Science and Mathematics Education Centre

An Interpretive Study of the Role of Teacher Beliefs in the Implementation of Constructivist Theory in a Secondary School Mathematics Classroom

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ABSTRACT

This thesis addresses the internationally recognised problem of transforming classroom teaching that is underpinned by transmissionist epistemologies, especially in the field of mathematics education. A constructivist-related theory of pedagogical reform was derived from the research literature in the fields of teacher cognition and conceptual change theory for the purpose of facilitating the radical reconstruction of teachers' centralist classroom roles and the development of pedagogies aimed at enhancing students' conceptual development. The thesis reports a collaborative action research study which was conducted with a teacher of high school mathematics who attempted to develop a constructivist-oriented teaching approach in his Grade 12 mathematics class.

An interpretive research approach (Erickson, 1986) was employed to generate an understanding of the complex network of teacher beliefs and their constraining influence on the radical reform of the teacher's centralist pedagogy. An emergent research focus was the efficacy of the theory of pedagogical reform, especially its underpinning constructivist theory which shaped my discourse with the teacher during the collaborative phase of the study. Data were obtained from classroom participant-observations, teacher and student interviews, the teacher's report on the study, and an application of the repertory grid technique that was designed to investigate the teacher's implicit pedagogical beliefs.

The major pedagogical reforms that resulted from the collaborative phase of the study included: (1) the teacher's adoption of the role of *teacher as learner*; and (2) the refinement, rather than radical reform, of his centralist classroom role of *teacher as informer*. Although the reforms provided enriched opportunities for the development of students' algorithmic abilities, they largely failed to enhance students' mathematical conceptual development.

Interpretive analyses were conducted from multiple constructivist-related perspectives (radical constructivism, social constructivism, critical constructivism) in order to generate a better understanding of the narrow scope of the pedagogical reforms. These analyses revealed (1) that rationalist preconceptions of mathematics and mathematical cognition underpinned important aspects of both the theory of pedagogical reform and the teacher's refined centralist classroom role of *teacher as interactive informer*, and (2) that the teacher's technical rationality, which was buttressed by personally constraining beliefs, maintained the ascendency of his technical curriculum interests and sustained his centalist classroom role of *teacher as controller*.

The results of this thesis suggest strongly, therefore: (1) that pedagogical reforms which are based largely on a cognitivist theory of constructivism are susceptible to being subsumed by a powerful *technical-rationalist* ideology, and (2) that the hegemonic nature of this ideology is capable of disempowering reform-minded teachers from realising the practical viability of their constructivist-related ideals.

The thesis recommends that future constructivist-related pedagogical reform in high school mathematics be based on a critical constructivist perspective which focuses attention on the curriculum interests that govern teaching and learning activities. In particular, it is recommended that reform-minded teachers establish a critical classroom discourse which aims to attain a balanced rationality by making visible and subject to critical examination the hidden frames of reference that constitute the prevailing rationality of the traditional mathematics classroom.

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Anne and Nell provided me with the emotional support that I needed during those seemingly endless hours of solitary confinement and self-indulgence in my study. Although the thesis kept me away from many pleasurable family pastimes it also has benefitted greatly from my family involvement. As a parent, I have come to understand the process of teaching and learning from a more human-centred perspective, and recognise the importance of nurturing the child's natural desire for inquiry and understanding with humour, patience, provocation and empathy.

THESIS CONTENTS

SECTION ONE

INTRODUCTION

Section One of the thesis comprises a single introductory chapter which provides an overview of the research design for an investigation of the feasibility of utilising the metaphor of *learning as construction* as a referent for the radical reconstruction of a teacher's transmissionist pedagogy. This section introduces the thesis as an interpretive research study of a collaborative research endeavour between Ray, a teacher of high school mathematics, and myself, a teacher educator.

CHAPTER ONE

RESEARCH DESIGN OF THE STUDY: AN OVERVIEW

INTRODUCTION

This thesis reports a research study that was designed to investigate the feasibility of utilising the metaphor of *learning as construction* for resolving a highly significant epistemological problem associated with traditional classroom styles of teaching school mathematics, namely, the problem of *teacher centralism*. In particular, the thesis presents an interpretive study of a collaborative research endeavour between Ray, a teacher of high school mathematics, and myself, a teacher educator, which aimed, initially, (1) to develop a constructivist-related pedagogy, and (2) to investigate the constraining role of teacher pedagogical beliefs.

Chapter One presents an overview of the main principles of the design of the research study reported in this thesis. The first section discusses the nature of the internationally recognised problem of teacher centralism which is associated with outmoded classroom learning environments in school science and mathematics, and identifies its epistemological underpinnings in terms of the metaphors of *teaching as transmission* and *learning as absorption*. The metaphor of *learning as construction* is proposed as an alternative epistemology for the teaching of school science and mathematics. The section concludes with a statement of the main research problem of the thesis: *How can the problem of teacher centralism be resolved in mathematics education, especially the problem of transforming radically teachers' transmissionist epistemologies*. The research problem is elaborated in a discussion on the resilient nature of teachers' role-determining beliefs, especially the constraining influence of teachers' role-determining beliefs on attempts to decentralise teachers' classroom roles, and the initial research question of the study is posed: *How do teachers'* *role-determining beliefs constrain the process of constructivist-related pedagogical reform?*

The second section of Chapter One argues that, although there is a popular perception amongst modern science and mathematics educators that the metaphor of learning as construction is a panacea for resolving the problem of teacher centralism, the constructivist metaphor is the subject of divergent and conflicting interpretations. This section explicates the nature of the constructivist perspective which was employed in the initial design of this study, namely, *the first principle of constructivism* and *the prior knowledge corollary*. It is argued, in relation to research on teaching, that at the time when this thesis was conceptualised this constructivist perspective provided a viable referent for the radical reconstruction of teachers' transmissionist epistemologies. Chapter Two discusses fully the ideas presented in this section, and utilises them for the development of a constructivist-related theory of pedagogical reform. However, it is acknowledged that the research problem was not resolved successfully during the collaborative phase of the study, and that, subsequently, a second research question emerged and directed the post-collaborative phase of the study: *To what extent did the cognitivist theory of constructivism used in the thesis constrain the process of pedagogical reform?* Summaries are presented of multiple constructivist-related perspectives which performed a major role in addressing the second research question. These perspectives, which are based on metaphysical and sociocultural principles of constructivist theory, and which serve as interpretive frameworks for further data analysis, are discussed fully in Section Three of the thesis.

The third section establishes the prospects of *collaborative research* as an appropriate means of (1) facilitating teachers' classroom-based attempts to undertake constructivist-related pedagogical reform, and (2) investigating the role of teachers' pedagogical beliefs in constraining the reform process. The context of the collaborative research relationship between Ray and me is discussed, especially the goals of our mutual research agenda which aimed to develop teaching strategies for enhancing students' mathematical conceptual development opportunities. The main goals of my constructivist discourse with Ray, which are fully discussed in Chapter Three, are outlined in this section.

The final section argues that Erickson's (1986) *interpretive* research approach, which I adapted for this study, is highly congruent with the goals of collaborative research, and was appropriate for addressing both the initial and emergent research questions of this study. The data collection methods are discussed, especially (1) the role of my subjectivity as a researcher in refocusing the post-collaborative phase of the study, and (2) the methodological strategies which I employed to optimise evidentiary adequacy and minimise researcher bias in the collection of data. Finally, I outline the data analysis strategies which I employed (1) to formulate and warrant the assertions which are presented in Chapters Four to Nine of the thesis, and (2) to report the assertions in a form which enables the reader of the thesis to understand the case, and to judge the validity of my interpretative analysis.

TEACHER CENTRALISM: A PROBLEMATIC EPISTEMOLOGY

In 1986, at the time of conception of this study, a recently published major review of research on teaching and learning mathematics (Romberg & Carpenter, 1986) criticised strongly the traditional style of teaching mathematics found to be widespread in schools throughout the United States of America. Romberg and Carpenter (1986) argue that research has shown that the traditional mathematics classroom is characterised by "extensive teacher-directed explanation and questioning followed by student seatwork on paper-and-pencil assignments" (p. 851). The review concludes that teacher-centred classroom learning environments such as this are underpinned by epistemological assumptions that recent research in cognitive science has found to be untenable.

Three serious limitations of traditional mathematics teaching are noted. First, in traditional classrooms, mathematics is promoted as a *record of knowledge* which is divorced from processes of inquiry by fragmentation into subjects, topics, lessons, facts and skills. Second, daily lessons are geared towards students passively absorbing the record of knowledge, rather than meaningfully constructing their own mathematical knowledge in relation to their established cognitive frameworks. Third, teachers' roles are determined largely by managerial concerns for covering the syllabus, maintaining control and order, and instructing the whole class, rather than by considerations of a conception of mathematical knowledge or an understanding of how individual learning occurs. The image of traditional mathematics teaching that is evoked by Romberg and Carpenter (1986) is one of classrooms in which teachers' epistemological conceptions are governed by the complementary metaphors of *teaching as transmission of knowledge* and *learning as absorption of knowledge*.

A Contemporary Problem

Recent reports on the future of mathematics teaching in schools in the United States of America confirm that overcoming the epistemological problem of teacher centralism continues to be a central focus of educational reform. The report of the National Research Council (1989, p. 57) describes traditional mathematics teaching in terms of an authoritarian *broadcast* metaphor in which *teachers prescribe* and *students transcribe*. The Council argues that the dominant mode of lecturing and listening in American classrooms underpins an image of mathematics as an established and immutable doctrine, and fails to help students to develop higher-order thinking skills. The report of the National Council of Teachers of Mathematics (1989, p. 3) argues for a set of standards for modern mathematics teaching in relation to the mathematical literacy needs of contemporary *information society*, and criticises the epistemology of prevalent school teaching approaches which were designed traditionally to meet the needs of an outmoded industrial age.

The epistemological problem of teacher centralism is not confined to the teaching of mathematics in American schools. The report of the American Association for the Advancement of Science (1989) argues that, in the interests of promoting scientific literacy for a postindustrial society, modern science teaching should abandon its traditional focus on quantity of information presented by teachers, and should adopt a more appropriate focus on the quality of students' understanding. Recently, the National Association for Research in Science Teaching and the National Science Foundation organised a national conference which focused on the need for science curriculum reform. The report (Shymansky & Kyle, 1991) is critical of the "archaic ritual of transmission and acquisition of knowledge" which, it claims, "is not able to provide students with the science and technology education requisite for future human needs" (p. 6). These reports are critical of the outmoded image of both science and learning — static, passive activities — that is portrayed by traditional science teaching which is underpinned by transmission and absorption metaphors.

In Australia, educational reform of school mathematics teaching has similar goals to those expressed in American reports, namely, the restructuring of traditional mathematics classroom environments in ways that address the evolving needs of modern technological society. Although the Australian Education Council's forward looking national statement on mathematics for Australian schools (1990) does not address directly the problems associated with traditional classroom teaching, it is clear that the traditional epistemological problem of teacher centralism remains to be resolved:

> The view of mathematics and the approaches to teaching and learning suggested in this Statement have implications for how mathematics learning is supported. For example, they suggest a classroom learning environment which encourages practical activity, the appropriate use of technology, and discussion. Mathematics can no longer be regarded as a chalk and talk subject from the perspective of the teacher or as a textbook, pencil and paper subject from the perspective of the students.
>
> (Australian Education Council, 1990, p. 22)

Constructivism: An Alternative Metaphor

At the time of conception of this thesis, a *constructivist* metaphor of learning — knowledge is constructed, or built, by the individual learner — was fuelling recommendations for research on resolving the epistemological problem of teacher centralism. For example, the review of research on teaching and learning mathematics by Romberg and Carpenter (1986) concludes that attempts to reform school mathematics should be underpinned by cognitive science research which "shows that learning proceeds through construction, not absorption" (p. 868). The constructivist metaphor of learning is very prominent in the review, and is employed to focus attention on the need for teaching to take account of the structure of children's knowledge which they bring with them to the classroom. The constructivist metaphor is contrasted with the *absorption* metaphor of traditional teaching:

> The picture of how students learn based on current research is quite different from that assumed in traditional classrooms. This perspective is that learners construct knowledge, they do not simply absorb what they are told. (Romberg & Carpenter, 1986, p. 859)

From a constructivist perspective, therefore, the problem of teacher centralism is articulated as a problem of teacher epistemology. Traditional teaching is characterised in terms of a transmission metaphor which interprets the traditional predominance of the didactic teaching activity of information dissemination as a teacher intention to transmit, transfer, or transplant, knowledge intact from their own minds into the minds of students (Bodner, 1986; Pope & Gilbert, 1983; Pope & Keen, 1981; Prawat, 1989). Metaphors of learning which are complementary to the transmission metaphor of teaching include *learning by absorption* (Clements & Battista, 1990; Romberg & Carpenter, 1986), *learner as tabula rasa*, or *blank mind* (Gergen, 1992, February; Gilbert, Osborne & Fensham, 1982; Gilbert & Watts, 1983), *learner as blank slate* and *learner as an empty gallery waiting to be filled* (Cobern, in press; von Glasersfeld, in press), and *knowledge as a* *commodity that can be transferred from mind to mind* (Bettencourt, in press; Taylor, 1991).

The Research Problem

The constructivist metaphor of learning provides a radically new perspective on the problem of teacher centralism in mathematics classrooms. The constructivist metaphor focuses attention on the content, or conceptual structures, of students' thinking and on the important role of teachers' epistemologies in facilitating students' conceptual development. However, a consideration of this metaphor raises the research problem, which is addressed in this thesis, of *how teachers' transmissionist epistemologies can be reconstructed in accordance with a constructivist metaphor of learning.*

Because the constructivist metaphor is associated with learning, rather than with teaching, it does not prescribe constructivist-related teaching approaches. Romberg and Carpenter (1986) make the point that there is an outstanding need for research on the nexus between constructivist-related theories of learning, about which much is known, and teaching, about which little is known:

> We currently know a great deal more about how children learn mathematics than we know about how to apply this knowledge to mathematics instruction. Research is clearly needed to explore how knowledge of children's learning of mathematics can be applied to the design of instruction. (Romberg & Carpenter, 1986, p. 859)

In a review of the constructivist-related research on student conceptions in science and mathematics, Confrey (1990), who is somewhat critical of the generality of Romberg and Carpenter's (1986) claim, nevertheless supports the principle that much more research on constructivist-related teaching is needed. This thesis was designed to address this outstanding need in the field of mathematics education.

The Resilience of Teacher Beliefs

When this thesis was designed initially, research on students' and teachers' cognition provided insights into the prospective difficulties of reconstructing teachers' well-established epistemologies. Research on students' conceptual frameworks in science education was reviewed for analogous insights into teacher conceptual change strategies. Research on teacher cognition was reviewed in order to learn more about the nature of teachers' established role-determining beliefs. This research is discussed fully in Chapter Two where a constructivist-related theory of radical pedagogical reform is developed.

Research on students' conceptions in science indicates that students' beliefs about the nature of the world, especially their experientially-based intuitive conceptual frameworks, can be very resilient to teachers' scientifically-based rational explanations. For example, research in science education has revealed that students can be reluctant to abandon their experientially-determined Aristotelian conceptions of motion in favour of the scientifically validated Newtonian framework. Arguing by analogy, it seems that teachers' long-established and intuitive conceptualisations of their centralist classroom roles might be equally resilient to rational demands for radical pedagogical reform.

A major review of research on teacher cognition (Clark & Peterson, 1986) classifies two types of pedagogical beliefs held by teachers. The first type — *explicit beliefs* — are readily identifiable in teachers' daily discourse, and comprise explicit propositions that attribute, for example, students' academic success or failure to students' characteristics or teachers' endeavours. The research on attribution theory has been well-established since the early 1970s. Research on teacher cognition (Clark & Peterson, 1986; Eisenhart, Shrum, Harding, & Cuthbert, 1988; Nespor, 1987; Tobin & Espinet, 1987) identifies also *implicit beliefs* which teachers' hold at a subconscious level, and which provide a psychological context for their classroom roles. Research on curriculum innovation (Munby, 1982, 1983, 1984; Olson, 1980, 1981) has shown that teachers' implicit theories and beliefs which determine their centralist classroom roles can undermine curriculum reform attempts towards establishing less teacher-centred classroom environments.

The Initial Research Question

The research on students' conceptions in science, and research on teacher cognition, especially in the context of curriculum reform, provide a compelling case for the need to account for teachers' role-determining pedagogical beliefs, especially in the type of research discussed in this thesis which sought to reconstruct radically a teacher's established transmissionist epistemology. The research suggests that teachers are likely to be largely unaware of their well-established transmissionist epistemologies, and that the implicit nature of their epistemologies is likely to contribute to their resilience to radical reconstruction. Because of the paucity of research on teacher cognition, the question of *how do teachers' role-determining beliefs constrain the process of constructivist-related pedagogical reform* was an apposite research question to be addressed by this thesis.

CONSTRUCTIVISM:
A REFERENT FOR RADICAL PEDAGOGICAL REFORM

This thesis constitutes an early attempt to respond to the challenge to investigate the prospects of a constructivist metaphor of learning for reforming radically mathematics classroom learning environments in Australian secondary schools. The thesis reports the design and implementation of a constructivist-related theory of radical pedagogical reform. The theory, which is elaborated in Chapters Two and Three of this thesis, was designed in relation to a conception of constructivism which was popular amongst science and mathematics education researchers in the late 1970s and early 1980s, and which has become increasingly popular in the 1990s.

The Rise of Constructivism

Although science education research and, to a much lesser extent, mathematics education research have utilised constructivist perspectives to investigate students' conceptual understandings during the past 15 years, at the time of conception of this thesis the term *constructivism* was not widely known outside of specialist research groups. Noddings (1990) reminds us of the recency of its ascension to centre stage in the debate over educational reform, especially in mathematics education:

> In 1985 . . . one hardly ever heard of the word *constructivism*. That has somehow changed. . . . The word has become a battle cry for a reconsideration of our problems and our best road toward the solution. (Noddings, 1990, p. 2)

In the 1990s, interest in the constructivist metaphor has burgeoned amongst educational researchers. Examples of this widespread international interest include: (1) a recently published special monograph edition of the *Journal for Research in Mathematics Education* (Davis, Maher, & Noddings, 1990) which has as its central theme constructivist views of teaching and learning mathematics; (2) a monograph on constructivist perspectives on science and mathematics education (Tobin, in press) commissioned by the American Association for the Advancement of Science; (3) a monograph on Australasian perspectives on learning science as a personal construction (Northfield & Symington, 1991) recently published by the national Key Centre for School Science and Mathematics, in Australia; (4) an international symposium on alternative epistemologies in education at the University of Georgia, in 1992 (Steffe, in press); and (5) a topic group on constructivist teaching and learning approaches in mathematics at the Seventh International Congress of Mathematics Education in Quebec, in 1992 (Malone & Taylor, in press).

Perhaps the most significant appropriation of the constructivist metaphor has occurred by education policy makers who wish to reform radically the perceived malaise in school science and mathematics.

Nowhere is this more evident than in the United States of America where a constructivist metaphor of learning is promoted in the reports of august bodies such as the National Council of Teachers of Mathematics (1989), the National Council of Education (1989), the American Association for the Advancement of Science (1989), and the Holmes Group (1990), whose recommendations for the reform of teaching school mathematics and science, of teacher education, and of educational research are based firmly on constructivist metaphors of learning. The suggestion by the President of the National Association for Research in Science Teaching, in a recent edition of NARST News (Yeany, 1991), that constructivism might be a unifying theme for the restructuring of science education, indicates the extent to which the constructivist metaphor has attained prominence amongst American science educators.

Recently, a constructivist metaphor has been adopted by the Australian Education Council (1990) for the purpose of articulating recommendations for the national reform of school mathematics education in Australia.

The Many Faces of Constructivism

Although the constructivist metaphor has become very attractive as a prospective panacea for the ills of education, the meaning of the term *constructivism* is problematic. The world-wide growing interest in the constructivist metaphor has been accompanied by an articulation of a wide range of constructivist epistemologies for science and mathematics education. In other words, constructivism is being constructed differently according to a wide range of interests in the nature of its role in education.

Examples of recent schools of constructivist thought include: *cognitive constructivism* as a social learning theory (Zimmerman, 1981); *critical constructivism* in mathematics teacher education (Taylor, 1991); *contextual constructivism* in science education (Cobern, in press); *cultural constructivism* (Scott, Cole, & Engel, 1992); *C_1 and C_2 constructivism* in mathematics and mathematics education (Lerman, 1989); *ecological constructionism* in educational research (Steir, 1992, February); the *generative learning model* for science education (Osborne & Wittrock, 1985); an *information-constructivist perspective* on higher-order knowledge (Stahl, in press); *personal construction of knowledge* in science education (Driver, 1988); *personal constructionism* in science education (Northfield & Symington, 1991); *radical constructivism* in mathematics education (von Glasersfeld, 1991); *social constructivism* as a philosophy of mathematics education (Ernest, 1991); *social constructivism* in science and mathematics teacher education (Tobin, 1990c); *social constructionism* and language in education (Gergen, 1992); and a *social constructivist perspective* on mathematics education (Bauersfeld, 1992).

Many of the recently articulated constructivist epistemologies derive from similar theoretical foundations, and are complementary to the extent that they have accommodated similar theoretical frameworks from the fields of cultural anthropology or literacy studies. Others, however, are perceived by their proponents to be in philosophical or sociological conflict. For example, personal constructivism has been criticised as being *pragmatic* (Bettencourt, in press) and *naive* (Steir, 1992; von Glasersfeld, in press) because of its failure to address metaphysical principles about the issue of truth, and as *rationalist* (Solomon, 1987) because it has not addressed the social context of learning.

The 1990s signifies an era in which the power of the constructivist metaphor has become widely recognised, and has been embraced internationally by science and mathematics educators. Heinrich Bauersfeld (1992, February) reminds us of its popularity when he claims that the initial statement 'I am a constructivist' has become a kind of an academic lip service in many disciplines. However, the multitude of complementary and conflicting interpretations of the term constructivism is problematic. References to constructivism are no longer sufficiently meaningful (if ever they were!) to assure an unambiguous interpretation without a qualifying statement of the type of constructivism that is intended.

Therefore, in order to avoid semantic confusion, the nature of the constructivist perspective which was employed in the design of the theory of pedagogical reform, and which was implemented during the collaborative phase of this study, is discussed briefly in this section.

Initial Constructivist Perspective

The constructivist perspective that shaped the initial design of this study was conceived largely in relation to cognitive science interpretations of Piaget's *cognitive constructivism* (Shuell, 1986; Smock, 1981; Zimmerman, 1981), and the *personal constructivist* tradition in science education which investigates children's alternative conceptions and is concerned with conceptual change models of science teaching (Bodner, 1986; Driver & Easley, 1978; Driver & Erickson, 1983; Driver & Oldham, 1986; Gilbert, Osborne, & Fensham, 1982; Gilbert & Watts, 1983; Gunstone, Champagne, & Klopfer, 1981; Head, 1982; Hewson & Hewson, 1983; Novak, 1978, 1988; Osborne & Wittrock, 1985; Pines & West, 1986; Pope & Gilbert, 1983; Posner, Strike, Hewson, & Gertzog, 1982; Prosser, 1985; Shuell, 1987).

The First Principle of Constructivism

A common feature of the constructivist metaphor of learning which was employed by these researchers is the principle that knowledge is constructed in the mind of the learner, rather than transferred intact from the mind of the teacher. Recently, von Glasersfeld (1988) has called this the first principle of constructivism: "Knowledge is not passively received either through the senses or by way of communication, but is actively built up by the cognising subject" (p. 83).

The Prior Knowledge Corollary

The prior knowledge corollary of the first principle of constructivism — *learning is a process of conceptual development which is intimately related to learners' extant conceptual frameworks* — was the chief focus of the personal constructivists, especially those who advocated a pedagogical concern for reconstructing students' misconceptions, preconceptions, or alternative frameworks.

Constructivist-oriented science educators departed from the Piagetian *stage theory* tradition of identifying generalised cognitive structures, or *content independent* forms of thought, and drew upon conceptually-oriented theories of cognition. In particular, Ausubel's *theory of cognitive learning,* which focuses on learners' pre-existing conceptual structures, or the ideas that students hold already about natural phenomena which they use to make sense of everyday experiences, became popular amongst constructivist science educators (Head, 1982; McClelland, 1982; Novak, 1978; Pines & West, 1986; Raths, 1971; Scott, 1987). Ausubel's theory was promoted as a viable alternative for constructivists who wished to abandon Piaget's *stage theory* of cognitive development. Novak (1978) describes Ausubel's theory as a model of cognitive development "that is not stage dependent but rather dependent on the framework of specific concepts and integrations between these concepts acquired during the active life-span of the individual" (p. 26).

At the time that this study was designed, another highly influential theory underpinning the *context-dependent* view of cognition of constructivist-oriented science educators was Kelly's *personal construct psychology* and its associated metaphor of *man-the-scientist*. Kelly's metaphor emphasises the role of *personal construction* in the development of both scientific community knowledge and children's attempts to make sense of their experiences of the world (Pope & Gilbert, 1983; Pope & Keen, 1981):

> Kelly's main emphasis is on the uniqueness of each person's construction of the world and the construct systems each will evolve and continue to evolve in order to import meanings of their experiences. . . . [T]his constructivist view of knowledge lends support to teachers who are concerned with the investigation of students' views, who seek to incorporate these viewpoints within the teaching-learning dialogue, and who see the importance of encouraging

> students to reflect upon, and make known, their construction of some aspect of reality. (Pope & Gilbert, 1983, p. 197)

In mathematics education, the rhetoric of constructivist-oriented researchers was very similar to that of science educators. For example, Resnick (1983) claimed that instruction should acknowledge explicitly students' prior knowledge and help students to confront their *naive theories*; and Novak (1986) associated constructivist instruction with developing students' *metacognitive strategies* that help them to become empowered learners who can better *learn how to learn* mathematics.

The constructivist-related theory of pedagogical reform which is outlined below, and which is discussed fully in Chapters Two and Three of this thesis, is based on the first principle of constructivism and the prior knowledge corollary. This constructivist perspective highlights the need to develop pedagogies which take account of students' extant conceptualisations and which address directly their conceptual development needs and difficulties.

A Constructivist-Related Theory of Pedagogical Reform

At the time of conception of the research study reported in this thesis, research on conceptual change teaching in science education, research in the field of the philosophy of research on teaching, and research on teacher cognition provided a constructivist-related framework for developing a theory of pedagogical reform which addressed teachers' transmissionist pedagogical beliefs. This research is discussed fully in Chapter Two, and is reported briefly in this section.

In the early 1980s, research on facilitating radical conceptual change amongst students of school science established conditions of conceptual change which, it was assumed, were similar to the conditions which precipitated major paradigmatic changes amongst scientists. Posner, Strike, Hewson, and Gertzog (1982) specified that in order for students to undertake radical reconstruction of their conceptual frameworks it is necessary for students to experience dissatisfaction with their extant ideas, and to experience the intelligibility, plausibility, and fruitfulness of alternative (scientific) concepts.

Research on teaching suggests establishing very similar conditions for facilitating the reconstruction of teachers' pedagogical beliefs (Fenstermacher, 1979, 1980, 1986; Nespor, 1987; Shulman, 1986). In brief, the research argues that teachers must be enabled concomitantly: (1) to become reflexively aware of their extant subjective pedagogical beliefs; (2) to undertake investigations to disconfirm their viability; (3) to develop alternative pedagogical ideas; and (4) to experience their success in practice. The research argues that teacher educators have a key role to perform in establishing these conditions of pedagogical reform. On the one hand, teacher educators are encouraged to be persuasive about alternative pedagogical ideas but, on the other hand, they are warned against adopting the authoritarian role of expert knowers (Floden, 1985), and are directed to ensure that teachers maintain a critical attitude towards all forms of reform-oriented rhetoric, including their own (Olson, 1981).

The research on teacher cognition and research on teaching suggest that the constructivist metaphor of learning can serve as a referent for pedagogical reform in a number of ways. Firstly, the prior knowledge corollary of the first principle of constructivism, which focuses attention on learners' extant conceptualisations, suggests that the starting point of pedagogical reform should be the explication of teachers' established transmissionist pedagogical beliefs. Secondly, the constructivist metaphor of learning can provide teachers with an alternative set of criteria for determining the viability of their established transmissionist pedagogies. Thirdly, the constructivist metaphor can serve as a referent for teachers' development of teaching strategies oriented towards the conceptual development needs of students. Fourthly, the constructivist metaphor suggests that, because teachers have the most direct access to their own pedagogical beliefs, they should adopt the role of researcher on their own pedagogical thoughts and practice. Therefore, in relation to a constructivist-related theory of radical pedagogical reform, the constructivist metaphor of learning can serve as a referent for both the processes and goals of teachers' pedagogical reforms.

In the context of the research study reported in this thesis, the constructivist metaphor of learning served as a referent for the articulation of a theory of pedagogical reform which is discussed fully in Chapter Two. The theory of pedagogical reform shaped my collaborative role as a facilitator of Ray's constructivist-related pedagogical reform in the context of his Grade 12 mathematics class. The nature of our collaborative research endeavour is discussed in the next section of this chapter.

The Emergent Research Question

On completion of the collaborative phase of the study, however, it became apparent that the constructivist-related theory of pedagogical reform had failed to transform radically Ray's centralist pedagogy (see Chapter Five). Rather than continuing with my original plan of conducting follow-up fieldwork for the purpose of determining the stability of Ray's constructivist-related pedagogical beliefs, I developed a second research question: *To what extent did the cognitivist theory of constructivism used in the thesis constrain the process of pedagogical reform?*

The final section of this chapter argues that research questions which emerge during the course of a study and in response to unpredicted events are a characteristic of interpretive research. The second research question complemented my initial research question about the constraining role of teacher beliefs. It aimed to generate a better understanding of both the constraining nature of Ray's pedagogical beliefs and the constructivist perspective which underpinned my constructivist-related theory of pedagogical reform. The second research question was formulated in relation to additional constructivist-related perspectives which provided interpretive frameworks for the collection and analysis of data in the post-collaborative phase of the study, which is reported in Section Three of the thesis.

Multiple Constructivist Perspectives

Multiple constructivist-related perspectives — *radical constructivism, social constructivism, critical constructivism* — were adopted in order to address the emergent research question. These perspectives are based on the following metaphysical and sociocultural principles of constructivism which are discussed in detail in Section Three of the thesis.

The Metaphysical Principle

During the early 1980s, a small group of mathematics educators was pursuing constructivist-related research in close harmony with an additional constructivist principle associated with Piaget's *genetic* epistemology, and encapsulated in the quotation of Piaget that "intelligence organises the world by organising itself" (von Glasersfeld, 1983, p. 5). This additional metaphysical principle underpins a theory of *radical constructivism* which is promoted in the writings of Ernst von Glasersfeld (1981, 1983, 1984, 1985, 1986, 1987, 1989, 1990, 1991, 1992, in press), and which shaped his collaborative research with Les Steffe and colleagues (Steffe, von Glasersfeld, Richards, & Cobb, 1983). Von Glasersfeld (1988) describes the metaphysical principle of coming to know as an adaptive process in which an individual's cognition organises his/her subjective experiences, rather than replicates objective reality:

> The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1988, p. 83).

Kilpatrick's (1987) critique of the role of constructivism in mathematics education notes the acknowledgement of proponents of radical constructivism (Cobb, 1986; von Glasersfeld, 1985) that the first principle of constructivism was accepted much more widely among constructivists. Furthermore, although the metaphysical principle is alluded to in the research literature, especially in discussions about the nature of science, the first principle served overwhelmingly as the epistemological rationale of the conceptual change research in science education which was conducted in the early 1980s.

The significance of the metaphysical principle of constructivism was overlooked during the collaborative phase of this study. However, this principle is employed in Section Three of the thesis, which addresses the efficacy of the theory of pedagogical reform which precipitated Ray's reforms, in order to illuminate constraints to the development of constructivist-related pedagogies.

Sociocultural Principles

Recently, early constructivist perspectives have been criticised for their unduly individualist, mentalist or rationalist emphasis on the cognitive activity of the individual learner, and their concomitant failure to account for the sociocultural milieux which govern knowledge development (Confrey, 1990; Ginsburg, 1981; O'Loughlin, in press; Sigel, 1981; Solomon, 1987). The constructivist metaphor has been extended to account for important social and cultural aspects of learning.

In Section Three, this thesis extends beyond the radical constructivist perspective of von Glasersfeld in order to identify important sociocultural factors which were not addressed by the constructivist-related theory of pedagogical reform, and which were found to constrain Ray's development of a constructivist-related pedagogy. Chapter Seven adopts a *social constructivist* perspective for considering the constraining nature of the *microsocial* processes (Forman, 1990) of the classroom learning environment, and Chapter Eight adopts a *critical constructivist* perspective for considering the *macrosocial* constraints on Ray's constructivist-related pedagogical reform.

THE COLLABORATIVE RESEARCH ENDEAVOUR

The previous section proposes a theory of pedagogical reform for teacher educators who wish to create curriculum reform conditions which facilitate, from a constructivist perspective, teachers' radical reconstruction of their established transmissionist pedagogies. The theory establishes a context of collaboration between reform-minded teacher educators and teachers who have mutual interests in improving students' opportunities for conceptual development in science and mathematics, and highlights the role of teachers as researchers on their own teaching practice.

Recent reports on the need for radical curriculum reform in science education recognise the importance of the collaborative involvement of teachers in shaping the reform process (American Association for the Advancement of Science, 1989; Shymansky & Kyle, 1991). A *collaborative approach* to classroom-based research, which involves teachers as researchers on their own practice, is well-recognised for the benefits that can accrue to both educational research and teaching practice (Kyle & McCutcheon, 1984; Saphier, 1982; Shymansky & Kyle, 1991; Watt & Watt, 1982), and suggests that this form of research can attain the dual goals of facilitating teachers' pedagogical reform, and investigating the constraints on the reform process. The value of collaborative research on curriculum reform in science education has been widely heralded, especially *collaborative action research* which aims to facilitate teachers' reconstruction of their pedagogical theories and beliefs:

> The collaborative action research model brings together teachers, researchers, staff developers, and others interested in the reform process for the purpose of improving practice. Its goal is to empower teachers to become self-reflective researchers, that is, practitioners who can examine their own practice critically. . . . [It] enables teachers to clarify, modify, and elaborate the theories that inform their instruction. Thus, it offers a method for testing and improving educational practices. (Shymansky & Kyle, 1991, p. 15)

In the research study reported in this thesis, a collaborative research relationship was established between Ray, a high school teacher of mathematics, and myself, a teacher educator, for the initial purposes (1) of reforming his teaching in accordance with a constructivist metaphor of learning, and (2) of investigating the major constraints on constructivist-related pedagogical reform, especially the constraining role of Ray's pedagogical beliefs. This section focuses on the collaborative phase of the study, and discusses the context of the collaborative research relationship between Ray and myself, and the constructivist discourse which governed the relationship.

The Context of the Collaboration

Ray and I negotiated our collaborative research agenda and engaged in a constructivist-oriented discourse in the context of his daily professional activities as a teacher of high school mathematics. Our research interests were shaped by theoretical concerns about the constructivist metaphor of learning as a referent for pedagogical reform, and by practical concerns for the efficacy of Ray's teaching in his Grade 12 mathematics class. A description of the professional setting within which the collaboration between Ray and I occurred provides a context for interpreting the outcomes of the study discussed in Sections Two and Three.

Ray and the Research Site

At the commencement of the study, Ray (a pseudonym used to protect his identity) was a high school teacher of 13 years professional teaching experience. His major teaching areas were biology and physics, which he had taught to all grades at the high school level. He had completed a university degree in biology with minor studies in chemistry and first-year mathematics and physics, but had not obtained a preservice teacher education qualification. Prior to his current appointment he had taught in three other schools in the metropolitan area since graduating from university.

The school which had appointed Ray was a long-established non-government institution funded largely by the Catholic Church. It is located in the metropolitan area of one of Australia's coastal cities, and enrols only female students. The school provides a combination of academic and personal development programs for Grades 8 to 12. At the time of the study, the mathematics department of the school was administered by a specially appointed senior teacher whose role was to ensure that the syllabus was implemented suitably by the teachers in the mathematics department. The content of the Grades 11 and 12 syllabus is prescribed by the Secondary Education Authority, which is a state government organisation with responsibility for administering university entrance courses and examinations for schools in both the government and independent sectors of the state.

Ray had been appointed by the school 18 months previously to assist with the teaching of the two-year university entrance courses in physics and mathematics to the Grade 11 and 12 classes. Ray was responsible to the head of the mathematics department for ensuring that his Grade 11 and 12 classes were well prepared for the Secondary Education Authority's university entrance examinations which are administered at the end of Grade 12. In addition, the school's continuous assessment policy required Ray to ensure that he covered a minimum part of the content of the syllabus in preparation for summative topic tests which were designed by the school and administered during an examination week every three months.

In the previous year, Ray's first year at the school, he had taught mathematics to a Grade 11 class. That had been his first experience at teaching mathematics, apart from some private coaching. At the time of the study, Ray was teaching the second year of the mathematics course to a Grade 12 class which contained about a third of the students from the previous Grade 11 class. The collaborative phase of this study was conducted in Ray's Grade 12 class.

Clearly, Ray was teaching out of field. He was very inexperienced at teaching mathematics, and was unfamiliar with most of the students in the class. However, in other respects he was an experienced high school teacher. Ray was eager for professional development and, at the time of the study, had enrolled as a part-time student in a postgraduate teacher education course in a local university where I worked as a teacher educator. One of the requirements of the course was to conduct a research project. Part of Ray's interest in participating in the study reported in this thesis was to conduct a research project which could be reported as part of the requirements for his postgraduate course.

The Collaborative Research Agenda

Ray and I met in the second year of his course and, because we shared an interest in the prospects of a constructivist metaphor for improving the learning opportunities of students in high school mathematics, I agreed to collaborate with him on his research project. The first step was to negotiate a research agenda that would serve both our interests.

One of the main characteristics of collaborative action research is the involvement of the teacher in establishing the research agenda. The major role of the teacher in planning the research study is considered to be a solution to the ethical dilemmas of traditional research approaches which objectify teachers and serve largely the interests of academic researchers (Brickhouse, 1991). The collaborative involvement of the teacher in establishing a research agenda in relation to his/her problematic teaching practice also is recognised as an important step towards developing teacher empowerment and intellectual autonomy (Carr & Kemmis, 1986; Kemmis & McTaggert, 1988; McNiff, 1988; McTaggert, 1991). However, the nature of collaborative action research relationships is not well-defined (Oja & Smulyan, 1989), and can give rise to ethical dilemmas about the locus of control of the research agenda, and the interests served by the research (Brickhouse, 1991).

Ray's Research Goals

In this study, Ray and I negotiated our common research agenda. Ray nominated his Grade 12 class as a research site because of his concern about the poor academic performance of many students on recent topic tests. He believed that the class might be a fruitful source of mathematical misconceptions which he hoped, with my assistance, to identify and rectify. I agreed to assist Ray with the tasks of identifying students' mathematical misconceptions and developing teaching strategies for resolving them.

My Research Goals

Also, I expressed an interest in taking the opportunity of pursuing my research interest in the role of teacher beliefs in shaping teachers' classroom roles, and suggested that our mutual interest in resolving students' misconceptions might be helped if I focused also on the relationship between Ray's pedagogical thoughts and practice. He agreed to collaborate with me on this task.

Initially, therefore, my research agenda had two major goals. The first was to facilitate Ray's development of a constructivist-related pedagogy which addressed students' conceptual development needs and difficulties. I pursued this goal by implementing the constructivist-related theory of pedagogical reform discussed in Chapter Two. The enactment of this theory in my discourse with Ray is discussed in Chapter Three, and is reviewed below in this section. My second goal was to investigate the constraints which restricted the nature and scope of Ray's pedagogical reform process, especially the constraining nature of Ray's pedagogical beliefs. The second goal, which was expressed as the initial research question of this thesis, was achieved by employing an *interpretive research* approach which was based largely on methodological principles enunciated in Erickson's (1986) review of qualitative research, and which is discussed in the next section of this chapter.

A Constructivist Discourse

The collaborative phase of the study occurred in the first year of the study, in 1987. Ray and I focused our pedagogical reform efforts on his Grade 12 mathematics class during a 10-week period of 24 consecutive lessons. During this period, I attempted, with recourse to the constructivist-related theory of pedagogical reform discussed above, to facilitate Ray's development of a pedagogy oriented towards students' conceptual development needs. Chapter Three discusses the nature of the constructivist-related discourse in which I engaged Ray during the collaborative phase of the study.

In brief, my discourse with Ray was designed to focus his pedagogical attention on the nexus between three interrelated issues, namely: (1) the problematic nature of students' extant mathematical conceptualisations; (2) the constraining nature of Ray's predominantly didactic classroom discourse; and (3) the need for a pedagogy which facilitated students' higher-order conceptual development. I adopted a predominantly interrogatory style of discourse which aimed to assist Ray to become aware of the nature of his established pedagogical assumptions, and to challenge him to reflect critically on the efficacy of his centralist classroom role, especially in relation to students' conceptual development needs and difficulties. My discourse aimed also to convince Ray of the feasibility of the constructivist metaphor of learning, and to assist Ray's development, from a constructivist perspective, of his *pedagogical concept knowledge* (Hand & Treagust, in press).

Not only did the daily discourse between Ray and me serve the purpose of facilitating the reconstruction of Ray's centralist pedagogy, but it enabled me also to address the initial research question of the study. The next section discusses the interpretive research approach which I employed throughout the study, and explains how my discourse with Ray enabled me to investigate the constraining nature of his pedagogical beliefs on the process of pedagogical reform.

INTERPRETING PEDAGOGICAL REFORM

The previous section argues for a collaborative research approach between teacher educators and classroom teachers which has the dual aims of improving educational practice and investigating important issues associated with the related innovations. These dual aims of collaborative research raise the perplexing methodological question of how inquiry can be conducted on the *moving target* of pedagogical innovation, especially when the main focus is on the unobservable and dynamic process of a teacher's reconstruction of pedagogical beliefs? This section discusses the interpretive research approach of this study which I employed in relation to (1) the initial research question of the role of Ray's pedagogical beliefs in constraining the constructivist-related process of pedagogical reform, and (2) the emergent research question of the efficacy of my theory of pedagogical reform.

An Interpretive Research Approach

In order to understand the dynamic relationship between Ray's pedagogical conceptions and his classroom practice, I employed an *interpretive research* approach which was based largely on the qualitative methods of participant observational research on teaching reviewed by Frederick Erickson (1986).

Interpretive research is relatively new in the field of research on teaching (Erickson, 1986), and is somewhat confusingly described as: *case study* research (Benson, 1989; Merriam, 1988); *constructivist* research (Guba, 1991; Lincoln, 1991; Magoon, 1977; Schwandt, 1990, 1991); *critical ethnography* (Anderson, 1989), *ethnomethodology* (Livingstone, 1987); *ethnographic* inquiry (Chilcott, 1987; Eisenhart, 1988; Gallagher, 1984; Rist, 1982; Taft, 1988); *naturalistic* inquiry (Guba, 1978; Guba & Lincoln, 1985, 1988; Phillips, 1992; Welch, 1983), *naturalistic-qualitative* inquiry (Dorr-Brem, 1985); *participant observational* research (Ball, 1988); *qualitative* research (Buchmann & Floden, 1989; Jacob, 1987; Lythcott & Duschl, 1990; Miles & Huberman, 1984; Osborne, 1987; Rogers, 1984; Spector, 1984), *qualitative-interpretive* research (Husen, 1988); and *triangulation* (Denzin, 1988; Mathison, 1988). Although many of these approaches differ in their disciplinary traditions, ontological premises, and fields of application (Spector & Glass, 1991a, 1991b), and employ different research methods, they tend to be constituted by a common epistemological perspective.

The distinctive characteristic of interpretive research is its concern with generating understandings about the significance of what is happening in a particular social setting, such as a classroom, from the perspectives of the participants. In particular, interpretive research includes the interpretive researcher as a participant in the research site, and highlights the *intersubjective* nature of interpretive research activity:

> The research questions posed by interpretivists are intended to get at intersubjective meanings of participants' and researchers' worlds. The questions ask first, What is going on here? and second, What intersubjective meanings underlie these *goings on* and render them reasonable? (Eisenhart, 1988, p. 104)

Because of its special interest in the *meaning-perspectives of the particular actors* (Erickson, 1986, p. 121) in relation to individual and collective actions and interactions, interpretive research is an appropriate research approach to employ in studies of the relationship between teachers' pedagogical thoughts and classroom actions. Erickson (1986) advocates interpretive research for the study of pedagogy, because the research questions which it addresses are "questions of basic significance in the study of pedagogy. They put mind back in the picture, in the central place it now occupies in cognitive psychology" (p. 127). Gallagher's (1991) report on the use of interpretive research in science education indicates its concern with teacher cognition:

> [I]nterpretive research seeks to improve teaching by helping teachers (and researchers) better understand the nature of their work and the meaning they give to it. Thus, interpretive researchers have given much attention to study of teachers' thinking in recent years.
> (Gallagher, 1991, p. 9)

Forms of interpretive research are being employed extensively in constructivist-related studies in science education. Ten years ago Gilbert and Watts (1983) prefaced their review of constructivist-related research of students' personal conceptions with an account of the alternative *verstehen* research paradigm concerned with generating understanding, or interpretation. Recently, interpretive research approaches have been employed in constructivist-related research on teacher epistemologies, beliefs and pedagogical perspectives (Benson, 1989; Tobin & Espinet, 1990; Tobin, Kahle & Fraser, 1990), and in teacher education programs which aim to promote critical, reflective thinking about teachers' own work (Cronin-Jones, 1991), and are advocated as a method of assessment of teaching and learning in science education (Bettencourt, 1991). Eisenhart (1988) reports that interpretive research approaches are being employed in constructivist-related research in mathematics education, especially in studies of teacher beliefs.

An interpretive research approach is highly congruent with the goals of collaborative action research described in the previous section. The development of a *noncoercive, mutually rewarding* relationship with a teacher, which is based on trust and rapport, is advocated by Erickson (1986, p. 142) for the purpose of obtaining valid insights into the teacher's view. As well, Erickson advocates the democratic involvement of the teacher as coresearcher in framing research questions and conducting the research enterprise. The interpretive teacher-researcher role complements the university-based researcher's role as a consultant. The high degree of interaction that occurs between the participants of interpretive research provides optimal conditions for mutual learning aimed at constructivist-related pedagogical reform (Tobin, 1992; Tobin & Jakubowski, 1990; Tobin & Ulerick, 1989), and for the type of collaborative action research that was employed in this study:

> [I]nterpretive research not only helps researchers to learn about teachers' thoughts, beliefs, and values, but it illuminates them for the teachers themselves, thus allowing teachers to become more reflective about their own work. (Gallagher, 1991, p. 7)

This section discusses the interpretive research approach employed in the study reported in this thesis. In particular, the data collection and data analysis methods are explained, and my attempts to optimise the validity of my interpretations of the data are discussed.

Data Collection

The methodological task of collecting data in interpretive research requires the interpretive inquirer to maintain a research focus on the interpretive frameworks of all participants, including his/her own frame of reference, and to maintain a developmental relationship between research questions and data collection processes:

> [T]he task of fieldwork is to become more and more reflectively aware of the frames of interpretation of those we observe, and of our own culturally learned frames of interpretation we brought with us to the setting. . . . [T]he central issue of method is to bring research questions and data collection into a consistent relationship, albeit an evolving one. (Erickson, 1986, p. 140)

In this study, there were two main phases to the collection of interpretive data. The first phase — the *collaborative phase* of the study — occurred during the first year when Ray and I engaged in a process of negotiating constructivist-related pedagogical reform in his Grade 12 mathematics class. During this 10-week period, my main interpretive activities were, initially, to understand the nature of Ray's established centralist pedagogy, especially his transmissionist epistemology and, subsequently, to interpret the nature and extent of the constructivist-related reform of Ray's pedagogy which resulted from our collaborative research endeavour. The interpretive frame of reference, especially my a priori constructivist epistemology which I employed to make sense of data obtained from classroom observations and teacher and student interviews, is outlined earlier in this chapter, and is discussed in detail in Section Two of this thesis.

At the commencement of this study, I had planned to conduct follow-up fieldwork for the purpose of determining the extent to which Ray's constructivist-related pedagogical reforms were sustained in the *post-collaborative* phase of the study. However, because of the unexpected result of the collaborative phase (see Section Two), namely, that Ray's constructivist-related pedagogical reforms had been highly constrained by his established pedagogical beliefs, the follow-up fieldwork served a different purpose.

My main interpretive activity in the post-collaborative phase of the study was (1) to understand better the nature of Ray's pedagogical beliefs which constrained the reform process, and (2) to understand the extent to which my a priori constructivist-related theory of pedagogical reform had served to constrain the reform process. During this phase of the study, I obtained interpretive data from a six-week period of participant observation in Ray's Grade 9 mathematics class, and from teacher interviews. In order to make sense of these data, I referred to the educational research literature and adopted multiple interpretive frameworks which are discussed in Section Three of this thesis.

Researcher Subjectivity

The interpretive framework of the researcher performs a significant role in all aspects of interpretive inquiry, especially in the collection and analysis of data (Erickson, 1986). Eisenhart (1988) describes the activity of *researcher introspection* which "involves the researcher herself or himself reflecting on the research activities and context" (p. 106). It is important, therefore, that the researcher's interpretive frames of reference are presented in an explicit form, and are subject to critical examination (Hutchinson, 1988):

> Since observations are theory-laden, data collection can only be subjective. . . . To be critically subjective, then, is to be conscious of one's beliefs, values, and epistemologies and construct alternative explanations that go beyond those which are associated with the beliefs to which highest value is assigned. The adoption of multiple theoretical perspectives in the interpretation of data and triangulation can assist interpretive researchers in overcoming problems of bias. (Tobin, 1991, p. 203)

Of significance to this study was my constructivist epistemology, especially its role in shaping my constructivist-oriented discourse with Ray during the collaborative phase of the study. In an attempt to identify the full impact on Ray's pedagogical reforms of my epistemological beliefs, I engaged in two *researcher retrospection* activities in writing this thesis.

The first was to make explicit the initial theoretical framework which influenced my discourse with Ray, and which constituted a frame of reference for my interpretation of the efficacy of Ray's pedagogical innovations. Chapters Two and Three in Section Two present accounts of my initial constructivist epistemology and related theory of facilitating classroom-based pedagogical reform. The second strategy was to examine critically the limitations of my initial theoretical framework in order to provide a more comprehensive account of the nature of the constraints to constructivist-related pedagogical reform experienced during the collaborative phase of the study. In Section Three of this thesis, I discuss the efficacy of my constructivist-related theory of pedagogical reform from contemporary multiple theoretical perspectives.

Evidentiary Adequacy

Erickson (1986, p. 140) warns of five major types of evidentiary inadequacy in relation to the inferences drawn from interpretive data: inadequate amounts of evidence; inadequate variety in kinds of evidence; faulty interpretive status of evidence; inadequate disconfirming evidence; and inadequate discrepant case analysis for adjusting major assertions. This section discusses how, in this study, I employed methodological strategies for avoiding these evidentiary inadequacies and, thereby, ensured that the assertions presented in Sections Two, Three, and Four of this thesis are adequately warranted. These strategies involved addressing issues of ethics and equity, and adopting an inquiry approach to data collection.

Ethics and Equity

The previous section discusses the establishment of a collaborative research relationship between Ray and me in relation to our mutual research interests. This relationship was both beneficial to the research and problematic. On the positive side, it facilitated entry to the research site. The school's administration approved readily of the research, and the students in Ray's Grade 12 class seemed welcoming of me, especially in my role as a highly available tutor during class time. Ray explained to both parties that our purpose was to study his teaching and improve its effectiveness.

Our collaborative research relationship seemed to be based on non-coercion, trust and a positive interest in pursuing mutual goals. At times, during recorded interviews, I asked Ray about his perception of the nature of our collaborative research relationship. Although I probed him for any negative aspects, such as coercion or undue demands on my behalf, he denied continually their existence and asserted that our relationship had been positive and rewarding, that he had not experienced any lack of control in relation to his classroom planning and decision-making, and that he found my presence to be professionally stimulating (Interview 1 June 1987, pp. 13, 14; Interview 7 September 1988, pp. 14, 15).

Nevertheless, from my perspective, our relationship became problematic during the post-collaborative phase of the study when I observed Ray's teaching from a non-interventionist perspective, and experienced an ethical dilemma. On these occasions, my research interest was not to intervene in Ray's teaching, but to assess the longer-term impact of the study on Ray's pedagogical beliefs and practice. Although I attended Ray's lessons as a participant observer, our out-of-class discussions were restricted to an exploration of his extant pedagogical beliefs. The daily analysis of fieldnotes was not couched in the type of constructivist discourse which sought to facilitate pedagogical reform. Rather, I was seeking to confirm or refute general assertions about the relationship between Ray's pedagogical beliefs and classroom practice which had been inferred from the data obtained during the collaborative phase of the study.

I did not feel comfortable with my non-interventionist role. My involvement seemed to be predominantly one of self-interest and, although Ray had agreed to participate, it seemed to me to be a rather inequitable form of collaboration. Was this the guilt response of an expert knower wishing to intervene with advice on how to teach more effectively? I prefer to believe that it was a response evoked by a sense of allegiance to a formerly more equitable partnership that thrived on critical discourse. This experience highlighted, for me, the very real prospects of the collaborative research label being misappropriated by educational researchers concerned only with their own research agenda.

Because Ray and I were coresearchers, the data which we collected were shared between us. However, early in the study, I experienced an ethical dilemma about sharing data on students' perceptions. Because students perceived me to be both genuinely interested in their learning difficulties and readily available to provide tutorial assistance, they began to provide me with confidential criticism about the efficacy of Ray's teaching. Initially, when Ray asked what students had been telling me, I found it difficult to reply without breaching students' confidence. I discussed the problem with Ray and we decided that I would provide him only with written feedback on students' perceptions, and would not identify individual students. We extended this policy to the results of recorded student interviews in which all students had agreed to participate (see Appendix C). In order to enhance students' willingness to express their critical opinions about the impact of Ray's teaching on their learning, I guaranteed to preserve their anonymity.

Brickhouse (1991) describes another ethical dilemma in relation to interpretive reports on collaborative action research studies of teaching. In reports on these studies, especially single-authored reports such as this thesis, there is a danger that the researcher's interpretation will mask the teacher's interpretation of his/her own experiences. Brickhouse (1991) believes, from an ethical perspective, that the teacher's *voice* should be heard in the report:

> When reporting teachers' actions in the classroom, it seems important that the reasons for those actions should be carefully probed and . . . the researcher should resist reinterpreting a teacher's experience to the exclusion of the teacher's own interpretation. . . . [T]he teachers' voices should be heard in the report; the report should not entirely consist of the researcher superimposing his or her own voice onto that of the teacher. Such reporting negates the purpose of any genuine dialogue between researchers and participants.
>
> (Brickhouse, 1991, p. 55)

In this study, this dilemma has been addressed by incorporating in this thesis (1) verbatim extracts from Ray's report on the study (Fleming, 1988), and (2) verbatim extracts of interview transcripts. My account of the impact of the study on Ray's pedagogical thoughts and actions which is discussed in Chapter Five, attempts to present Ray's pedagogical perspective. Although subsequent chapters present my increasingly extensive interpretations of Ray's pedagogy, I have attempted to incorporate important aspects of Ray's perspective on the nature and effectiveness of his pedagogical reforms.

An Inquiry Approach

Erickson (1986, p. 143) nominates three methodological issues that should be addressed by the interpretive researcher from the commencement of data collection in order to optimise evidentiary adequacy, and minimise the problem of researcher bias. Firstly, it is necessary to identify the full range of role relationships and meaning-perspectives amongst the participants. Secondly, it is necessary to collect evidence of a wide range of instances of events in order to establish their typicality. Thirdly, events at one system level should be examined from the contexts of multiple system levels. These methodological issues were addressed in this study by: (1) allocating sufficient time for fieldwork; (2) employing triangulation methods; (3) progressively focusing on issues of central concern; and (4) deliberately seeking disconfirming evidence of key assertions.

Fieldwork Time. Erickson's (1986) conditions for optimising evidentiary adequacy and minimising researcher bias require the researcher to spend sufficient time in the field collecting data. During the 10-week collaborative phase of the study, I attended 24 consecutive lessons in Ray's Grade 12 mathematics class. In the post-collaborative phase of the study, in the following school year, I attended 14 consecutive lessons presented by Ray to his Grade 9 mathematics class, in the same school. During both periods of fieldwork, my participant-observer role enabled me to record observations of teacher and student activities and interact as a tutor with students. The data were recorded in fieldnotes to which data from other sources were added. Rather than including in this thesis the vast amount of fieldwork data which were collected during the study, a sample of the fieldwork data and preliminary data analyses associated with the first two lessons in Ray's Grade 12 class has been included in Appendix A.

Erickson (1986) argues that adequate time is required also to reflect on the daily fieldnotes prior to returning to the field for further data collection. Reflecting on the data provides the opportunity for the researcher to determine continuously the research focus for the next data collection activity:

> A fundamental principle is that this subsequent reflection and write-up, which usually takes at least as long as the time spent initially in observation, needs to be completed before returning to the field setting to do further observations. . . . Write-up stimulates recall and enables the researcher to add information to that contained in the unelaborated raw notes. Write-up also stimulates analytic induction and reflection on relevant theory and bodies of research literature.
>
> (Erickson, 1986, p. 144)

Immediately on return from Ray's class, each day, I analysed my fieldnotes and recorded the analysis in the form of written *Reflections*. My Reflections contained descriptive summaries of the main aspects of the lesson, posed questions about Ray's intentions, and inferred assertions about the nature, and impact on student learning, of Ray's pedagogical beliefs and practice (see, for example, Appendix A).

On return to the school, each day, I gave a copy of my Reflections to Ray, and asked him to verify the contents and respond to the questions. Ray and I discussed his lesson plans prior to each class and, after each lesson, we discussed his response to my Reflections of the previous day's lesson. Ray's responses were recorded in my fieldnotes and, subsequently, incorporated into my next set of Reflections. The Reflections provided a focus for my subsequent classroom observations which sought evidence to support or refute tentative assertions about the nature of Ray's pedagogical beliefs which gave rise to his classroom actions.

Triangulation. Denzin (1988) argues that the application and combination of several research methodologies in the study of the same phenomenon in the social sciences is necessary for overcoming the biases of a single method, and for constructing sound interpretations of the social world. Denzin (1988, p. 512) identifies four types of triangulation which are relevant to this study. *Data triangulation,* which involves multiple data sources and locations, is evidenced in this study by obtaining data from: (1) two principal sources, namely, Ray and the students in his mathematics classes; (2) the same classroom setting at a number of different times in the day, week, month and year; (3) different classroom settings —Grades 9 and 12 — separated both in space and time; and (4) non-classroom settings, when teacher and student interviews were conducted. *Investigator triangulation,* which involves multiple investigators, was achieved by maintaining a largely equitable and ethical collaborative research relationship between Ray and me throughout the study.

Theory triangulation, which consists in using multiple theories to interpret the same phenomenon, occurred in two ways. Firstly, when the data on Ray's pedagogical beliefs were interpreted, two theories of the nature of beliefs were employed. The analysis of Ray's established pedagogical beliefs in Chapter Four employed metaphor as an analytical tool. The analysis of Ray's reformed pedagogical beliefs, reported in Chapter Five, employed the repertory grid technique which assumes that beliefs are propositional in nature. Secondly, when the post-collaborative phase of the study was conducted, data were interpreted from multiple interpretive frames of reference (see Section Three).

The use of multiple research methods, or *methodological triangulation,* was achieved by employing participant classroom observation, structured and unstructured interviews, and document analysis of textbooks and curriculum materials. In the case of unstructured teacher interviews, my intention was to engage Ray in *an interesting conversation* which was designed to assist us "to learn more about how the actors in the setting perceive their environment, understand their actions, and anticipate the views and behaviours of others" (Rist, 1982, pp. 443-444).

In these ways, triangulation contributed to a sound interpretation of the data corpus in this study. However, rather than adopt an objectivist position of assuming that triangulation results in eliminating bias and revealing the truth, the notion of triangulation employed in this study is one of obtaining several levels of inconsistent, convergent, and contradictory evidence for constructing *plausible explanations* (Mathison, 1988). The process of formulating plausible explanations, or warranted assertions, is discussed below.

Progressive Focusing. One of the main methodological strategies employed in this study was *progressive focussing* (Parlett & Hamilton, 1977). Initially, my research focus was general and sought to understand macrosocial issues, such as: students' seating patterns and friendship groups, the organisational nature and duration of learning activities, the content of the planned and implemented curricula, and the pedagogical content of whole-class teacher-student discourse. Subsequently, I focused on microsocial issues, such as: Ray's perspectives, expectations and idealisations in relation to the quality of textbooks, the abilities and classroom roles of students, the professional task of teaching, and his professional accountability; individual students' perspectives and expectations in relation to Ray's teaching, other students' classroom roles, and personal difficulties experienced in learning mathematics; and the quality of individual students' mathematical conceptualisations.

Throughout the data collection phases of the study, my research focus continuously *zoomed in* on microsocial issues and *zoomed out* on macrosocial issues. For example, my understanding of the opportunities which Ray provided for students to engage in mathematical conceptual development during class was based on data obtained from (1) a macrosocial focus on teacher-student and student-student classroom discourse in the multiple contexts of whole-class activities, small-group activities and individual seatwork activities, and (2) a microsocial focus on the mathematical conceptualisations of individual students.

Disconfirming Evidence. Erickson (1986, pp. 139-143) warns about the methodological problem of *premature typification* which occurs when the researcher maintains a focus on (1) frequently recurring, rather than atypical, events, and (2) evidence which confirms, rather than disconfirms, initial assertions. In this study, both confirming and disconfirming evidence were sought continuously throughout the study, and discrepant cases were analysed and used to qualify key assertions. Two examples of the important role performed by deliberately obtained disconfirming evidence follows.

Firstly, as a result of maintaining a research focus on the full range of students' mathematical conceptualisations during the collaborative phase of the study, rather than focusing only on student misconceptions, I identified a higher-achieving minority group in addition to the lower-achieving majority group of students. Consequently, my assertions about the negative influence of Ray's pedagogy on students' learning outcomes, which are reported in Section Two, are restricted to the former group, rather than generalised to the class as a whole.

Secondly, because my research focus on Ray's epistemology sought evidence continuously throughout the study of the existence of both the residual nature of his former transmissionist epistemology and his newly developed constructivist epistemology, I avoided the self-fulfilling prophesy of finding evidence only of a constructivist-related pedagogy. In the second year of the study, my deliberate attempts to obtain evidence of the failure of radical pedagogical reform resulted in evidence which warrants the assertion that Ray's transmissionist epistemology was largely sustained throughout the study (see Assertion Six in Chapter Five). This evidence stimulated me to refer to the educational research literature in an attempt to understand better the limitations of my constructivist-related theory of pedagogical reform. Consequently, the interpretive analysis presented in Section Three was conducted on the entire data corpus (Fieldnotes, Reflections, interviews, earlier reports on the study), and important assertions were developed about the prospects of future constructivist-related pedagogical reform.

Data Analysis

Although the analysis of intepretive data occurs continuously throughout the data collection phase of interpretive research studies, as evidenced by the evolving relationship between research questions and data collection (Erickson, 1986; Rist, 1982; Spector, 1984), the reporting of the study constitutes a unique opportunity to conduct an analysis on the entire data corpus. The particular interest of the report's main audience will prefigure, to an extent, both the content of the analysis and the style of report. However, Erickson (1986) argues that there are important analytical methods which must be observed to enable the reader "to understand the case and judge the validity of the author's interpretive analysis" (p. 146). This section discusses the main analytical methods employed in the reporting of this study which should, according to Erickson:

> . . . allow the reader to experience vicariously the setting that is described, and to confront key assertions and analytic constructs . . . to survey the full range of evidence on which the author's interpretive analysis is based . . . to consider the theoretical and personal grounds of the author's perspective as it changed during the course of the study. (Erickson, 1986, p. 145)

This discussion focuses on two major methodological issues, namely, the formulation of the warranted assertions presented in the subsequent chapters of this thesis, and the nature of their evidentiary warrants.

Formulating Warranted Assertions

During the course of fieldwork, the interpretive researcher generates *empirical assertions* in relation to his/her evolving research questions (Erickson, 1986, p. 146). Other interpretive researchers refer to the generation of *categories, themes,* and subsequent *hypotheses* (Spector, 1984), or to *grounded theory* (Glaser & Strauss, 1967). On return from the field, the researcher conducts a search of the entire data corpus — fieldnotes, interview transcripts, site documents — looking for both confirming and disconfirming evidence in order to warrant the assertions.

Erickson (1986) reports that assertions can vary in scope and in level of inference. As the fieldwork progresses, and the data corpus increases in size, the researcher reviews earlier assertions in response to new data. Earlier low-inference *subassertions* can be modified, especially in response to disconfirming evidence or discrepant cases and, later in the study, can be subsumed by *general assertions* which have been generated to account for patterns of generalisation at higher levels of inference which apply across a substantial proportion of the data corpus. Erickson (1986) argues that the most plausible assertions are warranted by many instances of items of data from multiple data sources, and by their ability to account for both frequently occurring and rare events. However, plausibility should not be interpreted as proof:

> The aim is to persuade the audience that an adequate evidentiary warrant exists for the assertions made, that patterns of generalization within the data set are indeed as the researcher claims they are.
>
> (Erickson, 1986, p. 149)

In this study, my classroom observation data were recorded in fieldnotes which I analysed immediately on return from the field, each day. This analysis was directed by tentative assertions which I had formulated during class or on my way back to the office. My fieldnotes comprised both *instances of action,* which had occurred in Ray's classroom, and *instances of comments on the significance of these commonplace actions,* which I had elicited from Ray during debriefing sessions after each lesson, and from both Ray and his students during a series of recorded interviews throughout the study. Erickson (1986, p. 149) describes these data as the basic units of interpretive analysis. My daily data analysis sought to confirm or disconfirm assertions by seeking patterns of generalisation within the fieldnotes and site documents (textbooks, worksheets, curriculum guides, tests). The analysis of trends in the data comprised both qualitative commentary and quantitative summaries of the frequency of events, and were recorded as Reflections (see, for example, Appendix A).

As the study progressed, I examined continuously my earlier assertions by reviewing both the analytical Reflections and fieldnotes. The transcripts of teacher and student interviews (see, for example, Appendix B) served as major sources of data for retrospective assessment of my assertions, especially after the completion of each phase of fieldwork when more time was available for protracted periods of data analysis. The examination of assertions continued well beyond the final data collection phase of the study. The formulation and testing of general assertions (i.e., Assertions One to Eleven) was facilitated by my subsequent holistic perspective on the data corpus which enabled *key linkages* (Erickson, 1986, p. 148) to be substantiated across a wide range of data contained in multiple data sources. These general assertions were tested further in relation to alternative theoretical perspectives (see Chapters Six, Seven and Eight) which I had developed as a result of my continuous reading of the educational research literature, and my preparation of papers for presentation at international conferences (Taylor, 1990a, 1991; Taylor & Fleming, 1987; Taylor & Williams, 1992) and for publication (Taylor, 1990b, in press).

Finally, the general assertions of greatest relevance to the research questions of the study (i.e., Assertions Five to Eleven in Chapters Five to Eight) were synthesised into a single *global assertion* (see Chapter Nine). This *strongest assertion* constitutes the highest level of generalisation whose scope of applicability extends "across the widest possible range of sources and kinds of data within the case" (Erickson, 1986, p. 148). The concept of a global assertion is similar to the concept of emergent *grounded theory* of Glaser and Strauss (1967) which, Merriam (1988) explains, is a theory that is grounded in the data and "seeks to explain a large number of phenomena and tell how they are related" (p. 146). The global assertion of this study shares with grounded theory the major characteristics of *parsimony* and *power*, that is, it constitutes a smaller set of higher level concepts which have high explanatory power within the case.

Reporting Warranted Assertions

Erickson (1986) describes the aims in writing the report of an interpretive research study as clarification and justification of the assertions. These aims can be achieved by including three major types of content in the report. The core of the report is *particular description,* which comprises analytic narrative and quotations extracted from fieldnotes, and which serves to illustrate instances of social action. Evidence of the generalisability of patterns of social action within the study is provided by *general description* which can contain synoptic surveys such as frequency tables (see, for example, the discussion on lessons one and two in Chapter Four). The particular and general descriptive evidence is presented within the author's interpretive frame of reference which is denoted by *interpretive commentary*. Interpretive commentary contains instances of comments, obtained from the participants in the study, of the significance of social actions, and "points the reader to those details that are salient for the author, and to the meaning-interpretations of the author" (Erickson, 1986, p. 152). The interpretive commentary in a report also should indicate the inevitable changes in the author's perspective which occurred as the study progressed.

In Section Two of this thesis, Chapters Two and Three discuss fully the initial interpretive frame of reference with which I designed and conducted the collaborative phase of the study and analysed the results. This interpretive framework incorporated a constructivist epistemology which was based on the first principle of constructivism and the prior knowledge corollary.

Chapters Four and Five utilise this initial interpretive framework to discuss the results of the collaborative phase. Each chapter presents one or more general assertions about key issues associated with Ray's pedagogical actions, either before or after the process of pedagogical reform (i.e., Assertions One to Six). The evidence which warrants the assertions about the nature and scope of Ray's pedagogical reforms comprises a combination of particular and general descriptions of (1) Ray's pedagogical interests and beliefs, and classroom actions, and (2) students' perspectives, and classroom actions. The interpretive commentary, which was governed by my initial constructivist perspective, contains evidence of both student and teacher pespectives on the significance of key events which were observed to occur in the classroom.

However, the assertions of Section Two were formulated and substantiated wholly within the interpretive framework which was established at the commencement of this study, and necessarily present a restricted interpretation of the pedagogical reform process. In Section Three of this thesis, multiple interpretive frames of reference, which are based on contemporary theories of constructivism, are adopted in order to generate a better understanding of the results discussed in Section Two. Multiple interpretive frameworks are employed in Chapters Six, Seven and Eight for the purpose of discussing the evidentiary warrants for general assertions (i.e., Assertions Seven to Eleven) which propose explanations for the lack of efficacy of the initial constructivist perspective of the study for facilitating a radical reconstruction of Ray's transmissionist pedagogy. The interpretive commentary includes both particular and general descriptive evidence from both the collaborative and post-collaborative phases of the study.

In Chapter Nine, the results of the study are synthesised and presented in the form of a single *global assertion*. This assertion addresses a research question which arises from the results of the study and which constitutes a synthesis of the two research questions of the study: *Is the first principle of constructivism an adequate referent for pedagogical reform which aims to resolve the problem of teacher centralism and develop a pedagogy aimed at improving the mathematical conceptual development of students of high school mathematics?* The purpose of this assertion is to focus the reader on the general implications of the results of this case study for similar, but unspecified, future cases of constructivist-oriented pedagogical reform of the traditional high school mathematics classroom. The assertion constitutes emergent, *grounded theory* (Glaser & Strauss, 1967) whose transferability to other situations requires empirical investigation. The concept of external validity which governed the formulation of the global assertion is discussed in Chapter Nine.

SECTION TWO

CONSTRUCTIVIST PEDAGOGICAL REFORM

Section Two of the thesis, which comprises Chapters Two to Five, addresses the initial research question: *How do teachers' role-determining beliefs constrain the process of constructivist-related pedagogical reform?* A constructivist-related theory of pedagogical reform is developed (Chapter Two), and its collaborative implementation in Ray's Grade 12 mathematics classroom is discussed (Chapter Three). From the perspective of the constructivist theory which guided the pedagogical reform process of this thesis, an interpretive analysis of Ray's pedagogy which had been established prior to the commencement of the collaborative phase of the study is presented (Chapter Four). Finally, an interpretive analysis is presented of the main outcomes of the implementation of the theory of pedagogical reform in Ray's Grade 12 mathematics classroom (Chapter Five).

CHAPTER TWO

A CONSTRUCTIVIST-RELATED THEORY OF PEDAGOGICAL REFORM

INTRODUCTION

This chapter reports on the development of a constructivist-related theory of pedagogical reform that was designed to resolve the problem of *teacher centralism* which is discussed in Chapter One. The theory aims to facilitate teacher development of constructivist-related pedagogies for high school mathematics. The theory was enacted in the form of a *constructivist discourse* between Ray and me during the collaborative phase of the study. Chapter Three discusses the main features of the discourse which attempted to facilitate Ray's development of a pedagogy oriented towards the conceptual development needs of students in his Grade 12 mathematics class.

The first section of this chapter considers the problematic nature of teachers' established pedagogical beliefs, especially their role in sustaining teachers' centralist classroom roles. The literature on students' alternative conceptual frameworks in science education is reviewed, and the resilient nature of students' intuitive beliefs is highlighted as an analogous problem of constructivist-related pedagogical reform. Research on the nature and influence of teachers' pedagogical beliefs is reviewed, especially teachers' implicit, role-determining beliefs. It is argued that constructivist-related pedagogical reform requires a *post-rationalist* approach to reconstructing teachers' major role-determining beliefs, especially their transmission-related epistemological beliefs which underpin their centralist classroom roles.

The second section develops a constructivist-related theory of pedagogical reform aimed at facilitating a radical reconceptualisation of teachers' centralist classroom roles. Research on conceptual change teaching in school science is reviewed, and conditions for establishing conceptual change in students' intuitive *lifeworld* beliefs are discussed.

Arguing by analogy, the findings of research on conceptual change teaching are combined with complementary research on teacher cognition, and a theory of pedagogical reform is established for shaping my collaborative facilitation of Ray's constructivist-related pedagogical reform.

THE ROLE OF TEACHER BELIEFS

Chapter One argues that a transition to constructivist-related pedagogies which directly address students' conceptual development needs requires a transformation of teachers' well-established centralist pedagogies, especially their transmissionist epistemologies. This requirement raises questions about the most appropriate means of facilitating teachers' epistemological reconstructions. However, an important prior consideration is the nature of teachers' role-determining beliefs, especially their susceptibility to change. This section explores the nature of teachers' pedagogical beliefs, especially their influence on curriculum innovation, and their susceptibility to reconstruction.

Conceptual Change Teaching: An Analogous Problem

The challenges involved in facilitating teachers' epistemological reconstructions can be appreciated by considering an analogous pedagogical challenge confronting modern science teachers who aim to transform their students' well-established beliefs about the natural world.

Traditional science teaching has involved teachers in the provision of scientific explanations of natural phenomena, and has focused teachers' pedagogical concerns mainly on developing strategies for presenting rational explanations in *interesting* ways. Students' pre-teaching beliefs about the nature of the physical world have received very little pedagogical recognition, apart from their sometimes comically inaccurate nature — the recounting of students' examination *howlers* has long been part of teachers' folkloric conversations.

Recently, however, research in science education has found that traditional *rationalist* teaching approaches, which inform students that their seemingly viable interpretations of the natural world should be abandoned in favour of counter-intuitive, scientifically acceptable alternatives, are far from successful. Despite teachers' careful rational explanations of the scientific version of a phenomenon, many students' *preconceptions*, or experientially-based intuitive beliefs, tend to persist (Gilbert, Osborne, & Fensham, 1982; Osborne & Wittrock, 1983; Pines & West, 1986).

For example, many students of school science remain unconvinced about Newtonian dynamics when much of their immediate experiences of how physical objects move can be explained to their satisfaction by their well-established Aristotelian conceptions (Champagne, Klopfer & Gunstone, 1981). The persistence of naive conceptual frameworks also has been detected among the academically elite, including those in university undergraduate science courses (Thorley, 1986). Especially compelling evidence of the persistence of naive conceptualisations is presented in a series of video-recorded interviews that examine the relationship between didactic teaching methods and students' conceptualisations of physical phenomena, such as the occurrence of seasonal variations in the earth's climate and lunar eclipses of the sun (Schneps, undated).

However, the fact that a student of science expresses the scientifically *correct* view of a phenomenon does not mean necessarily that the student is committed to that view. Students are capable of espousing scientific principles while harbouring contradictory theories of *real-life* events (Pines & West, 1986). This problem highlights the sometimes tenuous nature of students' formal, or officially sanctioned, scientific knowledge, especially when it conflicts with the intuitive *lifeworld knowledge* that constitutes the personally viable perspectives of their everyday experiences (Pusey, 1987).

Clearly, students are capable of resisting teaching attempts to abandon their well-established and intuitive commitments to naive, or outmoded, conceptualisations of the natural world. Rationalist arguments, alone, seem to be less than effective in promoting the reconstruction of many students' natural, world-view conceptualisations.

The challenge for modern science teaching, which aims to facilitate students' scientific conceptual development, therefore, is to develop teaching strategies which address directly students' pre-instructional, resilient conceptual frameworks, especially frameworks which harbour viable, but scientifically naive, intuitive beliefs. In other words, the appropriate *starting point* for modern science teaching is the extant intuitive beliefs of students, rather than the scientific beliefs of teachers.

The analogous problem of facilitating students' conceptual change in science teaching highlighted the need, in this study, to address the well-established, intuitive frameworks of pedagogical beliefs which underpin teachers' centralist classroom roles. A pedagogical reform strategy which provides teachers with only rational explanations for adopting counter-intuitive, constructivist-related teaching approaches seemed likely to be highly susceptible to the predominant influence of their well-established pedagogical beliefs, especially their transmissionist epistemologies.

Teachers' Pedagogical Beliefs

Evidence of the predominant influence of teachers' established role-determining, intuitive beliefs has been reported as a curriculum implementation problem (Munby, 1982, 1983, 1984; Olson, 1980, 1981), and has given rise to research on the nature and influence of teachers' role-determining theories and beliefs which has been reviewed by Clark and Peterson (1986).

A curriculum implementation problem occurs when the reforms of curriculum designers require teachers to transform their classroom roles, especially if a radically counter-intuitive reconceptualisation of teachers' roles is prescribed. The strength of teachers' commitment to their long-held beliefs in their centralist classroom roles should not be underrated. These beliefs have been formed and reinforced during a life-time of schooling, commencing with their early childhood classroom experiences, and culminating in their teaching careers, and are likely to be so deeply *ingrained* that they underpin a normative world-view of classrooms and schooling. In other words, a centralist classroom role for many teachers is an unquestioned, natural reality.

The recent trend in curriculum reform towards constructivist-related approaches to the teaching and learning of school science and mathematics (Cobb 1989; Cobb, Wood, & Yackel, 1990, in press; Driver, 1988, 1990; Driver & Oldham, 1986; Ernest, 1991; Steffe, 1989, 1991; von Glasersfeld, 1991, 1992, in press) has heightened the importance of developing an effective approach for changing teachers' beliefs about their centralist classroom roles.

At the commencement of this study, educational research was beginning to investigate the nature and influence of teachers' cognition on their classroom roles. A major review of teacher thinking (Clark & Peterson, 1986) argues that teachers' beliefs and theories directly affect many aspects of their professional work, including lesson planning and the decisions which they make during classroom interactions with students. The review classifies teachers' pedagogical beliefs and theories as being either *explicit* or *implicit*.

Explicit Beliefs and Attributions

Clark and Peterson's (1986) review of teachers' thought processes highlights the growing attention being paid to the importance of teacher's theories and beliefs, especially teachers' attributions of the causes of students' academic performance. Research on teachers' attributions has grown out of research on students' attributions, and has been based largely on the attribution theories of Cooper and Burger (1980), Bar-Tal and Darom (1979), Frieze (1976), Weiner (1974, 1979), Weiner, Frieze, Kukla, Reed, Rest, and Rosenbaum (1971), and Weiner and Kukla (1970).

The research reviewed by Clark and Peterson (1986) has assumed that teachers' attributions directly affect their perception of the academic prospects of different types of students, and give rise to teachers' expectations of differential academic performance amongst a class of students. Furthermore, it is assumed that teachers' expectations of students' differential academic performance are communicated to students, and affect students' own attributions and academic performance. Of central concern to research is the relationship between teacher's attributions, their sense of responsibility for students' performance, and the actions that they take to improve students' academic achievement:

> An implicit assumption of researchers has been that if teachers fail to accept responsibility for students' successes or failures, and thus fail to see a relationship between their behaviour and students' performance, they would be less likely to work to improve these students' performance in the classroom. Thus researchers have been concerned with factors that affect teachers' attributions and, in particular, the extent to which teachers accept responsibility for students' successes or failures. (Clark & Peterson, 1986, p. 282)

Clark and Peterson's (1986) review associates teachers' attributions with the Nisbett and Ross (1980) definition of theories and beliefs as "reasonably explicit propositions about the characteristics of objects or object classes" (p. 281) (see Chapter Five for a discussion of the concept of propositional beliefs). This definition implies that teachers are aware of their attributions and that they might readily be articulated as propositional statements, especially with assistance from a researcher. The review reports the findings of a wide range of educational research which has employed questionnaires and simulation methods to explicate teachers' attributions.

Research on teachers' attributions has revealed that many teachers attribute students' academic success or failure to factors such as student characteristics, the quality of instruction, the nature of the subject, or chance (Ruthven, 1987), depending on their belief about the causes of individual students' academic behaviour. It is not uncommon for teachers to express a belief in the inherently limited academic capabilities of lower-achieving students, and attribute the success of higher-achieving students to the merits of their instruction (Clark & Peterson, 1986). For example, a study of teachers' attributions and beliefs about girls, boys, and mathematics (Fennema, Peterson, Carpenter & Lubinski, 1990) concludes that teachers held gender-stereotypical beliefs about the causes of first-grade girls' and boys' academic performance and achievement. Teachers believed that the causes of academic success and failure were different for boys and girls. However, the research was unable to conclude that boys' and girl's differential academic achievement was caused by teachers' differential expectations and teaching behaviour.

Clark and Peterson (1986) argue that the speculative nature of much of the research on the relationship between teachers' attributions and students' academic achievement warrants closer investigation, especially in classroom settings:

> Research is needed that moves from laboratory settings . . . to real-world classroom settings in which researchers study teachers' attributions as part of teachers' on-going thoughts and actions during everyday teaching. (Clark & Peterson, 1986, p. 285)

In this study, one of the main research foci was the nature of Ray's explicit theories and beliefs about the prospects of pedagogical reform in his own classroom. An examination of Ray's attributions of (1) his own classroom role and (2) students' academic performance provides a significant contribution to understanding the nature and scope of Ray's constructivist-related pedagogical reforms (see Chapter Eight).

Implicit Theories and Beliefs

On the other hand, research on teacher thinking has revealed that teachers develop and hold implicit theories on a range of pedagogical issues, and that these theories play an important part in their everyday judgements and interpretations (Clark, 1988). However, these implicit theories are not readily classifiable:

> [T]eachers' implicit theories tend to be eclectic aggregations of cause-effect propositions from many sources, rules of thumb, generalizations drawn from experience, beliefs, values, biases, and prejudices. . . . [T[hese systems are not clearly articulated or codified by their owners. (Clark, 1988, p. 6)

Clark and Peterson's (1986) review of teacher thinking emphasises the importance of explicating teachers' *implicit* pedagogical theories and beliefs which shape their classroom roles and provide *psychological contexts* for their lesson planning and interactive classroom decisions:

> For the individual teacher this psychological context is thought to be composed of a mixture of only partially articulated theories, beliefs, and values about his or her role and about the dynamics of teaching and learning. The purpose of research on teachers' implicit theories is to make explicit and visible the frames of reference through which individual teachers perceive and process information.
> (Clark & Peterson, 1986, p. 286)

More recently, Tobin and Espinet (1987) argue that the teaching routines developed over time by experienced teachers are based largely on subconscious beliefs about teaching and learning. These partially articulated beliefs form stable perspectives through which individual teachers *filter* information conveyed by curriculum developers and researchers (Eisenhart, Shrum, Harding, & Cuthbert, 1988; Nespor, 1987).

Clark and Peterson's (1986) review reports on a number of small-scale studies of teachers' implicit pedagogical theories and beliefs (or *personal perspectives, principles of practice, construct systems, conceptual systems, practical knowledge*) which had been conducted by employing research approaches such as ethnographic inquiry, clinical interviews, stimulated recall, and the repertory grid technique. The review summarises the results of these studies of teachers' implicit theories about their work as follows: (1) teachers' implicit theories are varied; (2) an individual teacher's implicit theories are small in number; (3) teacher's implicit theories focus on student characteristics and states, teacher states, and the structure and organisation of the subject matter; (4) there is not always a high correspondence between teachers' espoused beliefs and classroom behaviour, and the correspondence is moderated by factors (e.g., time available, student ability, mandated curriculum, resources) which are beyond Ray's control; and (5) curriculum implementation difficulties can be associated with conflicting beliefs about the nature of good teaching held by teachers and curriculum developers.

The research on teachers' implicit pedagogical beliefs had major implications for the design of this study. The research indicates that a reliance on a research approach which focuses only on the articulation of teachers' explicit pedagogical beliefs as a means of investigating the major epistemological conceptualisations underpinning their classroom roles is likely to result in little more than an understanding of the surface features of conceptual *icebergs*. This metaphor highlights the need to investigate the hidden features of teachers' conceptual systems which lie in the subconscious *depths* beneath their conscious awareness.

Because the study reported in this thesis focused on Ray as a curriculum innovator, the research on teachers' implicit theories and beliefs which addresses the problem of curriculum innovation was especially salient.

Problem of Curriculum Innovation

A study by Olson (1980, 1981) of the implementation in British schools of an innovative student-centred science curriculum — the Schools Council Integrated Science Project (SCISP) — found that the process of innovation was characterised by teachers' unwillingness to abandon their *strong influence* on classroom events. Olson (1980, p. 8) concludes that teachers' *translated* the innovative curriculum principles into more familiar teaching practices consistent with their established centralist classroom roles, as they struggled to overcome perceived dilemmas with the requirements of their new roles. In particular, teachers acted with the intention of minimising the conflict and uncertainty associated with the increased *diffuseness* of their new roles.

The *top-down* rationalist approach of the SCISP curriculum reform program, which had delivered ready-made curriculum products for implementation by classroom teachers, had two main problems: (1) it ignored the *complex stable systems* in which teachers operate, and the *subtle interconnections* between their pedagogical goals, expectations and techniques; and (2) it assumed that the rationale of the innovation was universally *transparent* and, therefore, not susceptible to differential interpretations by teachers. Olson (1980, pp. 3-4) concludes that the teachers translated the jargon of the new curriculum into their own established pedagogical language. The translation was inevitable because of the cultural embeddedness of meaning — an innovation is in the eye of the beholder — and the cultural differences that existed between the curriculum designers and teachers. Olson (1980) recommends that the focus of curriculum reform should be on the *meaning systems* of teachers, rather than on the adoption process:

> The focus on adoption of innovation distracts from the existence of a functioning system of constructs and the need for an adequate representation of that system. (Olson, 1980, p. 3).

Munby (1982, 1983, 1984) reports, in the form of case studies, teachers' implicit theories of teaching, which he describes in terms of five construct categories: (1) student learning and developmental goals; (2) student involvement; (3) teacher control and authority; (4) student needs and limitations; and (5) motivation. Munby's research demonstrates both the idiosyncratic and powerful nature of teachers' pedagogical beliefs and principles, and helps to explain the inevitability of differential interpretations and implementations by teachers of a nominally common curriculum.

Both Munby's (1982, 1983, 1984) and Olson's (1980, 1981) research highlight the strong influence of teachers' implicit pedagogical theories and beliefs on curriculum implementation and innovation. This research strongly suggests that curriculum innovation, especially innovation which requires a radical change in teachers' established centralist classroom roles, should involve the individual teacher in a professional development process of critical reflection on the effectiveness of his or her partially articulated pedagogical theories and beliefs which are likely to resist changes in their classroom roles. Clark and Peterson (1986) conclude their review of research on teacher thinking with a brief portrait of *maturing professional teachers* who "as researchers on their own teaching effectiveness" (p. 293), maintain a state of critical awareness of the relationship between their pedagogical theories and classroom practice.

In this study, Ray's implicit pedagogical theories and beliefs were of central concern in the pedagogical reform process which was based on a collaborative research approach that promoted a teacher-as-researcher role. Of chief concern were Ray's epistemological beliefs.

Epistemological Beliefs

The analysis of Ray's established centralist pedagogy, reported in Chapter Four, reveals that one of his major role-determining pedagogical beliefs was that knowledge could be transmitted from teacher to students. Although this transmission-oriented belief per se was not expressed by Ray, its existence was inferred from an examination of the relationship between Ray's teaching strategies and discourse, especially in situations where Ray expressed his pedagogical priorities in the context of a *teaching as journey* metaphor. Ray's implicit epistemological belief was consistent with his priority interest in *covering* the syllabus, and his explicit belief in the inevitability of failure of many lower-achieving students. In other words, Ray's implicit epistemological belief was well-buttressed by other pedagogical beliefs and priority interests.

It was clear that the challenge of resolving the problem of teacher centralism required a constructivist-related theory of pedagogical reform for changing Ray's implicit, role-determining beliefs, especially beliefs which were antithetical to a constructivist theory of knowledge, and which enabled him to make sense of the most important aspects of his established centralist classroom role.

It seemed highly likely, therefore, that one of the major constraints to the successful facilitation of the desired constructivist-related pedagogical reform would be the influence of the implicit epistemological beliefs which underpinned Ray's centralist pedagogy. Because of their implicit nature, epistemological beliefs are likely to be less than highly susceptible to rational arguments for radically changing a well-established, centralist classroom role. It seemed likely, therefore, that pedagogical reconstruction efforts which relied solely on rational arguments for major changes in Ray's *natural* classroom role would experience a very short *half-life*.

For this study, the main problem to be resolved in facilitating constructivist-related pedagogical reform, therefore, was to develop a *post-rationalist* approach to professional development, an approach which would enable Ray to engage deliberatively in a process of reconstruction of the well-established, implicit pedagogical beliefs which underpinned his centralist classroom role, especially the transmission-oriented epistemological beliefs which are antithetical to a constructivist theory of knowing.

A THEORY OF PEDAGOGICAL RECONSTRUCTION

The preceding section argues, from a constructivist perspective, that the problem of resolving teacher centralism is a problem of facilitating reconstruction of teachers' conceptualisations of their classroom roles. In this study, a theory of pedagogical reconstruction was required that would obtain an intellectual commitment to a constructivist epistemology, and a complementary change in teaching strategies. This section reviews analogous research on conceptual change teaching in school science, and research on teacher cognition, and develops a constructivist-related theory of pedagogical reform aimed at facilitating a radical reconstruction of teachers' centralist classroom roles.

Conceptual Change Teaching: An Analogous Approach

The challenge to facilitate radical pedagogical reconstruction amongst teachers can be likened to the challenge in science education to facilitate conceptual change in students of school science.

Since the 1970s, research in science education has focused a constructivist perspective on the influence on learning of students' preconceptions of the natural world. A major research thrust has been the identification of students' pre-instructional *alternative frameworks* (Driver & Easley, 1978; Driver & Erickson, 1983). Although this research is strongly influenced by cognitive psychology, it departs from the Piagetian tradition of identifying generalised cognitive structures, or *content independent* forms of thought (e.g., *stage theory*), and draws upon Ausubel's theory of pre-existing conceptual structures (Novak, 1977) and Kelly's theory of *man-the-scientist* (Pope & Gilbert, 1983).

The field of conceptual change research in science has been concerned with the role of *context-dependent* knowledge on the learning of science, and has focused on the personal theorising and hypothesizing of individual students. By the commencement of this study, this research was flourishing internationally (see reviews by Gilbert & Watts, 1983; Pfundt & Duit, 1987).

Conceptual Assimilation

A common feature of constructivist epistemologies underpinning conceptual change research is their concern with the existing conceptual framework that a learner inevitably brings to bear on new learning experiences. The totality of an individual's prior *lifeworld* experiences constitutes a conceptual frame of reference for perceiving and making sense of new phenomena. New concepts are constructed to deal with cognitive perturbations arising from novel problematic experiences, and are assimilated into an individual's existing conceptual framework.

The meaningfulness, usefulness, and ultimate viability of a new concept depends on the extent to which it has been integrated into the network of existing concepts, and is available for future application. West's (1982) *evolutionary* conceptual change is a process of accretion, or building on, of new concepts to students' existing conceptual frameworks; the primary concern is with the meaningful *linkage* of new concepts.

Radical Reconstruction

However, on other occasions, a radical restructuring of an entire conceptual framework, or accommodation, might be required. For example, the shift from an experientially-based, intuitive Aristotelian perspective to a seemingly counter-intuitive Newtonian perspective seems to require a radical reconstruction of an entire conceptual framework (Champagne, Klopfer, & Gunstone, 1985). Without a radical reconstruction, the Aristotelian conceptual framework tends to persist, and continues to provide the most sensible, albeit naive or outmoded, interpretation of phenomena in the students' lifeworld. A major pedagogical challenge for teachers of school science, therefore, is to develop teaching strategies which facilitate radical restructuring of students' naive conceptual frameworks.

One of the pre-eminent pedagogical theories for facilitating radical conceptual change amongst science students has been proposed by the research group of Posner, Strike, Hewson, and Gertzog (1982). Their research is related directly to the epistemological priorities of Rosalind Driver's research (Driver, 1988; Driver & Oldham, 1986), and draws on the field of the philosophy of science to explain learning in terms of a process of conceptual change which they liken to a *paradigm shift* in the community of scientists.

From the viewpoint of the philosophy of science, major conceptual change for a learner is analogous to a paradigm change or scientific revolution (Kuhn, 1970), or to a change in research programs (Lakatos, 1970) for a scientist. When scientists are faced with challenges to their basic assumptions, they must develop new theories, or central commitments, and a new way of perceiving the world. However, scientists are unlikely to make major changes to their theories until they believe that less radical changes will not work. Such a belief is precipitated by an accumulation of unsolved problems or anomalies, and a loss of faith in the capacity of the current theories to solve these problems.

The Posner et al. (1982) research group set out specific conditions for facilitating radical conceptual change, or Piagetian-type accommodation, in students' central, content-related conceptions of the natural world. The main emphasis of their teaching approach is to identify students' preconceptions, organise *cognitive conflict* learning experiences to assist students to reconstruct their conceptions towards a scientifically acceptable view, and evaluate and monitor students' conceptual change processes. Similar strategies have been advocated by Nussbaum and Novick (1980) in Israel, and Osborne (1981) in New Zealand.

Posner et al. (1982) stipulate the following specific conditions of conceptual change: (1) students should experience *dissatisfaction* with their existing conceptions and, subsequently, should be provided with opportunities to develop alternative (scientific) conceptions that they perceive to be (2) *intelligible*, (3) *plausible*, and (4) *fruitful*. The researchers propose that the implementation in science classrooms of four conditions of conceptual change will result in the abandonment of students' naive conceptual frameworks, and the effective construction of the scientific concepts *embedded* in teachers' instructional goals. Evidence of the effectiveness of this theory of radical conceptual change has been reported by Gunstone, Champagne, and Klopfer (1981), Hewson and Hewson (1983), and, with qualifications about the differential nature of success amongst students, by De Jong and Gunstone (1988).

The Role of Metacognition

More recently, however, science educators have warned that establishing the Posner et al. (1982) conditions of conceptual change might be problematic, especially the condition of *dissatisfaction*, which requires "a powerful new experience to overthrow all the earlier positive experience of the usefulness of the old notion" (White, 1987, p. 11). Also, it has been argued that *fruitfulness* is a difficult condition to establish because of its context dependence. For example, students might perceive a new concept to be fruitful in a school context, rather than in an out-of-school context (White, 1987). Another major problem for achieving the condition of *fruitfulness* is that "episodic memory is plastic" (p. 11), and that recollection of evidence, therefore, might be changed to fit the naive conception, rather than vice versa. White (1987) suggests that the development of *metacognitive* strategies, which involve reflection on the meaningfulness of new knowledge in relation to established beliefs, might assist the conceptual change process.

The view about the importance of addressing students' metacognition when teaching for conceptual change is highlighted in Gunstone's (1990) review of 10 years of classroom investigation research into the conceptions that students bring to science classrooms. In the review, metacognition is defined as an amalgam of learners' conceptions about teaching and learning, their awareness of the purposes of learning activities, and the control which they exert over their learning actions. Gunstone (1990) argues that reconstruction of students' concepts or beliefs requires an accompanying reconstruction of their metacognition, or ideas and beliefs about the processes of learning and teaching, which enables "acceptance of the worth of constructivist based classroom approaches before [students] can benefit from these" (p. 18). Teaching approaches designed to reconstruct students' metacognitions would involve facilitating students' recognition and evaluation of their existing views, and their conscious decisions to reconstruct them.

Pedagogical Reconstruction Goals

The analogous approach of facilitating radical conceptual change in science students serves to highlight the importance of creating favourable conditions for facilitating teachers' pedagogical reconstructions. The analogy suggests a parallel strategy for overcoming the problem of teacher centralism. First, teachers should be given the opportunity to experience the problematic nature of their existing centralist pedagogies. Second, opportunities are required for teachers to reflect on, and be convinced of, the prospects of the metaphor of learning as construction. The third element of the strategy is to provide opportunities for teachers to design, trial, and evaluate teaching strategies consonant with the principles of a constructivist-related epistemology. Finally, it is important that teachers experience the success of their alternative teaching strategies for overcoming the problems associated with their established centralist pedagogies.

The feasibility of this four-part strategy of pedagogical reconstruction for overcoming the problem of teacher centralism is further developed below in relation to research in the complementary fields of teacher cognition and teacher education.

Evaluating Established Pedagogy

The first step towards teachers' pedagogical reconstruction is their realisation and explication of the pedagogical beliefs, especially key intuitive beliefs, which underpin their centralist roles. Fenstermacher (1979, 1980, 1986) argues that teachers should become *reflexively aware* of the *subjectively reasonable* beliefs or *practical arguments* that underpin their teaching practice, and undertake investigations to confirm or disconfirm them. Disconfirmation would constitute compelling rational grounds for pedagogical change. This process seems to be analogous to the student conceptual change approach which proposes that students' experience *dissatisfaction* with their existing conceptual frameworks.

Nespor (1987) responds directly to Fenstermacher's ideas and extends them with the suggestion that, because beliefs are self-contained *conceptual systems*, rather than logical propositions, the transformation of teachers' beliefs and practice will occur "only if alternative or new beliefs are available to replace the old" (p. 326). The *alternative beliefs* are necessary to stimulate the cognitive process of transformation that Nespor describes as a *gestalt shift*, as distinct from the rational process of argumentation implied by Fenstermacher.

These arguments were combined in the context of this study to establish the first goal of the constructivist-related theory of pedagogical reform, namely:

- *To enable teachers to determine, from the vantage point of a constructivist-related perspective, the problematic nature of their established centralist pedagogical beliefs and teaching practices.*

Establishing Theoretical Viability

But what is the genesis of teachers' alternative beliefs: how do their new ideas originate, and how do they achieve the status of beliefs? One answer to this question is suggested by Floden's (1985) argument that teacher educators should try to persuade teachers to make pedagogical changes, but not by recourse to their authority. He argues in favour of the use of rhetoric by teacher educators that enables student-teachers to hold a new belief "for reasons that are both sound and are thought by the student to be good reasons and that . . . are reasons that the teacher also would use to justify the belief. . . . [T]he teacher must be ready to answer students' questions about why beliefs should be held" (p. 30).

The rhetorical process should not be corrupted by resorting to the authority-laden myths of research as a body of universally applicable truths or to the teacher educator as an expert knower. Rather, the teacher should be encouraged to adopt a critical approach to the rhetoric of both the teacher educator and research reports and, for that matter, to the *rhetorical facade* that obscures her/his own intuitive pedagogical beliefs (Olson, 1981, p. 273).

As a prospective facilitator of teachers' pedagogical reconstruction, therefore, I conceived my role as an advocate who aims to negotiate a consensual viewpoint in relation to the value of new pedagogical ideas. I perceived my main goal to be similar to that of the conceptual change teacher who wishes to create the conditions of *intelligibility* and *plausibility* of a new conceptual framework for students of school science. Therefore, in the context of this study, a second goal of the constructivist-related theory of pedagogical reform was formulated:

- *To establish a dialectical discourse that recognises teachers' rationality and promotes critical analysis aimed at assisting them to establish the theoretical viability of their new constructivist-related pedagogical ideas.*

Determining Practical Viability

A further step in the genesis of new pedagogical beliefs is the opportunity for teachers to assess the viability of their new ideas in the context of their own classrooms. Ruthven (1987) argues that, because teachers' classroom expertise is largely tacit and grounded in experience rather than in pedagogical theory, changes in teachers' pedagogical perspectives are likely to be a consequence of successful changes in their classroom practice. This argument is similar to Nespor's (1987) claim that the transformation of teachers' beliefs cannot be achieved by rational argument alone, and suggests that opportunities are required for teachers to experience the success, or otherwise, of their innovative teaching strategies. This requirement is similar to the conceptual change teaching condition of providing opportunities for science students to experience the *fruitfulness* of an alternative conceptual framework.

However, it is essential that teachers do not assess the efficacy of their innovations in terms of criteria associated with their established centralist pedagogies. For example, there is little point in teachers judging students' engagement in group discussions in terms of their expectations of students' behaviour in whole-class, teacher-directed discussions. The development of constructivist-related criteria is necessary in order that teachers assess their innovative teaching strategies in terms of their potential to facilitate constructivist-related goals, such as students' successful conceptual development. Thus, a third goal of the constructivist-related theory of pedagogical reform was formulated:

- *To assist teachers to establish the practical viability of their constructivist-related beliefs and pedagogical innovations.*

Taken together, the three goals of constructivist-related pedagogical reform, discussed above, highlight the importance of a discourse for focusing teachers' attention on the problematic nature of their established centralist pedagogies, and facilitating their development of a constructivist-related pedagogy. Chapter Three discusses the nature of the discourse which I established with Ray during the collaborative phase of the study.

SUMMARY

This chapter commences by considering the problem of reconstructing teachers' established centralist pedagogies. It is argued that an analogous challenge faces science teachers who wish to reconstruct students' preconceptions of the natural world. The failure of *rationalist* teaching approaches to reconstruct the intuitive, experientially-based conceptual frameworks of many science students has been demonstrated by constructivist-oriented research. Similar problems have beset curriculum developers who wish teachers to implement *student-centred* curricula that require the adoption by teachers of less centralist classroom roles. Research has indicated that teachers' implicit, role-determining pedagogical beliefs are one of the major constraints to curriculum reform. In the context of this study, it was concluded that a *post-rationalist* approach would be required to facilitate a major reconstruction of Ray's transmissionist epistemology.

In the second section, a constructivist-related theory of pedagogical reform is developed for the purpose of facilitating a major reconstruction of teachers' centralist classroom roles. Arguing by analogy with conceptual change research in science education, the feasibility is established of a constructivist-related theory of pedagogical reconstruction. The theory is further elaborated in relation to research on teacher cognition, and my collaborative role as a facilitator of radical pedagogical reform is articulated in terms of three goals. The first goal involves assisting teachers to become aware, from a constructivist perspective, of the problematic nature of their established centralist classroom roles, especially in relation to students' conceptual development needs and difficulties. The second goal is to persuade teachers of the *theoretical viability* of principles associated with the metaphor of learning as construction. The third goal is to enable teachers to investigate, from a constructivist perspective, the *practical viability* of their pedagogical innovations.

The constructivist-related theory of pedagogical reform was enacted in the form of a discourse between Ray and me during the collaborative phase of the study. Chapter Three discusses the main features of the discourse which was designed to facilitate Ray's development of a pedagogy oriented towards the conceptual development needs of the students in his Grade 12 mathematics class.

CHAPTER THREE

THEORY IN ACTION: A CONSTRUCTIVIST DISCOURSE

INTRODUCTION

During the collaborative phase of the study, my discourse with Ray served the dual purposes of (1) implementing the theory of pedagogical reform, which is discussed in Chapter Two, in order to facilitate Ray's development of a constructivist-related pedagogy, and (2) enabling me to investigate the constraining nature of Ray's pedagogical beliefs, especially his personal epistemology, and to monitor changes to his pedagogy as the study progressed. Although both purposes were embedded in my discourse with Ray, this chapter discusses how the discourse was enacted in the interests of facilitating pedagogical reform. Chapters Four and Five address the interpretive outcomes of my discourse.

The first section of this chapter discusses how my discourse with Ray was facilitated by means of written Reflections, debriefing sessions, and interviews. Section two discusses one of the main foci of my discourse with Ray, namely, the constraining nature of Ray's didactic classroom discourse. The theoretical framework employed to analyse this aspect of Ray's classroom discourse comprises (1) a four-part classification of the organisational nature of students' classroom activities, and (2) the phenomenon of *target students*. This section discusses the descriptive and interrogatory forms of my discourse with Ray. The interrogatory nature of my discourse with Ray is explained in relation to the goal of facilitating Ray's critical reflection, from a constructivist perspective, on the efficacy of his established pedagogical beliefs and practices.

Section three discusses a second major focus of my discourse with Ray, namely, the development of a conceptually-oriented pedagogy aimed at identifying and resolving students' conceptual development difficulties, and facilitating students' higher-level conceptual development. This section discusses how my constructivist perspective, which comprised the first principle of constructivism and the prior knowledge corollary (see Chapter One), was articulated in terms of meaningful learning rhetoric and an information-processing metaphor of human cognition. In particular, the ways in which my constructivist discourse was designed to stimulate the development of Ray's *pedagogical concept knowledge* is explained.

MODES OF DISCOURSE

During the collaborative phase of the study, my daily discourse with Ray was enacted in both verbal and written modes. Immediately after returning from the field, I reflected on my fieldnotes which described salient aspects of selected teaching and learning activities that I had observed during lessons. I recorded my reflections in the form of a written commentary, and provided copies of these *Reflections* to Ray on a regular basis. The content of the written Reflections of a particular lesson depended on which of the three pedagogical reform goals, discussed in Chapter Two, constituted the main focus of the collaborative intervention, at that time.

Usually, my written Reflections contained an analysis of (1) the syllabus-related mathematical content of the lesson, (2) the organisational nature of classroom activities, (3) students' conceptual development difficulties, and (4) questions related to important issues that were designed to engage Ray in a process of critical reflection. Later in the study, the Reflections included (5) assertions, from a constructivist perspective, about the efficacy of Ray's teaching, and (6) constructivist-related suggestions for pedagogical reform. These aspects of my discourse are explained in the next sections of this chapter.

As well, Ray and I held debriefing sessions immediately after the completion of most lessons. These sessions provided opportunities to engage in a verbal discourse with Ray on issues of immediate concern, especially in relation to the teaching innovations which he had introduced in the preceding lesson. During these sessions, we discussed our perceptions of the impact on student learning of Ray's innovations and, at times, collaboratively planned teaching strategies for the next lesson. I made written records of these discussions immediately after each debriefing session and appended them to my written Reflections.

Other opportunities for advocating constructivist-related principles in my discourse with Ray occurred during the series of seven interviews (see, for example, Appendix B). The first interview, which was conducted during the collaborative phase, explored Ray's beliefs about the teaching innovations which he had trialled, considered the dynamics of our collaborative relationship, and discussed the pedagogical implications of the first principle of constructivism.

My discourse with Ray was designed to focus his pedagogical attention on the nexus between two interrelated issues, namely: (1) the constraining nature of Ray's didactic classroom discourse, especially the minimal opportunities it provided for Ray to monitor students' conceptual development, and for students to develop their conceptual understandings; and (2) the problematic nature of students' extant conceptual frameworks, and the development of a constructivist-related pedagogy aimed at facilitating students' conceptual development, especially a pedagogy which incorporated students' prior mathematical knowledge. The following sections discuss how my discourse with Ray focused his attention on these pedagogical issues.

A FOCUS ON TEACHER DISCOURSE CONSTRAINTS

The role of classroom discourse in promoting critical, reflective thinking and a more student-centred and participatory classroom environment has been identified as being central to conceptual change teaching. Gilbert and Watts (1983) list specific classroom discourse strategies in which teachers use language as a *tool* to promote thinking, rather than as a means of transmitting meaning. Examples of classroom discourse strategies in school science include increasing the proportion of student talk, exploratory use of language by teachers, teachers' use of metaphors and analogies, and teachers and students negotiating the purpose of practical work. However, as discussed below, research has revealed that classroom discourse also can underpin the problem of teacher centralism, especially teacher-directed discourse which is conducted in a predominantly whole-class mode and which promotes the active role of target students.

The constraining influence of the didactic nature of Ray's classroom discourse constituted an important focus of my discourse with Ray during the collaborative phase of the study. Evidence of the didactic nature of Ray's classroom discourse was obtained by adopting an observational framework which indicated the duration of various modes of classroom organisation during each lesson. This analysis of general organisational trends was supplemented by an analysis of the active presence of target students in whole-class discourse.

Modes of Classroom Organisation

I employed an observational framework to facilitate my recording and analysis of Ray's classroom discourse and related teacher and student activities, especially activities which might be constraining students' conceptual development. The chosen framework was based on Walter Doyle's (1980) classification of the organisational nature of students' classroom activities, and Gallagher and Tobin's (1987) extension of Doyle's classification for science and mathematics classrooms. My observational fieldnotes described Ray's classroom discourse and its impact on students' participation in teaching activities in terms of the following four categories.

Whole-class Non-interactive Mode

In this traditional mode of teaching, the teacher talks uninterruptedly to the whole class. Students are required to look and listen and record notes based on the teacher's presentation. For most of the time, there is little or no teacher-student or student-student verbal interaction. Occasionally, the teacher might direct a question to the whole class, or a student might seek clarification. However, this teaching mode usually is non-interactive and lecture-oriented, and features the teacher as the primary source of information.

Whole-class Interactive Mode

Traditionally, this highly interactive mode is characterised by frequent teacher-student verbal exchanges in a whole-class forum. The most common form is teacher-directed, question-and-answer episodes in which questions are directed to either the whole class or individual students. In both cases, the whole class is required to attend to exchanges in which only one person at a time is permitted to speak. Although the teacher remains the primary source of information, students might share this role, from time to time.

In contrast to its traditional nature, the whole-class interactive teaching mode can provide opportunities for a more student-centred classroom discourse in which a larger number and wider range of students are selected to participate in question-and-answer episodes, and many more students are encouraged to initiate their own inquiries of the teacher, or respond to other students' queries and assertions. These opportunities might facilitate more responsive teaching that accounts for a greater range of students' conceptual needs and interests.

Individual Seatwork Mode

Students engage in seatwork activities and consult either the teacher or their neighbouring students during this traditional accompaniment to whole-class teaching. Students are occupied individually with reading a text, copying notes from the board, or completing teacher-prescribed exercises in pursuit of correct answers. The teacher's role is to provide individual assistance to students, usually by engaging in relatively private consultations in response to students' requests. Traditionally, individualisation involves differential rates of completion of the same work by different students.

However, this teaching mode can provide opportunities for the teacher to obtain more information about the conceptual frameworks of individual students, and respond with teaching strategies that account for a greater range of conceptual development needs and difficulties.

In this study, teacher-student interactions in this mode were not monitored because (1) it was difficult for me in a participant-observer role to eavesdrop on private teacher-student discussions, and (2) during this teaching activity my main role was to interact with individual students for the purpose of interpreting their conceptual frameworks. However, evidence obtained from teacher interviews (see Chapter Four) and the surprised nature of Ray's responses to indications of students' conceptual difficulties (see Chapter Two), suggest that, during private consultations, Ray's centralist classroom role was characterised by his priority interest in telling students how to obtain correct answers, rather than in diagnosing their conceptual frameworks.

Groupwork Mode

This teaching activity involves small groups of between two and six students. Traditionally, students are required to work cooperatively to complete highly prescribed tasks, involving collection and analysis of data, especially in science laboratory classes. The teacher's role is to monitor the groups and intercede for the purpose of clarifying task requirements or resolving difficulties.

Groupwork, however, can provide opportunities for students to participate collaboratively in more student-centred activities, such as designing investigations, or negotiating the resolution of group problems or investigations. Through expressing and defending their own ideas and critically reflecting on, and challenging the ideas of, others in the group, students might experience enhanced opportunities to evaluate the viability of their own conceptualisations and reconstruct their knowledge in a relatively supportive environment, especially compared with the high-risk environment of the whole-class discourse which, in traditional classrooms, is dominated by the interests of higher-achieving students. Through questioning, diagnosing and prompting, the teacher can adopt a more facilitative role in small-group settings.

The Target Student Phenomenon

Tobin and Gallagher (1987) found that, in many teacher-centred classes, classroom discourse serves mostly the interests of a small group of higher-achieving *target students* who monopolise whole-class verbal interactions with the teacher: "Compared to others in the class, target students asked more questions of the teacher, were called on to respond to higher cognitive level questions, and received higher quality feedback" (p. 61).

Tobin and Gallagher's (1987) research, which was conducted in high school science classrooms, found that target students who were selected by the teacher tended to be higher-achieving students. Gallagher and Tobin (1987) found also that teachers paced their teaching on the responses of target students who provided teachers with the rewarding experience of obtaining more correct answers than was possible from less able students, and enabled teacher's to *cover* more syllabus *content*. The researchers asserted that target students enjoyed rewarding relationships with their teachers, and that their nurturing with higher-order questions and more personal attention in class was especially detrimental to the educational prospects of the less-able, *non-target students* who were largely excluded from these advantages.

Research has found that the target student is a widespread phenomenon which occurs also in high school mathematics classes, and that target students dominate small-group activities as well as whole-class interactions (Tobin, Espinet, & Byrd, 1987). A study by Tobin and Malone (1986, 1989) of target student activity in mathematics classes suggests that target students might be utilised by teachers who are teaching out of field and who have a lack of *content* knowledge. In this case, target students would serve to focus whole-class discussion on correct answers and reduce the risk for the teacher associated with dealing effectively with student responses that were incorrect or partially correct.

Tobin and Malone (1986) conclude that target students exist in classes for a variety of reasons, and that their existence underpins teachers' centralist classroom roles. It is likely, therefore, that teachers might be reluctant to minimise target student involvement. However, in order to assist teachers to recognise the existence of target students, and determine whether their involvement should be curtailed, teachers are likely to need assistance:

> In these circumstances, specific feedback on target student involvement in the teacher's classroom and professional advice on teaching could produce an environment which is conducive to reflection on current teaching practice and consideration of the efficacy of alternative patterns of involving students in learning tasks. (Tobin & Malone, 1986, p. 26)

Because Ray was teaching out of field, it was conceivable that he might have utilised target students to *buttress* his less than adequate content knowledge. The phenomenon of target students in classroom discourse, therefore, was an apposite focus for this study, and constituted one of the components of my observational framework.

Descriptive Evidence

During the collaborative phase of the study, I provided Ray with descriptive evidence of the didactic nature of his classroom discourse. The evidence indicated the predominance of whole-class modes of classroom organisation, and the dominance of whole-class discourse by target students. The data were analysed in both both quantitative and qualitative forms. An example of a quantitative analysis of the organisational nature of the teaching activities of a whole lesson is displayed in Fig 1 (see Chapter Four). Early in the study, I provided this type of analysis to Ray on a regular basis (see, for example, Reflections, Appendix A). The analysis serves to highlight the large proportion of class time occupied by teacher talk, and provides compelling evidence of the didactic nature of Ray's classroom discourse and his centralist classroom role. Quantitative changes in pedagogical emphasis, as more student-centred teaching innovations were evaluated by Ray, also were indicated by this type of analysis.

In addition, I provided Ray with qualitative analyses of his classroom discourse. For example, in the early lessons, I recorded in fieldnotes the following evidence of the centralist nature of Ray's classroom discourse:

- Lecture style format for duration of lesson — teacher-centred teaching with boardwork.
- Interaction between teacher and target students through question and answer, plus a few other students.
- Main pace of lesson driven by target students, but slowed from time to time in response to slower students' lack of comprehension, detected in questions directed to selected students. (Reflections 4 May 1987, Appendix A, p. 333)

An Interrogatory Discourse

My discourse with Ray, however, was far from being didactic. Although I presented him with descriptive evidence of classroom activities, one of the main characteristics of my discourse was its interrogatory style. The purpose of adopting an interrogatory style of discourse early in the collaborative phase was to facilitate Ray's examination of the viability of his established centralist pedagogy from a constructivist perspective. The interrogatory aspect of my discourse was designed to facilitate a process of critical reflection akin to Schon's (1983) *reflection-on-action*, or Olson's (1981) *dialectical approach*, in which the implications for practice of alternative pedagogical principles serve to challenge established pedagogical ideas and reshape a teacher's established language of practice:

> Common — and perhaps long unexamined — practice can be challenged by new images whose practical implications can be worked out first with teachers in their language. By trying the new ideas and discussing them with innovators, teachers can build the experiential base from which to develop a more powerful language. The growth of this language is a dialectical process. The new experience shows the old language to be inadequate and the old

> language helps teachers discuss the potential implications of the new practice. (Olson, 1981, p. 272)

The questions that I addressed to Ray, in debriefing sessions, in interviews, and in my written Reflections, were premised on constructivist principles. The discourse was designed with Ray's autonomy and integrity as paramount interests. It was consistent with a constructivist epistemology to enable Ray to become a critical and self-regulated learner, rather than a subject of the deliberations and predilections of myself as an expert knower. An implicit goal of my modelling of critical questions, which required Ray to generate empirical evidence for the substantiation of his assertions about the efficacy of his classroom role, was that Ray would adopt a similar style of self-discourse and, thereby, maintain a critical pedagogical perspective beyond the completion of the collaborative phase of the study.

For example, my written Reflections provided Ray with the following questions which were designed to engage him in a self-discourse on the problematic issue of students' comprehension:

- How many students are not keeping up with the pace of [your] presentation?
- How do these students manage to understand the mathematical content of the lesson?
- How aware [are you] of the ability range of the students and what prior knowledge they have?
- What factors determine the pace of [your] presentation?
- Why [do you] adopt a predominantly teacher-centred whole-class teaching style?

(Reflections 4 May 1987, Appendix A, p. 321)

Thus, a combination of compelling descriptive evidence of the didactic nature of Ray's classroom discourse, and questions that sought to explicate the underpinning rationale of Ray's teaching strategies, especially in relation to the limited availability of opportunities for the conceptual development of many students, fuelled my discourse with Ray during the collaborative phase of the study.

A FOCUS ON STUDENTS' CONCEPTUAL DEVELOPMENT

Another important focus of my discourse with Ray was the development of a conceptually-oriented pedagogy aimed at identifying and resolving students' conceptual development difficulties, and facilitating students' higher-level conceptual development. This process was facilitated by my advocacy of constructivist-related pedagogical principles as a plausible rationale for developing innovative teaching strategies. It was important that the discourse should enable Ray to perceive constructivist-related principles as providing a theoretically viable basis for his development of innovative teaching strategies. The *first principle of constructivism* and the *prior knowledge corollary* became the primary theme of my discourse with Ray. This section discusses how my constructivist perspective was articulated in terms of meaningful learning rhetoric and an information-processing metaphor of human cognition.

Meaningful Learning Rhetoric

The first principle of constructivism (see Chapter One) is highly congruent with Ausubel's theory of meaningful verbal learning which underpins much of the constructivist-related research in science education and mathematics education that informed this study. Novak (1978) claims that the most important idea in Ausubel's (1968) theory is his assertion that: "The most important single factor influencing learning is what the learner already knows" (p. 4).

Of the seven key concepts of Ausubel's *assimilation theory of learning* outlined by Novak (1978), the concepts of *subsumption* and *progressive differentiation* were incorporated into the constructivist discourse of the collaborative phase of this study. The process of meaningful learning results in subsumption, rather than the simple accretion, of new knowledge. Subsumption differs from Piaget's stage-dependent concept of *assimilation* in that new knowledge is *linked* specifically to relevant concepts embedded already in a conceptual network, or cognitive structure which, in turn, continues to develop as new concepts become more elaborated and new linkages form between concepts in the network. As well, *meaningful forgetting* of previously meaningfully constructed concepts does not impair the subsequent development of related concepts because of the continued existence of *residual* knowledge of conceptual relationships.

These Ausubelian ideas found expression in my discourse with Ray in a number of forms. For example, terms such as *linkage* and *integration* were employed to refer to the cognitive activity of making sense of new mathematical ideas. These terms served as signifiers of the first principle of constructivism, especially the prior knowledge corollary. However, before these terms are illustrated, a key metaphor that helped to give expression to the Ausubelian ideas is discussed. This metaphor had a major impact on Ray's conception of a constructivist epistemology, and on the reconstruction of his pedagogy.

An Information-Processing Metaphor

The first principle of constructivism seems to be highly compatible with theories of cognition based on information-processing psychology, The rhetoric of many constructivist-oriented researchers in both science education and mathematics education is permeated with references to the metaphor of *cognition as information-processing*. The main signifier of this metaphor in the discourse of proponents is the frequent occurrence of the term *information*, and of phrases that indicate its cognitive processing, such as *information is transformed, information is encoded, information is stored,* and *information is retrieved*.

Driver and Erickson (1983) identify information-processing psychology as the underpinning theoretical basis of models of conceptual change teaching in research reported by groups such as Champagne, Klopfer, and Gunstone (1982), and Posner, Strike, Hewson, and Gertzog (1982). In particular, the *generative learning model* of Osborne and Wittrock (1983, 1985) incorporates ideas from both constructivist traditions (e.g., Ausubel, 1960; Claxton, 1984; Kelly, 1955) and information-processing traditions (e.g., Larkin & Rainard, 1984; Newell & Simon, 1972) of cognitive psychology. A major emphasis is placed on learners' selection of *sensory input* from the environment and generation of a wide array of *firm links* with existing *stored information*:

> The essence of the generative learning model . . . is that the brain is not a passive consumer of information. Instead it actively constructs its own interpretations of information, and draws inferences from them. . . . The stored memories and information processing strategies of the brain interact with the sensory information received from the environment to actively select and attend to the information and to actively construct meaning. (Osborne & Wittrock, 1983, p. 492)

In a recent review of research on knowing mathematics in elementary schools, Putnam, Lampert, and Peterson (1990) report that most current theories of problem solving are based on information-processing models of human cognition. In particular, Resnick (1983) expresses constructivist-related theory in terms of an information-processing framework, and advocates teaching strategies that present carefully tailored mathematical *information* which would facilitate students' restructuring of their prior knowledge as they struggle to invent new knowledge structures in problem solving situations:

> People also . . . acquire from outside certain new facts and information about mathematics operations or conventions of representations. . . . [T]hese externally given pieces of information take on significance — indeed are retained — only to the degree that they are incorporated by the learner into organized and interconnected systems of knowledge. . . . [A]ttention will have to be given to the kinds of representations and routines that the learner will be able to incorporate into existing knowledge structures, and that will invite the transformation of linkages involved in constructing knowledge. (Resnick, 1980, p. 214)

Resnick (1983) characterises learners of science and mathematics as constructors of their own understandings whose *relational* understandings depend on their abilities to link new information meaningfully to what they know already. Resnick (1980) believes that it is important, therefore, that teachers present new information which can "make contact with already stored knowledge" (p. 236).

During the collaborative phase of the study, the information processing metaphor was connoted in my discourse with Ray by the frequent use of the term *information*. This term was embedded in the meaningful learning rhetoric, discussed above, in order to focus Ray's attention on the relationship between students' prior mathematical knowledge and their new mathematical experiences. This combination of terminology in my discourse with Ray served to articulate the first principle of constructivism.

Conceptual Development Discourse

During the collaborative phase of the study, the rhetoric associated with theories of meaningful learning and an information-processing metaphor helped me to express pedagogical implications of the first principle of constructivism and the prior knowledge corollary. My discourse with Ray was designed to stimulate the development of his *pedagogical content knowledge* (Shulman, 1987; Shulman & Sykes, 1986). This type of teacher knowledge has been identified worldwide, in the fields of both science education and mathematics education, as being of central importance to the facilitation of students' conceptual development (Arzi, White, & Fensham, 1987; Grossman & Richert, 1986; Kennedy, 1990; Wubbels, Korthagen, & Broekman, 1991). Consequently, my discourse aimed to focus Ray's pedagogical attention on the development of teaching strategies aimed at (1) identifying and resolving the problematic nature of students' extant conceptual frameworks, and (2) providing opportunities for students to integrate meaningfully new concepts with their existing conceptual frameworks.

Identifying Students' Prior Knowledge

My discourse with Ray focused on the opportunities available for students to recall and review important antecedent mathematical concepts related to current teaching activities in order to facilitate meaningful linkage with newly forming concepts. The identification of students' extant conceptual frameworks, especially the experientially-based preconceptions that are brought into class, is one of the main principles of modern constructivist-related curriculum development (Driver, 1990; Steffe, 1989).

In this study, my discourse with Ray was designed to highlight the problematic preconceptions and naive learning strategies and priorities which constrained students' ongoing conceptual development. Chapter Four discusses the extant conceptual frameworks of students in Ray's Grade 12 class, and indicates widespread developmental difficulties amongst the lower-achieving majority of the class. This section discusses ways in which my discourse focused Ray's attention on the problematic nature of students' conceptual frameworks.

Degrees and Radians as Conceptual Referents

For example, I found that the conceptual development of many students was being constrained by their failure to abandon naive conceptual frameworks associated with the use of the *degree* as a unit of measure, in favour of developing more powerful conceptual frameworks based on *radians*. The use of radians as a higher-order conceptual referent facilitates the sketching and manipulation of trigonometric graphs, and the graphical solution of intersecting graphs of linear and trigonometric functions. The following descriptive evidence of students' specific conceptual difficulties associated with the naive learning strategy of employing a *degree*-based conceptual framework was conveyed to Ray in the early lessons:

> Students' difficulties were associated with:
>
> - Calculating the period.
> - Sketching the *tan* function, in particular.
> - Interpreting the effect of variables on the shape of the graph (i.e., $y = asinbx$).
> - Choosing appropriate scales for sketching two intersecting graphs.
>
> (Reflections 11 May 1987, p. 1)

This evidence was accompanied by the following written questions, which were designed to focus Ray's attention on the specific conceptual development problems associated with the failure of many students to utilise radians as a conceptual referent, and to stimulate Ray's development of conceptually-oriented teaching strategies for assisting students to overcome the problem:

> To what extent have [student] misunderstandings in the [previous] Radians topic affected their understanding of sketching *trig* functions [in this topic]? (Reflections 11 May 1987, p. 3)

Cartesian Coordinate System as a Conceptual Referent

The identification of students' inadequate antecedent knowledge of the *Cartesian coordinate system* provides another example of the way in which my discourse focused Ray's attention on important aspects of students' prior knowledge. At the time, students were engaged in activities that required sketching *quadratic* functions and inferring the influence on shapes of graphs of the variables in the general equation $y = a(x-h)^2 + k$. This teaching activity was a preparation for the next topic Completing the Square. The following combination of descriptive evidence and critical questions was provided to Ray:

> [S]ome students seemed unable to answer simple questions regarding the Cartesian coordinate system (e.g., what *y-value* does a point on the *x-axis* have?). To what extent does inadequate knowledge of the Cartesian coordinate system affect a student's ability to think meaningfully about the mathematical content that is modelled within that framework?
> [You] stated that students *should know* how to do the work — *complete the square* and solve *quadratic* functions. Implicit in the statement is reference to students' standards, that is, the quality of knowledge/ information! What if the students do not have that quality/quantity of information, or have it but can't access/recall it?
> (Reflections 22 May 1987, pp. 3 & 5)

Integrating New Concepts

My discourse with Ray focused also on opportunities available for students to construct meaningful relationships between their prior knowledge and newly forming concepts. The discourse referred to processes of *linking, integrating* and *consolidating* new concepts.

Sometimes my discourse highlighted general pedagogical principles. For example, the following extract from the Fieldnotes contains both an assertion about the didactic nature of Ray's classroom discourse and suggestions for developing a more conceptually-oriented teaching approach:

> Students were trying to relate new maths content . . . to their previous knowledge, as evidenced by questions such as *What is this called again? We've done this in [Grade 10]!* What teaching strategies would enable students to meaningfully relate newly encountered content to similar prior knowledge? (Reflections 22 May 1987, p. 4)
>
> *Assertion*
>
> The teaching strategies are not facilitating . . . integration of new maths content information with existing maths content knowledge, for most students. Many students are failing to construct . . . new maths content knowledge because the teaching strategies are not accounting for students' existing maths content knowledge.
>
> *Suggestions*
>
> Reflect on your major role as information provider and adjust it to become more responsive to students' learning needs. For example, question-and-answer technique — elicit both recall and comprehension responses from a range of students (in terms of ability). (Reflections 5 June 1987, pp. 2-3)

At other times, my discourse highlighted concept-specific pedagogical principles. For example, the following extract of the Fieldnotes refers to a 12-minute episode of Ray's whole-class, non-interactive explanation of the solution of a *linear programming* problem on which students had been working in small groups for the previous 36 minutes. The account presents my constructivist interpretation of the pedagogical principles underpinning the teaching activity, questions about the pedagogical appropriateness of the activity, and suggestions for a more conceptually-oriented teaching approach which were included to stimulate Ray's reflection:

> [Your] information [to students] provided a link between students' knowledge of the procedural rules and students' recently constructed knowledge of *graphical regions* and their background knowledge of the Cartesian coordinate system.
> *Question 1*
> How effective, in terms of students' constructing their own understanding/concepts/knowledge, was your presentation of your interpretation of the graph?
> *Question 2*
> How else might you have facilitated students' development of their own . . . interpretations?
> *Suggestions*
> Make more use of question-and-answer (verbal or written) to facilitate students' qualitative interpretations. Give them the opportunity to utilise their own knowledge to create further understanding, especially when dealing with background knowledge and recently (partly) constructed knowledge. Example 1: ask them further questions about the graphical representation of the problem solution -- for them to reflect on before next lesson (to interpret other areas, calculate other costs). Example 2: provide them with the graphical representation of an unencountered problem solution and seek their interpretation of it. (Reflections 16 June 1987, pp. 2-5)

My discourse with Ray also enabled him to consider the prospects of a conceptually-oriented pedagogy by assisting him to experience alternative teaching strategies. For example, I designed a student assignment aimed at overcoming the *algorithmic stranglehold* of Ray's general problem-solving strategy for *linear programming* problems. The assignment (Reflections 22 June 1987, p. 14) contains exercises that require students to reconsider a recently completed problem, and further develop their conceptualisations of the solution of the problem. In particular, the assignment promotes an investigation of the graph as a source of further information about the original problem, in an attempt to assist students to develop a more extensive set of relationships between recently constructed concepts and their extant conceptual frameworks.

It was intended that the assignment serve as an exemplar of a conceptually-oriented approach to teaching for this topic. In addition to facilitating students' conceptual development, the assignment served another important pedagogical purpose. I analysed the results and discussed with Ray the nature and scope of students' conceptual difficulties. The analysis indicated that Ray's recent teaching had overlooked important inadequacies in students' conceptual frameworks. Students' unfamiliarity with *set* terminology and their inadequate conceptualisations of *inequalities*, in both algebraic and graphical forms, were severely constraining their conceptual development.

SUMMARY

This chapter discusses the implementation of the constructivist-related theory of pedagogical reform, which is developed in Chapter Two, through the agency of my discourse with Ray. The discourse was designed to engage Ray in the development of a constructivist-related pedagogy. The discourse took the form of a frequent exchange of written Reflections which contained descriptive summaries of salient issues evident in my observations of classroom activities, and questions that were designed to focus Ray's critical reflective thinking on the efficacy of his pedagogical beliefs and practices. Both quantitative and qualitative analyses were conducted on classroom observation records, and both written and verbal exchanges were maintained between us during the collaborative phase of the study.

One of the major foci of my discourse with Ray, especially early in the collaborative phase of the study, was the didactic nature of Ray's classroom discourse. Ray's ongoing classroom discourse was analysed in terms of an observational framework that highlighted the large amount of class time occupied by teacher talk, and the active role of *target students* in dominating whole-class verbal interactions with the teacher. My discourse with Ray yielded descriptive evidence of the minimal opportunities that his classroom discourse provided (1) for students to undertake conceptual development during class, and (2) for him to monitor students' conceptual development. Also, my discourse was couched in an interrogatory style for the purpose of stimulating Ray's critical reflection on the problematic nature of his pedagogical assumptions, and the efficacy of his established pedagogy for providing students' with conceptual development opportunities during class.

The ultimate purpose of my discourse with Ray was to facilitate his development of a conceptually-oriented pedagogy based on the first principle of constructivism and the prior knowledge corollary. Later in the collaborative phase, the rhetoric of my discourse comprised key terms associated with both Ausubel's theory of meaningful learning and an information-processing metaphor of human cognition. The discourse aimed to focus Ray's pedagogical attention on the problematic nature of students' mathematical conceptualisations, and to stimulate the development of his pedagogical content knowledge for the purpose of providing opportunities for students (1) to recall and review important antecedent mathematical concepts, (2) to abandon naive conceptions, and construct higher-level conceptual frameworks, and (3) to integrate meaningfully their newly constructed mathematical concepts with their extant mathematical knowledge. My discourse broadened to include constructivist-related critique of the efficacy of Ray's pedagogical strategies, and constructivist-related pedagogical strategies for enhancing students' conceptual development activities.

My discourse with Ray served two complementary purposes. This chapter discusses the enactment of my discourse with Ray which served the purpose of facilitating our mutual research goal of constructivist-related pedagogical reform. The second purpose of my discourse with Ray, which is addressed in the next three chapters, was to investigate the constraining nature of his role-determining pedagogical beliefs. Chapter Four presents an interpretive account of

Ray's established pedagogy prior to his introduction of constructivist-related pedagogical innovations.

CHAPTER FOUR

RAY'S ESTABLISHED PEDAGOGY: A STARTING POINT FOR PEDAGOGICAL REFORM

INTRODUCTION

In addition to facilitating Ray's pedagogical reforms, my constructivist discourse with Ray enabled me to investigate his pedagogical beliefs and practices, and to monitor changes associated with his development of a constructivist perspective on teaching. Prior to a discussion of the outcomes of the pedagogical reform process, which is the subject of subsequent chapters, it is important to understand the nature of Ray's established pedagogy which constituted the starting point for the pedagogical reform process.

Chapter Four discusses the main characteristics of Ray's pedagogy which had been established prior to the commencement of the study, and which remained relatively undisturbed during the early part of the collaborative phase. The results of interpretive analyses of data which were obtained during the early part of the collaborative phase of the study are reported in the form of Assertions One, Two, and Three in this chapter. Rather than including in this thesis the vast amount of fieldwork data which were collected during the study, a sample of the fieldwork data and preliminary data analyses associated with the first two lessons in Ray's Grade 12 class has been included in Appendix A.

The purpose of the first section of Chapter Four is to examine the main pedagogical interests and beliefs associated with Ray's conceptualisation of his established classroom role. The important role of metaphor in conceptualising teachers' classroom roles is discussed in relation to recent research on teaching, and a rationale is developed for a metaphorical analysis of Ray's daily discourse. The subsequent data analysis focuses on aspects of Ray's discourse which signify his main epistemology and syllabus-related pedagogical priorities, and discusses their implications for Ray's conceptualisation of his centralist classroom role.

The second section identifies *knowledge delivery* as the main theme of Ray's teaching strategies, and discusses the key features of his *teacher as informer* classroom role. An interpretive analysis of Ray's classroom teaching actions is presented. The interpretation draws on the theoretical framework of the first section of Chapter Three, and focuses on: (1) the organisational nature of students' classroom activities; (2) the role of target students in whole-class discourse; and (3) the pedagogical focus and mathematical content of lessons.

The third section discusses the constraining influence on students' conceptual development of Ray's established centralist pedagogy. The interpretive analysis focuses on the relationship between: (1) the restricted nature and source of Ray's *pedagogical concept knowledge,* especially his knowledge of students' mathematical conceptualisations; and (2) the problematic nature of students' mathematical conceptual development difficulties, and the inadequacy of Ray's pedagogy for resolving them. The interpretation draws on the theoretical framework of the second section of Chapter Three.

Finally, in section four, students' perceptions of the problematic impact on their learning of Ray's established centralist pedagogy are discussed, and the resilient nature of Ray's pedagogical beliefs is highlighted. The data on students' perceptions were obtained directly from students during classroom observations, and from an interview which I conducted with Ray during the collaborative phase of the study (i.e., Interview 1).

RAY'S CLASSROOM ROLE CONCEPTUALISATION

A major research focus of this study was the role of Ray's pedagogical beliefs in determining the nature and scope of his constructivist-related pedagogical reform. It was desirable to obtain an understanding of Ray's established pedagogical beliefs before the process of pedagogical reconstruction had commenced. However, an intrusive investigation of the type that was used later in the study to elicit propositional statements of belief (see Chapter Five) was deemed inappropriate at the commencement of the study for two reasons. Firstly, the collaborative research approach (see Chapter One) necessitated the establishment of an empathetic relationship between Ray and me prior to conducting relatively intimate and potentially threatening inquiries into Ray's personal epistemology. Secondly, the study's interpretive research approach (see Chapter One) required the documentation of practical classroom action within which to interpret Ray's propositional statements associated with his espoused pedagogical beliefs.

The solution to this problem was to adopt the relatively less intrusive research approach of analysing the metaphorical content of Ray's classroom discourse which I had recorded in my fieldnotes during the initial period of the collaborative phase of the study. This section presents an analysis of Ray's discourse which indicates his priority pedagogical interests and beliefs. The purpose of the analysis is to understand the major issues associated with Ray's conceptualisation of his established classroom role and, in particular, to construct a model of Ray's personal epistemology. Firstly, however, a case is argued for the reliability of this type of interpretive analysis.

Metaphor: An Interpretive Framework

The pervasiveness of metaphor in our conceptualisations of both ourselves and the world in which we live has been highlighted by the theories of Black (1979), Schon (1979), and Lakoff and Johnson (1980). Rather than a mere rhetorical linguistic device, metaphor is central to our conceptual systems. We invoke metaphors to order and structure our experiences as we attempt to make sense of our interactions with the world (Johnson, 1987). Examples of powerful metaphors which shape both our everyday thoughts and actions — metaphors we live by — include: *argument is war, time is money, mind is a machine, visual fields are containers,* and *ideas are commodities* (Lakoff & Johnson, 1980).

A specially powerful, but subtle, *conduit* metaphor governs popular perceptions of the nature of language (Lakoff & Johnson, 1980), and has important epistemological implications for education. The conduit metaphor is comprised of the metaphors that (1) ideas (or meanings) are objects, (2) linguistic expressions are containers, and (3) communication is sending. The *conduit* metaphor is associated closely with the *transmission* metaphor which has been identified (see Chapter One) as representing an epistemological conceptualisaton that is antithetical to that represented by the *constructivist* metaphor of learning. The *conduit* and *transmission* metaphors imply that teaching is a matter of transferring thoughts and feelings via linguistic communications such as speech or graphics — the *containers* of meaning — from the mind of the teacher to the mind of the learner whose complementary role is defined in terms of passive reception and extraction.

Explanatory Power and Accessibility

Of particular interest to this thesis is the growing research evidence of the explanatory power of metaphor in relation to teachers' pedagogies, and of the ready accessibility of metaphor in teachers' daily discourse. Recent research has highlighted the power and simplicity of metaphor. For example, Dickmeyer (1989) argues that the explanatory power of metaphor lies in its ability to characterise complex conceptual systems in simple and familiar terms:

> Metaphors are thus an important first step in understanding a complex system. We reduce the complexity to one or two key and important features. We then find a physical system that exhibits those few key features, and we draw the analogy. Our understanding is broadened because our powers to observe are increased. When we can focus on a few key features, we can then begin to see in greater detail how a system is operating. (Dickmeyer, 1989, p. 152)

In educational research, studies of teachers' thinking have produced evidence that teachers make sense of their classroom roles in terms of key metaphors. For example, Munby's (1986) study of the role of metaphors in the speech of a junior high school teacher found that the teacher conceived of teaching in terms of a spatial metaphor, namely, *the lesson as a moving object,* which was related to the powerful *conduit* metaphor of communication and the associated metaphor of *mind as a container.* However, because the data for Munby's (1986) research were obtained only from interview transcripts, the implications for classroom practice of his inferences require further substantiation.

Substantiation of the role of metaphor in explaining the relationship between teachers' thinking and classroom practice has been provided by the results of recent case studies by Tobin (1990b, 1990d, 1991b), Tobin and Ulerick (1989), and Tobin, Rennie, and Fraser (1990). These studies found that teachers metaphorical conceptions of their classroom roles have a strong bearing on the way in which the curriculum is implemented in classrooms. For example, these studies revealed that teachers' classroom roles were governed by metaphors such as *teacher as captain of the ship, teacher as entertainer, teacher as resource, teacher as preacher, teacher as intimidator, teacher as movie director, teacher as comedian, teacher as miser,* and *teacher as saintly facilitator.* The metaphors dominated teachers' speech about teaching, and seemed to govern both their management and teaching strategies. This research found also that experienced teachers conceptualised their classroom roles in terms of more than one metaphor, and that metaphors are context dependent and can be *switched* to suit the appropriate context.

The Tobin et al. case studies provide strong evidence that teachers' role-determining metaphors are powerful, or higher-order, conceptualisations which organise individual pedagogical beliefs into coherent belief systems which direct teachers' classroom actions. In other words, teachers' classroom roles are conceptualised primarily in terms of key metaphors, and each metaphor is associated with a discrete set of pedagogical beliefs which governs what teachers endeavour to do during classroom activities.

Research strongly suggests, therefore, that the researcher who identifies the key features of a teacher's metaphorical conceptualisations of his or her classroom role is well-equipped to explore in greater detail the complexities of the teacher's pedagogical belief system and its relationship with patterns of observable classroom-based teacher actions.

Another attraction of this preliminary research focus is the relative ease of obtaining an understanding of teachers' metaphorical conceptions of their classroom roles. Munby (1986) argues that teachers' metaphors are more readily identifiable and are more reliable indicators of teachers' conceptualisations because of their *natural*, or uncontrived, presence in teachers' daily discourse. By contrast, the propositional form of teachers' statements of their pedagogical beliefs results from less reliable *ex post facto* attempts at articulation, especially when research methods, such as the repertory grid technique, are used. However, rather than serving to dispel research attempts to elicit teachers' pedagogical beliefs in the form of propositional statements — a method employed later in this study (see Chapter Five) — Munby's argument strengthens the case for an additional focus on metaphor, especially in the early stages of a study when a largely non-interventionist research approach is desirable.

However, Munby's (1986) case for the superiority of metaphorical analysis must be tempered by the inherent limitations of metaphor. Although the simplicity of metaphor enhances its explanatory power for revealing higher-order conceptualisations, this feature also restricts the usefulness of metaphor for generating understanding of the complexity of its constituent belief systems:

> The main limitation of metaphors is in their inherent simplification. . . . Even if we use a dozen metaphors, each in turn ignores some factors that, through interaction, limit our ability to comprehend the system (Dickmeyer, 1989, p. 152)

Nevertheless, the research evidence of both the explanatory power and accessibility of metaphor provides a strong case for conducting an analysis of teachers' role-determining metaphors as a first step towards developing a more detailed understanding of their pedagogical thinking and its influence on their classroom roles.

The identification and analysis of metaphors embedded in teachers' discourse has become an important analytical tool for constructing models of teachers' pedagogical beliefs and personal epistemologies, and has revealed how teachers make sense of their teaching roles. In the case of Ray, this section provides evidence to support the following assertion.

Assertion One
Ray's established classroom role was conceptualised in relation to two highly congruent metaphors — *the syllabus as a conceptual map* and *teaching as a journey* — which signify a nexus between Ray's syllabus-related priority interests and his transmissionist epistemology.

The validity of these metaphorical conceptualisations, which were inferred from initial fieldwork data, is strengthened by the identification, in Chapters Five to Eight, of congruent metaphors, images, beliefs, and interests which constrained the nature and scope of Ray's constructivist-related pedagogical reforms.

Syllabus as a Conceptual Map

On completion of Lesson Seven, I asked Ray about the factors which determined his pedagogical priorities. He identified the following six factors:

1. The established sequence of the syllabus.
2. The need to complete predetermined *blocks of content* for the school-based topic tests.
3. The need to complete the year's syllabus in time for sufficient end-of-year examination preparation.
4. His perception of the relative *ease* of the *content*.
5. His perception of the relative importance of specific *content* for enhancing students' general understanding of the *content* area.
6. His perception of lack of student understanding of important ideas.

(Fieldnotes 18 May 1987, p. 10)

These six factors constitute a pedagogical framework which governed the pedagogical reform process of this study. Their influence on the nature and scope of Ray's constructivist-related pedagogical reform is discussed in Chapter Five. The first three factors relate specifically to Ray's perception of the requirements of the syllabus. The remaining factors are associated with an espoused concern for student understanding. However, the following sections in Chapter Four indicate that Ray's established centralist pedagogy took very little account of students' conceptual development. In particular, Ray's conception of students' learning needs was determined largely by his objectivist epistemology which directed his pedagogical attention towards his own mathematical knowledge, and away from students' conceptualisations.

Content Equated with Concept

A *semiotic* interpretation (Eco, 1976; Groisman, Shapiro, & Willinsky, 1991; Lemke, 1985, 1990; Solomon, 1988) of Ray's juxtaposition of the terms *syllabus* and *content* in the above listing of his pedagogical priorities suggests that he might have equated implicitly the textual content of the syllabus document, that is, the powerful codified signs and symbols which constitute the content of the text, with mathematical knowledge *per se*, or that which is signified by the textual content. In other words, Ray might have perceived the text of the syllabus document to be a *container* of new mathematical ideas. Congruent with this semiotic interpretation is the metaphor of *the syllabus as a conceptual map* whose textual features — topics, algorithms, diagrams, solution strategies — bear one-to-one correspondences with specific mathematical conceptualisations and, consequently, constitute objective forms of mathematical knowledge.

This metaphorical interpretation implies that Ray's epistemology conceived students' new mathematical knowledge to be *contained* in sources external to students, such as the text of syllabus documents or textbooks. A likely consequence of externalising, or objectifying, mathematical knowledge in this way is an implicit belief in its availability for communication, or transmission, to students. Ray's apparent concern for student understanding, therefore, might be interpreted as a concern for his accurate transmission to students of prescribed forms of syllabus-constitutive mathematical knowledge.

However, because of the brevity of the syllabus documents (see Fieldnotes 1 May 1987, Appendix A, pp. 316-317), it cannot be argued that Ray perceived the text to be a *template* of students' new mathematical knowledge. Rather, it is more likely that he perceived the syllabus text to be only partly constitutive of the requisite new mathematical knowledge. In other words, Ray perceived the syllabus as a *large-scale conceptual map* which constituted the general form and scope of students new mathematical knowledge, rather than its detailed features. The detailed features were furnished by Ray's explication of his own mathematical knowledge. This argument is strengthened in Chapter Six (see Assertion Ten) which argues that Ray sustained a formalist preconception of mathematics which regards the written syntax of mathematics as the objects of mathematical cognition, and ignores the meanings attributed by students to the syntax.

The above analysis suggests that, at the commencement of the study, Ray held a conceptualisation of *the syllabus as a conceptual map* and that, consequently, Ray conceived of his classroom role as: (1) drawing upon his own mathematical knowledge; (2) representing it in accordance with the prescribed form (or *content*) of the syllabus text; and (3) communicating, or transmitting, it to the whole class. This metaphor conceptualises a didactic and centralist classroom teacher role, and an objectivist conception of the nature of mathematical knowledge; a characteristic which evokes *transmission* and *conduit* metaphors of linguistic communication. The *syllabus as a conceptual map* metaphor highlights the role of the syllabus text (and textbook text) as the main epistemological referent, locates the teacher's mathematical knowledge at the centre of his pedagogical frame of reference, and marginalises students' conceptual frames of reference.

Teaching as a Journey

Another of Ray's daily pedagogical priorities was expressed by his *rule of thumb* which specified a minimum rate of delivery of his syllabus-related knowledge in the shorter-term framework of a 45-50 minute lesson. In response to my question (accompanying the Lesson Two fieldnotes) about the factors that determined the pace of his whole-class presentations, Ray replied: "Student progress partly, but mainly what I think we should be able to cover in that period. I try to introduce at least one new concept [each lesson]" (Fieldnotes 4 May 1987, Appendix A, p. 332). This syllabus-related teaching priority was maintained for the duration of the study, and its impact on the pedagogical reform process is discussed in Chapter Five.

Ray's references to *progress* and *covering* the syllabus are indicative of an epistemological metaphor of *teaching as a journey* through a *landscape* of pre-existing, objective knowledge. This metaphor is highly congruent with the *syllabus as a conceptual map* metaphor, and with Ray's priority interest in maintaining a predetermined syllabus-related teaching schedule. The *teaching as a journey* metaphor conceptualises the teacher's classroom role as a *navigator* whose main pedagogical interest is to direct the class along a predetermined route in accordance with a fixed timetable (i.e., the syllabus), and to provide didactic descriptions of significant features of the landscape (i.e., new concepts). The students' complementary role is largely one of attentiveness, compliance with teacher management directives, and accurate reproduction of the prescribed forms of the teacher's descriptions (i.e., new mathematical knowledge).

The *teaching as a journey* metaphor further highlights an objectivist conception of the nature of mathematical knowledge, which casts both teacher and students as *tour leader* and *tourists*, respectively, in a *landscape* of pre-constructed conceptual forms. A major implication of the metaphor is that neither the teacher nor students has a legitimate role as owners or artificers of any of the features of the landscape. The most that students can expect is an acknowledgement of their ability to recount accurately its prescribed features.

The following extracts of Ray's whole-class discourse illustrate his major concerns with students making both conceptual *observations* and efficient *progress* in relation to the syllabus *content*, and provide compelling evidence that the *teaching as a journey* metaphor was well-embedded in Ray's established centralist pedagogy:

> I want you to see the relationship between [*sin (-x)* and *sin(x)*]. . . . Let's move on then. (Fieldnotes 4 May 1987, Appendix A, pp. 325-326)
>
> We're running short on time. . . . I want to press on with something else. (Fieldnotes 11 May 1987, p. 4)
>
> I don't want to hold up things. . . . I have other things to do. (Fieldnotes 12 May 1987, p. 4)
>
> OK, move on to the hyperbola. . . . I know we're going a bit quick today. . . . We'll look at parabolas tomorrow. (Fieldnotes 18 May 1987, pp. 2, 4, 6)
>
> What I'd like to do now is to look at quadratic functions. (Fieldnotes 19 May 1987, p. 1)
>
> I thought we'd made good progress. . . . I'll just whip through the exam paper. (Fieldnotes 22 May 1987, pp. 2, 4)

There are strong similarities between the *teaching as a journey* metaphor, associated with Ray's established centralist classroom role, and the two metaphors which Olson (1981, pp. 266-268) associates with high influence science teaching. Olson reports that the *teacher as prime mover* role served an *editorial* function, and involved transmitting information to students to ensure that they "ended up with the right information and the correct ideas" (p. 267). The *teacher as navigator* was a complementary role which served a controlling function, and ensured that the lesson *did not go astray* (p. 267) or was implemented in accordance with its predetermined goals. These metaphors describe a centralist teacher role which closely associates learning activities with teacher planning and classroom action, and which is well-defined and unambiguous. Although Olson's metaphors are similar to the *teaching as a journey* metaphor described above, they do not focus sufficient attention on the important role-determining nature of teacher epistemology.

The argument about the relationship between Ray's objectivist conceptions of mathematics and his transmissionist epistemology is strengthened in Chapter Six (see Assertions Seven and Eight). Subsequent sections in this chapter describe classroom teaching manifestations of Ray's transmissionist epistemology, and provide substantial evidence of the major role played by the *teaching as a journey* and *syllabus as a conceptual map* metaphors in shaping Ray's established centralist pedagogy. However, before this discussion is commenced, an important pedagogical consequence of Ray's metaphorical conceptualisation of *teaching as a journey* is described.

Failure to Keep Up

Ray's priority pedagogical interest in *covering* the syllabus *content* enabled him to rationalise, as inevitable, the ultimate failure of a significant proportion of the class, especially students who were unable to *keep up* with the pace of the *journey* through the syllabus. These sentiments are illustrated in the following excerpts of Ray's written responses to questions accompanying Lesson Two fieldnotes:

> How many students are not keeping up with the pace of [your] presentation?
> Probably about a third.
> How do these students manage to understand the mathematical content of the lesson?
> They probably don't — we can't wait indefinitely for them.
> (Fieldnotes 4 May 1987, Appendix A, p. 332)

It should not be inferred from this quotation, however, that Ray had a callous disregard for the academic welfare of lower-achieving students. On the contrary, he made himself generally available at the end of lessons and conducted after-school classes for students who wanted tutorial assistance which was unavailable to them during lessons. Nevertheless, this brief anecdote indicates the extent to which a teacher's syllabus-related pedagogical priorities can result in the abandonment of students' conceptual development interests, especially students who are unable to *keep up* with the pace of the *teaching as a* *journey* through the syllabus, and serves as a warning of the difficulty of achieving pedagogical reform which attempts to relocate students' conceptual development needs to the centre of pedagogical considerations.

RAY'S TEACHER AS INFORMER CLASSROOM ROLE

This section presents an analysis of Ray's established classroom teaching practice, and illustrates the practical manifestations of Ray's objectivist epistemology and syllabus-related pedagogical priorities, which were discussed in the previous section. Evidence is presented which warrants the following assertion:

Assertion Two

Ray's established classroom role of *teacher as informer* was concerned with ensuring that students' reproduced accurately Ray's transmitted mathematical knowledge.

Ray's established centralist pedagogy was characterised by *show-and-tell* teaching conducted largely in a whole-class forum. Lessons were modelled on a whole-class oriented *transmission-teaching cycle,* and the predominant teaching strategies included didactic explication of new theory, *target student* activity, *confirmatory practice,* and *reproductive questioning*. This teaching approach promoted an active, centralist classroom role for Ray as the primary agent of delivery of his syllabus-related knowledge, and a complementary classroom role for students as passive receivers of Ray's knowledge. Strong teacher control of classroom activities determined a *lock step* approach to classroom learning. Ray provided only limited opportunities for students to engage in individual or small-group learning activities. Teacher-talk dominated classroom discourse, and few opportunities were provided for students actively to participate in whole-class discussion. The following analysis of Lesson One and Lesson Two illustrates Ray's centralist *teacher as informer* role in shaping a whole-class oriented learning environment (see Appendix A for fieldwork data obtained for Lessons One and Two).

The Case of Lessons One and Two

At the commencement of the study, the class recently had completed a unit of study concerned with *radian measures.* At the beginning of Lesson One, Ray introduced a new unit of study which combined three syllabus-prescribed topics: *Relations and Functions, Circular Functions,* and *Sketching Graphs.* Ray talked uninterruptedly to the whole class for most (65%) of the 46-minute lesson. Table A1 (see Appendix A) indicates the type of teaching activities which comprised Lesson One. In particular, the table shows how Ray organised the classroom, the pedagogical focus of his teaching, and the mathematical content of the lesson. After a 13-minute introduction, during which time Ray explained several important background concepts that were prescribed in the Grade 11 mathematics syllabus, the remainder of the lesson comprised repetitions of a three-step *transmission-teaching cycle* which comprised: (1) teacher whole-class explication of new theory; (2) individual student seatwork practice of a single exercise; and (3) teacher whole-class explication of the exercise solution. Apart from a two-minute episode of teacher-directed general questioning of the whole class, very little class discussion occurred during the remainder of the lesson.

During the first 15 minutes of Lesson Two, Ray conducted a whole-class question-and-answer episode (see Table A2 in Appendix A) which focused on the correct solutions to the five homework exercises which he had set in the previous lesson. This episode was followed by teaching sequences exemplified in Lesson One. The major difference between the first two lessons was the higher degree of verbal interaction between Ray and the class during whole-class teaching activities in the second lesson. During explications of new theory, Ray asked general questions of the whole class as he worked through homework and practice exercises, and responded to questions from a few students who sought clarification. However, the extra time spent on whole-class interactions was at the expense of individual seatwork activities which occupied only one minute of the total lesson time.

The pattern of lessons comprising one or more transmission-teaching cycles, observed during the first two lessons, typified Ray's pedagogy for at least the first seven lessons observed during the study (Fieldnotes 1-7, 1987). Although Ray began to introduce major constructivist-related teaching innovations after that time (see Chapter Five), the main variation in his general teaching approach was observed to be changes in the relative emphasis of the individual teaching steps in the transmission-teaching cycle. For example, the rate of *coverage* of Ray's syllabus-related knowledge decreased, and more class time was allocated for completing practice exercises. Nevertheless, Ray largely maintained his established centralist classroom role in controlling the substance and timing of the explication of his syllabus-related knowledge.

New Theory Explications

During Ray's explications of new theory to the whole class, students mostly were occupied with relatively passive *look-and-listen* activities. They listened to Ray's verbal explanations and copied his blackboard notes into their files. Because Ray's blackboard notes and sketches were drawn hastily with little attention given to their teaching effectiveness, students were faced with a difficult task of interpreting and recording significant aspects of the notes and, simultaneously, trying to make sense of Ray's verbal explanations.

The difficulty of this task was further compounded by Ray's tendency not to refer to specific parts of the student text in relation to the new theory that he was presenting. Ray utilised the text mainly as a source of practice exercises. His whole-class presentations were based largely on his own mathematical knowledge and, to a small extent, on the student textbook. Ray referred to the student text prior to lessons on occasions when he was unfamiliar with a particular topic. He did not specifically refer students to the textbook during his presentations, but occasionally selected homework exercises from it. Ray provided the following written response to questions about the the adequacy and role of the main text book (Mason & Broom, 1974):

> I think many students see [the text] as too complicated as it tends to be too long-winded. They often use it as a source of questions (in class and homework), also for formulae but, I suspect, not for theory.
>
> (Fieldnotes 8 May 1987, p. 11)

Although Ray expected students to initiate their own reading of the text, he regarded this to be a supplementary, out-of-class activity. Five of the higher-achieving students indicated to me that students' use of the text varied from "don't read Mason and Broom" to "usually read Mason and Broom if [the work] is unclear", and that its helpfulness varied from "usually helps" to "not helpful" (Fieldnotes 11 May 1987, p. 4). The majority of these students tended to consult the text only occasionally and found it to be of limited help.

Because Ray's centralist pedagogy promoted the teacher as the primary source of new theory, many students' lesson notes served as their primary out-of-class source of information, especially during homework. In a written response to a question, accompanying the Lesson Three fieldnotes, about the teaching role of his blackboard notes, Ray indicated both the important role and problematic nature of students' notes:

> Are you satisfied with the quality of students' note-taking from your . . . presentations? How important are students' notes in developing understanding? Is your presentation conducive to student note-taking?
>
> Maybe not. Perhaps I should make it more clear (to myself as well as students) what is essential to take down and what is not. I guess their notes are/should be quite important, as what they actually write in front of them must contribute significantly to their understandings. My presentation is often not conducive to note-taking.
>
> (Fieldnotes 8 May 1987, p. 11)

Most teacher-initiated whole-class discussions occurred within a teaching framework designed to serve Ray's main interest in delivering the syllabus. Ray's whole-class teaching approach provided only narrow *windows* of opportunity for students to participate actively in whole-class discussion. The opportunities mostly were dominated by a small number of higher-achieving students who served to support Ray's centralist teaching role. Most students chose not to participate in whole-class discussions, and adopted passive roles.

Target Students

Ray interacted regularly with only four or five students during whole-class teaching activities; generally, the same few students were observed to engage in teacher-directed whole-class question-and-answer episodes during the first few lessons. Tobin (1990b) and Tobin and Gallagher (1987) identified *target students* as those students who are regularly called on by the teacher, or who self-nominate, to answer most of the questions in whole-class discussions. *Target students* serve an important role in teacher-centred classrooms. Usually they are the higher-achieving students, and provide teachers with indicators of the success of their teaching strategies. Tobin and Gallagher (1987) found that teachers rely on this information to evaluate their explanations and determine the optimum rate of progress through the syllabus. Consequently, teaching *driven* by *target students* tends to serve largely the interests of a minority group of higher-achieving students. The target student phenomenon is discussed in more detail in Chapter Three.

During the first three lessons, the participation of students in whole-class discussions was limited mostly to a few of the higher-achieving students. The following exchange provides an example of Ray interacting with a high-achieving *target student* (S_{tg}) for the purpose of introducing new theory to the whole class. In this episode, Ray's pedagogical goal was to introduce the periodic nature of the *tangent function* in terms of the relationship between the *sine function* and *cosine function* (which recently had been the focus of teaching), especially the concept of *amplitude* and the rule for calculating the *period*. Ray had sketched a graph of the general *tangent function* on the blackboard, and had given a *unit circle* explanation of the function:

Ray: You can't talk about the *amplitude*, it's undefined.
Stg: And the *period* of *tan is pi*?
Ray: Yes.
Ray: Why is the *period* of *tan* only *pi*?
S1: Because it's only *90 degrees.*
S2: Because it's not a full cycle.
Ray: No!
Stg: [Gives an explanation in terms of sine and cosine].
Ray: [Reiterates *target student* explanation].
Ray: If you want a rule for the *period* of *tan*?
Stg: *Pi* over *b radians*!
Ray: *Pi* over *b*, instead of *two pi* over *b*!

(Fieldnotes 4 May 1987, Appendix A, p. 328)

In this episode, the *target student* assisted Ray in introducing new theory to the whole class by performing a leading teaching role. The *target student* previously had encountered the new theory in a more advanced mathematics class from which she recently had been transferred, and seemed to be seeking confirmation of her knowledge in the exchange with the teacher. For the *target student* only a minimal risk was associated with participation in a whole-class exchange. For Ray, the *target student* served as an ideal agent for explicating new theory to the whole class. From time to time, however, moderate achieving students also served as target students, especially when Ray wanted to gauge a wider degree of student understanding of his presentation of new theory (Reflections 4 May 1987, Appendix A, p. 333).

Generally, the role of *target students* was complementary to Ray's centralist teaching role. They facilitated the presentation of new theory to the whole class, and provided indicators of the extent to which the presentations were comprehensible to the higher-achieving students.

Confirmatory Practice

Ray's whole-class explications of new theory were punctuated with opportunities for students to *practice* their newly acquired knowledge. Ray referred to the need for practice on a number of occasions during the first four lessons (Fieldnotes: 4 May 1987, Appendix A, p. 326; 8 May 1987, pp. 4, 6; 11 May 1987, pp. 1, 4). The pedagogical purpose of practice can be explained in terms of the *transmission metaphor*. New theory is transmitted by the teacher to the whole class, and opportunities are provided at regular intervals for students to confirm their acquisition of Ray's knowledge. The opportunities consist of completing *confirmatory practice exercises* in which the new theory is applied to specific instances. The teacher's task is to determine the appropriate junctures in his whole-class presentations for students to undertake confirmatory practice.

Ray's use of practice exercises also had a management dimension: it facilitated his control of the class. Ray's established teaching habit was to prescribe common practice exercises for the whole class and provide brief periods of class time for their completion by students. During the first lesson, three periods of individual seatwork were allocated after periods of whole-class presentation of new theory (see Table A1, Appendix A, p. 320). On each occassion, a single practice exercise was prescribed. In Lesson Two, practice exercises were completed mostly in a whole-class interactive mode as Ray both presented new theory and collaboratively illustrated its application while employing the *reproductive questioning* described in the next section.

Sometimes practice exercises were prescribed for students to complete for homework. At the commencement of the next lesson, Ray provided opportunities for students to confirm the extent to which they had successfully acquired the new theory by conducting whole-class interactive explanations of the correct solutions. The first 15 minutes of Lesson Two illustrates this point (see Table A2, Appendix A, p. 334).

A further role for confirmatory practice of acquired knowledge occurred when students' responses to Ray's whole-class questions indicated widespread difficulty in obtaining correct answers to homework exercises. For example, during Lesson Three, Ray distributed a worksheet containing a number of practice exercises similar to those that recently had been completed in class, and announced to the whole class: "I've got a sheet for more practice because I think you need more practice" (Fieldnotes 8 May 1987, p. 6). This anecdote highlights Ray's habit of justifying publicly the role of practice exercises as an end in themselves, rather than as a means to a more pedagogically meaningful end, such as the development of specific mathematical conceptualisations.

At the commencement of the study, each *chunk* of new theory presented by Ray was accompanied by a small number of practice exercises which students completed, either during class or for homework, apparently in order to confirm the success of their acquisition of Ray's knowledge. However, opportunities for students to discuss the outcomes of their attempts were limited both in nature and scope. As the next section indicates, whole-class discussion focused largely on responses to Ray's reproductive questioning associated with the explication of new theory and the provision of correct answers to practice exercises.

Reproductive Questioning

In addition to interacting mostly with higher-achieving students, another feature of Ray's centralist teaching approach which contributed to minimal participation of most students in whole-class discussion was the predominantly reproductive nature of his questions. That is, questions mostly were designed to capture students' attention or obtain correct answers to practice exercises. While reflecting on his established questioning style during an interview Ray observed:

> Questions asked in lecture style were more rhetorical questions and more questions that I'm waiting to hear the answer that I want, just to try and engage the students for a while. . . . I know what answer I want to hear, and if I don't hear it I most probably just say 'Well, no, not quite — someone else', until I get the right answer; and [then I] continue on. . . . [My questions were designed] to test whether they've been listening. (Interview One 12 June 1987, p. 9)

The introduction to Lesson Two, which focused on the explication of correct solutions to homework exercises set at the end of the first lesson, illustrates the reproductive nature of Ray's whole-class questioning approach. The following exchange between Ray and several students (S) related to the third homework question which required a calculation of the period and amplitude of the function *y = 2/3 sin(-3x/2)* and a sketch of its graph:

Ray: Nina, what's the period of [*y = 2/3 sin(-3x/2)*]?
S_1: *Two pi* over one and a half!
Ray: How is that represented?
S_1: Two quarters!
Ray: Invert and multiply! [Writes *2pi/3/2, 4pi/3* on board]
S_2: Isn't it negative?
Ray: You can't have a negative period; give the absolute value!
Ray: What's it look like? How do you draw it? Where do you start? [Draws axes on board, and sketches graph in response to students' suggestions].
S_3: Go down!
Ray: Right through four *pi* over three. [Explains theory presented in previous lesson about *sin(-x) = -sin(x)*]
Ray: The amplitude is two-thirds. You get one and a half cycles in.
(Fieldnotes 4 May 1987, Appendix A, pp. 322-323)

This exchange illustrates Ray's intention to reproduce the correct solution to the exercise rather than explore the rationales underpinning students' responses. Students' short-answer responses were treated largely as *stepping stones* on a path to the correct solution. Blatantly incorrect responses evoked either a reminder of a procedural rule associated with a formula (e.g., "invert and multiply", "give the absolute value"), or a reiteration of Ray's explication of new theory presented in the previous lesson. This exchange was typical of most teacher-initiated whole-class discussions of practice exercises associated with the new theory which Ray recently had explicated.

RAY'S PEDAGOGICAL CONCEPT KNOWLEDGE

This section takes a closer look at the nature of Ray's pedagogical concept knowledge, that is, his knowledge of students' mathematical conceptualisations and of teaching strategies aimed at enhancing students' conceptual development. A similar type of teacher knowledge — *pedagogical content knowledge* — has been identified worldwide, in the fields of both science education and mathematics education, as being of central importance to the facilitation of students' conceptual development (Arzi, White, & Fensham, 1987; Grossman & Richert, 1986; Kennedy, 1990; Shulman, 1986b, 1987; Shulman & Sykes, 1986; Wubbels, Korthagen, & Broekman, 1991).

The term *pedagogical content knowledge,* however, implies that teachers' knowledge of pedagogy can be combined with their knowledge of the specific content of the syllabus, and that the resultant teaching strategies will be relevant to individual students' conceptual development needs. From a constructivist perspective, however, it is necessary for teaching strategies to be informed by a knowledge of individual students' conceptual development needs and difficulties; a form of teacher knowledge which requires teachers to focus their pedagogical attention on constructing better understandings of individual students' knowledge. Also, from a constructivist perspective, the term *content knowledge* signifies an objectivist conception of knowledge which connotes that the objects of mathematical cognition exist in syllabus documents. In this study, therefore, the term *pedagogical concept knowledge* is preferred because it promotes explicitly a form of teacher knowledge that relates directly to individual students' extant mathematical conceptualisations (Hand & Treagust, in press).

This section presents evidence which warrants the following assertion about Ray's established pedagogical concept knowledge.

Assertion Three

Ray's centralist classroom role constrained the development of his pedagogical concept knowledge which incorporated very little understanding of students' conceptual development needs and difficulties, and failed to account for students' mathematical conceptual difficulties.

Students' Prior Knowledge

It has been established in the previous section that, at the commencement of the study, Ray sought to obtain very little information about the nature of students' conceptual development difficulties. His teaching approach was largely delivery-oriented, and he interacted with only a few students in whole-class discussions. Ray's prime pedagogical concern focused on students' accurate reproduction of solutions to practice exercises. He rarely investigated the nature of incorrect student responses to his whole-class questions. Incorrect responses usually were ignored, or elicited a reminder of the correct answer, or caused Ray to review the appropriate theory. He rarely explored the underlying conceptual issues that prompted the occasional student-initiated request for clarification during his whole-class explications of new theory. Indications of student uncertainty usually elicited another whole-class explanation. Ray largely was unaware of the higher-order conceptual difficulties that rarely constrained many students' attempts to construct their mathematical knowledge in response to whole-class presentations and practice exercises.

Students' prior knowledge seemed to be of little pedagogical concern to Ray. Although Ray's introduction of a new topic was prefaced with revision of antecedent mathematical theory, his pedagogical concern was focused mainly on what students *should* have known as a result of their previous learning experiences, rather than on the quality of students' actual knowledge. For example, at the commencement of Lesson One, Ray talked uninterruptedly to the whole class for 13 minutes about a range of important concepts which were related to the new topic and specified in the Grade 11 syllabus (see Table A1 in Appendix A). In a written response to my question (accompanying Lesson Two Fieldnotes) about his awareness of students' prior knowledge, Ray indicated that he was aware of deficiencies in *basic* knowledge: "I'm aware of prior knowledge they should have. But many do not [have it]. There are some gaps in their knowledge of basics — for example, fractions" (Fieldnotes 4 May 1987, Appendix A, p. 332).

In his response, Ray made no mention of deficiencies in students' higher-level concepts that might hinder their construction of knowledge associated with his presentations of new theory. Indeed, Ray's transmission-oriented teaching approach seemed to assume that students' more recent learning experiences had been mostly successful, and that brief reminders would suffice to help them recall appropriate knowledge. This assumption was reflected in Ray's comments to the whole class during presentations of new theory and in relation to the completion of practice exercises. For example, in Lessons One, Two, and Seven, Ray directed the following remarks to the whole class:

> You've done it before [i.e., sketching *f(x) = sin(x)*], therefore . . . I'll not go into this in great detail. . . . I'm sure most of you are familiar with these trig functions already [i.e., *sine, cosine, tan*]. . . . You most probably know how to draw the graph [of *f(x) = sin(2x)*]. Draw a few points and verify it for yourselves.
>
> (Fieldnotes 1 May 1987, Appendix A, pp. 309-310)
>
> You won't have had too much trouble with the first two [i.e., homework exercises: sketching *y = 4 sin(4x)* and *y = 1/3 sin(x/2)*].
>
> (Fieldnotes 4 May 1987, Appendix A, p. 322)
>
> I know we're going a bit quick today but none of this [i.e., sketching *linear* and *hyperbolic functions*] should be new to you. . . . I hope that brought back a few memories [i.e., sketching *linear, hyperbolic* and *circular functions*]. (Fieldnotes 18 May 1987, pp. 4, 5)

Apart from limited student responses to his whole-class reproductive questions, Ray's understanding of the success of individual student attempts to construct their mathematical knowledge was dependent mostly on longer-term indicators such as the results of topic tests. Nevertheless, in a discussion after the completion of Lesson Three (Fieldnotes 8 May 1987, p. 9), Ray claimed that these sources of information provided him with a "pretty good understanding" of students' knowledge. This claim contrasts sharply with the self-critical remarks made by Ray about his established pedagogy a few weeks later, when he reflected that he had not been in the habit of:

> . . . [R]eceiving much feedback on student understanding. I wasn't investigating or delving into their specific difficulties or their specific concepts, constructs. . . . Rather, I was just seeing that they can't do it and [was] telling them how to. I thought that I must rectify this — I must tell them how it should be done. (Interview 1 12 June 1987, p. 6)

Students' Conceptual Difficulties

During the first seven lessons, Ray attempted to *cover* the two syllabus-prescribed topics Circular Functions and Graphs. However, widespread conceptual difficulties existed amongst the class, from the outset, and constrained the ability of many students to make sense of Ray's presentations, and to complete successfully the practice exercises. Because Ray's major pedagogical concern focused on student reception of new theory and reproduction of correct answers to practice exercises, he was unaware of the existence of these conceptual difficulties during the first few lessons.

In my participant-observer role, I detected students' conceptual difficulties during whole-class discussions, and confirmed their widespread existence during individual seatwork periods while providing tutorial assistance to individual students. I reported evidence of these conceptual difficulties to Ray in my daily written Reflections.

The first indication of Ray's awareness of students' conceptual difficulties was contained in his written response to my question which accompanied the Lesson Two Reflections:

> How well are students (not the target students) understanding the basic concepts of your teaching? (Refer to student's difficulties in *Reflections 8/5*). (Reflections 8 May 1987, Appendix A, p. 335)
>
> It appears that there are a lot of students not fully understanding (or not at all) some of the concepts. (Fieldnotes 11 May 1987, p. 7)

The Reflections referred to in the above question contain the following evidence of conceptual difficulties detected amongst six students, five of whom were *average* achievers in relation to the whole class:

> Difficulties experienced by selected students included:
>
> - Finding values of the function for *-x* (e.g., $y = tan(-x)$).
> - Identifying the cycle of a graphical function, and relating it to its period (especially $y = tan(x)$).
> - Understanding the radian-degrees equivalence for *x-axis* values.
> - Understanding the nature of the y-axis scale.
> - Relating the *tan* graph to the unit circle.
> - Using a procedure to sketch functions.
>
> (Reflections 8 May 1987, p. 1)

The following section focuses on the general nature of students' conceptual difficulties detected during the first seven lessons, and discusses the inadequacy of Ray's established pedagogy for providing opportunities for students to resolve their conceptual difficulties.

Radians as a Conceptual Referent

One of two major constraints to students' ability to sketch graphs of trigonometric functions and find graphical solutions to equations was their inadequate conceptualisation of *radian measures.* Students queried continually Ray's whole-class demonstrations of computational procedures which involved the use of radians as a unit of measure, and many attempted to circumvent the need to think in terms of *radians* by using the more familiar *degree* as a unit of measure.

For example, for the duration of the topic, many students attempted to avoid employing the procedure for sketching graphs, introduced in Lesson One, which involved determining *significant features* (e.g., *the period, x-axis* and *y-axis intercepts*) of a graph by calculating the values of *special angles* expressed in radian units (see Reflections 1 May 1987, Appendix A, p. 318). Students continued successfully to employ their established strategy of labelling the *x-axis* with degree-based *units* and using calculators to compute the corresponding *y-axis* values in order to locate coordinate points for sketching graphs. This naive, but successful, alternative strategy obviated Ray's requirement that graphs be sketched in terms of radian measures which involved algebraic expressions containing the term *pi*.

The use of radian measures requires the ability to perform arithmetic calculations on algebraic expressions that sometimes contain fractions. For example, a practice exercise in Lesson One required the calculation of the period (| *2pi/b* |) of the function *y = 2/3sin(-3x/2)*, where *b* had a value of -3/2 (Fieldnotes 1 May 1987, Appendix A, p. 314). The whole-class discussion of the correct solution in Lesson Two revealed students' conceptual difficulties in manipulating the fraction (Fieldnotes 4 May 1987, Appendix A, p. 322). Although Ray identified students' prior knowledge of fractions to be problematic, his established teaching rarely addressed the problem, beyond providing a reminder of the appropriate arithmetic rules.

The problematic nature of calculating algebraic expressions, especially in relation to radian measures, largely was not addressed in Ray's teaching. He seemed to assume that students' prior knowledge of radian measures, obtained in recent lessons, was sufficient to enable students successfully to manipulate radians, and use them as a conceptual referent for developing their knowledge of trigonometric functions. Research has shown, however, that algebraic symbol manipulation is more conceptually complex than arithmetic calculations involving only numerals, because the former requires an ability to tolerate *lack of closure* (Collis, 1975), or to deal with a *generalised number* that does not resolve to a single numerical value (Kuchemann, 1978). These abilities have been identified as necessary for a student to make the significant conceptual advance from arithmetic to algebraic conceptualisations.

In the case of radian measures, calculators are of limited use in computing solutions to exercises such as the one discussed above, unless students choose to use the conceptually less complex degree as a referent. Many students in Ray's class avoided the more algebraicallyoriented and conceptually-demanding radian referent in favour of the more familiar, numerically-oriented degree referent. The lack of viability of this option becomes apparent, however, when more conceptually advanced problems are encountered.

By Lesson Three, the degree-referent students' alternative strategy for sketching graphs could no longer successfully be employed. Practice exercises required the calculation of points of intersection of graphs (i.e., the graphical solution of simultaneous equations) of linear and trigonometric functions. For example, the following practice exercise was prescribed by Ray in Lesson Three: "Solve the following simultaneous equations graphically *(0≤x≤2pi)*, where $y = x$, $y = tan(x)$" (Fieldnotes 8 May 1987, p. 7).

The solution required the use of radians as a referent. In order that both functions could be sketched, the *x-axis* must be labelled in radian units, rather than in degree units. This strategy proved to be problematic for students who previously had persisted with the use of degrees as a referent.

The widespread conceptual difficulty associated with radians continued to constrain students' conceptual development at least until the second last lesson of the topic (i.e., Lesson Six) when seven students accepted Ray's offer of special assistance for those who were "having trouble with the use of radians on the x-axis" (Fieldnotes 15 May 1987, p. 1). At the same time, a number of other students who had elected not to accept Ray's offer of special assistance were observed to be experiencing problems associated with radian measures.

Unit Circle as a Conceptual Model

A second major constraint to students' ability to sketch and manipulate graphs of trigonometric functions was the conceptual difficulty associated with understanding the *unit circle* conceptual model. Ray frequently employed this model for the purpose of explaining fundamental characteristics of, and relationships between, the trigonometric functions, especially the general effect on graphical representations of changes in variables contained in algebraic expressions used to represent trigonometric functions.

In particular, many students sustained conceptual difficulties with sketching the inverted graphs of *negative functions.* Observations of whole-class discussion of answers to practice exercises at the commencement of Lesson Two, and of students' attempts to complete practice exercises in ensuing lessons, indicated that unresolved conceptual difficulties associated with the unit circle existed amongst the class. Many students seemed unable to apply the *inverse function* rules, $f(-x) = f(x)$ and $f(-x) = -f(x)$, to trigonometric functions in order to determine changes in the general shape of graphs. Students seemed confused about the relationship between a rule, such as $sin(-x) = -sin(x)$, and the procedure for determining the general shape of a graph such as $y = 2/3\ sin(-3x/2)$, in relation to the graph of $y = sin(x)$ (Fieldnotes 4 May 1987, Appendix A, p. 322-323).

Although Ray's whole-class questioning indicated that most higher-achieving target students were able to recall accurately the *inverse rules* and provide correct answers to practice exercises, many lower-achieving students continued to employ an alternative naive strategy. They preferred to use calculators to compute numerical values of coordinate points in order to plot graphs of inverse functions, rather than employ qualitatively the inverse rules to determine the general shape of graphs. This calculator dependency was apparent also in the analysis of the conceptual constraint associated with the use of radian measures for sketching graphs.

Although the alternative strategy was used relatively successfully by students to find correct answers to practice exercises, it was very inefficient, especially when only a general graphical shape was required. More importantly, however, use of the alternative strategy obviated the need for qualitative considerations of the general effect of altering the magnitude or sign of variables in algebraic expressions that represented trigonometric functions. Students who continued to employ the alternative, naive strategy, therefore, failed to engage in opportunities to develop more powerful conceptualisations of the general characteristics of trigonometric functions which required the visualisation of changes in graphical representations.

However, Ray's whole-class presentations of new theory did not emphasise the significance for students' conceptual development of a qualitative approach to thinking about the relationship between algebraic and graphical representations of trigonometric functions. The main emphasis of Ray's teaching approach was on the memorisation and application of rules and, in the case of inverse functions, a further rule-based unit circle explanation of the derivation of the inverse function rules.

For example, Ray first presented a unit circle explanation of the derivation of the inverse function rule *sin(-x) = -sin(x)* to the whole class in Lesson One, and extended this explanation to the inverse rules for cosine and tangent functions in later lessons. The main emphasis of Ray's teaching approach to introducing the role of the negative sign in Lesson One is encapsulated in the following summary:

1. Ray stated the rule: *the whole function will be inverted about the x-axis.*
2. He responded to a student query about the irrelevance for this concept of the modulus sign in the expression for the period.
2. He directed the class to sketch the graphs of *y = sin(x)* and *y = sin(-x)*.
3. He explained, in terms of a diagram of the *unit circle model*, that *sin(x)* is positive in the first quadrant, but that *sin(-x)* is negative in the fourth quadrant. Also, he referred to a sketch of the function *y = sin(x)* and stated that *sin(-90°) = -1*.
4. He asked the class how to complete the rule *sin(-x) = ?*
5. He received an incorrect answer (*sin(-x) = sin(x)*), and responded by reiterating the *unit circle* explanation, and elaborating the *quadrant (ACTS) rule* for determining the sign of a function.
6. He stated the rule: *sin(-x) = -sin(x)*.
7. He set homework practice exercises on sketching graphs with positive and negative values of *x*.

(Fieldnotes 1 May 1987, Appendix A, pp. 312-314)

During the above 11-minute teaching episode, Ray talked almost uninterruptedly to the whole class. Teacher-student interactions were restricted to a single student question and a single teacher question. Ray's teaching was characterised by an explanation of his understanding of the concept in terms of rules to be memorised. The only rationale that Ray provided to underpin the significance of the concept of *graphical inversions* was a further rule-based unit circle explanation which proved, initially at least, to be less than entirely helpful to students, especially those who were unable to respond correctly to Ray's whole-class reproductive questions.

Ray assumed that the unit circle provided a necessary and sufficient basis upon which to promote both the significance and conceptual foundations of concepts associated with trigonometric functions. This assumption was consistent with the syllabus document which stated briefly the content of the *Circular Functions* topic area: "Cosine and sine functions (defined with respect to real numbers and the unit circle), tangent function; amplitude, periodicity" (Fieldnotes 1 May 1987, Appendix A, p. 317).

Although unit circle explanations continued to underpin Ray's presentations of new theory in subsequent lessons, the significance of this conceptual model as a basis of conceptual development remained unexamined. Ray's teaching seemed to be based on an assumption that the syllabus-prescribed unit circle definition of *trigonometric functions* was conceptually transparent and, therefore, could serve as an instructional starting point for conceptual development, rather than as an instructional end-point for a synthesis of a range of related conceptions recently developed by students.

STUDENTS' PERCEPTIONS OF RAY'S TEACHING

In determining the impact on student learning of Ray's established centralist pedagogy, it is important not to overlook students' perceptions. It has been argued that Ray's expository, rule-based teaching approach best served (1) Ray's primary interest in delivering the syllabus, and (2) the interests of a small group of higher achieving *target students* who were able to *keep up* with Ray's whole-class presentations. Evidence of widespread conceptual difficulties amongst the lower-achieving majority of students has been discussed in relation to the assertion that Ray's established centralist teaching approach did not enable these students to obtain optimal benefits from classroom learning activities. This section presents evidence to support the following assertion about students' perceptions of Ray's established teaching strategies:

Assertion Four

Most of the students perceived the classroom learning environment to be problematic: they experienced difficulties in constructing meaningful mathematical conceptions and were concerned about their low levels of academic achievement.

At the commencement of the study, informal discussions with students during individual seatwork activities and at the end of lessons revealed students' concerns about their unexpectedly poor results in topic tests, especially compared with their academic performance in the previous year. Most students had been taught by a different mathematics teacher in Grade 11, and several students expressed concern about Ray's *university-style lecturing* (Fieldnotes 4 May 1987, Appendix A, p. 330).

During an interview, Ray recounted how a group of about 10 students (out of a total of 27) had approached the school's Senior Master of Mathematics shortly after this study began in order to express their concerns about their poor academic performance. In a subsequent meeting with Ray, the students expressed their concerns. Ray's general impressions of this meeting are contained in the following extract of the interview:

> What impression did you get about why they complained?
>
> By the end of the time I had been talking to them, the biggest impression I got was that I wondered why they'd complained at all. It was a bit hard to put my finger on it. I did ask them but, now that I think of it, their responses weren't all that convincing. They said

> things like: 'We felt that we weren't learning some of the material', or that 'We were having difficulty understanding'. Something like that.
>
> And that it had been an on-going problem?
>
> Yes
>
> Did they say for how long? Since the beginning of the year or just this topic?
>
> No, since the beginning of the year, particularly as the first topic kind of went right over their heads, in a way. . . . And it hit them hard. . . . And then they did badly in the final test.
>
> How representative was that group, ability-wise, of the rest of the class?
>
> It was representative of the lower half of the ability. . . . The top eight girls in the class weren't represented there.
>
> So the representation was of about two-thirds of the class then?
>
> Yes. . . . In ability terms, yes.
>
> (Interview One 12 June 1987, pp. 1-3)

Clearly, a large proportion of the class was harbouring serious concerns about the impact of Ray's teaching on their academic performance, especially their ability to understand Ray's whole-class presentations. Ray recounted a student's expressed concerns that indicated both a lack of opportunity to participate actively in whole-class discussions or privately consult the teacher, and the unsatisfactory social dynamics amongst the class:

> One problem that some of them saw . . . was that the class is too big, and . . . that it's difficult to get individual attention. . . . They're very aware of a kind of clique system within the class. . . . I'm not very aware of it but apparently they are. Certain people sit with certain other people and they know their little group and they hardly mix with the other girls, or hardly know them. (Interview One 12 June 1987, p. 3)

Ray recounted students' suggestions for improving the learning environment. These suggestions focused on the creation of opportunities to interact with both the teacher and other students:

> Maybe the good kids, the stronger students could do some work on their own or even go to the library while I took a smaller group in more of a tutorial situation. And a couple suggested changing seating arrangements. I couldn't quite see what that had to do with it, but I think they saw it as getting more into a communal aspect of learning, I suppose. (Interview One 12 June 1987, p. 5)

It is likely that the predominance of teacher-centred whole-class activities and the active presence of higher-achieving target students in whole-class discussions were sources of major discontent for the lower-achieving students. Capturing teacher attention by breaking the stranglehold of dominant students, and personalising the classroom environment seemed to underpin students' suggestions for reform. Ray reported that another student had stated that mathematics was "a rather boring subject" and had suggested that "problem solving" and "working in small groups" might be more interesting (Interview One 12 June 1987, p. 5).

From the above interview excerpts, it is clear that poor communication between the teacher and students had contributed to disparate perceptions of the classroom learning environment. Ray's predominantly centralist classroom role was preventing him from detecting students' perceptions of the need to adopt a more *student-centred* teaching approach. Indeed, his established centralist pedagogy seemed almost impervious to student evidence of the need for reform. He expressed surprise about students' concerns and tended to deny the credibility of students and the plausibility of their suggestions. Despite compelling evidence that most students were experiencing difficulties with their learning, and were seeking changes to the learning environment, Ray seemed to resist strongly the notion that his established pedagogy was a major contributing factor to the problems alluded to by the students.

SUMMARY

An analysis of Ray's discourse indicates that his established pedagogy was governed by two highly congruent metaphorical conceptualisations — *the syllabus as a conceptual map*, and *teaching as a journey*. These classroom role-related metaphors suggest that Ray's pedagogy was governed by a nexus of syllabus-related interests and an objectivist epistemology. It seems plausible to infer that Ray's conceptualisation of *the syllabus as a conceptual map* is indicative of a personal epistemology which (1) identifies the textual content of the syllabus document as the main pedagogical frame of reference, and (2) identifies the teacher's mathematical knowledge as the direct source of students' mathematical knowledge. This assertion is strengthened by the assertion, in Chapter Six, of Ray's formalist preconception of mathematics. It seems plausible to infer also that Ray's conceptualisation of *teaching as a journey* provided him with a compelling rationale for: (1) *covering* the syllabus content; (2) controlling students' classroom activities; and (3) justifying the underachievement of many students who failed to *keep up* with the predetermined pace of the journey. A pedagogical implication of these role-determining metaphors is a behaviourist conception of students' roles as acquiring the teacher's knowledge by emulating his manipulation of mathematical syntax.

In practice, Ray's *teacher as informer* classroom role seemed to be determined by his syllabus-related priority interests, and to be designed to facilitate students' expeditious and accurate reproduction of his own transmitted mathematical knowledge. Whole-class teaching, which featured teacher explications, was the dominant teaching mode. The active presence of higher-achieving *target students*, and the reproductive nature of Ray's questions combined to provide only limited opportunities for the lower-achieving majority of students to participate actively in public attempts to facilitate their own conceptual development. During class, opportunities for private consultations with Ray were very restricted by the predominance of whole-class activities.

During whole-class discussions, it was commonplace for a few higher-achieving students to provide the sequential *missing steps* within the structure of Ray's explication of new theory and solutions to practice exercises. Ray was not in the habit of encouraging students either to seek clarification or to explain their alternative solutions. His usual mode of response to individual students' queries or incorrect responses was a reiteration of an explanation or rule-based procedure for obtaining a correct solution. Diagnosis of the nature and causes of students' learning difficulties was not on Ray's whole-class teaching agenda.

Because of the limited nature and scope of the interactions between teacher and students, Ray's pedagogical concept knowledge incorporated very little understanding of students' conceptual development needs and difficulties. Ray was unable, therefore, to design teaching strategies which might pre-empt, or overcome, many conceptual difficulties commonly experienced by students. Although Ray was an experienced teacher, in many respects he had the pedagogical concept knowledge of a neophyte teacher of Grade 12 mathematics.

Furthermore, a combination of Ray's pedagogical disregard for the state of students' prior mathematical conceptualisations, together with a reproduction-oriented promotion of the *end-product* of his own syllabus-related mathematical conceptualisations, resulted in a teaching approach which constrained the conceptual development of many students, and failed to enhance the development of their mathematical thinking. Many students maintained their naive mathematical conceptions and failed to develop higher-level conceptualisations.

Many students, especially the lower-achieving majority, perceived Ray's teaching approach to be problematic because it did not seem to be serving their conceptual development needs. They experienced on-going difficulties in understanding Ray's whole-class presentations. Students' suggestions for improvement focused on reshaping the general social dynamics of the classroom and increasing opportunities for private consultations with the teacher. These suggestions are consistent with breaking higher-achieving students' stranglehold on the teacher and decentralising teaching.

CHAPTER FIVE

CONSTRUCTIVIST PEDAGOGICAL REFORM: RAY'S REFINED CENTRALIST CLASSROOM ROLE

INTRODUCTION

Chapters Two and Three present an account of the development and implementation of a constructivist-related theory of pedagogical reform aimed at resolving the problem of teacher centralism in high school mathematics classrooms. An analysis of Ray's established centralist pedagogy, which is presented in Chapter Four, highlights the ways in which his pedagogical beliefs shaped his centralist classroom role and constrained both his pedagogical knowledge development and students' mathematical conceptual development.

Chapter Five employs the interpretive framework which is discussed in previous chapters, and reports the main results of the collaborative phase of the study. Interpretive analyses of Ray's reformed pedagogy are reported as Assertions Five and Six, and indicate Ray's development of a constructivist perspective on learning and the subsequent refinement, rather than radical transformation, of his centralist classroom role.

The first section discusses the interpretive framework of the chapter, particularly methodological aspects of the interpretive research approach which is presented in Chapter One. The major sources of interpretive data are identified, and methods of data analysis are outlined. In particular, a rationale is presented for the use of the *repertory grid technique* as a method of inferring teachers' implicit role-determining beliefs. Previous applications of this research methodology are reviewed, important conditions for conducting the interviews are discussed, and the methodological procedures employed in this study are outlined. Further details of the repertory grid procedures for obtaining and analysing data, the preliminary outcomes of the data analysis, and interview transcripts are contained in Appendix B.

The second section discusses Ray's development of a critical awareness of the problematic nature of his established centralist pedagogy, especially in relation to students' outstanding learning difficulties. Assertion Five argues that Ray's identification of a pedagogical need to understand better students' short-term learning outcomes and his development of a constructivist-related perspective on learning resulted in his adoption of a new classroom teaching role of *teacher as learner*.

The third section discusses how Ray resolved the apparent conflict between his constructivist-related belief in the first principle of constructivism and his established transmissionist epistemology. Assertion Six argues that Ray's constructivist-related pedagogical innovations constitute a refinement, rather than a radical reconstruction, of his centralist classroom teaching role of *teacher as interactive informer*.

INTERPRETIVE FRAMEWORK

This section discusses important methodological issues concerning the collection, analysis, and interpretation of data which are presented as evidentiary warrants for the assertions of this chapter.

Ray's Interpretations

In Ray's report on the study (Fleming, 1988), which was written during the 12-month period proceeding the collaborative phase of the study, Ray identifies himself as 'the researcher' and myself as 'the observer'. This terminology appears in this chapter in verbatim quotations which have been extracted from Ray's report. Although the terminology might confuse the reader of this thesis, it has been retained because it serves an important validation role in relation to the interpretive data analysis.

In Chapter One, it was argued that in reports on collaborative research the teacher's *voice* should not be masked by the researcher's interpretations. The identification of the teacher's interpretations is regarded by Brickhouse (1991) as an important ethical issue which can enhance the evidentiary adequacy of assertions which are inferred from interpretive data. In this report, therefore, verbatim quotations extracted from Ray's account of the study serve to provide evidence of Ray's interpretation of the pedagogical reform process in which he was engaged. Of special merit is evidence of Ray's adoption of the role of researcher. This evidence highlights the collaborative nature of the relationship between Ray and myself, and provides evidence of Ray's sense of control and ownership in relation to the conduct and outcomes of the study.

Students' Perceptions

Students' perceptions of the nature of the changes to Ray's classroom role, and of their impact on students' learning activities, were obtained in recorded interviews. The methodology and preliminary outcomes of the data analysis are reported in Appendix C. The results of the interviews are incorporated into the interpretive analyses of Ray's pedagogical reforms in this chapter.

Interpretive Data Sources

The evidence which warrants the assertions presented in this chapter was obtained from the application of an interpretive research approach (see Chapter One). Although the *repertory grid technique,* which is discussed below, was the main source of data, other data sources, including observational fieldwork data, additional teacher interviews, and Ray's report on the study, supplement the analysis. Of particular significance are data obtained from interviews (Interview 6 and Interview 7) which were conducted after the completion of the second period of fieldwork in Ray's Grade 9 class. The interviews were conducted for the purpose of obtaining a better understanding of the network of beliefs that underpinned Ray's reconstructed pedagogy.

In addition to the use of the repertory grid technique, which assumes that beliefs are propositional in nature, Ray's pedagogical beliefs were inferred also from metaphors contained in records of his discourse. In this chapter, interview transcripts are analysed for evidence of metaphors associated with Ray's explication of his classroom role, especially metaphors associated with Ray's epistemology and syllabus-related priority interests. Chapter Four presents a rationale for the interpretive analysis of metaphorical content of speech.

Repertory Grid Technique

Mary Black's (1974) review of the literature of cognitive anthropology on the nature of beliefs and belief systems utilises Goodenough's (1963) definitions as a major organisational framework. In brief, Goodenough proposed that beliefs are deeply embedded in cultural knowledge structures, often are held unconsciously, shape the interpretation of experiences, and are capable of being expressed in the form of "propositions held to be true" (Black, 1974, p. 512). The propositional nature of beliefs is utilised also in cognitive science (Abelson, 1979). In education, similar definitions of the propositional nature of teachers' pedagogical beliefs appear in the research reported by Eisenhart, Shrum, Harding, and Cuthbert (1988), Fenstermacher (1986), Nespor (1987), Nisbett and Ross (1980), Munby (1982, 1984), Olson (1980, 1981), and Tobin, Kahle, and Fraser (1990). In this study, the following definition of a belief was adopted: *a proposition, or statement of relation among things, accepted as true.*

Widespread acceptance of the propositional nature of teachers' beliefs has been accompanied by the adoption of a popular research method — the *repertory grid technique* — for investigating the relationship between teachers' pedagogical beliefs and classroom practice (Nespor, 1987; Munby, 1982, 1984; Olson, 1980, 1981; Tobin, Kahle & Fraser, 1990). This research method is based on the *personal construct theory* of George Kelly (1955), and has been described by Fransella and Bannister (1977) as a method of exploring the structure and content of personal theories.

In this study, a series of unstructured interviews was conducted shortly after the completion of the collaborative phase of the study in order to determine the nature of Ray's pedagogical beliefs, especially implicit beliefs which had a major impact on the reconstruction of his classroom role and teaching practice. The articulation of implicit beliefs was expected to provide substantive evidence of the extent to which Ray had developed a constructivist epistemology. Of particular interest was the identification of beliefs that constrained the development of Ray's constructivist-related pedagogy. The unstructured interviews were modelled on the repertory grid technique for investigating implicit theories and beliefs (see Appendix B).

The use of this research method is well-described in the educational research literature. Olson (1980, 1981) employed the repertory grid technique to examine how teachers construe their teaching roles, especially in the context of challenges by innovative curriculum doctrines to teachers' established pedagogical construct systems. The research method enables a collaborative researcher — for heuristic and non-manipulative purposes — to penetrate the *rhetorical facade* and reveal teachers' *tacit knowledge* and the *deep structure* of practice, in order to assist teachers to become aware of, and to examine, their implicit pedagogical beliefs:

> In short, the approach allows access to how individuals involved perceive their work. . . . The process of construct elicitation . . . [is] like a somewhat unstructured interview whose direction cannot be preplanned, the upshot of which is an instrument developed on the spot, unique to the individual and capable of providing some insight into how that person construes the area of interest defined by the investigator. (Olson, 1980, p. 5)

Munby (1982, 1984) adapted the repertory grid technique, and used it to make explicit the dominant beliefs and principles, or *repertories of understanding*, which constitute a *teacher's world* and underpin their pedagogical pragmatism, especially in relation to their planning and teaching. Munby (1984) argues that the technique provides a qualitative research method which elicits teachers' fundamental beliefs and principles with greater integrity than do more traditional quantitative research approaches, such as questionnaires and rating scales. These approaches often reflect the beliefs of the instrument designer rather than the respondent, and impose an interpretation whose validity is more closely associated with the instrument's validation sample than with the individual respondent. By contrast, the repertory grid technique is an adaptable technique which can attend to "the uniqueness of an individual within a particular environment" (Munby, 1984, p. 29).

Use of the repertory grid technique requires the investigator to establish important conditions which (1) allow teachers "to speak for themselves in their own language", and (2) allow "their true feelings to emerge" (Olson, 1981, p. 262). As well, it is important that (3) the teachers' responses are focused on their "immediate and personal experiences", and that (4) the teachers' responses represent their true beliefs, rather than what they perceive to be *socially acceptable* (Munby, 1982, p. 218). The procedure adopted in this study was designed to establish these conditions. Appendix B contains a description of the five-step methodological procedure of the repertory grid technique employed in this study, and a preliminary analysis of both the repertory grid data and subsequent teacher interviews.

In brief, the repertory grid technique employed in this study involved (1) an interview (Interview 3) to elicit Ray's listing of classroom teaching and learning *events* that he would prefer to see taking place in the best possible version of his classroom, (2) a second interview (Interview 4) in which Ray arranged the events (recorded on individual cards) into meaningful *groups* which he labelled, (3) a third interview (Interview 4a) in which Ray recorded (numerically), in each cell of a two-dimensional matrix, or repertory grid, his perception of the strengths of the relationships between the teaching and learning events and groups, (4) a factor analysis of the completed repertory grid, and (5) a fourth interview (Interview 5) which aimed to identify the nature of Ray's role-determining pedagogical beliefs which were represented by the statistical factors. The preliminary results of the repertory grid analysis are presented in Appendix B. In the following sections of this chapter these results have been synthesised with data from other sources.

TEACHER AS LEARNER

Ray's development of a constructivist-related perspective on the teaching-learning process in his classroom resulted in a major change in his classroom teaching role during the collaborative phase of the study. Ray identified a need to adopt a *teacher as learner* role which would enable him to construct a better understanding of students' learning outcomes, especially in the shorter-term framework of a lesson. This section discusses the following assertion about the development of Ray's new constructivist-related classroom teaching role of teacher as learner:

Assertion Five
Ray developed a critical perspective on the efficacy of his established centralist pedagogy, and a constructivist perspective on student learning; his subsequent *teacher as learner* classroom role constituted a pedagogical focus on understanding better students' short-term learning difficulties.

A Critical Perspective

Prior to the study, Ray had become reconciled to "accepting that a proportion of the class would fail to understand some, or even most, of the concepts presented by the teacher" (Fleming, 1988, p. 37). However, subsequent evidence of widespread student learning difficulties and discontent compelled Ray to accept that his predominantly lecture style of presentation was not facilitating effective learning in a class comprising students of wide-ranging ability:

> As a result of discussions with the observer [sic] and a consideration of the observer's fieldnotes it became apparent that many students had inadequate understanding of recently taught concepts and, in many cases, students had inadequate background knowledge.
>
> (Fleming, 1988, p. 36)

In response to my inquiries about the established pattern of teacher questioning, Ray focused his attention on the nature of the questions which he asked students, especially in the whole-class forum. He found that he was unable to explain satisfactorily his practice of seeking responses mostly from a small number of high achieving *target students* (see Chapter Three):

> Because the researcher's teaching style had been a conventional *chalk and talk* lecture style presentation, with questions usually restricted to students who regularly volunteered answers (*target students*; Tobin, 1985), the researcher had previously not detected such detailed individual student learning difficulties (except in tests and exams where there was relatively little opportunity for immediate remediation, questioning or clarification). (Fleming, 1988, pp. 36, 37)

Ray realised that the pace of his whole-class presentations was determined largely by his perception of the learning needs of target students, and that he was obtaining very little feedback from most of the class. Also, he began to realise that "his own mathematical knowledge was not being adopted by the students" (Fleming, 1988, p. 37).

Self-Evaluation

Ray's evolving critical perspective prompted self-questions about the efficacy of his established classroom role. He became engaged in a process of critical self-inquiry in which he began to identify and question the assumptions underlying his established teaching practice:

> The researcher was, by the third week of the study, beginning to become more critical of his own actions, asking himself questions such as: 'What is my purpose in doing this?', 'Could this be done in a better way?', 'Have I covered too much too quickly?', 'Have I questioned a range of students?', 'How many of the students understand the main points of the lesson?'. (Fleming, 1988, p. 48)

Ray's critical reflections stimulated his attempts to record a critical account of his established teaching style. A commentary in his report on the study indicates a growing awareness of constructivist-related principles:

> I could best describe my teaching style as being characterised by the following:
>
> - Mainly teacher-centred, didactic, and providing information in terms of how I understand the mathematical content — giving methods for solving problems and rules for approaching problems.
> - No particular attention was paid to linking new information with the previous lesson's content other than to make the presentation logical and sequential. A lesson would consist of giving solutions to homework problems and then moving on to an explanation of the next new bit. I always tried to introduce some new material every lesson.
> - I used questioning fairly frequently but the quality of the questioning was not good because I was not using the questions to provide myself with information on student knowledge, rather as a means of engaging students (if briefly) in the presentation of the material. I wanted to elicit the (correct) answer to the question in order to continue. As a consequence, I probably failed to recognise the *incorrect* answers in terms of providing feedback on student (mis)understanding (i.e., on the nature of the students' constructed knowledge). The questions were not usually aimed at investigating students' difficulties in understanding but rather a desire for a correct response.
> - Related to the above point is the fact that I frequently chose *target students* (especially more capable) students to get quick and correct responses. Sometimes I would choose average ability students to test whether a representative student understood the concept.
> - I rarely referred to the text except for selection of homework problems.
> - My explanations were usually model solutions without reference to student difficulties. (Fleming, 1988, pp. 44, 45)

Ray realised that presentations of his own mathematical knowledge emanated largely from within his own *interpretive framework*, and took very little account of the range of students' interpretive frameworks. He became aware of Lee Shulman's (1987) categorisation of teacher knowledge (see Chapter Four), and attributed the problematic nature of his established teaching style to a teacher-centred perspective which had failed to allow him to account for students' attempts to construct their own mathematical knowledge. He became aware of the need to improve his *pedagogical content knowledge*:

> Although the researcher's own mathematical knowledge may have been adequate, his knowledge of how to present mathematical knowledge in ways which are most easily understood by the students and to modify that presentation according to the needs of students was inadequate for many students. The term *pedagogical content knowledge* has been used to describe this particular type of teacher's knowledge. (Fleming, 1988, p. 46)

A Constructivist Perspective

Ray's realisation of the inadequacy of his pedagogical content knowledge resulted in a significant change in his established classroom teaching role. Ray conceived of a need to adopt the role of *teacher as learner* which would enable him to become more knowledgeable about students' mathematical knowledge development, especially in the shorter-term:

> [There is] a need for the teacher to obtain feedback on the success or failure of students' short-term learning activities. That is, the teacher should closely monitor the effectiveness of students' attempts to construct their own mathematical knowledge. (Fleming, 1988, p. 50)

At about the same time, Ray generated a constructivist perspective on student learning:

> A number of factors, including discussions with the observer and acquaintance with some of the research literature on the theory of

> constructivism, led the researcher to appreciate that students construct their own versions of mathematical knowledge; that mathematical knowledge is not transferred per se from the mind of the teacher to the mind of the student. This realisation had important implications for the researcher in terms of his teaching style. (Fleming, 1988, pp. 71-72)

During interviews in the post-collaborative phase of the study, Ray explained the nature of his constructivist-related classroom teaching role:

> I'm interested in finding out what the students know . . . how they understand things. And in order to do that I need to monitor what they're doing and what they're thinking. And I can do that by listening to their responses to questions, looking at their work. . . . I need to know . . . what they understand . . . what they don't understand, where they're going wrong, what difficulties they're having with a concept . . . [by] monitoring their progress.
>
> (Interview 4, September 1987, Appendix B, pp. 341-342)
>
> The way I put [constructivism] into practice, or make use of it myself, is that I make a lot of effort to try to understand the way in which students are comprehending the work. I try and get a picture of their perspective of the particular work we're doing.
>
> (Interview 7, September 1988, p. 6)

There is little doubt that Ray's new constructivist-related belief in monitoring closely individual student understanding signalled a major new classroom teaching role: an investigative role which Ray adopted relatively early in the collaborative phase of the study. Ray's explanation of the reflective nature of his investigative teaching role, especially during whole-class discussions, seems very similar to the process of reflection-in-action described by Schon (1983):

> The teacher continually investigates the nature of students' understanding or constructed knowledge . . . [T]he teacher is, therefore, kind of taking a dual role. He is acting as an observer of the interaction as well as participator in it, and he is modifying his role according to the results of the observations . . . about student engagement, student

> understanding. . . . [T]he teacher needs to observe the dynamics of the process, to obtain feedback on student learning, and not just students' ability to perform algorithms . . . using specific questioning techniques to investigate the nature of students' knowledge, to assess student constructs. (Interview 2, August 1987, p. 14)

Investigation Strategies

Ray developed strategies for investigating students' mathematical knowledge. A major strategy which enabled Ray to enact his teacher as learner role was teacher questioning. Ray introduced a more inquiry-oriented style of whole-class discourse in which teacher questioning performed a major role. In his account of the study, Ray cited Minstrell's (1984) teacher questioning guidelines for assisting students to overcome their *alternative conceptions*. Ray adapted the guidelines in response to his perceived constructivist-related need to improve the adequacy of his knowledge of students' learning difficulties:

> During this study a constructivist framework was used to . . . develop appropriate teaching strategies. For example, one of these strategies was based on the use of questioning techniques. The questions were asked frequently at whole-class level, were designed to engage as many students as possible, and usually were single step and sequential (progressing logically) in nature. The questions provided both the teacher and students with a continuous source of feedback on the development of their new mathematical knowledge.
>
> (Fleming, 1988, p. 21)

Ray intended to avoid "superficial questions such as those which require single-word answers in order to complete a sentence for the teacher" (Fleming, 1988, p. 54), to direct questions to more students, and to probe student understanding.

Ray devised additional investigation strategies for enabling him to learn about individual students' learning difficulties during class. He increased the amount of individual student seatwork activity by designing student worksheets which provided him with increased opportunities to move around the class and consult individual students. At the commencement of lessons, Ray reviewed students' homework which he had set at the end of the previous lesson, and questioned students' problem solutions.

The investigation strategies which Ray employed to obtain a better understanding of students' learning difficulties served an additional purpose, namely, to enable Ray to provide enhanced opportunities for students to construct their own mathematical knowledge. The next section discusses how Ray's investigation strategies resulted in a refinement of his established classroom role of *teacher as informer*.

TEACHER AS INTERACTIVE INFORMER

The previous section argues that, during the collaborative phase of the study, Ray generated a belief in the first principle of constructivism, namely, that knowledge is not passively received, but is built up actively by the learner, and that the cognitive process of knowledge construction is dependent on the learner's prior knowledge. However, in this section, further analysis indicates that Ray sustained an apparently contradictory epistemological belief in his role as the primary source of students' new mathematical knowledge, and a priority pedagogical interest in facilitating students' accurate reproduction of his own mathematical knowledge. This section presents evidence which warrants the following assertion about Ray's reformed classroom role and pedagogical innovations:

Assertion Six

Ray's transmissionist epistemology subsumed his constructivist-related belief in accounting for students' prior mathematical knowledge, and resulted in a refined centralist classroom role of *teacher as interactive informer* which was favourably perceived by students but largely failed to enhance their mathematical conceptual development.

A Prior Knowledge Focus

Ray identified a constructivist-related need for his teaching to take account of students' prior mathematical knowledge. He believed that he should adopt a classroom role which facilitated students' validation of their extant mathematical knowledge and its meaningful integration with their newly constructed mathematical knowledge:

> [There is] a need for the teacher to assist students to review and validate their prior (background) knowledge. [There is] a need for the teacher to provide opportunities to assist students to assimilate newly encountered mathematical knowledge with their prior mathematical knowledge. That is, to cognitively link new knowledge with existing mathematical knowledge and to consolidate the linkage in the mind of the learner. In this way a framework of knowledge is constructed in the students' mind which is both internally logically consistent and externally valid. (Fleming, 1988, p. 50)

Ray's realisation of the need to facilitate students' mathematical knowledge construction resulted in a significant refinement of his established *teacher as informer* classroom role which is described in Chapter Four. One of Ray's major innovations was his specially designed worksheets which commenced with a focus on students' prior mathematical knowledge and its relationship with the new concept to be developed. For example, the Grade 12 worksheet *Graphing Inequalities* commenced with revision exercises designed to assist students to recall their prior knowledge of the *Cartesian Coordinate System* (Fieldnotes 8 June 1987, p. 7). Ray continued to develop prior-knowledge-related student worksheets in the post-collaborative phase of the study (see Assertion Eight in Chapter Six).

Another prior-knowledge-related innovative strategy which Ray introduced into his teaching of the Grade 12 class was *mental tests* which he conducted with the whole class at the commencement of lessons. Usually, the test questions comprised the homework exercises from the previous lesson. The subsequent whole-class discussion of students' answers enabled Ray to determine whether a general need existed for further *remedial* teaching of previously taught concepts:

> This provided a useful source of feedback for students on the development of their own understanding as well as a means of recalling, and establishing continuity with, the content of the previous lesson/s. (Fleming, 1988, p. 57)

On several occasions, Ray abandoned his lesson plan and conducted extensive whole-class instruction of a previously taught concept when it appeared that an inadequate level of general understanding existed amongst the class. However, Ray discontinued the use of mental tests in the post-collaborative phase of the study.

Prior to interpreting the significance of Ray's constructivist-related pedagogical innovations, however, it is necessary to understand how he reconciled newly generated constructivist-related beliefs with an established transmissionist epistemology which locates the genesis of students' mathematical knowledge in the mind of the teacher, defines the goal of teaching as students' accurate reproduction of the teacher's knowledge, and underpins a centralist classroom role for the teacher.

Transformation of Teacher Knowledge

The key to understanding Ray's epistemological rationalisation is to understand his interpretation of the concept of knowledge construction. In his report on the study Ray explains, in the form of a *knowledge-transformation* model, his epistemological conception of his reformed classroom role (Fleming, 1988, p. 15).

Although Ray apparently no longer believed that knowledge could be transmitted directly to students, he continued to conceive of external agents, especially the teacher, as the primary source of students' new mathematical knowledge. Ray's image of a knowledge transformation model (see Figure 1) underpinned his explicit epistemological belief that one of his main instructional roles was to *transform* his mathematical knowledge into *information*, and engage students in learning activities that would facilitate their accurate *transformation* of the (transmitted) information into their own personal knowledge. Ray's constructivist-related beliefs were confined to the interface between students' extant mathematical knowledge and newly received mathematical information. Ray conceived of students' knowledge construction, therefore, as a prior-knowledge-dependent process of information transformation. In other words, Ray's established transmissionist epistemology was sustained, and subsumed his newly developed constructivist-related beliefs.

As a result, Ray conceptualised his key classroom role to be the *exposure* of externally-generated mathematical information in a form that would optimise students' opportunities of *constructing* their new prior-knowledge-dependent mathematical knowledge:

> [T]he process of actually exposing the student to new information . . . is crucial. . . . [T]his is where concepts are introduced to students, hopefully by linking to something else, not just [by] snatching them out of the blue. (Interview 4, September 1987, Appendix B, p. 340)

Ray's established centralist classroom role of *teacher as informer* (see Assertion Two in Chapter Four) had been largely sustained, albeit in a new guise which was articulated in terms of constructivist-related terminology. In Ray's personal lexicon new terms, such as *information* and *transformation*, served as synonyms for *knowledge* and *transmission*, respectively. Ray's established pedagogical priority of facilitating students' accurate reproduction of his own mathematical knowledge was highly congruent with his knowledge-transformation model, and underpinned the design of his major teaching innovations during the collaborative phase of the study.

Chapter Six discusses Ray's knowledge-transformation model from the perspective of the philosophy of mathematics, and argues that the model implies an outmoded objectivist conception of mathematics which separates mathematical objects from mathematical cognition. Furthermore, it is argued that the constructivist-related theory of pedagogical reform, which governed my discourse with Ray during the collaborative phase of the study, promoted unwittingly an objectivist conception of mathematics and served to sustain Ray's transmissionist epistemology.

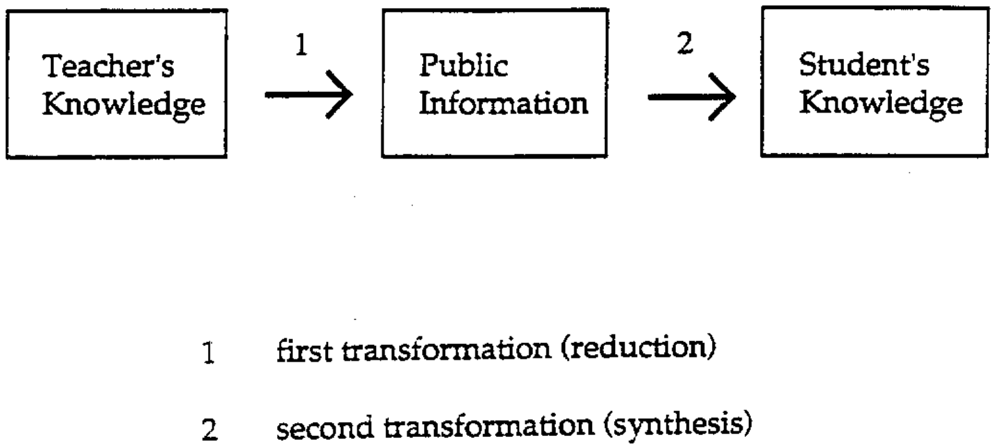

Figure 1. Ray's knowledge-transformation model

Ray's Knowledge-Reproduction Pedagogy

Assertion Five argues that Ray's adoption of a more interactive approach to teaching enabled him to monitor a greater range of student learning, and to generate a better understanding of students' shorter-term knowledge-related learning difficulties. Ray perceived a need for a pedagogical approach that would enable him to utilise his better understanding in order meet his priority pedagogical goal, namely, to facilitate more effectively students' endeavours to reproduce his own mathematical knowledge. Consequently, Ray extended his interactive classroom role of *teacher as learner* in order to account for students' learning difficulties:

> I'm trying to . . . modify or adjust the way I'm teaching a particular topic to meet the requirements that I perceive as a result of more continuous and a greater amount of feedback from the students.
>
> (Interview 1, June 1987, p. 7)

Ray refined his centralist classroom role by adopting the additional role of *teacher as interactive informer*. In accordance with his knowledge-transformation model (see Figure 1), Ray believed that it was important to transform his own mathematical knowledge into a form of information that was more user-friendly to students. He believed that he could utilise his better understanding of students' attempts to construct their knowledge by providing more *student sensitive* explanations:

> I'd certainly say that I think that if my explanations are sufficiently student sensitive and explicit, then that will minimise the students' difficulty in developing their content knowledge, or it would enhance their development of content knowledge.
>
> (Interview 6, August 1988, p. 10)

Inquiry-Oriented Discourse

However, Ray no longer believed in simply telling, or directly transmitting, his knowledge to students. Rather, he believed that teacher-directed *inquiry-oriented* discourse provided the best opportunities for students to develop new mathematical knowledge:

> [Introducing new concepts is] . . . mainly a teacher-initiated activity. . . . I might provide that information in an inquiry-based manner, so that I get the students thinking about some question, and then present information that answers the question, or a technique with which to handle the question. I might do this on the blackboard or I might use worksheets to introduce a topic. (Interview 4, Appendix B, p. 340)
> On the [black]board, and if I'm dealing with an individual . . . I try to let the student, or the class, discover the particular rule or the particular method that I'm using, with some guidance from me of course. . . . The main purpose is to allow the student to see for themselves . . . or even partly discover for themselves, the explanation or the method that is being explained, or that we're talking about.
>
> (Interview 6, August 1988, p. 6)

Ray's inquiry-oriented discourse was designed to stimulate students' to anticipate his presentation of new information, whether it be presented verbally to the whole class or to individual students, or in specially designed student worksheets.

One of Ray's inquiry-oriented discourse strategies for conveying new information to the class was teacher questioning. Although Ray had used whole-class questioning in the past, it had served a different purpose. Assertion Two (in Chapter Four) argues that Ray's *reproductive* questioning was designed primarily to capture students' attention and obtain correct answers. Ray explained, in terms of his constructivist-related perspective, that the purpose of his inquiry-oriented questioning technique was to focus student thinking on the relationship between their prior mathematical knowledge and the new concept which he was explaining to the class:

> [I am concerned with] avoiding the so-called *yoyo* type questions, asking good quality questions that require a student answer, and listening to the student answer; not just asking a question for the sake of a break or for the sake of pseudo student input. . . . . When you're doing newer material . . . that forms the opportunity to link the new material with the old, with their pre-existing knowledge by questioning and opening up the areas. I suppose that the idea of getting things ready to take on board the new information is a constructivist idea, isn't it? (Interview 1, June 1988, p. 11)

In private consultations with students, during periods of individual seatwork activity, Ray reported that he employed inquiry-oriented questioning for the purpose of intervening in students' knowledge construction processes and providing corrective instruction:

> Rather than saying: 'Look you've got that wrong, Here's the right answer, or Here's how you do it . . . I'll show you again the way I do it'; now I'll say: 'What went wrong? How did you get here?' So, I'll ask the student to explain their line of reasoning, if they've got one, and then I'll, perhaps, interrupt at a point where they've gone wrong or got some false method or impression, and then I try and correct them.
> (Interview 7, September 1988, p. 17)

Ray's inquiry-oriented discourse maintained his centralist classroom role as primary source of students' new mathematical knowledge, albeit in a more student-centred manner which attempted to account for student's prior mathematical knowledge.

Reductionism

Ray's priority pedagogical goal was for students' to understand his inquiry-oriented explanations of new theory and methods of obtaining solutions to practice exercises. During the collaborative phase of the study, Ray argued that a *reductionist* teaching approach, which provided students with *short tasks*, was congruent with a constructivist perspective on teaching:

> If you are going to teach for concept development . . . a lecture style presentation, I'm sure, is not the best way of disseminating information. So, I felt it would be a better means of doing it if [learning activities were] broken up more, if there were short tasks to be completed which involved the students more actively. . . . Then we would, perhaps, have a look at what they had done, discuss it . . . and I could walk around and have a look at what they were doing at the same time. (Interview 1, June 1987, p. 7)

Ray decided that a reductionist approach to transforming his mathematical knowledge would yield smaller quanta of mathematical information that were more user-friendly for students. In the post-collaborative phase of the study, he argued that reductionist instructional design which incorporated inquiry-oriented discourse obviated the problem of teaching from within his own interpretive framework:

> [W]hen I explain something, whether it be to an individual or to the whole class, I try not simply to present my understanding of it, my explanation of the whole problem because, often, I've found that it may go over their heads if I explain it in the way I understand it. . . . So, I start explaining it, trying to expose the explanation bit by bit, asking for student responses along the way. (Interview 6, August 1988, p. 6)

Whole-Class Instruction. Ray applied reductionist principles to his inquiry-oriented discourse, especially when addressing the whole class. In his report on the study, Ray cited the following two examples of his new style of questioning which was designed especially for students for whom understanding more complex questions was problematic:

> Rather than ask: 'What is the period of the function $y = 3\ sin(2x)$?'
> (which is how the researcher [sic] would have asked such a question with his initial teaching style), the question was phrased in several, more manageable steps:
>
> 'Where do we look to find the period of the function?'
> 'How do we use the *2* to find the period?'
> 'What is the period of this function?'
>
> [Rather than ask] 'Without reference to your calculator or a graph, determine whether tan 200° is positive or negative', the question was rephrased and stated in the following form:
>
> 'Quickly sketch a unit circle'
> 'Where is the radius vector when *Ø =200°*?'
> 'Sketch in the referent triangle'
> 'Is the adjacent (horizontal side) positive or negative?'
> 'Is the opposite (vertical side) positive or negative?'
> 'What is the definition of tan in terms of these sides?'
>
> (Fleming, 1988, pp. 54-55)

For Ray, a reductionist approach provided optimal opportunities for him to obtain important *feedback* during whole-class discourse on the effectiveness of his interactive informer role:

> I think that everyone doing a short section or a short step in a question, and then asking the students for answers and then seeing if the whole class or most of the class have got the same answer or have got it right . . . is a good way of assessing progress.
>
> (Interview 5, November 1987, Appendix B, p. 354)

One of the advantages of the worksheets was that they enabled Ray to monitor more accurately students' learning outcomes, and to make better informed decisions about students' conceptual readiness for new learning activities:

> The researcher [sic] was better informed about the readiness of the class to continue with further instruction on new content.
>
> (Fleming, 1988, p. 57)

Assertion Ten (in Chapter Eight) argues, however, that Ray's technical curriculum priority interests prevailed and that Ray's perception of the need to *cover* the syllabus compromised his belief in the need to allow students more time to complete learning activities and develop their conceptual understandings.

Finally, Ray's use of the blackboard for whole-class presentations became more organised and systematic as a result of his reductionist pedagogical focus, and enabled students to record important information, rather than relying only on their memory of the teacher's verbal explanations. For example, at the commencement of Lesson 10, Ray drew a *knowledge-utilisation map* on part of the board and, while explaining its significance to the class, he recorded a set of procedures for its implementation on an adjacent part of the board (Fieldnotes 2 June 1987, p. 2). On another occasion, Ray recorded a sequence of *Linear Programming Rules* on the board prior to setting a small-group, problem-solving activity (Fieldnotes 15 June 1987, p. 5). Students seemed to have little difficulty in transcribing accurately Ray's notes into their files.

Student Worksheets. Ray applied reductionist principles also to the design of student worksheets. He spent many hours designing worksheets for use in class. The worksheets provided carefully sequenced instructions and seatwork exercises. The main design criterion of the worksheets was the organisation of lesson content into a series of tasks, each of which could be completed independently by students in a relatively short time:

> The purpose of these worksheets was to provide students with a step-by-step approach to problem solving which would encourage them to

> think more clearly, logically, and sequentially, and prevent students from becoming confused about what to do next. (Fleming, 1988, p. 57)

For example, the Grade 12 student worksheets *Graphing Quadratic Functions* and *Graphing Any Quadratic Function* comprised carefully sequenced exercises, commencing with sketching graphs in the form $y = ax^2$, and ending with sketching graphs of quadratic functions expressed in the form $y = (x-h)^2 + k$ (Fieldnotes: 19 May 1987, p. 6; 22 May, 1987, p. 7).

The classroom implementation of student worksheets provided opportunities for Ray to move around the class and monitor students' attempts to understand the *smaller steps* in the development of a theory or the solution of a problem:

> By leading them through it in smaller steps, you can follow their progress more readily and see when they go wrong and see what they have difficulties with. Whereas, when you present the whole thing . . . if they can't do it at the end [then] you don't know; it's not as easy to identify where things have gone wrong in their understanding.
>
> (Interview 1, June 1987, p. 8)
>
> The worksheets also had the advantage of enabling the students and teacher to readily identify specific points of difficulty.
>
> (Fleming, 1988, p. 57)

The reductionist nature of the worksheets enabled Ray to identify more readily students' learning difficulties and to provide tutorial assistance to individual students. Ray reported that his role during individual seatwork activities was:

> [T]o identify difficulties and, maybe, talk them through with the student in a little bit more detail than would be possible . . . during the whole class engagement. (Interview 2, August 1987, p. 7)

Ray's pedagogical innovations were designed in accordance with his knowledge-transformation model (see Figure 1) which defined his main classroom role as the primary source of students' new mathematical knowledge. This model sustained Ray's transmissionist epistemology and his centralist classroom role throughout the collaborative phase of the study. Although Ray's constructivist-related pedagogical innovations — inquiry-oriented discourse, reductionist design principles — were designed to facilitate students' prior-knowledge-related construction of their mathematical knowledge, their main purpose was to enhance Ray's transmission of his mathematical knowledge to the class. Although Ray's classroom role appeared to become much more decentralised, he maintained the locus of instructional control by means of his worksheets which were designed to enable students to reproduce more effectively Ray's own mathematical knowledge.

Nevertheless, the increased amount of individual seatwork activity during class provided enhanced opportunities for students to: (1) focus their attention on evaluating the viability of their antecedent mathematical knowledge, (2) discuss their ideas and learning difficulties with fellow students and the teacher, and (3) exercise more control over the rate of generation of their new mathematical knowledge.

Perceptions of Effectiveness

The constructivist-related goals of this study, which are discussed in Chapter Three, require the effectiveness of Ray's pedagogical innovations to be assessed in relation to the extent to which they enhanced students' mathematical conceptual development.

In his report on the study, Ray asserted that the constructivist-related changes which he had made to his teaching had been effective for enhancing students' *acquisition* of mathematical knowledge (Fleming, 1988, pp. 60-61). However, his assertion is supported by evidence which warrants a lesser assertion that: (1) Ray's pedagogical innovations enabled him to generate a better understanding of students' shorter-term learning outcomes, and to design teaching strategies which were more informed about students' prior-knowledge-related learning difficulties; and (2) students' held favourable perceptions of the impact on their learning of the pedagogical innovations. The lesser assertion is warranted by evidence which acknowledges the perceived effectiveness of Ray's newly adopted constructivist-related role of *teacher as learner* and his refined centralist classroom role of *teacher as interactive informer*.

Ray's Perspective

Ray perceived his pedagogical innovations to have had a positive impact on both himself and the students. He perceived an improvement in students' attitudes towards his teaching, and believed that he had developed a more productive relationship with students. He experienced greater confidence in his own teaching ability, and had become more enthusiastic about the prospects of teaching more effectively:

> - Students seemed better motivated, more enthusiastic about maths and more optimistic about their own progress.
> - Lessons seemed to run more smoothly and successfully.
> - The researcher felt more *in touch* with the students' mathematical development and more confident in his own ability to present mathematical content in a way that students could understand.
> - The researcher developed a new-found enthusiasm for teaching as a result of his new (constructivist) perspective of the teaching/ learning process. (Fleming, 1988, p. 61)

Ray believed that the most significant of his pedagogical innovations was his inquiry-oriented classroom discourse which enabled him to generate a better understanding of students' understandings, and to provide more relevant information about new concepts:

> I think the most significant thing . . . is that I'm increasingly listening to the students' answers and looking for answers which really indicate their understanding, rather than just looking for an answer that indicates they've got something right or they haven't got it right. So, it would be the quality of the questioning and, consequently, the information available from the answers, and then what to do about

> that. . . [W]hat I really wasn't doing [before] was investigating, or delving, into their specific difficulties or their specific concepts. . . . Rather, [I was] just seeing: 'Right, they can't do it'; [and then saying] 'I'll tell them how to do it!' (Fleming, 1988, p. 62)

Ray believed also that his specially designed student worksheets provided opportunities which enabled him to diagnose more readily students' learning difficulties, and to intervene at strategic stages in the development of students' understandings:

> I've felt that it was quite effective on most occasions. There were one or two that didn't quite work out the way I had thought but, still, that's not necessarily a bad thing. . . . You can see the kids doing stuff bit by bit, taking smaller steps. (Interview 1, June 1987, p. 8)

Students' Perceptions

The lesser assertion about the effectiveness of Ray's constructivist-related pedagogical innovations also is consistent with evidence of students' positive perceptions of changes in the classroom learning environment which had occurred during the collaborative phase of the study.

I conducted interviews with almost all of the 24 lower-achieving students in Ray's Grade 12 class; only the three highest achievers intentionally were excluded (see Appendix C). The results indicate that changes made to Ray's teaching were perceived to be beneficial by a large majority of these students. In particular, three-quarters of the students claimed consequential improvements in their ability to understand mathematics, and half indicated increased confidence in their ability to understand mathematics. Only one student reported a negative impact of the changes. The most popular changes perceived by students included a more positive teacher attitude and more helpful instruction. Many students cited Ray's specially designed instructional worksheets as being beneficial.

Students' Conceptual Development.

However, evidence of the effectiveness of Ray's pedagogical innovations for enhancing students' conceptual development is overwhelmingly negative, especially in relation to the lower-achieving majority of students who constituted the main focus of the collaborative phase of the study.

There is evidence that, in general, students' learning outcomes were enhanced by Ray's pedagogical innovations. In his report on the study, Ray referred to a positive trend in students' recent topic test results as evidence of the effectiveness of his pedagogical innovations. Prior to the collaborative phase of the study, the class mean scores on topic tests one and two were 35% and 55%, respectively. Since the commencement of the study, class mean scores on tests three, four, and five were 65%, 66%, and 64%, respectively (Fleming, 1988, p. 67). It could be argued that these results demonstrate a general improvement in students' learning outcomes. However, as Ray (Fleming, 1988) points out, one must be aware of the "pitfalls of drawing conclusions from these results (namely, because of different difficulty levels of *topics* and of [test] *items*)" (p. 67).

A favourable interpretation of the topic test results is that they indicate that students had become more able to recall and manipulate successfully mathematical algorithms in order to solve single-answer exercises. This assertion is strengthened by Assertion Eight (in Chapter Six) which argues that Ray's formalist conception of the nature of mathematics resulted in a major pedagogical emphasis on the recall and manipulation of mathematical algorithms in accordance with predetermined deductive rules of logic. The improvements in students' learning outcomes indicated by the topic test results are likely to have been improvements in algorithmic ability, rather than in conceptual understanding.

For example, an analysis of the semester one school-based examination, Inequalities and Linear Programming (see Fieldnotes 26 June 1987, p. 6), suggests that the emphasis of the test was on measuring students' *algorithmic ability*, that is, their ability to recall and apply procedural rules to single-answer problems (see Reflections 26 June 1987, p. 1). However, the class mean score of 56% with a standard deviation of 17% suggests that students' algorithmic ability had not improved significantly as a result of Ray's pedagogical innovations. Only four students achieved a score of greater than 70% (see Reflections 26 June 1987, p. 2).

A more meaningful measurement of students' learning outcomes was obtained towards the end of the collaborative phase of the study. I designed an assignment on the recently completed topic of Linear Programming for the purpose of evaluating students' conceptual understandings (Fieldnotes 22 June 1987, p. 14). The analysis of the results indicates widespread conceptual difficulties amongst a majority of the class, and criticises the predominantly algorithmic nature of Ray's teaching which had failed to promote conceptual development (Reflections 30 June 1987, pp. 1-5). The results indicate that the constructivist-related goal of teaching for conceptual development had not been achieved for a significant proportion of the lower-achieving majority of students whose earlier conceptual difficulties are discussed in Assertion Three (in Chapter Four).

Consequently, it can be concluded that the evidence presented in this section warrants only the lesser assertion about the perceived effectiveness of Ray's refined centralist classroom role. This conclusion is further substantiated by Assertions Ten and Eleven (in Chapter Eight) which argue that Ray compromised his constructivist-related ideals, and did not allow students' adequate opportunities to control their classroom learning activities in the interest of learning with understanding.

SUMMARY

This chapter presents an interpretive analysis of Ray's major pedagogical innovations, and assesses the extent to which he transformed his centralist classroom role. In particular, the chapter identifies the constraining role of Ray's epistemological beliefs which limited the nature and scope of his constructivist-related pedagogical reforms.

A major constructivist-related change in Ray's pedagogy was his adoption of the role of *teacher as learner* which resulted in: (1) the development of a critical awareness that his teaching was failing to resolve widespread conceptual difficulties amongst the class; and (2) the development of a constructivist perspective on students' prior-knowledge-related construction of new mathematical knowledge. Ray developed a more interactive and inquiry-oriented teaching approach which was designed to enable him to generate a better understanding of students' extant mathematical knowledge and shorter-term learning outcomes. Ray's major pedagogical innovations included an inquiry-oriented classroom discourse with which he probed students' understandings, and longer periods of individual seatwork activity which allowed him to focus more closely on individual student learning.

However, Ray's constructivist-related pedagogical innovations resulted in a refinement, rather than a radical transformation, of his centralist classroom role. Ray's transmissionist epistemology subsumed his belief in the first principle of constructivism, and maintained his role as the primary source of students' new mathematical knowledge. His innovative inquiry-oriented classroom discourse and reductionist instructional strategies were designed largely to facilitate students' prior-knowledge-related interpretations of Ray's explications of his own mathematical knowledge. Chapter Six argues that Ray's *knowledge transformation* model reflects an objectivist epistemology which is attributable to his *realist* conception of the nature of mathematics.

Ray's refined *teacher as interactive informer* classroom role was perceived by most of the lower-achieving students to be a significant improvement which had had a beneficial impact on their attitudes and, to a somewhat lesser extent, on their abilities to understand. Also, Ray believed that, as a result of the study, his teaching had improved significantly. Nevertheless, measures of students' conceptual understandings, which were obtained towards the end of the collaborative phase of the study, indicate ongoing and widespread conceptual difficulties amongst the class. Assertion Eight (in Chapter Six) provides evidence to support the assertion that the discrepancy between teacher and students' favourable perceptions of the pedagogical innovations and students' unfavourable learning outcomes is a consequence of Ray's *formalist* conception of the nature of mathematics which maintained a major pedagogical focus on students' algorithmic ability.

SECTION THREE

CONSTRAINTS TO CONSTRUCTIVIST PEDAGOGICAL REFORM: MULTIPLE PERSPECTIVES

Section Three of the thesis, which comprises Chapters Six, Seven, and Eight, addresses both the initial research question of the thesis (see Section Two) and an interpretive research question which emerged during the collaborative phase of the study, namely: *To what extent did the cognitivist theory of constructivism used in this thesis constrain the process of pedagogical reform?* This section adopts multiple constructivist-related interpretive frameworks in order (1) to generate a better understanding of the nature of Ray's role-determining pedagogical preconceptions, and (2) to examine the efficacy of the cognitive constructivist perspective which precipitated Ray's pedagogical innovations. Interpretive analyses are presented of additional data which were obtained during the post-collaborative phase of the study in Ray's Grade 9 mathematics class.

Ray's beliefs in the objectivist nature of mathematics are assessed from a philosophy of mathematics perspective, and their subsumption of the first principle of constructivism is examined from a radical constructivist perspective on learning mathematics (Chapter Six). Next, Ray's rationalist beliefs in the nature of mathematical cognition are examined from a social constructivist perspective, and the narrow rationalist focus of the cognitivist theory of constructivism is highlighted (Chapter Seven). Finally, a critical constructivist perspective is employed to examine the constraining role of Ray's curriculum-related pedagogical beliefs, especially the beliefs which maintained his centralist classroom role of *teacher as controller* (Chapter Eight).

CHAPTER SIX

A RADICAL CONSTRUCTIVIST PERSPECTIVE ON RAY'S RATIONALIST PRECONCEPTIONS OF MATHEMATICS

INTRODUCTION

During the collaborative phase of the study, a constructivist-related theory of pedagogical reform was enacted in my daily discourse with Ray (see Chapters Two and Three). Assertions Five and Six (see Chapter Five) indicate the constraining influence of Ray's established pedagogical beliefs on the nature and scope of his constructivist-related pedagogical innovations.

Chapter Six assesses the extent to which the theory of pedagogical reform, which was based on the first principle of constructivism, also contributed to the restricted transformation of Ray's centralist classroom role. In particular, this chapter examines the assimilation of the first principle of constructivism by Ray's established epistemology, and the role of his objectivist preconceptions of the nature of mathematics. The results of interpretive data analyses (see Chapter One) are reported in the form of Assertions Seven and Eight.

The first section constitutes the interpretive framework of the chapter, and presents a review of research literature in the field of the philosophy of mathematics. Two outmoded foundational philosophies of mathematics which have had a major impact on modern school curricula — *realism* and *formalism* — are discussed, and their pedagogical implications are outlined. A modern countervailing philosophy of mathematical cognition — *radical constructivism* — also is reviewed, and its pedagogical implications are discussed in relation to recently established research and teaching programs in school mathematics.

The second section extends the interpretive analysis of Ray's reformed pedagogy (see Chapter Five) and, from a radical constructivist perspective, examines the metaphor of *cognition as information processing* which I employed in my constructivist discourse with Ray during the collaborative phase of the study. The question of the extent to which this metaphor served to reaffirm Ray's transmissionist epistemology is addressed by examining the relationship between Ray's knowledge transformation model and his objectivist preconception of the realist nature of mathematics.

The third section comprises an interpretive analysis of Ray's post-collaborative pedagogy. The analysis adopts a radical constructivist perspective, and examines Ray's pedagogical implementation of the prior knowledge corollary of the first principle of constructivism. The question of the extent to which Ray's formalist preconception of the nature of mathematics subsumed his newly developed prior-knowledge-related belief is addressed with reference to his objectivist preconception of *the syllabus as a conceptual map* (see Assertion One in Chapter Four).

AN INTERPRETIVE FRAMEWORK

The interpretive framework of this chapter commences with a review of research literature which argues that outdated foundationalist philosophies of mathematics underpin modern pedagogical models of university and school mathematics. In particular, the review discusses the problematic pedagogical implications of objectivist epistemologies which are associated with realist and formalist conceptions of mathematics. The non-foundational metaphysical principle of radical constructivism, which is congruent with the first principle of constructivism that was employed in this study (see Chapter One), and which underpins recent pedagogical models of school mathematics, is discussed as an alternative epistemology of mathematical cognition.

According to modern philosophers of mathematics (Codd, 1988; Davis & Hersch, 1981, 1986; Kitcher, 1984), *rationalist* philosophers, including the historical figures of Plato, Spinoza, Descartes, Leibniz, and Kant, believe that human *Reason* is an innate trait which acts independently of sensory perception and enables the human mind to perceive a priori truths that are independent of the world and of all human experience:

> Reason, as the term is used in the Platonic and Kantian traditions, is interlocked with the notions of truth as correspondence, of knowledge as discovery of essence. (Rorty, 1982, p. 172)

Rationalist conceptions of mathematics are associated with historical and philosophical attempts to establish the absolute certainty of the foundations of mathematics. Two of the most popular foundationalist philosophies of mathematics which continue to exert a strong influence on both mathematicians and the pedagogy of modern school mathematics are Platonism and formalism (Hersch, 1986).

A Realist Perspective on Mathematics

Davis and Hersch (1981) describe a Platonist, or realist, view as a "belief in the existence of ideal entities, independent of or prior to human consciousness" (p. 331). A realist perspective on mathematics elevates mathematical reason to the status of providing a *God's-eye* view of the universe of *pure forms,* which include geometrical shapes and numbers. The aim of the rational mind, therefore, is to transcend the misleading evidence of the senses and to perceive the *natural* world of mathematical objects. Davis (1986) outlines a set of objectivist beliefs which constitute a Platonist perspective on the nature of the mathematical realm which resides outside of human culture and history:

> 1. The belief in the existence of certain ideal mathematical entities such as the real number system.
> 2. The belief in certain modes of deduction.
> 3. The belief that if a mathematical statement makes sense, then it can be proven to be true or false.
> 4. The belief that fundamentally, mathematics exists apart from the human beings that do mathematics. Pi is in the sky.
>
> (Davis, 1986, p. 165)

In the nineteenth century, Platonism was manifested in the myth that (Euclidian) geometry provided the only true knowledge of physical space, and enabled science to 'reveal' the apparently natural laws of the Newtonian clockwork universe, especially the (fictitious) force of gravity (Kline, 1953). Mathematical knowledge was regarded as being objectively true for everyone because it was a priori, that is, pre-existing, ahistorical, and independent of experience:

> Euclidian geometry served as the supreme exemplar of the possibility of certainty in human knowledge. (Davis & Hersch, 1981, p. 331)

During the late nineteenth century, however, the invention of non-Euclidean geometries — *space-filling curves* and *continuous nowhere differentiable curves* (Hersch, 1986, p. 15) — undermined the Platonist view of mathematics. It was realised that mathematical theories of physical space, or *mathematical spaces*, are subjective constructions, rather than objective descriptions, which derive from experience and which constitute theoretical frames of reference for the scientific examination of physical space (Kline, 1953). In particular, it was realised that the axioms which underpin the geometrical theorems (e.g., the concepts of a straight line as a stretched string on a flat surface, or a great circle on a spherical surface, or a ray of light in interplanetary space) are subjective constructs that are embedded in human experience and embodied in cultural conventions.

Nevertheless, the Platonist myth continues to remain a popular philosophy in modern mathematics. Hersch (1986) and Davis and Hersch (1981) claim that Platonism is the underpinning, albeit covert, philosophy of most mathematicians, despite the ascendency of empiricist epistemologies amongst the scientific community:

> The basis of Platonism is the awareness we all have that the problems and concepts of mathematics exist independently of us as individuals. . . . [T]he myth of Platonism . . . remains alive because it corresponds to something real in the daily experience of the mathematician.
>
> (Hersch, 1986, p. 18)

The Platonist myth of mathematics remains viable because it explains the compelling common perception that mathematics is independent of the minds of knowers, that it has an ontological reality, or is *practically real* (Goodman, 1986). Nevertheless, Platonism fails to provide a plausible account of how mathematicians obtain access to knowledge in the platonic realm, and does not provide an adequate account of the internal dynamics of mathematics or its practical utility:

> [P]latonism fails to account adequately for the utility of mathematics, its relations with science, human activity or culture, and the genesis of knowledge. (Ernest, 1991, p. 30)

Pedagogical Implications of Realism

The main pedagogical implication of Platonism is the promotion by teachers of a realist perspective on the nature of mathematics. This perspective is constituted by an objectivist belief in (1) the unquestioned certainty of mathematics, and (2) the independence of mathematical objects in relation to students' prior experiences. The realist myth promotes an image of the separateness of mathematics and mathematical cognition, and underpins pedagogical assumptions that disembodied mathematical facts are knowable by the disciplined individual cognitive exercise of reason. Platonist philosophy is expressed in the realist language of mathematics teachers who talk about the *facts* of arithmetic, the properties of geometrical shapes, and the laws of the number system. For example, the Pythagorean theorem and the prime factorisation theorem often are presented as *true* statements about right triangles and integers, respectively (Davis & Hersch, 1981).

A Formalist Perspective on Mathematics

An alternative rationalist view of mathematics became predominant at the beginning of the twentieth century, in response to the *crisis in foundations* of mathematics. The aim of the formalists was to re-establish the absolute certainty of mathematics that had been lost in the demythologising of Platonism, especially in response to the discovery of non-Euclidean geometries (Davis & Hersch, 1981; Hersch, 1986; Lakoff, 1987). Formalism became the established alternative to Platonism.

Formalism abandoned the notion of mathematics as comprising self-evident truths or descriptions of reality and, in essence, viewed mathematics as the science of rigorous proof. Formalism characterises mathematics as the application of formal mathematical logic for the purpose of deducing *content-free* theorems which await non-mathematical or scientific interpretation in relation to the world of human experiences. Philosophers of mathematics explain the essence of Formalism:

> [T]he formalist interpretation tried to make [mathematics] safe by turning it into a meaningless game.The *proof-theoretic program* comes into action only after mathematics has been coded in a formal language and its proofs written in a way checkable by machine. As to the *meaning* of the symbols, that becomes something extra-mathematical. (Hersch, 1986, p. 16)

> In popular terms, formalism is the view that mathematics is a meaningless formal game played with marks on paper, following rules. . . . Hilbert's formalist programme aimed to translate mathematics into uninterpreted formal systems. (Ernest, 1991, p. 10)

In other words, formalism views mathematics as a meaningless game of logical deduction that utilises symbols and strings of symbols which, in themselves, have no mathematical meaning. Meaning for symbols and theorems is available, but only from *metamathematical,* or *extra-mathematical,* interpretations. Lakoff (1987), whose critique of the misappropriation of mathematical logic to justify objectivist approaches to linguistics and the philosophy of language, argues that formalism separates the syntax of the formal *language* of mathematics, which is viewed by objectivist philosophy as a natural language — rather than an artificial human construct — from the semantics of the language which gives meaning to the uninterpreted symbols of the syntax. The impoverished view of mathematical thinking which results from a formalist perspective — a concern only for the construction of formal mathematical proofs — is criticised by Lakoff (1987) who argues that formalism is buttressed by an objectivist philosophy, and incorrectly equates formal mathematical logic with human reasoning, rather than with a "limited form of reasoning used by mathematicians to construct mathematical proofs" (p. 224).

However, the formalist view of mathematics has lost credibility amongst mathematicians and philosophers of mathematics, especially since Godel's Incompleteness Theorem (Goodman, 1986; Hersch, 1986). The formalist view of mathematics also has begun to lose favour because formalism is not perceived to be a comprehensive description of mathematical knowledge, especially the mental activities of mathematicians when they are engaged in working with mathematical ideas. Hersch (1986) draws an analogy with music to highlight the inadequacy of mathematical formalism as a comprehensive representation of mathematical thinking. The lack of correspondence between the written symbols of a musical score and the musical thoughts of a composer is analogous to the lack of correspondence between the symbol system of formalism and the thoughts of mathematicians. Mathematical thinking involves *intuitive reasoning* which cannot be formalised. Intuition is plagued with uncertainties and, therefore, mathematical thinking also is uncertain, fallible, tentative, and evolving.

Pedagogical Implications of Formalism

Davis and Hersch (1981) and Hersch (1986) claim that there are two main pedagogical implications of a formalist view of mathematics. First, there is a central concern for formal proof and the transformation of symbols — formalist textbooks identify mathematics in terms of axiomatic presentations. Second, there is a disregard for interpretation, which is represented by pictures, diagrams, mental imagery, and applications, and for the reasons why a particular problem might be interesting. The dominant style of presentation of mathematics as *definitions and proofs* in mathematics journals and textbooks, and the importation of *set-theoretic notation* and *axiomatics* into high school curriculum during the 1960s, are evidence of the contemporary dominance of formalist conceptions of mathematics. In universities, a common measure of mathematical achievement is the explication of proofs. At the high school level, the formalist style is manifested in a fusion of set theory, algebra, and analysis, which is known as the *new maths*; the dominant pedagogical concern is for reproduction rather than for the conceptualisations of students.

In Chapter One, one of the major problems of traditional school mathematics was identified in numerous reports on the state of mathematics education to be the curriculum requirement that students undertake continuously sets of single-answer practice exercises which require only algorithmic manipulation for their solution.

From a semiotic perspective (Lemke, 1985, 1990; Solomon, 1988), the textual content of the syllabus, textbook, or teacher's blackboard notes are codified signs which signify, but do not bear a one-to-one correspondence with, the mathematical conceptualisations that are located in the minds of the authors. A pedagogical concern only with students' *algorithmic ability*, that is, their accurate manipulation of these signs (e.g., solving algebraic equations, transforming algebraic expressions), therefore, fails to address the interpretations and meanings which students attribute to their mathematical activities. Furthermore, a false sense of the certainty of mathematics obtains from the correspondence between students' answers and those of the textbook.

Platonism and formalism were derived in attempts to attain certainty in mathematical knowledge which was regarded to be a priori and objective. On the one hand, Platonism evokes an abstract, objective reality which transcends human existence, and which lies waiting to be discovered by means of pure reason, rather than by empirical experiences. On the other hand, formalism evokes a world of mathematical objects in the form of meaningless symbols whose manipulation is subject to a priori rules of deductive logic which are equated with human reasoning. Both perspectives represent objectivist epistemologies which conceive of mathematical objects as existing independently of human thought. Platonism and formalism maintain a powerful symbiotic relationship in the minds of contemporary mathematicians:

> [T]he typical *working mathematician* is a Platonist on weekdays and a formalist on Sundays. That is, when he is doing mathematics, he is convinced that he is dealing with an objective reality whose properties he is attempting to determine. But then, when challenged to give a philosophical account of that reality, he finds it easiest to pretend that he does not believe in it at all. (Hersch, 1986, p. 11)

A Radical Constructivist Perspective

In addition to the first principle of constructivism, which states that knowledge is not passively received but is constructed actively in the mind of the learner on the basis of the learner's extant conceptual framework (see Chapter One), Ernst von Glasersfeld's (1981, 1984, 1986a, 1986b, 1987, 1988, 1989, 1990a, 1990b, 1991, 1992, in press) *radical constructivism* propounds a metaphysical principle which is based on Piaget's biological model of genetic epistemology and its concern for the epistemic being's conceptual *fitness*, or adaptive *viability*. The metaphysical principle of radical constructivism eschews traditional notions of Truth which are associated with objectivist epistemologies:

> The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability; cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1990b, p. 23)

> Radical Constructivism . . . is a theory of *active knowing*, rather than a traditional theory of knowledge or epistemology. From this standpoint, as Piaget maintained fifty years ago, *knowledge serves to organize experience*, not to depict or represent an experiencer-independent reality. (von Glasersfeld, 1991, p. xix)

A radical constructivist perspective counters the accusation that it is little more than *subjectivism*, or *solipsistic idealism* — that reality is an idiosyncratic personal construct — by acknowledging both the existence of an objective reality and the intersubjective role of communication in negotiating relatively stable interpretations of the nature of reality. However, while acknowledging the existence of objective reality, a radical constructivist *theory of knowing* (von Glasersfeld, 1990b, 1991) claims that an individual has no access to the world other than through their senses and their conceptually-laden perspectives. Knowing, or cognition, involves the *abstracting of regularities* (von Glasersfeld, 1990b, p. 24) from experience, and an evaluation of their viability for rendering experience personally sensible, or meaningful. Tobin (1990) explains this subjective epistemic process in terms of the cognising being's adaptation to the constraints of objective reality:

> Experience involves an interaction of an individual with events, objects or phenomena in the universe; an interaction of the senses with reality. The result of the interaction is an image of reality, a personal construction which fits the external reality but is not a match. The senses are not conduits to the external world through which truths are conducted into the body. Because the senses of humans are embodied, all experiences are subjective. Objectivity, which is an assumed part of epistemologies such as logical positivism, is not possible for cognizing beings. (Tobin, 1990, p. 30)

The metaphysical principle of radical constructivism has important implications for the conceptualisation of the disciplines of science and mathematics, and for the claims to objectivity, certainty, and truth which are made by their proponents. The apparent objective reality of mathematics, which is experienced as the compelling certainty of mathematical *facts* such as the *real* number system, is explained from a radical constructivist perspective as the experience of *consensual domains*, or socially-determined conventions, which represent the (cognitive) activities of mathematicians, teachers, and other authority figures who practice mathematics:

> The certainty of mathematical *facts* springs from mathematicians' observance of agreed-on ways of operating, not from the nature of an objective universe. (von Glasersfeld, 1991, p. xvi)

The overwhelming experience of certainty in the consensual domain of mathematics is based on experience of an intricate and extensive network of non-arbitrary conceptual relations which constitute the rules of the *rational game* of mathematics and which, at less abstract levels such as counting, have a meaningful connection with everyday experience. The internal consistency of the game and its apparent reflection of experiential reality, however, do not constitute evidence of the ontological status of mathematics (von Glasersfeld, 1990b, p. 25).

A radical constructivist perspective regards as antithetical objectivist conceptions of the nature of mathematics. In particular, the metaphysical principle of constructivism rejects: (1) the Platonist perspective, or *metaphysical myth* (von Glasersfeld, 1990b, p. 20), that mathematics comprises transcendental truths about an objective universe, and that mathematicians discover ahistorical and depersonalized characteristics of an objective reality by transcending rationally their everyday experiences; and (2) the formalist perspective which objectifies mathematics by positing an uninterpreted syntax of formalist mathematical *language* which is independent of semantics, or human meaning:

> The alternative of Platonism and formalism comes from the attempt to root mathematics in some nonhuman reality. If we give up the obligation to establish mathematics as a source of indubitable truths, we can accept its nature as a certain kind of human mental activity. . . . Human beings live in the world and all their ideas ultimately come from the world in which they live — refracted through their culture and history, which are in turn, of course, ultimately rooted in man's biological nature and his physical surroundings. Our mathematical ideas fit the world for the same reason that our lungs are suited to the atmosphere of this planet. (Hersch, 1978, pp. 21-23)

Pedagogical Implications of Radical Constructivism

The central pedagogical focus of radical constructivism is a concern for *understanding*, or *consensual fit*, rather than for observable competent performance. Von Glasersfeld (1990b) contrasts the cognitive interest of radical-constructivist-oriented teaching with the behaviourist-oriented interest of training:

> [Teaching] aims at the students' conceptual fit with the consensual domain of the particular field, a fit which, from the teacher's perspective, constitutes understanding. [Training] aims at the students' behavioural fit which, from the teachers' perspective, constitutes acceptable performance . . . [I]nstruction that focuses on performance alone can be no better than trivial. (von Glasersfeld, 1990b, p. 26)

In mathematics education, pedagogical models based on radical constructivism (Cobb, 1985b, 1986a, 1987; Wheatley, 1990) identify knowledge as a subjective sense-making activity located in learners' minds, and focus on the *fitness* of learners' mathematical concepts for making sense of new and problematic experiences. These models eschew objectivist epistemologies which equate mathematical knowledge with the truth of an objective reality, locate new knowledge in sources external to the learner, and focus on conveying that knowledge into learners' minds.

Objectivist assumptions that certain knowledge can be transmitted in the speech of teachers underpin transmissionist pedagogies. However, a radical constructivist perspective views linguistic communication as inherently and inescapably indeterminate (von Glasersfeld, 1989, 1990b). Meaning is not conveyed in speech patterns or words, but is constructed actively by the listener on the basis of their extant conceptual frameworks. Understanding another person's speech is a subjective and interpretive process that is subject to ambiguity, especially when a conversation focuses on abstract matters:

> [T]he physical signals that pass between communicators — for instance the sounds of speech or the visual patterns of print in linguistic communication — do not carry what is ordinarily considered meaning. . . . [T]o understand what someone has said or written means to have built up a conceptual structure that, in the given context, appears to be compatible with the structure the speaker had in mind. This compatibility, as a rule, manifests itself in no other way than that

> the receiver says and does nothing that contravenes the speaker's expectations. (von Glasersfeld, 1990a, pp. 35, 36)

A radical constructivist view of linguistic communications helps to explain why teacher pronouncements of the wrongness of students' mathematical ideas, and the reiteration of clarifying explanations, are likely to be ineffective for many students. Students are able to interpret teacher communications only from within their extant conceptual frameworks. Radical constructivism requires teachers to attempt to infer the nature of students' conceptual structures and operations (Confrey, 1991; Steffe, 1991), and provide learning activities that facilitate their purposeful reconstruction, rather than assuming that *clearer* explanations of the teacher's conceptual structures will enable students more readily to *see* the teacher's intended meaning.

A formalist perspective on mathematics is congruent with the behaviourist-oriented interest of training which ignores students' unobservable conceptual operations and focuses primarily on students' observable manipulation of the symbols of operations. From a semiotic perspective (see Assertion One), Cobb (1987) argues that a formalist view of mathematics — a system of syntactical rules for manipulating symbols which are regarded as mathematical objects — ignores the issue of students' beliefs and motivations, and focuses primarily on the observable aspects of students' mathematical behaviour:

> The view of mathematics as a formal domain and the concomitant focus on signifiers rather than signifieds bear the hallmarks of the observer's perspective. The adult learner can experience conventional mathematical symbols as concrete things-in-themselves when he or she has constructed the relevant mathematical objects and is completing a routine task. (Cobb, 1987, p. 11)

In other words, the formalist teacher focuses on symbol manipulation, operates as though students already are in possession of the meanings associated with the symbols, and assumes that the meaning of the symbols is transparent to students. However, from the students' perspective the symbols appear often to be meaningless. A formalist view of mathematics underpins the much criticised contemporary mathematics education pedagogy which comprises teacher explanations of definitions, theorems, and proofs and subsequent student practice. Cobb (1987) associates this type of pedagogy with Skemp's (1976) *instrumental* learning, and empiricist and behaviourist traditions of psychology.

By contrast, a radical constructivist pedagogy identifies students as inquiry-oriented problem solvers whose mathematical objects of reflection are the *objectified concepts* which comprise students' own cognitive environments (Cobb, 1985b, 1986a; 1987; Wheatley, 1990). From this perspective, learning is an inherently subjective process of reflecting on the adequacy of existing knowledge structures, restructuring in order to neutralise cognitive perturbations, and determining the viability of the new knowledge structures:

> [A]ll learning is necessarily problem solving. . . . And the [cognitive] system solves its problems not by peeking around its concepts and *seeing* what is *really there.* Instead, it does so by reorganizing its own sensory-motor and conceptual activity. . . . This is the primary reason why radical constructivists focus on students' activities rather than on abstract relationships that they can *see* in students' environments as sources of knowledge. It is also why they believe that constructs such as reflective abstraction are essential to the explanation of learning.
>
> (Cobb, 1987, p. 29)

In particular, the metaphysical principle of radical constructivism highlights the *adaptive function* of cognition in relation to the experiential world of the individual student, and emphasises students' personal sense-making, or interpretive, activities. Radical constructivism requires teachers to abandon objectivist notions of mathematics and transmissionist epistemologies, adopt students' conceptual frameworks as their main pedagogical referents, and aim at facilitating students' development of *autonomous conceptual understanding* :

> The task of education, then, can no longer be seen as a task of conveying ready-made pieces of knowledge to students, nor, in mathematics education, of opening their eyes to an absolute mathematical reality that pervades the objective environment like a crystalline structure independent of any mathematician's mental operations. Instead, it becomes a task of first inferring models of the students' conceptual constructs and then generating hypotheses as to how the students could be given the opportunity to modify their structures so that they lead to mathematical actions that might be considered compatible with the instructor's expectations and goals.
>
> (von Glasersfeld, 1990a, pp, 33-34)

The *teacher as learner* metaphor connotes an appropriate classroom role for teachers who wish to begin the process of implementing radical constructivist theories of knowing in their mathematics classes by decentralising their pedagogical frames of reference. In this study, Ray's constructivist perspective resulted in his adoption of a *teacher as learner* classroom role (see Assertion Five in Chapter Five). However, the first principle of constructivism failed to provide Ray with a rationale for abandoning his objectivist conception of the nature of mathematics. The next two sections discuss the problems encountered in this study of facilitating radical pedagogical reform in the face of realist and formalist conceptions of the nature of mathematics.

THE EPISTEMOLOGICAL PROBLEM OF REALISM

During the collaborative phase of the study, I incorporated the metaphor of *cognition as information processing* into my discourse with Ray in order to facilitate his conceptualisation of the prior knowledge corollary of the first principle of constructivism, namely, that learning is a process of conceptual development which is related intimately to learners' existing conceptual frameworks (see Chapter Three). The metaphor was adapted from information-processing models which were popular amongst educational researchers who were working towards conceptualising science education (Champagne, Klopfer, & Gunstone, 1982; Driver & Erickson, 1983; Osborne & Wittrock, 1983, 1985; Posner, Strike, Hewson, & Gertzog, 1982) and mathematics education (Putnam, Lampert, & Peterson, 1990; Resnick, 1980, 1983) from constructivist perspectives. This section presents evidence which warrants the following assertion about the problematic impact of the metaphor on Ray's pedagogical reforms:

Assertion Seven

The metaphor of *cognition as information processing,* which highlights the prior knowledge corollary of the first principle of constructivism, was congruent with Ray's preconception of the realist nature of mathematics, and served to reaffirm his objectivist epistemology and centralist classroom role of *teacher as interactive informer.*

The Case Against Information Processing Models

In a review of the implications of information-processing psychology for mathematics education Cobb (1987) argues, from a radical constructivist perspective, that objectivist assumptions underpin the main information-processing models of human cognition which he identifies as (1) *production systems* models, and (2) *schema theorists* models. He argues that production systems models of information-processing represent learning as a process of encoding information which has been *deposited* into students' working memories.

In these models, the teacher's knowledge is coded and transmitted by means of the media of speech and visual imagery. This epistemological perspective is consonant with the role of the teacher as direct source of student knowledge, and invokes a realist view of mathematical objects as elements of a mind-independent and pre-existing mathematical reality which the teacher helps students to *see* by providing informative explanations (Cobb, 1987).

Cobb's (1987) critique of information-processing models of human cognition recommends a constructivist pedagogy which addresses the student's (or *actor's*) mathematical perspective, rather than the teacher's (or *observer's*) perspective. This shift in pedagogical perspective seems to be the main weakness of information-processing models of human cognition which are elaborated by the Piagetian-oriented schema theorists. Although the schema theorists profess the prior knowledge corollary of the first principle of constructivism, and proffer student-centred pedagogies which involve *concrete* learning aids such as analogies and blocks and rods, their instructional strategies are underpinned by metaphysical realist assumptions which signify the objectified existence of mathematical knowledge.

According to Cobb (1987), the schema theorists, including Resnick (1983) and Rumelhart and Norman (1981) whose rhetoric influenced the development of the information-processing metaphor employed in this study (see Chapter Three), fall foul of Bereiter's (1985) *learning paradox*. The schema theorists assume that students are capable of *seeing* the abstract mathematical representations that the teacher points out; a cognitive process which requires the student to have made already the construction in order to be able to *see* it:

> To suggest that pointing to a relationship or correspondence the adult can *see* in an analogy or instructional representation will somehow cause the child to make the desired construction is to argue that the child's constructive activity is precipitated by the product of that activity. (Cobb, 1987, p. 27)

Cobb's (1987) critique of the schema theorists provides further support for the validity of the inference made in Chapter Four about Ray's established role-determining conception of *teaching as a journey*. The prevalence in Ray's classroom discourse of references to students *seeing* mathematical relationships was interpreted as early evidence of his established objectivist epistemology.

Ray's Knowledge Transformation Model

In this study, an information-processing metaphor was aimed at focusing Ray's pedagogical attention on the *cognitive interface* between students' extant conceptual frameworks and their newly constructed knowledge. In practice, the articulation of the metaphor in my discourse with Ray involved the frequent use of the term *information*, and phrases such as *the linking of new information*, and *the integration of new maths content information*.

Ray reconceptualised his classroom role in terms of a metaphor of *teacher as interactive informer* (see Assertion Six in Chapter Five). In his report on the study (Fleming, 1988), Ray represents his constructivist-related epistemology in diagrammatic form (see Figure 1 in Chapter Five), and describes it as a process of *knowledge transformation*. In Ray's model, the teacher's knowledge is *transformed* into user-friendly *public information* in a process which involves a reduction of his complex knowledge structures into elemental components. Subsequently, the public information is transmitted to students whose task is to *decode* the information and transform it into their own *private knowledge*, a process which is dependent on students' prior knowledge.

Ray's epistemological model appears to share the main objectivist features of production systems models of human information processing. In Ray's model, the cognitive processes of coding and decoding are represented as the *first* and *second transformations*, respectively. These cognitive processes, which are indicated as operating in a uni-directional manner, serve to convey the teacher's knowledge, albeit in the form of *public information*, directly to students. For Ray *information* has, in part, replaced *knowledge*, but in terms of their epistemological significance the two terms appear to be synonymous.

Ray's knowledge-transformation model is susceptible, therefore, to Cobb's (1987) critique of production systems models of information processing. In particular, Ray's model *reifies* mathematical knowledge. That is, knowledge is conceptualised as an objective commodity which can be transmitted to students' minds from an external source, such as a teacher. From a radical constructivist perspective, one of the pedagogical problems associated with the reification of knowledge is the shift of pedagogical focus away from the meanings and processes of sense-making that underlie students' knowledge representations. Although Ray professed a constructivist-related belief in facilitating students' prior-knowledge-related construction of new mathematical knowledge, in practice his major pedagogical emphasis continued to be the presentation of his own mathematical knowledge in a manner that would enhance students' attempts at its accurate reproduction (see Assertion Six in Chapter Five). In other words, Ray's conceptual framework continued to serve as the main referent for his constructivist-related instructional strategies.

It seems, therefore, that the objectivist *cognition as information processing* metaphor is strongly implicated in the development of Ray's knowledge-transformation epistemology, and in his refined centralist classroom role of *teacher as interactive informer*. From a radical constructivist perspective, the information-processing metaphor, which construes knowledge development as the acquisition of information from sources external to the learner, appears to have reaffirmed Ray's preconception of the objectivist nature of mathematical knowledge (see Assertion One in Chapter Four), and continued to sustain his classroom role as the primary source of students' new mathematical knowledge. Although Ray's constructivist-related perspective viewed each student's prior mathematical knowledge as being located within their own cognitive environment, he conceptualised the genesis of students' new mathematical knowledge as a transformation of information which emanates from external sources, such as the teacher, and which is pregnant with objective meaning.

The corollary of the first principle of constructivism, which highlights the central role of prior knowledge in the construction of new knowledge, is of central importance to a constructivist-related theory of pedagogical reform. However, this study has shown how readily the corollary can be subsumed by an objectivist epistemology, especially when a realist preconception of the nature of mathematics prevails.

By contrast, the metaphysical principle of radical constructivism, which eschews the philosophy of objectivism, locates the genesis of new mathematical knowledge entirely within the cognitive environment of the individual learner. The metaphysical principle of radical constructivism preserves the integrity of the first principle of constructivism, which is susceptible to being misconstrued by a realist philosophy of the nature of mathematics, and has an important role in a constructivist-related theory of pedagogical reform in mathematics education.

THE EPISTEMOLOGICAL PROBLEM OF FORMALISM

Assertion One (see Chapter Four) argues that, at the commencement of the study, Ray conceptualised the syllabus in terms of the metaphor of *syllabus as a conceptual map*. In particular, it is inferred that the text of the syllabus document (and textbook) served Ray as the main epistemological referent for determining the form and scope of the mathematical knowledge (or *content*) to be transmitted to the class. From a radical constructivist perspective, this metaphor is indicative of an objectivist epistemology which locates knowledge in sources external to cognising beings.

In the first section of this chapter it is argued that a formalist conception of the nature of mathematics views the written symbols of mathematical discourse as mathematical objects. This objectivist conception of mathematics seems to be congruent with the metaphor of *syllabus as a conceptual map*, and raises questions about (1) the extent to which Ray's sustained objectivist epistemology was underpinned by a formalist preconception of the nature of mathematics, and (2) the interaction between his formalist preconception and constructivist-related beliefs. This section addresses these questions by examining Ray's post-collaborative pedagogy, and presents evidence which warrants the following assertion:

Assertion Eight
Ray's belief in the prior knowledge corollary of the first principle of constructivism was subsumed by his preconception of the formalist nature of mathematics which continued to underpin his objectivist epistemology, and resulted in learning activities of impoverished conceptual significance.

A Synopsis of 14 Lessons on Factorisation

In the second year of the study, I observed Ray's teaching of his Grade 9 mathematics class for a period of 14 consecutive lessons. During the six-week observation period, instruction focused on the topic of Factorisation. The purpose of the fieldwork was to investigate further the nature of Ray's pedagogical beliefs and their relationship with his teaching practice, in order to understand better the constraining influence of my theory of pedagogical reform which had been implemented during the collaborative phase of the study.

The main research question which focused my observations concerned the nature of Ray's pedagogical implementation of his espoused belief in the prior knowledge corollary of the first principle of constructivism. I wanted to know how Ray's teaching strategies accounted for students' extant conceptual frameworks, that is, how his teaching identified students' conceptual needs and difficulties, and facilitated their resolution. The following interpretive analysis draws on data recorded in fieldnotes and interview transcripts.

Introduction of the Topic

The first three of the 14 observed lessons comprised a review of a recent topic test which included the topic of Simplification. The subsequent 11 lessons focused on the topic of Factorisation. Ray's continuing objectivist epistemology was signified by a *teaching as a journey* metaphor which was implied by his explanation of his main pedagogical goals:

> [T]o be able to progress through [the topic of Factorisation] in a . . . focused way . . . looking at one type of problem at a time, and then gradually expanding . . . or developing [the concept of factorisation] as we go. (Interview 6, August 1988, p. 12)

Ray's introduction of the concept of factorisation exemplified his *teacher as interactive informer* classroom role which he had developed during the collaborative phase of the study in the preceding year (see

Assertion Six in Chapters Five). Initially, Ray explained the concept of factorisation in terms of the closely related topic of Simplification, which had been the main focus of instruction in the immediately preceding lessons, and in relation to a simple arithmetic analogy.

Halfway through Lesson 4, Ray announced to the whole class that "We're going to start something new now" (Fieldnotes 19 August 1988, p. 3). He wrote the term *Factorising* on the blackboard, and referred to part of the immediately prior problem of simplifying the expression $4(x+2)$, which he wrote in the expanded form $4x+8$. Ray announced that factorising was the *exactly opposite process* of simplification. The class was assigned a practice exercise which required the identification of the factors of the numeral *12*. Next, Ray demonstrated the technique for writing the factorisation of the initial problem as $4(x+2)$, and emphasised that factorisation was *working backwards*.

Ray proceeded to stimulate students' anticipation of the development of the concept of factorisation by means of interactive, step-by-step, whole-class discussions of the solutions of increasingly complex exercises, during which time he responded to individual student queries. The whole class was directed to complete four exercises: $4y+12$, $15+3x$, $2x^2+6x$, $4x^2+10x$. After each exercise, Ray conducted a whole-class discussion of the correct answers, and responded to students' queries about the selection of a *highest common factor*. Before the end of the lesson, Ray reiterated the process of factorisation and emphasised its reverse relationship with the process of simplification.

Completing the Topic

For the duration of the next 10 lessons, students were introduced to a taxonomy of types of factorisation methods which included *common factors, perfect squares, grouped factors,* and *trinomials* (Fieldnotes 22 August - 6 September 1988). The class was directed to the textbook for both self-instruction and practice exercises. Initially, Ray believed the textbook to be an adequate source of self-instruction. However, in later lessons the nature of students' conceptual development difficulties indicated to Ray that the text did not provide adequate instruction on *perfect squares*, or on the relationships between various types of factorisation.

In Lessons 5-8, Ray allowed students to complete sets of exercises at individually self-determined rates. He attempted to facilitate students' self-management of their learning by providing *progress sheets* on which students individually could record their completed work. During most of these lessons, students were engaged in individual seatwork activities. The answers to the practice exercises were available in the back of the textbook. Students either consulted their friends or sat with their hands raised and waited for Ray who moved around the class and provided individual assistance on request (Fieldnotes 22-26 August 1988).

In Lesson 9, Ray regrouped the whole class, "to draw a few loose ends together" (Fieldnotes 29 August 1988, p. 2), and provided instruction on four methods of factorising *perfect squares* which had proved to be problematic for many students. In preparation for the topic test, Lessons 10-14 focused on methods of factorising trinomials and identifying individual types of factorisation methods (Fieldnotes 30 August - 6 September 1988).

For a part of most lessons, Ray conducted whole-class instruction for purposes of: providing information about factorisation methods (especially information about the relationships between different factorisation methods, and between the processes of factorisation and simplification); remediating students' problematic prior mathematical knowledge; demonstrating solutions to practice exercises; and providing corrective instruction for students' misconceptions. These whole-class sessions were mostly interactive, with only brief periods of uninterrupted teacher discourse.

Addressing Learning Difficulties

One of the major learning difficulties which was experienced by most students was the identification of types of factorisation methods required to solve practice exercises. Ray addressed this problem by producing a *Summary and Revision* taxonomy-style handout which labelled and described each of the main types of factorisation method. The handout was distributed to the class in Lesson 10 (Fieldnotes 30 August 1988, p. 2).

Other learning difficulties which disabled students from completing successfully the prescribed practice exercises were associated with lack of ability to recall accurately, and apply, previously learned algebraic rules for manipulating indices, directed numbers, and algebraic equations. Ray assisted students to overcome these difficulties by producing handouts which contained lists of salient algorithms (Fieldnotes: 16 August 1988, p. 7; 23 August 1988, p. 5).

Ray's Formalist Pedagogy

Within the classroom environment and for the duration of the 14 lessons, a formalist view of the mathematics topic of factorisation prevailed. The concept of factorisation was introduced within the narrowly-defined formalist context of its symbolic relationship to the concept of simplification, and its development was confined entirely to a conceptual environment which was represented by algebraic symbols and deductive rules of logic. The text of both the syllabus document and the textbook provided a formalist frame of reference for Ray's pedagogy. The manipulation of algorithmic expressions according to predetermined deductive rules of logic, which were presented as various factorisation methods, constituted the entirety of students' mathematical activity.

No attempt was made during the topic, either by Ray or the textbook authors, to enable students to conceive of an enhanced significance of their developing concepts of factorisation by, for example, establishing meaningful relationships with other mathematical or scientific concepts, or reflecting on relevant aspects of their everyday experiences. Ray's instruction promoted an image of mathematical conceptual development as the accurate memorisation of predetermined algorithms which were expressed in algebraic terminology;

memorisation was facilitated by solving known-answer practice exercises.

Thom (1971) accuses curriculum developers of unwarranted *modernism* for their inclusion of algebraic theories and structures into school mathematics on the basis of their *technical utility* to specialist mathematicians. He argues that not even the elitist interests of selecting the most able students are well served by dogmatic teaching "where scholarly excellence is defined as exact and rapid memorisation of given material" (p. 70). Thom identifies modern algebra as an exemplar of technical utility and pedagogical futility because of the trivial nature of the problems available to students:

> [W]hile there are geometry problems there are no algebra problems. A so-called algebra problem can only be a simple exercise requiring the blind application of arithmetical rules and of a pre-established procedure. With rare exception one cannot ask a student to prove an algebra theorem; either the requested answer is almost obvious and can be arrived at by direct substitution of definitions, or the problem falls into the category of theoretical algebra and its solution exceeds the capacities of even the most gifted student. Exaggerating only slightly, one can say that any question in algebra is either trivial or impossible to solve. By contrast the problems of geometry present a wide range of challenges. (Thom, 1971, p. 70)

Thom's comments about the trivial nature of algebraic problems seem apposite to the context of conceptual insignificance in which factorisation was taught in Ray's Grade 9 class. The concept of factorisation was not promoted as being significant for purposes other than preparation for the forthcoming topic test requirements. These requirements were synonymous with the development of skill in reasoning deductively from unjustified premises to known-answer conclusions within a formalist context. The sets of practice exercises were designed to develop these skills, and the topic test was designed to assess the extent of their development.

For students, the significance of the concept of factorisation was to be realised, therefore, in the experience of certainty associated with the justification of the conclusions of deductively valid arguments, rather than with the justification of their premises (Giere, 1984). Although devoid of prospective significance outside of mathematical logic, it seems likely that practice exercises were designed to enable students to enjoy the significant experience of the certainty of mathematics — a major goal of the formalist program in mathematics.

However, the problem of conceptual insignificance was less apparent in the later lessons of Ray's Grade 12 mathematics class. Ray provided opportunities for students' to develop mathematical concepts in a relatively rich conceptual context that included problem-solving in everyday contexts. For example, several Linear Programming lessons (Lessons 14-16) provided opportunities for students to develop their knowledge of graphical solutions of algebraic equations by engaging in structured collaboration and role-play in small groups for the purpose of solving realistic problems experienced in manufacturing industry (see Fieldnotes 15-16, 19 June, 1987). From the perspective of radical constructivist pedagogy (Wheatley 1990), it is likely that these problems provided opportunities for students to experience an enhanced sense of purpose in relation to their mathematical problem solving activities.

The prior knowledge corollary of the first principle of constructivism, which influenced Ray's pedagogical reform, seems to have been subsumed by his formalist perspective on the nature of mathematics. Although Ray monitored students' prior mathematical knowledge and designed instructional strategies for assisting students to resolve their learning difficulties, his pedagogical interest was confined to a formalist context. Ray's verbal and written interventions comprised reiterations of the algorithms which students seemed unable to recall from previous classroom learning experiences. At best, Ray's constructivist-related perspective provided a reactive pedagogical interest in students' extant conceptual frameworks:

> I really started off doing factorisation, and then checking out their background as we were going, and then realising 'Ah, there are some

> weaknesses here!' [My instructional strategies were based on] finding out their mistakes, finding their weaknesses, by looking at the mistakes they're making, discovering gaps in their knowledge.
>
> (Interview 7, September 1988, p. 8)

Data obtained during the six-week period of observations in the post-collaborative phase of the study provides compelling evidence, therefore, that the constructivist-related theory of pedagogical reform had failed to account for Ray's formalist preconception of the nature of mathematics. In particular, the prior knowledge corollary of the first principle of constructivism seems to have been subsumed by Ray's formalist preconception. Ray's pedagogical interest in students' prior knowledge was restricted to a reactive remediation of the deficiencies in students' observable attempts to recall and apply antecedent symbolic structures in a formalist context of impoverished conceptual significance.

The formalist text of the syllabus document and textbook appears to have served as Ray's major pedagogical referent. Ray's conceptualisation of *the syllabus as a conceptual map*, which had been inferred at the commencement of the study (see Assertion One in Chapter Four), seems to have been sustained beyond the end of the collaborative phase of the study. From a radical constructivist perspective, Ray's instructional strategies were concerned primarily with facilitating students' *behavioural fit* (von Glasersfeld, 1990b), that is, with enabling students to demonstrate correct manipulation of the mathematical algorithms, rather than with the significance of students' autonomous conceptual understandings.

SUMMARY

This chapter extends the interpretive analysis of Chapter Five of Ray's reformed pedagogy for the purpose of assessing the efficacy of the constructivist-related theory of pedagogical reform which was implemented in the collaborative phase of the study. A review of the research literature on the philosophy of mathematics reveals that realist and formalist conceptions of the nature of mathematics are major historical sources of influence on the pedagogy of modern school mathematics. These outmoded objectivist conceptions differentiate between mathematical objects and mathematical cognition, and are associated with transmissionist epistemologies which this study sought to radically reconstruct.

From a radical constructivist perspective, an interpretive analysis of Ray's post-collaborative pedagogy confirms that he had sustained a transmissionist epistemology, and that it was associated with his realist preconception of the nature of mathematics. Assertion Seven argues that the metaphor of *cognition as information processing,* which I employed in my discourse with Ray for the purpose of articulating the prior knowledge corollary of the first principle of constructivism, was perceived by Ray to be consistent with his objectivist conception of the teacher as the primary source of students' new mathematical knowledge, albeit in the covert form of information provider.

Although Ray professed a belief in facilitating students' prior-knowledge-related construction of new mathematical knowledge, he continued to believe that its genesis occurred in external sources such as the teacher, and that his role was to transmit his own mathematical knowledge in a more user-friendly form. Chapter Five indicates that Ray's more interactive and inquiry-oriented classroom discourse and reductionist approach to instructional design served this purpose.

An interpretive analysis of Ray's post-collaborative pedagogy, from a radical constructivist perspective, indicates also that Ray's formalist preconception of the nature of mathematics had contributed to his sustained transmissionist epistemology. Ray's formalist preconception resulted in his priority pedagogical concern with transmitting to students the syntax of the symbolic language of mathematics, rather than with students' semantic interpretations. Assertion Eight argues that, although Ray believed in the prior knowledge corollary of the first principle of constructivism and implemented it in his teaching, his formalist view of mathematics subsumed this belief. Although Ray's pedagogy had a prior knowledge orientation, it focused entirely on algorithmic manipulation, and provided students with learning experiences of impoverished conceptual significance.

The interpretive analyses in this chapter provide compelling evidence that the constructivist-related theory of pedagogical reform which was enacted in my discourse with Ray: (1) contributed to sustaining Ray's realist view that mathematical objects exist independently of cognising beings; and (2) failed to account for Ray's formalist view that the mathematical objects are constituted by the objective symbolic structures which are recorded in the text of syllabus documents and textbooks. Because the theory of pedagogical reform failed to account for these objectivist preconceptions of mathematics, Ray's perception was sustained of the importance of (1) his centralist classroom role, which enabled him to control the delivery to students of his objective mathematical knowledge, and (2) students' roles as compliant replicators of Ray's objective mathematical knowledge.

Ray's realist and formalist views of mathematics constitute a rationalist conception of mathematical cognition, and resulted in Ray's recognition of the cognitive activity of the individual student as the subject of his constructivist-related pedagogical innovations. Chapter Seven adopts a *social constructivist* perspective and examines the implications of this issue for constructivist-related pedagogical reform.

CHAPTER SEVEN

A SOCIAL CONSTRUCTIVIST PERSPECTIVE ON RAY'S RATIONALIST PRECONCEPTION OF MATHEMATICAL COGNITION

INTRODUCTION

The previous chapter argues that the theory of pedagogical reform which shaped my constructivist discourse with Ray failed to address Ray's realist and formalist preconceptions of the nature of mathematics. These unchecked objectivist preconceptions subsumed Ray's newly developed belief in the constructivist nature of students' knowledge development and sustained his established transmissionist epistemology and centralist classroom role.

Chapter Seven assesses the extent to which the theory of pedagogical reform contributed also to the restricted transformation of Ray's centralist classroom role by failing to account for his preconception of the *rationalist* nature of mathematical cognition. It is argued that the theory of pedagogical reform was unable to provide a compelling rationale for a pedagogical interest in key social and emotional dimensions which are constitutive of students' construction of their mathematical knowledge. The results of an interpretive data analysis are reported in the form of Assertion Nine.

The first section constitutes the interpretive framework of the chapter, and presents a review of recent research literature on *social constructivist* perspectives on learning, especially in the fields of school science and mathematics. The review focuses on educational research which examines socio-cultural and socio-emotional influences on students' classroom actions. The pedagogical implications of the metaphor of *learning as social construction* are discussed in the context of research on the teaching and learning of school mathematics.

The second section extends the interpretive analysis of Ray's reformed pedagogy (see Chapter Five) and, from a social constructivist perspective, examines the efficacy of my collaborative attempts to persuade Ray to transform his individualist pedagogy by adopting a pedagogical interest in the social context of learning mathematics and facilitating small-group classroom learning activities. The interpretive analysis incorporates data obtained from classroom observations during the collaborative phase of the study, and from an interview with Ray which I conducted in the post-collaborative phase.

INTERPRETIVE FRAMEWORK

The interpretive framework of this chapter commences with a brief review of recently articulated social constructivist perspectives on school science and mathematics which criticise as unduly rationalist earlier constructivist-related pedagogical perspectives on individual student cognition. The influence on student learning of classroom-based socio-emotional factors is discussed, and it is argued that teachers of science and mathematics have an important classroom role to perform in facilitating an emotional climate that serves the conceptual development needs of all students. The pedagogical implications of social constructivist perspectives on mathematics education are discussed in relation to major research programs in elementary school mathematics.

Social Constructivist Perspectives on Learning

The theory of pedagogical reform which shaped the collaborative phase of this study was derived largely from the constructivist perspective which underpinned the research of science educators who, during the 1970s and early 1980s, were engaged in research on children's science-related conceptions and on instructional models for facilitating conceptual change amongst students of school science (see Chapters One and Two). Much of this research was founded on a constructivist perspective which Joan Solomon (1987) labelled recently as the *personal constructivist position* because of its close association with the personal construct theory of cognitive psychologist George Kelly.

Solomon's (1987) critique of the personal constructivist position argues that conceptual change instruction in science education is underpinned predominantly by *rationalist* pedagogical interests which are evidenced by pedagogical *individualism* and a concomitant lack of concern for the social and emotional contexts within which knowledge is constructed by learners. In particular, these pedagogies harbour *mentalist* assumptions that "each person makes up hypotheses, and then rejects or refines them as a result of his own private experience" (Solomon, 1987, p. 66), and fail to address seriously the social influences acting on the learning process.

From the sociological perspective of Berger and Luckmann (1966), Solomon (1987) argues that social influences are a major determinant of all forms of learning, including school knowledge development, and that meaning is as much a social construct as it is a personal construct. Reality is constructed *intersubjectively*, that is, it is socially negotiated between *significant others* who are able to share meanings and social perspectives of a common *lifeworld knowing* in which "the essential criterion is no longer the internal logic of the explanation but that it should be recognised and shared with others" (Solomon, 1987, p. 67).

Solomon (1987) argues that the rationalism of personal constructivist pedagogies, especially that of Strike and Posner (1982) (whose theory of conceptual change influenced strongly the theory of pedagogical reform of this study — see Chapter Two), have only a peripheral interest in the central role of social communication in the construction of children's culturally shared lifeworld knowing. Although personal constructivists acknowledge the importance of group work in the construction of students' knowledge, very little pedagogical concern has been shown for the dynamics of groups which shape the construction of personal meaning. The persistence of students' alternative conceptual frameworks, "after careful refutation . . . by experiment or argument", is cited by Solomon (1987, p. 66) as evidence of the lack of success of personal constructivist pedagogies in addressing important aspects of children's lifeworld knowing.

Recently, Bruner (1986) criticised the implicit individualism of his earlier theory of *discovery learning* which was underpinned by psychological theories of cognitive development that affirmed the individual as a private self who is free of cultural definition, and who constructs a reality solely on the basis of private encounters with exemplars of natural states. Bruner's reconstructed form of discovery learning is based on an ontological conviction, similar to the metaphysical principle of radical constructivism, that "there is no *aboriginal* reality against which one can compare a possible world in order to establish some form of correspondence between it and the real world" (p. 46), and acknowledges explicitly the importance of the activities of social negotiation in a cultural context:

> My model of the child in those days was very much in the tradition of the solo child mastering the world by representing it to himself in his own terms. . . . I have come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make his knowledge his own, but that he must make it his own in a community of his own who share his sense of belonging to a culture. It is this that leads me to emphasize not only discovery and invention but the importance of negotiating and sharing. (Bruner, 1988, p. 127)

Social constructivist perspectives have been applied also to mathematics education. Heinrich Bauersfeld's (1988) mathematics education research program, which is concerned with the impact of an innovative mathematics curriculum on students' thought and language, has changed from a focus on the *subject-matter* knowledge of individual students to a focus on the social reality of the classroom which teacher and students construct jointly. Bauersfeld's (1988) perspective, which is shaped by the metaphysical principle of radical constructivism (see Chapters One and Six), regards the individual learner's construction of mathematical knowledge to be linked inextricably to culture, whether the learner is a professional mathematician or a student of mathematics. The intersubjectivity of socially shared knowledge that results from the unifying bonds of culture and language saves the individual's subjective constructions of viable hypotheses of the world from arbitrariness:

> [L]earning is characterised by the subjective reconstruction of societal means and models through the negotiation of meaning in social interaction and in the course of related personal activities. New knowledge, then, is constituted and arises in the social interaction of members of a social group (culture), whose accomplishments reproduce as well as transmute the culture (e.g., of the mathematical community, of teacher and students of a class, etc.).
>
> (Bauersfeld, 1988, p. 39)

The research program in mathematics education led by Paul Cobb at Purdue University was founded on a radical constructivist perspective (Cobb 1985a, 1985b, 1986b), and has acknowledged recently the importance of the socio-cultural context of the classroom learning environment for the construction of the individual learner's mathematical conceptualisations (Cobb, 1989; Cobb, Wood, & Yackel, 1990, in press; Cobb, Wood, Yackel, & McNeal, 1992; Cobb, Yackel, & Wood, 1992, in press):

> Analyses that focus solely on individual children's construction of mathematical knowledge tell only half of a good story. The issue that needs to be addressed is the form that the process of mathematical acculturation should take and how it can be coordinated with what is known about the cognitive processes by which individuals construct mathematical knowledge. (Cobb, 1989, p. 34)

These social constructivist perspectives on learning argue against constructivist-oriented rationalist conceptions of cognition which give rise to pedagogies that focus on the cognitive activity of the individual learner and ignore the socio-cultural context of learning.

The Socio-Emotional Context of Learning

Although the role of reason in the learner's construction of knowledge remains a central tenet of contemporary theories of constructivism, the influence of social interaction on the emotional state of the individual learner has been acknowledged recently as being of central importance.

For Bruner (1988), the cognitive activity of discovery learning is more than the *pure reason* of rationalism. His claims about the pedagogical importance of emotion in relation to the cognitive processes of knowledge construction are based on a conviction that emotion is linked inextricably to both cognition and action:

> Emotion is not usefully isolated from the knowledge of the situation that arouses it. Cognition is not a form of pure knowing to which emotion is added. And action is a final common path based on what one knows and feels . . . the three are constituents of a unified whole. To isolate each is like studying the planes of a crystal separately, losing sight of the crystal that gives them being. (Bruner, 1986, p. 118)

According to sociologists Berger and Luckmann (1966), the emotionally laden nature of children's identification with *significant others* (or parents) as they undergo *primary socialisation*, contributes to the construction of their lifeworld knowing, or prevailing subjectivities. By contrast, *secondary socialisation*, which involves the acquisition of role-specific knowledge in institutions such as schools, involves less subjective inevitability and, therefore, is experienced as being less compelling. The processes of secondary socialisation are less emotionally charged and require only a partial commitment of the self. One of the implications of this theory for school science and mathematics is that a student's peer-group social relations might be more influential in determining commitment to new ideas than the rationality of the teaching-learning process which is governed by the teacher.

This implication was investigated for its relevance to science education in Solomon's (1987) review of theories of social research, social psychological perspectives (e.g., symbolic interactionists, neo-Piagetians), and research into interactive aspects of school learning. The review concludes that social interactions amongst groups of students in the classroom are a primary determinant of learning outcomes, and suggests that social interactions might provide opportunities for engendering a much stronger commitment to new ideas than was possible from only rational considerations of empirical data obtained in practical, or laboratory, work:

> – [S]ocial constructions of meaning within science practical groups might prove to be more important than the experimental results, and more even than the mental constructs of individuals.
>
> (Solomon, 1987, p. 69)

On the other hand, social influences have been found to constrain the quality of knowledge constructed in the classroom. Bauersfeld (1988) analysed patterns of discourse between teachers and students of primary school mathematics, and found a major pedagogical problem associated with the *covert social structure* of the classroom. Implicit expectations and obligations are shared by teachers and students, and shape their mutual social interactions. In question-and-answer routines the social interaction between teacher and student regulates the student's (and teacher's) sense-making processes and, in protracted interactions, a *funnel pattern* can result in the generation of narrow and impoverished meaning:

> [T]he learner's adaptive efforts towards an acceptable use of mathematical symbols and language are bound to generate context- and problem-specific routines and skills rather than insight, self-confidence, flexible strategies, and autonomy. The mathematical logic of an ideal teaching-learning process thus becomes replaced by the social logic of this type of interactions. (Bauersfeld, 1988, p. 38)

In particular, students' conceptions of the nature of mathematical activities, and of mathematics, can be constrained by their willing adherence to the established social norms of discourse in the classroom. The report on patterns of social interaction in primary school classrooms by Cobb, Wood, Yackel, and McNeal (1992) indicates that traditional *school mathematics*, which emphasises following procedural instructions rather than constructing meaning, promotes social norms that reward children who behave cooperatively and provide responses that are compatible with the teacher's expectations. Children become adept at acting in accord with teachers' implicit expectations for student compliance in order to experience a sense of effectiveness in relation to their membership of the classroom community. The strength of this emotional need is such that students willingly mask their lack of understanding and deny themselves opportunities of constructing richer meaning by failing to contest the apparently irrational nature of the teacher-directed discourse in which they are participating:

> [T]he children acted as pragmatists and strove to be effective in the social setting of the classroom by attempting to complete tasks and answer questions in ways that were compatible with the teacher's expectations. . . . [T]he possibility that arithmetic could be anything other than procedural instructions did not arise. . . . [M]athematics is reduced to an activity that involves constructing associations between signifiers that do not necessarily signify anything beyond themselves.
>
> (Cobb, Wood, Yackel, & McNeal, 1992, pp. 585-587)

The research by Cobb, Wood, Yackel, and McNeal (1992) has demonstrated how, for students, the significance of complying with teacher-determined classroom social norms outweighs the significance of developing autonomous conceptual understandings. These results suggest that many lower achieving students find the emotional payoff gained from social conformity to be sufficiently compelling that they participate willingly and unquestioningly in mathematical activities which they experience as irrational. Furthermore, these results help to explain why children are prepared to subjugate their autonomous intellectual development to the pedagogical priorities of an apparently "meaningless formal game played with marks on paper, following rules" (Ernest, 1991, p. 10). In other words, many children prefer to subscribe to classroom social norms which determine a formalist learning environment in which the predominant learning experience is one of impoverished conceptual significance (see Assertion Eight in Chapter Six) in order to experience the emotional satisfaction of being accepted as a successful social member of the classroom.

A similar problem has been identified in science classrooms by Trumbull (1987) who argues that teachers' rationalist language patterns have a constraining influence on the *emotional climate* of the classroom and, consequently, on personal meanings that are constituted in classroom communications. Classrooms that are dominated by teacher control interests generate emotional climates that constrain the social interactions necessary for rational inquiry, and induce students to *play it safe* by following the rules and memorising passively. Because emotions strongly influence cognition, pedagogies which ignore the affective domain will affect adversely the meanings generated by students. The resolution to the problem of rationalism is to develop constructivist-related pedagogies which aim at establishing appropriate emotional climates which facilitate meaningful discourse between teachers and students:

> We must learn to work with people before we can work with cognition. If we cannot create the emotional climates necessary for communication - thinking and speaking - all our cognitively oriented work will be wasted. (Trumbull, 1987, p. 493)

Recent constructivist-oriented research on learning argues that the individual learner is not well-served by pedagogies which are underpinned by rationalist conceptions of cognition. The rationalist view of the learner as *cognitive isolate* is a myth which is associated with objectivist conceptions of the nature of mathematics or science. Learning occurs in a social context. Learners respond emotionally to their social contexts, and their emotional states influence their cognitive activity.

Pedagogical Implications of Social Constructivism

Constructivist pedagogies that endeavour to take account of the socio-cultural aspects of knowledge construction are the subject of on-going research programs. The common feature of the contemporary constructivist pedagogies in mathematics education, which are discussed below, is their incorporation of the metaphysical principle of radical constructivism (see Chapters One and Six). Grayson Wheatley's *problem centred learning* is formulated in relation to a radical theory of constructivism which rejects all notions of objectivism. However, in the case of Paul Cobb's research group, and after Goodman (1986), Platonism regains a degree of respectability because of the pedagogical need to account for mathematical experiences from which it is inferred that mathematics is *practically real.*

Problem Centred Learning

A radical constructivist epistemology underpins Wheatley's (1990) problem centred learning pedagogy for school science and mathematics. Learning is a process of *cognitive adaptation* whose goal is to neutralise perturbations in the mind of the individual learner that arise through interactions with *the world* (Steffe, 1988). The cognising subject's successful resolution of cognitive perturbation is achieved by means of the restructuring of his/her conceptual schemes, and is explained in terms of making sense of problematic experiences, or constructing personally viable meaning. In terms of Piaget's metaphor of biological evolution, the organism re-establishes its viability in relation to the constraints of its physical environment (von Glasersfeld, 1981, 1989). The construct of *subjective viability* replaces the construct of objective truth:

> From an explorer who is condemned to seek *structural properties* of an inaccessible reality, the experiencing organism now turns into a builder of cognitive structures intended to solve such problems as the organism perceives or conceives. . . . What determines the value of the conceptual structures is their experiential adequacy, their goodness of *fit* with experience, their *viability* as means for the solving of problems, among which is, of course, the never-ending problem of consistent organization that we call *understanding*. This view of knowledge . . . will shift the emphasis from the student's *correct* replication of what the teacher does, to the student's successful organization of his or her *own* experience.
>
> (von Glasersfeld, 1983, pp. 5, 6)

One of the main pedagogical goals of problem centred learning is to design learning tasks that students experience as being problematic, or perturbing, and significant. Another important goal is to provide opportunities that facilitate students' purposeful and meaningful resolution of their problems. Because scientific and mathematical knowledge is a learner activity conducted in a social setting, rather than an independent body of knowns, "social interaction plays an important role" (von Glasersfeld, 1989, p. 129):

> It might be more representative of radical constructivism to say that individuals co-construct knowledge. Thus meanings stem from the consensual domains of individuals. . . . In the classroom it is important that students talk science and mathematics to provide opportunities for meaning to be negotiated and consensus reached.
>
> (Wheatley, 1991, p. 6)

In problem centred learning classes students work collaboratively in small groups on specially designed tasks. The whole class re-convenes for teacher-facilitated *sharing* sessions which involve whole-class group presentations and discussion. During collaborative learning and sharing sessions, an ethos of social interaction is established:

> In order to do mathematics and science, students and teachers must learn how to carry on a scientific discussion. The conversation goes on in public and is learned by participating. . . . Class discussion initiates *conversations* which students then learn to carry on within ourselves. . . . Class discussions, where students share their solutions, provide a forum for students to construct explanations of their reasoning. In the process of telling others how they thought about a problem, students elaborate and refine their thinking and deepen their understanding.
>
> (Wheatley, 1991, p. 18)

The teacher's authority is vested in the class as a community of validators, and she avoids *teaching by imposition* (Bishop, 1985) in order to foster students' intellectual autonomy. Students' *wrong* answers do not signal direct teacher intervention. Rather, the students are expected to attempt to understand a wide range of perspectives, and to accept the responsibility for negotiating a consensual viewpoint. However, a consensual view that indicates students have failed to construct the appropriate conceptualisations signals a challenging need for the teacher to design further problematic activities:

> Identifying tasks that embody the central ideas of the discipline and that will be problematic for students is quite challenging. . . . In considering students' thinking, we must learn to look at the world through the student's eyes. Rather than being considered mistakes to be corrected, student *errors* are rich sources of information about children's thinking. They indicate the meanings children have given to the associated ideas. The issue is not what procedures and knowledge they have amassed but what concepts they have constructed, the cognitive level at which they are operating, their beliefs, and their intentions. Selection of problematic activities requires knowledge of each student's level of functioning.
>
> (Wheatley, 1991, p. 14)

Inquiry Mathematics

Although radical constructivism purports to address the social context of learning, particularly the process of linguistic communications (von Glasersfeld, 1983, 1989), it is criticised for its *subjective idealism* which defines all knowledge as individual, rather than as social or cultural (Gruender, 1989). Recently, however, Cobb (1989), Cobb, Wood, and Yackel (in press), and Cobb, Yackel, and Wood (1992, in press) have elaborated radical constructivism from an anthropological perspective in order to accommodate (1) the intersubjective context of mathematical cognition, and (2) the commonly held Platonist belief that we discover, rather than invent, mathematical truth and mind-independent mathematical reality (see Chapter Six). The resultant constructivist-related pedagogy is called *inquiry mathematics*.

Negotiation amongst a community of learners is at the heart of the *inquiry* mathematics tradition, following Richards (in press), which Cobb, Yackel, and Wood (in press) contrast with the *school mathematics* tradition that continues to pervade most of mathematics education. The patterns of interaction between teacher and students in the two traditions are distinctly different. In the school mathematics classroom, the teacher asks questions to assess students' knowledge of specific mathematical information, students respond by attempting to provide correct steps in a procedure, and the teacher evaluates the answer in terms of their procedural validity.

By contrast, in an inquiry mathematics class, the teacher's questions are open-ended, students' responses constitute explanations and justifications of their thinking, the central emphasis is on the multiplicity of possible solution strategies, rather than the correct answer, and the teacher and students actively constitute a community of validators of *mathematical truth*. Therefore, the mathematical meanings and actions of teacher and students cannot be predetermined immutably, but emerge in the course of communication (Cobb, Wood, & Yackel, in press; Wood, 1991; Yackel, 1991):

> The classroom setting should be designed as much as possible to allow students to do their own negotiating and institutionalizing — in short, their own truth-making. This approach contrasts sharply with traditional instruction in which students are presented with codified, academic formalisms that, to the initiated, signify communally-sanctioned truths that have been institutionalized by others.
>
> (Cobb, 1989, p. 38)

The teacher's role involves mutually establishing with children a social classroom climate based on an interlocking set of obligations and expectations that solution methods, rather than answers, are of central importance. Mathematics is viewed as mathematical activity in which meaning is *interactively constituted* during social interaction. Students' roles involve explaining and justifying their own solution methods, and listening to, comparing, evaluating and challenging other children's explanations:

> Potential learning opportunities arise when children listen to and try to make sense of solution methods other than their own . . . as children attempt to extend their conceptual framework to encompass

> the thinking of others, as they resolve conflicts that arise when several different solutions lead to the same result, as they attempt to take the perspective of others, and as they critically evaluate others' explanations. (Yackel, 1991, p. 19)

In addition to students' mathematical meanings, the second theme of classroom discourse, *talking about* mathematical meanings, involves teacher and students in continuous renegotiations of the social norms of the classroom. The two themes, taken together, constitute an evolving classroom culture whose constitutive rationality is analogous to that of a scientific research community:

> The notion of the classroom community's evolving culture is compatible with the view that what counts as science and rationality cannot be specified in terms of ahistorical criteria (Barnes, 1982; Bernstein, 1983; Rorty, 1980). Instead, scientific knowledge and the encompassing research community are continually reconstituted together, with each informing the other.
> (Cobb, Wood, & Yackel, in press)

However, the analogy between an inquiry mathematics classroom and a scientific, or mathematical, research community is not complete because of the necessary *power imbalance* between teacher and students, in relation to whose interpretation ultimately becomes the taken-as-shared mathematical reality. Although inquiry teachers' main intention is to elicit students self-generated solution methods, rather than explicitly tell them the answers, teachers do not relinquish their authority for legitimating students' contributions in relation to the mathematical norms of both the classroom and wider society. However, the inquiry teacher expresses her authority more indirectly than does the school mathematics teacher who acts as the sole validator of her students' mathematical interpretations:

> [A] primary way in which the teacher expressed her authority in action was by initiating and guiding the interactive constitution of [situations for explanation and justification], and by subtly legitimizing certain of the children's mathematical constructions but not others . . .

> [students'] acculturation into the mathematical ways of knowing occurred as the teacher capitalized on their autonomous constructions to guide the constitution of taken-as-shared mathematical meanings and practices . . . [nevertheless] the teacher and children were mutually orienting each other in a taken-as-shared mathematical reality.
>
> (Cobb, Yackel, & Wood, in press)

By complementing *cognitive* constructivism with an anthropological perspective on the teaching and learning of mathematics, Cobb (1989) hopes to account for both the common Platonist experience that mathematical knowledge is independent of the mind of the knower, and the perceived solipsistic position of radical constructivism that locates knowledge exclusively in the mind of the knower. Cobb's (1989) anthropological perspective regards mathematical knowledge as being continually regenerated and modified by the "coordinated actions of members of communities" (p. 40). Mathematical truth, therefore, is relative to the cultural context in which it is realised and practised, rather than absolute and ahistorical in the Platonist sense .

Furthermore, from an anthropological perspective, the viability of an individual's conceptual developments is influenced by their attempts to *fit* their actions to the actions of others by means of negotiation. Negotiation contributes to establishing *institutionalized mathematical practices*, which constitute the intersubjective *consensual domain* of the group and which, reflexively, constrain individual mathematical activity and sense-making:

> This position might at first seem paradoxical; mathematical meaning can be in the world (experiential), in the individual's head (cognitive), and in social interaction (anthropological). This apparent paradox is the result of one attempt to cope with an omnipresent if implicit complementarity in mathematics education theorizing. . . . Such paradoxes are not, of course, unique to mathematics education but pervade our everyday lives. (Cobb, 1989, p. 41)

The *problem centred learning* and *inquiry mathematics* research programs, in which social-constructivist-related pedagogies have been developed for elementary school mathematics classes, emphasise the importance of negotiation amongst students in both small-group and whole-class forums. From a social constructivist perspective on learning, classroom discourse serves the purpose of enabling students to co-construct their mathematical knowledge in an interactive social learning environment which provides opportunities for students: (1) to undertake collaboratively meaningfully problematic mathematical activities; (2) to explain and justify their evolving mathematical conceptions; (3) to challenge other students' explanations, and to be challenged; (4) to negotiate a consensual domain of mathematical truth; and (5) to negotiate the social norms of classroom discourse upon which truth is established. In these ways, classroom discourse serves the important purpose of enabling students to be reflexively aware of the viability of their own knowledge, especially in relation to an intersubjective consensual domain of mathematical truth. The teacher's classroom role is to provide learning activities which meet the conceptual development needs of students, and to facilitate an interactive and inquiry-oriented classroom discourse.

THE EPISTEMOLOGICAL PROBLEM OF RATIONALISM

During the collaborative phase of the study, I attempted to persuade Ray to decentralise his pedagogical focus on the whole class by suggesting that he introduce small-group learning activities. This section adopts a social constructivist perspective, and examines the efficacy of the theory of pedagogical reform for facilitating Ray's pedagogical interest in the social context of students' mathematical cognition. Evidence is presented which warrants the following assertion about the inadequacy of the theory for addressing Ray's rationalist preconception of students' mathematical cognition:

Assertion Nine

The theory of pedagogical reform was based on a *cognitive constructivist* perspective which focused exclusively on the cognitive activity of the individual student, and served to reaffirm Ray's rationalist preconception of the nature of mathematical cognition

A Synopsis of Ray's Rationalist Pedagogy

From the commencement of the study, it was apparent that many students in Ray's Grade 12 class did not want to work in social isolation. At the commencement of lessons, many students relocated their single-person desks in order to sit in small friendship groups of between two and six students. However, a few students chose not to form friendship groups and worked in relative social isolation, never speaking to other students and speaking rarely with the teacher. Ray enforced only rarely the school policy that individual student desks should remain in serried rows. Although he permitted students to sit next to their friends, he insisted that the room be reorganised before students departed at the end of each lesson.

During class, students' proximity to their friends enabled them to interact for various purposes. However, students consulted each other mostly about their work, especially when they experienced difficulties obtaining correct answers to practice exercises during brief periods of individual seatwork activity early in the study.

Generally, students' desired a more collaborative and socially harmonious classroom learning environment in which they could receive more individual assistance from the teacher (see Assertion Four in Chapter Four). However, the predominance of teacher-directed whole-class activities which were dominated by target students (see Assertion Two in Chapter Four) prevented students from realising their goal.

Socialisation by Pedagogical Default

As the study progressed, Ray increased the amount of individual seatwork activity. In the Grade 12 class, teacher-designed worksheets provided a *self-paced* mode of learning (see Assertion Six in Chapter Five) which Ray believed would facilitate students development as *independent* learners (see Assertion Ten in Chapter Eight). The worksheets were designed for students to work in class independently of both the teacher and other students.

During periods of whole-class instruction, Ray tolerated students' interactions with their neighbours. However, on one occasion during a Grade 12 class, Ray admonished a student for talking to a friend while he was addressing the whole class. The student replied "I'm just explaining to her", which indicates that she was helping her friend to understand Ray's explanation of a problem (Fieldnotes 4 May 1987, Appendix A, p. 327). On another occasion Ray expelled, for the remainder of the lesson, a student from the Grade 9 class after several of his requests to "be quiet" had not been heeded (Fieldnotes 29 August 1988, p. 6). However, these disciplinary actions were infrequent. Most of the time, Ray displayed a tolerant attitude towards the active *pockets* of socialisation which occurred in discrete parts of the classroom. Students interacting with other students during whole-class activities was tolerated rather than encouraged.

As Ray's constructivist-related teaching style developed, he stimulated more frequent participation in whole-class discussion amongst a wider range of students than previously had been the case (see Assertion Six in Chapter Five). Ray's more interactive classroom discourse consisted of asking questions of students during whole-class instruction. Sometimes a single student was asked a series of questions. Sometimes several students participated, in turn, in responding to Ray's questions. Although increased numbers of students participated in whole-class discourse, they did so as individuals rather than, in an organised pedagogical sense, as representatives of group viewpoints or interests.

Ray's whole-class questions seemed to be designed primarily for him to determine the adequacy of individual students' understandings (see Assertion Five in Chapter Five) or to stimulate students' anticipation of his subsequent presentation of new information (see Assertion Six in Chapter Five), rather than for students to engage in discussions about the rationales of their conflicting interpretations of problem-solutions or theory development. Of course, this is not to deny that the dyadic teacher-student discourses might not have served, vicariously, to engage other students in reflective self-discourse. However, the main point is that Ray's more interactive teaching style continued to emphasise the learning of mathematics in terms of an individualistic cognitive pursuit.

Although Ray's constructivist-related teaching style was more interactive, socialisation amongst students continued to occur by default, rather than by pedagogical design. During either whole-class instruction or individual seatwork activities, Ray's predominant pedagogical focus continued to be fixed firmly on students' working individually to develop their own mathematical knowledge.

During the collaborative phase of the study, Ray developed a constructivist perspective which was based on the first principle of constructivism (see Assertion Five in Chapter Five), and which seems to have reaffirmed his established rationalist conception of students' mathematical cognition. One of the main characteristics of the first principle of constructivism is its emphasis on the psychological nature of knowledge development, and its corresponding implication that each student is a cognitive isolate whose knowledge is socially relevant only to the extent that s/he has a commonly identifiable misconception. The psychological emphasis is strengthened by the metaphor of *cognition as information processing* which I employed in my discourse with Ray (see Chapter Three), and which focused on the interface between the individual student's prior knowledge and the reception of *information* emanating from external sources (see Assertion Seven in Chapter Six). Ray's constructivist perspective did not, therefore, provide a compelling rationale for the adoption of innovative instructional strategies, such as small-group learning, which require students to interact for the purpose of co-constructing new mathematical concepts.

Ray's major constructivist-related pedagogical innovations were based on his rationalist conception of students' mathematical cognition, and were designed to provide students with user-friendly information and opportunities to obtain individual teacher assistance during class. Nevertheless, Ray was aware that the successful implementation of the latter instructional strategy was dilemmatic:

> I think individual attention is fairly important in this style, and if you've got a class in the thirties, well the average student receives, maybe, an average of one minute of your attention per period, on a one-to-one basis. Well, if they get that much they're lucky because some of the time may be devoted to whole-class instruction. So, there may be thirty minutes available for situations where I can give one-to-one instruction. (Interview 7, September 1988, p. 11)

Academic Motivation: An Unconceived Teacher Role

Another significant characteristic of Ray's rationalist pedagogical perspective was his lack of interest in the socio-emotional context of learning. Ray's rejection of the notion that his classroom role could be extended to enhance students' academic motivation, especially amongst the lower achieving majority of students, is attributed to his professed belief that student self-motivation is a personality trait which is established at elementary school, and which is largely beyond his influence as a high school teacher (see Assertion Eleven in Chapter Eight). However, this belief seems to conflict with Ray's belief in facilitating students' development as independent learners (see Assertion Ten in Chapter Eight).

The apparent conflict does not exist, however, if Ray's main pedagogical interest is underpinned by a rationalist conception of mathematical cognition. Rationalism gives rise to a narrow epistemological concern with reason, and its pedagogical manifestation is a focus on the interface between human reasoning and the objective knowledge of the discipline (see Chapter Six), rather than on the emotional experiences of students as they attempt make sense of their conceptual developments in relation to intersubjectively-determined consensual domains of mathematical truth. Therefore, from Ray's rationalist perspective, teaching in accordance with a constructivist metaphor of learning was achieved by his provision of enhanced learning opportunities for the individual student. However, if the individual student is not sufficiently self-motivated to participate in these enriched opportunities then, from Ray's rationalist perspective, that is a problem associated largely with the character of the student, rather than with the role of the teacher. Generally, Ray's constructivist-oriented classroom role focused on creating a learning environment that was conducive to the interests of individual students who were adequately self-motivated.

A Brief Experiment in Social Constructivism

Nevertheless, Ray's perspective on his classroom teaching role changed from one of hard-line rationalism for a brief period in the Grade 12 class. During the collaborative phase of the study, one of the constructivist-related innovations introduced by Ray into several lessons required students to engage in small-group learning activities. Prior to this innovation I had been suggesting to Ray that he adopt this form of instruction in order to address the differential learning needs of students, rather than continue to provide generalised whole-class instruction. The rationale for my suggestion was based on a constructivist perspective which viewed knowledge construction as a personal process that is located in the mind of the individual student, rather than a public process of knowledge transmission from teacher to students (see Chapter One).

Prior to the commencement of Lesson 14 (Fieldnotes 15 June 1987, p. 1), Ray informed me that he had designed *group-based problem-solving* activities which he intended to introduce to the class. In his report on the study (Fleming, 1988), Ray explained that these activities had several purposes:

- To engage students in a cooperative decision-making process. The researcher felt that it may have enhanced cohesiveness within the class. . . . [There was] evidence that students felt that cohesiveness within the class was not as high as they would have preferred.
- To provide students with the experience of analysing a problem and recognising the steps required for its solution.
- To expose students to alternative ways of thinking — to allow them to see that other students may approach a task differently and to assess the best method from a range of options.

- As a change from individual work. Variety helps to maintain enthusiasm and interest. (Fleming, 1988, p. 58)

It appears that Ray had decided in favour of this pedagogical innovation partly because of his concern for enhancing the socio-emotional context of the classroom learning environment which students had reported as problematic (see Assertion Three in Chapter Four). In particular, Ray was responding to a perceived need to improve student cohesiveness and their enthusiasm and interest in learning mathematics.

Ray had designed two small-group activities for the topic of Linear Programming which he had adapted from a resource book, *Mathematics 1: Examples for Years 11 and 12* (Hanrahan, undated, p. 44). The following example describes the nature of the first activity:

> The students were divided into groups of five or six and individual instructions were given, one to each group member, on a slip of paper. The purpose of the individual slips was to *break the ice* within the group and to initiate the activity on a cooperative basis which would, hopefully, continue throughout the session. The student receiving the slip labelled *manager* would, at least initially, define the problem and coordinate the solution. (Fleming, 1988, p. 59)

For most of Lesson 14, however, Ray informed the class about a set of instructional procedures which he had designed in the form of a handout to assist students to solve problems associated with the current topic of Linear Programming. The final 10 minutes of the lesson was occupied by the first small-group activity which required the class to form into groups of five or six students, and to collaborate to solve a real-life problem associated with an industrial manufacturing context (see Fieldnotes 15 June 1987, p. 7). Students continued with the activity during the next lesson (Lesson 15) and, after its completion, were allocated a second small-group activity (see Fieldnotes 16 June 1987, p. 4). The entire 107 minutes of the second double lesson were devoted to this activity and to a subsequent whole-class discussion of the outcomes of each group.

My Reflections of that lesson (16 June 1987, pp. 1-5) indicate that, during the small-group activities, students engaged in the tasks with apparent enthusiasm, despite initial reservations and conceptual difficulties, and constructed solutions to the problems. The problems were solved in a social context which required students to reflect on their initial understandings of the problem, and negotiate a consensual understanding of both the problem and its solution. Within the groups, negotiations proceeded informally and spontaneously, and were unconstrained by teacher-determined procedural rules of discourse. Ray served as a consultant to groups.

The subsequent whole-class discussion provided opportunities for various solution methods to be presented by group representatives. However, because of the dominance of Ray's *teacher as interactive informer* role, there were no opportunities for students to explore the alternative solution methods. Negotiation of students' alternative solutions took the form of teacher-directed questioning which sought to elicit students' knowledge of steps in the *correct* solution method. Ray did not enable students to request justifications of the solutions of other groups, or to express reservations about their own solutions in view of other groups' solutions.

Ray assumed that the resolution of students' conceptual difficulties was the primary responsibility of the teacher, rather than of the class as a community, and sustained the role of chief validator of students' mathematical knowledge. His instructional priority was to legitimise the correct problem solutions. The activity of student negotiation was an incidental pedagogical goal.

Perceived Effectiveness

At the commencement of Lesson 16, Ray sought students' opinions on the value of the small-group activities. The following exchange between Ray and individual students ($S_1 \ldots S_7$) occurred in a whole-class forum, and indicates strong support from amongst the class for the value of the activities. However, Ray seemed more interested in expressing his lack of satisfaction with students' progress, and with their failure to implement the procedural rules which he had elaborated during Lesson 14:

Ray	How was group work? Did you enjoy it?
S_1	You get to know different people.
Ray	Mathematically, apart from social aspects?
S_1	Good.
S_2	It makes you think more, compared with the text book formula.
S_3	You can . . . incorporate all ideas.
S_4	You learn from other people's ideas.
Ray	Is it quicker on your own?
S_5	No.
S_6	Definitely not.
S_7	[On your own is] longer.
Ray	Some groups took a long time . . . [and were] inefficient, [and] not systematic. Today . . . [there is] a much simpler problem. Do it in conjunction with the rules. Some people are not using them. You don't need the rules, but I want you to notice how the steps in the [handout] parallel the rules. Refer to the rules while you're doing the problem.

(Fieldnotes 19 June, 1987, p. 1)

Ray believed that his experiment in small-group learning had been relatively successful, and he cited his perception of students' positive attitudes and student-student collaboration as evidence to support his assertion. He reported favourably on students' engagement in the activity, and judged it to be a worthwhile innovation, although he added a caveat about the inefficiency of the learning process:

> Although the activity was not very efficient in terms of the quantity of problems attempted . . . the researcher [i.e., Ray] considered the exercise to be worthwhile as it provided a variety of experiences not normally available in the maths classroom. (Fleming, 1988, p. 60).

Nevertheless, Ray abandoned this form of social-constructivist-oriented instruction, and returned to a more traditional format of individual seatwork activities interspersed with whole-class instruction. For Ray, the main problem of small-group learning activities was its inefficiency. Reflecting on the lesson, Ray complained that "only two problems had been covered" and "that students had not been systematic in their approaches to solving the problems" (Fieldnotes 19 June, 1987, p. 5).

The Predominance of Ray's Rationalist Pedagogy

Ray's syllabus-related interests prevailed, and small-group learning was abandoned for the remainder of the collaborative phase of the study, despite evidence of its efficacy after only a brief trial. The abandonment of small-group learning was permanent; there was no evidence of it in Ray's Grade 9 class in the following year. Almost a year later, Ray reflected on the ephemeral nature of his interest in this form of learning:

> It was just an idea I had. It wasn't a strong permanent commitment to that style of teaching. Although, I think it was quite a useful exercise to do those fairly complex problems in a group so that some of the students could see how others nutted them out. Whereas, doing simple one-line . . . problems in factorisation I couldn't really see much benefit in doing that in a group situation. Despite the fact that I know students work in pairs and they help one another.
>
> (Interview 7, September 1988, p. 13)

The time-consuming nature of Ray's group-based problem solving instruction seemed to compromise his *teacher as controller* and *teacher as interactive informer* classroom roles (see Assertion Six in Chapter Five and Assertion Ten In Chapter Eight). However, this chapter has provided evidence that one of the main reasons for Ray's abandonment of small-group learning was the failure of my theory of pedagogical reform to address Ray's rationalist preconception of the nature of mathematical cognition. During the collaborative phase of the study, my discourse with Ray was founded on a constructivist perspective which emphasised a pedagogical interest in the cognitive activities of the individual student (see Chapter Three). The rationalist nature of this *personal constructivist* perspective on science education has been criticised by Solomon (1987) and abandoned by Bruner (1986). In mathematics education, cognitive constructivist interests have been subsumed by social constructivist pedagogical perspectives in the research programs of Bauersfeld (1988), Wheatley (1991), and Cobb (1989).

My cognitive constructivist perspective did not address important social and emotional dimensions of mathematical knowledge construction. This rationalist epistemological perspective was ill-equipped to capitalise on the positive results of Ray's brief experiment in social constructivism, and did not provide a compelling rationale for Ray to sustain a pedagogical interest in the social context of learning mathematics, and in the facilitation of students' intersubjective negotiation of their mathematical knowledge. Although small-group learning seemed to both Ray and me to be a sound pedagogical idea, the explicit social-constructivist-related rationale which was required to argue its case, especially in relation to Ray's conflicting priority interest in *covering* the syllabus, was unavailable to us during the collaborative phase of the study. A rationalist pedagogy which was based on a predominant and preconceived interest in individual student cognition remained as the preferred *default option* for Ray's constructivist-related pedagogical reforms.

SUMMARY

This chapter presents a social constructivist interpretation of key aspects of Ray's reconstructed pedagogy in an attempt to identify the constraining influence of the theory of pedagogical reform of this study on Ray's pedagogical transformation during the collaborative phase of the study.

The review of research literature, which serves as an interpretive framework for the chapter, reveals that recently articulated metaphors of *learning as social construction* emphasise the centrality of social and emotional influences on cognition. On the one hand, social and emotional influences can act in concert with rationalist beliefs, and contribute to (1) the perpetuation of objectivist conceptions of the nature of mathematics, and (2) the maintenance of pedagogies which sustain the centralist role of teachers, especially as the sole validators of scientific or mathematical truth. On the other hand, social and emotional influences can be harnessed pedagogically in the interests of promoting an explicitly non-rationalist, dialectical relationship between individual knowledge and the cultural knowledge of the classroom community.

From a social constructivist perspective, the intersubjective process of negotiation, which both constrains and validates community knowledge generated by professional scientists and mathematicians, is a feasible analogue for classroom social negotiations. From a pedagogical perspective, interpersonal negotiation amongst students can promote explanation and justification of personal ideas, and can serve to involve the class as a community of learners who seek to construct personally viable meanings in relation to the range of perspectives generated by other students. The teacher's role is to provide learning activities that stimulate students' to undertake inquiry or problem centred learning, promote a primary emphasis on solution methods, rather than correct answers, and facilitate classroom discourse that elicits a full range of students' personally constructed meanings, and negotiates consensual views that are congruent with the views of wider society.

However, this chapter argues that the *cognitive constructivist* perspective, which served as an epistemological referent for my discourse with Ray during the collaborative phase of the study, was founded on a rationalist conception of mathematical cognition. Students' conceptual development was perceived to be an individual cognitive activity which occurred in the contexts of an emotional vacuum and relative social isolation. During the collaborative phase of the study, this constructivist perspective served to sustain Ray's preconception of the rationalist nature of mathematical cognition, and reaffirmed his view that students' social and emotional development were issues which lay beyond his role as a classroom teacher. Furthermore, it underpinned his belief that classroom social interactions amongst students were activities which should be tolerated in the interests of social harmony, rather than incorporated into teaching activities. Ray's cognitive constructivist pedagogical perspective was highly congruent with his idealisation of students as independent learners, and focused his constructivist-related pedagogical innovations on the learning activities of the individual student.

Nevertheless, in the interests of enhancing the socio-emotional learning environment, Ray experimented with small-group learning activities in the Grade 12 class. Although the small-group activities were experienced positively by both the teacher and students, the largely successful pedagogical experiment was discontinued permanently because of Ray's perception of its inefficiency in relation to his syllabus-related priority interests. However, this chapter argues that one of the main reasons for Ray's abandonment of small-group learning was the lack of a compelling social-constructivist-related rationale for (1) enhancing Ray's perception of the significance of the positive results of the experiment, and (2) providing a better understanding of the dynamics of whole-class discourse for facilitating students' intersubjective interpretations of their mathematical experiences. The cognitive constructivist pedagogical perspective which underpinned the theory of pedagogical reform in this study was unable to provide this rationale.

CHAPTER EIGHT

A CRITICAL CONSTRUCTIVIST PERSPECTIVE ON RAY'S TECHNICAL RATIONALITY

INTRODUCTION

Chapters Six and Seven examine Ray's pedagogical reforms from radical and social constructivist perspectives, respectively. It is argued that the constructivist-related theory of pedagogical reform which shaped my discourse with Ray during the collaborative phase of the study failed to account for key epistemological problems associated with Ray's objectivist preconceptions of the nature of mathematics (see Assertions Seven and Eight) and his rationalist preconception of mathematical cognition (see Assertion Nine). As a consequence, Ray's reformed pedagogy continued to be conceptualised largely in terms of the transmission-oriented epistemological conceptions of *teacher as interactive informer* and *students as cognitive isolates.*

Chapter Eight adopts a critical constructivist perspective and examines the constraining role of Ray's technical rationality on his pedagogical reforms. In particular, the chapter focuses on the rationalisation of the conflict between Ray's constructivist-related ideals and his technical curriculum priority interest in *covering* the externally mandated syllabus. The results of interpretive analyses of Ray's pedagogy are presented in the form of Assertions Ten and Eleven.

The first section constitutes the interpretive framework of the chapter, and presents a review of research literature in the field of critical curriculum theory. The review discusses the pervasiveness of the professional *culture of positivism* in prefiguring the epistemological interests of modern curriculum developers and classroom teachers. The review focuses on Jurgen Habermas' *theory of human cognitive interests,* and discusses the hegemonic nature of the ideology of *technical rationality* which constitutes a modern form of positivism which is embedded in the structural foundations of education, and which is associated with an interest in control. Finally, the prospects are discussed of a *critical constructivist* perspective on curriculum for emancipating teachers and students from the disempowering constraints of technical rationality which underpins the concept of curriculum-as-product, promotes an objectivist epistemology, marginalises the professional authority of teachers, and defines students' classroom roles in terms of intellectual passivity and social compliance.

The second section discusses Ray's rationalisation of the conflict between his constructivist-related ideal of facilitating student autonomy and his technical curriculum priority interest in maintaining control of the classroom learning environment for the purpose of *covering* the syllabus. Assertion Ten argues that Ray's belief in student autonomy was subsumed by his technical rationality and resulted in a practically viable conception of student autonomy which was congruent with his prevailing classroom role of *teacher as controller*.

The third section argues that, although Ray's technical curriculum interests prevailed, they continued to conflict with his constructivist-related ideals of facilitating students' autonomy and learning with understanding. Assertion Eleven argues that Ray's personally constraining beliefs in the nature of students and the professional culture of teaching largely reconciled the conflict in favour of his technical curriculum interests. These beliefs helped to rationalise the ascendency of Ray's *teacher as controller* classroom role which denied many students the opportunity for autonomy and enhanced conceptual development, and disempowered Ray from believing in the possibility of a participatory role in deconstructing the technical rationalist culture of professional education.

INTERPRETIVE FRAMEWORK

There is a growing perception in the field of curriculum studies that the profession of education is failing to meet the evolving needs of modern post-industrial western society because the *modus operandi* of education continues to be underpinned largely by an anachronistic *technical rationality*. It is argued that technical rationality, which continues to thrive in the professional institutions of western society, is a legacy of outmoded *positivist* philosophies of science which have dominated the intellectual and economic development of the industrialised world during the nineteenth and twentieth centuries (Apple, 1990; Giroux, 1981, 1983; Habermas, 1970, 1972, 1978, 1984; Schon, 1983). The following review of the educational research literature illuminates the constraining impact of positivism on professional practice in education, and argues for a *critical constructivist* perspective on pedagogical reform.

The Decline of Positivist Science

The philosophy of positivism became the dominant form of scientific rationality during the nineteenth century. Positivism is an epistemology which asserts that only empirical scientific rationality can result in certain knowledge about objective reality. In this regard, positivism shares with rationalism (see Chapters Six and Seven) a *correspondence theory of truth*, that is, that scientific and mathematical knowledge can achieve the status of absolute, or universal, truth. Whereas rationalism asserts that reason is the only basis of valid knowledge of reality, positivism, in the form of empirical scientific inquiry, regards observation and experimentation to be central to discovering and describing the natural laws of the universe (Abercrombie, Hill, & Turner, 1984; Reber, 1985).

As early as the 1930s, the Institute for Social Research (or the Frankfurt School) regarded positivism to have distorted science (Giroux, 1983). It was argued that positivism had subordinated reason to the extent that scientific knowledge was regarded as the only reliable form of knowledge. This subordination had separated science from the question of ends and ethics, and subsumed science within an empirical and quantitative methodology which served as an instrument for the economic and technological growth of modern industrialised society. The resultant *technocratic* view of science rejected subjectivity, and prevented science from adopting a self-critical stance in relation to its own ideological presuppositions. In other words, positivism had colonised reason:

> Questions concerning the genesis, development and normative nature of the conceptual systems that select, organise and define the facts appear to be outside the concern of positivist rationality. Since it recognises no factors behind the fact, positivism freezes both human beings and history. . . . By not reflecting on its paradigmatic premises, positivist thought ignores the value of historical consciousness and consequently endangers the nature of critical thinking itself.
>
> (Giroux, 1983, pp. 13-14)

A Habermasian Perspective

The discreditation of positivism as a viable philosophy of science resulted from widespread criticism amongst philosophers of science during the second half of the twentieth century. The philosopher Jurgen Habermas joined the debate as an ardent critic of positivism. Habermas (McCarthy, 1985) argues against the *scientistic and objectivist illusion* of positivism which reifies scientific knowledge by assuming that the *discovered* laws of the natural world are independent of the thinking subject who conceives the scientific problem, the method, and the experiment, and creates the knowledge. From a positivist perspective, scientific rigour is attained by eliminating human judgement:

> One half of the underlying assumption [of positivist science] is that knowledge is always reducible to the totality of discovered properties of the object world. The other half is that the *subject* — the actor, the creator, the knower, the inventor, the scientist — is at worst a pollutant in his own purely objective world, or at best, a ghost in the machine of science and that must be methodically controlled and, so far as is possible, eliminated. (Pusey, 1987, p. 21)

The critical social theory of Habermas challenges the fundamental tenets of positivism in three ways. First, Habermas identifies the central role of subjectivity in constituting and validating knowledge, especially scientific knowledge derived by empirical means. In a Kantian sense, "knowledge is necessarily defined both by the objects of experience and by a priori categories and concepts that the knowing subject brings to every act of thought and perception" (Pusey, 1987, p. 22). This view is in close accord with the radical constructivist position of von Glasersfeld (see Chapter Six).

Second, Habermas departs from the individualism of Kant (and the personal constructivist position — see Chapter Seven), and identifies the sociocultural and historical contexts of knowing. From a sociocultural perspective, Habermas aims to show that "there is no knower without culture, and that all knowledge is mediated by social experience" (Pusey, 1987, p. 23). Indeed, knowing and understanding are intersubjectively grounded in the patterns of ordinary language usage which are shared in everyday communication. This view is congruent with social constructivist perspectives (see Chapter Seven). Thirdly, and most distinctively, knowing in all domains, including science, constitutes a rational process of critically reflecting on the problematic nature of extant knowledge, rather than on nature itself.

In addition to Habermas, both the subjective and intersubjective constitution of scientific knowledge is emphasised by modern philosophers of science, such as Popper, Toulmin, Kuhn, and Lakatos (Egar, 1989; Nussbaum, 1990; Phillips, 1987; Pitt, 1990). Although important differences exist between the views of these philosophers, especially in regard to the ontological status of scientific knowledge (Lakatos & Musgrave, 1970; McCarthy, 1985), elaborating these differences is beyond the scope of this thesis.

Nevertheless, central to critiques of positivism is the premise that empirical inquiry is conducted within an interpretive frame of reference, and that scientific knowledge is constructed rather than discovered — observation is *theory-impregnated* and is made meaningful and significant from within a biologically and culturally conditioned *horizon of expectations* (McCarthy, 1985, p. 45).

Constructivism: An Alternative Epistemology

In education, the traditional epistemology of positivism has become less popular since critical debate about the benefits of the alternative epistemology of constructivism has raised general awareness of the methodological and pedagogical limitations of traditional epistemological assumptions, especially in the related fields of educational research and science education.

Educational Research

Educational researchers have recognised that the coupling of *psychological behaviourism* and the philosophy of positivism created a scientific rationality which dominated educational research and curriculum theory until the late 1960s. Keeves (1988) describes the subsequent historical shift of emphasis in educational research that has continued since the 1970s when the supremacy of the *rationalist* paradigm (Guba & Lincoln, 1985) began to be challenged by *critical theory* and the *humanistic* perspectives which have originated from the newly reconstituted social sciences:

> In the humanistic or *naturalistic* approach . . . it is assumed that there exist multiple realities which, in the main, arise from the constructions generated in the minds of investigators. The observations made by different investigators are dependent on the different meanings and interpretations they ascribe to reality.
>
> (Keeves, 1988, p. 18)

Chapter One of this thesis discusses the *interpretive* research approach which was employed in this study. This approach is based on epistemological assumptions that are congruent with naturalistic, or constructivist, perspectives on the development of knowledge in the social sciences, and which are antithetical to a positivist epistemology.

Science Education

Nussbaum (1990) argues that, in science education during the 1960s, the dominant form of scientific rationality was influenced strongly by empiricist and rationalist views of the nature of scientific knowledge. For the behaviourists (or empiricists), incorrect learning outcomes were labelled *misconceptions*, and their source was attributed to faulty observations or inductive procedures. For the adherents of the Piagetian theory of developmental stages (or Kantian rationalists), incorrect learning outcomes also were labelled misconceptions, and their source was attributed to either the cognitive immaturity of the individual student or the incorrect application of formal logical operations.

According to Nussbaum (1990), the emergence of constructivist approaches to instruction in school science, which occurred during the 1970s, is ascribed to a generally increased awareness of Kuhn's (1962) social constructivist philosophy of science, and to the coupling of cognitive psychology with research on students' conceptions and conceptual change. In science education, constructivism has emerged as a popular philosophy that is antithetical to positivism:

> Thus, Constructivism replaced the *absolutism* of the Empiricist and Rationalist traditions. Constructivism presupposes that theory precedes observations, and that observations can be selected and conducted only through theoretical expectations. . . . Therefore our own conceptual theories determine how we perceive the world. . . . [H]owever, we can always break out of our framework, by constructing a new one. (Nussbaum, 1990, p. 281)

Chapter One of this thesis discusses the constructivist perspective which underpins the theory of pedagogical reform which I employed in this study, and which is the subject of critique in Chapters Six, Seven, and Eight. This constructivist perspective was adapted largely from science education where constructivist approaches to instruction provide a *student-centred* perspective on the nature of incorrect learning outcomes, and construe students as "active thinkers who construct personal meaning which, in turn, can help them to form conceptual frameworks, or paradigms" (Nussbaum, 1990, p. 285).

Positivism: The Culture of Modern Education

Despite the apparent demise of positivism as a viable philosophy of science (Bernstein, 1976), the growing interest in humanistic and critical inquiry among educational researchers, and the increasing orientation towards constructivist epistemologies in science education and mathematics education, positivist rationality continues to exert a strong influence over the field of education, in general, and classroom practice, in particular.

Positivism and School Science

Research has demonstrated that modern school science continues to be underpinned by a positivist view of science which is embedded in textbooks and curricula. Cawthorn and Rowell (1978) analysed school science textbooks and concluded that science texts promote implicitly an objectivist image of scientific knowledge. Many texts imply that the historical discovery of scientific knowledge had occurred by means of the application of Francis Bacon's inductivist *scientific method,* which is a "well-defined quasi-empirical process" that comprises "observation and experiment; inductive generalization; hypothesis (the formulation of general scientific statements or laws); attempted verification; proof or disproof; objective knowledge" (p. 33).

Pitt (1990) argues that in school science the prevailing practice of teaching the historical record of scientific knowledge as certain truths, the continued separation of the sciences in school curricula, and the lack of emphasis on the social activities of modern scientists, combine to promote a positivistic image of science and, therefore, a reductionist view of scientific knowledge.

The prevalence of the *positivist ideal* in school science is highlighted also by Michael Apple's (1990) criticism of most primary and secondary school science programs which present a *consensus theory of science* that ignores the serious disagreements, controversies, competition and conflicts amongst the scientific community which fuel the progress of science. The standard of *vulgar objectivity* promoted by school science masks the personal and political influences that shape the activities of the scientific community, and leads students towards adopting a neutral political stance in relation to the role of science in society.

The Structural Foundations of Education

The prevalence of positivist rationality is not restricted, however, to the narrow domain of school science. Extensively argued claims of the pervasiveness of positivism in the structural foundations of the field of education help to explain its predominance as an epistemology of classroom practice. A brief discussion of the claims of Donald Schon (1983), Henry Giroux (1983, 1981), and Shirley Grundy (1987), serves to illuminate both the pervasive and problematic nature of technical rationality in relation to educational practice, especially its role in promoting the conception of *teaching as control.*

Schon's Technical Rationality. According to Schon (1983), *technical rationality* extends beyond the confines of the classroom and serves as the dominant epistemology of practice of the education profession, in particular, and of the major institutionalised professions of Western society, in general. Historically, technical rationality *colonised* the professions as they obtained entry into the nineteenth and early twentieth century universities which harboured an ethos of technical rationality for the pursuit of scientific inquiry.

Schon (1983) argues that, towards the end of the twentieth century, the predominance of technical rationality as an *epistemology of professional practice* has resulted in a crisis of confidence and legitimacy amongst the major professions — law, medicine, engineering, education — which are unable to provide sufficient accounts of the artistic and intuitive actions of their practitioners in dealing with increasingly important nontechnical tasks that are characterised by their complexity, instability, uncertainty, and conflicting values (Richardson, 1990).

Technical rationality underpins an *instrumental* form of professional practice which is characterised by values of control, distance, and objectivity, and by interests in the rigorous application of expert knowledge, with its basis in scientific research, to the solution of well-defined problems of practice. As an epistemology of professional practice, however, technical rationality has little to offer in the context of *problem setting,* that is, in the subjective, ambiguous, and uncertain *design* process of defining problems and determining the ends to be achieved.

In the case of education, Schon (1983) argues that schools have been organised traditionally to serve the priority technical interests of transmitting *privileged* knowledge. Teachers have been cast in the role of technical experts who, like other professionals, expect their *clients* to be deferential, respectful, and willing to accommodate, unquestioningly, the authoritative expert's professional prescriptions. Teachers' autonomy, however, is little more than mythical. Their authority has been based on their role as agents of a locus of control that resides external to the classroom. A *dynamically conservative* hierarchical system of bureaucratic controls, buttressed by notions of objectivity and quantitative measures of performance, constrains teachers to perform as functionaries, and marginalises their authority. The spatial and temporal organisation of schools serves to control the classroom actions of both teachers and students in the interests of the efficient transmission of expert knowledge.

According to Schon (1983), teachers are largely excluded from the fundamental design process of defining the nature of the educational problem which professional practice is to address, and of determining the means for achieving its resolution. Teachers are required to adopt the narrowly defined technical role of implementing the curriculum *means* that have been designed to achieve predetermined teaching-learning *ends*. Furthermore, the technical epistemology of professional educational practice provides little scope for its critical examination, especially by teachers within schools. Conflicts of interest between the humanistic values of teachers and the technical values of the institution are resolved largely within the prevailing framework of technical rationality which serves as the constraining standard of normality:

> Curriculum and lesson plans, as well as measures of performance and rewards and punishments, emanate from a center and are imposed on teachers at the periphery. . . . In the control of both students and teachers, a high priority is placed on *objectivity*. It is considered important to achieve quantitative measures of proficiency and progress which are independent of individual judgements. . . . Quantitative measures permit the system of control, and the other systems that depend on it, to take on an appearance of consistency, uniformity, precision, and detachment. (Schon, 1983, p. 330)

Giroux's Technocratic Rationality. Giroux's (1981) perspective on the development of the field of education during the twentieth century is similar to Schon's foundational view. The establishment of a firm scientific foundation for the discipline of education was achieved by importing a positivist rationality from allied disciplines:

> Drawing heavily upon a positivist rationality that had become the dominant theoretical underpinning of allied disciplines such as sociology, psychology, and the natural sciences, American educational theory and research became firmly entrenched within an instrumentalist tradition that defined progress as technological growth and learning as the mastery of skills and the solving of practical problems. (Giroux, 1981, p. 5)

In the case of teacher education, Giroux (1981) describes the prevailing *culture of positivism*, or *technocratic rationality*, in terms of a metaphor of *school as industry* which portrays schools as factories and students as raw products. The main implication of this conception is that the implicit purpose of teacher education is to prepare prospective teachers for teaching roles designed to meet the economic imperatives of an industrial society which requires a continual input of socialised and technically proficient young workers. One of the main characteristics of technocratic rationality in teacher education is a definition of society that emphasises consensus, equilibrium, and order, and avoids addressing issues of social conflict, power, control, and competing socio-economic interests. The socialising effect of this *overly integrated* perception of society is to passify both teachers and students. Giroux (1981), after Lundgren (1979), criticises the disempowering nature of the predominant *cultural reproduction* interests of teacher education which promote social conformity rather than criticism and, consequently, silence the intellectual authority of students' own voices:

> Absent from [a cultural reproduction] view is any attempt to question the nature of and quality of the society that teacher-education programs so cheerfully support. . . . [T]he concept of socialization supports a passive view of students and an overly integrated perception of society. . . . Knowledge in the technical rationality view is defined and used so as to be separated from the lived histories and biographies not only of teachers but of students. . . . This means that teachers are trained neither to recognize nor use the cultural capital of others as a central category for dialogue and personal affirmation in their teaching. The result is a form of pedagogical violence that prevents teachers from establishing conditions which allow students to speak with an authentic voice. (Giroux, 1981, pp. 152-155)

Grundy's Technical Curriculum Interest. Grundy's (1987) critical analysis of teachers' curriculum reform attempts adopts, as an interpretive framework, Habermas' (1972) theory of human knowledge-constitutive interests, and reveals the predominance of the *technical cognitive interest* in determining the nature of modern school curricula and classroom practice. According to Habermas, there are three fundamental human cognitive interests — the *technical* interest, the *practical* interest, and the *emancipatory* interest — which give rise to three distinctive forms of knowledge-constitutive action — empirical-analytic, historical-hermeneutic, and critical, respectively.

The technical cognitive interest arises from a fundamental need of the human species to survive and reproduce itself, and is manifested in attempts to control and manage the environment. The technical interest underpins the empirical-analytic sciences which are concerned with discovering natural laws, and formulating *instrumental* means of action designed to attain predetermined and quantifiable ends, not only in relation to technological development, but also in social spheres of action such as education. The technical interest is an interest in control:

> [T]he technical interest is: a fundamental interest in controlling the environment through rule-following action based upon empirically grounded laws. . . . [T]he objectives model of curriculum design is informed by a technical cognitive interest . . . an interest in controlling pupil learning so that, at the end of the teaching process, the product will conform to the *eidos* (that is, the intentions or ideas) expressed in the original objectives. (Grundy, 1987, p. 12)

According to Grundy (1987), the *technically informed curriculum,* which comprises predetermined content, or learning objectives, is designed to be implemented in learning environments in which students' behaviour and learning are strongly controlled by the teacher. Students have little power to determine their own learning objectives and, together with the teacher, are caught up in a seemingly inexorable process of attaining predetermined learning outcomes of a *product oriented* curriculum. The technical interest not only prefigures the form of the curriculum process, and objectifies students, but promotes also an objectivist form of knowledge.

In the resulting curriculum culture of positivism, knowledge is viewed as "sets of rules and procedures or unquestionable truths. Knowledge is regarded as a commodity, a means to an end" (Grundy, 1987, p. 34). This instrumentalist view of knowledge leads to externally controlled *agricultural-botanical* and *industrial* models of curriculum evaluation (Lawton, 1980) which are designed to measure the learning product. Not only do these technical forms of evaluation break the intimate nexus between learning and assessment, but they define teachers' classroom practice in terms of the quantitative products of their actions, and promote a mechanistic, skills-based view of teaching, which leaves little room for moral judgements or critical analysis. Because the "power both to determine and to judge what teachers and learners must do is vested elsewhere" (Grundy, 1987, p. 38), the technically informed curriculum removes the control of the teaching-learning process from teachers and students, and teachers become de-skilled pedagogically.

According to philosophers of science, the influence of positivism as a philosophy of science declined during the twentieth century. However, according to critical curriculum theorists such as Schon, Giroux, and Grundy, modern positivism, in various forms of technical rationality, has remained well-entrenched as a foundational philosophy in the field of education, and as an epistemology of professional practice. An important question for constructivist-related pedagogical reform, therefore, concerns the prospects of removing the stranglehold of technical rationality on educational thinking and practice. The resilience of positivism, therefore, is the next important issue to consider. In this regard it is helpful to examine the notion of positivism as an ideology which instils in its adherents a sense of a natural and ineffable world view.

Technical Rationality: A Hegemonic Ideology

In the context of examining the implications for curriculum of *positivist ideology*, Grundy (1987), after Habermas (1970), identifies *ideology* as a set of ideas of a powerful group in a culture which dominate the perceptions and actions of the majority of less powerful members of the culture. Ideology has political overtones to the extent that the interests and ideas of one group have power to determine the thinking of other members of the group. Of particular interest is the means by which the ideology of the dominant group *colonises* the minds of the majority.

Grundy (1987) adopts Gramsci's (1971) concept of *hegemony* to describe the dominance or imposition of the ideology of a powerful group, and its unquestioned acceptance by less powerful members. The imposition of ideology is not necessarily consciously enacted, in an overtly conspiratorial sense. Rather, the imposition takes the form of promoting, as commonsense understandings, specific interpretations of (physical and social) reality that are congruent with the shared interests of the dominant group. The conspiracy theory of imposition is invalid to the extent that the members of the dominant group also have been subjected to an historical process of ideological hegemony, and are acting in concert to sustain a shared frame of reference whose parameters are located in their own subconsciousness. It is more accurate to claim, therefore, that the prevailing interests of the dominant group prefigure covertly which interpretations of the experiences of the less powerful majority will be given credence, or validity. Apple (1990), after Gramsci (in Williams, 1976), describes the colonising nature of ideological hegemony:

> [H]egemony acts to *saturate* our very consciousness, so that the educational, economic and social world we see and interact with, and the commonsense interpretations we put on it, becomes the world *tout court*, the only world. . . . It refers to an organised assemblage of meanings and practices, the central, effective and dominant system of meanings, values and actions which are *lived*. (Apple, 1990, pp. 5, 6)

Hegemonic ideology constitutes a dominant interpretive framework, or world view, which shapes the social conscience of cultural groups. Although its existence might remain undetected in the subconsciousness of the individual, ideology shapes the individual's fundamental beliefs and values, and gives rise to particular sets of actions which are validated in relation to established social norms. The hegemony of ideology is complete when the set of ideas, values, and associated actions are experienced by the individual as *natural*, rather than as culturally constructed. A powerful hegemonic ideology, therefore, masks the constructed nature of patterns of conventional beliefs and actions, commands an ethos of social conformity and rejects, as unnatural, alternative beliefs, values, and actions, especially those that threaten to disrupt the established social order:

> It is the trick of ideology to make that which is cultural, and hence in principle susceptible to change, appear natural, and hence not open to change at all. So cultural constructions are represented as natural laws. . . . [I]t is natural that a secondary school day should comprise eight forty-minute periods. (Grundy, 1987, p. 107)

The cognitively embedded nature of ideology has been attested to by cultural anthropologists. Clifford Geertz (1973) describes the concept of ideology as *schematic images* of social order which determine the social interactions amongst members of a culture, and which define social consciousness, or reality. Ideologies provide mental frameworks for making sense of social situations, and make it possible to act purposefully within them. Ideologies constitute patterns of belief and value, and command a strong attitudinal commitment from their adherents:

> Whatever else ideologies may be . . . they are, most distinctively, maps of problematic social reality and matrices for the creation of collective conscience. (Geertz, 1973, p, 220)

In Karl Mannheim's view (Feinberg, 1975), ideology is a set of organising principles that evolve out of attempts to understand the social world. Ideology is linked, therefore, to the individual's personal history of ideas and experiences, and provides him/her with a reliable basis for understanding the present and predicting the future. However, ideology does not exist as a network of logical propositions that can be examined readily. Rather, ideology exists in the form of *images* which constitute the individual's reality. From an outsider's point of view the individual's ideology might seem to be only one imaginative construction among many possible constructions, but from the individual's point of view the image is experienced not as an image but as an accurate description of reality. The individual's persistent and *quasi-sacred* image of the social world serves as a *mental grid* of the possible and the desirable, and constrains the range of actions that the individual deems to be sensible:

> To uncover an ideology is to reveal what people have accepted as possible and thereby to reveal the limits that they have placed on their own actions. (Feinberg, 1975, p. 6)

The concept of hegemonic ideology highlights the importance of understanding the historical role of professional institutions, such as universities and schools, in promoting covertly the ideas and interests of dominant groups in society, determining the social consciousness of teachers and students, and constraining the classroom social interactions which constitute the process of knowledge construction.

The hegemonic and ideological nature of technical rationality has been a central focus of recent critiques of modern educational theory and practice. Schon (1983, p. 34) argues that the *intellectual hegemony* of positivism constrains institutionalised professions, such as education, to serve predominantly technical interests. Giroux (1981, pp. 37-59) criticises the *cultural hegemony* of positivist rationality which underpins modern curriculum development, teacher education programs, and classroom teaching, and argues that its hegemony is extended through the *hidden cultural messages* that shape the social classroom roles of students. Apple's (1990, p. 1-25) critique of the role of technical ideology in curriculum identifies teachers as agents of *ideological hegemony* whose classroom actions give legitimacy to positivist epistemologies and serve to perpetuate the reproduction of the extant inequitable social order of society.

A Critical Constructivist Perspective

In an attempt to develop a theory of pedagogical reform aimed at overcoming the stranglehold of technical rationality, which underpins the predominant Tylerian (1949) *objectives* model of *curriculum-as-product*, Grundy (1987) adopted Habermas' (1972) *theory of knowledge-constitutive interests*, and analysed the curriculum conceptions of teachers who were engaged in pedagogical reform. Grundy's (1987) analysis indicates that teachers conceive of curriculum according to the nature of their dominant cognitive interests — technical, practical, or emancipatory — and that each interest gives rise to a distinctive type of pedagogical rationality. Whereas a technical interest gives rise to the objectivist conception of curriculum-as-product, discussed above, practical and emancipatory interests give rise to alternative conceptions of *curriculum-as-practice* and *curriculum-as-praxis*, respectively.

Practical Curriculum Interest

Grundy's (1987) practical curriculum interest conceives of knowledge and curriculum as a process of social interaction aimed at generating understanding, or sense-making, and gives rise to a conception of curriculum-as-practice. At the centre of a curriculum informed by practical curriculum interests is a concern with the *intersubjective* process of interpretation, personal judgement, and reflective deliberation, rather than with the objective products of learning. This curriculum is concerned with human interactions in which teacher and students are regarded as subjects engaged in sense-making activities. The teacher is interested in developing the judgement-making skills of his/her students, as part of an overall goal of personal development and improvement. The curriculum content is justified in terms of moral criteria associated with the *good* of students, and is selected on the basis of facilitating interpretation and meaning-making, rather than rote learning of pre-specified skills. The teacher is an interpreter of curriculum documents, rather than a technical implementer, and is an active participant in curriculum decision-making:

> The teacher whose work is informed by practical judgement will be concerned that the interactions of the classroom environment provide appropriate opportunities for learning. The learning moment *is its own end.* (Grundy, 1987, p. 63)

Grundy's (1987) practical curriculum interest is consonant with the constructivist perspective which underpins the theory of pedagogical reform in this study (see Chapter One). Both interests promote a primary pedagogical focus on the cognitive state of the individual learner (whether teacher or student), and both are concerned with enabling the learner to make sense of their learning experiences. The main difference between the two interests is that Grundy's practical curriculum interest focuses also on the role of negotiation in the intersubjective construction of knowledge. In this respect, the practical curriculum interest is highly congruent with the social constructivist perspective discussed in Chapter Seven, which reveals also that the constructivist-related theory of pedagogical reform of this study failed to address adequately the important social context of students' knowledge development activities.

According to Grundy (1987), however, the attainment of practical curriculum interests constitutes a limited pedagogical reform goal which is likely to be frustrated by the predominance of teachers' extant technical curriculum interests. Furthermore, the full realisation of practical curriculum interests is a necessary, but insufficient, condition for the development of teachers and students as autonomous intellectuals who are free from the unduly constraining influence of the hegemonic ideologies which underpin modern school curricula and which distort classroom communication.

Emancipatory Curriculum Interest

Grundy's (1987) emancipatory curriculum interest aims to transcend the dialectical relationship between technical and practical curriculum interests. At the centre of a curriculum informed by an emancipatory interest is a counter-hegemonic concern for liberating teachers and students from the disempowering constraints of the ideology, or dominant ideas and values, that underpins the curriculum and defines the social norms of the classroom.

However, the hegemonic nature of ideology makes this a very difficult task. Ideology constitutes teachers' and students' subjectivities, and serves as an invisible interpretive framework for making (restricted) sense of the teaching-learning process, and of the prospects of changing the *natural order* of the classroom. Ideology is subtly but effectively propagated in the *hidden curriculum* of rituals which socialise teachers and students into the dominant order of schooling (McLaren, 1986). To the extent that the prevailing ideology remains unrecognised, teachers and students will be unable to discern its socially constructed nature and will remain disempowered and unwitting agents of its propagation.

According to Grundy (1987), emancipation requires the development of a *critical consciousness* which (1) recognises the culturally constructed nature of the education enterprise, and (2) fuels debate about the fundamental assumptions and taken-for-granted interests that underpin the culture of teaching and learning, and that give rise to social inequalities and inequities. In practice, teachers and students share the locus of control of their mutually constructed and intersubjective knowledge, and struggle collaboratively to make sense of both the perceived world and the ideology that constrains their perceptions and actions. Thus, emancipation commences with *enlightenment* about the nature of ideological distortion, and continues with political action aimed at reforming the social structures that constrain the emancipatory interests of teachers and students.

To what extent is it possible for a pedagogical reform process to liberate teachers and students from the pedagogically distorting influence of a pervasive ideology that promotes a culture of schooling based on technical curriculum interests? Grundy acknowledges that emancipation might not be an immediately attainable pedagogical reform goal, and that the development of a critical consciousness amongst teachers and students is the most feasible goal in the shorter-term. The process of liberating students and teachers from the *false consciousness* which distorts the constitutive communications of the teaching-learning process is a collaborative enterprise in which teacher and student roles are expanded to incorporate the metaphors of *teacher as student* and *students as teachers.*

In *critical pedagogical practice* — a form of *action research* operating in an emancipatory mode — teachers and students not only engage actively in processes of reflective deliberation, personal judgement-making, and interpretation, in accord with practical curriculum interests, but also they bring a critical focus to bear on their social interactions and social contexts of learning. With recourse to *critical social theories* teachers and students participate in ideology critique for the purpose of identifying "the constraints imposed upon their practices by social structures and interactions which are informed by interests in domination and control" (Grundy, 1987, p. 146). In these new roles, teachers and students move strategically towards sharing authority and control in relation to the social construction of their own subjective and intersubjective knowledge.

The hegemonic ideology of technical rationality, which underpins much of the professional practice of modern education, and which prefigures teachers' technical curriculum interests, requires a *critical constructivist* perspective on pedagogical reform. A critical constructivist perspective combines practical and emancipatory curriculum interests, and focuses pedagogical attention on the nexus between the subjective and intersubjective constitution of the classroom learning environment. In relation to pedagogical reform, a critical constructivist perspective adopts not only the radical and social constructivist perspectives on learning which are discussed in Chapters Six and Seven, but also addresses the constraining ideology which shapes the curriculum and prefigures covertly the objectivist epistemology of much professional practice. A critical constructivist perspective on pedagogical reform is concerned with facilitating teachers' (and students') realisation of the *hidden curriculum* of professional practice which they perpetuate unwittingly, and which serves to disempower them from developing as autonomous intellectuals.

TEACHER AS CONTROLLER: A PRIORITY TECHNICAL INTEREST

Previous chapters have focused on the conflict between Ray's constructivist-related and transmission-oriented epistemological beliefs, especially the role of his objectivist preconceptions of the nature of mathematics and rationalist preconception of mathematical cognition in refining his centralist classroom role. However, another major source of influence constrained the nature and scope of Ray's pedagogical reforms, namely, Ray's technical curriculum priority interests.

This section adopts a critical constructivist interpretive framework which argues that objectivist epistemologies and technical curriculum priority interests are complementary components of a widespread technical rationality of professional practice. The interpretive framework is employed to analyse further the process of rationalisation of Ray's conflicting pedagogical beliefs and to generate a better understanding of the major constraints to constructivist-related pedagogical reform.

In Ray's ideal classroom two apparently conflicting scenarios occur. First, students behave as autonomous learners and exercise a high degree of control over their learning activities. Second, the teacher maintains a priority interest in *covering* the syllabus, and exercises strong control over classroom learning activities. This section discusses the following assertion about the way in which Ray's reformed pedagogy rationalised these conflicting beliefs:

Assertion Ten

Ray's prevailing technical rationality sustained his priority interest in *covering* the syllabus; in practice his constructivist-related ideal of student autonomy was subsumed by his technical rationalist belief in his centralist classroom role of *teacher as controller*.

Ray's Ideal of Student Autonomy

In his report on the study, Ray identified a need to facilitate students' abilities to exercise more control over their own learning, especially in relation to problem solving and evaluation activities:

> [There is] a need for students to assume a high degree of control over their own, individual learning, and to develop as independent learners. The teacher should assist students to develop an awareness of their learning processes, such as those involved in problem solving, and to begin to recognise and diagnose their own learning difficulties.
>
> (Fleming, 1988, pp. 49-50)

Ray described a need to transfer the *locus of control* within the classroom from the teacher to the students (Fleming, 1988, p. 75). He explained that, ideally, students should perceive a sense of ownership in relation to their learning processes, rather than regard learning as something "vested in and originating from the teacher" (p. 74). One of the outcomes of the study was Ray's realisation of the importance of students' developing the attributes of: (1) *independence*, that is, "the ability and confidence to be self-motivating and capable of accessing information from a variety of sources"; (2) *responsibility*, that is, "the realisation by students that they are responsible for their own learning; that the learning itself belongs to the student rather than to the teacher", and (3) *questioning*, that is, "the students develop the confidence and ability to formulate their own questions as part of their problem solving technique" (Fleming, 1988, p. 75).

Ideally, Ray would facilitate student autonomy by using a *self-study* approach which would allow students to *work at their own pace*, and enable him to move around the room and provide tutorial assistance. Ray believed that a self-study teaching approach would be more satisfying for students because some could be *going faster* rather than "sitting around waiting for the rest of the class to progress, and would allow other students who are having difficulty with a section to spend more time on it, and get help from the teacher":

> [A]n alternative would be to have a program. Students are handed a program and told they have to work through . . . a series of problems. Then they could do that at their own pace, and they could then ask the teacher for help when required, on an individual basis. So that the student would be setting their own pace. . . . I like that kind of role. I prefer that kind of role. (Interview 7, September 1988, p. 4)

Ray valued students who participated in whole-class discussion, especially students who initiated their own involvement by asking questions of an inquiry-oriented nature:

> With the students initiating their own involvement that also refers to things like them asking not only questions about the current tasks in hand — *How do you do it? I can't do it!* — but questions that might be leading somewhere, like *What would happen if . . . ?*
> (Interview 5, November 1987, Appendix B, p. 350)

Ray's Role of Teacher as Controller

During the collaborative phase of the study, however, Ray maintained a technical curriculum priority interest in *covering* the syllabus. At the end of the collaborative phase, interviews associated with the repertory grid technique identified the predominantly *top-down* nature of Ray's preferred classroom management role, especially in relation to his interest in *covering* the syllabus:

> I place a high emphasis on the teacher being in control of the classroom, and the rate of learning and the activities that are going on . . . and when a new topic will be started. . . . I have always taken for granted . . . that that's the way you teach, or that's the teacher's role or prerogative, to pace the class; almost like you want to push them along, but not too fast and not too slow. . . . I believe that the teacher has to determine when new information is going to be provided, and the nature of it. In other words, when to move on and expose students to the next step . . . controlling the rate.
>
> (Interview 5, November 1987, Appendix B, pp. 354 & 356)

At the end of the second period of fieldwork, I asked Ray about the most important aspects of his classroom role in relation to teaching a Grade 9 mathematics class. He replied: "Controlling what's going on in terms of what work is being done. Controlling the pace of it. Giving explanations for it" (Interview 6, August 1988, p. 2). Clearly, Ray had sustained a belief in the primacy of his role in controlling the classroom learning environment, a role which was highly compatible with his technical curriculum priority interest which required students to undertake common instructional activities:

> Students are pretty much doing the same thing as one another at the same time. . . . They might be graphing a *quadratic function*. They might have a certain time to do it, and then we might look at how far they've got, or what they've done.
>
> (Interview 5, November 1987, Appendix B, p. 351)

Student Compliance

Ray's continuing ownership of the locus of control of instructional activities, and the resultant *lockstep* approach to students' learning activities, was predicated on his expectation that students' interests would be well-served if they complied with his instructional prescriptions. Student compliance was the hallmark of Ray's ideal classroom environment. His language was replete with references to students' compliance and industriousness, and to his role in controlling their activities. In Ray's ideal classroom:

> [Students would be] working and interested in something. They're on-task . . . doing what they were supposed to be doing . . . rather than doing some other maths work. . . . I want them to be doing what I have set. (Interview 5, November 1987, Appendix B, p. 349)

Ray believed that, during class, students should be engaged at all times in completing the activities which he had prescribed. During teacher-directed, whole-class activities students should be: "[I]nvolved in what's going on, thinking about [the teacher's questions], formulating questions . . . attempting to understand . . . rather than simply copying stuff down from the board" (Interview 4, September 1987, Appendix B, p. 342). When students were working individually or in small groups, they should be focused on the learning activities which he had set: "[W]ork is set in class and students do it. This happens every period. . . . [T]hey either work on their own, or they may discuss things with their neighbours . . . or there may be pre-organised group work" (Interview 4, September 1987, Appendix B, p. 342).

Ascendency of Ray's Control Interest

Although Ray was willing to assert that students should operate in class as autonomous learners, in practice he provided limited opportunities for them to do so. Ray's belief in student autonomy was based on a narrow interpretation of a form of autonomy which was to be exercised within the framework of teacher-prescribed activities which served his technical curriculum priority interests.

Student Decision Making. For example, Ray did not believe that student autonomy should be extended to student participation in decision-making about the nature of classroom learning activities. He regarded such participation as second-guessing the next activity in the prescribed sequence of syllabus topics, and expressed his belief in the absurdity of such a role for anyone other than the teacher:

> I don't believe the students are so good at initiating their own learning that they can do it entirely. I think they need guidance and . . . in many cases, they won't know what it is that they want to know next, or need to know next, or what follows logically from where you are in the syllabus. (Interview 5, November 1987, Appendix B, p. 351)

Lock-step Learning. Another example of Ray's control interest overriding his student autonomy interest was the implementation of his *self-study* approach in the classroom. Ray designed student worksheets for his Grade 12 and Grade 9 classes for the purpose of enabling students to undertake periods of individual seatwork activity during class time (see Assertion Six in Chapter Five). In practice, however, Ray exercised strong control over the nature and scope of students' learning activities. The worksheets presented the Grade 12 class with a common set of instructional activities which all students were required to complete in the same time period. Although Ray allowed differential rates of completion of common learning activities amongst the Grade 9 students, after several lessons he regrouped the whole class and reviewed the full range of learning activities, regardless of the fact that many students had not completed them.

Ray's conflicting pedagogical beliefs in student autonomy and teacher control were resolved, in practice, in favour of the latter. Ray's prevailing role of *teacher as controller* and his requirement of student compliance are indicative of a technical rationality which: (1) presents a consensual theory of mathematics (cf. Apple, 1990) that ignores the historical and socio-cultural development of mathematics amid a community of mathematicians whose interests and theories often conflict and compete for acceptance; and (2) is based on a cultural reproduction model of learning (cf. Giroux, 1981) which eschews critical classroom discussion of the relevance and purpose of school mathematics, especially in relation to the *lived histories* and *biographies of students*, and promotes an intellectually disempowering role of *students as consumers* of the prescribed and seemingly objective and ahistorical truths of mathematics.

Ray's technical rationality subsumed his constructivist-related ideal of student autonomy and resulted in a narrow conception of autonomy which was highly congruent with his centralist classroom role of *teacher as controller*. This predominant role enabled Ray to redefine unilaterally the social norms of the classroom and to require students to comply unquestioningly with increased amounts of individual seatwork activity which were prescribed by his specially designed worksheets.

PERSONAL CONSTRAINTS TO PEDAGOGICAL REFORM

The analyses of Ray's reformed pedagogy, which are presented in the preceding section and in previous chapters, argue that Ray's constructivist-related beliefs were subsumed by his transmissionist epistemology and technical curriculum priority interests. It can be inferred from these analyses that the process of subsumption was straightforward and unproblematic. However, this section argues that the rationalisation of Ray's centralist pedagogy was not a simple matter. Throughout the study, Ray retained constructivist-related ideals of facilitating student autonomy and learning with understanding. These theoretically viable and preferred beliefs continued to conflict with the practically viable pedagogical beliefs which shaped Ray's centralist classroom role, especially his technical curriculum priority interest in *covering* the externally mandated syllabus.

This section extends the critical constructivist interpretive analysis of the previous section, and presents evidence which warrants the following assertion about the conservative role of Ray's personally constraining beliefs in reconciling the conflict between his constructivist-related ideals and his technical curriculum priority interest:

Assertion Eleven
Ray's personally constraining beliefs in the nature of students and the professional culture of teaching largely reconciled the conflict between his constructivist-related ideals and technical curriculum priority interest, and reaffirmed his centralist classroom teaching role.

Legitimation of Ray's Technical Rationality

The previous section argues that, during the post-collaborative phase of the study, it was apparent that Ray's *technical curriculum interests* (Grundy, 1987) had been sustained, and were conflicting with his constructivist-related ideals. Ray's discourse on his classroom teaching role was replete with references to his concerns for *leading* and *making progress*, and indicated that he had sustained an earlier conception of *teaching as a journey* (see Assertion One in Chapter Four). It was evident that Ray had sustained an overriding concern with *covering* the syllabus:

> I feel that I'd like them to be working more efficiently . . . progressing more quickly, and I would like to see the class moving along together. . . . [T]here's still this underlying thing that we must press on. We've been doing this for the last two weeks and we haven't done anything new and we haven't got anywhere. There's still this feeling that you want to do something new and progress through the syllabus.
>
> (Interview 6, August 1988, pp. 2 & 11)

In relation to teaching the Grade 9 class, Ray believed that he was accountable to the school authorities for *covering* the syllabus in accordance with the school's curriculum and assessment policy, and that this accountability outweighed his responsibility for providing meaningful learning opportunities for all students:

> [B]ecause we're working on Unit Curriculum . . . if you're going to give the student a certain grade for a certain unit, they're supposed to

> have covered that unit — 80 percent of that unit, 80 percent of the contents. You're allowed some flexibility but you're still supposed to guarantee that they've covered 80 percent of the content as specified in that unit. (Interview 7, September 1988, p. 10)

> [I]f at the end of the year you said to your senior master: 'I've only done a quarter of the first unit, but everyone can do it really well' . . . then [he/she] wouldn't be very impressed. Whereas, [it would be more acceptable] if I said: 'I've kept up with the top class. We've done all this. No one can actually do anything, but we've gone through the syllabus. They've seen it on the board. They've been exposed to it.'
> (Interview 6, August 1988, p. 11)

The ascendency of Ray's technical curriculum interests was based on his perception of the need to subscribe to the technical rationality of the professional culture of teaching which was legitimated by the school authorities. Ray's classroom role of *teacher as controller* enabled him to maintain the integrity of his technical curriculum interests by controlling the delivery of the syllabus. However, his technical curriculum interests continued to conflict with his constructivist-related ideals.

Ray's Conflict of Interests

During the collaborative phase of the study, Ray had recognised the persistent nature of the conflict between his constructivist-related ideals and his technical curriculum interests, and had explained that the only available option was to achieve a compromise:

> [T]he conflicting goals of covering the syllabus and developing understanding . . . [are] . . . always a compromise we're sort of stuck with. (Interview 2, August 1987, p. 14)

A year later, Ray reiterated his belief in the need to compromise, that is, to adopt a pragmatic approach when dealing with this conflict of pedagogical interests:

> [T]here is a conflict . . . and the answer is: What I do is a compromise between the two extremes. (Interview 7, September 1988, p. 9)

Ray believed that pedagogical compromise was akin to *steering a middle course* (Interview 6, August 1988, p. 11) between two opposing views or priorities. He seems to have regarded his conflicting pedagogical beliefs as being analogous to social beings who are capable of negotiation and consensus building. He implies that the resolution, or compromise, of the dispute is one to which both parties can subscribe with integrity, and that their mutual expectations can be fulfilled. Ray seems to have subscribed to a model of industrial reconciliation for dealing with the *cognitive dispute* between his constructivist-related belief in facilitating students' learning with understanding and his technical curriculum interests. A compromise based on *fair play* has a socially acceptable sound to it. In negotiating a social consensus, this concept has a high probability of evoking an empathetic response — a sense of desirability. Ray appears to have made recourse to this concept in negotiations with himself as he constrained the practical viability of his constructivist-related ideals. But, did the compromise result in equitable opportunities for student learning?

Ray perceived the actual compromise to entail directing the class to proceed with new learning activities when he had adjudged that the majority of the class had achieved a *reasonable* degree of conceptual readiness:

> I'm pacing the progress through the syllabus, to a degree, [in relation to] the speed at which the students are coping with the work or beginning to comprehend it. . . . I try to progress at such a rate that most of the students have a reasonable grasp of what we've done before we move onto anything else. (Interview 6, August 1988, p. 2)

In practice, Ray's delivery of the syllabus slowed and more time was available for whole-class discussion and individual seatwork activities (see Assertion Six in Chapter Five). Nevertheless, Ray abandoned his *self-paced* teaching strategy in favour of whole-class teacher presentations of common syllabus *content*, especially when he perceived that his technical curriculum interest in *covering* the syllabus was threatened by lack of *progress*. Although he interpreted his pragmatism as *steering a middle course*, it entailed a continuous abandonment of his constructivist-related ideals of facilitating student autonomy and learning with understanding in favour of his technical curriculum priority interest. In other words, Ray's constructivist-related ideals failed to attain a sustained practical viability.

Although Ray's much more interactive style of teaching had enabled him to monitor more closely individual student's learning needs and difficulties, and make better informed decisions about students' readiness to *progress* (see Assertion Five in Chapter Five), it resulted also in his growing awareness of the range of abilities amongst the class, and the pedagogical futility of wanting to see the class *moving along together*. He realised that the imposition of a lockstep approach to curriculum implementation resulted in a major pedagogical problem:

> The problem is that . . . even though it's a graded class there is a wide range of ability . . . and they don't all progress at the same rate. . . . They bring to the class a range of abilities and motivations and you try and lock them into all coping the same. . . . [T]here is always going to be two or three that probably are not ready to proceed.
>
> (Interview 6, August 1988, pp. 5-9)

Ray acknowledged that, in practice, it was difficult to reconcile the conflict between his constructivist-related ideal in a self-paced instructional approach which aimed at facilitating students' learning with understanding and his technical curriculum interest in *covering* common syllabus content. He explained that his practice of implementing a lockstep approach in order to satisfy the school curriculum and assessment policy was "cosmetic . . . somewhat of a deception" (Interview 7, September 1988, pp. 9-10) because he was aware that student understanding would be compromised:

> [I]f you force them to all go at the same rate, some will be understanding the work perfectly, and others will be incompletely understanding it and, so, you can say: 'Well, ok, I've got all my class working together, and we've covered this material, and we've all reached the same point in the same time'. But, I think that's just an illusion, and what's happening to each individual student is very different — one from another. So, whether they seem to be in step with one another is only a cosmetic thing, I think.
>
> (Interview 7, September 1988, p. 9)

Ray's Conciliatory Beliefs: Personal Constraints

Ray attempted to reconcile the conflict between his technical curriculum priority interest in implementing with high fidelity the externally mandated syllabus, which required him to maintain the locus of instructional control, and his constructivist-related ideals of facilitating students' autonomy and learning with understanding, which required an instructional focus on the learning needs and difficulties of the individual student. Ray's attempts to reconcile the conflict in favour of his technical curriculum interests involved him in making recourse to beliefs in constraints which he attributed to the nature of both students and the teaching profession.

Tobin (1990c) differentiates between *personal constraints* and *cultural constraints* as classes of beliefs that prevent teachers from implementing preferred pedagogical changes. A teacher's personally constraining beliefs result largely from the individual teacher's idiosyncratic constructive processes of sense-making, and are more susceptible to change than are culturally constraining beliefs. Culturally constraining beliefs result from teachers' enculturation into the teaching profession, and are professionally shared beliefs which are reaffirmed continuously in the daily social discourse between teachers. Culturally constraining beliefs, such as technical rationality, which serve to frame implicitly the normative beliefs, values and actions of teachers may be regarded as constituting a hegemonic ideology (Grundy, 1987) which acts largely at a subconscious level to constrain the nature and scope of teachers' attempts to redefine radically their well-established centralist classroom roles.

In this study, there is evidence that Ray held personally constraining beliefs which performed a major role in supporting his technical rationality which, in turn, constrained the nature and scope of his constructivist-related pedagogical reforms. Ray's personally constraining beliefs served to reconcile the conflict between his constructivist-related ideals and technical rationality and contributed, therefore, to sustaining his centralist classroom role. From Ray's perspective, however, the constraints which he experienced were not of his own construction, and were perceived to be real. Their reality was affirmed by Ray's attribution of the constraints to characteristics which, he believed, inhered in students and in the profession of teaching, and over which he had no control.

Students' Expectations

Ray attributed two major constraints to students. The first constraint concerned students' expectations of his classroom role. Although Ray professed a belief in a more decentralised classroom role for himself, he continued to believe that students' expectations of a centralised teacher role provided a major obstacle to constructivist-related pedagogical reform. He attributed these expectations to students' primary school experiences which, he claimed, were a *very difficult thing* to change. Ray explained that students would not regard as viable a decentralised classroom teacher role, and confirmed that his whole-class presentations catered for these perceived student expectations:

> Students are very set in their ideas, or their ways of going about things, which I think they would have inherited in primary school. They learnt at primary school . . . to become very teacher-dependent. . . . I find it's a very difficult thing to change at that stage. . . . [I] think they'd say 'Oh, this teacher doesn't teach you anything!' I think that's very much their attitude, that the teacher has to teach you. . . . Well, in a way it would be true that the teacher is not teaching them. . . . Perhaps it's not [true]; perhaps it shouldn't be [perceived to be true].
>
> (Interview 6, August 1988, pp. 3-4)

As well as attributing pedagogical reform constraints to students, Ray seemed not to be convinced of the practical viability of a decentralised classroom teaching role when he explained that in this role "in a way it would be true that the teacher is not teaching them" (p. 4). Ray seems to have sustained a belief, which he had professed earlier in the study, in the primacy of a centralised classroom role:

> I kind of feel that if I'm going to be there I should be doing something positive. I guess that is a thing with teachers. You feel . . . probably quite wrongly . . . that [if] you're engaged in whole-class instruction, [then] you're doing what you're paid to do, [that is] you're fulfilling your role more so than if students were working individually.
>
> (Interview 2, August 1987, p. 8)

Ray's recourse to a constraint associated with students' expectations provided him with grounds for partly justifying the lack of practical viability of a decentralised teaching role. However, Ray did not offer evidence to support his assertion about the nature of students' expectations. On the contrary, student interviews (see Assertion Four in Chapter Four) revealed widespread support for a decentralised classroom teacher role. Furthermore, many students identified Ray's specially designed worksheets — a self-instructional strategy — as being one of the main benefits of the changes to his teaching (see Appendix C). Ray's perception of the constraining nature of students' expectations did not seem, therefore, to be based on empirical evidence. Rather, it seems to have been a personally constraining belief which sustained the practical viability of Ray's technical curriculum interests.

Students' Self-Motivation

The second student attribute that Ray believed to be constraining the practical viability of his constructivist-related ideals was students' lack of academic self-motivation. During the second year of the study, Ray expressed a concern that, although most of the Grade 9 students were interested and keen to engage in classroom learning activities, he wanted the class to be "working more efficiently . . . progressing more quickly . . . moving along together" (Interview 6, August 1988, p. 1).

However, Ray claimed that student motivation was problematic for realising this goal:

> [S]tudent motivation is a problem from time to time, or in certain students. In general, the class is . . . reasonably well-motivated considering their ability. . . . [A]bout 80 percent of the class are . . . fairly to even highly motivated. (Interview 6, August 1988, p. 1)

Ray implied that at least 20 percent of the Grade 9 class was poorly self-motivated, and was not engaging in the work at a satisfactory rate. Ray's attribution of inadequate self-motivation as the cause of students' academic underachievement, is similar to his earlier assertion (see Assertion One in Chapter Four) that about one third of the Grade 12 class were destined to fail because they were unable to keep up with the pace of his whole-class presentations.

Ray's belief that the enactment of his constructivist-related ideal of *self-paced* learning requires students who are *ideally motivated* (Interview 7, September 1988, p. 9) implies that students' motivational development should precede, rather than be a consequence of, constructivist-related teaching. Although he acknowledged a responsibility for trying to motivate students, Ray's perception of the nature of motivational strategies was limited to strategies associated with his technical curriculum interest in controlling students' classroom learning behaviour, rather than with stimulating their conceptual development. For example, Ray asserted that although he was unwilling to coerce students to learn he was prepared "to set certain deadlines as a kind of motivation incentive for the less well-motivated students in the class" (Interview 7, September 1988, p. 9).

Ray's belief in his powerlessness to overcome the perceived constraint of inadequate academic self-motivation amongst students was evident in his use of the metaphors of *hitting my head against a brick wall* and *leading a horse to water* which indicate that Ray believed that trying to motivate students was a futile exercise:

> I sort of feel that you can lead a horse to water. I sort of can't be bothered. . . . I feel by [Grade] 9, and certainly by [Grade] 10, it's about

> time they started to be a little bit self-motivated . . . [I]f they're not starting to be by then, I really don't feel there's much hope for them, and I feel it's a bit of a case of hitting my head against a brick wall.
>
> (Interview 7, September 1988, p. 12)

However, Assertion Seven (in Chapter Six) argues that Ray's formalist preconception of the nature of mathematics resulted in learning activities of impoverished conceptual significance for the Grade 9 class. It is likely, therefore, that students' apparent lack of academic self-motivation resulted from their experience of a lack of mathematical meaningfulness as they continued to manipulate algorithmic expressions in accordance with predetermined rules for the immediate purpose of deriving solutions to known-answer exercises, and for the longer-term purpose only of preparing for summative assessment. Furthermore, because Ray held a rationalist conception of mathematical cognition, which resulted in a pedagogical focus only on the reasoning ability of the individual student (see Assertion Ten in Chapter Seven), he did not conceive of a teaching role which addressed the socio-emotional aspects of students' learning.

Ray's attribution of students' inadequate academic self-motivation to their predetermined personalities seems, therefore, to be misconceived. By attributing students' unwillingness to learn to a constraint associated with the inherent nature of students, Ray avoided attributing responsibility for students' underachievement to the technical curriculum priority interest which underpinned his instructional strategies. Ray's perception of the constraining nature of students' personalities seems to have been based largely on a misconception of the significance of students' observable classroom behaviour, and seems to have been a personally constraining belief which contributed to sustaining the practical viability of Ray's technical curriculum interests.

The Profession of Teaching

Although Ray acknowledged that he felt compromised by his pragmatic pedagogy, which resulted from compromising his constructivist-related ideals in favour of his technical curriculum priority interest in *covering* the syllabus, he perceived the feeling to be a problem of relatively minor concern which was common to the professional culture of teaching:

> I guess [compromising] should [cause me problems], and it does to a degree. But, I guess it's the sort of thing you learn to live with, and most teachers live with it without probably even realising it. . . . I feel compromised by having classes of 36, too. There's lots of factors that I feel are compromises in the teaching situation.
>
> (Interview 7, September 1988, pp. 10-11)

It seems, therefore, that the significance of the conflict between Ray's constructivist-related ideals and technical curriculum priority interest was diminished by his recourse to a belief in the inevitability of feeling compromised by constraints associated with normative requirements of functioning as a member of a teaching profession. In other words, feeling compromised by another conflict in his belief system which was caused by a constraint that he perceived to lie beyond his influence was not an unusual or significant event for Ray. It seems likely that Ray's reconciliatory belief in the inevitability of being compromised served to reduce, to a tolerable level, the daily emotional discomfort associated with feeling compromised by his pedagogical pragmatism.

To the extent that Ray continues to accept the personally constraining belief in the inevitability of a form of pedagogical pragmatism which denies many students the opportunity for autonomy and enhanced conceptual development (see Assertion Six in Chapter Five), he is likely to continue to perceive the technical rationalist culture of teaching to be immutable, and fail to seek a participatory role in its evolution towards practical and emancipatory curriculum interests (Grundy, 1987). Ray's beliefs in the constraining nature of students' apparently predetermined personalities and the inevitability of pedagogical pragmatism are likely to continue to disempower him from believing in the possibility of creating a curriculum that is conducive to the implementation of his constructivist-related ideals.

SUMMARY

This chapter extends the analyses of previous chapters which interpret the nature and scope of Ray's constructivist-related pedagogical reforms from radical and social constructivist perspectives. This chapter focuses a critical constructivist perspective on Ray's reformed pedagogy, and assesses the constraining role of his technical rationality. In particular, the chapter examines Ray's rationalisation of the conflict between his technical curriculum priority interest in *covering* the syllabus, which underpinned his *teacher as controller* classroom role, and his constructivist-related ideals of facilitating student autonomy and learning with understanding. The interpretive analysis contributes to a better understanding of the major constraints to achieving the constructivist-related pedagogical reform goals of this study.

The review of educational research literature in the field of critical curriculum theory argues that, for most of the twentieth century, the field of education has been dominated by a culture of positivism which was imported from allied professional disciplines, especially science, and which has been enshrined as the dominant epistemology of professional educational practice. Although positivism has been discredited as a philosophy of science, it has continued, in the form of *technical rationality,* to underpin educational research, teacher education, curriculum policy and, subsequently, classroom practice. Technical rationality is based on an objectivist epistemology associated with the empirical-analytic sciences which have been harnessed by the technological growth imperatives of western industrialised society in the interests of economic development.

Imported into education, technical rationality separates knower from known, promotes an unproblematic and reductionist view of knowledge and its relationship with society, and constitutes a conservative interest in social control and cultural reproduction. Technical rationality underpins a model of curriculum which defines an instrumentalist view of *knowledge as a commodity,* and prefigures learning outcomes. A technically informed curriculum objectifies both knowledge and students and, by means of an external locus of

curriculum and evaluation control, marginalises the authority of teachers and undermines their professional autonomy. In the classroom, a technical curriculum interest requires the teacher to assume the locus of instructional and management control, and to focus his/her pedagogical attention largely on the delivery of a highly prescribed curriculum in accordance with predetermined learning objectives. Students' roles are defined in terms of knowledge consumption and social compliance, and the opportunity for both teachers and students to develop as autonomous intellectuals is minimised.

In the case of Ray, his technical curriculum priority interest in implementing with high fidelity the externally mandated syllabus in both the Grade 9 and Grade 12 mathematics classes conflicted with his constructivist-related beliefs in facilitating student autonomy and learning with understanding. However, Ray's prevailing technical rationality sustained his priority interest in *covering* the syllabus. In practice, his constructivist-related ideal of student autonomy was subsumed by his technical rationalist belief in his centralist classroom role of *teacher as controller*. Ray equated student autonomy with individual seatwork activity which he facilitated by means of his specially designed worksheets. Although students had increased opportunities to undertake individually periods of self-paced learning in class, their learning activities were highly prescribed and, overall, they were required to maintain lockstep *progress* through a common syllabus.

However, Ray's more interactive classroom role of *teacher as learner* enabled him to generate a better understanding of the diversity of individual student learning outcomes and difficulties, and he realised the pedagogical futility of requiring a lockstep approach to learning. In other words, his technical curriculum priority interest conflicted with his constructivist-related ideals, especially the ideal of enabling students to learn with understanding. Nevertheless, Ray believed that he was compelled by the school authorities to maintain the ascendency of his technical curriculum interest. He maintained a pragmatic pedagogy which compromised his constructivist-related ideals.

Ray's beliefs in constraints to constructivist-related pedagogical reform, which he associated with students and the professional culture of teaching, enabled him largely to reconcile the conflict of interests and rationalise his pedagogical pragmatism. However, the constraints which Ray perceived to inhere in students and the culture of teaching were unfounded and untested personal beliefs, rather than real attributes. Firstly, although Ray believed that students' expected him to maintain a centralist classroom teacher role, interview data indicated that the majority of students had favourable perceptions of his decentralised pedagogical reforms, and preferred to use the worksheets rather than listen to his whole-class presentations.

Secondly, Ray attributed students' underachievement and lack of learning interest to poor academic self-motivation, and distanced himself from the possibility of a motivational teaching role by attributing the problem to students' predetermined personalities. However, classroom observation data indicate the formalist nature of Ray's pedagogy, especially in the Grade 9 class, which resulted in learning activities of impoverished conceptual significance for students. It is highly likely that students' lack of academic motivation resulted from their extended experiences of the lack of meaningfulness associated with (1) the process of manipulating algebraic algorithms in accordance with predetermined rules for the immediate purpose of solving sets of known-answer exercises, and (2) preparing for an examination which tested their ability to repeat this process.

Thirdly, Ray believed that the experience of feeling compromised as a result of compromising his constructivist-related ideals in favour of his technical curriculum priority interests was a relatively insignificant consequence of the inevitability of conforming to the prevailing cultural norms of the teaching profession which were legitimated by the school authorities who employed him. Ray's personally constraining belief in the inevitability of feeling compromised has the potential to disempower him from struggling against the prevailing technical rationality of the culture of teaching and seeking to renegotiate a conception of curriculum which embodies his constructivist-related ideal of students learning with understanding.

SECTION FOUR

CONCLUSION

Section Four of the thesis comprises a single concluding chapter (Chapter Nine) which presents a review and a synthesis of the results of the study in the form of a single general assertion. The general assertion addresses the following research question which constitutes a synthesis of the two research questions that are addressed in Sections Two and Three of the thesis: *Is cognitive constructivism an adequate referent for pedagogical reform which aims to resolve the problem of teacher centralism by developing a pedagogy aimed at improving the mathematical conceptual development of students of high school mathematics?*

The general assertion argues that the cognitivist theory of constructivism as described in this thesis is an inadequate referent for radical pedagogical reform because of its inability to address powerful teacher preconceptions which prefigured the social reality of the traditional mathematics classroom of this study.

Finally, this section proposes that future pedagogical reform which aims to emancipate both teachers and students from the disempowering effects of the hidden frames of reference which constituted the social reality of the traditional mathematics classroom of this study should adopt a critical constructivist perspective on classroom discourse.

CHAPTER NINE

CRITICAL CONSTRUCTIVISM: A REFERENT FOR FUTURE PEDAGOGICAL REFORM

INTRODUCTION

This final chapter serves both to review and synthesise the results of the study which are presented in earlier chapters, and to provide a forward-looking perspective on the prospects of constructivist theory as a useful referent for pedagogical reform of the traditional high school mathematics classroom.

The first section constitutes the interpretive framework of the chapter and provides a rationale for the generalisability of the results of the study. The problematic concept of external validity is discussed in relation to research involving interpretive case studies, and a definition of external validity is generated for this study.

The second section presents a synopsis of Chapters One to Eight of the thesis. The major research problem, the initial research question, the main research design characteristics, and early assertions about the problematic nature of the teacher's pedagogical reforms are discussed briefly in relation to the initial theoretical framework of the thesis. Subsequently, the expanded theoretical framework of the thesis, which was designed in response to an emergent interpretive research question, is summarised. Assertions which illuminate the nature of the teacher's sustained centralist classroom role and the lack of efficacy of the initial theoretical framework of the study are outlined.

The third section addresses the question of whether the cognitive constructivist perspective on learning which underpinned the theory of pedagogical reform of this study and precipitated the teacher's pedagogical innovations is an adequate referent for pedagogical reform. This question constitutes a synthesis of the two research questions which governed the interpretive data collection and analysis phases of the study. The question is addressed by a single general assertion about the problematic relationship between cognitive constructivist theory and key pedagogical beliefs which underpin the centralist classroom role of the traditionalist mathematics teacher. The general assertion is illustrated by a review of the results of this case study, and by a discussion of the possible implications of the results for future attempts at constructivist-oriented pedagogical reform.

The final section of this thesis addresses the question of the prospects of future constructivist-oriented pedagogical reform for overcoming key pedagogical beliefs that, as the results of this study suggest strongly, underpin the centralist classroom role of the traditionalist teacher of school mathematics. It is proposed that future pedagogical reform incorporates a critical constructivist perspective on the prevailing rationality of the traditional mathematics classroom. It is argued that a critical classroom discourse amongst teachers and students is required in order to attain a more balanced rationality. A critical discourse, which aims to make visible and subject to critical examination the largely hidden frames of reference (e.g., technical rationality, myths of realism and rationalism) that constitute the established social reality of the classroom learning environment, would enable an emancipatory interest to flourish in the midst of technical curriculum priority interests.

INTERPRETIVE FRAMEWORK

Central to the aim of this final chapter is the notion of the external validity of the results of the interpretive case study which have been presented in previous chapters. The concept of external validity which was adopted in this study governs the stance of the thesis towards the question of the generalisability of the results for future research studies and other classroom situations. This section discusses important aspects of the concept of external validity and concludes with a definition that governs both the formulation of the general assertion of the study and the discussion of the implications for future research that are presented in subsequent sections.

Interpretive research does not subscribe to the statistical notion of *external validity* which is concerned with making general inferences about a population from which a purportedly representative research sample has been carefully selected (Tuckman, 1978; Wiersma, 1985). However, the issue of the external validity of interpretive case studies, that is, "the extent to which the findings of one case study can be applied to other situations" (Merriam, 1988, p. 173), is problematic.

There is a range of opinions amongst interpretive-qualitative-ethnographic researchers about the need to reconceptualise the traditional notion of external validity, and about what best constitutes appropriate criteria. For example, Merriam's (1988) review of case study research in education includes the following conceptualisations of external validity: Cronbach's (1970) concept of *working hypotheses* which guide the conduct of future case studies; Eisner's (1981) concept of *transferability* of findings to new situations; Stake's (1978) notion of *naturalistic generalisation* which guides the search for similarities in new contexts; and the concept of *reader* or *user generalisability* which makes the reader of the report responsible for the generalisability of the case study results (Walker, 1980; Wilson, 1980). Furthermore, Merriam (1988) concludes that the generalisability of the results of a case study can be improved by adhering to the following conditions: providing rich *thick description* (cf. Geertz, 1973) that facilitates the readers' perception of the *transferability* of the results (Lincoln & Guba, 1985); establishing the *typicality* of the case in relation to other possible situations (Goetz & LeCompte, 1984); and conducting *cross-site* or *cross-case* analysis to establish the *range of generality* of the results of a case (Miles & Huberman, 1984).

Because the interpretive research design of the study reported in this thesis was based largely on the qualitative methods of participant observational research on teaching which are advocated by Erickson (1986), it is appropriate that Erickson's notion of external validity is examined in more detail. Erickson (1986) draws on research theory from the fields of anthropology and linguistics and argues, after David Hamilton (1980), that, although some aspects of any teaching situation are unique, other aspects are generic and display "universal properties of teaching" (Erickson, 1986, p. 130). However, these universals are manifested in the concrete, rather than the abstract, features of the case. Therefore, the primary interest of the interpretive researcher is with the *particularisability* of the case, rather than the generalisability. For Erickson (1986), *concrete universals*, that is, that which can be generalised to other cases, may be inferred only from cross-case analyses.

Nevertheless, the issue of the generalisability of the interpretive case study results arises for Erickson (1986) from a consideration of the interests of the prospective readers of the interpretive research report. Erickson (1986) argues that the report can best satisfy the interests of the *general scientific community*, or fellow researchers, and the practical interests of teacher educators amongst the *general community of practitioners* (i.e., groups for whom this thesis has been written) by optimising the internal validity of the data collection and analysis of the study, and by transferring the responsibility for generalisability of the results of the study to the reader:

> The reader must examine the circumstances of the case to determine the ways in which the case fits the circumstances of the reader's own situation. (Erickson, 1986, p. 153)

Clearly, it is beyond the scope of this thesis to optimise the external validity of the results of this case study by conducting further case studies and cross-case analysis. That is a research activity for the future. On the other hand, to assign the responsibility for determining the generalisability of the results of the study to the reader on the basis that this case study is uniquely determined seems less than satisfactory, especially in view of: (1) the international significance of the research problem (i.e., overcoming the problem of teacher centralism); (2) the generic nature of the major variables examined (e.g., teacher epistemology, pedagogical reform, constructivist theory, conceptual development, high school mathematics); and (3) the typicality of the major characteristics of the case (e.g., externally-mandated curriculum and assessment policy designed to prepare students for university, academically underachieving students, a relatively experienced teacher).

The question of the external validity of the results of this interpretive case study has been addressed in two ways. Firstly, after Erickson (1986), the procedures which were adopted for optimising internal validity (and, therefore, external validity) are discussed in detail in Chapter One. Secondly, the general assertion of the study, which is discussed in the next section, is presented as tentative theory which is grounded in the data of this study (cf. Glaser & Strauss, 1967) but which is expressed in a general form that implies its transferability to other situations. The term *assertion* has been used in preference to *theory* because it is intended to signify that this emergent grounded theory requires testing in other educational contexts in order to establish its cross-contextual viability.

Although the typicality of this case study has been established, the relative uniqueness of the case remains as a contextually constraining aspect of the general assertion. In the interests of informing future users of the general assertion the relatively unique major characteristics of this case should be borne in mind, namely: (1) that the teacher was out-of-field, that is, he was not teaching his normal teaching subject, (2) that the teacher's reform-mindedness was fuelled by his postgraduate part-time studies, (3) that the teacher's pedagogical reforms were precipitated by an intensive period of collaboration with a teacher educator, and (4) that the school in which this study was conducted enrolled only female students.

However, the extent to which this case is typical of other cases is an issue which is best decided by the reader of this thesis. Sufficient contextual data about the participants and the research site have been presented throughout the thesis and, of special importance, the theoretical framework which precipitated the teacher's pedagogical reforms has been explicated and subjected to detailed critical examination.

SYNOPSIS OF CHAPTERS ONE TO EIGHT

The study reported in this thesis was designed to address the internationally significant educational problem of reforming radically the traditional centralist pedagogies of teachers of school science and mathematics. Since the study was conceived, in 1986, the epistemological underpinnings of the problem of teacher centralism — *teaching as transmission, learning as absorption* — have received increasing criticism from both educational researchers and national committees of inquiry into the future of school science and mathematics (see Chapter One).

The main research problem which was addressed by this thesis is: *How can the problem of teacher centralism be resolved in mathematics education, especially the problem of transforming radically teachers' transmissionist epistemologies.* The problem was addressed by investigating the feasibility of employing an alternative epistemological referent of *learning as construction* for the radical reform of the centralist pedagogy of a teacher of high school mathematics.

This thesis reports an *interpretive* analysis (Erickson, 1986) of a *collaborative research* study which was conducted by Ray, a reform-minded teacher of high school mathematics with an interest in developing a constructivist-related pedagogy, and myself, a university-based teacher educator with interests in teacher cognition and constructivist-related pedagogical reform. Ray's research interest arose from his major concern about solving the immediate, practical problem of the underachievement of the majority of students in his Grade 12 mathematics class.

My research interest in teacher cognition gave rise to the initial research question of the thesis which asks: *How do teachers' role-determining beliefs constrain constructivist-related pedagogical reform?* Ray and I agreed on a collaborative research agenda which was designed to address our mutual research interests, and began a two-year study which commenced with a collaborative phase in his Grade 12 mathematics class in which was being implemented a State-mandated curriculum and assessment policy (see Chapter One).

Initial Theoretical Framework

The initial theoretical framework of this thesis (see Chapters Two and Three) was based on a synthesis of ideas associated with: (1) constructivist-related research on student conceptual change in science education; (2) research on the constraining role of teacher cognition in curriculum reform, especially teachers' implicit theories and beliefs; (3) Ausubel's theory of meaningful verbal learning; (4) information-processing psychology; (5) research on the role of target students during classroom discourse; and (6) research on teachers' organisation of classroom activities.

A constructivist-related theory of pedagogical reform was developed to facilitate the radical reconceptualisation of Ray's centralist classroom role, especially his implicit role-determining pedagogical beliefs (see Chapter Two). This constructivist perspective was designed to be implemented in a process of collaborative research in which my collaborative role as a facilitator of Ray's pedagogical reform was defined in relation to three goals. It was my intention to engage Ray in a dialectical discourse which aimed: (1) to facilitate his awareness, from a constructivist perspective, of the problematic nature of his established centralist pedagogy; (2) to enable him to construct a theoretically viable constructivist pespective on teaching and learning, and (3) to enable him to establish the practical viability of his constructivist-related pedagogical ideas in the context of his classroom practice.

The theory of pedagogical reform underpinned my discourse with Ray during the collaborative phase of the study. In particular, my discourse was designed to focus Ray's pedagogical attention on: (1) the constraining nature of his established classroom discourse, especially in relation to the problematic nature of students' mathematical conceptualisations; and (2) the development of a more conceptually-oriented pedagogy that accounted for students' extant mathematical knowledge (see Chapter Three).

The initial theoretical framework of the thesis also constituted an observational framework for interpretive data collection and analysis. The interpretive analyses resulted in assertions about: (1) Ray's established centralist classroom role (see Assertions One to Four in Chapter Four), (2) the major constructivist-related changes in his centralist classroom role (see Assertion Five in Chapter Five), and (3) the impact of his transmissionist epistemology on the main pedagogical innovations which he introduced into the Grade 12 mathematics class during the collaborative phase of the study (see Assertion Six in Chapter Five).

The interpretive research design of the study enabled an emergent research question to be formulated in response to an early assertion that Ray had refined, rather than transformed radically, his centralist classroom role. In particular, it was argued that Ray's pedagogical reforms largely failed to improve students' mathematical conceptual development, and that the nature and scope of the reforms were constrained by Ray's sustained transmissionist epistemology (see Assertion Six in Chapter Five). The second research question asked: *To what extent did the theory of pedagogical reform used in the thesis constrain the process of pedagogical reform?*

An Expanded Theoretical Framework

The initial theoretical framework of the thesis was expanded by incorporating multiple constructivist-related perspectives — *radical constructivism, social constructivism,* and *critical constructivism* — which enabled more insightful understandings to be generated about the pedagogical beliefs which constituted Ray's transmissionist epistemology (see Chapters Six, Seven and Eight). Further interpretive data were obtained in the post-collaborative phase of the study in relation to Ray's teaching of a Grade 9 mathematics class in which a school-mandated curriculum and assessment policy was being implemented.

Subsequent interpretive analyses resulted in assertions about the nature of Ray's constraining pedagogical beliefs and the lack of efficacy of the initial theoretical framework of the thesis. In particular, it is argued that the constructivist perspective which underpinned the theory of pedagogical reform was unable to account for: (1) Ray's preconceptions of the realist and formalist nature of mathematics (see Assertions Seven and Eight in Chapter Six); (2) Ray's rationalist preconception of the nature of mathematical cognition; and (3) Ray's technical curriculum priority interests (see Assertions Ten and Eleven in Chapter Eight).

The next section presents a synthesis of assertions presented in earlier chapters, and considers the implications of the results of this case study for other educational settings which might share the major characteristics of this case.

COGNITIVE CONSTRUCTIVISM: AN INADEQUATE REFERENT FOR PEDAGOGICAL REFORM

The theory of pedagogical reform which shaped the collaborative phase of the study that is reported in this thesis was based largely on an amalgam of Ausubel's cognitive theory of constructivism (Novak, 1978) and the extensive personal constructivist tradition in science education (see Chapter One). This cognitivist theory of constructivism embodies *the first principle of constructivism* (von Glasersfeld, 1988), and focuses pedagogical interest on the central role of prior knowledge in the individual learner's construction of new concepts.

This section addresses the following question which arises directly from the results of the study and constitutes a synthesis of the two research questions of the study: *Is cognitive constructivism an adequate referent for pedagogical reform which aims to resolve the problem of teacher centralism and develop a pedagogy aimed at improving the mathematical conceptual development of students of high school mathematics?* The question is addressed by the following general assertion which constitutes emergent grounded theory (cf. Glaser & Strauss, 1967).

General Assertion

Pedagogical reform which is based only on a cognitivist theory of constructivism, especially a theory that emphasises the central role of prior knowledge in the individual learner's construction of new concepts, might result in little more than a refinement of the classroom role of the traditionalist teacher of high school mathematics whose teaching is underpinned by the following key pedagogical beliefs:

- **Realist and formalist preconceptions of mathematics which objectify mathematical knowledge by separating knower from known, trivialise the subjectivity of the individual student, sustain the teacher as the primary source of students' new mathematical knowledge, and focus pedagogical attention on the development of algorithmic ability;**
- **A rationalist preconception of mathematical cognition which focuses pedagogical attention on individual student cognition and fails to account for sociocultural and socioemotional contexts of the intersubjective construction of mathematical knowledge;**
- **A technical rationality which determines a priority pedagogical interest in *covering* common syllabus *content* in a fixed time, enables constructivist-related pedagogical ideals to be compromised, sustains the centralist classroom role of *teacher as controller*, and disempowers teachers and students from developing as autonomous intellectuals.**

This assertion is illustrated below in the case of Ray whose pedagogical reforms were constrained by key pedagogical beliefs which the cognitivist theory of constructivism of this study was unable to address. The following review focuses on the role performed by Ray's pedagogical beliefs in subsuming his prior-knowledge-oriented constructivist perspective and maintaining an instructional focus on students' algorithmic ability rather than on their conceptual development. Major implications of the results of this case study are considered for future pedagogical reform amongst traditionalist teachers of high school mathematics, especially in classrooms in which externally-mandated curriculum and assessment policies are implemented.

The Resilience of Ray's Pedagogical Beliefs

The results of this study indicate the resilient nature of the pedagogical beliefs which underpinned Ray's centralist classroom role and their role in subsuming his constructivist perspective on learning.

Teacher as Learner

Ray's newly developed constructivist perspective on learning emphasised the important role of students' prior knowledge in their attempts to construct new mathematical knowledge. Ray realised that he knew very little about students' extant mathematical knowledge, and he perceived a need to change his classroom role in order to create learning opportunities for himself. Ray adopted a *teacher as learner* perspective on his classroom role (see Assertion Five in Chapter Five).

One of the main changes in Ray's pedagogy involved his development of a more interactive and inquiry-oriented classroom role which enabled him to monitor more closely the success or failure of students' learning outcomes, and generate a better understanding of their mathematical knowledge. His investigative teaching strategies included diagnostic questioning of students, and extended periods of individual seatwork activity that enabled him to interact more with individual students. The main focus of Ray's learning interest was on the development of his own *pedagogical content knowledge*. That is, Ray wanted to incorporate knowledge of students' daily learning difficulties into his teaching of the *content* of the mathematics syllabus.

However, because of his formalist preconception of the nature of mathematics (see Assertion Eight in Chapter Six), Ray interpreted students' learning difficulties as inability to recall algorithms and predetermined problem-solving procedures, and inability to apply them correctly to single-answer practice exercises. Nevertheless, Ray's enhanced knowledge of students' daily learning difficulties fuelled his subsequent instructional innovations and resulted in a refinement of his centralist classroom role.

Teacher as Interactive Informer

As Ray learned more about students' daily problematic attempts to construct their mathematical knowledge, he developed innovative teaching strategies aimed at improving students' learning outcomes. These teaching strategies were based on Ray's belief in accounting for students' extant mathematical knowledge. However, Ray's prior-knowledge-related belief was conceptualised largely in terms of the metaphor of *cognition as information processing* which I had employed in my discourse with him during the collaborative phase of the study (see Assertion Seven in Chapter Six). This metaphor is based on models of information-processing psychology which have been criticised for their metaphysical realist epistemological assumptions (Cobb, 1987).

Ray's knowledge-transformation model (see Figure 1 in Chapter Five) indicates that Ray continued to conceive of the teacher as the primary source of students' new mathematical knowledge. In this epistemological model, teacher knowledge is transformed into public information which is transmitted to students whose corresponding role is to transform the received information into their own private knowledge. By focusing his pedagogical attention on the interface between students' extant mathematical knowledge and their transformative endeavours, Ray believed that he could learn more about students' learning difficulties and, subsequently, provide more user-friendly forms of information (i.e., his own transformed knowledge).

The resilience of Ray's transmissionist epistemology may be attributed to his realist preconception of the nature of mathematics which assumed the independent existence of mathematical knowledge and the ability of the teacher to transform and transmit it to students. The metaphor of *cognition as information processing*, which promoted Ray's prior-knowledge-oriented constructivist perspective, was highly congruent with his preconception of the realist nature of mathematics, and served to reaffirm his transmissionist epistemology and sustain his centralist classroom role (see Assertion Seven in Chapter Six).

Ray's instructional strategies for facilitating students' prior-knowledge-related understandings (i.e., their cognitive acts of information transformation) involved a combination of a reductionist principle of instructional design and inquiry-oriented discourse. From Ray's perspective, he reduced the amount of transmitted information into smaller quanta, or steps, and attempted to stimulate students' anticipation of its arrival by means of his inquiry-oriented discourse. For example, in the whole-class forum, Ray's questions were designed to provoke students' thinking prior to his exposition of new theory and, in the worksheets, a similar purpose was served by sequenced instructions, reminders of antecedent algorithms, and practice exercises.

Ray's epistemology continued to be based on conceptions of *teaching as transmission* and *learning as reception*, albeit in a covert form in which information serves as a synonym for knowledge, and required students to reproduce accurately his own (transformed) mathematical knowledge. During the collaborative phase of the study, Ray's realist preconception of mathematics subsumed his newly developed constructivist perspective, and maintained a pedagogical focus on students' prior-knowledge-related interpretations of Ray's explications of his own mathematical knowledge.

The main emphasis of Ray's reformed pedagogy continued to be on the quality of students' observable performance, or *behavioural fit* (von Glasersfeld, 1990b). By contrast, a radical constructivist perspective, which incorporates the metaphysical principle of constructivism (see Chapter Six), replaces the construct of objective truth by the construct of *subjective viability* (von Glasersfeld, 1983), and promotes a pedagogical focus on the fitness of students' conceptual understandings for making sense of new and problematic experiences. The pedagogical implications of addressing this construct are explained in Wheatley's (1990) *problem centred learning* model and the *inquiry mathematics* model of Cobb, Wood, and Yackel (in press), Cobb, Wood, Yackel, and McNeal (1991), Cobb, Yackel, and Wood (in press), Wood (1991), and Yackel (1991) (see Chapter Six).

Students as Cognitive Isolates

Not only did Ray's epistemology objectify mathematical knowledge, but also it assumed that mathematical cognition was akin to a Platonic process of pure reasoning which was divorced from emotional and social influences (see Assertion Nine in Chapter Seven). Constructivist perspectives which focus only on the cognition of the individual learner and which ignore the sociocultural and socioemotional contexts of learning have been criticised recently by science and mathematics education researchers (Bauersfeld, 1988; Bruner, 1988; Cobb, 1989; Solomon, 1987; Wheatley, 1990).

There were two main pedagogical consequences of Ray's rationalism. Firstly, Ray sustained a disinterest in facilitating a social classroom learning environment despite students' explicit appeals for more social cohesion in class (see Assertion Four in Chapter Four), and despite the positive outcomes of a teaching experiment which was designed to facilitate small-group collaborative learning (see Assertion Nine in Chapter Seven). Secondly, Ray maintained a disinterest in a classroom teaching role aimed at enhancing students' academic motivation. He attributed lack of academic self-motivation to students' predetermined personality traits, rather than to a lack of perceived meaningfulness associated with the algorithmic and individualistic emphasis of his formalist pedagogy (see Assertion Eleven in Chapter Eight). In practice, Ray's major pedagogical reforms were designed for the *student as cognitive isolate*. Collaborative learning and negotiation amongst students occurred largely by pedagogical default in Ray's mathematics classroom.

The cognitive constructivist perspective of the collaborative phase of this study, which promoted a narrow pedagogical focus on students' prior syllabus-related mathematical knowledge, was not well-equipped to provide a compelling rationale for Ray's adoption of a social constructivist perspective on learning. Indeed, this constructivist perspective promoted deliberately a pedagogical focus on the cognition of the individual student as an *antidote* to the whole-class orientation of Ray's established pedagogy.

Teacher as Controller

Ray's major pedagogical reforms also were constrained by an important teacher belief which had not been addressed by the initial cognitive constructivist perspective of this study. Ray's pedagogical innovations were constrained by his belief that they must be compatible with his established *teacher as controller* classroom role. This centralist classroom role maintained the integrity of Ray's technical curriculum priority interest in *covering* common syllabus *content* in a fixed time (see Assertion Ten in Chapter Eight).

However, Ray's implementation of his pedagogical innovations proved to be unduly time-consuming, and threatened his technical curriculum priority interest. For example, Ray's specially designed worksheets provided opportunities for students to undertake self-paced learning during extended periods of individual seatwork activity. Because many students chose to work at slower-than-normal rates, which suited better their individual learning needs (and, in some cases, their social agenda), within the period of several lessons the class was engaged in a range of topics. By the end of the week, Ray's established *lock-step* policy of *covering* common syllabus *content* appeared to be threatened.

However, practical conflicts between Ray's constructivist-related beliefs in facilitating student autonomy and learning with understanding and his technical curriculum priority interest were resolved in favour of the latter (see Assertion Ten in Chapter Eight). For example, Ray resolved the dilemma of wanting students both to *cover* common syllabus *content* and to work in class at their preferred rates by reverting frequently to his *teacher as controller* classroom role which required students to comply unquestioningly with Ray's unilateral classroom management decisions. At times, he stopped individual seatwork activity and focused the whole class on a selected site in the syllabus (usually a topic review section), regardless of the many students who had not completed the full range of topics. Although this strategy resulted in Ray compromising deliberately his constructivist-related ideals, especially his ideals of students learning with understanding and students as autonomous learners, the strategy enabled him to maintain the integrity of his technical curriculum priority interest.

The cognitive constructivist perspective of the collaborative phase of the study was ill-equipped to counter Ray's technical curriculum priority interest. Although he acknowledged that his enforcement of this interest did not serve the learning interests of many students, his prior-knowledge-oriented constructivist perspective was unable to provide a rationale for countering this powerful and externally legitimated pedagogical priority interest. The ascendency of Ray's technical curriculum priority interest was buttressed by his personally constraining beliefs about the nature of students and the disempowering nature of the professional culture of teaching (see Assertion Eleven in Chapter Eight).

From a critical constructivist perspective, which is based on Habermas' theory of human cognitive interests (see Chapter Eight: Grundy, 1987; Habermas, 1970, 1972, 1978), Ray's technical rationality prevailed, and his practical curriculum interest in facilitating students' learning with understanding was subsumed by his technical curriculum priority interest. Ray's *teacher as controller* classroom role enabled him to continue to act in accordance with his established perception of the prevailing cultural norms of the teaching profession which were legitimated by the school authorities who employed him.

The Elusive Goal of Enhanced Conceptual Development

Ray's pedagogical innovations were perceived favourably by both Ray and most of the lower-achieving students who constituted the majority of the class. Students claimed that the changes to Ray's teaching had had a beneficial impact on their attitudes and, to a somewhat lesser extent, on their abilities to understand mathematics (see Assertion Six in Chapter Five). The increased amount of individual seatwork activity and specially designed worksheets provided enhanced opportunities for students (1) to focus on important antecedent mathematical knowledge, (2) to discuss their ideas and learning difficulties with fellow students and the teacher, and (3) to exercise more classroom-based control over the rate of generation of their new mathematical knowledge. Furthermore, students were required no longer to spend most of each lesson trying to copy notes from the blackboard while listening to the teacher's whole-class expositions.

Although students were disposed favourably to Ray's refined centralist classroom role, his pedagogical innovations largely failed to enhance their conceptual development (see Assertion Six in Chapter Five). Ray's formalist conception of the nature of mathematics subsumed his prior-knowledge-related belief and maintained a dominant pedagogical emphasis on recalling prescribed algorithms and problem solving procedures, and applying them to single-answer practice exercises (see Assertion Eight in Chapter Six). The explicit purpose of these learning activities was to prepare students for school-based topic tests which were designed to measure algorithmic ability, rather than conceptual understanding. Although Ray's prior-knowledge-related pedagogical innovations might have enhanced students' algorithmic abilities, as indicated by topic test results, they contributed to the ongoing development of many students' impoverished mathematical conceptualisations, and failed to address widespread conceptual development difficulties amongst the lower-achieving majority of the class.

Implications for Future Pedagogical Reform

In the study reported in this thesis, the results of a collaborative attempt to reform radically a teacher's centralist pedagogy highlight the inadequacy of a theory of pedagogical reform which is based only on a cognitivist theory of constructivism, especially when an externally-mandated curriculum and assessment policy is being implemented. What are the implications of the results of this case study for future constructivist-oriented pedagogical reform in other traditional mathematics high school classrooms?

The results of this case study indicate, firstly, that a constructivist perspective which emphasises the central role of prior knowledge in the individual learner's construction of new concepts is susceptible to being subsumed by objectivist preconceptions of the nature of mathematics, especially by: (1) a realist preconception of the nature of mathematics which objectifies mathematical knowledge, and promotes a pedagogical focus on the cognitive interface between students' prior knowledge and their reception of new knowledge from external sources; and (2) a formalist preconception of mathematics which focuses on the syntax of mathematical language, and promotes a pedagogical focus on the prior-knowledge-related development of algorithmic ability, while ignoring the semantics of the language which are constituted by students' conceptualisations.

Secondly, the study has indicated that a cognitivist theory of constructivism which focuses pedagogical attention largely on the cognition of the individual learner can reinforce rationalist preconceptions of the nature of mathematical cognition. In particular, this constructivist perspective is susceptible to being construed as compatible with the image of the *student as a cognitive isolate,* and can fail, therefore, to provide a compelling rationale for developing a pedagogy which addresses important social and emotional aspects of the intersubjective construction of mathematical knowledge by the classroom community.

Thirdly, the results of the study indicate that newly developed pedagogical beliefs which are based on a cognitivist theory of constructivism are susceptible to being relegated to the status of highly desirable but untenable ideals which lack practical curriculum viability, especially when they conflict with technical curriculum priority interests which are legitimated by authorities external to the classroom. In particular, the compelling nature of the technical curriculum priority interest in *covering* common syllabus *content* can result in the deliberate compromise of constructivist-related pedagogical ideals.

Taken together, the results of this study suggest strongly that pedagogical reform of the traditional high school mathematics classroom which is based only on a cognitivist theory of constructivism, especially a theory which emphasises the central role of prior knowledge in the individual learner's construction of new concepts, might be constrained from attaining little more than minor classroom role changes for both teachers and students by teachers' powerful preconceptions of the nature of mathematics. Furthermore, the results of this study suggest strongly that the nature and scope of cognitive constructivist pedagogical reform might be highly constrained in classrooms which implement externally-mandated and highly prescribed curriculum and assessment policies that prefigure a technical curriculum priority interest in *covering* common syllabus *content*.

The study suggests further that, although traditionalist teachers' cognitive-constructivist-related pedagogical reforms might be narrow in scope, both teachers and students might harbour favourable perceptions of the educational significance of the innovations. For example, instructional innovations which enhance students' recall of previously encountered algorithms might be perceived to be a major improvement in comparison with formerly didactic and expository teaching styles. Furthermore, these favourable perceptions might be reinforced by the results of ongoing summative assessment procedures which are designed (unwittingly) to assess students' knowledge-reproduction behaviour.

Nevertheless, the results of this study indicate that a prior-knowledge-oriented pedagogical reform focus is unlikely to attain the elusive goal of enhancing students' mathematical conceptual development, especially amongst the lower-achieving majority of students whose learning interests tend to be masked from the teacher by (1) the activities of the higher-achieving minority of target students, and (2) the teacher's technical priority interest in delivering the *content* of the curriculum.

CRITICAL DISCOURSE: TOWARDS A MORE BALANCED RATIONALITY IN THE MATHEMATICS CLASSROOM

The results of the study reported in this thesis indicate the inadequacy of a cognitivist theory of constructivism as a referent for pedagogical reform which aims to enhance students' mathematical conceptual development. Although the first principle of constructivism focuses pedagogical attention on the important role of the individual learner's extant mathematical knowledge, as the sole referent of pedagogical reform it seems to offer little prospect for resolving the problem of teacher centralism. In particular, cognitive constructivism seems unable to provide a compelling rationale for addressing key pedagogical beliefs which might underpin the transmissionist epistemologies of traditionalist teachers of school mathematics.

This conclusion raises the following question which relates directly to the initial research problem of this thesis (see Chapter One) and which is addressed in this final section: *What are the future prospects of constructivist-oriented pedagogical reform for overcoming the contemporary and widespread problem of the traditional transmissionist style of teaching mathematics which characterises the traditional mathematics classroom?*

In this thesis two additional constructivist perspectives — radical constructivism (see Chapter Six) and social constructivism (see Chapter Seven) — were adopted as interpretive frameworks. These frameworks made visible for critical examination key pedagogical beliefs which underpinned the teacher's transmissionist epistemology and constrained the nature and scope of his pedagogical reforms. These constructivist perspectives seem to be appropriate referents for reform which aims to overcome the pedagogical problems associated with teachers' realist, formalist, and rationalist conceptions of mathematicas and mathematical cognition. Recently, radical and social constructivist perspectives have been amalgamated in order to provide theoretical frameworks for research programs in mathematics education on student learning (Bauersfeld, 1988, 1992; Cobb, Wood, & Yackel, 1990, in press; Cobb, Yackel, & Wood, 1992; Wheatley, 1991, in press) and teacher cognition (Tobin, 1990a, 1990b, 1990c, 1991a, in press; Tobin & Espinet, 1987, 1989).

However, the results of this study suggest strongly that, although radical and social constructivist perspectives appear to hold much promise for the reform of traditional mathematics teaching, the social reality of the traditional mathematics classroom is prefigured by a technical cognitive interest which conceives of the curriculum as a product to be delivered by the teacher. Therefore, unless pedagogical reform addresses the technical rationality of traditional teaching, the technical curriculum interest in *covering* the syllabus *content* seems likely to sustain the centralist classroom role of the *teacher as controller* and compromise pedagogical principles based on radical and social constructivist perspectives. Refinements of the social reality of the classroom, therefore, are likely to be restricted to the attainment of *practical* curriculum interests (Grundy, 1987) which are concerned with facilitating conceptual understanding by means of negotiation and consensus buiding. Reforms which are based on practical curriculum interests, however, are unlikely to address the problematic nature of the extant curriculum framework and its underpinning technical rationality.

Critical Constructivism

A constructivist perspective which addresses the problem of teacher centralism from a curriculum perspective is critical constructivism. This perspective, which was adopted as an interpretive framework in Chapter Eight, made visible for critical examination the teacher's technical rationality. The critical constructivist perspective of this thesis is based largely on Grundy's (1987) interpretation of Habermas' (1972) theory of human cognitive interests, and argues for the realisation of the *emancipatory* curriculum interest in order to overcome the disempowering effects of the technical rationality which underpins teachers' technical curriculum priority interests. However, critical curriculum theorists argue that technical rationality constitutes a powerful cultural ideology which extends throughout the profession of modern education and encompasses curriculum and assessment policy, teacher education, and classroom teaching practice, and whose hegemony depends on its invisible presence (Apple, 1990; Giroux, 1981; Schon, 1983).

Because the microculture of the school classroom is buttressed by the macrocultures of the institution and of the larger society whose interests it serves, it does not seem feasible to envisage constructivist-oriented pedagogical reform which aims, overnight, to reconstruct radically the social reality of the traditional mathematics classroom. Nevertheless, this type of classroom might serve as a site in which flourish emancipatory interests that, in the shorter term, contribute to a more balanced rationality.

Critical Classroom Discourse

The type of pedagogical reform which is envisioned is one which involves both the teacher and students in an ongoing collaborative endeavour which aims to make visible and subject to critical examination the hidden frames of reference which constitute the prevailing and constraining rationality of the traditional mathematics classroom. Reform-minded teachers might promote a critical awareness, and therefore attain a more balanced rationality, in their traditionally-oriented mathematics classrooms by adopting a critical constructivist perspective and establishing with students a critical discourse based on Habermas' (1984) notion of *the ideal speech situation*.

For Habermas (1984), human cognitive interests constitute forms of human knowledge, and arise out of the actions of speech (see Chapter Eight). Habermas argues that, although it is realized rarely, everyday speech is oriented towards achieving the ideal of a genuine consensus through discourse:

> The rationality proper to the communicative practice of everyday life points to the practice of argumentation as a court of appeal that makes it possible to continue communicative action with other means when

> disagreement can no longer be headed off by everyday routines and yet is not to be settled by the direct or strategic use of force
>
> (Habermas, 1984, pp. 17-18).

However, the attainment of genuine consensus amongst participants in a discourse requires that an ideal speech situation (i.e., an empowering environment that provides opportunities for truth, justice and freedom) be created; these are moral principles which are cornerstones of Habermas' notion of the *good life* (McCarthy, 1985, p. 90). In the ideal speech situation there is full reciprocity which is characterised by "an absence of external coercions and internal distortions" (Dews, 1992, p. 17). Attainment of the ideal speech situation requires, therefore, that participants be emancipated from the distorting effects on their own self-consciousness of externally imposed constraints.

It seems, therefore, that the ideal speech situation is marked by the following conditions:

1. There are no constraints (such as lack of time) upon discussion. Therefore, any participant has full opportunity to question the truth of another's argument in arriving at a consensus.

2. All participants have unimpaired self-representation. That is, they are willing to disclose their true intentions and motives; and they are willing to accept responsibility for their own actions, and may expect the same of others.

3. All participants are free of coercion (such as bullying), possess equal right to command others, and allow each other equal opportunity for self-expression.

Although the ideal speech situation constitutes a utopian perception of communication, it can serve as a referent for the establishment of a pragmatic form of classroom communication, namely, a critical discourse. A teacher's establishment of a critical discourse which aims to make visible and subject to critical examination the major constraining frames of reference which underpin the traditional culture of the mathematics classroom might serve as a starting point for the attainment of a balanced rationality. In relation to this thesis, a reform-minded teacher should attempt to establish a critical classroom discourse which reveals the prevailing rationalities that prefigure and constrain the teaching and learning activities of the classroom, and that restrict opportunities for the enhancement of students' mathematical conceptualisations and intellectual autonomy.

Revealing the Technical Curriculum Interest

In mathematics classrooms in which the teaching and learning activities are framed by an externally-mandated curriculum and summative assessment policy, one of the major constraints to achieving a balanced rationality is the teacher's perception of the need to ensure that students *cover* the common *content* of the syllabus. The teacher's main clasroom role becomes that of *teacher as controller*, and students are required to act as passive and compliant consumers. Although the teacher assumes the locus of control of classroom activities (e.g., design of instructional activities, management of classroom discourse, validation of students' knowledge), the teacher remains an agent of an external locus of control which resides with the curriculum policy makers. In these types of traditional curriculum systems, both the teacher and students are disempowered; the taken-as-natural curriculum frame of reference constrains the teacher and students to act in accordance with a predetermined and impoverished pattern of physical and cognitive activities.

Attaining a balanced rationality in the mathematics classroom requires the teacher to establish a critical discourse which reveals the culturally constructed nature of the curriculum and assessment policy, and identifies the policy makers and their technically-oriented assumptions about the nature and purpose of schooling, in general, and of school mathematics, in particular. For example, it might be the case that an externally-mandated assessment practice is based on assumptions about the desirability of normatively distributed scores and ranking of students. The constraints of this form of assessment (e.g., individual development subjugated to common content coverage in a fixed time frame, limited opportunities for high achievement, divorce of learning from assessment, control of learning vested in external authorities) should be revealed as a politically inspired frame of reference.

Nested Curriculum Policy. A balanced rationality might be promoted by student participation in the design of a *nested* curriculum policy, that is, an alternative policy which is designed to operate within the framework of the predominant technically-oriented curriculum policy and, simultaneously, to subvert its hegemony. For example, in addition to the externally-mandated assessment system, a classroom-based assessment system that serves better the daily interests and needs of both the teacher and students could be designed collaboratively by the class. From a critical constructivist perspective on learning, the collaborative process of classroom policy formulation would result in a critical discourse on: the nature and purposes of assessment and its relationship with learning; design criteria for assessing conceptual development (cf. algorithmic ability); the role of the individual student in determining the viability of his/her newly constructed knowledge, and the collaborative role of the classroom community in constituting a consensual domain (cf. teacher/textbook validation).

The type of classroom discourse which both critically appraises the predominant technical curriculum frame of reference and engages students in a process of designing a nested curriculum policy can make a significant contribution to the attainment of a balanced rationality in the mathematics classroom. Not only does this type of discourse provide opportunities to make visible a major frame of reference which otherwise would continue to disempower both teachers and students, but also it stimulates the development of a more democratic classroom culture which is conducive to the development of intellectual autonomy.

Revealing the Myths of Realism, Formalism, and Rationalism

The culture of the traditional mathematics classroom is shaped also by powerful myths of the objectivity of mathematics and the rationalist nature of mathematical thinking. These myths continue to be propagated by an ideology which determines that the teacher is the primary source of students' mathematical knowledge, and that the formal rules of deductive symbolic logic constitute mathematical thinking.

Attaining a balanced rationality in the mathematics classroom requires that the teacher establishes a critical discourse aimed at revealing the Platonist myth that underpins intuitive and popular perceptions of the objectivity, certainty, and truth of mathematics (see Chapter Six). A critical discourse might focus on: the issue of the apparent truth of mathematics, especially, for example, the perception that geometrical properties inhere in geometrical forms; the concept of theoretical frames of reference, especially their role in governing our perceptions of the natural world and in making cultural conventions appear to be natural; the derivation of frames of reference from bodily experiences, and the empirical nature of mathematics; and the intimate relationship between the fields of science and mathematics, particularly their complementarity in constructing (mythical) models of the natural world.

The teacher might engage students in this type of discourse during, for example, their study of the prescribed curriculum topics of two- and three-dimensional (Euclidean) geometry which focus on the properties and constructions of regular polygons and regular polyhedra. Students should be provided with opportunities to become aware of non-Euclidean geometries (e.g., Riemann, Lobatchevsky, Bolyai), and of the historical significance of these frames of reference in replacing the scientific myth of the Newtonian clockwork universe with Einstein's relativistic conception of the universe (e.g., the observer-dependent laws of space and time) (Kitcher, 1984; Kline, 1953, 1982). The realisation that the axioms and theorems of Euclidean geometry were formulated on the basis of human experience of a seemingly flat earth provides a compelling case for the culturally and empirically constructed nature of mathematical frames of reference. Similar realisations about the number system can be facilitated from both historical and ethnomathematical perspectives.

The apparent certainty of mathematics and rationalist nature of its genesis has been sustained by a more recently established myth, namely, that of formalism. The aim of the formalist program was to re-establish the certainty of mathematics that had been lost by the discovery of non-Euclidean geometries. Formalism abandoned the notion of mathematics as comprising accurate descriptions of reality and, in essence, viewed mathematics as the science of rigorous proof. From a formalist perspective, mathematics is the application of formal mathematical logic for the purpose of deducing *content-free* theorems which await non-mathematical or scientific interpretation in relation to the world of human experiences (Ernest, 1991; Lakoff, 1987; Davis & Hersch, 1981; Hersch, 1986).

Attainment of a balanced rationality in the mathematics classroom, therefore, requires the teacher to establish a critical discourse on: the distorted nature of mathematical learning experiences which are seemingly irrelevant to students' social and physical worlds; the nature of mathematical thinking, especially the difference between algorithmic ability and conceptual development; the extent to which the technical curriculum priority interest (e.g., time constraints, common syllabus content) results in algorithmic ability constituting the primary goal of school mathematics; the difference between mathematical codifications (e.g., the algebraic symbol x) and mathematical concepts (e.g., the concepts of *place holder*, *unknown number*, and *generalised number*); and the difference between observable mathematical behaviors and unobservable mathematical ideas.

Of special importance in the attainment of a balanced rationality in the mathematics classroom is the requirement for a critical discourse to focus on the relationship between students behaving in accordance with traditional teacher (and student) expectations (e.g., passive listening, uncritical acceptance of solution strategies, working in isolation, accepting lack of comprehension, an unchallenging attitude to discourse) and the self-suppression of intellectual development. In other words, attaining a balanced rationality requires that the teacher adds the formerly taken-as-granted social reality of the classroom to the curriculum agenda, and promotes a discourse which examines critically the constitutive roles of both the teacher and students in its propagation and reconstruction.

POSTSCRIPT

This final section argues that it is not feasible to contemplate the overnight radical restructuring of the traditional mathematics classroom culture, especially in classrooms where prevailing technical curriculum interests are legitimated by external authorities. The prospects of future pedagogical reform which is based on social and radical constructivist perspectives on learning are dependent, therefore, on (1) the adoption by reform-minded teachers of a critical constructivist perspective on the curriculum interests which govern the teaching and learning activities, and (2) the establishment of a critical classroom discourse which aims to attain a balanced rationality by making visible and subject to critical examination the hidden frames of reference that constitute the prevailing rationality of the mathematics classroom.

Rather than rejecting the established technical curriculum interest and advocating the immediate radical transformation of the social reality of the classroom in accordance with practical and emancipatory interests (a process which reinforces the *teacher as controller* classroom role), a process of transitional pedagogical reform is envisioned in which the technical curriculum interest is subsumed momentarily but frequently by practical and emancipatory curriculum interests which underpin critical discourse amongst teacher and students.

In the short-term, teachers and students might continue to act for strategic purposes (e.g., preparation for examinations) largely in accordance with the established goals of the traditional curriculum culture. However, from the moment that the hidden cultural frames of reference begin to become visible, teachers and students will become mutually aware of the boundedness of their conceptualisations and of the technical curriculum interests which have shaped them. New choices will emerge about the extent to which it is preferable or desirable to continue to comply unquestioningly and uncritically with established cultural goals. A pedagogy which promotes a discourse based on the critical appraisal of new choices will have begun not only to reconstruct radically the traditional microculture of the classroom, but it also will have germinated the seeds of the reconstruction of the macroculture of the educational institution and of society at large.

A critical classroom discourse constitutes the process of the emergence of a balanced rationality in the mathematics classroom and is, therefore, *in itself* a form of emancipation. With the establishment of critical discourse in the mathematics classroom, the true nature of mathematics might begin to be revealed:

> [M]athematics does appear to be the product of human, fallible minds rather than the everlasting substance of a world independent of man. It is not a structure of steel resting on the bedrock of objective reality but gossamer floating with other speculations in the partially explored regions of the human mind. (Kline, 1953, p. 481)

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APPENDIX A

FIELDWORK DATA AND PRELIMINARY DATA ANALYSIS
LESSONS ONE AND TWO
RAY'S GRADE 12 CLASS
1987

APPENDIX A

FIELDWORK DATA AND PRELIMINARY DATA ANALYSIS LESSONS ONE AND TWO, RAY'S GRADE 12 CLASS, 1987

Chapter One discusses the data collection and analysis activities which were conducted in accordance with the interpretive research approach of Erickson (1986). This appendix contains the following examples of fieldwork data and preliminary data analysis which were associated with the first two lessons observed in Ray's Grade 12 class in 1987:

(1) Fieldnotes which are records of data that were collected during daily participant-observation activities in Ray's class. The fieldnotes comprise: handwritten *descriptive analyses* of lesson content, pedagogical strategies, and classroom verbal interactions; copies of teacher-designed worksheets and other *handouts* which Ray provided to the class; after-lesson *debriefing notes* of informal interviews with Ray; records of Ray's *written responses* to questions which emerged from my reflective analyses of the fieldnotes; and curriculum policy *documents* collected from the school.

(2) Reflections which are records of preliminary data analyses that were conducted daily on the fieldnotes immediately on return from the field. Copies of the Reflections of each lesson were provided to Ray for his contemplation (see Chapter Three) and validation (see Chapter One). The Reflections comprise: *synoptic analyses* of lesson content (See Tables A1 & A2); *interpretive analyses* of pedagogical strategies and students' conceptual difficulties; and *questions* which were designed to stimulate Ray's development of a critical, constructivist perspective on the teaching and learning activities in his class.

Chapter Three discusses the theoretical framework which I employed to analyse classroom discourse, especially the four-part classification of the organisational nature of students' classroom activities. In relation to this classification system, the following codes were used in the fieldnotes and Reflections: WCNI = Whole-class Non-interactive; WCI = Whole-class Interactive; ISW = Individual Seatwork (see Chapter Three for details).

In addition, the following codes were used frequently: T = teacher, S = student, S's = students. Please note that the only alteration which has been made to the handwritten fieldnotes is the erasure of indicators of the teacher's true name (Ray is a pseudonym).

Fieldnotes
Lesson One, 1 May 1987

① Lesson 1 1/5

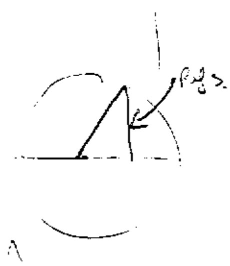
9.51
T. 'Reminder / review'
WCNI
ordered pr elements
rule of correspondence = linked 1st member of pair to 2nd

talks about diff b/w Relns / Fns. B.W (x , x)

Trig Fns of S, C, T — Unit ⊙
Graphs of S $f(x) = \sin x$.

9.53 "today look at variation of those"

fn is rule of corresp . . .
or
verbal def. of fn

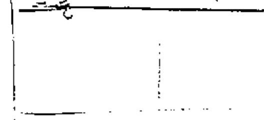


for ea pt. x — plot value of f(x)
i.e. sin x
pair pts on C.C. System
according to rule $f(x) = \sin x$

y is not f of x
looking at fn. $f(x) = \sin x$
plot x, value f(x) on y axis

9.55
push pts + hold them

mi Estimates for points



π, π/2.

Appendix A

Fieldnotes
Lesson One, 1 May 1987

②

find approp values sin x from tables or calculator

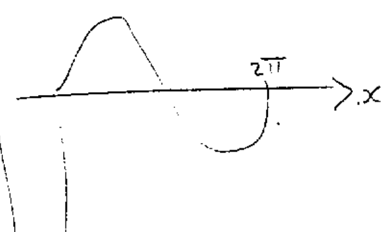
- fills in sin x col (expository)

Sin

x	sin x
0	
$\pi/4$	
$\pi/2$	
$3\pi/4$	
π	

draws curve, labels x axis

done it before ∴ no great detail



'Sig Features' where graph crosses axes, + max + min. good to know.

refers to unit ⊙ for max / min. values <1, >-1

10am (WCNI) Sure most of you are familiar c̄ these trig fns already

[sets up table
[fills in values

1st Qn sin $\frac{\pi}{2}$?

S' → 0

Special Angles

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0		—		—
cos	1	—			—
tan					—

Fieldnotes
Lesson One, 1 May 1987

(3)
can use sig. features to help find where graph crosses x + y axes

look at fn when varies

$f(x) = 3 \sin x$
$f(x) = \sin 2x$

; Q's by S's for clarification of x values in table.

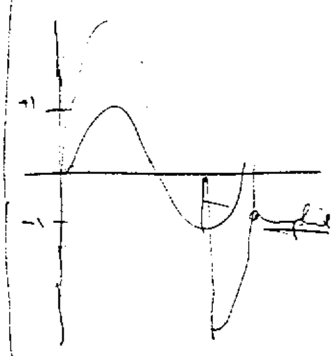
10.04 WCN1
Gen shape same, gen properties change

T. Q. What happens to height of gr. on v.axis for $f(x) = 3 \sin x$
– gen. answers from S's
T – right elaborates on answer

introduces term 'amplitude' of graph

no. of '3' affects amplitude

∴ const. placed in front of the fn ($\sin x$) is the amplit.



ISW 10.07 Draw graph of $f(x) = \sin 2x$
: Most probab. know – draw few pts + verify it for yourselves.

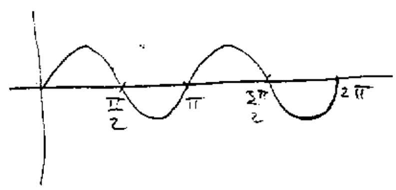
S'q re. parameters – radians?
T moves around class, checking indiv. S's

Fieldnotes
Lesson One, 1 May 1987

(4)

10.11 T: gives answer

way you found is, I hope, is to try some values



(S) used intuitive arg cf. algebraic.

T: Some checked on calculator to find $f(x)$.
idea
'period of fn.', 'periodic fn'

asks 1 S: why graph goes on indefinitely.

(WXI)? S: unit ⊙ answer.
13.15 T: elaborates — fns all repeat themselves

W
Student Q+A

Tg make gen. eqn.
$y = a \sin bx$

amplitude
↓
$y = a \sin bx$
↑

b influences the period
what is relnshp?

S_1: period $= \left|\frac{2\pi}{b}\right|$

Fieldnotes

Lesson One, 1 May 1987

⑤

1SW 10.17

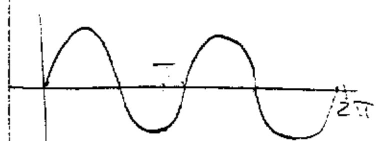
lets try graph $y = 2 \sin 3x$

use general rule

S: domain 360°?

T: ————→ $0 \leq x \leq 2\pi$

S's find difficult to articulate period relnshp.
after prompting → $\frac{1}{2}$, $\frac{1}{3}$
talk about sine curves occurring b/w 0 – 2π



10.23 WCNI

T: given ans.

refers to S's answers

gives process for finding x values – [proportional reasoning]

– from general rule ÷ [illegible] calculate period from b. (verbalised)

period $\frac{2\pi}{3}$

10.26 WCNI

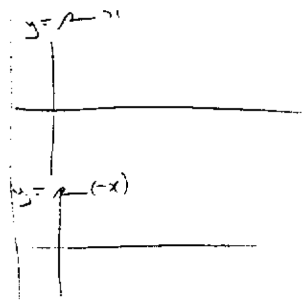
introduces – sign

"whole fn. will be inverted about the x axis"

S: a – x also value of period

T: put it there ∵ S' told me to

elaborates on role of || for period.



10.29 How does effect of ||

1SW

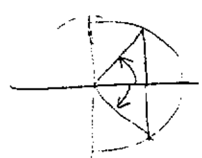
Appendix A

Fieldnotes
Lesson One, 1 May 1987

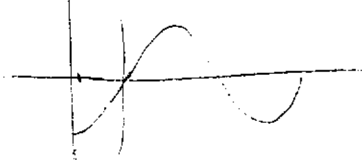
(6)

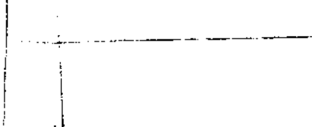
S's: How find -ve values? {sin -(-x)
S1: intuitive - just drew graph
S2: new from before - -
S3: with difficulty
S4: haven't yet

10 31 WCNI
gives unit ⊙ explanation



look at -ve region
for sin -x
sin (-90) = -1





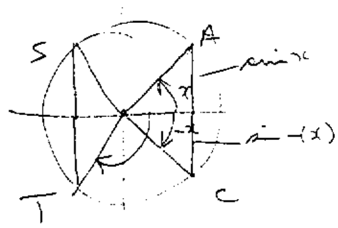
∴ sin(-x) = how shall I
write it

S: sin (-x) = sin x

T: No it doesn't.

sin(-x) = -sin x

WCNI
gives another 'unit ⊙'.
explanation
sin(-x) same 'magnitude'
but different sign
talks about sin (180-x).



Fieldnotes
Lesson One, 1 May 1987

(7)

10.37 gives verbal rules for -ve values of sin fn.

For think:

S.Q re. location of $-x$ on graph axes
T — -ve region / rotation.

1. $y = 4 \sin 4x$
2. $y = \frac{1}{3} \sin \frac{x}{2}$
3. $y = \frac{2}{3} \sin \frac{-3x}{2}$
4. $y = 2 \cos 3x$
5. $y = \cos(-\frac{x}{2})$

10.40 end.

Appendix A

Fieldnotes
Lesson One, 1 May 1987
Debriefing Notes

1 May, 1987

DISCUSSION WITH BF AFTER LESSON

1. Second year at school.
2. Taught only a few of the group last year.
3. Felt he hadn't covered much that lesson.
4. Believes 'Mason & Broom' contains adequate coverage of class work.
5. Intention is to teach 'Relations and Functions' concurrently with 'Circular Fn's' and 'Graphs'. Recently covered 'radian measures'. (ref. Maths 1 Syllabus)
6. Receptive to the notion that some students were confused over the concept of period, and the role of 'b' in the 'general rule' ($y = a \sin bx$), and in determining the x-intercepts from an application of the rule: 'period = $\frac{2\pi}{b}$'.

   Proportional reasoning may have a role to play - perhaps use TOLT on students.

BEFORE LESSON

Provided documentation: EDWA Maths 1 Syllabus p. 151
Maths 1 Year 12 General Objectives

Appendix A

Fieldnotes
Lesson One, 1 May 1987
Document 1: School Curriculum Policy

MATHEMATICS 1

YEAR 12

GENERAL OBJECTIVES

KNOWLEDGE OBJECTIVES

1 FACTS
To recall the facts and terminology of the course

2 CONCEPTS
To acquire the concepts of the course

3 RELATIONSHIPS
To understand the mathematical relationships of the course

4 SKILLS
To acquire the manipulative and computational skills of the course

5 USE
To use the facts, terminology, concepts, relationships and skills of the course in routine ways

PROCESS OBJECTIVES

6 COMPREHENSION
To comprehend information given in oral and written (including diagrammatic, graphical and tabular) forms

7 REPRESENTATION
To decide upon and use appropriate forms to represent mathematical data and relationships

8 GENERALIZATION
To make reasonable conjectures and attempt justification

9 APPLICATION
To apply suitable mathematical techniques and problem-solving strategies to routine and non-routine situations

10 COMMUNICATION
To communicate mathematical ideas and results, in oral and written (including diagrammatic, graphical and tabular) forms, showing method and reasoning where appropriate

SENSORI-MOTOR OBJECTIVES

11 DRAWING
To show facility in drawing diagrams and graphs

AFFECTIVE OBJECTIVES

12 INTEREST
To show interest in, and a positive attitude towards, mathematics and the learning of mathematics

13 PERSEVERANCE
To show a willingness to participate and persevere in the learning of mathematics

14 CONFIDENCE
To show confidence in his/her ability to use mathematics

15 RESPONSIBILITY
To show responsibility for his/her organization, presentation and learning of mathematics

16 INTERACTION
To interact in a constructive and co-operative manner with peers and teachers, and to assimilate praise and constructive criticism

Appendix A

Fieldnotes

Lesson One, 1 May 1987

Document 2: Secondary Education Authority Syllabus Policy

MATHEMATICS I (YEAR 12) — E201

INTRODUCTORY NOTE

It is presumed that students undertaking this course will have acquired the knowledge and skills necessary for the successful completion of the Year 11 Mathematics I course.

In the external examination, questions may be set on Year 12 material in which techniques from the Year 11 course are required.

For example, in a question testing correlation coefficients, students could be expected to calculate standard deviations.

SYLLABUS

1. SOLUTION OF ALGEBRAIC EQUATIONS

Solutions of simultaneous equations in two variables, the equations to be of the form $y = f(x)$ and $y = g(x)$. In the above f and g are restricted to linear, quadratic and reciprocal functions, and the systems of equations are restricted to those that are reducible to a single linear or quadratic equation.

2. LOGARITHMS

Logarithm; base.

Logarithms of products, quotients and powers of real numbers.

Solution of exponential equations of the from $a^x = b$ where a, b are positive constants.

3. SEQUENCES AND SERIES

Arithmetic and geometric sequences, nth term and sum to n terms. Sum to infinity of a geometric series whose common ratio is less than one in absolute value (intuitive treatment).

Summation notation.

Applications to geometric growth and decay problems, including compound interest, recurring decimals.

4. ANGLE

Circular arc length and angle measurement (radian measure). Angle measurement in degrees and its relation to radian measure.

5. CIRCULAR FUNCTIONS

Cosine and sine functions (defined with respect to real numbers and the unit circle), tangent function; amplitude, periodicity.

6. GRAPHS

Graphs of any of the following; linear, quadratic, reciprocal and exponential functions, circle centred the origin, trigonometric functions of the form $f(x) = a \cos bx$, $a \sin bx$ or $a \tan bx$.

(a) graphical solution of equations involving any pair,

(b) graphical solution of simultaneous equations in two variables involving any pair.

In (a), (b), refinement of an approximate solution by calculating appopriate values in a neighbourhood of the approximation.

7. INEQUALITIES

Graphical solutions of the following inequalities:

(a) $f(x) < g(x)$;

(b) $y < f(x)$;

(c) $x^2 + y^2 < a^2$, where a is a constant.

(d) simultaneous inequalities in two variables, the inequalities restricted to those of the form (b), (c).

In the above f, g may be any of the functions listed in section 2 Year 11, and $<$ may be replaced by $\leq, >$ or $\geq$. Application of the above to linear programming.

8. MEASURES OF RELATIONSHIP

Scatter diagram.

Covariance; correlation coefficient, with suitable computational formulae for simplification of calculations.

Effect of change of origin and/or scale on covariance and correlation.

Range of possible values of correlation coefficient; interpretation of correlation coefficeint as a measure of linear relationship.

Spurious cause and effect interpretation of correlation.

9. COUNTING TECHNIQUES

Addition and multiplication rules.

Permutations and combinations of distinct objects.

10. PROBABILITY

Addition rule for probabilities.

Mutually exclusive events.

Conditional probability; multiplication rule for probabilities.

Independent events.

Applications to include determination of probabilities by counting sample points in a finite equiprobable sample space either by listing outcomes or by use of suitable counting techniques.

11. NORMAL PROBABILITY DISTRIBUTION

Continuous probability distribution; normal distribution. Use of tables in calculation of probabilities associated with normal distributions.

EXAMINATION DETAILS

The examination will consist of one 3-hour paper. *Mathematics and Statistical Tables for School and University* by Hood and Storer or a combined book of *Mathematics and Statistical Tables and Chemical Data*, published by the Tertiary Institutions Service Centre or the Authority, should be taken into the examination.

In addition candidates should take into the examination a calculator. Calculators will not be made available at the examination; candidates must supply their own silent, hand-held, non-programmable calculator not having an alphabetical display.

Candidates will be permitted to take drawing instruments and templates into the examination.

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Appendix A

Reflections
Lesson One, 1 May 1987

1 May, 1987

REFLECTIONS 1

1. Predominantly teacher-centred whole-class non-interactive.

2m	(9.51- 9.53)	WCNI	'Revision' - Relns/Fns, Unit Circle, Trig Fns.
14m	(9.53-10.07)	WCNI	Introd. to graphing trig fns.
4m	(10.07-10.11)	ISW	Draw graph f(x) = sin 2x (amplitude).
4m	(10.11-10.15)	WCNI	Elaborate on answer to ISW.
2m	(10.15-10.17)	WCI	Theory on general equation y = a sin bx.
6m	(10.17-10.23)	ISW	Draw graph f(x) = 2 sim 3x (amplitude + period).
3m	(10.23-10.26)	WCNI	Elaborate on answer to ISW.
3m	(10.26-10.29)	WCNI	Theory on -ve sign f(x) = sin(-x) Answer student's question on period rule.
2m	(10.29-10.31)	ISW	Draw graph of f(x) = sin(-x).
6m	(10.31-10.37)	WCNI	Elaborate on answer to ISW.

WCNI	32m	70%
WCI	2m	4%
ISW	12m	26%
TOTAL	46m	100%

2. Assumed students had adequate prior knowledge largely from previous year (year 11) - continual reference to students knowing how to do ISW activities.

3. Theory was presented verbally with blackboard work supplementing presentation. No reference to text.

4. Major text used (?) 'Mason & Broom' and 'Hanrahan'.

5. Homework questions given by teacher on blackboard - no reference to students' texts.

6. Teacher presented information to students with little use of question and answer (ie. WCI - 4%) - Most questions were asked by (target?) students for clarification.

7. Students (sample) solved problems regarding amplitude and period and -ve x of function using intuitive methods rather than teacher prescribed method of finding the function values of selected values of 'x' and using the 'general rule'.

7a. Students' (sample) confusion over the concept of period and the role of 'b' in 'y = a sin bx'.

Reflections
Lesson One, 1 May 1987

7b. Teacher's explanation of the question:

'b influences the period - what is the relationship?'
Entirely verbal and not comprehensible to many students?
One student's answer taken:

'period = $\left|\frac{2\pi}{b}\right|$'

and not explained adequately?

7c. In ensuing ISW students (sample) could not articulate, in mathematical terms, the effect of 'b' on the period in their example $y = 2 \sin 3x$, or on the definition of period.

7d. Questionable as to whether the teacher's algorithm (given verbally) for finding the 'significant features' of the function was understood by students.

Appendix A

Reflections

Lesson One, 1 May 1987

Synoptic Analysis

Table A1

Nature and Duration of Lesson One Teaching Activities in Ray's Grade 12 Mathematics Class

Classroom Organisation	Duration (mins)	Pedagogical Focus	Mathematical Content
WI 1	2	Revision	*Relations* and *Functions*
WNI 1	11	Revision	Sketch of $f(x) = sinx$; *significant features*; *special angles*; *Unit Circle*
WNI 2	3	New theory	Sketch of $f(x) = asinbx$; *amplitude* & *period*
ISW1	4	Practice	Sketch of $f(x) = sin2x$
WNI 3	4	Solution	
WI 2	2	Poses problem	Influence of a and b on sketch of $y = asinbx$
ISW 2	6	Investigation	Sketch of $f(x) = 2sin3x$
WNI 4	3	Solution	
WNI 5	3	New theory	Influence of *negative* sign on $y = sin(-x)$
ISW 3	2	Practice	Sketch of $y = sin(-x)$
WNI 6	6	Solution and homework	

Note. WNI = Whole-class Non-Interactive (65%); WI = Whole-class Interactive (9%); ISW = Individual Seatwork (26%)

Reflections
Lesson One, 1 May 1987
Questions for Ray

QUESTIONS

1. How many students are not 'keeping' up with the pace of the teacher's presentation?
2. How do these students manage to understand the mathematical content of the lesson?
3. How aware is the teacher of the ability range of the students and what prior knowledge they have?
4. What factors determine the pace of the teacher's presentation?
5. Why does the teacher adopt a predominantly teacher-centred whole-class instructional style?

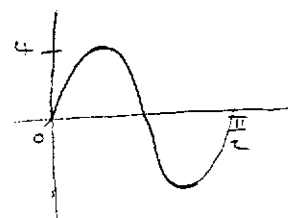
Fieldnotes
Lesson Two, 4 May 1987

①

Lesson 2

9.45 See how you got on 2 of Qns. You will have had some practice 2 & 2.

$y = 4 \sin 4x$



WCI

Q: what period of S?

S: ?

T: refers to → period $= \frac{2\pi}{b}$

should fit 2 complete cycles in domain 0 to 2π

T: Q of S on Ampl

S: 4π

T: what value of b

S: $\frac{1}{2}$

$y = \frac{1}{2} \sin \frac{x}{2}$

$\frac{1}{2} \times x$

T: now of b = ½

T: how much of fn will fit into dom S

S: $\frac{1}{2}$

T: Now what's period of ...

$y = \frac{2}{3} \sin -\frac{3x}{2}$

S: $\frac{2\pi}{1\frac{1}{2}}$

T: how express

S: $2\frac{1}{4}$'s

T: invert & mult

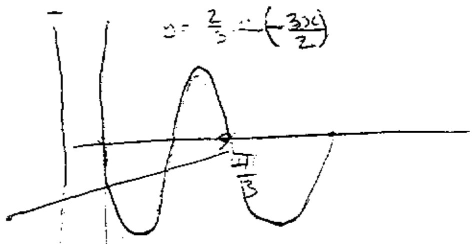
$\frac{4\pi}{3}$ $\frac{2\pi}{\frac{3}{2}}$

S: what is -ve?

T: can't have -ve period – give abs value

Fieldnotes
Lesson Two, 4 May 1987

(2)



T: what it look like
how draw
where stand

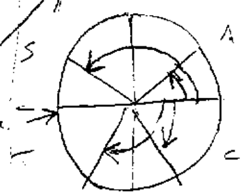
S: go down

T: right on $\frac{4\pi}{3}$

refers to prev. lesson

$\sin(-x) = -\sin x$

got a symmetry about there.



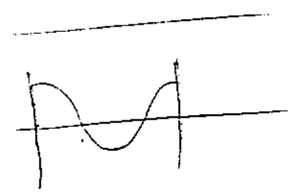
T: goes back to ex

T: angle $\frac{2}{3}$ 3rd, $1\frac{1}{2}$ cycles in

T: checks few S's

$y = 2\cos 3x$

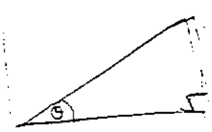
draws graph y=



T: 90°, or 2π "out of phase"

$\sin\theta = \cos$?

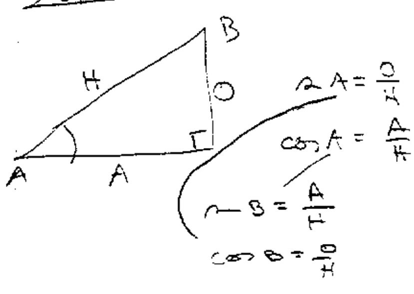
remembers relationship b/w side in a Δ?



S: say again please

T:

T: elicits S' responses



∴ sin one ∠ = cos other
+ vice versa
∴ "complement"

Fieldnotes
Lesson Two, 4 May 1987

(3)

T how rewrite ∠B (cos A ?
what does B equal
if replace B & put somet— in about A
B related to A

$A + B = 90°$

S's responses —
what does B equal

S — 90 − A $B = 90 - A$

T: cos $\Rightarrow \sin A = \cos(90 - A)$

T: shift 90° along axis : phase
sin graph same as cos gr
but moved along axis
S: "phase shift"

gesticulates on b'd re moving graph

T: not vital point, but if you can appreciate it.

pick up sig. pts from table

T: period of cos ?

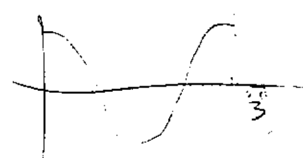
S: 2π/o

T: yes 2π

T: cos goes thru same natural cycle as sin

T: same rule applies $y = 2\cos 3x$

ampl and be 2
period $\frac{2\pi}{3}$
get 3 full cycles in



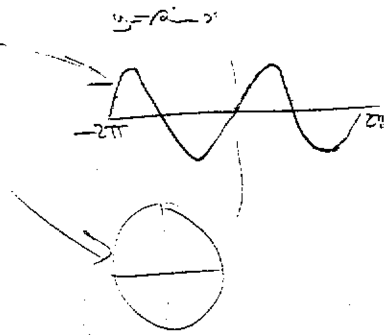
Fieldnotes
Lesson Two, 4 May 1987

(4)

T: all rules hold for sin hold for -x
However would look at something different.

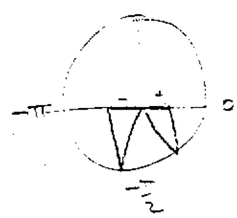
-ve part of graph -

going in -ve direction goes like this. Can verify using unit ○. Shows how



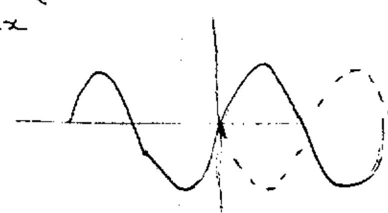
S: Qn?

T: no reason to assume sin of changes its shape in -ve region



10.03 Repeats with cos

Want you to see relationship b/w s(-x) + -sin x



superimpose -ve section over on +ve section see that
sin (-x) = -sin x

S: same for cos
T: correct - elaborates

Fieldnotes

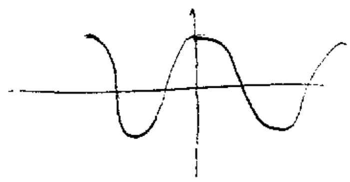
Lesson Two, 4 May 1987

5

Shows method — $\cos x = \cos(-x)$

& uses similar

argument →



(WC)

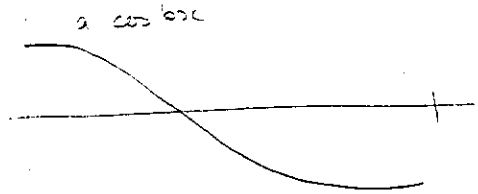
10.08

$y = \frac{3}{2}\cos -\frac{x}{2}$

a cos bx $\boxed{-\frac{3}{2}x}$

discusses

draws →



10.10 : moves round room

checking indiv pupils.

down 1 aisle.

10.13 lets move on then

little practice wouldn't

go wrong

they have finish

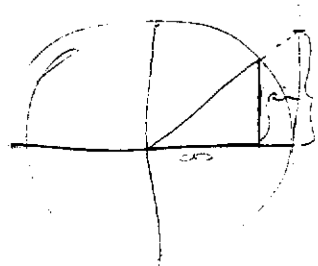
∴ principles are same

again let look at

unit O

draw tangent

talks about →



similar Δ's

unit length

etc

Fieldnotes

Lesson Two, 4 May 1987

(6)

T have ? done that too last

S's: yes

T: ok thought I'd ...

goes to unit ⊙

S: asks basic Qn about why sin + cos are used (90° △ ?)

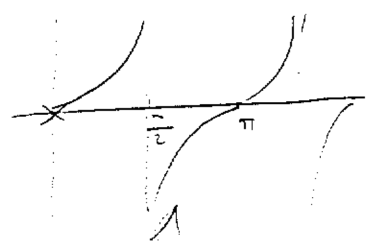
$\sin = \frac{opp}{hyp} = \frac{opp}{1}$

$\cos = adj$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

T: elaborates on relnshp b/w sin, cos, tan

discusses lengths of tangent line in unit ⊙ as θ gets v small + v lge — asks Q's of S's then draws



T: goes to unit ⊙ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ relnshp and discusses graph in $\frac{\pi}{2}$ to π

1018 — in 3rd Quad.
both S + C −ve
what does that make tan?

Asks S' to turn in

S: just explaining to her!

Fieldnotes
Lesson Two, 4 May 1987

T. elaborates on 4^{th} quad. (7)

10.25 can't talk about amplitude — amplitude undefined

S_{TA}: and period is π?

T: yes

T: why period of tan only π

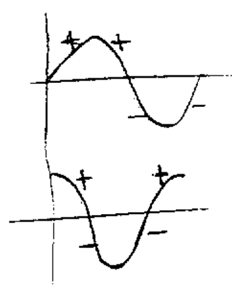
S: ∴ only 90° (naive)

S: ∵ not full cycle

T: no

S_{TA}: talks about sin/cos being −ve

T: tan goes +, −, +, −
all fns got repetition of shape
for each quadrant.
pattern already repeated within
itself : just by chance really



T: ? usual rule for period of tan?

S_{TA}: $\frac{\pi}{b}$ ~ — period $\frac{\pi}{b}$.

T: $\frac{\pi}{b}$ instead of $\frac{2\pi}{b}$ — $y = a \tan bx$

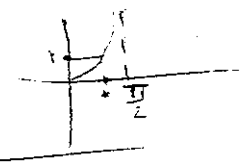
T: $y = 2 \tan x$
[illegible] pt will still have no effect

T: that's a rig. fig. for tan
S: yes

Fieldnotes
Lesson Two, 4 May 1987

(8)

S: asks clarification about finding 2tan2x, given x and uses $x = \frac{\pi}{4}$, tan = 1 example



T: × 2

S: oh, 2!

10.27

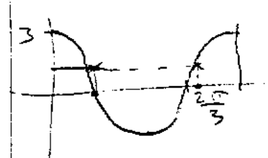
T: couple of other points
~~graph~~ describes meaning of them

graph — accurately, graph paper
sketch — general characteristics

S: use Mathaid

T: by graphing accurately draw gr of $3\cos\frac{x}{2}$ over cert. do—
we know am 3
per $\frac{2\pi}{3}$

→ find value of x when $3\cos\frac{x}{2} = 1.2$



T: how find value

S: —

T: yes, everyone u'stand that

$3\cos\frac{x}{3} = 1.2$

S: could be more than 1 value

T: yes got to look out for that

S: do we have to refine it

T: yes need Anne use calc to refine

Fieldnotes

Lesson Two, 4 May 1987

(9)

10.31 — Homework — $-\pi$ to π

find all values for which y equals these amounts

1) $y = \frac{1}{2} \cos \frac{x}{2}$ — $y = 0.3$

2) $y = 2 \tan \frac{-x}{2}$ — $y = -0.5$

3) $y = \tan 2x$ — $y = 5.0$

10.34 finish

Discussion with S' After Lesson

S: have to do work at home – with book

if don't visit see tchr after class

$\frac{1}{3}$ to $\frac{2}{3}$ class don't understand because of teacher's university-style lecturing

Fieldnotes
Lesson Two, 4 May 1987
Debriefing Notes

10

4 May, 1987

AFTER LESSON DISCUSSION

Centred on role of teacher as co-researcher and on possible focus. Discussed participation in pedagogical knowledge study and in student misconception study.

Discussed need for openness, commitment, criticism, and interpretation of data.

RE: LESSON

gt said that next lesson he would allow students to work, in class, on consolidating theory of last 2 lessons.

Asked what the girl student was telling me. I said I had asked her about:

1. Opportunities for students to get help from teacher.

2. Existence of tutorial classes.

3. Degree of understanding of the lesson.

Fieldnotes
Lesson Two, 4 May 1987
Ray's Written Responses to Questions of 1 May 1987

(11)

4 May, 1987

RESPONSES TO QUESTIONS

1. Probably about 1/3.
2. They probably don't - we can't wait indefinitely for them.
3. Fairly aware of ability range. Aware of prior knowledge they should have but many do not. Some have gaps in their knowledge of basics - eg. fractions.
4. Student progress partly, but mainly what I think we should be able to cover in that period. Try to introduce at least 1 'new' concept.
5. Has been my usual style. (Not that I think it's necessarily the best.) Don't really know any alternatives.

Appendix A

Reflections
Lesson Two, 4 May 1987

4 May, 1987

REFLECTIONS 2

1. Lecture style format for duration of lesson - teacher-centred instruction with boardwork.

2. Interaction between teacher and 'target' students through question and answer, plus a few other students.

3. Main pace of lesson 'driven' by target students, but slowed from time to time in response to 'slower' students' lack of comprehension, detected in questions directed to selected students.

4. Possible misconceptions arising from use of 'Unit Circle' model to explain:

   . periodicity of functions
   . phase differences between functions (sin, cos)
   . negative circular functions (sin -x)
   . derivation of tangent graph (ito sin and cos)

5. Tells students, from time to time, that they won't have had too much trouble with (homework) exercises.

6. Gives homework exercises (without reference to texts) to consolidate and link into next lesson?

7. Some students assisting friends to understand during the teacher presentation.

15m	9.45-10.00	WCI
10m	10.00-10.10	WCNI
1m	10.10-10.11	ISW
9m	10.11-10.20	WCNI
11 ~~9~~m	10.20-10.31	WCI
3m	10.31-10.34	WCNI
49m		

WCNI	WCI	ISW	TOTAL
22	24	1	47m
47%	51%	2%	100%

Reflections
Lesson Two, 4 May 1987
Synoptic Analysis

Table A2

Nature and Duration of Lesson Two Teaching Activities in Ray's Grade 12 Mathematics Class

Class-room Organis-ation	Duration (mins)	Pedagogical Focus	Mathematical Content
WI 1	15	Revision and homework	Sketching graphs; *significant points; Unit Circle; right triangle*
WNI 1	10	Revision	Influence of negative sign: $sin(-x) = -sinx; cos(-x) = cosx$
ISW 1	1	Practice	Sketch of $y = cos-(3/2)x$
WNI 2	9	New theory	Definition of *tan function (Unit Circle)*; sketch of *tan function*.
WI 2	7	New theory and practice	*Amplitude concept, period rule;* sketch of $y = 2tanx$
WI 3	4	Revision and new theory	Sketching vs graphing: graph of $3cos(x/2) = 1.2$, find x
WNI 3	3	Solutions and homework	

Note. WNI = Whole-class Non-interactive (45%); WI = Whole-class Interactive (53%)
ISW = Individual Seatwork (2%)

Appendix A

Reflections
Lesson Two, 4 May 1987
Questions for Ray

8 May, 1987

QUESTIONS

1. If the students can't do the homework questions and don't fully understand the teacher's explanation on the board, how else can they obtain solutions and understanding?

2. Is the text (Mason & Broom) adequate for the task of providing students with alternative instructional information?

3. What do the students think of Mason & Broom? Do they use it? How effective do they find it?

4. Are you satisfied with the quality of students' note taking from your 'A-V' presentations. How important are students' notes in developing understanding?

   Is your presentation conducive to students note taking?

5. How well are students (not the target students) understanding the basic concepts of your instruction? - refer to students' difficulties in 'Reflections 8/5'.

APPENDIX B

REPERTORY GRID ANALYSIS OF RAY'S ROLE-DETERMINING PEDAGOGICAL BELIEFS AND TRANSCRIPT OF TEACHER INTERVIEW 5

APPENDIX B
REPERTORY GRID ANALYSIS OF RAY'S ROLE-DETERMINING PEDAGOGICAL BELIEFS

This appendix presents a summary of the methodological procedures and results of the repertory grid technique which was employed in this study to determine Ray's implicit role-determining pedagogical beliefs. Table B1 provides a summary of the interviews which I conducted with Ray during the study. In particular, Table B1 indicates the four interviews associated with the repertory grid analysis (i.e., Interviews 3-5).

Table B 1

Teacher Interviews

Interview	Date	Type of interview	Interview record
Mid-point of collaborative phase of study			
1	12 June 1987 (after lesson 13)	general	tape & transcript
End of collaborative phase of study			
2	17 August 1987	general	tape & transcript
3	18 September 1987	RGT[a] 1 (elicit elements)	notes
4	22 September 1987	RGT 2 (elicit constructs)	notes
4a	12 October 1987	RGT 3 (construct repertory grid)	notes
5	3 November 1987	RGT 4 (discuss factors)	tape & transcript
Mid-point of post-collaborative phase of study			
6	19 August 1988	general	tape & transcript
End of post-collaborative phase of study			
7	13 September 1988	general	tape & transcript

[a]Repertory Grid Technique

REPERTORY GRID ANALYSIS

The following five-step procedure was adapted from the methodologies described by Munby (1982, 1984) and Olson (1980, 1981). The factor analysis procedure was modelled on the procedure described by Tobin, Kahle, and Fraser (1990). A summary of each of the five steps is presented.

STEP 1: ELICIT ELEMENTS OF REPERTORY GRID

At the commencement of Interview 3, I asked the following question of Ray:

> What sort of things might I see if I were to visit your classroom, say next week, and if it was your best class in terms of the best sorts of teaching you like to do. Use your language rather than mine. Tell me the sorts of things I would see in terms of brief statements like *the students are writing at their desks, the teacher is writing on the board, the students are working in groups.* (Interview 3)

I directed Ray to record his responses to the question on cards, and to explain each of his responses. The subsequent discussion (Interview 3) was audio-recorded. Table B2 shows the 39 event statements recorded by Ray.

STEP 2: ELICIT CONSTRUCTS OF REPERTORY GRID

Several days later, I conducted a second interview with Ray (Interview 4), and asked him to arrange the 39 cards into groups, and to explain his groupings. During this process Ray amended some of the statements he had recorded on the cards in step one. He arranged the 39 cards into the seven groupings shown in Table B3.

Table B2

Events Occurring in Ray's Best Classroom

Event	Description of Event
1	I ask questions in a whole class forum
2	Many students are prepared to respond to my whole-class questioning
3	Students may spontaneously ask questions generated by whole class discussions
4	I like to encourage involvement by everyone in the question and answer situation
5	I do some boardwork with a fair bit of question and answer interspersed
6	I like to use students' responses to direct my boardwork
7	I like to see written responses to monitor students' progress
8	The class may be invited to answer a question raised by another student
9	I like to get a lot of students' responses to a question to provide me with a picture of communal progress
10	Students responding in a whole class forum
11	Students' working on common content with frequent feedback
12	Students involved in small group problem-solving
13	I come round the class and help individual students
14	I come round the class and check progress how their concept is developing
15	I like to see students asking questions for help when they need it
16	I write on the board to make an important point
17	I write on the board to explain an example
18	I use worksheets to introduce a new topic
19	I use worksheets to consolidate a topic with practice questions
20	I use worksheets to provide a set of rules or strategies for problem-solving
21	I use boardwork to help stimulate student thinking
22	I like to see students coming up with spontaneous questions from their own study
23	I like to refer to last lesson's homework to possibly link to the present lesson
24	I give homework in order to provide practice work and extension
25	I like to see all students on-task and involved in the work that's going on
26	If students are not on-task I draw their attention to it
27	I like there to be a friendly atmosphere in the classroom; one of cooperation and informality
28	I like students to feel uninhibited to ask questions
29	I try to ask a variety of students' so they get used to answering questions
30	I try to use examples that are relevant, or have some meaning that students can relate to (e.g., draw analogies)
31	I try to visit students' individually and encourage them to ask for help
32	Students working individually on a problem
33	Students feel free to ask questions
34	I like to see a vigorous interchange between the teacher and students
35	I like to employ a lot of questions
36	I like to elicit a lot of student participation in responses to questions
37	I like students in the class to be thinking about the questions I ask
38	I like to see students asking questions of their own [of me]
39	I like students to have an opportunity to participate in a whole class forum

Table B3
Ray's Criteria for Grouping the 39 Events

Group	Criterion for Grouping	Event
A	Teacher (initiates) provides new information	13, 16, 17, 18, 19, 20, 21, 30
B	Students do work out of class	23, 24
C	Students initiate own involvement	3, 8, 10, 15, 22, 28, 37, 38
D	Teacher obtains feedback	6, 7, 9, 11, 14
E	Students engaged (in whatever activity)	1, 2, 4, 5, 29, 31, 34, 35, 36, 39
F	Students work individually or in small groups	12, 32
G[a]		25, 26, 27, 33

[a]These elements were not grouped.

The following extracts of Interview 4 indicate Ray's rationale for the label which he assigned to each grouping. A summary of Ray's verbatim explanation of the groupings — in the form of interview extracts — appears after each of the grouping labels.

Group A: Teacher (Initiates) Provides New Information

> This describes the process of actually exposing the student to new information, and this is crucial — it's a really distinct category. This is where . . . new concepts are introduced to students, hopefully by linking to something else, not just snatching them out of the blue. It could come about by student questioning, but I guess it's mainly a teacher-initiated activity, and I might provide that information by means of blackboard work. . . . I might present the information in an inquiry-based manner, so that I get the students thinking about some question and then present information that answers the question, or a technique with which to handle the question. I might do this on the blackboard or

I might use worksheets to introduce a topic. This all relates to providing information to the students. (Interview 4, 12 June 1987)

Group B: Students Do Work Out Of Class

I'm just talking about homework, basically, here. Students do homework. I set homework most lessons. . . . Sometimes it's exercises out of a text book, more often than not I suppose. Sometimes it might be in the nature of a worksheet that I've written, or an investigation, or some other kind of extension work. (Interview 4, 12 June 1987)

Group C: Students Initiate Own Involvement

There's not a lot I can do about that, I just like to see it. There's a bit I can do about it, I can encourage students to ask questions, that's what it's mainly about, I suppose, students coming up with questions of their own, because they don't understand something, or . . . they've thought of something that might be related, or because another student's asked a question I might ask if anyone else can answer it. That would be student-initiated . . . any activity that brings a student into contact with the teacher or the material . . . the subject matter.
Who's being involved there, the whole class, a single student?
Well, ideally the whole class will be involved. If a student asks a question, ideally, the whole class should be listening to the question, and if I reflect the question back to the whole class I'd like them to all be thinking about it. In this case [compared with teacher-initiated engagement] a student might initiate something.

(Interview 4, 12 June 1987)

Group D: Teacher Obtains Feedback

This overlaps with engagement because I obtain feedback by a process of engagement. What I'm interested here is finding out what the students know . . . how they understand things, and in order to do that I need to be able to monitor what they're doing and what they're thinking, and I can do that by listening to their responses to questions, looking at their work. . . . I need to know, want to know, what they understand . . . what they don't understand, where they're going wrong, what difficulties

they're having with a concept. . . . [It's a matter of] monitoring their progress so that I can modify my teaching strategy.
Does [the term feedback] have any action attached to it, in the sense that it's an action that you undertake or that the students undertake?
I guess, it's an action that I primarily undertake, I have to do things to get the feedback, to get the information, although sometimes it will be volunteered. (Interview 4, 12 June 1987)

Group E: Students Engaged (in Whatever Activity)

Students involved in what's going on, thinking about it, formulating questions about it, attempting to understand it . . . rather than simply copying stuff down from the board which they then try to understand later. . . . I'd rather they try to understand it at the time and ask questions, if they don't . . . to try to come to grips with the new material . . . attempt to understand it. It's partly a matter of attitude. I don't expect they'll all understand it first go. But I'd like them to be thinking about, trying to understand it, grappling with the concepts...rather than be copying down notes and thinking I haven't got a clue what this is about, I'll think about it later. . . . By engagement I'm referring to when I'm eliciting responses or when I'm kind of controlling what's going on, or maybe initiating what's going on, more to the point.

(Interview 4, 12 June 1987)

Group F: Students Work Individually or in Small Groups

This just describes what happens at certain times when work is set in class and students do it. This happens every period, for a greater or lesser period of time, or nearly period. They either work on their own, or they may discuss things with their neighbours, they usually do, or there may be a pre-organised group work . . . in a small group, on a specific task.
What do you mean by *working*?
Usually attempting to solve problems, answer questions, draw graphs or whatever, and maybe as part of that process they could be looking something up in their text book, looking through their notes. If they're working in small groups they would be discussing a problem.

(Interview 4, 12 June 1987)

Group G: Spare Elements

Ray was unable to assign a label for this grouping of events. Nevertheless, they were included in the repertory grid, and were assigned relationships with other groupings.

> By this, I mean the general way in which the students go about their work . . . cooperate. . . . The number of students who are on-task, is a measure of the thing I'm talking about. I like the students to feel comfortable and uninhibited in terms of asking questions. I think often . . . whether they do or don't is not so much a product of the class environment itself. [It] relates to student's own personalities. . . . Some students are forever asking questions and some never do, and I'm sure it's not because they haven't got questions or they understand everything perfectly. (Interview 4, 12 June 1987)

STEP 3: CONSTRUCT THE REPERTORY GRID

A repertory grid was constructed by arranging vertically the 39 statements of events, and arranging horizontally the six grouping labels (or constructs). I directed Ray to indicate his perception of the strength of the association between each event and each construct by assigning a score to each cell of the repertory grid. The following range of scores was available: 1 = definitely not associated; 2 = neutral; 3 = definitely associated. Table B3 indicates the completed repertory grid.

STEP 4: FACTOR ANALYSIS

A factor analysis was conducted on the repertory grid, with the constructs as variables, using the SAS *Factor procedure* (SAS Institute Inc., 1985) on the Curtin University Vax 3 mainframe computer. Initially, one-, two-, and three-factor solutions were obtained using the Principal Factors method for computing orthogonal factors. For each of the factor solutions, the factors were rotated using the Promax Rotation method for computing oblique factors. An analysis of the rotated factor patterns (i.e., standard regression coefficients), and of the variance explained by each factor after eliminating other factors, indicated that the best factor solution was obtained by a rotated factor pattern of three factors. Table B5 shows the final results. A variable which had a factor loading of magnitude greater than 0.40 was accepted as *belonging* to that factor (Ferguson, 1987). A *simple structure* principle was used to determine the best oblique factor solution:

> [S]imple structure . . . requires that no one factor should be related to all of the variables in the analysis. Rather, a simple structure solution would be seen when one factor might influence perhaps a fourth of the variables in the matrix and have essentially zero loadings on the rest of the variables. (Popham and Sirotnik, 1973, p. 261)

Table B5 indicates that the simple structure solution comprised non-zero loadings of variables on other factors. However, none of the other factor loadings was greater than the main factor loadings (all of which were greater than 0.40) and, in all cases, other factor loadings had regression coefficients of magnitude less than 0.30. It was concluded, therefore, that a satisfactory simple structure had been obtained.

Table B4
Completed Repertory Grid

	Criteria for Grouping					
Event	A	B	C	D	E	F
1	2	1	2	3	3	1
2	2	1	2	3	3	1
3	1	1	3	3	3	1
4	1	1	3	3	3	1
5	3	1	2	3	3	1
6	3	1	2	3	3	1
7	1	3	1	3	2	2
8	1	1	3	3	3	1
9	1	2	2	3	2	2
10	2	1	3	3	3	1
11	1	1	2	3	3	3
12	1	1	2	2	3	3
13	2	1	2	3	2	3
14	1	1	1	3	2	3
15	2	1	3	3	3	2
16	2	1	2	1	3	2
17	3	1	1	1	3	1
18	3	2	1	3	3	3
19	1	3	2	3	3	3
20	3	3	1	2	2	3
21	3	1	3	1	3	2
22	1	3	3	3	2	1
23	2	3	1	3	2	1
24	2	3	2	3	3	2
25	2	2	3	2	3	3
26	1	1	1	1	3	3
27	1	1	3	3	3	3
28	1	1	3	3	3	1
29	1	1	2	3	2	1
30	3	2	1	3	3	2
31	2	1	2	3	2	2
32	1	2	2	2	3	3
33	1	1	3	2	3	1
34	3	1	3	3	3	1
35	2	1	2	3	3	1
36	1	1	3	3	3	1
37	1	1	1	1	3	1
38	1	1	3	3	3	1
39	1	1	3	3	3	1

Table B5
Rotated Factor Pattern Standard Regression Coefficients for a Three-Factor Forced Solution for Ray's Repertory Grid Data

Grouping Criteria	Factor 1	Factor 2	Factor 3	Final Communality
A	0.02	0.07	**0.46**	0.17
B	**-0.59**	-0.02	-0.01	0.36
C	**0.40**	0.28	-0.26	0.49
D	-0.27	**0.49**	-0.10	0.29
E	**0.63**	-0.12	0.01	0.37
F	-0.25	**-0.48**	-0.18	0.27
% Variance[a]	9.6	3.5	2.1	19.6

Note. Boldface indicates factor loadings greater than 0.40 (Ferguson, 1987, p. 509)
[a]Variance explained by each factor eliminating other factors

Table B6
Factor Structure of Repertory Grid
Showing Groups of Constructs and Events

Factor	Criteria for Grouping	Event
Factor 1		
+++	Students engaged (in whatever activity)	1, 2, 4, 5, 29, 31, 34, 35, 36, 39
+	Students initiate own involvement	3, 8, 10, 15, 22, 28, 37, 38
---	Students do work outside of class	23, 24
Factor 2		
++	Teacher obtains feedback	6, 7, 9, 11, 14
--	Students work individually or in small groups	12, 32
Factor 3		
++	Teacher (initiates) provides new information	13, 16, 17, 18, 19, 20, 21, 30

Note. +/- signs indicate relative strengths of constructs.

STEP 5: INFER RAY'S IMPLICIT BELIEFS

The three-factor solution in Table B5 was interpreted in an interview with Ray (Interview 5). I discussed the results of the factor analysis (Table B6) with Ray, and asked him to explain each of the factor groupings. I employed a *probing* interview technique which aimed 'to probe for the beliefs and principles which give the best voice to the factors' (Munby, 1982, p. 219). The interview was audio-recorded and a transcript was made (see below). The following analysis presents a summary of propositional statements which are taken to represent Ray's individual pedagogical beliefs associated with each of the three factors.

Student Learning (Factor 1 Beliefs)

1. Student Compliance
The students' main role is to engage actively in the classroom-based learning activities prescribed by the teacher, in order to develop their understanding of the concept being presented at that time.

2. Ideal Student Attributes
The engagement of good students will be characterised by self-motivation and an attitude of inquiry.

3. Learning Constraints
Although differential rates of conceptual development will exist in the class, differential rates of progress through the syllabus cannot be tolerated because of the perceived constraints of external accountability for syllabus coverage and the limited abilities of students for self-determination.

Feedback (Factor 2 Beliefs)

4. Frequent Teacher Diagnosis of Students
Assessment of students' progress should be conducted frequently and informally by the teacher.

5. Teaching with Understanding
Student assessment is done most efficiently in the whole-class mode, by obtaining responses to teacher questions from a sample of key students.

The Teacher's Classroom Role (Factor 3 Beliefs)

6. Teacher Control Priorities
The teacher's main role is to exercise strong control over students classroom-based learning activities to ensure that both the learning

needs of most students and the requirements of the examination syllabus are satisfied.

7. Teacher Control of Syllabus Delivery

Learning needs and syllabus requirements are best fulfilled by the teacher presenting new content simultaneously to the whole class, and by ensuring that the whole class progresses in unison through the syllabus at an appropriate rate.

8. Constraints, Conflicts and Compromises

Ray experienced conflicts of interest and made compromises in relation to his constructivist perspective that favoured his perception of the constraints of both the syllabus and students' abilities.

TRANSCRIPT OF INTERVIEW 5

The transcript of the interview in which the factors were discussed and from which Ray's implicit pedagogical beliefs were inferred initially is presented below. The transcript contains important contextual data which are drawn on in the interpretation of Ray's pedagogical beliefs that is presented in Chapters Five to Eight. In this interview transcript the following codes are used: I = Interviewer; R = Respondent.

I Well, let's have a think about what each of those meant. Say, take the first factor. We'll just look at that alone in isolation. Now what did you mean by students' engaged?

R Well, basically they're working and interested in something, they're on task. So that, because these were categories of questions, categories of sort of observations weren't they?

I Yes.

R So when I group them together a lot of those observations or ideal kinds of behaviour mean that the students were doing what they were supposed to be doing?

I So in other words, you would like to see in a classroom students engaged in learning activities that you had set out?

R Yes.

I Okay. Rather than doing what?

R Rather than being off task in a rather . . . either talking to their neighbours or not doing the work or looking out the window, or as well, rather than doing some other maths work or catching up on something that they felt like doing, I'd rather them . . . I want them to be doing what I have set.

I Okay, that's interesting, so in other words, the direction and control is really laying with you in that regard and the students initiate their own involvement as a weaker correlation with this factor. So it's not so much you seeing the students as running the show, you're definitely in charge here and they're doing what they should do, but you're hoping that they would be initiating their involvement rather than having to be told.

R And with this one, with the students initiating their own involvement that also refers to things like them asking not only questions about the current tasks in hand, how do you do it, I can't do it or something like that, but questions that might be leading somewhere, like what would happen . . . I know how to do this but what would happen if something else. Ideally I'd like to think students were thinking broadly or starting to think of other possibilities instead of just simply dealing with current tasks.

I Okay, do you see any conflict then, or any potential conflict then, between the fact that you want to see the students very much engaged in the dynamics that you have established and the students having actually their own responsibility for being involved?

R No, not really, I can see what you are getting at.

I Tell me, because I'm not sure.

R Well, I think I can. I take it that you are saying that well if the students are going to be doing exactly what you want, then you set something and say okay I want everyone to solve this problem or everyone to attempt this calculation, or something, then how can they be initiating their own involvement at the same time as just following precise instructions and I guess that it is potentially an area of conflict, you, if you look at both of them in the extreme. I guess what I would like is the sort of compromise between the two. So the students doing what they are supposed to be doing, say solving a quadratic equation, but I would like to think that they might think other possibilities once they . . . particularly if we ask the questions in the right way, particularly if the design of the lesson or the explanation is such that it, what do you call it, unfolds, expository or something. So that . . . it's stepwise, you work along, say, let's take a simple example and then you take a more complex probably harder example and you look or introduce things bit by bit, then they might hopefully take the next step on their own and say what happens if the number in front of the x^2 is 2, or something like that.

I Oh, so what it sounds like you're saying is that you're providing the actual structure in which you wish to see students engaged but you're hoping that will generate enough momentum for self-motivation that they will go with it?

R Yes.

I Why does it negatively correlate with students do work out of class?

R No, I can't see why that's there.

I I wonder if its got something to do with your belief in your control of learning activities, so that factor 1 is something to do with what you believe in terms of how you control students work and if students are not working in class they are not working under your control. But you want some element of democracy in there as far as the students decision making is concerned and that's why factor 3 has a more positive correlation rather than a negative correlation. So does this mean that you're the despot who believes in democracy?

R I'm not sure.

I Factor 1 is to do with the principle which lies behind the way you want to see your classroom operate ideally and it seems that there is a very strong sense of teacher-centredness with respect to control of students' activities. Would you agree with that?

R Yes.

I You have made the statement to me in the past that you believe the teacher is paid to be somewhere towards the centre of action rather than say just being a tutorial resource who is at the disposal of the students' wants and needs. So I am beginning to see here, given this first factor, a belief in a central role for yourself in the classroom with respect to students' learning activities, which goes very much against the notion of students being independent learners in the classroom, independent learning takes place more outside the classroom.

I Is there a contradiction in having a belief about that teaching style when you consider the constructivist nature of learning?

R I don't think there is necessarily, because I think the teacher needs to, on the basis of even his constructivist approach, still needs to assess by feedback from the students where they are and what to do next, what course of action to take next. I don't believe the students are so good at initiating their own learning that they can do it entirely. I think they need guidance and, well, also in many cases they won't know what it is what they want to know next, or need to know next or that follows logically from where you are either in the syllabus or whatever.

I Is there a danger then that you never, in fact, given such a strong teacher-centred approach, give students the opportunity of developing those learning-to-learn skills? Learning to self-evaluate, learning to identify their own needs, and do they become too reliant on the central teacher?

R That's a hard question to answer, because it may vary a lot for each, may vary from student to student, but I think there has to be a sort of compromise position somewhere. If you just, say, look at one extreme where you just say, okay, we're going to learn about indices today, now start. You've got to give them some input somewhere or you've got to assess, or I believe these things, you've got to assess how well they've approached the task, maybe they can set their own goals in some ways, maybe they can determine their rate of progress, maybe they can determine their direction of progress, but I think only to a degree, or at least they may need redirecting.

I Will students readily be able to have their half of the bargain, given that a fairly strong teacher-centredness, in terms of direction and control, tends towards whole-class interactive, whole-class non-interactive type activities? And is that what, in fact, we are seeing when you say you see the students being engaged in the learning activity?

R Well, that they are doing pretty much the same thing as one another at the same time, I guess that's true.

I Contentwise you mean?

R Yes, contentwise. I mean they might all be graphing a quadratic equation or graphing a quadratic function, they might have a certain time to do it and then we might look at how far they've got or what they've done.

I So, this presumes a lockstep approach to content?
R Yes, to a degree I guess it does.
I Is that what you believe should be happening, or do you believe that students should be able to progress differentially with respect to content?
R Well, it would be nice if they could progress differentially I guess.
I But you don't think that they can and you'll work that way?
R Well I think the constraints of class size, syllabus, public exams, certainly inhibit that from happening, or from being able to happen.
I Okay, so are we seeing here with factor 1 a belief on your behalf that you are directing and controlling students en masse as they move together through the content and that you would hope that they can initiate some further involvement in the learning process within that context.
R Yes, that's where the differential between students will operate.
I Okay, just explain that differential in what regard?
R Well, I guess, inability, in the rate in which they can learn or in some cases whether they can learn a certain concept in a given time. Some students will grasp a concept quickly and then I would like to think that they would consider further developments of the concept, or maybe make further developments on their own, or pose questions about more complex examples perhaps. In the meantime other students will be struggling to grasp the basic concept.
I Okay.
R And I see that pattern as repeating itself throughout the year topic by topic, the same students are struggling to grasp the concept while others have got it under control and are capable of extension.
I Okay, and the teacher's role here?
R It's where I see the teacher's role as being one of sort of compromise, of leading the students through the topic at a rate which meets all the various constraints, that is, like rates of students, the varying rates of students' learning or the varying abilities, the necessity of getting, of actually feeling that you are progressing through a topic or the necessity of, in fact, progressing through a topic or covering material.
I Right, just for a contrast, would you see a Grade 8 maths class operating the same way? Given that you have similar constraints and similar range of student abilities?
R Yes, I guess I would pretty much in terms of covering content, I guess I would see it pretty much in a similar light.
I So that's really coming out as being a fundamental principle of yours isn't it, that all students cover the same content in the same time?
R Well, I don't think its so much a fundamental principle of mine, its not so much a belief, oh I suppose it's a belief, but I sort of see it as an externally imposed necessity.
I Okay, I can . . . you see that in Grade 11 and 12 where you are catering to a public examination and . . .
R Well, I would see it in Grade 8 in the same way because you have specified syllabus given to you, there is often some flexibility in those, but you're still expected, say by the senior teacher or whatever, to cover the syllabus material and not just to wander off on your own investigation.

I Does that accountability to the senior teacher exist also with Grade 11 and 12?

R Yes.

I So that's one of the constraints that you find yourself working under, a sense of accountability?

R Yes, I think so.

I Shall we have a look at the next factor, factor 2. Here we've got a reasonably strong positive correlation between teacher obtained feedback in the factor and almost the same negative correlation in size with students working individually or in small groups. Are we again seeing a conflict here similar to in factor 1?

R Yes, it looks a bit like that. Well it looks a bit like it in the sense that there is a conflict and the conflict seems to be between teacher initiated activities, maybe, and the activities in which the teacher is not involved directly.

I So, there seem to be two distinct activities happening, one where the teacher is very much involved. Can you describe the nature of the activity?

R The obtaining feedback or category of obtaining feedback.

I Where is it all happening, what situation?

R In a variety of situations I think it's an area that's a vital component of contructivist considerations, that the teacher needs to be able to assess the progress of the student on a fairly short-term basis and to do that we need to obtain feedback. That can be either written, looking at written answers that students had done, or asking them to maybe even mark their neighbour's answer or something like that, just for a short bit of work, or going around and asking students individually, while they are working, about their understanding or asking for students' responses in the whole-class situation.

I So there's a whole range of means of monitoring students' performance. With this factor it would seem to indicate that the students working individually or in small groups was a counter example of one of those, the counter example may be to whole class interactive. Would you think that was the way you prefer to obtain feedback? Is it the most efficient way of doing it?

R Well, I mentioned a variety of ways some of them might be more efficient than others, I can't really think of any other ways, or we have tests.

I Which is the most efficient way do you think?

R Of obtaining feedback?

I Yes.

R If you mean efficient in terms of time effectiveness from the whole class?

I Well, I suppose what I'm saying is what is the quickest and easiest way of getting feedback about the performance of individual students? How would you normally do that on a day to day basis?

R Well, by those things I said, ask a question of the class, get some responses or ask individual students, pick them out and ask them to explain something, if I choose 3 students and I know the sort of standard of the students or their ability in class, if I choose a student from the middle of the class and get a response from her, or do it for 3 or 4 students, that's a bit of a guide as to what I would probably expect the rest of the class to be knowing.

I Okay, so those methods you've just mentioned to me are examples of whole-class interaction where the teacher is controlling the discourse and usually front of the room orientated, everybody engaged in an activity that you were running, operating?

R Yes, that's true. Otherwise well just walking around and looking at students work or asking them in a one-to-one situation to explain what they are ding, that would probably be just as good.

I Would you have a preference for either of those two approaches, the monitoring students in the individual way by going around and watching what they are doing and establishing that situation in the first place so that you could do that, as opposed to whole-class orientation. Do you have a preference for those.

R Yes, I have a preference for sort of one-to-one situations wherever possible, but it's not always the quickest way of doing things because you can't get around to talk to students on a one-to-one basis very much, they are not going to get very much if you've got 30 students in a class, if you spent the whole class on one-to-one, then you're going to have a grand total of one minute with each student on the average.

I Right, so does that mean that you tended towards getting most of your feedback in the whole-class interactive mode?

R Yes, I get a lot of it like that. I think everyone doing a short section or a short question or a short step in a question and then asking the students for answers and then seeing if the whole class or most of the class have got the same answer or have got it right or whatever, that's quite a good way of assessing the progress.

I Okay, let's have the last factor, number 3. Teacher initiates or provides new information. Well that's fairly clear cut isn't it. What would you say was driving that action?

R Well I guess it's a bit like I mentioned before, the fact that I believe the teacher has to determine when new information is going to be provided and the nature of it. In other words, when to move on and expose students to the next step. Yes I see that as a teacher controlling the rate in a way.

I It goes back, to some extent, to the first factor where we were talking about a commonality of content amongst the students rather than the students being provided with opportunities to progress through the syllabus. You're really talking about, in this factor, that the teacher is controlling that by being the provider of that new information.

R It's not the only way you can run a class, you can have the self-paced learning.

I Yes, I think what we are interested in here in this situation is not so much what are all the possibilities, but what it is that you fundamentally believe in that drives your teaching practice the way it does without putting any judgement on it as to whether it's good or bad, but just that's the way it is, or why is it that way, I think that's what we're really getting at here.

R Yes.

I And you may see some dilemmas coming forward, there may be some conflicts or contradictions. That, again, is interesting but somewhat beside the point. Okay, so having gone through each of those separately is it possible to reflect on all three at the same time, if that's possible and see if there is any sort of differences or similarities between each of those.

R Yes, it's a bit hard because of the fact that two of them are negatively correlated so you've got to look at similarities and differences at the same time. I see all these six categories as fairly distinctively different and they are all aspects of what's going to be going on in a class.

I What's the impression that you get as a result of considering the emergence of these three factors? Is there anything that has emerged that is an insight to you now that you didn't really have an awareness of before?

R Yes, I guess it's, I find it surprising that these two factors correlate negatively . . .

I Which ones are they?

R Well v(2) in the first factor, that students do work out of class is sort of in conflict with the aim to have students engaged or to have students initiate their own involvement. I don't know why that's like that, it doesn't really make a lot of sense to me.

I Does it make sense if you try and look for a global picture in which both factors can coexist and I think I tried to paint one, I don't know if I was successful or not, but the fact that the first factor looks as though it could be something to do with the teacher being at the centre of the student's learning activities, and when the students are undertaking learning away from that situation there is somewhat of a conflict going on about your belief as to how effectively they can actually learn the work which you are setting.

R It's possible, I guess, because of the fact that I see homework largely as a revision and practice, consolidation of work done in the school . . .

I Rather than encountering new content. New content is usually, or should usually, be encountered in class under the teacher's control and initiation? Well that seems to make some sense. So factor 1 is really to do with the locus of control of learning activities lying with the teacher and factor 3 would support that.

R Yes, it means so you'd almost expect factor 3 to be, or you'd expect variable 1 to be mainly grouped with 5 and 3.

I Yes, is there any reason that it's different? I suppose factor 1 talks about students, each of the variables is a student orientated statement while factor 3 is a teacher orientated statement. Whilst they fit comfortably together they are different perspectives. This is what you believe the students should be doing in factor 1, factor 3 is what you believe you should be doing, but they do to some extent tie together under the umbrella.

R But the factor analysis doesn't take any account of sort of what the statements are does it?

I Not in terms of the actual meanings of the words no, but I just wonder . . . I mean to some extent its trying to explain something to do with the variance. This certainly isn't an infallible device, probably the benefit of it is more of a discussion that ensues rather than the actual factors that emerge.

R So that if you came out with sort of three positively correlated things then you could kind of group them and you could see some . . . kind of looking for a way of verbalising the factor or a way of rationalising the factor that those three things have been grouped together.

I Yes, so that's certainly the easiest option. I think that what we have here is something difficult to conceptualise because there are these negative correlations. I think I am seeing it clearly, now I may not be seeing it the way you're seeing it but I am seeing it and it makes sense to me and it made sense to me half an hour ago. I'm beginning to think that one of the major principles that drives your teaching with this class and maybe with other classes. . . . I don't know, is that you have a strong belief that you are the prime source of information, particularly new information and you believe that's your major role and that you see a need, therefore, to be in fairly tight control of the learning activities of the students in the classroom. Now, to some extent there is almost a dilemma there because, by the same token, you believe the students ought to be engaged in constructive learning activities which you should be monitoring to determine that they are engaged in them in a way that's beneficial to them. So again, what you're saying here is that you believe you should be monitoring that independent learning if you like and being in control of that independent learning. I think what I am beginning to see here is a belief that you should be in control of most aspects, whereas independent learning, or the process of construction of understanding, which is an individualised process can only be really in the control of the student, so I see a tension there between what you believe you should be doing and what is possible to do.

R I can see what you mean, but I think it's possible that both things can coexist. On the one hand we are talking about rates of progress say through a series of topics maybe. It's possible for everyone to be going through them more or less together, constructing their own knowledge at a particular time, if you like, I don't see the process of constructing the knowledge is not necessarily related to when they do it, so I think it is still possible to take a constructivist perspective without . . . that's not sort of destroyed by having control over what activities are going on and when. It might mean that some students, because of the lack of time or lack of opportunity to play around with the ideas for long enough, or something like that, that some students don't fully construct the knowledge.

I Do you think from what you've seen here that you see the initiative for teaching, which means providing the opportunities and monitoring students engagement, that initiative lies very much with the teacher in the classroom.

R Yes, from what's coming out of this, maybe it looks like that.

I Now, I'll go back to something I asked you before. Is there anything that strikes you as being new to yourself, that's emerged from this discussion, whether you agree or disagree with it?

R Well, maybe that idea that you just mentioned, that very fact that it seems like I place a high emphasis on the teacher being in control of the classroom, and the rate of learning and the activities that are going on, and so on, and when a new topic will be started etc.

I Have you never considered those aspects before?

R Yes, I suppose I have, I guess I have always taken them for granted that that's the way you teach or that's the teacher's role or prerogative to pace the class almost like you want to push them along but not too fast or not too slow sort of thing. I guess if you allow them to progress at their own rate well . . . there are some conflicts in this statement, but if you allow them to progress at their own rate well it sort of becomes chaotic, in a sense, that some kids are just not going to progress very fast at all. I know that maybe that's preferable . . .

I Are they likely to be totally off task?

R No I didn't really mean that, I just meant the rate of progress, I know you can't make, I know that's probably a delusion in a way to think that by moving the class along to the next topic every so often that you are making progress, maybe you would make more progress if students were allowed the opportunity to master one topic, I'm talking about the slower students, but . . .

I Could we look at it from the students point of view? The students are in a classroom where they feel they have very little control over what they are doing. Is that conducive to them developing a sense of responsibility for monitoring their own learning and for evaluating their own learning? Or do they see that responsibility is the teacher's and when the teacher doesn't do that for reasons such as there are so many students and not enough time, the students feel dissatisfied with the teaching because they think that the teacher should be doing it all for them.

R Yes, that's possible.

I I guess what I'm getting at now, this is going beyond this set of circumstances, the emphasis of . . . whole spectrum of beliefs . . . what is it you believe the students should be constructing their own learning. In order for students to do that there must be certain preconditions that exist. Clearly the students have to be self-motivated, clearly they have to see themselves as being capable of constructing their own understanding. I think what this leads towards is a situation where the students are actually, they have their traditional expectations reformulated, they've gone through school all their lives believing that the teacher is in control of their learning and they will continue to do so until someone actually stops and says "Hang on a minute, you're responsible for your own learning not me". Now until such time as they begin to be made aware that they do have the responsibility they will never seize upon it and they will always expect the teacher who is the central control figure is able to control everything including their learning.

R And when they don't learn, that's his fault, sort of thing.

I I'm just wondering whether, if you want to have a constructivist perspective, it has to go beyond simply the student-content interaction and have the teacher facilitating that to a management overview of the management of that interaction where the teacher has a certain role, the students have a certain role and the teacher might even monitor the student-content management interaction. I just wonder whether in considering constructivist theory or perspective, there is yet more for you to consider, and there are other aspects in which you might begin to reflect on: how to get the students to take on responsibility for themselves, and that would to some extent mean that you would have to shed some of the responsibility and what seems to come out here is that you have a very strong feeling for your involvement in most aspects of student's learning. Do you see that?

I Does it sound consistent with what we've been saying?

R Yes.

I Okay.

APPENDIX C

STUDENTS' PERSPECTIVES: METHODOLOGY AND RESULTS OF STUDENT INTERVIEW RAY'S GRADE 12 CLASS, JUNE 29-JULY 3, 1987

APPENDIX C

STUDENTS' PERSPECTIVES: RAY'S GRADE 12 CLASS JUNE 29-JULY 3, 1987

Towards the end of the 10-week collaborative phase of the study, I conducted interviews with students in order to determine their perceptions of the nature and value of the changes that Ray had made to his classroom teaching. This appendix discusses the methodology of the interviews and the evidence that warrants the following assertion.

Assertion
Most of the underachieving majority of Grade 12 students perceived that the changes made to Ray's teaching had enhanced their learning opportunities and activities.

INTERVIEW METHODOLOGY

This section discusses the rationale of the interview methodology. The discussion focuses on (1) the rationale for selecting the student sample, (2) the protocol employed for conducting the interviews, and (3) the format of the interviews.

Selection of Interviewees

Twenty (74%) of the 27 students in the class were interviewed. Nearly all of the 23 lower-achieving students were included in the interview sample. The three lower-achieving students who were omitted from the sample were absent from school when the interviews were conducted. In his report on the study (Fleming, 1988), Ray commented on the nature of the students who had been selected for interviews: "The entire ability range was represented, except for the *top end* of three high achievers" (p. 62).

The three highest achieving students were not selected because, at that time, I believed that their perceptions of the impact of instructional innovations which were designed in response to the perceived needs of the lower-achieving majority of students was not important. In retrospect, the decision not to interview the three highest achieving students was a mistake. I now believe that the successful development of a conceptually-oriented instructional approach should account for the interests of all students, and that a reliable measure of the degree of success of conceptually-oriented instructional innovations is the degree to which all students perceive changes to be favourable to themselves.

Interview Protocol

Interviews were conducted over consecutive days during normal class time, in a small room adjacent to the mathematics classroom. The duration of each interview was between 20-30 minutes. It is possible that students who had been interviewed discussed the interview proceedings with friends who were to be interviewed later, thus affecting the nature of their responses.

However, it seemed that the main threat to the reliability of interviewee responses would be caused by students' intentional avoidance of being critical of the teacher, in order to safeguard their perception of Ray's interests. Attempts were made to minimise this threat. In particular, Ray explained to the class that the interviews would be of benefit to himself and, ultimately, to the students because the results would assist him to determine whether the changes he had made to his teaching were worthwhile, and whether they should be continued or modified. Also, Ray assured the class that the results would have no impact on his employment prospects, and urged students to provide honest and forthright responses.

Students were interviewed in pairs in order to facilitate a more free-flowing discussion in which student's might trigger each other's responses. Another reason for interviewing pairs of students was to minimise the possibility of students experiencing anxiety, especially in relation to the audio-recording of interviews. It was assumed that students would feel less threatened by the presence of an active microphone if they were accompanied by a friend.

Students were guaranteed anonymity in relation to the reporting of the results of the interviews. Anonymity was preserved by (1) avoiding the use of students' names during the interview, (2) providing Ray with only the interview transcripts and withholding the cassette tapes, and (3) using unidentified interview extracts in reports on the study. Permission to record the interviews was sought, and obtained, from each student prior to switching on the tape recorder. Transcripts were produced by secretarial staff at Curtin University who remained unaware of the source of the tapes.

Interview Format

An interview format was designed to elicit students' perceptions of the nature of recent changes in Ray's classroom practice, and of the impact on their learning of the changes. In particular, the purpose of the interview was to investigate students' interpretations of the changes in the teaching, rather than affirm the *a priori* interpretations of Ray or myself. For this reason, students were not asked to comment on specific aspects of Ray's innovations.

For example, students were not asked to comment on changes in Ray's questioning techniques, whole-class presentations, or the introduction of worksheets. Rather, the questions required students to identify their personal experiences of the effect of changes in the teaching, to identify the nature of the changes, and to comment more specifically on the relationship between the changes and their impact on the classroom learning environment.

The questions comprising the interview format are presented in Figure C1. During the interviews, the general sequence of questions was maintained but the wording of questions sometimes was varied in attempts to maintain a naturalistic discourse, rather than a formalised interview.

1. How are you going with your current maths work on *Graphing Inequations* and *Solving Simultaneous Equations*?
2. Do you find this work easier or harder than the work that you were doing on *Radians* and *Sketching Sin and Tan Graphs*?
3. Compared with earlier in the year, has your ability to understand maths got better, or worse, or stayed the same?
4. Has your confidence in your own abilities in maths got better, or worse, or stayed the same?
5. Have you noticed any differences in the way your maths teacher teaches now, compared with earlier in the year?
6. What is the main difference in the way he teaches now, compared with before?
7. What other differences are there in your teacher's teaching?
8. What effects have changes in your teacher's teaching had on the whole class?
9. What effects on you have resulted from the changes in your teacher's teaching?
10. Is there anything that the teacher could do to help you improve further?

Figure C1. Student interview questions

INTERVIEW RESULTS

This section discusses the main results of the interviews, especially students' perceptions of the nature of recent changes in Ray's teaching, and the impact of changes on their confidence and ability to understand mathematics.

Understanding and Confidence

The first two questions were designed to focus students' thinking on the relative nature of their general learning experiences at the commencement of the study in comparison with their current work. Of particular interest to the study were students' responses to the questions that asked whether they had experienced any changes, in that period of time, in their ability to understand mathematics (question 3) or in their confidence in their own mathematical ability (question 4).

The results are summarised in Table C1, and indicate that three-quarters (15) of the students felt that their understanding had improved, and that half (10) had experienced an improvement in their confidence in their own abilities. A quarter of the sample (5) claimed that their understanding and confidence were unchanged, and one student reported a loss in confidence, although not in her ability to understand maths.

Table C1

Students' Perceptions of Changes in Ray's Teaching

Type of Effect	Understanding	Confidence
Improvement	15	10
No Change	5	9
Worse	0	1
Total	20	20

Changes in Teaching

Students were asked whether they had noticed any differences in the teaching since the beginning of the study (question 5) and, if so, to describe the nature of the differences (questions 6 & 7) and their effects on both the whole class (question 8) and on themselves (question 9). Questions nine and ten were designed (1) to elicit a broader range of responses than questions three and four, and (2) to serve as a *reliability* *check* on students' responses to questions three and four. All students claimed to have noticed differences in the teaching, mainly in Ray's attitude to students and in his classroom practice.

Teacher Attitude

Students reported that they perceived Ray to have a more sympathetic attitude to them; to be more interested in their welfare as students; to be more determined to ensure that students understood the work; and to be less assuming about students' prior mathematical knowledge.

Teacher Actions

Students perceived that Ray's classroom actions were more helpful to them. In particular, students perceived that Ray's explanations were much clearer; that he communicated with more students; that he was better organised; and that he was proceeding through the syllabus more slowly. In particular, 13 students mentioned the introduction of worksheets as being beneficial to them.

Whole-Class Effects

Nineteen of the 20 students perceived a positive effect on the whole class of the changes in Ray's teaching. In particular, students had noticed improvements in other students' attitudes towards Ray and towards studying the subject; that other students were enjoying maths more; that more students were participating in classwork activities; and that other students' mathematical understandings had improved.

Personal Effects

On the question of *what effects on you have resulted from the changes in your teacher's teaching*? (question nine), 17 of the 20 students reported a positive effect on either their attitudes or on their understanding. By contrast, several students reported that the changes in Ray's teaching had had little or no effect on themselves. These responses are highly consistent with responses to questions three and four, and provide evidence of the high reliability of students' responses, in general. Some typical student responses to question nine are presented below:

> I'm more confident because I understand the work better. . . . I feel that I can ask more questions now. (Student Interview, Tape 1, p. 5)
>
> I think it's made me more aware of what I've been doing wrong . . . and I've sort of tried to get my back up [to] start working harder for maths . . . and I think, well, [the teacher] can have a go, well, I can have a go at doing it. So, I feel that with him improving in his way, and I'm working harder, it's working. (Student Interview, Tape 1, p. 14)
>
> I find it easier to understand what he's saying. . . . Previously he just taught something and then you had to do it, and you'd go home and you'd sit there and stare at it and you wouldn't know what to do. . . . But, now because he's been giving us handouts and he's been explaining it through different things, its easier to answer things. (Student Interview, Tape 2, p. 12)
>
> I think everyone enjoys maths more because they know what they're doing and they can understand it. So, everyone comes into class with the attitude that we can do something today, not just sit there. . . . I enjoy going to maths now, knowing that I can do it. (Student Interview, Tape 3, p. 2)
>
> I think that I can learn more from the class. I don't need to go back home and try to understand everything from the book. . . . I find that it's much easier to understand what's going on in the class now than it was before. (Student Interview, Tape 3, p. 15)
>
> [I]t's really been good the way he's changed . . . in the sense [that] he's more helpful. . . . but I don't think I've benefited a lot by it because I always have to try and help myself after the lesson. It's not something where I can just pick it up, sort of thing. Even if I'm given the help I still have to work at it. (Student Interview, Tape 3, p. 19)

SUMMARY

The student interviews provided substantial evidence to support the assertion that students had perceived recent changes in the teaching to have enhanced the learning opportunities and activities of most of the class, especially the lower-achieving majority of students. In general, students perceived Ray to have a more positive attitude towards them, and that his teaching practice was providing better opportunities for facilitating their learning of mathematics. In particular, the worksheets were favourably perceived by many students. Most students reported a positive impact on their learning of the changes, especially in relation to their ability to understand and to their attitudes to learning.