Probabilistic Assessment of Financial Risk in E-Business Associations

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***Abstract*— Business activities are a result of carefully formed associations between different users in order to achieve certain pre-decided outcomes. Decision making in such associations is an important step and transactional risk analysis is one of the integral processes that facilitates this step. This paper presents an approach that determines the negative consequences (termed as financial risk) of forming e-business associations. Unlike other approaches, our model captures the different types of events and their uncertainties to determine the financial risk by using the convolution operator and expressing it as a probabilistic measure rather than as a crisp financial value. Such representation makes sense as the financial risk may be determined at a point of time in future where nothing is certain. Depending upon the complexity of the problem, we explain the different ways of using the convolution operator to determine the financial risk. The simulation result shows a better representation and understanding of the financial risk that will provide important inputs to the transactional risk analysis and the decision-making process.**

***Index Terms*—Transactional Risk, Performance Risk, Financial Risk, e-business, Dependent Events, Non-Dependent Events.**

I. Introduction

Current business trends and statistics show the growth of e-business or doing business electronically in the B2C and C2C interaction paradigm. Some of the primary factors responsible for this increase may be an improvement in the user's efficiency and productivity due to the ease with which activities can be carried out and tasks completed in less time, and a reduction of the delays associated with the conventional business methods. But apart from the benefits, business associations in such paradigms also produce concerns or the fear of loss in various aspects such as privacy of personal data, unauthorized use of personal information, secureness of the interaction medium, genuineness of the other user, successful completion and achievement of outcomes in the business association etc. Such fear of loss is termed 'risk' in the business association. Depending upon the type of loss being discussed, risk can be specialized so that its impact can be represented correctly according to the object of discussion. For example, the fear of the secureness of the interaction medium can be termed as security risk, the loss in personal information as privacy risk and there may be other types of risks such as legal risk, strategic risk, operational risk that may be possible in the business association according to the perspective being taken. In today's competitive business world, the aim of each user would be to avoid any such losses, while simultaneously achieving maximum returns and benefits. This is achieved by making an informed interaction-based decision, one step of which is risk analysis. Risk analysis is a process whereby the different types of risks are identified, their impacts assessed, and then managed up to a certain acceptable level of tolerance that may result in the successful completion of the business association. Various researchers in the literature have attempted to study the different types of risks and their impact on the successful completion of the business activity [1, 2]. But an important point to understand is that even though all these different types of risks may relate to the same business association, the analyses they represent are vastly different from each other, subsequently varying also the process of their analysis.

In this paper, we discuss transactional risk, which is one type of risk that should be considered when making a decision about initiating a business association. It is mentioned in the literature that the decision to buy is related to the cost-benefit analysis [3] and subsequently, transactional risk which indicates the chances of failure and the negative consequences of engaging in the business association, will be one of the key drivers in influencing the behavior of a user when engaging in a business activity. Hence, approaches are needed whereby these factors are identified, assessed and utilized for making an informed interaction-based decision. Our focus in this paper is on one sub-category of transactional risk, namely, the negative consequences or loss of engaging in a business association, which we term 'financial risk'. In this sub-category we focus on the risk identification and risk assessment phase in the process of risk analysis and propose a novel probabilistic-based approach to ascertain the financial risk in the business interaction. Such analysis is an important prerequisite to ascertain the beforehand the transactional risk in forming a business association. This paper is organized as follows: In Section 2, we discuss the importance of the analysis of transactional risk and differentiate it from the analysis of security and trust when making a decision about forming a business association. We then briefly discuss our previous work on transactional risk analysis and define the problem that is addressed in this paper. Related work from the literature according to the problem statement is then discussed. In Sections 3-5, we propose an approach to capture the different types of uncertain events and ascertain the financial resources to be kept at stake as a result of that. In Sections 6-7, we present our approach for determining the financial risk in forming a business association. In Section 8, we conclude the paper.

II. Problem Statement

A. Importance of considering Transactional Risk while forming an Informed Business Association

The outcome of a business association can be broadly classified into three types, which are (a) Positive (b) Neutral and (c) Negative. A positive outcome is when the interaction initiating user obtains more from its interaction than what it initially expected to achieve. A neutral outcome is when an interacting user gets exactly what it was expecting to achieve in its interaction; whereas, negative outcome is when the interacting user does not get what it was expecting to achieve as an outcome of its interaction. A business association may be formed between any two users in order to achieve an outcome which meets a combination of different criteria or factors, each of which will play a part in the successful completion of the business association. But each or some of these criteria may have uncertainty associated with them that may result in either their non-occurrence or occurrence not as desired, thereby producing a negative outcome. An analysis of Transactional Risk while decision-making determines the uncertainty that will produce a negative outcome in the interaction, and the impact or consequences that may be caused as a result of the desired outcomes not being achieved. Such an analysis of risk will significantly impact on the interacting user's informed decision-making about a business interaction. But by analyzing the literature, we note that despite its importance, the concept of transactional risk has often been confused with trust or security while forming an e-business association. Security in e-business association relates to the process of having a secure environment or a secure space in the open architecture of the web, where the customers exchange information with other entities or businesses. This is with an assurance that the information which they share and utilize is being sent only to, and by, the intended people, and that the information is not being changed in any way. Enhancing the security protects the users from intrusions, attacks from the outside world and safeguards them from vulnerabilities. In the literature, trust has been broadly discussed in two ways. Firstly the word 'trust' is taken to measure trust in the security-related aspects of the interaction. The concept of 'Trusted Computing' has been proposed in the literature in an attempt to solve some of the security problems encountered in today's world by adopting the security mechanisms. Secondly, in the context of an e-business association, the word 'trust' refers to the level of confidence that the user has in the other user's ability to deliver its desired outcomes through the interaction. Within the e-business context, for the user to have the necessary level of trust to commit itself to an interaction, it needs to believe that the other user can help it to achieve its desired outcomes. Hence, a clear distinction can be drawn between trust and security in the context of decision making in e-business associations. As with the cross relationship between security and trust, the same holds for the relationship of risk with security, and risk with trust as discussed in the literature. In the context of e-business associations, the analysis of risk in the area of security will ascertain only the security risks present in the interaction [4-6]. Such risks will not provide the meaningful information to the interacting user required for making an informed e-business association decision. This is because the level of risk determined in those areas is not synonymous with the level of risk to be determined (transactional risk) for decision making in an e-business association. Similarly, another confusion that arises in the literature is the relationship between trust and transactional risk. This relationship considers trust to be the authoritative concept to transactional risk, and decision-making can be carried out only on the level of trust as it also represents or nullifies the level of transactional risk in the interaction [7, 8]. But trust and transactional risk are different concepts which require different analysis according to the different areas of the interaction to which they are targeted. The analysis of trust in the interaction omits and does not represent the degree of failure and the magnitude of negative consequences to the interacting user as a result of engaging in the e-business association. Such concepts can be determined only by the analysis of transactional risk; omitting this while decision-making is equivalent to ignoring possible failure and the negative consequences of that interaction.

B. Subcategories for Transactional Risk Assessment in forming a Business Association

There are many different ways by which a business association between any two users is formed. In this paper, we consider that a business association is formed when a user (termed as the risk assessing agent) wants to achieve certain desired outcomes and in order to achieve these has to collaborate with another user (termed as the risk assessed agent) who has the capability to fulfill and to provide these. In exchange, the risk assessed agent receives an object of mutual consent from the risk assessing agent as decided between them for the services that it promises to provide in fulfilling its desired outcomes in the association. As mentioned earlier, the desired outcomes on which the business association is formed may comprise a collection of different criteria, each of which has to be achieved as desired for the successful completion of the interaction. Each of those criteria in the business association is termed the 'assessment criterion' and the collection of all the assessment criteria forms the 'expectations'. The expectations of the business association are established through a series of mutual negotiations between the interacting users so that what is expected and what has to be committed in the business association is clear and understood by all the users involved. The object which is exchanged between the interacting users in order to achieve the expectations can vary. In the context of discussion of this paper, we consider that the risk assessed agent interacts with a risk assessing agent in order to give its expectations, in exchange for the pre-decided monetary value. Hence, the object which the risk assessing agent has at stake is its monetary financial resources, of which it may subsequently experience the loss or negative consequences while interacting with a risk assessed agent,

Transactional risk plays an important part in such associations in the decision-making process. For it to make sense, the specific sub-categories according to which it should be analyzed are *'performance risk'* and *'financial risk'*. Performance risk represents the probability to which the risk assessing agent may not achieve its expectations in forming a business association with the risk assessed agent, whereas the financial risk represents the impact or the consequences to the risk assessing agent in financial terms as a result of failure of its business association. Each of these sub-categories represents important considerations like the likelihood and magnitude of experiencing a negative outcome in a business association. Hence, it is important to consider both of them when determining the transactional risk in forming a business association. From the definitions of transactional risk proposed in the literature, we note that not all of them take into consideration both these sub-categories. Some define it as the uni-subcategory outcome of the failure of the interaction [9-13], whereas some define it as the impact of occurrence [14-16]. Subsequently, the transactional risk determined by those approaches does not capture all the different types of uncertainties that may produce a negative outcome, and hence does not give a correct representation.

Our approach to ascertain the transactional risk, apart from considering both the sub-categories of performance risk and financial risk, also captures the context specific, assessment criteria specific and dynamic characteristics of risk in the business association. In our previous work, we proposed an approach by which the risk assessing agent after forming the expectations of its business association, considers the dynamic nature of time and determines beforehand the performance risk of a risk assessed agent on the Failure Scale [17]. The Failure Scale represents the different magnitudes of failures that are possible in a business interaction. The performance risk of the risk assessed agent is represented by the FailureLevel Curve (FLC) as shown in Figure 1. The FLC represents the magnitudes of occurrence of the different levels of failure that the risk assessing agent might experience in achieving its expectations with the risk assessed agent. This is by done by taking into consideration the variedness and uncertainty of failure throughout the duration of the business association. In this paper, we extend our framework for transactional risk assessment and present an approach whereby the risk assessing agent determines the consequences of failure or the financial risk in forming a business association.

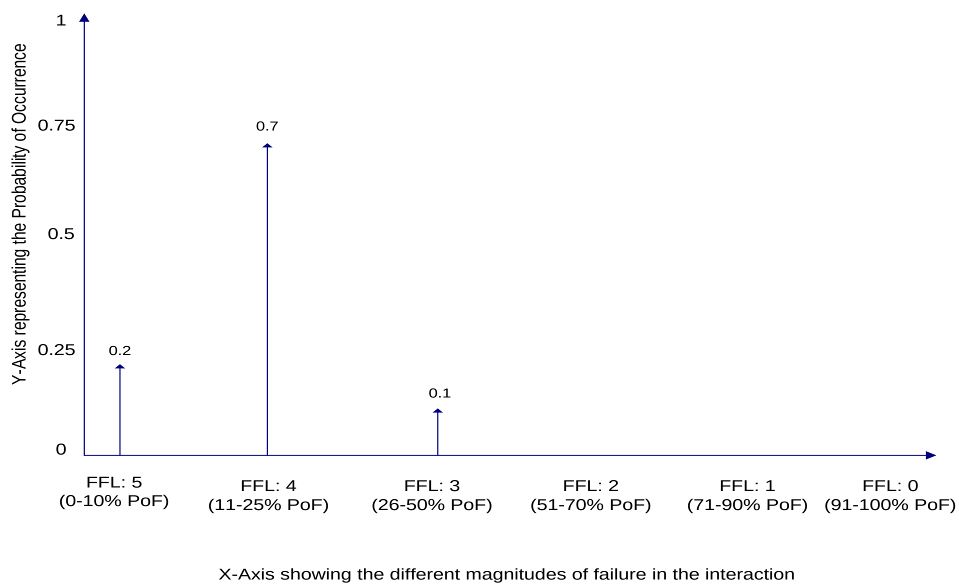

Figure 1: The FLC or Performance Risk of the Risk Assessed Agent in the Business Association

C. Identifying the different types of uncertain events

Transactional Risk assessment involves dealing with uncertainty and identifying those possible events that could be responsible for the occurrence of negative outcomes in an interaction. The uncertainty associated with an interaction can be classified into two broad categories. The first category relates to those events that a risk assessing agent wants to achieve as a result of its interaction and depends on the risk assessed agent to achieve them. We term such events 'Dependent events' of the business interaction. The uncertainty associated with such events is that the risk assessing agent is unsure whether the other agent will act and behave as promised, based on which it will achieve its desired outcomes. Failure to achieve these outcomes might result in the risk assessing agent experiencing failure in its interaction along with negative financial impact or consequences and hence, can be classified as undesired outcomes. The second category of uncertainty is associated with those events that are independent of the risk assessed agent, but which will have a direct impact on the successful completion of the interaction. We term such events as the 'Non-Dependent Events'. The uncertainty associated with such events is that the risk assessing agent is unsure of those events occurring as desired or planned in the expectations. Further, the occurrence of such events may be volatile and difficult to analyze beforehand according to a certain pattern or history, as can be done with dependent events. During transactional risk analysis in business associations, the variation in the occurrences of such events from expectations may also result in experiencing the financial consequences or impact and hence they too can be categorized as undesired outcomes. The financial loss experienced from such Non-Dependent Events is different from what could be experienced from the dependent ones and subsequently, such events should also be identified and assessed during transactional risk analysis in business interactions. To explain with an example, let us consider that risk assessing agent 'A' wants to choose a logistics company with which to form an association in order to move its goods from Milan, Italy to Perth, Australia during the time period 01/10/2008 - 15/10/2008. Before doing so, agent 'A' wants to analyze and consider the level of transactional risk while deciding which logistics company to choose from - risk assessed agents 'X', 'Y' and 'Z'. Let us consider that the assessment criteria formed in the expectations of agent 'A' during its interaction with the risk assessed agents are:

- The logistics company should pack the goods properly at the pick -up address (C1).
- The goods should reach the destination in 5 days (C2).
- The goods should reach the destination undamaged (C3).
- The logistics company should provide a track and trace facility (C4).
- The total cost of transferring the goods is 10,000 Euro (C5).
- During the time of transferring the goods, agent 'A' should pay the logistics company a total of 7,000 Euros at certain times during its interaction (C6).
- Once the goods have been delivered, agent 'A' will make the remaining payment of 3,000 Euros in equivalent Australian Dollars to the logistics company's branch office in Perth on 15/10/2008 (C7).

In the literature, approaches have been proposed which determine the level of financial loss or utility in failing to achieve a task. But these approaches consider only the assessment criteria for which the risk assessing agent depends on the other agent/s. By considering the present example, these approaches consider only assessment criteria C1-C4 which are dependent on the risk assessed agent's performance. The variability of assessment criterion C6 and the non-dependent nature of assessment criterion C7 over the time period of business association are not considered, as they are not related to the performance of the risk assessed agent and are not classified as 'threats' in the interaction. But the variability in the completion of assessment criterion C6 could result in the risk assessing agent experiencing variable financial loss and subsequently must be considered while determining the financial loss in the business activity. Further explanation is given in the next section. Assessment criterion C7 that comes under the category of non-dependent events can also pose a threat to the successful completion of the business interaction. A simple scenario to illustrate this is the variation in the exchange rate of the Australian Dollar against the Euro after agent 'A' initiates an interaction with a risk assessed agent. As a result, the amount which agent 'A' may have to pay to the logistics company as mentioned in assessment criterion C7 might be more than what was initially decided in the expectations. The extra investment which has to be made as opposed to what was decided initially can be considered as the financial loss. Subsequently, the risk assessing agent, when determining the financial risk, should also consider the level/s of loss and the variability in it from these types of uncertain events apart from considering just the loss that could be experienced from the uncertainty in the dependent events.

D. Related Work

The literature shows that financial risk has been studied in great detail in different areas of finance such as portfolio management, insurance, credit rating, stocks etc. At the same time, there is less literature or limited attention paid to equally important areas such as forming business associations over a virtual interacting medium. But as in other areas, financial risk is one of the important factors to be identified, assessed and considered for informed decision-making in business associations. In spite of its importance, the assessment techniques of financial risk of an area cannot be applied to business associations as the factors on which it is dependent may vary, thereby requiring a different assessment technique according to its interpretation. Hence, specialized techniques are needed that will assist in determining the financial risk according to its object of analysis in a specific area. As our focus in this paper is on business associations, in this section we limit our discussion to only those approaches from the literature that relate to financial risk or consequences of forming business associations. The importance of considering the consequences or impact of failure in terms of expected utility while decision-making in business associations is highlighted in the literature. But less work has been done to ascertain and quantify it according to the context specific and dynamic characteristics of transactional risk. In fact, most approaches attempt to either minimize or offset the level and impact of risk by the analysis of other concepts. For example, Aberer et al. [7] consider the possibility of agent 'A' cheating in the interaction, but they do not take into consideration the perceived risk while decision-making, which indicates the degree and magnitude of loss as a result of interacting with agent 'A' due to its cheating. Zheng et al. [18] consider the cost and utility function associated with an interaction while making a decision, but they associate the cost with the 'rewards' of the consequences of the decision. They do not consider the notion of loss associated with the uncertainty in the interaction and the consequences of a decision. Josang et al. [19] consider the 'possible harm' and 'negative consequences' as important factors while decision-making and propose an approach that determines the expected risk as a crisp financial amount depending upon the assessment impact probability [20]. Wang and Lin [21] state that trust represents the extent to which a party can depend on the other with relative assurance, even though 'negative consequences' are possible. But they do not propose an approach by which the negative consequences are quantified and considered while decision-making. Further in [22], Wang and Lin propose an approach that differentiates the transaction price of the interaction during trust evaluation. But this determination does not represent the level and degree of loss that can be experienced in forming an e-business association. Lam et al. [23] interpret risk as losing the deal when a buying agent based on its maximum amount selects an agent to deal with for the requested goods 'g'. But no approach is proposed by which it is quantified while decision-making. Furthermore, it is important that the financial risk in forming a business association be determined according to the time scenario in which it is going to be carried out. For example, if the time scenario of the business association extends to a point of time in the future, then it is important for the risk assessing agent to determine the variability in the nature of its investment over that period of time and ascertain the financial risk by taking into consideration its dynamic nature. On a general note, none of the approaches proposed in the literature which consider the consequence of failure of the business association determines the dynamic nature of investment according to the time scenario of the interaction. Approaches proposed by Wang and Lin [22, 24], Schlager et al. [5, 25], Lin and Varadharajan [4], Cvrcek and Moody [26], Lam et al. [23, 27] etc. even though they consider the total resources being invested in the interaction, they do not consider the dynamic nature by which those resources are invested in the different time scenarios of the interaction. So an approach is needed that would assist the risk assessing agent to capture the context specific and dynamic nature of its business association and determine beforehand the financial risk in forming a business association with a risk assessed agent. We define the problem statement that we intend to address in this paper in the next sub-section.

E. Problem Statement

The problem statement that we address in this paper is: The risk assessing agent forms a business association with another user to achieve certain desired outcomes in exchange for a pre-decided monetary value. To achieve a neutral outcome, it has to achieve its desired outcomes according to the financial resources and within the time period as decided in the expectations. But due to various types of uncertainties, it may not achieve these, thus resulting in financial loss to it. In this paper, we are interested in:

(a) capturing those different types of uncertainties;

(b) determining the probability of the risk assessing agent not achieving its desired outcomes within the time period and according to the financial resources decided in the expectations; and

(c) determining the different levels of financial losses that will represent the financial risk to it in forming a business association with a risk assessed agent.

Our approach to solving the abovementioned problem is as follows:

(a) Identify the accurate worth of financial resources that will be at stake in the business association.

(b) Identify and determine the different types of uncertainties that may lead the risk assessing agent in not achieving its desired outcomes in the time period and financial resources decided in the expectations.

(c) Capture the uncertainties identified in step 2 and determine their impact to ascertain the actual financial resources needed to be at stake in the business association as opposed to what was decided in the expectations.

(d) Determine the Financial risk from the actual financial resources to be kept at stake in the business association.

In the next sections, we propose an approach for ascertaining the financial risk in a business association by capturing the different types of uncertainties.

III. Determining The Financial Resources Required To Be At Stake Due To The Uncertainty From Dependent Events

For financial risk assessment, the risk assessing agent has to take into consideration the total time period during which it will possibly interact with the risk assessed agent. Normally, this may be a point of time in the future. But as discussed widely in the literature, risk is dynamic and it varies according to different periods of time. Subsequently, if the business interaction spans a long period of time, then it is important to take the dynamic characteristic of risk into consideration while ascertaining it over that period. Chang et al. [28] studied different factors according to which the time period of the interaction is divided, in order to take into consideration the dynamic nature of trust while ascertaining it. They proposed to first determine the total time period of the interaction, which is termed the 'time space', and then divide it into different non-overlapping, mutually exclusive parts, termed the ‘time slots’. We will utilize their approach in our framework to consider the dynamic characteristics of risk while ascertaining it over a given period of time. Depending on the number of time slots in the interaction, the risk assessing agent might have different levels of its financial resources at stake during different time periods of its interaction. The financial risk to the risk assessing agent is dependent on the level of financial resources that it has at stake within the time period of its interaction.

For example, from the interaction scenario discussed in the previous section, let us consider that agent ‘A’ determines the time period of its interaction to be from 01/10/2008 – 15/10/2008 with 5 time slots within the time space of its business activity. We term each time slot ‘t1’-‘t5’. While forming the expectations of the interaction, agent ‘A’ decides that it will invest 10,000 Euros in the interaction (assessment criterion C5). In other words, agent ‘A’ expects that with these financial resources it will achieve its outcomes from the logistics company, according to its desired outcomes. In the expectations, agent ‘A’ decides to pay 3,000 Euros in equivalent Australian Dollars to the logistics company’s branch office at Perth when the goods are delivered to it on 15/10/2008. This is done in the last time slot of the interaction’s time space, leaving the risk assessing agent ‘A’ having to pay the logistics company 7,000 Euros in the other time slots of the time space of the interaction as decided between them. Two possible scenarios arise according to how the risk assessing agent invests its resources. They are:

Scenario 1: The risk assessing agent ‘A’ invests its resources in the order of 500, 2,000, 2,500, 2,000 and 3,000 Euros in the time slots ‘t1’, ‘t2’, ‘t3’, ‘t4’ and ‘t5’ of the time space respectively.

Scenario 2: The risk assessing agent ‘A’ invests the total resources of 7,000 Euros in the time slot ‘t1’and 3,000 Euros in time slots ‘t5’ of the time space.

In each scenario, the amount from the risk assessing agent’s net resources at stake throughout the interaction varies. It should be noted that ‘resources at stake’ in the interaction is different from the resources which the risk assessing agent decides to invest in the expectations. The resources decided upon in the expectations represents the total net financial amount which will be invested by the risk assessing agent (which is 10,000 Euros in this case), whereas the resources at stake represents the actual worth of the financial amount that will be invested at a given point in time, according to the nature of the investment. Subsequently, the financial risk to the risk assessing agent in the interaction also varies depending upon how it has its resources ‘at stake’ in the interaction. To take this variation into consideration, the accurate worth of amounts that are at stake throughout the duration of the interaction from the net worth of resources decided in the expectations should be ascertained by plotting the Amount Invested Curve (AIC). The AIC is determined by the probability of a financial amount at stake in each time slot of the business interaction. Another important property and representation of the cumulative probabilistic model of the AIC is that it describes the probability of the net worth of the interaction to be at least equal to, or greater than, a certain amount throughout the duration of interaction. The AIC of scenarios 1 and 2 are shown in Figures 2 and 3 respectively.

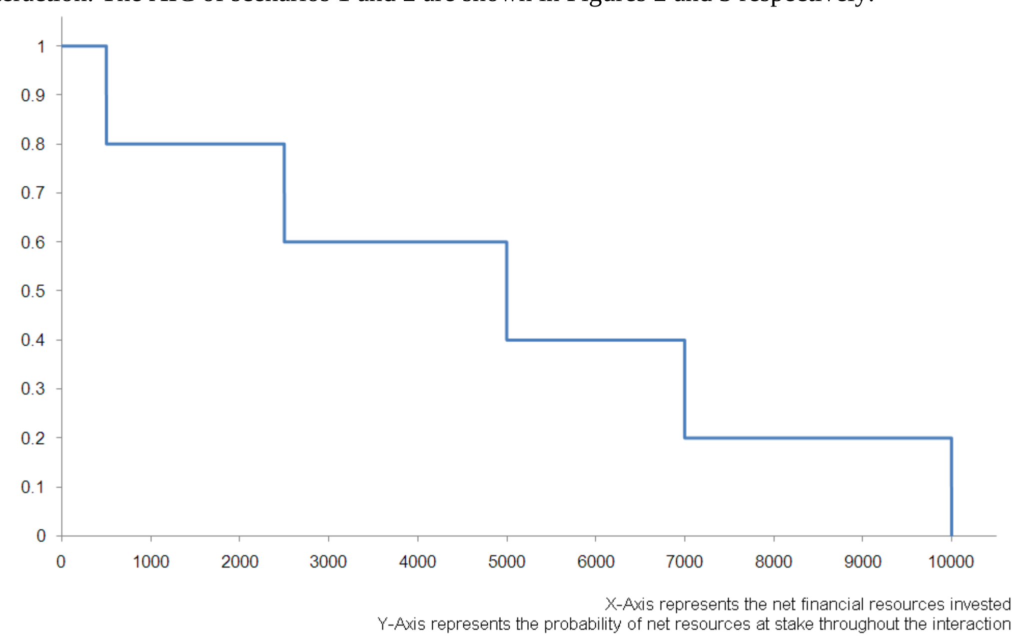

Figure 2: AIC according to the resources invested according to scenario 1

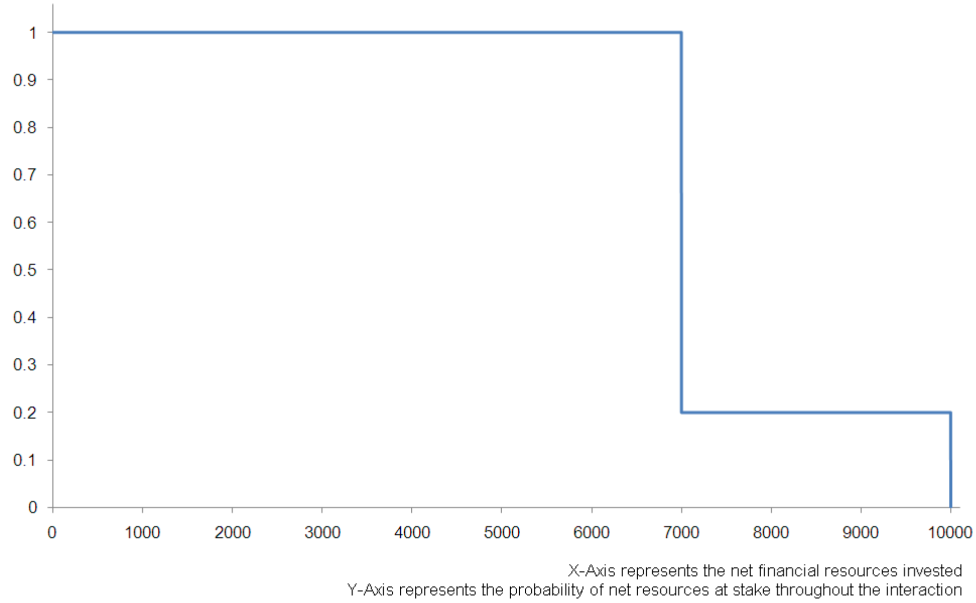

Figure 3: AIC according to the resources invested according to scenario 2

One of the characteristics of the business interaction is the possibility of the risk assessed agent being subjected to a level of performance risk, which is represented by its FLC shown in Figure 1. This will result in the risk assessing agent not attaining its objectives within the time period and in the invested resources, resulting in financial loss to it. To determine the financial loss from dependent events, the impact of the performance risk of the risk assessed agent (FLC) on the net resources at stake in the interaction should be determined. To achieve this, we propose the calculation of the Factual Amount Invested Curve (FAIC). The FAIC will represent the required resources and the probability of an amount from those resources to be kept at stake by the risk assessing agent, throughout the duration of its interaction with the risk assessed agent by considering its performance risk. This is in contrast to the AIC which shows the actual probability of the net resources at stake as decided in the expectations of the interaction between the two interacting agents. Hence, the FAIC is an extension of the AIC and the modification between the two curves reflects the impact of the performance risk (FLC) of the risk assessed agent on the resources initially decided upon and invested according to the expectations.

The impact of the performance risk of the risk assessed agent on the net resources at stake is determined by the mathematical operator 'convolution'. Convolution is a mathematical integral operator which expresses the amount of overlap and impact of one function as it shifts over the other. The FAIC for the period of interaction as a result of convolution of the AIC and the FLC of the risk assessed agent is determined by:

$$\text{FAIC}(x) = \sum_{i=1}^{n} p_i * \text{AIC}(x - FL_i) \qquad \text{for } (x - FL_i) \geq 0$$

$$\text{or,} \quad \text{FAIC}(x) = \sum_{i=1}^{n} p_i \qquad \text{for } (x - FL_i) < 0$$

where: n = the different levels of performance risk of the risk assessed agent,
x = the point at which the FAIC is to be ascertained,
FL_i = magnitude of failure of the performance risk level i,
p_i = magnitude of occurrence of the performance risk level i,
AIC $(x - FL_i)$ = Amount Invested Curve value at point $(x-FL_i)$.

Continuing the interaction scenario between agent 'A' and agent 'X', let us consider that the risk assessing agent 'A' determines the FLC of the risk assessed agent 'X' according to the assessment criteria C1-C4 of its expectations as shown in Figure 1. The FAIC for the time period of agent's 'A' interaction with agent 'X' by considering the AIC of Figure 2 is as shown in Figure 4.

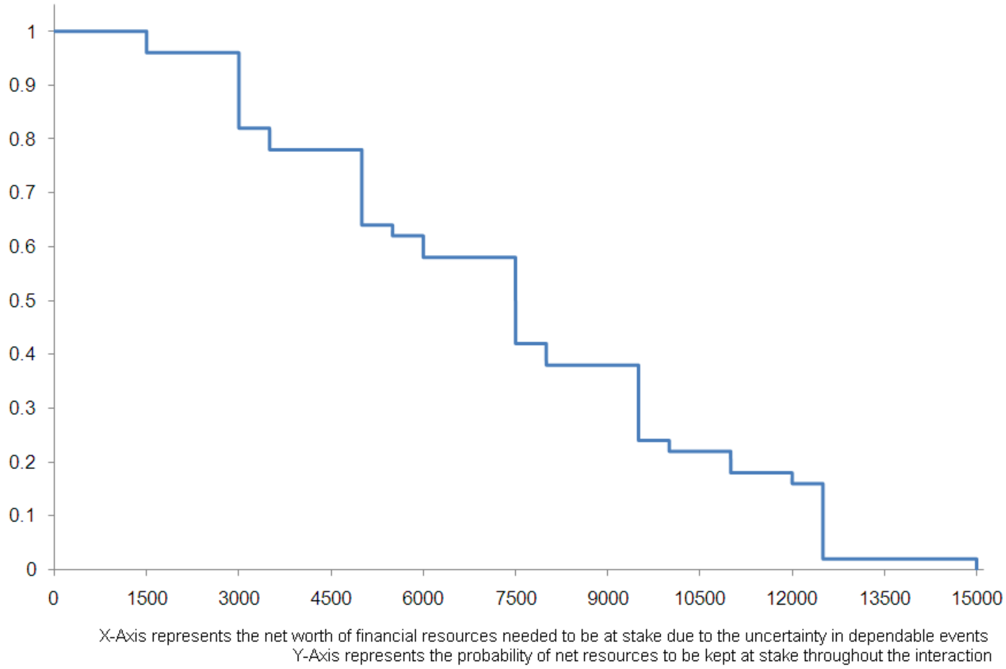

Figure 4: FAIC of the risk assessing agent 'A' interaction with agent 'X'

As can be seen from Figures 2 and 4, the FAIC is inflated compared to the AIC of the interaction. This inflation is due to the additional degrees of resources and the probability of these resources being kept at stake by the risk assessing agent, by considering the performance risk of the risk assessed agent. In certain cases, when the risk assessed agent does not have any performance risk in committing to the expectations of the risk assessing agent, then the FAIC will be the same as the AIC. This implies that the risk assessed agent will commit to the objectives and assessment criteria of the risk assessing agent within the time period and with the resources as decided in the expectations, thereby nullifying any financial loss from dependent events. But in cases where the risk assessed agent might have varying severities of performance risk in committing to the expectations of the risk assessing agent, then the FAIC of the interaction will be inflated compared with the AIC and the degree of inflation is dependent on the performance risk of the risk assessed agent.

In the next section, we will propose a methodology whereby the risk assessing agent can determine the financial resources to be kept at stake due to the non-dependent events.

IV. Determining The Financial Resources To Be Kept At Stake Due To The Uncertainty In Non-Dependent Events

It is important to note that the non-dependent uncertain events are specific to the business interaction for which they have to be determined. Subsequently, in order for the risk assessing agent to determine the impact of such events on the successful completion of its interaction, it should first identify them according to the specific characteristics of the interaction in question. Further in the paper, we term the Non-Dependent Events (NDE) in the interaction:

$$\{NDE_1, NDE_2, NDE_3, \ldots\ldots\ldots, NDE_n\}$$

where 'n' represents the number of Non Dependent Events identified in a business interaction.

Once the Non-Dependent Events have been identified, the uncertainty of their occurrence and their impact on the successful completion of the business association should be determined. By 'impact' we mean ascertaining the financial loss or consequences to the risk assessing agent as a result of variation in the occurrence from what was decided in the expectations. Unlike dependent events, the occurrence of Non-Dependent Events in a business interaction may be spontaneous and volatile with no certain pattern of occurrence, and as a result it is extremely difficult to have a distribution that models the trend in their occurrence during the interaction time period. But on the other hand, it is important for the risk assessing agent to account the uncertainty of such events to consider the expected financial loss from them while determining the financial risk for informed decision-making. So to capture them, in our approach we utilize an uncertainty propagation technique that models the different variations in the occurrence of a Non-Dependent Event from the one desired in the expectations. One such technique is the Monte Carlo method that is best utilized when the underlining model is complex, nonlinear and involves uncertain parameters, thereby making it best suited to our problem. It is a sampling method for analyzing uncertainty propagation that uses simulations of independent random numbers and models the uncertainty associated with the defined problem. It is not a model which gives a deterministic output at each simulation, but is a method for iteratively evaluating a deterministic model (usually the problem framework) using sets of random variables within the defined range as inputs [29]. By utilizing the Monte Carlo technique, we propose that the risk assessing agent determine the variations in the occurrence of a value of an event and their corresponding effect is determined on the output function. The value generated at each iteration will be according to a certain value of confidence interval, which has to be specified by the risk assessing agent. In our model, we consider the confidence interval value to be 99% during the simulations.

In the interaction scenario between agent ‘A’ and agent ‘X’, assessment criteria C7 is the Non-Dependent Event from the expectations of the interaction, which is scheduled to be completed at the end of the interaction. Let us consider that the risk assessing agent ‘A’ formed the expectations on 30/9/2008 by considering that 1 Euro (EUR) converts to 2.008 Australian Dollar (AUD). So according to the expectations, the risk assessing agent has to pay 6024 AUD to the logistics company’s branch office in Perth in order to successfully complete the business activity on 15/10/2008. But, by considering the recent financial situation in the world’s market, agent ‘A’ expects this to change. Let us consider that agent ‘A’, by considering the market volatility and by using existing approaches of time series analysis, determines the range of the input variable as {1.95, 2.49}. By running 5000 iterations of Monte Carlo simulation on this range with 99% confidence level, risk assessing agent ‘A’ will obtain the different output values that represent the impact of the Non-Dependent Event on its interaction. This result should be assimilated to determine the different levels of financial resources to be kept at stake from the variability in the occurrence of Non-Dependent Event and from that the financial loss should be determined. We will explain our approach to determine that in the next section.

V. Determining The Financial Loss Due To The Uncertainty In Non-Dependent Events

In the interaction scenario between agent ‘A’ and agent ‘X’, as the AIC and FAIC in Figure 2 and 4 respectively are determined in Euros, to maintain consistency we determine the financial loss from Non-Dependent Events too in Euros. The results obtained from the simulation in the previous step should be assimilated to plot the Factual Cost Curve (FCC) of its business interaction. The FCC as shown in Figure 5 represents the different levels of amounts with their probability that the risk assessing agent ‘A’ might end up paying to the risk assessed agent, due to the variation in the occurrence of the Non-Dependent Events. It is ascertained by determining the cumulative probability of an amount that the risk assessing agent may have to pay to the risk assessed agent from the simulation results.

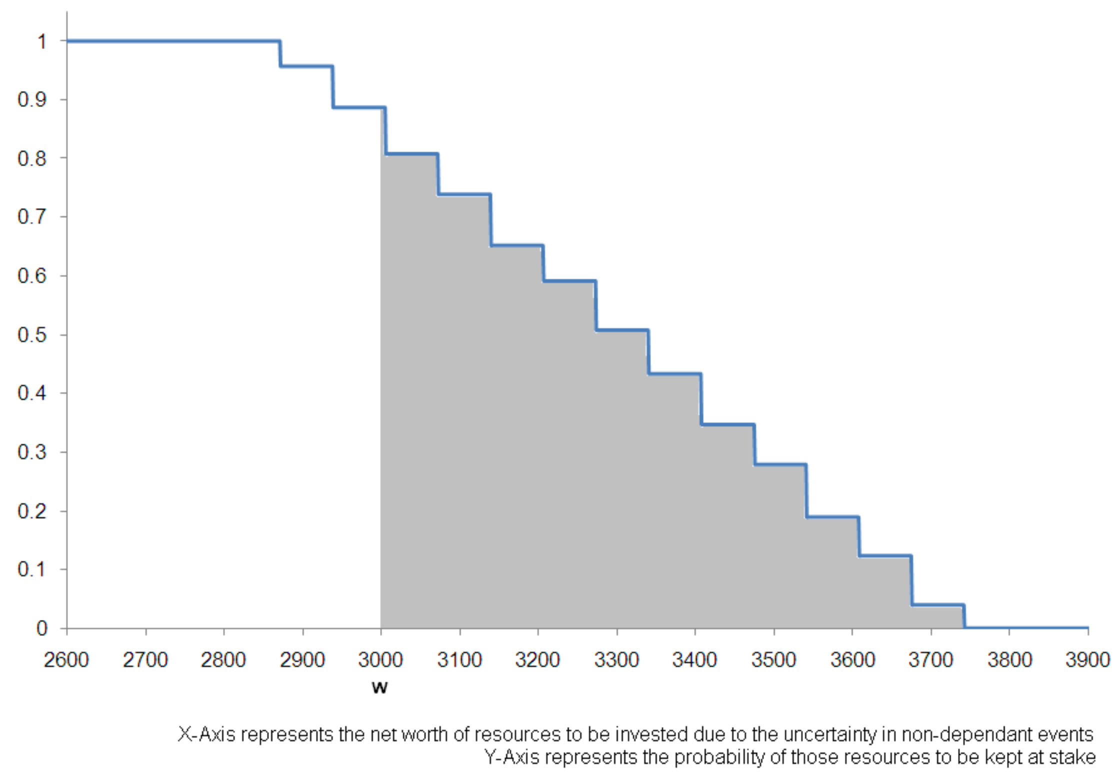

Figure 5: The FFC of the Business Association

According to the expectations, agent ‘A’ had to pay 6024 AUD or 3000 Euros to the logistic company upon receiving the goods, which is represented by point ‘w’ in Figure 5. But due to the uncertainty, it might have to pay additional levels of amounts as shown by the shaded part in Figure 5. In this paper, our focus is on risk assessment with the aim of determining the possible financial losses or extra financial resources to be kept at stake in a business interaction as an impact of risk. As such, we consider only the occurrences of negative outcomes of the events in the business activity that will result in financial loss in an interaction. The extra amount/s which agent ‘A’ might have to pay from what was decided in the expectations can be considered as the possible loss that could be incurred by it in the interaction. To determine the possible loss that could be experienced, we propose that agent ‘A’ should plot its maximum investment capacity (MIC) on the FCC. The maximum investment capacity of the risk assessing agent represents the financial amount that it had to pay to the risk assessed agent as decided in the expectations of its interaction. So any level of required investments after the risk assessing agent’s MIC on the FCC represents the possible loss it could experience. This is because that part of the curve represents the levels of financial resources and the probability that the risk assessing agent might have to pay and keep at stake in order to adhere to the expectations of the interaction, but theoretically those resources are beyond what was decided initially in the expectations. To determine the total financial resources to be kept at stake, the risk assessing agent should combine the shaded part of the FCC with the FAIC of the business interaction.

VI. Determining The Total Financial Resources To Be Kept At Stake In The Business Association

To combine the FAIC with the shaded part of the FCC of the interaction, the risk assessing agent should determine in crisp terms the different levels of extra financial amounts that it might have to pay to the risk assessed agent due to the Non-Dependent Events. In order to achieve this, we propose that:

- The risk assessing agent should determine the probability mass function (PMF) of the FCC in interacting with the risk assessed agent.
- The PMF of the FCC show the probability of the different levels of amounts that the risk assessing agent might have to pay to the risk assessed agent due to the Non-Dependent Events.
- It should then determine the point on the PMF of the FCC which represents its maximum investment capacity (MIC), termed ‘w’.
- The different levels of financial resources after point ‘w’ represents the degrees of extra financial resources that the risk assessing agent might have to pay to the risk assessed agent.
- The risk assessing agent, in order to determine accurately the extra levels of financial resources needed to be kept at stake, should normalize the part of the FCC after its maximum investment capacity so that the probability of occurrence of different levels of amounts after the MIC satisfies the equation:

$$\sum_{i=1}^{n} p(i) = 1$$

where: ‘i’ represents the different levels of financial resources,
‘n’ represents the number of financial levels after the MIC.

We term the normalized curve of the FCC after its maximum investment capacity the ‘Extra Investment Curve’ (EIC). The EIC represents the extra levels of financial resources with the probability of the risk assessing agent ending up paying those amounts to the risk assessed agent, in contrast to what was decided in the expectations. Utilizing the FCC of Figure 5, the EIC is as shown in Figure 6.

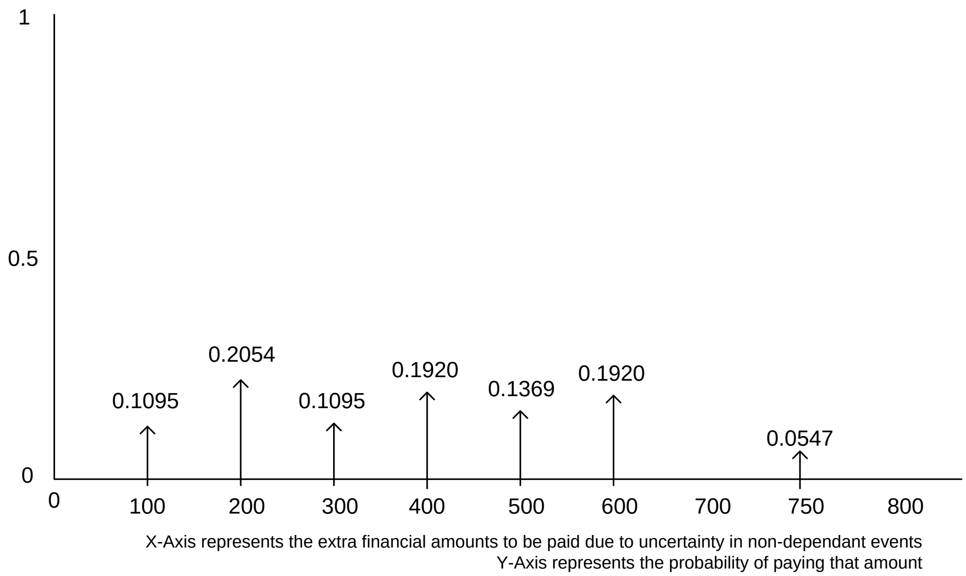

Figure 6: EIC of the Business Association after the MIC

In order for the risk assessing agent to determine the total financial resources that it needs to keep at stake, it should combine the EIC (shown in Figure 6) with the FAIC (shown in Figure 4) of the business association. We utilize the mathematical operator ‘convolution’ to achieve this and term the resulting curve the 'Total Factual Amount Invested Curve' (TFAIC) of the business association. There are various methods of convolving any two probabilistic functions. In this paper, we present two methods which are:

1. Conventional method
2. Cumulants method

A. Conventional Method to ascertain the TFAIC of the Business Association

Mathematically, the conventional convolution process can be described by the random variables of both the EIC and FAIC by:

$$\text{TFAIC} = \text{EIC} \oplus \text{FAIC}$$

where: FAIC = Factual Amount Invested Curve,
EIC = Extra Investment Curve,
TFAIC= Total Factual Amount Invested Curve.

Two scenarios arise while determining the FAIC in interacting with a risk assessed agent.

Scenario 1: There is only level of extra investment to be made by the risk assessing agent due to the Non-Dependent Events:

If there is only one level of investment to be made by the risk assessing agent due to the Non-Dependent Events, then the convolution of the EIC and FAIC is done by the recursive formulae:

$$\text{TFAIC}(x) = p_i * \text{FAIC}(x - l_i) \qquad \text{for } (x - l_i) \geq 0$$
$$\text{or,} \quad \text{TFAIC}(x) = p_i \qquad \text{for } (x - l_i) < 0$$

where: x = the point of financial amount at which the TFAIC is to be ascertained,
l_i = level of extra financial resource from the EIC,
p_i = probability of the risk assessing agent having to pay that extra level of financial resource,
FAIC $(x - l_i)$ = Factual Amount Invested Curve value at point $(x - l_i)$.

Scenario 2: There is more than one level of extra investment to be made by the risk assessing agent due to the Non-Dependent Events:

If there is more than one level of investment to be made by the risk assessing agent due to the Non-Dependent Events, then the convolution of the EIC and FAIC is done by the recursive formulae:

$$\text{TFAIC}(x) = \sum_{i=1}^{n} p_i * \text{FAIC}(x - l_i) \qquad \text{for } (x - l_i) \geq 0$$

$$\text{or,} \quad \text{TFAIC}(x) = \sum_{i=1}^{n} p_i \qquad \text{for } (x - l_i) < 0$$

where: n = the number of extra financial resources to be invested by the risk assessing agent,
x = the point of financial amount at which the TFAIC is to be ascertained,
l_i = level of extra financial resource from the EIC,
p_i = probability of the risk assessing agent having to pay that extra level of financial resource,
FAIC $(x – l_i)$ = Factual Amount Invested Curve value at point $(x – l_i)$.

B. Cumulant Method for ascertaining the TFAIC of the Business Association

The TFAIC of the interaction by using the cumulant method of convolution is determined by the properties known as the moments and cumulant of the distribution function of the EIC and the FAIC. Moments of a distribution function are termed ‘the expectations’ of different powers of the random variable. Alternately, they are a set of descriptive constants which are useful for measuring the properties and, in certain circumstances, for specifying the distribution function [30]. But they are not the best set or the only set of constants for representing the distribution function. Another series of the constants used to represent the distribution function are known as the cumulants (represented by κ). They have properties which can better describe the underlying distribution function and which are more useful from the theoretical standpoint. The cumulants of a distribution function are determined by its statistical moments. The cumulant method for convolution provides an alternative way to obtain the TFAIC of the interaction rather than by using the recursive conventional formulae. In the cumulant method, the convolution of the distribution functions is expressed as a sum of the individual cumulants of the random variables representing the distribution functions. So, by utilizing the cumulants method, the convolution of the EIC with the FAIC of the interaction is obtained by adding their corresponding individual cumulants. Based on the knowledge of the resulting obtained cumulants, the distribution function of the output function, the TFAIC is approximated by using the probabilistic functions. Of the available probabilistic functions, the one which best approximates the output distribution function from the knowledge of its cumulants is the Gram-Charlier Series Expansion. The Gram-Charlier series expansion approximates the cumulative distribution function of the TFAIC from the knowledge of its cumulants obtained as a result of convolution between the FAIC and the EIC of the interaction. A brief description of the series and how it is utilized to represent the TFAIC of the interaction is discussed in the next sub-section.

a) Gram-Charlier Series Expansion

Gram-Charlier series expansion is a probabilistic series expansion function from the knowledge of its cumulants. The Gram-Charlier series has the Normal distribution as its leading term, followed by terms containing the successive derivatives of the normal from order 3 onwards. The motivation for this series comes from the central limit theorem. This theorem basically states that a sum of any ‘n’ in independent random variables (satisfying certain conditions) is approximately normal distributed for sufficiently large n [31]. In Gram-Charlier expansion, certain coefficients are added to the normal to improve the approximation given by the normal distribution. The importance of these terms in the series decreases as the number of random variables involved in the sum increases. The standardized third cumulant of the series is proportional to the skewness of the resulting density function series with respect to the standard normal, whereas the standardized fourth cumulant is proportional to the kurtosis of the resulting density function series. The skewness of the density function shows the direction of its inclination, whereas the kurtosis shows the breadth of the resulting density function. The Gram-Charlier series expansion of the resultant distribution function is represented by:

$$\text{TFAIC}(z) = 1 - \int_{-\infty}^{z} N(z)dz + \sum_{k=3}^{5} \frac{G_k}{k!} N^{(k-1)}(z)$$

where: z = standardized variable,
G_k = standard cumulant of order ‘k’,
N(z) dz = standard normal,
$N^{(k-1)}(z) = (k-1)^{th}$ derivative of the standard normal.

The standard cumulant of order 'k' which is represented by G_k are the cumulants of the TFAIC of the interaction obtained as the sum of the resulting cumulants of FAIC with the EIC. The standardized variable is given by the expression:

$$z = \frac{x-\mu}{\sigma}$$

where: x = investment level for which the corresponding value on the TFAIC has to be determined,
μ = mean of the resulting random variable,
σ = standard deviation of the resulting random variable.

C. *Determining the TFAIC of the Interaction*

The TFAIC of the interaction by convolving the FAIC of Figure 4 with the EIC of Figure 6 is represented in Figure 7. The TFAIC is inflated compared with the FAIC due to the extra levels of financial resources needed to be kept at stake due to the Non-Dependent Events.

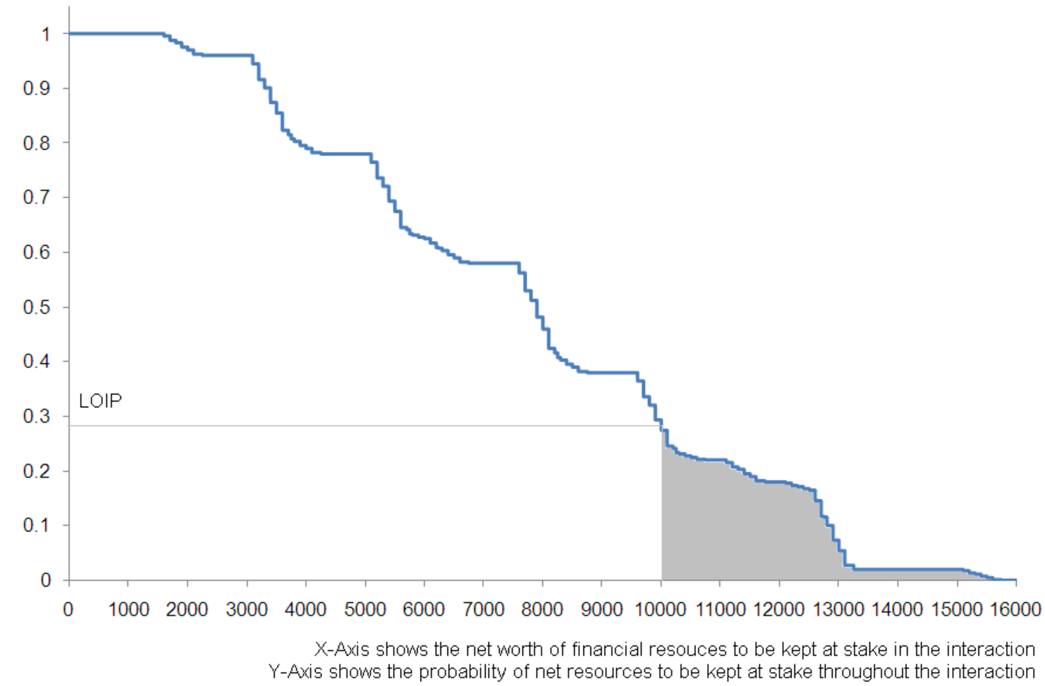

Figure 7: TFAIC in forming the Business Association

The higher the levels and probability of the resources in the EIC, the greater the inflated TFAIC will be compared with the FAIC. To determine the total financial risk in the interaction, we propose that the risk assessing agent should determine:

- Loss of Investment Probability (LOIP) at which it will not achieve the full benefit of the resources that it invests and has at stake while interacting with a risk assessed agent;
- Possible Consequences of Failure (PCF) which represents the levels of un-served investment or the degrees of the extra resources that it has to keep at stake in the interaction.

To ascertain these factors, the risk assessing agent has to first determine its 'maximum investment capacity' in the interaction. The maximum investment capacity of the risk assessing agent represents its maximum extent to which it can invest its resources while interacting with the risk assessed agent. In the next section, we will propose a methodology by which the risk assessing agent ascertains the abovementioned factors in order to determine the financial risk in interacting with the risk assessed agent.

VII. Determining The Financial Risk In Forming A Business Association

The 'Loss of Investment Probability' (LOIP) of the interaction gives the probability of the risk assessing agent not achieving the full benefit of its resources that it invests while interacting with a risk assessed agent. This arises due to:

a) the risk assessed agent not completing the risk assessing agent's objectives to its maximum investment capacity due to its performance risk; and

b) variation in the occurrence of Non-Dependent Events in the interaction.

So the LOIP index in the business association is the level to which extra financial resources need to be at stake in the interaction, after the MIC of the risk assessing agent. By definition of TFAIC, this ordinate is the probability of the corresponding level of amount needed to be at stake throughout the duration of the interaction, but this level and the levels of amounts after this point will not be kept at stake by the risk assessing agent as they are beyond its maximum investment capacity. So the LOIP in forming the business association is determined by:

$$\text{LOIP} = \text{TFAIC}(w)$$

where: w = the point immediately after the MIC of the risk assessing agent, and
TFAIC (w) = ordinate on the TFAIC after investing the total capacity of the risk assessing agent's resources in the interaction.

Continuing the example, risk assessing agent 'A' up to its maximum investment capacity of 10,000 Euros expects to achieve all of its expectations in interacting with the risk assessed agent 'X'. But there is the possibility of this not happening due to the performance risk of agent 'X' and the occurrence of Non-Dependent Events. As a result, the LOIP to the risk assessing agent 'A' is the ordinate of the TFAIC at point 'w', as shown in Figure 7. Once the risk assessing agent has ascertained LOIP in interacting with a risk assessed agent, it can then determine the possible consequences of failure in interacting with that agent.

The 'Possible Consequences of Failure' (PCF) in a business interaction represents the additional degree/s of resources needed to be kept at stake from its MIC. In other words, these are the un-served investments which theoretically are beyond the MIC of the risk assessing agent. To determine the different levels of net financial loss, the risk assessing agent should plot the cumulative probability function of the un-served amounts on the TFAIC. We term the cumulative probability function of the un-served amounts in the interaction shown in Figure 8 the 'Loss Curve'.

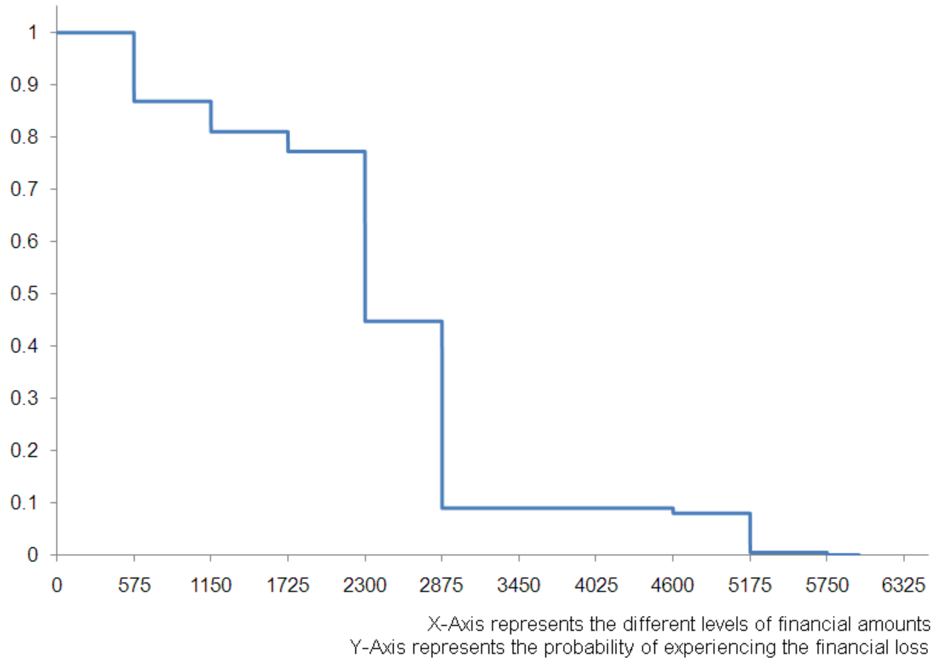

Figure 8: The Loss Curve of Agent 'A's interaction with Agent 'X'

The Loss Curve represents the different levels of financial loss that the risk assessing agent 'A' can experience in interacting with risk assessed agent 'X' by taking into account the different types of uncertainties associated with its interaction. This will provide an informed and better representation to agent 'A' about its business association with agent 'X'. If agent 'A' has to choose and decide from a set of agents one with whom to form a business association, then by using the determined analysis, it can ascertain the different levels and magnitudes of financial risk that it may experience in forming an association with each of them. This will provide significant inputs to it during the transactional risk analysis and decision-making phases. Further, such representation of financial risk in terms of the different levels of financial loss that could be experienced along with their probability would make sense, rather than representing it as a crisp measure if the business association is going to be formed or going to be carried until a point of time in the future, where nothing is certain. Approaches proposed in the literature determine the level of loss as a crisp financial amount that can be experienced in a business interaction. Value at Risk (VaR) is an approach that is a widely used measure of risk that represents the total financial loss that can be experienced over a time horizon with a given probability. But as the business interaction is going to be carried out at a future point in time where nothing is certain, it is quite difficult to determine beforehand and be certain of the crisp financial loss, in exact amounts, that a risk assessing agent can experience in its business interaction. Our proposed approach overcomes this and represents the different levels of financial loss along with the probability of experiencing these, by taking into consideration the different types of uncertainties in the business activity and according to the context specific, assessment criteria specific and dynamic nature of transactional risk. The analysis proposed in this paper, whereby the level of financial loss in the interaction is quantified, is novel and has not been discussed previously in the literature.

VIII. Conclusion

Transactional Risk is an important factor to be considered during the process of informed decision-making in an e-business association. In this paper, we proposed a probabilistic approach by which one sub-category of transactional risk, namely financial risk can be determined when forming a business association. Compared to the other approaches from the literature, our proposed approach takes into consideration the different types of uncertainties and then quantifies them to ascertain the different level/s of financial risk at any given point of time in the business association. Our future work is to consider the established level/s of performance risk and financial risk and determine the level/s of transactional risk in forming a business association. That will lead to making an informed interaction-based decision.

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