space. The well-used “50-minute hour” method - a scaling of productivity to account for losses in efficiency - also lacks researched credibility, and effectively becomes a simple scaling method (Gransberg 1996).

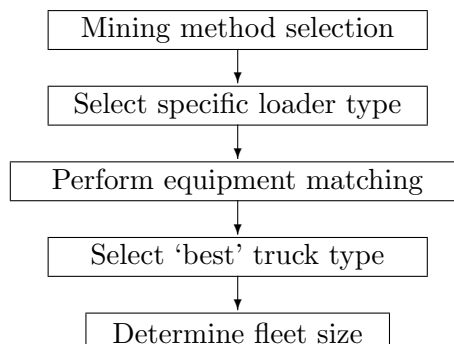


FIGURE 2.3: The classical equipment selection heuristic.

Markeset & Kumar (2000) argued for the use of *life cycle costing* (LCC) as an

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equipment selection method. This tool relies on the assumption that the equipment is kept for its entire life span, so no salvage is permitted. Unfortunately this means that the optimal replacement cycle cannot be determined which undermines the optimisation process. However this type of analysis may be useful in the determination of a cost per hour for equipment.

2.3.2 Statistical methods

Blackwell (1999) developed a *multiple linear regression* model to estimate the important equipment selection parameters that display great variation, such as truck cycle time, tire consumption, fuel consumption and truck operating hours. He argued that these parameters are usually estimated via simulation with questionable results due to “variations in truck power and load carried”. These parameters can then be used to determine an appropriate fleet of trucks and loaders through the use of the simple match factor heuristic above or other means. This method relies on the existence of large data sets for the appropriate parameters for the mine in question.

Griffis (1968) developed a heuristic for determining the truck fleet size using queuing theory. This extended the work by O’Shea (1964) outlined in Section 2.1.3 for calculating the productivity of a set of fleet options by estimating the truck arrival rates by Poisson distribution. Later, Farid & Koning (1994) used simulation to verify the equipment selection results of a queuing theory based on the Griffis and O’Shea works.

2.3.3 Optimisation techniques

The use of *integer programming* methods is well established in both the mining and construction operations (Jayawardane & Harris 1990). Integer programs have been used to create mining schedules (Dagdelen, Topal & Kuchta 2000, Dagdelen & Asad 2002, Johnson, Dagdelen & Ramazan 2002, Ramazan & Dimitrakopoulos 2003, Kumral & Dowd 2005) and for pit optimisation (Caccetta & Hill 2003). However, for equipment selection much of the focus is on project completion and dispatching or allocation (Ergelebi & Kirmanlı 2000). The models tend to assume given equipment types, rather than allowing the models to select these with the fleet size. Fleet homogeneity and restricted passes between loader and truck are also common constraints (Çelebi 1998) that have not been demonstrated to be sensible. In a departure from cost optimisation, some solution methods look at optimising productivity (Smith et al. 2000) and optimising equipment matching (Morgan 1994b). Since maximising productivity is different to minimising cost, such objectives are also useful in the construction industry. For example, in other formulations, “budgeting constraints” have been considered where the maximum permissible budget cash outlay for a given time period is an upper bound

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(Cebesoy et al. 1995). “Mutual exclusivity” is a common restriction that only allows one type to be used. Cebesoy et al. (1995) describes heterogeneous fleets as “unacceptable or even unthinkable”, although only anecdotal evidence has supported these claims to date.

Webster & Reed (1971) proposed an equipment selection model for general materials handling as an assignment problem. This model incorporates a predetermined utilisation matrix into the cost objective function, which is an achievement not yet accomplished in the mining industry. The objective function minimises the costs of the equipment according to a predetermined utilisation matrix. One of the limitations of this model is that it only allows one piece of equipment (and therefore only one type) to be allocated to a particular task. This quadratic integer program is intended for allocation rather than equipment selection in the way that we approach the problem in this thesis. It is primarily theoretical model: the authors admit that such a formulation is not capable of solving practical sized problems at the time of publication. In fact, a pre-determined utilisation matrix for a problem with 27 loaders (maximum age 20 years), 26 trucks (maximum age 15 years), 13 periods and maximum truck fleets of 10 and 30 for loaders and trucks would require a matrix of almost 1GB in size for the trucks alone. The model also does not account for truck and loader compatibility and does not attempt to ensure that productivity requirements are met. It is also restricted to a single period.

Hassan et al. (1985) extended Webster and Reed’s model to combine the equipment selection problem with the allocation problem. The objective function used in this model minimises the “sum of operating and capital costs over all [tasks] and equipment types”. It selects equipment to perform tasks, rather than satisfying productivity requirements. It does, however, ensure that the equipment performs these tasks in a nominated time-frame. For surface mining productivity requirements are considered important - time restrictions can be absorbed.

Cebesoy et al. (1995) developed a systematic decision making model for the selection of equipment types. The final step in the model made use of a binary integer program. This model considers a single period, single location mine with homogeneous fleets. Suitability matching of the equipment occurs prior to the solving of this model, presumably in the form of mining method selection. This model operates on the assumption that equipment is kept for its entire life, and that we pay the cost of running it for its entire life regardless of utilisation or the number of periods it is actually required. The objective function and constraints are very simple, reflecting the homogeneous fleet and single-location assumptions.

Michiotis, Xerocostas & Galitis (1998) developed a binary integer programming model to select equipment for a mine, which was paired with a layer stratification model. The equipment selection model minimises the time required for the extraction of the

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mine (in its entirety) as an objective function. It is not clear why this objective function is chosen, as the authors note that this time can be “estimated at the preliminary scheduling phase”. The model focuses on selecting appropriate equipment types that suit the geology of the mine, such as bench height, rock density and rock resistance. It is intended to reduce a large set of available equipment types to a satisfactory set from which optimal equipment selection can take place using other methods.

Edwards et al. (2001) used a linear programming model to select the optimum loader type for construction use. The model is not formally presented in their paper and so it is difficult to determine the decision variables used in the model. This model solves for very small time domains (such as 20 hours) and selects one fleet for the entire schedule. This means that salvage is not permitted and purchase occurs once at the beginning of the schedule. Due to the small time domain, the age of the equipment is not incorporated into the model and therefore equipment availability and its change with age is unaccounted for.

Other people have applied integer programming to this problem in a limited capacity. Jayawardane & Harris (1990) applied integer programming to optimise the movement of materials in the construction industry, incorporating project end dates. The developed model was intended to determine if the “available resources could be utilized toward the project completion within the target time” (Jayawardane & Harris 1990). They also consider soil compaction and swell factors. This model is designed to minimise the cost of moving the material, and allocate tasks to pre-existing fleets - in this sense it is not an equipment selection model.

2.3.4 Simulation

Simulation is a well used and notably powerful tool for the mining industry (Hall 2000). Although simulation is most effectively used in mining equipment selection to analyse the earth-moving system, some equipment selection solutions exist that use simulation models (Hrebar & Dagdelen 1979, Tailakov & Konyukh 1994, Marzouk & Moselhi 2004). Kannan (1999) provides some defined requirements and “success factors” for simulation programming, and a short but directed literature survey of simulation modelling in the construction industry.

Shi (1999) used simulation to produce large sets of data in order to train a neural network for predicting earthmoving production. Simulation is also a useful tool to observe the interactions of particular equipment. Schexnayder, Knutson & Fente (2005) describe a simulation model for predicting productivity, although this is not directly intended for equipment selection. On a simpler note, simulation can also be used to estimate a suitable truck cycle time (Frimpong, Changirwa & Szymanski 2003). However, without formal modelling of the effect of bunching for both homogeneous and

heterogeneous fleets, simulation results should be carefully interpreted.

2.3.5 Artificial intelligence

Artificial intelligence techniques are prevalent amongst large scale mining applications due to their ability to find feasible solutions within a short time (Clément & Vagenas 1994). The most common methods among the literature are the *expert system* (Denby & Schofield 1990), *decision support system* methods (Bandopadhyay & Venkatasubramanian 1987, Başçetin 2004) and genetic algorithms (Haidar & Naoum 1995, Haidar & Naoum 1996, Haidar, Naoum, Howes & Tah 1999, Marzouk & Moselhi 2002a, Marzouk & Moselhi 2002b, Xinchun, Weicheng & Youdi 2004). Genetic algorithms are a computer simulation technique which selects a solution after several generations of stochastic selection based on a fitness. In this sense they are designed to imitate evolution.

The expert systems approach, often preferred for complex systems, is a structured attempt to capture human expertise into an efficient program (Welgama & Gibson 1995). Amirkhanian & Baker (1992) developed an expert system for equipment selection in construction, incorporating 930 rules. These rules incorporate “information concerning a particular project’s soil conditions, operator performance, and required earth-moving operations”.

Ganguli & Bandopadhyay (2002) also developed an expert system for equipment selection. However, their method requires the user to input the “relative importance of the factors”, which is typically difficult to quantify and substantiate. Although the expert system method does not claim optimality of its solutions, it does highlight an important aspect of modelling equipment selection: the equipment subset to be considered in the model will be dependent on the soil and mining conditions. In this sense, rule-based pre-selection is a logical pre-process to any equipment selection model (Başçetin 2004).

Various decision support tools, such as analytical hierarchy process (Başçetin 2004) and expert systems (Amirkhanian & Baker 1992), apply priority to decisions for logic based heuristic solutions. These methods consider the entire process of equipment selection holistically, including site conditions, geology and environment, as well as equipment matching. Equipment matching is a step beyond merely considering compatibility, where equipment pairs can be ranked in suitability in a pre-optimisation process. However, these heuristic methods cannot claim optimality.

There are numerous examples of the application of *genetic algorithms* to the equipment selection problem. Naoum & Haidar (2000) developed a genetic algorithm model for the equipment selection problem. Their model incorporates the lifetime discounted cost of the equipment, which is attached to the assumption that the equipment is used

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from purchase until its official retirement age, and not sold or replaced before that time. It also requires an expert to select the loader type before the optimisation process begins, and is restricted to homogeneous fleets. The authors argue that intelligent search techniques are necessary because integer programming is incapable of solving a problem with more than one type of independent variable. While this is not true, such techniques may be required when constraints become nonlinear, which may arise due to queuing. Intelligent search techniques may also be useful for obtaining a good quality feasible solution where integer programming techniques cannot differentiate between clusters of near-optimal solutions.

Marzouk & Moselhi (2004) designed a bold model using simulation and genetic algorithms to simultaneously minimise two objectives: time and cost. Intended for the construction industry, their model provides a one fleet solution for the length of the entire project, which is not realistic for a multi-period mining operation with fluctuating production requirements.

2.4 Dispatching and allocation

The allocation of equipment is an important consideration for the satisfaction of productivity requirements. The objective of dispatch optimisation is to maximise the efficiency of the fleet at hand (Hagenbuch 1987). Truck dispatch systems apply linear programs dynamically to determine the minimum number of trucks required for a schedule, and dynamic programming is then used to determine allocation to mining locations (Blackwell 1999).

Easa (1988) developed two quadratic programming models for earthwork allocation. These models allow for linear unit cost functions of purchase and excavation for borrow pits, as opposed to constant cost functions. The operating cost of equipment can fluctuate with the age of the equipment, due to different levels of maintenance and running costs (Easa 1987).

Truck allocation is complicated by the presence of uncertainty in some parameters such as plant downtime, truck load and truck cycle time. Typically mathematical programming is used to solve the allocation problem, while heuristics are used for dispatch. Ta et al. (2005) developed a stochastic model that incorporates real-time data for allocation of the fleet. More recently, Karimi, Mousavi, Kaveh & Afshar (2007) addressed the uncertainty in parameters with a fuzzy optimisation model.

2.5 Discussion

With such a wide variety of methods applied to the problem, we can expect the quality and detail of the solutions to vary. For simple and fast models, shovel-truck productivity

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can obtain feasible solutions, although there is no guarantee of optimality. With the scale of operations involved in mining, sub-optimal solutions can equate to significant costs.

Mining method selection is an important step to consider before equipment selection. Ideally the mining method and equipment should be optimised together. Within the equipment selection literature, the popular techniques focus on decision support models, genetic algorithm approaches and integer programming models. Each has its own benefits - decision support models are capable of including a significant amount of expert detail, but cannot guarantee optimality; genetic algorithms can obtain feasible solutions to problems with a large number of variables and constraints, but also cannot guarantee optimality; integer programs can guarantee optimal solutions, but are computationally difficult to solve.

Although the applied literature for the surface mining equipment selection problem is diverse, there are common weaknesses amongst all of the presented models and techniques. Namely:

- **Pre-existing fleets:** Models that consider the existence of pre-existing equipment do not exist in the literature.
- **Fleet homogeneity:** There is no reason to believe that a mixed-type fleet performs worse than a homogeneous type fleet.
- **Equipment type pre-selection:** Loader (or truck) type pre-selection requires a highly skilled and experienced engineer to select a loader type based on geographical and geological information. This can be a time consuming task and optimality is unlikely. The equipment type selection should occur simultaneously with fleet size selection if optimality is desired.
- **Restricted passes:** Although there is a general preference for restricting the maximum number of passes from loader to truck, there is no evidence in the literature to support this constraint.
- **Political factors:** Political and social pressures can sometimes carry more weight than optimal cost considerations (Chanda 1995). These aspects are difficult to model and are simply excluded from the modelling process.
- **Bunching:** The effect of bunching is largely ignored, or is accounted for by a simple reducing factor (usually obtained through simulation or observation). However, without a formal modelling of the effect of bunching for both homogeneous and heterogeneous fleets, it is difficult to give too much credit to a simulation result.

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- **Operating cost:** The inclusion of sensible ownership, operating and maintenance costs may play a crucial role in the solution. Yet in integer programs the costs are often accepted as a constant input to which no further calculations are performed.
- **Objective choice:** Some research has modelled the equipment replacement problem but focuses on replacement time rather than optimising the type and number of truck/loader replacements.

Hajdasinski (2001) argued that while a particular model may produce an optimal solution, minor factors that were excluded from the model may prevent the solution from being realistic. In this case, they suggested that providing the next best solutions, even if they are suboptimal, is important for the scrutiny of management.

One of the greatest weaknesses of research in this area is the neglect of properly defining the problem that is being solved and listing all assumptions made in the process of acquiring a solution. Proper problem definitions and assumption lists allow the reader to quickly determine the applicability of the work to their own research area.

A program that allows the consideration of all sizes of equipment will test the ‘bigger is better’ notion that is prevalent amongst the mining community (Çebi, Köse & Yalçın 1994, Haidar et al. 1999). Accuracy of the formulation is also important: using approximations such as ‘average truck haulage distance’ over the life of the mine may be severely inadequate. The main objective of the program is to cut down on these undemonstrated and unrealistic assumptions and seek a solution that is either optimal or very near optimal. This means that we must consider the entire set of available equipment in order to select the best combination (Erçelebi & Kırmanlı 2000).

In this thesis we address several of these issues. Firstly we select an objective function that minimises the cost of materials handling, and through this, optimises the type and number of trucks and loaders in the solution. We carefully consider the manner in which operating costs change over the lifetime of the equipment and account for these changes where possible. We eliminate the restriction on the number of passes from the loader to the truck to enable the model to choose the most appropriate equipment match. Also, we simultaneously select the equipment types to ensure the validity of the optimisation process. We also consider the possibility of pre-existing equipment and allow for heterogeneous fleets. The models all consider multiple periods in a bid to obtain the cheapest overall materials handling cost. We consider two ways to represent the productivity requirements of the mine (namely single location and multiple location), and finally develop a model that accounts for equipment utilisation in the objective function.

Chapter 3

Match factor extensions

For the mining industry, the match factor ratio is an important indicator with a dual purpose: during the equipment selection phase, it can be used to determine an appropriate fleet size such that the truck fleet productivity matches that of the loader fleet; when a fleet is selected, the match factor can be used to estimate the relative efficiency of the fleet. Thus far this ratio has been restricted to homogeneous fleets - however, heterogeneous fleets are common in large scale mines. We present several extensions to the match factor ratio to allow consideration of heterogeneous fleets. The results of this chapter have been published in the International Journal of Mining, Reclamation and Environment (Burt & Caccetta 2007).

3.1 Introduction

The mining and construction industries have long held interests in determining the productivity or efficiency of a selected fleet of trucks and loaders. One way to study the efficiency of a fleet is to weigh the efficiencies of the truck fleet and loader fleet against one another. The *match factor* is the ratio of truck arrival times to loader service rates.

The aforementioned industries have used the match factor for many decades as an indicator of productivity performance. As defined by Morgan & Peterson (1968), the match factor ratio, $MF_{i,i'}$, for trucks of type i working with loaders of type i' is given as

$$MF_{i,i'} = \frac{t_{i,i'} x_i}{\bar{t}_X x_{i'}}, \quad (3.1)$$

where x_i is the number of trucks of type i , $x_{i'}$ is the number of loaders of type i' , $t_{i,i'}$ is the time taken to load truck type i with loader type i' , and \bar{t}_X is the average cycle time for all trucks. We provide a summary of the notation in section 3.6.

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This ratio has hitherto relied on the assumption that the truck and loader fleets are homogeneous. That is, all the trucks are of the same type, and all the loaders are of the same type. In reality, mixed fleets are common. Heterogeneous fleets can occur when equipment types are discontinued, equipment is superseded, or simply when a mixed fleet is cheaper than a homogeneous fleet. Heterogeneous fleets can occur when new equipment is purchased to work alongside pre-existing equipment. It is also possible that a heterogeneous fleet can represent a minimum cost equipment selection solution. In this chapter we propose new ways for defining match factor for heterogeneous fleets.

In particular, we:

- Present two ways to define match factor when heterogeneous truck fleets are present [Section 3.2];
- Present a new method to define match factor when heterogeneous loading fleets are operating [Section 3.3]; and
- Present a new method to define match factor when both truck and loader fleets are heterogeneous [Section 3.4].

The aim of this chapter is to provide extensions to the productivity and efficiency measures currently available in the literature. This will enable greater consideration of heterogeneous fleets.

3.2 Heterogeneous truck fleets

The fleet most likely to be heterogeneous is the trucking fleet. This is due to the large number of trucks required to meet production requirements, compared to a relatively small number of loaders. Also, although there may often be different types of loaders operating in a mine, they are often in different locations and so can't be considered as separate fleets.

We begin by considering the truck arrival rate in the case of a heterogeneous truck fleet with a homogeneous loading fleet.

Definition 3.2.1 *The **truck arrival rate**, A , for a heterogeneous truck fleet with homogeneous loading fleet is the ratio of the number of trucks to the truck cycle time:*

$$A = \frac{\sum_i x_i}{\bar{t}_X} ,$$

where x_i is the number of trucks of type $i \in \mathbf{X}$ (where \mathbf{X} is the set of all truck types), and \bar{t}_X is the average cycle time for all truck types.

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At this stage this rate is unaffected by the number of truck types, as we use an average truck cycle time. The loader service rate is the number of trucks that are served per second. The loader cycle time may vary between different truck types.

Definition 3.2.2 *The loader service rate, $D_{i'}$, for loader type i' is given by*

$$D_{i'} = \frac{x_{i'} \sum_i x_i}{\sum_i t_{i,i'} x_i},$$

where x_i is the number of trucks of type i , $x_{i'}$ is the number of loaders of type $i' \in \mathbf{X}'$, and $t_{i,i'}$ is the time required to load truck type i from loader type i' .

As the match factor is the ratio of truck arrival rate to loader service time, the match factor for heterogeneous truck fleets is easily derived from Definitions 3.2.1 and 3.2.2:

$$MF_{i'} = \frac{A}{D_{i'}} \tag{3.2}$$

$$= \frac{\sum_i x_i}{\bar{t}_X} \bigg/ \frac{\sum_i x_i x_{i'}}{\sum_i t_{i,i'} x_i} \tag{3.3}$$

$$= \frac{\sum_i t_{i,i'} x_i}{\bar{t}_X x_{i'}} \tag{3.4}$$

$$= \sum_i MF_{i,i'}. \tag{3.5}$$

It is clear that if only one truck type is operating in the fleet, then equation (3.4) will produce the same result as equation (3.1). Another way to think of this match factor for heterogeneous truck fleets is to add the individual match factors from each of the homogeneous sub-fleets. Note that the alternative method is only appropriate for the case of homogeneous loader fleets working with heterogeneous trucking fleets.

Sometimes we would like to use unique truck cycle times for different truck types in the fleet. This can occur when trucks have different routes. For example, consider a case where larger equipment is used to haul waste while smaller trucks are used to haul ore: the waste and ore may be sent to different locations, with significantly different cycle lengths. When individual truck cycle times are used, the times must be weight averaged to produce an accurate match factor. Equation (3.4) can be easily extended to account for unique truck cycle times.

Definition 3.2.3 The *average cycle time*, \bar{t}_X , is given by:

$$\bar{t}_X = \frac{\sum_i t_i x_i}{\sum_i x_i} ,$$

where t_i is the cycle time for truck type $i \in \mathbf{X}$ and x_i is the number of trucks of type i .

Now, substituting this new truck cycle time into equation (3.4), we have the following lemma:

Lemma 3.2.1 For heterogeneous truck fleets with individual truck cycle times, the *match factor* for homogeneous loader fleets of type $i' \in \mathbf{X}'$ can be represented by

$$MF_{i'} = \frac{(\sum_i x_i) (\sum_i t_{i,i'} x_i)}{x_{i'} \sum_i t_i x_i} . \quad (3.6)$$

3.3 Heterogeneous loader fleets

This section considers the case of mixed loaders in the fleet, while the trucks remain uniform in type. The time required to load a truck may be different for various types of loaders. The loader service rate is the number of trucks served in a defined time period. In a heterogeneous fleet, the time taken to serve a truck may differ between the varying loader types.

Lemma 3.3.1 The *loader service rate* for heterogeneous loader fleets working with truck type $i \in \mathbf{X}$ is given by

$$D_i = \sum_{i'} \frac{x_{i'}}{t_{i,i'}} . \quad (3.7)$$

Proof: The loader service rate is the ratio of the total number of trucks to the time required to serve them. We have $t_{i,i'}$ for several loader types i' and one truck type i . The number of trucks type i served by loader type i' in $t_{i,i'}$ time is:

$$\frac{1}{t_{i,i'}} .$$

Thus the total number of trucks served by all loader types in a unit of time is

$$D_i = \sum_{i'} \frac{x_{i'}}{t_{i,i'}} , \text{ as required.}$$

■

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Recall that the match factor is the ratio of truck arrival rate to loader service rate. The truck arrival rate is:

$$A_i = \frac{x_i}{t_i} ,$$

which gives the following theorem:

Theorem 3.3.2 *For heterogeneous loader fleets, the **match factor** for a homogeneous truck fleet of type $i \in \mathbf{X}$ is*

$$MF_i = \frac{x_i}{t_i \sum_{i'} \frac{x_{i'}}{t_{i,i'}}} . \quad (3.8)$$

When only one type of loader operates in the fleet, equation (3.8) reduces to equation (3.1). In the case of multiple dump locations or routes, equation (3.8) can be expanded to account for differing truck cycle times. First, we represent the average truck cycle time by:

$$\bar{t}_X = \frac{\sum_h t_{i,h} x_{i,h}}{x_i} \quad (3.9)$$

where $t_{i,h}$ is the cycle time for truck type i on route h , and $x_{i,h}$ is the number of trucks of type i working on route h . This gives the following corollary:

Corollary 3.3.3 *The **match factor** for heterogeneous loader fleets working with truck type i (with individual truck cycle times for trucks on route h) can be represented by*

$$MF_i = \frac{(x_i)^2}{\left(\sum_{i'} \frac{x_{i'}}{t_{i,i'}}\right) (\sum_h t_{i,h} x_{i,h})} . \quad (3.10)$$

3.3.1 Example

The following example calculates the match factor of a heterogeneous loader fleet. Table 3.1 outlines the equipment set.

	Equipment	Capacity (Tonnes)	Cycle Time (seconds)
22	Truck type A	150	1500
1	Loader type B	60	35
1	Loader type C	42	35

TABLE 3.1: Example data for a heterogeneous loader fleet with common truck cycle time.

The cycle time for the loader is the time taken for one full swing of the bucket. Some trucks may need several buckets to fill their trays. The first step is to determine

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the unique loading time for each truck. If the truck capacity is not a round multiple of the loader capacity, then we take another scoop if the capacity left is more than one third of the loader bucket size. This is because it takes almost the same amount of time to move a portion of a scoop as it does to move a full scoop (Gransberg 1996).

Truck type A and loader type B: $\frac{150}{60} = 2.5$ 3 swings $3 \times 35 = 105$ seconds

Truck type A and loader type C: $\frac{150}{42} = 3.6$ 4 swings $4 \times 35 = 140$ seconds

This gives the match factor:

$$\begin{aligned} MF_A &= \frac{x_A}{\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{A,i'}}} \\ &= \frac{22}{1500 \times \left(\frac{1}{105} + \frac{1}{140}\right)} \\ &= 0.88. \end{aligned}$$

This shows that the fleet is under-trucked. When a minimum cost fleet is desired, one would reasonably expect that under-trucking would provide better solutions than perfectly matching the fleets with a match factor of 1.

3.4 Heterogeneous truck and loader fleets

For heterogeneous truck and loader fleets, we consider the time required for each loader to serve the available truck fleet. This is equal to the sum of the number of trucks of type i multiplied by the time required to serve that truck type. We call this the *loading times*, $t_{i'}$, for each loader type i' .

Definition 3.4.1 *The time, $t_{i'}$, required for a loader of type i' to serve the entire fleet of trucks is*

$$t_{i'} = \sum_i t_{i,i'} x_i. \quad (3.11)$$

So the time taken for one loader to serve one truck is:

$$\frac{t_{i'}}{\sum_i x_i}. \quad (3.12)$$

Thus we obtain the following corollary:

Corollary 3.4.1 *The loader service rate for heterogeneous trucks and loaders is*

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given by

$$D = \sum_{i'} \frac{x_{i'} \sum_i x_i}{t_{i'}}. \quad (3.13)$$

As in Section 3.3, the truck cycle time is assumed to be constant for the entire truck fleet for that period.

Theorem 3.4.2 *The match factor for both heterogeneous truck and loader fleets can be represented by*

$$MF = \left(\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{i'}} \right)^{-1}. \quad (3.14)$$

Proof: We now consider the truck arrival rate for the entire fleet, given in definition 3.2.1:

$$\begin{aligned} MF &= \frac{A}{D} \\ &= \frac{\sum_i x_i}{\bar{t}_X} \times \frac{1}{\sum_i x_i \sum_{i'} \frac{x_{i'}}{t_{i'}}} \\ &= \frac{1}{\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{i'}}}, \text{ as required.} \end{aligned}$$

■

If we have unique truck cycle times, equation (3.14) can be easily extended to:

$$MF = \frac{1}{\left(\sum_{i'} \frac{x_{i'}}{t_{i'}} \right)} \sum_{i \in I} \frac{x_i}{(t_{i,I} x_{i,I})} \quad (3.15)$$

where $t_{i,I}$ is the unique truck cycle time for trucks in subset $I \in \mathbf{X}$.

We find an expression that is equivalent to equation (3.14) but may be simpler to implement in a spreadsheet by first observing that the loader service rate can be represented by:

$$D = \frac{\sum_{i'} x_{i'} \sum_i x_i \prod_{h \neq i'} (t_{i,h} x_i)}{\prod_{i,i'} (t_{i,h} x_i)}.$$

We take the truck arrival rate for the entire fleet from definition 3.2.1 to obtain the following theorem.

Theorem 3.4.3 *The match factor for both heterogeneous truck and loader fleets can be represented by*

$$MF = \frac{\prod_{i,i'} t_{i,i'} x_i}{\prod_{h \neq i'} (t_{i,h} x_i) \sum_{i'} x_i \bar{t}_X}. \quad (3.16)$$

When only one type of truck and one type of loader operate in the fleet, equations (3.14)-(3.15) reduce to equation (3.1), as expected.

3.4.1 Example

This example determines the match factor of a heterogeneous truck and loader fleet. Table 3.2 presents the data set.

	Equipment	Capacity (Tonnes)	Cycle Time (seconds)
15	Truck type A	150	1500
7	Truck type B	230	1500
1	Loader type C	60	35
1	Loader type D	38	30

TABLE 3.2: Example data for a heterogeneous truck and loader fleet with common truck cycle time.

The unique loading times for each truck are determined by the rule of thumb described in Section 3.3.

$$\text{Truck type A and loader type C: } \frac{150}{60} = 2.5 \quad 3 \text{ swings} \quad 3 \times 35 = 105 \text{ seconds}$$

$$\text{Truck type A and loader type D: } \frac{150}{38} = 3.9 \quad 4 \text{ swings} \quad 4 \times 30 = 120 \text{ seconds}$$

$$\text{Truck type B and loader type C: } \frac{230}{60} = 3.8 \quad 4 \text{ swings} \quad 4 \times 35 = 140 \text{ seconds}$$

$$\text{Truck type B and loader type D: } \frac{230}{38} = 6.1 \quad 6 \text{ swings} \quad 6 \times 30 = 180 \text{ seconds}$$

We calculate the *loading times*, $t_{i'}$, for each loader of type i' .

$$t_C = 15 \times 105 + 7 \times 140 = 2555$$

$$t_D = 15 \times 120 + 7 \times 180 = 3060$$

$$\begin{aligned} MF &= \frac{1}{\left[\frac{1}{2555} + \frac{1}{3060} \right] \times 1500} \\ &= 0.928 \end{aligned}$$

This solution is close to the theoretical perfect match of 1.0. This is a good result in terms of overall efficiency and productivity of the fleet. However, one should be aware that costing has not been considered in determining the match factor and so it is possible that the fleet would be cheaper to operate even if the match factor was lower.

3.5 Discussion

For the mining industry, the match factor ratio is an important indicator which we have extended for several likely circumstances, including heterogeneous truck and loader fleets with multiple routes. The match factor can be used to optimise the truck cycle time in order to gain maximal efficiency from the selected fleet. Alternatively, project managers may use the match factor formula to determine the ideal number of trucks in the fleet. The formulae presented in this chapter are less restricted in their choice of equipment, select mixed fleets to suit the productivity requirements and minimise materials handling expense.

It is interesting to note that the Morgan & Peterson (1968) match factor ratio excludes waiting and queuing times for trucks and loaders. This may be because the waiting time for a truck fleet is difficult to estimate without first knowing the size of the truck fleet. However, if we use the match factor ratio as an index of overall fleet efficiency, then it is acceptable to include waiting times that have been estimated by other methods.

The formulae presented in this chapter provide a sensible extension to the original equation and bring greater accuracy to the cases where mixed fleets operate together. All of these formulae can be implemented easily in spreadsheet software such as Microsoft Excel. Throughout the rest of this thesis, we employ the heterogeneous match factor ratios to indicate the overall efficiency of the selected fleets.

3.6 Summary of notation

\mathbf{X}	the set of all available truck types.
\mathbf{X}'	the set of all available loader types.
i	the truck equipment type index, $i \in \mathbf{X}$.
i'	the loader equipment type index, $i' \in \mathbf{X}'$.
$MF_{i,i'}$	the match factor for homogeneous truck and loader fleets of types i and i' respectively.
$MF_{i'}$	the match factor for heterogeneous truck fleets working with loader type i' .
MF_i	the match factor for heterogeneous loader fleets working with a homogeneous truck fleet of type i .
MF	the match factor for heterogeneous truck and loader fleets.
x_i	the number of trucks of type i .
$x_{i'}$	the number of loaders of type i' .
t_i	the cycle time for truck type i .
$t_{i'}$	the time required for a loader of type i' to serve the fleet of trucks.
$t_{i,i'}$	the the time taken to load truck type i with loader type i' .
\bar{t}_X	the average cycle time for all the trucks.
A	the truck arrival rate.
A_i	the truck arrival rate for truck type i .
$D_{i'}$	the loader service rate for heterogeneous truck fleets working with loader type i' .
D_i	the loader service rate for heterogeneous loader fleets working with truck type i .
D	the loader service rate for heterogeneous loaders working with a heterogeneous truck fleet.

Chapter 4

Single-location, multi-period equipment selection

In a simple surface mining scenario we can consider the mine to have one mining location and one dump-site connected by a lone truck route. Our objective is to determine a purchase and salvage policy for trucks and loaders such that the cost of materials handling is minimised over a multiple period schedule. This problem quickly becomes large scale when we consider large sets of trucks and loaders, and long schedules. The inclusion of pre-existing equipment leads to the possibility of heterogeneous fleets; the non-uniform behaviour of different equipment types (with respect to operating cost, availability and productivity) coupled with compatibility issues adds to the complexity of the problem. In this chapter, we present an integer program for equipment selection that incorporates heterogeneous fleets, pre-existing equipment and compatibility while optimising over multiple periods. We introduce a specialised linear constraint set to ensure satisfaction of production requirements. We also test the model on a case study and a set of synthesised test cases. The resulting model is a robust equipment selection tool that obtains optimal solutions quickly for large sets of equipment and long schedules.

4.1 Introduction

In the mining industry, materials handling represents a significant component of the operational cost. Extensive research has provided the industry with numerous feasible solution strategies, but the complexity of the problem has hindered optimal multi-period solutions. In this research, we provide a suitable equipment selection tool that will select the trucks and loaders for a multi-period mine at minimum cost.

In particular, we focus on developing a robust equipment selection tool for a single-location, multi-period mine. A single-location mine is a mine which has just one mining

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location with a single route to a lone dump-site (Figure 4.1). By limiting to a single-location mine we are in essence generalising the productivity requirements for the mine to just one location. By doing this it is possible that the selected fleet will be incapable of meeting a more complicated schedule. However, we consider the model in this chapter to be a tool for smaller mines or for mines where there is no significant difference in the mining locations and route lengths.



FIGURE 4.1: A simplified model of a mine with single-location, single-dump-site and a lone route connecting the two.

We wish to consider all planned time periods in the mining schedule to ensure that the selected fleet can cope with the changes that the planned schedule brings, rather than simply satisfying short-term requirements. In addition to this, we consider the possibility of pre-existing equipment. Due to the dynamic nature of mines and the subsequent demands placed on them, it is common for a mining schedule to change significantly after only a few years into the schedule. When this occurs, it may be necessary to re-perform the equipment selection. In this case, there would be a considerable amount of pre-existing equipment - some of which may be discontinued or superseded by better equipment. This also means that we may have heterogeneous fleets.

In addition to the purchase of equipment, we also consider the salvage of equipment for two reasons. Firstly, mines often operate for longer than the standard life of a piece of equipment. From this, we can expect that we will need to retire some of the equipment during the schedule. In particular, pre-existing equipment may already be close to retirement age. Secondly, mining schedules sometimes have significant productivity changes from period to period. The mining manager may wish to determine whether it is better to purchase equipment for short term periods, or to hire equipment over the peak periods. Allowing salvage permits them to investigate this.

In this chapter we develop an integer program with the following features:

- the inclusion of pre-existing equipment and heterogeneous fleets;
- a multi-period mining schedule;
- linear compatibility constraints answering satisfaction of the productivity requirements;

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- costing which is apportioned to the age of the equipment (in line with industry standards).

Many aspects of the presented model, such as the consideration of multiple periods and pre-existing equipment, are novel for the mining industry and ensure that the model is both a new and advanced equipment selection tool.

We formulate the model in section 4.2 by first listing the assumptions (section 4.2.1) before outlining the decision variables (section 4.2.2), followed by a derivation of the objective function (section 4.2.3) and constraints (section 4.2.4). We provide a summary of the notation and the complete model in sections 4.2.5 and 4.2.6. To test the model, we first run test cases to validate the model in section 4.4 before considering a mining case study in section 4.5. Finally, we discuss the results and opportunities to extend the work in section 4.6.

4.2 Problem formulation

Trucks and loaders only come in discrete quantities, so it is appropriate to track them using integer variables. A cost objective function can be linearised through careful definition of the variable indices, and productivity constraints are naturally linear. Therefore the equipment selection problem for surface mines is best expressed as a pure integer program.

4.2.1 Assumptions

The model is, by necessity, an abstraction of the real problem. Consequently we apply some assumptions and conditions to make the model solvable. For this formulation, the following assumptions apply:

Known mine schedule: We assume that an acceptable mine schedule has already been derived, and that the mining method has been selected. We start the equipment selection process with a sub-set of trucks and loaders that suit the particular mining scenario.

Single mining location: All the loaders and trucks are considered to operate as one fleet. That is, all loaders work in the same location, and all the trucks service all loaders. We consider the mine to have a single mining location, a single dump-site and a single haul route connecting the two.

Salvage: All equipment is salvageable at the start of each period at some depreciated value of the original capital expense. Any pre-existing equipment may be salvaged at the start of the first period.

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No auxiliary equipment: Auxiliary equipment, such as wheel loaders and small trucks, are not considered in this model. Although the cost of running auxiliary equipment may differ according to the overall fleet selection, for the purpose of this model this cost is considered trivial. Note that the cost of auxiliary equipment can be built into the operating cost if necessary.

Known operating hours: The operating hours of the mine are estimated by taking planned downtime, blasting and weather delays into account.

Single truck cycle time: Since we assume a single mining location, a single truck cycle time is used for all trucks. The cycle time is constant over a time period and is known for all periods. It accounts for factors that affect the truck performance, such as rimpull, rolling resistance, haul distance and haul grade.

Heterogeneous fleets: We allow different types of equipment to work side-by-side.

Fleet retention: All equipment is retained at the end of the last period. Optionally, we can relax this assumption and allow the model to salvage some or all equipment at the end of the final period.

Full period utilisation: Operating costs are charged as though the equipment has been fully utilised for the entire period in which it is owned.

The presented integer program assumes that some pre-processing of the available equipment has already occurred, and that equipment not satisfying the mining method or mine pit requirements are not included in the initial set of available equipment. The length of a time period as used in the model can be adjusted to any desired magnification, but for the purpose of the case study, is set to one year.

4.2.2 Decision variables and notation

Let the set of all truck types be \mathbf{X} , and the set of all loader types be \mathbf{X}' . We use i and i' to denote a single truck and loader type, respectively. To simplify the notation, we use e to represent a single type (which can be a truck or loader type), where $e \in \mathbf{X} \cup \mathbf{X}'$.

Suppose that there are K periods, M available truck types and N available loader types with maximum age L . The following six variable types for the single location model generate a total of $2(MKL + NKL) + K(2^N - 1)$ variables before variable reduction takes place.

Fleet purchase and operating variables

For this formulation we wish to capture the number of equipment, the equipment type, the age of the equipment and the periods in which the equipment is operating. We

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adopt three indexes to represent type (e), period (k) and age (l). The index e is drawn from the set of all available equipment types, $\mathbf{X} \cup \mathbf{X}'$; the index k is drawn from the set of all periods, $\{0, 1, \dots, K\}$; and the index l is drawn from the age range of equipment type e (which can vary significantly amongst types). The decision variable for equipment type e is:

$\mathbf{x}_e^{k,l}$: number of owned equipment units of type $e \in \mathbf{X} \cup \mathbf{X}'$ that are age l at the start of period k .

All equipment in ownership operate, so a value of 0 asserts that the equipment with those indices is not owned. The fleet purchase and operating variables are *general* integers.

Remark 4.2.1 *The index l represents the equipment age in operated periods. As each equipment type may have a unique number of operating hours per period (due to varying availability amongst equipment types), the value that l represents may not be uniform in hours between equipment types.*

Salvage variables

We use salvage variables to tell when a truck or loader is salvaged, and the period that it occurs. These variables are defined similarly to the purchase and operating variables:

$\mathbf{s}_e^{k,l}$: number of owned equipment units of type $e \in \mathbf{X} \cup \mathbf{X}'$ that are age l salvaged at the start of period k .

The salvage variables are also *general* integers.

Indicator variables

The compatibility constraints have an *or* condition. This means that either one or the other constraint will have dominance, but not necessarily both. We use this to ensure that the compatible truck fleet can match the productivity requirements of the mine *or* the productivity capabilities of the loader, whichever is less [Section 4.2.4]. In order to set up these constraints, we require a new 0-1 variable that selects the constraint which should dominate:

$\mathbf{h}_{i'}^k$: 0-1 variable forcing one constraint in two to be active for loader type i' in period k .

All the indicator variables are *binary* integers.

4.2.3 Objective function

In the mining industry, the viability of a mine depends on the efficiency of the equipment and its ability to meet production requirements at the lowest possible cost. Thus the profitability of a mine is intrinsically linked to the cost of the operating equipment: both capital and ongoing. Therefore, we consider the objective function as the cost of materials handling. More specifically, we are interested in the *net present value* (NPV) of the cost of materials handling for the life of the mine. We wish to consider the capital expense, the operating expense (per period of ownership), and the salvage value of the equipment with respect to a discount rate I .

First we consider the capital expense, which is a one-off cost incurred during the period of purchase. We represent the fixed cost of purchasing equipment of type e by F_e , and discount this purchase to the present using a discount factor, D_1^k :

$$D_1^k = \frac{1}{(1 + I)^k} \quad (4.1)$$

where k is the current period (starting from 0).

Note that pre-existing equipment does not incur a capital expense as it was purchased in a period not considered in this optimisation horizon. Thus the *total capital expense* for a truck or loader of type e is

$$\sum_{e,k} F_e D_1^k \mathbf{x}_e^{k,0}. \quad (4.2)$$

The operating expense reflects the cost of operating and maintaining the equipment. It takes into account varying maintenance expenses, availability and productivity levels, which are known to vary with the age of the equipment. In the mining industry, the typically nonlinear operating cost is simplified by creating a piece-wise linear function that is divided into cost-brackets (Figure 4.2).

The operating cost bracket size, denoted by B_0 , defines the size of the equipment age increments which estimate the cost of running equipment in a piecewise linear fashion. We use increments of 5000 hours for our analysis.

Let H^k be the operating hours for period k and a_e^l is the availability of equipment type e , aged l . The cost bracket in which equipment lies at the beginning of a period is given by $b(l) \in \{0, 1, \dots, s\}$, where $s - 1$ is the total number of cost brackets. We calculate the cost-bracket each period using:

$$b(l) = \frac{\sum_{h \leq l} a_e^{k,b(l-1)} H^k}{B_0}, \quad (4.3)$$

where the availability of the equipment is dependent on the cost-bracket in which the

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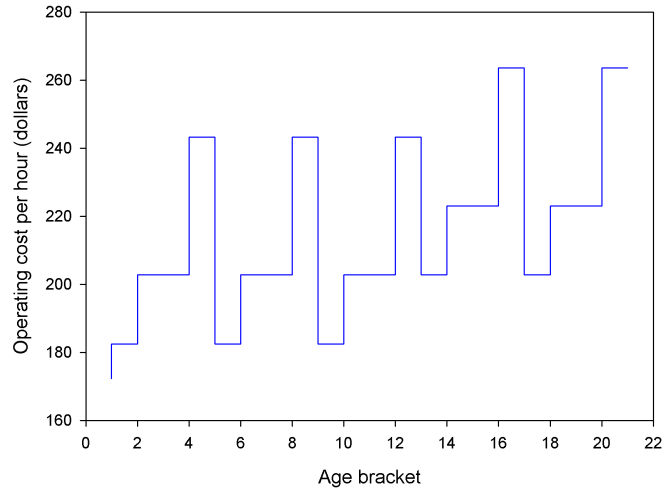


FIGURE 4.2: The variance of equipment operating cost against cost bracket. The rise in operating cost can reflect the increased maintenance expense or the time since the last overhaul. Large drops in the operating expense occur when an overhaul has taken place.

equipment lies. We assume that the operating cost is constant over the cost-bracket, so if a piece of equipment moves into a different age *cost bracket* within a period, the operating cost for the period should be appropriately split between these two brackets. We wish to determine the proportion of time that the equipment lies in this cost bracket, and the proportion of time that the equipment lies in the preceding cost bracket (Figure 4.3).

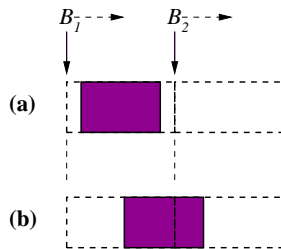


FIGURE 4.3: The two cases of equipment age landing between age brackets. For case (a), the equipment stays in the same age bracket for the entire period. In case (b), the equipment steps over into the next age bracket within the period.

We represent this proportion of time by the parameter $B_{h,e}^{k,l}$ for $h = 1, 2$, where h is the h th cost bracket that the equipment has landed within that one period. If we consider that the equipment could land in a maximum of two cost brackets within one

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period, then we have:

$$B_{1,e}^{k,l} = \begin{cases} 1 & \text{if } (b_e^{k,l} + 1)B_0 - \sum_k a_e^{k+P(e)} H^k > a_e^{k,l} H^k \\ \frac{(b_e^{k,l} + 1)B_0 - \sum_k a_e^{k+P(e)} H^k}{a_e^{k,l} H^k} & \text{otherwise} \end{cases}$$

and

$$B_{2,e}^{k,l} = 1 - B_{1,e}^{k,l}.$$

The variable cost, $V_e^{k,b(l)+h-1}$, is the cost per operated hour for equipment type $e \in \mathbf{X} \cup \mathbf{X}'$ in cost-bracket $b(l) + h - 1$ in period k . Hence, for the operating expense, we have the following expression:

$$\sum_{e,k,l,h} B_{h,e}^{k,l} D_1^k V_e^{k,b(l)+h-1} \mathbf{x}_e^{k,l}. \quad (4.4)$$

As we are minimising the cost of materials handling, we represent salvage by a negative expense. We apply a combined depreciation (at rate J per period) and NPV discount factor (at rate I per period), $D_2^{k,l}$, where:

$$D_2^{k,l} = \frac{(1 - J)^l}{(1 + I)^k}, \quad (4.5)$$

where l is the age of the equipment at the start of period k .

Remark 4.2.2 *The way that we have defined the decision variables “absorbs” the age of the equipment, and the period it is operating in, as an index. This prevents the NPV factor from becoming a function of the decision variables itself and subsequently deters nonlinearity in the objective function.*

Since F_e is the original capital expense, the salvage cost is:

$$- \sum_{e,k,l} F_e D_2^{k,l} \mathbf{s}_e^{k,l}. \quad (4.6)$$

Complete objective function

We wish to minimise the following objective function:

$$\text{minimise} \quad \sum_{e,k} F_e D_1^k \mathbf{x}_e^{k,0} + \sum_{e,k,l,h} B_{h,e}^{k,l} D_1^k V_e^{k,b(l)+h-1} \mathbf{x}_e^{k,l} - \sum_{e,k,l} F_e D_2^{k,l} \mathbf{s}_e^{k,l}.$$

The first term represents the cost of capital outlay for purchasing equipment; the second term captures the operating expense; and the third expression represents the

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salvage value of the equipment. In total there are $3(MKL + NKL) + MK + NK = O((M + N)KL)$ terms in the objective function (before variable reduction).

4.2.4 Constraints

Index restrictions

To reduce the total number of variables in our program, we restrict the age bracket of any piece of equipment to be no greater than the maximum age of the equipment type, which we denote as $L(e)$ for type e . In other words, we do not create a variable for equipment older than its maximum age. Furthermore, we restrict the age of any equipment that is not pre-existing to be no greater than the number of the current time period. This is because equipment cannot increase two age brackets in a single time period. However, we must also take into account the possibility of pre-existing equipment. If we define $P(e)$ to be the highest starting age of any pre-existing equipment of type e , then in time period k we only have to consider equipment up to age

$$L^k(e) = \min\{P(e) + k - 1, L(e)\}.$$

If there are no pre-existing equipment of type e , we set $P(e) = 0$. Whenever we sum

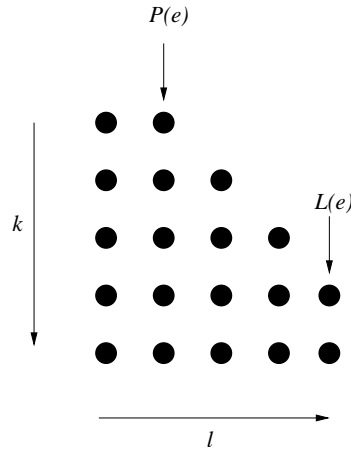


FIGURE 4.4: An illustration of the age indices considered for various time periods, with $P(e) = 2$, $L(e) = 5$. As the periods increase, the permitted age increases.

over l , we only need to sum up to $l = L^k(e)$ [Figure 4.4]. We note that as salvage occurs at the start of the period, salvage variables extend to $l = L^k(e) + 1$.

Productivity constraints

The simplest constraint in the model is the satisfaction of productivity requirements: the right quantities of materials must be handled to satisfy the mixing demands on

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the mill. To capture the productivity requirements in a constraint, we first consider the potential productivity of the loader, $P_{i'}^{k,l}$, when it is aged l in period k . We can determine this quantity by looking at the equipment availability ($a_{i'}^l$), capacity ($c_{i'}$) and cycle time ($t_{i'}^k$) where i' is a loader from \mathbf{X}' :

$$P_{i'}^{k,l} = \frac{a_{i'}^{b(l)} c_{i'}}{t_{i'}^k}. \quad (4.7)$$

For this formulation, availability is determined by the equipment's age. The age determines the age bracket under which the equipment falls. In turn, the age bracket determines an availability estimate, which represents the proportion of total time that the equipment is available to work. We have the following productivity constraints, where T^k is the productivity requirement for period k :

$$\sum_{i',l} P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l} \geq T^k \quad \forall k. \quad (4.8)$$

We similarly capture the productivity requirements for the trucks.

Compatibility constraints

We must ensure that the trucks and loaders used in a period are compatible with each other. However, we do not need to make all trucks compatible with all loaders; we must merely satisfy productivity requirements with the set of compatible trucks and loaders. To set this up as constraints, we first define the set $\mathbf{X}(i')$ to be the set of truck types which are compatible with loader type i' . Next we define a constraint that ensures that all equipment compatible with a particular loader type can satisfy the requirements:

$$\sum_{i \in \mathbf{X}(i'), l} P_i^{k,l} \mathbf{x}_i^{k,l} \geq \sum_l P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l} \quad \forall i' \in \mathbf{X}', k. \quad (4.9)$$

However, we must also consider the possibility that two loader types can be selected. Equation (4.9) only accounts for the case where all trucks are compatible with one loader, i' . We want to consider the loader type pairs case (i', h') and ensure that the compatible fleet of trucks can service both of these loaders together. We denote the union of the compatible truck fleets by the set $\mathbf{X}(i', h')$:

$$\sum_{i \in \mathbf{X}(i', h'), j, l} P_i^{k,l} \mathbf{x}_{i,j}^{k,l} \geq \sum_l (P_{i'}^{k,l} \mathbf{x}_{i',j}^{k,l} + P_{h'}^{k,l} \mathbf{x}_{h',j}^{k,l}) \quad \forall \{i', h'\} \subset \mathbf{X}', k. \quad (4.10)$$

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Similarly, we must allow the possibility of three types of loaders, (i', h', j') :

$$\sum_{i \in \mathbf{X}(i', h', j'), j, l} P_i^{k, l} \mathbf{x}_{i, j}^{k, l} \geq \sum_l (P_{i'}^{k, l} \mathbf{x}_{i', j}^{k, l} + P_{h'}^{k, l} \mathbf{x}_{h', j}^{k, l} + P_{j'}^{k, l} \mathbf{x}_{j', j}^{k, l}) \quad \forall \{i', h', j'\} \subset \mathbf{X}', k. \quad (4.11)$$

We let \mathbf{A} represent the set of all possible combinations of loaders, so that we can bring together constraints (4.9), (4.10) and (4.11) into a constraint set. Note that the assumption of full period utilisation may cause some problems with this constraint. In particular, we could be forcing the trucking fleet to exceed the productivity requirements of the mine - if the loaders exceed the productivity requirements we don't want to force the trucks to match them. We can rectify this by introducing an indicator variable, $\mathbf{h}_{A'}^k$, where $A' \subset \mathbf{X}'$, that will choose one of the following two constraints to dominate:

$$\sum_{i \in \mathbf{X}(A'), l} P_i^{k, l} \mathbf{x}_i^{k, l} \geq \sum_{i' \in A', l} P_{i'}^{k, l} \mathbf{x}_{i'}^{k, l} - M \mathbf{h}_{A'}^k \quad \forall A' \subset \mathbf{X}', k, \quad (4.12)$$

$$\sum_{i \in \mathbf{X}(A'), l} P_i^{k, l} \mathbf{x}_i^{k, l} \geq T^k \mathbf{h}_{A'}^k \quad \forall A' \subset \mathbf{X}', k. \quad (4.13)$$

Because we are taking the power set of \mathbf{X}' , this will generate $2K(2^N - 1)$ constraints (where K is the total number of periods and N is the number of loaders). For the 13 period, 27 loader case study this equates to 35 billion constraints. However, the number of loaders possible in the final solution will generally be much lower than the complete set. Thus we can limit the generation of constraints by allowing a maximum of α loader types. This will produce $2K \sum_{a=1}^{\alpha} \binom{n}{a}$ constraints. For the 13 period, 27 loader problem with a maximum of 4 loader types, this equates to just 542178 constraints.

It is not necessary to allow more types in the compatibility constraint than the number of different loader types in the optimal solution. Realistically, we expect that no more than three loader types will be selected when we have pre-existing loaders and no more than two loader types when we have no pre-existing loaders, although it is possible that more can be selected.

We further reduce the number of constraints in the model by entering this constraint set as *lazy constraints*. These constraints are only entered into the model if they are violated by the solution of the master model. This is a useful technique for eliminating a large set of inactive constraints from the model - potentially improving the computation time and solvability of the model.

Variable transition

Much of the linearisation in this model is due to the way that we have defined the decision variables: we capture the working period and age of each piece of equipment in respective indexes, k and l . It is important to establish the relationship between these two indexes. That is, as we increment through the periods, how does the age index, l , increase?

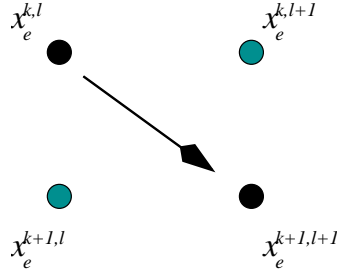


FIGURE 4.5: The relationship between the indexes k and l for the *variable transition* constraint in the single-location model.

In this model one period correlates with one period of equipment age [Figure 4.5]. Individual equipment types may age a different number of hours per period depending on the number of available hours. This level of detail is not lost in this formulation, but is abstracted to an integer age (relative to periods of use), l . We express this relationship for all equipment types in constraint (4.14):

$$\mathbf{x}_e^{k,l} = \mathbf{x}_e^{k-1,l-1} - \mathbf{s}_e^{k,l} \quad \forall \quad k > 0, l \in [1, L^k(e)], e. \tag{4.14}$$

That is, the equipment that we own in period k , $\mathbf{x}_e^{k,l}$, must be equal to the equipment that we owned in the previous period less the equipment that we salvaged at the start of this period. Note that this constraint set is only valid if the relationship between $l - 1$ and l is uniformly linear. In Chapter 6 we define a non-uniform relationship that leads to completely different constraints.

Forced salvage

The maximum age of each type of equipment can vary substantially, ranging from as low as 25,000 hours to 100,000 hours. When a piece of equipment reaches its maximum age, we want to force it to retire at the beginning of the following period. This is a particularly important consideration when including pre-existing equipment which may already be close to retirement age at the beginning of the schedule, or for when we wish to consider long mining schedules that breach the lifespan of the equipment.

$$\mathbf{x}_e^{k,l} = \mathbf{s}_e^{k+1,l+1} \quad \forall \quad l > L^k(e), e, k. \tag{4.15}$$

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However, during variable creation we can prevent over-age variables from existing in the first place, thus effecting forced salvage.

Salvage Restriction

In this formulation we aim to minimise the cost. However, we provide a means of creating profit: salvage of trucks and loaders. These variables must be carefully constrained to prevent the salvage of equipment that is not owned:

$$\mathbf{x}_e^{k-1,l-1} \geq \mathbf{s}_e^{k,l} \quad \forall \quad k > 0, l \in [1, L^k(e) + 1], e. \quad (4.16)$$

Also, we prevent the unbounded salvage variables from dominating:

$$\mathbf{s}_e^{k,0} = 0 \quad \forall \quad e, k. \quad (4.17)$$

Pre-existing equipment

A novelty of this model is the ability to include pre-existing equipment in the optimisation process. We achieve this by setting the truck or loader decision variable of the appropriate age to the number of pre-existing equipment of type e that are age $P(e)$, which we call $x_e^{P(e)}$.

$$\mathbf{x}_e^{0,P(e)} + \mathbf{s}_e^{0,P(e)} = x_e^{P(e)} \quad \forall \quad e \in \mathbf{P}. \quad (4.18)$$

Through this constraint we also permit the model to salvage the equipment immediately, which is the sole reason for forcing salvage to occur at the beginning of each period.

Pre-existing equipment that has breached the maximum age, $L(e)$, for its equipment type, e , must be salvaged immediately:

$$\mathbf{s}_e^{0,P(e)} = x_e^{P(e)} \quad \forall \quad e \in \mathbf{P}, P(e) > L^k(e). \quad (4.19)$$

4.2.5 Summary of notation

\mathbf{X}	set of all available truck types.
\mathbf{X}'	set of all available loader types.
i	truck type index, $i \in \mathbf{X}$.
i'	loader type index, $i' \in \mathbf{X}'$.
e	truck or loader type index, where $e \in \mathbf{X} \cup \mathbf{X}'$.
k	period index.
l	equipment age index.
b	current age bracket.
M	a large integer.
\mathbf{P}	set of all pre-existing equipment types.
\mathbf{A}	power set of all combinations of loader types, $\mathbf{A} = \mathcal{P}(\mathbf{X}')$.
$\mathbf{X}(A')$	set of all truck types compatible with loader set $A' \subset \mathbf{X}'$.
$\mathbf{X}(i')$	set of all truck types compatible with loader $i' \in \mathbf{X}'$, where $\mathbf{X}(i') \subseteq \mathbf{X}$.
K	total number of periods in the mining schedule.
$L(e)$	maximum age (in operating periods) for each truck and loader type, e .
$L^k(e)$	maximum age we need to consider in period k , $L^k(e) = \min\{P(e) + k - 1, L(e)\}$.
$\mathbf{x}_e^{k,l}$	number of trucks or loaders of type e that are selected in period k , aged l .
$\mathbf{s}_e^{k,l}$	number of trucks or loaders of type e that are salvaged at the start of period k , aged l .
$\mathbf{h}_{A'}^k$	indicator variable that decides dominance of one of two constraints for loader set $A' \subset \mathbf{X}'$ in period k .
$P(e)$	age of pre-existing equipment type e .
$x_e^{P(e)}$	number of pre-existing trucks or loaders of type e aged $P(e)$ at the start of the schedule.
F_e	fixed cost (capital expense) of obtaining equipment type e .
D_1^k	discount factor in period k .
$D_2^{k,l}$	discount factor and depreciation factor in period k when the equipment is aged l .
B_0	size of the cost brackets.
$B_{h,e}^{k,l}$	proportion of time equipment type e aged l spends in cost bracket $h \in \{1, 2\}$.
$V_e^{k,b(l)+h-1}$	variable expense for equipment type e in cost-bracket $b(l) + h - 1$ in period k .
$a_e^{b(l)}$	availability of equipment e in cost-bracket $b(l)$.
c_e	capacity of equipment e .
t_e^k	cycle time of equipment e in period k .
$P_e^{k,l}$	productivity of equipment e in period k at age l .
H^k	operating hours of the mine for period k .
T^k	required production of the mine (in tonnes) for period k .

4.2.6 Complete model

$$\text{Minimise} \quad \sum_{e,k} F_e D_1^k \mathbf{x}_e^{k,0} + \sum_{e,k,l,h} B_{h,e}^{k,l} D_1^k V_e^{k,b+h-1} \mathbf{x}_e^{k,l} - \sum_{e,k,l} F_e D_2^{k,l} \mathbf{s}_e^{k,l}$$

subject to

$$\sum_{i',l} P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l} \geq T^k \quad \forall k \quad (4.20)$$

$$\sum_{i,l} P_i^{k,l} \mathbf{x}_i^{k,l} \geq T^k \quad \forall k \quad (4.21)$$

$$\sum_{i \in \mathbf{X}(A'),l} P_i^{k,l} \mathbf{x}_i^{k,l} \geq \sum_{i' \in A',l} P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l} - M \mathbf{h}_{A'}^k \quad \forall A' \subset \mathbf{X}', k \quad (4.22)$$

$$\sum_{i \in \mathbf{X}(A'),l} P_i^{k,l} \mathbf{x}_i^{k,l} \geq T^k \mathbf{h}_{A'}^k \quad \forall A' \subset \mathbf{X}', k \quad (4.23)$$

$$\mathbf{x}_e^{k,l} = \mathbf{x}_e^{k-1,l-1} - \mathbf{s}_e^{k,l} \quad \forall k > 0, l \in [1, L^k(e)], e \quad (4.24)$$

$$\mathbf{x}_e^{k,l} = \mathbf{s}_e^{k+1,l+1} \quad \forall l > L(e), e, k \quad (4.25)$$

$$\mathbf{x}_e^{k-1,l-1} \geq \mathbf{s}_e^{k,l} \quad \forall k > 0, l \in [1, L^k(e) + 1], e \quad (4.26)$$

$$\mathbf{s}_e^{k,0} = 0 \quad \forall e, k \quad (4.27)$$

$$\mathbf{x}_e^{0,P(e)} + \mathbf{s}_e^{0,P(e)} = x_e^{P(e)} \quad \forall e \in \mathbf{P} \quad (4.28)$$

$$\mathbf{s}_e^{0,l} = x_e^{P(e)} \quad \forall e \in \mathbf{P}, P(e) > L(e) \quad (4.29)$$

$$\mathbf{x}, \mathbf{s} \in \mathbb{Z}^+$$

4.3 Implementation and validity

4.3.1 Reducing the total number of variables

The formulation of the variables for this model generates a lot of zero variables. For example, for the 40% depreciation problem in Section 4.4, 11006 variables are generated during implementation. Of these, only 146 are non-zero in the final solution. For example, consider the purchase variable $\mathbf{x}_e^{k,l}$. It is not necessary to create variables for the case where $l > k$ unless we have pre-existing equipment of age l .

With three indexes each, the truck and loader variables (both purchase and salvage) can be defined as three dimensional variable matrices (Figure 4.6). Consider the front face of the matrix to represent the increase of l periods of age (x -axis) with time period, k (y -axis). We know from constraint (4.14) that $l \leq k$, except where pre-existing equipment occurs. Thus we can exclude all variables where $l > k$ while noting and accounting for pre-existing equipment in the total set of variables. The resulting three dimensional matrix is shown in Figure 4.7 for $e \in \mathbf{P}$.

The total number of subsequent variables generated is almost half of the number

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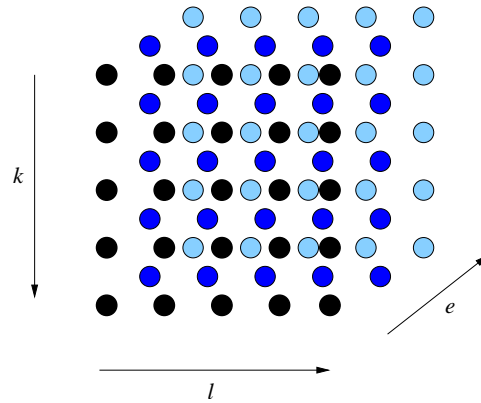


FIGURE 4.6: An illustrative three dimensional sketch of a variable matrix. Each node represents a variable with respect to time (k), age (l) and type (e).

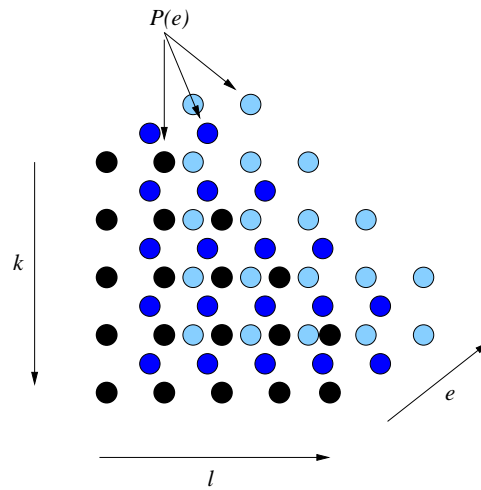


FIGURE 4.7: An illustrative three dimensional sketch of a variable matrix with redundant variables removed.

resulting if this approach had not been implemented.

4.3.2 Cplex and constraint order

We analysed the model in this chapter using *Ilog Cplex v11.0* libraries with *Ilog Concert Technology v2.5* objects. From observations, the best results are obtained from Cplex when the following are satisfied:

1. Each constraint is fully iterated before the next is considered.
2. The variables are entered into the constraints in the same order that they appear in the objective function.
3. Dominant coefficients for the variables are positioned before smaller coefficients of the same variable in the objective function

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Cplex creates a variable when it is entered into the objective function. Variables not appearing in the objective function are created afterward. Note that the order in which the variables appear in the objective function is dependent on the dominance of the coefficients of these variables as they are entered. That is, the largest coefficient entry for a particular variable acts as a magnet for all other entries of that same variable, and it is this entry position that will be recorded for all instances of that variable. For example, if we entered the string $2x_1 + x_2 + 40x_1$, then the order of the variables will be x_2, x_1 , because the dominant coefficient overrides the order of entry, and the first variable x_1 will be collected by the final x_1 entry with the dominant coefficient.

The order that the variables are entered into a constraint should match the order that they are entered into the objective function. Note that Cplex reduces all repeated variables, and the final position of the reduced variable in the objective function is related to the dominance of the coefficients. That is, the position with the greatest coefficient is the dominant and final position. This, coupled with the overall ordering of variables, has a significant impact on solution speed using the Cplex Optimizer. This phenomena is not unique to Cplex: the effect of rearranging variables or constraints has been noted in the past (Taha 1975).

4.4 Model testing

The parameters in this model are subject to uncertainty in two ways. Some parameters are known to be wild estimates, such as the depreciation of equipment; other parameters are certain to change once the schedule begins and new information comes to light, such as truck cycle time and productivity requirements. In sensitivity analysis, we are interested in the influence of these uncertain parameters on the model. Furthermore, we study the sensitivity of the model to pre-existing equipment, which will often introduce incompatibilities into the optimal solution - especially when the pre-existing equipment has been superseded or discontinued.

To do this, we created a 10-period test case with uniform productivity requirements (35 million tonnes) and uniform truck cycle times (30.75 minutes). In order to capture the replacement cycle of the equipment, we reduced the maximum age of the trucks to 35,000 hours. We did not include any pre-existing equipment in the test cases. Other important parameters and inputs are presented in Tables 4.1 - 4.3. Table 4.1 displays the maximum age for both the trucks and loaders available for selection by the model. This is the age at which the equipment must be salvaged if it is still in operation. Table 4.2 displays the compatibility matrix for the trucks and loaders, where a 0 denotes that the truck type is incompatible with the loader type, and a 1 indicates compatibility. Table 4.3 displays the availability multiplier which is applied to the equipment productivity. Similar multipliers are used to represent the cost of

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maintenance and the reduction in efficiency in the equipment.

We programmed the model in *C++* using *Ilog Concert Technology v2.5* objects, and solved the program with default IP algorithms in *Ilog Cplex v11.0*. We performed these tests on a Pentium 4 PC with 3.0GHz CPU and 2.5GB of RAM.

Depreciation

It is difficult to choose a sound depreciation value, as the prediction of the salvage value of equipment is highly dependent on variables outside the control of the mining company, such as the existence of an interested buyer. Therefore, we would like to vary the depreciation parameter and observe the change in the model. We expect to see a shift in the computation time that coincides with slight changes in the solution - computation time will increase when there are alternative solutions that are close to the optimal solution in terms of objective function value. We analysed the effect of depreciation on the 10-period test case under two conditions: we kept the adjusted maximum age of the trucks (reduced to 35000 each), and then we returned to the actual maximum age values. The difference between these two experiments is significant.

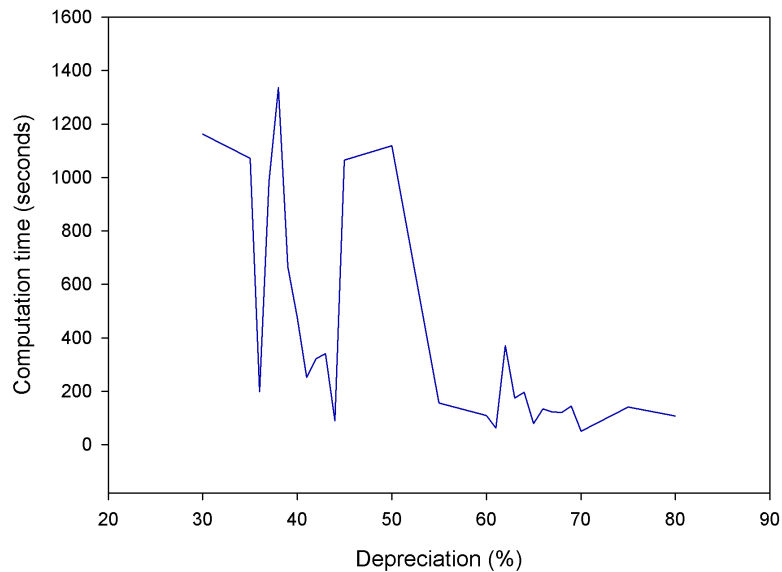


FIGURE 4.8: The effect of depreciation on computation time with adjusted maximum age for trucks.

Figure 4.8 depicts the change in computation time with varying depreciation when the maximum truck age is reduced to 35000 hours. We iteratively increased the depreciation in increments of 5%, but increased this resolution to 1% at points of interest. The times are unstable, although some asymptotic behaviour is notable as we head

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Truck type (i)	Capacity (tonnes)	Adj. Max. hours	$L(i)$	Loader type (i')	Capacity (tonnes)	$L(i')$
0	305	35000	75000	<i>Electric rope shovel</i>	0	100000
1	35	35000	25000		1	100000
2	50	35000	25000		2	100000
3	136	35000	25000		3	100000
4	92	35000	40000		4	100000
5	92	35000	40000		5	100000
6	136	35000	50000		6	100000
7	136	35000	50000		7	75000
8	177	35000	40000	<i>Hydraulic shovel</i>	8	35000
9	230	35000	45000		9	50000
10	136	35000	40000		10	50000
11	172	35000	40000		11	50000
12	172	35000	75000		12	50000
13	185	35000	65000		13	75000
14	218	35000	75000		14	50000
15	150	35000	50000		15	75000
16	150	35000	50000		16	75000
17	90	35000	50000		17	60000
18	90	35000	50000	<i>Hydraulic backhoe</i>	18	60000
19	218	35000	45000		19	60000
20	90	35000	40000		20	60000
21	90	35000	40000	<i>Front end loader</i>	21	35000
22	150	35000	40000		22	35000
23	150	35000	40000		23	40000
					24	25000
					25	25000
					26	40000
					27	45000

TABLE 4.1: Properties of trucks and loaders.

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Truck type	Loader type																											
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	0	0	1	1	1	0	1	0	1	0	1	0	1	1	0	1	1	1	1	0	1	0
2	0	1	0	0	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	1	0	0	1	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	0	0	1	0	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	0
5	0	1	1	0	1	1	0	1	1	1	1	1	1	0	1	0	1	1	1	0	1	1	1	1	1	1	0	0
6	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
7	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
8	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
11	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	0	1	1	1	0	0	1	1	1	1	1	0	0
21	0	1	1	0	1	0	0	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	1	1	1	1	0	0
22	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1

TABLE 4.2: Compatibility of trucks with loaders.

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		<i>Cost-bracket</i>																			
		1	2	3	4	5	6	7	8	9	10										
<i>Electric rope shovel</i>		0.9	0.9	0.9	0.9	0.89	0.88	0.87	0.86	0.85	0.84										
<i>Hydraulic shovel</i>		0.9	0.9	0.89	0.88	0.87	0.86	0.85	0.84	0.82	0.8										
<i>Front end loader</i>		0.9	0.88	0.86	0.84	0.82	0.85	0.83	0.81	0.79	0.8										
<i>Trucks</i>		0.92	0.92	0.91	0.91	0.9	0.9	0.89	0.89	0.885	0.881										
		11	12	13	14	15	16	17	18	19	20	21									
<i>Electric rope shovel</i>		0.83	0.82	0.81	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8									
<i>Hydraulic shovel</i>		0.78	0.76	0.74	0.72	0.7	0.7	0.7	0.7	0.7	0.7	0.7									
<i>Front end loader</i>		0.78	0.76	0.74	0.72	0.7	0.68	0.66	0.64	0.62	0.6	0.58									
<i>Trucks</i>		0.877	0.873	0.869	0.865	0.861	0.857	0.853	0.849	0.845	0.841	0.837									

TABLE 4.3: The availability of trucks and loaders for each 5000 hour age bracket.

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toward a depreciation of 0 and a depreciation of 100%.

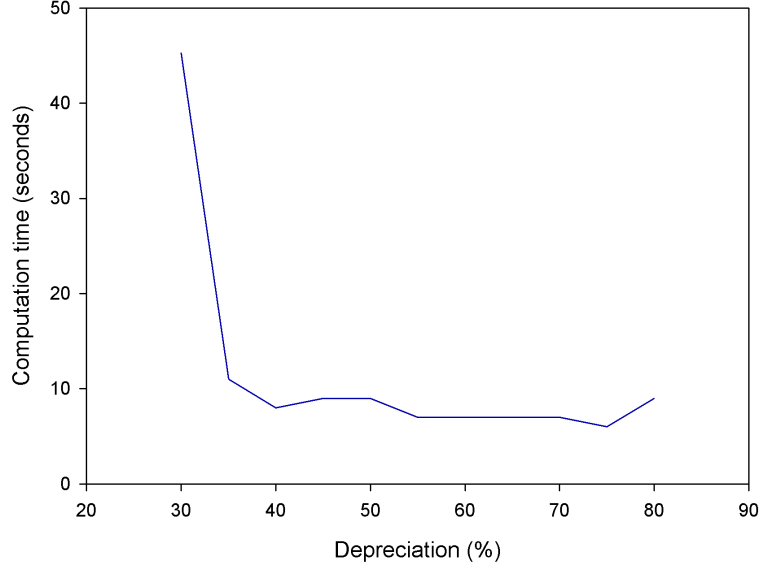


FIGURE 4.9: The effect of depreciation on computation time with actual maximum age for trucks.

When we return the truck maximum age to their actual values, a different story is seen (Figure 4.9): notably a stable behaviour. We explain this difference by observing the variation in actual maximum truck age - the actual values of this parameter distinguishes between trucks, as they vary from as low as 25,000 to 75,000. By setting the maximum age for all of the trucks to be the same value, we are removing a distinguishing factor.

Number of periods

The number of variables increases linearly with the number of periods, while the number of constraints increases quadratically [Figure 4.10]. Therefore we expect the computation time to increase nonlinearly as the number of periods increases (Table 4.4). Other factors, such as the maximum age of the equipment, can also contribute to large leaps in computation time.

Big- M value

The size of M is important for the first of the compatibility constraints:

$$\sum_{i \in \mathbf{X}(A'), l} P_i^{k,l} \mathbf{x}_i^{k,l} \geq \sum_{i' \in A', l} P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l} - M \mathbf{h}_{A'}^k \quad \forall A' \subset \mathbf{X}', k. \quad (4.30)$$

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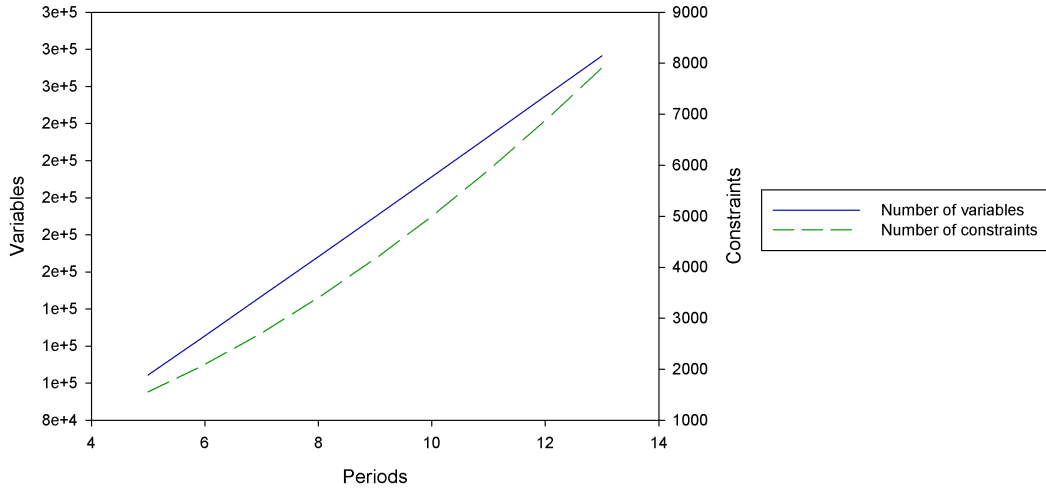


FIGURE 4.10: The increase of variables and constraints with the number of periods in the single-location model.

Obviously we need to set M large enough that it will dominate the $\sum_{i' \in A', l} P_{i'}^{k,l} \mathbf{x}_{i'}^{k,l}$ term when $\mathbf{h}_{A'}^k$ is 1. We are concerned, however, that setting it higher than necessary may add to computation time. To ensure that this does not happen, we tested values of M starting from 10000 and increasing by a factor of 10 to 10^{10} . This range encompasses values which are too small (10000 - 1000000) to be effective, to numbers that are too large for Cplex to accept (10^{10}). We did not expect the objective function value to differ in this experiment, and it did not (although it is possible it could become unstable in the nonsensical range). However the interesting result is that the computation time is not at all affected by variance in the value of the Big- M .

<i>Period</i>	<i>Variables</i>	<i>Constraints</i>	<i>Solution time (seconds)</i>
5	104420	1554	8
6	125608	2094	10
7	146896	2709	18
8	168284	3402	17
9	189766	4165	26
10	211342	4998	76
11	233012	5904	1723
12	254770	6876	6197
13	276612	7910	9092

TABLE 4.4: The number of variables and constraints for different numbers of periods for the test case.

Pre-existing equipment

An important aspect of this model is the ability to include pre-existing equipment. We test that this works by randomly selecting equipment types to be included as pre-existing equipment. Using the random number generator in Maple version 9.5, we selected three loader types from 0 to 27 with age selected randomly from 1 to 100,000 hours; and five truck types from 0 to 24 with age randomly selected from 1 to 60,000. The result is presented in Table 4.5. Note that we denote a truck of type i by T_i , and a loader of type i' by $L_{i'}$.

Equipment id.	L_{17}	L_8	L_1	T_1	T_1	T_7	T_3	T_9
Quantity	1	1	1	1	1	1	1	1
Capacity (tonnes)	42	19	25	35	35	136	136	230
Age (hours)	30539	22086	21769	17917	19462	35308	7350	10087

TABLE 4.5: The randomly selected pre-existing equipment for the 10-period test case.

We ran this trial varying the depreciation parameter by increments of 10 between 30% and 70%, using the actual maximum truck age [Table 4.6]. The time to obtain the optimal solution decreases as the depreciation parameter increases, suggesting the that solution becomes easier to find when the depreciation parameter is large. Interestingly there appears to be no marked difference between the 50% to 70% range. It is difficult to generalise the cause of this result from one experiment.

Depreciation (%)	Time (seconds)	Quality	Solution
30	139.953	Optimal	1.14664×10^8
40	98.203	Optimal	1.14849×10^8
50	35.375	Optimal	1.14901×10^8
60	31.641	Optimal	1.14914×10^8
70	33.500	Optimal	1.14917×10^8

TABLE 4.6: The 10-period test case with pre-existing equipment, varying depreciation and actual maximum age for all equipment.

We study the solution when the depreciation parameter is set to 60% [Figure 4.11] and compare it to the solution with no pre-existing equipment [Figure 4.12]. In the solution with pre-existing equipment, we immediately salvage two of the pre-existing loaders and purchase a new loader of type 4. This is because none of the pre-existing loaders are compatible with pre-existing truck type 1, so the solution includes the purchase of a new loader that is compatible. The cheap type 15 medium capacity trucks are purchased for the bulk of the fleet work as in the solution with no pre-existing equipment. A lone truck of type 6 is purchased in period 6. This truck type

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has a smaller capacity than the favoured type 15, and would be selected to fulfill a small variation in productivity requirements.

Certainly the solution with no pre-existing equipment appears to be simpler than the solution with pre-existing equipment. This may have been exacerbated by the randomness of the pre-existing equipment - the pre-existing equipment is unlikely to be the same as the optimal equipment purchases. The pre-existing equipment also leads to greater heterogeneity in the fleet, and more purchase/salvage activity through-out the schedule. The former result is clearly due to the presence of mixed types from the beginning. The latter behaviour can be explained by the presence of old equipment from the start of the schedule - this equipment may be forced to retire earlier in the schedule than a new piece of equipment would.

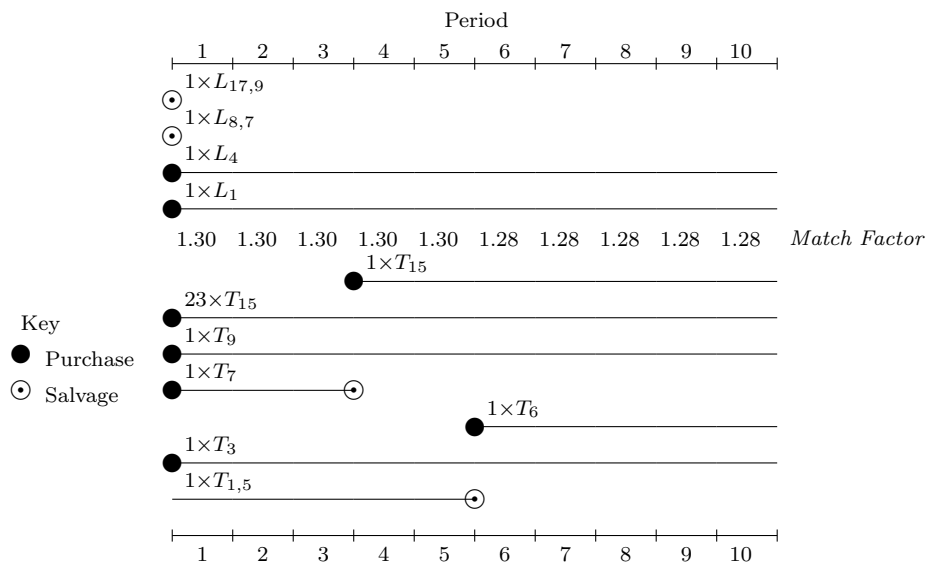


FIGURE 4.11: The 10-period test case with pre-existing equipment when the depreciation parameter is set to 60%.

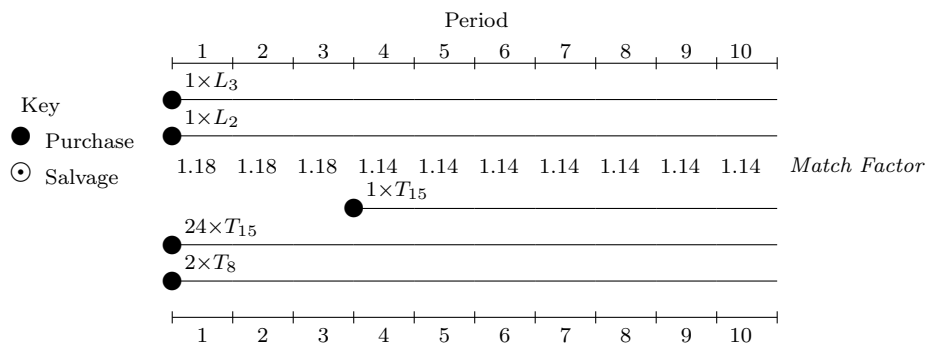


FIGURE 4.12: The 10-period test case with no pre-existing equipment when the depreciation parameter is set to 60%.

These solutions demonstrate that the model accounts for pre-existing equipment

and its corresponding age, and checks for compatibility of the selected fleets.

Productivity requirements and truck cycle time

We can expect that the variation of the productivity requirements over the schedule will affect computation time and the solution, as the changes in requirements coincides with the declining ability of the equipment to meet requirements. A similar situation occurs with truck cycle time, which also has an effect on the fleet’s ability to meet requirements. We begin this analysis with a uniform truck cycle time. In effect, the truck cycle time is constant, and so the purchase and salvage behaviour can be attributed to shifts in the productivity requirements or the reduced efficiency of the equipment.

We now study the following three shapes of the productivity requirements: uniform, linearly decreasing and parabolic. A flat productivity requirement is interesting because we expect to be able to see the effect of decreased equipment efficiency over time. This is the effect of the availability of the equipment.

We study linearly decreasing productivity requirements because a naive expected behaviour of the production schedule is to extract a lot of material early in the schedule; the quantity of material extracted slowly decreases as the pit largens and the trucks must travel further to reach it [Figure 4.4]. Although this is not really what happens, we study it out of interest.

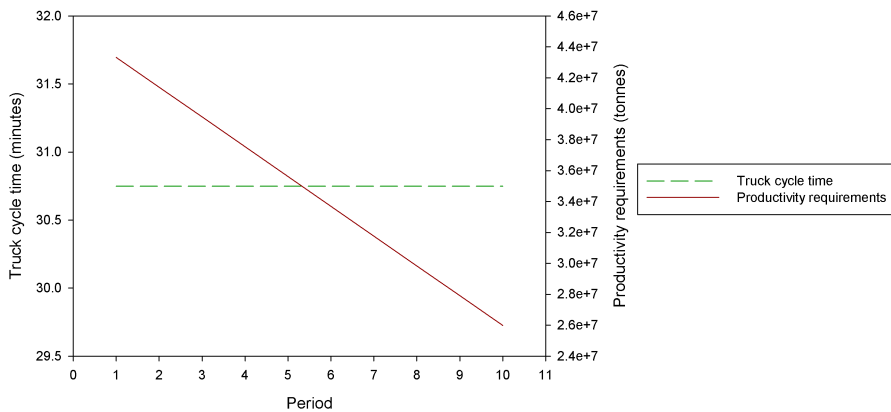


FIGURE 4.13: The linear productivity requirements used for the 10-period test case.

A more realistic shape to consider is a parabolic productivity schedule [Figure 4.14]. We study this last of all.

As a final experiment, we flatten the productivity requirements and set the truck cycle times to be increasing linearly [Figure 4.15]. This is a realistic expectation of the truck cycle time, which will increase as the pit deepens and widens.

We consider each of these test cases with varying depreciation parameter (in in-

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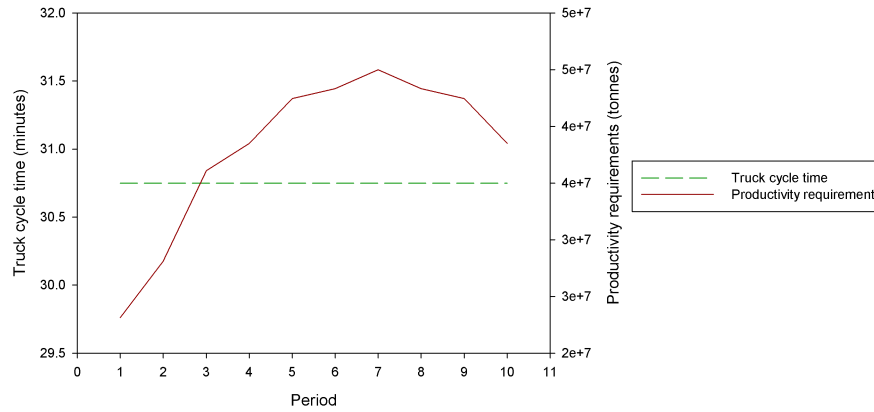


FIGURE 4.14: The parabolic productivity requirements used for the 10-period test case.

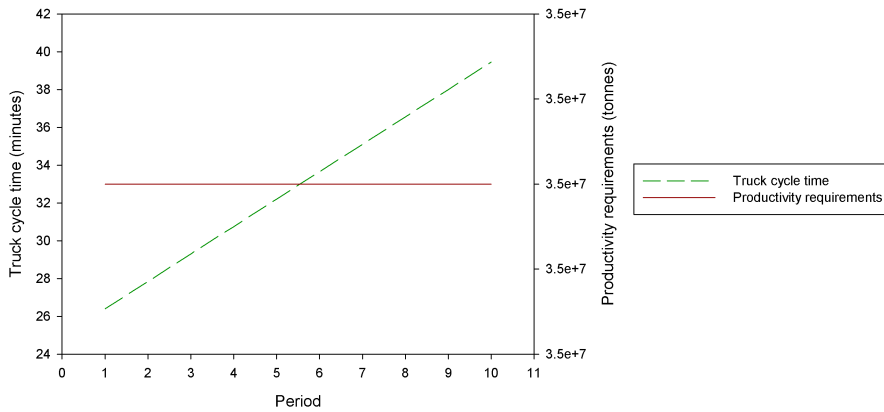


FIGURE 4.15: The linear truck cycle times used for the 10-period test case.

crements of 10%), from 30% to 70%. Any depreciation outside of this range can be considered unrealistic for this type of analysis. The results are presented in Table 4.7.

It is interesting to note that the model solved slowest when both the productivity requirements and truck cycle times were flattened; the best results were obtained when the productivity requirements were increasing linear while the truck cycle times were flattened. To understand this difference more fully, we look at the solutions.

Figure 4.16 shows the solution for the flat productivity requirements and flat truck cycle time for 50% depreciation. This is a very ‘clean’ solution, employing only two loader types to neatly fill productivity requirements, and two truck types to work alongside the loaders. All equipment is purchased in the first period, with the exception of one truck that is added in period-4 to offset the decreasing availability of the trucks.

We compare this solution with that obtained with linear productivity requirements and flat truck cycle time [Figure 4.17]. In contrast, this solution looks very ‘messy’

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<i>Production</i>	<i>Cycle Time</i>	<i>Depreciation</i>	<i>Quality</i>	<i>Time (s)</i>
flat	flat	50%	Optimal	1119
linearly decreasing	flat	50%	Optimal	8
parabola	flat	50%	Optimal	37
flat	flat	30%	Optimal	1162
flat	flat	40%	Optimal	476
flat	flat	50%	Optimal	1119
flat	flat	60%	Optimal	109
flat	flat	70%	Optimal	51
linearly decreasing	flat	30%	Optimal	10
linearly decreasing	flat	40%	Optimal	6
linearly decreasing	flat	50%	Optimal	8
linearly decreasing	flat	60%	Optimal	6
linearly decreasing	flat	70%	Optimal	4
parabola	flat	30%	Optimal	181
parabola	flat	40%	Optimal	17
parabola	flat	50%	Optimal	37
parabola	flat	60%	Optimal	10
parabola	flat	70%	Optimal	9
flat	linearly increasing	30%	Optimal	3573
flat	linearly increasing	40%	Optimal	321
flat	linearly increasing	50%	Optimal	40
flat	linearly increasing	60%	Optimal	20
flat	linearly increasing	70%	Optimal	26

TABLE 4.7: The results summary for the 10-period test cases with varying production requirement and flat truck cycle times.

with at least one salvage occurring in every period. Furthermore, this solution selects two different loaders: one 80-tonne capacity loader and one 48-tonne capacity loader. We have different loader types because of the starting productivity requirements, which suggests that the ease with which the latter solution was obtained is related to the limited number of loader combinations that could satisfy the productivity requirements. We tested this idea by increasing the flattened productivity requirements to the same starting level as the linearly decreasing productivity requirements. We were disappointed to find that the 50% depreciation problem with flat productivity requirements (40 million tonnes) took a sluggish 399 seconds to solve compared to the solutions with different productivity requirement shape.

With so many parameters in the model, it is difficult to determine the exact cause of the results in Table 4.7. However, it is clear that more complex productivity requirements are easier to solve than simple or constant requirements. Furthermore, it

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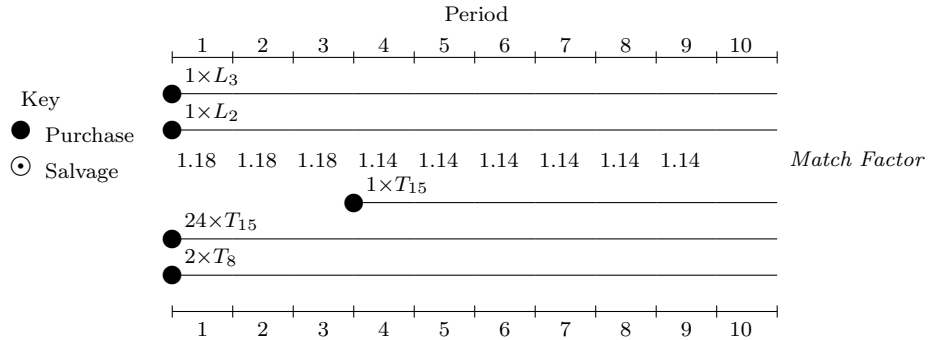


FIGURE 4.16: The solution for the 10-period test case with flat productivity requirements and flat truck cycle time for 50% depreciation.

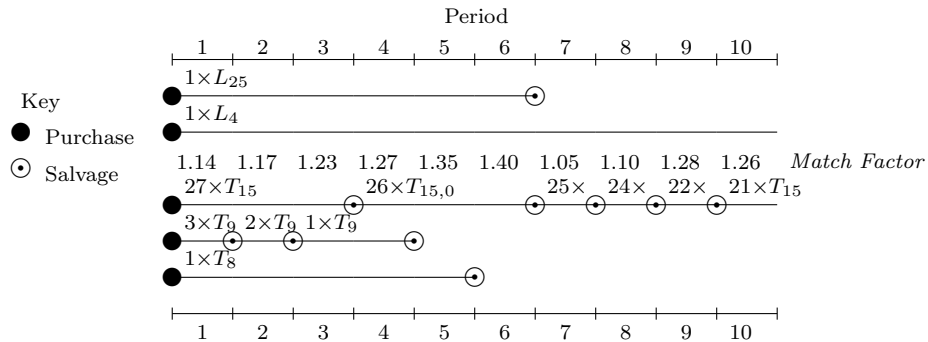


FIGURE 4.17: The solution for the 10-period test case with linear productivity requirements and flat truck cycle time for 50% depreciation.

appears that increasing the depreciation parameter highlights the financial differences between decisions, thus making the model easier to solve and improving computation time.

4.5 Case study

Our industry partner provided data for a complete case study, which included a list of available equipment, mine data, equipment performance data, compatibility data and pre-existing equipment lists. We started out with 24 truck types and 27 loader types, but whittled them down to 8 truck and 20 loader types once equipment unsuited to the mine was removed. The production requirements and truck cycle times are provided in Table 4.9, while the truck cycle times and productivity requirements are shown in Figure 4.18. No lease equipment was included in the set. Cost data and equipment identities were provided, but cannot be presented here for confidentiality reasons. The unit for production is one million tonnes while the unit for truck cycle time is minutes.

We begin the mining schedule with some pre-existing equipment [Table 4.8]. That is, eleven 172 tonne trucks of varying age in hours; and three loaders, namely two 34

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tonne hydraulic shovels and one 42 tonne hydraulic shovel. The age of this equipment is presented in Table 4.8.

Equipment id.	L_7^P	L_7^P	L_{17}^P	T_{12}^P	T_{12}^P	T_{12}^P
Quantity	1	1	1	3	2	6
Capacity (tonnes)	34	34	42	172	172	172
Age (years)	16	17	16	7	8	11

TABLE 4.8: Pre-existing equipment for the 13-period case study.

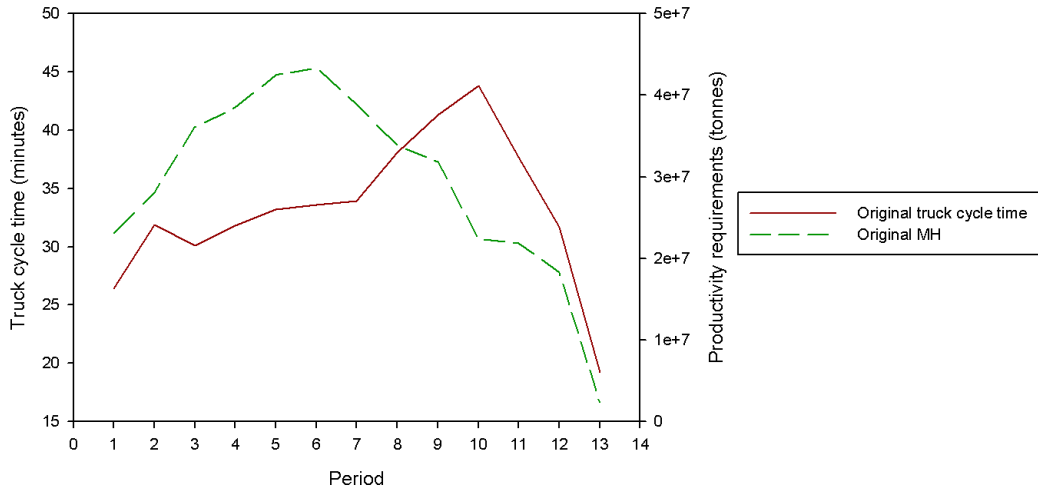


FIGURE 4.18: The production requirements and truck cycle times for the 13-period case study.

In this case study, we make the following assumptions:

- The mine is removing ore and waste, and operates under a loader-truck system.
- The operating hours of the mine is 7604 hours for each period.
- The loaders are selected from a set of 20 loader types.
- The trucks are selected from a set of 8 truck types.
- There are $K = 13$ periods in total, each of length 1 year.

We have the following parameters:

- The cost-bracket length, B_0 , is 5000 hours.
- The interest rate for all periods is 8%.
- The depreciation rate varies from 40% to 70%.

	<i>Period</i>												
	1	2	3	4	5	6	7	8	9	10	11	12	13
<i>Production (million tonnes)</i>	23	28	36	38	42	43	39	34	32	22	22	18	2
<i>Truck cycle time (minutes)</i>	26.4	31.9	30.1	31.8	33.2	33.6	33.9	38.1	41.3	43.8	37.7	31.7	19.2

TABLE 4.9: Production requirements and truck cycle times for the 13-period case study.

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We implemented the 13-period case study with 276612 variables but just 7910 constraints [Table 4.10].

Depreciation (%)	Time (s)	Objective function (\$)
40	3255	1.25806×10^8
50	9873	1.26075×10^8
60	3336	1.26212×10^8

TABLE 4.10: Summary of results for the 13-period case study with varying depreciation.

Convergence for this problem was swift, reaching within 1% of optimality in just 15 seconds. Figure 4.19 depicts the rate at which the *best integer* solution converges with the *best node* solution. The *best integer* solution is the best found feasible integer solution and acts as an upper bound on the optimal objective function value. The *best node* solution is the best found continuous solution which is found by relaxing the integer assumption and solving the equivalent linear program. This is a lower bound on the optimal solution. The optimal solution of $\$1.25806 \times 10^8$ was obtained after 3255 seconds (2.5 hours) for the 40% depreciation problem [Figure 4.20].

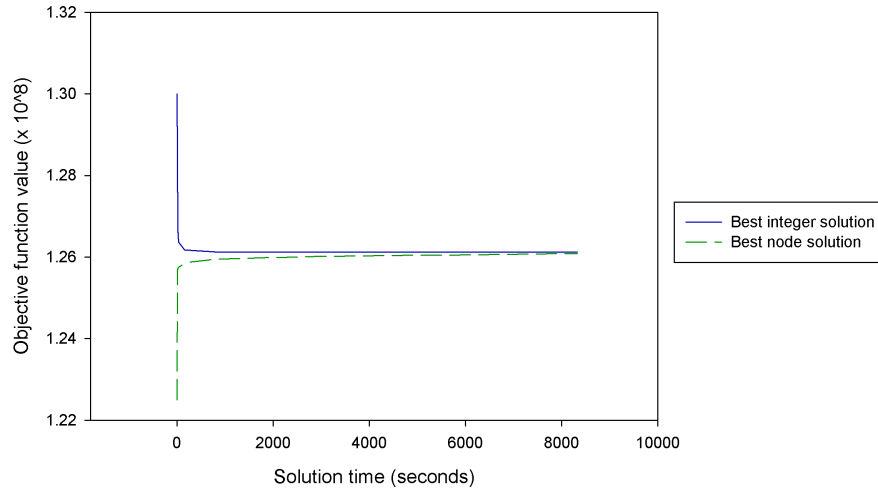


FIGURE 4.19: Convergence of the 13-period case study for the single-location model.

In our solution, two 60-tonne loaders were selected over the course of the schedule. Five different types of trucks were selected to work with these loaders: a 136-tonne truck, three 177-tonne trucks, two large 230-tonne trucks, 11 pre-existing trucks and twenty 150-tonne trucks. The match factor indicates that we always have an excess of trucks, and that the loaders will work almost constantly. This reflects the significantly higher cost of operating loaders.

Our industry partner derived three retrospective solutions for this case study. The

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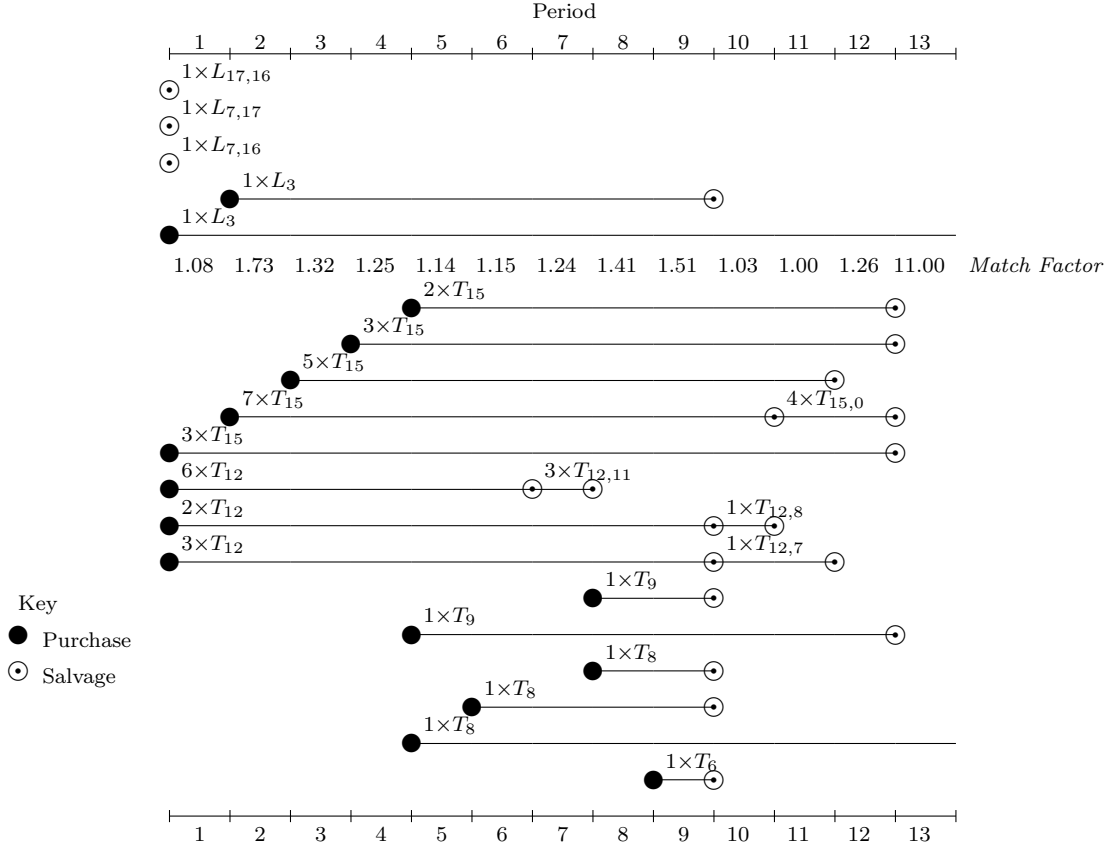


FIGURE 4.20: The optimal solution for the 13-period case study for the single-location model.

first is the solution provided by an inhouse equipment selection spreadsheet tool. The loader solution kept the youngest pre-existing loader with capacity $34T$, and purchased two new $40T$ loaders. The truck solution is presented in Table 4.11. This solution

	Periods												
	1	2	3	4	5	6	7	8	9	10	11	12	13
<i>Pre-existing 172T</i>	11	11	11	11	11	11	11	11	11	11	11	11	11
<i>New 172T</i>	5	5	5	5	5	5	5	5	5	5	5	5	5
<i>New 172T</i>	0	2	2	2	2	2	2	2	2	2	2	2	2
<i>New 172T</i>	0	0	0	0	1	1	1	1	1	1	1	1	1

TABLE 4.11: The retrospective truck purchase and salvage policy for the 13-period case study.

totalled $\$1.51483 \times 10^8$. The solution provided by our integer programming model improved this by 17.7% - an increase in profit of $\$26,876,000$.

A second solution was produced by an equipment selection manager (for the industry partner), who recommended the salvage of all pre-existing loaders. One $42T$ loader and two $57T$ loaders were to be purchased. The same truck solution as presented in Table

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4.11 was to be used. This solution cost $\$1.55241 \times 10^8$, using the cost function provided in this chapter [Section 4.2.3]. Our model solution provides an improvement of 18.75%, which amounts to $\$29,116,000$.

A third solution was provided, which was the actual solution adopted. All pre-existing loaders were salvaged and three 57T loaders were purchased. The truck purchase and salvage policy from Table 4.11 was used. This solution cost $\$1.66550 \times 10^8$. Our model improved this solution by 24.3%, amounting to a $\$40,425,000$ cost difference.

Clearly, our solution (although more complicated) is much cheaper, indicating the advantages of applying an integer programming model to this problem.

4.6 Discussion

For this project, we aimed to derive an integer programming model that would act as a reliable tool in the equipment selection process. In this chapter, we have presented a computationally fast integer program that can outperform industry generated solutions substantially.

We have placed particular importance on pre-existing equipment and compatibility in this model. Ensuring compatibility of multiple types of equipment is a unique aspect of this model that allows greater freedom for the equipment selection manager for two reasons:

1. the manager may consider purchasing equipment that is different to any of the pre-existing fleet;
2. the manager may consider purchasing mixed fleets that better suit the productivity requirements of the mine, and consequently may achieve lower operating expenses.

The ability to include pre-existing equipment in the equipment selection process is novel for the mining industry.

Another important advantage of this model is that it appropriates the operating cost across brackets. That is, within one period, if a piece of equipment graduates to the next cost bracket (relative to its age), then the operating cost will reflect this in a proportional manner.

The solutions presented in this chapter test the “bigger is better” philosophy that is commonplace in the mining industry. In our study, bigger equipment is considered too costly to select all the time. Instead the solutions tend toward moderate sized equipment that can adapt to changing productivity requirements and truck cycle times. This result suggests that bigger machines provide a cost benefit only if they can be fully utilised for the entire period.

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Although the assumptions contribute to the tractability of the model, they can also detract from its realism. In particular, the assumptions of *full period utilisation* and *generalised production requirements* are abstractions of the problem, designed to create a simple model.

The validity of the assumption of full period utilisation is wholly dependent on the management practices of the mine. Some mines may only operate the equipment as it is needed, while others will operate all equipment that is owned but at lesser capacity. The full period utilisation assumes the latter is taking place. This can lead to peculiar solutions, such as having many different truck types (up to 5 in some cases), with some equipment salvaged after only one year of ownership. This can be attributed to the assumption that any equipment that is owned is operated for the entire period. This forces the selection of truck types that closely fit the productivity requirements, and with the presented data set this sometimes results in many truck types being selected. However, hiring equipment may offset this problem, and encourage the model to select a more manageable variety of truck types. We develop a different interpretation of utilisation in the *utilised-cost, single-location, multi-period model* in Chapter 6.

The productivity constraints assume that no bunching occurs and the selected fleets operate efficiently (though not necessarily to full capacity). This is not an accurate abstraction of reality. For example, if we had 20 trucks operating in a fleet with one loader operating to full capacity and we added 5 more trucks, then the truck cycle time would increase due to bunching. Thus, with our present model of the problem, the productivity of the system will be inaccurately overestimated in this case. In actuality, the bunching is a function of the fleet solution. It would be a great challenge to pursue an integer programming model that can capture this type of cyclic relationship.

The assumption of *generalised productivity requirements* is restrictive in the sense that this model cannot be applied to a mine with an intricate system of loading locations and trucking routes with guaranteed satisfaction of productivity requirements. This model is therefore useful for mining schedules that are not fully developed, or for mines that do not have much movement between alternate dump-sites.

An obvious improvement is to relax the assumption of *generalised productivity requirements* by allowing multiple loading sites, dumping sites and truck routes. This is the challenge that we address in the forthcoming Chapter 5.

Chapter 5

Multiple-location, multiple-period equipment selection

Materials handling in surface mines can be complicated by the presence of multiple mining locations, multiple truck routes and multiple dump-sites. Even small mines can have multiple locations or routes in operation simultaneously. Differences in routes can accrue substantial cost differences if the cycle time is very different. This can lead to the selection of a fleet that is not capable of meeting productivity requirements if we do not account for these location differences. We derive a mixed-integer linear programming model for heterogeneous equipment selection in a surface mine with multiple locations and a multiple period schedule, while allocating the equipment to locations. We test the model on two case studies which demonstrate promising results for large sets of equipment and some long term schedule scenarios.

5.1 Introduction

A typical surface mine may have several mining locations, several dump-sites or several routes from a location to a dump-site. Ideally, we would like to capture these multiple locations in an equipment selection model to ensure that productivity requirements can be met by the selected fleet. An optimal equipment selection solution must allocate the equipment to these locations to ensure satisfaction of local production requirements. In spite of the prevalence of multiple trucking routes and dump-sites in surface mines, the multiple location problem has not been addressed in the truck and loader equipment selection literature. This may be because the consideration of multiple periods and multiple locations exacerbates what is already a large scale problem.

In this chapter we address the truck and loader equipment selection problem for

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multiple location, multiple period mines [Figure 5.1].

Remark 5.1.1 A *location* represents a place in the mine where loaders can operate; alternatively, the location can encapsulate the mining location, truck and dump-site.

The way in which we define the locations and routes is not significant to the outcome of the model presented in this chapter. During our analysis of the model, we consider two case studies that view location in two different ways. The first case study considers the mining location to be a position for the loader that is significantly different to another position. For the second case study a location represents the mining location, the route and the dumpsite. The second scenario occurs when the mining schedule is less detailed, and productivity requirements are only provided for the mining locations rather than mining locations and dumpsites.

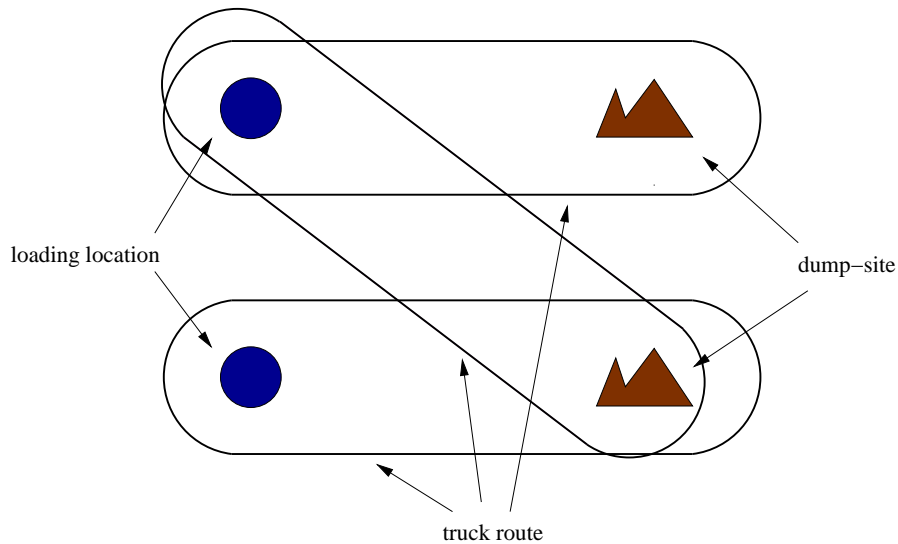


FIGURE 5.1: A multiple-location mine model with 2 loading locations, 2 dump-sites and 3 truck routes.

The consideration of multiple locations or routes introduces the need to allocate equipment to locations. Taking a simple approach as in Chapter 4, we could keep the $x_e^{k,l}$ variables as is and introduce a fourth index, location j . This approach is problematic in that trucks and loaders will be allocated to locations for the entire length of each period, which is unrealistic. Instead, we introduce continuous variables for allocation which represent the proportion of time the vehicles work at a particular location. In this chapter we also:

- Consider the inclusion of pre-existing equipment and heterogeneous fleets;
- Consider a multiple-period, multiple-location mining schedule;

- Derive an allocation policy for trucks to routes and loaders to locations. Surprisingly, an equipment selection model that also allocates equipment has not been considered before in the mining literature.

We outline the model in Section 5.2. We begin by considering the assumptions (Section 5.2.1) and the decision variables (Section 5.2.2), before deriving the objective function (Section 5.2.3) and constraints (Section 5.2.4). We then provide a summary of the notation (Section 5.2.5) before summarising the complete model (Section 5.2.6). To test the model, we consider two surface mining case studies (Sections 5.3 and 5.4).

5.2 Problem formulation

As with the single-location model in Chapter 4, integer programming is an appropriate modelling method for the multi-location problem. However, the allocation variable must be non-integer as it isn't effective to allocate a piece of equipment to a single location or route for the entire period if the period length is substantial, such as one year, and the equipment is not required. We can adequately capture the allocation of trucks to routes and loaders to loading locations in continuous variables. Therefore the multi-location equipment selection problem for surface mining is best modelled as a mixed-integer linear program.

5.2.1 Assumptions

The model presented in this chapter is a conceptual extension of the model presented in Chapter 4. Therefore we retain all assumptions from that chapter except the assumptions of *Generalised productivity requirements* and *Truck cycle time* [Section 4.2.1], which are relaxed.

Multiple locations: We consider multiple loading locations and multiple trucking routes. The selected fleet is free to move about these locations and routes as they are needed as deemed by the allocation policy.

Multiple truck cycle times: Each location may have a different truck cycle time.

Known mine schedule: We assume that an acceptable mine schedule has already been derived, and that the mining method has been selected. We start the equipment selection with a sub-set of trucks and loaders that suit the particular mining scenario.

Salvage: All equipment is salvageable at the start of each period at some depreciated value of the original capital expense. Any pre-existing equipment may be salvaged at the start of the first period.

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No auxiliary equipment: Auxiliary equipment, such as wheel loaders and small trucks, are not considered in this model. Although the cost of running auxiliary equipment may differ according to the overall fleet selection, for the purpose of this model this cost is considered trivial. Note that the cost of auxiliary equipment can be built into the operating cost if necessary.

Known operating hours: The operating hours of the mine are estimated by taking planned downtime, blasting and weather delays into account.

Heterogeneous fleets: We allow different types of equipment to work side-by-side.

Fleet retention: All equipment is retained at the end of the last period. Optionally, we can relax this assumption and allow the model to salvage some or all equipment at the end of the final period.

Full period utilisation: Operating costs are charged as though the equipment has been fully utilised for the entire period in which it is owned.

5.2.2 Decision variables and notation

Fleet purchase and tracking variables

As in Chapter 4, we need to capture the number of equipment, the equipment type, the age of the equipment and the periods in which the equipment is operating. This variable set is important for tracking whole equipment units while other variables are used to allocate equipment. Recall that we adopted three indexes to represent type (e), period (k) and age (l). The purchase and tracking variable for equipment is:

$\mathbf{x}_e^{k,l}$: number of equipment of type e in period k that are age l .

Allocation variables

We use continuous variables to allocate equipment to routes or locations for a proportion of the total time. These variables must be indexed to the period, equipment type, location or route and equipment age:

$\mathbf{f}_{e,j}^{k,l}$: number of equipment of type e , age l , that are allocated to route j in period k .

We note that as these variables are continuous we may allocate partial trucks to a route, for example, which denotes that a truck spends a partial amount of time on that route.

Salvage variables

Recall from Chapter 4 that we use salvage variables to retain important information, such as which specific truck or loader is salvaged:

$\mathbf{s}_e^{k,l}$: number of equipment of type e salvaged in period k while age l .

5.2.3 Objective function

Now that we have introduced a utilisation variable, $\mathbf{f}_{e,j}^{k,l}$, we would ideally like to bring it into the objective function to account for operating expense accurately. However, we cannot account for the cumulative age of a piece of equipment without accounting for individual machines. Although it seems simple, this step alone is quite difficult to model and we make this extension in Chapter 6. For now, we have retained the assumption of *Full period utilisation* from Chapter 4. Placing the utilisation variable into the objective function would imply that the equipment is only ageing by quantity $\mathbf{f}_{e,j}^{k,l}$ in each period, which is contradictory to our assumption of *Full period utilisation*. Therefore we only use the tracking variables in the objective function. We use the same objective function as for the single-location model in Chapter 4, presented in Section 4.2.3.

Complete objective function

$$\text{Minimise} \quad \sum_{e,k} F_e D_1^k \mathbf{x}_e^{k,0} + \sum_{e,k,l,h} B_{h,e}^{k,l} D_1^k V_e^{k,b(l)+h-1} \mathbf{x}_e^{k,l} - \sum_{e,k,l} F_e D_2^{k,l} \mathbf{s}_e^{k,l}.$$

5.2.4 Constraints

Productivity constraints

We must satisfy the productivity requirements for each location or route. As before, the production capability of a piece of equipment is determined by its availability, capacity and cycle time:

$$P_e^{k,l} = \frac{a_e^{b(l)} c_e}{t_e^k}. \quad (5.1)$$

Again, for this formulation availability is determined by the equipment's age. We require the loaders to satisfy the productivity demand, $T_{j'}^k$, at location j' :

$$\sum_{i',l} P_{i'}^{k,l} \mathbf{f}_{i',j'}^{k,l} \geq T_{j'}^k \quad \forall \quad k, j'. \quad (5.2)$$

Similarly, we require the trucks to satisfy the demand for each route or dumpsite,

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j :

$$\sum_{i,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq T_j^k \quad \forall k, j. \quad (5.3)$$

The trucks must also match the productivity demand of the mining locations to which route j connects to:

$$\sum_{i,l,j \in \mathbf{C}(j')} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq T_{j'}^k \quad \forall k, j'. \quad (5.4)$$

Compatibility constraints

We must satisfy productivity requirements with the set of compatible trucks and loaders for each location. To set this up as a constraint set, recall that we define the set $\mathbf{X}(A')$ to be the set of truck types which are compatible with loader types $A' \subset \mathbf{X}'$. For each constraint, we are only interested in the routes, j , that connect to the location j' . Therefore we consider the routes that are $j \in J(j')$, where J is the set of locations. Again, we let \mathbf{A} represent the set of all possible combinations of loaders:

$$\sum_{i \in \mathbf{X}(A'), j \in J(j'), l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i' \in A', l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \quad \forall A' \subset \mathbf{X}', k, j. \quad (5.5)$$

As we are now using a utilisation variable, these are the only compatibility constraints that we require. Since A' comes from the power set of \mathbf{X}' , this will generate $KJ(2^N - 1)$ constraints (where K is the total number of periods and J is the total number of locations). For the 13 period, 27 loader problem with 5 locations, this equates to 8724 million constraints. However, for a given case study the number of loaders possible in the final solution will generally be much lower than the complete set. In this case we can limit the generation of constraints to a maximum of α loader types. This will produce $KJ \sum_{a=1}^{\alpha} \binom{n}{a}$ constraints. For a 13 period, 27 loader problem with a maximum of 4 loader types and 5 locations, this equates to just 13 million constraints. Further ways to reduce this number of constraints are discussed in Section 4.2.4. We choose to enter these constraints into the model as *lazy constraints*.

Restricted allocation

We link the equipment tracking variables, $\mathbf{x}_e^{k,l}$, to the allocation variables $\mathbf{f}_{e,j}^{k,l}$ by placing an upper bound on the allocation:

$$\mathbf{x}_e^{k,l} \geq \sum_j \mathbf{f}_{e,j}^{k,l} \quad \forall e, k, l. \quad (5.6)$$

MULTIPLE-LOCATION MODEL

Variable transition

As we have extended the model in Chapter 4 for this multi-location model, the variable transition constraints are exactly the same (the tracking variables do not track the location in which the equipment works):

$$\mathbf{x}_e^{k,l} = \mathbf{x}_e^{k-1,l-1} - \mathbf{s}_e^{k,l} \quad \forall \quad k > 0, l > 0, e. \quad (5.7)$$

Forced salvage

We force the retirement of old equipment with the following constraint:

$$\mathbf{x}_e^{k,l} = \mathbf{s}_e^{k+1,l+1} \quad \forall \quad l > L(e), e, k < K. \quad (5.8)$$

However, during variable creation we can prevent over-age variables from existing in the first place, thus effecting forced salvage.

Salvage Restriction

We repeat the salvage restriction constraints from the previous model by ensuring that we do not salvage any equipment that we did not own in the previous period:

$$\mathbf{x}_e^{k-1,l-1} \geq \mathbf{s}_e^{k,l} \quad \forall \quad k > 0, l > 0, e. \quad (5.9)$$

Also, we prevent the unbounded salvage variables from dominating:

$$\mathbf{s}_e^{k,0} = 0 \quad \forall \quad e, k. \quad (5.10)$$

Pre-existing equipment

In this model we also consider pre-existing equipment, and so this constraint is the same as in Chapter 4. We only need to consider pre-existing trucks and loaders which are drawn from the subset $\mathbf{P} \subset \mathbf{X} \cup \mathbf{X}'$.

$$\mathbf{x}_e^{0,P(e)} + \mathbf{s}_e^{0,P(e)} = x_e^{P(e)} \quad \forall \quad e \in \mathbf{P}. \quad (5.11)$$

Pre-existing equipment that has exceeded the maximum age, $L(e)$, for its equipment type, e , can be salvaged immediately:

$$\mathbf{s}_e^{k,P(e)} = x_e^{P(e)} \quad \forall \quad e \in \mathbf{P}, k, P(e) > L(e). \quad (5.12)$$

5.2.5 Summary of notation

\mathbf{X}	set of all available truck types.
\mathbf{X}'	set of all available loader types.
i	truck type index, $i \in \mathbf{X}$.
i'	loader type index, $i' \in \mathbf{X}'$.
e	equipment type index, $e \in \mathbf{X} \cup \mathbf{X}'$.
j	dumpsite/route location index.
j'	mining location index.
k	period index.
l	equipment age index.
\mathbf{P}	set of all pre-existing equipment types.
\mathbf{A}	power set of all combinations of loader types, $\mathbf{A} = \mathcal{P}(\mathbf{X}')$.
$\mathbf{X}(A')$	power set of all truck types compatible with loader set $A' \subset \mathbf{X}'$.
K	total number of periods in the mining schedule.
J	total number of locations and/or routes in the mining schedule.
$L(e)$	maximum age (in operating hours) unique for each truck and loader type, e .
$L^k(e)$	maximum age we need to consider in period k , $L^k(e) = \min\{P(e) + k - 1, L(e)\}$.
$\mathbf{x}_e^{k,l}$	number of equipment type e selected in period k , aged l .
$\mathbf{f}_{e,j}^{k,l}$	number of equipment type e selected in period k , aged l sent to work on route j .
$\mathbf{s}_e^{k,l}$	number of equipment type e salvaged in period k , aged l .
$x_e^{P(e)}$	number of pre-existing equipment type e aged $P(e)$ at the start of the schedule.
F_e	fixed cost (capital expense) of obtaining equipment type e .
D_1^k	discount factor in period k .
$D_2^{k,l}$	discount factor in period k and depreciation factor for equipment aged l .
$B_{h,e}^l$	proportion of time equipment type e aged l spends in cost bracket $h \in \{1, 2\}$.
$V_e^{k,b(l)}$	variable expense for equipment type e , aged $b(l)$ in period k .
$P_e^{k,l}$	productivity of equipment e , in period k at age l .
T_j^k	required productivity of the mine (in tonnes) for period k at location j .

5.2.6 Complete model

$$\text{Minimise} \quad \sum_{e,k} F_e D_1 \mathbf{x}_e^{k,0} + \sum_{e,k,l,h} B_{h,e}^{k,l} D_1^k V_e^{k,b(l)+h-1} \mathbf{x}_e^{k,l} - \sum_{e,k,l} F_e D_2^{k,l} \mathbf{s}_e^{k,l}$$

subject to

$$\sum_{i,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq T_j^k \quad \forall k, j \quad (5.13)$$

$$\sum_{i',l} P_{i'}^{k,l} \mathbf{f}_{i',j'}^{k,l} \geq T_{j'}^k \quad \forall k, j' \quad (5.14)$$

$$\sum_{i,l,j \in \mathbf{C}(j')} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq T_{j'}^k \quad \forall k, j' \quad (5.15)$$

$$\sum_{i \in X(A'), j \in X(j'), l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i' \in A', l} P_{i'}^{k,l} \mathbf{f}_{i',j'}^{k,l} \quad \forall A' \subset \mathbf{X}', k, j' \quad (5.16)$$

$$\mathbf{x}_e^{k,l} \geq \sum_j \mathbf{f}_{e,j}^{k,l} \quad \forall e, k, l \quad (5.17)$$

$$\mathbf{x}_e^{k,l} = \mathbf{x}_e^{k-1,l-1} - \mathbf{s}_e^{k,l} \quad \forall k > 0, l \in [1, L^k(e)], e \quad (5.18)$$

$$\mathbf{x}_e^{k,l} = \mathbf{s}_e^{k+1,l+1} \quad \forall l > L(e), e, k \quad (5.19)$$

$$\mathbf{s}_e^{k,0} = 0 \quad \forall e, k \quad (5.20)$$

$$\mathbf{x}_e^{k-1,l-1} \geq \mathbf{s}_e^{k,l} \quad \forall k > 0, l \in [1, L^k(e) + 1], e \quad (5.21)$$

$$\mathbf{x}_e^{0,l} + \mathbf{s}_e^{0,l} = x_e^{P(e)} \quad \forall e \in \mathbf{P}, P(e) > 0 \quad (5.22)$$

$$\mathbf{s}_e^{0,P(e)} = x_e^{P(e)} \quad \forall e \in \mathbf{P}, P(e) > L(e) \quad (5.23)$$

$$\mathbf{x}, \mathbf{s} \in \mathbb{Z}^+$$

$$\mathbf{f} \in \mathbb{R}^+$$

5.3 Computational results: First case study

Our industry partner provided this case study from a new mining operation. Consequently, no retrospective data was available.

5.3.1 Locations and routes

The mine for this case study has eight loading locations - four mining locations (L_1, L_2, L_3, L_4) and four stockpiles (S_1, S_2, S_3, S_4). The stockpiles are used to meet ore grade mixing constraints at the mill. The mixing constraints are not considered in this model, as they are assumed to be pre-optimised when the mining schedule was produced. There are also four dumpsites (D_1, D_2, D_3, D_4), which includes one mill (D_3). Figure 5.2 depicts the scheduled locations and routes for this mine. There are 13 routes in total, shown

MULTIPLE-LOCATION MODEL

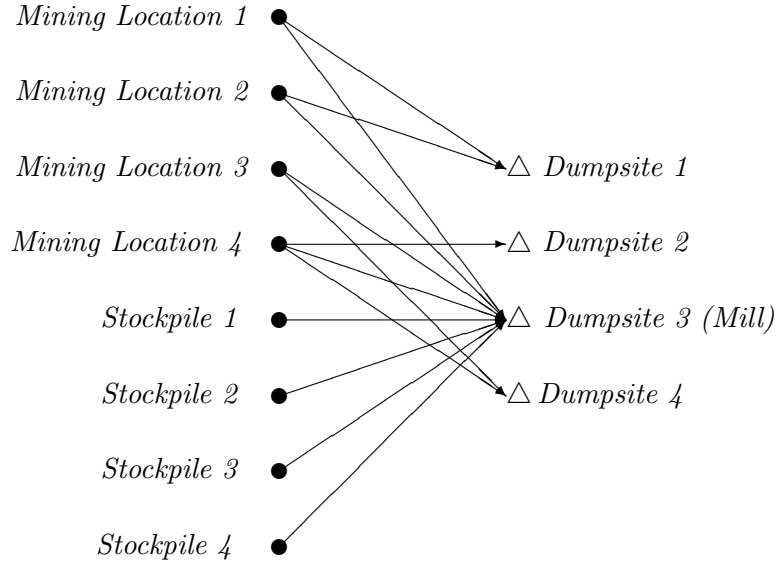


FIGURE 5.2: Routes from mining locations to dumpsites for the first case study.

in Figure 5.3.

5.3.2 Production requirements

Our industry partner provided the production requirement data for both the mining locations and the truck routes [Tables 5.1 and 5.2]. We combined the location data to create a production requirement graph for the mine over all periods [Figure 5.4]. We also created a relationship array to denote which routes correspond to which locations [Table 5.3]. We use this array for setting up the compatibility constraints. Our industry partner also provided pre-estimated truck cycle times for each route [Table 5.4].

5.3.3 Case-specific parameters

This case study considers a surface mine operating under a truck-loader hauling system, and mines ore and waste in an open pit. We know the following:

- The mine operates for 7604 hours in each period.
- The loaders are selected from a set of 20 loader types.
- The trucks are selected from a set of 8 truck types.
- There are 13 periods, K , each of length 1 year.
- The cost-bracket partition, B_0 , is 5000 hours.
- The discount rate for all periods is 8%.

MULTIPLE-LOCATION MODEL

<i>Route</i>	<i>Period</i>												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2598	0	0	0	0	0	0	0	0	0	0	0	0
2	455	0	0	0	0	0	0	0	0	0	0	0	0
3	203	10141	7659	0	0	0	0	0	0	0	0	0	0
4	741	7060	2964	0	0	0	0	0	0	0	0	0	0
5	221	827	5928	12797	9919	0	0	0	13990	13990	13990	5184	11
6	2592	22935	41035	26948	19873	0	0	0	8901	18191	12589	5572	32
7	0	650	0	0	0	0	0	0	0	0	0	0	0
8	0	270	809	0	0	0	0	0	0	0	0	0	0
9	0	1142	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	737	4071	12990	12890	13990	0	0	0	7713	0
11	0	0	0	17051	24126	8984	8583	12836	0	0	0	6251	0
12	0	0	0	0	0	799	0	0	0	0	0	0	0
13	0	0	0	0	0	0	798	0	0	0	0	0	0

TABLE 5.1: The production requirements for the truck routes for the first case study.

<i>Route</i>	<i>Period</i>												
	1	2	3	4	5	6	7	8	9	10	11	12	P13
1	3053	0	0	0	0	0	0	0	0	0	0	0	0
2	944	17201	10624	0	0	0	0	0	0	0	0	0	0
3	2813	23762	46963	39746	29792	0	0	0	22891	32181	26579	10756	43
4	0	650	0	0	0	0	0	0	0	0	0	0	0
5	0	270	809	0	0	0	0	0	0	0	0	0	0
6	0	1142	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	17789	28197	22773	21474	26826	0	0	0	13964	0
8	0	0	0	0	0	0	798	0	0	0	0	0	0

TABLE 5.2: The production requirements for the mining locations for the first case study.

<i>Location</i>	<i>Routes</i>												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	1	1	0
8	0	0	0	0	0	0	0	0	0	0	0	0	1

TABLE 5.3: Active routes for the first case study.

Period	Routes												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	35.56	35.17	38.49	37.89	11.23	10.62	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	38.03	37.42	14.97	17.23	16.05	60.00	20.00	0.0	0.0	0.0	0.0
3	0.0	0.0	40.09	40.77	25.02	22.08	0.0	60.00	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	27.18	25.09	0.0	0.0	0.0	34.39	34.27	0.0	0.0
5	0.0	0.0	0.0	0.0	28.58	26.25	0.0	0.0	0.0	26.57	27.38	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	32.20	33.51	37.43	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	34.48	35.71	0.0	15.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	37.66	38.76	0.0	0.0
9	0.0	0.0	0.0	0.0	32.64	29.41	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	36.42	32.47	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	38.53	36.49	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	43.54	43.57	0.0	0.0	0.0	39.52	46.67	0.0	0.0
13	0.0	0.0	0.0	0.0	46.93	45.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 5.4: Truck cycle times for the first case study. A value of 0 indicates that the route is not in use in that period.

MULTIPLE-LOCATION MODEL

1. *Mining Location 1* \longrightarrow *Dumpsite 3*
2. *Mining Location 1* \longrightarrow *Dumpsite 1*
3. *Mining Location 2* \longrightarrow *Dumpsite 3*
4. *Mining Location 2* \longrightarrow *Dumpsite 1*
5. *Mining Location 3* \longrightarrow *Dumpsite 3*
6. *Mining Location 3* \longrightarrow *Dumpsite 4*
7. *Stockpile 1* \longrightarrow *Dumpsite 3*
8. *Stockpile 2* \longrightarrow *Dumpsite 3*
9. *Stockpile 3* \longrightarrow *Dumpsite 3*
10. *Mining Location 4* \longrightarrow *Dumpsite 3*
11. *Mining Location 4* \longrightarrow *Dumpsite 4*
12. *Mining Location 4* \longrightarrow *Dumpsite 2*
13. *Stockpile 4* \longrightarrow *Dumpsite 3*

FIGURE 5.3: The routes from mining locations to dumpsites for case study one.

- The depreciation rate is set to 50%.
- The maximum value for any truck variable is 30.
- The maximum value for any loader variable is 10.

5.3.4 Solutions

We implemented this case study on a Pentium 4 PC with 3.0GHz and 2.5GB of RAM. The model was implemented in C++ using *Ilog Concert Technology v2.5* objects and *Ilog Cplex v11.0* libraries to solve the problem. The printable linear program contained 63433 variables and 19366 constraints for the 13-period, 8 truck types and 20 loader types problem. This model demonstrated a slower convergence than the single-location problem, indicating a potential symmetry problem that has been magnified by the increase in scale.

MULTIPLE-LOCATION MODEL

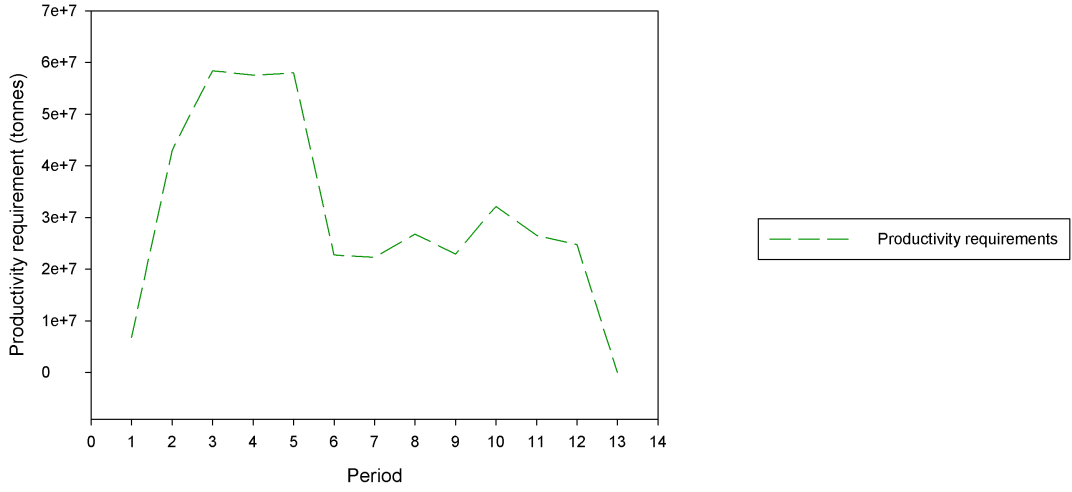


FIGURE 5.4: Production requirements for the 13 period case study.

We set up the 13-period problem with just 15571 constraints before the compatibility constraints were taken into account. We successively solved and checked infeasibility of the compatibility constraints 21 times, adding a total of 3795 compatibility constraints that were violated along the way.

After 7.5 hours of algorithm run-time, we obtained a solution within 3% of the optimal solution for the 50% depreciation case study [Table 5.5]. When the algorithm was permitted to run for a longer period, the computer memory was exhausted. This indicates a cluster of similar solutions near the optimal solution, making it difficult for the algorithm to differentiate between the solutions. Generally this occurs when we have several similar pieces of equipment and several alternate locations to which they must be allocated. When this allocation can be achieved in many ways without significantly altering the objective function, we have created a cluster of solutions. Typically these solutions do not differ much in terms of the objective function value.

The purchase and salvage policy for this multi-location mine is complicated by the productivity requirements, which contain several significant changes from period to period [Figure 5.4]. This leads to short-term ownership of some trucks, for example a

<i>Periods</i>	<i>Variables</i>	<i>Constraints</i>	<i># solves</i>	<i>Time (seconds)</i>	<i>Quality</i>	<i>Solution</i>
10	39855	9814 + 3319	26	5643	3%	1.26292×10^8
11	47166	11599 + 3043	15	3979	3%	1.31263×10^8
12	55032	13521 + 4043	33	17656	3%	1.37168×10^8
13	63433	15571 + 3795	21	26662	3%	1.37249×10^8

TABLE 5.5: The results summary for the first case study with varying periods.

MULTIPLE-LOCATION MODEL

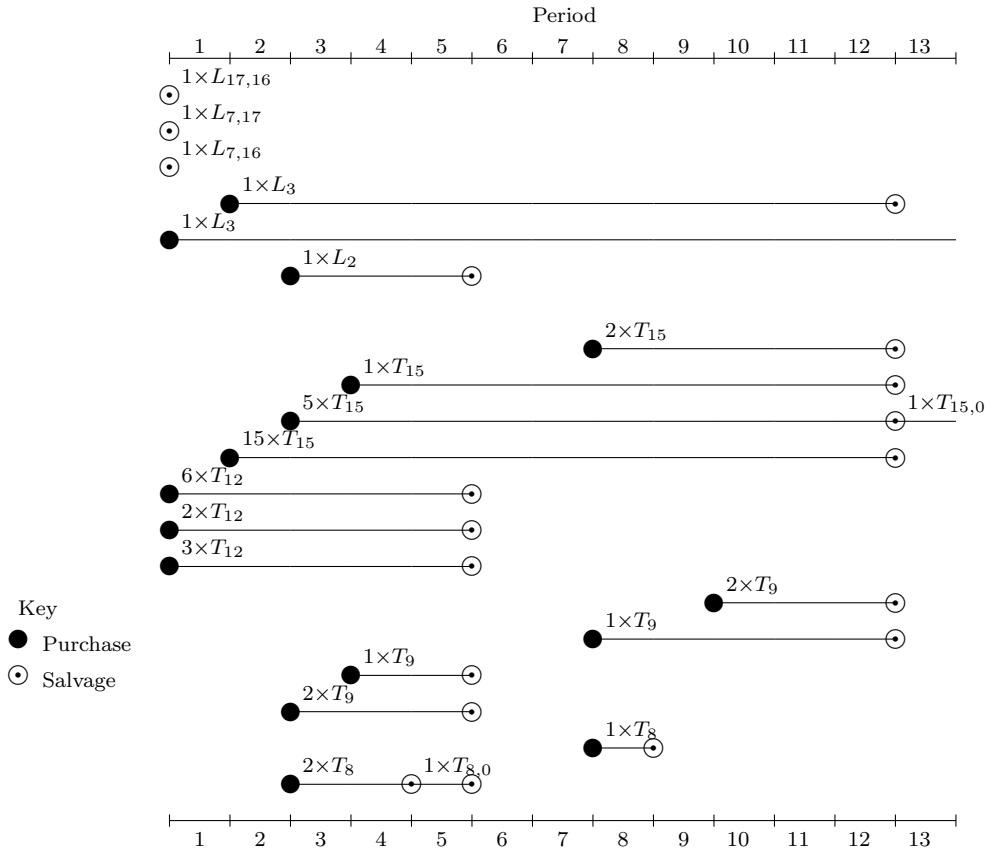


FIGURE 5.5: The 3% optimal 13-period solution for case study one with the depreciation parameter set to 50%.

type-8 truck was purchased in period 8 and salvaged at the start of period 9 [Figure 5.5].

The allocation policies are helpful tools because they guarantee that the selected fleet can cope with the requirements of the multiple locations. We know that the equipment is not necessarily working to maximum capacity. However, the allocation policy always allocates 100% of the equipment time across the locations. This implies that there is some flexibility in the allocation policy. Hence, although this solution is presented as the optimal solution, there will be numerous alternate optimal solutions. That is, the allocation policy is not unique. It would be simple to create a spreadsheet that can reflect the flexibilities in the policy and allow dynamic changes in the policy without affecting the objective function value.

In the allocation policy for this case study, we represent the age of the equipment in parentheses as an equipment tracking tool. That is, the age of the equipment is an important factor in the cost of operating the equipment, and so it is relevant to allocate the correct age equipment as dictated by the policy [Tables 5.6, 5.7, 5.8 and 5.9].

MULTIPLE-LOCATION MODEL

Routes	Period						
	1	2	3	4	5	6	7
0	1.81 $T_{12}(7)$ 6.00 $T_{12}(11)$						
1	0.35 $T_{12}(7)$						
2	0.17 $T_{12}(7)$	9.42 $T_{15}(0)$	6.00 $T_{12}(13)$ 0.97 $T_{15}(0)$				
3	2.00 $T_{12}(8)$	5.90 $T_{12}(12)$	0.68 $T_{12}(9)$ 2.00 $T_{12}(10)$				
4	0.05 $T_{12}(7)$	0.18 $T_{12}(8)$ 0.09 $T_{12}(12)$	3.70 $T_{15}(0)$	2.65 $T_9(1)$ 1.50 $T_8(1)$ 6.73 $T_{15}(2)$	0.42 $T_{12}(12)$ 6.00 $T_{12}(15)$		
5	0.61 $T_{12}(7)$	2.32 $T_{12}(8)$ 2.00 $T_{12}(9)$ 4.87 $T_{15}(0)$	2.00 $T_9(0)$ 2.32 $T_{12}(9)$ 0.32 $T_{15}(0)$ 15.0 $T_{15}(1)$ 0.99 $T_8(0)$	1.00 $T_9(0)$ 2.00 $T_9(1)$ 3.00 $T_{12}(10)$ 1.97 $T_{12}(11)$ 6.00 $T_{12}(14)$	3.00 $T_{12}(11)$ 1.58 $T_{12}(12)$ 1.00 $T_{15}(1)$ 5.00 $T_{15}(2)$ 0.67 $T_{15}(3)$		
6		0.31 $T_{15}(0)$					
7		0.40 $T_{15}(0)$	1.00 $T_8(0)$				
8		0.50 $T_{12}(8)$					
9				0.50 $T_8(1)$ 0.03 $T_{12}(11)$	2.67 $T_{15}(3)$	12.84 $T_{15}(4)$	13.115 $T_{15}(5)$
10				1.00 $T_{15}(0)$ 5.00 $T_{15}(1)$ 8.27 $T_{15}(2)$	11.7 $T_{15}(3)$	1.00 $T_{15}(2)$ 4.26 $T_{15}(3)$ 2.16 $T_{15}(4)$	1.00 $T_{15}(3)$ 5.00 $T_{15}(4)$ 1.59 $T_{15}(5)$
11						0.74 $T_{15}(3)$	
12							0.30 $T_{15}(5)$

TABLE 5.6: The truck allocation policy for case study one with 13 periods and 50% depreciation (first 7 periods).

MULTIPLE-LOCATION MODEL

Routes	Period												
	8	9	10	11	12	13							
0													
1													
2													
3													
4	1.00 $T_9(1)$ 15.00 $T_{15}(7)$	1.00 $T_9(6)$ 5.00 $T_{15}(7)$ 6.85 $T_{15}(8)$	2.00 $T_9(1)$ 1.00 $T_9(3)$ 2.00 $T_{15}(3)$ 1.00 $T_{15}(7)$ 5.00 $T_{15}(8)$ 0.83 $T_{15}(9)$	2.00 $T_9(1)$ 1.00 $T_9(3)$ 2.00 $T_{15}(3)$ 1.00 $T_{15}(7)$ 5.00 $T_{15}(8)$ 0.83 $T_{15}(9)$	2.00 $T_9(2)$ 1.00 $T_9(4)$ 0.72 $T_{15}(4)$ 1.00 $T_{15}(8)$	0.01 $T_{15}(10)$							
5	2.00 $T_{15}(1)$ 1.00 $T_{15}(5)$ 5.00 $T_{15}(6)$	2.00 $T_9(0)$ 1.00 $T_9(2)$ 2.00 $T_{15}(2)$ 8.15 $T_{15}(8)$	14.2 $T_{15}(9)$	6.16 $T_{15}(10)$	0.99 $T_{15}(10)$								
6													
7													
8													
9	1.00 $T_8(0)$ 1.00 $T_9(0)$ 10.6 $T_{15}(6)$	1.28 $T_{15}(4)$ 6.43 $T_{15}(10)$											
10	2.00 $T_{15}(0)$ 1.00 $T_{15}(4)$ 5.00 $T_{15}(5)$ 4.37 $T_{15}(6)$	5.00 $T_{15}(9)$ 2.41 $T_{15}(10)$											
11													
12													

TABLE 5.7: The truck allocation policy for the first case study solution with 13-periods and 50% depreciation (last 6 periods).

<i>Locations</i>	<i>Period</i>						
	1	2	3	4	5	6	7
0	0.13 $L_3(0)$						
1	0.04 $L_3(0)$	0.70 $L_3(0)$	0.43 $L_3(2)$				
2	0.12 $L_3(0)$	0.21 $L_3(0)$	0.51 $L_2(0)$	1.00 $L_2(1)$	0.33 $L_2(2)$		
		0.76 $L_3(1)$	1.00 $L_3(1)$	0.93 $L_3(3)$	1.00 $L_3(3)$		
			0.56 $L_3(2)$				
3		0.26 $L_3(0)$					
4		0.01 $L_3(0)$	0.05 $L_2(0)$				
5		0.05 $L_3(0)$					
6				0.73 $L_3(2)$	0.67 $L_2(2)$	0.93 $L_3(4)$	0.89 $L_3(5)$
					0.68 $L_3(4)$	0.13 $L_3(5)$	
7							0.03 $L_3(5)$
8							

TABLE 5.8: The loader allocation policy for the first case study with 13 periods and 50% depreciation (first 7 periods).

MULTIPLE-LOCATION MODEL

<i>Locations</i>	<i>Period</i>					
	8	9	10	11	12	13
0						
1						
2		0.34 $L_3(7)$	0.38 $L_3(8)$	0.26 $L_3(9)$	0.47 $L_3(11)$	0.01 $L_3(12)$
		0.97 $L_3(8)$	1.00 $L_3(9)$	1.00 $L_3(10)$		
3						
4						
5						
6	0.13 $L_3(6)$				0.07 $L_3(10)$	
	1.00 $L_3(7)$				0.53 $L_3(11)$	
7						
8						

TABLE 5.9: The loader allocation policy for the first case study with 13 periods and 50% depreciation (last 6 periods).

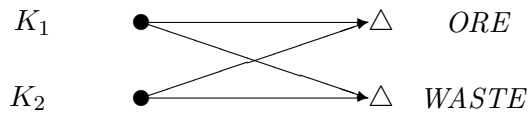


FIGURE 5.6: The locations for case study two.

5.4 Computational results: Second case study

Our industry partner provided a second case study from a mine that was scheduled to open in 2007. Equipment selection for this mine has not yet taken place, and so retrospective data (in the form of a comparative solution) is not available. This mine begins with no pre-existing equipment.

5.4.1 Locations and routes

The locations for this mine are defined differently to the first case study. For this mine the location encompasses the mining location, the route and the dump-site. This is a convenient way to define the locations if there are few overall mining locations and dump-sites.

For example, location one includes the K_1 mining location and the route to the ORE dump-site. The actual location of the dump-site is irrelevant as long as we have sufficiently accurate truck cycle time estimates. The mine for this case study has two loading locations (K_1, K_2). There are also two dump-sites ($ORE, WASTE$). There are 4 locations in total, represented by the arrows in Figure 5.6.

5.4.2 Production requirements

This new mine is simpler than the first in terms of the number of mining locations. The overall production requirements change at a moderate rate [Table 5.10]. However, the estimated truck cycle times, also provided by the industry partner, demonstrate great variability from period to period, and also between locations [Table 5.11]. For example, the smallest truck cycle time is 2.64 minutes, while the longest is 23.82 minutes.

5.4.3 Case specific parameters

This case study considers a mine operating under a truck-loader hauling system, and mines ore and waste in an open pit. We have the following:

- The mine is removing ore and waste, and operates under a shovel-truck system.
- The operating hours of the mine is 7604 hours for each period.

MULTIPLE-LOCATION MODEL

<i>Location</i>	<i>Period</i>								
	1	2	3	4	5	6	7	8	9
1	51701	668713	1602777	1463289	2227402	1657740	2031283	2230192	2474503
2	2082800	8843227	7294449	5837564	5356417	8051374	4582733	4666783	2001808
3	0	0	1189184	1120125	337101	1073245	1106596	455511	235810
4	0	9412391	9063476	10593225	11355932	4091566	530276	84354	94315

TABLE 5.10: The production requirements for the mining locations for the second case study.

MULTIPLE-LOCATION MODEL

<i>Period</i>	<i>Location</i>			
	1	2	3	4
1	8.24	2.64	0.0	0.0
2	8.3	3.48	0.0	8.24
3	9.28	3.84	5.74	10.23
4	10.52	4.88	8.73	10.45
5	11.16	6.01	10.38	12.6
6	12.47	7.23	11.71	16.72
7	12.05	8.49	13.82	19.6
8	15.77	10.11	15.49	21.37
9	17.74	12.05	16.52	22.82

TABLE 5.11: The truck cycle times for the second case study.

- The loaders are selected from a set of 20 loader types.
- The trucks are selected from a set of 8 truck types.
- There are $K = 9$ periods in total, each of length 1 year.
- The cost-bracket length, B_0 , is 5000 hours.
- The interest rate for all periods is 8%.
- The depreciation rate is set to 50%.
- The maximum value for any truck variable is 30.
- The maximum value for any loader variable is 5.

5.4.4 Solution

<i>Periods</i>	<i>Variables</i>	<i>Constraints</i>	<i># solves</i>	<i>Time (seconds)</i>	<i>Quality</i>	<i>Solution</i>
7	9100	4242 + 2044	15	5331	Optimal	1.88599×10^7
8	11648	5473 + 1484	9	12049	Optimal	1.97785×10^7
9	14502	6858 + 2380	16	19477	3%	2.05244×10^7
9	14502	6858 + 2240	15	122458	—	—

TABLE 5.12: The results summary for the second case study solutions with varying periods.

The optimal 9-period solution is given in Figure 5.7. This problem was implemented with 14502 variables and 6858 constraints, and was solved 16 times after adding 2380 compatibility constraints. It is interesting to note the significant decrease in the truck

MULTIPLE-LOCATION MODEL

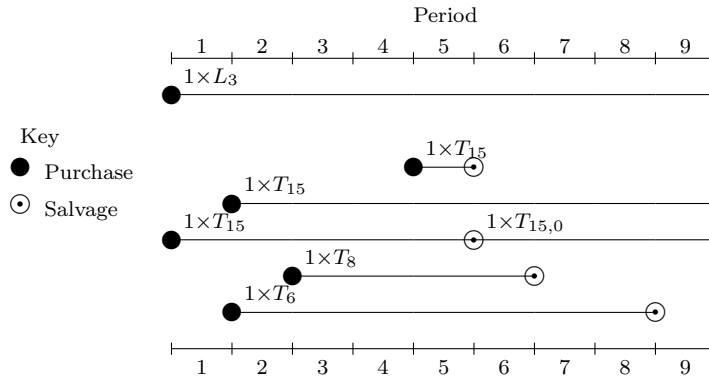


FIGURE 5.7: The second case study 9-period purchase and salvage policy with depreciation 50%.

fleet size of this case study compared to the other, in spite of similar production requirements. This is due to the comparatively short truck cycle times.

We ran the 9-period problem for 34 hours before the computer memory was exhausted, and although the optimality gap achieved a little less than 3%, the problem did not solve to optimality.

The truck allocation solution is presented in Table 5.13. It is difficult to identify the slackness in the allocation table because the model did not motivate a minimum utilisation value - it does not cost more (in terms of the objective function value) to allocate trucks to locations for more time than necessary. The loader allocation solution splits one loader across 2 mining locations [Table 5.14]. This is unrealistic if we take the size of the loader into account, and the time required to dismantle or move the loader.

5.5 Discussion

In this chapter, we extended the results from Chapter 4 to account for multiple mining locations or routes. This proved more challenging than simply adding a new index to the purchase and salvage variables; we introduced a new allocation variable which dictated the proportion of total time that the equipment works at each location. This variable turned the model into a mixed-integer program, which are in general more difficult to solve than linear programs. Many good algorithms exist for pure integer programs that enable fast solution times for large problems, as we saw in Chapter 4, but these do not extend to MILPs.

However, although the continuous variables may have increased the difficulty of the problem, they brought the advantage of a flexible allocation policy. This is a useful tool for mining engineers, who may take the allocation policy as further evidence that the selected fleet would be able to perform the required tasks under uncertainty, or simply

MULTIPLE-LOCATION MODEL

<i>Locations</i>	<i>Period</i>								
	1	2	3	4	5	6	7	8	9
1	0.01 $T_{15}(0)$ 0.14 $T_{15}(1)$	1.00 $T_{15}(1)$	1.00 $T_6(1)$	0.44 $T_{15}(3)$	0.51 $T_8(2)$	0.51 $T_{15}(4)$	0.62 $T_{15}(5)$	0.89 $T_{15}(7)$	0.11 $T_{15}(7)$ 1.00 $T_{15}(8)$
2	0.99 $T_{15}(0)$ 0.97 $T_6(0)$	0.58 $T_8(0)$	0.58 $T_8(0)$	0.14 $T_{15}(2)$ 0.56 $T_{15}(3)$	0.88 $T_6(3)$	1.00 $T_8(3)$ 0.26 $T_{15}(4)$	1.00 $T_6(5)$ 0.74 $T_{15}(6)$	0.07 $T_6(6)$ 1.00 $T_{15}(6)$ 0.11 $T_{15}(7)$	0.74 $T_{15}(7)$
3			0.23 $T_{15}(1)$	0.24 $T_{15}(2)$	0.07 $T_8(2)$	0.23 $T_{15}(4)$ 0.21 $T_{15}(5)$	0.38 $T_{15}(5)$	0.19 $T_6(6)$	0.10 $T_{15}(7)$
4		0.03 $T_6(0)$ 1.00 $T_{15}(0)$ 0.86 $T_{15}(1)$	0.42 $T_8(0)$ 0.77 $T_{15}(1)$ 1.00 $T_{15}(2)$	0.62 $T_{15}(2)$ 1.00 $T_8(1)$ 0.44 $T_9(2)$	0.12 $T_6(3)$ 0.41 $T_8(2)$ 1.00 $T_{15}(0)$ 1.00 $T_{15}(3)$ 1.00 $T_{15}(4)$	1.00 $T_6(4)$ 0.79 $T_{15}(5)$	0.26 $T_{15}(6)$	0.73 $T_6(6)$	0.06 $T_{15}(7)$

TABLE 5.13: The 9-period truck allocation policy for the second case study, 50% depreciation.

MULTIPLE-LOCATION MODEL

<i>Locations</i>	<i>Period</i>								
	1	2	3	4	5	6	7	8	9
1	0.01 $L_3(0)$	0.03 $L_3(1)$	0.07 $L_3(2)$	0.06 $L_3(3)$	0.09 $L_3(4)$	0.07 $L_3(5)$	0.08 $L_3(6)$	0.09 $L_3(7)$	0.10 $L_3(8)$
2	0.09 $L_3(0)$	0.36 $L_3(1)$	0.30 $L_3(2)$	0.24 $L_3(3)$	0.23 $L_3(4)$	0.33 $L_3(5)$	0.19 $L_3(6)$	0.20 $L_3(7)$	0.08 $L_3(8)$
3			0.05 $L_3(2)$	0.05 $L_3(3)$	0.01 $L_3(4)$	0.04 $L_3(5)$	0.05 $L_3(6)$	0.02 $L_3(7)$	0.01 $L_3(8)$
4		0.38 $L_3(1)$	0.39 $L_3(2)$	0.44 $L_3(3)$	0.47 $L_3(4)$	0.17 $L_3(5)$	0.02 $L_3(6)$	0.01 $L_3(7)$	0.01 $L_3(8)$

TABLE 5.14: The 9-period loader allocation policy for the second case study, 50% depreciation.

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use the policy as a guide to manage the fleet.

We increased the scale of the problem significantly by including multiple locations in the model. However we were able to solve two case studies for the entire length of the schedule, which is a major achievement. In part, the solvability of this model has been achievable because we eliminated the Big- M constraints that were necessary in the single-location model (Chapter 4). Also, we continued to make use of the lazy constraint technique to add the compatibility constraints, which helped to keep the total number of constraints and the size of the model relatively small.

One major weakness in this model is the symmetry which prevents the branch-and-bound algorithm from swiftly converging on an optimal solution. To remedy this, we would need to define a rule or constraint that allocates equipment in the case of a tie. However, it is difficult to define a such a rule without being able to identify individual equipment. A similar issue is discussed in Chapter 6, although in that case it is simpler to define the rule without undermining the optimisation process.

These multi-location models have selected what appears to be an excessive number of truck types for the optimal solution - industry reports that it is unusual to have more than three types of trucks. This suggests that the models are not reflecting the true penalties associated with the fixed costs of owning equipment. Our data did not contain any cost that would account for auxiliary equipment, spare parts, or the training of artisans for multiple equipment types. These penalties can easily be introduced as a fixed cost, if they are known, or can be estimated from historical data.

However, the optimal solution may still contain multiple truck types after these costs are taken into account. This is because the optimisation process occurs under the assumption that the mining schedule is fixed - the solution typically selects a mix of truck volumes in order to make up the required productivity as closely as possible. Yet we know historically that any schedule is dynamic and subject to change. This prompts a need for good retrospective studies that consider the ancillary costs as accurately as possible (including auxiliary equipment). By considering a series of case studies that have been fully implemented we can analyse the true costs associated with selecting multiple truck types.

As the data for the case studies were limited or unknown (such as the requirement for stockpiles to have their own loaders, and the congruency of mining at each location) the solution sometimes depicts one loader that moves from location to location. This may not be realistic and a penalty can be introduced to prevent such solutions.

It is clearly an important issue to properly account for the utilisation of the equipment - both in terms of the operating cost each period and in terms of tracking the equipment age (when measured in terms of operated hours). If we are interested in accounting for the utilisation of the equipment, then we must account for each piece of equipment individually. While this increases the scale of the problem, it also opens

MULTIPLE-LOCATION MODEL

up avenues for reducing the symmetry of the problem using symmetry-breaking constraints. We build upon these ideas in the following chapter to develop a utilisation cost, multi-period model for a single-location mine.

Chapter 6

Utilisation cost, single-location, multi-period equipment selection

When performing equipment selection, we can best account for the operating cost by considering the utilised hours of the equipment. In a surface mine, equipment is often not utilised to full capacity and not accounting for this difference may lead to inferior solutions. In operations such as this, the cost of operating equipment depends on the age of the equipment while the utilisation of equipment is usually based on the equipment cost. The co-dependency of the age of the equipment and the utilisation has provided a barrier to tractable equipment selection models. That is, equipment is rarely utilised in a linear way, causing the ageing of the equipment (when considering the total hours utilised) to be non-uniform. In our bid to address this issue, we consider a single-location multiple-period mine. We present a mixed-integer linear program that achieves optimal equipment selection and accounts for the equipment utilisation. This model considers pre-existing equipment and allows for heterogeneous fleets. We also introduce linear constraints that relate the utilisation variable to a non-uniform piecewise linear age function. The resulting solution is a purchase and salvage policy for a multiple-period schedule mine, together with an optimised utilisation policy.

6.1 Introduction

In a surface mining operation, the operating cost of equipment is often represented by a *cost per hour*. The cost of running equipment may change with the age of the equipment, usually reflecting maintenance expenses [Figure 6.1]. When accounting for cost in an equipment selection model, we are thus interested in determining the **utilisation** and cumulative utilisation (or the equipment age) so that we may best account for the cost of the equipment.

The models presented in Chapters 4 and 5 have adhered to the assumption of

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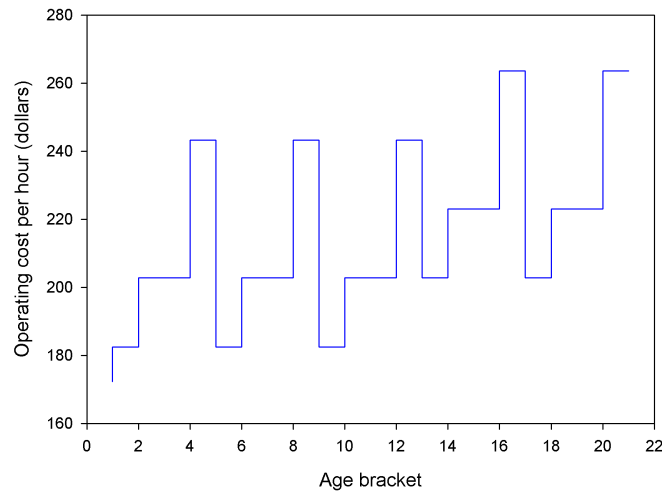


FIGURE 6.1: The variance of equipment operating cost against cost bracket. The rise in operating cost can reflect the increased maintenance expense or the time since the last overhaul. Large drops in the operating expense occur when an overhaul has taken place.

full period utilisation [Section 4.2.1]. By this assumption, if equipment is owned in a particular period then we assume that it was utilised to the fullest possible extent in this period. Clearly it is not ideal to be charging full utilisation if the equipment is not operating to capacity - we will favour the selection of equipment that is slightly cheaper to run for the entire period but not necessarily cheaper to run if charged by utilisation. In addition, in previous chapters this assumption may have been responsible for the selection of multiple truck types in the optimal solutions. This may have been caused by the discrepancy between the fleet productivity levels and the required productivity levels: the optimiser would rather select a small cheap truck to fill a gap than pay for a full period of a more expensive but larger truck.

In this chapter, we discard the assumption of full period utilisation and keep track of how much each equipment is actually utilised. This allows us to bring a utilisation variable into the objective function, thus preventing overstatement of operating cost. By doing this we can:

- estimate salvage value for the current period;
- track how close the equipment is to its enforced retirement age; and
- maintain the equipment according to its maintenance schedule.

However, the mine requirements may change from period to period, so the utilisation of any piece of equipment will change too. This means that the cumulative utilisation of a

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piece of equipment may be a non-uniform piecewise linear function. Thus our challenge is:

To account for utilisation when it is a non-uniform piecewise linear function.

As equipment is discrete, it is appropriate to use integer variables to keep track of the purchase and salvage of trucks and loaders. Ideally, we would therefore like to express all our variables as integers to take advantage of algorithms available to solve a pure integer program. In this vein, we could represent the utilisation of equipment by an integer variable containing the number of hours that that equipment has worked. However, it is more natural to formulate utilisation as the proportion of total available time spent working, which would require a continuous variable.

We know from previous chapters that the productivity constraints are naturally linear and that the cost objective function can be linearised through careful definition of the variable indexes. Therefore, the equipment selection problem with utilisation cost objective can be best expressed as a mixed-integer linear programming model. To help us deal with the complexities of this model, we only consider a single-location mine [Figure 6.2]. We represent a single-location mine by a loading location connected to a dump-site by a single truck route.



FIGURE 6.2: A single-location mine model.

In this chapter, we:

- Consider the inclusion of pre-existing equipment and possibly heterogeneous fleets. If equipment selection is performed part-way through the mining schedule, then a pre-existing fleet must be considered and we must allow the tool to select different types of equipment if the current equipment is obsolete. Neither pre-existing equipment nor heterogeneous fleets have been previously considered in a surface mining equipment selection optimisation model.
- Consider a multiple-period mining schedule. The productivity requirements of the mine and the truck cycle time can both change significantly over time, and we wish to optimise the selected fleet over all time periods rather than considering each period individually. Although this seems like an obvious consideration for optimisation, multiple-period models are not common in the mining literature.

- Introduce a set of linear integer constraints that account for non-uniform piecewise linear ageing of the equipment.

We formalise the model in Section 6.2. First, we list the assumptions associated with our solution (Section 6.2.1). We then describe the decision variables (Section 6.2.2) before deriving the objective function (Section 6.2.3) and constraints (Section 6.2.4). We provide a summary of notation in Section 6.2.5 and then state the complete model (Section 6.2.6). To test the model, we first validate the model with a test case before considering a surface mining case study in Section 6.3 that we solve to optimality over 4 periods and to near optimality for 5 periods. In this same section we also perform sensitivity analysis of the most influential parameters in the model. Finally, we discuss opportunities for extending this work in Section 6.4.

6.2 Problem formulation

6.2.1 Assumptions

The model presented in this chapter is in part an extension of the models presented in Chapters 4 and 5. Therefore we retain all assumptions from Chapter 4 except the assumption of *Full period utilisation* [Section 4.2.1]. The following assumptions apply:

Known mine schedule: We presume that an acceptable mine schedule has already been derived, and that the mining method has been selected. We start the equipment selection process with a sub-set of trucks and loaders that suit the particular mining scenario.

Single mining location: For this model, all the loaders and trucks are considered to operate as one fleet. That is, all loaders work in the same location, and all trucks service all loaders. We consider the mine to have a single mining location, a single dump-site and a single haul route connecting the two.

Salvage: All equipment is salvageable at some depreciated value of the original capital expense. Any pre-existing equipment may be salvaged at the start of the first period.

No auxiliary equipment: Auxiliary equipment, such as wheel loaders and small trucks, are not considered in this model. Although the cost of running auxiliary equipment may differ according to the overall fleet selection, for the purpose of this model we consider this cost to be trivial. Note that the cost of auxiliary equipment can be built into the operating cost if necessary.

Known operating hours: The operating hours of the mine are estimated by taking planned downtime, blasting and weather delays into account.

Single truck cycle time: Since we assume a single mining location, a single truck cycle time is used for all trucks. The cycle time is constant over a period and is known for all periods. It accounts for factors that affect the truck performance, such as rimpull, rolling resistance, haul distance and haul grade.

Heterogeneous fleets: We allow different types of equipment to work side-by-side.

Fleet retention: All equipment is retained at the end of the last period. Optionally, we can relax this assumption and allow the model to salvage some or all equipment at the end of the final period.

Age brackets: We count the age of the equipment in terms of hours utilised. To reduce the number of variables, and to bring the variable structure in line with industry standards, we divide the age into brackets.

6.2.2 Decision variables and notation

Recall that the set of all truck types is \mathbf{X} , and the set of all loader types is \mathbf{X}' . We use i and i' to denote a single truck and loader type, respectively. To simplify the notation, we use e to represent a single type (which can be a truck or loader type), where $e \in \mathbf{X} \cup \mathbf{X}'$.

Fleet purchase and tracking variables

We must manage whole units of equipment. For this purpose we define the following binary variables for tracking the equipment:

$\mathbf{x}_{e,j}^{k,l}$: 0-1 selection of one piece of equipment of type e with identification number j at the start of period k where the equipment is in age bracket l .

We must now track equipment individually, which sets all tracking (and salvage) variables to **binary variables** rather than general integer variables as in previous chapters. We use the index j to track individual equipment which is now necessary to calculate individual equipment age. Equipment can now be identified by two indices, (e, j) .

We would like to use the index l to denote the age of the equipment, in time periods. However, this is impossible, as the equipment age depends on the number of hours the equipment has been utilised, rather than the amount of time since its purchase. One way to do this is by letting l denote utilisation in hours. However, this would lead to a large number of unused variables. Since, in the mining industry, the equipment costings are typically divided into age brackets (such as 5000-hour brackets), we can let l denote the utilisation, counted in age brackets. This reduces the number of variables in the model drastically.

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The utilisation is permitted to change each period, creating a non-uniform piece-wise linear ageing function. As we will see, this complicates the variable transition constraints if we wish to retain linearity in them.

Remark 6.2.1 *Equipment may age in the following two ways: $\mathbf{x}_{e,j}^{k,l} \longrightarrow \mathbf{x}_{e,j}^{k+1,l}$ if the cumulative utilised hours does not exceed the age bracket l at the start of period $k + 1$; $\mathbf{x}_{e,j}^{k,l} \longrightarrow \mathbf{x}_{e,j}^{k+1,l+1}$ if the cumulative utilised hours does exceed the age bracket l at the start of period $k + 1$ [Figure 6.3].*

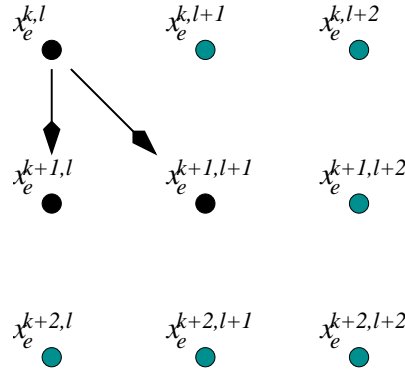


FIGURE 6.3: An illustration of the transition alternatives from period to period. If we own a piece of equipment in age bracket l in period k , then in period $k + 1$ this equipment could be in age bracket l or $l + 1$ depending on how much it was utilised in period k .

Remark 6.2.1 shows that a piece of equipment may stay in the same age bracket or graduate to the next age bracket from one period to another. This changes the way that we need to deal with the constraints, such as *variable transition*, from the previous two models in this thesis.

Fleet salvage variables

We define the salvage variables:

$\mathbf{s}_{e,j}^{k,l}$: 0-1 indicator for equipment with identification number j , of type $e \in \mathbf{X} \cup \mathbf{X}'$ salvaged at the start of period k while in age bracket l .

Note that the equipment operates for another period from the last \mathbf{x} variable before salvage can take place. Consequently, the equipment may move into the next age bracket as the salvage occurs. Alternatively it may be salvaged from the current age bracket [Figure 6.4]. We can cover both cases by saying that a salvage occurs in time period k when $\mathbf{x}_{e,j}^{k-1,l} - \mathbf{x}_{e,j}^{k,l} - \mathbf{x}_{e,j}^{k,l+1} = 1$. For pre-existing equipment only, a salvage at the start of the schedule occurs when $\mathbf{x}_{e,j}^{0,l} = 0$.

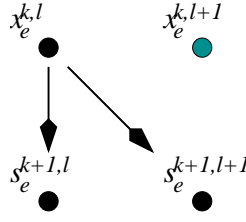


FIGURE 6.4: An illustration of the salvage variable transition alternatives. If we own a piece of equipment in age bracket l in time period k , then we may salvage that equipment in the following period from age bracket l or age bracket $l + 1$, depending on how much the equipment was utilised in period k .

Equipment utilisation variables

We define a continuous variable (restricted to $[0, 1]$) to represent the utilisation of the equipment in terms of proportion of available time.

$\mathbf{f}_{e,j}^{k,l}$: utilised proportion of total time that equipment with identifier j , of type e , in age bracket l will work in period k .

6.2.3 Objective function

As in previous chapters, we wish to minimise the cost of materials handling. Costs can be incurred by capital purchases and operating expenses, while salvaging equipment can create revenue.

First we consider the capital expense, which is a one-off cost at the start of the period. To find out when we have purchased a truck or loader, we look at the tracking variables. For the case $k = 0$ this is straightforward – we simply look at the variable $\mathbf{x}_{e,j}^{0,0}$ (while accounting for any pre-existing equipment that may happen to fall into that age-bracket). However, for $k > 0$ the use of this term may result in overcounting. This is because it is likely for equipment to remain in the $l = 0$ age bracket for more than one period. For example, if the equipment is purchased in period $k = 0$, we may have $\mathbf{x}_{e,j}^{0,0}$, $\mathbf{x}_{e,j}^{1,0}$, and $\mathbf{x}_{e,j}^{2,0} = 1$ for the same e and j .

We solve this problem by counting a purchase, for a truck or loader of type e with identification j , as being made in period k if

$$\sum_l \mathbf{x}_{e,j}^{k,l} - \sum_l \mathbf{x}_{e,j}^{k-1,l} + \sum_l \mathbf{s}_{e,j}^{k,l}$$

is 1. The first term counts if we own the equipment in period k . The second term subtracts any equipment that we already owned. The third term prevents miscounting in the case where equipment was owned in the previous period but not owned in this period. Note that pre-existing equipment does not incur a capital expense as it was purchased at a time not considered in the optimisation period.

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As in previous chapters, we denote the capital expense for equipment type e by F_e , and we discount future costs by multiplying by a net present value discount factor, D_1^k . Thus the total capital expense for a truck or loader of type e with identification j is

$$F_e D_1^0 \mathbf{x}_{e,j}^{0,0} + \sum_{k>0} F_e D_1^k \left(\sum_l \mathbf{x}_{e,j}^{k,l} - \sum_l \mathbf{x}_{i,j}^{k-1,l} + \sum_l \mathbf{s}_{e,j}^{k,l} \right)$$

for all equipment types, e , and identification numbers, j .

To represent the operating expense, we look at the utilisation variables to find the amount of time that each piece of equipment is used. As with previous chapters, we denote the operating expense (per hour) of equipment type e in period k and age bracket l by $V_e^{k,l}$. We also discount the variable expense to the net present value. Therefore, the total operating cost for an equipment of type e with identification j is

$$\sum_{k,l} V_e^{k,l} D_1^k \mathbf{f}_{e,j}^{k,l}.$$

Recall that for the salvage terms we make use of a combined depreciation and net present value discount factor, $D_2^{k,l}$. Since F_e is the original capital expense, the salvage 'cost' associated with equipment of type e and identification j is

$$- \sum_{k,l} F_e D_2^{k,l} \mathbf{s}_{e,j}^{k,l}.$$

Complete Objective Function

We wish to minimise:

$$\begin{aligned} \sum_{e,j} F_e D_1^0 \mathbf{x}_{e,j}^{0,0} + \sum_{e,j,k>0} F_e D_1^k \left(\sum_l \mathbf{x}_{e,j}^{k,l} - \sum_l \mathbf{x}_{i,j}^{k-1,l} + \sum_l \mathbf{s}_{e,j}^{k,l} \right) \\ + \sum_{e,j,k,l} V_e^{k,l} D_1^k \mathbf{f}_{e,j}^{k,l} - \sum_{e,j,k,l} F_e D_2^{k,l} \mathbf{s}_{e,j}^{k,l}. \end{aligned}$$

The first two terms represent the cost of capital outlay for the fleets; the next term represents the operating expense in terms of utilised hours; and the final term denotes the salvage cost of the fleet.

6.2.4 Constraints

Productivity constraints

Recall that the production capability, $P_e^{k,l}$, of a piece of equipment of type e , aged l in period k , is determined by its availability (a_e^l), capacity (c_e) and cycle time (t_e^k). The productivity constraint now deviates from previous chapters where full utilisation was

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assumed. In this model, we account only for the utilised productivity of the equipment, rather than the potential productivity. This relaxation of the *full period utilisation* assumption allows the model to keep equipment on hand without necessarily using that equipment in every period. This gives us the following productivity constraints (where T^k is the total mine productivity requirement for period k , $i \in \mathbf{X}$ and $i' \in \mathbf{X}'$):

$$\sum_{i,j,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq T^k \quad \forall k, \quad (6.1)$$

$$\sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \geq T^k \quad \forall k. \quad (6.2)$$

However, we need only generate these constraints for all loaders if we include the following balance equations:

$$\sum_{i,j,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} = \sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \quad \forall k, i \in \mathbf{X}, i' \in \mathbf{X}'. \quad (6.3)$$

At this stage, there is no obvious benefit for including this set of equations. However, this constraint is useful for reducing the total number of constraints generated by the compatibility constraints in the proceeding section.

Compatibility constraints

We must ensure that the trucks and loaders used in a period are compatible with each other. However, we do not need to make all trucks compatible with all loaders: we must merely satisfy productivity requirements with the set of compatible trucks and loaders.

Again, we let \mathbf{A} represent the set of all possible combinations of loaders:

$$\sum_{i \in X(A'), j, l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i' \in A', j, l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \quad \forall A' \subset \mathbf{X}', k. \quad (6.4)$$

As a power set this will generate $k(2^n - 1)$ constraints. For the 13 period, 27 loader problem this equates to 1744830451 constraints. We recognise that for a given case study the number of loaders possible in the final solution will be much lower than the complete set. In this case we can limit the generation of constraints to a maximum of α loader types. This will produce $k \sum_{a=1}^{\alpha} \binom{n}{a} \frac{n!}{a!(n-a)!}$ constraints. For the 13 period, 27 loader problem with a maximum of 4 loader types, this equates to just 271089 constraints.

Even this quantity of constraints will create computer memory issues for the Cplex optimiser. Therefore we solve the model with only the first level compatibility constraint (where A' is just one loader loader type), and only add the remaining constraints

into the model post hoc if they are violated by the solution before re-solving.

Variable transition

Time periods and age brackets do not necessarily increase in tandem, so we know that whenever a time period is incremented, a piece of equipment may remain in the same age bracket, or it may graduate to the next age bracket. Note that even if it is utilised at full capacity, it cannot increase two age brackets. This is because of the chosen age-bracket size (5000 hours) compared to the maximum utilised hours of the equipment (typically 3500). Taking into account the possibility of salvage, this corresponds to the constraints (where $L^k(e) = \min\{P(e) + k - 1, L(e)\}$ and $P(e)$ is the highest age of pre-existing equipment type e at the start of the schedule):

$$\mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall \quad k > 0, l \in [1, L^k(e) - 1], e, j \quad (6.5)$$

$$\mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall \quad k > 0, l = L^k(e) - 1, e, j \quad (6.6)$$

$$\mathbf{x}_{e,j}^{k-1,l-1} \leq \mathbf{x}_{e,j}^{k,l-1} + \mathbf{s}_{e,j}^{k,l-1} + \mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \quad \forall \quad k > 0, l \in [1, L^k(e) - 1], e, j. \quad (6.7)$$

Constraint (6.5) ensures that we do not own or salvage equipment in period k if we did not at least own the equipment in the previous period $k - 1$. Constraint (6.6) is an ammendment of constraint (6.5) for the variables that lie on the diagonal edge of the variable matrix. Constraint (6.7) ensures that if we own a piece of equipment in period $k - 1$, the we must at least own the equipment in the next period, k , in either the same or next age-bracket, or we must salvage the equipment in the next period, k , in either the same or next age-bracket.

As noted before, we only set one $\mathbf{x}_{e,j}^{k,l}$ to be 1 for any particular piece of equipment and time period:

$$\sum_l \mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq 1 \quad \forall \quad k, e, j. \quad (6.8)$$

Age bracket graduation

The age bracket that a piece of equipment is in, l , is dependent on the accrued utilised hours of the equipment. For this model we represent the operation hours of the mine by O^k , and use age brackets of size B_0 hours.

Remark 6.2.2 *The accumulated utilised hours in age brackets of a piece of equipment at the start of period k is given by*

$$\sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0},$$

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where e is the equipment type and j is the individual equipment identification number.

Remark 6.2.3 If ϵ is an arbitrarily small continuous number and the equipment is in age bracket l at the start of period k (equivalent to $\mathbf{x}_{e,j}^{k,l} = 1$), then

$$l \leq \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \leq l + 1 - \epsilon.$$

Lemma 6.2.1 If we represent the accrued utilised hours of equipment type e with identification j in period k and age bracket l by:

$$\sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0},$$

then

$$l \leq \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \leq l + 1 - \epsilon$$

is true when $\mathbf{x}_{e,j}^{k,l} = 1$ if

$$\sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \geq M \mathbf{x}_{e,j}^{k,l} + l - M \quad \forall \quad k > 0, (e, j) \notin \mathbf{P}, l \quad (6.9)$$

and

$$\sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \leq M + ((l + 1) - M - \epsilon) \mathbf{x}_{e,j}^{k,l} \quad \forall \quad k > 0, (e, j) \notin \mathbf{P}, l, \quad (6.10)$$

where M is a large integer and \mathbf{P} is the set of pre-existing equipment.

Proof: The accumulated utilised hours, $\sum_{h < k, l} \mathbf{f}_{e,j}^{h,l} O^h$, is equivalent to the equipment age in hours for period k . Thus $\sum_{h < k, l} \mathbf{f}_{e,j}^{h,l} O^h \geq 0$. Note that $\mathbf{x}_{e,j}^{k,l}$ is a binary variable.

If $\mathbf{x}_{e,j}^{k,l} = 0$, then (6.9) becomes

$$\begin{aligned} \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} &\geq M \mathbf{x}_{e,j}^{k,l} + l - M \\ &= l - M \\ &\geq -M. \end{aligned} \quad (6.11)$$

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The inequality (6.11) is true since $\sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \geq 0$. (6.10) becomes

$$\begin{aligned} \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} &\leq M + ((l + 1) - M - \epsilon) \mathbf{x}_{e,j}^{k,l} \\ &= M. \end{aligned} \tag{6.12}$$

The inequality (6.12) is true since M is an arbitrarily large integer.

If $\mathbf{x}_{e,j}^{k,l} = 1$, then (6.9) becomes

$$\begin{aligned} \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} &\geq M \mathbf{x}_{e,j}^{k,l} + l - M \\ &= M + l - M \\ &= l. \end{aligned} \tag{6.13}$$

The inequality (6.13) is true from Remark 6.2.3. (6.10) becomes

$$\begin{aligned} \sum_{h < k, l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} &\leq M + ((l + 1) - M - \epsilon) \mathbf{x}_{e,j}^{k,l} \\ &= M + l + 1 - M - \epsilon \\ &= l + 1 - \epsilon. \end{aligned} \tag{6.14}$$

The inequality (6.14) is also true from Remark 6.2.3. ■

Lemma 6.2.1 excludes pre-existing equipment because the cumulative utilisation is calculated slightly differently for this case. However, the age bracket constraints must also hold for pre-existing equipment. If we define $P(e, j)$ for $(e, j) \in \mathbf{P}$ to be the age in hours of the pre-existing equipment of type e and identification j , then at time period k the accumulated utilised hours of this piece of equipment is

$$\sum_{h < k, l} \mathbf{f}_{e,j}^{h,l} O^h + P(e, j).$$

Constraints (6.9) and (6.10) can then be repeated for $(e, j) \in \mathbf{P}$ by replacing the accumulated utilised hours by the above term.

Salvage restriction

Each individual piece of equipment can only be salvaged once:

$$\sum_{k,l} \mathbf{s}_{e,j}^{k,l} \leq 1 \quad \forall \quad e, j. \quad (6.15)$$

We also cannot salvage equipment which is not owned (in the previous period):

$$\mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall \quad k > 0, l > 0, e, j. \quad (6.16)$$

We do not permit an identification number to be re-used once the equipment has been salvaged:

$$\sum_{h < k, l} \mathbf{s}_{e,j}^{h,l} + \sum_l \mathbf{x}_{e,j}^{k,l} \leq 1 \quad \forall \quad k, e, j. \quad (6.17)$$

Finally, we give a value to the unbounded salvage variable:

$$\mathbf{s}_{e,j}^{0,0} = 0 \quad \forall \quad \{e, j\} \notin \mathbf{P}. \quad (6.18)$$

Pre-existing equipment

We set each pre-existing equipment to be either selected or salvaged in the first period:

$$\mathbf{x}_{e,j}^{0,l} + \mathbf{s}_{e,j}^{0,l} = 1 \quad \forall \quad (e, j) \in \mathbf{P}, l = P(e, j). \quad (6.19)$$

Symmetry-breaking constraints

In creating many variables that are identical save for the identification number, j , we have created large clusters of solutions that are effectively permutations of each other. This is not ideal computationally. However, we can eliminate this redundancy if we define some simple rules regarding which identification numbers to use first. We nominate that we would like to keep the equipment with the lowest identification the longest, bringing us to our final constraint for this model:

$$\mathbf{x}_{e,j}^{k,l} \geq \mathbf{x}_{e,j+1}^{k,l} \quad \forall \quad (e, j) \notin \mathbf{P}, k, l.$$

6.2.5 Summary of notation

\mathbf{X}	set of all available truck types.
\mathbf{X}'	set of all available loader types.
i	truck type index, $i \in \mathbf{X}$.
i'	loader type index, $i' \in \mathbf{X}'$.
e	truck or loader type index, $e \in \mathbf{X} \cup \mathbf{X}'$.
j	individual machine identification index.
k	time period index.
l	equipment age bracket index.
M	a large integer.
ϵ	arbitrarily small number.
\mathbf{P}	set of all pre-existing equipment tuples.
\mathbf{A}	power set of all combinations of loader types, $\mathbf{A} = \mathcal{P}(\mathbf{X}')$.
$\mathbf{X}(A')$	power set of all truck types compatible with loader set $A' \subset \mathbf{X}'$.
$P(e)$	highest starting age (in hours) for pre-existing equipment type e .
$P(e, j)$	starting age (in hours) for pre-existing equipment type e with identification j .
$L(e)$	maximum age bracket for equipment type e .
$L^k(e)$	maximum age bracket we need to consider in period k , $L^k(e) = \min\{P(e) + k - 1, L(e)\}$.
$\mathbf{x}_{e,j}^{k,l}$	indicator variable for whether the truck or loader with type e and machine identification j is selected in period k , while in age bracket l .
$\mathbf{f}_{e,j}^{k,l}$	proportion of time that the truck or loader with type e and machine identification j is utilised in period k , while in age bracket l .
$\mathbf{s}_{e,j}^{k,l}$	indicator variable for whether the truck or loader with type e and machine identification j is salvaged at the start of period k , while in age bracket l .
F_e	fixed cost (capital expense) of obtaining equipment type e .
D_1^k	net present value discount factor for period k .
$D_2^{k,l}$	combined net present value discount factor and depreciation factor for period k for equipment aged l .
$V_e^{k,l}$	variable expense for equipment with type e in period k , at age l .
$P_e^{k,l}$	productivity of equipment type e in period k , at age l .
T^k	required productivity of the mine (in tonnes) for period k .

6.2.6 Complete model

$$\begin{aligned} \text{Minimise } \sum_{e,j} F_e D_1^0 \mathbf{x}_{e,j}^{0,0} + \sum_{k>0,e,j} F_e D_1^k \left(\sum_l \mathbf{x}_{e,j}^{k,l} - \sum_l \mathbf{x}_{e,j}^{k-1,l} + \sum_l \mathbf{s}_{e,j}^{k,l} \right) \\ + \sum_{k,e,j,l} V_e^{k,l} D_1^k \mathbf{f}_{e,j}^{k,l} - \sum_{k,e,j,l} F_e D_2^{k,l} \mathbf{s}_{e,j}^{k,l} \end{aligned}$$

subject to

$$\sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \geq T^k \quad \forall k \quad (6.20)$$

$$\sum_{i,j,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} = \sum_{i',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \quad \forall k \quad (6.21)$$

$$\sum_{i \in X(A'),j,l} P_i^{k,l} \mathbf{f}_{i,j}^{k,l} \geq \sum_{i' \in A',j,l} P_{i'}^{k,l} \mathbf{f}_{i',j}^{k,l} \quad \forall A' \subset \mathbf{X}', k \quad (6.22)$$

$$\sum_{h<k,l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \geq M \mathbf{x}_{e,j}^{k,l} + l - M \quad \forall k > 0, (e,j) \notin \mathbf{P}, l \quad (6.23)$$

$$\sum_{h<k,l} \frac{\mathbf{f}_{e,j}^{h,l} O^h}{B_0} \leq M + ((l+1-\epsilon) - M) \mathbf{x}_{e,j}^{k,l} \quad \forall k > 0, (e,j) \notin \mathbf{P}, l \quad (6.24)$$

$$\sum_{h<k,l} \frac{\mathbf{f}_{e,j}^{h,l} O^h + P(e,j)}{B_0} \geq M \mathbf{x}_{e,j}^{k,l} + l - M \quad \forall k > 0, (e,j) \in \mathbf{P}, l \quad (6.25)$$

$$\sum_{h<k,l} \frac{\mathbf{f}_{e,j}^{h,l} O^h + P(e,j)}{B_0} \leq M + ((l+1-\epsilon) - M) \mathbf{x}_{e,j}^{k,l} \quad \forall k > 0, (e,j) \in \mathbf{P}, l \quad (6.26)$$

$$\mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall k > 0, l \in [1, L^k(e) - 1], e, j \quad (6.27)$$

$$\mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall k > 0, l = L^k(e) - 1, e, j \quad (6.28)$$

$$\mathbf{x}_{e,j}^{k-1,l-1} \leq \mathbf{x}_{e,j}^{k,l-1} + \mathbf{s}_{e,j}^{k,l-1} + \mathbf{x}_{e,j}^{k,l} + \mathbf{s}_{e,j}^{k,l} \quad \forall k > 0, l \in [1, L^k(e) - 1], e, j \quad (6.29)$$

$$\sum_l \mathbf{x}_{e,j}^{k,l} \leq 1 \quad \forall k, e, j \quad (6.30)$$

$$\mathbf{f}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k,l} \quad \forall k, l, e, j \quad (6.31)$$

$$\mathbf{s}_{e,j}^{k,l} \leq \mathbf{x}_{e,j}^{k-1,l} + \mathbf{x}_{e,j}^{k-1,l-1} \quad \forall k > 0, l > 0, e, j \quad (6.32)$$

$$\mathbf{s}_{e,j}^{0,l} \leq 1 \quad \forall l > L(e), \in \mathbf{P} \quad (6.33)$$

$$\sum_{h<k,l} \mathbf{s}_{e,j}^{h,l} + \mathbf{x}_{e,j}^{k,l} \leq 1 \quad \forall k, e, j, l \quad (6.34)$$

$$\mathbf{s}_{e,j}^{0,0} = 0 \quad \forall \{e, j\} \notin \mathbf{P} \quad (6.35)$$

$$\mathbf{x}_{e,j}^{0,l} + \mathbf{s}_{e,j}^{0,l} = 1 \quad \forall (e, j) \in \mathbf{P}, e, l \quad (6.36)$$

$$\mathbf{x}_{e,j}^{k,l} \geq \mathbf{x}_{e,j+1}^{k,l} \quad \forall (e, j) \notin \mathbf{P}, k, l \quad (6.37)$$

$$\mathbf{x}, \mathbf{s} \in \{0, 1\}$$

$$\mathbf{f} \in [0, 1]$$

6.3 Computational results

6.3.1 Test case

There are two modes of model validation that we analyse here. In the first instance, we ensure that the model behaves as we expect. Secondly, we show that the model adequately captures the real world problem.

To ensure that the model behaves as we expect, we created a test case that show that the following actually occurs:

- Purchase and salvage can occur in any time period;
- The productivity requirements are met;
- The fleets are sufficiently compatible (subject to productivity requirements);
- The age index, l , is actually capturing the cumulative utilisation;
- Equipment does not age more than one bracket at a time;
- Equipment is salvaged at most once and only if it was owned in the previous period;
- Equipment is not used beyond its retirement age;
- Pre-existing equipment is either kept or immediately salvaged.

Therefore we need the test case to run for such a length of time that some equipment reach the end of their replacement cycle. We must also include pre-existing equipment, and start from a set of trucks and loaders that are not all compatible. To do this, we created a test case with two types of trucks and two types of loaders. We introduced four pre-existing trucks of type 0, one pre-existing loader of type 0, and two pre-existing loaders of type 1. We set up simple mine schedule characteristics such as constant productivity requirements at a relatively low rate of 4 million tonnes per period; and a constant truck cycle time of 10.2 minutes. Also, we simplified the remaining parameters in the model, such as constant operating expense over the entire schedule, and unvarying availability of equipment. Lastly, we reduce the maximum life of the trucks to 35000 hours to capture the replacement cycle of the equipment in the solution.

We programmed the model in *C++* using *Ilog Concert Technology v2.5* objects, and solved the program with default MILP algorithms in *Ilog Cplex v11.0*. We implemented

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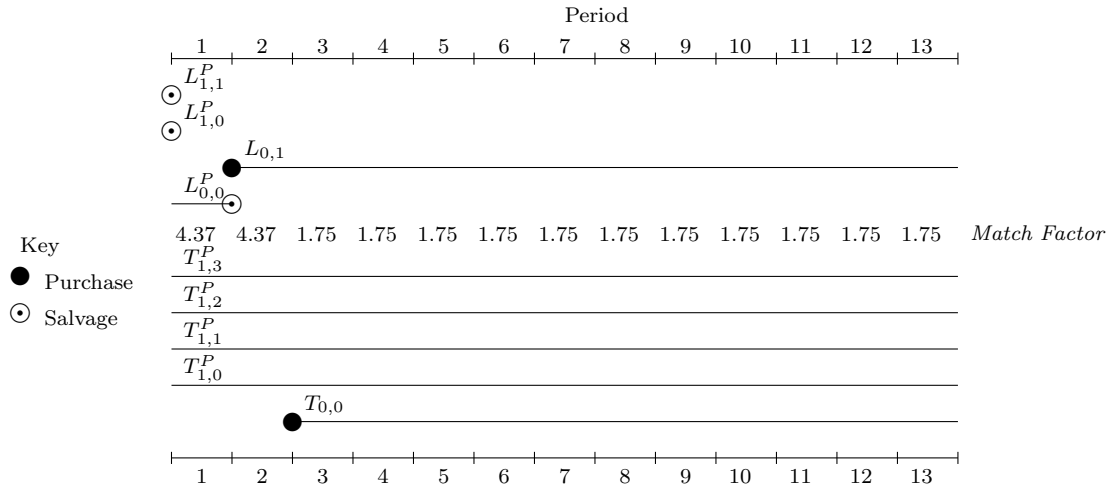


FIGURE 6.5: The optimal purchase and salvage policy for the 13-period validation test case.

the validation problem with 14496 variables and 24280 constraints. We included only two levels of the compatibility power constraint. This is possible because the optimal solution contains only two loader types, so a higher level is redundant. The problem solved using the default Cplex parameters in 1893 seconds (32 minutes). In the solution, one new loader was purchased at the beginning of period 2 and one new truck was purchased at the beginning of period 3 [Figure 6.6]. All pre-existing trucks are kept for the entire schedule; the two pre-existing loaders (type 1) are salvaged at the beginning of the schedule while the loader of type 0 is retained for only one period.

In Table 6.1 we sum the utilised hours of the new truck to represent the age of the equipment over the accumulating periods. By dividing by the size of the bracket (5000), we calculate the age of the truck in terms of age brackets. By comparing this value to the solution value, we can determine whether the implemented model is behaving as we expect. We can see that these values match, which indicates that the age bracket index, l , is effectively capturing the cumulative utilisation of the equipment. We can also see that, as we expect, the equipment never ages more than one bracket at a time. Thus we are satisfied that we have implemented the model correctly.

6.3.2 Case study

We implemented this model on the multi-period surface mining case study studied in Chapter 4. We start by analysing the first four periods of this thirteen period case study. The 4-period model contained 65940 variables and 119656 constraints before the higher level compatibility constraints were added. After performing preliminary tests, we chose a depreciation value of 53% which obtained the fastest solution time. The *Cplex* optimiser found the optimal solution in 1582 seconds (27 minutes) with an

$T_{0,0}$	Period												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Actual age (hours)	0	0	2299.46	4598.92	6898.37	9197.83	11497.3	13796.7	16096.2	18395.7	20695.1	22994.6	25294
Actual age (brackets)	0	0	0	0	1	1	2	2	3	3	4	4	5
Solution age (brackets)	0	0	0	0	1	1	2	2	3	3	4	4	5

TABLE 6.1: The validation of the matching of ageing index with actual cumulative age.

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overall fleet cost of $\$5.34279e^7$. We analyse this solution first. After the initial solve, no higher level compatibility constraints were found to be breached and the solution was accepted as optimal. The solution converged to within 3% of optimality after just one minute of computation, taking the remaining 26 minutes to close this small gap [Figure 6.6].

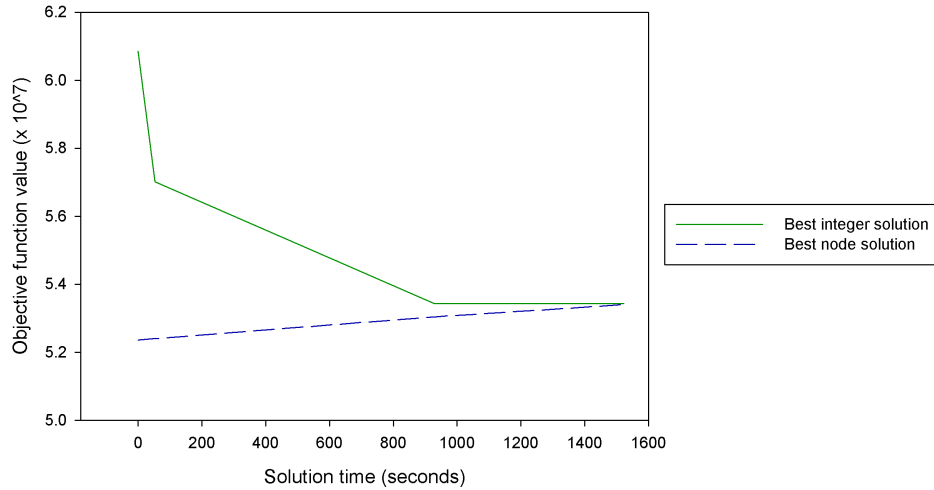


FIGURE 6.6: Convergence of the 4-period utilisation cost model, with the depreciation parameter set to 53%.

In the optimal solution, all the pre-existing loaders were salvaged: reflecting both their age and their low compatibility with other “cheap” trucks [Figure 6.7]. Two new loaders were purchased: one loader of type 3 in period 1, and one loader of type 25 in period 2. The presence of only two loader types in the solution calls a need for up to only two levels of compatibility constraint. In the truck fleet, three types were selected overall: the type-12 pre-existing trucks were kept for the entire four periods; one of the medium-sized type-8 trucks was purchased in period-1 and nine of the super-sized type-9 trucks was purchased in period-2. This reflects both the increasing demands of the production schedule and increased truck cycle times, and the declining performance of the older pre-existing trucks.

One note-worthy aspect of this solution is that there are still three truck types being selected. As an optimal solution, this goes against the intuition of the mining engineer, who has a greater sense of the hidden costs of operating multiple types of equipment. Unfortunately these costs were not visible to the model, and this is reflected in the types of solutions it finds. This clearly reveals a need for better cost estimates for heterogeneous fleets.

The match factor for each period reveals that the fleet is well balanced, and that the truck fleet has some slackness. Again, this reflects the greater cost of running the

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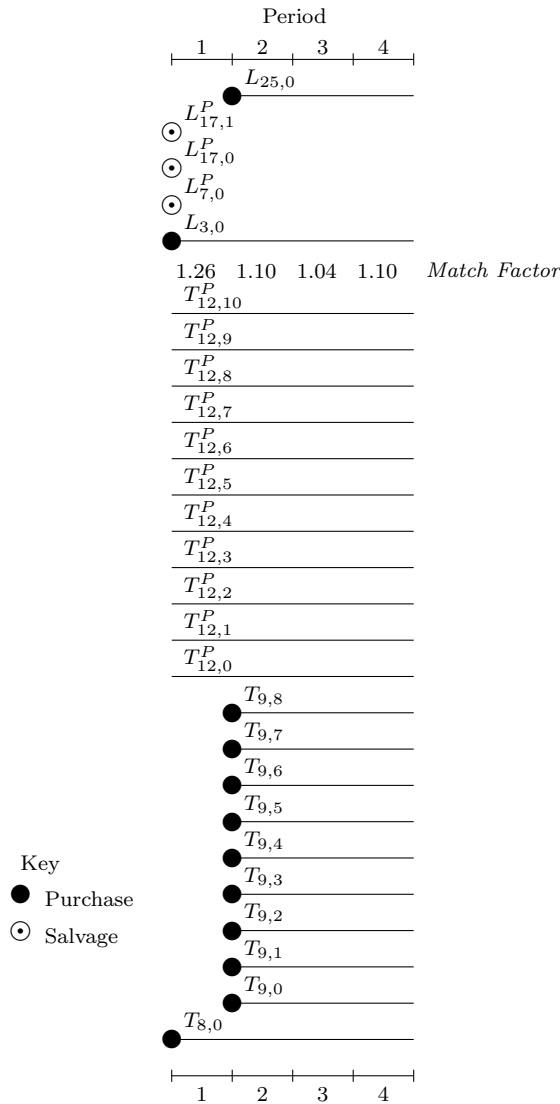


FIGURE 6.7: The optimal solution for the 4-period case study with the utilisation model. The depreciation parameter is set to 53%.

loaders over the trucks.

An important part of the way we modelled this problem is the optimisation of the utilisation of the equipment [Table 6.2]. That is, as we are accounting for the way that the equipment is used in each period, we are able to optimise the allocation of equipment to tasks. This means that in the solution we also obtain an optimised utilisation policy. In this policy it is clear to see which pieces of equipment are not fully utilised. This provides the mining manager with some flexibility, and a clear indication of where these flexibilities occur in the fleet.

We implemented the 5-period problem with 95550 variables and 174037 constraints. After 70926 seconds (19.7 hours) of computation we obtained a solution with optimality

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<i>Equipment</i>	<i>Period</i>			
	1	2	3	4
$T_{8,0}$	1	1	1	1
$T_{9,0}$	0	1	1	1
$T_{9,1}$	0	1	1	1
$T_{9,2}$	0	1	1	1
$T_{9,3}$	0	1	1	1
$T_{9,4}$	0	1	1	1
$T_{9,5}$	0	1	1	1
$T_{9,6}$	0	1	1	1
$T_{9,7}$	0	1	1	1
$T_{9,8}$	0	1	1	1
$T_{12,0}$	0.935386	0.4821	0	1
$T_{12,1}$	1	0	1	1
$T_{12,2}$	1	0	1	1
$T_{12,3}$	1	0	1	1
$T_{12,4}$	1	0	1	1
$T_{12,5}$	1	1	0	0.919593
$T_{12,6}$	1	1	0.232463	1
$T_{12,7}$	1	0.0112244	1	1
$T_{12,8}$	1	1	1	1
$T_{12,9}$	1	0.0112244	1	1
$T_{12,10}$	1	1	1	1
$L_{3,0}$	0.852581	1	1	1
$L_{25,0}$	0	0.0772999	0.709341	0.89751

TABLE 6.2: The optimal utilisation policy for the 4-period case study, when the depreciation parameter is set to 53%.

gap of 1.10%, with an objective function value of $\$7.09709e^7$. In terms of a mining operation where some aspects of the schedule are expected to change over time, this is not an unreasonable gap. We observe that the final solution (with optimality gap 1.10%) was found after 17647 seconds, although it took a further 60000 seconds to close the remaining 0.4% [Figure 6.8].

It is interesting to observe the differences between the 4-period solution and the 5-period solution [Figure 6.9]. Instead of salvaging all the pre-existing loaders, we opt to keep the loader of type 17 and purchase two new type-2 loaders over the first two periods. This is related to the choice to purchase a fleet of type-15 trucks in the first period while keeping all the pre-existing trucks of type 12 - in the 4-period optimal solution, no type-15 trucks were purchased at all. This highlights the need to optimise over the entire schedule, as the solutions for shorter periods can be quite different and lead to worse long-term solutions.

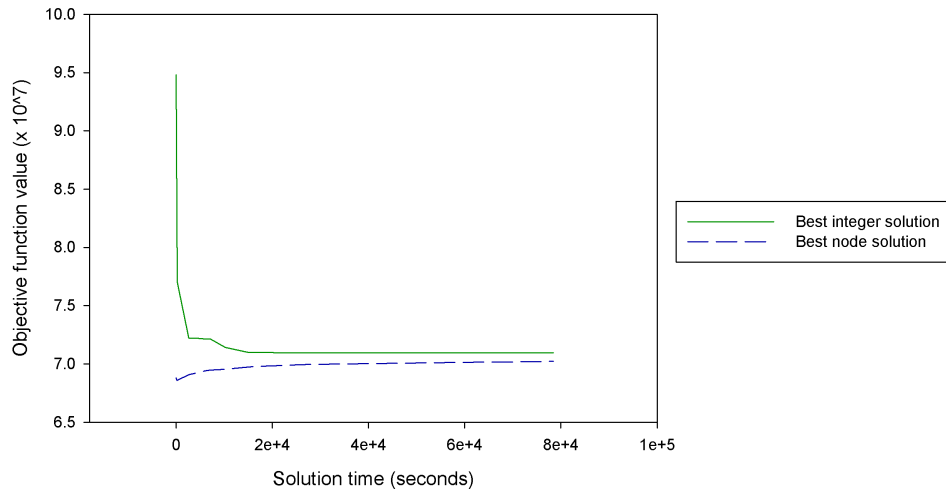


FIGURE 6.8: Convergence of the 5-period utilisation cost model, with the depreciation parameter set to 53%.

<i>Periods</i>	<i>Depreciation (%)</i>	<i>Time (seconds)</i>	<i>Quality</i>	<i>Solution</i>
4	35	2444	Optimal	5.33485×10^7
4	40	2608	Optimal	5.33968×10^7
4	45	2457	Optimal	5.34175×10^7
4	50	2717	Optimal	5.34258×10^7
4	51	2721	Optimal	5.34266×10^7
4	52	2274	Optimal	5.34273×10^7
4	53	1588	Optimal	5.34279×10^7
4	54	1764	Optimal	5.34284×10^7
4	55	1985	Optimal	5.34287×10^7
4	60	2386	Optimal	5.34297×10^7

TABLE 6.3: The 4-period case study solutions with varying depreciation.

6.3.3 Sensitivity analysis

The parameters in this model are subject to uncertainty in two ways. Some parameters are known to be estimates, such as the depreciation of equipment; some parameters are certain to change once the schedule begins and new information comes to light, such as truck cycle time and productivity requirements. In our sensitivity analysis, we are interested in the influence of these uncertain parameters on the robustness of the model. We begin by studying the behaviour of the model at differing values of depreciation [Table 6.3].

We observe an interesting phenomenon here where there exists a critical value near 53% where the solution time drops drastically [Figure 6.10]. However, the purchase and

UTILISATION COST MODEL

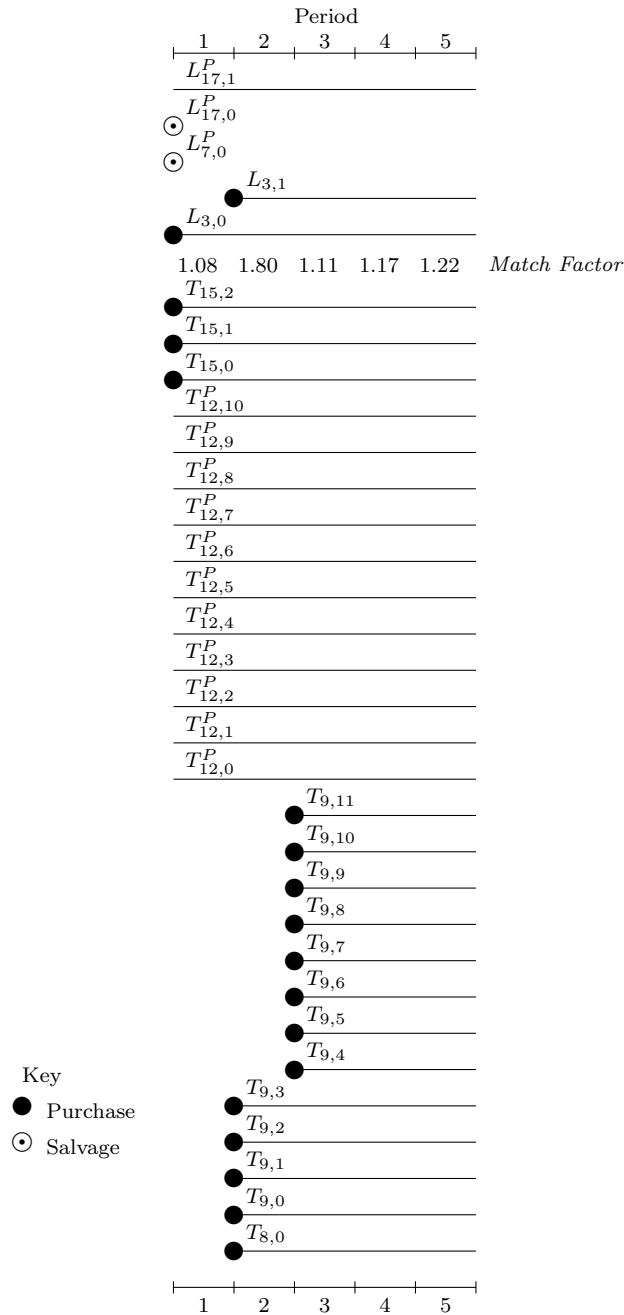


FIGURE 6.9: The solution for the 5-period case study with optimality gap 1.10% for the utilisation model. The depreciation parameter is set to 53%.

salvage policies are identical across all depreciation values tested. Instead of differing fleets, this critical point marks the decision to significantly alter the utilisation of several pre-existing trucks [Table 6.4]. Interestingly, the objective function values for the 4-period case study converge as the depreciation value increases [Figure 6.11].

Next we study the influence of truck cycle time on the solution when the production requirements are uniform over the entire schedule to 36 million tonnes per period.

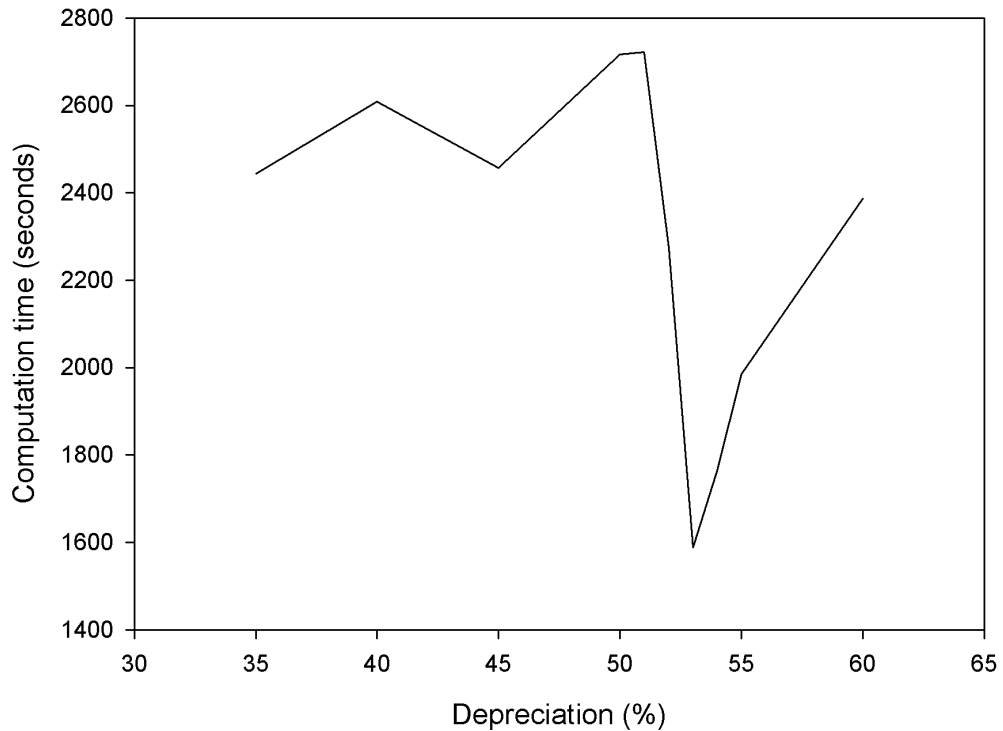


FIGURE 6.10: The varying solution times with depreciation for the 4-period case study.

Thus we expect any variability in the fleet to be exclusive to two factors: ageing of the equipment and truck cycle time. In a 4-period solution, the effects of ageing are not apparent [Figure 6.12]. This test case solved in 1576 seconds, demonstrating that a simplified production schedule does not necessarily simplify the problem. The decreasing slackness in the utilisation policy hints at the increasing truck cycle time, but otherwise the fleet is fairly stable [Table 6.5]. In fact, with less volatility in the schedule, the purchase policy is also more stable than the case study solution - only one loader type is used.

We repeat this experiment for productivity requirements by flattening the truck cycle time over the entire schedule and observing the behaviour of the solution. We set the truck cycle times over the four periods to 30.75 minutes. In this case, we expect the purchasing behaviour to closely reflect the demands from the productivity requirements.

We obtained the optimal solution to this test case in 5781 seconds (97 minutes) [Figure 6.13]. This indicates the increased complexity of the problem when the production requirements vary from period to period - there are many options to increase the size of the fleet to satisfy a small change in productivity, and many of these options still will be heterogeneous fleets. This is further confirmed by observing the level of

UTILISATION COST MODEL

<i>Equipment</i>	<i>Period</i>			
	1	2	3	4
$T_{8,0}$	1	1	1	1
$T_{9,0}$	0	1	1	1
$T_{9,1}$	0	1	1	1
$T_{9,2}$	0	1	1	1
$T_{9,3}$	0	1	1	1
$T_{9,4}$	0	1	1	1
$T_{9,5}$	0	1	1	1
$T_{9,6}$	0	1	1	1
$T_{9,7}$	0	1	1	1
$T_{9,8}$	0	1	1	1
$T_{12,0}$	1	0	1	0.919593
$T_{12,1}$	1	0	0.210014	1
$T_{12,2}$	1	0	1	1
$T_{12,3}$	0.935386	0.4821	0	1
$T_{12,4}$	1	0	1	1
$T_{12,5}$	1	0.0112244	1	1
$T_{12,6}$	1	1	1	1
$T_{12,7}$	1	0.0112244	1	1
$T_{12,8}$	1	1	1	1
$T_{12,9}$	1	1	0.0224488	1
$T_{12,10}$	1	1	1	1
$L_{3,0}$	0.852581	1	1	1
$L_{25,0}$	0	0.0772999	0.709341	0.89751

TABLE 6.4: The utilisation policy for the 4-period case study with 53% depreciation.

heterogeneity in the fleet: two loader types and four truck types. The changes in the utilisation policy can be clearly noted to follow the changes in the production schedule [Table 6.6]. That is, as the actual productivity requirements increase, so too does the utilisation of loaders until the fourth period where almost all equipment is used fully.

Other parameters that have great influence on the scale of the problem include:

- number of available truck types
- number of available loader types
- number of periods
- number of identification numbers per equipment type
- ϵ (the chosen tolerance value)
- number of age brackets (which corresponds to the Big- M value in the model)

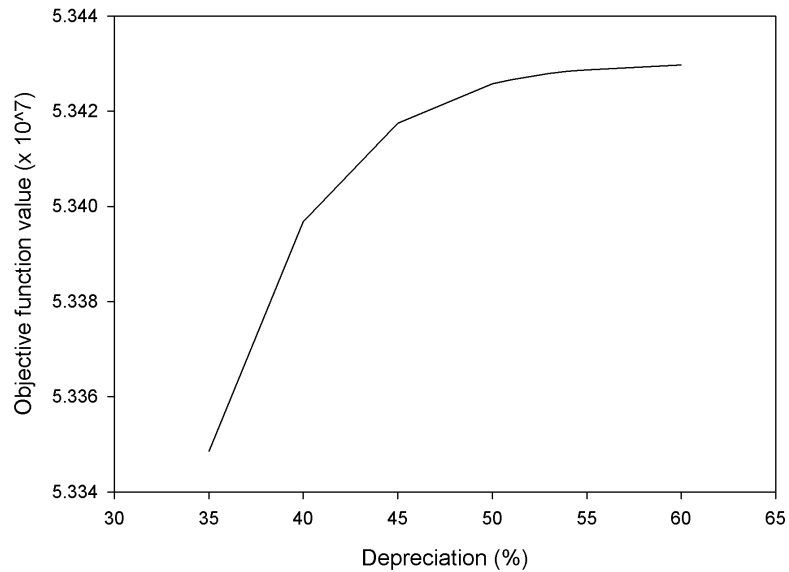


FIGURE 6.11: The varying objective functions with depreciation for the 4-period case study.

Clearly, reducing the overall number of trucks, loaders and periods will improve the tractability and computation time of the model. As we observed with the 5-period problem, increasing the scale of the problem by increasing the number of periods, trucks or loaders only renders the problem too large to solve. We can reduce the number of identification numbers per equipment type to the optimal number and gain significant reductions in computation time. However, to do this we need to know the optimal number of identifications *a priori*, and this information is only gleaned after solving the problem. Before solving the problem, we must be cautious that we do not over-constrain the problem by setting the number of identifications at too low a level.

We expect the value of ϵ to have an impact on computation time (with a noticeable effect on solution only with large values of ϵ). We also expect that there is a critical value of ϵ that relates to the stability of the solution - as it is used in the age-bracket constraint, ϵ is scaled by the size of the age bracket (in this case study, 5000). We are therefore interested in studying the behaviour of the model as ϵ decreases in size. We do this by observing the changes in computation time *and* objective function value as we reduce ϵ . Figure (6.14) depicts the predicted unstable behaviour, before the solution stabilises at $\epsilon = 0.000000001$ [Table 6.7].

The Big- M value needs only be large enough to guarantee the validity of the age-bracketing constraints. Therefore, the number of age brackets corresponds to the Big- M value in the model. While this number is fixed by the policies of the mine, we are interested in studying the robustness of the model subject to different age bracket

UTILISATION COST MODEL

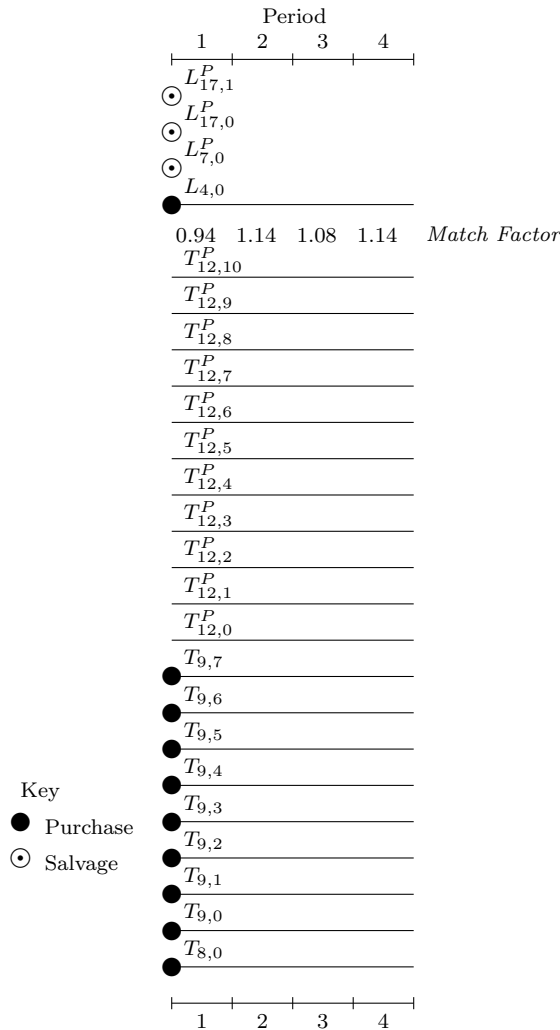


FIGURE 6.12: We flatten the productivity requirements for the 4-period case study to observe the fleet changes due to ageing and truck cycle time.

values. To do this we vary M size in increments of 5. We expect the computation time to increase with the size of age bracket, reflecting the increased difficulty in solving the linear program associated with the Big- M value. Again we study the problem with depreciation set to 53% and ϵ set to 0.00000001. We vary the size of M in four subsequent tests from 15 to 30 [Table 6.8].

We can see through the analyses of these uncertain parameters that the model is reasonably robust - the solutions themselves are stable and the main aspect affected by the uncertainty is computation time.

Other parameters subject to less variability (and are therefore not considered as interesting for our analysis) are:

- NPV discount rate

UTILISATION COST MODEL

<i>Equipment</i>	<i>Period</i>			
	1	2	3	4
$T_{8,0}$	1	1	1	1
$T_{9,0}$	1	1	1	0.829834
$T_{9,1}$	1	1	1	1
$T_{9,2}$	1	1	1	1
$T_{9,3}$	1	1	1	1
$T_{9,4}$	1	1	1	1
$T_{9,5}$	1	1	1	1
$T_{9,6}$	1	1	1	1
$T_{9,7}$	1	1	1	1
$T_{12,0}$	0.0224488	1	1	1
$T_{12,1}$	0.0224488	1	1	1
$T_{12,2}$	0.705255	1	0.317194	1
$T_{12,3}$	0.0224488	1	1	1
$T_{12,4}$	0.179235	0.843213	1	1
$T_{12,5}$	1	1	0.252479	1
$T_{12,6}$	1	1	1	1
$T_{12,7}$	1	1	1	1
$T_{12,8}$	1	1	1	1
$T_{12,9}$	1	1	1	1
$T_{12,10}$	1	1	1	1
$L_{4,0}$	0.998269	0.998269	0.998269	0.998269

TABLE 6.5: The utilisation policy for the 4-period problem with flattened productivity requirements.

- operating hours of the mine
- compatibility of equipment
- availability of equipment
- fixed cost of equipment
- equipment capacity
- operating cost
- maximum age of equipment

6.4 Discussion

In this model, we observe some interesting results. Initially we expected that by accounting for utilisation in the objective function we would eliminate the tendency to

UTILISATION COST MODEL

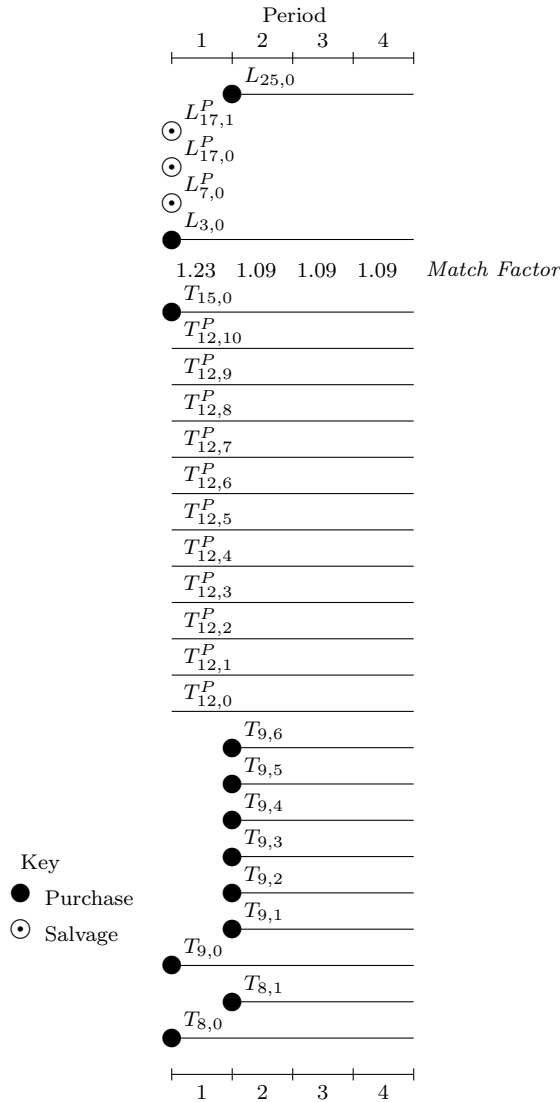


FIGURE 6.13: We flatten the truck cycle times in the 4-period case study.

purchase multiple truck types. However, as we saw with the case study results, this is not the case. This suggests that high volatility in production requirements is best dealt with high heterogeneity - a fleet with mixed sizes is more flexible to changes in demands than a fleet of all the same size. This makes sense analogously if we consider that two 50 cent pieces cannot be used to closely match prices that are higher than 50 cents and less than 1 dollar in the same way that one 50 cent, two 20 cent and one 10 cent coins can.

As the equipment operating cost is now calculated on hours used, it is no longer necessary for owned equipment to operate in every period as before. This reduces the need to purchase equipment for short periods of time and then salvage it, resulting in a more accurate model than the single-location model presented in Chapter 4.

An interesting observation was the critical phenomena for the depreciation param-

UTILISATION COST MODEL

<i>Equipment</i>	<i>Period</i>			
	1	2	3	4
$T_{8,0}$	1	0.0112244	1	1
$T_{8,1}$	0	1	1	1
$T_{9,0}$	1	0.0112244	1	1
$T_{9,1}$	0	1	1	1
$T_{9,2}$	0	1	1	1
$T_{9,3}$	0	1	1	1
$T_{9,4}$	0	1	1	1
$T_{9,5}$	0	1	1	1
$T_{9,6}$	0	1	1	1
$T_{12,0}$	0.697495	0	1	0.90094
$T_{12,1}$	1	0.98417	0.0382786	1
$T_{12,2}$	1	0	1	1
$T_{12,3}$	1	0	1	1
$T_{12,4}$	1	0	1	1
$T_{12,5}$	1	1	1	1
$T_{12,6}$	1	1	1	1
$T_{12,7}$	1	1	1	1
$T_{12,8}$	1	1	1	1
$T_{12,9}$	1	1	0.427323	1
$T_{12,10}$	1	1	1	1
$T_{15,0}$	1	1	1	1
$L_{3,0}$	0.852581	1	1	1
$L_{25,0}$	0	0.0772999	0.709341	0.89751

TABLE 6.6: The utilisation policy for the 4-period test case with flattened truck cycle times.

eter - corresponding with a shift in utilisation policy. This result, along with other sensitivity analysis, demonstrates that the model is robust to uncertainty, and that the main factor affected by uncertainty is computation time - a pleasing result.

It is clear that the problem of equipment selection for surface mines is not trivial due to the dependencies of costs on the age of the equipment, which may be non-uniform. We had some success in improving the solvability of our model by introducing symmetry-breaking constraints and by with-holding higher-level compatibility constraints as lazy constraints. However, for this model to be useful in practice we would like to see greater scalability and the potential to consider large mining schedules (with up to, say, 20 periods). Greater scalability would enable us to consider a longer number of mining and dumping locations: allowing us to solve the model in a more realistic setting.

In spite of this shortcoming, we are satisfied that we adequately captured the non-uniform piece-wise linear ageing of the equipment within linear constraints. Further,

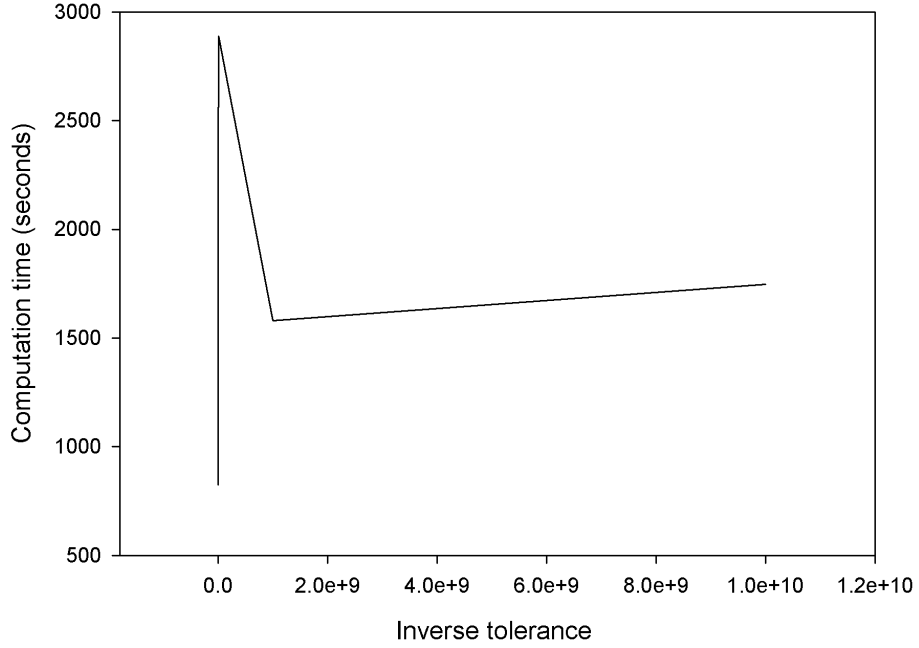


FIGURE 6.14: The varying solution times as we vary the tolerance values (note that the tolerance values are inverted here).

<i>Periods</i>	ϵ	<i>Time (seconds)</i>	<i>Quality</i>	<i>Solution</i>
4	1e-10	1748	Optimal	5.34279×10^7
4	1e-9	1579	Optimal	5.34279×10^7
4	1e-8	1538	Optimal	5.34279×10^7
4	1e-7	2889	Optimal	5.34279×10^7
4	1e-6	2232	Optimal	5.34279×10^7
4	1e-5	2314	Optimal	5.34279×10^7
4	1e-4	824	Optimal	5.34279×10^7
4	1e-3	2080	Optimal	5.3428×10^7

TABLE 6.7: The variance of computation times and objective function value when we vary ϵ for the 4-period case study solutions with depreciation at 53%.

we have presented a model that considers pre-existing equipment and heterogeneous fleets, allows equipment to be salvaged, and outputs a utilisation policy, and a purchase and salvage policy. This model is robust to the uncertainty of a mining schedule, and provides the mining manager with a set of optimal decisions based on the best available information at the time of solution.

UTILISATION COST MODEL

<i>Periods</i>	<i>Cost-brackets</i>	<i>Time (seconds)</i>	<i>Quality</i>	<i>Solution</i>
4	15	10801	Optimal	5.12685×10^7
4	20	11154	Optimal	5.12685×10^7
4	25	13065	Optimal	5.12685×10^7
4	30	13065	Optimal	5.12685×10^7
4	40	5057	Optimal	5.12685×10^7

TABLE 6.8: The variance of computation times and objective function value when we vary ϵ for the 4-period case study solutions with depreciation at 53%.

Chapter 7

Conclusions

This thesis addressed the equipment selection problem from an optimisation perspective. Our first contribution was the collation and organisation of a large body of literature (Chapter 2). The edges of mining method selection and equipment selection have been blurred in the literary evolution. We categorised the literature based on the actual achievements and types of solutions obtained. Furthermore, we brought together the relevant works from shovel-truck productivity and surface mining equipment selection. Previously these two streams have seen little inter-referencing.

Pre-existing equipment is common in mines when further equipment selection is required and it seems a short logical step to include this equipment in the process. A major contribution of this work begins with the consideration of such equipment. The inclusion of older trucks and loaders which may be discontinued or superseded will inevitably lead to heterogeneous fleets. In Chapter 3 we extended the match factor ratio to heterogeneous fleets. While the homogeneous match factor ratio has been studied to some extent, this heterogeneous ratio is original.

We presented three well behaved integer programming models for equipment selection. All of these models force the retirement of equipment that has reached its maximum age. This is particularly important when considering pre-existing equipment, or long schedules that exceed the lifetime of new equipment. The first of these models, in Chapter 4, considered a single-location, multi-period mining schedule with pre-existing equipment. This model assumed full period utilisation and generalised productivity requirements in a bid to achieve a fast and general equipment selection tool. An innovative aspect of this model is the satisfaction of production requirements under compatible fleets. This model is useful for mines that work all owned equipment all the time as a policy, and whose mined area is small or localised.

In Chapter 5 we presented an extension to this model by including multiple locations. This allows the consideration of multiple dumpsites, multiple routes and multiple mining locations. Consequently, this model also provides an allocation policy. In spite

CONCLUSIONS

of the significant increase in the scale of the problem (by the introduction of multiple periods), we achieved near optimal solutions for practical sized mining schedules using the default settings in Cplex.

The models in Chapters 4 and 5 both assume full period utilisation. We relax this assumption in Chapter 6 where we present an MILP for single-location, multi-period, utilised cost based equipment selection. This model incorporated several novel constraints that account for the non-uniformity in the relationship between the periods of the mine and the rate of age of the equipment. Although we were only able to achieve optimal solutions for 5 period problems, this model can be used to provide very detailed allocation policies for a four-term one-year schedule or simply to improve on the accuracy of the results in previous models for a 5-period (5-year) schedule.

During the course of this work we identified several problems of interest that extend the work completed here. An important problem arising from this collection of work is the combined mining method selection and equipment selection problem.

Problem 1 *Seek an optimisation model for the combined mining method selection and equipment selection problem.*

Given the influence of the pit optimisation on both mining method selection and equipment selection, one could consider mining method selection, pit optimisation and equipment selection performed together.

Problem 2 *Seek an optimisation model for the combined pit optimisation, mining method selection and equipment selection problem.*

In a similar vein, we may be able to improve global solutions by creating the production schedule at the same time as selecting the equipment. This may help to reduce the short life cycle of the equipment that was evident in our solutions.

Problem 3 *Seek an optimisation model for the combined production scheduling and equipment selection problem.*

The effect of truck-loader interactivity on efficiency is not well understood in the industry, and has the potential to underpin the optimisation process. We discussed that queuing and bunching is important for the accurate estimation of cycle times. Sound queuing and bunching models would therefore be a valuable asset for the mining industry.

Problem 4 *Develop a general queuing and bunching model for the estimation of truck cycle time.*

Often the queuing and bunching of trucks is dependent on the types of equipment and also the fleet size. It may be possible that such a model can be integrated with an equipment selection model.

CONCLUSIONS

Problem 5 *Develop an integrated queuing, bunching and equipment selection model.*

A potential drawback in both of these models is the selection of multiple truck types in the optimal solutions. It is not known if such solutions could prove more costly if the mining schedule changes significantly. To study this we should compare retrospective case studies with these optimal solutions over a fully implemented mining schedule. Unfortunately data of this quality was not available for the research presented in this thesis.

Problem 6 *Determine the relative costs associated with selecting multiple truck types over the full dynamic schedule of a surface mine.*

The importance of accurate cost data cannot be over-emphasised. Equipment with different levels of accuracy in the data are incomparable. Equipment with poor data should not be included, as the results could be devastating if this equipment were selected and the costs increased significantly.

This model may also be extended to multi-locations, although if the same concepts as were developed here are used this would require an exorbitant number of variables.

Problem 7 *Extend the utilised cost based equipment selection model to multiple locations.*

The work we have achieved in this thesis is clearly only the beginning of an optimisation approach for this challenging, large-scale, mining problem.

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